No Topography approach for Stokes-Helmert's geoid modelling:

results for an area in the Canadian Rockies

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Outline

- Motivation of the study
- “Standard” two-space approach

versus

- New three-space approach

- Numerical results
- Discrepancies
- Comparison with GPS-leveling data
- Summary and work in progress

The Helmert disturbing potential

$$T^h(r,\Omega) = T(r,\Omega) - V^t(r,\Omega) + V^{ct}(r,\Omega) - V^a(r,\Omega) + V^{ca}(r,\Omega)$$

$$\forall r > r_g : \nabla^2 T^h(r,\Omega) = 0$$

The NT disturbing potential

$$T^{NT}(r,\Omega) = T(r,\Omega) - V^t(r,\Omega) - V^a(r,\Omega)$$

$$\forall r > r_g : \nabla^2 T^{NT}(r,\Omega) = 0$$

Fund. grav. equation at earth's surface

$$\Delta g(r_t,\Omega) = -\frac{\partial T(r_t,\Omega)}{\partial n} + \gamma\left[r_t - \varsigma(r,\Omega),\phi\right]^{-1} \frac{\partial \gamma(r,\phi)}{\partial n} T(r_t,\Omega)$$

Conversion of free-air anomaly to Helmert anomaly

1. On the surface

$$\Delta g^{h}(r,\Omega) = \Delta g^{FA}(r,\Omega) + \frac{\partial\left[V^{t}(r,\Omega) - V^{ct}(r,\Omega)\right]}{\partial r} + \frac{\partial\left[V^{a}(r,\Omega) - V^{ca}(r,\Omega)\right]}{\partial r} + \frac{2}{r}\left[V^{t}(r,\Omega) - V^{ct}(r,\Omega)\right] + \frac{2}{r}\left[V^{a}(r,\Omega) - V^{ca}(r,\Omega)\right] + \varepsilon$$

<u>Residual</u> quantities utilized!!!

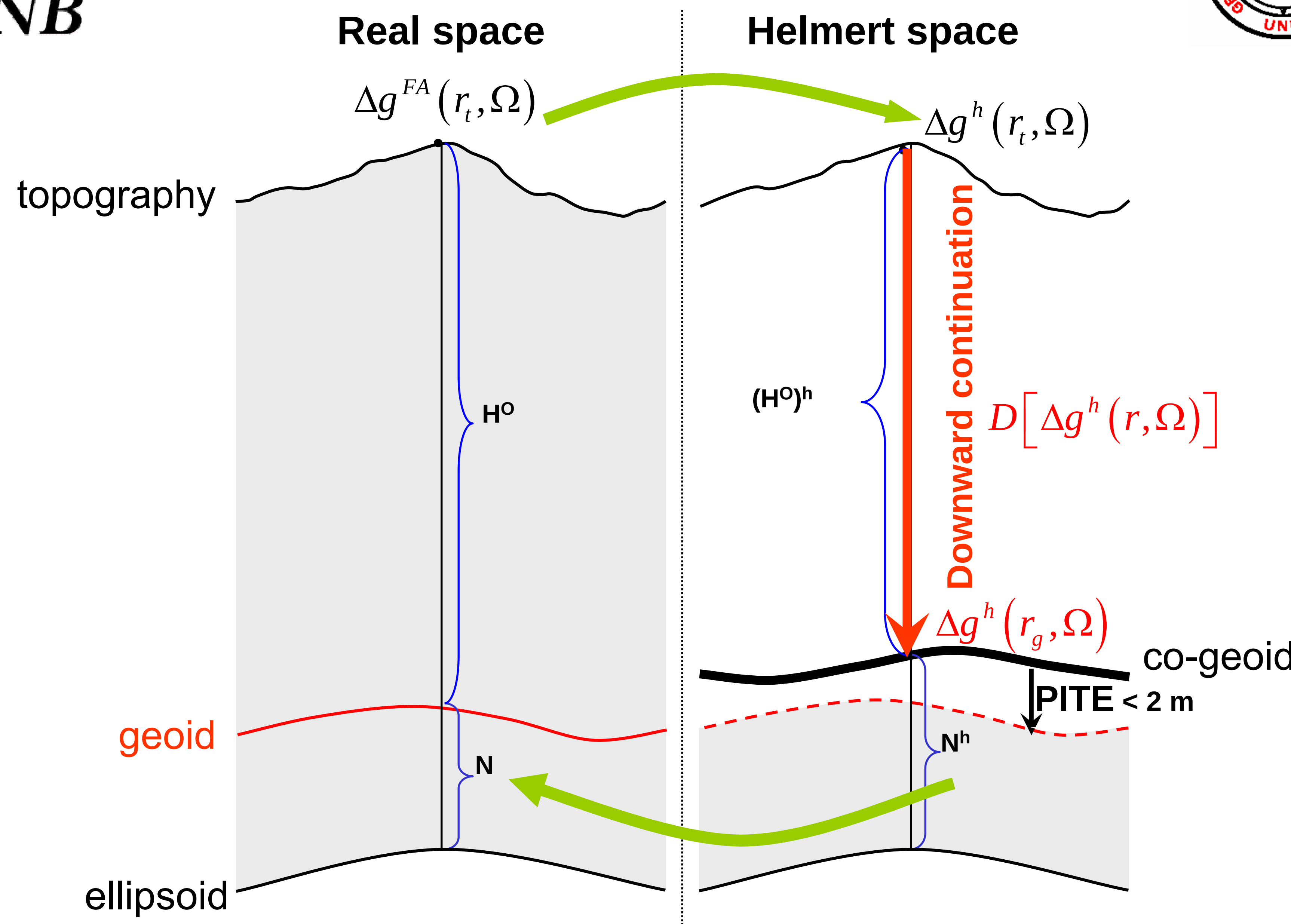
$$\nabla^{2}\left[r\Delta g^{h}(r,\Omega)\right] = 0 \qquad \forall r \geq r_{g}$$

Harmonic!!!

