Innovation

Modeling Photon Pressure

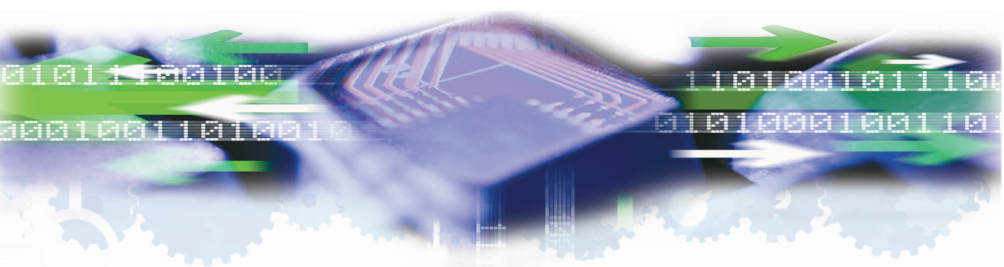
The Key To High-Precision GPS Satellite Orbits

Marek Ziebart, Paul Cross, and Sima Adhya University College London

"Photons have mass?! I didn't even know they were Catholic." — Anonymous

Actually photons have no mass, but that does not mean they cannot affect GPS satellite orbits. GPS satellites operate in a harsh, radiation-filled environment 20,000 kilometers above the surface of the Earth. Solar radiation pressure — the force due to the impact of solar photons and the related effects of anisotropic thermal re-radiation and albedo are all tiny forces and yet they have a strong perturbing effect on the GPS satellite orbits. Predicting how GPS satellites will move in space relies upon understanding and modeling these effects, and the accuracy of these predicted orbits underpins the entire system for positioning, velocity determination, and a host of other applications. This month's Innovation *column examines the significance of these forces and how they can be modeled.*

Marek Ziebart is a research fellow at the Department of Geomatic Engineering, University College London (UCL). His main research areas are orbit determination problems, non-conservative force modeling and space-borne GPS attitude control. Dr. Ziebart co-authored the article "Over the Silk Road: Bringing Satellite Imagery Down to Earth" in the April 2001 issue of GPS World. *Paul Cross is the Leica Professor and head of the Department of Geomatic Engineering at UCL. He has been involved in GPS research since about 1980, concentrating mainly on high precision applications in engineering and geophysics. Dr. Cross is a member of the* GPS World *Editorial Advisory Board. Sima Adhya has degrees in Natural Science and Space Science from Cambridge and University College London respectively. She is currently reading for a Ph.D. in satellite geodesy and astrodynamics at UCL. Her principal research interests are high precision orbit determination and GNSS system design.*



A fundamental component of the Global Positioning System is the calculation of a highly accurate predicted orbit for each of the constellation spacecraft on a regular basis. In general, the parameters used to describe these orbits are not stable and need to be constantly updated. Being able to predict changes, and also being able to calculate the orbit after the fact, relies upon a combination of range observations and force modeling. A whole range of different forces, with varying magnitudes, act on the GPS satellites. **Table 1** lists these forces in decreasing order of magnitude along with crude estimates of the accelerations they cause.

There are also many smaller forces, such as drag effects, that are too small to worry about for current techniques.

While the modeling of gravitational forces is well understood, the following forces are more problematic (see **Figure 1**):

- Solar radiation pressure (SRP), that is, the force produced by the impact of electromagnetic radiation from the Sun striking the spacecraft.
- Albedo, the force due to electromagnetic radiation reflected by the Earth which is supplemented by thermal radiation emitted by the Earth, and
- Thermal re-radiation (TRR) forces, caused by anisotropic radiation of heat from the spacecraft.

These are non-conservative forces (NCF). This means that they change the energy state of the spacecraft, and for this reason they are important agents in changing the evolution of the orbit's parameters in both the short and the long term.

Despite having tiny magnitudes, SRP and the other non-conservative forces cause large changes to the spacecraft orbit. See the sidebar **"How big are these radiation forces?"** to get a "ball-park" feel for how big these forces are, and what effect they have on the orbit.

Overall force modeling is used in three areas of GNSS: design, operation and scientific analysis.

At the design stage we need to understand how changes in the spacecraft structure (e.g. size of the solar panels, materials used for component shielding) affect the orbit dynamics. These systems are designed to provide global coverage, and this entails being able to predict how the orbits change over a number of years. Moreover, as the orbit decays over time fuel must then be expended in firing thrusters to bring the satellite trajectory back within its design threshold. Only a limited amount of fuel can be carried, and therefore, optimizing the cost of launching the satellite (which is strongly mass-dependent) against the mass of fuel available at the start of the spacecraft's operational lifetime requires detailed understanding of how the orbit is going to evolve.

