The 5th Canadian Geoid Workshop
Ottawa, Canada, May 16 - 17, 2001

Computational Methods for the Discrete Downward Continuation of the Earth Gravity

J. Huang

Geodetic Survey Division

Outline

- Poisson's kernel modification
- The discrete Poisson downward continuation (DDC) and Moritz's analytical downward continuation (ADC)
- Downward continuation of the refined Bouguer anomaly

Poisson's Kernel Modification

Poisson's integral:

$$\Delta g^{l}(r,\Omega) = \frac{R}{4\pi\, r}\int_{\Omega_0'} K_I(r,\psi,R)\,\Delta g^{l}(R,\Omega')\,d\Omega' + F^{l}_{\Delta g,I}(r,\Omega)$$

The spheroidal Poisson kernel:

$$K_{WG}(r,\psi,R) = K(r,\psi,R) - \sum_{n=0}^{l}(2n+1)\left(\frac{R}{r}\right)^{n+1} P_n(\cos\psi)$$

The modified spheroidal Poisson kernel:

$$K_{VK}(r,\psi,R) = K(r,\psi,R) - \sum_{n=0}^{l}\frac{2n+1}{2}\, t_n(H,\psi_0) P_n(\cos\psi)$$

The Far Zone Contribution

The far zone contribution:

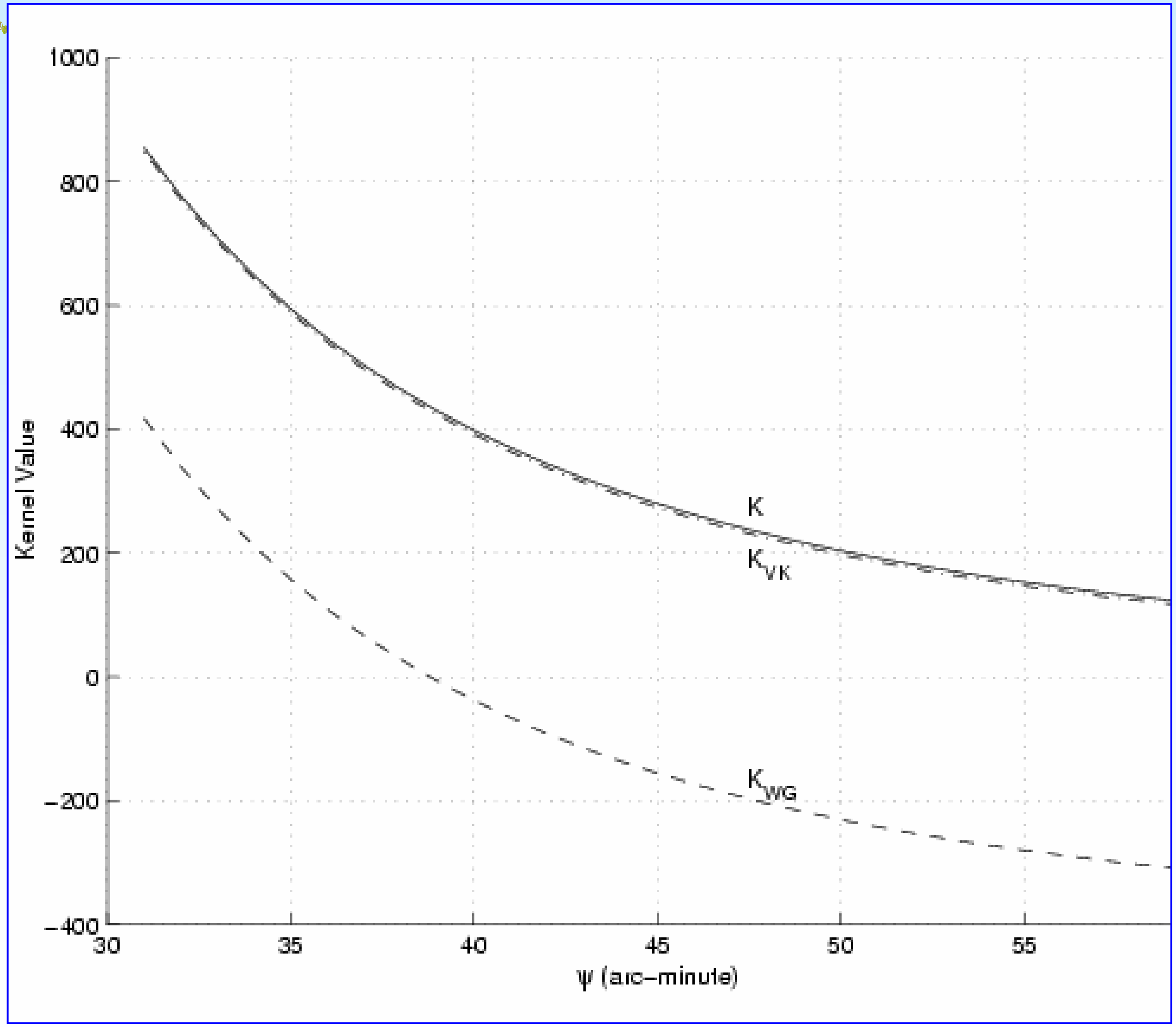
$$F_{\Delta g}^{l}(r,\Omega) = \frac{R\gamma}{2r} \sum_{n=l+1}^{\infty} (n-1) Q_n^I(H,\psi_0) \sum_{m=-n}^{n} C_{nm} Y_{nm}(\Omega)$$

