Compilation of a map of vertical crustal movements in Eastern Canada using polynomial spline

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Overview

In order to obtain representations of the recent Vertical Crustal Movements (VCM) in Eastern Canada, algebraical polynomial surfaces of different orders were computed based on sea level data records from 17 reliable tide gauges and 2972 filtered relevelled segments of the first-order Canadian levelling network in the region. The resulting velocity surfaces indicated only the crude features of VCM mostly because of the sparsity of data coverage. In an attempt to obtain a more physically meaningful VCM map, without increasing the degree of polynomials, we decided to divide Eastern Canada into two zones: Maritimes zone and Southern St. Lawrence River zone. The border of these two zones is dictated by the actual data distribution and the preliminary knowledge of the geodynamics of the area (See figure 1). The vertical movement was then represented by a different polynomial surface in each zone. The polynomials were joined together at the interval knots along the border in such a way that a certain degree of smoothness (differentiability) of the resulting function was guaranteed. In this paper, we use polynomial spline function to produce such a continuous surface.

In general, if we divide the area of study into *m* zones and the degree of the polynomials is *n*, the resulting function is a polynomial spline function of degree *n* with *m* zones. A given spline polynomial in the *m*-th zone looks as follows:

$$V_m(x, y) = \sum_{i,j=0}^{n} c_{ij,m}(x - x_m)^i (y - y_m)^j \tag{1}$$

where V_m is the algebraic least squares velocity surface for zone *m*, fitted to the desired data located at (*x, y*) in an arbitrary selected local horizontal coordinate system and (x_m, y_m) is the knot located in the predefined border between two zones.

In this study, the polynomials have to satisfy the following conditions:

$$\text{i) } V_m(x_m, y_m) = V \tag{2.a}$$

$$\text{ii) } V_m(x_{m+1}, y_{m+1}) = V_{m+1}(x_{m+1}, y_{m+1}) \tag{2.b}$$

$$\text{iii) } V'_m(x_{m+1}, y_{m+1}) = V'_{m+1}(x_{m+1}, y_{m+1}) \tag{2.c}$$

$$\text{iv) } V''_m(x_{m+1}, y_{m+1}) = V''_{m+1}(x_{m+1}, y_{m+1}) \tag{2.d}$$

Conditions (2.a) make sure that the spline fits to the knot points. The second condition (2.b) ensures that the spline is continuous everywhere in the region. Conditions (2.c) and (2.d) ensure that the polynomial spline is continuous in slope and curvature respectively throughout the region spanned by the data points. The appropriate degree of the velocity surface was determined by testing the predicted error (a posteriori standard deviation) and the capability of the surface to portray the main features.

Results

Several tests were made to determine the appropriate degree of the velocity surface to be computed. All polynomial degrees yielded an a-posteriori variance factor equal varying from 8 to 8.5. The value n=4 was finally selected as the highest degree compatible with data distribution.

The map of vertical crustal movements in Eastern Canada produced by spline polynomials is shown in Figure 1.

