UNB application of Stokes-Helmert's approach to geoid computation

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Outline

- Formulation of the appropriate BVP
- Determination of the boundary values
- Rigorous treatment of topo-effects + DWC
- Modified Stokes's formula
- Practical results & problems
- Current developments
- Conclusions

Unknown function **Region of interest**

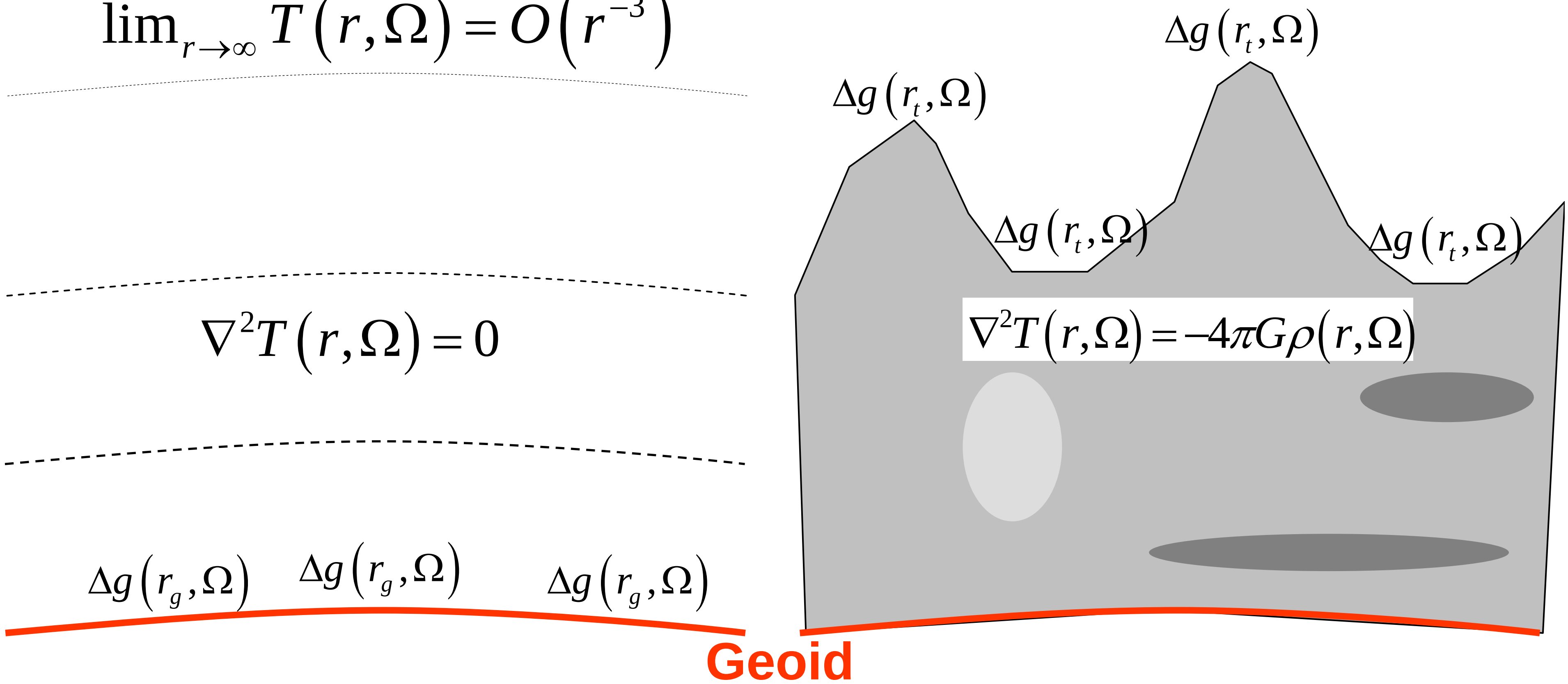
$$T(r,\Omega) = W(r,\Omega) - U(r,\Omega) \;,\quad \forall r \geq r_g \;,\; \Omega \in \Omega_0 .$$