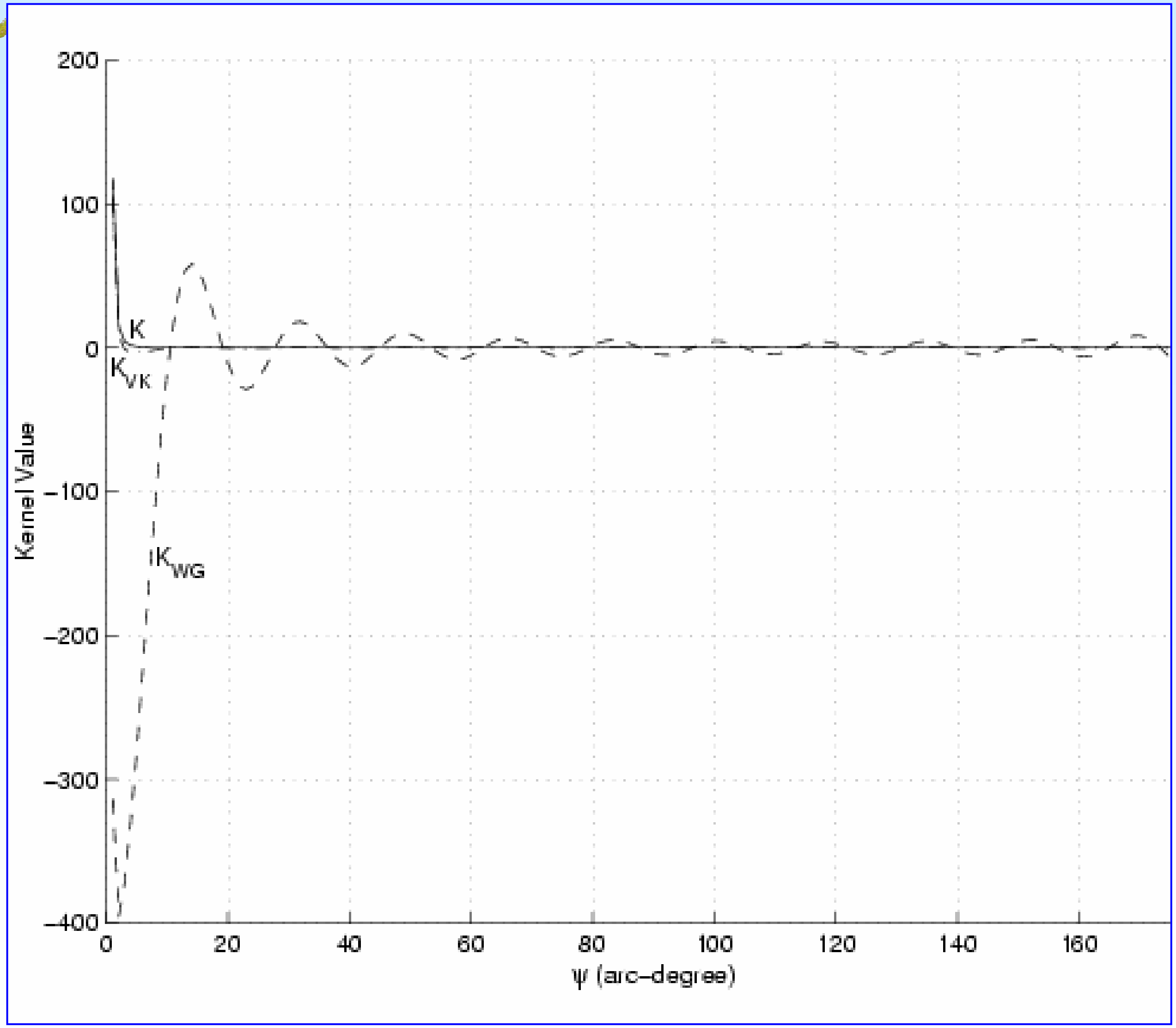
The truncation error coefficients:

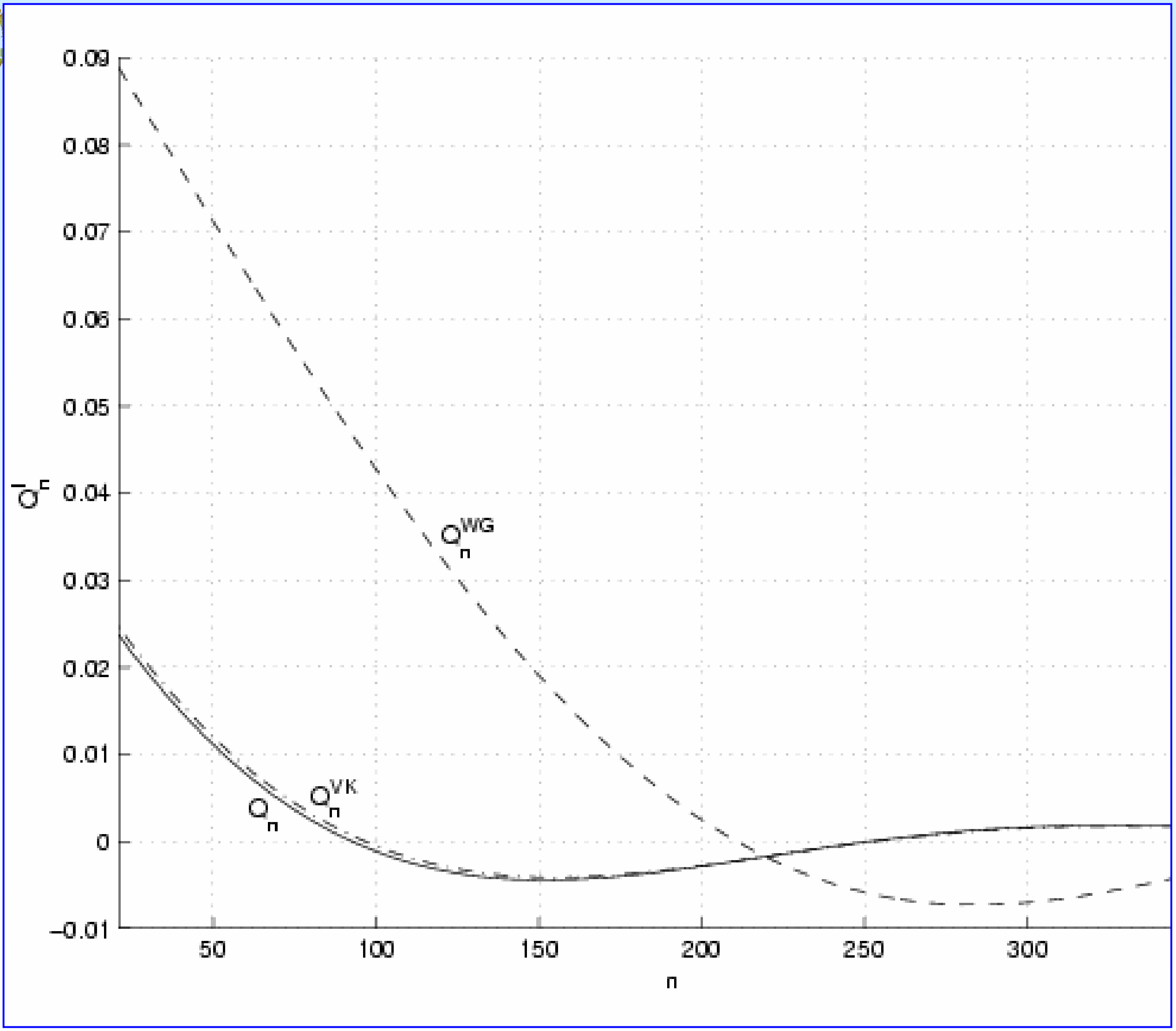
$$Q_n^I(H,\psi_0) = \int_{\psi_0}^{\pi} K_I(r,\psi,R) P_n(\cos\psi)\sin\psi \, d\psi$$

The standard deviation:

