Effect of topographical density on geoid in the Canadian Rocky Mountains

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Abstract

Gravity reduction from the Earth's surface to the geoid requires a knowledge of topographical mass density. However, in practice the constant density (2.67 g/cm^3) is mostly used to approximate the actual density because of the difficulty and complexity of obtaining the actual density. This approximation introduces errors in the reduced gravity, and consequently, in the geoid. Recently, the Geographical Information System (GIS) was introduced as an efficient tool to geo-reference actual bedrock densities to digital geological maps.

As a part of the effort towards the construction of the 'one centimeter geoid' for Canada, the effects of lateral topographical density variation on gravity and geoid were investigated in the Canadian Rocky Mountains. Density values were estimated from the geological maps of Canada and the US and bedrock density tables compiled for the use in the ArcView GIS. The $5' \times 5'$ mean and point topographical effects were computed from height and density data available on a $30'' \times 60''$ grid. The mean direct (topographical) density effect (DDE) on gravity ranges between -4.5 $mGal$ and 2.3 $mGal$ (mean of 0.008 $mGal$), at the Earth's surface, and from -12.7 $mGal$ to 9.8 $mGal$ (mean of 0.007 $mGal$), at the geoid. The secondary indirect (topographical) density effect (SIDE) on gravity varies between -8 μGal and 5 μGal. The primary indirect (topographical) density effect (PIDE) on geoid changes from -2.5 cm to 1.7 cm (mean of 0.2 cm). The total topographical density effect on the geoid ranges between -7.0 cm and 2.8 cm (mean of -0.8 cm). Our results suggest that the effect of topographical density lateral variations is significant enough and ought to be taken into account for the determination of the one centimetre geoid.

Key words: Geoid, Gravity, Density, the Rocky Mountains

1 Introduction

The existence of topography and atmosphere violates the requirements for the Stokes boundary value problem. However, Helmert's 2nd condensation method can be applied to satisfy the requirements (see Figure 1). In the Figure 1, H^0 and H^N stand for the orthometric height and the normal height, respectively. The superscript h indicates the specified object is defined in Helmert's space in which the topography has been removed and condensed by applying Helmert's 2nd condensation method. PI(T)E represents the (primary) indirect effect (Heiskanen and Moritz, section 3-6, 1981).

Helmert's 2nd condensation method has been used to determine the geoid in Canada and the US (Vaníček et al. 1995; Véronneau 1996; Smith and Milbert 1999). This approach conceptually consists of the following steps (Najafi, 1996; Vaníček et al. 1999):

1. Transformation of the 'observed gravity anomaly' Δg_t on the Earth's surface from the real space into the Helmert gravity anomaly Δg_t^h, in Helmert's space.

2. Downward continuation of Δg_t^h to the Helmert co-geoid.

3. Solution of the boundary value problem in the Helmert space, i.e., solution for the Helmert co-geoid using the generalized Stokes formula (Vaníček and Sjöberg 1991).

4. Transformation of the co-geoid in Helmert's space to the geoid by evaluating the primary indirect topographical effect (PITE).

The transformation of the gravity anomalies from the real space to Helmert's space is given by (omitting atmospherical effects) (Vaníček et al. 1999)

$$\Delta g^h(r_t, \Omega) = \Delta g(\Omega) + \frac{2}{R} H(\Omega) \Delta g^B(\Omega) + \delta A(r_t, \Omega) + \delta\gamma(r_t, \Omega), \qquad (1)$$

where Δg is the free-air gravity anomaly, the second term is a correction for the difference between the quasigeoid and the geoid, δA is the direct topographical effect (DTE), $\delta\gamma$ is the secondary indirect effect (SITE), Ω is the geocentric angle denoting the pair (θ, λ), the spherical co-latitude and longitude, r_t is the radius of a point on the Earth's surface, H is the orthometric height, and Δg^B is the simple Bouguer gravity anomaly. DTE is a correction to gravity for shifting the topographical mass to the Helmert Layer. SITE is a correction to gravity for the change of the telluroid due to shifting the topographical mass. Sjöberg (Eqs. (70)-(73),2000) formulates an identical transformation to Eq. (1), but the second term of Eq. (1) is not explicitly given.