The operational stage requires orbit prediction. The better we understand all the physical mechanisms affecting the satellite's trajectory, the better we can predict the orbit. In determining the broadcast ephemerides, the Master Control Station estimates the values of two parameters related to SRP for each spacecraft. Even in space-based augmentation systems such as WAAS (Wide Area Augmentation System), the modeling of the tiny SRP forces is important. All real-time GPS applications rely fundamentally upon the accuracy of the predicted orbit.

Scientific analysis involves post-processing of the received signals and is used in measuring geodynamic phenomena

Force	Acceleration (meters per second squared)
Earth gravity modeled as a point mass	6.1×10^{-1}
Earth gravity oblateness modeled by the J2 coefficient	1.0×10^{-4}
Lunar gravity	3.9×10^{-6}
Solar gravity	1.0×10^{-6}
Summed effect of Earth gravity field, coefficients 2,1 to 4,4	2.2×10^{-7}
Solar radiation pressure	7.2×10^{-8}
Summed effect of Earth gravity field, coefficients 5,0 to 8,8	5.9×10^{-9}
Albedo (or Earthshine)	1.5×10^{-9}
Thermal re-radiation	1.4×10^{-9}
Solid Earth tide, raised by the Moon	1.3×10^{-9}
Solid Earth tide, raised by the Sun	4.5×10^{-10}
Venus gravity	1.1×10^{-10}

TABLE 1 Forces acting on GPS satellites

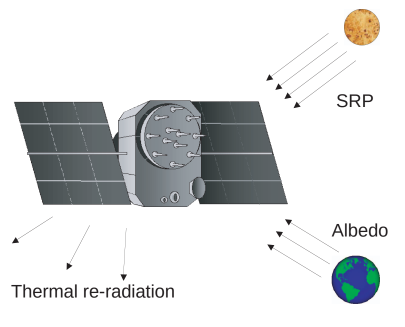


FIGURE 1 GPS satellite orbits are perturbed by the impact of photons in solar radiation as well as sunlight reflected from the Earth and also by the emission of photons in satellite heat dissipation.

such as post-glacial rebound, plate tectonic motion and volcano magma chamber inflation. Such ultra-high precision applications of GPS observables require positional accuracy of a few millimeters over length scales of 100-1000 kilometers. To this end, enormous effort has been expended in improving the accuracy of GPS orbits by trying to improve the accuracy of the force modeling.

So, how can we deal with the problem?

Data and Parameters

Somehow we need to build up a model of the non-conservative forces that are affecting an orbit. The following parameters are important in this process:

- Solar irradiance — the amount of solar electromagnetic radiation (in watts) passing through one square meter at one astronomical unit (AU, approximately equal to the semimajor axis of the Earth's orbit) from the Sun.
- Spacecraft attitude — this governs which parts of the spacecraft are illuminated by the Sun, and which parts are in shadow, which parts are heating up, and which parts are cooling down.
- Optical parameters of spacecraft surface materials — typically these are the reflectivity and specularity coefficients of each spacecraft surface component; see the sidebar **"Terminology"** (next page) for definitions.
- Spacecraft structural details – the size and shape of the structure, which parts are static and which parts can move.
- Thermal conductivity and emissivity of structural elements.

How can we assign values to these parameters or create models for them?

Solar Irradiance. The total solar irradiance (TSI) is measured directly in the space environment by a number of sensors on probes such as SOHO (Solar and Heliospheric Observatory) or ERBS (Earth Radiation Budget Satellite). Because of this, we know that the solar irradiance does vary as a f unction of the solar cycle (the solar cycle lasts approximately 11 years) over a range of 1.7 watts per meter squared, with an approximate mid-range value of 1369 watts per meter squared. This variation introduces only very small changes to the SRP force calculated using TSI, of the order of 0.1% over the entire solar cycle.

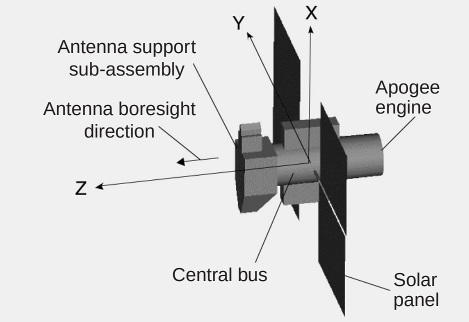


FIGURE 2 A GLONASS IIv satellite illustrates the spacecraft body-fixed system.

Spacecraft Attitude. The spacecraft attitude has two design constraints that can be used in a model. Firstly, the antenna boresight is constrained to point at the geocenter in order to distribute evenly the GPS signal over the hemisphere that is visible to the satellite. Secondly, the solar panels are oriented to point continually towards the Sun. The spacecraft body-fixed system (BFS) is then related to the instantaneous geocentric position vector of the satellite center of mass as follows:

Let

$\mathbf{r}$ = Earth-centered inertial (ECI) position vector of the spacecraft center of mass at time t

$\mathbf{s}$ = ECI position vector of the Sun at time t

$\mathbf{p}$ = vector from the spacecraft (or probe) to the Sun.