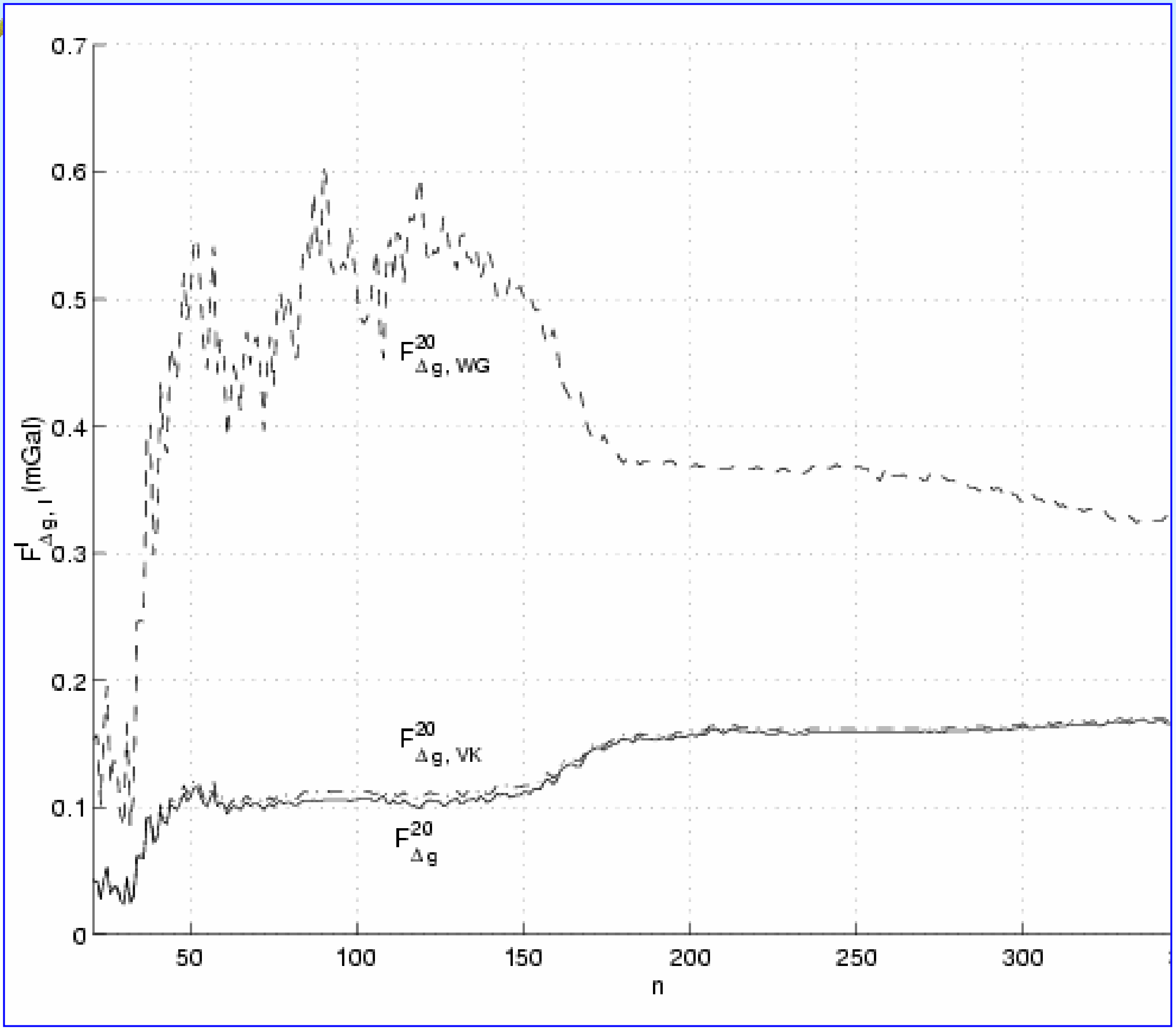
$$m_F^l(r,\Omega) = \frac{R\gamma}{2r} \left(\sum_{n=l+1}^{\infty} (n-1)^2 (Q_n^I)^2 \sum_{m=0}^{n} dC_{nm}^2 Y_{nm}^2 \right)^{\frac{1}{2}}$$


1000
800
600
400
200
0
−200
−400
Kernel Value
30
35
40
45
50
55
ψ (arc−minute)
K
K_VK
K_WG

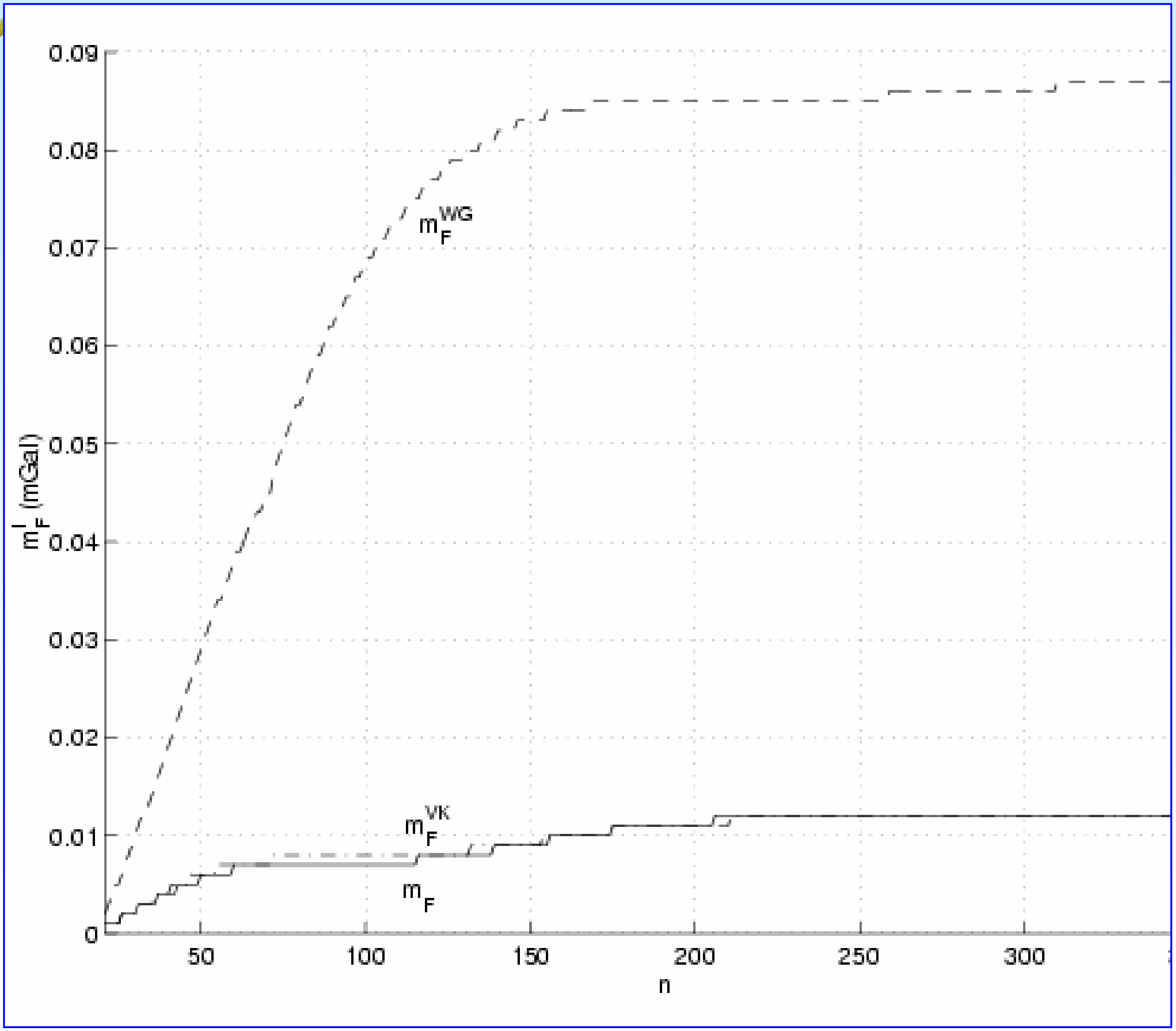

200
100
0
-100
-200
-300
-400
K
K_VK
K_WG
Kernel Value
0
20
40
60
80
100
120
140
160
ψ (arc-degree)



