GEOID-QUASIGEOID CORRECTION IN FORMULATION OF THE FUNDAMENTAL FORMULA OF PHYSICAL GEODESY

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ABSTRACT

To formulate the fundamental formula of physical geodesy at the physical surface of the Earth, the gravity anomalies are used instead of the gravity disturbances, because the geodetic heights above the geocentric reference ellipsoid are not usually available. The relation between the gravity anomaly and the gravity disturbance is defined as a product of the normal gravity gradient referred to the telluroid and the height anomaly according to Molodensky's theory of the normal heights (Molodensky, 1945; Molodensky et al., 1960). Considering the normal gravity gradient referred to the surface of the geocentric reference ellipsoid, this relation is redefined as a function of the normal height (Vaníček et al., 1999). When the orthometric heights are practically used for the realization of the vertical datum, the geoid-quasigeoid correction is applied to the fundamental formula of physical geodesy to determine the precise geoid.
Theoretical formulation of the geoid-quasigeoid correction to the fundamental formula of physical geodesy can be found in Martinec (1993) and Vaníček et al. (1999). In this paper, the numerical investigation of this correction at the territory of Canada is shown and the error analysis is introduced.

Keywords: Geoid-Quasigeoid Correction, gravity, height.

1. BASIC THEORY

The gravity disturbance $\delta g[r_t(\Omega)]$ referred to the Earth's surface $\forall\, \Omega \in \Omega_0 :\; r_t(\Omega) \cong r_g(\Omega) + H^{O}(\Omega)$, where $\forall\, \Omega \in \Omega_0 : r_g(\Omega)$ denotes the geocentric radius of the geoid surface, is defined by (Heiskanen and Moritz, 1967, Eq. 2-146)

$$\forall\, \Omega \in \Omega_0 :\; \delta g[r_t(\Omega)] = g[r_t(\Omega)] - \gamma\,[r_t(\Omega)], \tag{1}$$

where $g(r,\Omega)$ is the actual gravity, and $\gamma\,(r,\phi)$ is the normal gravity of the geocentric reference ellipsoid. The normal gravity is defined according to Somigliana – Pizzetti's theory of the normal gravity field generated by the ellipsoid of revolution (Pizzetti, 1894 and 1911; Somigliana, 1929).

A geocentric position is represented by the geocentric radius r; $r \in \Re^{+}\ \left(\Re^{+} \in \langle 0, +\infty)\right)$, and the geocentric spherical coordinates ϕ and λ; $\Omega = (\phi, \lambda)$, $\Omega \in \Omega_0\ \left(\Omega_0 \in \langle -\pi/2 \le \phi \le \pi/2;\, 0 \le \lambda \le 2\pi \rangle\right)$.

Since the evaluation of the normal gravity $\gamma\,[r_t(\Omega)]$ at the Earth's surface would require the knowledge of the geodetic height $h(\Omega)$ above the geocentric reference ellipsoid, the gravity disturbance $\delta g[r_t(\Omega)]$ is transformed into the gravity anomaly $\Delta g[r_t(\Omega)]$ according to the following equation (Vaníček et al., 1999)

$$\forall\, \Omega \in \Omega_0 :\; \Delta g[r_t(\Omega)] = \delta g[r_t(\Omega)] + \gamma\,[r_t(\Omega)] - \gamma_o(\phi) - \left.\frac{\partial \gamma\,(r,\phi)}{\partial\, \mathrm{n}}\right|_{r=r_o(\phi)} H^{N}(\Omega). \tag{2}$$

In Eq. (2), $H^{N}(\Omega)$ denotes the normal height, $\gamma_o(\phi)$ is the normal gravity referred to the surface of the geocentric reference ellipsoid $\forall\, \phi \in \langle -\pi/2, \pi/2 \rangle : r_o(\phi)$, and $\partial\gamma\,(r,\phi)/\partial\, \mathrm{n}$ is the normal gravity gradient.

2. GEOID-QUASIGEOID CORRECTION

When the orthometric heights $H^{O}(\Omega)$ are used instead of the normal heights $H^{N}(\Omega)$, the geoid-quasigeoid correction is applied to Eq. (2). To define this correction, the commonly used Helmert's orthometric heights $H^{O}(\Omega)$ and Molodensky's normal heights $H^{N}(\Omega)$ are briefly introduced.