Then,

$\mathbf{p} = \mathbf{s} - \mathbf{r}$.

The spacecraft Z-axis, $\hat{\mathbf{z}}_{sc}$, points along $-\mathbf{r}$, hence

$$\hat{\mathbf{z}}_{sc} = -\frac{\mathbf{r}}{r}$$

and the spacecraft Y-axis, $\hat{\mathbf{y}}_{sc}$, points in a direction perpendicular to both $\hat{\mathbf{z}}_{sc}$ and $\mathbf{p}$, hence

$$\hat{\mathbf{y}}_{sc} = \frac{\hat{\mathbf{z}}_{sc} \times \mathbf{p}}{\left|\hat{\mathbf{z}}_{sc} \times \mathbf{p}\right|}$$

Finally, the spacecraft X-axis, $\hat{\mathbf{x}}_{sc}$, is orthogonal to the other two axes, completing the right-handed system.

So, given the geocentric position vectors of the satellite and the Sun, this procedure yields the BFS axial unit vectors in terms of ECI vector components.

Under these attitude constraints the BFS X-Z plane nominally includes the Sun and is, to first order, also a plane of symmetry for the spacecraft (see **Figure 2**). As a result of the attitude control algorithm, the profile of the spacecraft presented to the Sun over most

How big are these radiation forces? What effect do they have?

Go to the kitchen and pick up 100 grams of something. The force that you feel pushing down on your hand is about one newton at sea level. Divide that by two and imagine how much smaller the force has become. Now divide the original one newton force by 10,000, and try to imagine how much it would push your hand. That is the magnitude of the first order force due to SRP acting on a spacecraft of mass circa 1,500 kilograms. Now make that tiny force 100 times smaller still (one millionth of a newton!) and that is the magnitude of the force that needs to be modeled accurately for precise orbit determination. This is equivalent to the force of just two grains of salt pushing down on your hand! This is at the level where thermal re-radiation, albedo and subtle SRP effects become important.

So, what would happen to your predicted orbit if you were to ignore NCF? The second order differential equation of motion of the satellite is:

$$\ddot{\mathbf{r}} = -\frac{GM\mathbf{r}}{r^3} + \mathbf{a}_{perturbing} \text{ where } -\frac{GM\mathbf{r}}{r^3}$$

is the acceleration due to the Earth mass modeled as a point and $\mathbf{a}_{perturbing}$ comprises all the accelerations due to perturbing forces.

If this equation were solved numerically including models for forces due to lunar, solar and planetary gravitation, solid Earth tides and other small forces, as well as SRP effects, and then we did the same calculation but ignoring NCF, the difference between these two trajectories after one revolution around the Earth (about 12 hours for the GPS satellites) would be between 100 and 200 meters in the along-track direction. In practice, the process of orbit prediction does involve estimating empirical parameters, so even if SRP effects were ignored then the modeling would pick up some of the effect. However, from the above it is clear that over time SRP effects have a big influence on the spacecraft trajectory.

of its mission lifetime varies uniquely as a function of the so-called Earth-probe-Sun (EPS) angle. This is the angle between the BFS Z-axis (which points to the Earth) and the 'Probe-Sun' vector (see **Figure 3**). Some SRP models use the EPS angle as an independent variable for computation of the force.

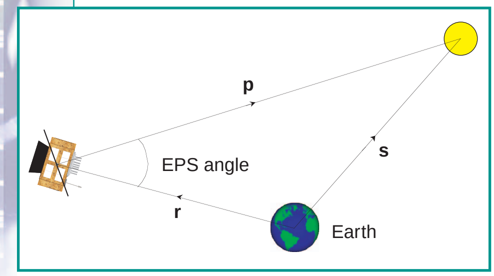


FIGURE 3 The Earth-probe-Sun (EPS) angle is the angle between the Earth and Sun directions as viewed from a spacecraft or probe.

The only problem we may have to deal with using the approach above is that it is based on a nominal attitude algorithm, as opposed to some measurements that might give us an estimate of the true attitude of the spacecraft. While it might be possible to estimate attitude variations within the orbit determination process, in practice it is difficult to de-correlate other anomalous, un-modeled forces from the true variations in attitude. Unpredictable attitude variations can give rise to the so-called Y-bias. The more predictable and deliberately inserted yaw bias also changes the physical attitude, albeit by a very small amount (yaw biases are only applied to Block II and Block IIA spacecraft).

Optical Properties of Spacecraft Surface. The interaction of solar radiation with the spacecraft can be modeled using coefficients that describe how much radiation is absorbed, how much is reflected and in what way. These can be expressed in various ways, but one common system uses reflectivity and specularity. See the "Terminology" sidebar for details. Provided we know the material type, standard measured values of the coefficients can be used. Once the surface has been exposed to the space environment, these optical properties may change slightly, tending towards more diffuse reflection as the surface becomes pitted by galactic cosmic rays. However, caution is needed in deciding whether these coefficients are changing or not – some secular changes in the spacecraft dynamics are due to other causes, such as space vehicle out-gassing.

