A LEAST SQUARES ADJUSTMENT FOR LONG BASELINE INTERFEROMETRY

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PREFACE

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A LEAST SQUARES ADJUSTMENT FOR LONG BASELINE INTERFEROMETRY

by

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ABSTRACT

Long baseline interferometry software and data, developed by the Canadian L.B.I. group at York University, has been combined with a least squares adjustment package. The options have been implemented to accept an input of both weighted parameters and functional parameter constraints. The results are then analysed statistically, including a chi-square goodness-of-fit test on the residuals, a rejection criteria for residual outliers, and a chi-square test on the variance factor.

The package has been developed with close regard to computer economy. Computer storage space has been reduced by 60% and processing time has been reduced by 96% compared with the previously used maximum likelihood adjustment routines. This increase in efficiency has resulted in an ability to input a large number of observations and, accordingly, in an improvement in accuracy.

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ACKNOWLEDGEMENTS

This thesis was supervised by Dr. P. Vanicek. It formed a continuation in studies on geodetic applications of L.B.I. being carried out at York University, Toronto, by Dr. W.H. Cannon, Dr. R.B. Langley, and W.T. Petrachenko. This group developed the software and, together with astronomers at the Herzberg Institute of Astrophysics, Ottawa, undertook the observations which have been used by the author.

Continued assistance has been rendered by many individuals. Mrs. L. Mills has assisted as a computer consultant. T. Wray has given mathematical guidance. W.T. Petrachenko demonstrated the use of various packages. R.R. Steeves has given advice and allowed adaption of his programs. Dr. R.B. Langley has instigated advice and programs, all by correspondence. Fellow graduate students have also given assistance to this work.

Financial assistance to the author through this period of study has been made available through graduate teaching and research assistantships from the Surveying Engineering Department. The National Research Council forwarded funds towards L.B.I. research.

The typing was carried out by Mrs. S. Burgess.

The author extends his gratitude to these individuals and organisations, as well as to others who may have assisted, but space prohibits recognition.

CHAPTER 1

INTRODUCTION

This thesis describes a least squares adjustment package written for one specific geodetic method: long baseline interferometry (L.B.I.). The importance of L.B.I. to the determination of geophysical and geometrical properties of the earth has been extensively discussed by such authors as Jones [1969], Meeks [1976], Cannon [1978], and Shapiro [1978]. Only a brief description is thus given of L.B.I. principles sufficient to outline the L.B.I. process and the specific computational problems. The main concern has been to produce an efficient adjustment and statistical testing package to process L.B.I. observations. The routines developed from this work form a contribution to the Canadian L.B.I. software system [Cannon, 1978; Langley, 1979].

The initials V.L.B.I. will be encountered in some literature, the V standing for "very". There is no implied difference between V.L.B.I. and L.B.I. except that L.B.I. tends to be used by the Canadian workers centred at York University in Toronto. The other main groups working on L.B.I. are the "East Coast Group" which includes the Massachusetts Institute of Technology, in

Cambridge, Massachusetts, the Haystack Observatory, Westford, Massachusetts, and Goddard Space Flight Center, Greenbelt, Maryland. The "West Coast Group" is based at the Jet Propulsion Laboratory, Pasadena, California. A European group is centred at Bonn, West Germany.

The historical background to the Canadian system is that radio astronomers at the Herzberg Institute of Astrophysics in Ottawa, the Appleton Laboratory in the United Kingdom, and the University of Toronto, have developed instrumentation to study compact extragalactic radio sources. These organisations are concerned mainly with astrophysics. Use of the Canadian observations as a geodetic tool was initiated by the Geodetic Survey of Canada [Jones, 1969], and was continued by a group at York University in Toronto.

The group at York University have developed software to determine parameters of geodetic interest that uses a maximum likelihood adjustment. The maximum likelihood routines were considered very expensive to use on the computer. The central processor unit (C.P.U.) requirement in time and immediate access store space were restrictively high to the extent that from an observation period involving 5,700 observations a sample of only 180 were processed to give results [Langley, 1979]. A more efficient adjustment package would allow the economical use of the full set of observations, and correspondingly a decrease in the standard error of results, since accuracy of a set of independent observations is proportional to the square-root of the number of observations.

The aim of this thesis has been to produce an efficient least squares adjustment package. The options have been implemented to accept an input of both weighted parameters, and functional parameter constraints. Statistical analysis used includes a chi-square goodness-of-fit test on the residuals, a rejection procedure for residual outliers, and a chi-square test on the variance factor.

A data set of 180 observations was processed at York University using the established computer package, including the maximum likelihood adjustment. This package, the data set of 180 observations, and the full data set of 5,700 observations were then transferred to the University of New Brunswick. The author reproduced the output computed at York with the maximum likelihood adjustment. A least squares adjustment was then used to produce the same results, but at a more economical level. As a result the immediate access store requirement was reduced by 60%, and C.P.U. time was reduced by 96%. The full data set of observations was then adjusted and standard errors were found to be reduced by a factor of approximately five. The author's least squares adjustment routines were thus considered ready to be used in any subsequent L.B.I. observation set analysis.

Chapter 2 describes the basic principles of L.B.I., showing the mathematical models used in the adjustment, and summarising the observing process. Chapter 3 shows the derivation of the least squares adjustment equations, and Chapter 4 outlines the statistical tests available in the routines. Chapter 5 comments on some attributes of the author's computer subroutines, especially those which have allowed the reported savings in computer storage space and C.P.U. time. A comparison of results between the maximum likelihood and the least squares adjustment routines, including results from a full observation set, is given in Chapter 6. Chapter 7 concludes with recommendations for future work.

CHAPTER 2

A BRIEF INTRODUCTION TO L.B.I.

This chapter gives a summary of L.B.I. as used for geodesy. Some aspects of the radio sources are discussed. Definitions are given of the observables: delay and fringe frequency. Parameters which are typically resolved such as the baseline components, source directions, and clock polynomial coefficients are outlined. Simplified descriptions are given of the L.B.I. models used in the adjustment process, and also given is a limited description of the L.B.I. observing and processing sequences.

2.1 The Source of the Radio Signal

In L.B.I. observations are made of the signal emitted from compact extragalactic radio sources which, for astrophysical purposes, can be classified into quasars, Seyfert galaxies, and BL Lac type objects. Definitions of these source types are beyond the scope of this thesis. The sources are situated at extragalactic distances allowing an assumption of being at infinity. Angular size and proper motion are negligible to the extent that the sources may be considered as points fixed on the celestial sphere. These sources can thus be useful to define a stable celestial reference frame.

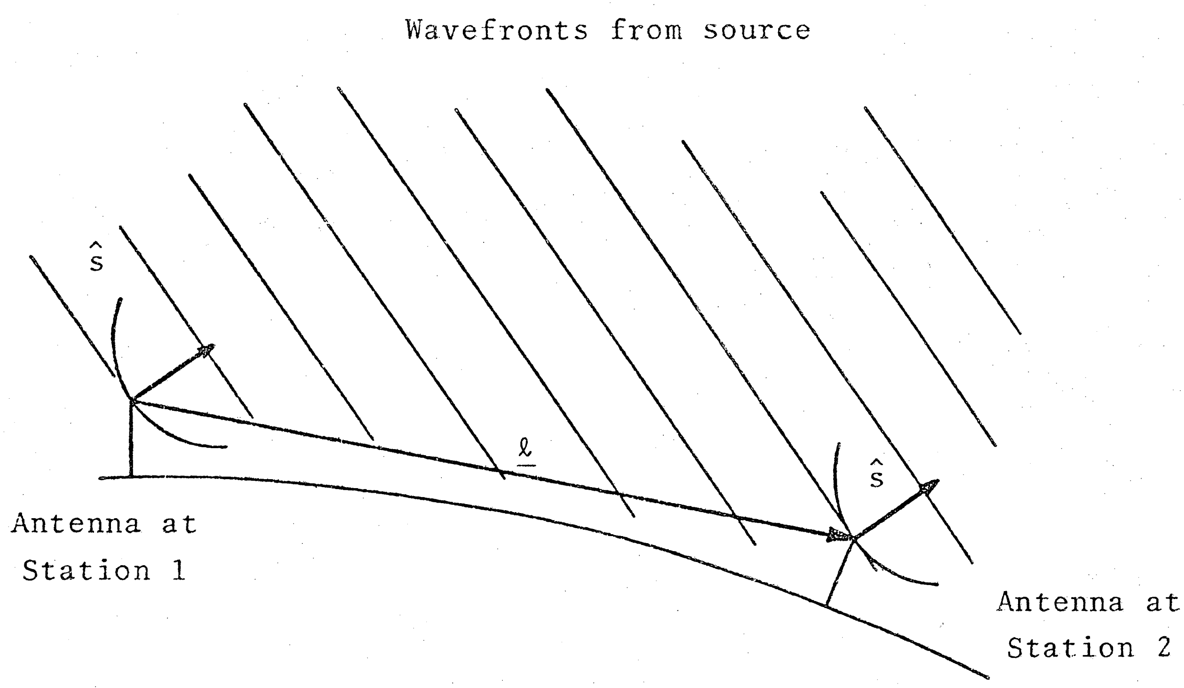
Since the radio signal received is weak directional antennae of dimensions between twenty and forty metres in diameter are commonly used for reception. The signal reaches the earth in the form of plane wave-fronts because of the sources being sited at such large distances. It is these plane wave-fronts which, on being received by pairs of antennae, give the L.B.I. observations.

2.2 Definitions of Observations of Delay and Fringe Frequency

An L.B.I. baseline, shown in Figure 2.1, is defined as the vector between two antennae which record the plane wave-fronts from a source. Delay is the time taken for a particular wave-front to pass between the two antennae. Because the earth is rotating both antennae will be moving and introducing a Doppler shift to the recorded signal at each station. Fringe frequency is the difference in Doppler shifts of the recorded signal at each station. Delay and fringe frequency are the observations of interest to geodesy and for the respective instant of time express a relationship between the baseline vector and the direction to the source.

2.3 Resolvable Parameters

Parameters which may be deduced from L.B.I. include the three dimensional baseline vectors, and the directions to the sources. The absolute position of the baseline



$\hat{s}$ ... unit vector in direction of source

$\underline{\ell}$ ... baseline vector

Figure 2.1. L.B.I. Baseline Receiving Plane Wave-fronts.

vector cannot be resolved from L.B.I., so baseline results are usually given as differences in three dimensional cartesian coordinates of the respective stations. Source directions are conveniently expressed in the form of right ascension and declination.

A third set of parameters are clock polynomial coefficients. A polynomial is used to model the difference in time as given by the clocks at each station. The incoming signal at each station is recorded on magnetic tape with precise time marks given by a clock. These clocks will have errors, and these errors will not be constant. Absolute error cannot be detected, only the difference between the two clocks affect L.B.I. observations. A polynomial in time is thus used to represent the error difference between the two clocks: an epoch difference gives a zero order polynomial, a rate difference implies a first order polynomial, and a difference in acceleration gives a second order polynomial. The order of the clock polynomial should represent the instability of the clock mechanisms, but the exact modelling is unknown and either an order is assumed, or a search is made with varying orders of polynomial. The polynomial fit which forms a minimum of the sum of the squares of the weighted residuals can be accepted as the best model.

Clock polynomial coefficients are not directly useful to geodesy, but their values do indicate the stabilities of the clocks, and their correct modelling is important to yield parameters which are directly useful to geodesy.

There are other parameters which can be determined by L.B.I. such as the earth's rotation, and polar motion, but these are held fixed in the model routines used by the author.

2.4 The L.B.I. Models

Models are mathematical relationships between sets of parameters and observations. They are used to derive the solution equations for the parameters. In L.B.I. there are two classes of models: a non-linear parametric model relating observations and parameters, and a linear model relating only parameters.

The non-linear parametric class of model can be derived from Figure 2.2, showing a baseline of length $|\underline{\ell}|$ between stations 1 and 2. The source is essentially at infinity in the direction of unit vector $\hat{s}$. The angle θ is between the directions to the source and of the baseline vector.

In Figure 2.2 the wave path difference ($c\tau$) is the distance travelled by a wave-front between the two L.B.I. stations. This shows the delay observation. The speed of light is c and the value of delay is τ. The formula for the geometric value of delay can be deduced:

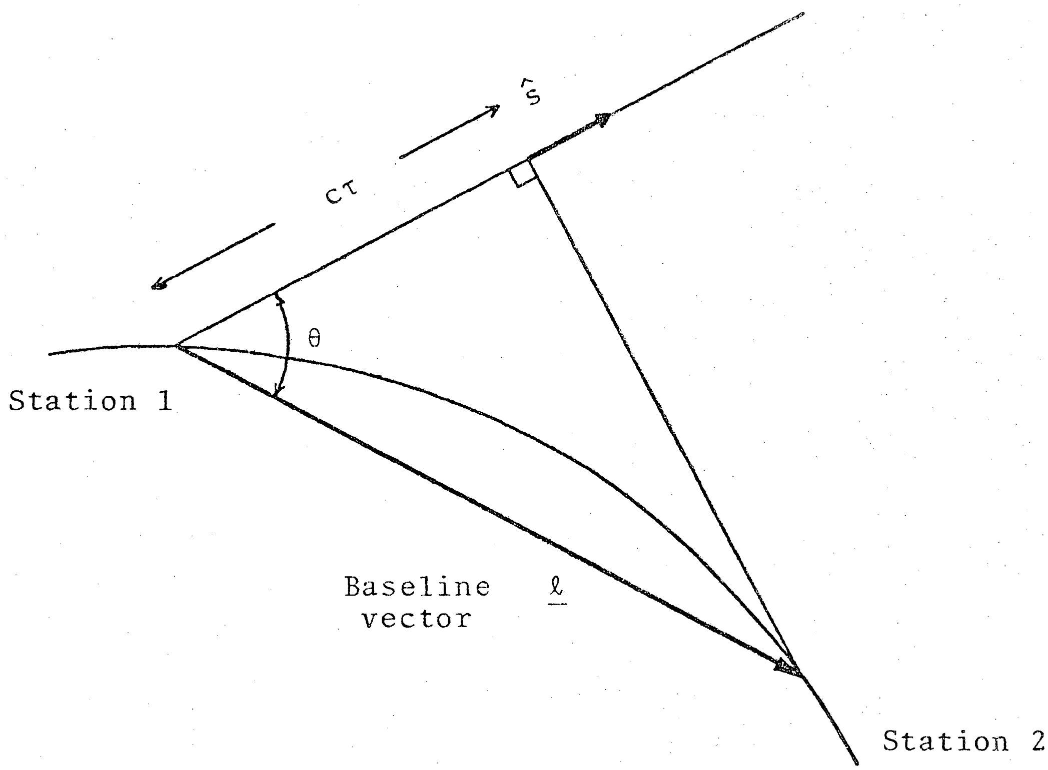


Figure 2.2. Two L.B.I. Receivers Observing a Source.

$$\cos\theta = \frac{c\tau}{|\ell|} \tag{2.1}$$

$$\tau = \frac{|\ell|}{c}\cos\theta \tag{2.2}$$

The observed value of delay is measured from the difference in the times, as given by the two clocks, that the wave-front is received at each antenna. The measured delay thus involves a polynomial to model the clock's error difference. The same symbol τ can be used for measured delay.

$$\tau = \frac{|\ell|}{c}\cos\theta + a_0 + a_1 t + a_2 t^2 + \ldots \tag{2.3}$$

The argument of the polynomial is time (t), and the coefficients are a_i, $i = 0,1,2,\ldots$

Fringe frequency has been defined as the difference in Doppler shift of the received signals at the two stations. This is equivalent to the rate of change of cycles of the received signal along the wave path difference. The number of cycles of the received frequency along the wave path difference equals frequency multipled by delay:

$$\text{cycles} = f\tau \tag{2.4}$$

Measured fringe frequency (F) equals the rate of change of the number of these cycles.

$$F \doteq f \frac{\partial \tau}{\partial t} \tag{2.5}$$

Differentiating equation (2.3) with respect to time

$$F = -\frac{f}{c} \, |\underline{\ell}| \, \sin\theta \, \frac{d\theta}{dt} + f(a_1 + 2a_2 t + \ldots) \tag{2.6}$$

The clock polynomial coefficients of the fringe frequency observation equation (2.6) are in theory functions of their respective coefficients in the delay observation equation (2.3). This could be implied as a constraint into the adjustment or could be allowed to vary, but subsequently checked to validify the adjustment.

