ANALYSIS OF DEFORMATION SURVEYS - A GENERALIZED METHOD

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PREFACE

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ANALYSIS OF DEFORMATION SURVEYS – A GENERALIZED METHOD

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PREFACE

This report is an unaltered printing of the author's Ph.D. thesis of the same title, which was submitted to this department in April, 1983.

The thesis advisor was Professor Adam Chrzanowski. The assistance rendered by others is given in detail in the acknowledgements.

Any comments communicated to the author will be greatly appreciated.

ABSTRACT

A generalized approach to deformation analysis has been developed and successfully applied to five examples of monitoring networks within the activity of the "ad hoc" committee on the analysis of deformation surveys of the International Federation of Surveyors (FIG).

The approach is applicable to any type of geometrical analysis, both in space and in time domain, including detection of unstable points in reference networks, and determination of strain components and relative rigid body motion within relative networks. It allows utilization of not only geodetic observations, but also physical-mechanical measurements. Functional relations between deformation parameters and various types of observables have been developed. The approach is capable of handling any datum defects and configuration defects in monitoring networks. The problem of datum defects has been approached through the projection theory in the parameter space.

The generalized approach consists of three basic processes: preliminary identification of deformation models, estimation of the deformation parameters and diagnostic checking of the models. The MINQUE principle has been adopted for the assessment of multi-epoch observations prior to the final adjustment of monitoring networks. A method of iterative weighted projection in the parameter space has been created for the identification of the deformation models in space domain. Formulation and computation strategies for the estimation of the deformation parameters are provided in great detail. The statistic for testing linear hypotheses in the General Gauss-Markoff Model has been formulated using the theory of vector spaces and, from this statistic, all the hypothesis tests used in the different phases of deformation analysis were derived. Compared with other methods, the generalized approach permits a systematic step-by-step analysis of deformations.

During the development of the generalized approach some problems encountered by other authors have been rectified. Examples of the problems and their solution are also given in the thesis.

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Without omitting distant supports from my colleagues and my wife in Wuhan College of Geodesy, Photogrammetry and Cartography, they share my duties both in the public and in the family, for which I will forever feel obliged.

After day and night work in the past two and a half years, I have benefitted much from my stay in this well-known department of surveying engineering. All the above will be deeply engraved on my memory. Without these supports and assistances, this thesis would have not existed. Therefore, if I may take the liberty, I would like to dedicate this tiny work to the friendship between the scientists in the Department of Surveying at the University of New Brunswick and in the Wuhan College of Geodesy, Photogrammetry and Cartography.

CHAPTER 1

INTRODUCTION

1.1 Background

Deformation measurements are of major importance in the broad spectrum of activities covered by surveying engineering. Man-made structures must be supervised during their lifetime; man-induced ground movements, for example, ground subsidence due to mining exploitation, withdrawal of oil or underground water, or construction of large reservoirs, have to be controlled; accumulation of tectonic stress near the active plate boundaries has to be monitored. With technological advancements in the construction of sensitive engineering projects, with increasing exploitation of minerals beneath populated areas, and with growing interest in the studies of crustal movements, requirements both in accuracy and in the frequency of resurveys for deformation monitoring are increasing.

Essentially, there are both practical and scientific reasons for the study of deformations. Practical reasons include checking the stability of a structure, assessment of the degree of geological hazards, and detection of the precursors of earthquakes or failure signals of structures. The anticipated cumulative monetary losses from earthquakes in California alone are estimated to exceed $2 billion by the year 2000 (The Committee on Geodesy and the Committee on Seismology of the United States, 1981). At the same time, the same cost is estimated for damages due to the ground subsidence in coal mining areas in the U.S. (Chrzanowski and Faig, 1982). In terms of human suffering, more than 3000 lives were lost in the accident of the Vaiont dam in Italy in 1963 (Whitten, 1982). Scientific reasons include a need for a better understanding of the deformation mechanism, to test new theories which are applied to the design of structures, and to establish prediction methods. It can be said that any object, natural or manmade, undergoes changes in space and time. Through the study of deformation measurements our knowledge about behavior of deformable bodies will be greatly improved.

Surveying engineers and scientists have played an important role in the field of deformation measurements. When people, in the 1940s and 1950s, reasoned about the control of structures (e.g., dams), they emphasized geodetic triangulation as the only tool of research (Giussani, 1981). Even today, geodetic methods are still widely used to obtain the global status of a deformable body, although many specialized instruments have been developed for collecting specific information. A few years ago a report of the U.S. Academy of Sciences contained the statement: "Virtually, everything we know about the nature of strain build-up that leads to earthquakes in the western United States comes from geodetic studies that began in the late 1800s" (Whitten, 1982). That statement was made by a non-geodesist and, thus, those of us working in the field of deformation measurements find satisfaction in quoting it.

Depending on their extent, deformation measurements may be categorized as being of a local, regional, continental or global scale (Whitten, 1982). Investigation of deformations which occur in man-made structures or in localities of coal mining, petroleum production, extensive water pumping for industry, and so forth, is of a local nature. Study of the deformation of the earth-crust near the plate boundaries is of a regional nature, and is usually carried out using special networks, for example, a fault-crossing network to monitor the creep between two tectonic blocks along the fault. Information about crustal movements on a continental scale can be obtained from the resurveys of national or continental geodetic networks. Geodetic space techniques, for example, very long baseline radio interferometry and satellite laser ranging, provide deformation data of global extent, such as polar motion, variation of the earth's rotation, and relative motion between tectonic plates.

The study of deformation, from the point of view of time variation, can be classified into three groups (Welsch, 1981b): static model, in which examination of the existence or non-existence of deformation in the local domain is the only aim; kinematic model, in which not only the spatial characteristics of deformation but also its temporal attributes are of major concern; dynamic model, in which transformation of observed data, a time series, to the frequency domain is the usual practice.

Compared with other types of surveys, deformation measurements have the following characteristics.

(1) Higher accuracy requirement. For example, according to Savage (1978), long-term deformation rates in seismic zones (such as the San Andreas Fault) are less than 3 parts in 10^7 per year. This might be considered as a threshold level for a long-term horizontal crustal deformation rate. In engineering projects, an accuracy of $\pm$ 1 mm or higher might be a typical requirement (see Miechelev et al. (1977)).

(2) Repeatability of observations. The periods of resurveys range from seconds to years, depending on the rate of deformation.

(3) Integration of different types of observations. Here not only geodetic methods should be considered but also physical-mechanical instrumentation, e.g., pendula, tiltmeters, strain meters, mechanical and laser alignment, hydrostatic levels and others in order to get more complete information.

(4) Sophisticated analysis of the acquired data in order to avoid the misinterpretation of measuring errors as deformation and local phenomena as a global status.

(5) Greater requirement of interdisciplinary knowledge for appropriate physical interpretation. Geodesists have a good knowledge of data acquisition and other specialists, e.g., geophysicists, and civil engineers, are well acquainted with the behaviour of the deformable body. Efficient cooperation between them is indispensable in order to successfully interpret the measured results. Thus, a mutual understanding (use of the same technical language) is a major requirement.

A considerable effort has been made in the last few years in the development of new methodologies and new instrumentation to satisfy the above requirements. More attention is being paid to the analysis of deformation surveys than ever before. In 1978, Federation Internationale des Geometres (FIG) Commission 6 created an "ad hoc" committee on the analysis of deformation measurements, which involved many well-known scientists under the chairmanship of Dr. Chrzanowski, in order to prepare a proposal for guidelines and specifications for all computational procedures connected with deformation measurements. At the 3rd FIG Symposium on Deformation Measurements held in Budapest, Hungary, in August, 1982, the committee was expanded from the original five research centres (Delft, Fredericton, Hannover, Karlsruhe and Munich) to sixteen, involving 40 scientists. Research projects have been set up for the (Chrzanowski and Secord, 1983):

(1) optimization and design of monitoring networks with geodetic and non-geodetic observables;

(2) evaluation of the observation data (including correlation of observations), detection of outliers, and systematic errors;

(3) geometrical analysis of deformations;

(4) physical interpretation of deformations.

The Department of Surveying Engineering at the University of New Brunswick, referred to as the "Fredericton group", is a member of the FIG "ad hoc" committee. For a number of years, the Department has been involved in research projects related to the development of new deformation surveying techniques and new methods for the analysis of deformation surveys. One of the projects included the establishment of test networks in seismically active areas in Peru (Chrzanowski et al., 1978; Dennler, 1980) to monitor tectonic faults by means of micro-geodetic observations. One network, known as the Huaytapallana network, was established in 1975 and has been remeasured four times.

The second network, known as the "Pitec" network, was established in 1978 and has been remeasured twice. Results of measurements of the Huaytapallana network have been used by the FIG "ad hoc" committee as one of their examples for a comparison of different approaches to the analysis of deformation surveys (Chrzanowski, 1981b).

Two other examples used in the comparison studies of the FIG "ad hoc" committee were: a small geodetic network (reference network) for the observation of deformations of a power dam (Lohmühle dam) in Europe; and a simulated network, prepared at the Geodetic Department of the Delft University of Technology crossing a simulated tectonic fault. The results of the comparison of the analysis of the above two networks were discussed in reports by Heck (1982) and by Chrzanowski and Secord (1983).

In addition, five sets of data have been prepared for further comparison. They are: two sets of simulations of the 2-D relative network; the Hollister network, a 2-D trilateration for monitoring tectonic movements across the San Andreas fault in California; the Cossonay network in Switzerland, a 3-D triangulateration with spirit levelling for monitoring a land slide area; and the Adamow mine network in Poland, a 3-D monitoring of an open pit mine.

The author has actively been involved in the work of the "Fredericton group" within the FIG committee since 1980 and has participated in the evaluation of the results of the above three comparison examples. Recently, three sets of new data have been analysed by the author. This thesis has evolved from the author's contributions to the research work of the "Fredericton group".

1.2 Identification of Problems and Scope of the Thesis

Generally, in deformation measurements by geodetic methods, whether they are performed for monitoring engineering structures or ground subsidence in mining areas or tectonic movements, two basic types of geodetic networks are distinguished (Chrzanowski, 1981b):

(1) absolute networks in which some of the points are, or are assumed to be, outside the deformable body (object) thus serving as reference points (reference network) for the determination of absolute displacements of the object points;

(2) relative networks in which all the surveyed points are assumed to be located on the deformable body.

In the first case, the main problem of deformation analysis is to confirm the stability of the reference points and to identify the possible single point displacements caused, for instance, by local surface forces or wrong monumentation of the survey markers. Numerous approaches have been suggested by different authors to determine the stability of the reference points, for instance, methods developed by Pelzer (1974), Heck et al. (1977), Lazzarini (1977), Polak (1978), van Mierlo (1978), Niemeier (1981), just to mention a few. A comparison of some of the methods has been a subject of studies of the aforementioned special committee (Chrzanowski, 1981b) of Commission 6 of FIG. Once the stable reference points are identified, the determination of the geometrical state of deformation of the deformable body is rather simple.

In the case of relative networks, deformation analysis is more complicated because, in addition to the possible single point displacements like in the reference network, all the points undergo relative movements caused by strains in the material of the body and by relative rigid translations and rotations of parts of the body if discontinuities in the materials, for instance, tectonic faults, are present. The main problem in this case is to identify the deformation model, i.e., to distinguish, on the basis of repeated geodetic observations, between the deformations caused by the extension and shearing strains, by the relative rigid body displacements, and by the single point displacements.

For example, the geodetic monitoring of tectonic movements is usually done with relative geodetic networks unless extraterrestrial observations are included in the network. Thus the problem of identification of the deformation model is of primary importance in the analysis.

As far as strain analysis is concerned, the computational procedures are well known and have been applied in mechanical and structural engineering as well as in the analysis of tectonic movements for many years. A brief review of some basic works on the subject of the strain analysis of geodetic observations is given by Vaníček and Krakiwsky (1982). Recent papers by Margrave and Nyland (1980), Snay and Cline (1980), Savage et al. (1981), Prescott (1981), and Chrzanowski and Chen (1982) serve as a good sample of different approaches being used by different authors in the strain analysis of tectonic movements. The approaches can be classified into two basic types (Chrzanowski and Chen, 1982):

(1) raw-observation approach;

(2) displacement approach.

The first approach is based on the calculation of the strain components directly from differences in the repeated observations. In the second approach the strain components are calculated from differences in adjusted coordinates (displacements) of the geodetic points.

In both approaches, if the number of repeated observations or the number of derived displacements at discrete points is larger than the number of unknown deformation parameters, then the least-squares fitting of a deformation model is performed to yield the parameters. The displacement approach is favoured by many authors, for instance by Bibby (1975), Brunner (1979) and Chrzanowski and Chen (1982). Its main advantage is the possibility of using all observables in the strain analysis, even if they differ from one epoch of observations to another, as long as they can be reduced to the same geodetic datum and can be used in the calculation of displacements. On the other hand, the raw-observation approach requires that the same observables and the same geometry of the network in each epoch be maintained and utilized. Some other advantages of the displacement approach are also mentioned in Chapter 4. However, the displacement approach may be inconvenient to use if the geodetic network has configuration defects (lack of geometrical ties between the observables).

In addition, usually more than one deformation model can be fitted to the observation data which leads to the question of which of the models is the "best". Other difficulties and ambiguities in the deformation analysis develop when different minimum constraints must be used in the least-squares adjustments of individual epochs due to changes in the network geometry or when the geodetic observations must be combined with other types of observables such as tilt, strain, and alignment measurements.

In order to overcome the above problems, a generalized approach to the analysis of deformation surveys as part of a so-called "Fredericton approach" (Chrzanowski, 1981b) has been developed by the author and successfully applied to evaluate the aforementioned comparison examples. The aims of this thesis are the:

(1) development of the theoretical foundations of the approach;

(2) full evaluation of the generalized approach;

(3) clarification of some practical problems which cause ambiguities;

(4) discussion of other aspects concerning deformation analysis which, in the author's opinion, should be included in the entirety of the "Fredericton approach".

Deformation measurements encompass a very broad spectrum ranging from the study of the deformation of the earth to the monitoring of engineering structures; from the investigation of dynamic phenomena to checking the stability of the points in reference networks. As suggested in the title of the thesis, only the analysis of deformation measurements is discussed. Furthermore, the focus is set on the analysis of small deformations, where sophisticated analysis is highly required. Discussions on deformations of continental and global extent, as well as deformations of a dynamic nature are not included in this thesis.

1.3 Organization of the Contents and Summary of the Contributions

As a guide to the reader, a logical train of thought in the organization of the contents is presented in this section. At the same time, the contributions in each chapter which the author has deemed significant, are listed.

A deformed body in a geometric sense can be characterized by some deformation parameters. Determination of the parameters is one of the main tasks of deformation measurements. Usually the deformation parameters must be derived from some other physical quantities (e.g., changes in distances, angles) observed in the deformation surveys. A bridge between deformation parameters on one side and observed quantities on the other side is made through a deformation model. In Chapter 2, deformation and its monitoring, basic deformation parameters are first defined, followed by a comprehensive summary of monitoring techniques and methodologies coupled with their achievable accuracies, from traditional surveying instrumentation to special electro-mechanical devices; from geodetic means to photogrammetric methods; from conventional geodetic surveys to space techniques. Finally the general functional relations between the deformation parameters and various types of observables are developed.

If the deformation model were exactly known; the observations did not contain outliers and systematic errors; and the accuracy of the observations and their possible correlations within each epoch or between epochs were given, then computation of the deformation parameters would have been simply a matter of applying the principle of least squares to the above established model. Unfortunately, this is not reality. Hence, chapters 3, 4 and 5 are indispensable, as they not only lay down a firm foundation for Chapter 6, but also contribute to the knowledge in three corresponding aspects:

(1) evaluation of the observation data;

(2) solution to the datum defect problem;

(3) statistical tests in data processing and deformation analysis.

Chapter 3 is devoted to the assessment of observation data. In deformation analysis, an important step is firstly the assessment of the observations, on which the adjustment is based. Information regarding the behaviour of the instruments used and the influence of the environmental conditions on the observations can be obtained. Coupled with the adjustment, statistical inference pertaining to the significance of the computed deformation parameters can be drawn. Chapter 3 begins with a survey of the methods of estimation of variance and covariance components. Then the Minimum Norm Quadratic Unbiased Estimation (MINQUE) principle and its properties are introduced, which the author suggests can be used as a general tool in the assessment of observation data. Some developments are made in this chapter, including the creation of a general error structure model for the assessment of deformation surveys, and extension of the computation procedures to the condition adjustment. Finally, some results from the assessment of the observations in the aforementioned "Pitec" network are presented.

Chapter 4 is devoted to the datum defect problem, since geodetic deformation monitoring networks are mostly free networks. The study of the datum defect problem provides a better understanding of the attributes of the displacements of the points and their covariance matrix obtained from the adjustments referred to a certain datum, which avoids committing the mistakes in deformation analysis; it also provides a good picture of the displacement field by selecting an appropriate datum, making it easier to identify the deformation model in space domain or the suspected unstable points of a reference network. The main development lies in the solution to the datum defect problem using projection theory in parameter space and in the linkage of this method with the method of similarity transformation developed by Baarda (1973) and the method of generalized inverse. A thorough discussion of the relation between the datum defect problem and deformation analysis is also made.

Statistical testing is especially important in deformation analysis. Screening of the observations for outliers and systematic errors, diagnostic checking of the deformation models, and examination of the significance of the derived deformation parameters, are to a large extent based on statistical tests. In Chapter 5 the statistic of linear hypothesis testing in the General Gauss-Markoff Model, where the configuration matrix may be deficient in rank and the dispersion matrix is possibly singular, is developed using the theory of vector spaces. The adoption of geometrical language provides better illustration and simplification. As a special case of general hypothesis testing, the statistic for outlier detection is formulated, which is valid for multiple outlier detection as well as for the case in which the observations are not independent. It has been demonstrated that the existing techniques for outlier detection (data snooping by Baarda (1968); τ-test by Pope (1976); t-test by Heck (1981)) are again special cases of this statistic. Under a derived statistic the techniques for diagnostic checking of deformation model are discussed in more detail.

The generalized approach to deformation analysis is developed by the author in Chapter 6, where the basic philosophy behind the approach and its capability are first described. The approach consists of three basic processes: preliminary identification of the deformation models; estimation of the deformation parameters; and diagnostic checking of the models. After the mathematical model for the generalized approach is given, the three processes are fully evaluated in Sections 6.3, 6.4, and 6.5. A method of iterative weighted projection in the parameter space is developed in Section 6.3.2 for the preliminary identification of the deformation model in space domain. To estimate the deformation parameters, formulation and computation strategies are discussed in greater detail in Section 6.4.1. Under the title of "Remarks" (6.4.3) some ambiguities existing in the literature are pointed out; a numerical demonstration and theoretical analysis to remove the ambiguities are also given. Finally, as a summary of Chapter 6, a diagram showing the computation and statistical testing procedures is presented.

As illustrative examples of application of the developed approach as well as within the aforementioned projects under the activities of the FIG "ad hoc" committee, the results of the analysis of the Huaytapallana network, three sets of the simulation network, and a mine monitoring network are contained in Chapter 7.

Chapter 8 provides an overview of the analysis and interpretation of deformation measurements. Three methods -- interpretation by statistical method, deterministic method, and combination of both methods -- are introduced. A flowchart showing the different interpretation methods and their interactions is developed.

The thesis terminates with Chapter 9, in which some conclusions are drawn and the recommendations for further study are made.

CHAPTER 2

DEFORMATION AND ITS MONITORING

Under the action of forces (body forces or surface forces) a deformable body undergoes changes in its shape and position. These changes occur either gradually or suddenly. The determination and interpretation of the changes are the main goal of deformation surveys. In this chapter a brief review of the basic deformation parameters is first given; then monitoring methods, techniques and their achievable accuracies are summarized; finally, the functional relations between the deformation parameters and the observables are developed.

2.1 Basic Deformation Parameters

The deformation of a body is completely determined by the displacements of the particles in the body (Wempner, 1973). Let the position vector of a particle p in a three-dimensional Cartesian coordinate system (x,y,z) before and after deformation be $\underline{r}_p$ and $\underline{r}'_p$ respectively (the notations used in this thesis are given in Appendix

I), then $\underline{r}'_p$ can be, in general, expressed as

$$\underline{r}'_p = \underline{r}'_p(x_p, y_p, z_p; t) \quad . \qquad (2\text{-}1)$$

Equation (2-1) asserts that the terminal position of the particle depends on its initial coordinates and time. So does the displacement vector $\underline{d}_p$ of the particle p, written as

$$\underline{d}_p = \underline{r}'_p - \underline{r}_p = \underline{d}_p(x_p, y_p, z_p; t) \quad . \qquad (2\text{-}2)$$

Although the relation (2-1) is in general very complicated in the theory of elasticity, it can be decomposed into three parts (Sokolnikoff, 1956): translation and rotation of the body as a whole, and pure deformation. Denoting the vector of rigid body translation by $\underline{t}^T = (t_x, t_y, t_z)$, the matrix of the rotation of the rigid body by R and the remaining part, pure deformation, by $\underline{\tilde{d}}_p$, one can write

$$\underline{r}'_p = \underline{t} + R\,\underline{r}_p + \underline{\tilde{d}}_p \qquad (2\text{-}1')$$

or

$$\underline{d}_p = \underline{t} + (R-I)\underline{r}_p + \underline{\tilde{d}}_p \quad . \qquad (2\text{-}2')$$

Rigid body motion (translation and rotation) is of minor concern in the theory of elasticity, but from the engineering point of view, it is of the same importance as pure deformation. The creep of a gravity dam may be more dangerous than its elastic deformation. Monitoring the rate of the translation of tectonic "blocks" along the fault may be of the same interest as monitoring the strain accummulation within each block. Rigid body motion leaves unchanged the length of every vector joining a pair of points within the body, but pure deformation causes the extension of lines and the distortion of the angles between the lines in the body. Generally speaking, the nature and severity of the deformation vary from point to point. In order to describe the deformation in the vicinity of a point, the concept of strain is employed. Since the displacements of the points of a deformable body are very much smaller compared with the dimension of the body in this study, differentiating (2-2) with respect to three coordinate axes results in the infinitesimal non-translational deformation tensor (e.g., Ramsay (1967)):

$$E = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{pmatrix} = \begin{pmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{pmatrix} \qquad (2\text{-}3)$$

where u, v, w are the three components of the displacement vector in the x, y, and z directions, respectively. The deformation tensor E is asymmetric and therefore can be decomposed into a symmetric part and a skew symmetric part by means of the relation $E = \frac{1}{2}(E + E^T) + \frac{1}{2}(E - E^T)$, that is to say,

$$E = (\varepsilon_{ij}) + (\omega_{ij}) \qquad (2\text{-}4)$$

with

$$\varepsilon_{ij} = 1/2\ (e_{ij} + e_{ji}) \qquad (2\text{-}5a)$$

$$\omega_{ij} = 1/2\ (e_{ij} - e_{ji}) \qquad (i,j = x,y,z) \qquad (2\text{-}5b)$$

where (ε_{ij}) is called the strain tensor and (ω_{ij}) represents the rotation (Sokolnikoff, 1956). The diagonal elements ε_{ij} designate elongation in the corresponding direction, called extensional strain, and the off-diagonal elements characterize distortion of the angles between initially corresponding lines, called shear strain. Figure 2.1 depicts the geometrical interpretation of the strain components, where

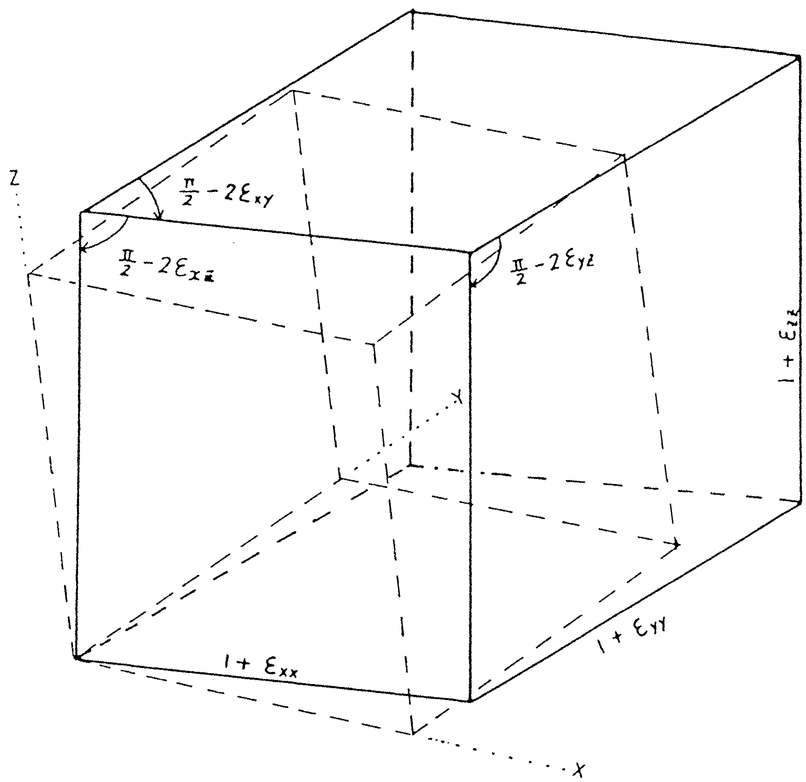


Fig. 2.1

Geometrical Interpretation of the Strain Components the original body plotted with dashed lines is a cube with edges of unit length and where its deformed state plotted with solid lines is a parallelepiped.

Since the displacement vector $\underline{d}$ in (2-2) depends on position and time, the components of the strain tensor and the rotation are, in general, position and time dependent. Taking partial derivatives of them with respect to time, the strain rate $\dot{\varepsilon}_{ij}$ and the rate of the rotation components $\dot{\omega}_{ij}$ can be obtained. The rotation components ω_{ij}, on the other hand, can be partitioned into two parts: the position independent ω^{o}_{ij} and the position dependent ω'_{ij}. The former corresponds to rigid body motion, but the later designates the local twists. When the different parts of the deformable body experience the same deformation independent of their position, the deformation is homogeneous, in which ε_{ij} is invariant over the whole body and corresponds only to rigid body motion.

The strain tensor provides enough information to compute the strain components for any direction. The computation procedure is a simple matter of transformation. Let ξ, η, ζ be the axes of the new coordinate system and have the direction cosines in the original system x,y,z as (a_{11},a_{12},a_{13}), (a_{21},a_{22},a_{23}) and (a_{31},a_{32},a_{33}) respectively, then these direction cosines form an orthogonal matrix $A = (a_{ij})$. Analogous with the law of error propagation in the theory of errors, the strain tensor in the ξ,η,ζ system, denoted by (ε'_{ij}), can be calculated from

$$(\varepsilon'_{ij}) = A\ (\varepsilon_{ij})\ A^T \qquad (2\text{-}6)$$

which is in coincidence with the results given by Wempner (1973), who derived them from the definition of each strain component.

Now, it is possible to show that there are always three mutually perpendicular lines at each point of the undeformed body which remain perpendicular in the deformed one. The directions of these lines are called principal directions. By definition, the shear strains associated with the principal directions are zero. The computation of principal strains and principal directions can be carried out by eigenvalue decomposition, which can be found in any standard textbook on linear algebra. In (2-6) if A is a matrix such that (ε'_{ij}) becomes diagonal, then the diagonal elements of (ε'_{ij}) are principal strains, and the row vector of A represents the principal direction. The principal strains are the extremal values of the extensional strains at the point. In general, one principal strain is the maximum extensional strain at the point and another is the minimum. A special case arises if three principal strains are equal. In this case the extensional strain is the same in all directions and shear strain vanishes for all directions. Every direction is a principal direction. An infinitesimal sphere of the medium would dilate (or contract) in a spherically symmetric manner.

Another special case arises if all the principal strains are equal except one. Then all lines in the plane formed by two principal directions having the same principal strain experience the same extensional strain, and the shear strain vanishes for any direction in this plane. An infinitesimal cylinder with its axis along another principal direction lengthens (or shortens) and dilates (or contracts) radially, but remains cylindrical.

It is also possible to show that the shear strain has a stationary value for the pair of orthogonal lines which bisect the principal lines. Denote the three principal directions by the ξ, η, ζ axes and the corresponding principal strains by

$$\lambda_1,\ \lambda_2,\ \lambda_3 \quad (\lambda_1 \geq \lambda_2 \geq \lambda_3)$$

then the stationary values of shear strain for a pair of orthogonal lines bisecting the ξ, η axes, η, ζ axes and ξ,ζ axes are $\frac{1}{2}(\lambda_1 - \lambda_2)$, $\frac{1}{2}(\lambda_2 -$) and $\frac{1}{2}(\lambda_1 - \lambda_3)$, respectively. The maximum shear strain is $\frac{1}{2}(\lambda_1 - \lambda_3)$ (Wempner, 1973).

In addition, some deformation parameters derived from the strain tensor often appear in the technical literature.

i) Dilatation Δ or the divergence of the displacement vector $\underline{d}$ (Sokolnikoff, 1956):

$$\Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} \tag{2-7}$$

which expresses the change in volume per unit volume. Note that some authors give a different definition of dilatation as $\Delta = \frac{1}{2}(\varepsilon_{xx}+\varepsilon_{yy})$ in the two-dimensional case (e.g., Bibby, 1976).

ii) Two components of shear strain (Frank, 1966; Bibby, 1976):

$$\gamma_1 = (\varepsilon_{xx} - \varepsilon_{yy}) \tag{2-8}$$

$$\gamma_2 = 2\,\varepsilon_{xy} \tag{2-9}$$

where is sometimes called the engineering shear strain component (Mase, 1970). The quantities γ_1 and γ_2 denote the angular change of right angles whose initial sides are oriented northeast and east (the x direction), respectively. γ_1 and γ_1 are also called pure shear and simple shear, respectively (Borre, 1978).

Another way to present shear strain is in the form of the total shear

$$\gamma = \sqrt{(\gamma_1^{\,2} + \gamma_2^{\,2})} \tag{2-10}$$

and its azimuth

$$\tan(2\psi) = \gamma_1/\gamma_2 \tag{2-11}$$

The quantity ψ, called the direction of maximum right-lateral shear strain, specifies the orientation of the right angle that experiences the maximum angular change. The quantity γ specifies the magnitude of the angular change for this right angle (Snay et al., 1982).

Thus, the basic deformation parameters are given. They are: rigid body translation, rigid body rotation (or relative translation and rotation of one "block" with respect to another), strain tensor and rotation components. If a time factor is involved, the derivative of the above quantities with respect to time is used instead.

Acquisition of these parameters is one of the main goals of deformation monitoring. Different methodologies and techniques have been developed for this purpose. A comprehensive summary of them is necessary because it is the opinion of the author that a successful analyst of deformation surveys should have a good knowledge of data acquisition techniques.

2.2 Deformation Monitoring

From the point of view of mechanics, the state of a deformable body may be static, or kinematic, or dynamic. Subsidence of the ground due to the withdrawal of minerals, and strain accummulation near active plate boundaries have a kinematic nature. Vibration of a building due to wind, oscillation of a bridge due to dynamic loading, and deformation of the earth due to tidal force and sea tide loading are some examples of dynamic movements. Selection of monitoring techniques depends heavily on the magnitude and rate of the deformation. The design of a monitoring system is a very important step in deformation measurement. Satisfactory results could not be obtained if the monitoring scheme was poor. Compared with the design of an ordinary geodetic network, the design of the monitoring system is more complicated, involving different types of observables. As far as optimization of a geodetic monitoring network is concerned, it differs from the conventional network in the accuracy criterion, plan of the configuration, and consideration of reliability criterion. Although optimization of geodetic networks has been extensively discussed (Grafarend et al. 1979; Schmitt, 1982), little has dealt specifically with monitoring networks, especially prior to Niemeier and Rohde (1982) who considered levelling networks for the monitoring of vertical crustal movements. A discussion of this problem is given in a separate publication (Chen et al., 1983).

In the following, different monitoring techniques and their achievable accuracies are reviewed and classified.

2.2.1 Conventional geodetic methods

Conventional geodetic methods have been widely used in deformation surveys for a long time. The reasons can be summarized as follows:

(a) they supply the global status of a deformable body;

(b) they contain the scheme of self checking the results and are capable of evaluating the measuring accuracy globally;

(c) they provide versatility and suitability to any environmental and operative situation.

In opposition to these undeniable merits, some drawbacks are: complexity of measurement requires the presence of many operators for several days; it is very difficult and expensive to adopt the geodetic methods for continuous monitoring. Thus, the methods provide measurements of deformation at a number of discrete points and epochs of time.

As mentioned in Chapter 1, geodetic monitoring networks are usually divided into relative networks and reference networks. The former provides the change in relative positions of the points, and the latter serves as a reference frame to which the displacements of object points are referred. A confirmation of the stability of reference points is the only aim of the measurements in the reference network.

Horizontal triangulation monitoring networks have been gradually replaced by trilateration or triangulateration networks because the accuracy of angle measurements has remained from 0.5" to 1" for the first-order observation scheme, but the accuracy of distance measurements has been much improved. Among the short range EDM instruments for the control of engineering structures the Mekometer and Tellurometer MA-100 are the most accurate. The first can be used to measure distances up to 3 km with a standard deviation of ±0.2 mm ±(1 to 2) ppm (0.2 mm is constant error and 1 to 2 ppm is scale error); the second has a measuring range of up to 1.5 km with a standard deviation of ±1.5 mm ±2 ppm (Blachut et al., 1979). For medium ranges of up to 30 km distance measurements, the U.S. Geological Survey successfully uses a laser Geodolite and samples the meterological data using an aircraft along the line of sight to determine the atmospheric refractive index, and a standard deviation of ±3 mm ±0.2 ppm has been reported (Savage and Prescott, 1973). Recent progress in distance measurements is the development of the multiple wavelength distance--measuring equipment, e.g., Terrameter (Hugget, 1982), where an accuracy in the order of 0.1 ppm can be achieved.

First-order geodetic levelling constitutes the basic method of studying vertical movements. Its main merit lies in its high precision. Typical accuracy is about $(0.7 \sim 1.0)$ mm/$\sqrt{L}$ (L - distance of levelling in kilometers), but the field procedure is time consuming, especially in hilly terrain. Accumulation of systematic errors, which may lead to misinterpretation of the reported deformation surveys (see Jackson et al. (1980); Strange (1981)) still requires more investigation. In order to overcome the troublesome aspect of geodetic levelling, a modified method based on a leap-frog trigonometric levelling has been recently developed and tested at UNB. An accuracy 3 mm/km is easy to achieve, and the method can be used for monitoring purposes in mountainous areas (Chrzanowski 1983).

2.2.2 Photogrammetric methods

Both terrestrial and aerial photogrammetry have been extensively used in the determination of deformations of large structures (e.g., Brandenberger and Erez (1972); Faig (1978); Veress and Sun (1978)) and ground subsidence (Faig and Armenakis, 1982). The advantages of photogrammetric monitoring are:

(a) simultaneous determination of the deformation of any points on a deformable body;

(b) provision of complete and instantaneous information in three-dimensional space;

(c) reduction of field work to a minimum;

(d) a spatial model of the deformable body can be recovered at any time.

The accuracy of photogrammetric point determination has been much improved in the past few years, which makes it attractive for high precision deformation measurements. The main reason for this development is the successful refinement of the mathematical model by using self-calibration techniques (e.g., Moniwa (1977)), including checks to guarantee the reliability of the results. The accuracy reaches standard deviations of 2-3 m in the image coordinates (Förstner and Schroth, 1982) or in the order of 10^{-5} of object-camera distance (ASP, 1980).

2.2.3 Specialized monitoring devices

Whereas conventional geodetic instrumentation is usually used only at discrete epochs of time, some other instruments can be installed permanently in the area of interest to provide continually information at one site.

Strainmeters measure the change $\Delta\ell$ in the distance ℓ between two points. Then strain in a particular direction can be calculated as $\varepsilon = \Delta\ell/\ell$. Strain-meters can be classified into mechanical strainmeters and laser interferometers (the Committee on Geodesy and the Committee on Seismology of the United States, 1981). The "Distinvar" developed by CERN and the "Distometer" produced by Kern are of the mechanical type. Their reported accuracy is about 0.05 mm over a range of 100 mm for and 50 m for (Keller, 1978; CERN, 1979). A fundamental progress in building strainmeters comes with the use of the laser as a coherent light source, extending the technique of optical interferometry to encompass a much greater distance. Presently, laser strainmeters or laser interferometers can be successfully operated up to 100 m in the open atmosphere (e.g., HP5525B model) and 1 km or longer in a controlled atmosphere, with an uncertainty of measurement of about 10^{-7} (e.g., Schuler (1981)). In a vacuum tube an accuracy of $4 \cdot 10^{-10}$ is achievable (Chrzanowski, 1981a).

The tilt of a structure or ground can be measured either by using:

(a) hydrostatic levelling; or

(b) electrolytic or electro-mechanical tiltmeters; or

(c) vertical alignment (using either mechanical pendula or optical plumb lines to measure displacements from the vertical).

The achievable accuracy of the tilt measurement varies from one type of instrument to another, for example (Chrzanowski, 1981a), tiltmeters:

(a) electrolytic type, the standard deviation $\sigma \doteq 0.5''$;

(b) electro-mechanical type, $\sigma = 0.001'' \sim 1''$;

hydrostatic levels:

(a) water type, $\sigma = 0.03$ mm/40 m;

(b) mercury type, $\sigma = 0.001''$

Alignment is a special procedure for measuring the relative displacements in a critical direction. Wire alignment was adopted in the 1960s for monitoring horizontal displacements of dams. A recent development is in the use of laser alignment in deformation surveys. Using Chrzanowski et. al., (1972) as the foundation, the author (Chen, 1980; Chen, 1981) has developed a laser alignment system for dam control. In this system diffraction zoneplates were installed in the gallery of the dam as permanent targets, the laser beam was modulated mechanically to increase the signal-noise ratio, and an automatic detector was developed. The results from the field test and from a comparison with other methods over two years indicate that an accuracy of 10^{-6} or higher over a distance of up to 1 km in open air is achievable.

The above electro-mechanical instruments and alignment devices have two advantages:

(1) they do not need any long and complex measurement procedures; and

(2) they may easily be used for monitoring with automatic data recording and with telemetric data acquisition (Chrzanowski et al., 1980; Pelzer, 1980).

2.2.4 Space techniques

An important element in monitoring strategy, especially in tectonic deformation monitoring, is the forthcoming availability of space methods. Very long baseline interferometry (VLBI) and satellite laser-ranging techniques appear to be capable of determining the difference between the positions of any two points with an accuracy of a few centimetres (e.g., Rogers et al. (1978); Smith, et al. (1979); Ma (1981). Signals from the satellites of the Global Positioning System (GPS) could be used to measure the baseline vector components with an accuracy of a few millimetres for baselines of a few kilometres and of a few decimetres for baseline lengths of up to 5000 km (Langley et al., 1982).

A well-known project proposed by Kumar and Mueller (1978) was to utilize orbital stations for a study of deformations in the area of the San Andreas fault. Under this project a laser range-finder would be installed aboard an orbital station, while ground stations (75 stations as a variant) would be equipped only with passive reflectors. With a rangefinder ensuring an accuracy of 5 to 10 cm, observations over one day may determine distances between ground stations with an error of 2 to 3 cm.

A group of Canadian scientists have proposed a NASA crustal dynamics project in Canada (Langley, 1982, personal communication). Under this proposal the deformation of the interior of the North American plate and the regional deformation at the boundary of the North American and Pacific plates would be monitored using a NASA transportable long baseline interferometry (LBI) data system. The LBI stations would be distributed over Canadian territory, forming triangles. The strain accumulation and vertical movement across each triangle would be determined from annual LBI observations. In order to appropriately interpret the results, the possible local movements of the observing sites would be modelled using the conventional geodetic monitoring networks with an accuracy of 1 to 2 cm.

2.3 The Functional Relationship between the Deformation Parameters and Observed Quantities

Our objective is to determine the deformation parameters defined in Section 2.1 from the observations made in at least two different epochs. The observables may be in the form of coordinates, coordinate differences, azimuths, horizontal angles, vertical angles, distances, tilts and strains. The observations of horizontal and vertical alignments, and geodetic levelling or hydrostatic levelling may be categorized as coordinate differences.

The observables are related to the deformation parameters through a deformation model:

$$\ell(t) = \ell(t_o) + \Delta\ell(x,y,z;\ t-t_o;\ \underline{e}) \quad , \qquad (2\text{-}12)$$

where (t) is the observation made at time t; (t_o) is the initial observation at time t_o; x,y,z are local Cartesian coordinates of the points to which the observable $\ell(t)$ is related; $\underline{e}$ is a vector of the deformation parameters, in which the strain tensors and the rotation components are arranged in a vector form, to be included. The deformation model is, in general, very complicated, so that in practice it should be approximated by a simplified function.

Let the displacement field of a deformable body be approximated by a general polynomial:

$$\underline{d}(x,y,z;\ t-t_o) = \sum_{k=1}^{m_t} \sum_{j=0}^{m_z} \sum_{i=0}^{m_y} \sum_{h=0}^{m_x} X^h\ Y^i\ Z^j\ (t-t_o)^k \underline{C}_{hijk} \qquad (2\text{-}13)$$

where $\underline{d}$ is the displacement vector of a point located at (x,y,z) at time t with respect to time t_o; m_x, m_y, m_z, m_t are maximum power in x,y,z and $(t-t_o)$ respectively; $\underline{C}_{hijk}$ is a vector of the unknown coefficients, which have three components corresponding to the displacements in x,y,z direction. Using matrix notation, (2-13) can be rewritten as

$$\underline{d}(x,y,z;\ t-t_o) = B(x,y,x;\ t-t_o)\ \underline{c} \quad , \qquad (2\text{-}14)$$

where B is a 3 by m matrix whose elements are functions of position and time, $\underline{c}$ is an m-vector of unknown coefficients with

$$m = 3(m_x+1)(m_y+1)(m_z+1)m_t \quad .$$

Some simple models can be deduced from (2-13). If the deformation is of linear time variation, then k = 1; if a homogeneous deformation is assumed, then $m_x = m_y = m_z = 1$ and $h+i+j\leq 1$ in one term; if only the horizontal deformation is under investigation, then $m_z = 0$

and the vector $\underline{d}$ has only two components; if one analyses vertical movements, $\underline{d}$ has only one component representing subsidence or uplift of the ground; furthermore if one assumes that the subsidence or uplift is independent of the terrain elevation, then $m_z = 0$, and the model is reduced to that given in (Vaníček et al., 1979).

In reality a deformable body may be noncontinuous, consisting of separate continuous blocks. Thus, the form of the general polynomial (2-13) varies from one case to another. But the expression (2-14) will not lose its generality. From here on, the model (2-14) will be called the deformation model. The deformation model (2-14) is regarded as a medium, from which the deformation parameters can be derived and to which the observation quantities are related.

Let us express the model (2-14) more explicitly as

$$\underline{d} = \begin{pmatrix} u(x,y,z;\ t-t_o) \\ v(x,y,z;\ t-t_o) \\ w(x,y,z;\ t-t_o) \end{pmatrix} = \begin{pmatrix} B_u(x,y,z;\ t-t_o)\ \underline{c}_u \\ B_v(x,y,z;\ t-t_o)\ \underline{c}_v \\ B_w(x,y,z;\ t-t_o)\ \underline{c}_w \end{pmatrix} . \qquad (2\text{-}14')$$

The observable quantities are related to the model (2-14') in the following manner.

(1) Observed coordinates.

From equation (2-2), the position of point p at time t is expressed as

$$\underline{r}_p(t) = \underline{r}_p(t_o) + B(x_p,y_p,z_p;\ t-t_o)\ \underline{c} + \underline{v}(t) \quad , \qquad (2\text{-}15)$$

where $\underline{v}(t)$ is a vector of random errors.

(2) Observed coordinate differences.

<u>Levelling</u> observes the coordinate difference in the z direction. In Section 2.2, hydrostatic levelling is considered as a tiltmeter. This is the case when the distance between the two points is small in comparison with the dimension of the body under study. If it is not, the observation of hydrostatic levelling should be treated as that of a coordinate difference. The same idea is applied to the pendulum observations and the observations of interferometry (as distance observations).

Let levelling be run between points P and Q at time t, then

$$\ell(t_o) = \ell(t_o) + \{B_w(x_Q,y_Q,z_Q;\ t-t_o) - B_w(x_P,y_P,z_P\ t-t_o)\}\ \underline{c}_w + v(t) \quad (2\text{-}16)$$

A <u>pendulum</u> observation gives the coordinate difference between two points, the one is the suspending point Q, while the other is point P, moving in the z direction. Therefore,

$$\ell_x(t) = \ell_x(t_o) + \{B_u(x_Q,y_Q,z_Q;t-t_o) - B_u(x_Q,y_Q,z_P;t-t_o)\}\ \underline{c}_u + v_x(t)$$

$$\ell_y(t) = \ell_y(t_o) + \{B_v(x_Q,y_Q,z_Q;t-t_o) - B_v(x_Q,y_Q,z_P;t-t_o)\}\ \underline{c}_v + v_y(t) \quad (2\text{-}17)$$

where ℓ_x, ℓ_y are the observations in the x and y direction, respectively. If the two axes of the pendulum observation devices are not parallel to the axes of the coordinate system, a transformation should be performed. The same idea is used below when it is necessary.

An <u>alignment</u> observation provides the coordinate difference at point P with respect to two base points Q_1 and Q_2. Assume that the alignment is carried out in the plane $z = z_c$ along line the $y = y_c$; then

$$\ell(t) = \ell(t_o) + \{B_v(x_P,y_c,z_c;t-t_o) - (1-k)B(x_Q,y_c,z_c;\ t-t_o) - - kB_v(x_Q,y_c,z_c;\ t-t_o)\}\underline{c}_v + v(t) \quad , \quad (2\text{-}18)$$

where $k = (x_P - x_{Q_1})/(x_{Q_2} - x_{Q_1})$.

(3) Azimuth and horizontal angle.

Assume that the points P and Q are located in a deformable body. The distance S between the two points, the azimuth α, and the vertical angle β from P to Q are observed; then

$$\begin{pmatrix} x_Q - x_P \\ y_Q - y_P \\ z_Q - z_P \end{pmatrix} = S \begin{pmatrix} \cos\beta & \cos\alpha \\ \cos\beta & \sin\alpha \\ \sin\beta & \end{pmatrix} \tag{2-19}$$

From (2-19), $\tan(\alpha) = (y_Q - y_P)/(x_Q - x_P)$. Differentiating this expression, one obtains

$$\Delta\alpha = \frac{\cos\alpha}{s.\cos\beta}(v_Q - v_p) - \frac{\sin\alpha}{s\cdot\cos\beta}(u_Q - u_p)$$

where u_P, v_P and u_Q, v_Q are the components in x,y of the displacement at points P and Q respectively. Thus,

$$\begin{aligned} \alpha(t) = \alpha(t_o) &+ \frac{\cos\alpha}{S\cos\beta}\{B_v(X_Q, Y_Q, Z_Q;\ t-t\ ) - B_v(X_p, Y_p, Z_p;\ t-t_o)\}\cdot C_v \\ &- \frac{\sin\alpha}{S\cos\beta}\{B_u(X_Q, Y_Q, Z_Q;\ t-t_o) - B_u(X_p, Y_p, Z_p;\ t-t_o)\}\cdot C_u \\ &+ v(t) \end{aligned} \tag{2-20}$$

The observation of a horizontal angle is simply expressed as the difference of the azimuths.

(4) Distance observation.

From (2-19), one can express the change in the distance S as

$$\Delta S = \cos\beta\ \cos\alpha\ (U_Q - U_p) + \cos\beta\ \sin\alpha\ (V_Q - V_P) + \sin\ \ (W_Q - W_P)$$

Therefore

$$\begin{aligned} S(t) = S(t_o) &+ (\text{Cos}\beta\ \text{Cos}\alpha,\ \text{Cos}\beta\sin\alpha,\ \sin\beta)\{B(X_Q, Y_Q, Z_q;\ t-t_o) \\ &- B(X_p, Y_p, Z_p;\ t-t_o)\}\underline{C} + v(t) \end{aligned} \tag{2-21}$$

(5) Strain observation.

Strain observations may be considered as a special case of distance measurements, in which points P and Q are close enough so that the term $\{B(x_Q,y_Q,z_Q;\ t-t_o) - B(x_P,y_P,z_P;\ t-t_o)\}\underline{c}$ in (2-21) can be replaced by its differential. Thus the expression for a strain observation at time t is

$$\ell(t) = \ell(t_o) + (\cos\beta\ \cos\alpha,\ \cos\beta\ \sin\alpha,\ \sin\beta)\ \underline{E}\ (\cos\beta\cos\alpha,\ \cos\beta\ \sin\alpha,\ \sin\beta)^T + v(t) \quad , \qquad (2\text{-}22)$$

where the deformation tensor E can be found from (2-25)

(6) Vertical angle observation.

From (2-19), one obtains

$$\Delta\beta = \left(-\frac{\sin\beta\ \cos\alpha}{s},\ -\frac{\sin\beta\ \sin\alpha}{s},\ \frac{\cos\beta}{s}\right)(u_Q-u_p,\ v_Q-v_p,\ w_Q-w_p)^T$$

Therefore

$$\beta(t) = \beta(t_o) + \left(-\frac{\sin\beta\ \cos\alpha}{s},\ -\frac{\sin\beta\ \sin\alpha}{s},\ \frac{\cos\beta}{s}\right)[B(x_Q,\ y_Q,\ Z_Q;\ t-t_o) - B(x_p,\ y_p,\ z_p;\ t-t_o)]\cdot \underline{c} + v(t) \qquad (2\text{-}23)$$

(7) Tilt observation.

A tilt observation, again, might be considered as a special case of a vertical angle observation when points P and Q are close enough. Therefore, the expression for a tilt observation is

$$\ell(t) = \ell(t_o) + \{\cos\alpha\frac{\partial Bw}{\partial x} + \sin\alpha\frac{\partial Bw}{\partial y}\}\underline{c}_w + v(t) \qquad (2\text{-}24)$$

The developments made above are an extension of Reilly's work (1981; 1982), where the treatment was restricted to homogeneous strain only. In the real world, a nonhomogeneous case with discontinuities is more likely. This is contained in matrix $B(x,y,z;\ t-t_o)$.

After the vector of coefficients $\underline{c}$ of the deformation model (2-14) is estimated from the observation quantities, the deformation parameters can be calculated. The coefficients which are independent of position represent the rigid body translation. With the relations (2-3) and (2-4), the deformation tensor reads

$$E = \left(\frac{\partial B}{\partial x} \cdot \underline{c} \ \vdots\ \frac{\partial B}{\partial y} \cdot \underline{c} \ \vdots\ \frac{\partial B}{\partial z} \cdot \underline{c}\right) \tag{2-25}$$

Consequently, the strain tensor is

$$(\varepsilon_{ij}) = \tfrac{1}{2}(E + E^T) \quad , \tag{2-26a}$$

and the rotational part of the deformation is

$$(\omega_{ij}) = \tfrac{1}{2}(E - E^T) \quad . \tag{2-26b}$$

The position-independent part in (2-26b) corresponds to rigid body rotation. Taking the derivative of ε_{ij} and ω_{ij} with respect to time, one obtains the strain rate $\dot{\varepsilon}_{ij}$ and rotation rate $\dot{\omega}_{ij}$.

Based on the above developments, the problem of deformation analysis seems solved. But this is true only when

(1) the deformation model is known, including identification of the unstable points in a reference network;

(2) the accuracies or the weight relationship of different types of observations are given;

(3) no outliers and systematic errors exist in the observations.

The ideal condition is never fulfilled. In reality geodetic monitoring networks are mostly free networks, suffering from datum defects, therefore the coordinates and displacements of points are unestimable quantities; the network comprises different types of observables whose accuracies are not well defined; the deformation model is not fully understood or can even be completely unknown; and so forth. All this shows the need for the following three chapters, which contribute to the solution of these problems in deformation analysis.

CHAPTER 3

ASSESSMENT OF THE OBSERVATION DATA IN DEFORMATION SURVEYS USING THE MINQUE PRINCIPLE

3.1 Estimation of Variance and Covariance Components

3.1.1 Definition of the problem

Deformation monitoring networks usually involve different types of observables. A typical example is a triangulateration geodetic monitoring network, which comprises measurements of both angle and distance. The measurement of the same observable might be made by different instrumentation. Moreover, each type of observable may be contaminated with errors having several components corresponding to different characteristics. Distance by EDM equipment can serve as a good example--each measured distance contains a constant error and a scale error. Estimation of unknown parameters, e.g., coordinates of points, in a least-squares adjustment requires a knowledge of the variances of the observations or at least their weight relationship, and even their mutual correlations. Frequently, however, these weights are not known adequately, and a hypothetical establishment of them will inevitably lead to systematic deviation in the results. In addition, statistical inference about the significance of the computed deformation parameters (displacement, strain or strain rate, relative movement between blocks, and so forth) demands appropriate assessment of the observations. Moreover, the investigation of the variances and covariances of observations can aid further design by supplying information on the behaviour of the instruments and on the influence of the environmental conditions.

Surveyors, before performing the adjustment of a network, usually acquire the variances of the observations from one of the following sources:

(1) accuracy of the instruments as claimed by the manufacturers;

(2) analysis of the observations prior to the network adjustment, e.g., estimation of angle accuracy from the Ferrero formula, or distance accuracy from the "double observation method";

(3) separate adjustments of the network using individual groups of observations;

(4) trial and error method, in which different combinations of the suspected variances of the observations are entered into the adjustment until the a posteriori variance factor $\hat{\sigma}_0^2$ passes the test on its compatibility with the a priori one (Dennler, 1980).

Obviously, there are some limitations to the above methods. The accuracy of an instrument claimed by the manufacturer is generally an average one and may significantly differ from the actual. Methods (2) and (3) may not always be possible and may not take full advantage of the available observations. For example, when the accuracy of the angle measurements of the aforementioned Pitec network in Peru, described below, was assessed from the Ferrero formula, only 40% of the observations could be used (Dennler, 1980). The last method usually requires many combinations of the suspected values of the error components and suffers by not having a very clear theoretical background. Therefore, it becomes necessary to suggest a more general method.

3.1.2 A general model and summary of the estimation methods

Let us consider a linear model of the parametric adjustment:

$$\underline{\ell} = A\underline{x} + \underline{v} \quad , \tag{3-1}$$

where $\underline{\ell}$ is an n-vector of observations, $\underline{x}$ is a u-vector of unknown parameters, A is the first-order design matrix of $n \times u$, and $\underline{v}$ is an error vector which, in general, can be written as

$$\underline{v} = U_1\underline{\xi}_1 + U_2\underline{\xi}_2 + \dots + U_k\underline{\xi}_k = U\underline{\xi} \quad , \tag{3-2}$$

where $\underline{\xi}_i$ is a c_i-vector, U_i is an $n \times c_i$ matrix, $n = \sum_1^k c_i$,

$U = (U_1 \mid U_2 \mid \dots \mid U_k)$, and $\underline{\xi}^T = (\underline{\xi}_1^T \mid \underline{\xi}_2^T \mid \dots \mid \underline{\xi}_k^T)$.

