Precise Point Positioning

A Powerful Technique with a Promising Future

Sunil Bisnath and Yang Gao

The main goal of this article is to describe the current performance of what has become known as the precise point positioning (PPP) technique, and to discuss the future potential of the technique, along with its technical limitations. We begin with a review of the current state of PPP, covering performance and usage. We then discuss current technical limitations of the approach,

MORE THAN 10 YEARS AGO in an Innovation column, I wrote, "Although RTK is the latest word, or should we say acronym, in GPS positioning, it will not be the last. Scientists and engineers will continue to invent faster, more accurate, more convenient, and more reliable ways to use GPS in navigation, surveying, and a host of other areas, some of which we haven't even dreamt of yet."

In the intervening decade, RTK — or real-time kinematic — positioning has become an industry standard procedure in surveying, machine control, and other high-precision applications. RTK makes use of carrier-phase and pseudorange measurements recorded at a (usually) fixed reference location with known coordinates and transmitted in real time to a user's rover receiver using a radio link of some kind. The rover processes the double differences of observations between satellites and receivers to determine its coordinates with better than 10-centimeter accuracy. It can do this successfully if it can resolve the integer ambiguities in the carrier-phase measurements. Ambiguities are the bane of carrier-phase positioning. They must be resolved to turn carrier-phase measurements into unbiased range measurements.

INNOVATION INSIGHTS
with Richard Langley

Is there a viable alternative to RTK?

In RTK positioning, the ability to resolve ambiguities is determined by many factors, such as the distance between the reference station and the rover and atmospheric effects. RTK is a much more efficient technique than the earlier developed (but sometimes still used) post-processing surveying techniques. However, it does require an investment in reference station infrastructure or the purchase of commercial RTK services.

Is there a viable alternative to RTK? In this month's column, we take a look at the technique of precise point positioning (PPP). Like RTK, PPP makes use of ambiguous carrier-phase measurements but only from the user's receiver. Rather than measurements from a reference receiver, it needs ultra-precise (and accurate) satellite orbit and clock information such as that provided by the International GNSS Service. Currently, there are issues with how long solutions take to converge and the difficulty in resolving the ambiguities, for example, but research is targeting these and other practical issues. How close is PPP to prime time? Read on.

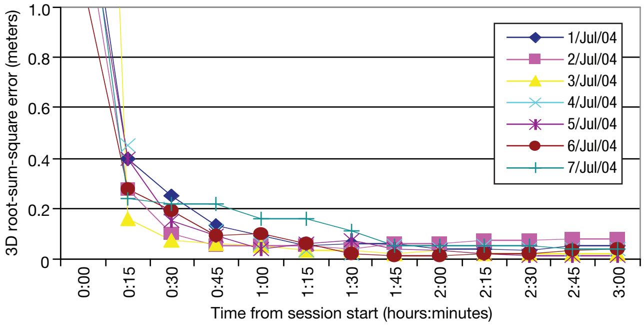
"Innovation" is a regular column that features discussions about recent advances in GPS technology and its applications as well as the fundamentals of GPS positioning. The column is coordinated by Richard Langley of the Department of Geodesy and Geomatics Engineering at the University of New Brunswick, who welcomes your comments and topic ideas. To contact him, see the "Contributing Editors" section on page 4.

including solution convergence period, accuracy, and integrity of solutions. The next section considers potential improvements upon the current approach, in terms of integer ambiguity resolution; integration with other data, for example, from real-time kinematic (RTK) solutions or an inertial navigation system (INS); and the use of other external modeling data, such as atmospheric refraction models. Equally important are PPP-infrastructure challenges, including the availability of precise satellite orbits and clock offsets, precise orbit and clock prediction, real-time dissemination of predicted orbits and clocks, and reference frame realizations. Given the upcoming great changes due to GPS modernization and the development of other global navigation satellite systems (GNSS), we would be remiss not to speculate on the potential significant positive impacts of these added signals on future PPP performance. Finally, we end with a rather provocative discussion of the potential of PPP to perform in a similar manner as the RTK technique.

