On the removal of the effect of topography on gravity disturbance in gravity data inversion or interpretation

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Abstract: Four types of topographical corrections to gravity disturbance (to actual gravity) are investigated, distinguished by the density (real or constant–model) and by the lower boundary (geoid or reference ellipsoid) of the topographical masses. Each type produces a specific "topo–corrected gravity disturbance", referred to as "NT", "NTC", "NET", and "NETC". Our objective is to compare the four types and to study the physical meaning, in the light of gravity data interpretation/inversion, of these four gravity disturbances. Our method of investigation is the decomposition of the actual potential and actual gravity. It is shown, that the "NETC gravity disturbance" – i.e. the gravity disturbance corrected for the gravitational effect of topographical masses of constant density with the topo–surface as the upper boundary and the reference ellipsoid as the lower boundary – is rigorously and exactly equal to the gravitational effect of anomalous density inside the entire earth, i.e., below the topo–surface. The regions, where the four types of the topo–corrected gravity disturbance are harmonic, are studied also. Finally some attention is paid to areas over the globe, where geodetic (ellipsoidal) heights of the topo–surface are negative, with regard to the evaluation of the topo–correction and in the context of the gravimetric inverse problem. The strategy for the global evaluation of the "NETC topo–correction", or its evaluation in areas with negative ellipsoidal topography, is presented.

Key words: potential, gravitational effect, reference ellipsoid, geoid, density contrast, topographical effect, topo-correction

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1. Introduction

In geophysics, particularly in gravimetry, one is concerned with acquiring some knowledge on the underground geological structure, or at least on some of its elements, given in terms of the anomalous mass distribution (density contrast). In practice gravity (the magnitude of the gravity vector) is observed, which is physically associated with (actual) gravity potential. If actual gravity is interpreted or inverted, real underground density is sought. It is, however, more advantageous to formulate the inverse problem in terms of anomalous gravity, which is physically associated with the disturbing (anomalous) potential, thus seeking the underground anomalous density.

The disturbing potential is formed based on the selected (model) normal potential. The anomalous density is formed based on a (model) reference density. Inevitably there arises a link, which must be physically meaningful, between the reference density and the normal potential. The normal potential, following the most basic concepts of geodesy and geophysics, is generated by unspecified (undefined) normal masses (normal mass density) within the normal (reference) ellipsoid. Although the hypothetical normal masses inside the normal ellipsoid generate the normal potential, they are not uniquely determinable from the apriori chosen external normal potential. If looking for a solution to the normal mass distribution inside the reference ellipsoid, one faces a non–unique inverse problem (e.g., *Vaníček and Kleusberg, 1985*). Nevertheless, a specific solution – a model of normal density inside the normal ellipsoid, such that satisfies the external normal potential – can be selected (e.g., *Tscherning and Sünkel, 1980*) and used as reference density, in order to define the anomalous density inside the ellipsoid.

So far we have obtained reference mass distribution inside the reference ellipsoid. But what about the topographical masses above the ellipsoid? These masses reach altitudes of almost 9 kilometers. Shall we aim at getting rid of them and of their effect on gravity, or do we want to define reference density also in the region above the ellipsoid and below the topo–surface, that would define anomalous density distribution, which would also become the target of our investigation in terms of gravity inversion/interpretation? We shall look into this issue herein. Notice, that if we choose a model reference (background) density of topographical masses (such as a constant density, horizontally stratified, or laterally varying density), the gravitational potential of the model reference density of the topo–masses is not part of the normal potential. On that account, it must be treated separately.

At present, gravity observation positions are commonly referred either in "sea–level" (orthometric or normal) heights that refer to the "sea level" (geoid or quasigeoid, which is referred to a reference ellipsoid) as a vertical datum, or in geodetic (ellipsoidal) heights that refer to the reference ellipsoid as a vertical datum, the horizontal position being given by latitude and longitude. Having two vertical datums in use, a question arises: Should the lower boundary of the topo–masses be defined as the "sea level" or as the reference ellipsoid? We will investigate this topic.

Hence, when anomalous gravity data derived from the disturbing potential, such as the gravity disturbance or gravity anomaly, are to be used in gravity inversion or interpretation, the objective of which is to find anomalous density below the topo–surface, the gravitational effect of the topographical masses must be accounted for, as it is too significant to be ignored. The treatment of the effect of topography is known as topographical correction applied to observed gravity data. In this paper we shall deal exclusively with gravity disturbances, paying no attention to gravity anomalies, although gravity anomalies are extensively used in geophysical applications. The objective here shall be to explore (revisit) the topographical gravitational effect on the gravity disturbance, in order to properly understand the interpretation of the "topo–corrected gravity disturbance".

2. Theoretical background

We shall base our investigations on the concepts of the theory of the gravity field (e.g., *Kellogg, 1929; MacMillan, 1930; Molodenskij et al., 1960; Grant and West, 1965; Heiskanen and Moritz, 1967; Bomford, 1971; Pick et al., 1973; Moritz, 1980b; Vaníček and Krakiwsky, 1986; Blakely, 1995; Vaníček et al., 1999*). Only some concepts, of particular importance in the context of our study, will be highlighted below. Hereafter all the discussed quantities will be considered as already properly corrected for the effects of the atmosphere, tides, and all the other smaller temporal effects.

We shall refer the discussed quantities to a geocentric geodetic coordinate system, using geodetic coordinates – geodetic (ellipsoidal) height h, geodetic latitude ϕ, and geodetic longitude λ (e.g., *Vaníček and Krakiwsky, 1986*, Section 15.4), cf., also Section 1 in *Vajda et al. (2004)*. If compared with the geocentric spherical coordinate system, we note that the geodetic latitude differs from the spherical latitude. Sometimes the height of a point above "sea level" – orthometric height H (above the geoid), or normal height H^N (above the quasigeoid) – shall be used in addition to the ellipsoidal (geodetic) height. The ellipsoidal height can be evaluated as, cf. e.g. Fig. 5–1 on p. 180 in *Heiskanen and Moritz, (1967)*,

$$h \cong H + N \quad (a), \qquad h \cong H^N + \zeta \quad (b), \tag{1}$$

where the geoidal height N (height anomaly ζ) is the height of the geoid (quasigeoid) above the reference ellipsoid. For brevity we shall often denote the horizontal position in geodetic coordinates as $\Omega \equiv (\phi, \lambda)$. Often we shall evaluate our quantities of interest in spherical approximation, cf. e.g. *Moritz, 1980b*, p. 349. In spherical approximation the geocentric distance is $r = R + h$, R being the mean earth's radius. For the vertical derivatives we have in spherical approximation $\partial/\partial n = \partial/\partial h = \partial/\partial r$, where $\partial/\partial n$ is the derivative in the direction of the outward normal to the actual equipotential surface at the evaluation point, $\partial/\partial h$ is the derivative with respect to the geodetic height of the evaluation point, i.e., the derivative in the direction of the outward normal to the reference ellipsoid passing through the evaluation point, and $\partial/\partial r$ is the radial derivative, i.e., the derivative in the direction of the geocentric distance of the evaluation point.