Ideal world

$$N(\Omega) = \frac{T(r_g,\Omega)}{\gamma_0(\phi)}$$

Real world

$$\lim_{r\to\infty} T(r,\Omega) = O\left(r^{-3}\right)$$



Helmertization of the disturbing potentia[l]

$$T^h(r,\Omega) = T(r,\Omega) - V^t(r,\Omega) + V^{ct}(r,\Omega) - V^a(r,\Omega) + V^{ca}(r,\Omega)$$

$$\nabla^2 T^h(r,\Omega) = 0$$

Fundamental gravimetric equation

$$\Delta g(r,\Omega) = -\frac{\partial T(r,\Omega)}{\partial n} + \gamma\left[r - \varsigma(r,\Omega), \phi\right]^{-1} \frac{\partial \gamma(r,\phi)}{\partial n} T(r,\Omega)$$

Conversion of free-air to Helmert's anomaly

$$\Delta g^h(r,\Omega) = \Delta g^{FA}(r,\Omega) + \frac{\partial\left[V^t(r,\Omega) - V^{ct}(r,\Omega)\right]}{\partial r} + \frac{\partial\left[V^a(r,\Omega) - V^{ca}(r,\Omega)\right]}{\partial r} +$$

$$+ \frac{2}{r}\left[V^t(r,\Omega) - V^{ct}(r,\Omega)\right] + \frac{2}{r}\left[V^a(r,\Omega) - V^{ca}(r,\Omega)\right] + \varepsilon_{\delta g} + \varepsilon_n$$

Harmonic!!! $\nabla^2\left[r\Delta g^h(r,\Omega)\right] = 0$

“Spherical” topographic effects

Planar

$$V^t(r_t,\Omega) \to \infty \qquad A(r_t,\Omega) = 2\pi G\rho H$$

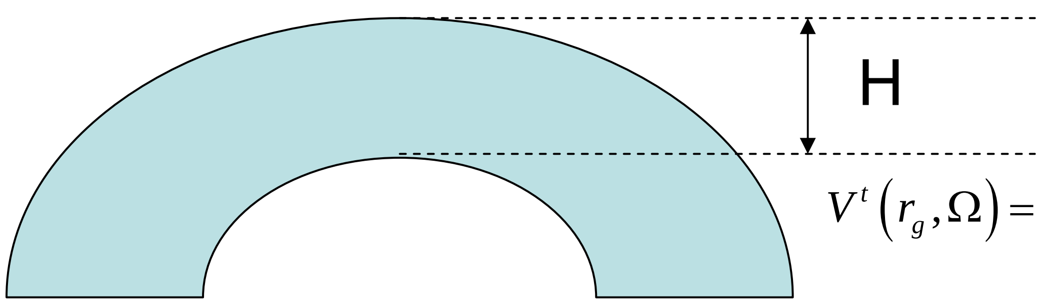
$$V^t(r_g,\Omega) \to \infty \qquad A(r_g,\Omega) = -2\pi G\rho H$$

Disqualifies for the harmonization of the Earth gravity field!!!