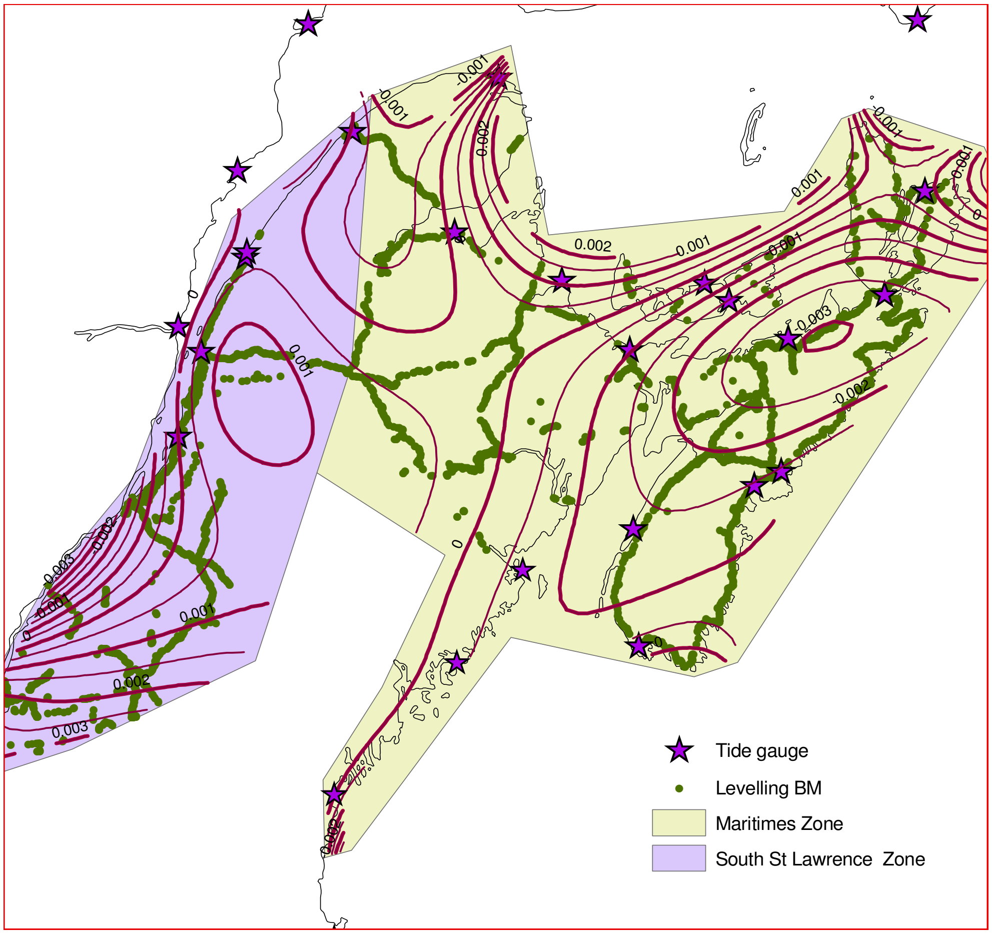

Figure 1. Map of vertical crustal movements in Eastern Canada compiled using cubic polynomial spline. Contours are in m/year. Splines used for joining the VCM surfaces.

The solution is evidently much generalized. The map depicts clearly the zero line of the postglacial rebound. The zero line follows the St. Lawrence River (Figure 1). Present-day radial displacement predictions due to postglacial rebound over North America computed using VM2 Earth model and ICE-4G adopted ice history show a zero line very similar to ours along the St. Lawrence River. (See Peltier (2004) for ICE-4G model predictions). The general Northwest Southeast trend of vertical crustal movements is consistent with the predictions of Glacial Isostatic Adjustment models**.**

With respect to the individual features on the map, the subsidence in the Maritimes, predominantly in Nova Scotia and eastern New Brunswick, is due to postglacial rebound. This area lies immediately outward the region that was covered by the Laurentide Ice Sheet at the last glacial maximum (See Peltier (1994) for maps of surface ice cover from last glacial maximum to the present). As the Laurentian ice started to decay, this area began to collapse. The map of VCM in this area reflects this phenomenon and is also compatible with the recent map of gravity changes, as presented by Pagiatakis et al. (2003).

The pattern shown in the north eastern margin of the former Laurentide ice sheet (the border of which has been postulated to have been parallel to St Lawrence River) is complicated due to the probable discontinuities of the crust in this zone. The map seems to reflect the concentration of seismicity in the Lower St Lawrence Zone, which opens new doors into the study of geodynamics of this complex area. (See Lamontage (2003) for the definition of Lower St. Lawrence Seismic Zone).

The earlier reported uplift of the northern New Brunswick and the subsidence of the South St. Lawrence River (Carrera et al., 1990) are here more sharply defined.

Conclusions

This study shows that polynomial spline surface can represent the available data in a unified map. The local pattern of the map gives more details of the Southern St Lawrence River region as compared to our previous maps. However, the computed a-posteriori variance factor value of 8.2 indicates the probability of the existence of some shorter wavelength features that could not be modelled by a surface of such a low degree. Increasing the number of intervals (zones) in the area of computation might be a solution for representing shorter wavelength features of VCM which would be the next step in our study. Compilation of a unified map which satisfies all the observations and is consistent with most of the geological evidences requires further investigations.

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