0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
−0.01
50
100
150
200
250
300
n
$\bar{Q}_n$
Q_n^{WG}
Q_n^{VK}
Q_n



0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
50
100
150
200
250
300
n
$F^{l}_{\Delta g,\,l}$ (mGal)
$F^{20}_{\Delta g,\,WG}$
$F^{20}_{\Delta g,\,VK}$
$F^{20}_{\Delta g}$



0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
50
100
150
200
250
300
n
m_F^l (mGal)
m_F^{WG}
m_F^{VK}
m_F

Moritz's Analytical Downward Continuation

(Moritz, 1980, Section 45)

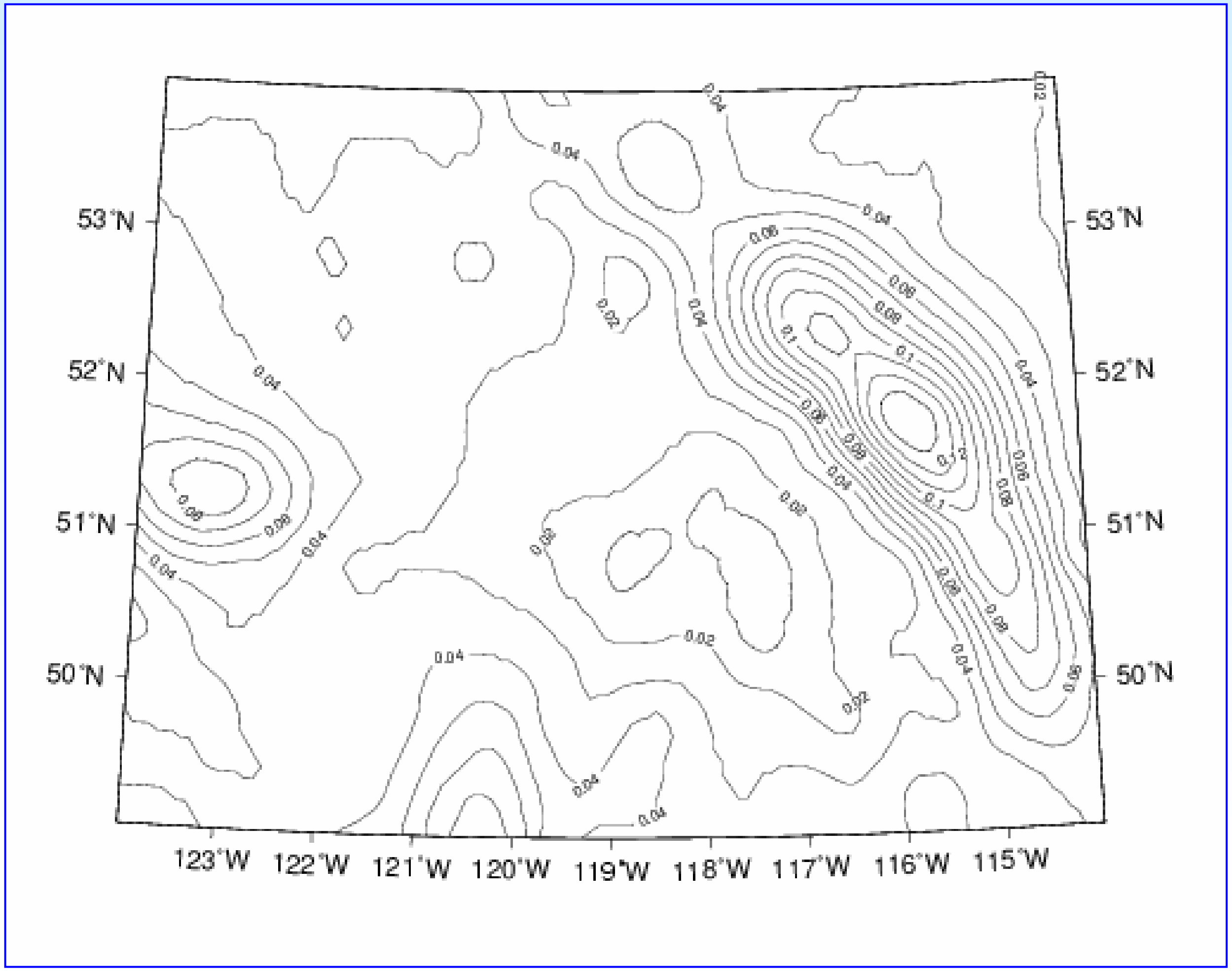
$$\Delta g(r_g, \Omega) = \sum_{n=0}^{\infty} g_n,$$

$$\begin{aligned} g_0 &= \Delta g(r_t, \Omega), \\ g_1 &= -H \cdot L_1(g_0), \\ g_2 &= -H \cdot L_1(g_1) - H^2 \cdot L_2(g_0), \\ &\ldots \\ g_n &= -\textstyle\sum_{r=1}^{n} H^r \cdot L_r \cdot g_{n-r}. \end{aligned}$$

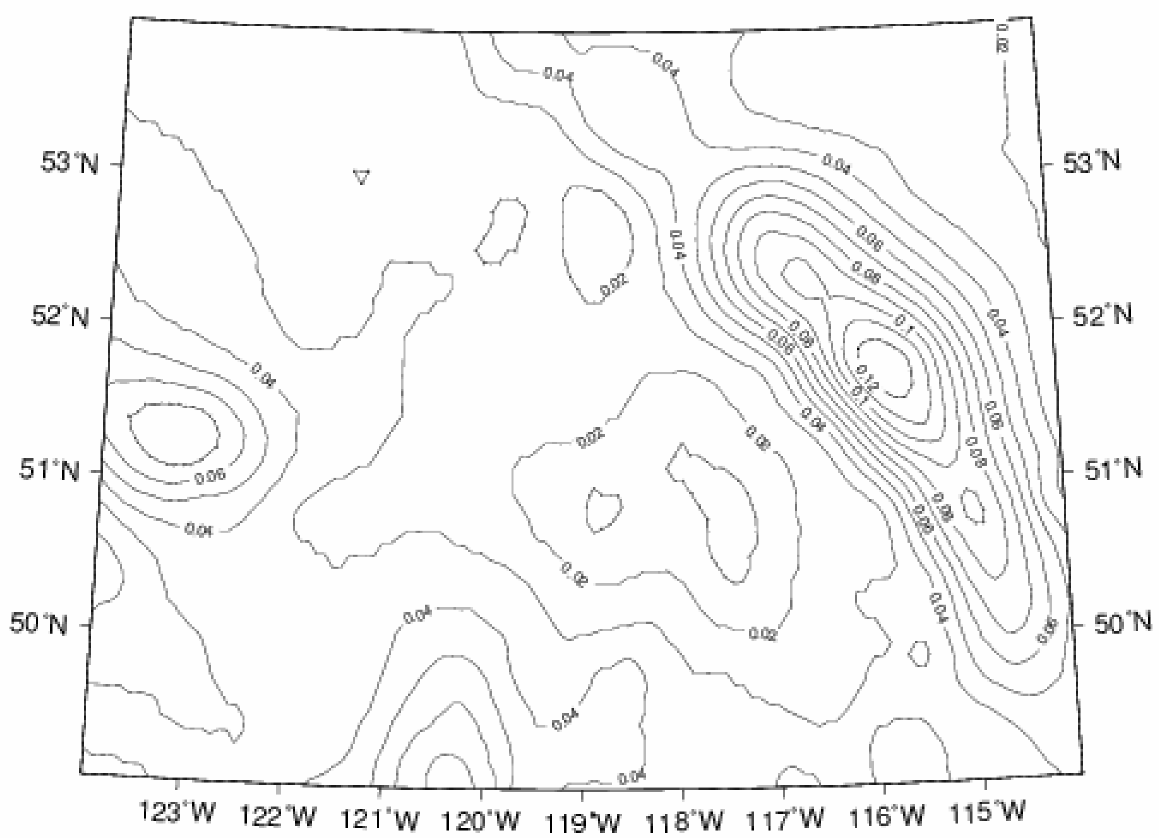
$$L(g) = \frac{R^2}{2\pi} \int_{\Omega'} \frac{g - g_p}{d^3} d\Omega' - \frac{1}{R} g_p,$$