2. Downward continuation

$$\Delta g^{h}(r_{g},\Omega) = \Delta g^{h}(r_{t},\Omega) + D\left[\Delta g^{h}(r_{t},\Omega)\right]$$

"Standard" two-space scenario



Sequential conversion:

From free-air anomaly to NT-anomaly…

1. On the surface

$$\Delta g^{NT}(r,\Omega) = \Delta g^{FA}(r,\Omega) + \frac{\partial V^t(r,\Omega)}{\partial r} + \frac{\partial V^a(r,\Omega)}{\partial r} + \frac{2}{r}V^t(r,\Omega) + \frac{2}{r}V^a(r,\Omega) + \varepsilon$$

$$\nabla^2\left[r\Delta g^{NT}(r,\Omega)\right] = 0 \qquad \forall r \geq r_g$$

Harmonic!!!

2. Downward continuation

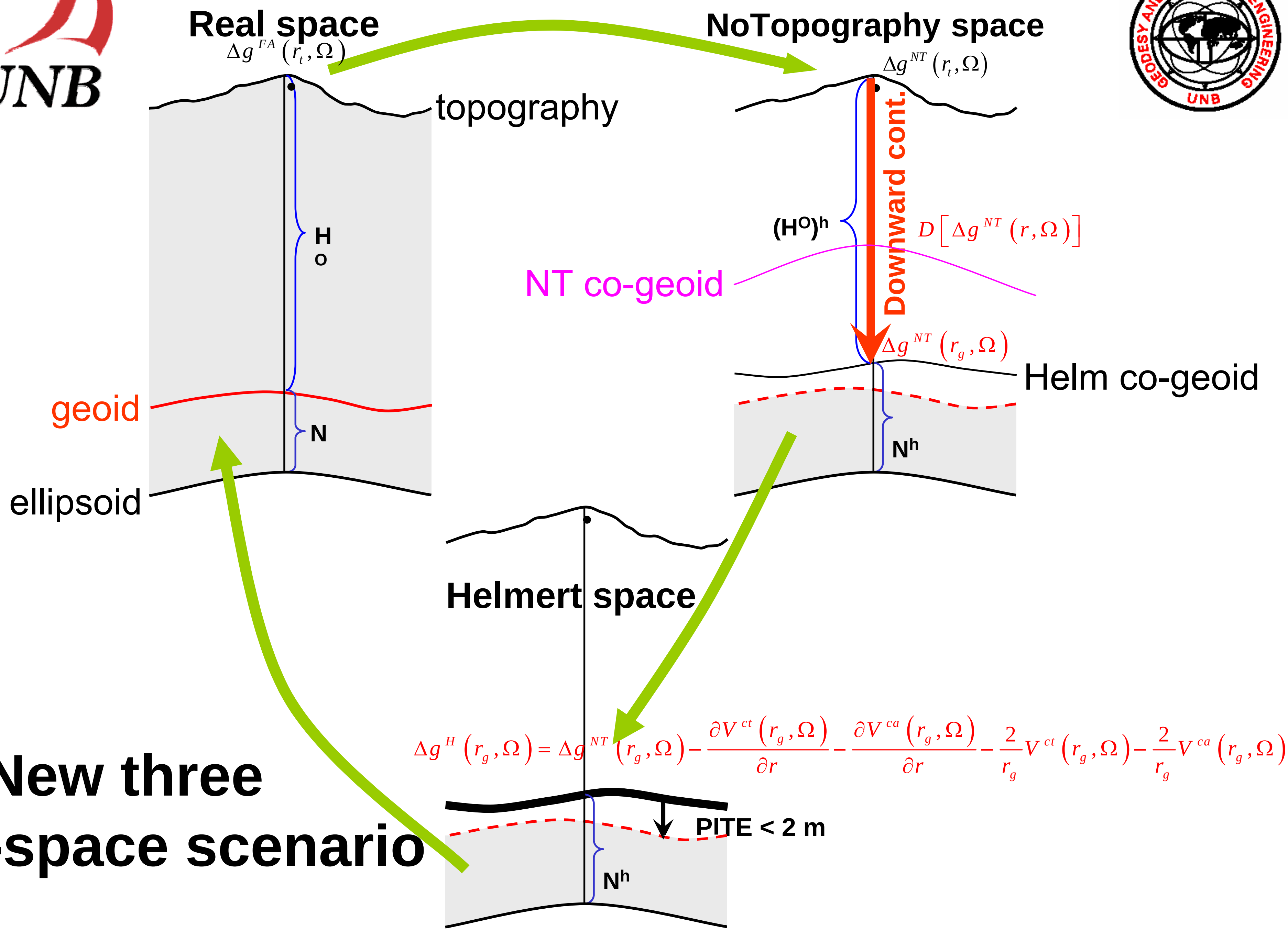
$$\Delta g^{NT}(r_g,\Omega) = \Delta g^{NT}(r,\Omega) + D\left[\Delta g^{NT}(r,\Omega)\right]$$