Current Status

This section is designed to summarize current PPP performance using a number of metrics, and to set the technique's impact within the context of the wider field of positioning and navigation. What we mean by PPP is the state-space solution to the processing of pseudorange and carrier-phase measurements from a single GNSS receiver, utilizing satellite constellation precise orbits and clock offsets determined by separate means. Typically, a dual-frequency GNSS receiver is used with dual-frequency code and phase measurements linearly combined to remove the first-order effect of ionospheric refraction. The real-valued carrier-phase ambiguity terms are estimated from the measurement model. The tropospheric refraction is also estimated, along with the receiver position and ambiguity parameters from the measurements. PPP using a single-frequency GNSS receiver has also been investigated with great promise for certain applications. However, we will not discuss these further in this article; see Further Reading for publications reporting developments in single-frequency PPP and other advances in the technique. To achieve the best position accuracy possible from PPP, effects such as carrier-phase wind-up, transmitter-antenna phase offset, solid Earth tides, and the contribution of ocean tide loading must be corrected using models. Residual terms such as receiver noise and multipath are generally ignored or minimally handled using stochastic procedures.

A unique aspect of PPP is that it is an area of research being actively pursued by academia, government, and industry, in concert and individually. As is typical, early development occurred in research settings for scientific goals. Governments, as service providers, have in some cases engaged in providing PPP services to the public, given the socioeconomic benefits. Industry has embraced and advanced the technology to better serve its clients. The results are: 1) rapid development and use of PPP in a variety of application areas, and 2) significant overlap between the three sectors in terms of research and development, and service models. The latter point will be discussed further in the infrastructure section.



▲ **FIGURE 1** St. John's (STJO) PPP solution convergence on seven sequential days.

Performance Specifications

The standard metrics used here to describe the performance of conventional PPP services are: accuracy, precision, convergence period, availability, and integrity. With PPP solutions showing very little in the way of biases — typically a few centimeters at the most — there is very little difference between the accuracy and precision metrics. In terms of north, east, and up component accuracies at the 1-sigma level, PPP is able to provide few-centimeter-level results in static mode and decimeter-level results in kinematic mode; both could be achieved in either post-mission analyses or in real time.

The convergence period, namely the length of time required from a cold start to a decimeter-level position solution, is typically about 30 minutes under normal conditions and will be significantly longer before the position solution can converge to the few-centimeter level, if at all. This period is determined by the measurement strength of the observables for a GPS-only solution, the geometry of the problem, and the redundancy available for the estimation problem. Initial solutions rely almost exclusively on noisy pseudorange measurements, the uncertainties of which are magnified via the ionosphere-free linear combination. The availability of solutions is usually high, given that application areas for this technique are open sky, continuously unobstructed environments. Otherwise PPP would typically not be used (at its current level of development). Finally, the availability of integrity measures for the PPP solution is considered. Aside from filter covariance estimates, quantitative quality measures of the obtained results are limited. For example, knowledge of biases in corrections such as precise satellite orbit and clock products, the potential for biases in estimated coordinates, and measurement outliers are typically not considered (that is, not rigorously specified and accounted for) in solutions.

Use and Applications

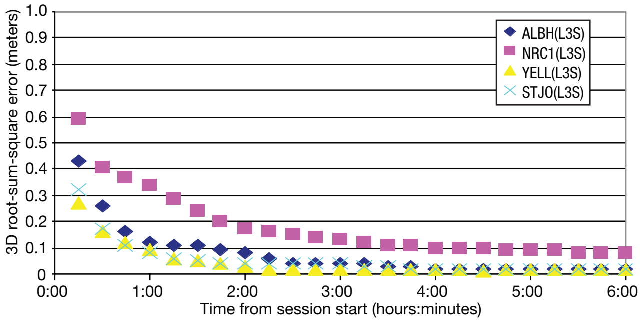
PPP can be used for the processing of static and kinematic data, both in real time, if the dissemination mechanisms are in place to construct, transmit, receive and process precise satellite orbit and clock products, and in post-processing mode. The caveat for all such usage is that there needs to be uninterrupted GNSS signal availability, as loss of tracking lock on a minimum number of satellites requires processing filter re-initialization, resulting in tens of minutes of greater than decimeter-resolution positioning, until filter re-convergence. This constraint severely limits the utility of PPP, insofar as it can only be robustly (that is, successfully) used in environments with continuous open sky coverage.