Spacecraft Description

In the body-fixed frame, as each satellite moves through its orbit, the Earth is continually on the Z-axis and the Sun apparently rises and sets above the Z-Y plane. Apart from eclipse seasons (which are discussed later), when the EPS angle varies from close to zero through to 180 degrees, this apparent movement of the Sun is always less than 180 degrees in any one orbit. In the BFS frame the only motion of the space vehicle is the rotation of the solar panels about the Y-axis as they attempt to keep track of the Sun. For this reason it makes sense to model the variations in NCF effects in the BFS frame, and therefore the spacecraft structure should be described in terms of BFS co-ordinates. SRP can be modeled a priori along the Z and X axes, provided that the profile of the spacecraft surface components as seen from the Sun can be computed. One problematic issue here is accounting for the shadowing of one component by another. Similarly, albedo effects are functionally related to the variations in the spacecraft profile as seen from the Earth along the Z-axis.

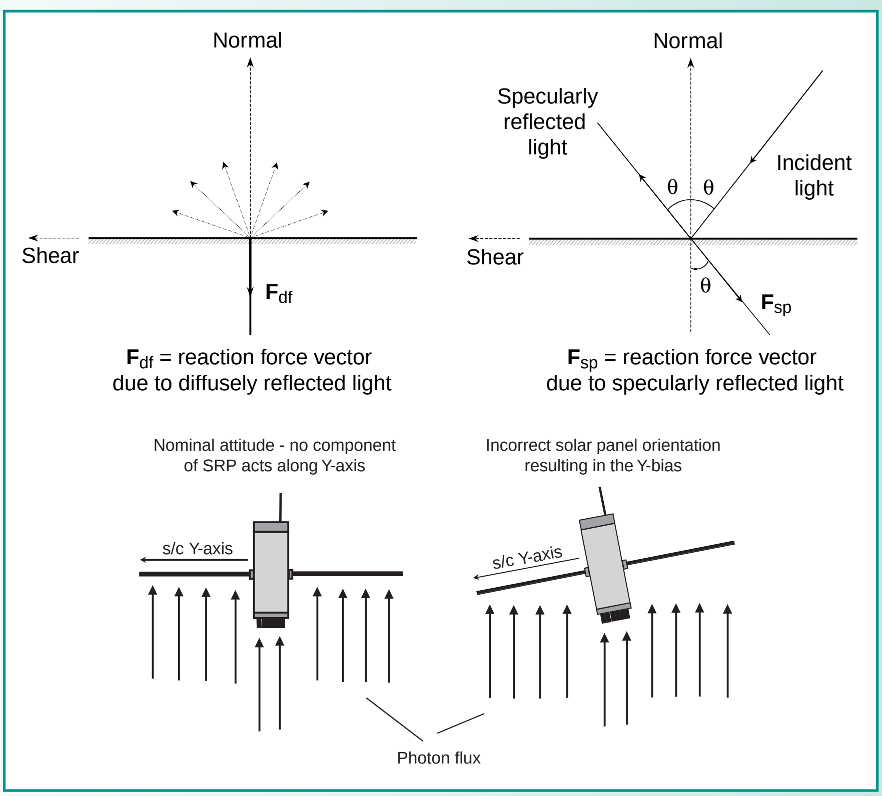
Thermal Properties. Modeling the thermal state of the satellite requires knowledge of the conductivity and emissivity of the spacecraft materials. Thermal conduction is the process of heat transfer by molecular motion. The thermal conductivity of a material gives an indication of how well the heat energy is transferred and it depends on chemical composition, physical structure, state of the material and temperature (satellites in orbit endure an extreme temperature range of 130K - 350K). The emissivity of a body is the ratio of the radiation actually emitted by a surface and the radiation that would have been emitted from

Terminology

Reflectivity (ν) – the proportion of radiation incident on a surface that is reflected, the reflected radiation being separated into diffuse (scattered) and specular (beamed) components.

Specularity (μ) – the proportion of reflected radiation that is reflected specularly. Specular reflection implies that the surface behaves like a perfect mirror.

Y-bias – a force acting along the spacecraft BFS Y-axis and believed to derive from NCF effects. A likely mechanism for the Y-bias is due to non-orthogonality of the solar panels with respect to the solar photon flux, as a result of attitude bias or variations. However, another possible contribution could come from heat dissipation effects of payload components.



Reflected radiation from a spacecraft may be separated into diffuse and specular components. If a spacecraft's solar panels are not oriented precisely orthogonal to the photon flux, an anomalous bias force is generated along the spacecraft Y-axis.

a perfect blackbody (a perfect blackbody absorbs all incident radiation, and thus has a reflectivity of zero).