When three stations simultaneously observe a single source further constraints may be imposed on the adjustment. The differences in clock errors around the three baselines sum to zero. This is implied by the summation around the three baselines of each respective order of polynomial coefficient to zero [Langley, 1979].

$$\sum_{j=1}^{3} a_i^j = 0 \tag{2.7}$$

The number of the baseline is j, and i the order of the polynomial.

Previous estimates of parameters can be introduced into the adjustment as parameter constraints. A source position, for example, can be set equal to a pre-determined value with a weight representing the amount of confidence in the value. This is essentially an observation of a parameter.

There are two models in the L.B.I. adjustment. The first describes the expressions for the measured observations of delay and fringe frequency. The second describes the constraints which may be imposed on the parameters. A relationship between parameters which is known to be true is termed a functional parameter constraint, while an estimation of a parameter with a weight is termed a weighted parameter constraint [Mikhail, 1970]. In the author's adjustment the functional parameter constraints are not rigorously applied, but are included as observations of parameter relationships with high weights. This is further discussed in Chapter 5.

Equations (2.3) and (2.6) are simplifications of the equations used in the York L.B.I. software [Langley 1979; Cannon 1978]. The reduction phase of the process involves a tropospheric correction. The York software model includes the effects of the retarded baseline, precession, nutation, polar motion, solid earth tides and the variation of UT1-UTC.

The constraints on the clock polynomials are only correct with perfect instrumentation and a simplified earth model. The delay clock polynomials are due to a combination of the atomic frequency standard and the clock, while the fringe frequency "clock" polynomials are due to a combination of the atomic frequency standard and the oscillator. The sum to zero around a three-baseline array is not always implied because of the effect of the retarded baseline.

2.5 The L.B.I. Observing and Correlation Process

An L.B.I. observation period may last for several days with perhaps an observation every minute. There is thus a large number of observations and computer control and magnetic tape storage is required to process the data. Two antennae simultaneously record the signal from the same source. The received signal band at each antenna is translated to a lower frequency band to allow recording on magnetic tape together with accurate timing records.

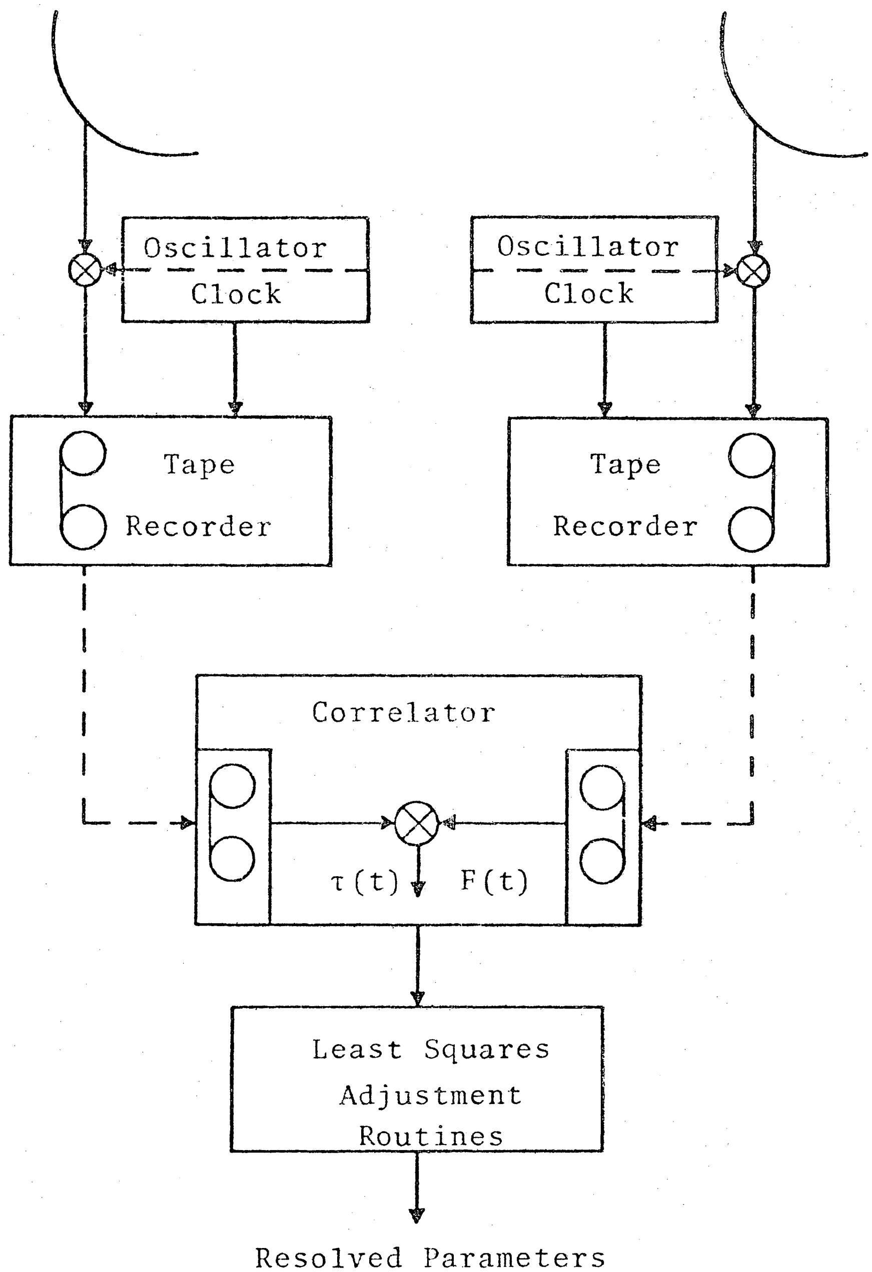


Figure 2.3. The L.B.I. Process.

At a later date two tapes for each baseline are played back at the correlator facility. Using the recorded time signals one tape is delayed with respect to the other until obtaining a maximum correspondence between the two signals. The observations of delay and fringe frequency for an instant of observation time are abstracted and recorded on magnetic tape. The final processing stage is an adjustment using the delay and fringe frequency observations to resolve the parameters of baseline vector, source direction, and clock polynomial coefficients.

CHAPTER 3

LEAST SQUARES ADJUSTMENT

In this chapter the least squares solution is derived from the two classes of L.B.I. models. The least squares solution gives estimates for the parameters which minimise the summation of the squares of the weighted residuals [Mikhail, 1976]. The true parameters cannot be deduced, but least squares gives a best estimate of parameters. The derivation uses the Lagrange method. The covariance matrix of the results is deduced, and the expression is given for the variance factor.

Symbols used in this chapter are underlined capital letters (e.g. $\underline{A}$) for a matrix, and underlined lower case letters for a vector (e.g. $\underline{x}°$, $\underline{\delta}$).

3.1 Derivation of the Least Squares Equations

3.1.1 Input for the adjustment

A major input into the adjustment is the vector of observations pertaining to the first model of Chapter 2 (2,3), (2.6), and its covariance matrix ($\underline{C}_\ell$). The second model outlined in Chapter 2 involves the constraint observation vector ($\underline{\ell}_x$) and its covariance matrix ($\underline{C}_x$).

The two mathematical models for L.B.I. relate the parameters and the observations.

$$\underline{F}_1(\underline{x}, \underline{\ell}) = \underline{0} \tag{3.1}$$

$$\underline{F}_2(\underline{x}, \underline{\ell}_x) = \underline{0} \tag{3.2}$$

The observations will have errors, so the true observations $(\hat{\underline{\ell}}, \hat{\underline{\ell}}_x)$ are given as the observation plus the residual.

$$\hat{\underline{\ell}} = \underline{\ell} + \underline{v} \tag{3.3}$$

$$\hat{\underline{\ell}}_x = \underline{\ell}_x + \underline{v}_x \tag{3.4}$$

The à priori parameter vector $(\underline{x}^o)$ is the initial guess of the parameters. Added to the parameter increments $(\underline{\delta})$ gives the correct parameters.

$$\underline{x} = \underline{x}^o + \underline{\delta} \tag{3.5}$$

The adjustment will give an estimation of the parameter increment vector.

3.1.2 A Taylor's expansion of the models

The models are currently in a form expressing the true parameters and true observations. The à priori parameters and observations, and the parameter increments and residuals, can be involved using a Taylor's expansion, but neglecting second order terms.

$$\underline{F}_1(\underline{x},\underline{\ell}) = \underline{F}_1(\underline{x}^o,\underline{\ell}) + \left.\frac{\partial \underline{F}_1(\underline{x}^o,\underline{\ell})}{\partial \underline{x}^o}\right|_{\underline{x}^o,\underline{\ell}} \cdot \underline{\delta}$$

$$+ \left.\frac{\partial \underline{F}_1(\underline{x}^o,\underline{\ell})}{\partial \underline{\ell}}\right|_{\underline{x}^o,\underline{\ell}} \cdot \underline{v} \qquad (3.6)$$

The misclosure vector is

$$\underline{w}_1 = F_1(\underline{x}^o,\underline{\ell}) \qquad (3.7)$$

The first design matrix, sometimes termed the A matrix is

$$\underline{A}_1 = \frac{\partial \underline{F}_1(\underline{x}^o,\underline{\ell})}{\partial \underline{x}^o} \qquad (3.8)$$

The second design matrix, or B matrix is

$$\underline{B}_1 = \frac{\partial \underline{F}_1(\underline{x}^o,\underline{\ell})}{\partial \underline{\ell}} . \qquad (3.9)$$

The model can thus be expressed:

$$\underline{w}_1 + \underline{A}_1 \underline{\delta} + \underline{B}_1 \underline{v} = \underline{0} . \qquad (3.10)$$

A similar expression can be derived for the second model.

$$\underline{w}_2 + \underline{A}_2\,\underline{\delta} + \underline{B}_2\,\underline{v}_2 = \underline{0}\,. \tag{3.11}$$

Neglecting the second order terms in the Taylor's expansion can falsify the derived equations. If the model is linear, then the second and higher order terms will be zero. With a non-linear model, but with à priori parameters selected as close to the true parameters, then the second and higher order terms will approach zero. In general one would continue iterations of the adjustment using updated parameter values until the iterations cease to significantly change the results.

3.1.3 The least squares solution by the Lagrange method

The sum of the weighted squares of the residuals can be expressed in matrix form for the two models:

$$\underline{v}^t\,\underline{P}_\ell\,\underline{v} \tag{3.12}$$

$$\underline{v}_x^t\,\underline{P}_x\,\underline{v}_x \tag{3.13}$$

where $\underline{P}_\ell$ and $\underline{P}_x$ are the weights of the respective observations.

The variation function is formed:

$$\phi = \underline{v}^t \underline{P}_\ell \underline{v} + \underline{v}_x^t \underline{P}_x \underline{v}_x$$

$$+ 2\underline{k}_1^t(\underline{A}_1 \underline{\delta} + \underline{B}_1 \underline{v} + \underline{w}_1)$$

$$+ 2\underline{k}_2^t(\underline{A}_2 \underline{\delta} + \underline{B}_2 \underline{v}_x + \underline{w}_2) \qquad (3.14)$$

k_1 and k_2 are column vectors of Lagrange correlates which will be determined. The extremal value of the variation function is found by differentiating with respect to the unknowns ($\underline{v}$, $\underline{v}_x$, $\underline{\delta}$, $\underline{k}_1$ and $\underline{k}_2$) and equating each derivative to a zero vector.

$$\frac{\partial\phi}{\partial\underline{v}} = 2\underline{v}^t \underline{P}_\ell + 2\underline{k}_1^t \underline{B}_1 = \underline{0}$$

$$\therefore \underline{P}_\ell \underline{v} + \underline{B}_1^t \underline{k}_1 = \underline{0} \qquad (3.15)$$

$$\frac{\partial\phi}{\partial\underline{v}_x} = 2\underline{v}_x^t \underline{P}_x + 2\underline{k}_2^t \underline{B}_2 = \underline{0}$$

$$\therefore \underline{P}_x \underline{v}_x + \underline{B}_2^t \underline{k}_2 = \underline{0} \qquad (3.16)$$

$$\frac{\partial\phi}{\partial\underline{\delta}} = 2\underline{k}_1^t \underline{A}_1 + 2\underline{k}_2^t \underline{A}_2 = \underline{0}$$

$$\therefore \underline{A}_1^t \underline{k}_1 + \underline{A}_2^t \underline{k}_2 = \underline{0} \qquad (3.17)$$

$$\frac{\partial\phi}{\partial\underline{k}_1} = \underline{A}_1 \underline{\delta} + \underline{B}_1 \underline{v} + \underline{w}_1 = \underline{0} \qquad (3.18)$$

$$\frac{\partial\phi}{\partial\underline{k}_2} = \underline{A}_2 \underline{\delta} + \underline{B}_2 \underline{v}_x + \underline{w}_2 = \underline{0} \qquad (3.19)$$

Simultaneous solution to equations (3.15) through (3.19) is the least squares solution. The result gives the minimum of the sum of the squares of the weighted residuals since the second derivatives of (3.15) and (3.16) are positive through the definition of the weight matrices $\underline{P}_\ell$ and $\underline{P}_x$ being positive definite.

A hypermatrix expression is formed for the simultaneous equations to be solved:

$$\begin{bmatrix} \underline{P}_\ell & \underline{0} & \underline{B}_1^t & \underline{0} & \underline{0} \\ 0 & \underline{P}_x & \underline{0} & \underline{B}_2^t & \underline{0} \\ \underline{B}_1 & \underline{0} & \underline{0} & \underline{0} & \underline{A}_1 \\ \underline{0} & \underline{B}_2 & \underline{0} & \underline{0} & \underline{A}_2 \\ \underline{0} & \underline{0} & \underline{A}_1^t & \underline{0} & \underline{0} \end{bmatrix} \begin{bmatrix} \underline{v} \\ \underline{v}_x \\ \underline{k}_1 \\ \underline{k}_2 \\ \underline{\delta} \end{bmatrix} + \begin{bmatrix} \underline{0} \\ \underline{0} \\ \underline{w}_1 \\ \underline{w}_2 \\ \underline{0} \end{bmatrix} = \underline{0} \qquad (3.20)$$

The method of partitioning of matrices is used where, given

$$\begin{bmatrix} \underline{D} & \underline{E} \\ \underline{E}^t & \underline{F} \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \begin{bmatrix} \underline{d}_1 \\ \underline{d}_2 \end{bmatrix} = \underline{0} \qquad (3.21)$$

then

$$\underline{x}_1 = -\underline{D}^{-1}(\underline{E}\ \underline{x}_2 + \underline{d}_1) \qquad (3.22)$$

and

$$(\underline{F} - \underline{E}^t\ \underline{D}^{-1}\ \underline{E})\underline{x}_2 - \underline{E}^t\ \underline{D}^{-1}\ \underline{d}_1 + \underline{d}_2 = \underline{0} \qquad (3.23)$$

where matrix $\underline{D}$ is not singular. Equation (3.20) is solved:

$$\begin{bmatrix} \hat{\underline{v}} \\ \hat{\underline{v}}_x \end{bmatrix} = - \begin{bmatrix} \underline{P}_\ell & \underline{0} \\ \underline{0} & \underline{P}_x \end{bmatrix}^{-1} \begin{bmatrix} \underline{B}_1^t & \underline{k}_1 \\ \underline{B}_2^t & \underline{k}_2 \end{bmatrix} \tag{3.24}$$

$$\therefore \hat{\underline{v}} = -\underline{P}_\ell^{-1} \underline{B}_1^t \underline{k}_1 \tag{3.25}$$

$$\hat{v}_x = -\underline{P}_x^{-1} \underline{B}_2^t \underline{k}_2 \tag{3.26}$$

Further partitioning will result in expressions for all the variable parameters.

$$\hat{\underline{k}}_1 = \underline{M}_1^{-1} (\underline{A}_1 \hat{\underline{\delta}} + \underline{w}_1) \tag{3.27}$$

where

$$\underline{M}_1 = \underline{B}_1 \underline{P}_\ell^{-1} \underline{B}_1^t \tag{3.28}$$

$$\hat{\underline{k}}_2 = \underline{M}_2^{-1} (\underline{A}_2 \hat{\underline{\delta}} + \underline{w}_2) \tag{3.29}$$

where

$$\underline{M}_2 = \underline{B}_2 \underline{P}_x^{-1} \underline{B}_2^t \tag{3.30}$$

$$\hat{\underline{\delta}} = -(\underline{A}_1^t \underline{M}_1^{-1} \underline{A}_1 + \underline{A}_2^t \underline{M}_2^{-1} \underline{A}_2)^{-1} (\underline{A}_1^t \underline{M}_1^{-1} \underline{w}_1 + \underline{A}_2^t \underline{M}_2^{-1} \underline{w}_2) \tag{3.31}$$

The 'hat' symbol above the solution vectors signifies that these are only the best estimates as given by the least squares solution. Another defined solution might give different results, and neither may be the true results. The normal equation matrix $(\underline{A}_1^t \underline{M}_1^{-1} \underline{A}_1 + \underline{A}_2^t \underline{M}_2^{-1} \underline{A}_2)$ is seen to be the summation of normal equations due to the two respective models. The vector of constant terms $(\underline{A}_1^t \underline{M}_1^{-1} \underline{w}_1 + \underline{A}_2^t \underline{M}_2^{-1} \underline{w}_2)$ is similarly the summation of the vectors due to the two models.