Spherical

$$V^t(r_g,\Omega) = 4\pi G\rho RH + O(H/R)$$

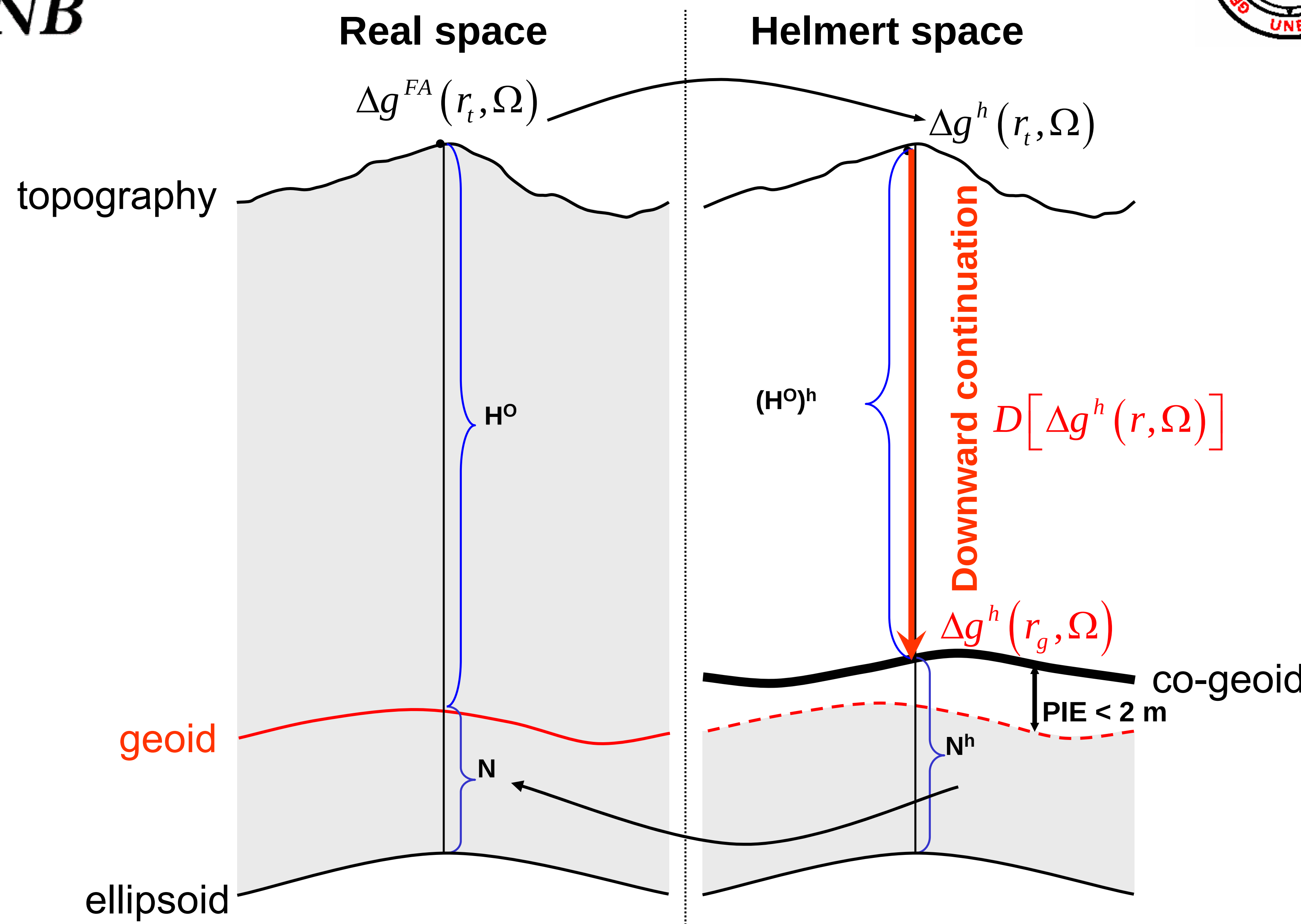
$$A(r_t,\Omega) = 4\pi G\rho H + O(H/R)$$



$$A(r_g,\Omega) = 0$$

$$V^t(r_g,\Omega) = 4\pi G\rho RH + O(H/R)$$

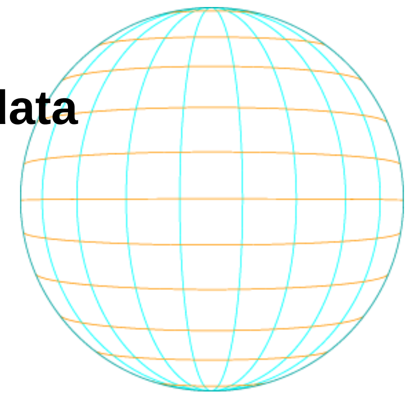
Rigorous far-zone contribution
Meaningful condensation models: preserves mass or mass-centre