It is possible to measure these coefficients experimentally, and these would be used in the process of spacecraft design and manufacture. Their values are assumed to be constant though they may change slightly with time as the surface of the satellite degrades and this may lead to possible errors in the models.

A GPS satellite's attitude changes slowly, therefore in orbit it is heated unevenly since only the Sun-facing side receives direct solar radiation and, due to the complex shape of a real satellite, some parts are shadowed by other parts. This results in an uneven temperature distribution and since the amount of energy radiated by a surface is dependent on its temperature, this leads to anisotropic emission of radiation.

Any heated surface which radiates loses energy in the form of photons. This energy loss causes a reaction force against the radiating surface (see **"Theoretical Background"** sidebar). This force must be integrated over the whole surface to calculate the total force. To do this, the temperatures at each point on the surface must be known.

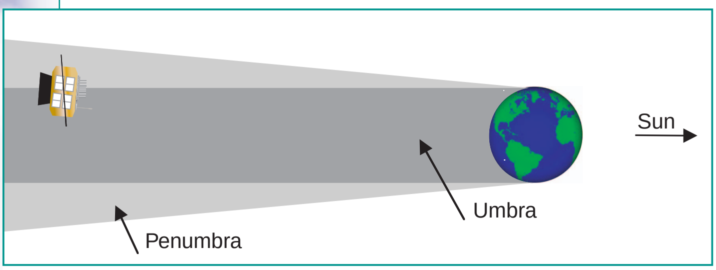


FIGURE 4 At certain times of the year, GPS satellites find themselves in the Earth's shadow for a short period during each orbit. During an eclipse, the Earth partially (while the satellite is in the penumbra) or completely (while it is in the umbra) blocks the Sun's photons from reaching the satellite.

Eclipse Seasons

During eclipse seasons, the satellites pass into the shadow of the Earth for a short period (up to about one hour) in each 12-hour orbit. The shadow region is divided into two parts, the umbra and the penumbra (see **Figure 4**). An eclipse season for a particular orbital plane lasts for between four and eight weeks and occurs twice per year.

As soon as the satellite passes into the penumbra two things happen. First, the SRP force is effectively turned off. However, TRR forces still act while the temperature of the spacecraft drops exponentially. The second occurrence is that the attitude control system (ACS) solar sensors lose sight of the Sun. Prior to 1994, this would cause the spacecraft to yaw wildly about the BFS Z-axis. Since then, to make this movement more predictable, a yaw-bias has been applied by the GPS control segment. Once the satellite emerges from the shadow crossing, it begins its recovery to attain nominal attitude once again. This so-called "midnight turn" can take from zero to forty minutes, depending on the satellite attitude at the start of the

Theoretical Background

James Clerk Maxwell, the Scottish physicist, first showed the theoretical basis for radiation pressure in 1871. The Russian physicist, Pyotr Nicholaievich Lebedev, demonstrated experimental evidence in 1900. Ernest Nichols and Gordon Hull in the United States also independently showed this in 1901. Electromagnetic radiation possesses momentum. This is described in Albert Einstein's special theory of relativity in that:

$$E = \left[(cp)^2 + (m_0 c)^2\right]^{1/2}$$

where

E = energy of the particle
p = momentum magnitude
m_0 = rest mass
c = speed of light in a vacuum.

For a photon $m_0 = 0$, and hence $E = cp$. Einstein, building on ideas developed by Max Planck, also proposed a corpuscular theory of light in which each photon has an energy proportional to its frequency (ν) such that $E = h\nu$, where h is Planck's constant. From which it is seen that $p = h\nu/c$. So, the photon's momentum is proportional to its frequency. If such a photon is absorbed by some surface, the momentum is transferred to the body.

Let the average number of solar photons of frequency ν, striking a unit surface area per second at 1 AU, be $n(\nu)$. The change in momentum per unit time per unit area is therefore $n(\nu)hn/c$. And as first proposed by Isaac Newton, the rate of change of momentum of a body is equal to the applied force. So, integrating over the electromagnetic spectrum, we have for the magnitude of the force:

$$F_{\text{due to absorbed radiation}} = \int n(\nu) h\nu/c \; d\nu$$

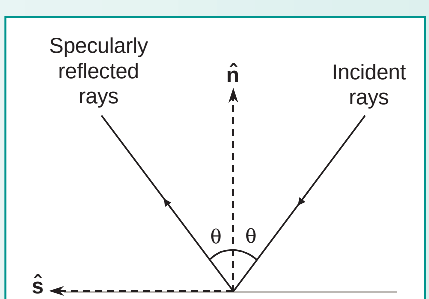
But

$$\int n(\nu) h\nu \; d\nu$$

is just the solar irradiance (W) in watts per square meter. Therefore, at 1 AU, the force per unit area (pressure) due to absorbed radiation is

$F = W/c$.