The fundamental formula for a definition of the orthometric height reads (Heiskanen and Moritz, 1967)

$$\forall\,\Omega\in\Omega_{O}:\qquad H^{O}(\Omega)=\frac{C[r_t(\Omega)]}{\bar{g}(\Omega)},\qquad(3)$$

where $C[r_t(\Omega)]$ is the geopotential number, and $\bar{g}(\Omega)$ is the mean value of the gravity along the plumbline between the geoid and the Earth's surface.

For the Helmert (1890) orthometric height, the mean value of the gravity $\bar{g}(\Omega)$ is evaluated using Poincaré-Prey's gravity gradient (Heiskanen and Moritz, 1967, Eq. 4-25)

$$\forall\,\Omega\in\Omega_{O}:\ \bar{g}(\Omega)\cong g[r_t(\Omega)]-\frac{1}{2}\frac{\partial g(r,\Omega)}{\partial H}\bigg|_{r=r_t(\Omega)}H^{O}(\Omega)$$
$$\approx g[r_t(\Omega)]-\frac{1}{2}\frac{\partial\gamma(r,\phi)}{\partial\,\mathrm{n}}\bigg|_{r=r_t(\Omega)}H^{O}(\Omega),$$
$$-2\pi\,\mathrm{G}\,\rho_{o}H^{O}(\Omega),\qquad(4)$$

where $g[r_t(\Omega)]$ is the observed gravity at the Earth's surface, G is Newton's gravitational constant, and ρ_o is the mean topographical density $\rho_o=2.67\,[\mathrm{g.cm}^{-3}]$.

Molodensky's normal height $H^{N}(\Omega)$ reads (Molodensky, 1945)

$$\forall\,\Omega\in\Omega_{O}:\qquad H^{N}(\Omega)=\frac{C[r_t(\Omega)]}{\bar{\gamma}(\Omega)}.\qquad(5)$$

The mean value of the normal gravity $\bar{\gamma}(\Omega)$ along the ellipsoidal normal between the surface of the geocentric reference ellipsoid and the telluroid $\forall\,\Omega\in\Omega_{O}$: $r_o(\phi)+H^{N}(\Omega)$ is given by

$\forall\,\Omega\in\Omega_{O}$:

$$\bar{\gamma}(\Omega)\cong\gamma\left[r_o(\phi)+H^{N}(\Omega)\right]-\frac{1}{2}\frac{\partial\gamma(r,\phi)}{\partial\,\mathrm{n}}\bigg|_{r=r_o(\phi)+H^{N}(\Omega)}H^{N}(\Omega)$$
$$-\frac{1}{4}\frac{\partial^2\gamma(r,\phi)}{\partial\,\mathrm{n}^2}\bigg|_{r=r_o(\phi)+H^{N}(\Omega)}\left[H^{N}(\Omega)\right]^2.\qquad(6)$$

The difference between the normal height and orthometric height, i.e., the **geoid-quasigeoid correction**, can be found beginning with the following relation

$$\forall\,\Omega\in\Omega_{O}:\ H^{N}(\Omega)-H^{O}(\Omega)=H^{O}(\Omega)\frac{\bar{g}(\Omega)-\bar{\gamma}(\Omega)}{\bar{\gamma}(\Omega)}$$
$$\cong H^{O}(\Omega)\frac{\bar{g}(\Omega)-\bar{\gamma}(\Omega)}{\gamma_o(\phi)}.\qquad(7)$$

Assuming

$\forall\,\Omega\in\Omega_{O}$:

$$\frac{1}{4}\frac{\partial^2\gamma(r,\phi)}{\partial\,\mathrm{n}^2}\bigg|_{r=r_o(\phi)+H^{N}(\Omega)}\left[H^{N}(\Omega)\right]^2\approx 0,$$
$$\frac{1}{2}\frac{\partial\gamma(r,\phi)}{\partial\,\mathrm{n}}\bigg|_{r=r_t(\Omega)}H^{O}(\Omega)-\frac{1}{2}\frac{\partial\gamma(r,\phi)}{\partial\,\mathrm{n}}\bigg|_{r=r_o(\phi)+H^{N}(\Omega)}H^{N}(\Omega)\approx 0,$$
$$(8)$$

the difference between the mean gravity $\bar{g}(\Omega)$ and the mean normal gravity $\bar{\gamma}(\Omega)$ in Eq. (7) becomes

$$\forall\,\Omega\in\Omega_{O}:\ \bar{g}(\Omega)-\bar{\gamma}(\Omega)\cong g[r_t(\Omega)]-\gamma\left[r_o(\phi)+H^{N}(\Omega)\right]$$
$$-2\pi\,\mathrm{G}\,\rho_o H^{O}(\Omega).\qquad(9)$$

