TREATMENT OF SEA-LEVEL RECORDS IN MODELLING LINEAR VERTICAL CRUSTAL MOTION

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ABSTRACT

For modelling linear vertical crustal movements over Canadian territory, we use the vertical velocity surface in the form of a two-dimensional algebraic polynomial. This is a known technique that allows us to combine “point velocities”, computed from tide gauge records, with scattered segments of relevellings and water transfers supplying the information on “velocity differences”. Here we concentrate on the question of how best to incorporate the “point velocity values” into the mathematical model. It has been repeatedly pointed out that the standard deviations of individual linear trends (point velocities) of tide-gauge records are significantly larger than the corresponding standard deviations of trend differences (velocity differences) between close-by tide gauges. This is due to a high degree of coherence between sea level

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variations at close-by sites; a large portion of these variations disappears when the records are differenced. This behaviour offers an alternative, and better, way of treating sea level trends: use only one trend value as a point velocity input and difference the rest to obtain velocity differences. We show the use of regional correlation matrices and correlation coefficient confidence intervals for selecting the optimal pairing of sites, i.e., a tree diagram for optimal differencing, that gives the most precise and accurate velocity differences to be used in the modelling.

1. INTRODUCTION

The technique described in this paper was devised for and implemented in the compilation of the most recent map of recent vertical movements in Canada [Carrera et al., 1990]. Other aspects of this work are described in another paper in this volume [Carrera and Vaníček, 1994] and we shall not discuss them here. Instead, we shall concentrate directly on the treatment of tide-gauge records, known also as sea-level records.

The point velocity at the tide-gauge location is given by the linear trend fitted to the record, corrected for the linear trend of the sea level itself (eustatic water rise). To determine this linear trend, one can use different temporal sampling. Extensive tests we had conducted have confirmed the soundness of the established practice of using monthly averages, which are optimal for this purpose: there is practically no leakage from the suppressed high frequencies into the linear trend, yet there are enough data points to determine the trend quite reliably.

Typically, monthly averages of sea level oscillate within the range of about 0.5 metre throughout the years of observations. Since the linear trend we are interested in is typically a fraction of a centimetre per year, it is important from the precision and accuracy point of view, to have as long records as possible and relatively small oscillations. Many studies have been conducted with the aim of reducing the oceanic 'noise' in the tide-gauge records (cf. Rossiter [1972]; Lennon [1978]; Vaníček [1978]) and thus reducing their variability. Such a reduction requires a good knowledge of the variable atmospheric and oceanic behaviour and thus a large amount of observed atmospheric and oceanic data. These data are unfortunately not available for all the tide gauges one wishes to use.

There is another well documented feature of tide-gauge records: their striking similarity when they are obtained at two near locations (cf. Vaníček [1978]). This spatial coherence is indeed caused by common atmospheric and oceanic 'noise', and this feature can also be used to get a higher accuracy. Clearly, when records coming from two adjacent coastal locations are differenced, the oceanic noise is significantly reduced, and the linear trend of the record difference can be determined to a significantly better accuracy (cf. Sjøberg [1987]). This trend is, of course, nothing else but the linear velocity difference between the two points in question. This difference can then be accommodated within the mathematical model the same way as the velocity difference obtained from repeated water transfers, or repeated levellings.

Differencing of sea-level records to detect recent vertical crustal movements has been used by several researchers in the past. In Japan, differencing has been applied extensively by, e.g., Kawasumi and Omote [1950], Tsumura [1957, 1963, 1970], and it has been frequently used to verify the results obtained from geodetic levelling [Kato and Tsumura, 1979]. Some of the clearest examples of the power of this technique can be found in the determination of co-seismic movements as demonstrated by Tsubokawa et al. [1964] and reviewed by Rikitake [1976]. The only study known to the authors, of an attempt to define spatial boundaries of regions for the purpose of processing tide-gauge record differences, is that of Kato [1983], who divided the Japanese territory into nine regions according to their common oceanographic and tectonic behaviour. Differencing of sea-level records has also been practised in Scandinavia by, e.g., Sjøberg [1987], or Vermeer et al. [1988].

Let us just point out, that this procedure not only increases the precision as measured by the value of the standard deviation of the trend, but also reduces the danger of potential bias in the trend due to long-periodic oceanic noise, i.e., increases the accuracy [Sturges, 1987]. Figure 1 shows the record from Halifax together with the differenced records from Halifax and Charlottetown for the common period of time. The reduction of the noise level is clearly visible; the ratio of the standard deviations of the trends computed from the two records is 1.55. Also a significant whitening can be seen in the spectra of the differenced records, compared to the spectra of the individual records, cf., Figure 2 showing the spectra of the records in Halifax and that of the difference Halifax - Charlottetown. Note that the differencing reduces naturally even the two most predominant periodicities, i.e., the annual and the semi-annual.

The following strategy of using sea-level trends in the compilation of a map of recent vertical crustal movements thus suggests itself: at each detached coastline, only the longest and therefore most reliable record should be used in the point mode and the rest of the records should be used in the differenced mode. In reality, however, the records have very different lengths, and when they are differenced, only the common portions are used in the trend determination. The differencing should thus be done judiciously, so that utmost information is preserved when differencing the records. A reasoned optimal selection of candidate pairs for the differencing is what constitutes the contribution of this paper.