This expression gives the force acting on a unit area due to absorbed radiation falling perpendicularly onto a surface as a function of the solar irradiance parameter.



Snell's law predicts that the angle of specularly reflected rays with respect to the normal direction equals the angle of the incident rays.

It can be shown that this leads to the following functions, which model the force acting on a spacecraft surface component due to SRP:

$$F_{\hat{n}} = -P\{(1+\mu\nu)\cos\theta + (2/3)\nu(1-\mu)\}$$

$$F_{\hat{s}} = P\{(1-\mu\nu)\sin\theta)\}$$

P = A W cos θ / c
A = surface area of component
W = solar irradiance
θ = radiation angle of incidence
ν = reflectivity of component
μ = specularity of component
c = vacuum speed of light

One additional model is often used, and this relates the temperature of a spacecraft surface component (typically a solar panel) to the force normal to the surface caused by the radiation of heat:

$$f_{\text{thermal}} = \frac{2\sigma A}{3c} \varepsilon T^4 \hat{n}$$

where the additional parameters are
σ = Stefan-Boltmann constant
ϵ = emissivity
T = temperature

The factor 2/3 arises in this equation due to the diffuse nature of the heat radiation; a similar term arises in the SRP functions.

maneuver. This is problematic in that the attitude is unpredictable. In general, even with the post-processed International GPS Service (IGS) orbits, the orbital precision for satellites in eclipse season is somewhat degraded. These problems map into greater uncertainties for GPS applications.

Modeling Methods

To date, most modeling has concentrated on solar radiation pressure (as opposed to explicitly modeling TRR and albedo effects as well). There are three main methods used to calculate SRP models:

- Analytical methods
- Analytical methods with empirical scaling or augmentation
- Empirical methods.

In the purely analytical methods, the theoretical ideas presented in the sidebar are used to compute models based on the structural, nominal attitude and optical properties of the spacecraft alone. The early analytical models for GPS, termed the ROCK series (after Rockwell International), were typical of this approach, which works well provided that the spacecraft structure is quite simple. Later versions of the ROCK models used basic thermal re-radiation modeling to augment the SRP component of the force model. The ROCK 42 model was adopted as an International Earth Rotation Service standard in 1996. The main drawback of the approach used in computing the ROCK models is that it becomes cumbersome when the spacecraft structure becomes complicated. Lockheed Martin, the designer and manufacturer of the Block IIR satellites, has been continually improving its NCF models, and the latest versions include thermal modeling for payload components. More recently, we have developed a high precision approach to analytical model computation based on pixel array methods that can be used easily with very complicated structures, and this is described in the following section.

In post-processed orbit determination applications, typically those carried out by the various agencies contributing to the IGS orbit, empirical scaling or augmentation parameters related to NCF can be estimated as part of heavily over-determined global network analyses. These approaches are mainly characterized by the methods developed at the Centre for Orbit Determination in Europe and at the Jet Propulsion Laboratory. Such methods generally start with the ROCK models and estimate parameters that correct for the perceived deficiencies in the a priori analytical models. While these methods do result in very precise orbits that are of enormous benefit to the global scientific community, the empirical terms tend to soak up any unmodeled forces that affect the orbit, including the effect of satellite mass changes as orbit adjustment fuel is consumed. Hence the variation in the empirical terms does not necessarily help us to understand better the physical mechanisms that drive the true trajectories. Having said this, the well known (but mysterious) Y-bias was first discovered through empirical methods, and this has spawned much useful research. Because of this drive to improve the post-processed precise orbits, little work has been done on the problem to develop high precision analytical modeling techniques that might aid design and operational applications. All the methods involving empirical estimation rely upon a large data volume for success.

Finally, there are purely empirical approaches where the knowledge of the spacecraft structure and attitude is effectively ignored. These methods include a number of parameters to which no particular physical meaning can be attributed, and these effectively soak up any unmodeled force effects.

Pixel Array Methods. As we try to model the forces on real spacecraft, the complexity of the spacecraft's physical form causes difficulties. The main problems that have to be overcome involve accounting for the changing profile of the spacecraft in the course of its orbit, and the way in which these variations change the amount of solar radiation incident on any particular part of the spacecraft structure. The method we have adopted to solve this problem is to simulate the flux of electromagnetic radiation from the Sun using a pixel array (see **Figure 5**). The pixel array is rotated around a computer simulation of the spacecraft, using a body-fixed coordinate system, in accordance with the spacecraft attitude control algorithm. This simulates the effect of the changing geometry of the EPS system. Any part of the spacecraft that would change its orientation in this system as a function of the Sun's position can be adjusted accordingly. The SRP interaction of the photon flux with the spacecraft is calculated by projecting the pixel array onto the spacecraft simulation at discrete points in the periodic EPS geometry. This process results in a series of data points giving the acceleration of the spacecraft due to SRP along its BFS X and Z-axes as a function of the EPS angle. The SRP model is formed by fitting a Fourier series to the data, using the EPS angle as the independent variable. The variations in the solar irradiance are modeled by taking a nominal value at one astronomical unit, and then scaling it based on the actual distance of the spacecraft from the Sun at any point in its trajectory.