The expression on the right-hand side of Eq. (9) is approximately equal to the simple Bouguer gravity anomaly $\Delta g^{SB}[r_t(\Omega)]$ (Martinec, 1993)

$\forall\,\Omega\in\Omega_{O}$:

$$\Delta g^{SB}[r_t(\Omega)]=g[r_t(\Omega)]-\gamma_o(\phi)-\frac{\partial\gamma(r,\phi)}{\partial\,\mathrm{n}}\bigg|_{r=r_o(\phi)}H^{O}(\Omega)$$
$$-2\pi\,\mathrm{G}\,\rho_o H^{O}(\Omega).\qquad(10)$$

Regarding Eq. (10), the geoid-quasigeoid correction from Eq. (7) finally takes the following form

$$\forall\,\Omega\in\Omega_{O}:\ H^{N}(\Omega)-H^{O}(\Omega)\cong H^{O}(\Omega)\frac{\Delta g^{SB}[r_t(\Omega)]}{\gamma_o(\phi)}.\qquad(11)$$

Substituting Eq. (11) to Eq. (2), the gravity anomaly $\Delta g[r_t(\Omega)]$ becomes

$\forall\,\Omega\in\Omega_{O}$:

$$\Delta g[r_t(\Omega)]=\delta g[r_t(\Omega)]+\gamma[r_t(\Omega)]-\gamma_o(\phi)$$
$$-\frac{\partial\gamma(r,\phi)}{\partial\,\mathrm{n}}\bigg|_{r=r_o(\phi)}H^{O}(\Omega)\left[1+\frac{\Delta g^{SB}[r_t(\Omega)]}{\gamma_o(\phi)}\right]$$
$$=\Delta g^{FA}[r_t(\Omega)]-\frac{1}{\gamma_o(\phi)}\frac{\partial\gamma(r,\phi)}{\partial\,\mathrm{n}}\bigg|_{r=r_o(\phi)}$$
$$\times H^{O}(\Omega)\,\Delta g^{SB}[r_t(\Omega)],\qquad(12)$$

where $\Delta g^{FA}[r_t(\Omega)]$ stands for the free-air gravity anomaly

$\forall\,\Omega\in\Omega_{O}$:

$$\Delta g^{FA}[r_t(\Omega)]=g[r_t(\Omega)]-\gamma_o(\phi)-\frac{\partial\gamma(r,\phi)}{\partial\,\mathrm{n}}\bigg|_{r=r_o(\phi)}H^{O}(\Omega).\qquad(13)$$

Applying the spherical approximation (Heiskanen and Moritz, 1967, Eq. 2-150)

$$\forall\,\Omega\in\Omega_{O}:\qquad -\frac{1}{\gamma_o(\phi)}\frac{\partial\gamma(r,\phi)}{\partial\,\mathrm{n}}\bigg|_{r=r_o(\phi)}\cong\frac{2}{\mathrm{R}},\qquad(14)$$

Eq. (12) is subsequently rewritten as

$\forall\,\Omega\in\Omega_{O}$:

$$\Delta g[r_t(\Omega)]=\Delta g^{FA}[r_t(\Omega)]+\frac{2}{\mathrm{R}}H^{O}(\Omega)\,\Delta g^{SB}[r_t(\Omega)].\qquad(15)$$

The second term on the right-hand side of Eq. (15) defines the **geoid-quasigeoid correction to the fundamental formula of physical geodesy** (Vaníček et al., 1999)

$$\forall\,\Omega\in\Omega_{O}:\qquad \chi[r_t(\Omega)]\cong\frac{2}{\mathrm{R}}H^{O}(\Omega)\,\Delta g^{SB}[r_t(\Omega)].\qquad(16)$$

3. ERROR ANALYSIS

The accuracy estimation of the geoid-quasigeoid correction $\chi[r_t(\Omega)]$ to the fundamental formula of physical geodesy depends on the accuracy of the orthometric height, gravity and topographical density.