It is assumed that the elements of $\underline{\xi}_i$ are uncorrelated with common mean and variance σ_i^2, and the elements of $\underline{\xi}_i$ and $\underline{\xi}_j$ are also uncorrelated. Then, the dispersion of $\underline{v}$ is

$$\Sigma = E\{\underline{v}\underline{v}^T\} = U \cdot D\{\underline{\xi}\} \cdot U^T = \sum_1^k \sigma_i^2 V_i \quad , \tag{3-3}$$

with $V_i = U_i U_i^T$. $E\{\cdot\}$ and $D\{\cdot\}$ the expected value and dispersion, respectively.

In the most general case, in which Σ may contain variances and covariances, (3-3) can be rewritten as

$$\Sigma = \sum_{1}^{k} \theta_i T_i \quad , \tag{3-4}$$

where θ_i are the distinct variance and covariance components, and T_i depends on the model. The problem on hand is to estimate the unknown parameters $\underline{x}$, $\underline{\theta}$ from the observation vector $\underline{\ell}$ when the design matrices A and T_i are given.

There is considerable literature on the variance components model in statistics, most of which is applied in the biological and behavioural sciences. Comprehensive reviews are given by Searle (1971b), Kleffe (1977), and Harvill (1977). Essentially, there have been three approaches. One is based on the assumption that the variable is normally distributed, in which case the likelihood function can be written in terms of variance-covariance components and mean values. Then the maximum likelihood estimates can be obtained by setting the partial derivative of the likelihood function with respect to unknown parameters $\underline{x}$, $\underline{\theta}$ equal to zero and solving the equations for $\underline{x}$ and $\underline{\theta}$ simultaneously. The equations involved in this approach are complicated and have to be solved by iterative techniques. Furthermore, very little is known about the properties of the maximum likelihood estimators in this case (Rao, 1971a). Another approach is through the analysis of variances, in which variance-covariance components are obtained by making calculated mean squares of some kind equal to their expectation values and solving linear equations for these components. Different methods along this line can be found in Searle (1971b). As pointed out by Rao (1971a), the theoretical basis of the approach is not clear, and the procedures suggested are ad hoc in nature, and much seems to depend on intuition. The third approach is based on optimization theory. The most popular method of estimating θ in an optimal way is the minimum norm quadratic unbiased estimation (MINQUE) principle proposed by Rao and developed in a series of his papers (1970, 1971a, 1972, 1973). This estimation principle has the advantage of working without additional distributional assumptions. The method provides conventional estimators for the situation with the usual assumptions and offers a procedure of estimation for the less obvious cases. Unfortunately, as in some other methods of estimation, a somewhat troublesome aspect of the MINQUE is that negative estimates of variances may arise. If this happens, one may infer that a negative estimator corresponds to a small positive true value or that the assumed model is not correct (Rao, 1972). But this is a rather arbitrary and symptomatic treatment. Therefore, many authors (e.g., Horn et al. (1975); Brown (1977); Rao et al. (1977)) have made contributions to overcome this drawback by disregarding certain properties of the MINQUE, e.g., the condition for unbiasedness.

In geodetic science, this problem has attracted the attention of geodesists only quite recently, although Helmert had proposed a method in 1924 (Grafarend, 1982). Among the publications, Kubik's approach (1970) is based on the maximum likelihood principle; Ebner (1972), Förstner (1979), Grafarend et al. (1980), and Welsch (1978; 1981a) extended the Helmert principle; Schaffrin (1981) has used the method of the best quadratic unbiased estimation (BQUE).

The Helmert method is equivalent to the MINQUE as far as the estimation of variance components is concerned. But they differ when the covariance components are to be estimated. In general, the Helmert method gives unbiased estimators but not the "best". On the other hand, the BQUE is a special case of the MINQUE, when the variables are assumed to be normally distributed. Therefore, it becomes obvious that the MINQUE principle should be adopted in this study. The possibility of providing negative values for the estimates of θ_i will seldom occur in geodetic applications because of the much smaller number of variance-covariance components to be estimated compared with that of observations. The reason is that the estimated variance-covariance components are random variables, and their variances become small in comparison with their expected values when a relatively large number of observations is available, hence negative estimates would appear very rarely.

3.2 The MINQUE and its Properties

Let us assume that we wish to estimate a linear function of variance-covariance components $\underline{p}^T\theta$, where $\underline{p} = (p_1, p_2, \ldots, p_k)^T$ is known, with the quadratic form $\underline{\ell}^T B \underline{\ell}$. A symmetric matrix B should be determined so that it satisfies the following conditions:

(1) Invariant of translation of the unknown parameters $\underline{x}$, i.e., for any u-vector $\underline{x}_o$ the relation $(\underline{\ell} - A\underline{x}_o)^T B(\underline{\ell} - A\underline{x}_o) = \underline{\ell}^T B \underline{\ell}$ holds, which implies $BA = 0$.

(2) Unbiasedness, i.e., $E\{\underline{\ell}^T B \underline{\ell}\} = \underline{p}^T \underline{\theta}$. Since

$$E\{\underline{\ell}^T B \underline{\ell}\} = Tr\{B\ E\{\underline{\ell}\underline{\ell}^T\}\} = Tr\{BA\underline{xx}^T A^T + B\Sigma\}$$

$$= Tr\{B \sum_1^k \theta_i\ T_i\}$$

$$= \sum_1^k \theta_i\ Tr\{BT_i\}$$

unbiasedness will be satisfied when $Tr\{BT_i\} = p_i$.

(3) Minimum norm. There exist two estimators of $\underline{p}^T\underline{\theta}$: one is the proposed quadratic form $\underline{\ell}^T B \underline{\ell} = \underline{v}^T B \underline{v} = \underline{\xi}^T U^T B U \underline{\xi}$, and the other is a so-called "natural one", that is, when the variable $\underline{\xi}$ is known, the natural estimator of $\underline{p}^T\underline{\theta}$ is $\underline{\xi}^T \Delta \underline{\xi}$, where Δ is diagonal with elements $(p_i/c_i)I_i$. The difference between the above two quadratic forms is $\underline{\xi}^T(U^T BU - \Delta)\underline{\xi}$ and is minimized by minimizing the Euclidean norm $||U^T BU - \Delta||$. It has been proved that minimizing this norm is equivalent to minimizing Tr$\{BTBT\}$, where

$T = \sum_1^k T_i$, (Appendix II.1). A similar derivation of the MINQUE for covariance components was given in Rao (1972).

Summarizing the above results, one can define the MINQUE as follows: a quadratic function $\underline{\ell}^T B \underline{\ell}$ is said to be the MINQUE of $\underline{p}^T\underline{\theta}$ if B is such that Tr$\{BTBT\}$ is minimum subject to $BA = 0$ and Tr$\{BT_i\} = p_i$ $(i = 1,2,\ldots,k)$.

The extreme problem with linear constraints can be solved by the well-known method of Lagrange. The solution reads (Rao, 1970)

$$B = \sum_1^k \lambda_i R T_i R \quad , \tag{3-5}$$

where $R = T^{-1}Q = Q^T T^{-1}$, $Q = (I - P_A)$ and P_A is the projection operator onto space S(A), generated by the column vectors of matrix A, i.e., $P_A = A(A^T T^{-1} A)^{-1} A^T T^{-1}$. The Lagrange multipliers λ_i in (3-5) are obtained from

$$\sum_1^k \lambda_i \ \mathrm{Tr}\{RT_i RT_j\} = p_j \ , \quad j = 1,2,\ldots,k \quad . \tag{3-6}$$

Therefore, the MINQUE of $\underline{p}^T\underline{\theta}$, denoted by $\underline{p}^T\underline{\hat{\theta}}$, is

$$\underline{p}^T\underline{\hat{\theta}} = \underline{\ell}^T B \underline{\ell} = \underline{\ell}^T Q^T(\sum_1^k \lambda_i T^{-1} T_i T^{-1}) Q \underline{\ell}$$

$$\underline{\hat{v}}^T(\sum_1^k \lambda_i T^{-1} T_i T^{-1}) \underline{\hat{v}} \quad , \tag{3-7}$$

where $\underline{\hat{v}}$ is the residual vector, calculated from the least-squares adjustment using the weight matrix T^{-1}. In matrix form, (3-6) and (3-7) become

$$S\underline{\lambda} = \underline{p} \quad , \tag{3-6'}$$

and

$$\underline{p}^T\underline{\hat{\theta}} = \underline{\lambda}^T\underline{q} \quad , \tag{3-7'}$$

where $S = (S_{ij})$, $S_{ij} = \mathrm{Tr}\{RT_i \ \ RT_j\}$ and $q_i = \underline{\hat{v}}^T T^{-1} T_i T^{-1} \underline{\hat{v}}$, termed the subquadratic form of the residuals. The variance-covariance components can be computed from (3-6') and (3-7') as

$$\underline{\hat{\theta}} = S^{-1}\underline{q} \quad . \tag{3-8}$$

Alternative derivations of the MINQUE can be found in Mitra (1971), Pringle (1974), and Pukelsheim (1976).

Let θ_i^o be an approximate value of θ_i. Then $T_i^o = \theta_i^o \, T_i$ and

$D\{\underline{v}\} = \sum_1^k \gamma_i T_i^o$, where $\gamma = \theta_i / \theta_i^o$, the scaled variance-covariance components to be estimated. Figure 3.1 shows the computational procedure.

Some properties of the MINQUE are as follows.

(1) Additive. If S_1 and S_2 are the MINQUE of $\underline{p}_1^T\underline{\theta}$ and $\underline{p}_2^T\underline{\theta}$ respectively, then $(S_1 + S_2)$ is the MINQUE of $(\underline{p}_1 + \underline{p}_2)^T\underline{\theta}$ (Rao, 1970).

(2) Invariant under a linear non-singular transformation of the variables (Rao, 1971a).

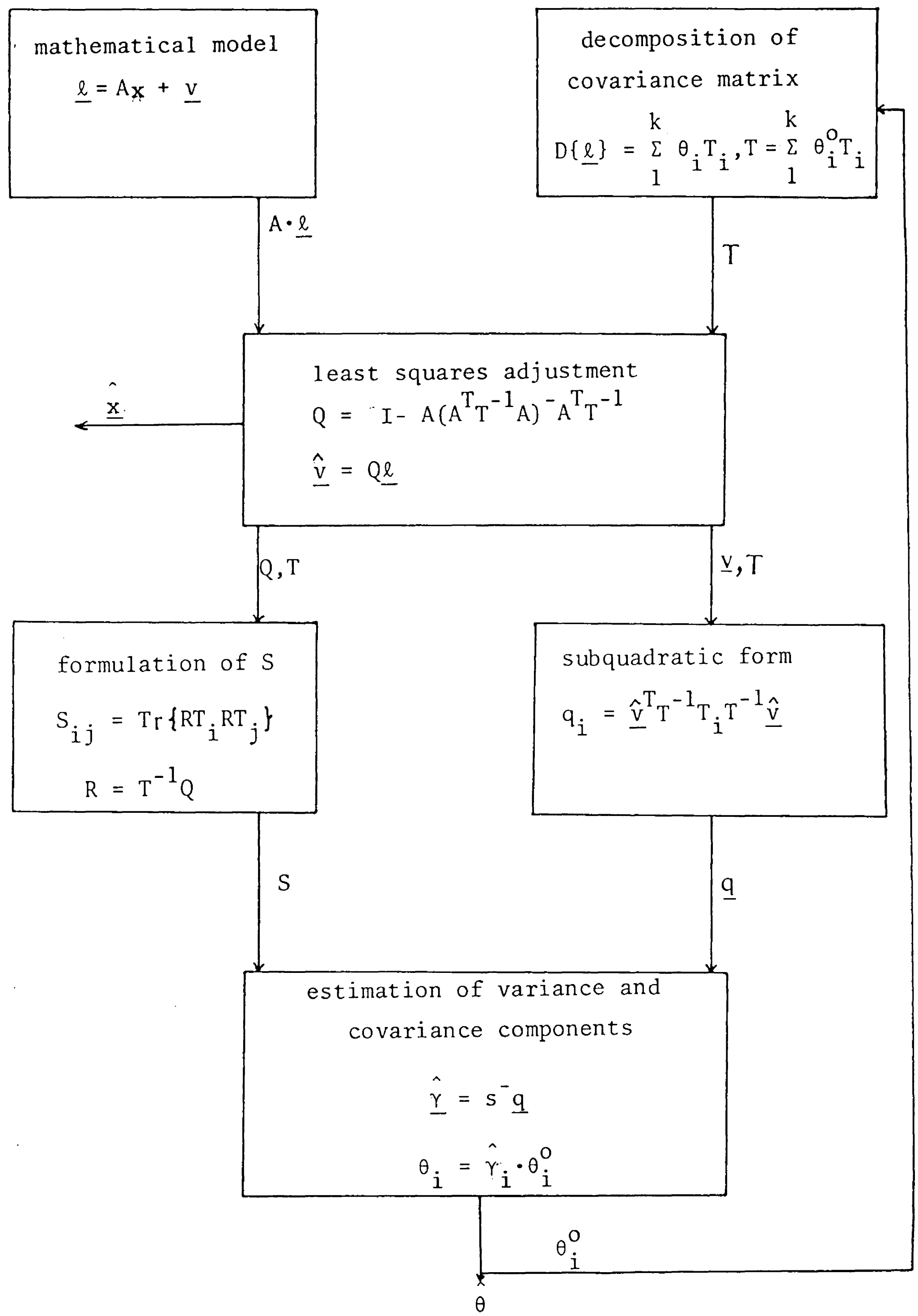


Fig. 3.1: The procedure of computation of variance-covariance components.

(3) Identical with the best quadratic unbiased estimation when the variables are normally distributed (Lamotte, 1973).

(4) Coincident with Helmert type estimation of variance components (Grafarend et al., 1980), but different from that of covariance components (Appendix II.2).

(5) Asymptotically equivalent to the maximum likelihood estimator when the variables of the model are assumed to be normally distributed (Brown, 1976).

(6) $\hat{\underline{\theta}}$ is unbiased when S in (3-8) is regular (Appendix II.3).

3.3 Application of the MINQUE to the Assessment of the Observations in Deformation Surveys

3.3.1 Extension of the MINQUE to Condition Adjustment

The original derivation of the MINQUE is based on model (3-1), parametric adjustment. But in surveying practice the adjustment of a network may be performed by the condition method, whose mathematical model reads

$$C\underline{v} + \underline{w} = \underline{0} \quad ,$$

$$D\{\underline{v}\} = \sum_{1}^{k} \theta_i T_i \quad , \qquad (3\text{-}9)$$

where C is a configuration matrix of order r × n, $\underline{v}$ is an n-vector of residuals, $\underline{w}$ is an r-vector of misclosures, and the others have been defined before. From the least squares method comes the relation (Mikhail, 1976) CA = 0, where A is the design matrix in the parametric adjustment of the same problem. In a vector space, if the inner product is defined as $(\underline{x}, \underline{y}) = \underline{x}^T T^{-1} \underline{y}$, then $Z = TC^T$ forms the orthogonal complement of the space S(A) spanned by the column vectors of A, since $Z^T T^{-1} A = CTT^{-1} A = 0$. In (3-5), P_A is the projection operator onto space S(A), and $Q = I - P_A$ is the orthogonal projection operator onto its orthogonal complement S(Z). Thus

$$Q = Z(Z^T T^{-1} Z)^{-1} Z^T T^{-1} = TC^T (CTC^T)^{-1} C \quad . \tag{3-10}$$

The corresponding expressions in (3-6') and (3-7') will be changed to

$$R = T^{-1} Q = C^T (CTC^T)^{-1} C \quad , \tag{3-11a}$$

$$S_{ij} = \mathrm{Tr}\,\{RT_i RT_j\} \quad , \tag{3-11b}$$

$$q_i = \underline{w}^T (CTC^T)^{-1} CT_i C^T (CTC^T)^{-1} \underline{w} \quad . \tag{3-11c}$$

It can be seen from (3-11) that the inversion of T is not needed. This may be useful in some problems where T is not block diagonal and involves a large dimension matrix.

3.3.2 A general error structure model in deformation surveys

Deformation surveys involve multiple epochs of heterogeneous observations. Possible damage to benchmarks, extension or reduction of a monitoring network, as well as the utilization of newly developed instrumentation make the same repeated observation program unrealistic, that is, the first- and second-order designs may change from epoch to epoch. Thus, the function model of the adjustment for m epochs is given by

$$\begin{pmatrix} \underline{\ell}_1 \\ \underline{\ell}_2 \\ \cdot \\ \cdot \\ \underline{\ell}_m \end{pmatrix} = \begin{pmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ & & & \\ & & & \\ 0 & 0 & \cdots & A_m \end{pmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \cdot \\ \cdot \\ \underline{x}_m \end{pmatrix} + \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \\ \cdot \\ \cdot \\ \underline{v}_m \end{pmatrix} \tag{3-12}$$

or more compactly as

$$\underline{\ell} = A\underline{x} + \underline{v} \quad , \tag{3-13}$$

where $\underline{\ell}_i$ is an n_i-vector of observations, $\underline{v}_i$ is an n_i-vector of errors, $\underline{x}_i$ is a u_i-vector of unknown parameters, and A_i is the first-order design matrix of $n_i \times u_i$. The unsubscripted variables represent the corresponding block of vectors or matrices. Since the same instruments may be used in different epochs and similar environmental conditions may have a common influence on the observations, the possiblity of stochastic dependency among the observations, not only in the same epoch but also between different epochs, exists. The stochastic model is expressed by the expectation value $E\{\underline{v}\} = 0$, $E\{\underline{\ell}\} = A\underline{x}$ and dispersion matrix

$$D\{\underline{v}\} = E\{\underline{v}\underline{v}^T\} = \Sigma = (\Sigma_{ij}) \quad i,j = 1,\ldots,m \quad , \tag{3-14}$$

with Σ_{ij} being a covariance matrix between $\underline{\ell}_i$ and $\underline{\ell}_j$. As in model (3-2) each covariance matrix may contain several variance- covariance components to be estimated, say k_{ij} components for Σ_{ij}, i.e.,

$$\Sigma_{ij} = \sum_{\ell=1}^{k_{ij}} \theta_{ij}^{\ell} \, T_{ij}^{\ell} \quad .$$

Therefore, the dispersion matrix Σ can be decomposed into

$$\Sigma = \sum_{i \leq j} \sum_{\ell=1}^{k_{ij}} \theta_{ij}^{\ell}(1 + \delta_{ij})^{-1}(\underline{e}_i\underline{e}_j^T \quad T_{ij}^{\ell} + \underline{e}_j\underline{e}_i^T \quad T_{ji}^{\ell}) \quad , \tag{3-15}$$

in which δ_{ij} is a Kronecker symbol (i.e., $\delta_{ij} = 1$ when $i = j$, otherwise $\delta_{ij} = 0$) and H_{ij}^{ℓ} is defined as

$$H_{ij}^{\ell} = \begin{pmatrix} 0 & \cdots & & 0 \\ \cdots & 0 & T_{ij}^{\ell} & \cdots \\ 0 & & & 0 \end{pmatrix} \qquad (i,j = 1, 2, \ldots, m)$$

with dimension being $\sum_1^m n_i$. When $\ell = 1$, the model (3-15) corresponds to the so-called generalized multi-variate regression model of heterogeneous observations developed by Schaffrin (1981). But $\ell = 1$ implies that there is only one variance component in one epoch and one correlation coefficient between epochs to be estimated. This will not be the case in general since a monitoring network usually includes different types of observables, each of which may have one or more error components to be estimated, and correlation coefficients between epochs will be different for different types of observations. As soon as the stochastic model (3-15) is ready, the computation of θ_{ij}^{ℓ} (total number $k = \sum_{i \leq j} k_{ij}$, $i,j = 1,\ldots,m$) is just a straightforward matter following Figure 3.1. Model (3-15) may be reconstructed by defining

$$\theta = (\theta_{11}^1, \theta_{11}^2, \ldots, \theta_{11}^{k_{11}}, \theta_{12}^1, \theta_{12}^2, \ldots, \theta_{12}^{k_{12}}, \ldots, \theta_{mm}^{k_{mm}}) := (\theta_1, \theta_2, \ldots, \theta_k),$$

$(\tilde{H}_{11}^1, \tilde{H}_{11}^2, \ldots, \tilde{H}_{11}^{k_{11}}, \tilde{H}_{12}^1, \tilde{H}_{12}^2, \ldots, \tilde{H}_{mm}^{k_{mm}}) := (T_1, T_2, \ldots, T_k)$, where

$\tilde{H}_{ij}^{\ell} = (1 + \delta_{ij})^{-1}(H_{ij}^{\ell} + H_{ij}^{\ell})$, then

$$\Sigma = \sum_1^k \theta_i T_i = \Sigma(\theta) \quad . \tag{3-16}$$

If an approximate value θ_i^o is assigned to θ_i, the least-squares adjustment is performed using $\Sigma^{-1}(\theta^o)$ as a weight matrix. Subsequently, the following quantities can be calculated:

$$R = \Sigma^{-1}(\theta^o) - \Sigma^{-1}(\theta^o)A(A^T\Sigma^{-1}(\theta^o)A)^{-1}A^T\Sigma^{-1}(\theta^o)$$

$$q_i = \underline{v}^T\Sigma^{-1}(\theta^o)T_i^o\Sigma^{-1}(\theta^o)\underline{v}$$

$$S_{ij} = \mathrm{Tr}\,\{RT_i^oRT_j^o\}$$

and the scaled variance-covariance components $\underline{\gamma}$ are estimated from

$$\underline{\hat{\gamma}} = S^{-1}\underline{q} \quad .$$

Thus, the estimated variance-covariance components are

$$\hat{\theta}_i = \hat{\gamma}_i \theta_i^o \quad (i = 1,2,\ldots,k) \quad .$$

3.4 Assessment of the Observations of the Pitec Network --An Example

The Pitec network was established by the University of New Brunswick in 1977 (Chrzanowski et al., 1978) to monitor tectonic movements in Peru. The network is located about 10 km due east of Huaraz in central north Peru at an average elevation of 4000 m. The configuration of the network is shown in Figure 3.2. The distances were measured using HP 3800A and AGA 12 EDM instruments, and backward and foreward observations were made with different instruments during the same observation campaign. The angles were measured using a Wild T2 with independent angle method of four sets (Dennler, 1980).

In the assessment of the observations, it is assumed that the measured angles and distances are all independent and have the variances θ_β^2 for the angle, and $\alpha_{HP}^2 + b_{HP}^2\ S^2$ and $a_{AGA}^2 + b_{AGA}^2\ S^2$ for the distances measured with the HP 3800A and the AGA 12 respectively, where S is the measured distance. Then the model (3-3) can be constructed as

$$\Sigma = a_{HP}^2 \begin{pmatrix} I_{n_1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + b_{HP}^2 \begin{pmatrix} D_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + a_{AGA}^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & I_{n_2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$+ b_{AGA}^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & D_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \theta_\beta^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_{n_3} \end{pmatrix} \qquad (3\text{-}18)$$

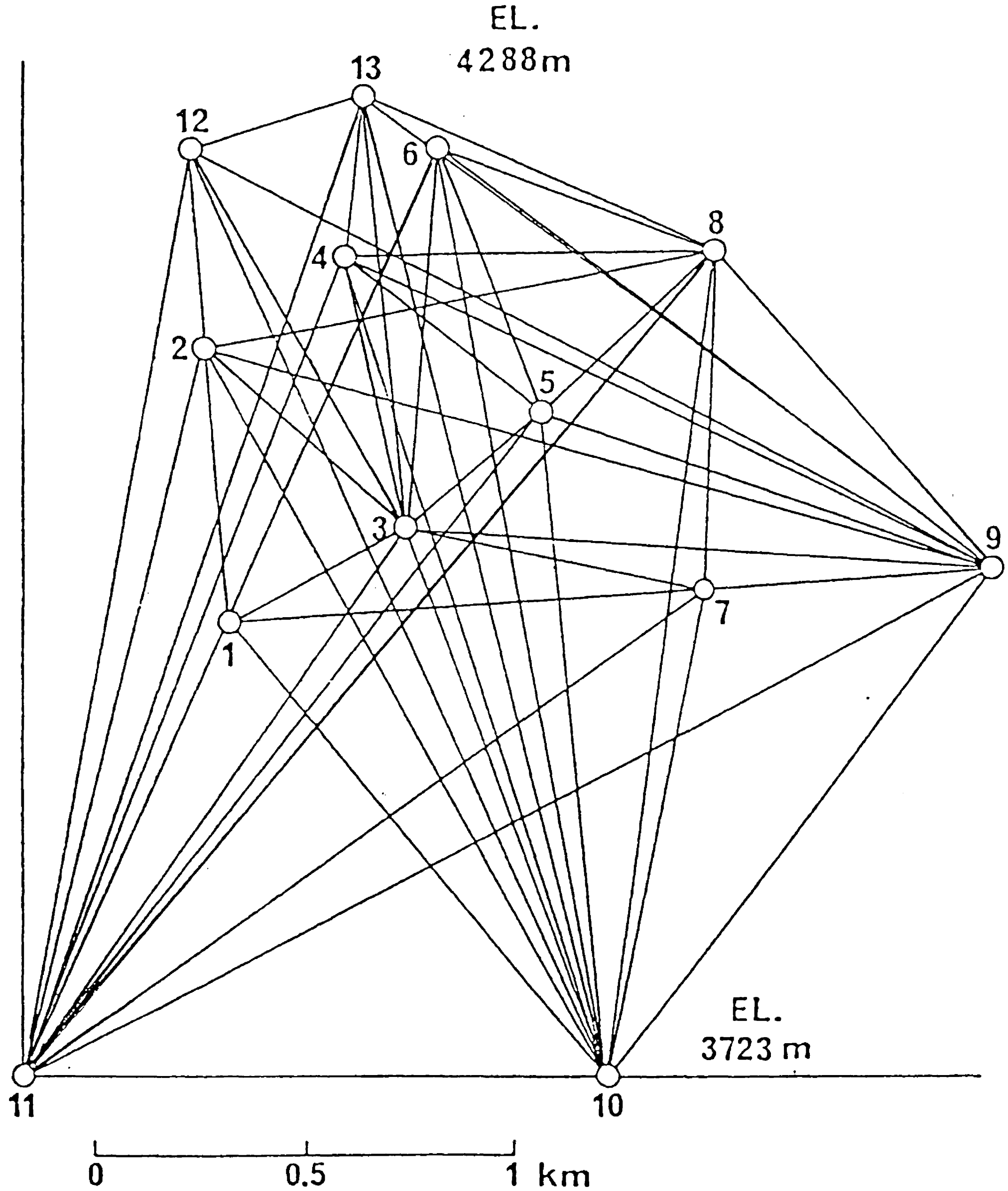


Fig. 3.2: The Pitec Network.

where I_{ni} is an identity matrix with dimension n_i of the number of observations, D_i is a diagonal matrix with non-zero elements being S_i^2. The estimated values of those variance components using the MINQUE principle are presented in Table 3.1. For the sake of brevity, the data used and the program developed will not be included in the thesis. (The same policy is adopted in the following chapters.)

	a_{HP} (mm)	b_{HP} (ppm)	a_{AGA} (mm)	b_{AGA} (ppm)	θ_β (")
MINQUE method	2.6	4.2	4.5	3.6	3.2
Trial and error method *	2.0	2.0	4.0	2.0	3.0
Accuracy claimed by manufacturer	3.0	7.0	5.0	5.0	-

* The values are taken from Dennler (1980).

TABLE 3.1

The Variance Components of the Observations in the Pitec Network.

From the assessment of the observation data of the Pitec network, the following remarks can be made:

(1) The computation procedures using the MINQUE are straightforward. The iteration process converges very rapidly. Table 3.2 shows the iteration process. Actually, the values are acceptable after the first iteration.

Iteration	a_{HP} (mm)	b_{HP} (ppm)	a_{AGA} (mm)	b_{AGA} (ppm)	θ_{β} (")
0	1.0	1.0	1.0	1.0	1.0
1	2.6	4.2	4.6	3.3	3.2
2	2.5	4.2	4.4	3.6	3.2
3	2.7	4.2	4.5	3.5	3.2
4	2.6	4.2	4.5	3.6	3.2

TABLE 3.2

The Iteration Process in the Estimation of Variance Components of the Observations of the Pitec Network.

(2) The estimated variance components for distance measurements using the MINQUE principle, in comparison with the value obtained by the trial and error method, are closer to those given by the manufacturers. The scale errors are smaller than those claimed by the manufacturers which was to be expected because the surveying site is located at high altitude, thus the environmental conditions have less influence.

(3) Components b_{HP} and b_{AGA} reflect only the random part of the scale errors of the EDM instruments. The systematic part of the scale errors cannot be estimated from this free network. Moreover, the two error components of the EDM-measured distances cannot be separated if the distances in a network are more or less uniform.

At the end of this chapter it should be stressed that the principle of the MINQUE seems to be as fundamental as the least-squares method of estimation, where no assumptions about the distribution of the variable is made. Although the method was proposed in 1970, application to geodetic practice is only beginning, and further investigation is required.

CHAPTER 4

DATUM DEFECT PROBLEM AND ITS RELATION TO DEFORMATION ANALYSIS

4.1 Datum Defect Problem

Geodetic deformation monitoring networks are mostly free networks, suffering from datum defects (Van Mierlo, 1978), that is, the network can be freely translated, rotated or scaled in space. These defects, of course, could be removed by external observations. For example, the orientation of a network can be determined by measuring a gyro-azimuth or an astronomic azimuth. However, the precision possible from such external observations is, at present, low in comparison to the precision possible from the internal observations of a network; in addition, the cost of acquiring such external observations is too high to be economically justified. Therefore, external observations are not used frequently. The quantities which define the network in space are known as datum parameters (Krüger, 1980). The datum parameters for different types of geodetic networks are listed in Table 4.1.

type of network	datum defects	datum parameters
horizontal network		
a) triangulation net	4	2 translations
		1 rotation
		1 scale
b) trilateration net	3	2 translations
		1 rotation
vertical network	1	1 translation
three-dimensional net	6	3 translations
	(+1)	3 rotations
		(1 scale)

TABLE 4.1

Datum Parameters of Networks.

Let $\underline{\ell}$ be a n-vector of observations whose dispersion matrix is Σ, $\underline{x}$ be a u-vector of unknown coordinates, and A be a n by u configuration matrix or the first-order design matrix. The linear model of the parametric adjustment of the monitoring network reads:

$$E\{\underline{\ell}\} = A\underline{x} \quad ,$$
$$D\{\underline{\ell}\} = \Sigma = \sigma^2 Q \quad . \qquad (4\text{-}1)$$

The information about datum defects, their number and types, is embedded in the configuration matrix A. Note that $\underline{x}$ should not be confused with the coordinates in x direction of a Cartesian coordinate system. This should be clear from the context. If a network is complete, i.e., without configuration defects, the number of rank defects of A is equal to the number of datum defects of the network, denoted by d_D, and the basis of the null space of A characterizes the datum defects. It is easy to find a u by d_D matrix whose column vectors generate the null space of A by purely geometrical considerations. Considering a three-dimensional network, which comprises m points, one such matrix has, for the maximum case of seven datum defects, the structure

$$H = \begin{pmatrix} \underline{e} & \underline{0} & \underline{0} & \underline{0} & \underline{z}_o & -\underline{y}_o & \underline{x}_o \\ \underline{0} & \underline{e} & \underline{0} & -\underline{z}_o & \underline{0} & \underline{x}_o & \underline{y}_o \\ \underline{0} & \underline{0} & \underline{e} & \underline{y}_o & -\underline{x}_o & \underline{0} & \underline{z}_o \end{pmatrix}_{3m \times 7} \qquad (4\text{-}2)$$

with $\underline{e}$ being a vector of the form

$$\underline{e}^T = (1, \ldots, 1) \qquad (4\text{-}3)$$

and $\underline{x}_o$, $\underline{y}_o$, $\underline{z}_o$ being vectors of the approximate coordinates with respect to the centroid of the network, i.e.,

$$\left.\begin{aligned} X_{io} &= X_i - \frac{1}{m} \sum_1^m X_i \\ Y_{io} &= Y_i - \frac{1}{m} \sum_1^m Y_i \\ Z_{io} &= Z_i - \frac{1}{m} \sum_1^m Zi \end{aligned}\right\} , \qquad (4\text{-}4)$$

where x_{io}, y_{io}, z_{io} are the ith elements of $\underline{x}_o$, $\underline{y}_o$, $\underline{z}_o$ respectively, and x_i, y_i, z_i are the approximate coordinates of point i in a local Cartesian coordinate system. The first three columns of H correspond to the free translations in x, y, z, directions; the second three columns take care of the rotations of the network with respect to the centroid around x, y, z, axes respectively; the last column accounts for the change in scale. For other types of geodetic networks, such a matrix H can be produced from (4-2) by simply deleting some irrelevant columns. For instance, for a vertical network $H = \underline{e}$, and for a horizontal trilateration network

$$H = \begin{pmatrix} \underline{e} & \underline{0} & -\underline{y}_o \\ \underline{0} & \underline{e} & \underline{x}_o \end{pmatrix} \quad . \tag{4-5}$$

Since the matrix H generates the null space of A, the relation

$$AH = 0 \tag{4-6}$$

holds.

The normal equation for model (4-1) reads

$$N\underline{x} = \underline{w} \quad , \tag{4-7}$$

with $N = A^TQ^{-1}A$, $\underline{w} = A^TQ^{-1}\underline{\ell}$. Due to the rank deficiency in A, the coefficient matrix N of the normal equations is singular and shares the same null space as A, which can be proved by verifying $NH = 0$ using relation (4-6). Therefore, there exists an infinite number of vectors $\underline{x}$ which satisfy the equation (4-7). In general, the solution to (4-7) can be expressed by (Bjerhammar, 1973):

$$\hat{\underline{x}} = N^-\underline{w} + (I - N^-N)\underline{z} \quad , \tag{4-8}$$

where N^- is a generalized inverse, and $\underline{z}$ is an arbitrary u-vector. The solution is, in general, biased, i.e., $\hat{\underline{x}}$ does not belong to the estimable quantities. However, a linear function $\underline{p}^T\underline{x}$ of unknown parameters $\underline{x}$ will be invariant of the choice of g-inverse of N provided that the condition $p^T(I - N^-N) = 0$ is satisfied, where $\underline{p}$ is a u-vector of scalars. The quantities $\underline{p}^T\underline{x}$ are said to be estimable.

It is well known that derived spatial angles, distances, or distance ratios (if no distance was measured in a network) are estimable or datum independent (Grafarend and Schaffrin, 1974), but the coordinates of the points or the azimuths are not. Neither are their variances and covariances. Obviously, in deformation analysis, the computed deformation parameters should be estimable, otherwise the conclusion will be biased due to the choice of the datum.

The coordinates of points are the quantities that geodesists and surveyors would like to work with because of some appealing benefits: the formulation of the deformation model is very transparent and easily schematicized for the computation; the plot of the results is straight forward and lucid (Caspary, 1982). However, some scientists argue against the use of the coordinates in deformation analysis, especially in the study of tectonic movements because they consider that this introduces a datum problem and involves some difficulty in assessing the precision of the derived strain tensor (Reilly, 1982). Therefore, study of the datum defect problem is indeed significant and possesses the following purposes:

(1) to better understand the attributes of the coordinates of the points and their covariances derived from the adjustment of a free network, which avoids committing the mistakes in deformation analysis;

(2) to provide a good picture of the displacement field by selecting an appropriate datum, which makes it easier to identify the deformation model in the space domain or the suspected unstable points of a reference network.

Definition of the datum of a network means specification of a datum equation $D^T\underline{x} = 0$ with $r\{D\} = d_D$. However, how the problem of the datum defect is solved and what should be taken into consideration in deformation analysis concerning this problem are the topics of the next two sections. The main development in Section 4.2 lies in the solution of the datum defect problem using projection theory in a parameter space and in the linkage of this method with the methods of similarity transformation and generalized inverse. A thorough discussion of the relation between the datum defect problem and deformation analysis is made in Section 4.3.

4.2 Solution to the Datum Defect Problem Using Projection Theory in the Parameter Space

As mentioned in Section 4.1, the coefficient matrix of normal equations is singular due to datum defects. The singularity problem can be solved by means of projection theory in the parameter space, which offers a better understanding of the problem.

Let E^u be a u-dimensional Euclidean space with the inner product being defined as $\langle \underline{x}, \underline{y} \rangle = \underline{x}^T \underline{y}$. Then all the $\underline{x}$ satisfying the equation (4-7) lie in a hyperplane Ψ (or linear variety):

$$\psi = N^- \underline{w} + S(H) \quad , \qquad (4\text{-}9)$$

where S(H) denotes the subspace spanned by the column vectors of H. A case of three-dimensional Euclidean space is portrayed in Figure 4.1. In E^u datum equation $D^T \underline{x} = \underline{0}$ defines a subspace $E^q \subset E^u$ with $q = u - d_D$ and its orthogonal complement is subspace S(D) with dimension d_D, generated by the column vectors of D. The solution to the datum defect problem can be understood as projecting any solution $\underline{x}$ parallel to S(H) onto the subspace E^q (see Figure 4.1). Intuitively, it is clear from Figure 4.1 that the projected vector $\hat{\underline{x}}_1$ on the subspace E^q is unique, whatever the original solution $\hat{\underline{x}}$ is. The theoretical proof of that is given in Note 3 below.

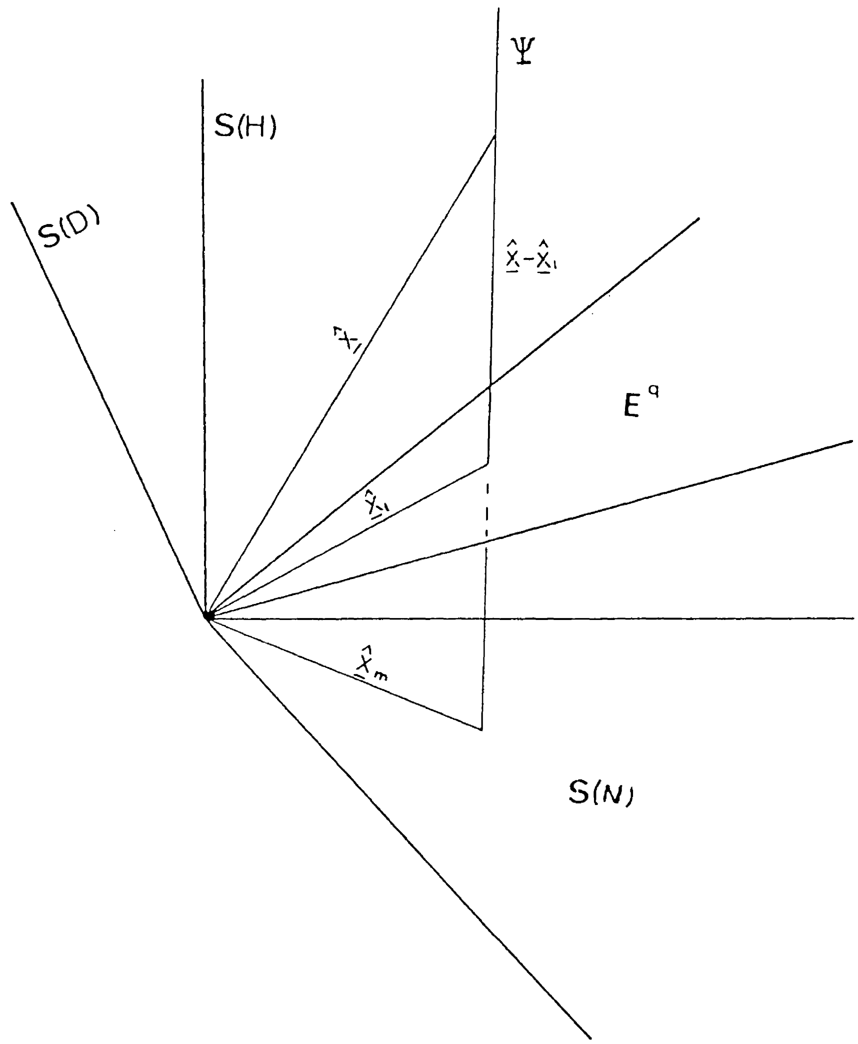


Fig. 4.1

Solution to the Datum Defect Problem Using Projection Theory in the Parameter Space.

The orthogonal projector in a vector space is widely discussed in the least-squares approximation theory, e.g., Luenberger (1969); Wells (1974); Adam (1980). However, the oblique projection operator, to be used here, is not very popular in surveying engineering, therefore more description is necessary. Further reference is made to Afrait (1957) and Rao (1974).

Let A and B be disjoint matrices, each with the same number of rows n, which span the whole Euclidean space E^n. By disjoint matrices is meant that the intersection of two spaces S(A), S(B) is a zero vector, symbolized by $S(A) \cap S(B) = \underline{0}$. Any vector $\underline{y}$ in $E^n(E^n = S(A) \oplus S(B))$ has the unique resolution:

$$\underline{y} = \underline{y}_1 + \underline{y}_2, \ \underline{y}_1 \in S(A), \ \underline{y}_2 \in S(B) \quad , \tag{4-10}$$

where $\oplus$ denotes the direct sum of subspaces. Then $P_{A/B}$ is said to be a projector onto S(A) parallel to S(B) if and only if

$$\underline{y}_1 = P_{A/B}\, \underline{y} \quad \text{for all } \underline{y} \in E^n \quad . \tag{4-11}$$

Since $\underline{y}_1 \in S(A)$ and $\underline{y}_2 \in S(B)$, they can be expressed as linear combinations of the bases of the corresponding space, i.e.,

$$\underline{y} = \underline{y}_1 + \underline{y}_2 = A\,\underline{a} + B\,\underline{b} \quad , \tag{4-12}$$

where a and b are arbitrary. Using (4-11) yields the following:

$$P_{A/B}(A\,\underline{a} + B\,\underline{b}) = A\underline{a} \quad \text{for all a and b} \quad ,$$

which results in

$$P_{A/B}A = A, \ P_{A/B}B = 0 \quad . \tag{4-13}$$

From (4-13), $P_{A/B}B = 0$, $P^T_{A/B}$ can be considered as the matrix of maximum rank of B. By denoting a matrix of maximum rank by C, such that $C^TB = 0$, $P_{A/B}$ can be expressed as $P_{A/B} = KC^T$ for some K. By substituting in $P_{A/B}A = A$, $KC^TA = A$ or $K = A(C^TA)^{-1}$, the representation of the projector $A_{A/B}$ is established:

$$P_{A/B} = A(C^TA)^{-1}C^T \quad . \tag{4-14}$$

An alternative representation of $P_{A/B}$ is given by Afrait (1957) as

$$P_{A/B} = A(A^TQ_BA)^{-1}A^TQ_B \quad , \tag{4-15}$$

with $Q_B = [I - B(B^TB)^-B^T]$. The result is established by verification of the condition that

$$P_{A/B}A = A \text{ and } P_{A/B}B = 0.$$

Now, the solution $\hat{\underline{x}}_1$ in the subspace E^q can be derived by using the oblique projector developed above. From Figure 4.1 $(\hat{\underline{x}} - \hat{\underline{x}}_1)$ is the projection of $\hat{\underline{x}}$ onto subspace S(H) parallel to E^q. Using (4-14) and considering $S(D) \perp E^q$, one can get

$$(\hat{\underline{x}} - \hat{\underline{x}}_1) = P_{H/D^\perp}\,\hat{\underline{x}} = H(D^TH)^{-1}D^T\hat{\underline{x}} \quad .$$

Hence

$$\hat{\underline{x}}_1 = (I - H(D^TH)^{-1}D^T)\hat{\underline{x}}$$

$$= P_{D^\perp}\cdot\underline{x} \ . \tag{4-16}$$

The alternative expression of $\hat{\underline{x}}_1$ can be obtained using (4-15). Note that Q_B in (4-15) is the orthogonal projector onto the subspace perpendicular to S(B). Thus

$$\hat{\underline{x}}_1 = [I - H(H^TD(D^TD)^{-1}D^TH)^{-1}H^TD(D^TD)^{-1}D^T]\hat{\underline{x}} \ . \tag{4-17}$$

As a special case, the projection onto the subspace S(N) parallel to S(H) is orthogonal because $S(H) \perp S(N)$. It has the shortest length among any other projected vectors. By the definition of the inner constraint solution, i.e., the solution having minimum Euclidean norm ($\|\underline{x}\|$ = min), such a projected vector, denoted by $\hat{\underline{x}}_{in}$, is the solution of the inner constraint. Therefore

$$\hat{\underline{x}}_{in} = (I - H(H^TH)^{-1}H^T)\hat{\underline{x}} \quad , \tag{4-18}$$

which suggests that the datum equation for the inner constraint solution has the form of

$$H^T\underline{x} = 0 \quad . \tag{4-19}$$

Note 1. The inner constraint solution (4-18) is a special case of (4-16), where D = H.

Note 2. The second term of the alternative expression (4-17) can be understood as an orthogonal projection of any solution vector $\underline{x}$ onto the subspace S(H) when the inner product in E^u is defined as $(\underline{x},\underline{y}) = \underline{x}^T w \underline{y}$ with $w = D(D^TD)^{-1}D^T$. This understanding is useful. When some components of $\underline{x}$ are more favored than the others, "heavier" weights can be imposed on the favored ones.

Note 3. A projector has the property that $P_{D_1^\perp/H} P_{D_2^\perp/H} = P_{D_1^\perp/H}$. This can be proved by verifying $(I - H(D_1^TH)^{-1}D_1^T)(I - H(D_2^TH)^{-1}D_2^T) = (I - H(D_1^TH)^{-1}D_1^T)$. This establishes the result that the projected vector $\hat{\underline{x}}_1$ on the subspace E^q is unique whatever the original solution $\hat{\underline{x}}$ is.

Note 4. If G and E are any other matrices such that S(G) = S(H), S(E) = S(D), then

$$(I - H(D^TH)^{-1}D^T) = (I - G(E^TG)^{-1}E^T) \quad . \tag{4-20}$$

So far, the general solution to the datum defect problem has been given. Koch (1982) has considered this problem from a different view, using a symmetric reflexive generalized inverse of N. Here the author has taken full advantage of projection theory in the parameter space, whose implications are essentially in terms of better insight and understanding of the mathematical formulations.

Another way to look at the datum defect problem is through Baarda (1973), using similarity transformations, which was further discussed and developed by van Mierlo (1980) and Molenar (1981). Given a vector of coordinates $\hat{\underline{x}}$, it is required to transform $\hat{\underline{x}}$ to a new one $\hat{\underline{x}}_1$, which satisfies the datum equation $D^T\hat{\underline{x}}_1 = 0$. Denote the datum parameters by a vector $\underline{t}$, whose components correspond to the columns of the matrix H in (4-2). For instance, for a horizontal trilateration network $\underline{t}^T = (t_x, t_y, t_r)$ whose components represent the translation of the network in x,y direction and rotation of the network. Then, the similarity transformation can be expressed as

$$\hat{\underline{x}}_1 = \hat{\underline{x}} + H\underline{t} \quad . \tag{4-21}$$

The parameters $\underline{t}$ are determined so that $\hat{\underline{x}}_1$ satisfies the datum equation, i.e., $D^T\hat{\underline{x}}_1 = D^T\hat{\underline{x}} + D^TH\underline{t} = 0$. Therefore,

$$\underline{t} = -(D^TH)^{-1}D^T\hat{\underline{x}} \quad . \tag{4-22}$$

Substituting (4-22) into (4-21), one obtains

$$\hat{\underline{x}}_1 = [I - H(D^TH)^{-1}D^T]\hat{\underline{x}} \quad , \tag{4-23}$$

which is the same as (4-16).

As we know, the normal equation (4-7) can be solved by a generalized inverse (g-inverse). There are many ways to compute a g-inverse, see, e.g., Rao and Mitra (1971), Blaha (1971), Mittermayer (1972), Welsch (1979). However, one is interested in knowing the attributes of the computed vector of coordinates $\hat{\underline{x}}$ and its covariance matrix. This is profitable in deformation analysis. The choice of a g-inverse implies the definition of a specified datum. Let us derive a general expression of the g-inverse, which is related to a particular datum. Denote the datum by equation $D^T\underline{x} = 0$, then the generalized inverse has the form

$$N_D^- = (N + DD^T)^{-1} - H(H^TDD^TH)^{-1}H^T \quad . \tag{4-24}$$

Proof. Since (N:D) is of full rank, $(N + DD^T)$ is nonsingular (Rao, 1973, p. 34). Accordingly, $(N + DD^T)^{-1}$ is one of the g-inverses of N, which is established by verification of the condition $N(N + DD^T)^{-1}N = N$ (Rao, 1973, p. 225). One solution to the normal equation (4-7) becomes $\underline{x} = (N + DD^T)^{-1}\underline{w}$. Using the formula (4-16), one obtains $\underline{x}_1 = [I - H(D^TH)^{-1}D^T](N + DD^T)^{-1}\underline{w} = N_D^-\underline{w}$ with

$$N_D^- = (I - H(D^TH)^{-1}D^T)(N + DD^T)^{-1} \quad . \tag{4-25}$$

Further consider that

$$(N + DD^T)(H(D^TH)^{-1}D^T) = DD^T \text{ (since } NH = 0) \quad ,$$

$$H(D^TH)^{-1}D^T = (N + DD^T)^{-1}DD^T \quad ,$$

$$D[(D^TH)^{-1}]^TH^T = DD^T(N + DD^T)^{-1} \quad ,$$

$$[(D^TH)^{-1})^TH^T = D^T(N + DD^T)^{-1} \quad .$$

Substituting this into (4-25),

$$H(D^TH)^{-1}D^T(N + DD^T)^{-1} = H(H^TDD^TH)^{-1}H^T \quad ,$$

which establishes (4-24).∥

Note that N_D^- is a reflexive inverse and that when D = H the inverse N_H^- becomes the pseudo-inverse of N, i.e., $N_H^- = N^+$. This is easy to verify from the definition of these two types of inverses.

Projection of the vector of coordinates $\hat{\underline{x}}$ from one datum to another is usually accompanied by the transformation of its cofactor matrix $Q_{\hat{\underline{x}}}$. Applying the law of error propagation to (4-16), the cofactor matrix of the projected vector $\hat{\underline{x}}_1$ is written as

$$Q_{\hat{\underline{x}}_1} = (I - H(D^TH)^{-1}D^T)Q_{\hat{\underline{x}}}(I - H(D^TH)^{-1}D^T)^T \quad . \tag{4-26}$$

On the other hand, since $\hat{\underline{x}}_1 = N_D^-\underline{w} = N_D^-A^TQ^{-1}\underline{\ell}$, by applying the law of error propagation one can obtain $Q_{\hat{\underline{x}}_1} = N_D^-NN_D^-$. Consequently,

$$Q_{\hat{\underline{x}}_1} = N_D^- \quad , \tag{4-27}$$

because N_D^- is a reflexive inverse.

Note that $Q_{\hat{\underline{x}}_1}$ possesses the same rank defects as $(I - H(D^TH)^{-1}D^T)$ and has the property

$$D^TQ_{\hat{\underline{x}}_1} = 0 \quad , \tag{4-28}$$

which will be useful in the following chapters.

4.3 Datum Defect Problem in Deformation Analysis

Without datum defects in a monitoring network, deformation analysis would be much simpler. Comparison of the calculated displacements with their accuracies would indicate whether the points have moved, and the identification of the deformation model would be easier because the obtained displacement field would not be distorted. Unfortunately, this is not so in reality, at least at present, due to the aforementioned reasons.

Let $\hat{\underline{x}}_1$, $\hat{\underline{x}}_2$ be the vectors of coordinates calculated from the adjustment of the first and the second epochs of observations, respectively, and $Q_{\hat{\underline{x}}_1}$, $Q_{\hat{\underline{x}}_2}$ be their cofactor matrices. The vector of displacements $\underline{d}$ are computed as the differences between the two vectors, i.e., $\underline{d} = \hat{\underline{x}}_2 - \hat{\underline{x}}_1$. If $\hat{\underline{x}}_1$ and $\hat{\underline{x}}_2$ are obtained from the adjustments referring to different datums, the quantity $(\hat{\underline{x}}_2 - \hat{\underline{x}}_1)$ will contain "false movements", caused by the different datum parameters. This may often happen in practice. For example, $\hat{\underline{x}}_2$ is calculated using the minimum constraints which differ from those used in the first epoch adjustment due to the damage of some points; $\hat{\underline{x}}_1$, $\hat{\underline{x}}_2$ are both calculated from inner constraint solutions, but some points are added in the second epoch; the observation schemes at two epochs of time are different, producing the change in the datum defects of the network. In these cases, projection of the calculated coordinates as well as their covariance matrices to the same datum is necessary. This can be achieved by means of the projection operator developed in Section 4.2.

The vector of displacements $\underline{d}$ and its cofactor matrix $Q_{\underline{d}}$ with respect to the datum defined by the equation $D^T\underline{x} = 0$ are computable from

$$\underline{d} = P_{D^\perp}(\hat{\underline{x}}_2 - \hat{\underline{x}}_1) \tag{4-29}$$

and

$$Q_{\underline{d}} = P_{D^\perp}(Q_{\hat{\underline{x}}_1} + Q_{\hat{\underline{x}}_2})P^T_{D^\perp} \quad , \tag{4-30}$$

where the projector $P_{D^\perp} = (I - H(D^TH)^{-1}D^T)$. Note that the matrices H and D should consist of the appropriate columns corresponding to the union of the datum parameters of two epochs and of the appropriate rows for the common points in two epochs. For instance, if a monitoring network was a triangulation network in the first epoch, but triangulateration network in the second epoch, then the datum equations imposed on the separate adjustments have rank of four and three respectively. To formulate the projection operator P_D, the datum equation possessing rank of four should be used.

In practice it often happens that one is interested in the investigation of a partial network. Let the vector of displacement $\underline{d}$ and its cofactor matrix $Q_{\underline{d}}$ be conformably partitioned as $\underline{d}^T = (\underline{d}_1^T \vdots d_2^T)$ and

$$Q_{\underline{d}} = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix} \quad ,$$

where $\underline{d}_1$ and Q_{11} are referred to the portion under investigation. In general, Q_{11} is regular and $\underline{d}_1$ contains translation, rotation components of the partial network with respect to the datum used in the adjustment. In order to study the change in the shape of this partial free network, for example, use of the quadratic form $\underline{d}_1^T Q_{11}^- \underline{d}_1$ tests the distortion of the partial network, the displacement vector $\underline{d}_1$ and the cofactor matrix Q_{11} must be transformed to those referring to a datum specified for that partial network. This can be achieved by means of a projector $P_{D^\perp}$:

$$\underline{d}_1' = P_{D^\perp}\,\underline{d}_1 \;, \quad Q_{11}' = P_{D^\perp}\,Q_{11}\,P_{D^\perp}^T \;,$$

where P_D is established using (4-16) and refers to the datum equation for the partial network.

Another problem in deformation analysis is to set up a criterion for the datum definition. Although the estimation of the deformation parameters using the method developed in Chapter 6 does not depend upon the choice of datum, careful selection of a datum will help a lot in the identification of a deformation model, which is an important step toward the estimation of deformation parameters.