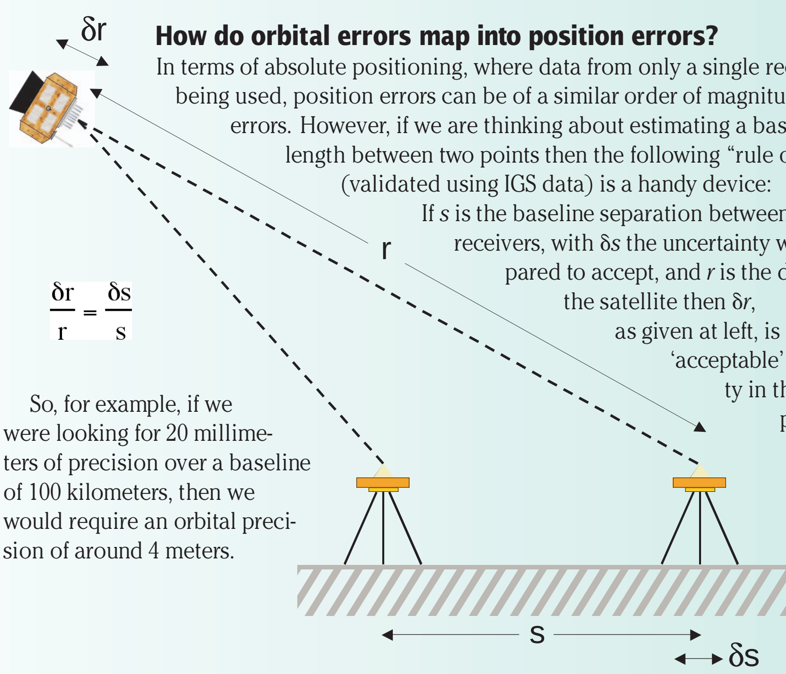
With knowledge of the precise shape and orientation of the spacecraft, the pixel array method can be used to calculate the dose rate of radiation incident upon each component, and with values for the conductiv-

How do orbital errors map into position errors?

In terms of absolute positioning, where data from only a single receiver is being used, position errors can be of a similar order of magnitude to orbit errors. However, if we are thinking about estimating a baseline length between two points then the following "rule of thumb" (validated using IGS data) is a handy device: If s is the baseline separation between two receivers, with δs the uncertainty we are prepared to accept, and r is the distance to the satellite then δr, as given at left, is the 'acceptable' uncertainty in the satellite position.

$$\frac{\delta r}{r} = \frac{\delta s}{s}$$

So, for example, if we were looking for 20 millimeters of precision over a baseline of 100 kilometers, then we would require an orbital precision of around 4 meters.



For a given satellite orbit error, δr, the error in the estimated baseline vector length, δs, in relative GPS positioning, is roughly proportional to the ratio of baseline length to the distance to the satellite.

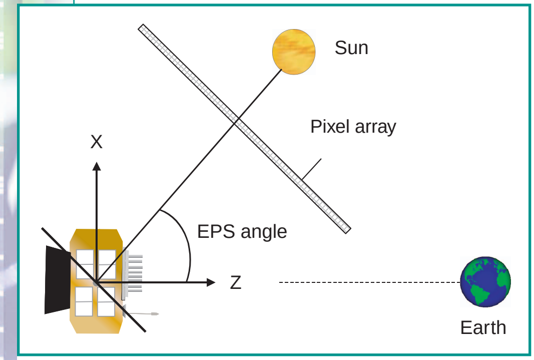


FIGURE 5 The effect of the Sun's photons on a spacecraft may be simulated using a pixel array which is rotated around a computer model of the spacecraft.

ity and emissivity finite element analysis (FEA) can be used to calculate the temperature at each point. The principle of FEA is based on the premise that an approximate solution to any complex problem can be reached by subdividing the problem into smaller more manageable (finite) elements. Using finite elements, solving complex partial differential equations that describe heat transfer mechanisms can be reduced to a set of linear equations that can easily be solved using the standard techniques of matrix algebra. So the principles above can be used to develop a thermal analysis model, using FEA to determine the temperatures of each part of the satellite in order to compute the force due to thermal re-radiation.

In a similar fashion these pixel array methods can be used to model albedo effects, although the variation in the flux of radiation from the Earth is much less predictable compared to solar irradiance, due to the effect of cloud cover.

Discussion

In post-processed applications where a precise orbit is needed, provided that a sufficiently dense tracking station network is available (such as the IGS network) then SRP effects can be accounted for empirically. However, if the number of tracking stations is reduced, simply applying the existing empirical methods to the computation for a new constellation does not solve the problem, and this is borne out by recent work in the International GLONASS Experiment 1998 (IGEX98) campaign. At the very least, starting the estimation process with a relevant analytical model provides a hypothesis that can be tested.