From Eq. (16) follows the relation between the actual error $\varepsilon_\chi[r_t(\Omega)]$ of the geoid-quasigeoid correction to the fundamental formula of physical geodesy and the error of the orthometric height $\varepsilon_{H^O}(\Omega)$ and the error $\varepsilon_{\Delta g^{SB}}[r_t(\Omega)]$ of the simple Bouguer gravity anomaly

$$\forall\,\Omega\in\Omega_O:$$

$$\begin{aligned}\varepsilon_\chi[r_t(\Omega)] &= \frac{\partial\chi[r_t(\Omega)]}{\partial H^O(\Omega)}\varepsilon_{H^O}(\Omega)+\frac{\partial\chi[r_t(\Omega)]}{\partial\,\Delta g^{SB}[r_t(\Omega)]}\varepsilon_{\Delta g^{SB}}[r_t(\Omega)]\\ &= \frac{2}{R}\Delta g^{SB}[r_t(\Omega)]\varepsilon_{H^O}(\Omega)+\frac{2}{R}H^O(\Omega)\varepsilon_{\Delta g^{SB}}[r_t(\Omega)]\\ &= \chi[r_t(\Omega)]\left(\frac{\varepsilon_{H^O}(\Omega)}{H^O(\Omega)}+\frac{\varepsilon_{\Delta g^{SB}}[r_t(\Omega)]}{\Delta g^{SB}[r_t(\Omega)]}\right). \qquad (17)\end{aligned}$$

Furthermore, the error $\varepsilon_{\Delta g^{SB}}[r_t(\Omega)]$ of the simple Bouguer gravity anomaly is expressed by

$$\forall\,\Omega\in\Omega_O:$$

$$\begin{aligned}\varepsilon_{\Delta g^{SB}}[r_t(\Omega)] &= \frac{\partial\,\Delta g^{SB}[r_t(\Omega)]}{\partial\, g[r_t(\Omega)]}\varepsilon_g[r_t(\Omega)]+\left[\frac{\partial\,\Delta g^{SB}[r_t(\Omega)]}{\partial H^O(\Omega)}\right.\\ &\left.+\frac{\partial\,\Delta g^{SB}[r_t(\Omega)]}{\partial\gamma[r_o(\phi)+H^O(\Omega)]}\frac{\partial\gamma[r_o(\phi)+H^O(\Omega)]}{\partial H^O(\Omega)}\right]\varepsilon_{H^O}(\Omega)\\ &+\frac{\partial\,\Delta g^{SB}[r_t(\Omega)]}{\partial\,\rho(\Omega)}\delta\rho(\Omega)\\ &\cong \frac{\partial\,\Delta g^{SB}[r_t(\Omega)]}{\partial\, g[r_t(\Omega)]}\varepsilon_g[r_t(\Omega)]+\frac{\partial\,\Delta g^{SB}[r_t(\Omega)]}{\partial H^O(\Omega)}\varepsilon_{H^O}(\Omega)\\ &+\frac{\partial\,\Delta g^{SB}[r_t(\Omega)]}{\partial\,\rho(\Omega)}\delta\rho(\Omega), \qquad (18)\end{aligned}$$

where $\varepsilon_g[r_t(\Omega)]$ is the actual error of the observed gravity at the Earth's surface. The laterally varying anomalous topographical density $\delta\rho(\Omega)$ is given by a difference of the lateral topographical density $\rho(\Omega)$ and the mean topographical density ρ_o (Martinec, 1993)

$$\forall\,\Omega\in\Omega_O: \qquad \delta\rho(\Omega)=\rho(\Omega)-\rho_o. \qquad (19)$$