3. On the boundary

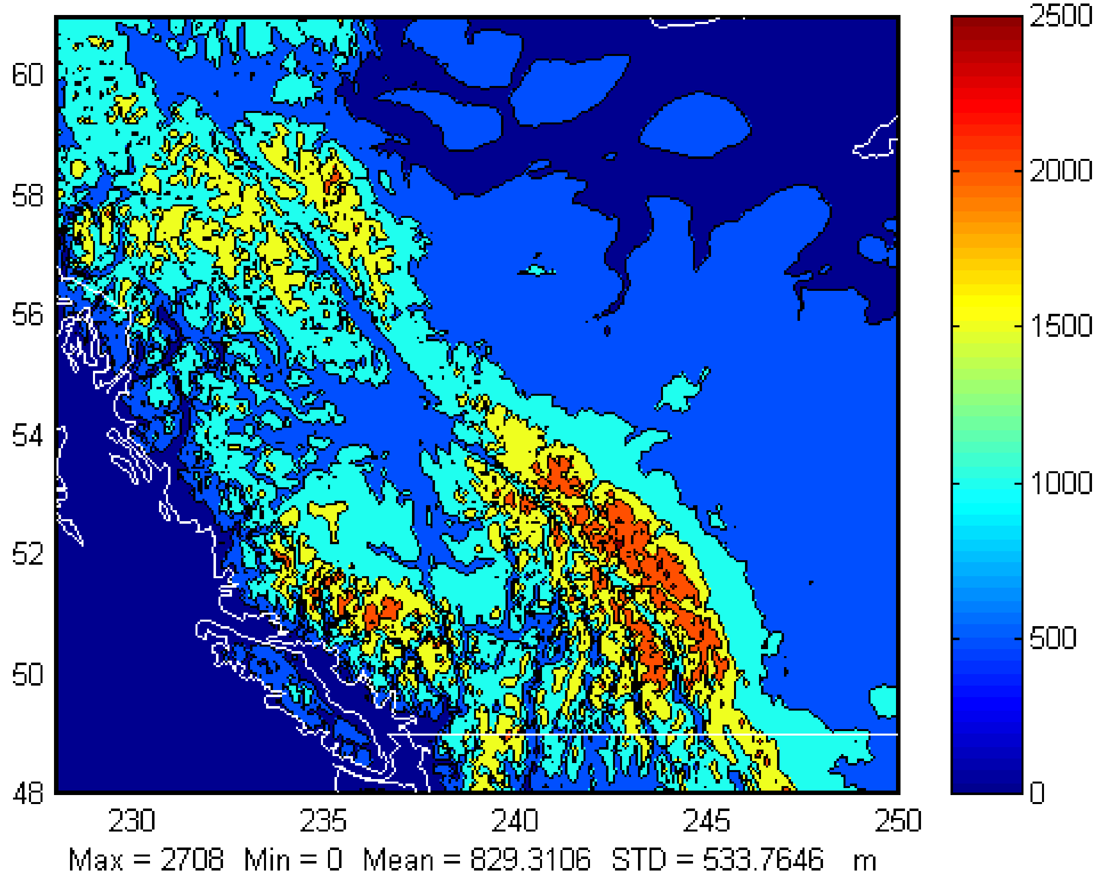
$$\Delta g^{H}(r_g,\Omega) = \Delta g^{NT}(r_g,\Omega) - \frac{\partial V^{ct}(r_g,\Omega)}{\partial r} - \frac{\partial V^{ca}(r_g,\Omega)}{\partial r} - \frac{2}{r_g}V^{ct}(r_g,\Omega) - \frac{2}{r_g}V^{ca}(r_g,\Omega)$$

… and to Helmert's anomaly on the geoid!!

New three -space scenario



Used datasets and test area



Data:

Mean Helmert & NT anomalies

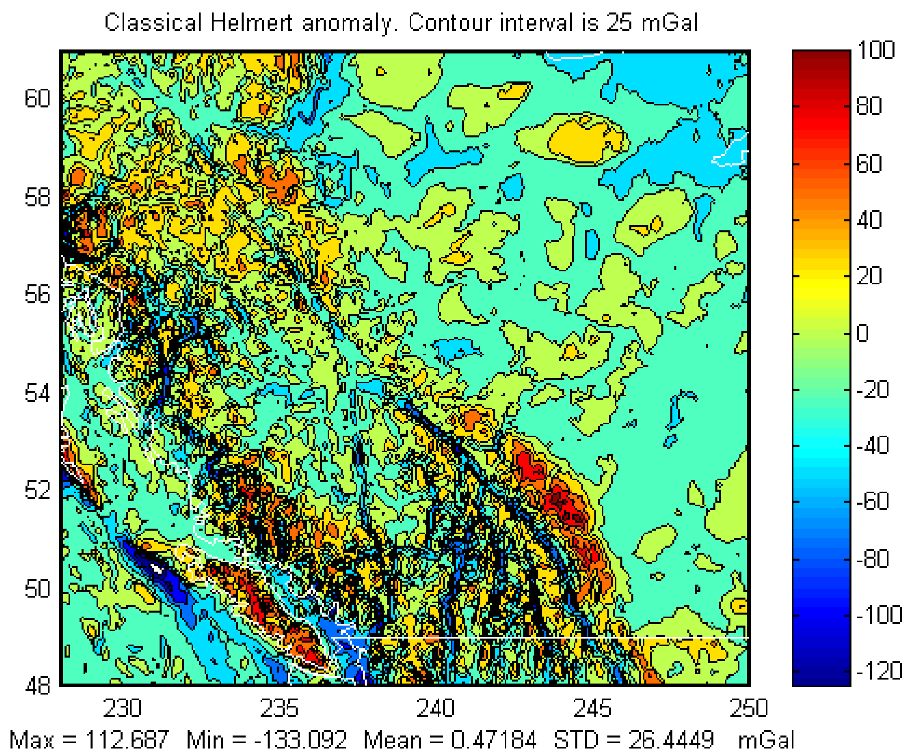
Gravity data resolution 5’x5’,

DTMs: 3”x3”; 30”x30”; 5’x5’;