One of the first uses of a prototype PPP approach was for the rapid post-processing of static geodetic data for establishing and updating reference-station coordinates or for crustal-deformation monitoring. Other scientific uses range from precise orbit determination of low-Earth-orbiting satellites for gravity field recovery to ocean buoy positioning for tsunami detection. The main commercial applications of PPP have been in the agricultural industry for precision farming; in marine applications for sensor positioning in support of seafloor mapping and marine construction, for example; and in airborne mapping.

Further growth in current active areas is underway, and the technique is making inroads into other application areas such as atmosphere remote sensing, precise time transfer, land surveying, construction, and military uses. Fundamentally, PPP is a viable option wherever precise positioning and navigation is required in isolated locations or expansive areas and reference station infrastructure is not available, or very costly to temporarily erect.

Technical Limitations

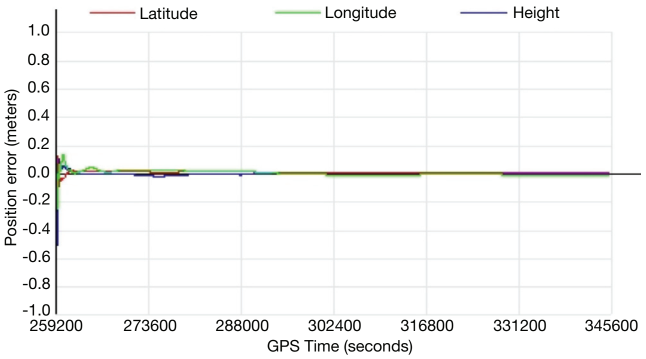
Although the PPP approach presents definite advantages for many applications in terms of operational flexibility and cost-effectiveness, its more widespread use is limited by convergence period, accuracy, and integrity issues.



▲ **FIGURE 2** PPP solution convergence for stations Albert Head (ALBH); National Research Council, Ottawa (NRC1), Yellowknife (YELL), and St. John's (STJO). L3S = ionosphere-free, static.

Convergence Period. PPP requires a long initialization period for phase ambiguities to converge to near constant values and for the solution to reach its optimal precision, taking full advantage of the precise but ambiguous carrier-phase observations. PPP convergence depends on a number of factors such as the number and geometry of visible satellites, user environment and dynamics, observation quality, and sampling rate. As these different factors interplay, the period of time required for the solution to reach a pre-defined precision level will vary.

Shown in **FIGURE 1** are the convergence times with respect



▲ **FIGURE 3** Static positioning using a dataset from Algonquin Park

to different position accuracies from processing data from an International GNSS Service (IGS) tracking station over seven consecutive days. The station data was collected at 30-second intervals, and each 24-hour session was processed using IGS precise orbits and satellite clock values at 5-minute epochs. The position error was computed every 15 minutes. The results show a high degree of day-to-day variability, despite the similar satellite geometry one would expect given a common daily session start time. For example, the solution crosses the 10-centimeter threshold within 30 minutes on several days, but also takes more time to do so on other days. Shown in **FIGURE 2** are the average weekly convergence time series for four different IGS tracking stations (separated by 1,000 kilometers or more and therefore subject to different satellite geometries). Significant differences in positioning accuracy and convergence time exist, likely caused by varying satellite geometry and/or station-specific tracking conditions, such as the multipath environment.