The error of the correction $\chi[r_t(\Omega)]$ caused by the inaccuracy of the normal gravity $\gamma[r_o(\phi)+H^O(\Omega)]$ due to the error $\varepsilon_{H^O}(\Omega)$ of the orthometric height is negligible

$$\forall\,\Omega\in\Omega_O:$$

$$\begin{aligned}&\frac{\partial\chi[r_t(\Omega)]}{\partial\Delta g^{SB}[r_t(\Omega)]}\frac{\partial\Delta g^{SB}[r_t(\Omega)]}{\partial\gamma[r_o(\phi)+H^O(\Omega)]}\frac{\partial\gamma[r_o(\phi)+H^O(\Omega)]}{\partial H^O(\Omega)}\varepsilon_{H^O}(\Omega)\\ &= -\frac{2}{R}H^O(\Omega)\frac{\partial}{\partial H^O(\Omega)}\left[\gamma_o(\phi)+\left.\frac{\partial\gamma(r,\phi)}{\partial\,n}\right|_{r=r_o(\phi)}H^O(\Omega)\right]\varepsilon_{H^O}(\Omega)\\ &\cong \frac{2}{R}\gamma_o(\phi)\,H^O(\Omega)\left[\frac{2}{a}\left(1+f+\frac{\omega^2a^2b}{GM}-2f\sin^2\phi\right)\right]\varepsilon_{H^O}(\Omega). \qquad (20)\end{aligned}$$

Therefore, the error $\varepsilon_g[r_t(\Omega)]$ of the observed gravity is practically equivalent to the error of the free-air gravity anomaly $\varepsilon_{\Delta g^{FA}}[r_t(\Omega)]$, so that $\varepsilon_g[r_t(\Omega)]\cong\varepsilon_{\Delta g^{FA}}[r_t(\Omega)]$.

In Eq (20), a, b are the semi-axes and f is the first numerical flattening of the geocentric reference ellipsoid, ω is the mean angular velocity of the Earth's spin, and GM stands for the geocentric gravitational constant.

Performing the partial derivatives of the simple Bouguer gravity anomaly $\Delta g^{SB}[r_t(\Omega)]$ with respect to the orthometric height $H^O(\Omega)$, gravity $g[r_t(\Omega)]$ and topographical density $\rho(\Omega)$, Eq. (18) takes the following form

$$\begin{aligned}\forall\,\Omega\in\Omega_O:\quad \varepsilon_{\Delta g^{SB}}[r_t(\Omega)] &= \varepsilon_g[r_t(\Omega)]-2\,\pi\,G\,\rho_o\varepsilon_{H^O}(\Omega)\\ &-2\,\pi\,G\,H^O(\Omega)\,\delta\rho(\Omega). \qquad (21)\end{aligned}$$

Substituting Eq. (21) to Eq. (17), the total differential holds

$$\forall\,\Omega\in\Omega_O:$$

$$\begin{aligned}\varepsilon_\chi[r_t(\Omega)] &= \left[\frac{\partial\chi[r_t(\Omega)]}{\partial H^O(\Omega)}+\frac{\partial\chi[r_t(\Omega)]}{\partial\,\Delta g^{SB}[r_t(\Omega)]}\frac{\partial\,\Delta g^{SB}[r_t(\Omega)]}{\partial H^O(\Omega)}\right]\\ &\times\varepsilon_{H^O}(\Omega)+\frac{\partial\chi[r_t(\Omega)]}{\partial\,\Delta g^{SB}[r_t(\Omega)]}\frac{\partial\,\Delta g^{SB}[r_t(\Omega)]}{\partial\,\rho(\Omega)}\delta\rho(\Omega)\\ &+\frac{\partial\chi[r_t(\Omega)]}{\partial\,\Delta g^{FA}[r_t(\Omega)]}\varepsilon_g[r_t(\Omega)]\\ &= \frac{2}{R}\left[\,\Delta g^{FA}[r_t(\Omega)]-4\pi\,G\,\rho_oH^O(\Omega)\,\right]\varepsilon_{H^O}(\Omega)\\ &-4\pi\,G\frac{[H^O(\Omega)]^2}{R}\delta\rho(\Omega)+\frac{2}{R}H^O(\Omega)\,\varepsilon_g[r_t(\Omega)]. \qquad (22)\end{aligned}$$

4. NUMERICAL INVESTIGATION

The geoid-quasigeoid correction $\chi[r_t(\Omega)]$ to the fundamental formula of physical geodesy is a linear function of the free-air gravity anomaly $\Delta g^{FA}[r_t(\Omega)]$, orthometric height $H^O(\Omega)$, and topographical density $\rho(\Omega)$ instead of the mean topographical density ρ_o only. The relation between the correction $\chi[r_t(\Omega)]$ and the free-air gravity anomaly is shown in Fig. 1.