Free network adjustment of a conventional geodetic network in the concept of inner accuracy has been rather widely accepted after many elucidating discussions in the last years concerning the statistical properties of the results (Schmitt, 1982). However, it does not necessarily provide the best result as far as the identification of the deformation model is concerned. This point has been investigated by Caspary and Chen (1981) and Prescott (1981). In deformation analysis, the definition of the datum by the inner constraint method suggests that:

(1) the summations of the displacement components of all the points are zero, i.e., $\sum_i u_i = \sum_i v_i = \sum_i w_i = 0$ or the centroid of the network is stationary, where u_i, v_i, w_i are the components of the displacement of point i in x,y,z direction respectively;

(2) no rotation of the network during two epochs of time around three axes of the coordinate system about the centroid takes place, i.e.,

$$\sum_i(y_{io}\,w_i - z_{io}\,v_i) = \sum_i(z_{io}\,u_i - x_{io}\,w_i) = \sum_i(x_{io}\,v_i - y_{io}\,u_i) = 0;$$

(3) there is no scale change between the two epochs, i.e.,

$$\sum_i (x_{io}\, u_i + y_{io}\, v_i + z_{io}\, w_i) = 0.$$

The above points are for the case of a three-dimensional geodetic monitoring network with seven datum parameters. The situation for other types of monitoring networks is easily deduced from this general case.

If all the points of the network are stable during two epochs of time, the calculated displacements are caused only by the random observing errors. Then the inner constraint method will provide a better datum, compared with the minimum constraint method because it weakly depends on any particular point and keeps the influence of the random observing error small. But if some points in the network have moved, then the inner constraint solution would smooth the real displacements and bias the datum, making identification of unstable points in a reference network and of a deformation model in a relative network more difficult. Having recognized this point, many scientists have developed different approaches.

The approach based on the examination of invariant quantities has been developed at UNB and used to select "best" minimum constraints for the adjustment of the monitoring network (Chrzanowski et al., 1981; Tobin, in prep.). In this approach the separate adjustments of two epochs of observations are performed, adjusted distances and angles from each point to the rest of the points of the network are calculated for both epochs, and the differences of these adjusted quantities and their variances are derived, which are invariant of the choice of the datum in the adjustment. The stability of any pair (in a distance) or of any triplet (in an angle) of points is examined by statistical testing of these differences. The failure of the test at a certain significance level is the suspicion that any of the members of the pair or triplet of points is significantly unstable. In order to identify the specific points, i.e., isolate them from the pair or triplet, a listing of observation difference failures is made at several levels, viz. at $\alpha = 0.10, 0.05$. The frequencies of failure for each point are tabulated. Then the points with the least frequency of involvement in failures of the statistical test are considered as the least unstable and so are used as control points serving as necessary minimum constraints of the adjustment. The method is successfully applied to preliminary identification of suspected moving points in several monitoring networks (Chrzanowski et al., 1982(a); Chrzanowski and Secord, 1983).

Another approach (Koch and Fritsch, 1981) is to define the datum step by step. Based on the global statistical test (see Chapter 5), some points, assumed to be stable, are selected to define the datum using the inner constraint method, i.e., translating, rotating and scaling (if no distance in the network is observed) the network so that the sum of the squared displacement components of those stable points is minimized. Then one of the points remaining in the network enters the list of the stable points if the statistical test on the assumption that all the points are stable is accepted, and the datum is redefined including that point in the inner constraint solution. This successive estimation of the coordinates with the test for stable points stops when all the stable points are used to define the datum and all hypotheses for additional stable points have to be rejected.

Rather than the trial and error approach, a new method for the location of the datum has been developed by Caspary and the author

(1981). In this method a special similarity transformation is used so that the sum of the length of final displacements of all the points is minimum. Assume that $\underline{d}$ is the 3m-vector of displacements in three-dimensional space and $\underline{d}_i$ is the 3-vector of displacement of the ith point. Then a special similarity transformation is carried out in such a way that the transferred vector of displacement $\underline{d}'$ satisfies the condition that $\sum_{1}^{m} \| \underline{d}_i' \| = \min$, where $\underline{d}' = \underline{d} + H\underline{t}$ with H and $\underline{t}$ being defined in (4-21). The datum defined in this manner is more robust to single point movement in a reference network than the inner constraint solution, particularly, when the network is comparatively small (Caspary and Chen, 1981).

In a relative monitoring network the definition of an appropriate datum will provide a good picture for the identification of the deformation model. A method has been created by Prescott (1981) for analysing the crustal movement along the San Andreas fault, where there is a priori knowledge that station motion will most likely be along the direction of the fault, say the y direction. Consequently, the datum equation is constructed so that the centre of the mass of the network remains stationary and the components of displacement normal to the preferred direction are minimized, that is, the matrix D of the datum equation $D^T x = 0$ has the form:

$$D = \begin{pmatrix} \underline{e} & \underline{0} & -\underline{y}_o \\ \underline{0} & \underline{e} & \underline{0} \end{pmatrix} .$$

However, a priori knowledge about a deformable body in many cases is either poor or does not exist at all. Thus, the creation of the deformation model becomes difficult. A method elaborating on the merits of the different approaches is developed in Chapter 6 and serves as a tool for a preliminary identification of the deformation model in the space domain.

CHAPTER 5

ASPECTS OF STATISTICAL TESTING IN DEFORMATION ANALYSIS

As pointed out by Pelzer (1971), statistical testing is especially important if geodetic networks are established for the detection of recent crustal movements or of deformation in man-made structures. Screening of the acquired data for outliers and systematic errors, diagnostic checking of the deformation models, examination of the significance of the derived deformation parameters and so on are based to a large extent on statistical tests.

It is well known that no specification of the probability distribution of the observations is required, except for their mean values, variances and covariances, if the least-squares principle is adopted in the processing of data. However, an exact statement of the probability distribution of the observations is necessary in order to draw a statistical inference. In surveying engineering the acquired data are most often normally distributed about their mean values (e.g., Baarda, 1967; 1976). Hence, the assumption of normality of the observations is made throughout this chapter.

For the sake of self-containment some basic attributes of the sample statistic for functions of normal random variables are presented in Section 5.1. Then a general test statistic for linear hypotheses valid for a model with a singular dispersion matrix for the observations and with rank deficiency in the configuration matrix is developed using the theory of vector spaces. As special cases of the general test statistic, the techniques for the detection of outliers and systematic errors in observations and the methods for the testing of deformation model are presented in Sections 5.3 and 5.4, respectively. Finally, the practical problem of choosing the significance level, α, for hypothesis testing is discussed.

5.1 Some Basic Attributes of the Sample Statistic for Functions of Normal Random Variables

Let $\underline{\ell} \overset{d}{\sim} N_n(\underline{\mu}, \Sigma)$ denote an n-vector of random variables following a normal distribution with expected value $\underline{\mu}$ and dispersion matrix Σ. Then its density function is defined as

$$(2\pi)^{-\frac{n}{2}} \, |\Sigma|^{-\frac{1}{2}} \exp\{-\frac{1}{2}(\underline{\ell} - \underline{\mu})^T \Sigma^{-1} (\underline{\ell} - \underline{\mu})\} \quad . \qquad (5\text{-}1)$$

However, the density function defined in (5.1) does not exist if the dispersion matrix Σ is singular, i.e., $r\{\Sigma\} = k < n$. A singular dispersion matrix implies that the random observations $\underline{\ell}$ are linearly dependent and accordingly some natural restrictions are imposed on the observations. Denote matrix $\Sigma^{\perp}$ of order nx(n − k) as the matrix of maximum rank of Σ, such that $\Sigma\Sigma^{\perp} = 0$, then $\underline{\ell}$ is restricted on the hyperplane $(\Sigma^{\perp})^T(\underline{\ell} - \underline{\mu}) = \underline{0}$. Khatri (1968) and Caspary (1983) express the density function of the observations $\underline{\ell}$ on this hyperplane as

$$\frac{(2\pi)^{-\frac{k}{2}}}{\sqrt{\lambda_1^2 \lambda_2^2 \cdots \lambda_k^2}} \exp\{-\frac{1}{2}(\underline{\ell}-\underline{\mu})^T \Sigma^{-}(\underline{\ell}-\underline{\mu})\} \qquad (5\text{-}2)$$

where the λ_i^2 are the nonzero eigen values of Σ, and k is called the rank of the distribution (Khatri, 1968).

A linear function $A\underline{\ell}$ also follows the normal distribution with expected value $E\{A\underline{\ell}\} = A\underline{\mu}$ and dispersion matrix $D\{A\underline{\ell}\} = A\Sigma A^T$. Two linear functions $A_1\underline{\ell}$ and $A_2\underline{\ell}$ are correlated with a covariance matrix $A_1\Sigma A_2^T$. If $A_1\Sigma A_2^T = 0$, then they are statistically independent. The quadratic functions of normal variables have the following properties (Rao, 1973):

i) The quadratic function $q = \underline{\ell}^T B\underline{\ell}$, where B is an arbitrary symmetric matrix, has an expected value of

$$E\{q\} = Tr\{B\Sigma\} + \underline{\mu}^T B\underline{\mu} \qquad (5\text{-}3a)$$

and a variance of

$$V\{q\} = 4\underline{\mu}^T B\Sigma B\underline{\mu} + 2Tr\{B\Sigma B\Sigma\} \quad . \qquad (5\text{-}3b)$$

ii) The quadratic form $q = (\underline{\ell}-\underline{\mu})^T B(\underline{\ell}-\underline{\mu})$ has a χ^2 distribution with degrees of freedom being $tr\{B\Sigma\}$ under the condition that $\Sigma B\Sigma B\Sigma = \Sigma B\Sigma$, which will reduce to $B\Sigma B = B$ if $|\Sigma| \neq 0$.

iii) Two quadratic forms $q_1 = (\underline{\ell}-\underline{\mu})^T B_1(\underline{\ell}-\underline{\mu})$ and $q_2 = (\underline{\ell}-\underline{\mu})^T B_2(\underline{\ell}-\underline{\mu})$ are statistically independent if either the condition that $\Sigma B_1\Sigma B_2\Sigma = 0$ in general or that $B_1\Sigma B_2 = 0$ when $|\Sigma| \neq 0$ is fulfilled. Furthermore, if both quadratic forms are independently χ^2 distributed with degrees of freedom being k_1, k_2 respectively, the ratio $T = (q_1/q_2)\cdot(k_2/k_1)$ follows a central F-distribution.

iv) The linear function $A\underline{\ell}$ and the quadratic form $(\underline{\ell} - \underline{\mu})^T B(\underline{\ell} - \underline{\mu})$ are statistically independent of each other if the condition $\Sigma B \Sigma A^T = 0$ is fulfilled.

There are four basic probability distribution functions widely used in surveying engineering, that is, the normal, "n"; the Fisher, "F"; the Student's, "t"; and the chi-squared, "χ^2" distributions. They are related in the following manner (Scheffé, 1959):

$$\chi^2(\alpha;df) = df \cdot F(\alpha;df,\infty) \tag{5-4a}$$

$$t(\frac{\alpha}{2};df) = \sqrt{F(\alpha;1,df)} \tag{5-4b}$$

$$F(1 - \alpha;df_1,df_2) = 1/F(\alpha;df_2,df_1) \tag{5-4c}$$

and for normal distribution (Baarda, 1968):

$$n(\frac{\alpha}{2}) = \sqrt{F(\alpha;1,\infty)} \tag{5-4d}$$

in which the degrees of freedom are denoted by df_i and significance level by α.

5.2 Formulation of the Test Statistic for the General Gauss-Markoff Model Using the Theory of Vector Spaces

Geodesists have developed a series of test statistics for different purposes using classical algebraic methods. Among others are the statistic by Caspary (1979), established from stepwise condition adjustment and parametric adjustment; the global congruency test by Pelzer (1971) for deformation analysis; data snooping by Baarda (1976), the τ-test by Pope (1976) and t-test by Heck (1981) for the detection of outliers. A general hypothesis testing for the Gauss-Markoff model has been given by Koch (1975) and that for the Gauss-Helmert model has been derived by Wolf (1980).

There is a more general case, viz. the General Gauss-Markoff Model

(GGM) (Rao, 1973). Let the triplet ($\underline{\ell}$, $A\underline{x}$, $\sigma^2 Q$) denote the stochastic model (4-1), i.e., $E\{\underline{\ell}\} = A\underline{x}$ and $D\{\underline{\ell}\} = \sigma^2 Q$. In the GGM, the configuration matrix A might be deficient in rank or the cofactor matrix Q might be singular due to the linear dependency of the observations or both. This may be the case in deformation analysis since

(1) the geodetic monitoring networks are mostly free networks, as discussed in Chapter 4, thus the configuration matrix A would not be of full rank;

(2) deformation parameters may be calculated from the adjusted coordinates (quasi-observations), whose covariance matrix may be singular due to datum defects or configuration defects.

Therefore, the test statistic for the GGM applicable in these instances should be developed. The mechanism of the following derivation is based on the theory of vector spaces. The adoption of geometrical language provides better illustration and simplification of the derivation.

As we know, examining the power characterictics of a test or constructing tests with certain desirable statistical properties requires the additional specification of the alternative hypothesis. Any statistical test must inevitably examine two hypotheses: the null hypothesis H_o, which would be conserved unless significant evidence is found to support its rejection; and the alternative hypothesis H_a, in favour of which the null hypothesis would be rejected. Following Searle (1971), the derivation of the statistic for hypothesis testing can be done by imposing on the parameters $\underline{x}$ in the GGM a linear constraint $H\underline{x} = \underline{w}$. Thus, H_o: $H\underline{x} = \underline{w}$ is the null hypothesis against the alternative hypothesis H_a: $H\underline{x} \neq \underline{w}$, where H is an $m \times u$ matrix of known coefficients and $\underline{w}$ is an m-vector of known elements.

In the theory of linear models, the original one, without any restriction on $\underline{x}$, is referred to as the full model, and that with the restrictions imposed on $\underline{x}$ is called the reduced model (Searle, 1971b). In the derivation of the test statistic, the full model reads ($\underline{\ell}$, $A\underline{x}$, $\sigma^2 Q$) and the reduced model reads ($\underline{\ell}$, $A\underline{x} | H\underline{x} = \underline{w}$, $\sigma^2 Q$) where H_o: $H\underline{x} = \underline{w}$ is the null hypothesis to be tested against the alternative hypothesis H_a: $H\underline{x} \neq \underline{w}$.

It should be stressed that systematic errors and outliers in the observations $\underline{\ell}$ are treated as model errors in this chapter since their existence makes the applied model poor in describing what actually has happened. Consequently, the derived test statistic is universally valid for any case.

5.2.1 Constrained least-squares estimation in vector spaces

The least-squares estimation in vector spaces without restrictions on parameters and with the configuration matrix being of full rank has been discussed in, e.g., Wells (1974) and Adam (1980). A more general case, that linear restrictions are imposed on parameters and the configuration matrix is of possible deficiency in rank, will be considered here. This establishes the preliminary step for the formulation of the test statistic in the GGM.

Consider the model:

$$(\underline{\ell},\ A\underline{x},\ \sigma^2 I) \text{ with } r\{A\} = r \leq u \quad , \tag{5-5}$$

where A is an n $\times$ u matrix. In an n-dimensional Euclidean space E^n, where the inner product is defined as $(\underline{x},\underline{y}) = \underline{x}^T\underline{y}$, the matrix A forms an r-dimensional subspace S(A), called the solution space, and its orthogonal complement $S(A^{\perp})$ is the condition space. The direct sum of these two subspaces establishes the sample space or observation space, i.e., $E^n = S(A) \oplus S(A^{\perp})$ (Wells, 1974).

In addition, if the homogeneous constraints $H\underline{x} = \underline{0}$ are imposed in model (5-5), where the $H\underline{x}$ are estimable functions, i.e., $S(H^T) \subset S(A^T)$, and $r\{H\} = h \leqslant m$, then the solution space is restricted to a subspace of S(A), denoted by Φ (see Fig. 5-1). The subspace Φ can be generated by the column vectors of the matrix $A(I-H^{+}H)$ and has dimension of $(r - h)$.

Proof. Any vector $\underline{a} \in S(A)$ can be expressed as $\underline{a} = A\underline{\beta}$ with $\underline{\beta}$ being some constant vector. Furthermore, if $\underline{a} \in \Phi \subset S(A)$ then $\underline{\beta}$ will satisfy the condition that $H\underline{\beta} = \underline{0}$ and can be represented by $\underline{\beta} = (I-H^{+}H)\underline{\gamma}$ with $\underline{\gamma}$ being an arbitrary vector. Hence, $\underline{a} = A(I-H^{+}H)\underline{\gamma}$, meaning that $\underline{a}$ lies in the space spanned by the column vectors of $A(I-H^{+}H)$. The dimension of the subspace Φ, denoted by $d\{\Phi\}$, is computable from the formula (5-6), (Rao, 1973)

$$d\{\Phi\} = r\{\binom{A}{H}\} - r\{H\} \qquad (5\text{-}6)$$

since $S(H^T) \subset S(A^T)$, (5-6) becomes

$$d\{\Phi\} = r\{A\} - r\{H\} = r - h \quad . \quad \| \qquad (5\text{-}7)$$

In the solution space S(A) the orthogonal complement of the subspace Φ, denoted by $\Phi^{\perp}$, can be generated by the column vectors of the matrix $A(A^TA)^{-}H^T$ because

$$H(A^TA)^{-}A^T \cdot A(I-H^{+}H) = H(I-H^{+}H) = 0 \; (\because S(H^T) \subset S(A^T)) \; .$$

Moreover, the non-homogeneous constraints $H\underline{x} = \underline{w}$ force the solution space of the model (5-5) to be limited in a hyperplane, denoted by Ψ, which is the translation of the subspace Φ and can be expressed as $\Psi = \underline{\psi} + \Phi$ with $\underline{\psi} \in \Psi$. One representation of $\underline{\psi}$ is $AH^{+}\underline{w}$. The proof follows the same line as the above, only $\underline{\beta} = H^{+}\underline{w} + (I-H^{+}H)\underline{\gamma}$ and $\underline{a} = AH^{+}\underline{w}$

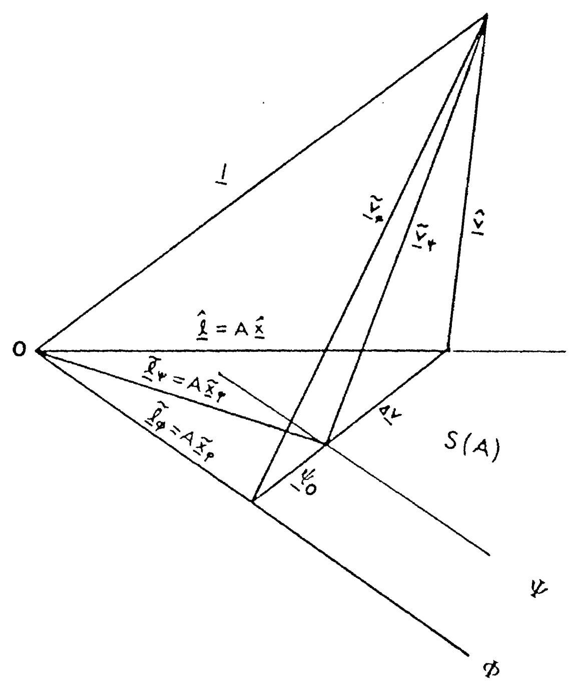


Fig. 5.1: Constrained Least-Squares Estimation in Vector Space.

$+ A(I-H^+H)\underline{y}$. This suggests that $\underline{a}$ must lie on the hyperplane $\Psi = \underline{\psi} + \Phi$.

As is known, the least-squares solution to model (5-5) is the orthogonal projection of $\underline{\ell}$ onto solution space S(A) and the residual vector $\hat{\underline{v}} = \underline{\ell} - A\hat{\underline{x}}$ is the projection of $\underline{\ell}$ onto the condition space $S(A^{\perp})$ (Wells, 1974). One representation of the orthogonal projection operators onto S(A) and $S(A^{\perp})$ are $P_A = A(A^TA)^-A^T$ and $P_{A^{\perp}} = (I-P_A) = I-A(A^TA)^-A^T$ respectively, which are symmetric, idempotent and unique for any choice of g-inverse of A^TA (Rao and Mitra, 1971). Moreover, if C is any other matrix such that S(C) = S(A), then $P_A = A(A^TA)^-A^T = C(C^TC)^-C^T$. A simpler form is $P_A = \bar{C}\bar{C}^T$ when $\bar{C}$ is the orthonormal basic spanning the solution space S(A).

The solution to model (5-5) with restrictions on parameters $H\underline{x} = \underline{0}$ can be obtained by the orthogonal projection of $\underline{\ell}$ onto the subspace Φ. The projection operator can be established in the same manner as above except the matrix A is replaced by $A(I-H^+H)$. Alternatively, $\underline{\ell}_{\phi}$ can be calculated by projecting $\hat{\underline{\ell}}$ onto space Φ since $(\tilde{\underline{v}}_{\phi} - \hat{\underline{v}}) \perp \Phi$. The projection of $\hat{\underline{\ell}}$ onto the subspace $\Phi^{\perp}$ establishes the vector $(\tilde{\underline{v}}_{\phi} - \hat{\underline{v}})$ as

$$(\tilde{\underline{v}}_{\phi} - \hat{\underline{v}}) = P_{\Phi^{\perp}} \cdot \hat{\underline{\ell}} = A(A^TA)^-H^T(H(A^TA)^-H^T)^-\ H\hat{\underline{x}} \quad . \tag{5-8}$$

Therefore

$$\tilde{\underline{\ell}}_{\phi} = A\tilde{\underline{x}}_{\phi} = A(I-(A^TA)^-H^T(H(A^TA)^-H^T)^-\ H)\hat{\underline{x}} \quad . \tag{5-9}$$

However, if the non-homogeneous constraints $H\underline{x} = \underline{w}$ are forced on the model (5-5), the least-squares solution $\tilde{\underline{\ell}}_{\psi} = A\tilde{\underline{x}}_{\psi}$ is obtained by projecting $\underline{\ell}$ onto the hyperplane Ψ. This can be expressed as

$$\tilde{\underline{\ell}}_{\psi} = \tilde{\underline{\ell}}_{\phi} + \underline{\psi}_o, \qquad \tilde{\underline{v}}_{\psi} = \tilde{\underline{v}}_{\phi} - \underline{\psi}_o \quad , \tag{5-10}$$

where $\underline{\psi}_o$ is the projection of any vector $\underline{\psi} \in \Psi$ onto the subspace $\Phi^{\perp}$.

Proof. From the theory of vector space (Luenberger, 1969), any vector $\underline{\ell}_\psi$ in the hyperplane Ψ can be decomposed into

$$\underline{\ell}_\psi = \underline{\psi} + \underline{\ell}_\phi, \ \forall\ \underline{\ell}_\psi \in \Psi,\ \forall\ \underline{\ell}_\phi \in \Phi,\ \underline{\psi} \in \Psi \quad . \tag{5-11}$$

Let $\tilde{\underline{\ell}}_\phi$ and $\tilde{\underline{\ell}}_\psi$ be the least-squares solutions in the subspace Φ and in the hyperplane Ψ, respectively. By definition comes the relation:

$$||\underline{\ell} - \tilde{\underline{\ell}}_\psi|| \le ||\underline{\ell} - \underline{\ell}_\psi|| \qquad \forall\ \underline{\ell}_\psi \in \Psi \quad , \tag{5-12}$$

Substituting (5-11) into (5-12) yields $||\underline{\ell} - \tilde{\underline{\ell}}_\phi - \underline{\Psi}|| \le ||\underline{\ell} - \underline{\ell}_\phi - \underline{\psi}||$ $\forall$ $\underline{\ell}_\phi \in \Phi$, which leads to (5-13) if a $\underline{\psi} \perp \underline{\ell}_\phi$, denoted by $\underline{\psi}_o$,

$$||\underline{\ell} - \tilde{\underline{\ell}}_\phi|| \le ||\underline{\ell} - \underline{\ell}_\phi|| \quad . \tag{5-13}$$

The inequality (5-13) suggests that if $\tilde{\underline{\ell}}_\phi$ is the least-squares solution in Φ, then so is $\tilde{\underline{\ell}}_\psi = \tilde{\underline{\ell}}_\phi + \underline{\psi}_o$ in Ψ. As mentioned above, one representation of $\underline{\psi}$ is $AH^+\underline{w}$. Therefore,

$$\begin{aligned} \underline{\psi}_o &= P_{\phi^\perp} \cdot AH^+\underline{w} \\ &= A(A^TA)^-H^T(H(A^TA)^-H^T)^-\ \underline{w} \quad . \ || \end{aligned} \tag{5-14}$$

Substituting (5-8), (5-9), (5-14) into (5-10), the following relations are established:

$$\tilde{\underline{\ell}}_\psi = A\tilde{\underline{x}}_\psi = A[\hat{\underline{x}} - (A^TA)^-H^T(H(A^TA)^-H^T)^-\ (H\hat{\underline{x}}-\underline{w})] \tag{5-15}$$

$$(\tilde{\underline{v}}_\psi - \hat{\underline{v}}) = A(A^TA)^-H^T(H(A^TA)^-H^T)^-\ (H\hat{\underline{x}}-\underline{w}) \quad . \tag{5-16}$$

5.2.2 The test statistic for the model ($\underline{\ell}$, Ax, σ^2I)

First, some statistical attributes of the vectors in the observation space are established.

(1) Any two vectors, $B_1\underline{\ell}$ and $B_2\underline{\ell}$, with B_1, B_2 being arbitrary symmetric matrices, are statistically independent if they are perpendicular to each other. The result is easy to verify. Since the orthogonality of $B_1\underline{\ell}$ and $B_2\underline{\ell}$ implies that $\underline{\ell}^TB_1^TB_2\underline{\ell} = 0$ leading to $B_1B_2 = 0$, so the statistical independency between them is established (see Section 5.1).

(2) The squared length of the residual vector follows a χ^2-distribution with degrees of freedom being the dimension of the space in which the vector lies.

Proof: Since a residual vector $\underline{\hat{v}}$ is the orthogonal projecton of $\underline{\ell}$ onto a subspace, expressed as $\underline{\hat{v}} = P\underline{\ell}$ with P being the projection operator, the squared length of the residual vector is $q = \underline{\ell}^T P^T P \underline{\ell} = \underline{\ell}^T P \underline{\ell}$. In terms of Section 5.1, the symmetry and idempotency of the orthogonal projecton operator P are the necessary and sufficient conditions for q to have a χ^2-distribution, in which case the degrees of freedom are r{P}, the dimension of the subspace in which $\underline{\hat{v}}$ lies. ||

Furthermore, the squared lengths of two residual vectors are independently χ^2-distributed if they are perpendicular to each other.

(3) The vector resulting from the sum of two vectors, one of which is non-random, has the same statistical attributes as does the constituent random vector, except for the expected value. For example, $\underline{\tilde{\ell}}_\psi = \underline{\tilde{\ell}}_\phi + \underline{\psi}_o$ and $\underline{\tilde{\ell}}_\phi$ in Figure 5-1 share the same statistical properties, except for their expected values, so do $\underline{\tilde{v}}_\psi$ and $\underline{\tilde{v}}_\phi$ because $\underline{\psi}_o$ is non-random.

As mentioned in the beginning of this section, hypothesis testing involves two models: the full model and the reduced model. The latter is produced by imposing the null hypothesis H_o:$H\underline{x} = \underline{w}$ as constraints on the former. If the H_o is correct, the results from the two models are statistically the same. In Figure 5-1, $\underline{\hat{\ell}}$ and $\underline{\hat{v}}$ are obtained from the full model, and $\underline{\tilde{\ell}}$ and $\underline{\tilde{v}}$, from the reduced model. Note that from here on the subscripts ψ or ϕ will be omitted. The squared lengths of $\underline{\hat{v}}$ and $\underline{\tilde{v}}$, denoted by R and R_1 respectively, follow χ^2-distributions with degrees of freedom being df = n − r{A}, df_1 = n −

$r\{A\} + r\{H\}$. Since $\hat{\underline{v}} \perp (\tilde{\underline{v}} - \hat{\underline{v}})$, R and $(R_1 - R)$ are statistically independent. Consequently, for the acceptance of the null hypothesis at certain significance level α, the test statistic T, which by definition would follow an F-distribution, should not exceed the critical value:

$$T = \frac{R_1 - R}{R} \cdot \frac{df}{df_1 - df} \leq F(\alpha;\ df_1 - df,\ df) \quad . \qquad (5\text{-}17)$$

Equivalently, the test statistic can be written in another way:

$$T' = \frac{R_1 - R}{R_1} \cdot \frac{df_1}{df_1 - df}$$

$$\leq \frac{df_1}{(df_1 - df) + df \cdot F(1 - \alpha;\ df,\ df_1 - df)} \qquad (5\text{-}18)$$

The expression (5-17) or (5-18) serves as the statistic for the general hypothesis testing. It can be seen from Figure 5-1 that the computation of $(R_1 - R)$ may be carried out in three ways, depending on the problems at hand.

(1) From the results of the adjustment of the two models:

$$R_1 - R = ||\Delta\underline{\hat{v}}||^2 = (\tilde{\underline{v}} - \hat{\underline{v}})^T (\tilde{\underline{v}} - \hat{\underline{v}}) \quad , \qquad (5\text{-}19)$$

or

$$R_1 - R = (\hat{\underline{x}} - \tilde{\underline{x}})^T A^T A(\hat{\underline{x}} - \tilde{\underline{x}}) \quad . \qquad (5\text{-}20)$$

(2) From the results of the adjustment of the full model $(\underline{\ell}, A\underline{x}, \sigma^2 I)$ only. Considering (5-16), the expression (5-19 can be rewritten as

$$R_1 - R = (H\hat{\underline{x}} - \underline{w})^T [H(A^T A)^- H^T]^- (H\hat{\underline{x}} - \underline{w}) \quad . \qquad (5\text{-}21)$$

Note that although $\hat{\underline{x}}$ may not necessarily be estimable because of the possible existence of rank defects in A, the quantity $(R_1 - R)$ is invariant of any solution of the normal equation of the full model.

(3) From the results of the adjustment of the reduced model ($\underline{\ell}$, $A\underline{x}$ | $H\underline{x}$ = $\underline{w}$, $\sigma^2 I$) only. It is clear from Figure 5-1 that $\Delta\hat{\underline{v}}$ is the orthogonal projection of $\tilde{\underline{v}}$ onto the subspace $\Phi^{\perp} \subset S(A)$. Thus

$$(\tilde{\underline{v}} - \hat{\underline{v}}) = A(A^TA)^- A^T\tilde{\underline{v}}$$
$$= A(A^TA)^- H^T[H(A^TA)^- H^T]^- H(A^TA)^- A^T\tilde{\underline{v}} \quad .$$

Accordingly,

$$R_1 - R = \tilde{\underline{v}}^TA(A^TA)^-A^T\tilde{\underline{v}}$$
$$= \tilde{\underline{v}}^TA(A^TA)^-H^T[H(A^TA)^-H^T]^- H(A^TA)^-A^T\tilde{\underline{v}}. \tag{5-22a}$$

Considering $\tilde{\underline{v}}$ Φ, which implies that $\tilde{\underline{v}}^TA(I - H^+H) = 0$, the formula (5-22a) can be expressed in another form:

$$R_1 - R = \tilde{\underline{v}}^TAH^+H(A^TA)^- H^+HA^T\tilde{\underline{v}} \quad . \tag{5-22b}$$

Consider an important case when the solution space S(A) can be explicitly expressed as the union of two subspaces, say $S(A_1)$ and $S(A_2)$, which corresponds to the full model:

$$\{\underline{\ell},\ (A_1 \mid A_2)\begin{pmatrix}\underline{x}\\ \underline{y}\end{pmatrix},\ \sigma^2 I\} \quad .$$

The null hypothesis $H_o:\underline{y} = 0$ suggests that the solution is restricted to the subspace $\Phi = S(A_1)$ and that the reduced model reads ($\underline{\ell}$, $A_1\underline{x}$, $\sigma^2 I$). Observe that the subspace $\Phi^{\perp}$ can be generated by the column vectors of the matrix $(I - A_1(A_1^TA_1)^-A_1^T)A_2$, and that $(I - A_1(A_1^TA_1)^-A_1^T) = Q\tilde{\underline{v}}$, the cofactor matrix of $\tilde{\underline{v}}$, which can be verified by applying error propagation to $\tilde{\underline{v}} = (I - A_1(A_1^TA_1)^-A_1^T)\underline{\ell}$. Therefore, the projection of $\tilde{\underline{v}}$ onto the subspace $\Phi^{\perp}$ is computable from

$$(\tilde{\underline{v}} - \hat{\underline{v}}) = Q_{\tilde{\underline{v}}}\ A_2(A_2^T\ Q_{\tilde{\underline{v}}}\ A_2)^-\ A_2^T\ Q_v\ \tilde{\underline{v}} \quad . \tag{5-23}$$

Since $\tilde{\underline{v}} \perp S(A_1)$, $Q_{\tilde{\underline{v}}} \cdot \tilde{\underline{v}} = \tilde{\underline{v}}$. Consequently,

$$(\tilde{\underline{v}} - \hat{\underline{v}}) = Q_{\tilde{\underline{v}}} \cdot A_2(A_2^T\ Q_{\tilde{\underline{v}}}\ A_2)^-\ A_2^T \cdot \tilde{\underline{v}}\ ,$$

and $(R_1 - R)$ can be written as

$$R_1 - R = \tilde{\underline{v}}^T A_2 (A_2^T Q_v A_2)^- A_2^T \tilde{\underline{v}} \quad . \qquad (5.24)$$

The expression (5-24) will be found useful in model diagnostics. The diagnostic techniques based on (5-24) could be called a generalized method of the analysis of residuals. As will be shown in the next section, all the techniques for the identificaton of outliers can be treated as special cases of (5-24).

5.2.3 The test statistic for the GGM

The principle of least squares in the processing of observation data was propounded by Gauss in 1795 (Krakiwsky, 1975) and Legendre in 1806 (Robinson, 1981) under the model

$$(\underline{\ell},\ A\underline{x},\ \sigma^2 I) \quad \text{with } r\{A\} = u \text{ (full rank)} \quad . \qquad (5\text{-}25)$$

Later, Markoff made a systematic presentation of the theory under the same model. Therefore, the model (5-25) is named after them as the standard Gauss-Markoff model. In 1934, Aitken extended the model to the case that $D\{\underline{\ell}\} = \sigma^2 Q$ with $|Q| \neq 0$; Bose was the first to consider the situation that $r\{A\} = r \leqslant u$; in 1964 Goldman and Zelen studied the case when the dispersion matrix of observations is singular; combining all the possible situations, Rao constructed the General Gauss-Markoff Model (GGM) (Rao, 1971c) as

$$(\underline{\ell},\ A\underline{x},\ \sigma^2 Q) \quad , \qquad (5\text{-}26)$$

with Q possibly being singular, and A possibly having deficiency in rank. Note that the model with linear restrictions on the parameters $\underline{x}$, i.e., $(\underline{\ell},\ A\underline{x}\,|R\underline{x} = \underline{c},\ \sigma^2 Q)$ is, indeed, a special case of the GGM since it can be expressed as $(\underline{\ell}_e,\ A_e\underline{x},\ \sigma^2 Q_e)$ with $\underline{\ell}_e^T = (\underline{\ell}^T : \underline{c}^T)$, $A_e^T = (A^T : R^T)$

and

$$Q_e = \begin{pmatrix} Q & 0 \\ 0 & 0 \end{pmatrix} \quad .$$

The least-squares solution to the GGM may be found in Rao (1973) and Hallum et al (1973).

However, it will be shown that other more complicated situations can be reduced to the model (5-5), $(\underline{\ell}, A\underline{x}, \sigma^2 I)$ with $r\{A\} = r \leq u$, for which the statistic of hypothesis testing has been developed in 5.2.2.

(1) Consider the model

$$(\underline{\ell}, A\underline{x}, \sigma^2 Q) \text{ with } r\{A\} = r \leq u \text{ and } |Q| \neq 0 \quad . \tag{5-27}$$

Let $Q^{1/2}$ be the square root of the positive definite matrix Q. Then, under the transformation $\underline{\ell}_t = Q^{-1/2}\,\underline{\ell}$ and $A_t = Q^{-1/2} A$, the setup (5-27) is reduced to $(\underline{\ell}_t, A_t\underline{x}, \sigma^2 I)$, coincident with the model (5-5).

(2) In the General Gauss-Markoff Model (5-26), the singularity of the dispersion matrix implies that the random variables $\underline{\ell}$ are linearly dependent. Let $\underline{f}_1, \ldots, \underline{f}_s$ be the eigenvectors corresponding to the non-zero eigenvalues $\lambda_1^2, \lambda_2^2, \ldots, \lambda_s^2$ of Q and $\underline{g}_1, \ldots, \underline{g}_d$ be $d = n - s$ eigenvectors corresponding to the zero root. Define the matrices $F = (\lambda_1^{-1}\underline{f}_1, \ldots, \lambda_s^{-1}\underline{f}_s)$, $G = (\underline{g}_1, \ldots, \underline{g}_d)$. Then the transformation $\underline{\ell}_f = F^T\underline{\ell}$, $\underline{\ell}_g = G^T\underline{\ell}$, $A_f = F^T A$, $A_g = G^T A$ leads to the model

$$E\{\underline{\ell}_f\} = A_f \circ \underline{x}, \qquad D\{\underline{\ell}_f\} = \sigma^2 I \tag{5-28a}$$

$$E\{\underline{\ell}_g\} = A_g \circ \underline{x}, \qquad D\{\underline{\ell}_g\} = 0 \quad . \tag{5-28b}$$

Since $D\{\underline{\ell}_g\} = 0$, $\underline{\ell}_g$ is a non-random vector, so that $\underline{\ell}_g = A_g \cdot \underline{x}$ become constraints on the parameters. Two cases exist:

(i) $S(A) \subset S(Q)$ in the GGM, then $G^T\underline{\ell} = G^T A\underline{x} = \underline{0}$, no constraints are imposed on the parameters $\underline{x}$. Hence the model (5-28) is reduced to $(\underline{\ell}_f$,

$A_f \underline{x}, \sigma^2 I)$.

(ii) S(A) does not belong to the subspace of S(Q), the setup (5-28a) is restricted by the condition that $A_g\underline{x} = \underline{\ell}_g$. As is known, a general solution of $A_g\underline{x} = \underline{\ell}_g$ is $\underline{x} = A_g^+ \underline{\ell}_g + (I - A_g^+ A_g) \underline{z}$, in which $\underline{z}$ is an arbitrary u-vector. Observe that $E\{\underline{\ell}_f - A_f A_g^+ \underline{\ell}_g\} = A_f(I - A_g^+ A_g)\underline{z}$ and denote $\bar{\underline{\ell}}_f = \underline{\ell}_f - A_f A_g^+ \underline{\ell}_g$, $\bar{A}_f = A_f(I - A_g^+ A_g)$ so that the model (5-28) becomes

$$(\bar{\underline{\ell}}_f, \bar{A}_f\underline{x}, \sigma^2 I) \quad , \qquad (5\text{-}29)$$

which again is the model (5-5).

The above discussion may be summarized by saying:

(1) All possible linear models can be reduced to the model $(\underline{\ell}, A\underline{x}, \sigma^2 I)$ with $r\{A\} = r \leq u$, for which the statistic for testing the general hypothesis has been developed in 5.2.2.

(2) The acceptance of the null hypothesis $H_o: H\underline{x} = \underline{w}$ can be tested using either

$$T = \frac{R_1 - R}{R} \cdot \frac{df}{df_1 - df} \leq F(\alpha;\ df_1 - df,\ df) \qquad (5\text{-}30a)$$

or

$$T' = \frac{R_1 - R}{R_1} \cdot \frac{df_1}{df_1 - df}$$

$$\leq df_1/[(df_1 - df) + df \cdot F(1-\alpha;\ df, df_1 - df)] \qquad (5\text{-}30b)$$

with $df = n - r\{A\}$, $df_1 - df = r\{H\}$. The formulae for the computation of all the quantities involved in the statistic are listed in Table 5-1.

Model	R	R_1	$\Delta R = R_1 - R$: Case 1 (adj. of the full and reduced model)	Case 2* (adj. of the full model)	Case 3** (adj. of the reduced model)
$(\underline{\ell}, A\underline{x}, \sigma^2 I)$ $r\{A\} = \gamma \leq u$	$\hat{\underline{v}}^T \hat{\underline{v}}$	$\tilde{\underline{v}}^T \tilde{\underline{v}}$	$(\tilde{\underline{v}} - \hat{\underline{v}})^T (\tilde{\underline{v}} - \hat{\underline{v}})$ $(\tilde{\underline{x}} - \hat{\underline{x}})^T A^T A (\tilde{\underline{x}} - \hat{\underline{x}})$	$(H\hat{\underline{x}} - \underline{w})^T [H(A^T A)^- H^T]^- \cdot$ $\cdot\ (H\hat{\underline{x}} - \underline{w})$	$\tilde{\underline{v}}^T A_2 (A_2^T Q_{\tilde{\underline{v}}} A_2)^- A_2^T \tilde{\underline{v}}$
$(\underline{\ell}, A\underline{x}, \sigma^2 Q)$ $r\{A\} = \gamma \leq u$ $\lvert Q \rvert \neq 0$	$\hat{\underline{v}}^T Q^{-1} \hat{\underline{v}}$	$\tilde{\underline{v}}^T Q^{-1} \tilde{\underline{v}}$	$(\tilde{\underline{v}} - \hat{\underline{v}})^T Q^{-1} (\tilde{\underline{v}} - \hat{\underline{v}})$ $(\tilde{\underline{x}} - \hat{\underline{x}})^T A^T Q^{-1} A (\tilde{\underline{x}} - \hat{\underline{x}})$	$(H\hat{\underline{x}} - w)^T [H(A^T Q^{-1} A)^- H^T]^- \cdot$ $\cdot\ (H\hat{\underline{x}} - \underline{w})$	$\tilde{\underline{v}}^T Q^{-1} A_2 (A_2^T Q^{-1} Q_{\tilde{\underline{v}}} Q^{-1} A_2)^- \cdot$ $\cdot\ A_2^T Q^{-1} \tilde{\underline{v}}$
$(\underline{\ell}, A\underline{x}, \sigma^2 Q)$, $r\{A\} = \gamma \leq u$, $\lvert Q \rvert = 0$: $S(A) \subset S(Q)$	$\hat{\underline{v}}^T Q^- \hat{\underline{v}}$	$\tilde{\underline{v}}^T Q^- \tilde{\underline{v}}$	$(\tilde{\underline{v}} - \hat{\underline{v}})^T Q^- (\tilde{\underline{v}} - \hat{\underline{v}})$ $(\tilde{\underline{x}} - \hat{\underline{x}})^T A^T Q^- A (\tilde{\underline{x}} - \hat{\underline{x}})$	$(H\hat{\underline{x}} - \underline{w})^T [H(A^T Q^- A)^- H^T]^- \cdot$ $\cdot\ (H\hat{\underline{x}} - \underline{w})$	$\tilde{\underline{v}}^T Q^- A_2 (A_2^T Q^- Q_{\hat{\underline{v}}} Q^- A_2)^- \cdot$ $\cdot\ A_2^T Q^- \tilde{\underline{v}}$
$(\underline{\ell}, A\underline{x}, \sigma^2 Q)$, $r\{A\} = \gamma \leq u$, $\lvert Q \rvert = 0$: $S(A) \not\subset S(Q)$	$\hat{\underline{v}}^T Q^- \hat{\underline{v}}$	$\tilde{\underline{v}}^T Q^- \tilde{\underline{v}}$	$(\tilde{\underline{v}} - \hat{\underline{v}})^T Q^- (\tilde{\underline{v}} - \hat{\underline{v}})$ $(\tilde{\underline{x}} - \hat{\underline{x}})^T \bar{A}^T Q^- \bar{A} (\tilde{\underline{x}} - \hat{\underline{x}})$	$(H\hat{\underline{x}} - \underline{w})^T [H(\bar{A}^T Q^- \bar{A})^- H^T]^- \cdot$ $\cdot\ (H\hat{\underline{x}} - \underline{w})$	$\tilde{\underline{v}}^T Q^- \bar{A}_2 (\bar{A}_2^T Q^- Q_{\tilde{\underline{v}}} Q^- \bar{A}_2)^- \cdot$ $\cdot\ \bar{A}_2^T Q^- \tilde{\underline{v}}$

* $\bar{A} = A(I - A_g^+ A_g)$.

** in case 3, only the special case (5-24) and its corresponding ones are listed; $\bar{A}_2 = A_2(I - A_g^+ A_g)$.

TABLE 5.1

The Formulae for the Computation of the Quantities Needed in the Test Statistic (5-30).

5.3 Detection of Outliers and Systematic Errors in Observations

Errors are often inherent in the observation data. The sources are diverse: either in the phase of data acquisition or in that of data transcription to computer-readable form; either caused by the influences of the environmental conditions or due to the defects of the instruments. Outlying and systematic errors in observations are of major concern to geodesists, for their existence may produce large falsification in the results. In the past few years more attention has been given to this problem. Screening of the observations is one of the research topics, set up by the FIG "ad hoc" Committee on deformation analysis (Chrzanowski, 1981b). Different techniques for outlier detection have been developed (e.g., Baarda, 1968; Pope, 1976; Stefanovic, 1978; Heck, 1981) and widely used in surveying and photogrammetry (e.g., El-Hakim, 1981). A comprehensive review is given by van Mierlo (1982) and Kavouras (1982). It is the purpose of this section to link the different techniques for outlier detection by providing a generalized method which is derived from the general test statistic developed in Section 5.2.

Let n-vector of observations $\underline{\ell}$ be partitioned into two groups: $\underline{\ell}_1$ and $\underline{\ell}_2$ with $\underline{\ell}_1$ of dimension n_1 free from outliers and $\underline{\ell}_2$ of dimension n_2, containing suspected outliers, denoted by vector $\underline{\delta}$. Then the mathematical model reads

$$\begin{pmatrix} \underline{\ell}_1 \\ \underline{\ell}_2 \end{pmatrix} + \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \end{pmatrix} = \begin{pmatrix} \tilde{A}_1 & 0 \\ \tilde{A}_2 & I \end{pmatrix} \begin{pmatrix} \underline{x} \\ \underline{\delta} \end{pmatrix} , \qquad (5\text{-}31a)$$

$$D\{\underline{\ell}\} = \sigma^2 Q , \qquad (5\text{-}31b)$$

where Q is assumed to be nonsingular. For outlier detection the null hypothesis is H_o: $\underline{\delta} = 0$ versus the alternative hypothesis H_a: $\delta \neq 0$. In the practice of outlier detection, the adjustment is performed for the reduced model, i.e., without considering outliers in the observations or, in other words, the restriction $\underline{\delta} = \underline{0}$ is forced on the model (5-31a). Let $\underline{\tilde{v}}^T = (\underline{\tilde{v}}_1^T : \underline{\tilde{v}}_2^T)$ be the vector of residuals, whose subvector corresponds to $\underline{\ell}_1$ and $\underline{\ell}_2$, and $Q_{\tilde{v}}$ be its cofactor matrix with

$$Q_{\underline{\tilde{v}}} = \begin{pmatrix} Q_{\underline{\tilde{v}}_1} & Q_{\underline{\tilde{v}}_{12}} \\ Q_{\underline{\tilde{v}}_{21}} & Q_{\underline{\tilde{v}}_2} \end{pmatrix} .$$

The test on the null hypothesis H_o: $\underline{\delta} = \underline{0}$, can be carried out using the statistic (5-30a) or (5-31b). The quantities involved can be computed from Table 5.1 as

$$R_1 = \underline{\tilde{v}}^T Q^{-1} \underline{\tilde{v}}$$

$$R_1 - R = \underline{\tilde{v}}^T Q^{-1} A_2[A_2^T Q^{-1} Q_{\tilde{v}} Q^{-1} A_2]^{-1} A_2^T Q^{-1} \underline{\tilde{v}}$$

$$df_1 - df = n_2,$$

with $A_2^T = (0 : I)$. This general method is capable of testing multiple outliers in correlated observations.

As a special case, if only one outlier is suspected of existing in the ith observation, then A_2^T is replaced by $\underline{e}_i^T = (0, 0, \ldots, 1, 0, \ldots, 0)$, ith unit-vector, and the corresponding formula reduces to

$$R_1 - R = \frac{(e_i^T Q^{-1} \underline{\tilde{v}})^2}{\underline{e}_i^T Q^{-1} Q_{\underline{\tilde{v}}} Q^{-1} \underline{e}_i} . \qquad (5\text{-}32)$$

Now, depending on how the denominator in (5-30a) and (5-31b) is calculated, the test statistic for flagging the outlier may have the following cases:

(1) When the a priori variance factor σ_o is available, it results in the method of data snooping by Baarda (1968):

$$T = \frac{(e_i^T Q^{-1} \underline{\tilde{v}})^2}{\underline{e}_i^T Q^{-1} Q_{\underline{\tilde{v}}} Q^{-1} \underline{e}_i \cdot \sigma_o^2} > F(\alpha;\ 1,\ \infty) \quad . \tag{5-33}$$

(2) When the a posteriori variance factor $\hat{\sigma}_o$ is used, the test statistic can be either

$$T = (e_i^T Q^{-1} \underline{\tilde{v}})^2 / [(\underline{e}_i^T Q^{-1} Q_{\underline{\tilde{v}}} Q^{-1} e_i) \cdot \hat{\sigma}_o^{*2}] > F(\alpha;\ 1,\ df_1 - 1)\ , \tag{5-34}$$

or

$$T = (e_i^T Q^{-1} \underline{\tilde{v}})^2 / (e_i^T Q^{-1} Q_{\underline{\tilde{v}}} Q^{-1} e_i) \cdot \hat{\sigma}_o^2]$$

$$> df_1 / [1 + (df_1 - 1)\ F(1-\alpha;\ df_1 - 1, 1)] \tag{5-35}$$

where

$$\hat{\sigma}_o^2 = \underline{v}^T Q^{-1} \underline{\tilde{v}} / df_1\ ; \quad \hat{\sigma}_o^{*2} = (\underline{v}^T Q^{-1} \underline{\tilde{v}} - (R_1 - R)) / (df_1 - 1),$$

with $df_1 = n - r\{(\begin{smallmatrix}\tilde{A}_1 \\ \tilde{A}_2\end{smallmatrix})\}$

In addition, if the observations are uncorrelated, then (5-32) reduces to $(R_1 - R) = \tilde{v}_i^2 / q_{ii}$, where q_{ii} is the ith diagonal element of $Q_{\tilde{v}}$ and $\tilde{v}_i$ is the ith component of $\underline{\tilde{v}}$. The statistics (5-34) and (5-35) become the t-test proposed by Heck (1981), and the τ-test by Pope (1976), respectively; namely,

$$\sqrt{T} = \frac{\tilde{v}_i}{\sqrt{q_{ii}} \cdot \hat{\sigma}_o^*} > t(\tfrac{\alpha}{2};\ df_1 - 1) \quad , \tag{5-34'}$$

and

$$\sqrt{T'} = \frac{\tilde{v}_i}{\sqrt{q_{ii}} \cdot \hat{\sigma}_o} > \tau(\frac{\alpha}{2}; df_1) \quad . \tag{5-35'}$$

With independence among the observations $\underline{\ell}$ or between $\underline{\ell}_1$ and $\underline{\ell}_2$, the quantity $(R_1 - R)$ can be simplified as

$$(R_1 - R) = \tilde{\underline{v}}_2^T \, Q_{\underline{v}_2}^{-1} \, \tilde{\underline{v}}_2$$

which has χ^2-distribution with degrees of freedom equal to n_2 and is called the partial quadratic form of residuals, in accordance with the result of Stefanovic (1978).

It is clear from the above discussion that the generalized method for outlier detection has covered all the situations or, in other words, the existing techniques for outlier detection are embedded in this generalized method.

Another group of errors in observations is systematic errors, which are in general difficult to model and perhaps more difficult to detect than outliers (Niemeier et al., 1982). But it is extremely important to discover and eliminate them in order to ensure that the derived deformation parameters are not caused by systematic errors in the observations. Successful localization of the systematic errors depends largely on the experience of an analyst, who should be well acquainted with the instrumentation used in the data collection and field operation as well as the theory of errors. Statistical testing on the significance of suspected systematic errors can follow the method developed in Section 5.2.

Assuming that the observations $\underline{\ell}$, or part of them, are contaminated by systematic errors $\underline{y}$, then the mathematical model reads

$$E\{\underline{\ell}\} = A\underline{x} + D\underline{y} \quad , \quad D\{\underline{\ell}\} = \sigma^2 Q \quad . \tag{5-36}$$

The null hypothesis is H_o: $\underline{y} = \underline{0}$ versus the alternative hypothesis H_a: $\underline{y} \neq 0$. From Section 5.2, the test statistic can be established in two ways: from the adjustment of the full model (5-36), in other words, additional unknown parameters $\underline{y}$ are introduced to the adjustment; from the adjustment of the reduced model or the model without additional parameters.

If the estimated $\hat{\underline{y}}$ and its cofactor matrix $Q_{\hat{y}}$ are obtained from the adjustment of the full model, the test statistic can be derived from Table 5.1, case 2, and the following inequality holds in favour of the null hypothesis

$$T = \frac{\hat{\underline{y}}^T \, Q_{\hat{y}}^{-1} \, \hat{\underline{y}}/q}{\hat{\sigma}_o^2} \leq F(\alpha;\ q,\ df) \quad , \tag{5-37}$$

with $\hat{\sigma}_o^2$ being the a posteriori variance factor, q being the dimension of vector $\underline{y}$ and $df = n - r\{(A \mid D)\}$. This is a well known test on the estimated parameters.

If the adjustment of the reduced model is performed, the method for testing systematic errors is focused on the analysis of the residuals. The vector of residual $\tilde{\underline{v}}$ and its cofactor matrix $Q_{\tilde{v}}$ are obtained from the adjustment of the model without introducing additional unknown parameters $\underline{y}$. Thus, from Table 5.1, Case 3, the test statistic (5-37) can be equivalently set up as

$$T = \frac{\Delta R/q}{(R_1 - \Delta R)/df} \quad , \tag{5-38}$$

with $R_1 = \tilde{\underline{v}}^T Q^{-1} \tilde{\underline{v}}$, $\Delta R = (R_1 - R) = \tilde{\underline{v}}^T Q^{-1} D[D^T Q^{-1} Q_{\tilde{\underline{v}}} Q^{-1} D]^{-1} D^T Q^{-1}\tilde{\underline{v}}$.

Some systematic errors cannot be discovered through the adjustment procedure, e.g., scale error of an E.D.M. instrument in a free network, therefore other measures, such as regular calibration of instruments, should be taken to eliminate the systematic errors as much as possible.

At the end of this section an example is cited to illustrate the importance of the analysis of measuring errors. There is an ongoing debate about the reported aseismic uplift in southern California. As reported by the Committee on Geodesy and Committee on Seismology of the United States (1981), levelling data showed that a large region of southern California was uplifted by as much as 450 mm between 1960 and 1974, then subsided to its previous elevation after 1974. The implied tilts were in the order of several microradians, well above the claimed accuracy of precise levelling. However, some scientists have recently suggested that the apparent uplift may be the result of height-dependent systematic error. Regardless of the outcome of these discussions, it is clear that the effects of systematic errors need to be studied thoroughly.

5.4 Hypothesis Tests on Deformation Models

The deformation model describing the changes of a body in space and time is generally not well known. Therefore, the main interest of an analyst is to select an appropriate model from among many feasible ones.