2.1. Potential, gravitation, and gravitational effect

The earth's *gravity potential* W is the sum of the *gravitational potential* V and the *centrifugal potential*. The real (actual) gravitational potential of the earth is generated by the real earth's mass distribution and can be computed via the Newton integral for potential (e.g., *Heiskanen and Moritz, 1967*, Eq. (1–11)). For the exact formulation of the Newton integral in geodetic coordinates see e.g. *Vajda et al. (2004)*. Here we shall use the spherical approximation of the Newton integrals expressed in geodetic coordinates (*Vajda et al., 2004*, Section 4), as a good enough approximation for our purposes, since the ellipsoidal correction to the spherical approximation is by three orders of magnitude smaller (*Novák and Grafarend, 2004*). Hence

$$\forall\ (h_P, \Omega_P): \quad V\,(h_P, \Omega_P) = G \iiint\limits_{Earth} \rho(h, \Omega) L^{-1}(h_P, \Omega_P, h, \Omega) \mathrm{d}\vartheta\,, \tag{2a}$$

where G stands for Newton's gravitational constant, $\rho(h, \Omega)$ is real mass density distribution inside the earth, i.e., below the topo–surface $h = h_T(\Omega)$, L is the 3–D Euclidian distance (e.g. *Vajda et al. 2004*, Eq. (22)) between integration points (h, Ω) and the evaluation point (h_P, Ω_P), and "*Earth*" denotes the domain (region) containing all the earth's non–zero mass density distribution (disregarding the atmosphere). Here $\mathrm{d}\vartheta = (R + h)^2 \cos\phi\ \mathrm{d}h\ \mathrm{d}\phi\ \mathrm{d}\lambda$ is the infinitesimal solid (volume) element in geodetic coordinates in spherical approximation. The integration over the entire earth means integrating in geodetic height from $-R$ to $h_T(\Omega)$, in geodetic latitude from $-\pi/2$ to $\pi/2$, and in geodetic longitude from 0 to 2π

$$\forall\ (h_P, \Omega_P):$$

$$V\,(h_P, \Omega_P) = G \int\limits_{-R}^{h_T(\Omega)} \iint\limits_{\Omega_0} \rho(h, \Omega) L^{-1}(h_P, \Omega_P, h, \Omega)\,(R + h)^2\,\mathrm{d}h\ \mathrm{d}\Omega\,, \tag{2b}$$

where Ω_0 is the full solid angle, and $\mathrm{d}\Omega = \cos\phi\ \mathrm{d}\phi\ \mathrm{d}\lambda$.

Gravity is the magnitude of the *gravity vector*. *Gravitation* is the magnitude of the *gravitation vector*. The difference between gravity vector and gravitation vector is the centrifugal acceleration vector. The earth–bound measuring device (land and ship–borne gravimeters) measures gravity, while the air–borne measuring device measures gravitation (upon removal of the on–flight accelerations). The measuring device is supposed to be always leveled along the actual equipotential surface passing through it; thus it measures the magnitude of the gravity vector at the observation point.

The *normal gravity potential* U is the sum of the *normal gravitational potential*, which we shall denote here non–standardly as V_0^E, and the already discussed *centrifugal potential*. The *normal gravity potential* is selected as a known, mathematically defined model of the earth's *actual gravity potential* (e.g, *Heiskanen and Moritz, 1967; Vaníček and Krakiwsky, 1986*). The *normal gravity potential* defines a *normal earth* in terms of its gravity field and its shape. In general, the normal field represents a spheroidal reference field, but most commonly used, due to simplicity, is the ellipsoidal normal field of the "mean earth ellipsoid" (ibid). The *mean earth ellipsoid* is geocentric, properly oriented, and "level" ("equipotential", cf. *Heiskanen and Moritz, 1967*, Section 2–7). That assures, that the ellipsoid as a coordinate surface (as a 3D datum for geodetic coordinates) is, at the same time, also the equipotential surface of the normal gravity field, such, that the value of the normal potential on the ellipsoid is equal to the value of the actual potential on the geoid. In this way a physically meaningful tie between (geodetic) coordinates and normal gravity is established.

Therefore, rigorously, and particularly valid for global studies, the gravity data will have a correct physical interpretation only if their positions are referred to a "mean earth ellipsoid". We say this because in geodesy, and in practice in some countries, often locally best fitting ellipsoids, that are not geocentric and/or not properly oriented (e.g., *Vaníček and Krakiwsky, 1986*) are in use as datums for geodetic coordinates. For instance *Hackney and Featherstone (2003,* Section 2.3*)* discuss the impact of the use of a local ellipsoid as a datum on normal gravity values.

According to Somigliana and Pizzetti (*Somigliana, 1929*) the normal gravitational potential tells us nothing about the mass density distribution inside the *normal ellipsoid*. Hence *normal ellipsoid* is understood to be a body with *unspecified density distribution* generating the *normal potential* exterior to the ellipsoid (external normal potential). The *external normal gravitational potential* is harmonic.

Normal gravity is the magnitude of the *normal gravity vector*. *Normal gravitation* is the magnitude of the *normal gravitation vector*. The normal potential and thus normal gravity is not defined inside the normal ellipsoid, unless some normal density distribution *is assumed* within the ellipsoid, which would generate the appropriate external normal gravitational potential. This issue will be addressed later on.

The *disturbing potential* is defined as the difference between the *actual* and the *normal potentials*

$$\forall\ (h,\Omega): \quad T(h,\Omega) = W(h,\Omega) - U(h,\Omega) = V(h,\Omega) - V_0^E(h,\Omega)\,. \tag{3}$$

The disturbing potential thus does not depend on the spin of the earth. It is harmonic outside the topo–surface, unless the topographic surface is bellow the ellipsoidal surface. What was said about the definition of normal potential below the surface of the reference ellipsoid, applies also to the disturbing potential below the surface of the reference ellipsoid. The disturbing potential is the fundamental quantity used for deriving *anomalous gravity*.

Under the term *gravitational effect*, denoted as A in the sequel, we shall understand the vertical component of an attraction vector, where the attraction is attributed to a specific part of the earth (a real (ρ), constant (ρ_0), or anomalous ($\delta\rho$) density distribution in a sub–region). Expressed in geodetic coordinates in spherical approximation (cf. *Vajda et al., 2004*, Sections 2.2, 3.2, and 4.2) it reads $\forall\ (h_P, \Omega_P)$:

$$A^{body}(h_P,\Omega_P) = -\frac{\partial V^{body}(h_P,\Omega_P)}{\partial h_P} = \\ = -G \iiint\limits_{body} \rho(h,\Omega)\frac{\partial L^{-1}(h_P,\Omega_P,h,\Omega)}{\partial h_P}\mathrm{d}\vartheta\,, \tag{4a}$$

$$A_0^{body}(h_P,\Omega_P) = -\frac{\partial V_0^{body}(h_P,\Omega_P)}{\partial h_P} = \\ = -G\rho_0 \iiint\limits_{body} \frac{\partial L^{-1}(h_P,\Omega_P,h,\Omega)}{\partial h_P}\mathrm{d}\vartheta\,, \tag{4b}$$

$$\delta A^{body}(h_P,\Omega_P) = -\frac{\partial \delta V^{body}(h_P,\Omega_P)}{\partial h_P} = \\ = -G \iiint\limits_{body} \delta\rho(h,\Omega)\frac{\partial L^{-1}(h_P,\Omega_P,h,\Omega)}{\partial h_P}\mathrm{d}\vartheta\,. \tag{4c}$$

2.2. Relation between interior masses and external potential

Let us have masses defined by a non–zero density distribution $\rho(h,\Omega)$ bound by a closed and smooth boundary surface S. These masses generate gravitational potential $V(h,\Omega)$, which is internal within the boundary and external outside the boundary. The external potential is harmonic and can be developed into the series of solid spherical harmonics (e.g. *Heiskanen and Moritz, 1967; Vaníček and Krakiwsky, 1986*). The relation between density distribution and gravitational potential (both internal and external) *is unique in the direction from density to potential.* It means that a particular density distribution generates one particular potential. The task of computing the potential from a density distribution is known as the *direct gravimetric problem* and is solved by means of the Newton integral for potential.

This does not hold true the other way round. The relation between an external potential and a density distribution *is non–unique in the direction from external potential to density.* There are many density distributions that generate the same external potential. It is known, that the task of computing the density distribution from an external gravitational potential, the so called *inverse gravimetric problem*, is a non–unique task. This is due to the fact, that there are many density distributions which generate a zero external potential. Such density distributions are known as *Schiaparelli's bodies* of "vanishing outer potential" (e.g., *Vaníček and Kleusberg, 1985*). Any of the Schiaparelli's bodies may be added (following the superposition principle) to the density distribution that generates a particular external potential resulting in a different density distribution, which also generates the same external potential. Although there are infinitely many Schiaparelli's bodies, not every density distribution is a Schiaparelli's body, of course.