Accuracy. The primary factors that limit the accuracy of PPP are the limited precision of current precise orbit and clock products and the effects of unmodeled error sources. PPP is able to provide few-centimeter-level results in static mode and decimeter-level results in kinematic mode. Shown in **FIGURE 3** are the positioning results of a high-quality, 24-hour, static dataset from an IGS station (Algonquin Park, or ALGO) using the Jet Propulsion Laboratory (JPL) real-time orbit and clock products. It can be seen that the coordinate estimates could converge to the centimeter level within 30 minutes. After convergence, all position coordinate components are accurate at the sub-centimeter level. The positioning-accuracy statistics (root mean square, bias, and standard deviation) are given in **TABLE 1**. Shown in **FIGURE 4** are the positioning results of a kinematic dataset acquired from an airborne platform flying at an altitude of approximately 250 meters above the ground at 50 knots. The precise orbit and clock corrections are again from JPL's real-time orbit and clock product. The short baseline, double-differenced, ambiguity-fixed position solutions were used as ground-truth, and the positioning accuracy statistics are given in **TABLE 2**. The results indicate that it takes about 20 to 30 minutes for the positioning solution to converge to the decimeter level.

	Latitude	Longitude	Height
RMS	0.9	1.0	0.7
Bias	0.8	0.3	0.0
STD	0.3	0.9	0.7

▲ **TABLE 1** Accuracy statistics for static positioning results (see Figure 3) after filter convergence (in centimeters)

	Latitude	Longitude	Height
RMS	2.8	6.8	4.9
Bias	−0.2	−1.5	−1.5
STD	2.8	6.7	4.6

▲ **TABLE 2** Accuracy statistics for kinematic positioning results (see Figure 4) after filter convergence (in centimeters)

In addition to further improvement of the precise orbit and clock products, the ability to exploit the integer property of phase ambiguities can further improve the obtainable position accuracy of PPP. Minor error sources, including initial satellite and receiver phase biases, would need to be estimated and removed in the measurement model, as they cannot be eliminated in undifferenced processing. We will elaborate on these effects later in the article.

Integrity. Integrity monitoring is an essential component of any positioning or navigation system. In PPP processing, some parameters are estimated while others are eliminated via estimates derived from a separate process without multiple solutions (in contrast to network RTK); therefore, providing integrity information for PPP single-receiver estimates is that much more important. A particular industrial service provider has clients who are willing to pay for two independent solutions: PPP and long-range RTK (with float ambiguities). The independent solutions can be compared to judge their accuracies.

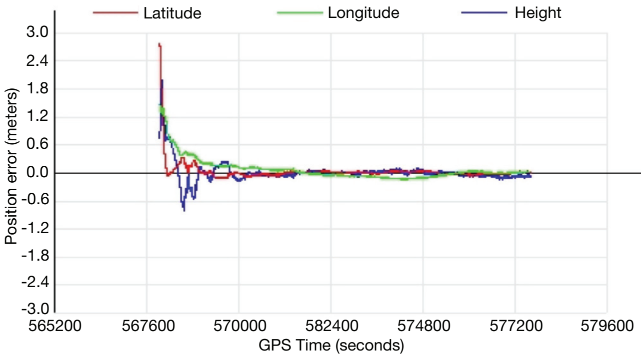
Obviously, post-fit residuals from a PPP solution can be analyzed to detect individual measurement outliers or more significant problems. Such assessment of residuals should be standard practice. More complex examples of integrity monitoring exist in other GNSS applications and should be considered for PPP processing. Potentially, a type of receiver autonomous integrity monitoring (RAIM) could be implemented to help screen PPP estimates. This would be a straightforward design, and would provide users with additional confidence in their PPP solutions beyond covariance estimates and post-fit residuals. More elaborate, it may be possible to contemplate a Wide Area Augmentation System-type of state-space grid error approach to evaluate PPP orbit and clock correction products, which could be specified when generated from tracking stations and integrated into the PPP processing.

Potential Improvement

Having looked at some of PPP's limitations, let's see how this technique might be improved.