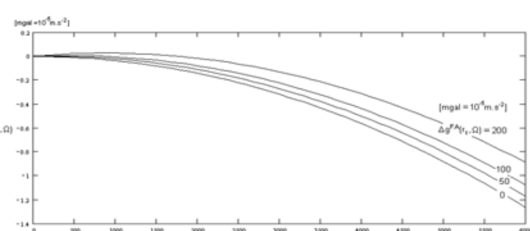

Figure. 1- Relation between the geoid-quasigeoid correction $\chi\left[r_t(\Omega)\right]$ to the fundamental formula of physical geodesy and the free-air gravity anomalies $\Delta g^{\mathrm{FA}}\left[r_t(\Omega)\right]$.

The error $\varepsilon_{H^{\mathrm{o}}}(\Omega) = 100$ [m] of the orthometric height causes the inaccuracy – 42 [μGal] of the geoid-quasigeoid correction $\chi\left[r_t(\Omega)\right]$ to the fundamental formula of physical geodesy for the height 6000 [m] (Fig. 2). The real topographical density $\rho(\Omega)$ varies from 1.0 (water) to 2.98 $\left[\mathrm{g.cm}^{-3}\right]$ (gabbro). When the real density of the topographical masses is considered disregarding existing water bodies, the variation of density is approximately within the interval $\delta\rho(\Omega) \in (-0.3, +0.3)$ $\left[\mathrm{g.cm}^{-3}\right]$ around the mean value ρ_{o}. The variation of the topographical density $\delta\rho(\Omega)$ can cause hundred of microgals error of the geoid-quasigeoid correction to the boundary value problem (Fig. 4). On the other hand, the inaccuracy of this correction due to the error $\varepsilon_{\Delta g^{\mathrm{FA}}}\left[r_t(\Omega)\right]$ of the free-air gravity anomalies is only a few microgals (Fig. 3).

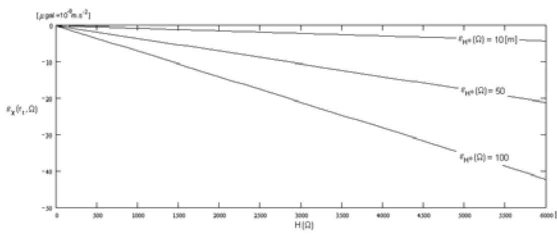

Figure. 2- Relation between the error of the geoid-quasigeoid correction $\varepsilon_{\chi}\left[r_t(\Omega)\right]$ to the fundamental formula of physical geodesy and the error $\varepsilon_{H^{\mathrm{o}}}(\Omega)$ of the orthometric height.

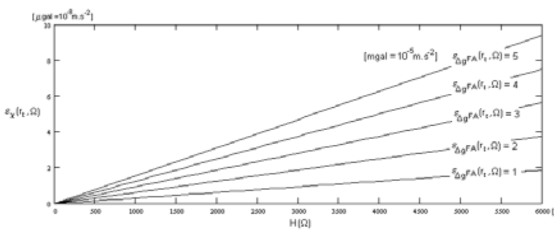

Figure. 3- Relation between the error of the geoid-quasigeoid correction $\varepsilon_{\chi}\left[r_t(\Omega)\right]$ to the fundamental formula of physical geodesy and the error $\varepsilon_{\Delta g^{\mathrm{FA}}}\left[r_t(\Omega)\right]$ of the free-air gravity anomaly.

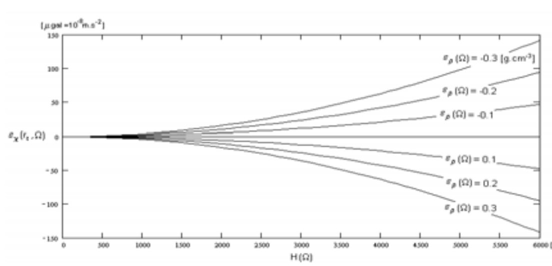

Figure. 4- Error $\varepsilon_{\chi}\left[r_t(\Omega)\right]$ of the geoid-quasigeoid correction to the fundamental formula of physical geodesy due to the variation of the topographical density $\delta\rho(\Omega) \equiv \varepsilon_{\rho}(\Omega)$.