2.3. Normal density. Normal potential and normal gravity – above and below the reference ellipsoid

Normal gravity above the reference ellipsoid can be computed from the *external normal potential* exactly, using either closed formulae (e.g. *Heiskanen and Moritz, 1967*, Section 6–2) or the series expansion with latitude and altitude terms (e.g. *Heiskanen and Moritz, 1967*, Section 6–3). The formulae are based on the four parameters of the reference ellipsoid, such as the GRS80 that has been used recently (*Moritz, 1980a; Groten, 2004*). Both sets of formulae make use of the *geodetic height* above the reference ellipsoid, also called the *ellipsoidal height*, and the *geodetic latitude.*

Rigorously the normal potential makes sense only outside the ellipsoid. It provides no unique knowledge about the interior mass density distribution of the normal ellipsoid. As a matter of fact, due to the non–uniqueness of the inverse gravimetric problem even for the normal potential, there are infinitely many *normal density distributions* generating the same external normal potential.

Since the *normal density* is unknown, the *internal normal potential* is also unknown, and so is the *normal gravity* inside the ellipsoid. This is an obstacle, which we wish to overcome somehow, in order to define the disturbing potential and *anomalous gravity* inside the ellipsoid. We also need to know the normal density inside the ellipsoid in order to define the reference and anomalous densities inside the ellipsoid. What we can do is to find one such normal density distribution within the ellipsoid that satisfies the external normal potential. This can be done (cf. *Tscherning and Sünkel, 1980*). Once we have the (particular solution to) *normal density* within the normal ellipsoid, we can compute the *internal normal potential* (via Newton integral for potential) and the *normal gravity vector* inside the normal ellipsoid (as the gradient of the normal potential).

2.4. Anomalous gravity

The objective of introducing the *normal potential* as a selected mathematical model of potential field is to produce small *anomalous*, also called *disturbing*, quantities by subtracting the normal quantities from real quantities. Since the anomalous quantities are by several orders of magnitude smaller than the real quantities, they are less susceptible to errors introduced by approximations. As a result, they are also easier to handle in computations. Having the pairs *actual potential* and *actual gravity*, *normal potential* and *normal gravity*, we would anticipate to encounter the pair *disturbing potential* and *anomalous (disturbing) gravity.* In fact, two such anomalous quantities have historically been used, the *gravity anomaly* and the *gravity disturbance.* Only the gravity disturbance will be discussed in the sequel.

2.5. Definition of the point gravity disturbance using actual gravity

The *gravity disturbance* is defined *at any point in space* (h, Ω) as the difference of *actual gravity* and *normal gravity* taken at the same point (e.g., *Heiskanen and Moritz, 1967*, Eq. 2–142)

$$\forall\,(h,\Omega):\quad \delta g(h,\Omega)=g(h,\Omega)-\gamma(h,\Omega)\,. \tag{5}$$

In its nature, gravity disturbance defined by Eq. (5) is a function of location, i.e., a point function. It can only be known at the point, where the actual gravity is known – it tells us nothing about the spatial behavior of the gravity disturbance, unless we can describe the spatial behavior of g. Note, that the definition is not restricted to the topo–surface or the geoid. The only requirement is that we know the actual and the normal gravity values at the point of interest. Notice, that the normal gravity must be evaluated at the point of interest. That requires the use of geodetic height in the expression for the normal gravity. The knowledge of the geodetic height of the evaluation point is thus requisite for obtaining the value of the gravity disturbance. Gravity disturbance is a realizable quantity wherever the geodetic height of the evaluation point is known.

The observation point may be located at the topo–surface (terrestrial measurements – land and ship–borne surveys), below the topo–surface (bore–hole measurements, sea bottom survey), and above the topo–surface (air–borne observations). Note, that in the case of air–borne observations the meter is not earth–bound and a proper care must be taken of the centrifugal acceleration, in order to transform the observed gravitation, after the removal of on–flight accelerations, into gravity.

The gravity disturbance is a scalar quantity. We shall not deal with the *gravity disturbance vector* in this paper. We shall just note that the absolute value of the gravity disturbance vector is not equal to the gravity disturbance as defined by Eq. (5). In spherical approximation it is the vertical (radial) component of the gravity disturbance vector, that is equal to the gravity disturbance as defined by Eq. (5).

Notice, that so far there was no need to use any *vertical gradient of gravity* ("free–air", "Bouguer", etc.), or the so called "*altitude correction*", in defining the *gravity disturbance*. The *correction* regarding the *effect of topographical masses* and the *continuation of the data to a reference surface* (if it exists!) are kept separate from this *generic definition* of the *point gravity disturbance*.

2.6. Definition of the gravity disturbance using disturbing potential

The *gravity disturbance* at any point (h, Ω) can be also defined by means of the *disturbing potential*, (e.g., *Heiskanen and Moritz, 1967*, Eq. 2–146 and p. 91) given by Eq. (3), as

$$\forall\ (h, \Omega): \quad \delta g(h, \Omega) = -\frac{\partial T(h, \Omega)}{\partial h}\,. \tag{6}$$

Rigorously, this definition of gravity disturbance is not identical with the point definition given by Eq. (5). It can be shown, making use of Eq. (5), that if neglecting the deflection of the vertical at the evaluation point (θ), i.e., neglecting the angle between the actual gravity vector at the evaluation point and the normal gravity vector at the (Helmert's) projection of the evaluation point onto the reference ellipsoid, as well as neglecting the deflection due to the curvature of the normal plumbline between the reference ellipsoid and the point of evaluation (ξ_N), i.e., neglecting the angle between the normal gravity vectors at the reference ellipsoid and at the evaluation point, cf. Fig. 1, the two definitions, Eqs. (5) and (6), become compatible. Figure 1 shows the projection of the directions onto a tangent plane to the unit sphere at the evaluation point. For altitudes say up to 12 km, covering ground– and air–borne gravity surveys, the neglect of the first deflection can cause an error in the (point) gravity disturbance of about 1 μGal (1μGal = 10 nms^{-2}) in flat terrain ($\theta \approx 10''$), of up to about 10 μGal in mountainous

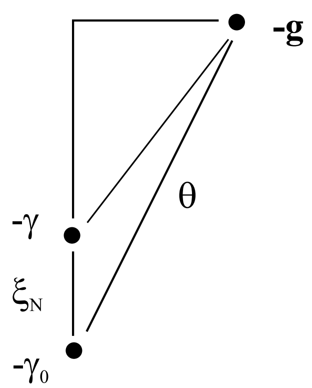

Fig. 1. The deflections of actual and normal vertical directions.

terrain ($\theta \approx 30''$), and is estimated to be at most 42 μGal ($\theta \approx 1'$), e.g. *Vaníček and Krakiwsky (1986)*, p. 96. The neglect of the latter deflection (ξ_N) can cause (for altitudes up to 12 km) an error in the (point) gravity disturbance at most 0.2 μGal.

Note, that the gravity disturbance defined in this section is known wherever the disturbing potential is known. The disturbing potential, however, is not an observable quantity (disregarding satellite altimetry and satellite tracking). Hence this definition will be useful only when working in a model space, where some assumptions about the mass density are made. The gravity disturbance (in spherical approximation and multiplied by r) is harmonic in the region where the disturbing potential is harmonic (e.g. *Heiskanen and Moritz, 1967*, Sections 1–18 and 6–6).

3. Topographical corrections to gravity disturbance

Since the generic gravity disturbance is computed as the vertical derivative of the disturbing potential, and the disturbing potential is the difference between actual and normal potentials, while the normal potential is generated by the normal density distribution of the normal ellipsoid, the generic gravity disturbance contains also a signal associated with the terrain morphology and the density distribution of the topographical masses. When studying the anomalous density underneath the earth's surface, the mentioned signal becomes unwanted. Hence the effect of topography must be corrected for, i.e., subtracted from the generic gravity disturbance, forming a "topographically corrected gravity disturbance".

Now the question arises: How should the topographical correction be applied? We have several options to do that, as will be examined below. Obviously the upper boundary of the topographical masses is the topo–surface. The lower boundary is not as obvious – should it be the geoid or the reference ellipsoid? We shall seek the answer to this question. Another question arises: Should we model the topographical masses using constant density, or should we try to evaluate the effect of topography using a model density distribution as close as possible to the real topographical density? We will deal with this question below, as well. For this latter point, we shall assume, that ideally we know the real topographical density distribution (although in reality we do not).