Where the requirement is to *predict* the satellite orbital trajectory, the application of an analytical model is yet more important. Operational systems are tracked by a relatively small number of ground stations and hence the number of observations used to help predict the orbits is correspondingly small. The more accurate the a priori SRP modeling, the more accurate is the predicted trajectory, with associated improvements in system performance for all real-time applications.

This is not to dismiss in any way the role of empirical methods in orbit determination. The combination of an analytical model, based on all the available structural and attitude data for the spacecraft, and a number of carefully selected empirical parameters is a powerful tool for calculating high precision orbits. However, the benefit of good a priori analytical modeling is that it enhances the ability to understand the system, and hence, predict how it will function over time. This can, in turn be exploited to either improve the accuracy of predicted orbits, or, maintaining a certain level of accuracy in the orbits, reduce the number of tracking stations required to support the system.

Improving the accuracy of the orbit reaps other benefits in that physical parameters, such as the wet tropospheric propagation delay, also become more accurately determined. These parameters are increasingly important in applications such as virtual RTK networks and interferometric synthetic aperture radar.

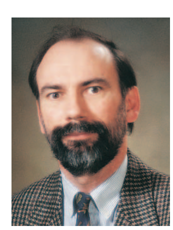
Conclusion

Solar radiation pressure, thermal re-radiation and albedo forces are tiny in magnitude, and yet have a strong perturbing effect on GPS satellite orbits. The modeling of these forces is important at the stages of system design, operation and scientific analysis.

Although much previous work has concentrated on empirical methods (where a large number of tracking stations are required), newer high precision analytical techniques make it feasible to model non-conservative force effects more accurately at the design and operational stages. This can be exploited to either

- increase the length of time over which a predicted orbit is valid, or
- decrease the number of tracking stations required for the system, and thereby reduce running costs.

As Prof. Richard Feynman once said, "It is only the principle of what you think will happen in a case you have not tried that is worth knowing about. Knowledge is of no real value if all you can tell me is what happened yesterday. It is necessary to tell what will happen tomorrow if you do something".



"Innovation" is a regular column featuring discussions about recent advances in GPS technology and its applications as well as the fundamentals of GPS positioning. The column is coordinated by Richard Langley at the University of New Brunswick. To contact him with comments or suggestions for future columns, see the "Columnists" section on page 4.

Further Reading

For an introduction to GPS satellite orbit modeling, see

- "GPS Satellite Orbits" by G. Beutler, R. Weber, U. Hugentobler, M. Rothacher, and A. Verdun in *GPS for Geodesy*, 2nd edition, edited by P.J.G. Teunissen and A. Kleusberg and published by Springer-Verlag, Berlin, Heidelberg and New York, 1998.

For a compendium of papers on GLONASS orbit modeling, see

- "Session 5: Orbit Determination" in *Proceedings of the International GLONASS Experiment, IGEX-98, Workshop*, Nashville, Tennessee, September 13-14, 1999, published by the International GPS Service Central Bureau, Jet Propulsion Laboratory, Pasadena, California, 2000, pp. 155-258. An on-line version of the proceedings is available at <http://igscb.jpl.nasa.gov/overview/ igex98.html>.

For a discussion of the ROCK4 (for the former Block I satellites) and ROCK42 (for the Block II satellites) solar radiation pressure models, see

- "Global Positioning System Radiation Force Model for Geodetic Applications" by H.F. Fliegel, T.E.Gallini and E.R.Swift in *Journal of Geophysical Research*, Vol. 97, No. B1, 1992, pp. 559-568.

For a discussion of solar radiation pressure modeling for the Block IIR satellites, see

- "Solar Force Modeling of Block IIR Global Positioning System Satellites" by H.F. Fliegel and T.E. Gallini in *Journal of Spacecraft and Rockets*, Vol. 33, No. 6, 1996, pp. 863-866.

For a discussion of solar radiation pressure stochastic modeling, see

- "A New Solar Radiation Pressure Model for GPS Satellites" by T. A. Springer, G. Beutler and M. Rothacher in *GPS Solutions*, Vol. 2, No. 3, 1999, pp. 50-62.

For further information on the use of finite element modeling in GPS satellite orbit determination, see

- "Thermal Force Modeling for Global Positioning System Satellites Using the Finite Element Method" by Y. Vigue, B.E. Schutz and P.A.M. Abusali in *Journal of Spacecraft and Rockets*, Vol. 31, No. 5, 1994, pp. 855-859.

For a discussion of the pixel array approach to solar radiation pressure modeling, see

- "Analytical Solar Radiation Pressure Model for GLONASS Using a Pixel Array" by M. Ziebart and P. Dare in *Journal of Geodesy*, Vol. 75, No. 11, 2001, pp. 587-599.