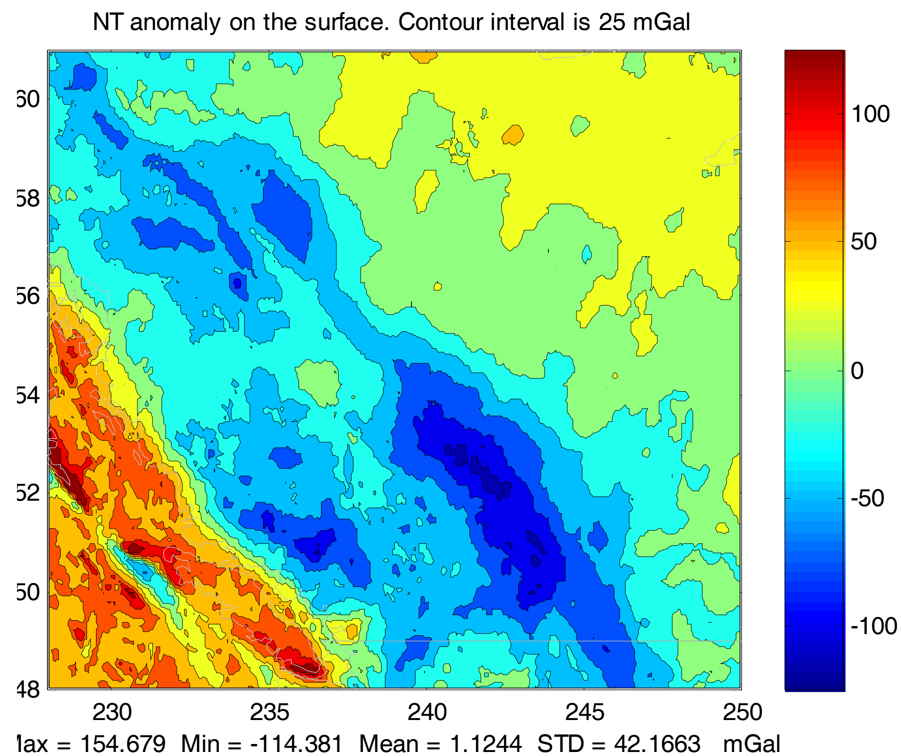
30’x30’ (far-zone/global contribution)