In this approach, topography affects geoid modeling through the terms DTE, SITE, and PITE. The evaluation of these terms requires a digital elevation model (DEM) and a digital topographical density model (DTDM). While DEMs are readily available with high resolution, this is not the case for the DTDM. Because of this, the constant topographical density (2.67 g/cm^3) is used in practice to approximate the real density. The real density varies from 1.0 g/cm^3(water) to 2.98 g/cm^3(gabbro). The use of the constant density introduces errors in the reduced Helmert gravity anomalies, and consequently, in the geoid.

Martinec (1993) showed theoretically that the lateral density variation of the topographical masses may introduce errors in the geoid at the decimeter level. Fraser et al. (1998) developed a prototype GIS-based system to calculate terrain corrections using the real topographical rock density values. They have shown that in the Skeena Region of British Columbia, Canada the terrain corrections to gravity can change by a few mGals when real topographical density is used. Subsequently, Pagiatakis et al. (1999) showed that the effect of lateral density variations on the geoid can reach nearly 10 cm in the Skeena Region BC and several millimeters in New Brunswick, where the terrain is moderate (hills). However in their study they considered only the effect of terrain corrections to gravity.

In this work, the topographical mass density variation effects on geoid were systematically investigated. The Canadian Rocky Mountains have the largest relief and density variations in Canada, thus, the geoid computation for this area is affected the most by the topographical density effects: the results shown here represent the largest effects on gravity and geoid in Canada.

2 Digital topographical density model

A digital topographical density model is a description of density distribution in the topography. Strictly speaking, a three-dimensional model would be needed to give accurate topographical density distribution. It would require however, a three-dimensional geological model of topography. At present, the available geological information in Canada and the US comes from (two-dimensional) geological maps and only lateral variation of density can thus now be modeled.

Geological Survey of Canada published in 1997 the Digital Geological Map of Canada, which displays bedrock formations at or near the earth surface. The bedrock units are grouped according to composition and geological age. The digital version of the geological map facilitates its use greatly, by allowing a direct import into a GIS. It is digitized from the geological map of $1:5,000,000$. About $16,000$ geometrical polygons are used to delimit bedrock units over Canada. The area of individual polygon varies from 0.02 km^2 to about 800,000 km^2 depending on the geological complexity of the region. These polygons form the fundamental density units. U.S. Geological Survey published a similar geological map in a digital version over the US in 1998.

Pagiatakis and Armenakis (1999) described the principles and the procedure of generating the two-dimensional topographical mass density map using the digital geological maps in a GIS. This procedure includes the following steps

1 Compilation of topographical mass density tables in which each geological unit is assigned a range of densities or a unique density value in terms of existing geological studies.

2 Assignment of the mean value of the density range as a representative density value to each geological unit.

3 Overlay of the topographical mass density tables onto the digital geological map layer to generate the geological density map.

In this study, we have used the density tables for Canada that were compiled by Fraser et al.(1998) and an approximate density table over the north-west part of the US that was compiled by Castle (Personal communication 1998). In order to characterize the errors inherent in the DTDM, a standard deviation was associated with each representative density value. By assuming the uniform distribution of densities over the density range within each geological unit, the standard deviation can be estimated by the following formula (Vaníček 1971 page 21):

$$\sigma_{\overline{\rho}} = q/\sqrt{3}, \tag{2}$$

where q is the half-range of the density within each geological unit. A DTDM and the associated standard deviations were generated on a 30" $\times$ 30" geographical grid in the area of the Rocky Mountains by rasterizing the geological maps of the density distribution and its standard deviation. This area covers the north-west part of the US and the south-west part of Canada ($49° - 62°$N, $221° - 261°$E, see Figures 2 and 3).