3.1.4 Solution simplification of a parametric model

Equation (3.31) is used in the author's adjustment routines, but with some simplification. Both $\underline{B}_1$ and $\underline{B}_2$ are negative unit matrices from their definitions of being the derivatives of the model with respect to the observations. The definition of a parametric model is that $\underline{B} = \underline{I}$ or $\underline{B} = -\underline{I}$.

$$\underline{B}_1 = -\underline{I} \tag{3.32}$$

$$\underline{B}_2 = -\underline{I} \tag{3.33}$$

If the constraints are not used, or not all of the parameters are involved in the constraints, then there are some modifications to the contributions due to the second model. Without constraints, these contributions reduce to zero. When only certain parameters are involved then only additions corresponding to those parameters are added to the normal equation matrix and vector of constant terms. Considering the addition of the second model in (3.31) and using $\underline{B}_2 = -I$ and $\underline{C}_x^{-1} = \underline{P}_x$, the additions become

$$\underline{A}_2^t \ \underline{P}_x \ \underline{A}_2$$

and

$$\underline{A}_2^t \ \underline{P}_x \ \underline{w}_2 .$$

The problem of zero diagonal elements of $\underline{P}_x$ is not encountered: terms are added to the normal matrix and the vector of constant terms as defined by the parameters used in the constraints.

3.1.5 The residuals

The equation for the residuals from the first model, from equations (3.25), (3.27) and (3.32) is

$$\hat{\underline{v}} = \underline{A}_1 \ \hat{\underline{\delta}} + \underline{w}_1 \qquad (3.34)$$

The residuals from the second model are derived from the computed value of the model misclosure:

$$\hat{\underline{v}}_x = \underline{F}_2(\hat{\underline{x}}, \underline{\ell}_x) \qquad (3.35)$$

3.2 Covariance Matrix of Parameters

The covariance matrix of the estimated parameters ($\underline{C}_{\hat{\delta}}$) is deduced from the covariance matrix of the observations using the covariance law. The covariance law in matrix form is given by

$$\underline{C}_{\hat{\delta}} = \underline{J}\ \underline{C}_{\ell}\ \underline{J}^t \tag{3.36}$$

where $\underline{J}$ is the Jacobian of transformation between the observations and the parameters.

$$\underline{\hat{\delta}} = \underline{F}(\underline{\ell}) \tag{3.37}$$

$$\underline{J} = \frac{\partial \underline{F}(\underline{\ell})}{\partial \underline{\ell}} \tag{3.38}$$

It is convenient to use equation (3.31) which has the vectors of constant terms, $\underline{w}_1$ and $\underline{w}_2$, as the variables to be transformed. From equations (3.7) and (3.9)

$$\underline{w}_1 = \underline{F}_1(\underline{x}^o, \underline{\ell})$$

$$\underline{C}_{w_1} = \frac{\partial \underline{w}_1}{\partial \underline{\ell}}\ \underline{C}_{\ell}\ \frac{\partial \underline{w}_2^t}{\partial \underline{\ell}} \tag{3.39}$$

$$\therefore\ \underline{C}_{w_1} = \underline{B}_1\ \underline{C}_{\ell}\ \underline{B}_1^t = \underline{M}_1 \tag{3.40}$$

$$\underline{w}_2 = \underline{F}_2(\underline{x}^o, \underline{\ell}_x)$$

$$\underline{C}_{w_2} = \frac{\partial \underline{w}_2}{\partial \underline{\ell}_x}\ \underline{C}_x\ \frac{\partial \underline{w}_2^t}{\partial \underline{\ell}_x} \tag{3.41}$$

$$\therefore \underline{C}_{w_2} = \underline{B}_2 \underline{C}_x \underline{B}_2^t = \underline{M}_2 \tag{3.42}$$

The covariance matrix of the parameters can be derived using equation (3.31) and the covariance matrices of the misclosure vectors (3.40) and (3.42).

$$\underline{C}_{\hat{\delta}} = \frac{\partial \delta}{\partial \underline{w}_1} \underline{M}_1 \frac{\partial \delta}{\partial \underline{w}_1}^t + \frac{\partial \delta}{\partial \underline{w}_2} \underline{M}_2 \frac{\partial \delta}{\partial \underline{w}_2}^t \tag{3.43}$$

This assumes zero correlation between the two misclosure vectors. Matrix manipulation can be shown to give

$$\underline{C}_{\hat{\delta}} = (\underline{A}_1^t \underline{M}_1^{-1} \underline{A}_1 + \underline{A}_2^t \underline{M}_2^{-1} \underline{A}_2)^{-1} \tag{3.44}$$

Noting that a covariance matrix is the inverse of the corresponding weight matrix, equation (3.44) is the inverse of the normal equation matrix, as given in equation (3.31).

3.3 The Variance Factor

The standard error of observations ($\underline{C}_\ell$) may not be known, but for a solution relative errors of observations ($\underline{P}_\ell^{-1}$) must be known for substitution into equation (3.31). Then the covariance matrix is known only to a scale factor.

$$\underline{C}_\ell = \sigma_o^2 \underline{P}_\ell^{-1} \tag{3.45}$$

This scale factor is termed the variance factor, and gives the standard error of an observation of unit weight as given by the weight matrix $\underline{P}_{\ell}$. The variance factor does not affect the estimation of the results for the parameters or the residuals, but it does scale the covariance matrices of the results.

It can be shown that the estimate of the variance factor equals [Mikhail, 1976]

$$\hat{\sigma}_o^2 = \frac{\underline{\hat{v}}\ \underline{P}_{\ell}\ \underline{\hat{v}} + \underline{\hat{v}}_x\ \underline{P}_x\ \underline{\hat{v}}_x}{\text{degrees of freedom}} . \tag{3.46}$$

The standard errors are known with observation sets used by the author [Langley, 1979], and present use of the variance factor, which is discussed further in Chapter 4 has been to check the validity of these standard errors and of the L.B.I. models.

CHAPTER 4

STATISTICAL ASSESSMENT OF RESULTS

Methods of statistical analysis of results used with the least squares routines are described in this chapter. The tests are based on the works of Hamilton [1964], Mikhail [1976], and Vanicek and Krakiwsky [1980], and are based on the confidence interval method. In general an hypothesis is made about a population and from a sample of this population a statistic is calculated which is tested at a particular confidence level. The confidence level is defined as $(1 - \alpha)$, where α is the significance level. The significance level is the probability of a type I error: the rejection of a true hypothesis.

The significance level can be varied, being an input value for a program run. The tests carried out by the program and described here are a test for normality of the residuals, a test on the variance factor, and a detection of outliers. The standard error of the unweighted delay and fringe frequency residuals, and the covariance between them are also evaluated by the program.

4.1 Test for Normality of the Residuals

The test for normality of the residuals is carried out because subsequent tests rely on the residuals being normally distributed. Different residuals will in general have different standard errors, and cannot be described as from the same normal distribution. Standardization will imply that all residuals have the same distribution and is achieved by division by the respective residual's standard error. The standardized residual is defined by

$$\tilde{v}_i = \frac{v_i}{\sigma_i} \tag{4.1}$$

where $\tilde{v}_i \overset{d}{\rightarrow} n(0,1)$.

The standardized residuals are grouped into classes according to value. From the standardized normal probability distribution function (p.d.f.) can be estimated the number which should be in that class. The summation of the squares of the difference between these values, divided by the estimated value is defined as the chi-square statistic [Hamilton, 1964].

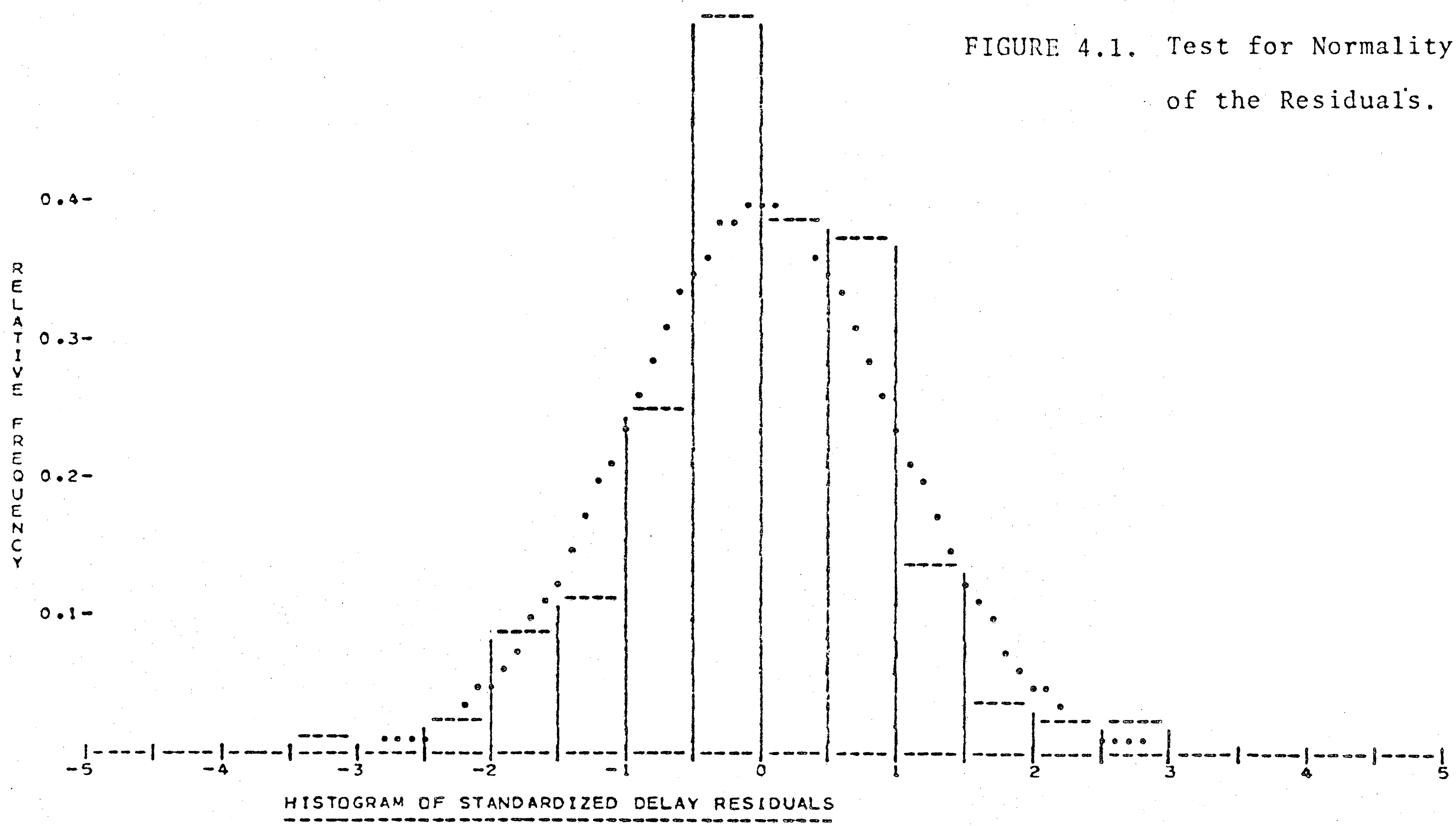
$$\chi^2 = \sum_{i=1}^{n} \frac{(a_i - e_i)^2}{e_i} \tag{4.2}$$

The observed number in the class is a_i, e_i is the expected number from the p.d.f., and n is the number of classes. The chi-square statistic is obtained at the α significance level with (n-1) or (n-2) degrees of freedom. The degree of freedom is (n-1) if the computation results ($\underline{x}$) were computed from the set of observations. A second degree of freedom is lost, giving (n-2) if the standard errors of observations are unknown. In this second case the variance factor has been estimated from the set of observations and used to scale the weight matrix (section 3.3). Hamilton [1964] writes that e_i should be at least five. The subroutines group the class intervals at the limits of the normal curve together until e_i is greater than five.

As a visual aid for checking the normality of the residuals the histograms of the standardized residuals of both delay and fringe frequency are printed, overlaid with the standardized normal p.d.f. (Figure 4.1).

Vanicek and Krakiwsky [1980] write that the distribution of the residuals will depend on what components of that curve are estimated from the observations. If the results ($\underline{x}$) are estimated then the residuals have a t-distribution, while if also the variance factor ($\hat{\sigma}_o^2$) is estimated then the residuals have a tau (τ) distribution. For large numbers of observations both these distributions approach a normal distribution. The author's routines thus compute the estimated numbers in each class from a normal p.d.f. This is considered acceptable because L.B.I. observation sets are usually in large numbers. The actual computation of the estimated standard error of a residual is computationally expensive, but Pope [1976] concludes

FIGURE 4.1. Test for Normality of the Residuals.



NUMBERS IN INTERVALS:
1 0/ 2 0/ 3 0/ 4 1/ 5 0/ 6 2/ 7 8/ 8 10/ 9 23/10 48/
11 35/12 34/13 12/14 3/15 2/16 2/17 0/18 0/19 0/20 0/
CHI-SQUARE STATISTIC: 0< 13.751<= 18.470 PASSES. NUMBER OF CLASSES: 8. DEGREES OF FREEDOM: 7.0

that for large numbers of observations it can be approximated by the standard error of the observation.

4.2 Chi-square Test on the Variance Factor

A check is made on the variance factor ($\hat{\sigma}_o^2$, section 3.3), or as it is also termed, the quadratic form of the residuals. When the observation standard errors are considered known, then the à posteriori variance factor should equal one. Discussed as from the least squares equations in section 3.3, the variance factor can also be deduced from the definition of the chi-square statistic. Hamilton [1964] writes that the sum of the squares of random variables each having a standardized normal distribution has a chi-square distribution.

$$\chi^2_{df} = \sum_{i=1}^{n} v_i^2 \, P_i = \sum_{i=1}^{n} \left(\frac{v_i}{\sigma_i}\right)^2 \qquad (4.3)$$

since $\frac{v_i}{\sigma_i} \xrightarrow{d} n(0,1)$.

The i^{th} residual is v_i and its weight is P_i. The number of observations is n. Observations include those of constraints. The degrees of freedom (df) equal the total number of observations from both models, minus the number of estimated parameters. The expected value of this statistic is the degrees of freedom. Failure of this test implies that the residuals do not have a normal distribution and can suggest that either the à priori standard errors of observations are incorrect, or that the L.B.I. models have errors.