Surface gravity anomalies

Standard Helmert

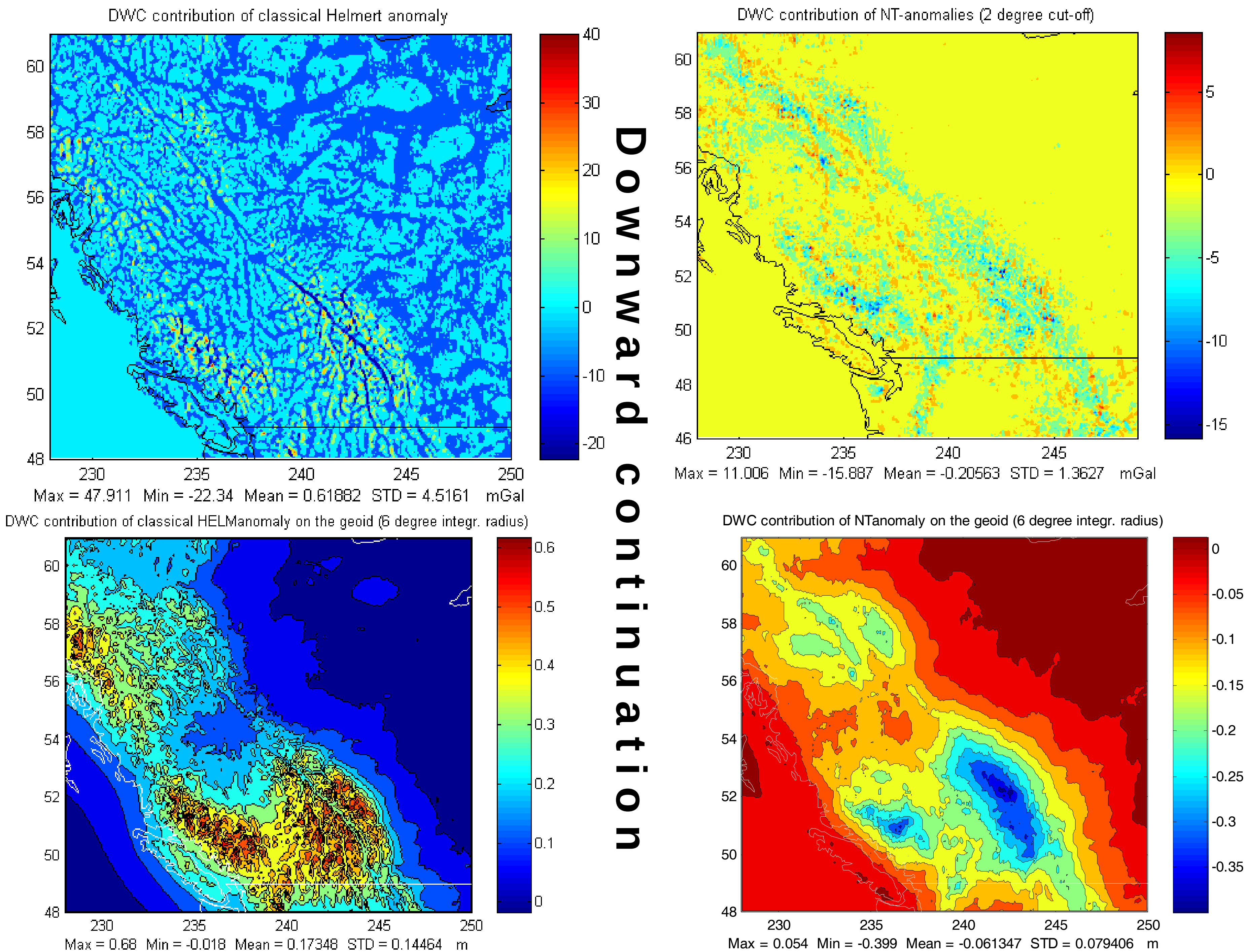


NT anomaly = *Spher. complete Bouguer anom.*



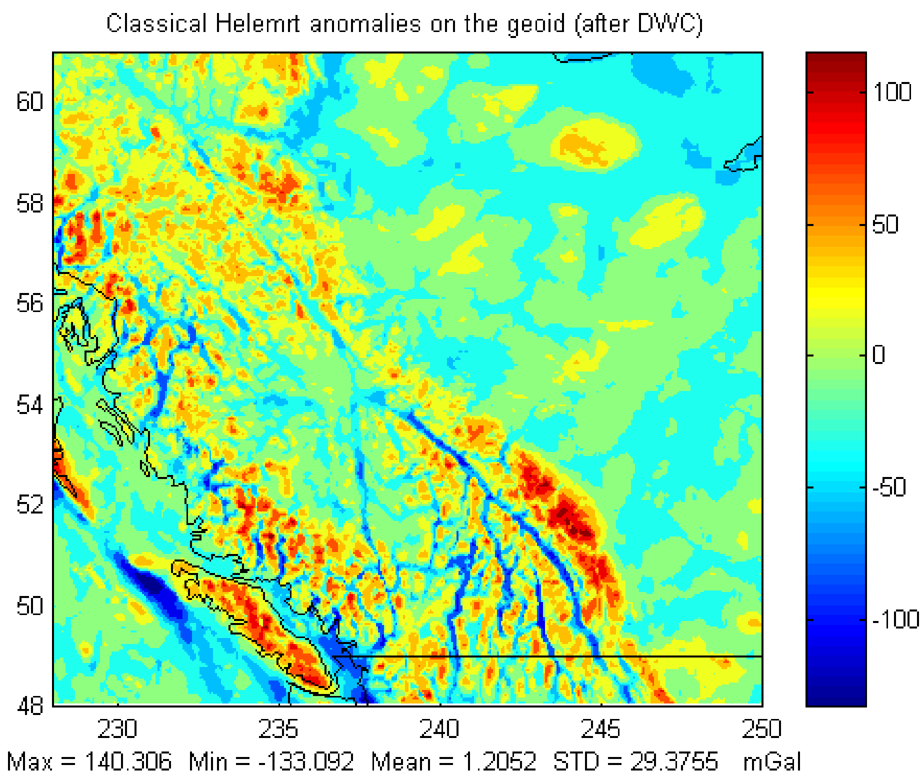
$$\Delta g^{h}(r,\Omega) = \Delta g^{FA}(r,\Omega) + \frac{\partial\left[V^{t}(r,\Omega) - V^{ct}(r,\Omega)\right]}{\partial r} + \frac{2}{r}\left[V^{t}(r,\Omega) - V^{ct}(r,\Omega)\right]$$

$$\Delta g^{NT}(r,\Omega) = \Delta g^{FA}(r,\Omega) + \frac{\partial V^{t}(r,\Omega)}{\partial r} + \frac{2}{r}V^{t}(r,\Omega)$$

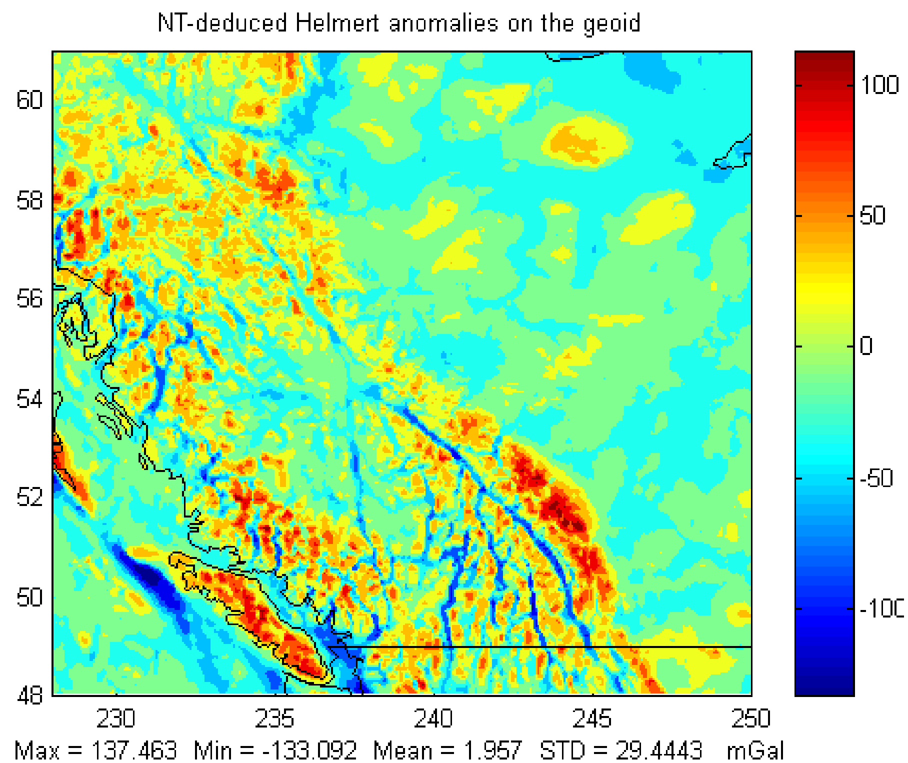

DWC contribution of classical Helmert anomaly
Max = 47.911 Min = -22.34 Mean = 0.61882 STD = 4.5161 mGal
Downward continuation
DWC contribution of NT-anomalies (2 degree cut-off)
Max = 11.006 Min = -15.887 Mean = -0.20563 STD = 1.3627 mGal
DWC contribution of classical HELManomaly on the geoid (6 degree integr. radius)
Max = 0.68 Min = -0.018 Mean = 0.17348 STD = 0.14464 m
DWC contribution of NTanomaly on the geoid (6 degree integr. radius)
Max = 0.054 Min = -0.399 Mean = -0.061347 STD = 0.079406 m