Real space
Helmert space
$\Delta g^{FA}(r_t, \Omega)$
$\Delta g^{h}(r_t, \Omega)$
topography
H^O
$(H^O)^h$
Downward continuation
$D\left[\Delta g^{h}(r, \Omega)\right]$
$\Delta g^{h}(r_g, \Omega)$
co-geoid
PIE < 2 m
geoid
N
N^h
ellipsoid

Stokes's solution

The original formula requires global coverage of gravity data

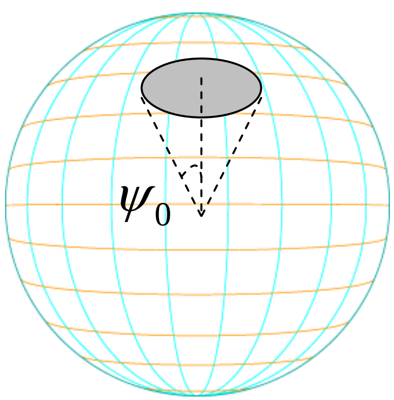


$$N(\Omega) = \frac{R}{4\pi\gamma_0(\phi)} \iint_{\Omega \in \Omega_0} \Delta g^h(R,\Omega') S(\psi(\Omega,\Omega')) d\Omega' + N_I(\Omega)$$

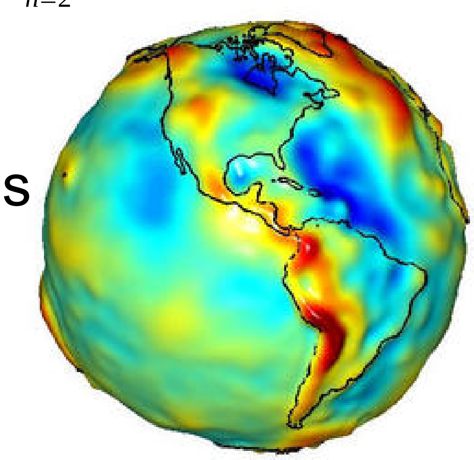
Combination of a high-degree reference field and GGM

$$N(\Omega) = \frac{R}{4\pi\gamma_0(\phi)} \iint_{\Omega \in \Omega_{\psi 0}} \left[\Delta g^h(R,\Omega') - \sum_{n=2}^{M} \Delta g_n^h(\Omega) \right] S^L(\psi(\Omega,\Omega')) d\Omega' +$$

$$+ N_I(\Omega) + \frac{R}{2\gamma} \sum_{n=2}^{M} \frac{2}{n-1} \Delta g_n^h(\Omega)$$