At the $(1-\alpha)$ confidence level the à posteriori variance factor $(\hat{\sigma}_o^2)$ is compared with the à priori variance factor (σ_o^2) by the bounds given by Vanicek and Krakiwsky [1980]:

$$\frac{df\ \hat{\sigma}_o^2}{\xi_{\chi^2_{df,1-\frac{\alpha}{2}}}} < \sigma_o^2 < \frac{df\ \hat{\sigma}_o^2}{\xi_{\chi^2_{df,\frac{\alpha}{2}}}} \qquad (4.4)$$

ξ_{χ^2} is the abscissa value of the χ^2 statistic corresponding to the degrees of freedom, and the respective probability.

4.3 Detection of Residual Outliers

A detection of residual outliers is carried out by the author's routines. The residuals are hypothesised to have a normal distribution and a residual not complying with a normal distribution can be rejected. The normal p.d.f. (Figure 4.2) shows that the probability of a residual plotting within the limits given by the critical values $(+c,-c)$ is $(1-\alpha)$. The probability is α of the residual lying outside this confidence region. Rejection of an observation whose residual plots outside the confidence region would only have an α probability of loosing

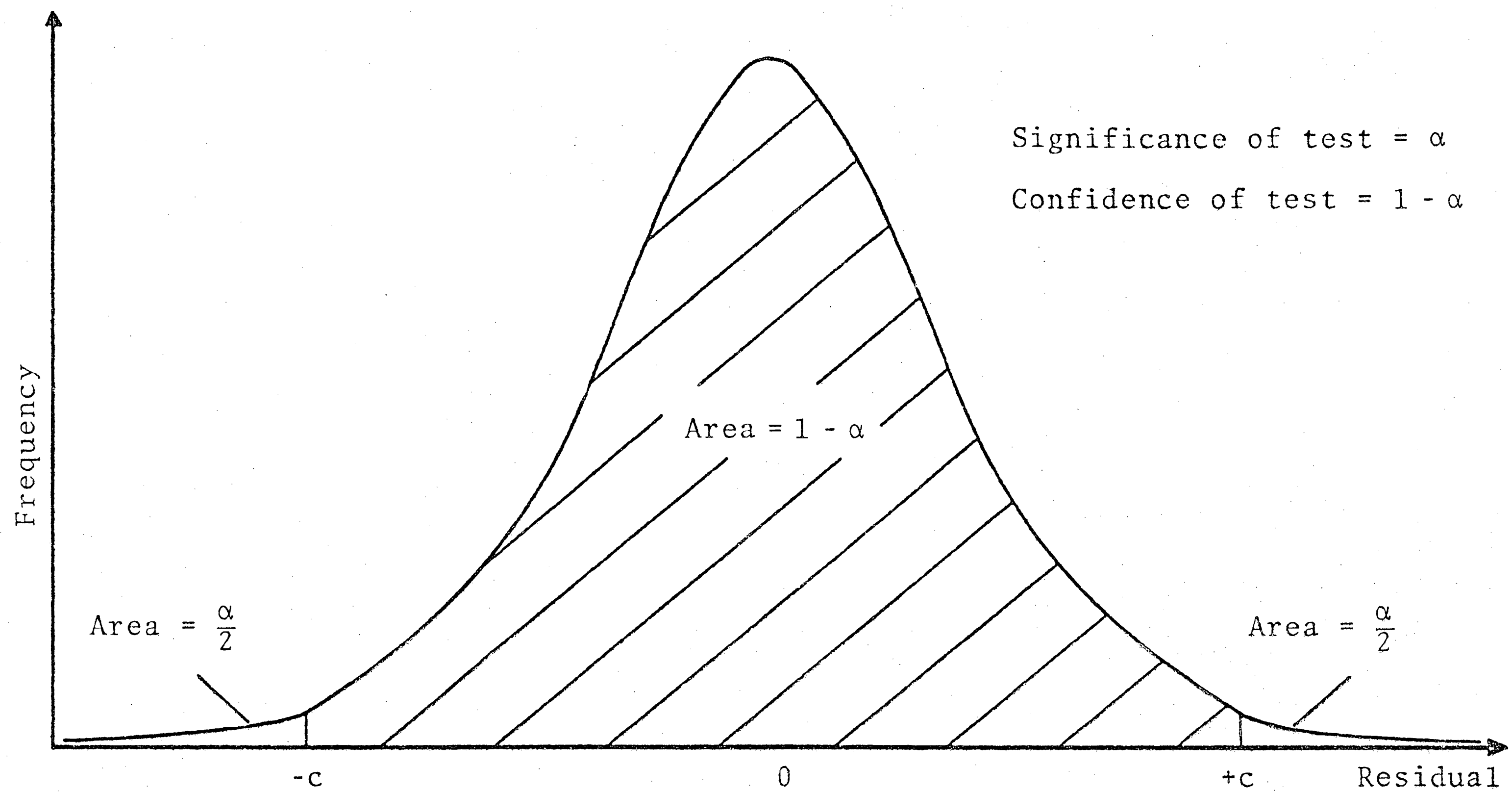


Figure 4.2. A Normal Probability Distribution Function (p.d.f.)

a good observation. At this expense, all gross errors should be eliminated.

In the test each residual is standardized by dividing by the standard error of the observation and compared with the critical value abstracted from a standardized normal p.d.f. at the α significance level. All residuals are plotted as a function of time of observation, and residuals that may be rejected are shown with an asterisk (Figure 4.3).

Outlying residuals may be specified within the context of the other residuals (max-test) or out of context [Krakiwsky, 1978; Vanicek and Krakiwsky, 1980]. The difference is outlined as the probability of one residual being within certain limits, compared with the probability of a large number of residuals being within the same limits. If the probability of one observation being within certain limits is the confidence level $(1-\alpha)$, then the simultaneous probability of n such occurrences equals

$$(1-\alpha)^n \simeq 1-n\alpha \qquad (4.5)$$

In the routines, if the probability of all n observations being within the confidence interval is required to be defined under the significance level then each residual is tested individually at a lower significance

FIGURE 4.3. Residual Plot.

level $(\frac{\alpha}{n})$.

4.4 Standard Error, and Covariance Between, Unweighted Fringe Frequency and Delay Residuals

The standard errors of the unweighted delay and fringe frequency residuals are evaluated using the formula

$$\hat{\sigma}_i = \sqrt{\frac{\sum_j v_{ij}^2}{df_i}} \qquad (4.6)$$

where $j = 1,2,\ldots,n$

n = number of the i^{th} type of observation

i = 1, 2 (delay and fringe frequency observations)

df_i = degree of freedom of the i^{th} type of observation.

The correlation between the two types of observation residual obtained for the same instant of time is calculated:

$$\hat{\sigma}_{ij}^2 = \frac{\sum_l v_{il} v_{jl}}{k} \qquad (4.7)$$

where i, j = delay, fringe frequency

k = number of time points with delay and fringe frequency observations

$l = 1,2,\ldots,k$.

These statistics were evaluated in the maximum likelihood adjustment. The standard error of the residuals can be compared with the à priori standard errors, and the covariance should approach zero.

CHAPTER 5

PROGRAMMING APPLICATIONS

The aim of this chapter is to assist in an understanding of the author's routines so that future users may be able to adapt and improve the present adjustment. This is achieved by outlining some specific computing methods used by the author. Most are applied to increase efficiency of the routines: compressing the A matrix, storing the A matrix on a sequential file, the iteration requirements, the use of station coordinates as parameters, and the method of imposing parameter constraints. An efficient method of detecting singularities in the normal equation matrix is also described.

5.1 Compressing the First Design (A) Matrix

There are many zeros in the A matrix because the partial derivatives of the model with respect to some of the parameters will be zero. This means that full storage of the A matrix, and numerical manipulations on that matrix will be wasteful on two accounts: much of the computer space will be storing zero, and there will be manipulations and additions involving zero. In the present form of the routines the maximum number of non-zero elements in one row of the A matrix is thirteen, while the number of columns in the A matrix is typically greater than thirty. This assumes a fourth order clock polynomial, six parameters corresponding to the two station positions, and two source parameters. Storing only non-zero elements in each row of the A matrix, as done in the least squares routines, is thus given the phase "compressing the A matrix".

An integer value for each observation gives the number of non-zero elements in the row of A pertaining to the observation (Figure 5.1). An integer vector contains a number for each non-zero element corresponding to the correct position in the row if the zero elements had been stored. Additions and multiplications can then be carried out efficiently manipulating with only non-zero elements. The true array position of the results are indicated by the integer vector of element positions.

5.2 Storing the A matrix

For a large number of observations, a few thousand of which is possible after only a few days of observations, the storage of even the compressed A matrix would be prohibitively expensive. Thus the A matrix is not stored in immediate access computer store. At first, in the author's routines each time that a row of the A matrix was required the row was again computed. This was found to be expensive in time, as the routines used to evaluate

i^{th} observation, row of A matrix

$[a_{i1}, a_{i2}, a_{i3}, 0, \ldots 0, a_{i7}, a_{i8}, a_{i9}, 0,$

$0, \ldots 0, a_{i17}, a_{i18}, 0, \ldots 0, a_{i29}, a_{i30}, 0, 0]$

Compressed row of A matrix

integer	integer vector
10	[1, 2, 3, 7, 8, 9, 17, 18, 29, 30]

Compressed row

$[a_{i1}, a_{i2}, a_{i3}, a_{i7}, a_{i8}, a_{i9}, a_{i17}, a_{i18}, a_{i29}, a_{i30}]$

FIGURE 5.1. The Compressed A Matrix.

the partial derivatives and computed observations consume large amounts of time.

The final procedure adopted was to store the partial derivatives in the compressed form with the integer vector of positions, the computed observations, and other necessary logistic information on a sequential disc file. An iteration, of course, requires complete re-evaluation of the A matrix, but comments on this are given in section 5.3. The residuals however, are computed very efficiently using the formula (3.34):

$$\hat{\underline{v}} = \underline{A}_1 \, \hat{\underline{\delta}} + \underline{w}_1$$

5.3 Iteration Requirement for a Solution

The first mathematical model (equations 2.3, 2.6) is nonlinear, and iterations of the computation with updated parameter values should be required until the absolute values of the increments approach zero. The author found, however, that a second iteration was never required. (For the definition of the i^{th} iteration it is considered that the first approximation ($\underline{x}^o$) on being updated by the first set of increments consistutes the first iteration.) The sensitivity of the model to à priori station coordinates and clock polynomial coefficients is low. The source positions are usually well known so often one iteration will suffice to give good results.

It is thus suggested that C.P.U. time can be economically decreased by using only one iteration. The ability for any defined number of iterations, or until the increments approach zero, is available in the author's routines.

5.4 Station Positions Used as Parameters Instead of Baseline Components

The maximum likelihood routines use the baseline components as parameters in the adjustment. The least squares adjustment uses the station coordinates, with one station fixed in space. This reduces the number of parameters, allowing savings in computer space and time, since for any number of baselines there is always an equal or lower number of adjustable stations. For example, with five stations, one is fixed giving four adjustable station sets of parameters. Using baselines, five stations would imply ten baseline sets of parameters. A set in each case would be the three-dimensional (X,Y,Z) coordinates.

The results are the same from either parameter definition used in the adjustment. L.B.I. can only detect coordinate differences, which are in effect the baseline components, so the least squares routines print out the differences in station coordinates for all combinations of baselines.

The covariance matrices of all baselines are evaluated applying the covariance law to the parameter covariance matrix ($\underline{C}_{\hat{\delta}}$, equation 3.44).

The parameter covariance matrix can be considered as composed of sub-matrices corresponding to parameter types, and their covariance sub-matrices.

$$\underline{C}_{\hat{\delta}} = \begin{bmatrix} \underline{C}_{s} & \underline{C}_{s,q} & \underline{C}_{s,c} \\ \underline{C}_{q,s} & \underline{C}_{q} & \underline{C}_{q,c} \\ \underline{C}_{c,s} & \underline{C}_{c,q} & \underline{C}_{c} \end{bmatrix} \qquad (5.1)$$

The parameter subsets:

s ... station coordinates

q ... source directions

c ... clock polynomial coefficients.

Baseline components can be deduced as a function of the station coordinates.

$$\underline{b} = \underline{F}(\underline{s}) \qquad (5.2)$$

where $\underline{b}$ ... vector of baseline components

$\underline{s}$ ... vector of station coordinates.

The covariance law (equation (3.36) is applied as in section 3.2 to give the covariance matrix of the baseline components.

5.5 Weighted Parameters and Functional Parameter Constraints

In section 2.4 are derived the two classes of constraint which may be implied in an L.B.I. adjustment: functional parameter constraints, and weighted parameter constraints. Mikhail [1976] gives the standard method of rigorously imposing the former class, using the notation of Chapter 3:

$$\underline{\hat{\delta}} = \underline{\delta}^1 - \underline{N}_1^{-1} [\underline{A}_2^t (\underline{A}_2 \; \underline{N}_1^{-1} \; \underline{A}_2)^{-1} (\underline{w}_2 + \underline{A}_2 \; \underline{\delta}^1)] \tag{5.3}$$

where $\underline{N}_1 = \underline{A}_1^t \; \underline{P}_\ell \; \underline{A}_1$

$\underline{\delta}^1 = -\underline{N}_1^{-1} \; \underline{A}_1^t \; \underline{M}_1 \; \underline{w}_1$.

The standard method of imposing weighted parameter constraints is to use an observation:

$$\underline{x} = \underline{\ell}_x \tag{5.4}$$

A weight reflects the amount of confidence in these parameter observations.

The least squares routines impose the functional parameter constraints in a method similar to the weighted parameters, but with a high weight reflecting the fact that these constraints are known to be true. The observation is of the form

$$\underline{F}(\underline{x}) = \underline{\ell}_x \tag{5.5}$$

since these constraints involve more than one parameter. Both classes of constraint can thus be included in the second model (equation 3.2) in the least squares adjustment.

The main reason for applying the constraints in the above manner is computer economy. Equation 5.3 is relatively uneconomic in the adjustment. Another reason is that the normal equation matrix ($\underline{N}_1$ in equation 5.3) is inverted without the constraints. It is possible that the normal equation matrix is ill-conditioned without imposing the functional parameter constraints. When observing to a single source for a long period the clock polynomial coefficients become highly correlated with the other parameters. This is a consequence of the information content of the observables [Shapiro, 1978]. The functional parameter constraints may reduce these correlations.

5.6 The "Googe Number" as an Indicator of Singularity

The normal equation matrix used in L.B.I. can be ill-conditioned. The various parameters have different scales, in that unit changes in different parameters will not cause similar changes in the variation function [Adby and Dempster, 1974]. Computer round-off errors may then affect the result. There may also be high correlations between parameters as the observing programme may have been designed for astrophysics, which involves observations to a single source for long periods of time. This can cause high correlations between parameters [Shapiro, 1978]. The problem of an ill-conditioned normal equation matrix may not be readily apparent, and computer round-off may even produce apparently good results.

The author has not completely resolved this problem, having experimented with scaling the matrix, and calculating the determinant, but an economical answer, giving directly the poorly determined parameter is the method of the Googe number [Schwarz, 1978]. This facility has been incorporated into the inversion routine.

The Googe number for each parameter expresses the dependence of that parameter with respect to the sub-space defined by the previously determined parameters. It is calculated by dividing the respective diagonal element of the normal matrix into the corresponding diagonal term of the Cholesky decomposed upper triangular matrix before this latter number has been square-rooted.

In the Cholesky inversion the normal matrix is decomposed into the upper triangular matrix $\underline{U}$, where

$$\underline{U}^t \; \underline{U} = \underline{N} \qquad (5.6)$$

The Googe number for the i^{th} parameter is defined as

$$g_i = \frac{u_{ii}^2}{n_{ii}} \qquad (5.7)$$

To appreciate the geometric evaluation of the Googe number of the i^{th} parameter one considers the first design (A) matrix,

$$\underline{A} = [\underline{A}_{i-1} \; \underline{a}_i \; \underline{A}_{u-i}]$$

where u = total number of parameters.