Gravity anomalies on the geoid

Standard Helmert



NT-deduced Helmert

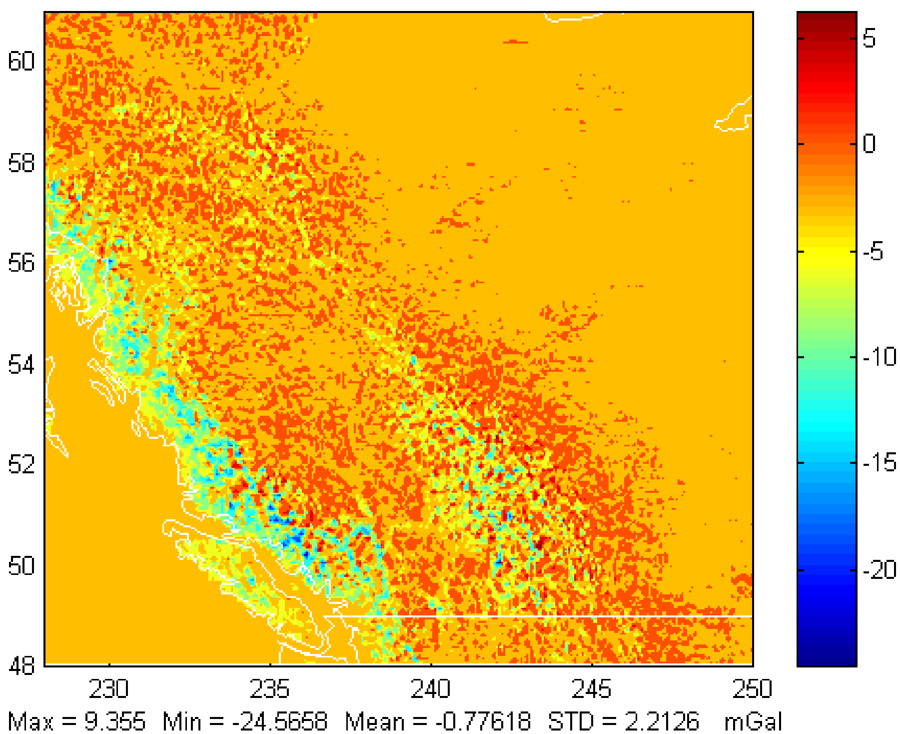


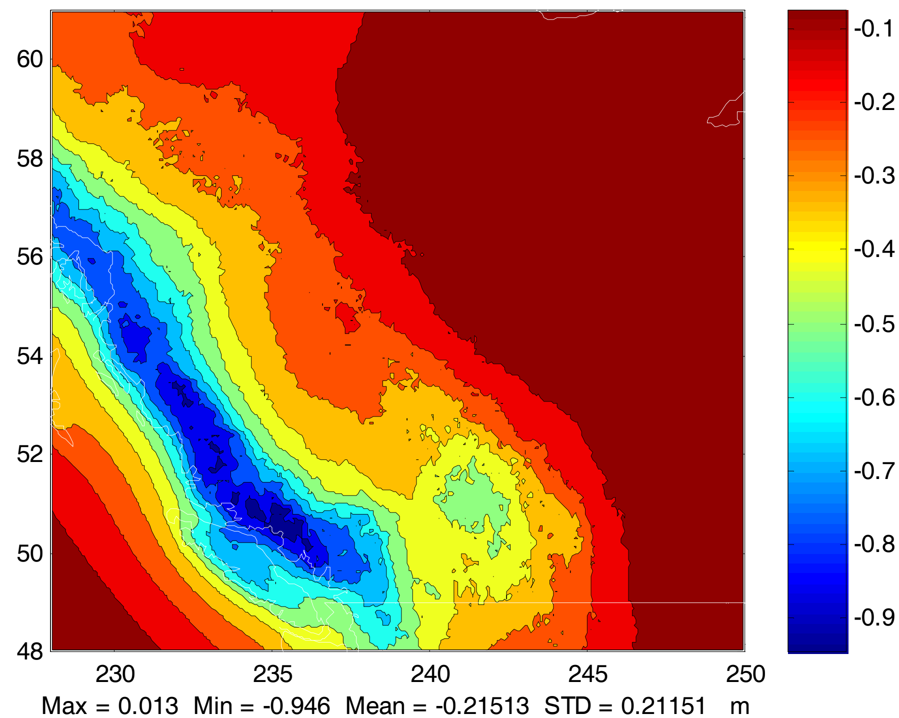
$$\Delta g^{h}\left(r_g,\Omega\right)=\Delta g^{h}\left(r_t,\Omega\right)+DWC^{h}$$

$$\Delta g^{H}\left(r_g,\Omega\right)=\Delta g^{NT}\left(r_g,\Omega\right)-\frac{\partial V^{ct}\left(r_g,\Omega\right)}{\partial r}-\frac{2}{r_g}V^{ct}\left(r_g,\Omega\right)$$

Discrepancies between the standard Helmert and NT approaches

The two approaches are theoretically equivalent, but ….