All in all, we will investigate four models of the topographical masses, as shown in Tab. 1, and thus four different topographical corrections. The objective of our investigation shall be to compare the four types of topo–corrected gravity disturbance, to study their physical meaning in terms of interpretation, and to find the right type of topographically corrected gravity disturbance that is exactly equal to the gravitational effect of all the anomalous masses inside the entire earth (below the topo–surface).

Tab. 1. Four models of topographical masses. The upper boundary is the topo–surface

model of topo–masses	**lower boundary**	**density**	**notation**
topography	geoid	real	T
topography of constant density	geoid	constant (model)	TC
ellipsoidal topography	ref. ellipsoid	real	ET
ellipsoidal topography of constant density	ref. ellipsoid	constant (model)	ETC

The term "topography" has been standardly used in geodesy and geophysics to describe topographical density distribution the lower boundary of which is the geoid. This brought us to introducing the term "ellipsoidal topography" to describe the topographical density distribution the lower boundary of which is the reference ellipsoid, in order to distinguish the two. The "ellipsoidal topography" is not to be understood as the topography of the ellipsoid, but as the topography reckoned from the ellipsoid.

3.1. Definition of reference (background) density and the anomalous density (density contrast) respective to it

If the gravimetric inverse problem is to be solved in terms of anomalous density (density contrast), then the reference (background) density must be defined apriori, to which the anomalous density is respective. Inside the normal ellipsoid the best choice is the reference density that is equal to the normal density $\rho_N(h, \Omega)$, cf. Section 2.3, which generates the normal potential. Between the reference ellipsoid and the topo–surface the simplest choice is a reference density that is constant, such as 2.6 g/cm^3. Of course there are other options for the choice of density between the reference ellipsoid and the topo–surface, such as some horizontally stratified density, or laterally varying density, etc., but for simplicity we shall stick here with the constant density model, cf. Tab. 2 and Fig. 2.

The anomalous density is then generally defined as $\delta\rho\,(h,\Omega) = \rho\,(h,\Omega) - \rho_R\,(h,\Omega)$. As was already discussed, our "Real Earth" is considered void of atmospherical masses, since proper atmospherical corrections are assumed to have been applied to observed gravity data. Thus all the real (and hence anomalous) mass density shall be distributed below the topo–surface.

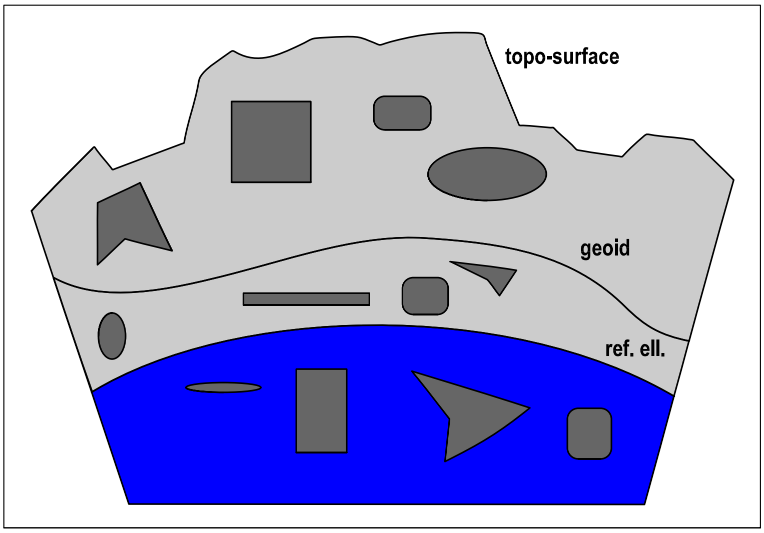

Fig. 2. Real Earth (schematically) – decomposition of real density distribution.

Tab. 2. Decomposition of real density into reference and anomalous densities

region of the earth	notation	reference density
within reference ellipsoid	E	$\rho_R(h,\Omega) = \rho_N(h,\Omega)$
between reference ellipsoid and geoid	EG	$\rho_R(h,\Omega) = \rho_0$
between geoid and topo–surface	GT	$\rho_R(h,\Omega) = \rho_0$

Sometimes we will also talk about the region between the reference ellipsoid and the topo–surface, denoted as ET, ET = EG ∪ GT, inside the geoid as a solid body (below the geoid as a surface), denoted as G, G = E ∪ EG, while the earth as a region will be denoted as "Earth", Earth = E ∪ EG ∪ GT.

3.2. Decomposition of actual gravitational potential and actual gravitation

In order to find which type of the topographical correction to gravity disturbance produces the topo–corrected gravity disturbance that is exactly equal to the gravitational effect of all the anomalous masses of the entire earth, we will make use of the decomposition of the actual gravitational potential of the earth (cf., *Vogel, 1982; Meurers, 1992*). We will decompose the actual potential according to the three regions of the earth as defined by Tab. 2, and within each region according to reference and anomalous densities, arriving at six terms (omitting the position argument (h_P, Ω_P)) as follows:

$$V = V_0^E + \delta V^E + V_0^{EG} + \delta V^{EG} + V_0^{GT} + \delta V^{GT}\,, \tag{7}$$

where the meaning of the terms is given by Tab. 3, and their expressions by Tab. 4.

Realizing that $\left(V - V_0^E\right)$ is the disturbing potential T (Eq. (3)), and changing the order of the terms on the right hand side, we can rewrite Eq. (7) as

$$T = \left(\delta V^E + \delta V^{EG} + \delta V^{GT}\right) + V_0^{EG} + V_0^{GT}\,. \tag{8}$$

Equation (8) represents the decomposition of the disturbing potential. The sum of the three terms in the round brackets is the potential of all the anomalous masses inside the entire earth, denoted as δV^{Earth}.

Now we apply the operator of the negative vertical derivative with respect to the geodetic height of the evaluation point $(-\partial/\partial h_P)$, to Eq. (8) to arrive at (again omitting the position argument (h_P, Ω_P))

$$\delta g = \left(\delta A^E + \delta A^{EG} + \delta A^{GT}\right) + A_0^{EG} + A_0^{GT}\,, \tag{9}$$

Tab. 3. The meaning of the decomposition terms of the actual gravitational potential

term	meaning
V_0^E	normal gravitational potential
δV^E	gravitational potential of anomalous masses inside the normal ellipsoid
V_0^{EG}	gravitational potential of masses of constant density between the reference ellipsoid and the geoid
δV^{EG}	gravitational potential of anomalous masses between the reference ellipsoid and the geoid
V_0^{GT}	gravitational potential of masses of constant density between the geoid and the topo–surface
δV^{GT}	gravitational potential of anomalous masses between the geoid and the topo–surface

Tab. 4. The expressions for the decomposition terms of the actual gravitational potential (in spherical approximation), where $L^{-1}\mathrm{d}\vartheta \equiv L^{-1}(h_P, \Omega_P, h, \Omega)\,(R+h)^2\,\mathrm{d}h\,\mathrm{d}\Omega$

term	definition
V_0^E	$V_0^E(h_P,\Omega_P) = G\int_{-R}^{0}\iint_{\Omega_0}\rho_N(h,\Omega)L^{-1}d\vartheta$
δV^E	$\delta V^E(h_P,\Omega_P) = G\int_{-R}^{0}\iint_{\Omega_0}\delta\rho(h,\Omega)L^{-1}d\vartheta$
V_0^{EG}	$V_0^{EG}(h_P,\Omega_P) = G\rho_0\int_{0}^{N(\Omega)}\iint_{\Omega_0}L^{-1}d\vartheta$
δV^{EG}	$\delta V^{EG}(h_P,\Omega_P) = G\int_{0}^{N(\Omega)}\iint_{\Omega_0}\delta\rho(h,\Omega)L^{-1}d\vartheta$
V_0^{GT}	$V_0^{GT}(h_P,\Omega_P) = G\rho_0\int_{N(\Omega)}^{h_T(\Omega)}\iint_{\Omega_0}L^{-1}d\vartheta$
δV^{GT}	$\delta V^{GT}(h_P,\Omega_P) = G\int_{N(\Omega)}^{h_T(\Omega)}\iint_{\Omega_0}\delta\rho(h,\Omega)L^{-1}d\vartheta$

where δg is the generic gravity disturbance (cf. Sections 2.5 and 2.6), and the meaning of the remaining terms is given by Tab. 5, while their expressions are given by Tab. 6.