Let $\underline{\ell}_i$ (i = 1, 2) be n_i-vectors of observations, $\underline{v}_i$ be vectors of residuals, A_i be the configuration matrices of $n_i \times u$, and $\underline{x}_i$ be u-vectors of coordinates of the points in a monitoring network. They are related by

$$\begin{pmatrix} \underline{\ell}_1 \\ \underline{\ell}_2 \end{pmatrix} + \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \end{pmatrix} = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \end{pmatrix} \quad , \tag{5-39}$$

with $D\{\underline{\ell}_i\} = \sigma^2 Q_i$. Here only two epochs of observations are considered, but the results can easily be extended to multiple epochs of observations (see Chapter 6).

Assume that the deformation model is $\underline{d} = (\underline{x}_2 - \underline{x}_1) = B \cdot \underline{c}$ where $\underline{c}$ is the vector of coefficients for the model and B is a proper matrix whose elements are functions of the positions of the points, as defined in Chapter 2. The deformation model is regarded as a null hypothesis and can be arranged in the form

$$H_o: \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \end{pmatrix} = \begin{pmatrix} I & 0 \\ I & B \end{pmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{c} \end{pmatrix} \quad , \tag{5-40}$$

or more compactly as

$$H_o: \underline{x} = U\, \underline{y} \quad . \tag{5-40'}$$

From the discussion in Section 5.2, the model (5-39) is regarded as the full model, and the model with the null hypothesis being imposed on it is the reduced one, which has the form

$$\begin{pmatrix} \underline{\ell}_1 \\ \underline{\ell}_2 \end{pmatrix} + \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \end{pmatrix} = \begin{pmatrix} A_1 & 0 \\ A_2 & A_2 B \end{pmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{c} \end{pmatrix} \quad . \tag{5-41}$$

From the adjustment of two modelss, (5-30a) or (5-30b) can be readily applied to test the appropriateness of the deformation model. The null Hypothesis is accepted if

$$T = \frac{R_1 - R}{R} \cdot \frac{df}{df_1 - df} \leq F(\alpha;\ df_1 - df,\ df)$$

where R and R_1 are the quadratic form of the residuals from the model (5-39) and (5-41) respectively; and

$$df = n_1 + n_2 - r\{A_1\} - r\{A_2\}\ , \tag{5-42a}$$

$$df_1 = n_1 + n_2 - r\{\begin{pmatrix} A_1 \\ A_2 \end{pmatrix}\} - n_{\underline{c}} \tag{5-42b}$$

with $n_{\underline{c}}$ being dimension of $\underline{c}$.

If the adjustment is performed only to the full model (5-39), $(R_1 - R)$ can be calculated from the formula in Table 5.1, Case 2.

In order to apply the statistic developed in Section 5.2, the null hypothesis H_o in (5-40') should be altered to the form $H\underline{x} = 0$. It is well known that the general solution to $H\underline{x} = 0$ is $\underline{x} = U\ \underline{y}$, where U is a matrix of rank $r\{U\} = 2u - r\{H\}$ such that $HU = 0$ (Rao and Mitra, 1971) and $\underline{y}$ is a vector having dimension $r\{U\}$, if U is of order $2u \times r\{U\}$. One representation of H is $H = (I - U(U^TWU)^- U^TW)$ with W being an arbitrary matrix such that $r\{U^TWU\} = r\{U\}$, which is easy to establish by verification of $HU=0$. Therefore, the null hypothesis (5-40') is equivalent to

$$H_o:\ H\underline{x} = 0 \quad \text{with } H = (I - U(U^TWU)^- U^TW)\quad . \tag{5-40"}$$

Thus the quantity $\Delta R = R_1 - R$ is computable from

$$(H\hat{\underline{x}})^T[H(A^TQ^{-1}A)^-H^T]^-H\hat{\underline{x}}\quad , \tag{5-43}$$

where $\hat{\underline{x}} = (\hat{\underline{x}}_1^T \ \vdots\ \hat{\underline{x}}_2^T)$, $A = \mathrm{diag}\ \{A_1,\ A_2\}$ and $Q = \mathrm{diag}\ \{Q_1,\ Q_2\}$.

In (5-40"), H can be considered as a projection operator onto the orthogonal complement of the space S(U). Hence, $H\hat{\underline{x}}$ is the projection of $\underline{\hat{x}}$, denoted by $\hat{\underline{v}}_x$ onto this space and $H(A^TQ^{-1}A)^-H^T$ is the cofactor matrix of $\hat{\underline{v}}_x$. Therefore, ΔR is equivalent to the quadratic form of the residulas in the following quasi-observation equations with weight matrix being diag $\{N_1, N_2\}$:

$$\begin{pmatrix} \underline{x}_1 \\ \\ \underline{x}_2 \end{pmatrix} + \begin{pmatrix} \underline{v}_{x_1} \\ \\ \underline{v}_{x_2} \end{pmatrix} = \begin{pmatrix} I & 0 \\ \\ I & B \end{pmatrix} \begin{pmatrix} \underline{x}_1 \\ \\ \underline{c} \end{pmatrix} \quad \text{with } P_x = \begin{pmatrix} N_1 & 0 \\ \\ 0 & N_2 \end{pmatrix} \quad . \tag{5-44}$$

that is,

$$\Delta R = \hat{\underline{v}}_{x1}^T N_1 \hat{\underline{v}}_{x1} + \hat{\underline{v}}_{x2}^T N_1 \hat{\underline{v}}_{x2} \quad . \tag{5-45}$$

So far the statistic for testing the deformation model is established. Some special cases are discussed below.

Suppose that it is to be tested that the network is congruent during two epochs of time, then the null hypothesis becomes $\underline{x}_1 = \underline{x}_2$ or $\underline{c} = 0$. Using (5-30a), one obtains the statistic

$$T = \frac{\Delta R}{R} \cdot \frac{df}{n_x} \overset{d}{\sim} F(n_x, df) \quad , \tag{5-46}$$

with

$$n_x = r\{A_1\} + r\{A_2\} - r\left\{ \begin{pmatrix} A_1 \\ \cdots \\ A_2 \end{pmatrix} \right\} \quad .$$

Using the model (5-44) with $\underline{c} = 0$, ΔR can be calculated from (5-45) as:

$$\begin{aligned} \Delta R &= \hat{\underline{v}}_{x_1}^T N_1 \hat{\underline{v}}_{x_1} + \hat{\underline{v}}_{x_2}^T N_2 \hat{\underline{v}}_{x_2} \\ &= (\hat{\underline{x}}_2 - \hat{\underline{x}}_1)^T (N_1(N_1 + N_2)^- N_2)(\hat{\underline{x}}_2 - \hat{\underline{x}}_1) \quad . \end{aligned} \tag{5-47}$$

If $\hat{\underline{x}}_1$, $\hat{\underline{x}}_2$ and $Q_{\hat{\underline{x}}1}$, $Q_{\hat{\underline{x}}2}$ are referred to the same datum, using the properties of the parallel sum of N_1 and N_2 (Rao and Mitra, 1971), (5-47) reduces to

$$\Delta R = (\hat{\underline{x}}_2 - \hat{\underline{x}}_1)^T (Q_{\hat{\underline{x}}_1} + Q_{\hat{\underline{x}}_2})^- (\hat{\underline{x}}_2 - \hat{\underline{x}}_1) \quad . \tag{5-48}$$

Consequently, the global congruency test (5-46) for this case becomes:

$$T = \frac{(\hat{\underline{x}}_2 - \hat{\underline{x}}_1)(Q_{\hat{\underline{x}}_1} + Q_{\hat{\underline{x}}_2})^-(\hat{\underline{x}}_2 - \hat{\underline{x}}_1)}{(R/df) \cdot r\{Q_{\hat{\underline{x}}_1}\}} \overset{d}{\sim} F(r\{Q_{\hat{\underline{x}}_1}\}, df)$$

which was first derived by Pelzer (1971).

ΔR can also be computed from the simultaneous adjustment of two epochs of observations. In the adjustment, two epochs of observations are merged for the one total adjustment with all the points being given, firstly, double designations except for the points used as minimum constraints, and then single designation. Two quadratic forms of the residuals R and R_1 are obtained from these two adjustments, respectively. Then $\Delta R = R_1 - R$.

One may be interested in testing the partial congruency of a network. Let us divide the network into two parts, part 1 and part 2. The vector of the coordinates is conformably denoted by $\underline{x}_1^T = (\underline{x}_{11}^T \mid \underline{x}_{12}^T)$ and $\underline{x}_2^T = (\underline{x}_{21}^T \mid \underline{x}_{22}^T)$. Then the deformation model for the congruency of part 1 becomes $\underline{x}_{11}^T = \underline{x}_{21}^T$. The corresponding null hypothesis reads:

$$H_o: \begin{pmatrix} \underline{x}_{11} \\ \underline{x}_{12} \\ \underline{x}_{21} \\ \underline{x}_{22} \end{pmatrix} = \begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ I & 0 & 0 \\ 0 & I & I \end{pmatrix} \begin{pmatrix} \underline{x}_{11} \\ \underline{x}_{12} \\ --- \\ \underline{c} \end{pmatrix} \quad .$$

The hypothesis tests on this deformation model follow the same procedures as the above.

Since the status of a deformed body is not well defined, different deformation models may be applied. Therefore, the question arises as to which model is the "best". There exist diverse criteria for the "best". Here we restrict ourselves to the criterion concerning the statistical test. More will be said in Chapter 6.

Assume that there are two models to be discriminated. One is of the form $B_1\underline{c}_1$ and the other $(B_1 \mid B_2)(\underline{c}_1^T \mid \underline{c}_2^T)$. Thus, the model (5-45) is rewritten as

$$\begin{pmatrix} \underline{x}_1 \\ \\ \underline{x}_2 \end{pmatrix} + \begin{pmatrix} v_{\underline{x}_1} \\ \\ v_{\underline{x}_2} \end{pmatrix} = \begin{pmatrix} I & 0 & 0 \\ \\ I & B_1 & B_2 \end{pmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{c}_1 \\ \underline{c}_2 \end{pmatrix} \text{ with } P_x = \begin{pmatrix} N_1 & 0 \\ \\ 0 & N_2 \end{pmatrix} . \quad (5\text{-}49)$$

The null hypothesis H_o: $\underline{c}_2 = \underline{0}$ is to be tested against the alternative hypothesis H_a: $\underline{c}_2 \neq \underline{0}$. From the results in Section 5.2, the quadratic form of the residuals $\tilde{\underline{v}}_x$ in the reduced model, denoted by R_{1x}, can be decomposed into two statistically independent parts: R_x, calculated from the full model (5-49), and

$$\Delta R_x = R_{1x} - R_x = \hat{\underline{c}}_2^T \, Q_{\hat{\underline{c}}_2}^{-1} \, \hat{\underline{c}}_2 \text{ with } \hat{\underline{c}}_2 \text{ and } Q_{\hat{\underline{c}}_2}$$

being the estimators and their cofactor matrix. Two statistics on the deformation models can be evaluated. First, the estimated parameters $\hat{\underline{c}}_2$ are significant at the $(1 - \alpha)$ level of confidence if

$$T = \frac{\Delta R_x}{R} \cdot \frac{df}{n_{c_2}} \geq F(\alpha; n_{c_2}, df) \quad , \qquad (5\text{-}50)$$

where R, df have been previously defined, and $n_{\underline{c}_2}$ is the number of coefficients $\underline{c}_2$. Second, introducing $\underline{c}_2$ into the deformation model will improve the modelling significantly if

$$T = \frac{\Delta R_x}{R_x} \cdot \frac{n_c}{n_{c_2}} \geq F(\alpha;\, n_{c_2},\, n_c) \quad , \qquad (5\text{-}51)$$

with $n_{\underline{c}}$ being the total number of coefficients $\underline{c}$.

Thus, through a series of hypothesis testing the postulated deformation models can be distinguished.

5.5 Discussion on the Selection of the Significance Level

A practical problem in hypothesis testing is the choice of the significance level α of the test. It has become customary in most cases in geodesy to use a fixed value of $\alpha = 0.05$ or $\alpha = 0.01$, say, but there is no unique way of fixing α. Generally, it is a matter of personal intuition to choose the significance level.

An attempt has been made to connect a global test or f-dimensional F test with a one-dimensional test by matching their significance levels α and α_o for the f-dimensional and one-dimensional tests respectively, so that the decision will be consistent in both tests. From the point of view of testing strategy, the global test seems a preliminarily diagnostic tool to find whether the one-dimensional test is worth trying; and the one-dimensional test is used to localize the component which is responsible for the failure of the global test. The test on the a posteriori variance factor $\hat{\sigma}_o^2$ versus the a priori one is a global test, but testing an individual residual for outlier detection is one dimensional; checking the significance of the estimated deformation parameters on the whole is of a global nature, but examining an individual parameter belongs to the one-dimensional test. There are several schools of thought on these aspects (see Miller

(1966)). Three methods can be found in surveying engineering applications.

The B-method proposed by Baarda (Baarda, 1968; van Mierlo, 1977; Kok, 1977) is to relate the significance level α for the f-dimensional F-test to the characteristic value $\lambda(\alpha_o, \beta_o, 1, \infty)$ of a one-dimensional test by choosing a common power of the test $\beta = \beta_o$ and keeping the non-centrality parameter λ constant for both types of test. Therefore, the significance levels α and α_o become interdependent, symbolically denoted as

$$\lambda = \lambda(\alpha_o, \beta, 1, \infty) = \lambda(\alpha, \beta, f, \infty) \quad . \qquad (5\text{-}52)$$

Given α_o and β, α can be calculated through the common λ.

The B-method was originally created to relate the probability of the test statistic for the global test on the a posteriori variance factor $\hat{\sigma}_o^2$ to the probability of the test statistic used in the data snooping strategy for outlier detection. The values $\alpha_o = 0.001$, $\beta = 0.80$ are suggested and the nomograms for the relation (5-52) are given in Baarda (1968). The extension of the basic idea to the general $F(\alpha; f_1, f_2)$ test is claimed by Heck (1982).

The S-method proposed by Scheffé (1959) is based on the simultaneous confidence region. Let $\underline{\hat{\theta}}$ be an f-vector of the least-squares estimates of $\underline{\theta}$, $\hat{\Sigma}_{\hat{\underline{\theta}}}$ be their covariance matrix, and $\hat{\sigma}_o^2$ be the a posteriori variance factor with degrees of freedom being df. The confidence ellipsoid is

$$(\underline{\hat{\theta}} - \underline{\theta})^T \hat{\Sigma}_{\hat{\underline{\theta}}}^{-1} (\underline{\hat{\theta}} - \underline{\theta}) \le f \, \hat{\sigma}_o^2 \, F(\alpha; f, df) \quad . \qquad (5\text{-}53)$$

The probability is $(1 - \alpha)$ that simultaneously for all the functions of $\underline{\theta}$, say any function ψ

$$\hat{\psi} - s\,\hat{\sigma}_{\hat{\psi}} \leq \psi \leq \hat{\psi} + s\,\hat{\sigma}_{\hat{\psi}} \quad , \tag{5-54}$$

where the constant s is

$$s = [f\, F(\alpha;\ f,\ df)]^{1/2} \quad . \tag{5-55}$$

The Bonferroni method is based on the first-order Bonferroni bounds. Denoting $[(\hat{\theta}_i - \theta_i)/\hat{\sigma}_\theta] = w_i$, the Bonferroni inequality tells us that the events $\{|w_i| < \xi,\ \forall\ i\}$ have the probability (Cook and Prescott, 1981):

$$(1 - \sum_i \alpha_i - \sum_{i<j} \alpha_{ij}) \geq \Pr\{|w_i| < \xi,\ \forall\ i\} \geq (1 - \sum_i \alpha_i) \quad , \tag{5-56}$$

if $\alpha_i = \Pr\{|w_i| \geq \xi\}$ and $\alpha_{ij} = \Pr\{|w_i| > \xi,\ |w_j| > \xi\}$ $(i \neq j)$. The equality sign holds for uncorrelated $\underline{\hat{\theta}}$. It may be required that the probability for a simultaneous test is $(1 - \alpha)$, then the significance level α_o for the individual test can be obtained from the relation:

$$\alpha \leq \sum_i \alpha_o \quad . \tag{5-57}$$

In practice, $\alpha = \sum_i \alpha_o$ is usually taken. The maximum τ-test for outlier detection by Pope (1976) is based on this method.

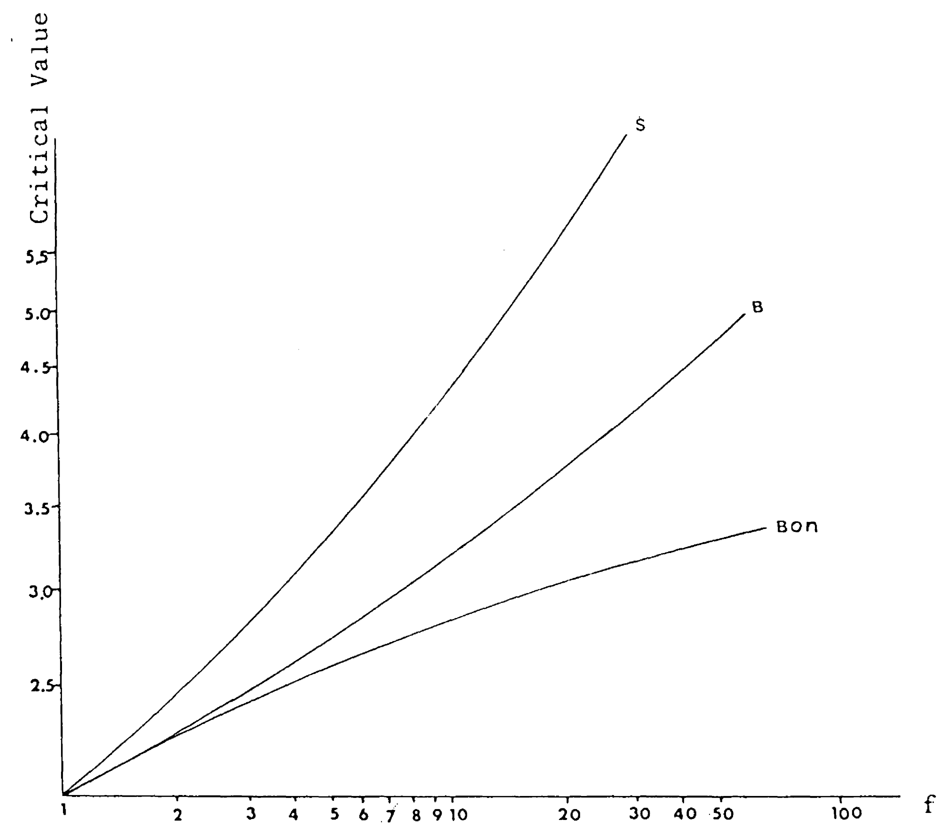
Because of the correlation between the parameters $\underline{\hat{\theta}}$ to be tested, the upper and lower Bonferroni bounds never coincide theoretically. But the practical usefulness of the lower bounds has been exploited by many statisticians (e.g., Quenouille, 1952; Stefansky, 1972; Cook and Prescott, 1981). Many simulation studies have shown that the upper and lower Bonferroni bounds differ little (John and Prescott, 1975).

In order to get rid of the correlation between the parameters $\underline{\hat{\theta}}$, an attempt is made to transform the correlated $\underline{\hat{\theta}}$ into a new set of uncorrelated parameters and then perform the statistical test on the new parameters. However, this does not make much sense, especially in deformation analysis, unless the transformed parameters have definite physical meaning and can be used to reply to the question asked in performing the statistical test.

The three methods are based on different philosophies. The B-method incorporates type I error and type II error, but the other two do not. The S-method projects a hyperellipsoid onto a subspace, while the Bonferroni method computes the lower bound of the joint probability. Figure 5.2 shows the relation between the critical value of the one-dimensional test for three methods and the degrees of freedom f in the multi-dimensional test when α is fixed.

Regarding the selection of the significance level, it is the philosophy of the author that different tests have different purposes. In deformation analysis, the global test may serve the investigation of global phenomena, but a one- or two-dimensional test is used to study the local behaviour of the deformable body. In many cases it may happen that the global test on all the points together fails at a certain significance level α, but the test on the individual points may pass even at the same significance level because of the existence of a deformation trend; conversely, the real movement of a single point may not be easily discovered by the global test unless a large significance level α is selected. Analogously, examining the deformation parameters as a whole may indicate significance, while not all the parameters may do so individually. Therefore, no attempt is made in this thesis to relate the one-dimensional test to the multi-dimensional test. Instead a fixed value of $\alpha = 0.05$ or $\alpha = 0.01$ is used.

At the end of this chapter it should be emphasized that the



S - S-Method

B - B-Method

Bon - Bonferroni Method

Fig. 5.2

The Relation Between the Critical Value of One-Dimensional Test and the Degrees of Freedom f in the f-Dimensional Test ($\alpha = 0.05$) for the Three Methods.

statistical test is a powerful tool in deformation analysis, but it is not a panacea generating all required remedies automatically. The logical judgement of a skillful analyst is always necessary. Blindly performing the statistical test without realizing the problem fully can never be condoned.

CHAPTER 6

A GENERALIZED APPROACH TO DEFORMATION ANALYSIS

6.1 Basic Philosophies, Criteria, and Procedures of the Approach

As mentioned in Chapter 1, data from deformation surveys have contributed to the understanding of the deformation mechanism and deformation processes. It is important to analyse the acquired data thoroughly for several reasons. First, the data may contain important scientific findings. Sometimes these findings are not evident in a cursory inspection. Second, different types of analyses may suggest important generalizations. Third, when new experiments are planned, the lessons learned from previous experiments should be used. The Committee on Geodesy and the Committee on Seismology of the United States (1981) have made an important recommendation: "theoretical studies and analysis of existing data should be given much higher priority than they currently enjoy."

Regarding the data, on the one hand it should be recognized that data acquisition is expensive and any observation, geodetic or physical-mechanical measurement, will contribute to the determination of deformation parameters. Therefore they should be fully utilized in the analysis. The approach based on direct comparison of raw observations may not take full advantage of the data unless the same observation scheme remains in all epochs. On the other hand, it should be recognized that the data, besides random measuring errors, may contain gross and systematic errors. Identification and elimination of them should be made prior to the estimation of deformation parameters in order to avoid misinterpreting measuring errors as deformation phenomena. As pointed out in Chapter 2, the deformation parameters could be directly estimated from the observations. The approach based on this idea, referred to as the simultaneous reduction method of strain analysis (Bibby, 1981), or the simultaneous adjustment of network and strain parameters, might make it more difficult to distinguish between the errors in the observations and the errors in the deformation model.

Regarding the construction of the deformation model, it should be recognized that the deformation model is usually not fully understood or it may even be completely unknown. Careful identification and checking of the models is necessary in order to avoid misinterpreting local deformation phenomena (e.g., a single point movement) as a global deformation. By directly setting a postulated model to the data one may miss an important deformation pattern. It should also be recognized that the deformation analysis, in many cases, is to answer some fundamental questions, not just to fit any polynomial to the data in order to use the data in the form of a function (rather than use tabulated data). Therefore, statistical testing should be performed on those parameters which have definite physical meaning, not on something else. This problem has been mentioned in Chapter 5.

Based on the above, the basic criteria and basic procedures of the approach are established.

In developing the generalized approach, the following criteria are set up.

(a) The approach should be applicable to any type of deformation, i.e., the same computational procedure should be used in the analysis of single point displacements in reference networks and in the analysis of rigid body displacements as well as in the determination of strain components in relative survey networks.

(b) The same approach should be used for one-, two-, and three-dimensional survey data for the determination of deformation parameters either in a local domain or in a time domain, or both.

(c) Any type of survey data, i.e., not only geodetic (distances, angles, etc.) but also physical-mechanical measurements of tilts, strains, pendula deviations, etc., should be utilized in a simultaneous analysis as long as the differences in the observed or quasi-observed (e.g., derived coordinates) quantities could be expressed as functions of relative displacements of the points at which the measurements were made.

(d) The approach should be applicable to any geometrical configuration of the survey network including incomplete networks with configuration defects. These isolated observations, which are not connected to other points of the network, would be taken as long as approximate coordinates of all survey stations are given in the same coordinate system.

(e) Different minimum constraints (including inner constraints) could be used in the numerical processing of each epoch of observations as long as the same approximate coordinates of points are used in each of the epochs.

Generally, the approach consists of three basic processes.

(1) Preliminary identification of deformation models.

(2) Estimation of the deformation parameters.

(3) Diagnostic checking of the models and the final selection of the "best" model.

Identification procedures are rough methods applied to a set of data to indicate the kind of deformation that warrants further investigation. The process having led to a tentative formulation of the deformation trend, then efficient estimates of deformation parameters or the coefficients of the model are obtained using the least-squares estimation techniques. After the parameters have been estimated, diagnostic checks are performed to determine the adequacy of the fitted model or to indicate potential improvements. Of course, the identification, estimation and diagnostic checking of the deformation model are not independent procedures, so they necessarily overlap. Therefore, these methods should be performed as an iterative three-step procedure.

6.2 Mathematical Model

As described in Chapter 2, a deformation model acts as a medium which links together the observed quantities on one side and the deformation parameters on the other side. In order to connect the observations to the deformation model, we can either

(a) express all the observed quantities in terms of the coordinates, even in the case of there being configuration defects in the network, and then transform the differences of the coordinates into the deformation model; or

(b) express each difference of the observed quantities in terms of the deformation model.

Let $\underline{\ell}_i$ be the n_i-vector of observations in epoch i (i=1,...,k), $\underline{x}_i$ be the u_i-vector of coordinates, and A_i be the $n_i \times u_i$ configuration matrix. In addition, assume that the deformation model is of the form $\underline{d}(x, y, z; t_i - t_1) = B(x, y, z; t_i - t_1)\underline{c}$ or, in short, $\underline{d}_i = B_i\underline{c}$, where $\underline{d}(x, y, z; t_i - t_1)$ is the vector of displacements at time t_i with respect to t_1; $B(x, y, z; t_i - t_1)$ is the matrix of the deformation model whose elements are functions of position and time; $\underline{c}$ is the vector of coefficients of the model to be estimated. Then the first case is symbolically represented by

$$E\{\underline{\ell}_i\} = A_i\underline{x}_i = A_i\underline{x}_1 + A_i B_i\underline{c} \quad , \tag{6-1}$$

and the second case by

$$E\{\underline{\ell}_i\} = E\{\underline{\ell}_1\} + A_i B_i\underline{c} \quad . \tag{6-2}$$

The approach based on (6-1) should be called the "coordinate approach" and that based on (6-2) the "observation approach". Using the principle of least squares, the coefficients $\underline{c}$ of the deformation model can be estimated from (6-1) or (6-2).

However, it has been emphasized in Chapter 2 and the beginning of this chapter that it is worthwhile to separate into two parts, if possible, the mathematical model which relates the observations to the deformation parameters. The first part is an adjustment model of a geodetic monitoring network for screening the observations for outliers and systematic errors as well as for statistical evaluation of the quality of the observations. In addition, the displacements calculated from the adjustment will give a better picture for identification of the deformation pattern. In the second part, transformation of the coordinate differences of the points into the deformation model is formulated.

6.2.1 The model of the adjustment of observations

Consider the n_i-vector of observations $\underline{\ell}_i$ in epoch i (i=1,2,...,k) such that

$$\begin{pmatrix} \underline{\ell}_1 \\ \underline{\ell}_2 \\ \vdots \\ \underline{\ell}_k \end{pmatrix} + \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \\ \vdots \\ \underline{v}_k \end{pmatrix} = \begin{pmatrix} A_{\eta_1} & & & & A_1 & & & \\ & A_{\eta_2} & & & & A_2 & & \\ & & \ddots & & & & \ddots & \\ & & & A_{\eta_k} & & & & A_k \end{pmatrix} \begin{pmatrix} \underline{\eta}_1 \\ \vdots \\ \underline{\eta}_k \\ --- \\ \underline{x}_1 \\ \vdots \\ \underline{x}_k \end{pmatrix} \qquad (6\text{-}3)$$

where $\underline{v}_i$ is the n_i-vector of residuals, $\bar{A}_i = (A_{\eta i} \mid A_i)$ is the $n_i \times u_i'$ configuration matrix, $\underline{x}_i$ is the u_i-vector of coordinates, and $\underline{\eta}_i$ is the vector of nuisance parameters (e.g., the orientation unknowns in direction observations, unknown scale factor in the EDM measuring distances). More compactly, (6-3) can be written as

$$\underline{\ell} + \underline{v} = (A_\eta \mid A) \begin{pmatrix} \underline{\eta} \\ \underline{x} \end{pmatrix} \qquad (6\text{-}3')$$

Its dispersion matrix reads

$$D\{\underline{\ell}\} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \cdots \Sigma_{1k} \\ \Sigma_{21} & \Sigma_{22} \cdots \Sigma_{2k} \\ \cdot & \\ \cdot & \\ \Sigma_{k1} & \Sigma_{k2} \cdots \Sigma_{kk} \end{pmatrix} = \sigma_o^2 \begin{pmatrix} Q_{11} & Q_{12} \cdots Q_{1k} \\ Q_{21} & Q_{22} \cdots Q_{2k} \\ \cdot & \\ \cdot & \\ Q_{k1} & Q_{k2} \cdots Q_{kk} \end{pmatrix} \quad (6\text{-}4)$$

where Σ_{ij} and Q_{ij} are the covariance matrix and cofactor matrix between the observations in epoch i and epoch j, respectively. The correlations between the observations not only in the same epoch but in different epochs may exist because of the use of the same instruments and the existence of similar environmental conditions. In order to provide the stochastic model (6-4), a technique to assess multi-epoch observations has been discussed in Chapter 3.

The model (6-3) can be solved using the least-squares adjustment technique. If $\Sigma_{ij} \neq 0$ $(i \neq j)$, simultaneous adjustment of multiple epoch observations is required. However, if $\Sigma_{ij} = 0$, which is the customary assumption in present practice because the study of correlations between the observations has just started, then the solution to the model (6-3) breaks down into a separate adjustment for each epoch of observations. The different methods of the least-squares adjustment technique are well synthesized in Krakiwsky (1975). But, the rank deficiency in the configuration matrix A_i calls for more discussion, which has been done in Chapter 4.

It is possible that Σ_{ii} is singular due to the existence of some functional relations among the observations. Then the solution in the sense that the best estimator of any estimable function $\underline{b}^T\underline{x}$ is $\underline{b}^T\hat{\underline{x}}$ is $\hat{\underline{x}} = (A_i^T Q_{ii}^- A_i)^- A_i^T Q_{ii}^- \underline{\ell}_i$ only when the condition that $S(A_i) \subset S(Q_{ii})$ is fulfilled (Rao, 1973). The singular dispersion matrix of observations seldom occurs in the adjustment of networks, hence the problem will not be fully discussed here but later, where the singular dispersion matrix of quasi-observations appears in the estimation of deformation parameters.

Following the present practice, a separate adjustment of each epoch of observations is performed. The normal equation for epoch i after the nuisance parameters are eliminated has the form:

$$N_i \underline{x}_i = \underline{w}_i \quad . \tag{6-5}$$

Using the method developed in Chapter 4, any specific solution $\hat{\underline{x}}_i$ and its cofactor matrix $Q_{\hat{\underline{x}}_i}$ are obtained. It has been proved that $Q_{\hat{\underline{x}}_i}$ is a reflexive inverse of N_i, that is, $r\{Q_{\hat{\underline{x}}_i}\} = r\{N_i\}$ and $Q_{\hat{\underline{x}}_i} \cdot N_i Q_{\hat{\underline{x}}_i} = Q_{\hat{\underline{x}}_i}$. Because $Q_{\hat{\underline{x}}_i}$ is singular, the conventional weight matrix of $\hat{\underline{x}}_i$, defined as $Q_{\hat{\underline{x}}_i}^{-1}$, does not exist. However, whatever the solution $\hat{\underline{x}}_i$ and $Q_{\hat{\underline{x}}_i}$ is, N_i is one of the g-inverses of $Q_{\hat{\underline{x}}_i}$. Therefore, we define N_i as the "weight matrix" of $\hat{\underline{x}}_i$. In addition, a monitoring network may be incomplete, suffering from configuration defects, N_i in (6-5) will have more rank defects if all the observations are entered into the formulation of the normal equation (6-5). The "weight matrix" for any solution $\hat{\underline{x}}_i$ is also defined as N_i for the same reason as above. It should be mentioned that the "weight matrix" here does not have physical meaning, and is used only for numerical treatment in the second step-- the computation of deformation parameters.

Another way to look at the problem of "weight matrix" is through the direct estimation of the coefficients of the deformation model from (6-1). Assuming that outliers and systematic errors have been excluded from the observations $\underline{\ell}_i$ and the cofactor matrix of $\underline{\ell}_i$ is Q_{ii}, then the normal equation for the model (6-1) reads

$$\begin{pmatrix} \sum_1^k N_i & \sum_2^k N_i B_i \\ \sum_2^k B_i^T N_i & \sum_2^k B_i^T N_i B_i \end{pmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{c} \end{pmatrix} = \begin{pmatrix} \sum_1^k \underline{w}_i \\ \sum_2^k B^T \underline{w}_i \end{pmatrix} \tag{6-6}$$

where $N_i = A_i^T Q_{ii}^{-1} A_i$, $\underline{w}_i = A_i^T Q_{ii}^{-1} \underline{\ell}_i$. Equation (6-6) is also the normal equation for the model:

$$E\{\hat{\underline{x}}_i\} = E\{\hat{\underline{x}}_1\} + B_i \underline{c} \quad , \tag{6-7}$$

with the "weight matrix" of $\hat{\underline{x}}_i$ being N_i.

In the adjustment step, the outliers and systematic errors are detected using the techniques discussed in Chapter 5. The outcomes are the vector of estimated coordinates $\hat{\underline{x}}_i$ and its "weight matrix" N_i or cofactor matrix $Q_{\hat{\underline{x}}_i}$.

6.2.2 The model of the estimation of deformation parameters

Let $\underline{y}_i$ (i=1,2,...,k) be the vector of observations in epoch i, including quasi-observations (e.g., the coordinates of points from an adjustment), physical-mechanical measurements and individual geodetic observations, and P_i be the weight matrix of $\underline{y}_i$. The weight matrix for the coordinates of points has been discussed in Section 6.2.1, and for the other observed quantities it is taken in the conventional way as the inverse of the covariance matrix. Because of datum and configuration defects in a monitoring network, the weight matrix P_i is in general considered singular (Chrzanowski et al., 1982a). The process of the least-squares determination of the coefficients of a deformation model,

$\underline{d}(x, y, z; t - t_1) = B((x, y, z; t - t_1)\underline{c}$, is based on the null hypothesis:

$$H_o: E\{\underline{x}_i\} = E\{\underline{x}_1\} + B(x, y, z; t_i - t_1) \cdot \underline{c} \quad ,$$

with $\underline{x}_i$ being the vector of coordinates of the points at epoch i. For the individual observation, it can be either transferred to the coordinates or expressed in the form of (6-2) depending on the problem on hand. This will be further discussed in Section 6.4. For the other physical-mechanical measurements, the expressions for this hypothesis have been derived in Chapter 2. In general, the above null hypothesis leads to the functional relation:

$$\begin{pmatrix} \underline{y}_1 \\ \underline{y}_2 \\ \\ \underline{y}_k \end{pmatrix} + \begin{pmatrix} \underline{\delta}_1 \\ \underline{\delta}_2 \\ \\ \underline{\delta}_k \end{pmatrix} = \begin{pmatrix} I & 0 \\ I & \bar{B}_2 \\ & \cdot \\ I & \bar{B}_k \end{pmatrix} \begin{pmatrix} \underline{\xi} \\ \underline{c} \end{pmatrix} \quad , \tag{6-8}$$

with weight matrix

$$P = \text{diag}\{P_1, P_2, \ldots, P_k\} \quad , \tag{6-9}$$

where the matrix $\bar{B}_i$ is a function of the position of points and time, but may differ from the matrix B in the deformation model depending on the type of $\underline{y}_i$ (coordinate or individual geodetic observation, or physical-mechanical measurements, or both); $\underline{\xi}$ is a vector of unknown constants; and $\underline{\delta}_i$ is a vector of residuals at epoch i. In order to keep the same population of vector $\underline{y}_i$ in each epoch, dummy observations with zero weights will be put in the place of the missing observations in the vector $\underline{y}_i$. Applying the principle of least squares to model (6-8), the normal equation reads:

$$\begin{pmatrix} \sum_{1}^{k} P_i & \sum_{2}^{k} P_i \tilde{B}_i \\ \sum_{2}^{k} \tilde{B}_i^T P_i & \sum_{2}^{k} \tilde{B}_i^T P_i \tilde{B}_i \end{pmatrix} \begin{pmatrix} \underline{\xi} \\ \underline{c} \end{pmatrix} = \begin{pmatrix} \sum_{1}^{k} P_i \underline{y}_i \\ \sum_{2}^{k} \tilde{B}_i^T P_i \underline{y}_i \end{pmatrix} \quad . \tag{6-10}$$

The coefficient matrix of the normal equation (6-10) is singular with rank defects $rd\{\sum_{1}^{k} p_i\} = d$, equal to the number of remaining datum defects and configuration defects which are not determined in at least one epoch. The number and types of defects can be figured out by purely geometrical consideration. Eliminating $\underline{\xi}$ from (6-10) allows the vector $\underline{c}$ and its accuracy to be calculated from

$$\hat{\underline{c}} = [\sum_{2}^{k} \tilde{B}_i^T P_i \tilde{B}_i - \sum_{2}^{k} \tilde{B}_i^T P_i (\sum_{i}^{k} P_i)^- \sum_{2}^{k} P_i \tilde{B}_i]^{-1}$$

$$[\sum_{2}^{k} \tilde{B}_i^T P_i \underline{y}_i - \sum_{2}^{k} \tilde{B}_i^T P_i (\sum_{1}^{k} P_i)^- \sum_{1}^{k} P_i \underline{y}_i] \tag{6-11a}$$

and

$$\hat{\Sigma}_{\hat{\underline{c}}} = \hat{\sigma}_o^2 [\sum_{2}^{k} \tilde{B}_i^T P_i \tilde{B}_i - \sum_{2}^{k} \tilde{B}_i^T P_i (\sum_{1}^{k} P_i)^- \sum_{2}^{k} P_i \tilde{B}_i]^{-1} \quad . \tag{6-11b}$$

Because the space generated by the column vector of the matrix $\sum_{2}^{k} P_i \tilde{B}_i$ is a subspace of that generated by $\sum_{1}^{k} P_i$, $\hat{\underline{c}}$ is invariant of any choice of the g-inverse of $(\sum_{1}^{k} P_i)$.

However, how to select the deformation models, what strategies should be taken in the formulation and computation of (6-11), and how to test the appropriateness of the selected model are the topics of the next three sections.

6.3 Preliminary Identification of Deformation Models

6.3.1 Deformation Models

A preliminary deformation model may come from

(a) an a priori knowledge, either assumed or expected from previous experience, of the behaviour of the deformable body;

(b) a graphical demonstration of the displacement field in the space domain or of displacements versus time.

When using the generalized approach in deformation analysis, the whole area covered by the deformation surveys is treated as a noncontinuous deformable body consisting of separate continuous deformable blocks. Thus the blocks may undergo relative rigid body displacements and rotation, and each block may change its shape and dimensions. In the case of single point movement, the given point is treated as a separate block being displaced as a rigid body in relation to the undeformed block composed of the remaining points in the network.

Deformation parameters and deformation models in the general case have been discussed in Chapter 2. For illustration, the situations in two-dimensional space are repeated here with some examples of typical deformation models. For each block the following deformation parameters in an x, y coordinate system will be considered: two components (a_o and b_o or c_o and g_o, etc.) of the rigid body displacement; rotation parameter $\omega(x, y)$; extension strain components $\varepsilon_x(x, y)$ and $\varepsilon_y(x, y)$; and shearing strain, $\varepsilon_{xy}(x, y)$.

The deformation of a block is completely determined by a displacement function $\underline{d}(x, y)$. If its two components $u(x, y)$ and $v(x, y)$, in x, y direction respectively, are given, then the strain components and differential rotation at any point can be calculated from the well-known infinitesimal strain-displacement relation (2-4) and (2-5):

$$\varepsilon_x = \frac{\partial u}{\partial x} \quad , \quad \varepsilon_y = \frac{\partial v}{\partial y}$$
$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \quad , \quad \omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad (6\text{-}12)$$

The displacement function is usually complicated, therefore it is approximated by a polynomial. From (2-13), a simplified polynomial in two-dimensional space can be written as

$$u(x, y) = \sum_{ij}\sum a_{ij} x^i y^j \qquad (6\text{-}13a)$$

$$v(x, y) = \sum_{ij}\sum b_{ij} x^i y^j \quad . \qquad (6\text{-}13b)$$

Depending on the selected deformation model, some of the coefficients in the polynomial (6-13a) and (6-13b) will vanish. Examples of typical deformation models are given below.

(a) Single point movement or a rigid body movement (Figure 6.1a) of a group of points (say, block B) with respect to a stable block (say, block A); the deformation model is

$$u(x_A, y_A)=0, \; v(x_A, y_A)=0, \; u(x_B, y_B)=a_o, \; v(x_B, y_B)=b_o \; , \qquad (6\text{-}14)$$

where (x_A, y_A) represents all the points located in block A; (x_B, y_B) all the points in block B.

(b) Homogeneous strain (Figure 6.1b) in the whole body without discontinuities; for the whole body, the linear deformation model is

$$u(x, y) = a_1 x + a_2 y \text{ and } v(x, y) = b_1 x + b_2 y \qquad (6\text{-}15)$$

which, after substituting (6-12) into (6-15) becomes

$$u(x, y) = \varepsilon_x x + \varepsilon_{xy} y - \omega y \qquad (6\text{-}15a)$$

$$v(x, y) = \varepsilon_{xy} x + \varepsilon_y y + \omega x \quad . \qquad (6\text{-}15b)$$

(c) A deformable body with one discontinuity, say, between blocks A and B, with different linear deformation of each block plus a rigid body movement of B with respect to A (Figure 6.1c)

$$u(x_A, y_A) = a_1 x + a_2 y$$

$$v(x_A, y_A) = b_1 x + b_2 y \qquad (6\text{-}16a)$$

$$u(x_B, y_B) = c_o + c_1 x + c_2 y$$

$$v(x_B, y_B) = g_o + g_1 x + g_2 y \quad . \qquad (6\text{-}16b)$$

In the above case, components Δx_i and Δy_i of a total relative dislocation at any point i located on the discontinuity line between blocks A and B may be calculated as (Chrzanowski and Chen, 1982):

$$\Delta x_i = c_o + (c_1 - a_1) x_i + (c_2 - a_2) y_i \qquad (6\text{-}17)$$

$$\Delta y_i = g_o + (g_1 - b_1) x_i + (g_2 - b_2) y_i \quad .$$

Usually, the actual deformation model is a combination of the above simple models or, if more complicated, it is expressed by non-linear displacement functions which require fitting of higher order polynomials. However, if no a priori knowledge about the expected model exists, then the demonstration of deformation trend is of great help in the identification of the deformation model.

In the study of tectonic movements near active plate boundaries, the dislocation model is of importance. Therefore, a brief review of this model is given in Appendix III.

It should also be mentioned that, in strain analysis of horizontal crustal movements, an alternative expression of (6-13) is the

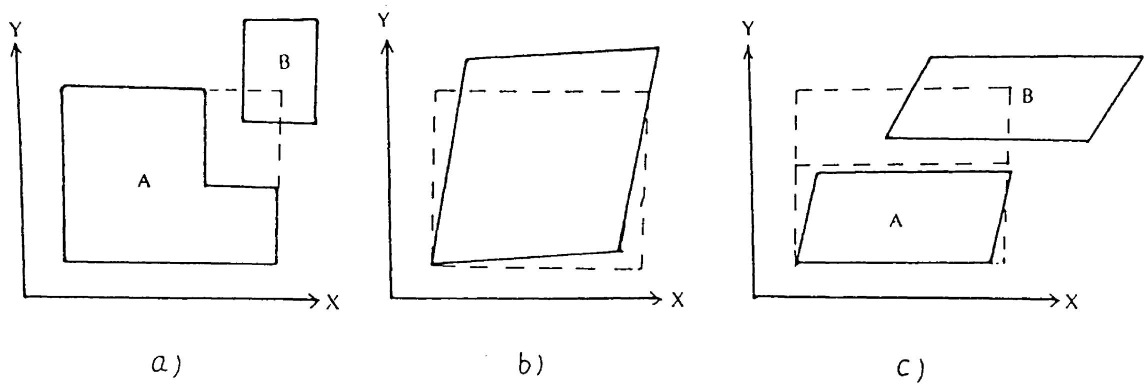


Fig. 6.1

Examples of Typical Deformation Models

use of complex function, see, e.g., Bibby (1973, Appendix I), and Schneider (1982).

6.3.2 Preliminary identification of deformation models in space using a method of iterative weighted projection in the parameter space

The analysis of pairs of epochs of observations is an indispensable part in deformation analysis for the following reasons.

(1) Single point movement in a reference network does not usually follow certain time functions, and therefore the main interest lies in the localization of unstable points between two epochs of time.

(2) An analyst of deformation measurements is often curious about what happened to the deformable body between the most recent surveying campaign and the previous one.

(3) Through the analysis of pairs of epochs of observations, the deformation trend in the time domain will be recognized.

An important step in the analysis of pairs of epochs of observations is to identify the deformation pattern in the space domain. Moreover, if the deformation is postulated to be of a linear nature in time, then all the observations made at different epochs of time can be reduced to the observed rate of change of the observation (Prescott et al., 1981). Therefore, analysis of multi-epoch observations becomes the estimation of the deformation rate. In this case the main task is again to identify the deformation pattern in space.

As discussed in Chapter 4, appropriate definition of a datum will aid in the identification of the deformation models. Some methods for this goal have already been discussed. The approach based on the examination of invariant quantities (derived distances and angles) by Chrzanowski (1981) and Tobin (1983), and the approach based on a special similarity transformation by Caspary and Chen (1981) were initiated for the analysis of unstable points in a reference network. The method proposed by Prescott (1981) is to obtain a particularly illustrative picture of displacements in a relative network when there is an a priori expectation that station motion will most likely be along a certain direction, e.g., along the direction of a fault. However, a priori knowledge about a deformable body is not always available. If it is, the step of preliminary identification of deformation models seems dispensable, and one can go directly to the step of the estimation of deformation parameters.

In order to provide a general tool for the purpose of preliminary identification of a deformation pattern, a method which coordinates the merits of different approaches is proposed. In this method the displacement field is obtained using an iterative weighted projection method as follows.

Let $\underline{d}$ be the vector of displacements of a monitoring network with respect to a certain datum. The projection of $\underline{d}$ onto another datum which is defined by datum equations $D^T\underline{d} = 0$ is computable from

$$\underline{d}_1 = (I - H(H^TWH)^{-1}H^TW)\underline{d} \quad , \tag{6-18}$$

where $W = D(D^TD)^{-1}D^T$, and H and D have been defined in Chapter 4. This problem differs from the one in Chapter 4 in that the datum equations are unknown and must be defined so that a clearer deformation pattern is provided. If all the points in a monitoring network are considered of the same importance in the definition of a datum, then W = I and the result is identical to that of the inner constraint solution or the Helmert transformation. However, the author focuses his interest on the case where some points in a reference network may not be stable, that is, all the points should not be considered of equal importance, or where some points in a relative network may be more likely to move in a certain direction, namely two components of the displacement of a point in a horizontal monitoring network should not be treated equally. Based on this idea, the weight matrix W is established iteratively. At the outset the projection of the displacement vector $\underline{d}$ is obtained with the weight matrix W = I, then in the (k + 1)th projection the weight matrix is defined as

$$W = \text{diag}\{1/\ |d_i(k)|\} \quad , \tag{6-19}$$

where $d_i(k)$ is the ith component of the vector $\underline{d}(k)$ after kth iteration. The iterative procedure continues until the differences between the successive estimated datum parameters approach to zero. During this iterative procedure, some $d_i(k)$ may come close to zero. This will cause numerical instabilities because the weight $W_i = 1/\ |d_i(k)|$ becomes very large. To avoid this problem, a lower bound is set. When $|d_i(k)|$ is smaller than the lower bound, its weight is set to zero. If in the following iterations the $d_i(k + 1)$ becomes significantly large again, the weights can change accordingly.

This method is based on the work of Schlossmacher (1973). Theoretically, the procedure provides a datum so that the first norm of the final projected displacement vector $\underline{d}(k + 1)$ is minimum.

In the numerical processing, two approximations are used: iteration is performed only a few times, say, 4 to 6 times; $[I - H(H^TW(k)H)^{-1}H^TW(k)]$ is treated as deterministic in the error propagation. As a matter of fact, $[I - H(H^TWH)^{-1}H^TW]$ is stochastic since W is the function of $\underline{d}$. However, these treatments are justified because the procedure is used only for preliminary identification of the deformation pattern, and W could be understood as the confidence that we have in each point.

The following are some examples which are used to illustrate the proposed method. Application to analysis of real monitoring networks is presented in Chapter 7.

In a simulated geodetic network (Figure 7.4), where the directions were measured at the first epoch and both the directions and the distances were measured at the second epoch (a detailed description of the network is in Chapter 7), 200 mm relative movement in the y direction of one block with respect to the other is introduced. Without considering measuring errors, the displacement fields using the proposed method and the method of the inner constraint solution are portrayed in Figure 6.2(a) and (b), coupled with the real displacement pattern with dashed lines. As one can see, the displacement field obtained from the proposed method is closer to the real situation, compared with the inner constraint solution.

The same network was simulated by Kok from the Geodetic Computing Centre of Delft University of Technology (the simulation parameters have not been given). Using the proposed method, the displacement field for one pair of epochs is presented in Figure 6.3. The graphical deformation pattern coincides well with the final numerical results: 8.8 cm in the x direction and 20.9 cm in the y direction of the relative movement of the block comprising points 3, 5, 11, 39, 41, with respect to the remaining part.

The method is also applied to the Lohmuhle network (Figure 6.4), a reference network of six points from which a number of targetted points on a dam were positioned. Point 5 was moved progressively in the

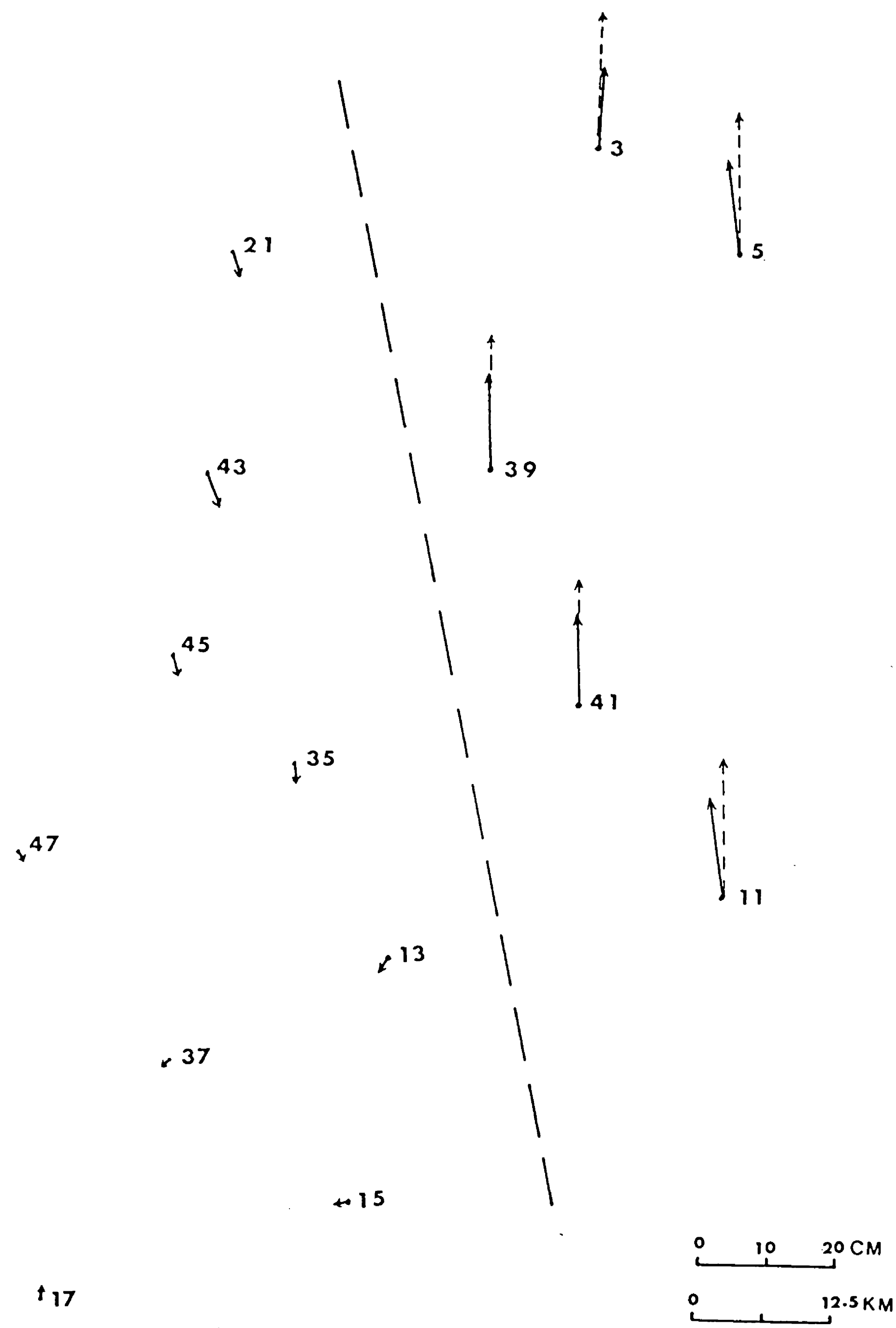


Fig. 6.2 (a): Displacement field using the method of iteractive weighted projection — A simulated example.

