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ESTIMATION OF IONOSPHERIC DELAY USING KRIGING AND ITS IMPACT ON WAAS AVAILABILITY

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1 INTRODUCTION

To safeguard airline navigation based upon the Global Positioning System (GPS), satellite-based augmentation systems (SBAS) have been developed to ensure the accuracy, integrity, availability, and continuity of user position estimates derived from GPS measurements. Currently the ionosphere remains the largest source of error for single-frequency users of the Global Positioning System (GPS). In the United States, the Wide Area Augmentation System (WAAS) measures ionospheric slant delays using multiple dual frequency receivers in a network of thirty-eight reference stations distributed across North America. WAAS computes ionospheric vertical delays at *ionospheric grid points* (IGPs) as specified by the WAAS Minimum Operational Performance Standards (MOPS)[1]. For each vertical delay at an IGP, WAAS also computes and broadcasts a safety-critical integrity bound called the Grid Ionospheric Vertical Error (GIVE). GIVEs must be sufficiently large to protect against delay estimate error due to ionospheric electron density gradients, both sampled and undersampled.

The sampling of ionospheric gradients is determined by the raypaths that connect satellites to receivers. A single raypath may be characterized, in part, by its *ionospheric pierce point* (IPP), *i.e.*, the location where a station-to-satellite raypath penetrates an infinitesimally thin shell at 350 km. When the IPP coverage near an IGP is sparse, the region is undersampled, and the corresponding GIVE must be inflated to bound gradient threats which may be present when the ionosphere is disturbed. The WAAS undersampled ionospheric threat model[2] consists of a table of values, designated $\sigma_{decorr}^{undersamp}$, that are used to determine the amount by which the GIVE is inflated. It is based upon historical WAAS observations from the twenty- one days during the last solar maximum that exhibited the highest levels of ionospheric disturbance (as indicated by the planetary K-index (K_p) and the disturbance storm time index (D_{ST}) metrics). To define the threat model, the worst case undersampled ionospheric gradient threats are determined as a function of metrics that describe the IPP distribution.

Initially in WAAS, the vertical delay estimate at each IGP was calculated from a planar fit of neighboring slant delay measurements, projected to vertical using the standard thin-shell obliquity factor. In WAAS Follow-On (WFO) Release 3, estimation of vertical delays will be performed by a method based upon *kriging*[3], a type of minimum mean square estimator adapted to spatial data, that originated in the mining industry in the 1950's. Kriging provides a smoothed image of a spatially distributed variable that has been sampled by irregularly spaced measurements. Compared to the planar fit model, the kriging model provides a better matchthe observed random structure of the vertical delays (or it can be tuned to match these data better). This paper discusses the kriging equations used at each IGP to estimate the vertical delay and its uncertainty, and it summarizes the resulting improvement in WAAS availability.

2 THE KRIGING EQUATIONS

Consider a set of N_{IPP} measurements whose IPPs lie in the vicinity of the v^{th} IGP. Let be I_{vIPP} a vector whose elements represent the corresponding vertical delay values, *i.e.,* slant delay measurements converted to vertical using the standard obliquity factor. The kriging estimate $\tilde{I}_{IGPv}$ of the ionospheric vertical delay at this IGP is calculated as[4]:

$$\tilde{I}_{IGPv} = W_v^T I_{vIPP} \tag{1}$$

where the vector of coefficients W_v is specified by:

$$W_v = [W_v - W_v G_v (G_v^T W_v G_v)^{-1} G_v^T W_v] c_v + W_v G_v (G_v^T W_v G_v)^{-1} [1 \quad 0 \quad 0]^T \tag{2}$$

The observation matrix G_v is defined as

$$G_v \equiv \begin{bmatrix} 1 & \Delta x_{1,v}^T \cdot \hat{E}_v & \Delta x_{1,v}^T \cdot \hat{N}_v \\ 1 & \Delta x_{2,v}^T \cdot \hat{E}_v & \Delta x_{2,v}^T \cdot \hat{N}_v \\ \vdots & \vdots & \vdots \\ 1 & \Delta x_{N_{IPP},v}^T \cdot \hat{E}_v & \Delta x_{N_{IPP},v}^T \cdot \hat{N}_v \end{bmatrix} \tag{3}$$

where $\hat{E}_v$ and $\hat{N}_v$ are the standard East and North unit vectors defined for the Up-East-North, (UEN) Cartesian coordinate system with its origin at the v^{th} IGP, and

$$\Delta x_{K,v} \equiv \begin{bmatrix} x_{IPP_K} \\ y_{IPP_K} \\ z_{IPP_K} \end{bmatrix} - \begin{bmatrix} x_{IGP_v} \\ y_{IGP_v} \\ z_{IGP_v} \end{bmatrix} \tag{4}$$

is the Euclidean vector describing the distance separating the K^{th} IPP from the v^{th}IGP in earth-centered, earth-fixed (ECEF) Cartesian coordinates. W_v is a weighting matrix used to assign appropriate weights to the individual measurements for the linear estimate and is defined as

$$W_v \equiv [M_v + C_v]^{-1} \tag{5}$$

where M_v is the N_{IPP} x N_{IPP} measurement noise covariance matrix and C_v is the N_{IPP} x N_{IPP} nominal ionospheric covariance matrix at the v^{th} IGP. The latter's elements are specified by

$$C_{v,KK} = \left(\sigma_{decorr}^{total}\right)^2 \tag{6}$$

$$C_{v,Kl} = \left(\left(\sigma_{decorr}^{total}\right)^2 - \left(\sigma_{decorr}^{nom}\right)^2\right) exp\left(-D_{v,Kl}/d_{decorr}\right) \quad \text{if } K \neq l \tag{7}$$