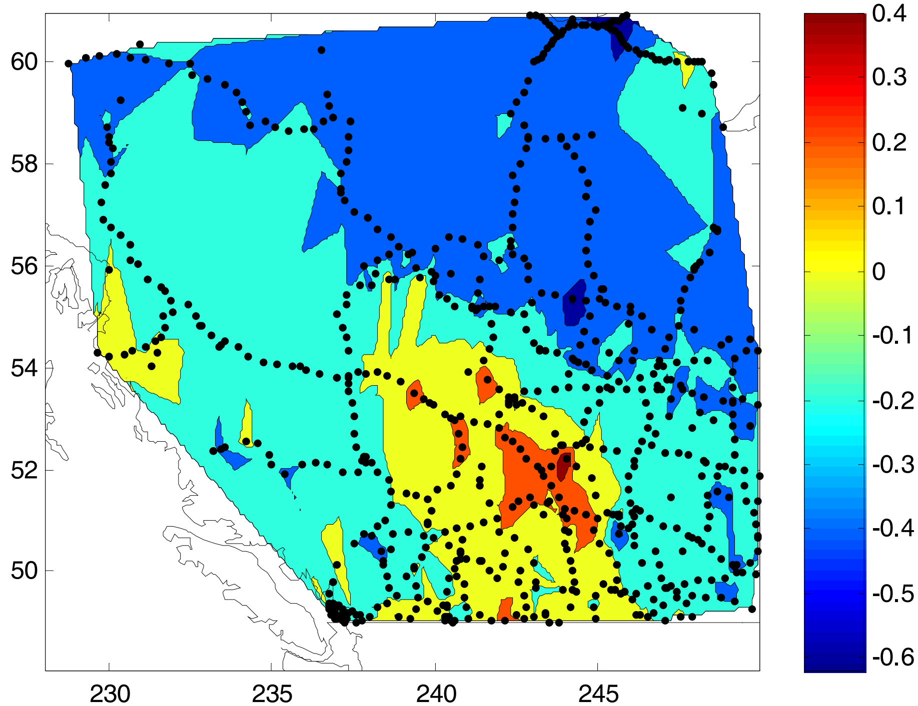
SCTC ???

DWC ?

$$-\frac{\partial V^{ct}\left(r_g,\Omega\right)}{\partial r} = BS - \iint_{\Omega_0} \frac{r^3(\Omega')-r^3(\Omega)}{3}\,\frac{\partial l^{-1}\left[R,\psi(\Omega,\Omega'),R\right]}{\partial r}\,d\Omega'$$

Comparison with GPS-leveling data.

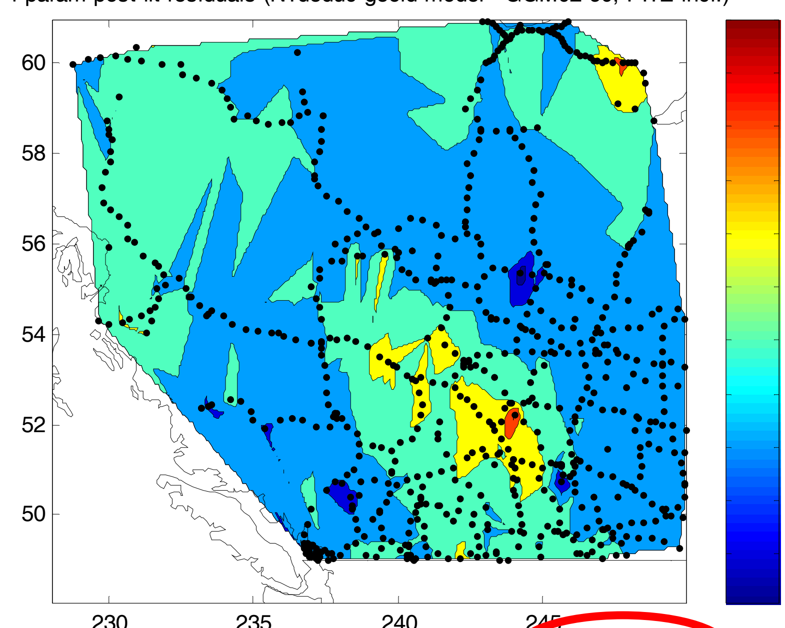
Pre-fit differ btw GPS-level & NTdeduc geoid model (GGM02-40,PITE,trunc.bias incl.)



Max = 0.6131 Min = -0.64452 Mean = -0.09581 STD = 0.16826 m

Low-land - STD < 5…10 cm,
Mountains - STD >13 cm

4-param post-fit residuals (NTdeduc geoid model - GGM02-60, PITE incl.)



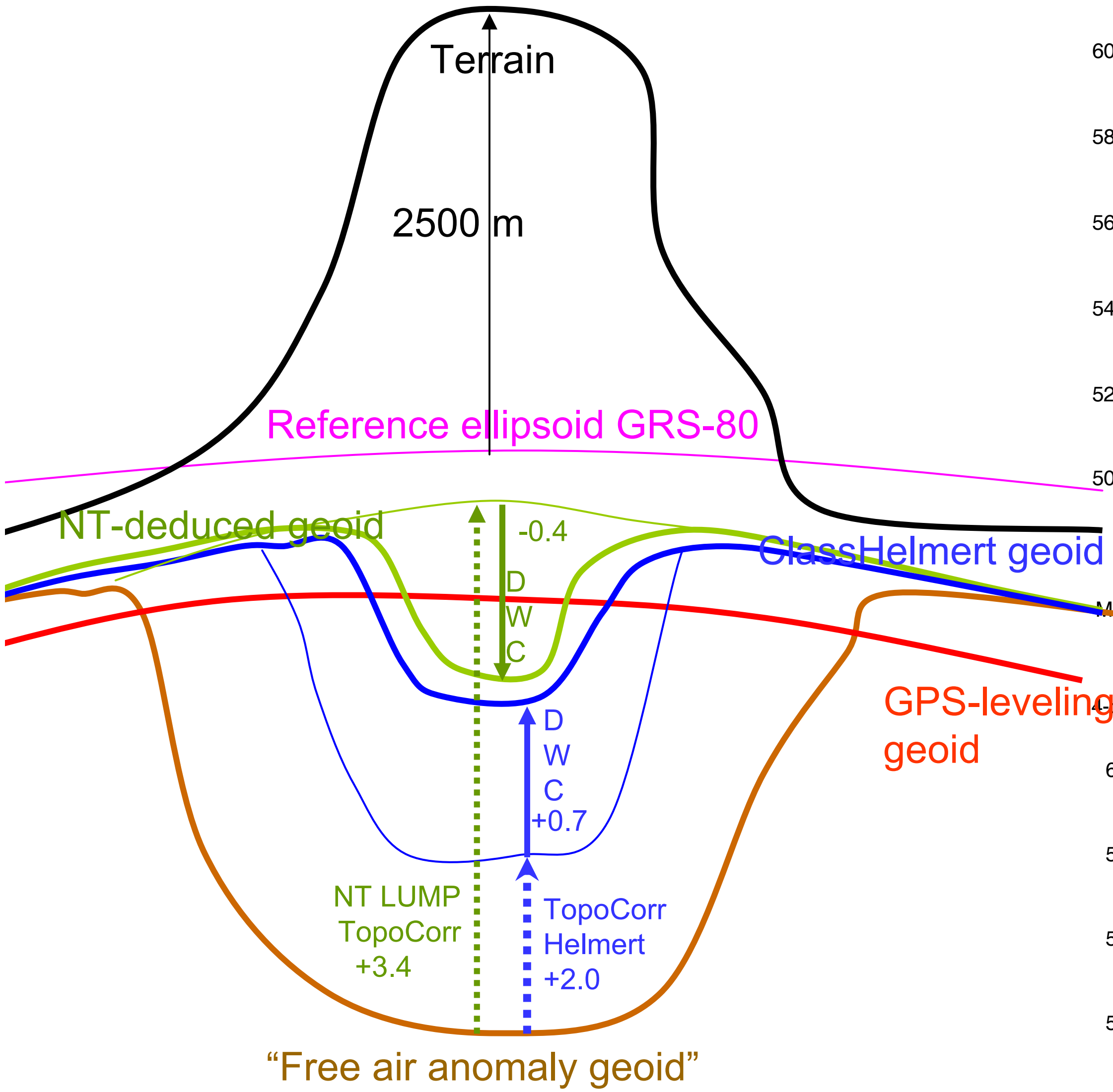
Max = 0.66 Min = -0.51465 Mean = 6.3007e-013 STD = 0.13067 m