The geoid-quasigeoid correction $\chi\left[r_t(\Omega)\right]$ to the fundamental formula of physical geodesy has been computed at the territory of Canada (Fig. 5). At this territory it varies from – 0.425 to + 0.019 [mGal], with the mean value equal to – 0.013 [mGal]. The magnitude of the correlation between this correction and the variation of the lateral topographical density $\delta\rho(\Omega)$ is between – 0.036 and + 0.032 [mGal], see Fig. 6.

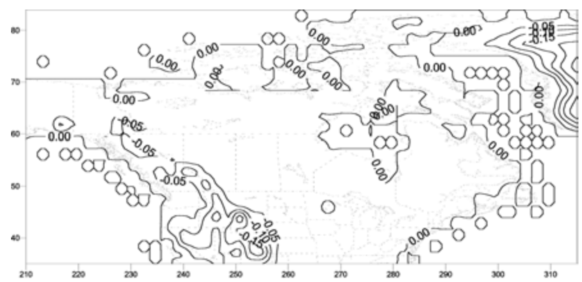

Figure. 5- The geoid-quasigeoid correction $\chi\left[r_t(\Omega)\right]$ to the fundamental formula of physical geodesy at the territory of Canada [mGal].

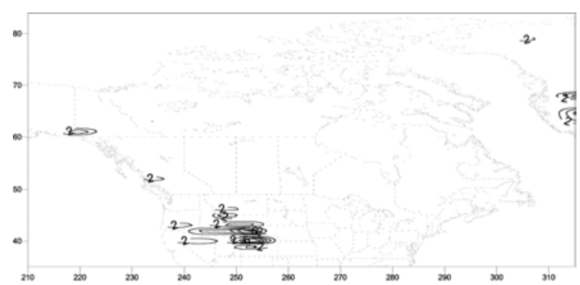

Figure. 6- Variation of the geoid-quasigeoid correction $\chi\left[r_t(\Omega)\right]$ to the fundamental formula of physical geodesy with lateral topographical density $\delta\rho(\Omega)$ [$\mu\mathrm{Gal} = 10^{-8}\,\mathrm{m.s}^{-2}$].

5. CONCLUSION

To increase the accuracy of the geoid-quasigeoid correction, the laterally varying topographical density can be used for the computation of the simple Bouguer gravity anomaly. According to the error propagation in Chapter 3, the change of the topographical density causes the largest error of the geoid-quasigeoid correction to the fundamental formula of physical geodesy. From the numerical result over the territory of Canada follows that the variation of this correction due to the anomalous topographical density is up to ± 40 microgals.

6. REFERENCES:

HEISKANEN, W.A., MORITZ, H., 1967. **Physical geodesy**. W.H. Freeman and Co., San Francisco.

HELMERT, F.R., 1890. Die Schwerkraft im Hochgebirge, insbesondere in den Tyroler Alpen. Veröff. Königl. Preuss. Geod. Inst., Vol. 1.

MARTINEC, Z., 1993. Effect of lateral density variations of topographical masses in view of improving geoid model accuracy over Canada. Final report of contract DSS: 23244-2-4356, Geodetic Survey of Canada, Ottawa.

MOLODENSKY, M.S., 1945. **Fundamental problems of Geodetic Gravimetry**. (in Russian), TRUDY Ts NIIGAIK 42, Geodezizdat, Moscow.

MOLODENSKY, M.S., YEREMEEV, V.F., YURKINA, M.I, 1960. **Methods for Study of the External Gravitational Field and Figure of the Earth**. TRUDY Ts NIIGAiK, 131, Geodezizdat, Moscow. English translat.: Israel Program for Scientific Translation, Jerusalem 1962.

PIZZETTI, P., 1894. Sulla espressione della gravità alla superficie del geoide, supposto ellissoidico. Atti R. Accad. Lincei, ser. V, V.3.

PIZZETTI, P., 1911. Sopra il calcolo teorico delle deviazioni del geoide dall` ellissoide. Atti R. Accad. Sci. Torino, V. 46.

SOMIGLIANA, C., 1929. Teoria Generale del Campo Gravitazionale dell'Ellisoide di Rotazione. Memoire della Societa Astronomica Italiana, IV. Milano.

VANÍČEK, P., HUANG, J., NOVÁK, P., PAGIATAKIS, S.D., VÉRONNEAU, M., MARTINEC, Z., FEATHERSTONE, W.E., 1999. Determination of the boundary values for the Stokes-Helmert problem, **Journal of Geodesy**, Vol. 73, Springer, pp. 180-192.