3 Mathematical formulation

The topographical mass density can now be expressed as the sum of the constant value $\rho_0 = 2.67 g/cm^3$ and the lateral variation $\delta\overline{\rho}(\Omega)$:

$$\overline{\rho}(\Omega) = \rho_0 + \delta\overline{\rho}(\Omega), \tag{3}$$

where the over-bar indicates that the lateral variation model of density is used. This equation means that the topographical mass density varies only with respect to horizontal locations. Martinec (1993) and Martinec and Vaníček (1994a, 1994b) derived formulae for the quantities of interest, namely the DDE, SIDE and PIDE defined as follows:

The *direct density effect* (DDE) - the part of DTE caused by lateral topographical mass density variation with respect to the constant density can be written as

$$\delta A_D(r_t, \Omega) = \quad G \int_{\Omega_0'} \delta\overline{\rho}(\Omega') \left[\frac{\partial \tilde{K}(r, \psi, r_t')}{\partial r} - \frac{\partial \tilde{K}(r, \psi, R)}{\partial r} \right.$$
$$\left. - R^2 \tau(H(\Omega')) \frac{\partial L^{-1}(r, \psi, R)}{\partial r} \right] \Bigg|_{r=r_t} d\Omega', \tag{4}$$

where

$$\tilde{K}(r, \psi, r') = \frac{1}{2}(r' + 3r\cos\psi) L(r, \psi, r') + \tag{5}$$
$$+ \frac{r^2}{2}(3\cos^2\psi - 1) \ln |r' - r\cos\psi + L(r, \psi, r')| + C,$$

$$\tau(H) \doteq H, \tag{6}$$

where r_t and r_t' are radii of the point of interest and the integration point respectively, ψ is the spherical distance between a point of interest and an integration point, R is the mean radius of the earth, G is the gravitational constant, $L(r,\psi,r')$ is the distance between (r,Ω) and (r,Ω'), and $\tilde{K}(r,\psi,r')$ is the primitive function of the Newtonian kernel $L^{-1}(r,\psi,r')$ with respect to r'. τ is a coefficient function of the surface mass density for the Helmert layer. Ω'_0 indicates the integration domain.

The *secondary indirect density effect* (SIDE) - the part of SITE caused by lateral topographical mass density variation with respect to the mean density reads

$$\begin{aligned}\delta\gamma_D(r_t,\Omega) = \quad & \frac{2G}{R}\int_{\Omega'_0}\delta\overline{\rho}(\Omega')\left[\tilde{K}(r_t,\psi,r'_t) - \tilde{K}(r_t,\psi,R)\right.\\ & \left.-R^2\tau(H(\Omega'))L^{-1}(r_t,\psi,R)\right]d\Omega'. \qquad (7)\end{aligned}$$

The *primary indirect density effect* (PIDE) - the part of PITE caused by lateral topographical mass density variation is

$$\begin{aligned}\delta N_D(\Omega) = \quad & \frac{G}{\gamma}\int_{\Omega'_0}\delta\overline{\rho}(\Omega')\left[\tilde{K}(R,\psi,r'_t) - \tilde{K}(R,\psi,R)\right.\\ & \left.-R^2\tau(H(\Omega'))L^{-1}(R,\psi,R)\right]d\Omega' \qquad (8)\end{aligned}$$

Integrals (4), (7) and (8) become singular when the point of interest coincides with the integration point, but the singularity is removable (see Martinec 1993).

The errors of the derived DDE, SIDE and PIDE can be estimated from the errors of topographical height and density data. Only the topographical density errors will be considered here while the errors in heights are not considered in this paper. To facilitate the derivation, let us write eqns. (4), (7) and (8) in their generic form:

$$\delta(r_t, \Omega) = c \int_{\Omega'} \delta\overline{\rho}(\Omega') D(r_t, \psi, r_t') d\Omega', \tag{9}$$

where $D(r_t, \psi, r_t')$ stands for the appropriate integration kernel. Then the individual error of any of these effects is given by

$$e(r_t, \Omega) = c \int_{\Omega'} \epsilon(\Omega') D(r_t, \psi, r_t') d\Omega', \tag{10}$$

where $\epsilon(\Omega')$ stands for the error of the topographical density of the integration point. The variance σ_δ^2 of δ can be expressed as (Heiskanen and Moritz 1967 7-74)

$$\sigma_\delta^2(r_t, \Omega) = c^2 \int_{\Omega'} \int_{\Omega''} \sigma_{\overline{\rho}}(\Omega', \Omega'') D(r_t, \psi', r_t') D(r_t, \psi'', r_t'') d\Omega'' d\Omega', \tag{11}$$

where $\sigma_{\overline{\rho}}(\Omega', \Omega'')$ is the covariance of the topographical density $\overline{\rho}(\Omega)$ between two integration points located at Ω' and Ω".