Ambiguity Resolution. Could convergence time be improved through stochastic model refinement and use of a higher sampling rate? No. It is only when we can exploit the integer property of ambiguities in PPP that we will have the potential to reduce convergence time to several minutes or even several seconds. In double-differenced GPS processing, where integer ambiguity resolution has been widely utilized, the double-differenced ambiguity parameters are integers, which can render the position solution accuracy to the few-centimeter level or better after they have been fixed to their correct integer values. For PPP with undifferenced observations, the ambiguity parameters are not integers as they are corrupted by the initial fractional phase bias in the GNSS satellites and receivers.

Significant progress is being made in understanding the characteristics and the estimation of the abovementioned initial phase biases. Recent undifferenced ambiguity resolution approaches involve a new model or reformulation of the ionosphere-free code and carrier-phase observation equations, which, when combined with the widelane-phase/narrowlane-code observable, permits resolution of the 86-centimeter and 11-centimeter ambiguities. The nature of the new model implies a clock, or clock-like, parameter estimated for each observable for both satellites and receivers. When applied to static PPP, there is little improvement provided by ambiguity resolution at the end of a 24-hour period. What ambiguity resolution does provide is the ability to reach similar levels of accuracy within much shorter observation periods. With 30-second test data, after 60 minutes 90 percent of horizontal positions are at the two-centimeter level or better with ambiguity resolution, compared to 10 centimeters without ambiguity resolution. In test data sets processed, researchers reported that 50 percent of the horizontal positions were at the 2-centimeter level after 10 minutes using ambiguity resolution.



▲ **FIGURE 4** Positioning using an aircraft dataset

Integration with RTK. Integration of PPP with network RTK techniques may lead to improved position accuracy and performance, particularly a reduction in convergence time. Some researchers have already started to investigate such an integration, and one commercial vendor has developed a global differential positioning system. As PPP can be an efficient alternative to RTK in certain applications, it is expected that more work will be carried out to investigate the seamless integration of PPP and RTK methods.

Integration with INS. The integration of stand-alone

Product		Accuracy	Interval	Latency
Final	Orbits	<5 cm	15 min	~13 days
	Clocks	<0.1 ns	5 min	
Predicted	Orbits	~10 cm	15 min	Real time
	Clocks	~5 ns	15 min	

▲ **TABLE 3** IGS orbit and clock products.

and double-differenced GNSS and INS has been extensively investigated in the past as the coupling has benefits such as improved cycle-slip detection, smoothed receiver trajectory, and increased reliability. An integrated PPP GPS/INS system has been developed to support georeferencing in airborne mapping, which offers similar performance to a differential GPS/INS system. An integration of PPP with INS can also reduce re-initialization time, since INS can supply accurate position and velocity information during short periods and subsequently reduce the position convergence time. This is particularly important for real-time kinematic applications, as frequent signal blockages are common in the field.

Ingestion of Precision Atmospheric Models. A tropospheric parameter unknown is usually estimated along with the position and ambiguity parameters from the measurements in dual-frequency-receiver-based PPP, while several ionosphere parameters are estimated in single-frequency-receiver-based PPP. Ingestion of precision atmospheric models can reduce the total number of unknown parameters that need to be estimated from the measurement model, potentially obviating the need for noise-propagating linear combinations of observables, and potentially improving positioning performance. As a result, the approach could reduce the convergence time of PPP. The challenge is that range corrections from such measurement and physics-based atmospheric models have to be very accurate, with accuracies better than a few centimeters.

Infrastructure Issues

Infrastructure refers to the satellite orbit and clock information products (and potentially, in the future, satellite bias terms for ambiguity resolution) being generated and used in PPP parameter elimination schemes, and the information and processes related to the collection, generation, and dissemination of these products.

Availability of Precise Orbits and Clock Offsets. Precise orbit and clock products have improved significantly in recent years. Although advanced modeling and sophisticated software are required for their generation, post-processed and predicted products are freely available over the Internet. As such, PPP processing can be performed by anyone who can develop the processing software or access one of several online processing engines.