$$L_n = \frac{1}{n!} L^n = \frac{1}{n} L L_{n-1}.$$

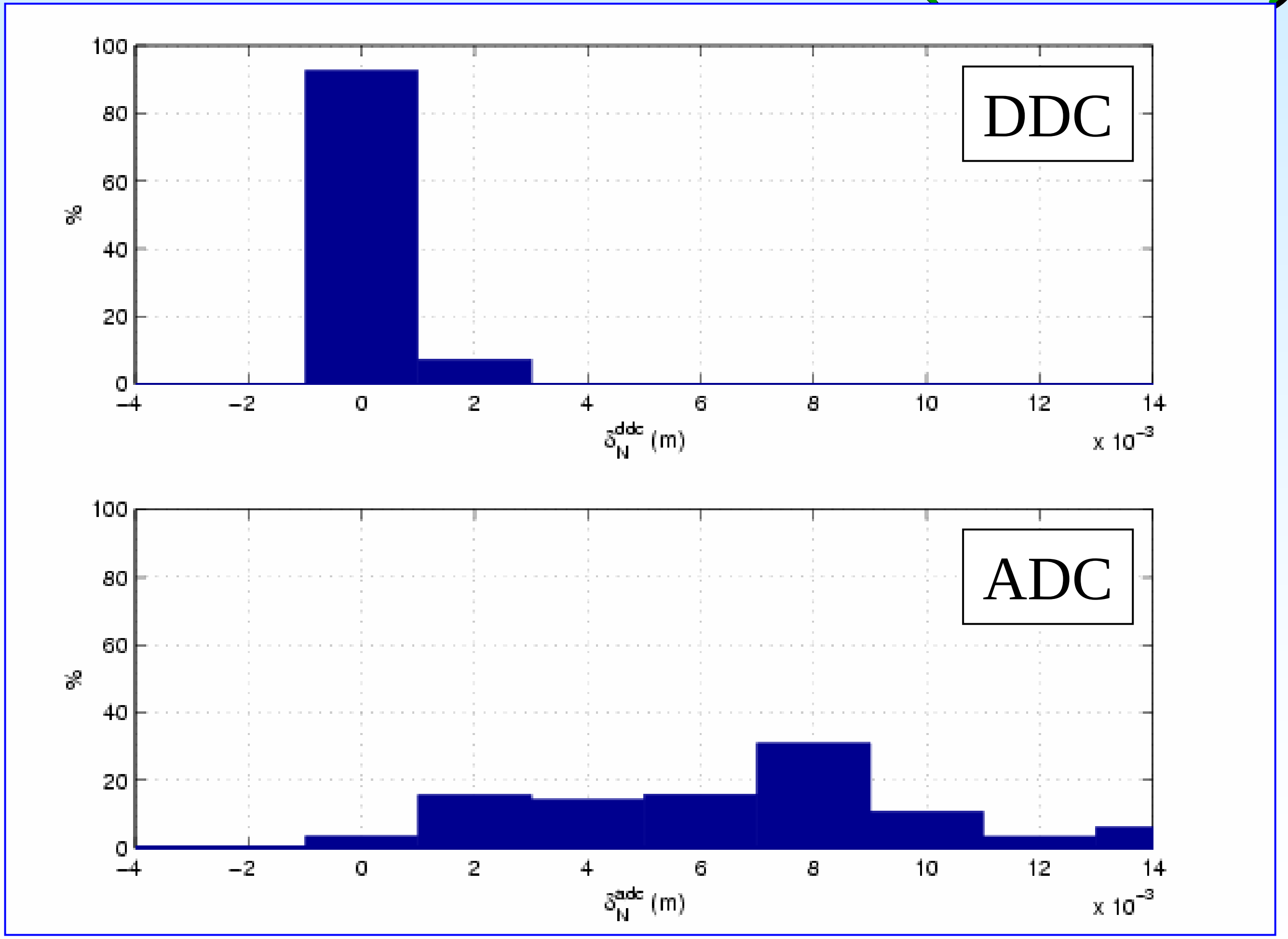
The DDC for the GPM98a Synthetic Data (degrees 21-1800)



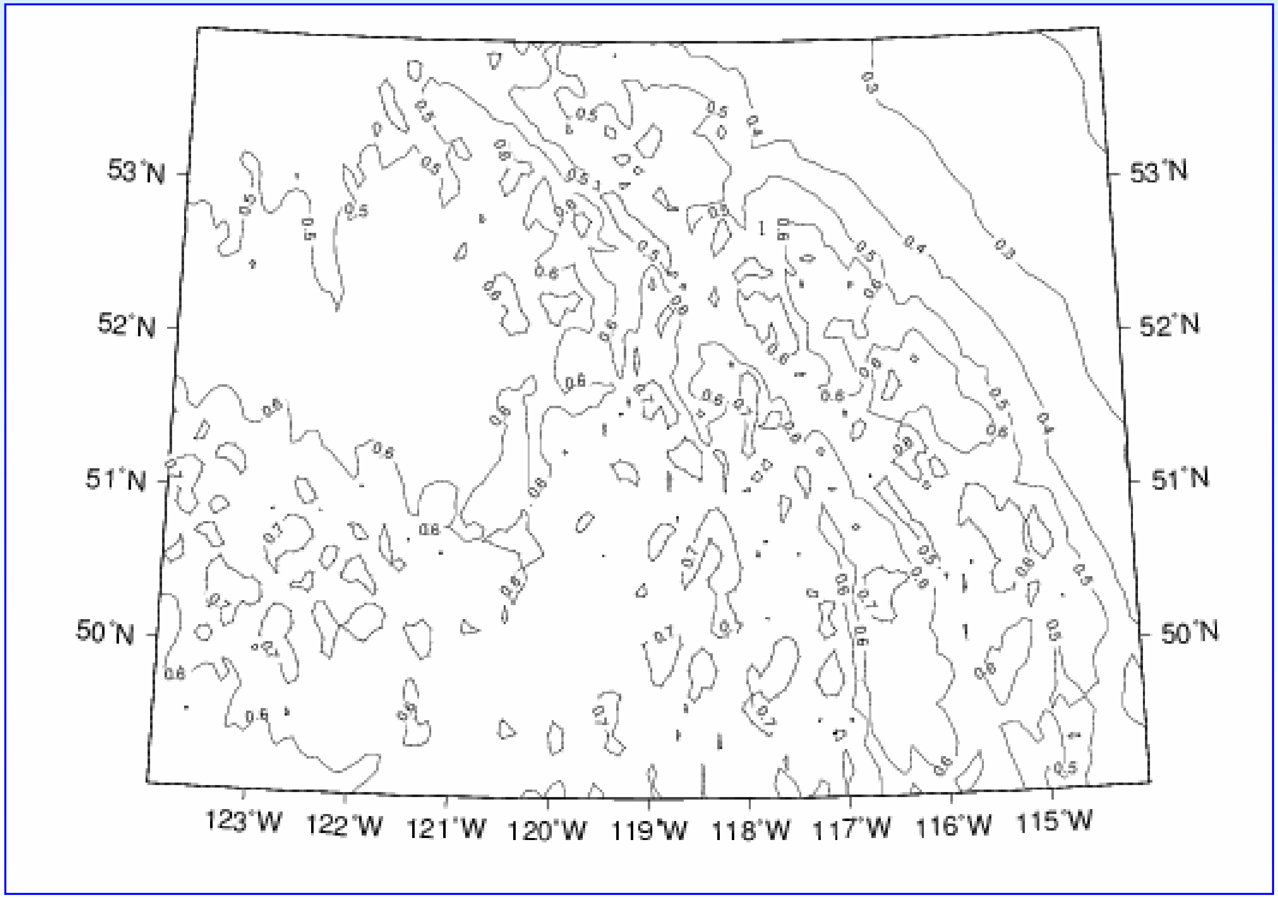
The ADC for the GPM98a Synthetic Data (degrees 20-1800)