The discrete form of eq. (11) can be written as

$$\sigma_\delta^2(r_t, \Omega_i) = c^2 \sum_j \sum_k \int_{\Delta S_j'} \int_{\Delta S_k''} \sigma_{\overline{\rho}}(\Omega_j', \Omega_k'') D(r_t, \psi_{ij}', r_t') D(r_t, \psi_{ik}'', r_t'') d\Omega'' d\Omega'. \tag{12}$$

where $\Delta S_j'$ and $\Delta S_k''$ represent the discrete surface elements corresponding to points Ω' and Ω'' respectively. If the errors between two different discrete cells are assumed to be independent, eq. (12) becomes

$$\sigma_\delta^2(r_t, \Omega_i) = c^2 \sum_j \sigma_{\overline{\rho}}(\Omega_j')^2 \left(\int_{\Delta S_j'} D(r_t, \psi_{ij}', r_t') d\Omega' \right)^2. \tag{13}$$

where $\sigma_{\overline{\rho}}$ is the standard deviation of topographical mass density.

Thus under the assumption of the uncorrelated errors between two different cells, the error variances of DDE, SITE and PIDE for discrete integral can be evaluated using the following discretized equation:

$$\sigma^2_{\delta A}(r_t,\Omega_i) = G^2 \sum_j \sigma^2_{\bar\rho}(\Omega'_j)\left(\left[\frac{\partial \tilde K(r,\psi_{ij},r'_t)}{\partial r} - \frac{\partial \tilde K(r,\psi_{ij},R)}{\partial r} - R^2\tau(H(\Omega'_j))\frac{\partial L^{-1}(r,\psi_{ij},R)}{\partial r}\right]\Bigg|_{r=r_t} \Delta S'_j\right)^2 , \tag{14}$$

$$\sigma^2_{\delta\gamma}(r_t,\Omega_i) = \left(\frac{2G}{R}\right)^2 \sum_j \sigma^2_{\bar\rho}(\Omega'_j)\left(\left[\tilde K(r_t,\psi_{ij},r'_t) - \tilde K(r_t,\psi_{ij},R) - R^2\tau(H(\Omega'_j))L^{-1}(r_t,\psi_{ij},R)\right]\Delta S'_j\right)^2 , \tag{15}$$

$$\sigma^2_{\delta N}(\Omega_i) = \left(\frac{G}{\gamma}\right)^2 \sum_j \sigma^2_{\bar\rho}(\Omega'_j)\left(\left[\tilde K(R,\psi_{ij},r'_t) - \tilde K(R,\psi_{ij},R) - R^2\tau(H(\Omega'_j))L^{-1}(R,\psi_{ij},R)\right]\Delta S'_j\right)^2 . \tag{16}$$

4 Numerical results

As in the case of the DTE and SITE, the DDE and SIDE should be added to the Helmert anomalies at the earth surface. If the mean Helmert anomalies are used, then mean values of the DDE and SIDE have to be evaluated; they should be computed for the cells of the same size as the mean Helmert anomalies represent. For Canada, the mean Helmert gravity anomalies for $5' \times 5'$ cells are being used to determine the geoid. Thus the mean values of $5' \times 5'$ DDE and SIDE must be evaluated and added to the gravity anomalies.

The PIDE is the correction to the geoid height that is evaluated as the point value, thus the point PIDE must be evaluated. In our computation, the $30'' \times 60''$ DTDM and DEM have been used as input data for the evaluation of the point PIDE in the spacing of $5' \times 5'$.