Tab. 5. The meaning of the decomposition terms of the generic gravity disturbance

term	meaning
δA^{E}	gravitational effect of anomalous masses inside the normal ellipsoid
A_0^{EG}	gravitational effect of masses of constant density between the reference ellipsoid and the geoid
δA^{EG}	gravitational effect of anomalous masses between the reference ellipsoid and the geoid
A_0^{GT}	gravitational effect of masses of constant density between the geoid and the topo–surface
δA^{GT}	gravitational effect of anomalous masses between the geoid and the topo–surface

Tab. 6. The expressions for the decomposition terms of the generic gravity disturbance (in spherical approximation)

term	definition
δA^{E}	$\delta A^{E}(h_P,\Omega_P) = -G\int\limits_{-R}^{0}\iint\limits_{\Omega_0}\delta\rho(h,\Omega)\dfrac{\partial L^{-1}}{\partial h_P}\,d\vartheta$
A_0^{EG}	$A_0^{EG}(h_P,\Omega_P) = -G\rho_0\int\limits_{0}^{N(\Omega)}\iint\limits_{\Omega_0}\dfrac{\partial L^{-1}}{\partial h_P}\,d\vartheta$
δA^{EG}	$\delta A^{EG}(h_P,\Omega_P) = -G\int\limits_{0}^{N(\Omega)}\iint\limits_{\Omega_0}\delta\rho(h,\Omega)\dfrac{\partial L^{-1}}{\partial h_P}\,d\vartheta$
A_0^{GT}	$A_0^{GT}(h_P,\Omega_P) = -G\rho_0\int\limits_{N(\Omega)}^{h_T(\Omega)}\iint\limits_{\Omega_0}\dfrac{\partial L^{-1}}{\partial h_P}\,d\vartheta$
δA^{GT}	$\delta A^{GT}(h_P,\Omega_P) = -G\int\limits_{N(\Omega)}^{h_T(\Omega)}\iint\limits_{\Omega_0}\delta\rho(h,\Omega)\dfrac{\partial L^{-1}}{\partial h_P}\,d\vartheta$

3.3. The "No Topography" earth model – the NT model space

After re–shuffling the terms in Eq. (9) we can write it as

$$\delta g - \left(A_0^{GT} + \delta A^{GT}\right) = \left(\delta A^{E} + \delta A^{EG}\right) + A_0^{EG}\,, \tag{10}$$

where $\left(A_0^{GT} + \delta A^{GT}\right) = A^{GT}$ is the gravitational effect of the real topographical density distribution between the geoid and the topo–surface

$$A^{GT}\left(h_P, \Omega_P\right) = -G \int\limits_{N(\Omega)}^{h_T(\Omega)} \iint\limits_{\Omega_0} \rho(h,\Omega) \frac{\partial L^{-1}}{\partial h_P} \left(R+h\right)^2 \mathrm{d}h\, \mathrm{d}\Omega\,. \tag{11}$$

The left hand side of Eq. (10) defines the gravity disturbance in the NT model space (*Vaníček et al., 2004*), i.e., the *NT gravity disturbance*

$$\begin{aligned}\delta g^{NT}\left(h_P, \Omega_P\right) = \delta g\left(h_P, \Omega_P\right) - A^{GT}\left(h_P, \Omega_P\right) = \\ = g^{NT}\left(h_P, \Omega_P\right) - \gamma\left(h_P, \Omega_P\right)\,,\end{aligned} \tag{12}$$

where $g^{NT}\left(h_P, \Omega_P\right) = g\left(h_P, \Omega_P\right) - A^{GT}\left(h_P, \Omega_P\right)$ is the actual gravity corrected for the effect of the topography of real density. The removal of the gravitational effect of the "T" from the actual gravity (along with the removal of normal gravity), or in other words the application of the NT topo correction to the generic gravity disturbance, transforms the "Real Earth" (Fig. 2) into the "No Topography" model earth (Fig. 3). The "NT model earth" consists of anomalous density below the reference ellipsoid, real density between the reference ellipsoid and the geoid, and zero density (no masses) above the geoid. Consequently, the NT gravity disturbance (in spherical approximation, multiplied by the geocentric radius r) is harmonic above the geoid.

On the right hand side of Eq. (10) the two terms in the round brackets amount to the gravitational effect of the anomalous density below the geoid, $\delta A^{G} = \left(\delta A^{E} + \delta A^{EG}\right)$, expressed as

$$\delta A^{G}\left(h_P, \Omega_P\right) = -G \int\limits_{-R}^{N(\Omega)} \iint\limits_{\Omega_0} \delta\rho(h,\Omega) \frac{\partial L^{-1}}{\partial h_P} \left(R+h\right)^2 \mathrm{d}h\, \mathrm{d}\Omega\,. \tag{13}$$

Equation (10) finally reads

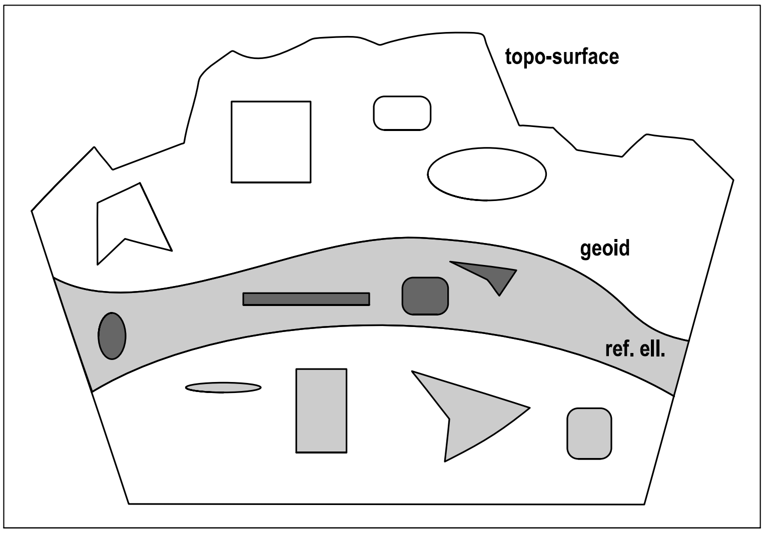

Fig. 3. The NT model space.

$$\delta g^{NT}(h_P, \Omega_P) = \delta A^{G}(h_P, \Omega_P) + A_0^{EG}(h_P, \Omega_P) \ , \tag{14}$$

which tells us that the NT gravity disturbance is equal to the gravitational effect of all anomalous masses below the geoid biased by the gravitational effect of masses of constant density between the reference ellipsoid and the geoid.

3.4. The "No Topography of Constant Density" earth model – the NTC model space

After re–arranging the terms in Eq. (9) differently, we can write Eq. (9) as follows

$$\delta g - A_0^{GT} = \left(\delta A^{E} + \delta A^{EG} + \delta A^{GT}\right) + A_0^{EG} \,. \tag{15}$$

The left hand side of Eq. (15) defines the gravity disturbance in the NTC model space, i.e., the *NTC gravity disturbance*

$$\begin{aligned}\delta g^{NTC}(h_P, \Omega_P) = \delta g(h_P, \Omega_P) - A_0^{GT}(h_P, \Omega_P) = \\ = g^{NTC}(h_P, \Omega_P) - \gamma(h_P, \Omega_P) \ ,\end{aligned} \tag{16}$$

where $g^{NTC}(h_P, \Omega_P) = g(h_P, \Omega_P) - A_0^{GT}(h_P, \Omega_P)$ is the actual gravity corrected for the effect of the topography of constant density. The removal of the gravitational effect of the "TC" from the actual (observed) gravity (along with the removal of normal gravity), or, in other words, the application of the NTC topo correction to the generic gravity disturbance, transforms the "Real Earth" (Fig. 2) into the "No Topography of Constant Density" model earth (Fig. 4). The "NTC model earth" consists of anomalous density below the reference ellipsoid, real density between the reference ellipsoid and the geoid, anomalous density between the geoid and the topo–surface, and zero density (no masses) above the topo–surface. Consequently, the NTC gravity disturbance (in spherical approximation, multiplied by r) is harmonic above the topo–surface only. It is not harmonic between the geoid and the topo–surface due to the presence of anomalous density in this region.