Precise Orbit and Clock Prediction Accuracy. As can be seen in **TABLE 3**, post-processed (final) orbit and clock IGS products are produced at quite a high level of accuracy. However, improvements in IGS predicted products for real-time usage is desirable to marginally improve PPP solutions. The greatest disparity between the final and predicted products is the 50 times worsening of the clock product coupled with a three times enlarging of the data rate. Comparable positioning accuracy, however, has been demonstrated using IGS final products and JPL real-time orbit and clock correction products.

Real-Time Dissemination of Predicted Orbits and Clocks. The production of orbit and clock information for real-time processing, hence prediction of quantities, is a major focus of current research efforts. The IGS, for example, has been studying the generation and dissemination of real-time data products for the past few years and has recently begun a real-time data products pilot project. A number of other institutes are now providing predicted orbits and clocks for, amongst other uses, real-time PPP.

Dissemination of products in the form of corrections can be done in a variety of ways that can primarily be grouped into satellite-based or Internet-based. Satellite-based transmission is the usual choice of commercial service providers, as the correction signals can be received by an antenna built into the GPS receiver package. Internet-based correction transmission can be made with much lower cost and therefore has been used as the model for academic prototypes, with great potential to be widely adopted in applications. A common dissemination protocol is the Networked Transport of RTCM via Internet Protocol (NTRIP).

The manner of development of PPP and PPP infrastructure leads to an interesting question of provider model: free versus paid real-time corrections. That is, an academic/government model of infrastructure support, or a commercial model of paid service provider.

Reference Frame. Satellite orbit and clock products refer to a particular realization of the International Terrestrial Reference Frame. This realization depends on data from numerous GNSS, very long baseline interferometry, and satellite laser ranging stations distributed heterogeneously around the world, and the non-uniform weighting of the varying-length data records from each station. When a new version of ITRF is established and published, and data products redefined with respect to that frame, PPP user coordinates are slightly changed. The end-user must be aware of this metadata and, furthermore, that the coordinates generated by PPP are referred to ITRF and that a coordinate transformation is necessary to bring the PPP solution into the “flavor” of coordinates the user requires such as the Canadian Spatial Reference System implementation of the North American Datum 1983 (NAD83[CSRS]).

Effects of GNSS Evolution. GPS-only PPP has its limitations, such as an insufficient number of visible satellites due to signal blockages and insufficient reliability for safety-of-life applications. An integration of GPS with other navigation systems, such as the Russian GLONASS or the future European Galileo and Chinese COMPASS systems, could provide many more observations and is expected to have a significant impact on position accuracy, reliability, and convergence time of PPP. Several researchers have compared GPS-only and GPS/Galileo-based PPP using simulations. The results demonstrate that a combined system can reduce convergence time by half over GPS-only PPP. For example, we demonstrated that combined GPS/GLONASS PPP greatly improves positioning accuracy and reduces convergence time (see **FIGURES 5** and **6**). These improvements are dependent on the enhanced level of satellite availability and geometry for position determination. Issues such as interoperability and compatibility, however, must be addressed for successful integration of data from hybrid navigation systems.

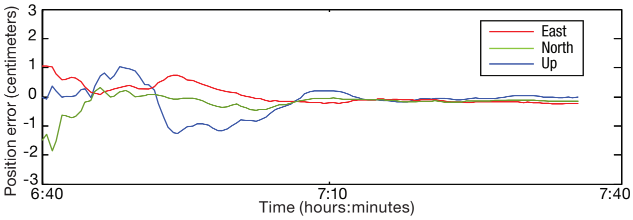
PPP versus RTK

Given the previous discussions about the capabilities and potential of PPP, an obvious topic to conclude this article is a comparison of PPP and the industry-standard RTK technique, and to answer the question: Can PPP ever replace RTK? To address this overall issue, we pose a series of logically ordered questions. Some queries may be answered quite readily, while others can only be partially addressed.