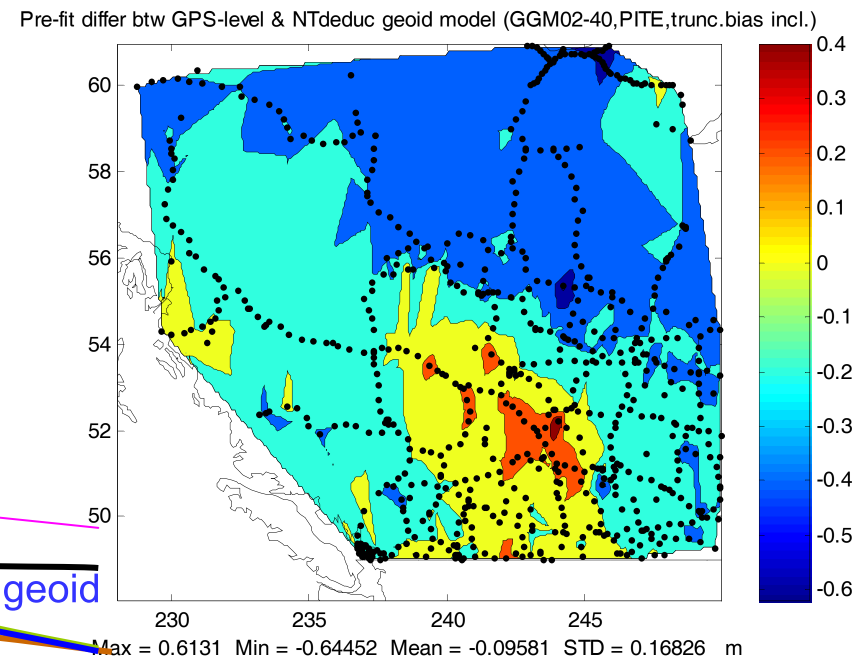
GRACE based GGM02
Modification degree M = L = 40

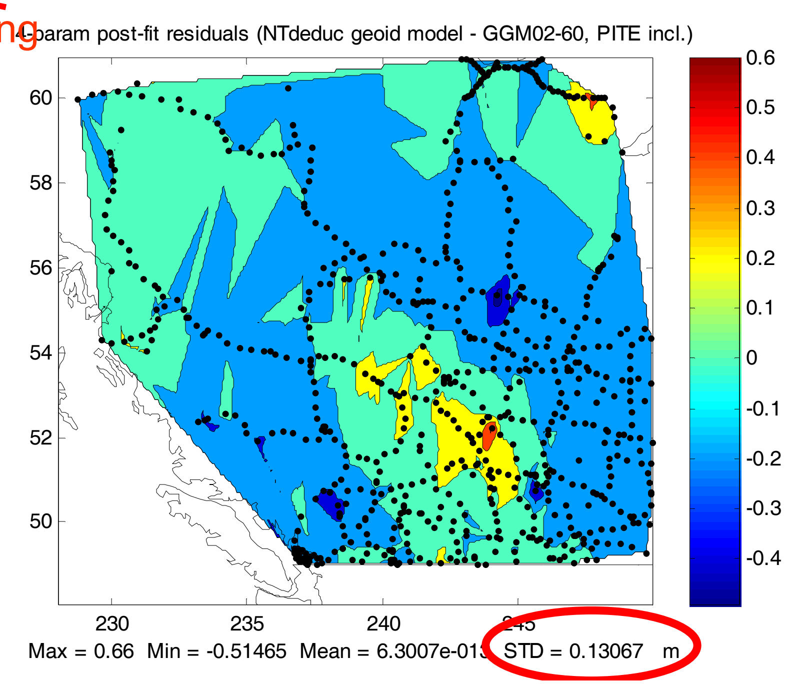
Statistics slightly better for NTdeduced models

UNB Summary and work in progress

- substantial (numerical) differences between the two- & three-space scenarios
- higher – resolution (2’x2’?) geoid models useful
- Laterally varying density
- CHAMP & GRACE-based geopotential models
- Synthetic data: AUS-SEGM







Comparison with GPS-leveling data:

Low-land - STD < 5…10 cm,

Mountains - STD 12-13 cm