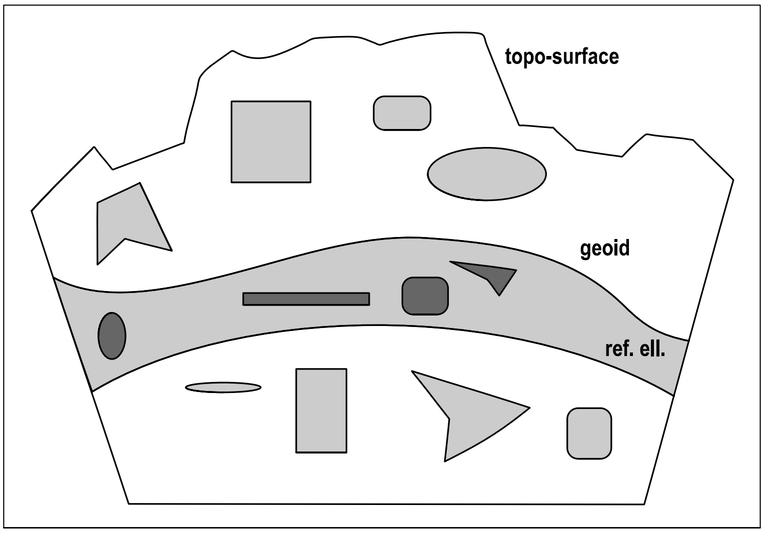

Fig. 4. The NTC model space.

On the right hand side of Eq. (15) the three terms in the round brackets amount to the gravitational effect of the anomalous density below the topo–surface (inside the entire earth), $\delta A^{Earth} = \left(\delta A^E + \delta A^{EG} + \delta A^{GT}\right)$, expressed as

$$\delta A^{Earth}(h_P, \Omega_P) = -G \int\limits_{-R}^{h_T(\Omega)} \iint\limits_{\Omega_0} \delta\rho(h, \Omega) \frac{\partial L^{-1}}{\partial h_P} (R+h)^2 \, \mathrm{d}h \, \mathrm{d}\Omega \,. \tag{17}$$

Equation (10) finally reads

$$\delta g^{NTC}(h_P, \Omega_P) = \delta A^{Earth}(h_P, \Omega_P) + A_0^{EG}(h_P, \Omega_P) \,, \tag{18}$$

which makes it clear, that the NTC gravity disturbance is equal to the gravitational effect of all anomalous masses inside the whole earth biased by the gravitational effect of masses of constant density between the reference ellipsoid and the geoid.

3.5. The "No Ellipsoidal Topography" earth model – the NET model space

After re–arranging the terms in Eq. (9) yet differently, we can write it as

$$\delta g - \left(A_0^{EG} + \delta A^{EG} + A_0^{GT} + \delta A^{GT} \right) = \delta A^E \,, \tag{19}$$

where $\left(A_0^{EG} + \delta A^{EG} + A_0^{GT} + \delta A^{GT} \right) = A^{ET}$ is the gravitational effect of the real topographical density distribution between the reference ellipsoid and the topo–surface

$$A^{ET}(h_P, \Omega_P) = -G \int\limits_{0}^{h_T(\Omega)} \iint\limits_{\Omega_0} \rho(h, \Omega) \frac{\partial L^{-1}}{\partial h_P} (R+h)^2 \, \mathrm{d}h \, \mathrm{d}\Omega \,. \tag{20}$$

The left hand side of Eq. (19) defines the gravity disturbance in the NET model space, i.e., the *NET gravity disturbance*

$$\begin{aligned} \delta g^{NET}(h_P, \Omega_P) &= \delta g(h_P, \Omega_P) - A^{ET}(h_P, \Omega_P) = \\ &= g^{NET}(h_P, \Omega_P) - \gamma(h_P, \Omega_P) \,, \end{aligned} \tag{21}$$

where $g^{NET}(h_P, \Omega_P) = g(h_P, \Omega_P) - A^{ET}(h_P, \Omega_P)$ is the actual gravity corrected for the effect of the ellipsoidal topography of real density. The removal of the gravitational effect of the "ET" from the actual (observed) gravity (along with the removal of normal gravity), or in other words the application of the NET topo correction to the generic gravity disturbance, transforms the "Real Earth" (Fig. 2) into the "No Ellipsoidal Topography" model earth (Fig. 5). The "NET model earth" consists of anomalous density below the reference ellipsoid, and zero density (no masses) above the reference ellipsoid. Consequently, the NET gravity disturbance (in spherical approximation, multiplied by r) is harmonic above the reference ellipsoid.

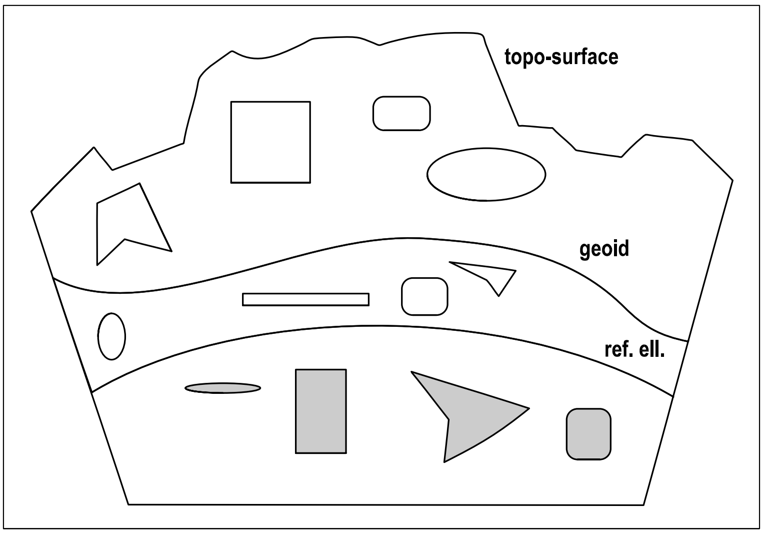

Fig. 5. The NET model space.

On the right hand side of Eq. (19) we have the gravitational effect of the anomalous density below the reference ellipsoid. Equation (19) finally reads

$$\delta g^{NET}(h_P, \Omega_P) = \delta A^E(h_P, \Omega_P) \ , \tag{22}$$

which claims, that the NET gravity disturbance is equal to the gravitational effect of all anomalous masses below the reference ellipsoid. The advantage of the NET gravity disturbance is that it is not biased by the gravitational effect of any masses between the reference ellipsoid and the geoid. The disadvantage though is that it is blind to any density contrast in the region between the reference ellipsoid and the topo–surface. Of course the NET topo correction needed for compiling (realizing) the NET gravity disturbance requires the knowledge of real density between the reference ellipsoid and the topo–surface, thus there is no need to solve for it. But in reality we are far from the satisfactory knowledge of the real topographical density between the reference ellipsoid and the topo–surface, so we better assume a background density in this region and solve for the anomalous density, as shall be done below.