Since the sub-space corresponding to the u-i parameters beyond the i^{th} are not involved, the normal matrix can be expressed, with convenient disregard of the weights:

$$\underline{N} = \begin{bmatrix} \underline{A}^t_{i-1} \underline{A}_{i-1} & \underline{A}^t_{i-1} \underline{a}_i & \underline{A}^t_{i-1} \underline{A}_{u-i} \\ \underline{a}^t_i \underline{A}_{i-1} & \underline{a}^t_i \underline{a}_i & \underline{a}^t_i \underline{A}_{u-i} \\ \underline{A}_{u-i} \underline{A}_{i-1} & \underline{A}_{u-i} \underline{a}_i & \underline{A}_{u-i} \underline{A}_{u-i} \end{bmatrix} \qquad (5.9)$$

In the process of the Cholesky decomposition up to the i^{th} column

$$\underline{U}_i = \begin{bmatrix} \underline{U}_{i-1} & (\underline{U}^t_{i-1})^{-1} \underline{A}_{i-1} \underline{a}_i \\ \underline{0} & u_{ii} \end{bmatrix} \qquad (5.10)$$

where

$$u_{ii}^2 = \underline{a}_i^t \underline{a}_i - \underline{a}_i^t \underline{A}_{i-1} \underline{U}_{i-1}^{-1} (\underline{U}_{i-1}^t)^{-1} \underline{A}_{i-1}^t \underline{a}_i \tag{5.11}$$

since

$$\underline{N}_{i-1}^{-1} = (\underline{U}_{i-1}^t \underline{U}_{i-1})^{-1} = (\underline{A}_{i-1}^t \underline{A}_{i-1})^{-1} \tag{5.12}$$

$$\therefore u_{ii}^2 = \underline{a}_i^t [\underline{I} - \underline{A}_{i-1} (\underline{A}_{i-1}^t \underline{A}_{i-1})^{-1} \underline{A}_{i-1}^t] \underline{a}_i . \tag{5.13}$$

The matrix in the square brackets of equation (5.13) is recognised as idempotent. Where $\underline{S}_{i-1}$ equals this idempotent matrix, multiplication will show

$$\underline{S}_{i-1}^2 = \underline{S}_{i-1} . \tag{5.14}$$

$\underline{S}_{i-1}$ is thus a projection operator. Some projection operators annihilate spaces [Jacobson, 1953]. Multiplication of

$$\underline{S}_{i-1} \underline{A}_{i-1} = 0 \tag{5.15}$$

shows that this projection operator annihilates, at least, the i-1 sub-space. Multiplication of any vector, for example $\underline{a}_i$, by $\underline{S}_{i-1}$, would result in the component of $\underline{a}_i$ which is the orthogonal complement to the i-1 sub-space. Thus

$$u_{ii}^2 = \underline{a}_i^t \, \underline{S}_{i-1} \, \underline{a}_i \qquad (5.16)$$

$$u_{ii}^2 = \underline{a}_i^t \, \underline{S}_{i-1}^t \, \underline{S}_{i-1} \, \underline{a}_i \qquad (5.17)$$

since $\underline{S}_{i-1}$ is symmetric.

$$\therefore \; u_{ii}^2 = (\underline{S}_{i-1} \, \underline{a}_i)^t (\underline{S}_{i-1} \, \underline{a}_i). \qquad (5.18)$$

Equation (5.18) is recognised as the dot product of the vector component of $\underline{a}_i$ which is orthogonal to the i-1 sub-space. The square of the complete length of the $\underline{a}_i$ vector is given by

$$n_{ii} = \underline{a}_i^t \, \underline{a}_i. \qquad (5.19)$$

The Googe number can thus be interpreted as the square of the sine of the angle of the i^{th} parameter vector with the i-1 parameter sub-space.

The Googe number should ideally equal one. The i^{th} parameter vector is then orthogonal to the i-1 sub-space. If equal to zero, then the i^{th} parameter vector is dependent on some previously determined parameters. The author's routines compare each Googe number to a tolerance value, and prints a warning if the parameter is ill-determined. Schwarz [1978] uses a comparison with

0.1×10^{-5}, but the author found a value of 0.1×10^{-3} was required to detect an ill-conditioned L.B.I. adjustment.

CHAPTER 6

RESULTS

The objectives of this thesis have been achieved, and an economical least squares adjustment of L.B.I. observations, with statistical evaluation of the results, has been developed. This chapter gives results of computations involving a full data set, and a 180 observation sub-set of that set. The 180 observation sub-set had been selected from the full set by Langley [1979] previous to being supplied to the author, and all observations with large residuals had been deleted. The full data set was reduced by the author, and observations with residuals greater than three times the standard error have been rejected to leave 4,300 from the original 5,700 observations.

Results are given in tabular form. The 180 data sub-set results from both the maximum likelihood adjustment and the least squares adjustment are shown in each table. This shows that the same results are produced, but more efficiently, by the least squares adjustment. Each table also gives the results of using the 4,300 data set, showing the increased accuracy of results obtained economically. Table 6.1 gives the comparison of computer space and C.P.U. time from the three adjustments. Table 6.2 shows the corresponding baseline results, Table 6.3 compares the source position results, and Table 6.4 gives the clock polynomial coefficients.

The parameters used in these adjustments are the same as used and described by Langley [1979]. The three antennae are at Algonquin Park (AR) Ontario, Owen's Valley (OV) California, and Chilbolton (CH) England. The baselines can thus be described by the initials AROV, ARCH, and OVCH. The sources are listed in Table 6.3, except 3C 273B which was held fixed. The fringe frequency clock polynomials were two first order on AROV, one second and one first order on ARCH, and one first order on OVCH. The delay polynomials were the same in number and order as the fringe frequency polynomials. For the reasons described in 2.4 independent coefficients were used for delay and fringe frequency.

The standard errors as shown for the 4300 observation set are not correct. The author assumed that the standard errors of observations were correct, while they should have been scaled by the variance factor. Too many outlying observations were rejected thus giving standard errors of parameters which were too optimistic. The differences in results shown between the 180 and 4300 observation sets do, however, agree at the two sigma level.

Table 6.1

Comparison of Computer Space and C.P.U. Time

Adjustment Number of Observations	Maximum Likelihood 180	Least Squares 180	Least Squares 4,300
Compiler	Fortran G	Fortran H	Fortran H
Link region	652 K	652 K	652 K
Link C.P.U. time	2.66 seconds	2.65 seconds	2.64 seconds
Go region	464 K	180 K	196 K
Go C.P.U. time	298.16 seconds	14.26 seconds	332.01 seconds

Computer: IBM 370/3032 with the VS2 operating system using almost completely double precision.

Table 6.2

Baseline Component Comparison

Adjustment Number of Observations	Maximum Likelihood 180	Least Squares 180	Least Squares 4,300
Baselines			
AROV			
X	-3 327 634.75 ± 0.36 m	-3 327 634.73 ± 0.33 m	-3 327 635.20 ± 0.07 m
Y	- 132 217.45 ± 0.30 m	- 132 217.45 ± 0.31 m	- 132 217.77 ± 0.06 m
Z	- 723 369.12 ± 3.96 m	- 723 369.13 ± 3.67 m	- 723 369.09 ± 0.79 m
ARCH			
X	3 090 274.89 ± 0.41 m	3 090 274.89 ± 0.40 m	3 090 275.36 ± 0.08 m
Y	4 245 482.03 ± 0.40 m	4 245 482.03 ± 0.41 m	4 245 482.17 ± 0.09 m
Z	381 826.39 ± 4.22 m	381 826.37 ± 3.85 m	381 821.70 ± 0.78 m
OVCH			
X	6 417 909.63 ± 0.89 m	6 417 909.62 ± 0.51 m	6 417 910.56 ± 0.11 m
Y	4 377 699.44 ± 0.78 m	4 377 699.49 ± 0.66 m	4 377 699.94 ± 0.14 m
Z	1 105 195.49 ± 5.20 m	1 105 195.50 ± 4.19 m	1 105 190.79 ± 0.87 m

Table 6.3

Source Position Comparison

Adjustment Number of Observations	Maximum Likelihood 180	Least Squares 180	Least Squares 4,300
Source			
0J 287			
Right ascension	$08^h\ 51^m\ 57^s.2517 \pm 0^s.0013$	$08^h\ 51^m\ 57^s.2517 \pm 0^s.0014$	$08^h\ 51^m\ 57^s.2529 \pm 0^s.0003$
Declination	$20°\ 17'\ 58''.372 \pm 0''.037$	$20°\ 17'\ 58''.373 \pm 0''.035$	$20°\ 17'\ 58''.399 \pm 0''.008$
4C 39.25			
Right ascension	$09^h\ 23^m\ 55^s.3215 \pm 0^s.0018$	$09^h\ 23^m\ 55^s.3215 \pm 0^s.0018$	$09^h\ 23^m\ 55^s.3205 \pm 0^s.0004$
Declination	$39°\ 15'\ 23''.603 \pm 0''.024$	$39°\ 15'\ 23''.603 \pm 0''.024$	$39°\ 15'\ 23''.605 \pm 0''.005$
3C 345			
Right ascension	$16^h\ 41^m\ 17^s.6100 \pm 0^s.0012$	$16^h\ 41^m\ 17^s.6100 \pm 0^s.0013$	$16^h\ 41^m\ 17^s.6104 \pm 0^s.0003$
Declination	$39°\ 54'\ 10''.803 \pm 0''.016$	$39°\ 54'\ 10''.804 \pm 0''.016$	$39°\ 54'\ 10''.823 \pm 0''.003$
BLLAC			
Right ascension	$22^h\ 00^m\ 39^s.3636 \pm 0^s.0014$	$22^h\ 00^m\ 39^s.3636 \pm 0^s.0014$	$22^h\ 00^m\ 39^s.3636 \pm 0^s.0003$
Declination	$42°\ 02'\ 08''.562 \pm 0''.014$	$42°\ 02'\ 08''.562 \pm 0''.014$	$42°\ 02'\ 08''.519 \pm 0''.003$

Table 6.4

Clock Polynomial Coefficient Comparison

Adjustment Number of Observations	Maximum Likelihood 180	Least Squares 180	Least Squares 4,300
Parameter No.			
AROV $a_2(F)$	$-(0.62 \pm 0.03) \times 10^{-8}$	$-(0.61 \pm 0.03) \times 10^{-8}$	$-(0.651 \pm 0.005) \times 10^{-8}$
$a_2(F\ 2nd)$	$-(0.50 \pm 0.02) \times 10^{-8}$	$-(0.50 \pm 0.02) \times 10^{-8}$	$-(0.521 \pm 0.003) \times 10^{-8}$
ARCH $a_2(F)$	$-(0.38 \pm 0.05) \times 10^{-8}$	$-(0.38 \pm 0.05) \times 10^{-8}$	$-(0.445 \pm 0.009) \times 10^{-8}$
$a_3(F)$	$-(0.44 \pm 0.07) \times 10^{-10}$	$-(0.44 \pm 0.08) \times 10^{-10}$	$-(0.247 \pm 0.014) \times 10^{-10}$
$a_2(F\ 2nd)$	$-(0.51 \pm 0.02) \times 10^{-8}$	$-(0.51 \pm 0.02) \times 10^{-8}$	$-(0.499 \pm 0.004) \times 10^{-8}$
OVCH $a_2(F)$	$-(0.75 \pm 0.61) \times 10^{-9}$	$-(0.76 \pm 0.33) \times 10^{-9}$	$-(0.166 \pm 0.061) \times 10^{-9}$
AROV $a_1(\tau)$	$(0.87 \pm 0.03) \times 10^{-6}$	$(0.87 \pm 0.03) \times 10^{-6}$	$(0.863 \pm 0.005) \times 10^{-6}$
$a_2(\tau)$	$-(0.58 \pm 0.09) \times 10^{-8}$	$-(0.58 \pm 0.09) \times 10^{-8}$	$-(0.573 \pm 0.018) \times 10^{-8}$
$a_1(\tau\ 2nd)$	$(0.63 \pm 0.01) \times 10^{-6}$	$(0.63 \pm 0.01) \times 10^{-6}$	$(0.628 \pm 0.002) \times 10^{-6}$
$a_2(\tau\ 2nd)$	$-(0.54 \pm 0.02) \times 10^{-8}$	$-(0.54 \pm 0.02) \times 10^{-8}$	$-(0.552 \pm 0.004) \times 10^{-8}$
ARCH $a_1(\tau)$	$-(0.44 \pm 0.05) \times 10^{-6}$	$-(0.44 \pm 0.05) \times 10^{-6}$	$-(0.342 \pm 0.010) \times 10^{-6}$

Table 6.4 - Continued

Adjustment Number of Observations	Maximum Likelihood 180	Least Squares 180	Least Squares 4,300
Parameter No.			
$a_2(\tau)$	$-(0.82 \pm 4.05) \times 10^{-9}$	$-(0.83 \pm 3.95) \times 10^{-9}$	$-(0.906 \pm 0.079) \times 10^{-8}$
$a_3(\tau)$	$-(0.12 \pm 0.08) \times 10^{-9}$	$-(0.12 \pm 0.08) \times 10^{-9}$	$(0.558 \pm 0.152) \times 10^{-10}$
ARCH $a_1(\tau \text{ 2nd})$	$-(0.65 \pm 0.01) \times 10^{-6}$	$-(0.65 \pm 0.01) \times 10^{-6}$	$-(0.644 \pm 0.002) \times 10^{-6}$
$a_2(\tau \text{ 2nd})$	$-(0.51 \pm 0.02) \times 10^{-8}$	$-(0.51 \pm 0.02) \times 10^{-8}$	$-(0.545 \pm 0.004) \times 10^{-8}$
OVCH $a_1(\tau)$	$-(1.18 \pm 0.01) \times 10^{-6}$	$-(1.18 \pm 0.01) \times 10^{-6}$	$-(1.178 \pm 0.002) \times 10^{-6}$
$a_2(\tau)$	$-(0.75 \pm 0.27) \times 10^{-9}$	$-(0.75 \pm 0.26) \times 10^{-9}$	$-(0.772 \pm 0.050) \times 10^{-9}$

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

An efficient least squares adjustment package has been produced and it is recommended that future analysis involves these routines. There are undoubtably changes that can be made to improve the routines and to suit specific customer requirements.

7.1 Analyse Full Sets of Observations

It is suggested that previous data sets in which only a small proportion of available observations were processed should be re-analysed. The author has carried out some experimentation with the full data sets and has found that the standard errors of results has decreased in proportion to the increase in number of observations. Lack of time has limited these experiments, but initial results do give cause for optimism with respect to accuracies which can be obtained using the full data sets.

7.2 Consistency in Accuracy Throughout the Model

Improvements in accuracy from the adjustment may cause some parts of the L.B.I. models to be deficient in attaining these accuracies. Langley [1979] reports the model to be accurate to the order of 10 centimetres. Polar motion is currently given a single set of values for a three or four day observation period. It is suggested that full sets of observations may cause such model parameter errors to be above the error level of the least squares adjustment. A thorough analysis of the model is thus required to ensure consistency in error level in the L.B.I. model.

7.3 Interactive Process Mode

The author's routines were processed in batch mode, but it might be more efficient to use a video display unit (V.D.U.), and possibly a fully interactive computation and storage process. While experimenting with data sets of 5,000 observations the author found the task of inspecting the residual plot and deleting outlying observations time consuming and prone to errors. To delete an observation cards had to be punched, and the reduced data set stored on disc. Problems were also found with the paper plot residual scale. Use of a V.D.U. should thus be able to improve efficiency in analysing data.

7.4 Data Storage on Direct Access File

The present use of a sequential file to store the data is a main cause for inefficiency in deleting outlying observations. A direct access file would allow an outlying observation to be marked while the residual is being computed, or, in an interactive process, on deletion from the V.D.U. screen inspection. Possibly a value over-printed in a particular column would show deletion. This could be involved with an ability of subtracting the effect of that observation from the current adjustment. Subsequent adjustments would check the deletion column of the observation storage line. An original copy of the unedited data would, of course, be stored, probably on a tape.