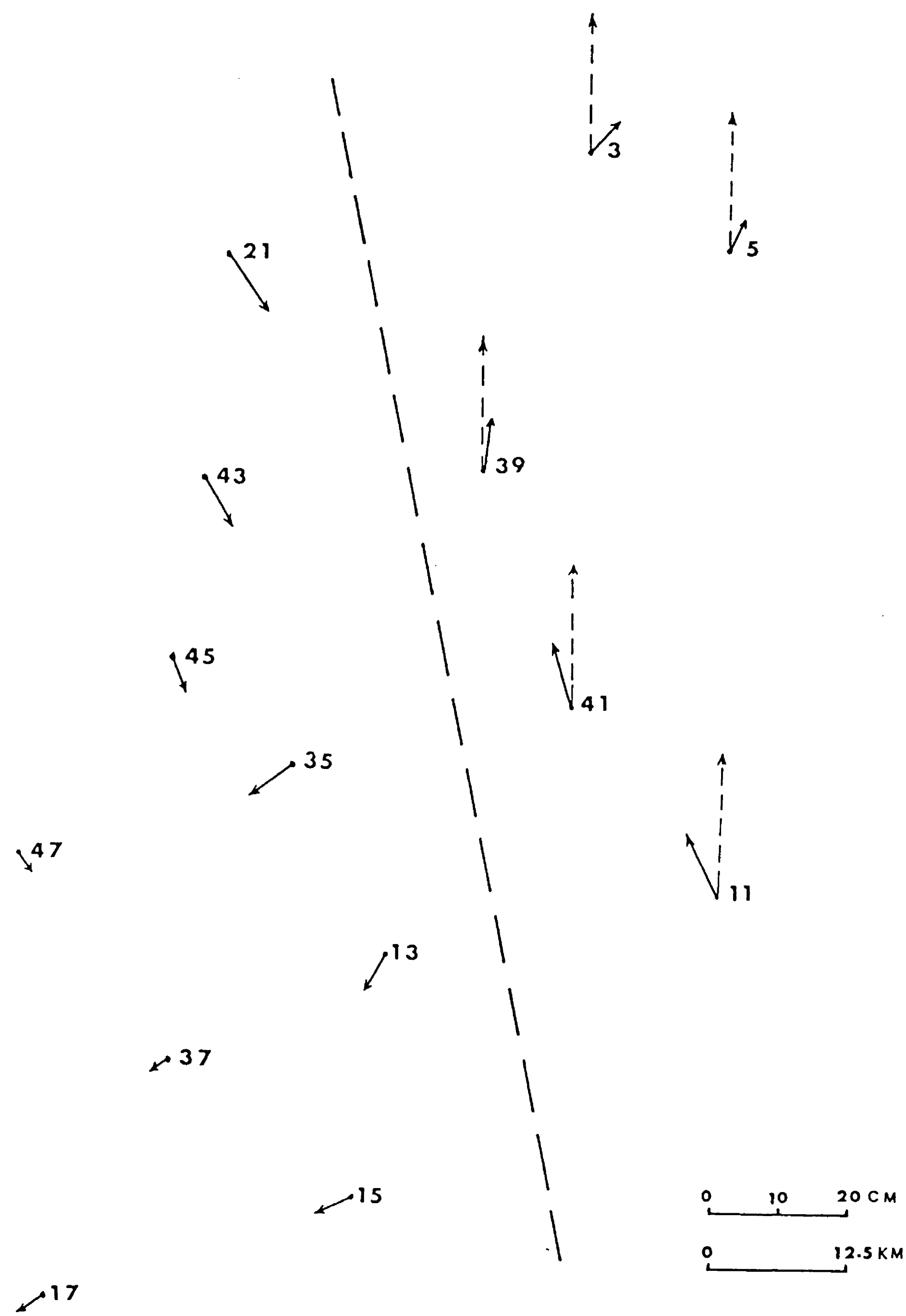


Fig. 6.2 (b): Displacement field using the method of inner constraint solution — A simulated example.

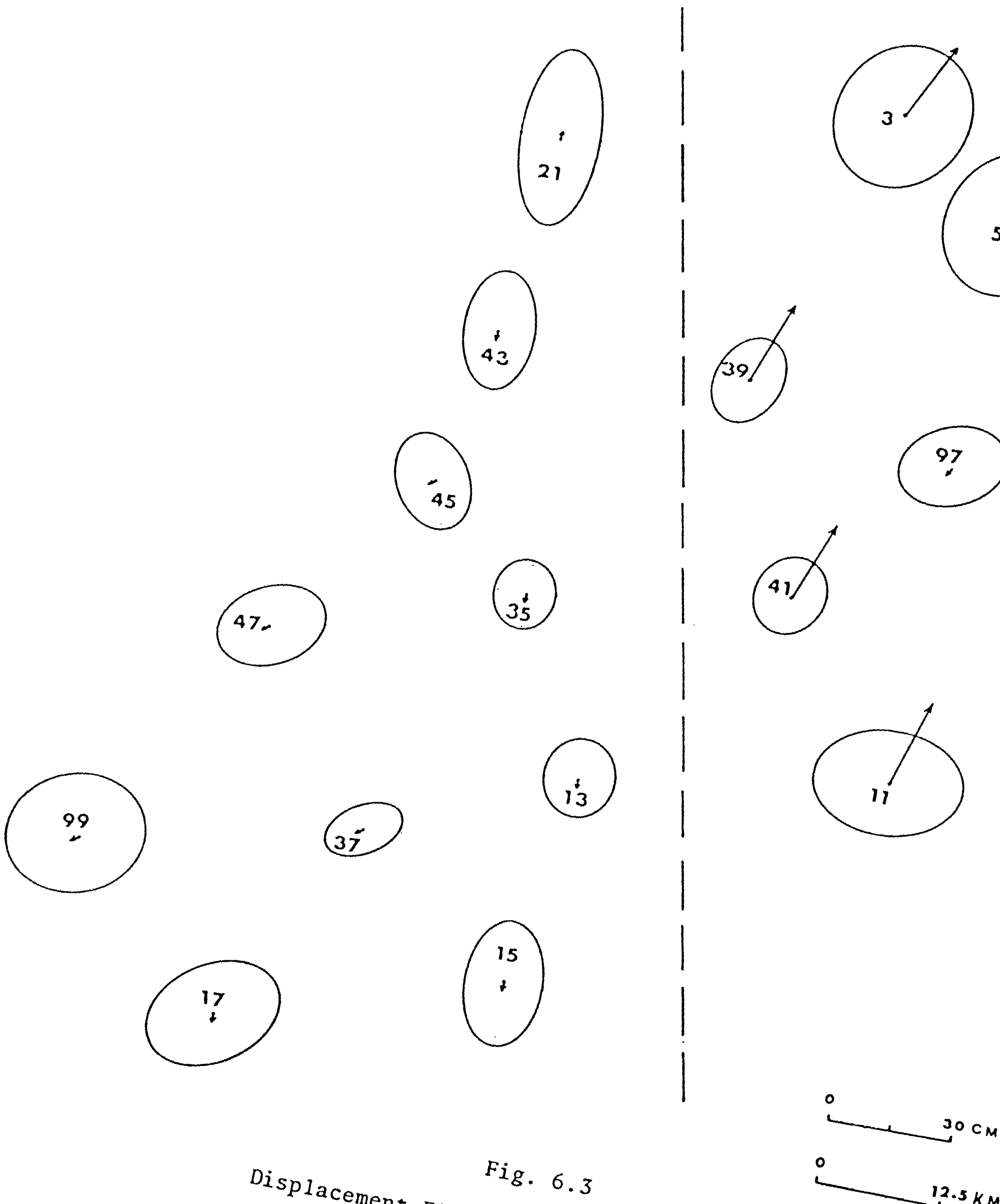


Fig. 6.3
Displacement Field of One Pair of Epochs of a Simulated Network Using the Method of Iterative Weighted Projection.

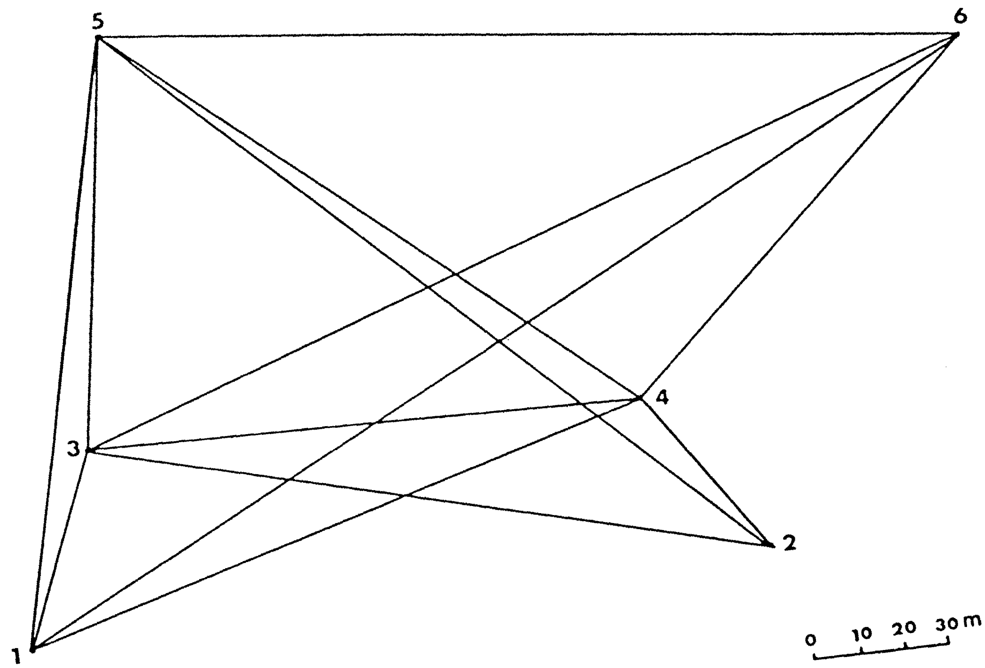


Fig. 6.4: The Lohmühle Network

y direction by 1 mm each time. Then the displacement field coupled with the error ellipses at 95% confidence level were calculated using the proposed method and the method of the inner contraint solution. If the displacement of a point is outside the confidence error ellipse, it would be considered as a suspected unstable point in the step of the estimation of deformation parameters. Table 6.1 summarizes the results.

Suspected	Simulated Displacement of Point 5 in mm									
Unstable Point*	1	2	3	4	5	6	7	8	9	10
1							0	0	0	0
2										0
3					0	0	0	0	0	0
4				0	0	0	0	0	0	0
5	X 0	X 0	X 0	X 0	X 0	X 0	X 0	X 0	X 0	X 0
6									0	0

* X: using the proposed method;
0: using the method of the inner constraint solution.

TABLE 6.1

Points Outside the 95% Confidence Region After a Simulated Displacement is Introduced to Point 5 in the Lohmühle Network.

From the above examples and discussions some remarks can be made.

(1) In the definition of a datum the proposed method is more robust and therefore provides a more realistic picture of the displacement field.

(2) The whole computation procedure can be performed automatically.

(3) This method differs from that of Caspary and Chen (1981) in the optimization criterion and computation method; the latter ends up with minimization of the sum of the length of displacements, but

the former approaches the minimization of the total sum of the absolute values of the displacement components.

(4) In a very special case, theoretically it may be possible that (H^TWH) in (6-18) is not of full rank, so no unique solution exists. But this will not occur in a real-world network.

6.3.3 Some considerations in identification of deformation models in time domain

As is well known, deformation of a body develops with time. The revelation of its temporal attributes is of the same importance as its spatial behaviour. Of course, this can be achieved only when multiple-epoch observations are available.

Obviously, nothing would prevent us from a simultaneous estimation of deformation model in space and in time. The mathematical model has been given in Section 6.2.2 and appeared in the work of Chrzanowski et al. (1982a; 1982b). However, in practice this simultaneous estimation could be performed only when the deformation models are identified preliminarily. Otherwise, only through the trial and error method can complication of the analysis arise and some important deformation phenomena may be missing. Thus, if possible, one should try to identify the deformation model in space and in time separately.

As mentioned in the last section, analysis of pairs of epochs of observations will reveal the deformation trend in the time domain. But one should be aware that only by analysing successive pairs of epochs small deformation patterns may not be disclosed. Comparison of the pair of the last and the first epochs can ascertain the phenomenon of deformation accumulation.

When the observations scatter in time, a method developed in the U.S. Geological Survey (Prescott et al., 1981) should be mentioned. In their method, all the observations of each line (trilateration network) are plotted as a function of time, and then a linear time function (without precluding the possibility of nonlinearity) is fitted to each of the plots. The slope of each fitted straight line is an estimate of the average rate at which the line was changing during the time period covered by the observations. The standard deviation in the rate is also calculated. Then the problem reduces to the estimation of the deformation rate.

6.4 Estimation of Deformation Parameters

6.4.1 Formulation and computation strategies

Regarding equation (6-11), three problems should be discussed in more detail. They are: formulation of vector $\underline{y}_i$ of the quasi-observations (or observations); establishment of the weight matrix P_i; and computation of $(\sum_1^k P_i)^-$.

The basic consideration of the formulation of $\underline{y}_i$ and P_i has been mentioned in Section 6.2.2. For illustration, some typical situations are simulated in Figures 6.5, 6.6, and 6.7. Suppose that Figure 6.5 represents a five-epoch observation scheme. In the first epoch only directions were observed; in the second and third epochs both angles and distances were observed; in the fourth epoch only distances were observed; and in the fifth epoch a traverse was measured. A

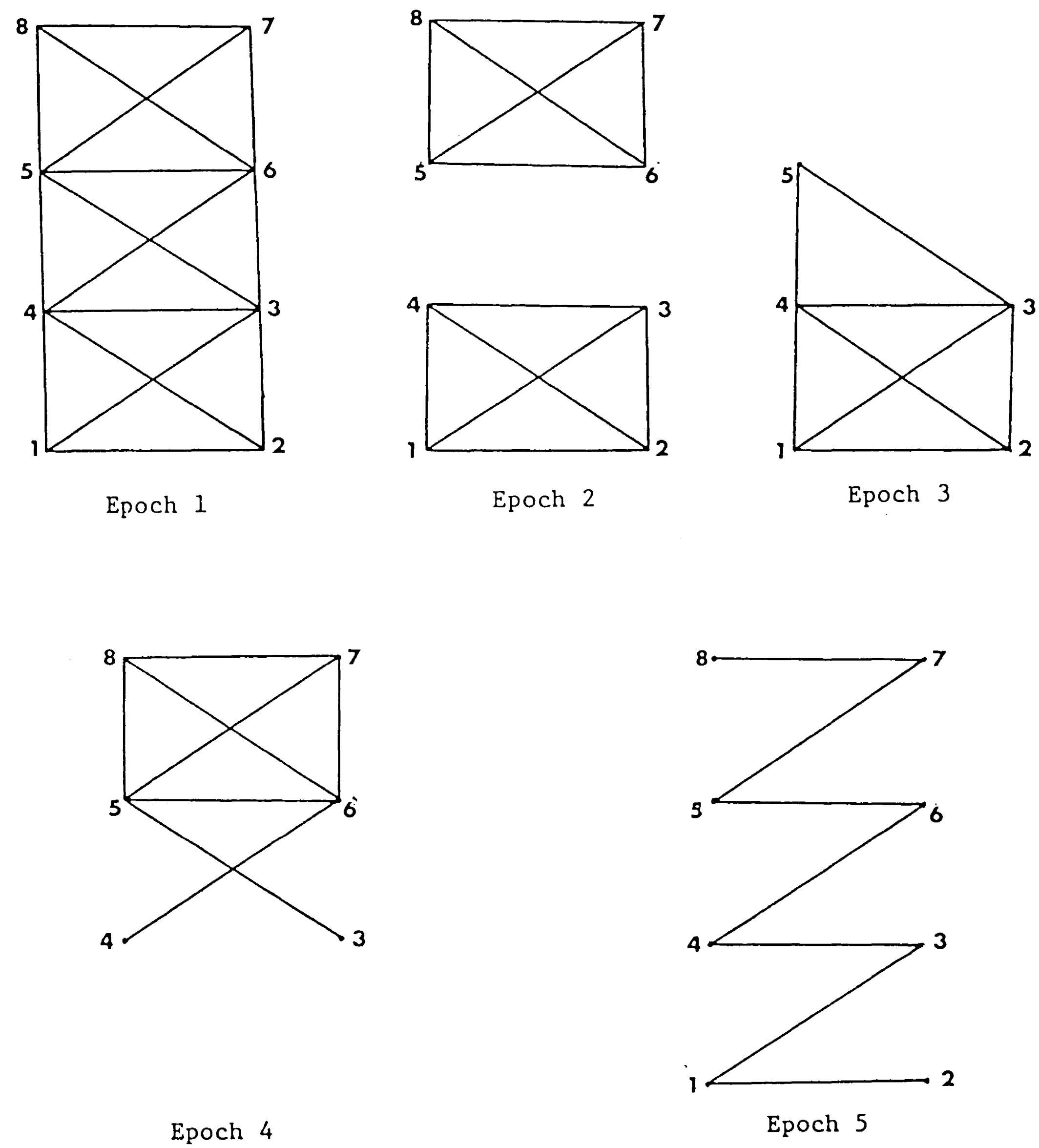


Fig 6.5: A simulated five-epoch monitoring scheme.

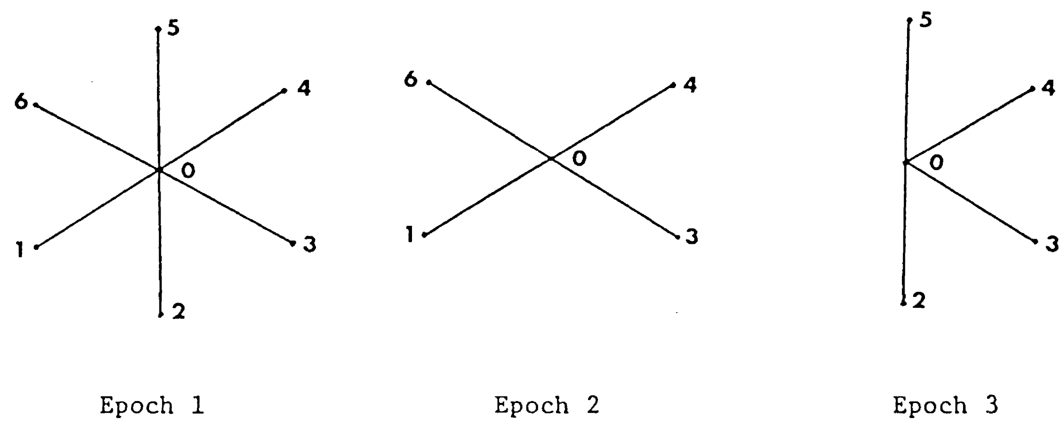


Fig. 6.6: A simulated three-epoch monitoring scheme.

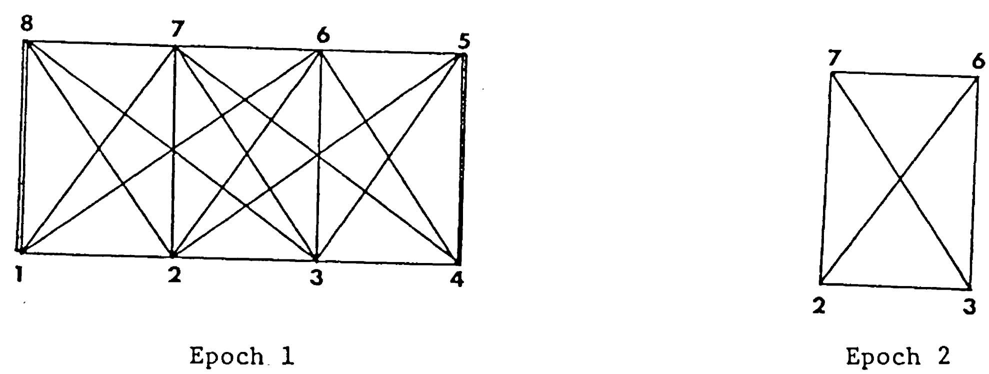


Fig. 6.7: A simulated two-epoch monitoring scheme.

separate adjustment of each epoch of observations or each configuration (e.g., epoch 2) is carried out using any available program. The coordinates of each point in a local Cartesian coordinate system and the coefficient matrices of the normal equations are obtained from the adjustment. To obtain the coordinates of points 3 and 4 in epoch 4, the approximate azimuths from point 5 to point 3 and from point 6 to point 4 are used. The coordinates of the points which do not appear in one epoch, for instance, points 6, 7, 8 in epoch 3 and points 1, 2 in epoch 4, are assigned with their approximate value. In epoch 5 no adjustment is performed, but the coordinates of the points can be calculated using point 2 and direction 2-1, say, as fixed.

As discussed in Section 6.2.2, the coefficient matrix of the normal equation is used as a "weight matrix". In epoch 1 the nuisance parameters of the orientation unknown should be eliminated from the normal equation. Using the notation defined in Section 6.2.1, the observation equation is

$$\underline{\ell}_1 + \underline{v}_1 = (A_{\eta 1} \;\vdots\; A_{x1}) \begin{pmatrix} \underline{\eta}_1 \\ \underline{x}_1 \end{pmatrix}$$

with $D\{\underline{\ell}_1\} = \Sigma_{11} = \sigma_o^2 Q_{11}$. The coefficient matrix of the normal equation reads

$$N_1 = \begin{pmatrix} A^T_{\eta_1} Q^{-1}_{11} A_{\eta_1} & A^T_{\eta_1} Q^{-1}_{11} A_{x_1} \\ A^T_{x_1} Q^{-1}_{11} A_{\eta_1} & A^T_{x_1} Q^{-1}_{11} A_{x_1} \end{pmatrix} = \begin{pmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{pmatrix} .$$

After eliminating the nuisance parameters $\underline{\eta}_1$, the coefficient matrix of the reduced normal equation becomes $\bar{N}_1 = N_{22} - N_{21} N^{-1}_{11} N_{12}$. This can be used as a "weight matrix" for epoch 1. The rank defects of $\bar{N}_1$ is 4 (datum defects). In epoch 2, the weight matrix can be constructed as $P_2 = \text{diag}\{N_2^1, N_2^2\}$, where N_2^1, N_2^2 denote the coefficient matrix of the normal equation for subnetwork 1 and 2 respectively. P_2 has rank defects of 6 (datum defects of two subnetworks). In epoch 3, $P_3 = \text{diag}\{N_3, 0\}$, where the zero submatrix cooresponds to points 6, 7, 8. P_3 has datum defects of 3 and configuration defects of 6. In epoch 4, $P_4 = \text{diag}\{0, N_4\}$, where the zero submatrix corresponds to points 1 and 2, and P_4 has datum defects of 3 and configuration defects of 6. In epoch 5, $P_5 = N_5$ having datum defects of 3.

When the monitoring network is a portion of a geodetic network, the "weight matrix" should be considered more carefully. Its datum defects, including the number and types of defects, should coincide with that of the original whole network. For instance, in Figure 6.7, at the first epoch is an old triangulation network, where the distances 4-5 and 1-8 are the base lines. The portion of this network was resurveyed at epoch 2. To determine the deformation parameters, the coordinates of points 2, 3, 6, 7 and their "weight matrix" should be available for both epochs. In the first epoch, the coordinates of these points can be obtained from the previous adjustment, but caution should be exercised in obtaining the "weight matrix". Denote the coefficient matrix of the normal equation of the whole network by N with

$$N = \begin{pmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{pmatrix}$$

where N_{22} corresponds to the portion of the network of interest. Then, the "weight matrix" for the first epoch is established as

$$\bar{N} = N_{22} - N_{21} N_{11}^{-1} N_{12} \quad . \tag{6-20}$$

It is easy to prove that $\bar{N}$ has the same datum defects as N. Two mistakes should be avoided. First, only the N_{22} is used. This means that the rest of the points of the network were held fixed, therefore the monitoring network becomes completely defined, without datum defects. Second, only the observations in that portion of the network are used in formulation of the "weight matrix". In this case, the number of datum defects would increase from 3 to 4, without scale factor, and full advantage of all the available observations would not be taken. Hence, the relative accuracy of the points in that portion of the network would decrease.

Another way to compute the "weight matrix" of the coordinates is through their cofactor matrix. Let $Q_{\underline{\hat{x}}}$ be the cofactor matrix of the coordinates $\underline{\hat{x}}$ of the points with respect to the datum defined by datum equation $D^T\underline{\hat{x}} = 0$ (see Chapter 4). It has been proved in Chapter 4 that $D^TQ_{\underline{\hat{x}}} = 0$ and $Q_{\underline{\hat{x}}} = (N + DD^T)^{-1} - H(H^TDD^TH)^{-1}H^T$. Hence the "weight matrix" can be calculated from

$$P = [Q_x + H(H^TDD^TH)^{-1}H^T]^{-1} - DD^T \quad . \qquad (6\text{-}21)$$

If Q_x is the "inner accuracy" (Pelzer, 1971), we have $H^TQ_{\underline{\hat{x}}} = 0$, then

$$P = Q^+_{\underline{\hat{x}}} = [Q_{\underline{\hat{x}}} + H(H^TH)^{-2}\, H^T]^{-1} - HH^T \quad . \qquad (6\text{-}22)$$

For a portion of a network, it is desirable to transform $Q_{\underline{\hat{x}}}$ into a new one which is referred to a datum specified for that portion of the network (see Section 4.3). The above formulae are still applicable with matrix H being formulated only for that portion of the network.

Brunner et al. (1981) used the formula $Q^+_{\underline{\hat{x}}} = \lim_{\delta\to 0}(Q_{\underline{\hat{x}}} + \delta^2 I)^{-1}$ to calculate the pseudo-inverse of a cofactor matrix in the computation of crustal strain, but he found that reducing δ had little change in the estimated strain components while their standard deviations changed sometimes by a comparatively large value. Therefore, the choice of the value of δ should be considered for individual cases. However, the formulae (6-21) and (6-22) provide a general and a rigorous way.

Figure 6.6 is a three-epoch observation scheme, where only distances were measured. One can either transform all the observations into coordinates, like model (6-1), or express each observed quantity in terms of the observations in the first epoch and the deformation parameters, like model (6-2). The normal equation for model (6-1) has been formulated in (6-6). In this example $\sum_1^3 N_i$ has rank defects 8 (3 datum defects and 5 configuration defects). In the second case, model (6-2) can be rewritten more explicitly as

$$\begin{pmatrix} \underline{\ell}_1 \\ \underline{\ell}_2 \\ \vdots \\ \underline{\ell}_k \end{pmatrix} + \begin{pmatrix} \underline{v}_1 \\ \underline{v}_2 \\ \vdots \\ \underline{v}_k \end{pmatrix} = \begin{pmatrix} I & 0 \\ I & A_2B_2 \\ \vdots & \vdots \\ I & A_kB_k \end{pmatrix} \begin{pmatrix} \underline{\xi} \\ \underline{c} \end{pmatrix} \tag{6-23}$$

with $P = \mathrm{diag}\{Q_{11}^{-1}, Q_{22}^{-1}, \ldots, Q_{kk}^{-1}\}$. The same formulation strategy as the above is adopted for the missing observations. Using (6-11), replacing $\underline{y}_i$ by $\underline{\ell}_i$, $\tilde{B}_i$ by A_iB_i and $\sum_1^k P_i$ by $\sum_1^k Q_{ii}^{-1}$, the coefficients $\underline{c}$ of the deformation model can be estimated. The advantage of this method is that no inversion of a singular matrix is needed because $\sum_1^k Q_{ii}^{-1}$ is of full rank. In this example, the "observation approach" may be more convenient. However, the "coordinate approach" allows one to utilize all the geodetic observations in the calculations of deformation parameters even if different observables appear in each of the repeated surveys.

It is worthwhile mentioning that when using model (6-1), vector $\underline{x}_1$ in all the epochs must refer to the same geodetic datum, otherwise the results will be distorted. Taking the well-known California monitoring networks as an example, where the distances are measured very precisely, the precision in a single observation of length is 3.6 mm for a 10 km line and 6.7 mm for a 30 km line (Prescott, 1981), but the accuracy of the heights of the stations is low: only $\pm$ 1 m (Prescott, 1982, personal communication). If $\underline{x}_1$ in (6-1) is referred to a horizontal geodetic datum, all the distances must be reduced to a common surface. In this case, a 30 km line with the height difference of 300 m between two terminal stations will be contaminated by a reduction error of about $\pm$ 10 mm, larger than the measuring error. Therefore, the model (6-1) is not suitable unless the conditions in note 3 of Section 6.4.2 are fulfilled, or one formulates this problem in three-dimensional space, i.e., vector $\underline{x}_1$ is referred to a three-dimensional coordinate system. But little will be gained from this type of formulation, hence the "observation approach" is much simpler. The generalized approach has been designed such that it can handle either or both types of formulation in a simultaneous solution.

In equation (6-11), a generalized inverse $(\sum_{1}^{k} P_i)^-$ has to be computed. There are many techniques to calculate a g-inverse of a singular matrix, for example, singular value decomposition, orthogonalization method, rank factorization (Rao and Mitra, 1971).

Here a simple method to compute a g-inverse of $(\sum_1^k P_i)$ is developed, which transfers the inversion of a singular matrix to the inversion of a non-singular one.

As mentioned in Section 6.2.2, the number of rank defects of $\sum_1^k P_i$ is equal to the number of remaining datum defects and configuration defects which are not determined in at least one epoch. In practice it is easy to figure out from a purely geometrical consideration. For example, in the five-epoch observation scheme (Figure 6.5) $\sum_1^k P_i$ has 3 datum defects and no configuration defects; but in the three-epoch observation scheme (Figure 6.6) $\sum_1^k P_i$, when using the "coordinate approach", has 5 configuration defects and 3 datum defects.

If matrix $(\sum_1^k P)$ has only datum defects remaining, computation of its generalized inverse is rather simple. One of the g-inverses of $(\sum_1^k P_i)$ can be calculated from

$$(\sum_1^k P_i)^- = (\sum_1^k P_i + HH^T)^{-1} \quad , \qquad (6\text{-}24)$$

where H is a matrix such that $S(H) \cap S(\sum_1^k P_i) = 0$. One representation of matrix H has been given in Chapter 4. The expression (6-24) can be proved by verifying $(\sum_1^k P_i)(\sum_1^k P_i + HH^T)^{-1}(\sum_1^k P_i) = (\sum_1^k P_i)$.

Furthermore, if the matrix $(\sum_1^k P_i)$ contains both datum defects and configuration defects, the matrix H should be augmented by a matrix

H_c taking care of the configuration defects so that $\bar{H} = (H \mid H_c)$. Then

$$(\sum_1^k P_i)^- = (\sum_1^k P_i + HH^T + H_c H_c^T)^{-1} \quad . \tag{6-25}$$

The matrix H_c^T can be considered as the configuration matrix of some pseudo-observations. After they are added into the configuration, the configuration defects will be removed. In Figure 6.5, assuming that epoch 2 and 3 form a two-epoch observation scheme, the matrix $(\sum_1^k P_i)$ contains one configuration defect. After adding a pseudo-distance 3-6, the combined figure is complete. Therefore H_c can be expressed as

$$H_c^T = (0, 0, \ldots, \cos\alpha_{3-6}, \sin\alpha_{3-6}, 0, 0, 0, 0, -\cos\alpha_{3-6}, -\sin\alpha_{3-6}, 0, \ldots, 0).$$

Actually, in the numerical computation both datum defects and configuration defects can be treated in the same way. Some pseudo-observations which remove both types of defects are introduced. Then the coefficient matrix of the normal equation for these pseudo-observations is formulated and added to $\sum_1^k P_i$. This results in a non-singular matrix. An example to demonstrate this idea is presented in Appendix IV.1.

6.4.2 Special case--analysis of a pair of epochs of observations

The analysis of a pair of epochs of observations is a special case of the general model (6-8). Its importance has been discussed in Section 6.3.2.

Let $\underline{y}_1$, $\underline{y}_2$ be two-epoch observations or quasi-observations. Then the mathematical models (6-8) and (6-9) reduce to

$$\begin{pmatrix} \underline{y}_1 \\ \\ \underline{y}_2 \end{pmatrix} + \begin{pmatrix} \underline{\delta}_1 \\ \\ \underline{\delta}_2 \end{pmatrix} = \begin{pmatrix} I & 0 \\ \\ I & \tilde{B} \end{pmatrix} \begin{pmatrix} \underline{\xi} \\ \\ \underline{c} \end{pmatrix} \tag{6-26}$$

and

$$P = \mathrm{diag}\{P_1, P_2\} \quad . \tag{6-27}$$

Applying the principle of least squares, the normal equation is as follows:

$$\begin{pmatrix} P_1 + P_2 & P_2\tilde{B} \\ \\ \tilde{B}^T P_2 & \tilde{B}^T P_2 \tilde{B} \end{pmatrix} \begin{pmatrix} \underline{\xi} \\ \\ \underline{c} \end{pmatrix} = \begin{pmatrix} P_1\underline{y}_1 + P_2\underline{y}_2 \\ \\ \tilde{B}^T P_2 \underline{y}_2 \end{pmatrix} \quad . \tag{6-28}$$

Eliminating $\underline{\xi}$, one gets

$$\underline{\hat{c}} = [\tilde{B}^T(P_2 - P_2(P_1+P_2)^- P_2)\tilde{B}]^{-1}[\tilde{B}^T(P_2\underline{y}_2 - P_2(P_1+P_2)^-(P_1\underline{y}_1 + P_2\underline{y}_2))] \quad . \tag{6-29}$$

Denote $P_2 - P_2(P_1 + P_2)^- P_2$ by $P_1 \mp P_2$, called the parallel sum of matrices. According to Rao and Mitra (1971), it has the following properties:

(1) $P_1 \mp P_2 = P_2 - P_2(P_1 + P_2)^- P_2$ is invariant for any choice of the g-inverse.

(2) $P_1 \mp P_2 = P_1(P_1 + P_2)^- P_2 = P_2(P_1 + P_2)^- P_1$.

(3) $S(P_1 \mp P_2) = S(P_1) \cap S(P_2)$.

(4) $P_1 \mp P_2 = [P_S(Q_1 + Q_2)P_S]^+$, where Q_1 and Q_2 are any inverse of P_1 and P_2 respectively, P_S is the orthogonal projection operator onto the space $S(P_1) \cap S(P_2)$.

From (1) and (2) the equation (6-29) becomes $\underline{\hat{c}} = [\tilde{B}^T(P_1 \mp P_2)\tilde{B}]^{-1} \cdot [\tilde{B}^T(P_1 \mp P_2)(\underline{y}_2 - \underline{y}_1)]$, which corresponds to an observation equation

$$(\underline{y}_2 - \underline{y}_1) + \underline{\delta} = \tilde{B} \cdot \underline{c} \tag{6-30}$$

with weight matrix $P_{\Delta y} = (P_1 \mp P_2)$.

Note 1: If $\underline{y}_i$ stands for observations $\underline{\ell}_i$, the model (6-30) becomes the "raw observation approach" (Chrzanowski and Chen, 1981), and

$$\hat{\underline{c}} = [B^T A^T (P_1 \mp P_2) AB]^{-1} [B^T A^T (P_1 \mp P_2)(\underline{\ell}_2 - \underline{\ell}_1)] \qquad (6\text{-}31)$$

where

$$P_1 \mp P_2 = Q_{11}^{-1} \mp Q_{22}^{-1} = (Q_{11} + Q_{22})^{-1} \quad ,$$

and all the notations have been defined in Section 6.2. If $\underline{y}_i$ stands for the coordinates $\underline{x}_i$, then model (6-30) is called the "displacement approach" (Chrzanowski and Chen, 1981), and

$$\hat{\underline{c}} = [B^T (P_1 \mp P_2) B]^{-1} [B^T (P_1 \mp P_2)(\underline{x}_2 - \underline{x}_1)] \qquad (6\text{-}32)$$

with

$$P_1 \mp P_2 = N_1 (N_1 + N_2)^- N_2 \quad ,$$

where N_i is the coefficient matrix of the normal equation in epoch i, which has been discussed in the last section.

Note 2: Since $\underline{x}_2$ and $\underline{x}_1$ may be calculated from the adjustments with different datums, $(\underline{x}_2 - \underline{x}_1)$ will contain the displacements caused by the translation, rotation, and scale change of the network, besides the true displacements. Nevertheless, these additional displacements will not have an effect on the estimated coefficient $\hat{\underline{c}}$ of the deformation model because they lie in the null space of $P_1 \mp P_2$. This indicates that whatever minimum constraint (or inner constraint) is used in the processing of each epoch of observations, a unique solution for $\underline{c}$ must be obtained. The concern expressed by Brunner (1979) and Brunner et al. (1981) that any constraint (minimum constraints) introduced into a deformation network can disguise the true deformation is not pertinent. This fact was first pointed out by Chrzanowski and Chen (1981). But caution should be exercised in the computation when the minimum constraint solution $\underline{x}_i$ and its cofactor matrix $Q_{\hat{\underline{x}}}$ are used instead of weight matrix N_i. This problem will be fully discussed in the next section (6.4.3).

Note 3: Let us compare the "raw observation approach" with the "displacement approach". Assume that both epochs have the same configuration matrix A, then $N_i = A^T Q_{ii}^{-1} A$ (i = 1,2). From (6-32) one gets

$$Q_{\hat{c}} = (B^T(P_1 \mp P_2)B)^{-1} = (B^T A^T Q_{11}^{-1} A(A^T(Q_{11}^{-1} + Q_{22}^{-1})A)^{-} A^T Q_{22}^{-1} AB)^{-1}. \quad (6\text{-}33)$$

Equation (6-33) is equivalent to $(B^T A^T (Q_{11} + Q_{22})^{-1} AB)^{-1}$, that is to say, the "raw observation approach" and the "displacement approach" yield identical results only when either

(1) $Q_{11} = k\, Q_{22}$ with k being a constant; or

(2) $r\{Q_{11}^{-1} + Q_{22}^{-1}\} = r\{A^T(Q_{11}^{-1} + Q_{22}^{-1})A\}$.

The first condition is obvious and the second one can be satisfied only when no redundancies are available, thus no adjustment is possible. The second condition can be proved as follows.

Proof: Under condition (2), $A(A^T(Q_{11}^{-1} + Q_{22}^{-1})A)^{-}A^T = (Q_{11}^{-1} + Q_{22}^{-1})^{-1}$ (Rao and Mitra, 1971). Thus $Q_{11}^{-1}A(A^T(Q_{11}^{-1} + Q_{22}^{-1})A)^{-}A^T Q_{22}^{-1} = Q_{11}^{-1}(Q_{11}^{-1} + Q_{22}^{-1})^{-1} Q_{22}^{-1} = (Q_{11} + Q_{22})^{-1}$. ||

A simple example to demonstrate the difference between both approaches when the above two conditions are not fulfilled is presented in Appendix IV.2.

6.4.3 Remarks

In practice, there exist some problems that need be clarified. The first problem is related to results reported by Prescott (1981):

> ...examination of a trilateration network near San Francisco Bay demonstrates the large effect that the choice of adjustment technique can have on the inferred relative motion of the two sides of the fault. The inner coordinates solution gave a rate of about 1 mm/yr, whereas the preferred outer coordinate solution rate was 36 mm/yr.

This result contradicts the statement made in note 2 of Section 6.3.2, that whatever minimum constraints are used in the processing of each epoch of observations the unique result for the deformation parameters must be obtained.

Another problem is related to the rotation parameter of the rigid body motion. It is well known that the rigid body rotation parameter ω cannot be determined from a trilateration monitoring network unless an external azimuth observation is made. Thus in the case of the lack of external orientation the omission of the ω component from a deformation model seems justified. This is true when the coefficients of the deformation model are estimated using the generalized approach or calculated from raw observations or from derived invariant quantities (derived angles, distances). However, if the deformation parameters are estimated from the displacements coupled with their covariance matrix, caution should be taken. The omission of the ω component may produce a biased result, which was first pointed out by Chrzanowski and Chen (1981).

In order to clarify these practical problems, some numerical examples are presented, followed by a theoretical study.

In the first example, the same monitoring network (Figure 6.8) used by Prescott (1981) in South Bay near San Francisco has been analysed using the survey data obtained from the American Geophysical Union. The line lengths were measured periodically between 1972 and 1980. A linear least-squares fit to all the observations of a single

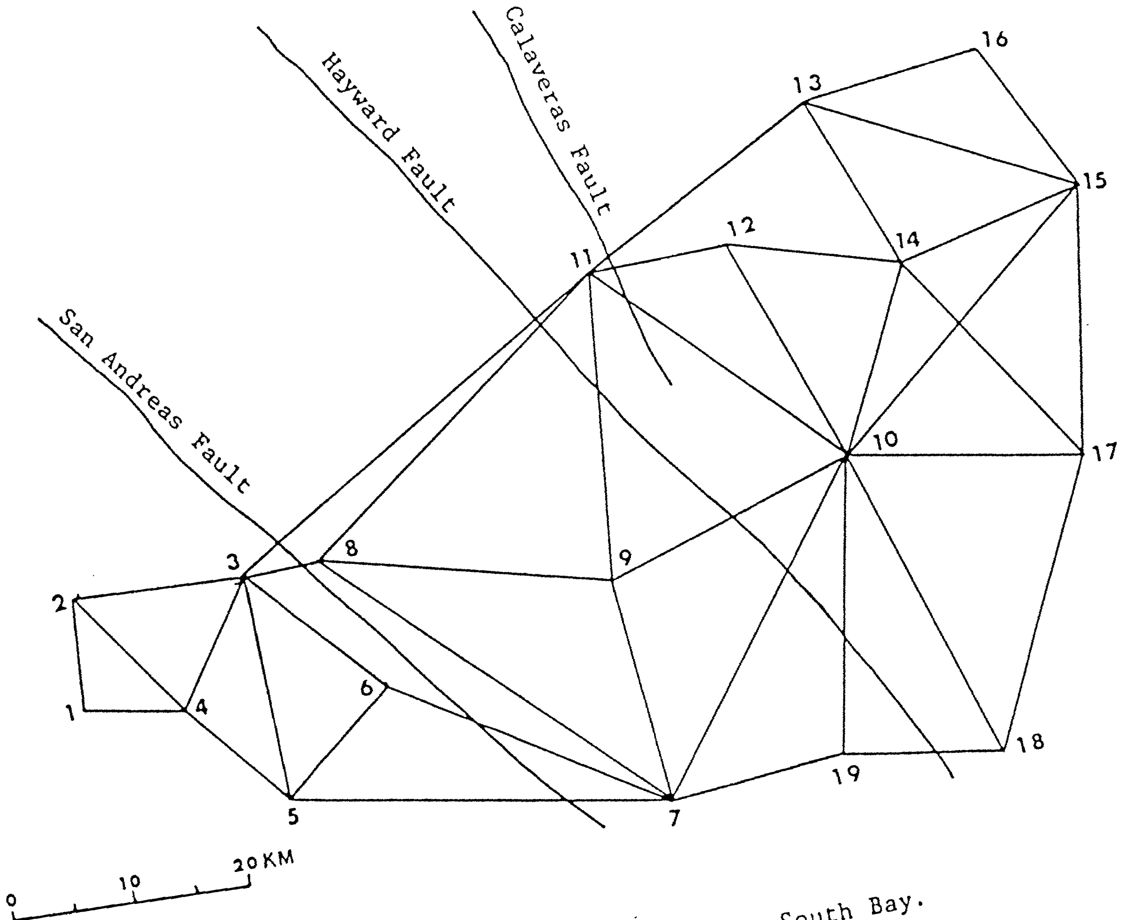


Fig. 6.8: The monitoring network in the South Bay.

line as a function of time was carried out to obtain the average rate of change for each line. These rates were then adjusted by three different methods: minimum constraints holding point 10 and the direction from point 10 to point 12 fixed; inner constraints; and a method of the "outer coordinate solution" proposed by Prescott (1981). Displacement rate vectors for the three adjustments are given in Figures 6.9 to 6.11.

Relative motion of the western block with respect to the eastern block of the network was determined using the generalized approach. The Heyward fault line was taken as the separation line of the relative movement (the same as used by Prescott). The results are shown in Table 6.2, where a and b are components of the relative motion rates along and perpendicular to the fault line respectively. The motion is right lateral strike slip.

Relative Motion Parameters	Inner Constraints	Outer Coordinate Solution	Minimum Constraints
a	12.8 mm/yr.	12.8 mm/yr.	12.8 mm/yr.
b	- 1.6 mm/yr.	- 1.6 mm/yr.	- 1.6 mm/yr.

TABLE 6.2

The Relative Motion Rate Between Two Tectonic Blocks in South Bay, Estimated from the Results Using Different Adjustment Methods.

As one can see, the parameters a and b are invariant of the choice of solution, although the displacement fields are quite different.

Prescott (1981) in his comparison directly used the displacements (their projections on the fault line) which obviously are datum dependent. Therefore, each of his solutions gave different results due to different rotations introduced to the displacement field.

Fig. 6.9: Displacement field for the minimum constraint solution holding point 10 and direction from point 10 to point 12 fixed.

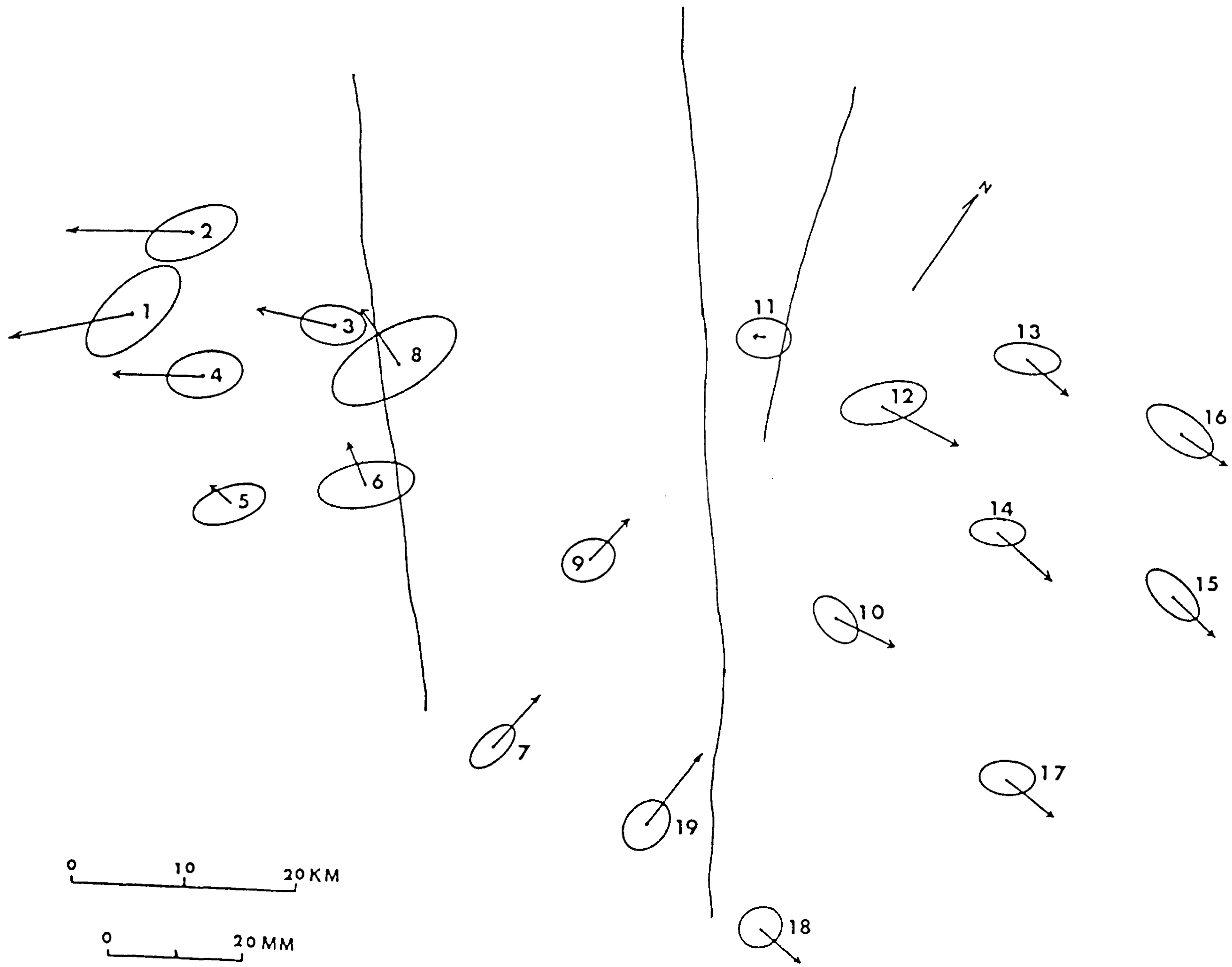


Fig. 6.10: Displacement field for the inner constraint solution.

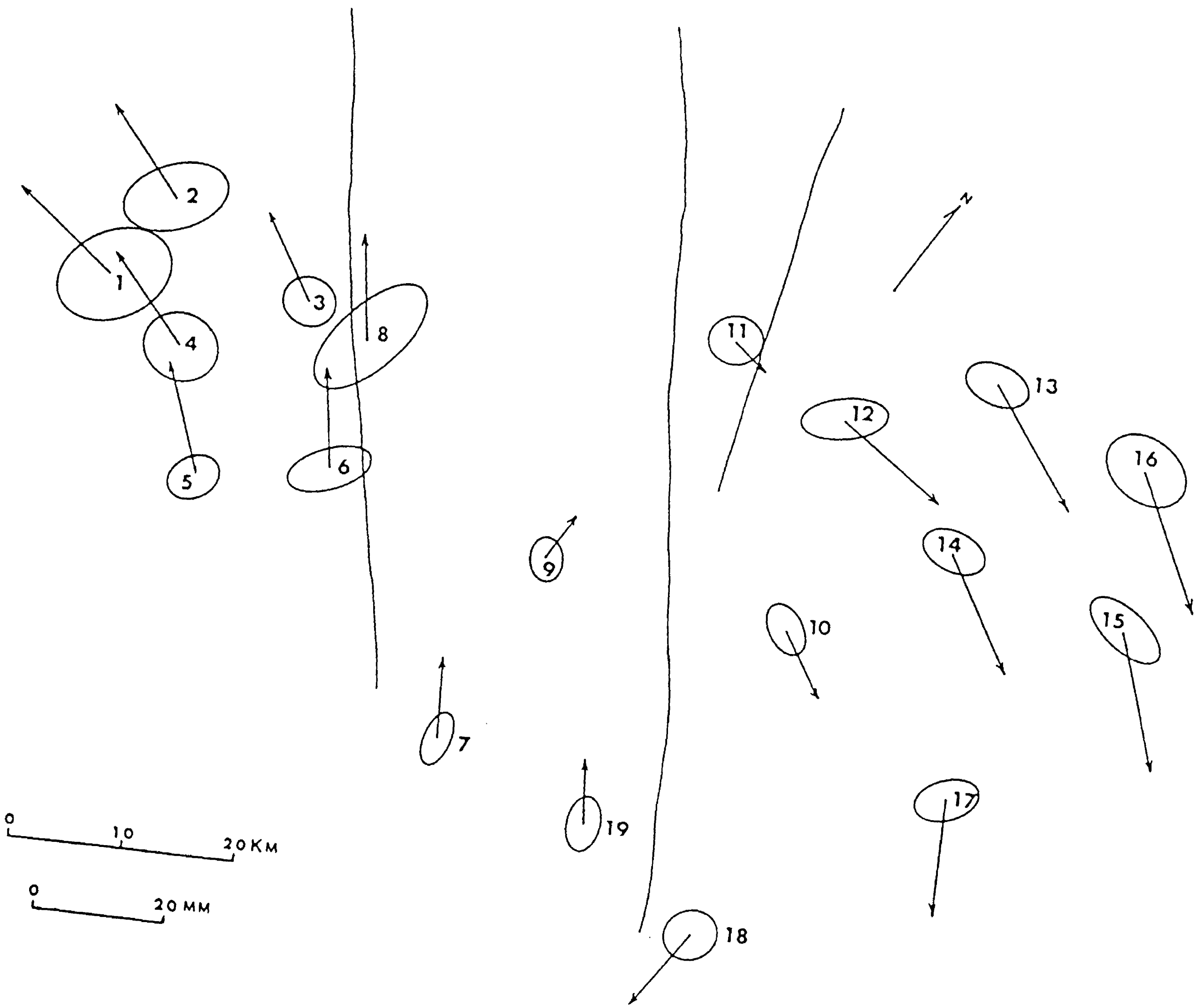


Fig. 6.11: Displacement field for the "outer coordinate" solution.

Let us consider in more detail the aforementioned second problem related to the rotation parameter of the rigid body motion. In the adjustment of a trilateration or triangulateration (mixed triangulation and trilateration) network, a point P_i and the direction from P_i to a second point P_j act as constraints on the degrees of freedom of the configuration. This is imposed by fixing the coordinates (x_i, y_i) of P_i which is then taken as the origin of the local coordinate system (Chrzanowski and Chen, 1982) and by assigning a very small variance to the azimuth α_{ij}. Consequently, when comparing the coordinates under the same constraints at two epochs, the displacement of P_j is confined to occur in the direction of the azimuth α_{ij} so that

$$dx_j/dy_j = \tan(\alpha_{ij}) \quad .$$

If, for instance, the displacement field is approximated by the model expressed by (6-15a) and (6-15b), then the observation equations for the displacement components of point P_j are:

$$dx_j + v_{xj} = \varepsilon_x x_j + \varepsilon_{xy} y_j - \omega y_j \quad ,$$

$$dy_j + v_{yj} = \varepsilon_{xy} x_j + \varepsilon_y y_j + \omega x_j \quad .$$

The very small variance of α_{ij} constrains these to have the relation

$$\frac{\varepsilon_x x_j + \varepsilon_{xy} y_j - \omega y_j}{\varepsilon_{xy} x_j + \varepsilon_y y_j + \omega x_j} = \tan(\alpha_{ij}) \quad ,$$

so that

$$\omega = \frac{1}{2}(\varepsilon_x - \varepsilon_y) \sin(2\alpha_{ij}) + \varepsilon_{xy} \cos(2\alpha_{ij}) \quad . \qquad (6\text{-}34)$$

Thus the variation associated with the change in minimum constraints is absorbed by the change in the value of ω, rendering the values of the other strain parameters invariant. If ω were omitted (considered as being zero) in the model, as is done by some authors when no absolute orientation of the network is available (no ties to external reference points), then the calculated strain parameters would vary with the choice of minimum constraints. The same applies to other deformation models, whenever the constrained direction α_{ij} is within the deformed part of the investigated body. For instance, in the case of rigid body displacements as shown in Figure 6.1a, if point i of the constrained direction would be in block A and point j in block B, then the equations (6-14) of the deformation model should be written in the form:

$$dx_A = -\omega y, \quad dy_A = \omega x, \quad dx_B = a_o - \omega y, \quad \text{and} \quad dy_B = b_o + \omega x \ .$$

Otherwise, the values of the parameters a_o and b_o would be dependent on the choice of the direction α_{ij} to be constrained. The rotation parameter ω in the above cases plays the role of a nuisance parameter.

Two-epoch observations from the aforementioned monitoring network in Peru (Figure 7.1) are used to demonstrate this point. The results with and without including ω are presented in Table 6.3(a) and (b), respectively. In Table 6.3(a) two deformation models are estimated. One is the homogeneous strain over the whole area; the other is the homogeneous strain in both blocks on opposite sides of the fault plus rigid body translation between the blocks. Parameters a and b are the two components of the translation in x and y directions, respectively.

Parameters	MODEL 1				MODEL 2	
	Holding Pt. 4 & 4-1	Holding Pt. 5 & 5-4	Holding Pt. 4 & 4-2	Inner Constraint	Holding Pt. 4 & 4-1	Holding Pt. 5 & 5-4
ε_x (10^{-6})	- 1.57	- 1.55	- 1.55	- 1.54	- 1.68	- 1.70
ε_{xy} (10^{-6})	- 1.66	- 1.62	- 1.64	- 1.64	- 0.96	- 0.94
ε_y (10^{-6})	- 2.72	- 2.73	- 2.67	- 2.67	- 0.40	- 0.38
a (mm)					- 1.20	- 1.20
b (mm)					- 1.80	- 1.75
ω (10^{-6})	+ 0.34	- 0.22	+ 0.24	+ 0.01	+ 0.20	- 0.38
ω^* (10^{-6})	+ 0.34	- 0.26	+ 0.22	0.00	+ 0.20	- 0.35

TABLE 6.3(a)

Parameters	Holding Pt. 4 and 4-1	Holding Pt. 5 and 5-4	Holding Pt. 4 and 4-2
ε_x (10^{-6})	- 1.62	- 1.14	- 2.15
ε_{xy} (10^{-6})	- 0.00	- 0.78	- 1.24
ε_y (10^{-6})	- 1.88	- 3.72	- 0.90

TABLE 6.3(b)

The Estimated Deformation Parameters from Two Epoch Observations in the Huaytapallana Network: (a) With and (b) Without Including the Rotation Parameter ω.