where

$$D_{v,Kl} \equiv \sqrt{\left(\Delta x_{K,v} - \Delta x_{l,v}\right)^T \left(\Delta x_{K,v} - \Delta x_{l,y}\right)} \tag{8}$$

and σ_{decorr}^{total} , σ_{decorr}^{nom} and d_{decorr} are parameters that specify the nominal ionospheric decorrelation. The elements in the vector describing the nominal covariance between the IGP and the N_{IPP} IPPs are defined as:

$$C_{v,K} = \left(\left(\sigma_{decorr}^{total}\right)^2 - \left(\sigma_{decorr}^{nom}\right)^2\right) exp\left(-d_{v,K}/d_{decorr}\right) \tag{9}$$

where

$$d_{v,K} = \sqrt{\Delta x_{K,v}{}^T \Delta x_{K,v}} \tag{10}$$

These equations reduce to the planar fit equations used in IOC when σ_{decorr}^{total} is equal to σ_{decorr}^{nom} . The uncertainty in the vertical delay estimate is given by

$$\sigma_{IGP}^2 = R_{irreg}^2 \left(W^T \cdot C \cdot W - 2W^T \cdot c + \left(\sigma_{decorr}^{total}\right)^2\right) + W^T \cdot M \cdot W \tag{11}$$

where R_{irreg}^2 is an inflation factor used to account for ionospheric and statistical uncertainty in the X^2goodness-of-fit statistic associated with the estimate (this equation reduces to the formal error variance when R_{irreg}^2 is set to 1).

3 THE WFO RELEASE 3 IONOSPHERIC THREAT MODEL

The ionospheric threat model is a two-dimensional overbound of raw values of $\sigma_{decorr,raw}^{undersamp}$ tabulated as a function of two metrics characterizing the IPP distribution near the IGP, namely, the fit radius (R_{fit}) and the relative centroid metric (*RCM*, *i.e.*, the centroid radius divided by the fit radius). Tabulation of raw data is performed using the following equation:

$$\sigma_{decorr,raw}^{undersamp}\left(R_{fit},RCM\right) = \max_{over\ K,T}\sqrt{\frac{\left|I_{IPP_K} - \tilde{I}_{IPP_K}\right|^2}{K_{undersampled}^2} - \sigma_{IPP_K}^2} \tag{12}$$

where I_{IPP_K} and $\tilde{I}_{IPP_K}$ are the measured and estimated values, respectively, of the vertical delay associated with the K^{th} IPP, $K_{undersampled}$ is a constant that translates the maximum residual into one-sigma numbers (its nominal value is 5.33), and $\sigma_{IPP_K}^2$ is similar to σ_{IGP}^2 but evaluated at the IPP position. The overbound is defined such that $\sigma_{decorr}^{undersamp}$ is monotonically increasing with respect to both IPP distribution metrics. Figure 1 shows the WFO Release 3 threat model.

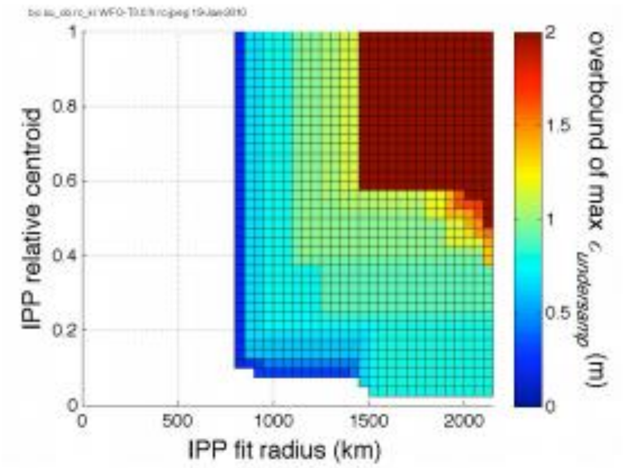

Figure 1: The WAAS WFO Release 3 ionospheric threat model based upon kriging.

CONCLUSIONS

The implementation of kriging improves system availability. At a specified user location, the user's Vertical Protection Level (VPL) is defined as the vertical region, centered on the user's location, in which the WAAS estimate of this location can be reliably assumed to lie. The Raytheon Service Volume Model (SVM) has been used to evaluate the fraction of the service volume for which a given aviation service is available, where availability is specified in terms of the fraction of the day when the Vertical Alert Limit (VAL) for the service bounds the user's VPLs. Implementing kriging improves the fraction of North America experiencing

100% availability under nominal ionospheric conditions from 88.8% to 92.2% for LPV service (VAL = 50 meters) and from 66.5% to 75.5% for LPV200 service (VAL = 35 meters). Under moderately disturbed conditions, the relative improvement is even greater: from 79.1% to 91.4% for LPV service and from 56.6% to 71.9% for LPV200 service.

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REFERENCES

[1] *Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment*, (WAAS MOPS), RTCA Document Number DO-229C, 28 November 2001.

[2] Paredes, E., Pandya, N., Sparks, L., Komjathy, A., "Reconstructing the WAAS Undersampled Ionospheric Gradient Threat Model for the WAAS Expansion into Mexico", *Proceedings of ION GNSS*, Savannah, GA, September 2008.

[3] Wackernagel, Hans, *Multivariable geostatistics: an introduction with applications*, third edition, New York: Springer-Verlag, 2003.

[4] Blanch, J., "Using Kriging to Bound Satellite Ranging Errors Due to the Ionosphere". Doctoral dissertation. Stanford University, December 2003.