7.5 Permanent Storage of the A Matrix

Permanent storage of the first design (A) matrix coefficients, with observations and logistic information, could be combined in an interactive process. The author's routines, having compressed the A matrix, could be adapted, and lead to even greater C.P.U. time efficiency. In an L.B.I. analysis many of the computer runs vary only in their use of different orders of clock polynomials. Time could be greatly reduced since this implies that exactly the same computations are carried out in each run to form most of the coefficients of the A matrix.

7.6 Comparison of Doppler Satellite and L.B.I. Coordinate Systems

Langley [1979] compared L.B.I. results with those of Doppler satellite and was able to deduce scale and orientation differences between coordinate systems as defined by the Bureau International de l'Heure (B.I.H.) and the United States Navy Navigation System. This comparison should be repeated with a re-evaluation of the L.B.I. observations using the least squares adjustment routines and the full set of observations.

7.7 L.B.I. Observing Programme for Geodetic Results

Observations used for L.B.I. using the Canadian observation system have been designed for the needs of astrophysics. This involves continued observations to a single source for many hours. This situation is not ideal for geodetic use of the observations, causing high correlations between parameters. Observations to sources in various positions on the celestial sphere for short periods provides better resolution for geodetic results. This is mainly a financial problem, but it is suggested that the full advantages of the whole L.B.I. system for geodesy can only be realised from a specifically geodetic observing programme.

7.8 Spectral Analysis of Residuals

Initial computations of full data sets shows the plots of residuals to display sinusoidal tendencies. It is suggested that a spectral analysis of the residuals might lead to an improvement of the L.B.I. model.

Atmospheric and oscillator effects in particular have possibilities for model improvements.

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APPENDIX 1

JOB CONTROL FOR I.B.M. 370/3032 AT U.N.B.

An example of the J.C.L. to run the least squares routines at U.N.B. The observations of delay and fringe frequency are stored on disc.

```
//LBIJOB      ,     , S=335,R=704,L=13,LC=0,ANAME
/*SETUP DISK=SEGEOM
//  EXEC FORTXCLG,RG=320K,RL=704K,
//  PARM.LKED='LIST,MAP,LET,SIZE=(650K,128K)'
//FORT.SYSIN DD *
                      MAIN ROUTINE
                     AND BLOCK DATA
/*
//LKED.SYSLIB DD DSN=DAVIDSON.LBI.X,DISP=SHR
//             DD DSN=DAVIDSON.ETIDE.X,DISP=SHR
//             DD DSN=SYS1.FORTXLIB,DISP=SHR
//             DD DSN=UNB1.FORTLIB,DISP=SHR
//             DD DSN=UNB1.IMSL.LOAD&LIB,DISP=SHR
//LKED.SYSUT1 DD SPACE=(TRK,(100,10))
//GO.FT06F001 DD SYSOUT=S
//GO.FT11F001 DD DSN=&&TEMPA,DISP=(NEW,PASS),UNIT=SYSDA,
//   SPACE=(TRK,(50,10)),DCB=(RECFM=VBS,LRECL=140,
                      BLKSIZE=7004)
```

```
//GO.SYSIN DD *
                         CONTROL DATA
/*
//GO.FT05F002 DD *
                        EARTH TIDE DATA
//GO.FT09F001 DD DSN=DAVIDSON.MAY77.REJ3P0,DISP=SHR
//
```

APPENDIX 2

INPUT DATA

Control data cards

The data cards and the variable names are described in the position order of the input pack.

1. Variables: MD, NPARAM, NUSED, NFIXED, NVARBL, NPLNS, MXEPOC, NCONS, INTS, NUPDT, NCDIM

   Format: (26I3)

   Definitions:

MD	Model number. Used to reference the program run.
NPARAM	Maximum parameter reference number. Parameter reference numbers are allocated according to parameter type. 1 - 30 station coordinates 31 - 50 source coordinates 51 - NPARAM clock polynomial coefficients.
NUSED	Total number of fixed and variable parameters used in the adjustment.
NFIXED	Number of fixed parameters.
NVARBL	Number of variable parameters.
NPLNS	Number of clock polynomials (Note: NPARAM = 50 + (NPLNS*5).

MXEPOC Maximum number of epochs in any baseline. An epoch is the start point of a clock polynomial.

NCONS Number of constraints

INTS Number of class intervals in histogram of residuals.

NUPDT Number of updated parameters.

NCDIM Dimension value for arrays used in connection with constraints.

Read in MAIN routine

Example card

0011000360050310100030000200060 20

2. Variables: ((IPARAM(K), ISTAT(IPARAM(K)), K=1, NUSED)

Format: (13(I4,I2))

Definitions:

IPARAM Vector of all used parameters. Values stored are the parameter reference numbers.

ISTAT Vector of status numbers for each parameter stored in reference value element, e.g.,

1^{st} station coordinate status number in ISTAT(1)

1^{st} source right ascension status number in ISTAT(31)

Status number 1 implies fixed parameter.

Status number 3 implies variable parameter.

Read in RDWRT

Example card

0001010002010003010004030005 03 . . . .

004301004401004503004603 . . . .

007703008103008203 009703

3. Variables: MTYPE, ITIDE, ISIGMA, IMAX, ISAME, ICORR

Format: (3(I1,1X),I2,1X,I1,1X,I1)

Definitions:

MTYPE — Model type, signifies types of observations used in adjustment.

- 3 .. fringe frequency and delay observations.
- 2 .. delay observations.
- 1 .. fringe frequency observations.

ITIDE — Signifies use of earth tide corrections.

- 0 .. no earth tide corrections.
- 1 .. earth tide corrections are applied.

ISIGMA — Indicates whether standard errors are used with observations.

- 0 .. no standard errors applied.
- 1 .. standard errors applied.

IMAX — Number of iterations (update of à priori parameters constitutes first iteration).

- 0 .. any number, until increments approach zero.
- 1 .. one iteration only
- n .. n iterations, or until increments approach zero.

ISAME Indicates whether delay clock polynomial coefficients are to be equal to their respective fringe frequency coefficients.

0 .. coefficients are equal.

1 .. coefficients are not equal.

ICORR Corrects time of observation by +1 second.

0 .. corrects by +1 second.

n .. any other number does not correct time.

Read in RDWRT

Example card

3 1 1 1 1

4. Variables: (XTRASM(K,1) K=1,8)

(XTRASM(K,2) K=1,8)

Format: 8 F 10.5

This set of cards apply extra standard errors, according to baseline (K) and observation type (1 or 2), which have been estimated using the variance factor. If ISIGMA equals 0, then these cards are omitted. Fringe frequency increases are given on the first card; delay increases are given on the second card. If only delay observations are used, then only the delay increases card is used.

Definition:

XTRASM Array of increases to standard errors of observations.

Read in RDWRT

Example card

0.0018 0.0016 0.0025

0.01 0.01 0.01

5. Variable: SESION

Format: (2A8)

Definition:

SESION Observation session name, using up to 16 letters.

Read in RDWRT

Example card

MAY 1977

6. Variables: OBSFRQ, JDJAN0

Format: (F10.5,T15,I10)

Definitions:

OBSFRQ Observing frequency (MHz)

JDJAN0 Julian Day January 0 at beginning of the year of the observations.

Read in RDWRT

Example card

10680.0 2443144

7. Variable: TOBS1

Format: F15.5

Definition:

TOBS1 Day of year immediately prior to all observations. Used as epoch day for the first clock polynomials of each baseline, and to initialise the earth tide routines.

Read in RDWRT

Example card

133.0

8. Variables: XPOLE, YPOLE, OMEGA, UTPOLY(K), K=1,3)

Format: (3D20.5/3D20.5)

Symbol / denotes card skip.

Definitions:

XPOLE X coordinate of polar motion, in seconds of arc.

YPOLE Y coordinate of polar motion, in seconds of arc.

OMEGA Rotation rate of earth, in radians per U.T. second.

UTPOLY UT1-UTC polynomial coefficients.

These values are taken from external information, e.g., B.I.H. [Langley, 1979].

Read in RDWRT

Example card

```
-0.139               0.482               7.292114897D-05
        2.330175D-04        -1.557534D-06        2.738229D-
```

9. Constraint cards. If NCONS equal 0, then none of these cards are used as input.

A. Variables: (NCONP(I), I=1, NCONS)

Format: (26I3)

Definition:

NCONP   Vector of number of constrained parameters in each constraint.

Read in RDWRT

Example card

3 3 3 1 1

B. Variables: ESTCON(I), SGMCON(I), (ICONS(I,J), J=1, NCONP(I))

Format: (D25.16,D10.3,9I5)

One card for each constraint equation.

Definitions:

ESTCON   Estimation of constraint.

SGMCON   Standard error of constraint.

ICONS   Each row gives the defined parameter numbers, and signs, used in a constraint.

Read in WDWRT

Example cards

```
0.0                         0.1D-8   52  -62   72
-0.24096185          D+0.4   0.5D-4    4
```

10. A priori station coordinates.

Input in the form of ellipsoidal coordinates on a local geodetic datum. The first card gives the number of stations, followed by three cards for each station.

A. Variable: NSTNS

Format: (I1)

Definition:

NSTNS Number of stations

Read in STNGEO

Example card

3

B. Variables: STNAM(I), RSURF1(I), RSURF2(I), DELTX(I), DELTY(I), DELTZ(I)

Format: (1X,2A8,3X,4A8,3(2X,F7.2))

Definitions:

STNAM	Vector (COMPLEX*16) of station names.
RSURF1, RSURF2	Vectors (COMPLEX*16) which, when together, give the geodetic reference surface of each station.

DELTX	X(m)	geocentric coordinates of origin of geodetic reference surface.
DELTY	Y(m)	
DELTZ	Z(m)	

Read in STNGEO

Example card

ARO 46M CLARKE ELLIPSOID OF 1866 N.A.D.

-27.0 +160.0 +180.0

C. Variables: EQTRAD(I), FLAT(I), SGN, IDLAT(I), IMLAT(I), RSLAT(I), IDLONG(I), IMLONG(I), RSLAT(I), HEIGHT(I)

Format: (1X,F11.6,5X,F10.6,5X,A1,I2,1X,I2,1X,F7.4, 5X,I3,1X,I2,1X,F7.4,5X,F8.3)

Definitions:

EQTRAD	Vector of equatorial radii of ellipsoids (Km)
FLAT	Vector of inverse of ellipsoidal flattenning (1/F)
SGN	Sign of latitude (+ or -)
IDLAT	degrees
IMLAT	minutes vectors of latitude
RSLAT	seconds
IDLONG	degrees
IMLONG	minutes vectors of longitude
RSLONG	seconds
HEIGHT	Vector of heights above ellipsoid (m)

Read in STNGEO

Example card

6378.2064 294.978698 +45 57 19.812

281 55 37.055 260.42

D. Variable: OFFSET(I)

Format: F8.2

Definition:

OFFSET Vector of offsets of antennae axes (m)

Example card

0.0

11. Variable: FOFSET(L)

Format: (F10.4)

Definition:

FOFSET Vector of frequency offsets for each baseline (Hz). One card for each baseline.

Read in RDWRT

Example card

0.0

12. Clock polynomial data

A set of cards for each baseline gives the number of clock polynomials for that baseline and if the number is greater than 1, the starting time epochs for the subsequent polynomials. The epoch of the first polynomial is TOBS1. The end card of this set gives the order of each polynomial. Polynomials are arranged first into fringe frequency and delay, then into baseline number, and finally into time of epoch.

A. Variable: NCP(L)

Format: (I1)

Definition:

NCP Vector of number of clock polynomials in each baseline.

Read in RDWRT

Example card

2

B. If NCP for the baseline is 1, there is not any following epoch card.

Variable: (EPOCHS(L,J), J=2, NCP(L))

Format: (4D20.10)

Definition:

EPOCHS Zero time for J^{th} polynomial on L^{th} baseline.

Read in RDWRT

Example card

134.7291666666667D00

C. Variables: (NPOLY(L), L=1), NPLNS)

Format: (26I3)

Definition:

NPOLY Vector of the order of the polynomial for each of the polynomials.

Read in RDWRT

Example card

1 1 2 1 1 1 1 2 1 1

13. A priori source positions.

First card gives the number of sources, followed by 1 card for each source. The maximum number is 10 sources. Not all input sources need be used in the adjustment: those used are defined as such by the input card of used parameters.

A. Variable: NSORCE

Format: (I2)

Definition:

NSORCE Total number of source positions.

Read in SOURCE

Example card

10

B. Variables: SCNAME(JSORCE),

IHOURA, IMINRA, SECRA,

SGN1, IDGDEC, IMNDEC, SECDEC

Format: (1X,A7,7X,I2,1X,I2,1X,F6.3,1X,A1,I2,1X,
I2,1X,F5.2)

Definitions:

SCNAME	Vector of source names.	
IHOURA	hours	Right ascension.
IMINRA	minutes	
SECRA	seconds	
SGN1	sign (+ or -)	
IDGDEC	degrees	Declination.
IMNDEC	minutes	
SECDEC	seconds	

Read in SOURCE

Example card

```
 0235+16       02 35 52.634 +16 24  4.01
```

14. Variables: NSKIP, NOBS, (NOBSLN(K), K=1, NBASE)

Format: (8I10)

Definitions:

NSKIP Number of observations at the beginning of data file to be skipped.

NOBS	Number of observations to be used from the data set.
NOBSLN	Vector of number of observations for each baseline.

Read in RDWRT

Example card

4607 1770 1834 1003

15. Update of parameters

If NUPDT=0, the following cards are not included. An input card is used for each parameter updated.

Variables: K, X(K)

Format: (I5,5X,D25.16)

Definitions:

K	Defined parameter number e.g. K = 1 - 30 station coordinates K = 31 - 50 source coordinates
X	Vector of parameter values. Units X(1) - X(30) km X(31) - X(50) degrees

Read in RDWRT

Example card

4 -0.2409618078849968D +04

16. Statistical information

A single input card with all variables as REAL*4.

Variables: AMAX1, AMAX2, ALPHA, XFIRST, YFIRST, YINC, CRIT

Format: (2A4, F7.4, 6F10.6)

Definitions:

AMAX1	Word definiting method of rejecting outliers.
AMAX2	i.e., DICTATED .. stated rejection criteria
	MAX .. max text
	NON-MAX .. non-max test.
ALPHA	Probability of a type I error, i.e., significance level of all statistical tests.
XFIRST	Addition to TOBS1, in hours, to give start time for each residual plot.
YFIRST	Left-hand side of residual plot scale.
YINC	Increment of residual scale per printer column.
CRIT	Defined criteria for outlier rejection. This value is used if AMAX1 = DICT, otherwise CRIT is statistically computed. Outlying residuals are denoted with an asterisk, but not subtracted from the solution.

Read in RDWRT

Example card

MAX 0.01 12. -0.06 0.0012

17. Delay and fringe frequency observations

Observations are inputted after the GO.FT09 J.C.L. card in the form of card images. A card, or a line in the storage file, exists for each observation time point.

Variables: IDYOBS, UTH, UTM, SNAME, BNAME, OBDLY, SGMDLY, OBFF, SGMFF

Format: (3X,I3,2F3.0,1X,A8,1X,A4,D18.10,F6.3, D18.10,F7.4)

Definitions:

IDYOBS	Day of observation.
UTH	Hour of observation.
UTM	Minute of observation.
SNAME	Source name.
BNAME	Baseline name.
OBDLY	Observed delay value (micro-seconds).
SGMDLY	Standard error of delay (micro-seconds) if equals 0.0, denotes rejected delay observation.
OBFF	Observed fringe frequency value (Hz).
SGMFF	Standard error of fringe frequency (Hz) if equals 0.0, denotes rejected fringe frequency observation.