Numerical tests show that the density effects on the geoid heights evaluated from integration over $1°$, $2°$ and $3°$ caps differ by less than 1 mm in absolute value even for an extreme density variation of 0.3 g/cm^3. Therefore, a spherical cap with the radius of $1°$ was used to evaluate the DDE, SIDE and PIDE. The far zone contributions of lateral density variation effects were not estimated due to lack of a global coverage of density data. Further studies will be needed to show the impact of the far zone.

As described above, the representative density value for each geological unit is taken as the mean value of the density variation range. Naturally, the random errors of the density estimates affect the estimates of DDE, SIDE and PIDE. Thus, as a part of this study, the standard deviations of the DDE and PIDE have been also estimated. The SIDE contributions are too small to be taken into account. Its amplitude is about 2.5% of the SITE, which is evaluated on the geoid (Vaníček et al., 1995). Consequently, the standard deviations of SIDE were not evaluated.

4.1 Mean direct density effect

When a mean value of the DDE is evaluated in a cell, a certain number of point values must be used to compute it. For example, a mean $5' \times 5'$ value can be evaluated by using 5, 10, 50 or even more point values within the $5' \times 5'$ cell. The question is: how many point values are required to give a sufficiently accurate mean value ? Since the final product is the geoid, it is appropriate to set the criterion in terms of geoid height accuracy. Figure 4 shows that for height data on $30'' \times 60''$ grid, 50 points regularly spaced within the $5' \times 5'$ cell, will generate accurate enough approximation to the mean DDE on a $5' \times 5'$ cell. The maximum difference between the geoid heights evaluated from the mean DDE values of 50 and 100 point values is 0.6 cm. The testing profile ($49°N$) passing through the Rocky Mountains suggests that when the point values are evaluated at a step equal to or smaller than the input height step, the geoid height can not be improved significantly. In Figure 4, the numeric labels stand for the numbers of point values used to evaluate the mean values.

The mean $5' \times 5'$ DDE values are summarized in Table 1. The DDE range is about 5% of the DTE range which is $[-54.3, 79.5] mGal$ (Vaníček et al., 1995). The DDE standard deviations were estimated only as point values based on eqn. (14) because of the sheer volume of computation for the mean standard deviations. These point values are theoretically greater than the standard deviations of the mean values. In terms of the standard deviation estimates, the DDE estimates appear statistically precise (see Figures 5 and 6). On the other hand, these point values may overestimate the accuracy of the point DDE because the topographical mass densities within the same geological unit are correlated positively.

After the downward continuation to the geoid using DOWN'97 software (Vaníček et al. 1997), the mean DDE at the geoid is between -12.7 $mGal$ and 9.8 $mGal$ with the mean of -0.007 $mGal$ (see Figure 7). More than 99% of DDE values are within the range of $[-4, 4]$ $mGal$. After conversion to geoid heights through Stokes's integration, the effect on the geoid reaches a few centimeters with a dominant short wavelength feature (see Table 2 and Figure 8). The DDE demonstrates a high correlation with the topographical density as shown in Figure 2.

4.2 Primary indirect density effect

Like the DDE, the PIDE affects the geoid at the decimeter level. The PIDE estimates are statistically precise as we can see from the comparison with their standard deviations (Figures 9 and 10). Its range roughly corresponds to 5% of the PITE range (Vaníček et al., 1995). While the PITE is always negative, the sign of the PIDE changes between the positive and negative due to the nature of the density variation. The PIDE is mainly characterized by the intermediate wavelength features which highly correlate to the topographical density.

4.3 Total lateral topographical density variation effect on the geoid

The sum of both effects (DDE and PIDE) on geoid heights have been evaluated and shown in Figure 11. No significant cancellation between the PIDE and the DDE can be detected in these results. Both, the short wavelength components from the DDE and the long wavelength components from the PIDE have been combined into the total effects. More than 99% of the values are between -3 and 3 centimeters (see Table 4). Only eight values are larger (in absolute value) than 5 cm.