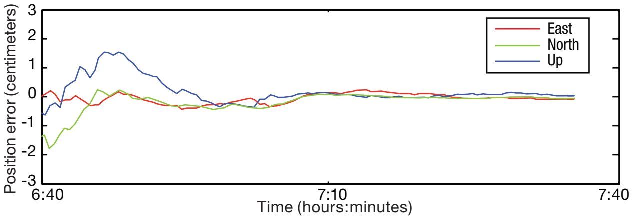
1. *Can PPP algorithms, data and operation be improved to the point where the technique obtains the same level of performance as RTK? And, if so, what specific improvements are required for PPP to perform like RTK?*

This result can be seen as a potential final objective of PPP algorithm research — and a recasting of the overall question. All of the measurement strength of the undifferenced PPP observables is used to determine: 1) few-centimeter-level position estimates, 2) with a few seconds worth of measurements, 3) without the need for a reference station. The utility of such a solution would be significant (see below).

The third characteristic (of removing the need for reference stations) is the one that would make PPP so appealing as compared to RTK. The first characteristic (of attaining centimeter-level positioning accuracy) may be possible, given recent research results related to ambiguity resolution of undifferenced observables. The potential exists to isolate and estimate initial fractional phase biases in order to isolate true integer ambiguities,



▲ **FIGURE 5** Simulated GPS-only PPP solution convergence period and accuracy performance



▲ **FIGURE 6** Simulated GPS/GLONASS PPP solution convergence period and accuracy performance

without over-parameterizing the processing model. The second characteristic (of attaining the desired accuracy with a few seconds of data) is perhaps the most challenging. Given the inherent weaker measurement models of PPP versus RTK (that is, less data in PPP), it will be difficult to estimate the correct biases and ambiguities, aside from the problem of performing this resolution quickly. Meeting the initialization and re-initialization challenge will be the most difficult hurdle for the PPP technique to receive greater industrial acceptance for real-time applications.

2. ***Can this objective be reached in the near future? In a cost-effective way? In a practical manner?***

If the objective of parity between techniques can be reached, it will require significant further algorithm development and perhaps more observables and independent data to reduce the processing-filter convergence period. This latter requirement would delay implementation of an RTK-like PPP processor. No infrastructure changes are required to estimate receiver and satellite code and phase biases, and the added complexity should not increase costs to service providers. If, as some research suggests, few-minute convergence periods are possible, we are hesitant to state that this limitation will be tolerated by many industries other than the ones currently using PPP.

3. ***Do we even want PPP to work like RTK?***

Though it may seem like an odd question, given the context of the discussion, it is a reasonable question to ask if, at least theoretically, PPP can perform at the level of RTK. The two approaches have been developed independently, for different purposes. This fact makes them very useful as independent, mutual integrity checkers for some scenarios.

4. ***If PPP can work like RTK, would it replace RTK, network RTK, or Differential GPS?***

We can speculate that PPP would first be used as a complementary solution to RTK in positioning and navigation work. But as the approach gained acceptance, it could replace RTK. The only caveat here is that a significant level of integrity would have to be guaranteed along with accurate PPP solutions to gain industrial acceptance.

5. ***How would science, industry, and society be affected?***

RTK-like PPP would positively affect activities in all of these areas. Relieved of RTK-baseline constraints, users would be able to perform few-centimeter-level positioning almost anywhere, though performance would be similar in urban areas where RTK networks are already established. That said, a two-system solution would enable new applications.

Conclusion

PPP is a work in progress. Already providing significant benefits to many user communities, PPP will continue to evolve and improve as researchers develop innovative techniques to finally solve the ambiguity resolution problem, and then proceed to further improve accuracy and convergence period — further expanding the technique's utility.

Acknowledgments

The authors acknowledge support from the Geomatics for Informed Decisions Network of Centres of Excellence and the Natural Sciences and Engineering Research Council of Canada. This article is based on the paper "Current Status of Precise Point Positioning and Future Prospects and Limitations" presented at the 24th General Assembly of the International Union of Geodesy and Geophysics held in Perugia, Italy, July 2–13, 2007.

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Further Reading

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FURTHER READING

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- **Precise Point Positioning and RTK Positioning**

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