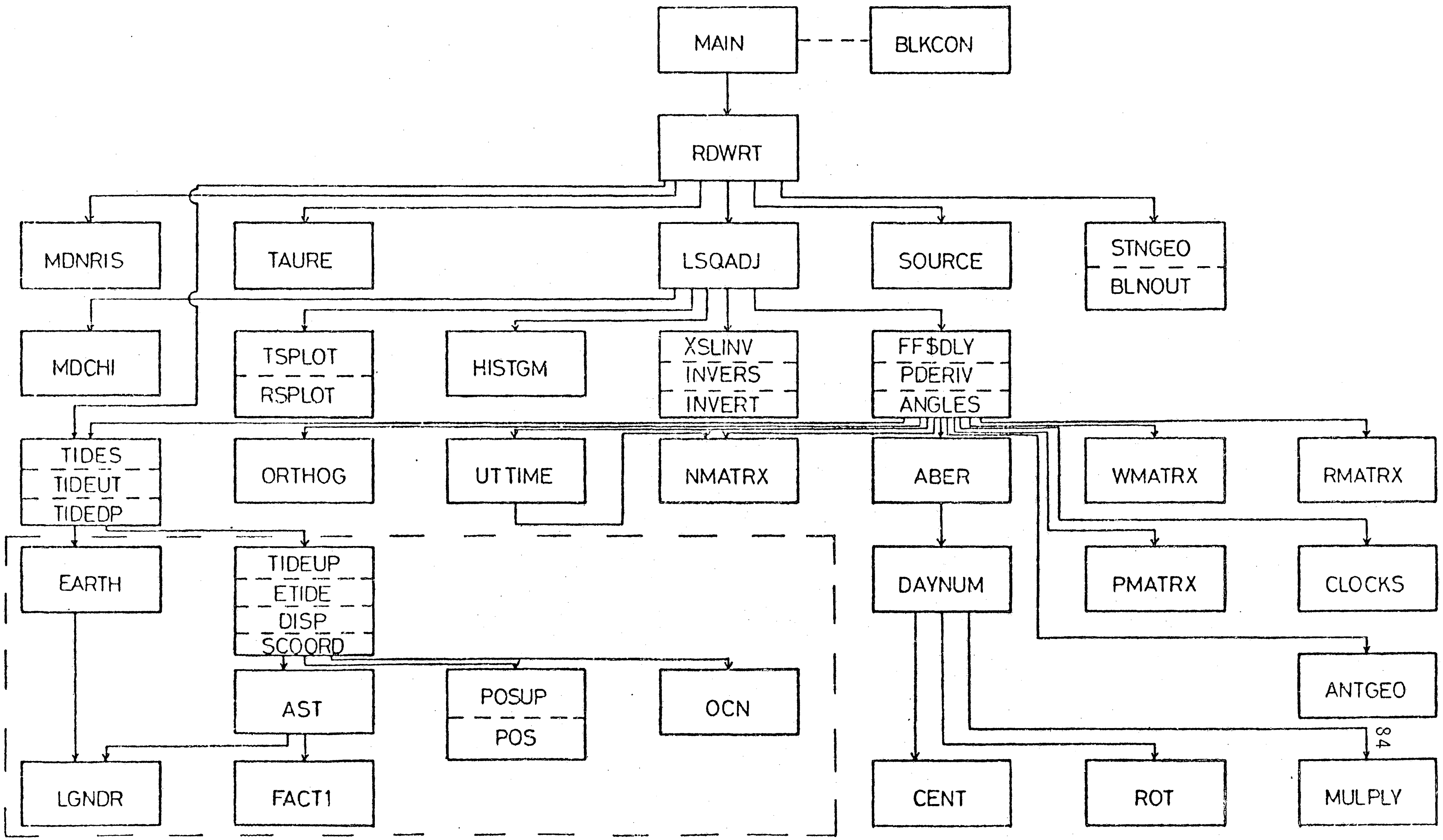
APPENDIX 3

CANADIAN L.B.I. ANALYSIS PROGRAM (MAY 1980)

A flow chart is shown of the fringe frequency and delay analysis program which uses the U.N.B. least squares adjustment. Card images are given of the routines which have been developed mainly at U.N.B. The remaining routines used in the Canadian L.B.I. analysis are the property of York University and are not produced in this appendix.