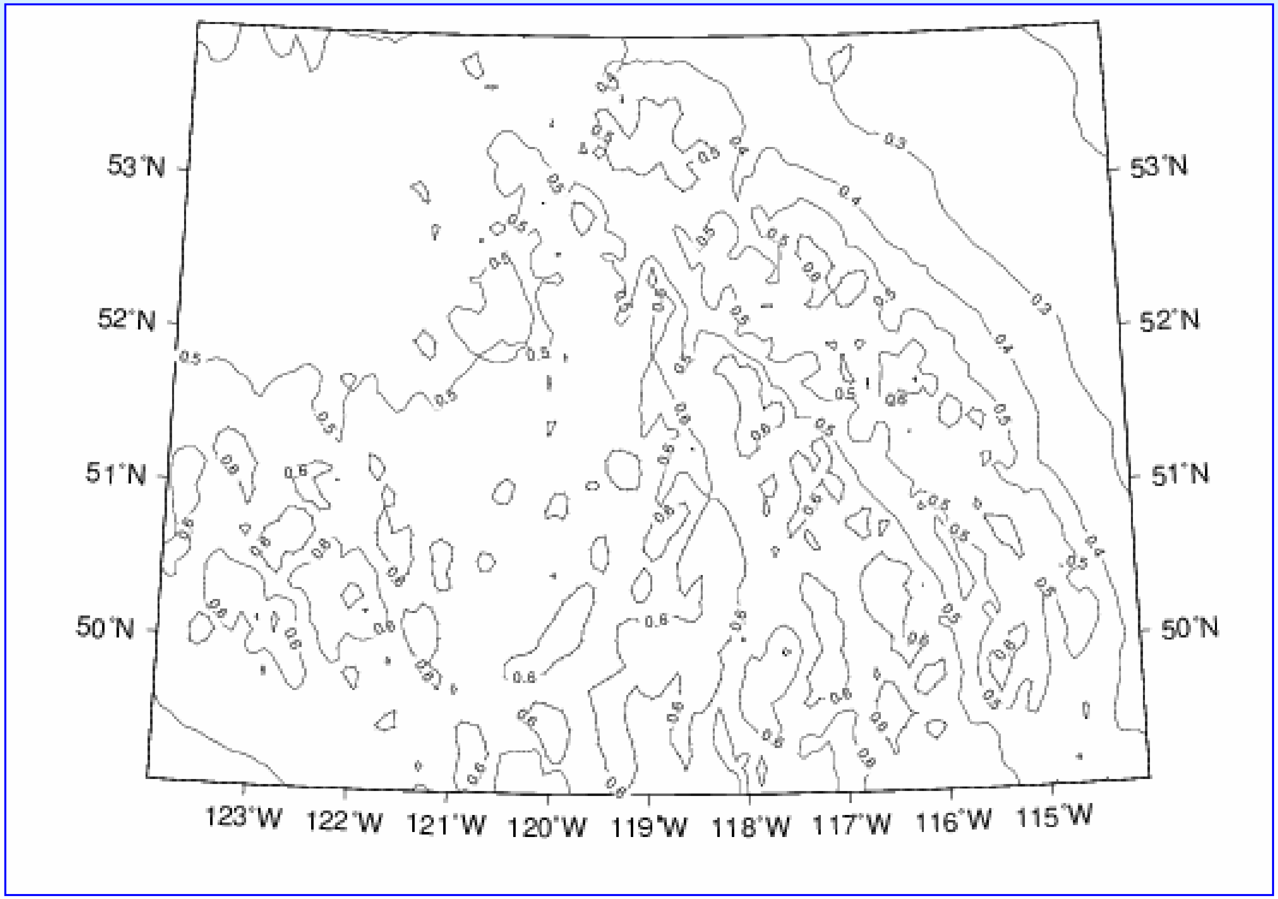
Errors of the DDC and the ADC for the GPM98a Data (21-1800)