Near and far-zone contributions

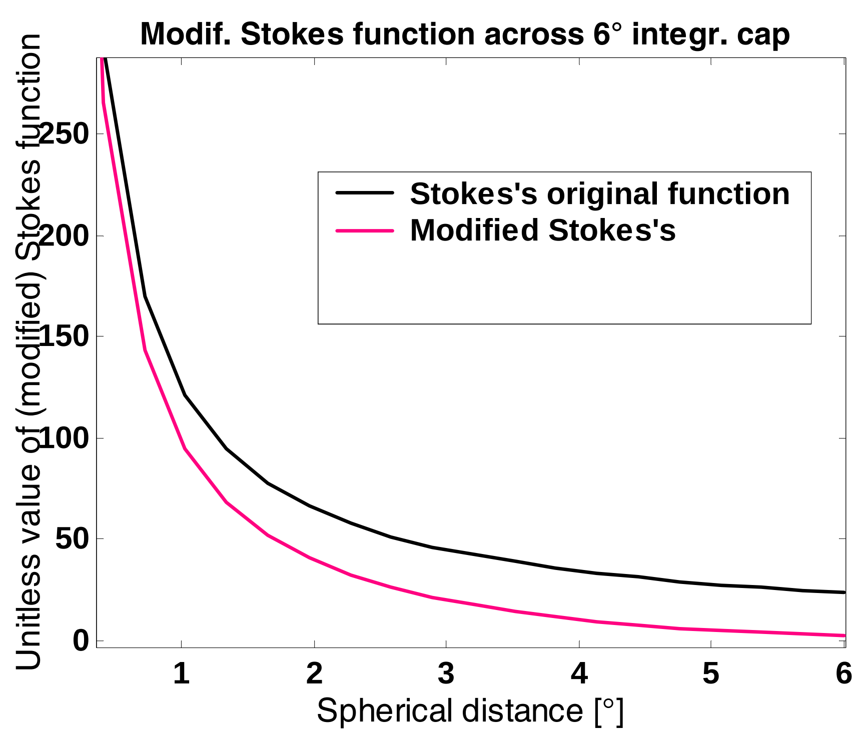


Modified Stokes's function

$$s_n \approx \frac{2}{n-1}$$

$$S^L(\psi(\Omega,\Omega')) = \sum_{n=2}^{\infty} \frac{2n+1}{n-1} P_n(\cos\psi) - \sum_{n=2}^{L} \frac{2n+1}{2} s_n P_n(\cos\psi)$$

$$S(\psi(\Omega,\Omega')) = \sum_{n=2}^{\infty} \frac{2n+1}{n-1} P_n(\cos\psi)$$

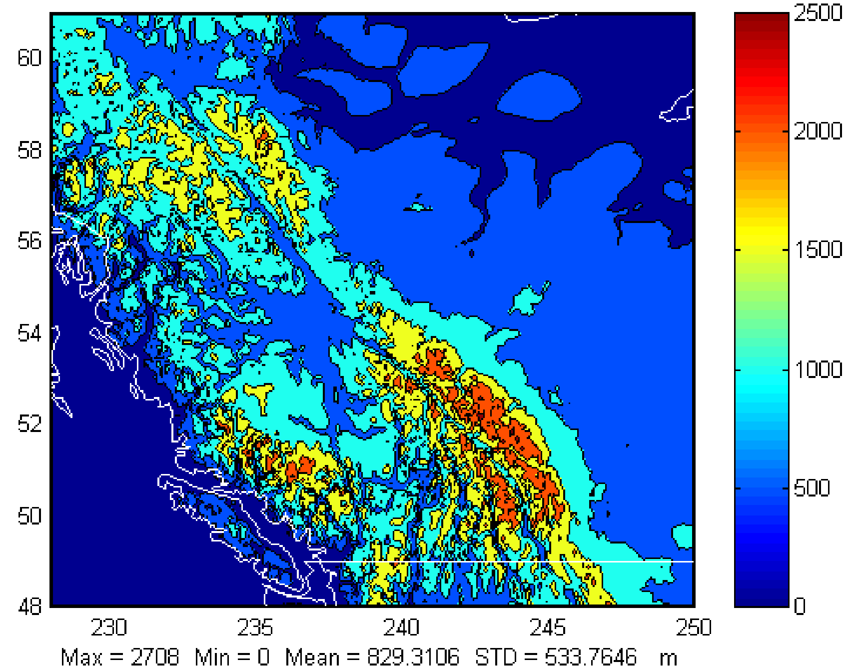


Truncation bias

$$\frac{R}{2\gamma} \sum_{M+1}^{120} Q_n^L \Delta g_n(\Omega)$$

Practical results & current developments

Comparison with GPS-leveling data:
Low-land - STD < 5…10 cm,
Mountains - STD 12-13 cm



Data:
Gravity data resolution 5’x5’,
DTMs: 3”x3”; 30”x30”; 5’x5’
<u>Mean</u> Helmert anomalies

Testing:

Synthetic data: AUS-SEGM
2’x2’ gravity data resolution
New anomaly type – NoTopography anomaly (smoother!)
Laterally varying density
CHAMP & GRACE-based geopotential models

Conclusions

If time permits….

Textbook in preparation

P. Vaniček, Z. Martinec, L. Sjöberg

Tentative title: "The Geoid"

Springer Verlag 2006 (?)