3.6. The "No Ellipsoidal Topography of Constant Density" earth model – the NETC model space

After again re–arranging the terms in Eq. (9), we can write it as

$$\delta g - \left(A_0^{EG} + A_0^{GT}\right) = \left(\delta A^E + \delta A^{EG} + \delta A^{GT}\right) . \tag{23}$$

where $\left(A_0^{EG} + A_0^{GT}\right) = A_0^{ET}$ is the gravitational effect of the constant topographical density distribution between the reference ellipsoid and the topo–surface

$$A_0^{ET}\left(h_P, \Omega_P\right) = -G\rho_0 \int\limits_0^{h_T(\Omega)} \iint\limits_{\Omega_0} \frac{\partial L^{-1}}{\partial h_P} \left(R + h\right)^2 \mathrm{d}h \, \mathrm{d}\Omega \, . \tag{24}$$

The left hand side of Eq. (23) defines the gravity disturbance in the NETC model space, i.e., the *NETC gravity disturbance*

$$\begin{aligned} \delta g^{NETC}\left(h_P, \Omega_P\right) = \delta g\left(h_P, \Omega_P\right) - A_0^{ET}\left(h_P, \Omega_P\right) = \\ = g^{NETC}\left(h_P, \Omega_P\right) - \gamma\left(h_P, \Omega_P\right) , \end{aligned} \tag{25}$$

where $g^{NETC}\left(h_P, \Omega_P\right) = g\left(h_P, \Omega_P\right) - A_0^{ET}\left(h_P, \Omega_P\right)$ is the actual gravity corrected for the effect of the ellipsoidal topography of constant density. The removal of the gravitational effect of the "ETC" from the actual (observed) gravity (along with the removal of normal gravity), or in other words the application of the NETC topo correction to the generic gravity disturbance, transforms the "Real Earth" (Fig. 2) into the "No Ellipsoidal Topography of Constant Density" model earth (Fig. 6). The "NETC model earth" consists of only anomalous density below the topo–surface. The NETC gravity disturbance (in spherical approximation, multiplied by r) is harmonic above the topo–surface only.

On the right hand side of Eq. (23) we have the gravitational effect of the anomalous density below the topo–surface (inside the entire earth),

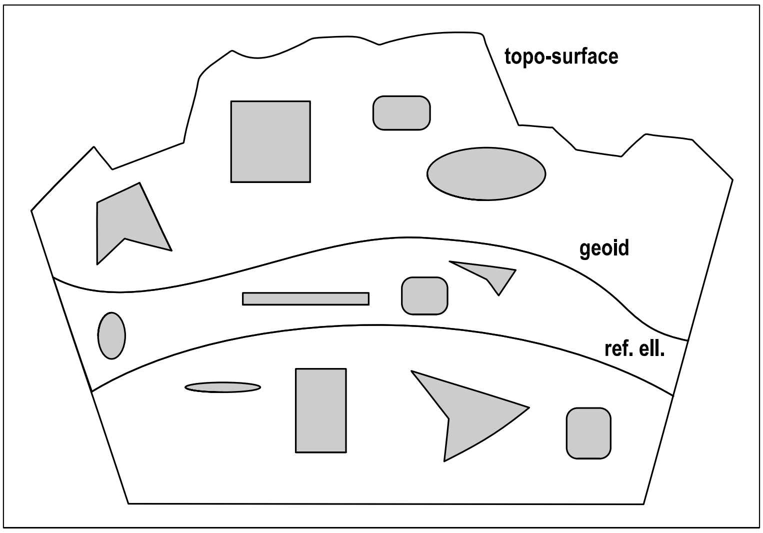

Fig. 6. The NETC model space.

$\delta A^{Earth} = \left(\delta A^{E} + \delta A^{EG} + \delta A^{GT}\right)$, given by Eq. (17). Equation (23) finally reads

$$\delta g^{NETC}(h_P, \Omega_P) = \delta A^{Earth}(h_P, \Omega_P) \ , \tag{26}$$

which proclaims, that the NETC gravity disturbance is exactly equal to the gravitational effect of all anomalous masses inside the whole earth. This is the right type of the gravity disturbance that we were looking for. The NETC gravity disturbance can be interpreted as the gravitational effect of all underground anomalous masses.

4. NETC model space – Negative topography

So far we have been ignoring the fact that the geoidal heights are negative in some areas over the globe, where the geoid dips below the reference ellipsoid, and also the geodetic heights of the topo–surface are negative in some areas over the globe, where the topo–surface dips below the reference ellipsoid. This fact must be treated properly in global applications or when working in areas with negative geodetic heights. In the sequel we shall focus on the NETC gravity disturbance only, and will try to handle the NETC topo correction properly, taking into account the negative ellipsoidal topography.

In order to treat the negative heights of the topo–surface, we introduce another reference surface – the surface of the inner quasi–ellipsoid (Fig. 7). This surface is defined as the surface the depth of which below the surface of the reference ellipsoid is constant (h^*) and as such is not an ellipsoidal surface – hence the name. The value of h^* is chosen equal to the maximum dip of the topo–surface below the reference ellipsoid over the entire globe. The reference ellipsoid and the surface of the inner quasi–ellipsoid make up the quasi–ellipsoidal layer of constant thickness h^*. The reference density within this layer is chosen constant and equal to that used for topographical masses of constant density, $\rho_R(h, \Omega) = \rho_0$, while the normal density within this layer is chosen as zero density, $\rho_N(h, \Omega) = 0$. Recall, that the reference density is used for defining the anomalous density (that is the subject of the gravimetric inverse problem), while the normal density generates the normal potential and the normal gravity. Henceforth the normal potential external to the reference ellipsoid is generated by a new normal mass density inside

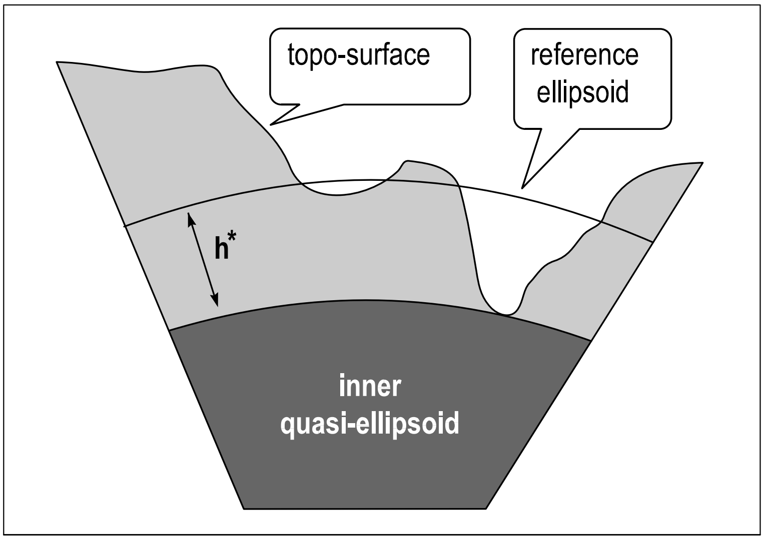

Fig. 7. The inner quasi–ellipsoid and the quasi–ellipsoidal layer of constant thickness h^*.

the inner quasi–ellipsoid. The argument, that was put forth in Section 2.3 for finding a particular solution to the normal density inside the normal ellipsoid may be extended to this new normal density inside the inner quasi–ellipsoid. The zero normal density in the quasi–ellipsoidal layer extends also the validity of the closed form or series expansion formulae for normal gravity for the realm of negative geodetic heights from the interval $h \in (0; -h^{*})$.

Now the role of the lower boundary of the topographical masses that was played by the reference ellipsoid in the case of the NETC model space is played by the surface of the inner quasi–ellipsoid, and the NETC topo correction becomes (with respect to Eq. (24) only the lower integral boundary changes from 0 to $-h^{*}$)

$$A_0^{ET}(h_P, \Omega_P) = -G\rho_0 \int\limits_{-h^*}^{h_T(\Omega)} \iint\limits_{\Omega_0} \frac{\partial L^{-1}}{\partial h_P} (R+h)^2 \, \mathrm{d}h \, \mathrm{d}\Omega \,. \tag{27}$$

For numerical aspects of evaluating the A_0^{ET} given by Eq. (27) refer to *Vajda et al. (2004)*. Note, that the presence of the quasi–ellipsoidal layer slightly modifies the reference density model of the earth used in Section 3.6 and given by Tab. 2. It will now become as specified in Tab. 7.