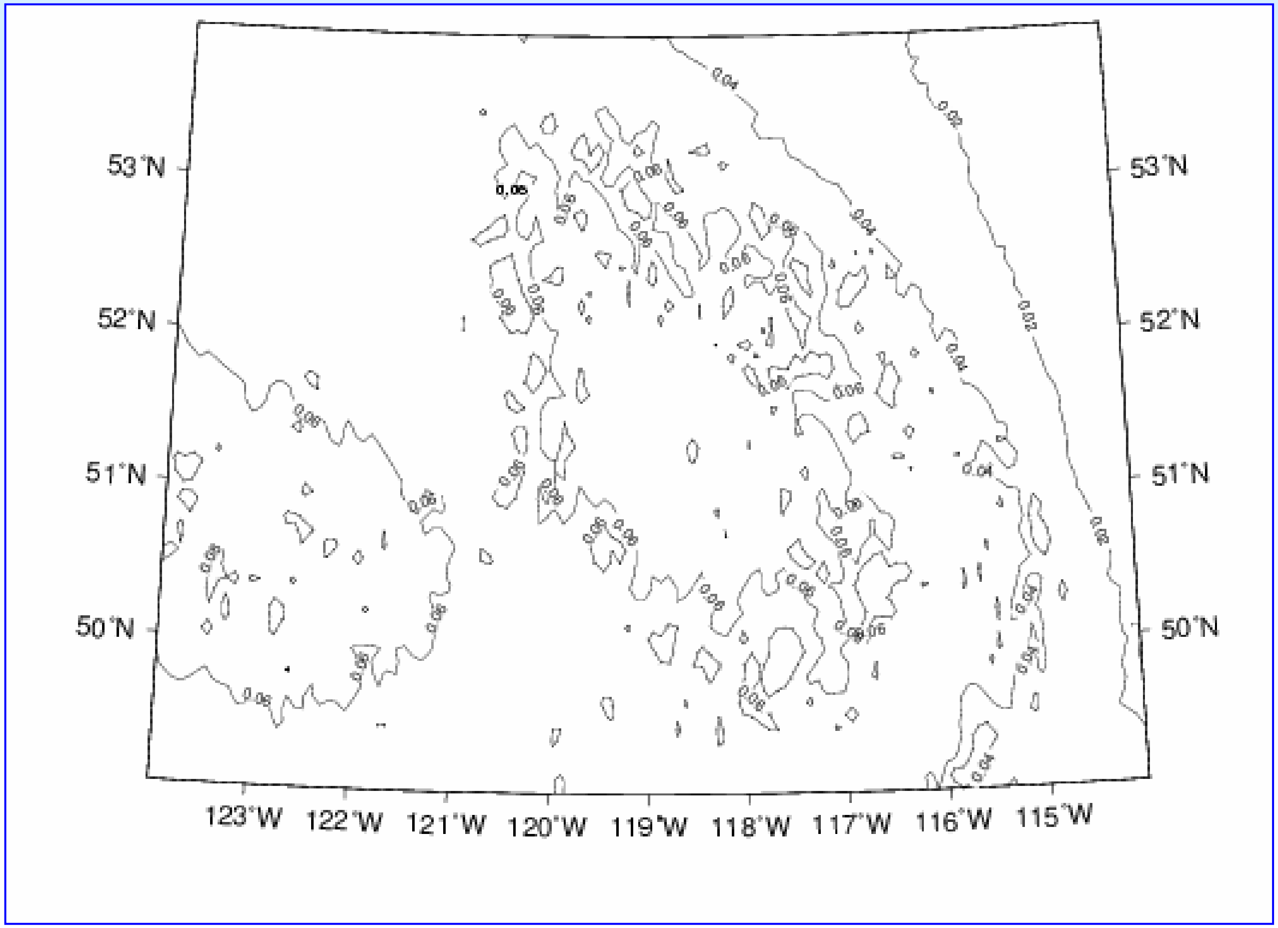
The DDC of the Residual Helmert Gravity Anomaly (above degree 20)



The ADC of the Residual Helmert Gravity Anomaly (above degree 20)



The Difference Between the DDC and the ADC



The Downward Continuation of the Refined Bouguer Anomaly

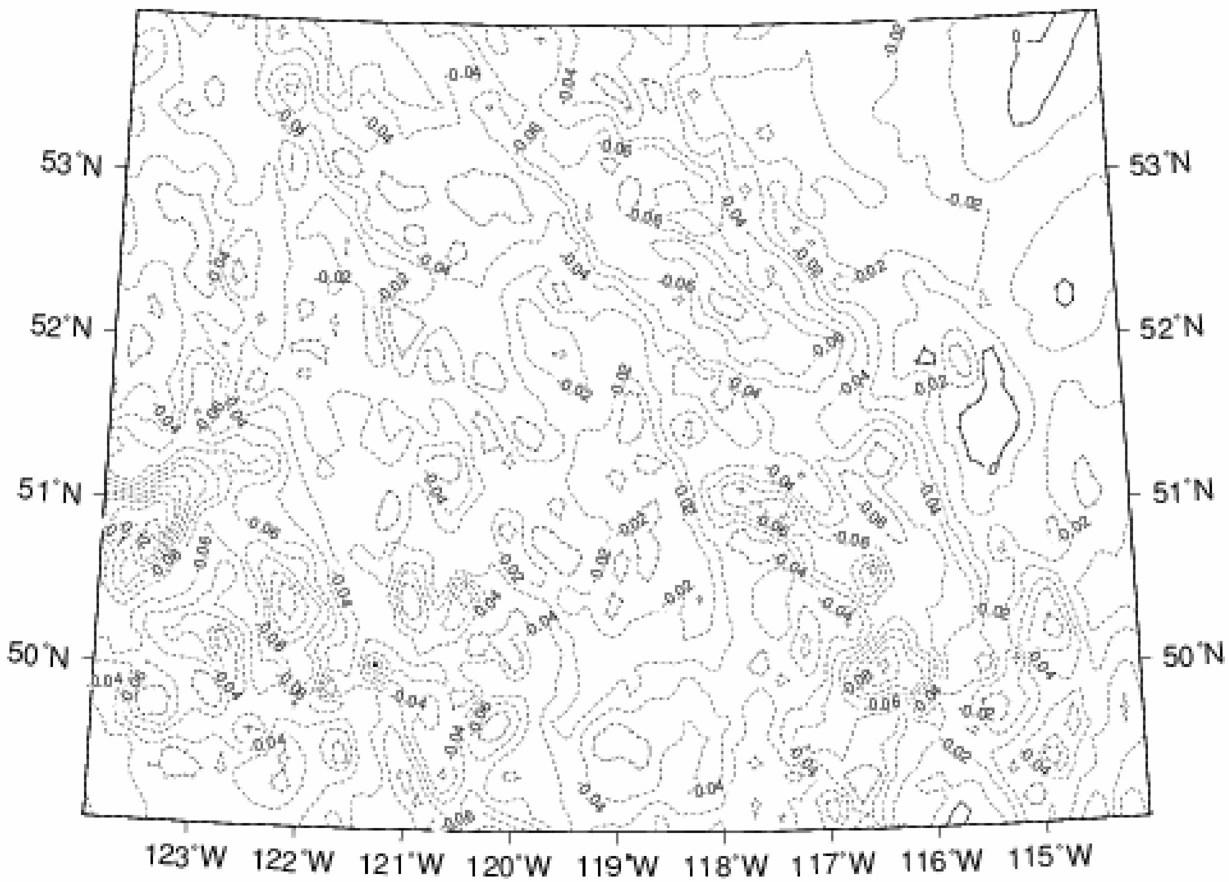
$$-\frac{\partial T^h(r,\Omega)}{\partial r} - \frac{2}{r} T^h(r,\Omega) = \Delta g^h(r,\Omega) + \varepsilon$$