Note that ω^* in Table 6.3(a) for the first model are calculated from (6-34) using the estimated values ε_x, ε_{xy}, ε_y, while ω are obtained from the least-squares estimation. For the second deformation model, ω^* is expressed as

$$\omega^* = \frac{1}{2}(\varepsilon_x - \varepsilon_y + \frac{a}{x_j} - \frac{b}{y_j}) \sin(2\alpha_{ij}) + \varepsilon_{xy}\cos(2a_{ij}) \quad ,$$

when the constrained azimuth crosses the fault, or is determined from (6-34) when the constrained azimuth does not cross the fault. The above contention has been further verified by Snay (1982, personal communication) who performed the test using a Utah data set.

Let us analyse this problem in a more general case. Assume that the deformation model $B\underline{c}$ is to be estimated from the mathematical model:

$$\underline{d} + \underline{v} = (\underline{x}_2 - \underline{x}_1) + \underline{v} = B\underline{c} \quad , \tag{6-35}$$

with

$$D\{\underline{d}\} = \sigma^2 Q_{\underline{d}} \quad ,$$

where $\underline{v}$ is the vector of residuals, $\underline{d}$ is the vector of displacements, and $Q_{\underline{d}}$ is its cofactor matrix with respect to a datum defined by datum equation $D^T\underline{x} = 0$ (see Chapter 4). Matrix $Q_{\underline{d}}$ is singular with rank defects being equal to the datum defects of the monitoring network. The singularity of $Q_{\underline{d}}$ implies that some natural constraints on the vector of displacements $\underline{d}$ exist. As discussed in Chapter 4, the following relation holds:

$$D^T\underline{d} = 0 \quad \text{and} \quad D^TQ_{\underline{d}} = 0 \quad . \tag{6-36}$$

Since $D\{D^T\underline{d}\} = D^TQ_{\underline{d}}D = 0$, $D^T\underline{d}$ is nonstochastic. Therefore, imposed on the deformation model is a restriction:

$$D^TB\underline{c} = \underline{0} \quad .$$

According to Rao and Mitra (1971), the unbiased estimators of (6-35) are obtained by minimizing $(\underline{d} - B\underline{c})^T\ Q_{\underline{d}}^-(\underline{d} - B\underline{c})$ subject to the constraints $D^TB\underline{c} = 0$. Obviously the estimated coefficients $\hat{\underline{c}}$ will change due to the choice of the datum equations in the least-squares processing of the observations. In order to obtain the invariant coefficients $\hat{\underline{c}}$, a set of nuisance parameters, $\underline{c}_*$, say, is introduced. The number and types of the parameter $\underline{c}_*$ should correspond to the defects of $Q_{\underline{d}}$. Therefore, (6-35) is modified as

$$\underline{d} + \underline{v} = H\underline{c}_* + B\underline{c} \quad , \tag{6-38}$$

with $D\{\underline{d}\} = \sigma^2 Q_{\underline{d}}$, where matrix H has been defined in Chapter 4. The constraints on the unknown coefficients, due to singularity of $Q_{\underline{d}}$, read

$$D^T H\underline{c}_* + D^T B\underline{c} = 0 \quad . \tag{6-39}$$

Expressing $\underline{c}_*$ as a function of $\underline{c}$ from (6-39) and substituting it into (6-38), one gets

$$\underline{d} + \underline{v} = (I - H(D^T H)^{-1} D^T) B\underline{c} = \tilde{B}\underline{c} \quad , \tag{6-40}$$

where $\tilde{B}$ is the projection of B onto the subspace E^r perpendicular to S(D) in Figure 4.1. It is easy to verify that $S(\tilde{B}) \subset S(Q_{\underline{d}})$ because $D^T\tilde{B} = D^T Q_{\underline{d}} = 0$, so that no more constraints on $\underline{c}$ exist. Applying the principle of least squares to the (6-40), the coefficients $\underline{c}$ are estimated from

$$\hat{\underline{c}} = (\tilde{B}^T Q_{\underline{d}}^- \tilde{B})^{-1} \tilde{B}^T Q_{\underline{d}}^- \underline{d} \quad , \tag{6-41}$$

which is invariant of any choice of g-inverse $Q_{\underline{d}}^-$.

As discussed in Chapter 4, another solution $\underline{d}_1$ $Q_{\underline{d}_1}$ are related to $\underline{d}$ and $Q_{\underline{d}}$ in the following manner:

$$\underline{d}_1 = P_{D_1^\perp} \underline{d} \quad \text{and} \quad Q_{\underline{d}_1} = P_{D_1^\perp} \cdot Q_{\underline{d}} \cdot P_{D_1^\perp}^T \quad ,$$

where $P_{D_1^\perp}$ is the projection operator with respect to datum equation $D_1^T \hat{\underline{x}} = 0$.

Similar to (6-40), the deformation model $B\underline{c}$ can be estimated from $\underline{d}_1$ and $Q_{\underline{d}_1}$:

$$\underline{d}_1 + \underline{v}_1 = (I - H(D_1^T H)^{-1} D^T) B\underline{c} = \tilde{B}_1\underline{c} \quad . \tag{6-42}$$

Let $\hat{\underline{c}}_1$ be the least-squares estimator from (6-42), then

$$\underline{\hat{c}}_1 = (B_1^T Q_{\underline{d}_1}^- B_1)^{-1} B_1^T Q_{\underline{d}_1}^- \underline{d}_1 \quad .$$

From the fact that $B_1 = P_{D_1^\perp} \cdot B$ and $P_{D_1^\perp}^T (P_{D_1^\perp} Q_{\underline{d}} P_{D_1^\perp}^T)^- P_{D_1^\perp} = Q_{\underline{d}}^-$ comes the relation:

$$\underline{\hat{c}}_1 = (B^T Q_{\underline{d}}^- B)^{-1} B^T Q_{\underline{d}}^- \underline{d} = \underline{\hat{c}} \quad .$$

Therefore, introducing a set of nuisance parameters $\underline{c}_*$ renders the invariance of the estimated coefficients of the deformation model.

However, if $Q_{\underline{d}}$ is obtained from the inner constraint solution, and the pseudo-inverse $Q_{\underline{d}}^+$ is used in the estimation of the deformation model, then introducing the nuisance parameters is not necessary because $Q_{\underline{d}}^+ = P_1 \mp P_2$ (see Section 6.4.2 (4)). But the above statement will not lose its generality. Only in this case are the estimated nuisance parameters zero if they are introduced into the model.

6.5 Checking the Deformation Models and Selecting the "Best" Model

The global appropriateness of a deformation model can be tested using the quadratic form of the residuals δ_i in (6-8). Let ΔR denote such a quadratic form, thus

$$\Delta R = \sum_1^k \underline{\delta}_i^T P_i \underline{\delta}_i \quad . \tag{6-43}$$

In order to obtain $\underline{\delta}_i$ the vector of unknown constant $\underline{\xi}$ is to be computed. This can be realized by

$$\underline{\xi} = (\sum_1^k P_i)^- [\sum_1^k P_i \underline{y}_i - (\sum_2^k P_i B_i) \cdot \underline{\hat{c}}] \quad , \tag{6-44}$$

where $\underline{\hat{c}}$ have been estimated from (6-11) and the other quantities involved have been calculated in the process of the estimation of the coefficients $\underline{c}$.

The degrees of freedom of the quadratic form ΔR are computable from

$$df_c = r\{P\} - u + d \quad , \tag{6-45}$$

where, as defined in Section 6.2, $P = \text{diag}\{P_1, P_2, \ldots, P_k\}$; u is the dimension of the unknown vector $(\underline{\xi}^T : \underline{c}^T)$ in (6-8); d is the rank defects of $(\sum_1^k P_i)$, equal to the number of remaining datum defects and configuration defects which are not determined in at least one epoch.

From the results in Section 5.4, the deformation model $B\underline{c}$ is globally acceptable at confidence level $(1 - \alpha)$ if the following inequality holds:

$$T = \frac{\Delta R}{R} \cdot \frac{df}{df_c} \leq F(\alpha;\ df_c,\ df) \tag{6-46}$$

or

$$T = \Delta R/(\hat{\sigma}_o^2 \cdot df_c) \leq F(\alpha;\ df_c,\ df) \quad , \tag{6-47}$$

with R being the quadratic form of the residuals obtained from the step of the adjustment of the observations (see model (6-3)), and $\hat{\sigma}_o^2 = R/df$, the a posteriori variance factor. If a monitoring network is adjusted separately for each epoch of observations, then $R = \sum_1^k \hat{\underline{v}}_i^T Q_{ii}^{-1} \hat{\underline{v}}_i$ and $df = \sum_1^k df_i$, where $\underline{v}_i$ is the vector of residuals and df_i is the number of degrees of freedom in epoch i.

Note 1: If the a priori variance factor σ_o^2 is used instead of the a posteriori one $\hat{\sigma}_o^2$, the expression (6-46) is replaced by

$$T = \frac{\Delta R}{df_c \cdot \sigma_o^2} \leq F(\alpha;\ df_c,\ \infty) \quad .$$

Note 2: When $\underline{c} = 0$, the test statistic (6-46) can be regarded as an extension of the global congruence test, which originated from Pelzer (1971; 1974) and was further developed by Niemeier (1981). In their developments, only the coordinates are considered. As suggested in the formulation of (6-8), coordinates and other observables enter into the global congruency test of multiple-epoch observations.

Note 3: Consider the simpler case, the analysis of two epochs of observations. The quadratic form ΔR can be derived from (6-30) as

$$\Delta R = (\underline{y}_2 - \underline{y}_1)^T P_{\Delta \underline{y}} (\underline{y}_2 - \underline{y}_1) - \hat{\underline{c}}^T Q_{\hat{\underline{c}}}^{-1} \hat{\underline{c}} \qquad (6\text{-}48)$$

where $Q_{\hat{\underline{c}}}$ is the cofactor matrix of the estimators $\hat{\underline{c}}$; and $P_{\Delta \underline{y}}$ is the weight matrix of $(\underline{y}_2 - \underline{y}_1)$, its formulation has been considered in Section 6.4.2.

As discussed in Chapter 5, the quadratic form ΔR and the quantity $\hat{\underline{c}}^T Q_{\hat{\underline{c}}}^{-1} \hat{\underline{c}}$ in (6-48) are statistically independent. Therefore, they can be used to test whether the introduction of the deformation model $B\underline{c}$ would reduce the noncongruency between two epochs significantly.

The significance of the individual parameters $\hat{c}_i$ or a group of u_i parameters, $\hat{\underline{c}}_i$ which is a subset of $\hat{\underline{c}}$, is revealed by testing the null hypothesis H_o: $c_i = 0$ or $\underline{c}_i = \underline{0}$ versus the alternative hypothesis Ha: $c_i \neq 0$ or $\underline{c}_i \neq \underline{0}$. Following the developments in Section 5.4, their significances are indicated by

$$\frac{\hat{c}_i^2}{\hat{\sigma}_o^2 q_{c_i}} > F(\alpha;\ 1,\ df) \quad \text{or} \quad \frac{\hat{c}_i}{\hat{\sigma}_o \sqrt{q_{c_i}}} > t(\tfrac{\alpha}{2};\ df) \quad , \qquad (6\text{-}49)$$

and

$$\frac{\hat{\underline{c}}_i^T \, Q_{\underline{c}_i}^{-1} \, \hat{\underline{c}}_i}{\hat{\sigma}_o^2 \cdot u_i} > F(\alpha;\, u_i,\, df) \quad , \qquad (6\text{-}50)$$

with q_{c_i} being the ith diagonal element of $Q_{\hat{\underline{c}}}$ and $Q_{\hat{\underline{c}}_i}$ being a submatrix of $Q_{\hat{\underline{c}}}$. The significance level α in all the tests are fixed, say α=0.05 or α=0.01. If the global test fails, localization in time domain or in the space domain should be performed. Displaying the residuals in space and in time will help in amelioration of the model (Chrzanowski and Chen, 1981). When a set of new parameters is added into the model, its significance in the reduction of the quadratic form of the residuals can be tested using the test statistic (5-51).

Because the behaviour of the deformable body is usually not completely known, there is often more than one possible model that may be appropriate. The choice of the "best" model has regard both for statistical significance and for physical appropriateness, which would have justified consideration a priori. The "best" model should possess at least one or a combination of the following characteristics (Himmelblau, 1970; Chrzanowski et al., 1982c):

(1) fewer number of coefficients consistent with reasonable error;

(2) simpler form consistent with reasonable error;

(3) rationale that is based on physical grounds;

(4) minimal error of fit;

(5) significance of the coefficients.

6.6 Summary of the Computation Procedures

Step 1. Assessment of the observations using the MINQUE principle to obtain the variances of observations and possible correlations of the observations within one epoch or between epochs, if the a priori values are not available.

Step 2. Separate adjustment of each epoch of observations if the correlations of the observations between epochs are negligible, otherwise simultaneous adjustment of multiple epochs of observations is required, for detection of outliers and systematic errors.

Step 1 and 2 overlap because the existance of outliers and systematic errors will influence the estimated variances and covariances and adopted variances and covariances of the observations will affect the outlier detection.

Step 3. Comparison of pairs of epochs; use of the method of iterative weighted projection to yield the "best" picture of the displacement field.

Step 4. Selection of deformation models based on a priori considerations and the displacement pattern.

Step 5. Estimation of the coefficients of deformation models and their covariances using all the information available.

Step 6. Global test on the deformation model; testing groups of the coefficients or an individual one for significance.

The above three steps should be considered as an iterative three-step procedure, so they necessarily overlap.

Step 7. Simultaneous estimation of the coefficients of the deformation model in space and in time if the analysis of pairs of epochs of observations suggests that it is worth doing so.

This simultaneous estimation must be performed if the observations are scattered in time. The iterative three-step procedure is still valid. The possible deformation models can be selected either based on a priori considerations or by plotting the observations versus time.

Step 8. Comparison of the models and choice of the "best" model; computation of deformation parameters.

Step 9. Graphical display of the selected deformation model.

Figure 6-12 shows the computation flowchart.

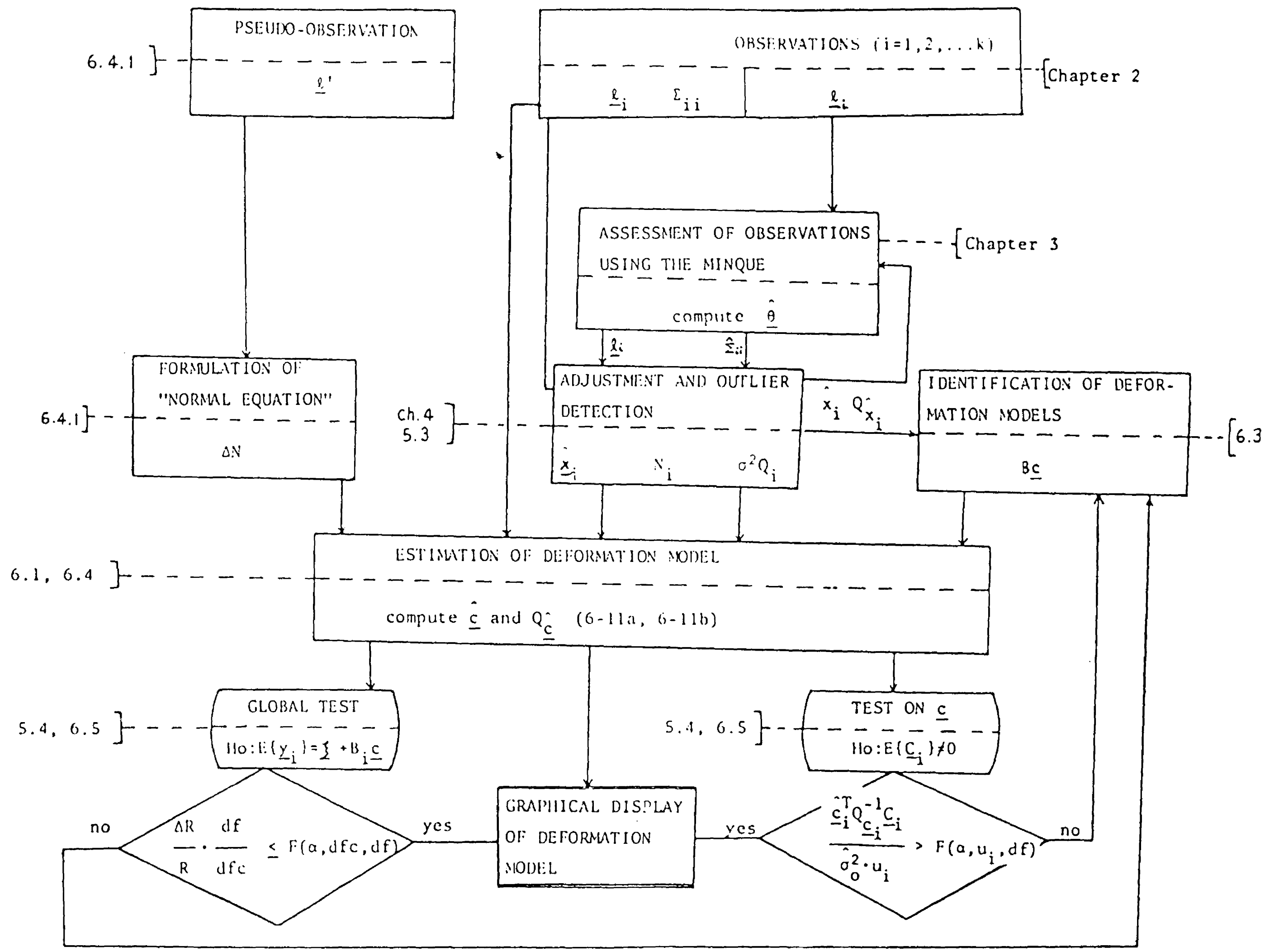


Fig. 6.12

Diagram of the Computation Procedures for the Generalized Approa ch to the Deformation Analysis

CHAPTER 7

APPLICATION OF THE GENERALIZED APPROACH TO THE ANALYSIS OF MONITORING NETWORKS

In the course of the development of the generalized approach to deformation analysis, three geodetic monitoring networks having different characteristics have been analysed within the activities of the aforementioned "ad hoc" committee of FIG. These networks were: the Huaytapallana network in Peru, provided by the Fredericton group of the FIG committee; a reference network for monitoring the Lohmühle dam, provided by the Hannover group; a simulated network across a geological fault, provided by the Karlsruhe group (the data was prepared at the Geodetic Department of the Delft University of Technology). Some of the results are presented in this chapter. In addition, three more sets of new data have been analysed by the author during the preparation of this thesis. They are: two sets of new simulations of the 2-D relative network, and a 3-D monitoring network in the open pit mine of "Adamów" in Poland.

7.1 Analysis of the Huaytapallana Monitoring Network

7.1.1 Description of the survey data

The Huaytapallana network (Figure 7.1) is located in the Huaytapallana mountain range of the Peruvian Andes, at an average elevation of 4500 m, and crosses a reverse fault which was activated by an earthquake in 1969 (Deza, 1971). At that time, a vertical displacement of 1.6 m and a horizontal strike-slip motion of 0.7 m were recorded. When designing the network and the survey (Chrzanowski et al., 1978; Nyland et al., 1979) no additional information on the expected deformations was available. The goal of this microgeodetic survey was to detect relative rigid body displacements of groups of points on both sides of the fault with a standard deviation in the order of 3 mm, or better, and to determine strain components with standard deviations in the order of 3×10^{-6}.

Due to the difficult topographic conditions only horizontal surveys were carried out, and for economical reasons only standard surveying equipment could be used. The eleven points of the network were monumented in rock outcrops using brass markers. Table 7.1 summarizes the type and number of observables and their estimated standard deviations in four epochs of observations.

7.1.2 Identification of the deformation models

Due to the lack of geophysical information on the expected deformation of the investigated area, several simple models were selected for further testing starting with the simplest assumption that no global deformations had taken place (dx = 0 and dy = 0 for the whole

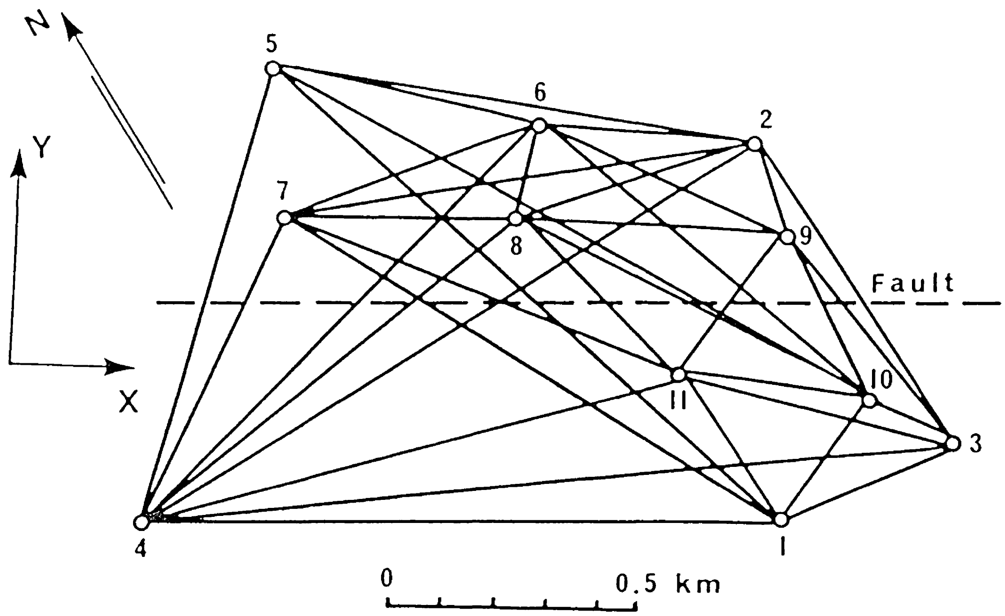


Fig. 7.1: The Huaytapallana Network.

Type of Observations	1975 No. of Observ.	1976 No. of Observ.	1977 No. of Observ.	1978 No. of Observ.
Angles	73 2.6"	81 2.2"	2 3.9"	--
Directions	--	--	91 2.8"	91 2.5"
Distances	60 4.0 mm	65 3.5 mm	74 2.7 mm	70 5.5 mm

Table 7·1: Type and standard deviations of observations - Huaytapallana Network.

area), followed by a model of the rigid body displacement along the fault line of the northern versus southern parts of the network, then a homogeneous strain model for the whole area, and then different homogeneous models on both sides of the fault. In addition, some more complicated models had been selected on the basis of the preliminary trend analysis.

Figures 7.2a and 7.2b give the examples of the displacement field in epochs 1978-1977, based on the method of the "best" minimum constraints and the method of iterative weighted projection, respectively. In this pair of epochs, points 4 and 11 indicated a movement separate from the remaining portion of the network, thus indicating the possibility of an additional discontinuity running from the fault line through the vicinity of point 1 and isolating these two points. This trend had also been confirmed by the examination of epochs 1976-1975.

An examination of the displacement field for other pairs of epochs also led to a suspicion that point 2 was unstable. Therefore, additional deformation models had been selected for further investigation which took under consideration the possibility of the separate rigid body displacements of points 4 and 11 as one block and point 2 as another block with respect to the remaining portion of the network.

7.1.3 Results and selection of the "best" model

When the network had to be analysed, the computer program for the generalized approach was not completed. Therefore the computations followed the procedures contained in Chrzanowski and Chen (1981), where

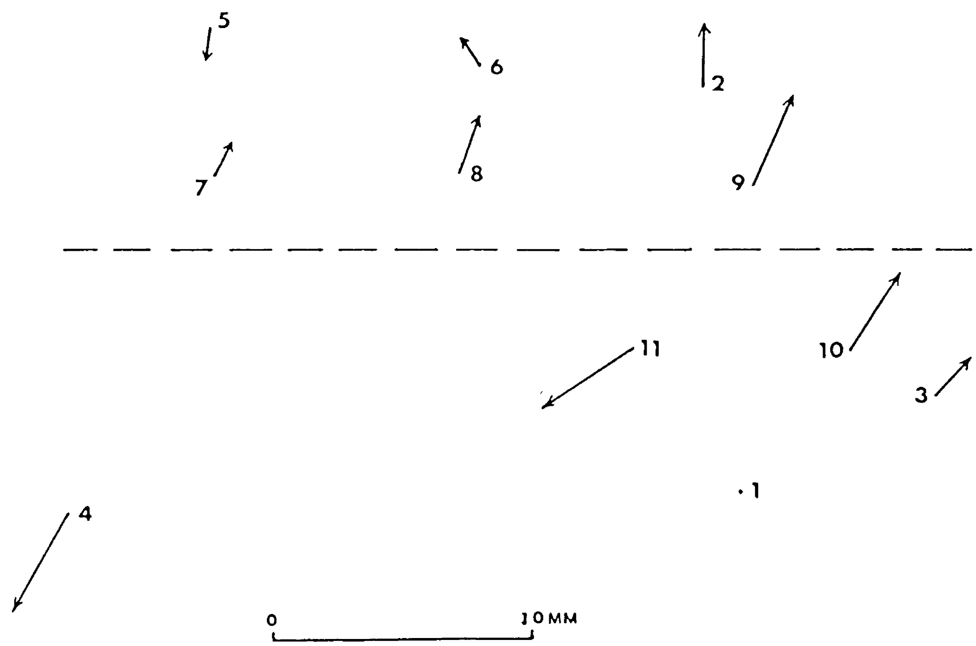


Fig. 7.2a: Displacement field for the Huaytapallana Network in epochs 1978-1977, using the "best" minimum constraints.

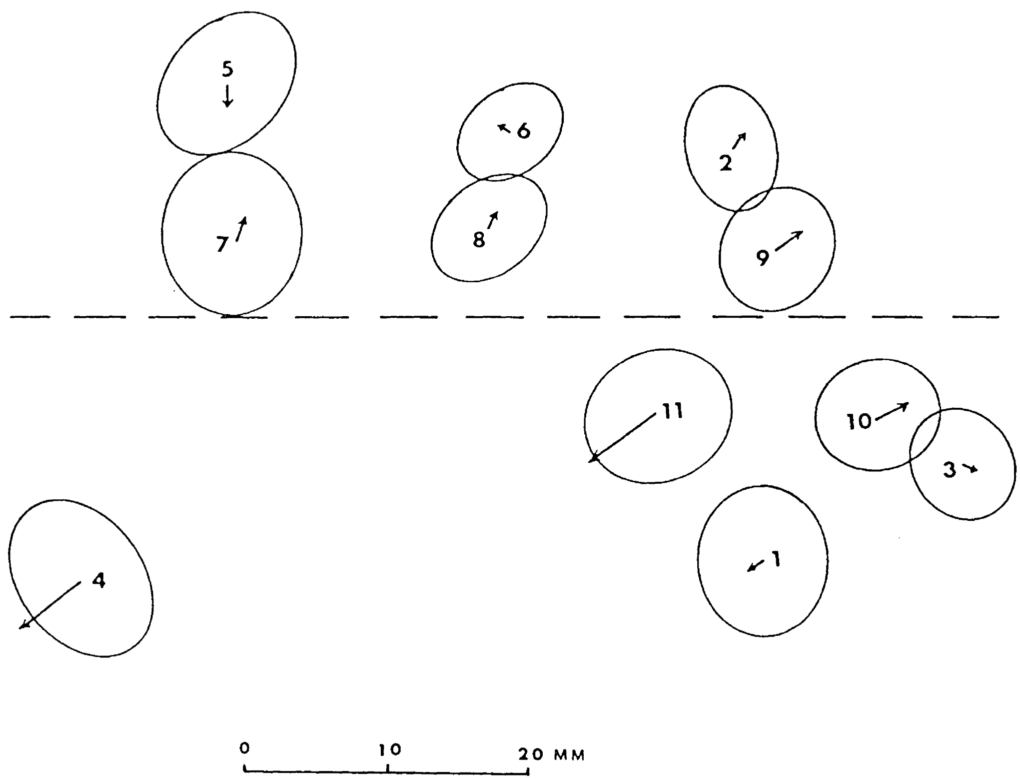


Fig. 7.2b: Displacement field for the Huaytapallana Network using the method of iterative weighted projection.

the rotation parameter ω was introduced as the "nuisance" parameter. A total of eight deformation models were fitted to the data and examined. Table 7.2 illustrates five models which had been accepted at the 95% confidence level by the global statistical test. Table 7.3 summarizes the results of the least-squares estimation of the deformation parameters and their confidence level at Pr%.

As one can see, the first model, "no global-deformation", had passed the global tests at the 95% confidence level. However, after a close examination of the results, model no. 5 had been accepted as the "best" on the basis of the indicated trend of displacements and high confidence levels (> 99%) of its parameters. Figure 7.3 gives a graphical display of the estimated deformations of the area covered by the survey network using the "best" deformation model. In this model, block C contains points 4 and 11 of the network and block B contains point 2. Most probably, the estimated relative displacements of the blocks, particularly of point 2, are of a local, non-tectonic nature, such as surface movements of the marked survey points. However, the movement of points 4 and 11 as one block, if not coincidental, may indicate an additional crustal discontinuity and an action of tectonic forces. Only additional future remeasurements of the network will allow for more concrete conclusions.

It is interesting to note that the survey data of the Huaytapallana network were also analysed by Margrave and Nyland (1980). They considered only the homogeneous strain model in their analysis. They concluded that between 1975 and 1976 the area of the survey experienced a left-lateral shear strain of about -3 microstrains, which was possibly associated with tensional straining perpendicular to the

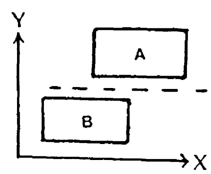
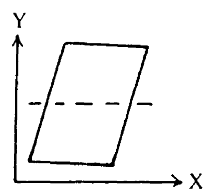
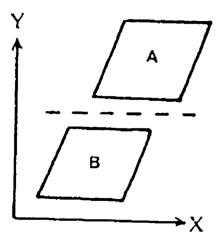
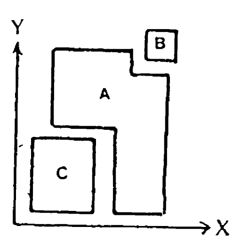
Model No.	Deformation Model	Global Tests: Epochs	Global Tests: $T^2 \lessgtr F_{95\%}$
1.	No global deformation $dx = 0$ $dy = 0$	'76-'75 '77-'76 '78-'77 '78-'75	1.48<1.63 1.38<1.63 1.44<1.63 0.80<1.63
2.	 $dx_A=0 \quad dx_B=a_o$ $dy_A=0 \quad dy_B=b_o$	'76-'75 '77-'76 '78-'77 '78-'75	1.35<1.69 1.39<1.68 0.92<1.68 0.60<1.69
3.	 $dx=\varepsilon_x x+\varepsilon_{xy} y+\omega y$ $dy=\varepsilon_{xy} x+\varepsilon_y y+\omega x$	'76-'75 '77-'76 '78-'77 '78-'75	1.32<1.71 1.35<1.71 1.15<1.71 0.68<1.71
4.	 $dx_A=\varepsilon_x x+\varepsilon_{xy} y-\omega y$ $dy_A=\varepsilon_{xy} x+\varepsilon_y y+\omega x$ $dx_B=a_o+\varepsilon_x x+\varepsilon_{xy} y-\omega y$ $dy_B=b_o+\varepsilon_{xy} x+\varepsilon_y y+\omega x$	'76-'75 '77-'76 '78-'77 '78-'75	1.43<1.76 1.51<1.76 0.91<1.76 0.67<1.76
5.	 $dx_A=0 \quad dx_B=a_o$ $dy_A=0 \quad dy_B=b_o$ $dx_C=c_o$ $dy_C=g_o$	'76-'75 '77-'76 '78-'77 '78-'75	0.44<1.73 1.00<1.73 0.37<1.73 0.69<1.73

Table 7.2: Deformation models and Global Tests

Model No.	Deformation parameters	1976-1975 e_i	Pr%	1977-1976 e_i	Pr%	1978-1977 e_i	Pr%	1978-1975 e_i	Pr%
1.	-	-		-		-		-	
2.	a_o [mm]	1.8	89%	-1.6	89%	-3.4	99%	-3.2	97%
	b_o [mm]	2.2	94%	0.0	1%	-3.7	99%	-1.8	75%
	$(a_o^2+b_o^2)^{1/2}$	2.9	93%	1.6	73%	4.2	99%	3.6	93%
3.	ε_x [µstrain]	-1.6	86%	-0.4	33%	1.6	81%	-0.4	21%
	ε_y [µstrain]	-2.6	80%	0.8	33%	3.8	89%	2.0	55%
	ε_{xy} [µstrain]	-1.6	91%	1.7	96%	2.1	94%	2.5	95%
4.	ε_x [µstrain]	-1.7	88%	-0.4	37%	2.0	90%	-0.2	11%
	ε_y [µstrain]	-0.4	10%	2.5	54%	-2.5	41%	-1.8	28%
	ε_{xy} [µstrain]	-0.8	36%	2.1	84%	-1.0	38%	1.1	39%
	a_o [mm]	1.3	51%	0.7	28%	-4.8	95%	-2.2	62%
	b_o [mm]	1.6	54%	1.2	46%	-4.4	88%	-2.7	63%
5.	a_o	-2.6	96%	3.7	99%	0.8	37%	0.0	1%
	b_o	-2.9	94%	1.2	58%	-0.4	16%	-1.5	54%
	$(a_o^2+b_o^2)^{1/2}$	3.9	99%	3.9	99%	0.8	15%	1.5	26%
	c_o [mm]	2.7	99%	0.3	22%	-4.8	99%	-1.4	65%
	g_o [mm]	1.2	63%	-0.0	1%	-4.0	99%	-2.9	91%
	$(c_o^2+g_o^2)^{1/2}$	3.0	99%	0.3	4%	6.2	99%	3.2	88%

Table 7.3: Results of the least squares fitting of deformation models - Huaytapallana Network.

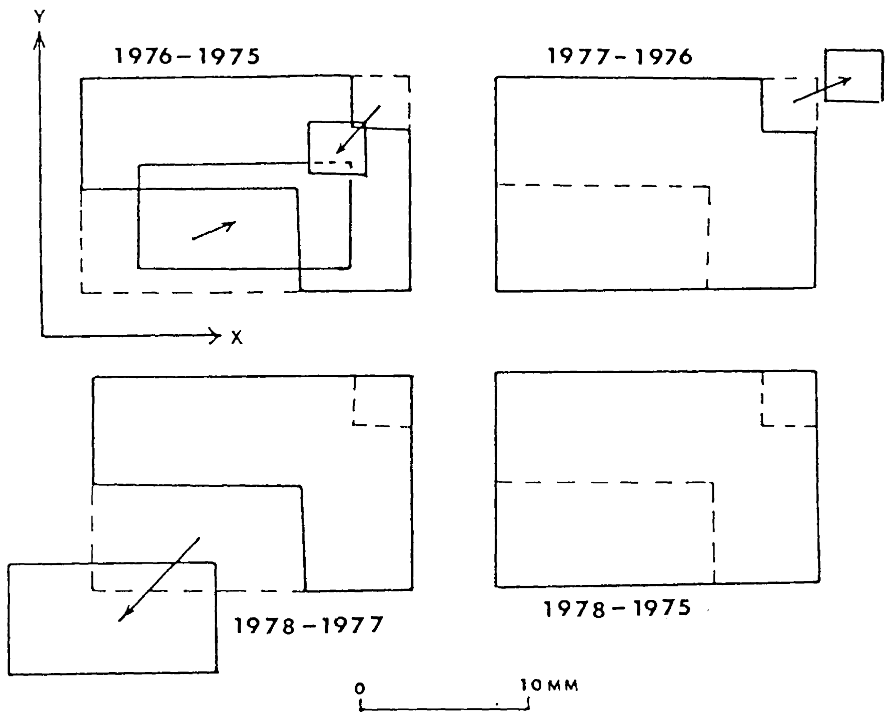


Fig. 7.3: Schematic representation of the "best" deformation model for Huaytapallana Network.

fault. From 1976 to 1978 a right-lateral shear strain of about +3 microstrains occurred in this area and was associated with a probable tensional straining parallel to the fault. In their analysis, Margrave and Nyland used the raw observations approach, thus they were not able to utilize all the observables which differed from one epoch to another. This may partially explain the numerical deviations from the values shown for model no. 3 in Table 7.3. However, the overall conclusions of both studies would be in agreement if model no. 3 were accepted as the "best" model. Since that was not the case, the comparison of the final results indicates how important it is to follow the proposed generalized approach to the analysis with a careful examination of more than just one deformation model.

7.2 Analysis of the Simulated Relative Network

7.2.1 First simulation

The network (Figure 7.4) was simulated as a part of a first-order horizontal control network crossing a geological fault. A total of three simulations of deformations of the network were prepared at Delft University of Technology.

The first simulation included two epochs of observations. In the first epoch, the network consisted of 16 points, which were connected by 70 direction measurements with equal a priori standard deviations of 0".32. At epoch 2, one point (point 9) has been added; moreover, points 7 and 19 have been replaced by new points 207 and 209 in the vicinity of the old ones. The observations in the second epoch included 70 direction measurements (uniform a priori standard deviation

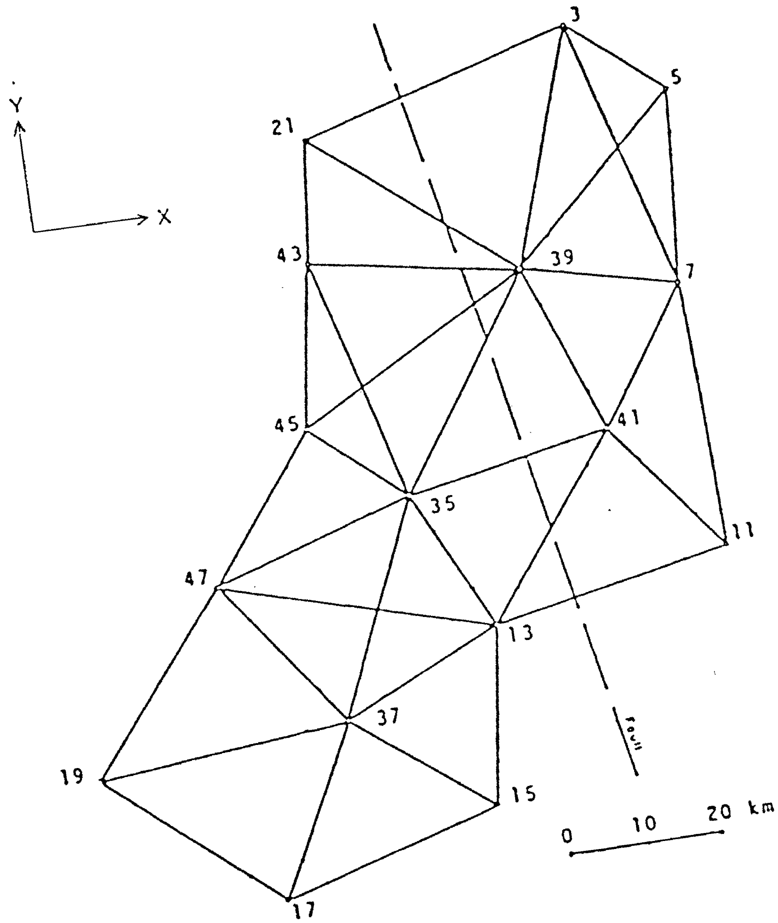


Fig. 7.4: Simulated Network.

of 0".32) and 37 distance measurements (standard deviation of 0.06 m). The simulated deformation model, which had not been known to the members of the FIG "ad hoc" committee before the analysis, consisted of single point displacement of points 3, 15 and 45, and a relative rigid body displacement of the two opposite parts of the network along the fault line.

Separate adjustments of the network were carried out using the available program, GEOPAN, at UNB, in which the technique for outlier detection is τ-max criterion. The adjusted coordinates and their covariance matrix with respect to a certain datum are obtained. Using the method of the iterative weighted projection discussed in Section 6.3, the displacement field and the error ellipses at α = 5% of the displacements of the points have been plotted in Figure 7.5.

A trend analysis of the displacement field led to a choice of several possible deformation models. The results are tabulated in Table 7.4. Although model 2 corresponds to the actually simulated deformations, it is not the "best" one because the global test fails and point 3 shows no significant movement. A total number of six models were attempted. Finally, model 6 has been selected as the "best model". The whole area experiences a homogeneous shear strain and additional movements exist--points 3 and 5 form a separate block and also single points 11, 15, and 45, each treated as a separate block undergoing a rigid body displacement plus the same shear strain. The "best" model is graphically represented in Figure 7.6

A comparison of the results with the other research groups (Heck, 1982; Chrzanowski and Second, 1983) is listed in Table 7.5.

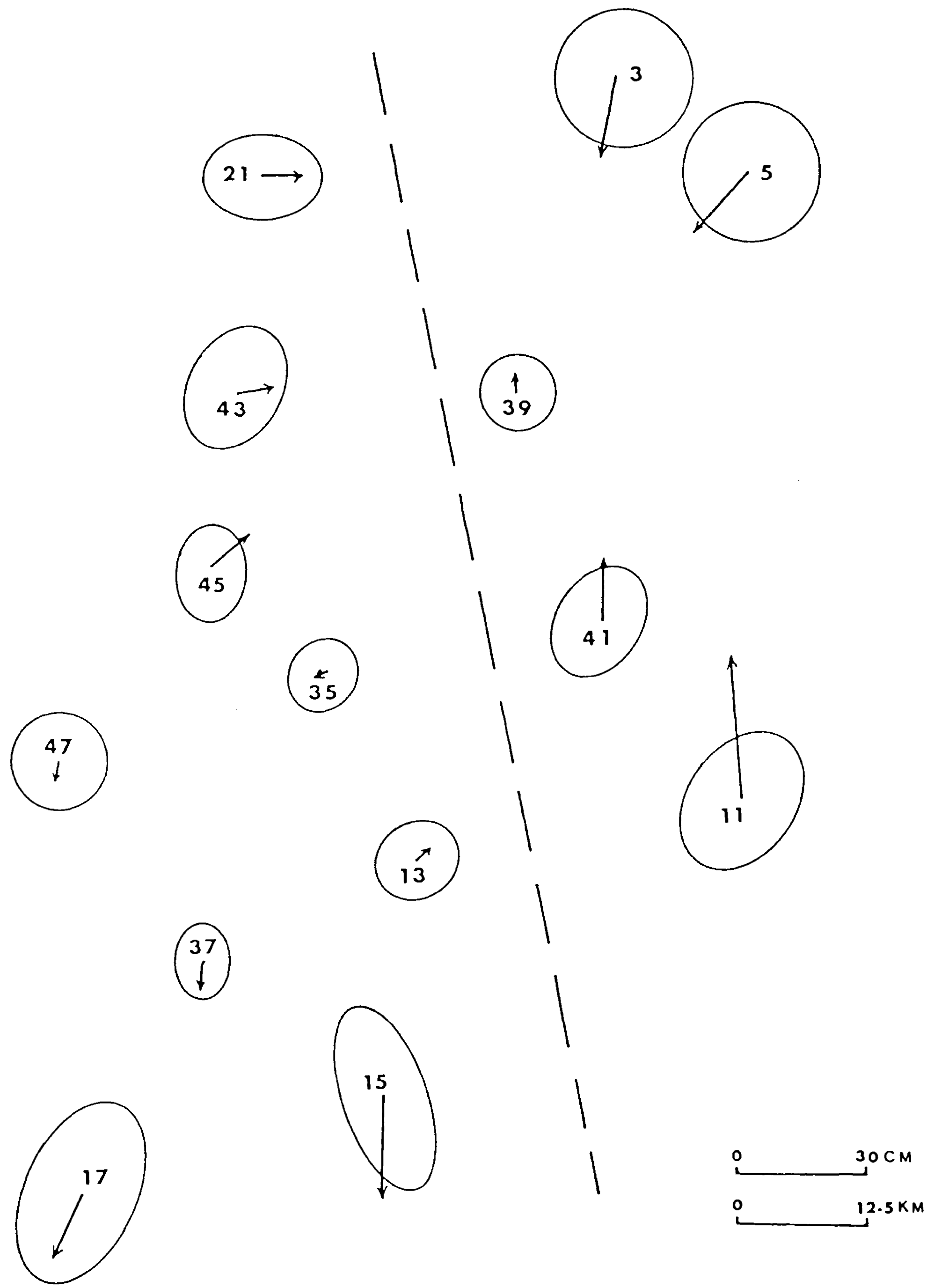


Fig. 7.5: Displacement Field and Error Ellipses (α = 5%) - The First Simulated Network.

No.	Deformation Model	Estimated Deformation Parameters and Their Statistics ($\alpha = 0.05$)			Global Test ($\alpha = 0.05$)
1	no significant deformation				3.24 > 1.64
2	relative movement of the Eastern vs. Western block plus single point displacements of pts. 3, 15, 45	a = 4.9 cm	b = 7.9 cm	(4.4 > 3.1)	2.06 > 1.80
		dx_3 = -3.5	dy_3 = -1.7	(0.7 < 3.1)	
		dx_{15} = -8.3	dy_{15} = -15.4	(7.36 > 3.1)	
		dx_{45} = 6.9	dy_{45} = 9.0	(10.0 > 3.1)	
3	pt. 3 and 5 form a block plus single point displacements of pts. 11, 15, 41, 45	a = -10.5 cm	b = -13.9 cm	(4.22 > 3.1)	1.07 < 1.84
		dx_{11} = -3.8	dy_{11} = 26.6	(11.5 > 3.1)	
		dx_{15} = -6.3	dy_{15} = -16.2	(6.9 > 3.1)	
		dx_{41} = -0.1	dy_{41} = 8.1	(2.5 < 3.1)	
		dx_{45} = 6.9	dy_{45} = 8.7	(9.5 > 3.1)	
4	pts. 3 and 5 form a block plus single point displacements of pts. 11, 15, 37, 45	a = -7.1 cm	b = -12.9 cm	(3.3 > 3.1)	1.13 < 1.84
		dx_{11} = -6.8	dy_{11} = 20.8	(9.2 > 3.1)	
		dx_{15} = -8.8	dy_{15} = - 19.4	(8.5 > 3.1)	
		dx_{37} = -0.9	dy_{37} = - 5.5	(2.0 < 3.1)	
		dx_{45} = 7.0	dy_{45} = 9.1	(10.2 > 3.1)	
5	pts. 3 and 5 form a block plus single point displacements of pts. 11, 15, 45	a = -7.1 cm	b = -12.9 cm	(3.3 > 3.1)	1.25 < 1.80
		dx_{11} = -6.7	dy_{11} = 20.4	(9.0 > 3.1)	
		dx_{15} = -7.2	dy_{15} = -15.9	(7.1 > 3.1)	
		dx_{45} = 6.9	dy_{45} = 9.2	(10.3 > 3.1)	

continued ...

No.	Deformation Model	Estimated Deformation Parameters and Their Statistics ($\alpha = 0.05$)			Global Test ($\alpha = 0.05$)
6	the whole body undergoing a homogeneous shear strain plus additional movements of the block (pt. 3 and 5) and of points 11, 15, 45	$a = -14.1$ cm	$b = -19.6$ cm	(6.8 > 3.1)	0.68 < 1.82
		$dx_{11} = -3.4$	$dy_{11} = 16.8$	(5.5 > 3.1)	
		$dx_{15} = -3.7$	$dy_{15} = -20.9$	(8.9 > 3.1)	
		$dx_{45} = 7.6$	$dy_{45} = 10.5$	(12.5> 3.1)	
		$\varepsilon_{xy} = 1.69$ μstrain	(critical value 1.1 μstrain)		

Note: 1. a, b are components of block translation in x, y direction respectively, dx_i, dy_i are components of ith point displacement (same notation will be used below);

2. ($\xi > \xi_\alpha$) where value ξ is the quadratic form of the displacement of a point and ξ_α corresponds the critical value at significance level $\alpha = 0.05$.

Table 7.4: The Estimated Deformation Models -- The First Simulated Network.

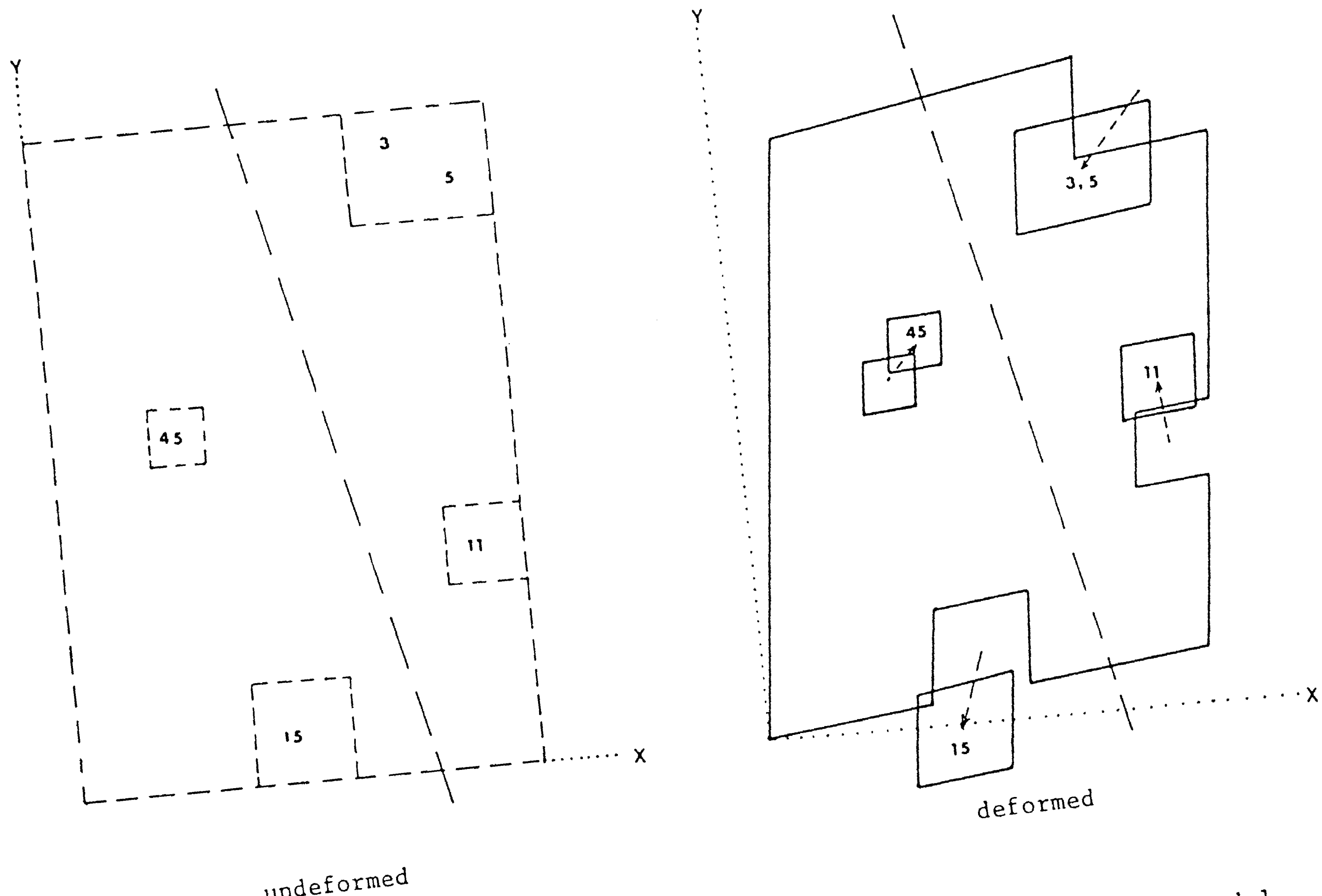


Fig. 7.6: Graphical Representation of Selected Deformation Model.
- The First Simulated Network.

	Single Point Displacement						Relative Movement of the Eastern vs. Western
Group	3*	5	11	15*	35	45*	Block*
Delft	-	-	yes	yes	yes	yes	-
Fredericton	yes**	yes**	yes	yes	-	yes	yes**
Hanover	-	-	yes	yes	-	yes	-
Karlsruhe	-	-	yes	yes	-	yes	-
Munich	yes	yes	yes	yes	-	-	-

* Models are underlying the simulation.
** The best model of the Fredericton Group treated points 3 and 5 as one block and the relative movement of the eastern block versus the western block was interpreted as a significant homogeneous shearing strain.

TABLE 7.5

Significant Deformation in the First Simulated Network--A Comparison.

As one can see, the Fredericton Group gave a better result. Some inconsistencies exist among the groups. The reason is due to the selection of the significance levels. Since the Hannover group and the Karlsruhe group use the global congruency test at 95% confidence level as the only criterion for testing single point movements, one should expect them to detect a fewer number of unstable points than the Fredericton group and the Munich group, which use $\alpha = 0.05$ significance level for testing single point movement and individual deformation parameters. The Delft group have their own philosophy in the selection of significance levels (see Chapter 5), so their results may be sometimes closer to the results of the Fredericton and Munich groups, but sometimes closer to the Hannover and Karlsruhe results, depending on the number of points in a network. The reasons for inclusion of this example here has been to give an explanation of why the different groups obtained different results and to test a newly developed program for the generalized appproach.

The first simulation was, obviously, not very successful because the simulated deformation parameters were too small in comparison with the accuracy of observations. Therefore, two more simulations of the same network have been prepared and their analyses follow.

7.2.2 Second simulation

The second simulation included three epochs of simulated observations (the nature of the deformation is unknown, the same for the third simulation). In all the epochs, distances and directions were observed, but their variances were not given. In the first epoch, the network consisted of 16 points, connected by 72 direction measurements and 34 distance measurements. In epochs 2 and 3, point 9 has been added and points 7 and 19 have been replaced by new points 97 and 99 in the vicinity of the old ones. The network contained 72 direction measurements and 37 distance measurements.

Because the a priori variances of the observations in the second simulation were not given, evaluation of the accuracy of the observations had to be performed before the final adjustment of the network. The minimum norm quadratic unbiased estimation principle (MINQUE) has been employed in the assessment of the observation data, which was discussed in Chapter 3. Since the measured distances in the network are more or less uniform, which differs from the Pitec network which was given as an example in Chapter 3, isolation of two error components of the measured distances, a constant error and an error proportional to the measured distance, is not pertinent. Thus, all the measured directions and distances in one epoch are assumed to have the same variances. The error structure for one epoch of observations $\underline{\ell}_i$ is

$$D\{\underline{\ell}_i\} = \sigma_s^2 \begin{pmatrix} I_s & 0 \\ & \\ 0 & 0 \end{pmatrix} + \sigma_r^2 \begin{pmatrix} 0 & 0 \\ & \\ 0 & I_r \end{pmatrix} ,$$

where σ_s^2 and σ_r^2 are the variances for the distances and directions; I_s and I_r are identity matrices with dimensions being equal to the number of the distances and directions, respectively. The estimated standard deviations are $\sigma_s = 0.026$ m, $\sigma_r = 0''.46$ for the first epoch, $\sigma_s = 0.10$ m, $\sigma_r = 0''.35$ for the rest.

Using the same method as in Section 7.2.1, the displacement field is plotted in Figure 7.7. Several possible models are estimated and the results are presented in Table 7.6.