Everything else remains the same as in Section 3.6. The approach that we just described assures that the NETC gravity disturbance (in spherical approximation, multiplied by r) remains harmonic everywhere above the topo–surface, even in areas, where the topo–surface dips below the reference ellipsoid. This is due to the fact, that the real density as well as the normal

Tab. 7. Reference and normal densities used for defining the NETC space accounting for negative ellipsoidal topography

region of the earth	**reference density**
inner quasi–ellipsoid	$\rho_R(h,\Omega) = \rho_N(h,\Omega)$
between the surfaces of the inner quasi–ellipsoid and the reference ellipsoid	$\rho_R(h,\Omega) = \rho_0$ $\rho_N(h,\Omega) = 0$
between the surfaces of the reference ellipsoid and the topo–surface	$\rho_R(h,\Omega) = \rho_0$

density is zero in the region between the reference ellipsoid and the topo–surface in the area of negative ellipsoidal topography.

5. Discussion, summary, and conclusions

We have investigated four types of the topographical correction to gravity (and hence to gravity disturbance), as shown in Tab. 8. These topo–corrections applied to the (generic) gravity disturbance, through their application to actual gravity (a topo–correction has no impact at all on the normal gravity), define four types of "topo–corrected gravity disturbance".

In global studies, or if working in areas with negative ellipsoidal heights of the topo–surface, the NETC topo correction must be computed by means of Eq. (27). In such cases the NETC gravity disturbance must be interpreted (inverted) in the light of Section 4, i.e., the density contrast is defined with respect to the reference density given by Tab. 7.

We have investigated the four types of topo–corrected gravity disturbance with the objective of finding which one would be equal exactly to the gravitational effect of all anomalous masses within the whole earth (below the topo–surface). We have used the reference density distribution below

Tab. 8. Four topographical corrections to gravity (to gravity disturbance) in a notation, where $\mathrm{d}\vartheta \equiv (R+h)^2\,\mathrm{d}h\,\mathrm{d}\Omega$

topo correction to gravity	definition
NT	$-A^{GT}(h_P,\Omega_P) = G\int\limits_{N(\Omega)}^{h_T(\Omega)}\iint\limits_{\Omega_0}\rho(h,\Omega)\dfrac{\partial L^{-1}}{\partial h_P}\,d\vartheta$
NTC	$-A_0^{GT}(h_P,\Omega_P) = G\rho_0\int\limits_{N(\Omega)}^{h_T(\Omega)}\iint\limits_{\Omega_0}\dfrac{\partial L^{-1}}{\partial h_P}\,d\vartheta$
NET	$-A^{ET}(h_P,\Omega_P) = G\int\limits_{0}^{h_T(\Omega)}\iint\limits_{\Omega_0}\rho(h,\Omega)\dfrac{\partial L^{-1}}{\partial h_P}\,d\vartheta$
NETC	$-A_0^{ET}(h_P,\Omega_P) = G\rho_0\int\limits_{0}^{h_T(\Omega)}\iint\limits_{\Omega_0}\dfrac{\partial L^{-1}}{\partial h_P}\,d\vartheta$

the topo–surface, given by Tab. 2 (or Tab. 7), to define the anomalous density. The method of our study was the decomposition of the actual potential and gravitation of the real earth. The results of the investigation are summarized in Tab. 9. Table 10 presents the regions of harmonicity of the four types of gravity disturbance.

The gravitational effect of masses of constant density between the reference ellipsoid and the geoid (A_0^{EG}) is found as an unwanted systematic bias when interpreting a topo–corrected gravity disturbance. To have a perception of its magnitude and spatial behavior, we give a numerical example of A_0^{EG} estimated for the area of Eastern Alps in Fig. 8. The illustrated A_0^{EG} was computed using a flat earth approximation and the FFT. The constant model topo–density used was 2.67 g/cm^3. This bias is a fairly long–wavelength signal. Perhaps in some local studies A_0^{EG} may be ne-

Tab. 9. The meaning (interpretation in terms of gravitational effects) of the four types of the topo–corrected gravity disturbance

relation between the gravity disturbance and the gravitational effect	**conclusion**
$\delta g^{NT} = \delta A^{G} + A_0^{EG}$	NT gravity disturbance is equal to the gravitational effect of all anomalous masses below the geoid biased by the gravitational effect of masses of constant density between the reference ellipsoid and the geoid
$\delta g^{NTC} = \delta A^{Earth} + A_0^{EG}$	NTC gravity disturbance is equal to the gravitational effect of all anomalous masses inside the whole earth biased by the gravitational effect of masses of constant density between the reference ellipsoid and the geoid
$\delta g^{NET} = \delta A^{E}$	NET gravity disturbance is equal to the gravitational effect of all anomalous masses below the reference ellipsoid
$\delta g^{NETC} = \delta A^{Earth}$	NETC gravity disturbance is equal to the gravitational effect of all anomalous masses inside the whole earth

Tab. 10. Harmonicity of the four types of gravity disturbance (in spherical approximation, multiplied by r)

quantity	is harmonic in the region
$r\delta g^{NT}$	above the geoid
$r\delta g^{NTC}$	above the topo–surface
$r\delta g^{NET}$	above the reference ellipsoid
$r\delta g^{NETC}$	above the topo–surface

glected as a trend of no interest.

The effort of topo–correcting the anomalous gravity quantities such as the gravity disturbance or gravity anomaly has historically been associated with the name "Bouguer". If we were to link this name with the topo–corrected gravity disturbance, it would fit best the NT topo–correction to gravity. Then the *NT gravity disturbance* would be referred to as the "Bouguer

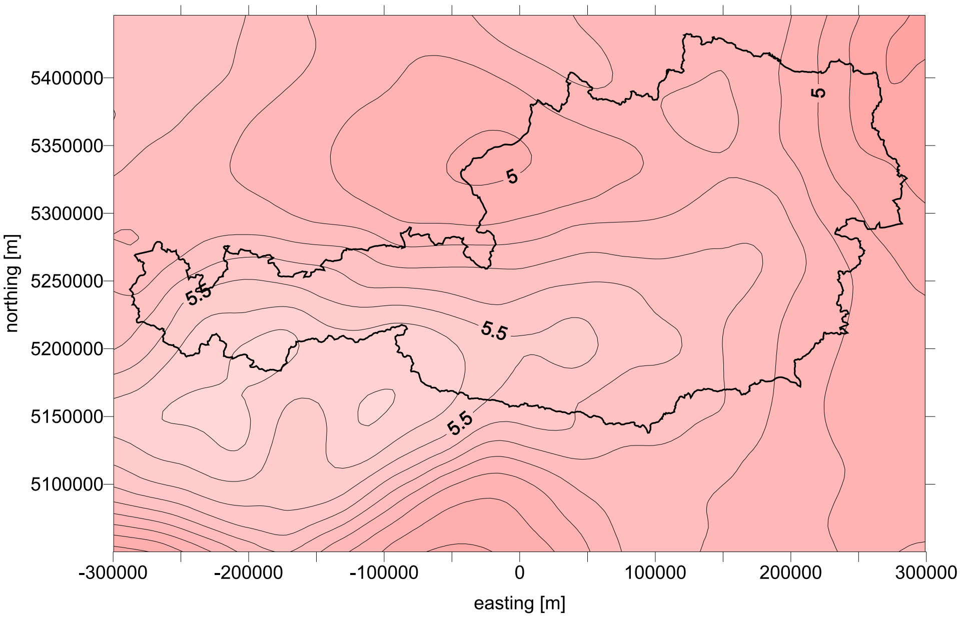

Fig. 8. An estimate of the A_0^{EG} systematic bias (mGal) for the area of the Eastern Alps. The border line of Austria is indicated.

gravity disturbance".

A similar study could be performed for the gravity anomaly. Note, that in the case of gravity anomaly, the application of a topographical correction (to actual potential and/or to actual gravity) affects not only gravity, but also the vertical displacement (via the disturbing potential in the Bruns equation) used for evaluating the normal gravity needed for compiling the gravity anomaly (e.g. *Vaníček et al., 1999; Vaníček et al., 2004*). For a detailed investigation regarding the NT–gravity anomaly the interested reader is referred to *Vaníček et al. (2004)*.

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