$$T^h(r,\Omega) = T(r,\Omega) - V^t(r,\Omega) + V^{ct}(r,\Omega)$$

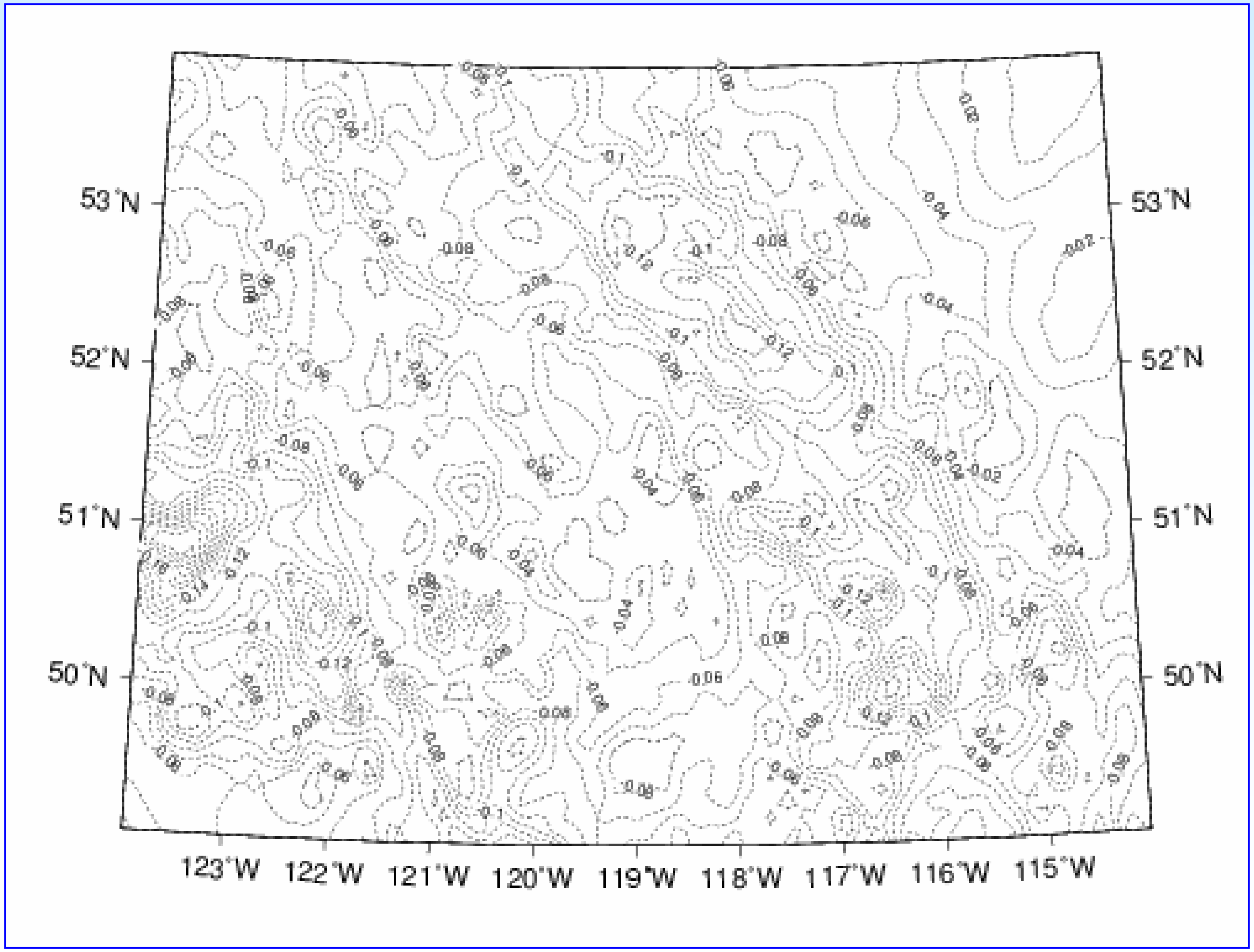
$$\Downarrow$$

$$-\frac{\partial T^{rb}(r,\Omega)}{\partial r} - \frac{2}{r} T^h(r,\Omega) = \Delta g^h(r,\Omega) + \frac{\partial V^{ct}(r,\Omega)}{\partial r} + \varepsilon$$

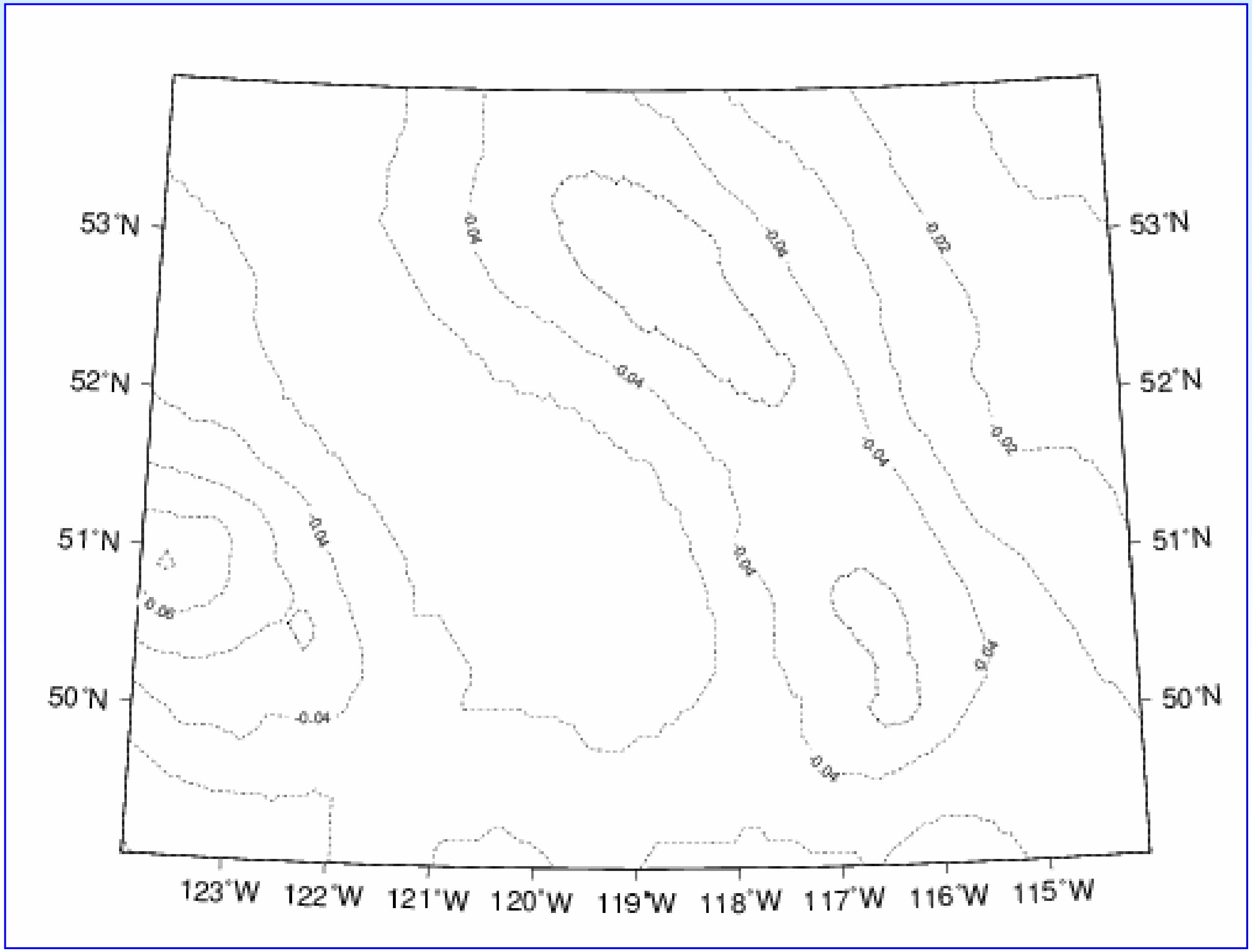
ψ_0 = 1 arc-degree



ψ_0 = 2 arc-degrees



The Difference Between the 1-deg. Cap and the 2-deg. Cap



Conclusions

- The standard Poisson kernel functions as well as the modified spheroidal Poisson kernel in reducing the far zone contribution.
- The analytical downward continuation agrees with the discrete Poisson's within 10% of the total downward continuation effect.
- The DC of the refined Bouguer gravity anomaly faces a challenge of evaluation.