In epochs 2-1, point 3 does not follow the deformation trend--the block comprising points 3, 5, 11, 39, 41 moves with respect to the other block, and point 45 shows significant single point movement. In epochs 3-2, the block continues to move, but points 3 and 11 undergo additional movements. Points 97 and 9 seem to be located in another block. In addition, points 15 and 45 are significantly unstable. In epochs 3-1, the block experiences rigid body displacement, but irregular movement of point 11 exists. Moreover, point 15 shows single point movement. Comparing these results, one can conclude:

(1) the eastern block without including points 9 and 97 demonstrates a continuous northeast movement;

(2) points 3 and 11 within this block undergo additional movements, with point 3 moving significantly in both pairs of 1-2 and 2-3 epochs but point 11 moving significantly only in epochs 2-3.

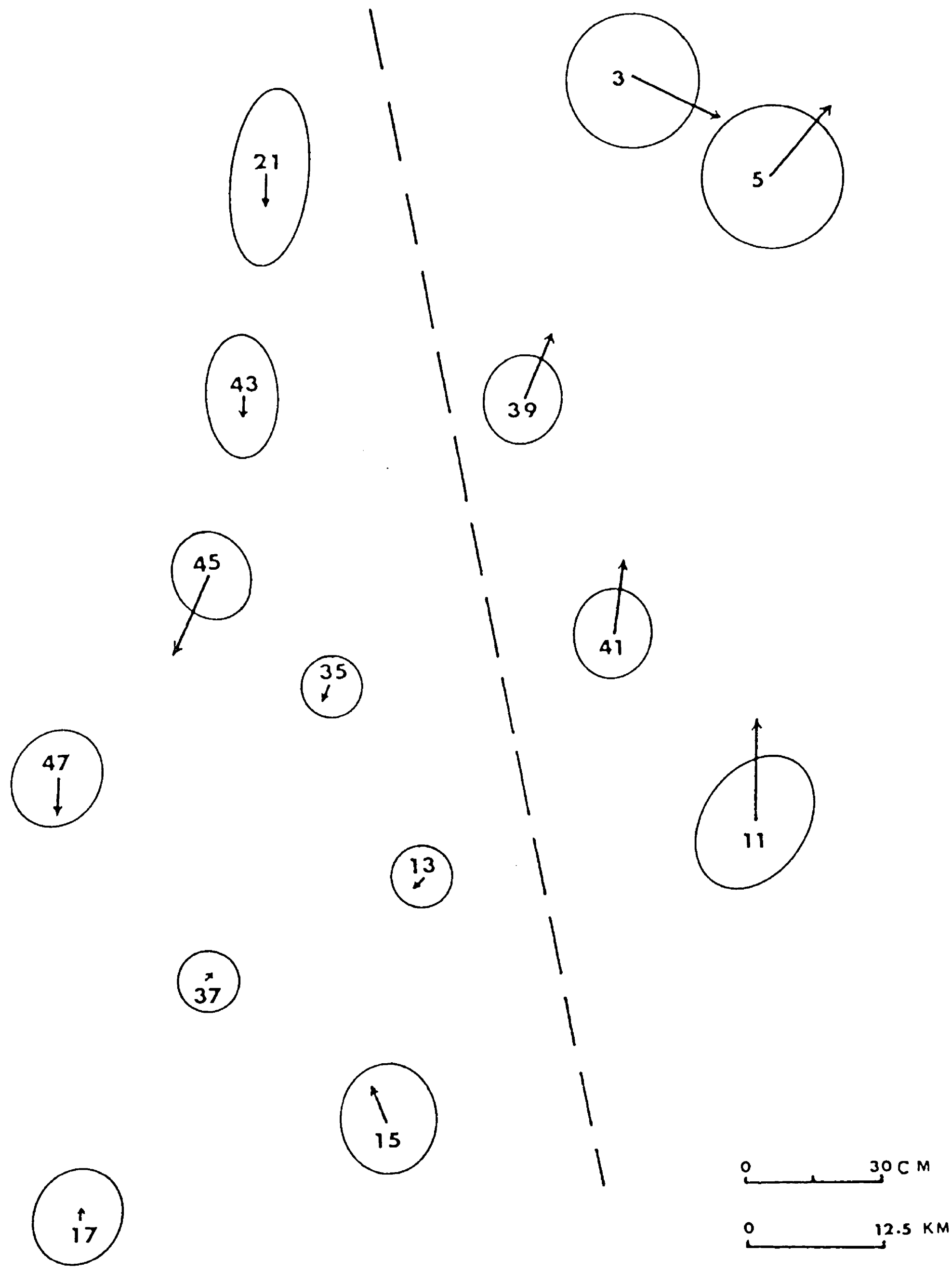


Fig. 7.7 (a)

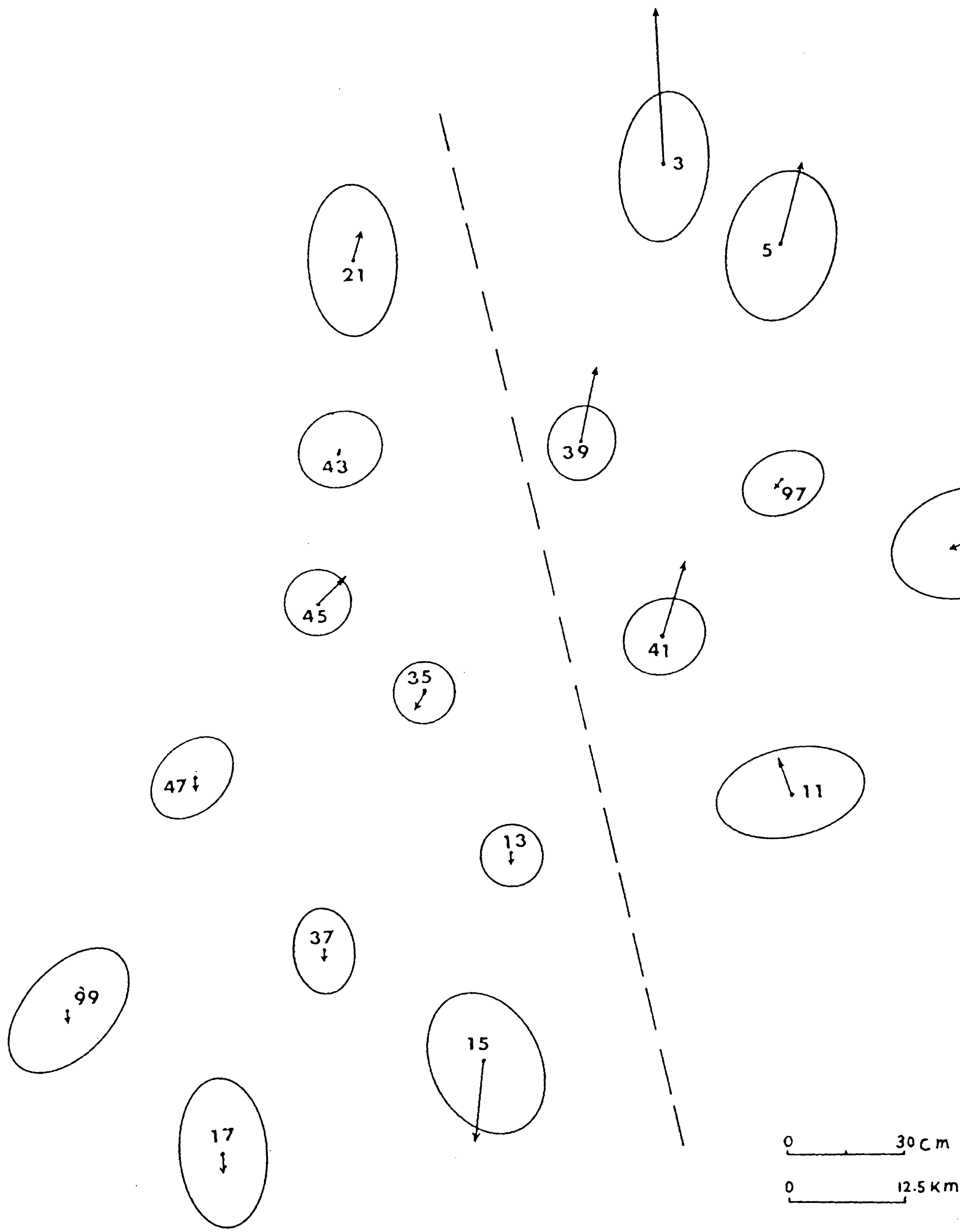


Fig. 7.7 (b)

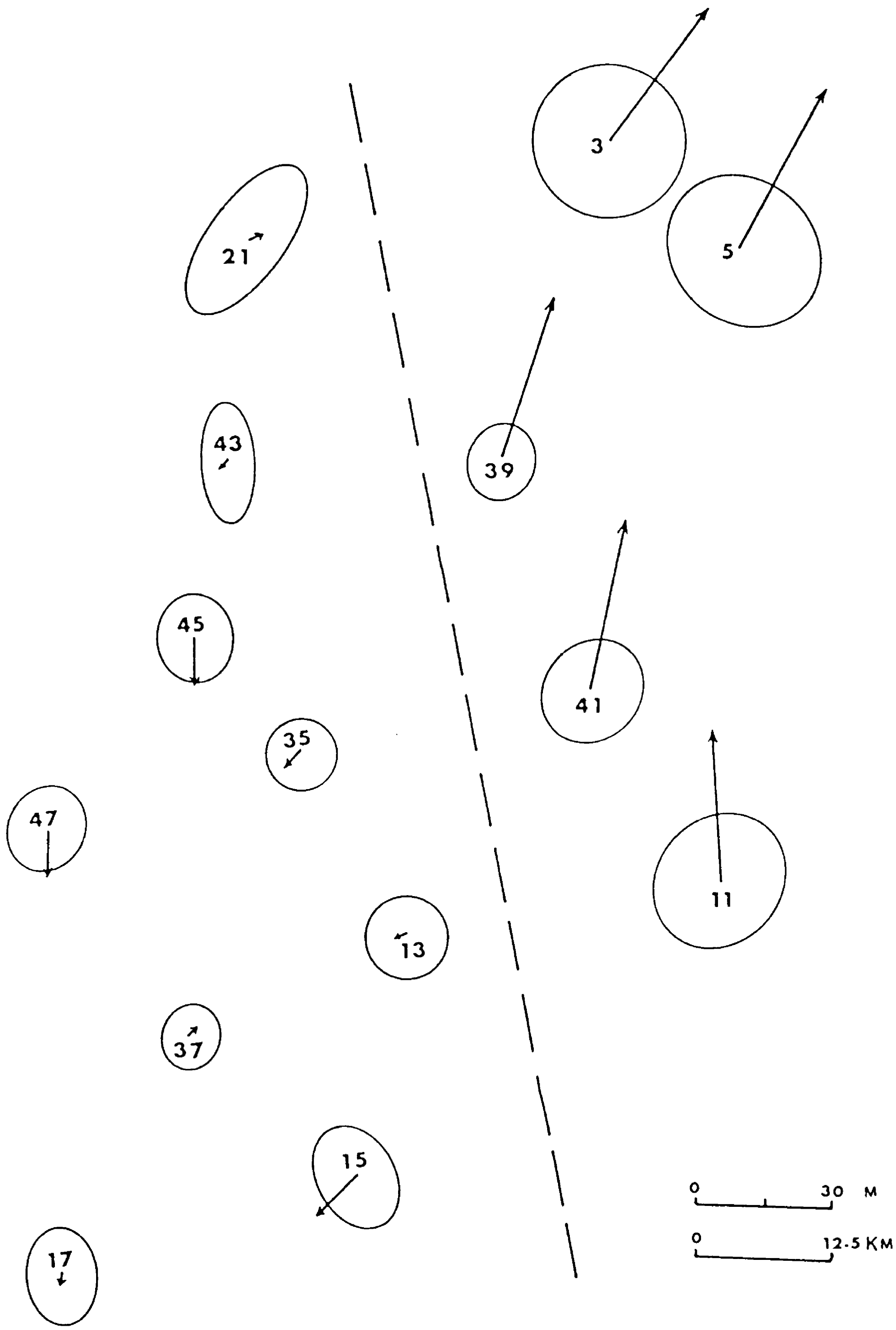


Fig. 7.7 (c)

Displacement Field and Error Ellipses (α = 5%) — The Second Simulated Network

(a) Epochs 2-1, (b) Epochs 3-2, (c) Epochs 3-1.

Epochs	No.	Deformation Model	Estimated Deformation Parameters and Their Statistics ($\alpha = 0.05$)			Global Test ($\alpha = 0.05$)
Epoch 2 - Epoch 1	1	no global deformation				5.99 > 1.60
	2	relative block movement of points 3, 5, 11, 39, 41 plus single point movements of pts. 3, 15, 45	a = 7.0 cm	b = 22.1 cm	(38.2 > 3.1)	0.71 < 1.73
			$dx_3 = 10.5$	$dy_3 = -25.8$	(26.6 > 3.1)	
			$dx_{15} = -3.8$	$dy_{15} = 7.1$	(2.4 < 3.1)	
			$dx_{45} = -7.4$	$dy_{45} = -12.5$	(16.8 > 3.1)	
	3	relative block movement of points 3, 5, 11, 39, 41 plus single point movements of pts. 3, 45	a = 7.3 cm	b = 22.2 cm	(38.5 > 3.1)	0.89 < 1.72
			$dx_3 = 10.1$	$dy_3 = -25.8$	(27.7 > 3.1)	
			$dx_{45} = -7.4$	$dy_{45} = -12.8$	(16.6 > 3.1)	
	4	relative block movement of point 3, 5, 11, 39, 41 plus single point movements of pts. 3, 21, 45, 47	a = 6.7 cm	b = 21.7 cm	(30.3 > 3.1)	0.82 < 1.75
			$dx_3 = 9.1$	$dy_3 = 26.3$	(29.2 > 3.1)	
			$dx_{21} = -1.2$	$dy_{21} = -5.0$	(0.28 < 3.1)	
			$dx_{45} = -7.5$	$dy_{45} = -14.0$	(18.5 > 3.1)	
			$dx_{47} = -1.3$	$dy_{47} = -7.3$	(2.0 < 3.1)	

continued ...

Epochs	No.	Deformation Models	Estimated Deformation Parameters and Their Statistics ($\alpha = 0.05$)			Global Test ($\alpha = 0.05$)
Epoch 3 - 2	1	relative block movement of pts. 3, 5, 11, 39, 41 plus single point displacements of pts. 15 and 45	$a = 5.2$ cm $dx_{15} = -2.9$ $dx_{45} = 8.9$	$b = 19.7$ cm $dy_{15} = -17.4$ $dy_{45} = 9.6$	(55.1 > 3.1) (8.7 > 3.1) (13.4 > 3.1)	1.86 > 1.65
	2	model 1 plus single point movements of pts. 3 and 11	$a = 6.6$ cm $dx_3 = -6.8$ $dx_{11} = -9.8$ $dx_{15} = -3.5$ $dx_{45} = 9.1$	$b = 19.9$ cm $dy_3 = 19.5$ $dy_{11} = -10.1$ $dy_{15} = -17.4$ $dy_{45} = 9.4$	(51.8 > 3.1) (14.9 > 3.1) (6.0 > 3.1) (8.7 > 3.1) (13.5 > 3.1)	0.2 < 1.72
Epoch 3 - 1	1	relative block movement of pts. 3, 5, 11, 39, 41 plus single point movements of pts. 15 and 11	$a = 13.0$ cm $dx_{11} = -11.7$ $dx_{15} = -6.8$	$b = 41.1$ cm $dy_{11} = -3.1$ $dy_{15} = -10.7$	(131.3 > 3.1) (4.0 > 3.1) (5.5 > 3.1)	0.83 < 1.72
	2	model 1 plus single point movement of point 45	$a = 13.1$ cm $dx_{11} = -11.7$ $dx_{15} = -6.7$ $dx_{45} = -1.4$	$b = 41.0$ cm $dy_{11} = -3.0$ $dy_{15} = -10.6$ $dy_{45} = -3.5$	(142.4 > 3.1) (4.0 > 3.1) (5.4 > 3.1) (0.9 < 3.1)	1.16 < 1.75

Table 7.6: The Estimated Deformation Model -- The Second Simulated Network.

(3) point 45 moved significantly between epoch 1 and epoch 2, but moved back (also significantly) between epoch 3 and 2; therefore no significant movement is detected from the results for epochs 3-1;

(4) point 15 has moved between epoch 3 and epoch 2.

The final deformation parameters are estimated from three epochs of observations simultaneously using the above detected movements as the final deformation models. The results are given in Table 7.7 and displayed graphically in Figure 7.8.

7.2.3 Third simulation

The third simulation also included three epochs. The configuration of the network and the number of observations are identical with the second simulation. Moreover, both simulations share the common first epoch. The whole procedure of analysis is the same as above. The estimated standard deviations are: $\sigma_s = 0.026$ m, $\sigma_r = 0''.46$ for the first epoch; $\sigma_s = 0.10$ m, $\sigma_r = 0''.35$ for the second and third epochs. The displacement field is plotted in Figure 7.9 and the results of the estimated deformation models are given in Table 7.8.

Examination of the results of epoch 2-epoch 1, epoch 3-epoch 2, and epoch 3-epoch 1 led to the conclusion that during the time interval of three epochs, the block consisting of points 3, 5, 11, 39, and 41 experiences rigid body displacement in respect to the block containing all other points. The results of simultaneous estimation of the deformation parameters from three epochs of observations are given in Table 7.9 and displayed in Figure 7.10.

Deformation Model	Rigid Body Motion of Eastern Block * (cm)	Point 3 ** (cm)	Point 11 ** (cm)	Point 15 (cm)	Point 45 (cm)	Global Test
Epoch 2 - Epoch 1	6.5 (2.6) ***	10.3 (4.4)			- 7.4 (2.4)	
	21.3 (2.5)	-25.4 (3.4)			-12.7 (2.8)	0.36
						<
Epoch 3 - Epoch 2	6.6 (2.2)	- 6.8 (5.2)	- 9.8 (3.7)	- 3.5 (3.7)	9.1 (2.4)	1.50
	19.6 (2.0)	19.6 (3.9)	-10.1 (4.2)	-17.4 (4.2)	9.4 (2.4)	

* Without including points 97, 9 .

** Besides following block movement, the points undergo additional displacements.

*** The value in parentheses are the standard deviations. The first value is the component in the x-direction and the second in the y-direction.

Table 7.7: Simultaneous Estimation of Deformation Model from Three Epochs of Observations -- The Second Simulated Network.

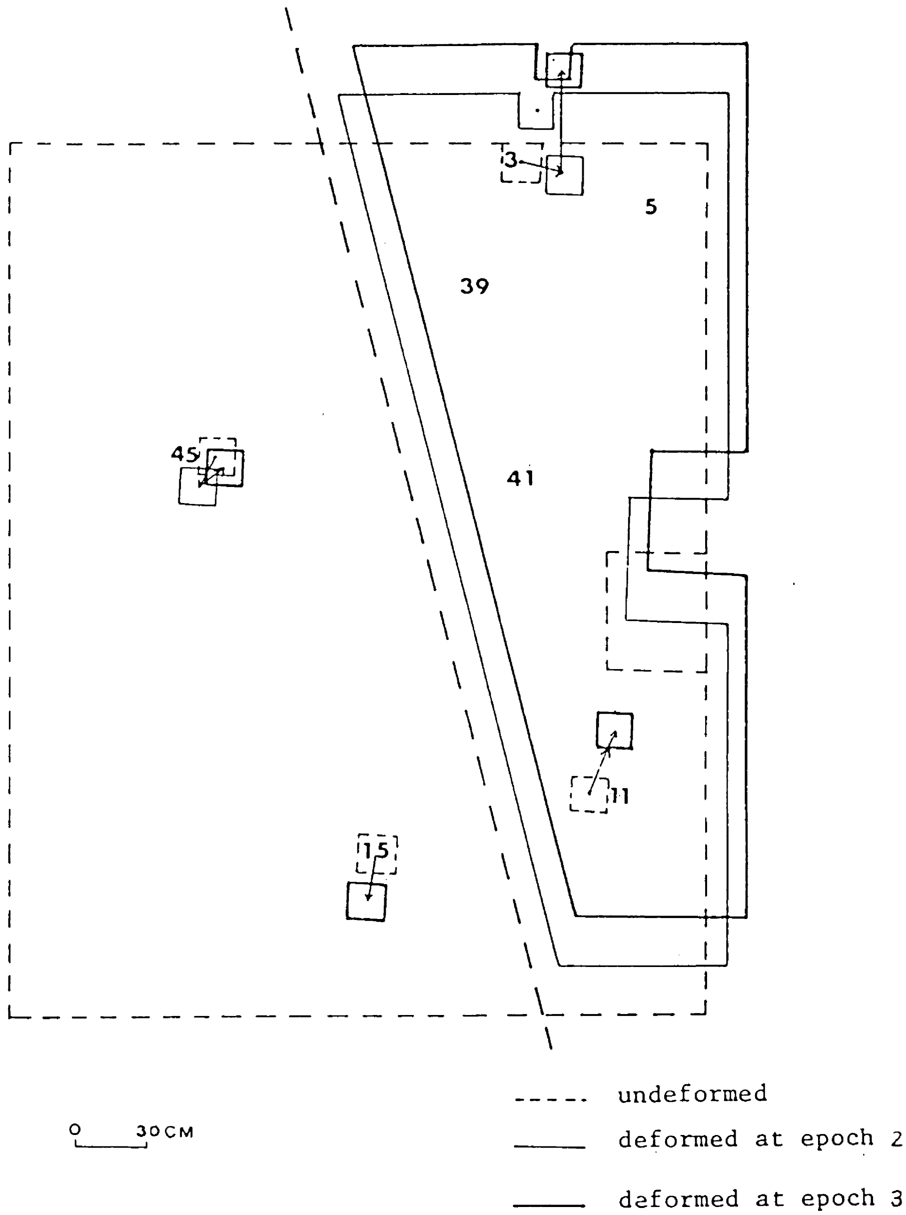


Fig. 7.8

Graphical Representation of the Final Deformation Model - The Second Simulated Network.

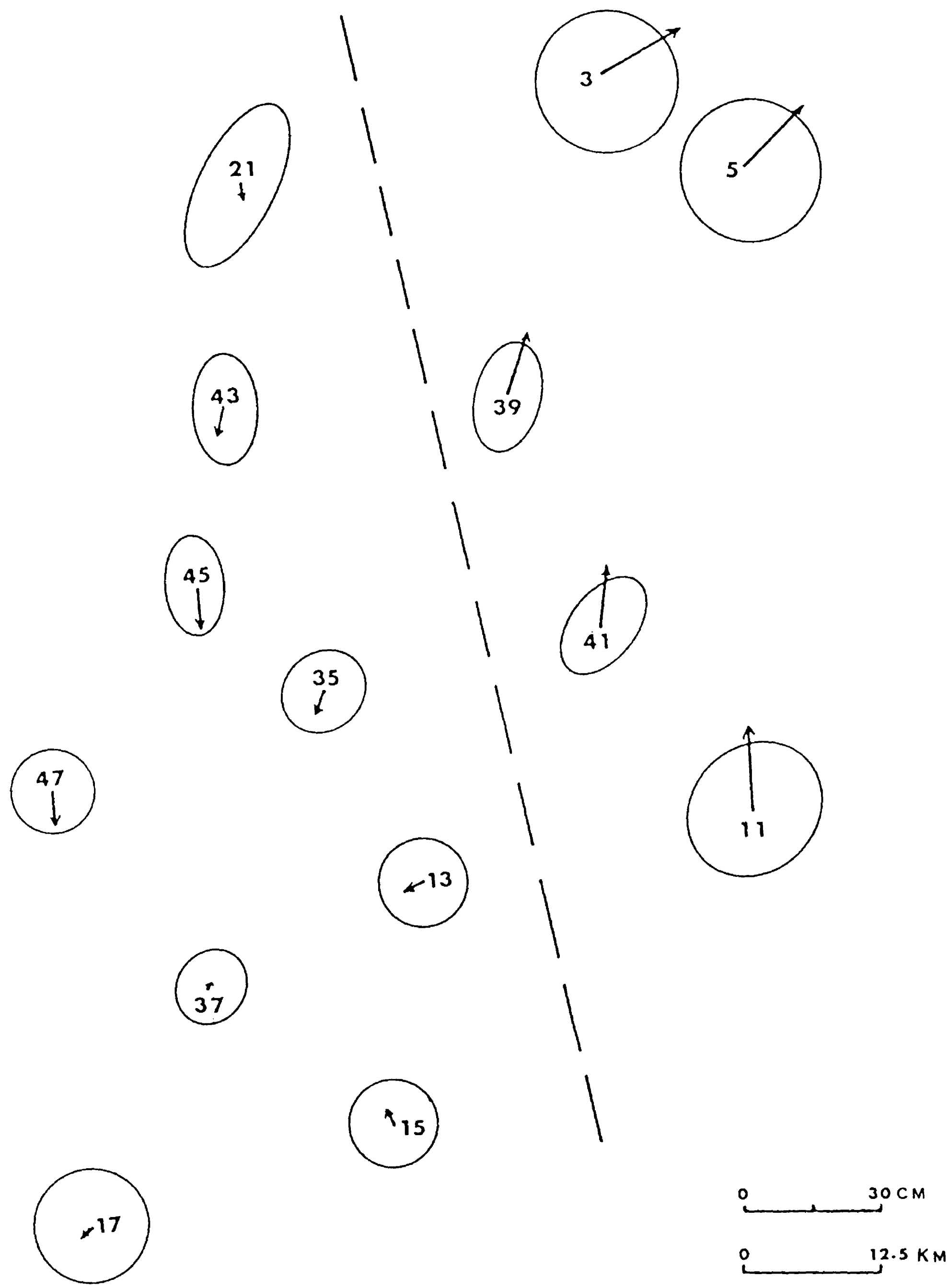


Fig. 7.9 (a)

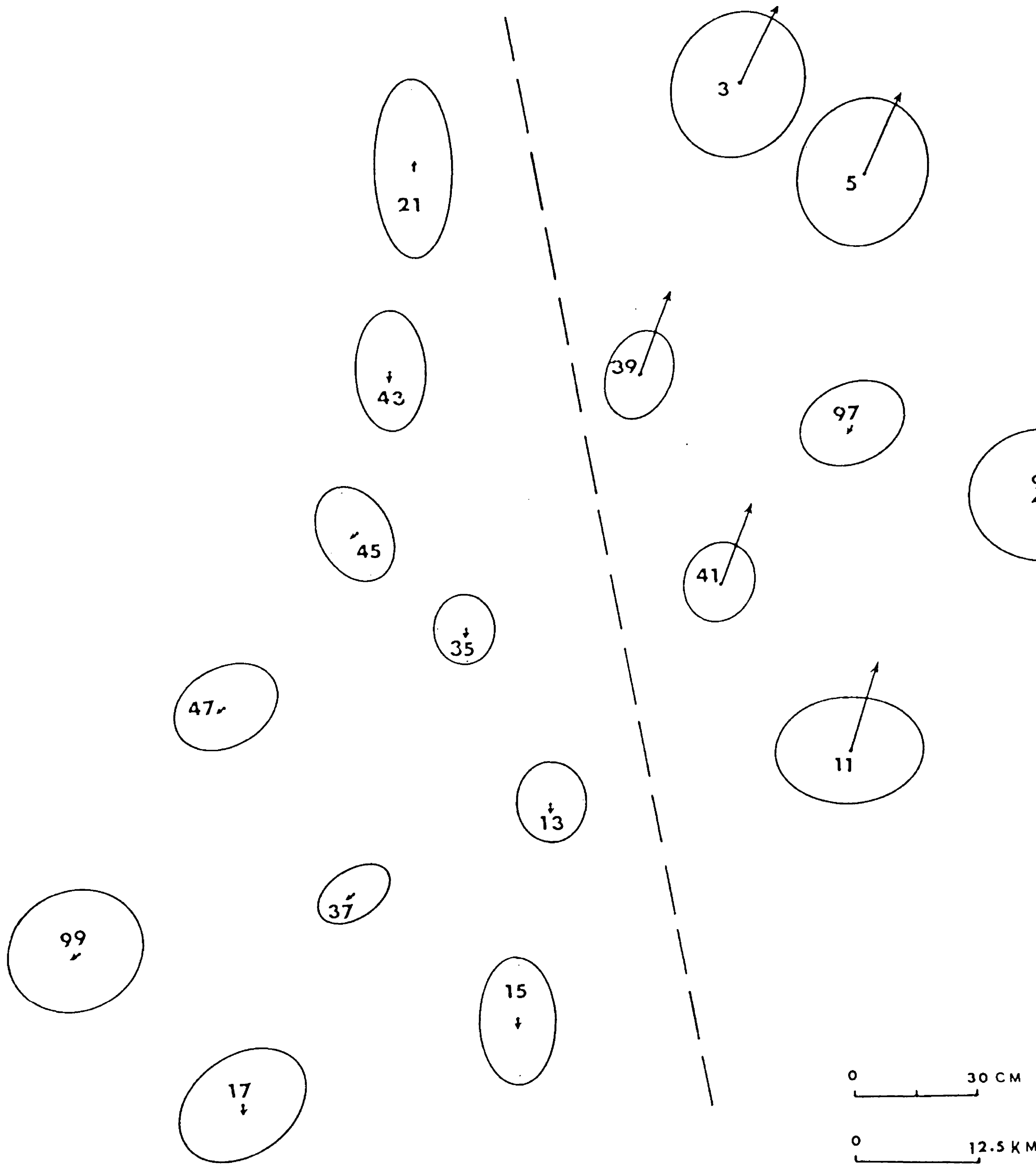


Fig. 7.9 (b)

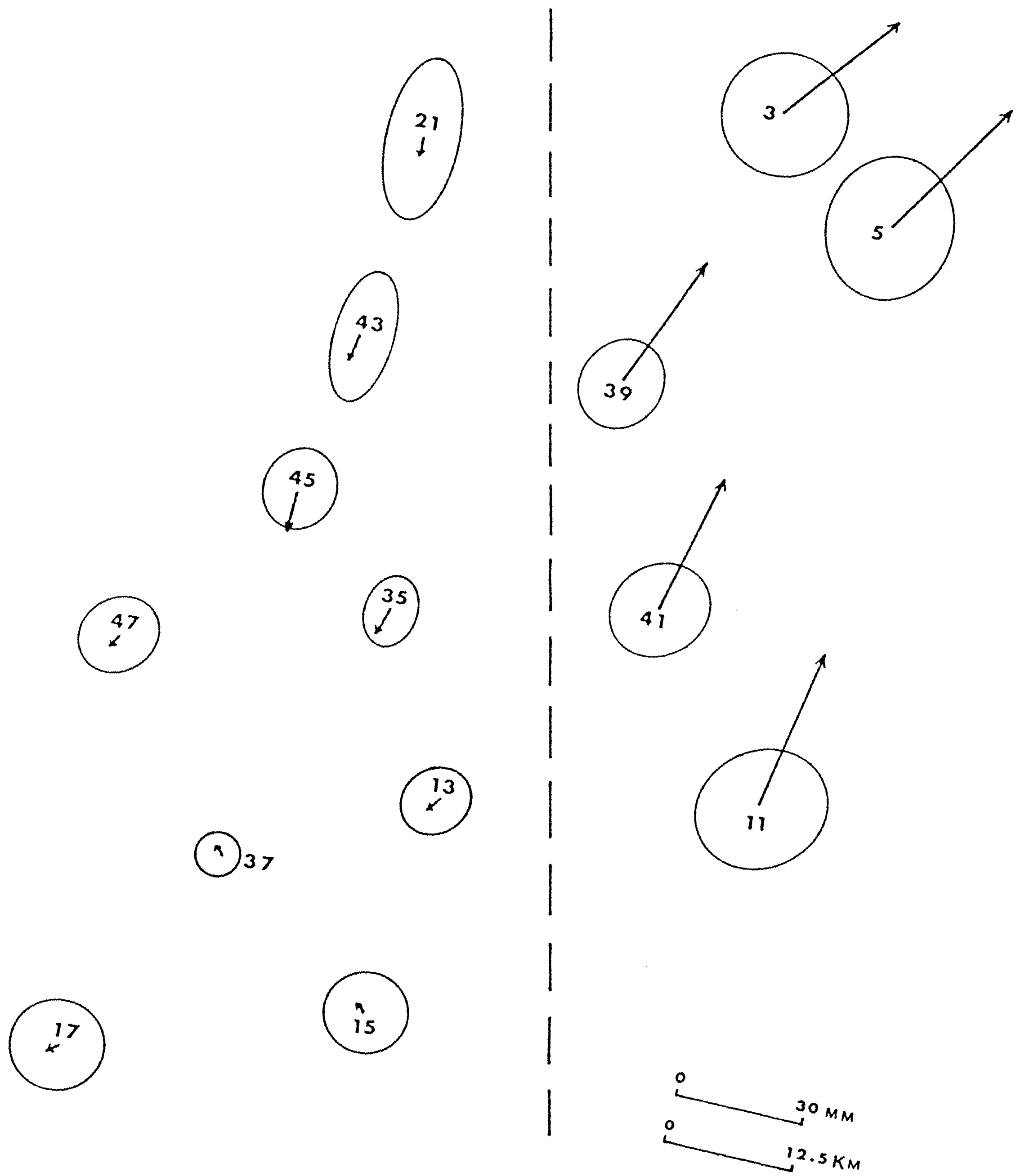


Fig. 7.9 (c)

Displacement Field and Error Ellipses (α = 5%) - The Third Simulated Network. (a) Epochs 2-1, (b) Epochs 3-2, (c) Epochs 3-1.

Epochs	No.	Deformation Model	Estimated Deformation Parameters and Their Statistics ($\alpha = 0.05$)			Global Test ($\alpha = 0.05$)
Epochs 2-1	1	relative block movement of pts. 5, 11, 39, 41, plus single point movements of pts. 3, 45 and 47	a = 5.6 cm	b = 21.7 cm	(35.6 > 3.1)	0.5 < 1.77
			dx_3 = 9.9	dy_3 = 15.6	(12.2 > 3.1)	
			dx_{45} = 1.5	dy_{45} = -5.5	(<u>1.7 < 3.1</u>)	
			dx_{47} = 0.5	dy_{47} = -6.2	(<u>1.5 < 3.1</u>)	
	2	relative block movement of pts. 3, 5, 11, 39, 41.	a = 6.3 cm	b = 21.4 cm	(36.7 > 3.1)	0.8 < 1.63
Epochs 3-2	1	relative block movement of pts. 3, 5, 11, 39, 41	a = 8.8 cm	b = 20.9 cm	(63.5 > 3.1)	0.1 < 1.63
Epochs 3-1	1	relative block movement of pts. 3, 5, 11, 39, 41	a = 15.3	b = 42.9	(151.6 >3.1)	0.8 < 1.63

Table 7.8: The Estimated Deformation Model -- The Third Simulated Network.

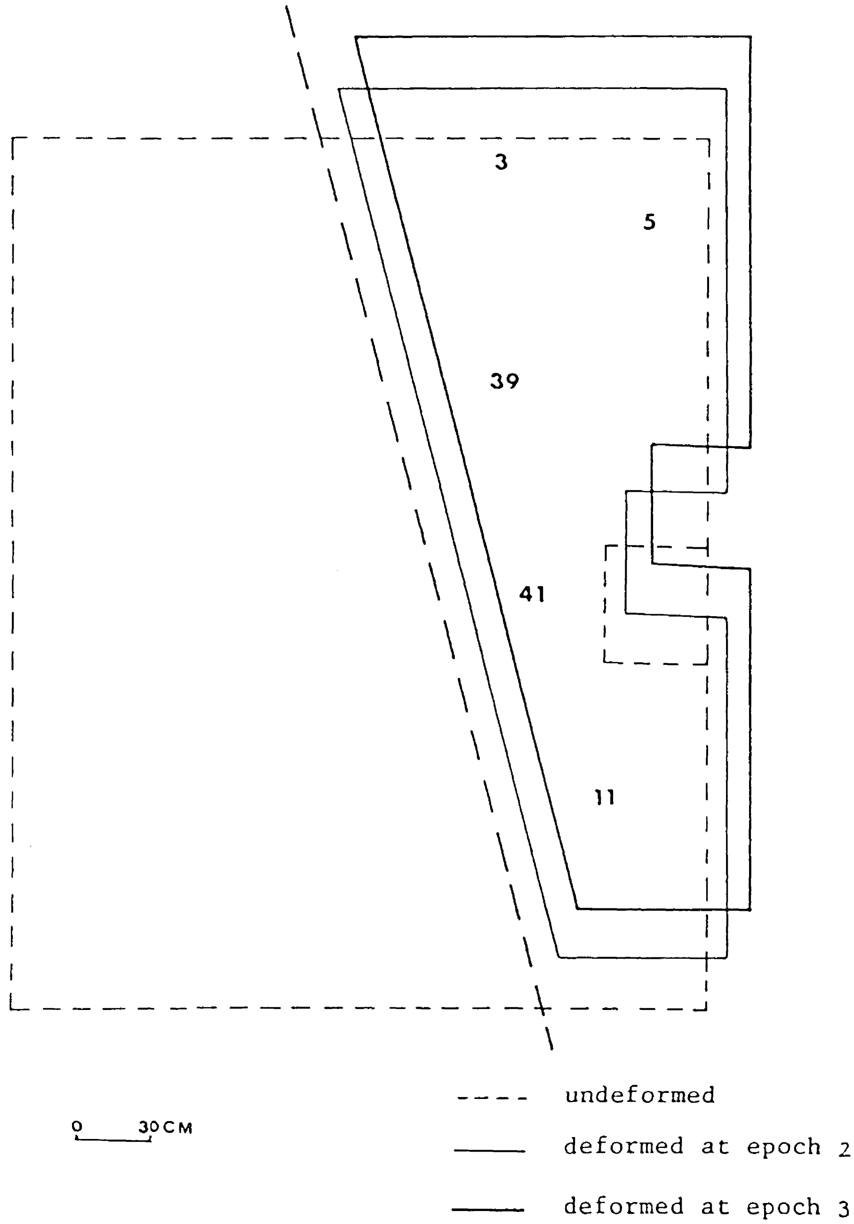


Fig. 7.10

Graphical Representation of the Final Deformation Model - The Third Simulated Network.

	Rigid Body Motion of Eastern Block*		
Deformation Model	x direction (cm)	y direction (cm)	Global Test
Epoch 2-Epoch 1	6.2 (2.4)**	21.5 (2.3)	0.3 < 1.42
Epoch 3-Epoch 2	8.7 (2.2)	21.0 (1.9)	

* Without including points 97 and 9.
** The values in parenthesis are the standard deviations.

TABLE 7.9

Simultaneous Estimation of the Deformation Model from Three Epochs of Observations--The Third Simulated Network.

7.3 Analysis of the 3-D Mining Network "Adamow"

7.3.1 Description of the network

A three-dimensional geodetic network (Figure 7.11) was established by the Institute of Geodesy and Applied Mathematics of the Academy of Agriculture, in cooperation with the Central Research and Design Institute for Open-pit Mining in Wroclaw, Poland (Cacon et al., 1982). The network is used to monitor rock mass deformations and landslide hazards.

The observation data of the network was provided by Cacon in Wroclaw and is used by the FIG "ad hoc" committee as a numerical example for the comparison of different approaches.

The data contains three epochs of observations, 1978, 1979, and 1980. The types and numbers of observations are summarized in Table 7.10.

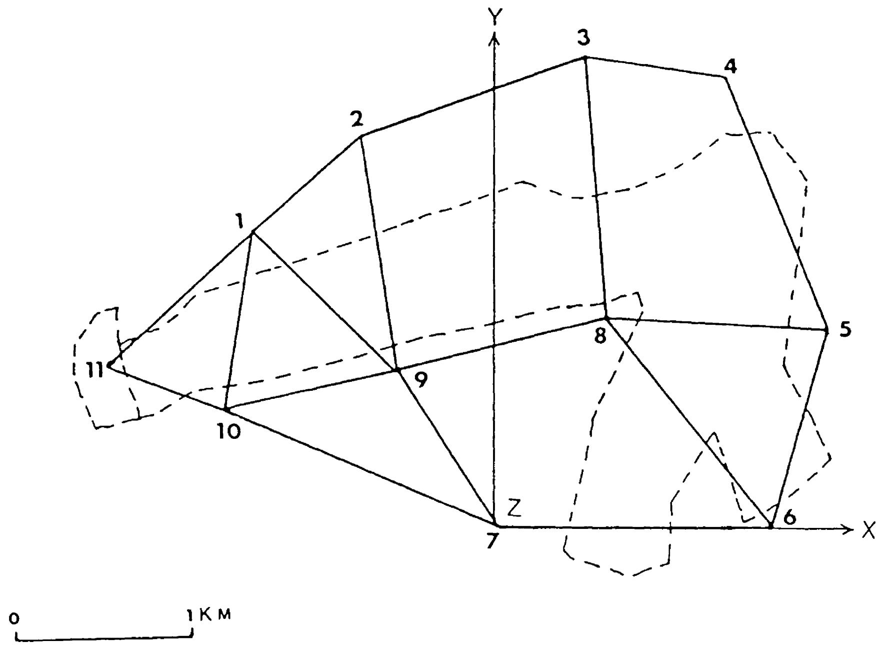


Fig. 7.11: Adamow Mine Network.

	Horizontal Angles	Vertical Angles	Spatial Distances	Levelling
1978	23	24	13	11
1979	23	28	13	12
1980	20	28	13	14

TABLE 7.10

Types and Number of the Observations in the Adamów Mine Network.

The horizontal angles and zenith distances were measured using a Wild T3 theodolite; spatial distances were measured using an EOK-2000 EDM instrument produced by Zeiss-Jena. Besides trigonometric levelling, geodetic levelling was run between the points using a Zeiss Jena level Ni-007. No detailed information on the field operation procedures and the accuracy achieved was given.

7.3.2 Adjustment of the network and preliminary identification of the deformation models

Due to the lack of information on the accuracy of the observations, the standard deviations for each type of observable were estimated using the MINQUE principle. The results are listed in Table 7.11.

	Horizontal Angles	Vertical Angles	Distance	Levelling
1978	1"2	3"6	0.015 m	2.3 mm*
1979	1"6	4"5	0.012 m	2.8 mm
1980	0"7	4"5	0.009 m	2.8 mm

* The standard deviation of each levelling line.

TABLE 7.11

The Standard Deviations of the Observations in the Adamów Mine Network.

The estimated standard deviations are compatible with the positioning accuracy of points using this type of spatial network, which is given in (Cacon et al., 1982) as

σ_x = (5.0 - 9.8) mm,

σ_y = (5.7 - 11.1) mm,

σ_z = (11.9 - 21.8) mm,

$\sigma_{z(H)}$ = (1.3 - 3.6) mm - spatial network completed with levelling.

The adjustment of the network was undertaken separately in the horizontal and the vertical because the vertical angles between the points in the network are small (the maximum is about 1^{o}). Thus the correlations between the adjusted horizontal position and the vertical position is negligible.

Using the method discussed in Section 6.3, the displacement fields coupled with the error ellipses for horizontal components and error bars for vertical components at 95% confidence level are plotted in Figures 7.12 to 7.14.

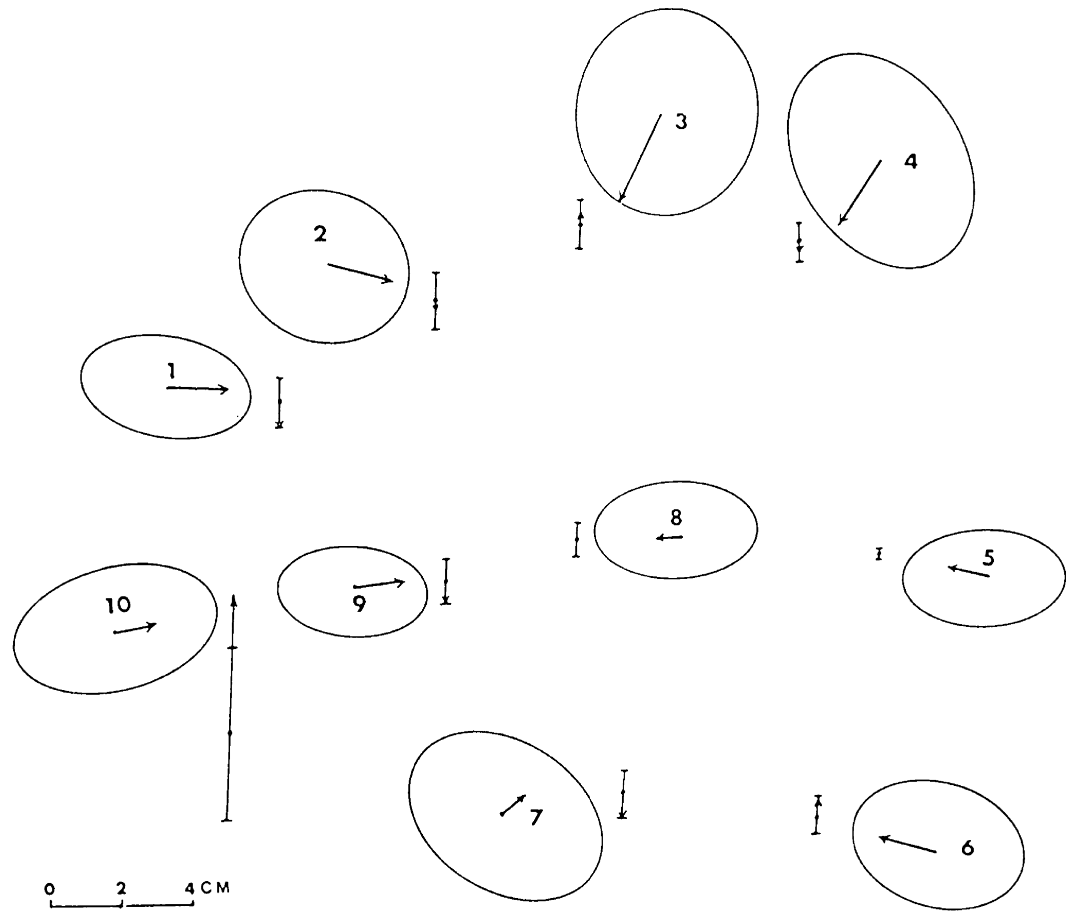


Fig. 7.12

Displacement Field for 1979-1978. Error Ellipses and Error Bars are Drawn at the 95% Confidence Level - The Adamow Mine Network.

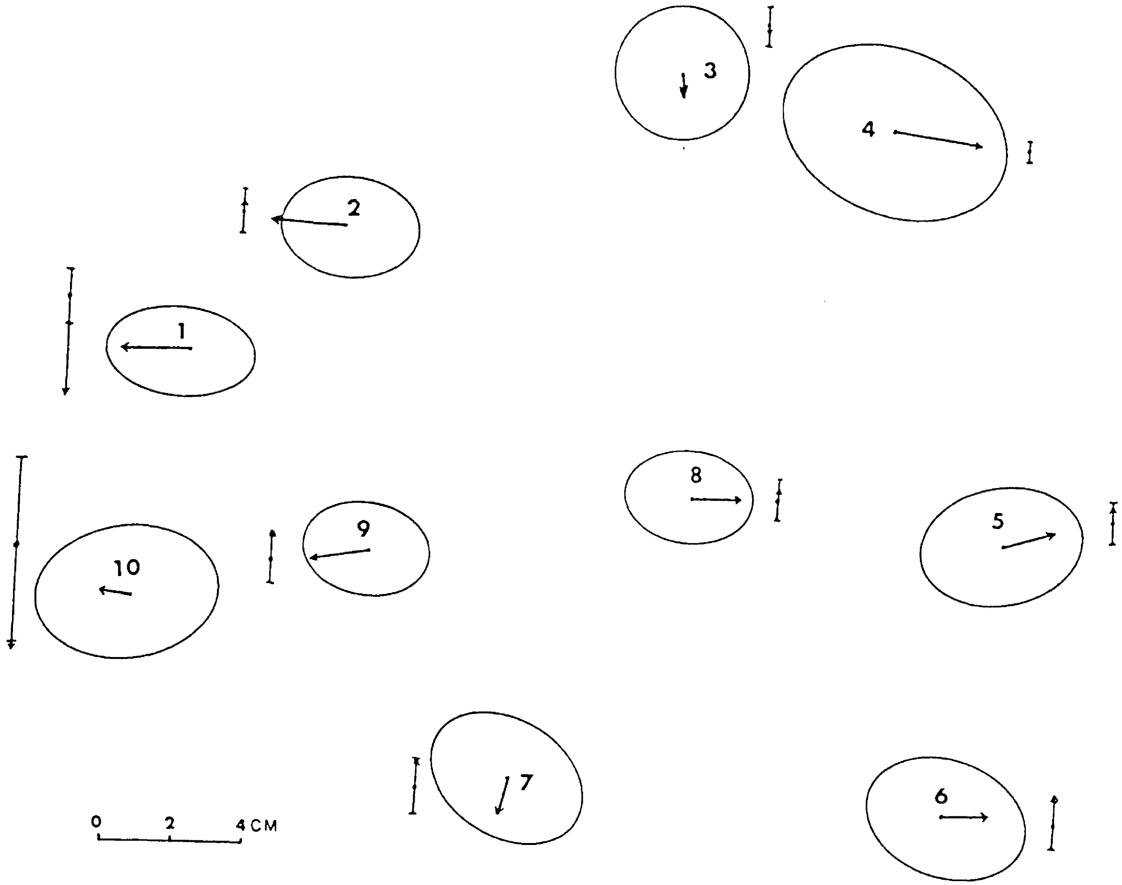


Fig. 7.13

Displacement Field for 1980-1979. Error Ellipses and Error Bars are Drawn at the 95% Confidence Level - The Adamow Mine Network.

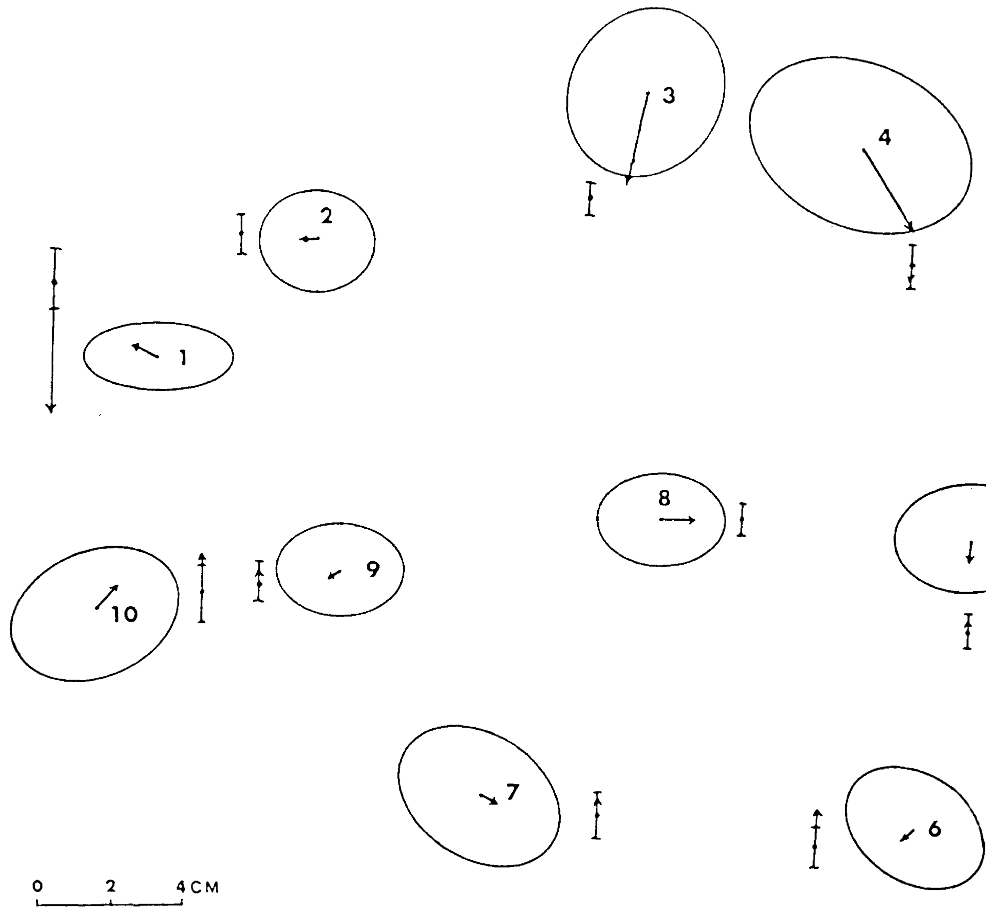


Fig. 7.14

Displacement Field for 1980-1978. Error Ellipses and Error Bars are Drawn at the 95% Confidence Level - The Adamow Mine Network.

7.3.3 Results

From the plotted displacement fields, various deformation models are estimated. The results are listed in Tables 7.12 to 7.15. It can be seen that as far as the horizontal movements are concerned, the scale of the EDM at epoch 2 seems different from that at epoch 1 and epoch 3. Thus, the estimated scale factors from epoch 79-78 and 80-79 have opposite signs and their values are more or less close to each other, but still under the critical values (the result from epoch 79-78 is very close to the critical value). This suspicion can only be confirmed by the field operators. Some deformation parameters are insignificant if they are estimated from successive pairs of epochs, but significant if they are estimated from the last epoch with respect to the first one. This may indicate that a deformation trend in time space exists particularly for points 3 and 4.

Therefore, the final deformation model is accepted with points 3 and 4 as not stable. They have a tendency to move horizontally, but show no significant vertical movement. As far as the vertical movements are concerned, point 6 shows uplift and point 1 shows subsidence. Point 10 seems to have irregularity, in 1979-1978 it uplifts but subsides mm in 1980-1979. In 1979, point 10 was not connected to other points by geometric levelling, only by trigonometric levelling. The lower accuracy of trigonometric levelling may be responsible for this irregularity. If we compare the results in 1980 with 1978, point 10 shows marginal significant uplift. The final estimation of the deformation parameters from a three-epochs simultaneous solution is presented in Table 7-16.

No.	Deformation Model	Estimated Deformation Parameters and Their Statistics ($\alpha = 0.05$)	Global Test ($\alpha = 0.05$)
1	no global deformation		0.58 < 1.86
2	the whole area is compressed or possible change in the scale of E.D.M.	K = -7.4 ppm (critical value at 95% confidence level: 7.6 ppm)	0.38 < 1.90
3	single point displacement of points 3 and 4	dx_3 = -7.7 mm dy_3 = -15.0 mm (1.9 < 3.2) dx_4 = -5.9 dy_4 = -7.9 (0.9 < 3.2)	0.45 < 1.93

Table 7.12: The Estimated Deformation Model between 1979 and 1978 -- the Adamow Mine Network.

No.	Deformation Model	Estimated Deformation Parameters and Their Statistics ($\alpha = 0.05$)	Global Test ($\alpha = 0.05$)
1	no global deformation		1.53 < 1.86
2	relative block displacement of points 3,4,5,6,8	a = 2.2 mm b = 4.2 mm (1.64 < 3.2)	1.52 < 1.92
3	possible change in the scale of E.D.M.	k = 4.5 ppm (critical value at 95% confidence level: 5.8 ppm)	1.47 < 1.90
4	single point movement of point 2	dx_2 = -8.0 dy_2 = 3.3 (1.92 < 3.2)	1.47 < 1.92
5	single point movement of point 4	dx_4 = 22.3 dy_4 = -21.1 (2.1 < 3.2)	1.47 < 1.92

Table 7.13: The Estimated Deformation Model between 1980 and 1979

-- the Adamow Mine Network.