2. CORRELATIONS

The first thing one has to consider when dealing with coherence, is the phase lag between the studied records. Our investigations have shown that for series of monthly means, the lag is always only a small fraction of a month and should thus be considered equal to 0. Figure 3 shows a typical plot of correlations as a function of the lag in months between Halifax and Charlottetown, one on the Atlantic coast, the other in the Gulf of Saint Lawrence (for location of these two stations see Figure 5). This plot illustrates the appropriateness of the zero lag.

Once the appropriate lag is determined, one can compute the Pearson linear correlation coefficient r for any pair of series. Denoting the corresponding parts of the two series (covering the time span of the shorter series and beginning with the same month) as:

$$\mathbf{y}^i = (y^i_1, y^i_2, \ldots, y^i_n) \quad \text{and} \quad \mathbf{y}^j = (y^j_1, y^j_2, \ldots, y^j_n), \qquad (1)$$

the correlation coefficient is given by

$$r_{ij} = \sum_{k} (y^{i}_{k} - y^{i})(y^{j}_{k} - y^{j}) \,/\, [\, \sqrt{\sum_{k} (y^{i}_{k} - y^{i})^{2}} \; \sqrt{\sum_{k} (y^{j}_{k} - y^{j})^{2}} \,], \qquad (2)$$

where y^{i} and y^{j} are the average values of the two series. Figure 4 shows the correlation coefficients between the sea-level record in Halifax, the longest record on the east coast of Canada, and all the other locations on the east coast. Note that the value for Halifax itself is, of course, equal to 1.

The accuracy with which the correlation coefficients are determined, can also be evaluated. The 95% confidence interval for r_{ij} is obtained by means of Fisher's z-transformations [Sachs, 1984]: the correlation coefficient r is first transformed into the z domain by

$$z = 0.5 \, \mathit{ln} \, [\, (1 + r)(1 - r)^{-1}], \qquad (3)$$

where the 95% confidence interval is given as

$$< -1.96\,(n - 3)^{-0.5},\ 1.96\,(n - 3)^{-0.5} > . \qquad (4)$$

This interval is then transformed back into the r-domain by the inverse Fisher transformation

$$r = [\, \exp(2z) - 1] \, [\, \exp(2z) + 1]^{-1}. \qquad (5)$$

Whereas the correlation coefficients indicate which pairs of records should be differenced preferentially, based on their proximity, these confidence intervals show which pairs of records have the longest common epochs. Thus they are the most suitable indicators as to how the pairs should be selected if we wish to use the largest amount of information.

3. CONSTRUCTION OF THE TREE

Having constructed the matrices of correlations and their confidence intervals, we can now use them to get the “tree”, i.e., to define the optimum topology for the differencing. First, the linear trend with the smallest standard deviation, usually coming from the longest record on the coast, is chosen to be used in the point velocity mode. The longest record can be assumed also to give the value of trend the least biased by any long term variations in oceanic and atmospheric influences. For the east coast of Canada, we selected Halifax, even though the Charlottetown record actually contains more monthly means and gives a marginally better trend standard deviation, because of the absence of any significant gaps. Thus we opted for the maximization of accuracy, rather than precision.

Then the adjacent locations that show the smallest confidence intervals for their correlation coefficients are selected to play a role of ‘nodes’ in the network of differenced velocities. The pairing of these ‘nodal tide gauges’ and adjacent tide gauges was then done on the basis of highest correlations. Figure 5 shows the actual tree for the east coast of Canada.

As a matter of curiosity, one may now compare the point vertical velocities and their standard deviations obtained from individual records, with those obtained from the point velocity in Halifax and the appropriate differenced velocities. These comparisons show either the suppression of bias or an improved accuracy. Let us select one example typical for many: Rustico (cf., Figure 5). The linear trend of the relatively short Rustico record (less than 16 years long) is -1.82 mm per year, with a standard deviation of 0.35 mm per year. When the point velocity at this point is derived from the point velocity in Halifax (-3.56 ± 0.08 mm per year), propagated through the differenced records Halifax-Charlottetown and Charlottetown-Rustico, we obtain a value of -3.28 ± 0.47 mm per year. This value is a lot closer to the point velocity in Charlottetown (-3.55 ± 0.11 mm per year) located just a few kilometres away and makes much more sense.

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Captions for figures

Figure 1. Sea level record for Halifax and the differenced record between Halifax and Charlottetown.

Figure 2. Least-squares spectra for the Halifax record and for the differenced record between Halifax and Charlottetown.

Figure 3. Correlation function between Halifax and Charlottetown.

Figure 4. Correlations between Halifax and all the other points on the northeastern seaboard of North America.

Figure 5. Topology of tide-gauge differencing in eastern Canada

1	Halifax	2	Charlottetown
3	Saint John	4	Eastport
5	Yarmouth	6	Boutilier Point
7	Pictou	8	North Sydney
9	Port aux Basques	10	Rustico
11	Lower Escuminac	12	Point au Pere
13	Point du Chene	14	Point Tupper
15	St John's	16	Dalhousie
17	Riviere Renard	18	Ste Anne des Monts
19	Baie Comeau	20	Tadoussac
21	Harrington Harbour	22	Lark Harbour
23	Sept Iles	24	St Jean Port Joli
25	Quebec	26	St Francois
27	Bar Harbor	28	Portland
29	Seavy I.	30	Boston
31	Woods Hole	32	Newport
33	Providence	34	New London
35	Bridgeport	36	Willets
37	Port Jefferson	38	New York
39	Montauk	40	Atlantic City
41	Lewes	42	Philadelphia
43	Kiptopeke	44	Solomon I.
45	Norfolk	46	Annapolis
47	Baltimore	48	Portsmouth