5 Summary

This investigation shows that the DDE and PIDE can reach about 5% of the DTE and PITE respectively. The SIDE is too small to be taken into account. Comparing the DDE and PIDE with their standard deviations, their estimates appear to be statistically precise. The total density variation effect on geoid heights ranges from -7.0 cm to 2.8 cm in the Canadian Rocky Mountains. It is evident that the introduction of the digital topographical density model will significantly improve the accuracy of the precise geoid evaluation. It should be pointed out that the DEM significantly affects the evaluation of the topographical mass density effects. The results presented in this paper may have underestimated the effects because the mean DEM of $30'' \times 60''$ is far insufficient to model the topography in the Canadian Rocky Mountains.

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Table 1: Direct topographical lateral density variation effects on gravity at the earth surface in mGal.

TERM	MIN	MAX	Mean	r.m.s.
DDE	-4.5	2.3	0.0	0.3
StD	0.0	2.4		0.1

Table 2: Direct and primary indirect lateral topographical density variation effects on geoid heights, in centimeters.

TERM	MIN	MAX	Mean	r.m.s.
DDE	-5.1	2.6	-1.0	1.3
PIDE	-2.5	1.7	0.2	0.5
StD	0.0	0.5		0.0
Total Effect	-7.0	2.8	-0.7	1.1

Table 3: Topographic mass density variation effects versus the total topographical effects using the actual mean density value. (Note: DTE, SITE and PITE by Vaníček et al. are computed by using the constant density 2.67 g/cm^3.)

TERM	MIN	MAX	RANGE
DTE(Vaníček et al., 1995)	-54.3 $mGal$	79.5 $mGal$	133.8 $mGal$
DDE	-4.5 $mGal$	2.3 $mGal$	6.8 $mGal$
SITE(Vaníček et al., 1995)	0 μGal	470 μGal	470 μGal
SIDE	-8 μGal	5 μGal	13 μGal
PITE(Vaníček et al., 1995)	-105 cm	1 cm	106 cm
PIDE	-2.5 cm	1.7 cm	4.2 cm

Table 4: Distribution of the total effect values caused by the lateral topographical mass density variation.

Range (cm)	Count	%	Range (cm)	Count	%
-7 to -6	4	0.06	-2 to -1	1800	25.04
-6 to -5	4	0.06	-1 to 0	3779	52.57
-5 to -4	8	0.11	0 to 1	890	12.38
-4 to -3	51	0.71	1 to 2	159	2.21
-3 to -2	479	6.66	2 to 3	15	0.21

Legends for Figures:

Figure 1 The real to Helmert space transformation and vice-versa.

Figure 2 The topographical lateral mass density map in the Rocky Mountains. The white color indicates water bodies. Unit: g/cm^3.

Figure 3 The standard deviation of the topographical mass density in the Rocky Mountains. The white color indicates water bodies. Unit: g/cm^3.

Figure 4 Geoid correction profiles due to the mean DDE computed from different number of point values. The label numbers associated with profiles are the numbers of point values adopted for the evaluation of the mean DDE within the $5' \times 5'$ cell.

Figure 5 The mean direct topographical lateral mass density variation effect on gravity at the Earth's surface in the Canadian Rocky Mountains in mGals.

Figure 6 The point standard deviation of the direct topographical lateral mass density variation effect in the Canadian Rocky Mountains in mGals.

Figure 7 The mean direct topographical lateral mass density variation effect on gravity on the geoid in the Canadian Rocky Mountains in mGals.

Figure 8 The mean direct topographical lateral mass density variation effect on geoid in the Canadian Rocky Mountains. The solid lines represent the positive contours, the dash lines represent the negative contours. Contour interval: one centimeter.

Figure 9 The primary indirect topographical mass density effect on the geoid in the Canadian Rocky Mountains. The solid lines represent the positive contours, the dash lines represent the negative contours. Contour interval: one centimeter.

Figure 10 The standard deviation of the primary indirect topographical lateral mass density variation effect in the Canadian Rocky Mountains in centimeters.

Figure 11 The total effect of the topographical lateral mass density variation effects on geoid in the Canadian Rocky Mountains. The solid lines represent the positive contours, the dash lines represent the negative contours. Contour interval: one centimeter.