No.	Deformation Model	Estimated Deformation Parameters and Their Statistics ($\alpha = 0.05$)	Global Test ($\alpha = 0.05$)
1	no global deformation		<u>2.29 > 1.86</u>
2	single point movements of points 3 and 4	dx_3 = -11.1 mm dy_3 = -26.1 mm (9.2 > 3.2) dx_4 = 15.7 mm dy_4 = -29.9 mm (4.6 > 3.2)	1.35 < 1.95

Table 7.14: The Estimated Deformation Model between 1980 and 1978 -- the Adamow Mine Network.

Deformation Model			Point 1 (mm)	Point 6 (mm)	Point 9 (mm)	Point 10 (mm)	Block of Pts 5,6,7,8,9 (mm)	Global Test (α=0.05)
79 - 78	1.	global congruency						3.4 > 2.1
	2.	single point movement	-7.0 (5.9*)		4.0 (4.6)	41.6 (26.6)		1.9 < 2.3
	3.	single point movement	-5.4 (5.4)			44.0 (26.6)		2.0 < 2.3
	4.	single point movement	-5.4 (5.4)	5.8 (4.6)		43.9 (26.6)		1.3 < 2.3
80 - 79	1.	global congruency						18.2 > 2.1
	2.	single point movement	-31.9 (6.0)		3.9 (4.8)	-35.0 (26.8)		1.1 < 2.5
	3.	single point movement	-31.9 (6.0)	2.9 (4.6)	3.9 (4.8)	-35.0 (26.8)		1.1 < 2.5
	4.	single point movement plus block movement	-31.5 (6.0)			-33.1 (27.0)	4.7 (5.0)	1.0 < 2.3
	5.	single point movement	-33.8 (5.6)			-37.1 (26.6)		1.4 < 2.3
80-78	1.	global congruency						29.1 > 2.1
	2.	single point movement	-38.5 (5.4)	8.3 (4.6)		7.5 (5.8)		1.1 < 2.3

* The values in round brackets are the critical values.

Table 7.15: The Estimated Deformation Models for Vertical Movements — the Adamow Mine Network.

Deformation Model	Point 3 dx (m)	Point 3 dy (mm)	Point 4 dx (mm)	Point 4 dy (mm)	Global Test
1979-1978	-9.9 (8.9)*	-17.1 (9.5)	-6.3 (13.8)	-9.4 (13.3)	0.82<1.58
1980-1979	-0.9 (7.9)	-9.0 (8.0)	22.2 (11.7)	-20.7 (11.5)	

TABLE 7.16(a)

Deformation Model	Point 1 (mm)	Point 6 (mm)	Point 10 (mm)	Global Test
1979-1978	-5.1 (2.7*)	5.5 (2.2)	44.1 (12.6)	0.72 < 1.80
1980-1979	-33.8 (2.6)	2.9 (2.7)	-37.3 (12.7)	

* The standard deviation.

TABLE 7.16(b)

Final Deformation Model--The Adamow Mine Network.
(a) Horizontal Movement; (b) Vertical Movement.

CHAPTER 8

AN OVERVIEW OF THE PHYSICAL INTERPRETATION OF DEFORMATION MEASUREMENTS

The physical interpretation of deformations is one of the research topics of the FIG "ad hoc" Committee (Chrzanowski and Secord, 1983). In this chapter a general insight into the interpretation of deformation surveys is given, which provides a guideline for further study.

8.1 Interpretation Methods and Their Interaction

A deformation survey is to serve one, or both, of the two main purposes:

(1) to give information on the geometrical status of a deformable body, the change in its position, shape and dimensions;

(2) to give information on the physical status of a deformable body, the state of internal stresses and the load-deformation relationship.

In the first case, information on the acting forces and stresses (force per unit area) and on the mechanical properties of the body are of no interest to the interpreter or are unavailable. The process of transforming the deformation measurements into the geometric status should be called geometrical analysis (Chrzanowski et al., 1982c). As a final result of the geometrical analysis, the relative movement of a single point or a group of points (a block), strain parameters and their time-dependent attributes are presented. The generalized approach to the geometrical analysis of deformation surveys has been developed in the previous chapters. From the outcome of the geometrical analysis, one may make a qualitative interpretation of the causes of the deformation.

In the second case, the process of deriving information on the load-deformation relation should be called physical interpretation (Chrzanowski et al., 1982c). Somewhat schematically, one can perform such interpretation by using either of the following.

(1) A statistical method (empirical method) which analyses the correlations between observed deformations (e.g., displacements) and observed loads (external and internal causes producing the deformation). These correlations can be obtained by performing statistical analysis of the past data. Therefore this method is of an a posteriori nature.

(2) A deterministic method which utilizes information on the loads, properties of the materials, geometry of the body and physical laws governing the stress-strain relationship. In contrast to the previous method, this one is of an a priori nature.

The distinction between the two methods should not be taken as absolutely clear cut. In fact each method includes a statistical and a deterministic part. The possible forms of the model sought under (1), relating the causative quantities to the response effects, are obtained by qualitative knowledge about the expected behaviour of the body; the model determined by the deterministic method may be further enhanced through the statistical method, for instance, calibration of the physical parameters of the material from the measured deformation quantities (Gicot, 1976).

The deterministic method provides the expected deformation from the measured causative quantities. If the difference between the expected deformation and the measured one is small, compared with the various errors and uncertainties which characterize the process, then the body behaves as expected, and the deterministic model is justified. Otherwise a search for the reasons for the large discrepancies should be undertaken and the model should be improved.

The statistical method establishes a prediction model. Using this model the forecasted deformation can be obtained from the measured causative quantities. A good agreement between the forecasts and the measurements then tells us that the deformable body behaves as in the past. Otherwise, as in the previous case, reasons should be found and the model should be refined.

The flowchart in Figure 8.1 summarizes the interpretation methods and their interactions.

8.2 Interpretation by Statistical Method

Interpretation by statistical method always requires a suitable

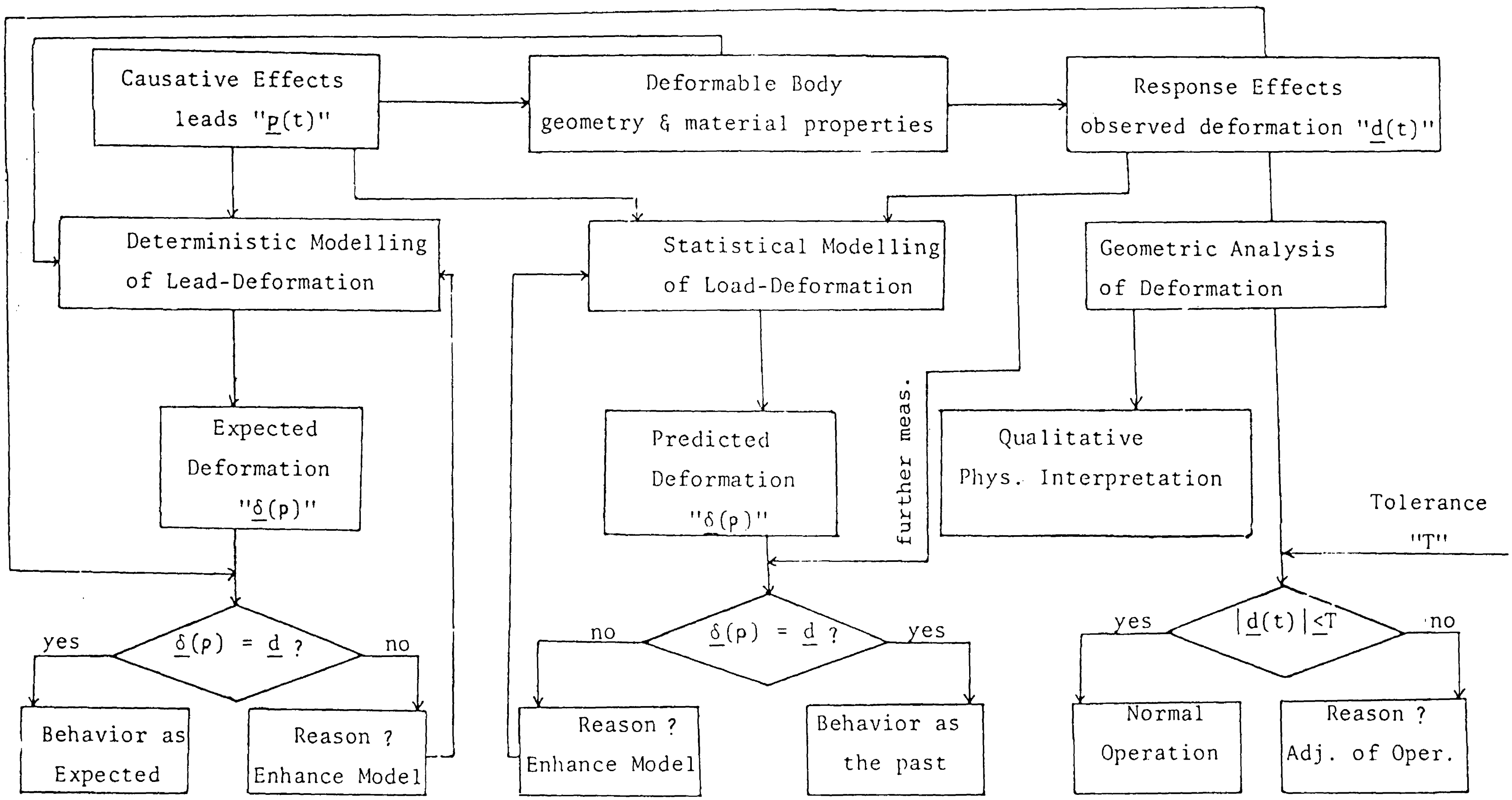


Fig. 8-1: Flowchart of the Deformation Interpretation.

amount of observations, both of causative quantites and of response effects. Let $\underline{d}$ be the vector of response effects. Then a functional relation between the "causes" and the "responses" can be established in the form

$$\underline{d} + \underline{v} = B\,\underline{c} \quad , \tag{8-1}$$

where $\underline{d}$ may be directly observed or the outcome of the geometrical analysis; $\underline{v}$ is a vector of errors. In contrast to geometrical analysis, the elements of the matrix B are functions of the causative quantities.

Different causative quantities may produce the deformations in different ways. Some effects can be approximated by polynomial functions, but others may be more adequately expressed as trigonometric functions, and so forth. All that is embedded in the matrix B. The vector $\underline{c}$ in (8-1), representing the magnitude of the effects, is to be estimated.

Let us take as an example the modelling of the response of a power dam to the causative effects of water pressure and air temperature. The horizontal displacement $d_i(t)$ of a point i can be expressed as a function of the water elevation h(t) in the reservoir and of air temperature $T(t - \tau)$, with τ being the response delay of the dam. Such a relation may have the form

$$d_i(t) = a_1 h(t) + a_2 h^2(t) + a_3 h^3(t) + b_1 T(t - \tau) \quad . \tag{8-2}$$

However, when the information on the air temperature is not available, the term $b_1 T(t - \tau)$ in (8-2) can be replaced by a trigonometric function. Since the trend of the temperature is typically periodical over a year, the relation (8-2) can be rewritten as

$$d_i(t) = a_1 h(t)+a_2 h^2(t)+a_3 h^3(t)+b_1 \cos\omega t+b_2 \sin\omega t \tag{8-2'}$$

where ω corresponds to the annual frequency; the coefficients a_i, b_i are unknown parameters to be estimated from the observations.

The main interest in the previous chapters was focussed on the geometrical analysis, but the procedures and methods of statistical testing are applicable to this case. The three basic steps are usually followed, that is, preliminary identification of the response model, estimation of unknown parameters, and diagnostic checking of the model.

This method of interpretation possesses some undeniable merits:

(1) knowledge of the mechanical properties of a deformable body is not required;

(2) good results from the point of view of prediction are usually obtained (ENEL, 1980).

But it also contains some undesirable features:

(1) A comparatively large amount of data about both causative and response quantities is needed in order to obtain a reliable model;

(2) less interest exists as far as research is concerned, for the deformable body in this case is acting only as a "black box".

8.3 Interpretation by Deterministic Method

In order to expand our knowledge about the behaviour of the deformable body, deterministic modelling of the load-deformation relation should be performed if possible.

As is well known, any real material will be deformed if an external force is applied to it. The external forces may be of two kinds (Timoshenko and Goodier, 1970): surface forces, i.e., forces distributed over the surface of the body, such as the pressure of one body on another; and body forces which are distributed over the volume of the body, such as gravitational forces, thermal stress or, in the case of a body in motion, inertial forces. Under the action of external forces, internal forces are produced between the parts of the body. The internal forces, when divided by unit areas of the surface on which they act, define the state of stress.

As discussed in Chapter 2, the deformation is fully described if six components of the strain tensor are known at every point of the deformable body. The six components consist of three components of normal strain, ε_x, ε_y, and ε_z, which describe change in the dimensions (elongations and compressions) at the point (x, y, z) along the x, y and z axes of the coordinate system, and three components of the shearing strain ε_{xy}, ε_{xz} and ε_{yz}, which describe change in the shape of the element (angular changes) in the corresponding planes of the coordinate system.

Similarly, the state of stress at any point of the medium is completely characterized by the specification of six components of stress tensor: three components of normal stress, σ_x, σ_y and σ_z; and three components of shearing stress σ_{xy}, σ_{xz} and σ_{yz}.

The relation between the strain tensor and stress tensor is governed by the generalized Hooke's law. For a homogeneous isotropic medium, the generalized Hooke's law can be written as (Sokolnikoff, 1956):

$$\sigma_{ij} = \lambda\delta_{ij}\Delta + 2\mu\varepsilon_{ij} \quad (i,j = x, y, z) \quad , \tag{8-3}$$

where δ_{ij} is the Kronecker symbol; λ, μ are called Lamé constants; and

$$\Delta = \varepsilon_x + \varepsilon_y + \varepsilon_z \quad . \tag{8-4}$$

For instance, $\sigma_{xy} = 2\mu\varepsilon_{xy}$, $\sigma_x = \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) + 2\mu\varepsilon_x$.

The elastic properties of materials are often described by

Young's modulus E and the Poisson ratio ν. Young's modulus represents the ratio of the tensile stress, say σ_x, to the extension ϵ_x, produced by the stress σ_x i.e., $E = \sigma_x/\epsilon_x$. The Poisson ratio ν denotes the ratio of the contraction, say in the y direction, of the linear elements perpendicular to the axis of the applied stress σ_x to the longitudinal extension ϵ_x, i.e., $\nu = \epsilon_y/\epsilon_x$. They are related to the Lamé constants in the following manner (Rockey et al., 1975):

(1) for a plane strain problem

$$\lambda = E \cdot \nu/(1 + \nu)(1 - 2\nu) \quad , \qquad (8\text{-}5a)$$

$$\mu = E/2(1 + \nu) \quad ; \qquad (8\text{-}5b)$$

(2) for a plane stress problem

$$\lambda = \nu E/(1 - \nu^2) \quad , \qquad (8\text{-}6a)$$

$$\mu = E/2(1 + \nu) \quad . \qquad (8\text{-}6b)$$

In order to determine the relation between external forces, state of stress, and displacements, the solution must satisfy the three basic conditions (Timoshenko and Goodier, 1970):

(1) the equilibrium of forces (external and internal),

(2) the compatibility of displacements, and

(3) the law of material behaviour (the stress-strain relation).

The compatibility condition requires that the deformed structure fits together, i.e., that deformations of its elements are compatible. The equilibrium condition and stress-strain relations are ensured through the following differential equations (Sokolnikoff, 1956):

$$\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + (\lambda + \mu)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z}\right) = - f_x$$

$$\mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + (\lambda + \mu)\left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z}\right) = - f_y \quad (8\text{-}7)$$

$$\mu(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}) + (\lambda + \mu)(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial z^2}) = - f_z$$

with u, v, and w being the components of displacement, and f_x, f_y and f_z being the components of body force.

Expression (8-7) can be rewritten more compactly as

$$L \cdot \underline{d} = - \underline{f} \quad , \qquad (8\text{-}7')$$

where $\underline{d}^T = (u, v, w)$, $\underline{f}^T = (f_x, f_y, f_z)$ and L is the corresponding differential operator. Summarily, the differential operator is obtained through the following three relations:

(1) The strain-displacement relation, which is briefly written as (see Chapter 2)

$$\underline{\varepsilon} = L_1 \cdot \underline{d} \qquad (8\text{-}8)$$

with

$$L_1^T = \begin{pmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix} ,$$

where $\underline{\varepsilon}$ is the vector of the strain components.

(2) The stress-strain relation (8-3), which is rewritten as

$$\underline{\sigma} = L_2 \, \underline{\varepsilon} \qquad (8\text{-}3')$$

with

$$L_2 = \begin{pmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\mu \end{pmatrix} .$$

(3) The equilibrium condition, which has the form (Sokolnikoff, 1956)

$$-\underline{f} = L_3 \cdot \underline{\sigma} \qquad (8\text{-}9)$$

with $L_3 = L_1^T$.

Therefore, equation (8-7') is equivalent to

$$-\underline{f} = L_3\, L_2\, L_1\, \underline{d} = L_1^T\, L_2\, L_1\, \underline{d} \quad . \qquad (8\text{-}7'')$$

and

$$L = L_1^T\, L_2\, L_1 \quad .$$

In principle, when the boundary conditions, either in the form of displacements or in the form of the acting forces, are given and the body forces are prescribed, the differential equations (8-7) are completely solved. However, direct solutions to (8-7) may be difficult if boundary conditions and the geometric shape of the body are complicated. The finite element method provides a powerful numerical method to solve the boundary value problems. The finite element technique gives an exact solution to the problem which approximates the differential equations (8-7).

The basic concept of the finite element method is that the continuum of the deformable body is replaced by an assemblage of individual small elements of finite dimensions which are connected together only at the nodal points of the elements. The elements may be of any shape but usually triangular or rectangular elements are chosen for two-dimensional analysis and rectangular or trapezoidal "bricks" are used in the three-dimensional solutions.

For example, in the two-dimensional analysis with triangular elements, the displacement function may be chosen as

$$u = a_1 + a_2 x + a_3 y$$

$$v = a_4 + a_5 x + a_6 y \quad . \tag{8-10}$$

Since both these displacements are linear in x and y, the displacement continuity between the adjoining elements for any nodal displacement is ensured. Thus the finite element model of a deformable body involves a piecewise polynomial interpolation. The nodal displacements define several displacement fields that are laid side by side. Let $\underline{d}_e$ be a vector of displacements for the three nodal points, i.e., $\underline{d}_e^T = (u_1, v_1, u_2, v_2, u_3, v_3)$ and $\underline{a}^T = (a_1, \ldots, a_6)$. Then, for an element, the expression (8-10) can be written as

$$\underline{d}_e = M_e \underline{a} \quad , \tag{8-11}$$

with M_e being a 6 by 6 matrix, easily obtained from (8-10). Applying the relation (8-8) to (8-10), we obtain the strain vector

$$\underline{\varepsilon}^T = (\varepsilon_x, \varepsilon_y, \varepsilon_{xy}) = (a_2, a_6, a_3 + a_5) \quad ,$$

or, in shorthand matrix notation,

$$\underline{\varepsilon} = N \, \underline{a} \quad . \tag{8-12}$$

Combining (8-11) and (8-12), the strain-displacement relation may be expressed briefly in the form:

$$\underline{\varepsilon} = N \, M_e^{-1} \, \underline{d}_e = \tilde{L}_1 \, \underline{d}_e \quad . \tag{8-13}$$

Considering (8-3'), the internal stresses are related to nodal displacements by

$$\underline{\sigma} = L_2 \, \tilde{L}_1 \, \underline{d}_e \quad . \tag{8-14}$$

Furthermore, the internal stresses are replaced with statically equivalent nodal forces $\underline{f}_e$, resulting in the relationship between the nodal forces and displacements (Rockey et al., 1975):

$$\begin{aligned} \underline{f}_e &= [\int \tilde{L}_1^T \, L_2 \, \tilde{L}_1 \, d(\mathrm{vol})] \, \underline{d}_e \\ &= k_e \, \underline{d}_e \end{aligned} \tag{8-15}$$

where k_e is called stiffness matrix of the element. Comparing (8-15) with (8-7'), one can recognize that the key step in the finite element method is the approximation of the differential operator L by a linear algebraic operator. This can be accomplished by linearizing L1. Linearizing L_1 is done by obtaining a displacement function such that the deformation of each element is required to be compatible with the deformation of every other element.

Once the stiffness matrices for all elements of the deformable body have been calculated, an overall structural stiffness matrix K is composed by a superposition of the stiffness matrices for all the elements, and the total equilibrium equation for the whole body is written

$$\underline{\tilde{f}} = K \cdot \underline{\tilde{d}} \quad , \tag{8-16}$$

where $\underline{\tilde{f}}$ is a vector of applied nodal loads in the whole body, and $\underline{\tilde{d}}$ is a vector of nodal displacements. The unknown displacements or unknown forces can be solved from (8-16).

Nowadays, since programs for the finite element computations are available, the interpretation of deformations by the deterministic method becomes realistic. If there exist several causative quantities, the deformable body may be treated as a linear system (ENEL, 1980). The effect of each causative quantity is calculated separately and then the total effect is obtained by a superposition of each effect.

At the end of this section it should be mentioned that only a brief introduction to the deterministic modelling and the finite element method is given so that surveying scientists and engineers might gain some appreciation of its utility and have an overview of the interpretation process. The deterministic modelling may find useful applications in deformation surveys, such as prediction of the extent of deformation for the zero-order design of a monitoring network, prediction of the maximum deformations for selecting the location of the object points of a monitoring network, and for designing the types and required accuracy of the deformation surveys, and physical interpretation of deformation surveys. A pilot study of its applications in surveying engineering has been done by Chrzanowski et al. (1983).

8.4 Interpretation by Combination of the Deterministic and Statistical Methods

Due to many uncertainties in the deterministic model of deformations, the theoretically calculated displacements $\underline{\delta}$ (or any other deformation quantities) will generally depart from the observed values $\underline{d}$ by $\underline{\Delta}$, i.e.,

$$\underline{\Delta} = \underline{\delta} - \underline{d} \quad . \tag{8-17}$$

The discrepancies may be due to:

- imperfect knowledge of the material properties, for example, errors in the elasticity constants;
- wrong modelling of the behaviour (elastic instead of plastic or creep neglected, etc.) of the material;
- errors in the thermal parameters of the material;
- approximation in calculations;
- measuring errors in $\underline{d}$;
- measuring errors in sampling and incomplete sampling of loading (causative) effects.

The investigation of the discrepancies is useful in gaining a better knowledge of the behaviour of the deformable body. Let Σ_Δ, Σ_d and Σ_δ be their covariance matrices, then

$$\Sigma_{\Delta} = \Sigma_{d} + \Sigma_{\delta} \quad . \tag{8-18}$$

In order to test whether the discrepancies $\underline{\Delta}$ are of a systematic nature, we have a null hypothesis H_o: $\underline{\Delta} = 0$ against an alternative hypothesis H_A: $\underline{\Delta} \neq 0$. The test statistic is

$$T = \underline{\Delta}^T \; \Sigma_{\Delta}^{-1} \; \underline{\Delta} \tag{8-19}$$

with a critical value: $\chi^2(\alpha;\ df)$ where df is the rank of Σ_{Δ} (degrees of freedom). If $T > \chi^2(\alpha;\ df)$ at the $(1 - \alpha)\%$ level of confidence, then H_o is rejected and a further search for an explanation of the discrepancies is required. At this stage the deterministic and statistical methods are combined for the interpretation of the deformation measurements. One way of doing this is to assume that the systematic discrepancies are caused, for example, only by the improperly chosen material parameters, say E and υ. In this case, new ("calibrated") values of the parameters can be estimated by applying the principle of least-squares:

$$\min_{E,\upsilon} \{(\underline{\delta} - \underline{d})^T \; C_{\underline{\Delta}}^{-1}(\underline{\delta} - \underline{d})\} \quad . \tag{8-20}$$

However, this method for calibration of the constants of the material properties may lead to physically unacceptable values of the calibrated quantities if the real reason for the discrepancy is of a different source. In such a case, one has to try another approach to the interpretation by using, for example, the statistical analysis of the discrepancies as discussed above in Section 8.2 in order to find the reason and then enhance the deterministic model.

Surveying engineers and scientists have not been very involved in the deformation interpretation which has usually been done by other specialists. This situation should be changed. As pointed out in

Chapter 1, surveyors have a good knowledge of data acquisition, therefore their involvement would contribute to the interpretation of deformation surveys. In addition, by participating in the interpretation of deformation surveys, surveyors would gain a good insight into deformation measurements, which would contribute to the optimal design of monitoring schemes.

CHAPTER 9

FINAL REMARKS, CONCLUSIONS AND RECOMMENDATIONS

Research carried out in the development of the "Fredericton Approach" to deformation analysis has resulted in a series of conclusions. In light of these conclusions and in looking ahead at further developments, some recommendations are coupled with the corresponding conclusions.

1. Deformation surveys are one of the most important activities in surveying, especially in engineering surveying. Their results are directly relevant to the safety of human life and engineering structures. Deformation surveys can provide not only the geometrical status of the deformed object, but also information on its response to loading stresses. This provides a better understanding of the mechanics of deformations and the checking various theoretical hypotheses on a behaviour of the deformable body.

The design of a monitoring scheme, the field operation campaign, and the analysis of the acquired data have to be performed with special care because deformation surveys usually require a higher accuracy than any other type of survey.

A look at the research activities in the surveying community reveals that optimization and design of monitoring schemes with geodetic and non-geodetic observables require further research, especially in engineering surveys where the design problem is complicated. A publication by the "Fredericton Group" (Chen et al., 1983) can serve as a starting point in this respect.

2. A thorough analysis of deformation surveys should be given a high priority. The reasons are that

(a) the data acquisition is costly, and therefore, the processing of these data should be performed very carefully;

(b) the data may contain important scientific findings and such findings cannot be revealed by a superficial inspection.

Different approaches to deformation analysis have been developed. A comparison of them is profitable and may lead to a development of general guidelines. Here the research methodology of the FIG Commission 6 "ad hoc" committee should be followed--the comparison of different approaches is based not only on theory, but also on real examples. It is very true that the numerical examples which have different characteristics are the best witnesses in testing the efficiencies and limitations of each approach.

3. Processing of deformation surveys can be distinguished into geometrical analysis and physical interpretation. Providing the geometrical status of a deformed body is the final goal of geometrical analysis, but looking deeply into the physical behaviour of the body, how the body responses to the applied stresses, is in the realm of physical interpretation. Until recently, the involvement of surveying

scientists and engineers in the analysis of deformation surveys has been limited mainly to the former. Their participation should be emphasized because of two significant aspects:

(a) contributing of their knowledge on the data acquisition and statistical testing to the interpretation process;

(b) allowing them to obtain a better understanding of the purpose of deformation surveys, which enables them to design better monitoring schemes.

The "Fredericton Group" should continue its activity in physical interpretation. Chapter 8 gives general guidelines in this aspect. Other references can be made to Fanelli et al. (1975), Fanelli (1979), ENEL (1980) and Chrzanowski et al. (1982c and 1983).

4. Deformation analysis involves two types of errors: the errors of observations, and the errors in deformation models. In order to avoid a misinterpretation of systematic errors or outliers in the observations as deformation phenomena, screening of the observations for outliers or systematic errors should be done prior to the estimation of the deformation models. Therefore, if possible, it is worth separating the whole process of deformation analysis into two steps: adjustment of the geodetic monitoring network and estimation of deformation parameters.

Direct estimation of deformation parameters from the raw observations is no problem mathematically, but it makes it more difficult to distinguish between the errors in the deformation model and the errors of the observations. This approach is mainly adopted in the study of tectonic movements. One reason is that some monitoring schemes do not allow for the performance of a network adjustment. Another reason is the attempt to avoid the datum defect problem in a monitoring network, which is reflected in the recent comments given in Reilly (1982):

> ...the determination of the local deformation of the earth's crust from repeated geodetic measurements can be approached in a number of ways. One is to compare the coordinates resulting from the separate adjustments of surveys made at different epochs. This introduces a datum problem and involves some difficulty in assessing the precision of the derived strain tensor.

Of course, his concern is not pertinent and is overcome by the generalized approach given in this thesis.

The coordinates are the quantities with which geodesists and surveyors would like to work, because of such appealing benefits as: the preliminary identification of deformation models is more intuitive when using adjusted coordinates than when using raw observations; the formulation of deformation models is very transparent and easily schematized. The datum defect problem should be solved rather than avoided.

5. In the practice of deformation analysis, there exist some ambiguities which are directly related to the datum defect problem. There are some who worry about the dependency of the derived deformation parameters on the choice of adjustment techniques; but others overlook the datum defect problem, and this produces biased results in the analysis.

A solution to the datum defect problem is the definition of a set of datum equations, say $D^T\underline{x}=0$ (Chapter 4). The solution vector $\hat{\underline{x}}$ and its cofactor matrix $Q_{\hat{\underline{x}}}$ will satisfy the relations, $D^T\hat{\underline{x}}=0$ and $D^TQ_{\hat{\underline{x}}}=0$. The reason for causing the above ambiguities comes from the negligence of these functional relations. A numerical demonstration and theoretical analysis in Chapter 6 have clarified these problems.

The datum defect problem can be approached in different ways: use of the generalized inverse of the coefficient matrix N of the normal equation, similarity transformation from one solution to another, and the projection method in a parameter space. The choice of g-inverses implies the definition of a special datum. One of the g-inverses, which corresponds to the datum equation, is $N_D^- = (N+DD^T)^{-1} - H(H^TDD^TH)^{-1}H^T$ with H being a matrix such that NH=0 (Chapter 4). The projection method projects any solution in the parameter space to the subspace defined by the datum equation. This is accomplished through the projection operator $P_{D^\perp} = (I - H(D^TH)^{-1}D^T)$. The results are equivalent to the S-transformation and the generalized inverse.

6. A deformation monitoring network usually includes different types of observables. Evaluation of their variances and covariances is indispensable step prior to the final adjustment of a network. Hypothetical establishment of them should be avoided because it inevitably biases the results.

The Minimum Norm Quadratic Unbiased Estimation principle (MINQUE) provides a general tool. This approach gives the conventional estimators for the simple cases and offers a procedure of estimation for more complicated ones. The principle of MINQUE seems to be as fundamental in nature as the least-squares method of estimation, where no assumptions on the distribution of the variables is needed. When the variables are postulated to follow a normal distribution, the approach provides results with minimum variance of a quadratic form.

The numerical examples presented in Chapters 3 and 7 show that the approach works well. In geodetic applications, the estimation process converges rapidly, usually two to three iterations are enough because the approximate values of variance components are always available from the type of instruments used and field operation procedures.

Concerning this topic, the author suggests the following.

(1) When a program for network adjustment is to be written, the program for estimation of variance components of the observations should be included as a subroutine.

(2) Statistical tests on the estimated variance-covariance components should be further evolved.

(3) Estimation of the covariances or correlations between the observations in geodetic applications should be further studied.

7. An important aspect in deformation analysis is statistical testing. All the hypothesis tests used in the different phases of deformation analysis are treated as a special case of a general hypothesis test developed in Chapter 5 which is valid for the general Gauss-Markoff Model (GGM). Derivation of a statistic for the GGM is facilitated using the theory of vector spaces. Three methods to compute the quantities involved in the statistic for examining a null hypothesis are distinguished and tabulated. Thus the statistical tests are easily understood and can be readily applied to various problems.

A practical problem in hypothesis testing comes from the selection of the significance level α. Up to now, there is no unique way to do that. A fixed value α for different tests is suggested. However, a separate research topic should be set for this problem. The direction may be from statistical decision theory (e.g., Clerici, et al. (1980)) by setting a more objective criterion.

8. Deformation parameters in the geometrical sense include single point movements, relative block movements, strain parameters, rotation components and their time derivatives. Determination of these parameters from the observables can be formulated in two ways: through the coordinates of the points (the "coordinate approach"); through the observations (the "observation approach").

The "coordinate approach" permits the utilization of all geodetic observations in the computation of deformation parameters even if different observables are measured in each of the repeated surveys and the network geometry is changed between the epochs. However, the "coordinate approach" requires that the coordinates in all the epochs should be referred to the same coordinate system and involves the inversion of a singular matrix caused by datum defects and configuration defects. The "observation approach" may be more convenient if the observations are scattered in space or in time, but requires that observables are the same in both epochs. In addition, the "observation approach" will be free from the systematic errors which are common in all epochs. If the same observables and geometry are maintained in both epochs, the two approaches produce identical results only when either

(1) the covariance matrices of the observations in both epochs are equal or proportional; or

(2) no redundancy in the network is available (no adjustment is possible).

The "generalized approach" is elaborated so that it can combine two types of approaches in a simultaneous solution.

9. Compared with other methods, the "generalized approach" has the following characteristics:

(a) Any observables can enter into the generalized approach. They may be the adjusted coordinates, individual geodetic observations or any physical-mechanical measurements of tilts, strains, pendula deviations, alignment observations, etc. In order to realize this point, two key measures are undertaken -- formulation of the weight matrix, and establishment of the functional relations between observables and deformation parameters.

The functional relations between different types of observables and deformation parameters are developed in Chapter 2, which is an extension of the work of Reilly (1981; 1982) and provides a general mathematical tool.

Due to the datum defects and configuration defects in a monitoring network, the covariance matrix of the adjusted coordinates is singular and varies from one solution to another. Thus, the conventional weight matrix, the inverse of the covariance matrix, does not exist. However, the "weight matrix" can be defined as one of the g-inverses of the covariance matrix such that it is invariant of any solution. Two methods to obtain such a matrix are discussed in Section 6.3.1. One is to use the coefficient matrix of the normal equations after certain irrelevant parameters are eliminated; another is to compute it from the covariance matrix using (6-21) and (6-22). Since the weight matrix so defined is invariant, the "generalized approach" provides the deformation parameters independent of any minimum constraints used in the numerical processing of each epoch of observations.

(b) The "generalized approach" is applicable to any type of deformation analysis. In this approach, the whole area covered by the deformation survey is treated as a noncontinuous deformable body consisting of separate continuous deformable blocks. Thus the blocks may undergo relative rigid body displacements and rotations, and each block may change its shape and dimensions. In the case of single point movement, the given point is treated as a separate block being displayed as a rigid body. Therefore, the approach is valid for the analysis of a reference network and for the analysis of rigid body movements between blocks as well as for the determination of strain components in a relative network.

(c) The "generalized approach" provides a systematic step-by-step analysis. Rather than the trial and error method, this approach is composed of three basic processes: preliminary identification of the deformation models, estimation of the deformation parameters, and diagnostic checking of the models. Although the three steps necessarily overlap, the strategies for each step are considered.

An iterative weighted projection method is proposed to create an appropriate displacement field for preliminary identification of deformation models in space. The numerical examples show that the projection method is more robust in the definition of a datum, and therefore makes the selection of the deformation model easier.

In order to compute a g-inverse of $\sum_{1}^{k} P_i$ (Section 6.4.1), a method is constructed which transfers the inversion of a singular matrix to the inversion of an non-singular one by introducing some pseudo-observations. The numerical examples indicate that the method is simple and rigorous.

Based on a firm theoretical foundation, hypothesis tests are formulated to test the global appropriateness as well as the local adequacy of the deformation model.

10. Application of the "generalized approach" to the analysis of five numerical examples shows that the method works well. Further work is to continue and to expand the application of the approach to other numerical examples set by the FIG Commission 6 "ad hoc" committee.

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Abbreviations

AVN	:	Algemeine Vermessungs-nachrichten
Bull. Geod.	:	Bulletin Geodesique
CIS	:	The Canadian Institute of Surveying
JASA	:	Journal of American Statistics Association
J. Geophys. Res.	:	Journal of Geophysics Research
ZfV	:	Zeitschrift fur vermessungswesen

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APPENDIX I

NOMENCLATURE

I.1. General Conventions

1. Vectors are lowercase letters underscored, e.g., $\underline{a}$.
2. Matrices are uppercase letters, e.g., A, or letters in parentheses, e.g., (a_{ij}); or diag $\{a_1, a_2 \ldots\}$ in the case of diagonal matrix with diagonal elements being $a_1, a_2, \ldots$, or diag $\{A_1, A_2, \ldots\}$ in the case of block diagonal matrix with A_i being a submatrix.
3. Terminologies are consistent with
   - i) Wells and Krakiwsky (1971) for the least squares adjustment;
   - ii) Rao (1973) for statistics;
   - iii) Chrzanowski (1981a) and Heck (1982) for deformation analysis.

I.2. Symbol Definition

$\subset$	inclusion or containment; is a subset of
$\not\subset$	no inclusion, or is not a subset of
$\in$	is an element of
$\cap$	intersection
$\cup$	union
:=	equal by definition

$\otimes$	Kronecker product
I	identity matrix
δ_{ij}	Kronecker symbol, $\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$

I.3. Notations and Operations on Matrices A and B

A^T	transpose of A
$\|A\|$	determinant of a square matrix A
$A^{\perp}$	a matrix of maximum rank such that $A^T A^{\perp}=0$
A^-	any generalized inverse of A (g-inverse) such that $AA^-A=A$
A^+	pseudo-inverse or Moore-Penrose inverse such that $AA^+A=A$, $A^+AA^+=A^+$, $(AA^+)^T=AA^+$, $(A^+A)^T=A^+A$
$S(A)$	linear vector space generated by the columns of A
$r\{A\}$	rank of A
$rd\{A\}$	rank defect of A
$Tr\{A\}$	trace of A, equal to $\sum_i a_{ii}$
$A \mp B$	parallel sum of A and B, defined by $A(A+B)^-B$
$P_{A/B}$	projection operator onto S(A) along (parallel to) S(B)

I.4. Notations and Operations on Spaces

E^n	Euclidean space with dimension n
$d\{\Phi\}$	dimension of space Φ
$\Phi^{\perp}$	orthogonal complement of space Φ
$\Phi \oplus \Psi$	direct sum of spaces Φ and Ψ, such that

$\Phi \oplus \Psi = \{\underline{\phi} + \underline{\psi} : \underline{\phi} \in \Phi, \underline{\psi} \in \Psi\}$, where $\Phi \cap \Psi = \{0\}$

I.5. Notations and Operations on Vectors and Random Variables

$\lvert\lvert \underline{x} \rvert\rvert$	the norm of, or the length of $\underline{x}$
$< \underline{x}, \underline{y} >$	inner product of $\underline{x}$ and $\underline{y}$
$\underline{\ell} \overset{d}{\sim}$	$\underline{\ell}$ is distributed as
$E\{\ell\}$	expected value of $\underline{\ell}$
$D\{\ell\}$	dispersion matrix of $\underline{\ell}$
$V\{\varepsilon\}$	variance of random variable ε
$Pr\{\xi > \xi_\alpha\}$	probability when $\xi > \xi_\alpha$

APPENDIX II

This appendix contains the mathematical proof of three statements in Chapter 3.

II.1. Minimizing the Euclidean Norm $\|U^TBU-\Delta\|$ is Equivalent to Minimizing Tr{BTBT} .

Proof. $\|U^TBU-\Delta\| = \mathrm{Tr}\{(U^TBU-\Delta)(U^TBU-\Delta)^T\}$ (definition of Euclidean norm)

$$= \mathrm{Tr}\{U^TBUU^TBU\}-2\mathrm{Tr}\{U^TBU\Delta\}+\mathrm{Tr}\{\Delta\Delta\}$$

Since $\mathrm{Tr}\{U^TBU\Delta\} = \mathrm{Tr}\{BU\Delta U^T\}$ and is diagonal with elements $(p_i/c_i)I_i$, $\mathrm{Tr}\{BU\Delta U^T\} = \mathrm{Tr}\{\ (p_i/c_i)BT_i\} = \mathrm{Tr}\{\Delta\Delta\}$ (consider $\mathrm{Tr}\{BT_i\} = p_i$) .

Therefore,

$$U^TBU- \quad = \mathrm{Tr}\{BTBT\} - \mathrm{Tr}\{\Delta\Delta\} \quad (\because T = UU^T) \quad .$$

Thus

$$\min_B \ U^TBU- \quad = \min_B \mathrm{Tr}\{BTBT\}$$

II.2. The MINQUE is Coincident with Helmert Type Estimation of Variance Components, but Different from that of Covariance Components.

Let us introduce the variance covariance component estimation of Helmert type. Note that the notations used here are consistent with those in Chapter 3 in order to compare with the MINQUE.

The variance covariance components of Helmert type are estimated from (Grafarend, et al., 1980):

$$\hat{\theta} = S^{-1}\underline{q} \qquad \text{(II-1)}$$

with

$$S_{ij} = \mathrm{Tr}\{Q^T E_i Q T_j\} \qquad \text{(II-2a)}$$

$$q_i = \hat{\underline{v}}^T E_i \hat{\underline{v}} \qquad \text{(II-2b)}$$

E_i is obtained from the block decomposition of the inverse T^{-1}, which corresponds to T_i, i.e., $T^{-1} = \sum_i E_i$. For example,

$$\Sigma = \begin{pmatrix} \sigma_1^2 Q_{11} & \sigma_{12} Q_{12} \\ \sigma_{12} Q_{21} & \sigma_2^2 Q_{22} \end{pmatrix} = \sigma_1^2 \begin{pmatrix} Q_{11} & 0 \\ 0 & 0 \end{pmatrix} + \sigma_{12} \begin{pmatrix} 0 & Q_{12} \\ Q_{21} & 0 \end{pmatrix} +$$

$$+ \sigma_2^2 \begin{pmatrix} 0 & 0 \\ 0 & Q_{22} \end{pmatrix}$$

$$= \sum_1^3 Q_i T_i \quad ,$$

and

$$T^{-1} = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}^{-1} = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}$$

$$= \begin{pmatrix} P_{11} & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & P_{12} \\ P_{21} & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & P_{22} \end{pmatrix}$$

$$= \sum_{1}^{3} E_i \quad .$$

If only variance components are to be estimated, i.e., T is block diagonal, then $E_i = T^{-1}T_iT^{-1}$ and

$$S_{ij} = Tr\{Q^T E_i Q T_j\} = Tr\{Q^T T^{-1} T_i T^{-1} Q T_j\} = Tr\{RT_i RT_j\} \qquad \text{(II-3a)}$$

$$q_i = \underline{\hat{v}}^T E_i \underline{\hat{v}} = \underline{\hat{v}}^T T^{-1} T_i T^{-1} \underline{\hat{v}} \qquad \text{(II-3b)}$$

Therefore, the variance components calculated from formula that from (II-1) is identical with (3-8). However, if variance-covariance components are to be estimated, i.e., T is not block diagonal, then $E_i \neq T^{-1}T_iT^{-1}$ and equalities (II-3a) and (II-3b) do not hold. Hence formula (II-1) is different from (3-8). This establishes the statement.

II.3. <u>is unbiased when S in (3-8) is Regular.</u>

Since $q_i = \underline{\hat{v}}^T T^{-1} T_i T^{-1} \underline{\hat{v}} = \underline{\ell}^T RT_i R\underline{\ell}$, its expectation value is

$$E\{q_i\} = Tr\{RT_i RE\{\underline{\ell}\underline{\ell}^T\}\}$$

$$= Tr\{RT_i R(A\underline{x}\underline{x}^T A^T + \Sigma)\}$$

$$= Tr\{RT_i R \sum_{1}^{K} \theta_i T_i\}$$

$$= (Tr\{RT_i RT_1\}, Tr\{RT_i RT_2\}, \ldots , Tr\{RT_i RT_k\}) \ \underline{\theta} \quad .$$

Thus

$$E\{\underline{\hat{\theta}}\} = E\{S^{-1}\underline{q}\} = \underline{\theta} \quad .$$

The proof is finished.

APPENDIX III

DISLOCATION MODELS

Dislocation models are used to relate surface displacements to the displacements on a fault. In dislocation theory, the earth is viewed as a perfectly elastic, isotropic, homogeneous half-space. Faults are represented as rectangles embedded in the half-space, and ground deformation corresponds to the elastic medium's response to slip on the rectangular surfaces (Snay, et al., 1982).

There are three elementary dislocation models, which represent rigid block motion, the strike slip displacements for a locked fault and for a surface fault (Savage, et al., 1973; Turcotte, et al., 1974; Brunner, et al., 1980). In a local Cartesian coordinate system, where the y-axis is perpendicular to the fault line and the x-axis is coincident with the fault line (see Fig III.1), the displacement u_i at a point (y_i) can be expressed as

$$u_i = \frac{b}{2} \operatorname{sign}(y_i) \qquad \text{(III-1)}$$

for rigid body motion;

$$u_i = \left(\frac{b}{\pi}\right) \tan^{-1}(y_i/D) \qquad \text{(III-2)}$$

for a locked fault and

$$u_i = \left(\frac{b}{\pi}\right) \tan^{-1}(D/y_i) \qquad \text{(III-3)}$$

for a surface fault, where b is the total relative displacement of the two sides of the fault and D is the depth of the locked fault or of the surface fault. The displacement and strain patterns for these models are plotted in Fig. III.1.

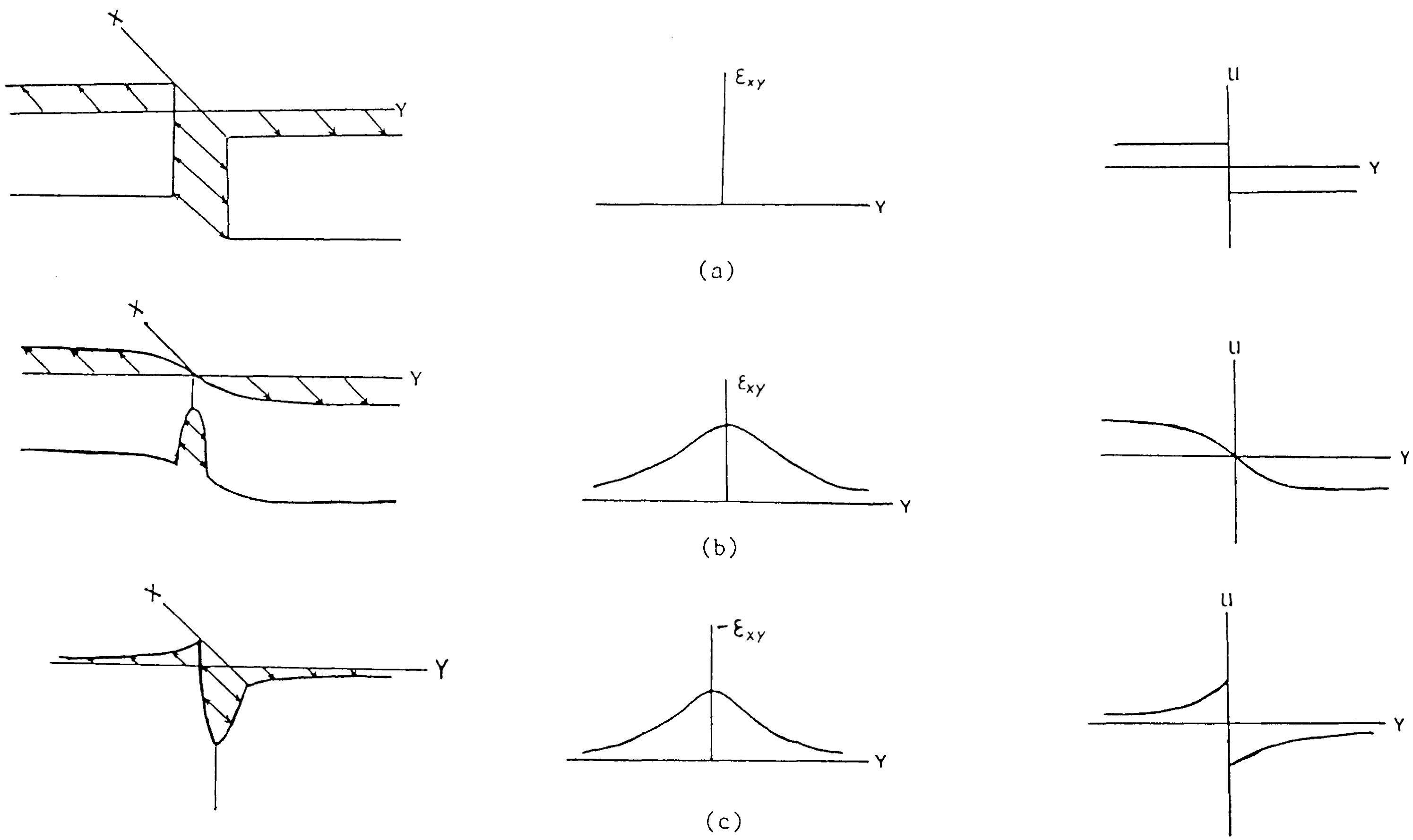


Fig. III.1

Dislocation Models (a) Rigid Body Motion; (b) Locked Fault; (c) Surface Fault.

APPENDIX IV

This appendix contains two examples which are relevant to the statements in Section 6.4.

IV.1. An Example to Demonstrate the Proposed Method for the Computation of $(\sum_i^K P_i)^-$ in (6-11).

Let Fig. IV.1 represents a two-epoch monitoring scheme, where only the distances were measured. Of course, in this example the use of the "observation approach" is much simpler, but for the illustration of the proposed method the "coordinate approach" is used. From the discussion made in 6.4.1, the coefficient matrix N_i of the normal equation is taken as the weight matrix P_i. Obviously, N_1 for epoch 1 (Fig. IV.1a) has three datum defects and two configuration defects; N_2 for epoch 2 (Fig. IV.2b) has three datum defects and three configuration defects. Combining the two epoch observations leads to Fig. IV.1c. Fig. IV.1c has three datum defects and one configuration defect, so does $(N_1 + N_2)$.

In order to transfer the inversion of singular matrix $(N_1 + N_2)$ to the inversion of a non-singular matrix, four pseudo-observations are added. They are: coordinates of point 1, azimuth from point 1 to point 2, distance from point 3 to point 5. These observation equations are as follows:

Observation equations for the coordinates:

$$x + v_x = dx_1$$

$$y + v_y = dy_1$$

with weights being

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Observation equation for the distance:

$$S+v_S = -\sin_{3-5} dx_3 - \cos_{3-5} dy_3 + \sin_{3-5} dx_5 + \cos_{3-5} dy_5$$

with weight of 1.

Observation equation for the azimuth:

$$\alpha + v = \frac{1}{S_{1-2}} (dy_2 - dy_1)$$

with weight being S^2_{1-2}. Here the selection of the weight does not make much difference, just from numerical consideration (compatible with the other observations). Then the coefficient matrix from these observations, denoted by N, is formulated in the conventional way and added to $(N_1 + N_2)$, resulting in non-singular matrix $N = (N_1 + N_2 + N)$. The inverse of N is a g-inverse of $(N_1 + N_2)$.

IV.2. A Numerical Example for the Comparison of the Displacement Approach" and the "Raw Observation Approach".

Fig. IV.2 is a monitoring trilateration network. In epoch 1, all the distances were measured with the same accuracy, but in epoch 2 the two diagonal distances were measured with accuracy one time higher than the others. Using two approaches, the cofactor matrix of the estimated strain parameters (εx, εy, εxy) of a homogenous deformation model is computed as follows.

Configuration matrix A and the matrix of deformation model B are:

$$A = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & +1 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

1) The "observation approach":

$$B^TA^T = \begin{pmatrix} 0 & 1 & 0 & 1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 0 & 1 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{2}} & \frac{2}{\sqrt{2}} \end{pmatrix}$$

$$P_1 = \text{diag}\ \{1,\ 1,\ 1,\ 1,\ 1,\ 1\}$$

$$P_2 = \text{diag}\ \{1,\ 1,\ 1,\ 1,\ 4,\ 4\}$$

$$P_{1\underline{+}2} = \text{diag}\ \{0.5,\ 0.5,\ 0.5,\ 0.5,\ 0.8,\ 0.8\}$$

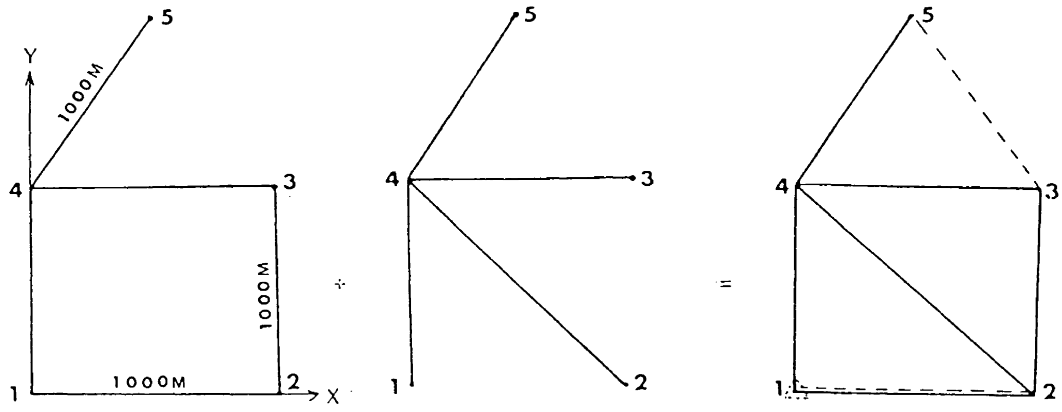
$$Q_{\underline{\hat{e}}}^{-1} = B^TA^T(P_{1\underline{+}2})AB = \begin{pmatrix} 1.8 & 0.8 & 0 \\ 0.8 & 1.8 & 0 \\ 0 & 0 & 3.2 \end{pmatrix} \qquad \text{(IV-1)}$$

2) The "coordinate approach":

$$N_1 = A^T A \quad , \quad N_2 = A^T P_2 A$$

$$Q_e^{-1} = B^T N_1 (N_1 + N_2)^- N_2 B = \begin{pmatrix} 1.93 & 0.93 & 0.0 \\ 0.93 & 1.93 & 0.0 \\ 0.0 & 0.0 & 3.20 \end{pmatrix} \qquad \text{(IV-2)}$$

Comparing (IV-1) and (IV-2), one can see that the "displacement approach" and the "raw observation approach" are not identical when the conditions in Section 6.4.2 are not fulfilled. The discrepancy of the obtained results using the two approaches depends on the strength of a network.



(a)
Epoch 1

(b)
Epoch 2

(c)

Fig. IV.1

A Simulated Monitoring Scheme for the Demonstration of the Proposed Method in a g-inverse of $(\sum_{i}^{k} P_i)$.

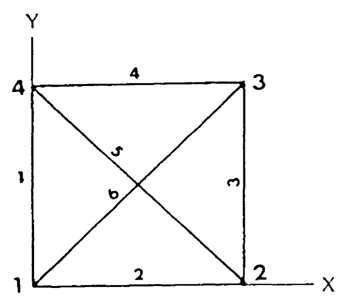


Fig. IV.2

A Simulated Network for the Comparison of the "Coordinate Approach" and the "Observation Approach".

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"Design considerations in deformation monitoring." (with M. Kavouras and J. Secord) To be presented at FIG-17th International Congress, paper 608.2, Sofia, June 1983.

"Use of finite element method in the design and analysis of deformation measurements." (with A. Chrzanowski and A. Szostak-Chrzanowski) To be presented at FIG-17th International Congress, paper 611.1, Sofia, June 1983.

"Investigation of congruency of a deformation network by minimizing the sum of the length of displacements." (in German) (with W. Caspary and R. Konig) To be presented at Seminar "Deformation Analysis" of FIG Commission 6, Munich, April 1983.