CANADIAN FRINGE FREQUENCY AND DELAY ANALYSIS PROGRAM

LEAST SQUARES ADJUSTMENT (MAY 1980)



```
C                                                                       CMAIN
C        MAIN                                                            MAIN
C                                                                       CMAIN
C CARD INPUT  TO START LEAST SQUARES ADJUSTMENT OF L.B.I. DATA.          CMAIN
C COMMON /ADJUST/ USED IN     RDWRT      LSQADJ                         CMAIN
C DIMENSIONS ARRAYS OF VARIABLE SIZE.           CALLS     RDWRT         CMAIN
C                                                                       CMAIN
C DIMENSIONS MUST MATCH INPUT CARDS:                                    CMAIN
C    X(NPARAM), CLPOLY(5,NPLNS), EPOCHS(10,MXEPOC), ICOL(NPARAM),       CMAIN
C    ISTAT(NPARAM), NPOLY(NPLNS),                                       CMAIN
C    IVARBL(   ), DELTA(   ), ATRW(   ), ANS(   ), ANORM(   ,   ), PACC(   ), CMAIN
C          ALL DIMENSIONED NVARBL                                       CMAIN
C    IPARAM(NUSED),                                                     CMAIN
C    ICONS(NCDIM,5), NCONP(NCDIM), SGMCON(NCDIM), ESTCON(NCDIM).        CMAIN
C                                                                       CMAIN
C D. A. DAVIDSON    MAY 1980                                            CMAIN
C                                                                       CMAIN
      IMPLICIT REAL*8(A-H,O-Z)                                           MAIN
      REAL*4 XFIRST,YFIRST,YINC                                          MAIN
      COMMON /ADJUST/ FOFSET(10),OBSFRQ,XTRASM(10,2),                    MAIN
     &SCNAME(10),VBNAME(10),                                             MAIN
     S XFIRST,YFIRST,YINC,                                               MAIN
     1 NOBSLN(10),NCONS,                                                 MAIN
     2 MTYPE,ISIGMA,NY,NSTNS,NSKIP,ICORR                                 MAIN
     3,SAPB(40)                                                          MAIN
      DIMENSION X(100),CLPOLY(5,10),EPOCHS(10,3),ICOL(100),ISTAT(100),   MAIN
     1 NPOLY(10),                                                        MAIN
     2 IVARBL(31),DELTA(31),ATRW(31),ANS(31),ANORM(31,31),PACC(31),      MAIN
     3 IPARAM(36),                                                       MAIN
     4 ICONS(20,5),NCONP(20),SGMCON(20),ESTCON(20)                       MAIN
      EQUIVALENCE (X(51),CLPOLY(1,1))                                    MAIN
      READ(5,5000) MD,NPARAM,NUSED,NFIXED,NVARBL,NPLNS,MXEPOC,NCONS,     MAIN
     1 INTS,NUPDT,NCDIM                                                  MAIN
 5000 FORMAT(26I3)                                                       MAIN
      CALL RDWRT(X,CLPOLY,EPOCHS,ICOL,ISTAT,NPOLY,NPARAM,NPLNS,          MAIN
     1 IVARBL,DELTA,ATRW,ANS,ANORM,PACC,                    NVARBL,      MAIN
     2 IPARAM,                                                           MAIN
     3 ICONS,NCONP,SGMCON,ESTCON,                           NCDIM,       MAIN
     5 MXEPOC,MD,NUSED,NFIXED,INTS,NUPDT                                 MAIN
      STOP                                                               MAIN
      END                                                                MAIN
      BLOCK DATA                                                         MAIN
C-----------------------------------------------------------------------CMAIN
C INITIALIZATION OF CONSTANTS IN COMMON BLOCK, BLKCON                   CMAIN
```

```
C                                                                       CMAIN
C       C         299792.5 KM/S                                         CMAIN
C       TWOPI   6.283185307179586                                       CMAIN
C       DEGRAD  0.017453292519943295                                    CMAIN
C       RADDEG  57.29577951308232                                       CMAIN
C       Q         1.00002098083496                                      CMAIN
C       RPOLE   6356.7747 KM                                            CMAIN
C       REQTOR  6378.160 KM                                             CMAIN
C       FLATNG  0.00033528919                                           CMAIN
C                                                                       CMAIN
C  R.B. LANGLEY - 1 DECEMBER 1977                                       CMAIN
C-----------------------------------------------------------------------CMAIN
      IMPLICIT REAL*8(A-H,O-Z)                                           MAIN
      COMMON /BLKCON/ C,TWOPI,DEGRAD,RADDEG,Q,RPOLE,REQTOR,FLATNG        MAIN
      DATA C,TWOPI,DEGRAD,RADDEG,Q,RPOLE,REQTOR,FLATNG/299792.5D0,       MAIN
     1 6.283185307179586D0, 1.745329251994329D-02, 57.29577951308232D0,  MAIN
     2 1.000020980834960D0, 6356.7747D0, 6378.160D0, 3.35289190D-03/     MAIN
      END                                                                MAIN

      SUBROUTINE HISTGM(MTYPE,NY,NHIST,VAL,INTS,NOB,ISIGMA,ALPHA)         HIST
C-----------------------------------------------------------------------CHIST
C                                                                       CHIST
C  HISTGM PLOTS THE HISTOGRAMS OF STANDARDIZED DELAY & FRINGE FREQUENCY CHIST
C  RESIDUALS OVERLAID WITH A NORMAL CURVE. A CHI-SQUARE GOODNESS-OF-FIT CHIST
C  TEST IS PERFORMED. IT IS BASED ON 'GODFIT' BY R.R.STEEVES IN 'GEOPAN'CHIST
C  CALLED BY         LSOADJ               CALLS      MONOR      MDCHI   CHIST
C  INPUT PARAMETERS                                                     CHIST
C  MTYPE   DENOTES OBSERVATIONS 1=FF,2=DLY, 3=FF AND DLY                CHIST
C  NY      NUMBER OF OBSERVATION TYPES IE. 1 OR 2                       CHIST
C  NHIST   TWO VECTORS OF CLASSES FOR DLY AND FF STANDARDIZED RESIDUALS. CHIST
C          MAXIMUM CLASS NUMBER = 40.                                   CHIST
C  VAL     VALUE OF CLASS WIDTH.                                        CHIST
C  INTS    NUMBER OF CLASS INTERVALS FOR HISTOGRAM. GOODNESS-OF-FIT TEST CHIST
C          WILL GROUP CLASSES AT LIMITS OF CURVE UNTIL EXPECTED NUMBER  CHIST
C          GT. 5. AND THEN USE THE HISTOGRAM INTERVALS                  CHIST
C  NOB     NUMBER OF OBSERVATIONS OF DLY AND FF.                        CHIST
C  ISIGMA  WHETHER STANDARD ERRORS APPLIED (=0 NO STD. ERR. =1 STD ERR  CHIST
C  ALPHA   PROBABILITY OF REJECTING A TRUE HYPOTHESIS                   CHIST
C  THERE ARE NO OUTPUT PARAMETERS.                                      CHIST
C  REFERENCE        D. A. DAVIDSON    M.SC.E.   THESIS U.N.B.           CHIST
C          D. A. DAVIDSON       MAY 1983                                CHIST
C                                                                       CHIST
C-----------------------------------------------------------------------CHIST
```

```
      REAL*4 ESTN(2),CHISQ(2),VAL,Y,P,ANUM,UPTO,DF,CONF,CHI
      INTEGER NOB(2),NVEC(53),ITOTI(2),ITOTO(2),MTYPE,NY,INTS
      INTEGER NFR(2,40),NHIST(2,40)
      LOGICAL*1 STRING(101),VLINES(101),SV(34)
      LOGICAL*1 VLINE/'|'/,BLANK/' '/,DOT/'.'/,HLINE/'-'/
      DATA SV/4*' ','R','E','L','A','T','I','V','E',' ','F','R','E','Q',
     &'U','E','N','C','Y',2*' '/
      DATA NVEC/22*0,4*1,2,2,3,4,4,5,6,8,9,10,12,14,16,17,19,21,23,25,27
     &,28,29,31,31,3*32,31/
C ZERO TOTALS OF EXTERNAL HISTOGRAM CLASSESS WHICH ARE GROUPED TO HAVE
C ALL EXPECTED CLASS NUMBERS GREATER THAN 5
      DO 1 J=1,NY
      ITOTI(J)=0
    1 ITOTO(J)=0
C FIND OUTSIDE CLASS TO SUM TO > 5
      I=0
    2 I=I+1
      Y=-5.0+VAL*FLOAT(I)
      CALL MDNOR(Y,P)
      ESTN(1)=P*FLOAT(NOB(1))
      IF(ESTN(1).LT.5.0) GO TO 2
      IF(NY.EQ.2) ESTN(2)=P*NOB(2)
C HISTOGRAM CLASS HEIGHT OF 'OUTER' CLASSES AND GROUPS THESE CLASSES FOR
C CHI-SQUARE G-O-F TEST. COMPUTES CHI-SQUARE STATISTIC FOR D. OF FREEDOM
      DO 3 JJ=1,I
      IOJJ=INTS-JJ+1
      DO 3 J=1,NY
      ANUM=FLOAT(NHIST(J,JJ)*80)/FLOAT(NOB(J))/VAL
      NFR(J,JJ)=ANUM+0.5
      ITOTI(J)=ITOTI(J)+NHIST(J,JJ)
      ANUM=FLOAT(NHIST(J,IOJJ)*80)/FLOAT(NOB(J))/VAL
      NFR(J,IOJJ)=ANUM+0.5
    3 ITOTO(J)=ITOTO(J)+NHIST(J,IOJJ)
      NCLASS=INTS-2*I+2
      DF=FLOAT(NCLASS-1)
      IF(ISIGMA.NE.1) DF=DF-1.0
      IF(DF.LT.1.0) GO TO 7
      CONF=1.0-ALPHA
      CALL MDCHI(CONF,DF,CHI,IER)
      DO 4 J=1,NY
      CHISQ(J)=(ITOTI(J)-ESTN(J))**2/ESTN(J)
    4 CHISQ(J)=CHISQ(J)+(ITOTO(J)-ESTN(J))**2/ESTN(J)
      GO TO 7
C HISTOGRAM CLASS HT. OF 'INNER' CLASSES. SUMS CHI-SQUARE STATISTIC.
```

```
    5 I=I+1
      IOJJ=IOJJ-1
      UPTO=P
      Y=-5.0+VAL*FLOAT(I)
      CALL MDNOR(Y,P)
      UPTO=P-UPTO
      DO 6 J=1,NY
      ESTN(J)=UPTO*FLOAT(NOB(J))
      ANUM=FLOAT(NHIST(J,I)*80)/FLOAT(NOB(J))/VAL
      NFR(J,I)=ANUM+0.5
      CHISQ(J)=CHISQ(J)+(NHIST(J,I)-ESTN(J))**2/ESTN(J)
      IF(IOJJ.EQ.I) GO TO 6
      ANUM=FLOAT(NHIST(J,IOJJ)*80)/FLOAT(NOB(J))/VAL
      NFR(J,IOJJ)=ANUM+0.5
      CHISQ(J)=CHISQ(J)+(NHIST(J,IOJJ)-ESTN(J))**2/ESTN(J)
    6 CONTINUE
    7 IF(I*2.LT.INTS) GO TO 5
C PLOTS HISTOGRAM, NORMAL CURVE, FROM TOP OF PAGE. PRINTS STATS.
      A=0.4
      B=0.3
      C=0.2
      D=0.1
      III=100/INTS-2
      IF(III.LT.1) III=1
      DO 22 J=1,NY
      MAX=50
      KK=1
      MAXIM=0
      I=51
      DO 20 JJ=1,101
      VLINES(JJ)=BLANK
   20 STRING(JJ)=BLANK
      WRITE(6,6000)
 6000 FORMAT('1')
      DO 9 JJ=1,INTS
      L=NFR(J,JJ)
      IF(L.LE.MAX) GO TO 8
      II=(JJ-1)*100/INTS+1
      VLINES(II)=VLINE
      VLINES(II+III+2)=VLINE
    8 IF(L.GT.MAXIM) MAXIM=L
      IF(MAXIM.GE.MAX) GO TO 10
      IF(MAXIM.LT.32) MAXIM=32
      L=MAX-MAXIM
```

```
      DO 9 JJ=1,L
    9 WRITE(6,6001)
 6001 FORMAT(' ')
      MAX=MAX-L
   10 WRITE(6,6002)(VLINES(JJ),JJ=1,101)
 6002 FORMAT(' ',6X,101A1)
      DO 21 JJ=1,101
   21 STRING(JJ)=BLANK
      IF(MAX.GT.32) GO TO 12
      IF(NVEC(I).NE.MAX) GO TO 12
   11 K=102-I
      STRING(I)=DOT
      STRING(K)=DOT
      I=I-1
      IF(NVEC(I+1).EQ.NVEC(I)) GO TO 11
   12 DO 14 JJ=1,INTS
      IF(NFR(J,JJ).NE.MAX) GO TO 14
      II=(JJ-1)*100/INTS+2
      IIP=II+III
      DO 13 L=II,IIP
   13 STRING(L)=HLINE
      VLINES(II-1)=VLINE
      VLINES(II+III+1)=VLINE
   14 CONTINUE
      IF(MAX.GT.32.OR.MAX.LT.8) GO TO 15
      WRITE(6,6003) SV(KK),(STRING(JJ),JJ=1,101)
 6003 FORMAT('+',1X,A1,4X,101A1)
      KK=KK+1
      IF(MAX.EQ.32) WRITE(6,6004) A
      IF(MAX.EQ.24) WRITE(6,6004) B
      IF(MAX.EQ.16) WRITE(6,6004) C
      IF(MAX.EQ.8) WRITE(6,6004) D
 6004 FORMAT('+',3X,F3.1,'-')
      GO TO 16
   15 WRITE(6,6005)(STRING(JJ),JJ=1,101)
 6005 FORMAT('+',6X,101A1)
   16 MAX=MAX-1
      IF(MAX.GT.0) GO TO 10
      WRITE(6,6002)(VLINES(JJ),JJ=1,101)
      WRITE(6,6006)
 6006 FORMAT('+',6X,20('|----'),'|'/' ',5X,'-5',8X,'-4',8X,'-3',8X,'-2',
     & 8X,'-1',9X,'0',9X,'1',9X,'2',9X,'3',9X,'4',9X,'5')
      IF(MTYPE*J/NY.GE.2) GO TO 17
      WRITE(6,6007)
```

```
6007 FORMAT('0',21X,'HISTOGRAM OF STANDARDIZED FRINGE FREQUENCY RESIDUA
     &LS',/' ',21X,52('-'))
      GO TO 18
   17 WRITE(6,6008)
6008 FORMAT('0',21X,'HISTOGRAM OF STANDARDIZED DELAY RESIDUALS',/' ',
     & 21X,41('-'))
   18 WRITE(6,6009)((JJ,NHIST(J,JJ)),JJ=1,INTS)
6009 FORMAT(2X,'NUMBERS IN INTERVALS:',(/' ',10(I2,I7,'/')))
      IF(DF.LT.1.0) GO TO 19
      IF(CHISQ(J).LE.CHI) WRITE(6,6010) CHISQ(J),CHI,NCLASS,DF
6010 FORMAT('  CHI-SQUARE STATISTIC: 0<',F7.3,'<=',F9.3,' PASSES. NUMBE
     &R OF CLASSES:',I5,',  DEGREES OF FREEDOM:',F5.1)
      IF(CHISQ(J).GT.CHI) WRITE(6,6011) CHISQ(J),CHI,NCLASS,DF
6011 FORMAT('  CHI-SQUARE STATISTIC: 0<',F7.3,'NOT <=',F9.3,' FAILS. NU
     &MBER OF CLASSES:',I5,',  DEGREES OF FREEDOM:',F5.1)
      GO TO 22
   19 WRITE(6,6012) DF
6012 FORMAT('  CHI-SQUARE GOODNESS OF FIT TEST WAS NOT PERFORMED: DEGRE
     &ES OF FREEDOM=',F6.3)
   22 CONTINUE
      RETURN
      END

      SUBROUTINE LSQADJ(X,ICOL,ISTAT,                          NPARAM,
     1 ANORM,ATRW,DELTA,PACC,ANS,IVARBL,                       NVARBL,
     2 IPARAM,                                                 NUSED,
     3 ICONS,NCONP,SGMCON,ESTCON,                              NCDIM,
     5 CLPOLY,NPOLY,           NPLNS,           EPOCHS,        MXEPOC,
     6 INTS                                                         )
C
C LSQADJ PERFORMS A LEAST SQUARES ADJUSTMENT OF L.B.I. OBSERVATIONS.
C HAS ABILITY OF CONSTRAINTS ON PARAMETERS. RESULTS, STANDARD ERRORS,
C AND STATISTICAL ANALYSIS ARE PRINTED
C CALLED BY          PDWRT
C CALLS              FF$DLY    PDERIV    XSLINV    INVERT    TSPLOT
C                    RSPLOT    MDCHI     HISTGM
C REFERENCE          D. A. DAVIDSON  M.SC.E. THESIS U.N.B.  1980
C WRITTEN BY D. A. DAVIDSON             MAY 1980
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*4 YSCALE(6),YFIRST,YINC,XFIRST,WR4(2)
      REAL*4 STD,VAL,CRIT,ALPHA,P,STAT1,STAT2
      INTEGER*2 SABB1,SABB2,SABB
```

```
      LOGICAL*1 REJCT(2)
      LOGICAL*1 STAR/'*'/,BLANK/' '/
      COMMON /ADJUST/ EOFSET(10),OBSFRQ,XTRASM(10,2),
     &SCNAME(10),VENAME(10),
     5 XFIRST,YFIRST,YINC,
     1 NOBSLN(10),NCONS,
     2 MTYPE,ISIGMA,NY,NSTNS,NSKIP,ICORR
     3,SABB(40)
      COMMON /LBIVAR/
     1RA(10),DEC(10),XBASE(10),YBASE(10),ZBASE(10),OFFSET(5),HEIGHT(5),
     2UTPOLY(3),XPOLE,YPOLE,OMEGA,TOBS1,
     3 NCP(10),JDJAN0,NBASE,NSORCE,ITIDE,ISAME,NOBS,IMAX
      COMMON /STNABB/SABB1,SABB2
      COMMON /STATS/ CRIT,ALPHA
      INTEGER NHIST(2,40),NOB(2)
      DIMENSION X(NPARAM),CLPOLY(5,NPLNS),EPOCHS(10,MXEPOC),NPOLY(NPLNS)
     1,ICOL(NPARAM),ISTAT(NPARAM),
     3 ANORM(NVARBL,NVARBL),ATRW(NVARBL),DELTA(NVARBL),PACC(NVARBL),
     4 PD(13),PF(13),NACOLD(13),NACOLF(13),IFROM(3),ITO(3),COVAR(3,3)
     5,ICUT(2),IY(2),SIGMB(3),SMUW(2),
     6 ANS(NVARBL),IVARBL(NVARBL),IPARAM(NUSED),
     7 NCONP(NCDIM),ICONS(NCDIM,5),SGMCON(NCDIM),ESTCON(NCDIM)
      EQUIVALENCE (PNAME,SABB1)
      DATA RADDEG /57.29577951308232/
      ISIGN(I)=I/IABS(I)
      ITNO=0
      VAL=10.0/FLOAT(INTS)
      DO 10 I=1,6
   10 YSCALE(I)=YFIRST+YINC*20.*(I-1)
      NAXIS=MTYPE/2+2
      LOOP=3*NSTNS
      DO 2 II=1,NVARBL
      IK=IVARBL(II)
      PACC(II)=3.370786516D-10
      IF(IK.LE.30)PACC(II)=1.D-5
      IF(IK.GT.30.AND.IK.LE.50)PACC(II)=2.8D-7
    2 CONTINUE
    3 ITCHEK=0
      ITNO=ITNO+1
      DO 5 II=1,NVARBL
      ATRW(II)=0.D0
      DO 4 JJ=1,LOOP
    4 ANORM(II,JJ)=0.D0
      DO 5 JJ=II,NVARBL
```

```
    5 ANORM(II,JJ)=0.D0
C
C INCREMENTS THE NORMAL MATRIX AND CONSTANT VECTOR FOR PARAMETER
C CONSTRAINTS
      IF(NCONS.EQ.0) GO TO 57
      DO 6 I=1,NCONS
      WCON=0.0D0
      NPARI=NCONP(I)
      SIGMSQ=SGMCON(I)**2
      DO 1 J=1,NPARI
      II=ICONS(I,J)
    1 WCON=WCON+DFLOAT(ISIGN(II))*X(IABS(II))
      WCON=WCON-ESTCON(I)
      DO 6 J=1,NPARI
      JJ=ICOL(IABS(ICONS(I,J)))
      DSGNJ=DFLOAT(ISIGN(ICONS(I,J)))
      ATRW(JJ)=ATRW(JJ)-DSGNJ*WCON/SIGMSQ
      DO 6 IK=1,J
      II=ICOL(IABS(ICONS(I,IK)))
      DSGNI=DFLOAT(ISIGN(ICONS(I,IK)))
    6 ANORM(II,JJ)=ANORM(II,JJ)+DSGNI*DSGNJ/SIGMSQ
   57 IF(NSKIP.EQ.0)GO TO 9
      DO 8 I=1,NSKIP
    8 READ(9,5001) JUNK
C
C PERFORMS A LOOP FOR EACH OBSERVATION. COMPRESSED A MATRIX COMPUTED
C AND STORED. NORMAL MATRIX AND CONSTANT VECTOR INCREMENTED.
C NB. NEGATIVE VALUE OF CONSTANT VECTOR FOR SOLUTION IN XSLINV
C
    9 DO 301 I=1,NOBS
   12 READ(9,5001) IDYOBS,UTH,UTM,SNAME,BNAME,OBDLY,SGMDLY,OBFF,SGMFF
 5001 FORMAT(3X,I3,2F3.0,1X,A8,1X,A4,D18.10,F6.3,D18.10,F7.4)
      UTC=UTH+UTM/60.D0
C      CORRECTION FOR 603US TIMING ERROR AT PLAYBACK
      UTC=UTC-603.D-6/3600.D0
      IF(ICORR.EQ.0)UTC=UTC+1.D0/3600.D0
      DO 13 J=1,NSORCE
   13 IF(SNAME.EQ.SCNAME(J))JSORCE=J
      DO 25 J=1,NBASE
   25 IF(BNAME.EQ.VBNAME(J))IBASE=J
      JBASE=IBASE*100
      DO 26 J=1,NSTNS
      IF(SABB1.EQ.SABB(8*J-7))JBASE=JBASE+J*10
   26 IF(SABB2.EQ.SABB(8*J-7))JBASE=JBASE+J
```

```
      CALL FF$DLY(X,EPOCHS,CLPOLY,FOFSET(IBASE),OBSFRQ,UTC,NPOLY,ICOL,
     1 ISTAT,JSORCE,JBASE,IDYOBS,NPARAM,MTYPE,NPLNS,NAXIS,MXEPOC,HRANGL,
     2 FFMODL,DYMODL)
      CALL PDERIV(PD,PF,NACOLD,NACOLF,IACOL,IACOLF)
      IF(MTYPE.EQ.2.OR.SGMFF.EQ.0.D0)GO TO 281
      WF=FFMODL-OBFF
      IF(ISIGMA.EQ.0)GO TO 28
      SGMFF=DSQRT(SGMFF**2+XTRASM(IBASE,1)**2)
      WF=WF/SGMFF
      DO 27 K=1,IACOLF
   27 PF(K)=PF(K)/SGMFF
   28 CONTINUE
      DO 29 II=1,IACOLF
      K=NACOLF(II)
      ATRW(K)=ATRW(K)-PF(II)*WF
      DO 29 JJ=II,IACOLF
      L=NACOLF(JJ)
      ANORM(K,L)=ANORM(K,L)+PF(II)*PF(JJ)
   29 CONTINUE
  281 IF(MTYPE.EQ.1.OR.SGMDLY.EQ.0.D0)GO TO 3011
      WD=DYMODL-OBDLY
      IF(ISIGMA.EQ.0)GO TO 31
      SGMDLY=DSQRT(SGMDLY**2+XTRASM(IBASE,NY)**2)
      WD=WD/SGMDLY
      DO 30 K=1,IACOL
   30 PD(K)=PD(K)/SGMDLY
   31 CONTINUE
      DO 32 II=1,IACOL
      K=NACOLD(II)
      ATRW(K)=ATRW(K)-PD(II)*WD
      DO 32 JJ=II,IACOL
      L=NACOLD(JJ)
      ANORM(K,L)=ANORM(K,L)+PD(II)*PD(JJ)
   32 CONTINUE
 3011 WRITE(11) IDYOBS,UTH,UTM,UTC,JSORCE,IBASE,
     1   WF,SGMFF,IACOLF,((PF(J),NACOLF(J)),J=1,IACOLF),
     2   WD,SGMDLY,IACOL,((PD(J),NACOLD(J)),J=1,IACOL)
  301 CONTINUE
      REWIND 9
      REWIND 11
      DO 45 II=1,LOOP
      IIP1=II+1
      DO 45 JJ=IIP1,NVARBL
   45 ANORM(II,JJ)=ANORM(II,JJ)+ANORM(JJ,II)
```

```
C
C FORMATION OF THE SOLUTION VECTOR (DELTA)
      NCODE=2
      CALL XSLINV(ANORM,ATRW,NVARBL,NVARBL,NCODE,ANS,DET,IDEXP,DELTA)
      WRITE(6,6006)DET,IDEXP
 6006 FORMAT(1H0,'DETERMINANT=',F15.6,'D ',I5,/,' INCREMENTS')
      WRITE(6,6002)(DELTA(II),II=1,NVARBL)
 6002 FORMAT(5D24.16)
      DO 46 II=1,NVARBL
      L=IVARBL(II)
      X(L)=X(L)+DELTA(II)
      ANS(II)=X(L)
      IF(DABS(DELTA(II)).GE.PACC(II))ITCHEK=ITCHEK+1
   46 CONTINUE
      WRITE(6,6001)ITNO
 6001 FORMAT(1H0,'ITERATION',I5,' PARAMETERS',/)
      WRITE(6,6002)(ANS(II),II=1,NVARBL)
      DO 47 J=1,NSORCE
      K=31+2*(J-1)
      RA(J)=X(K)
   47 DEC(J)=X(K+1)
      WRITE(6,6003)ITCHEK,IMAX
 6003 FORMAT(1H0,'ITCHEK',I5,'   IMAX',I5)
      IF(ITCHEK.GE.1.AND.(IMAX.EQ.0.OR.ITNO.LT.IMAX))GO TO 3
      WRITE(6,6000)
 6000 FORMAT(1H1)
      CALL INVERT(ANORM)
      WRITE(6,6017)
 6017 FORMAT(1H0,T15,'CORRELATION MATRIX          .....UPPER HALF DIAGONAL
     & ONLY SINCE SYMMETRIC'///)
      DO 77 I=1,NVARBL
   77 ATRW(I)=DSQRT(ANORM(I,I))
      DO 56 I=1,NVARBL
      DO 78 J=I,NVARBL
   78 ANS(J)=ANORM(I,J)/(ATRW(I)*ATRW(J))
   56 WRITE(6,6018) IVARBL(I),ANORM(I,I),(ANS(J),J=I,NVARBL)
 6018 FORMAT('OVARIANCE ',I5,D15.6,10X,'CORRELATIONS:',/(' ',20F6.2))
      SUMW=0.D0
      SSQR1=0.D0
      SSQR2=0.D0
      SMUW(1)=0.D0
      SMUW(2)=0.D0
      NOB(1)=0
      NOB(2)=0
```

```
      IOUT(1)=0
      IOUT(2)=0
      DO 58 I=1,NY
      DO 58 J=1,INTS
   58 NHIST(I,J)=0
C
C COMPUTATION OF RESIDUALS. LOOP PER BASELINE; INNER LOOP FOR EACH
C OBSERVATION. RESIDUALS ARE PLOTTED. HISTOGRAM CLASSES ARE INCREMENTED.
C SUM WEIGHTED RESIDUALS AND SUM UNWEIGHTED RESIDUALS ARE INCREMENTED
C
      IK=0
      DO 16 L=1,NBASE
      N1=IK+1
      IK=IK+NOBSLN(L)
      IF(MTYPE.EQ.2) GO TO 60
      WRITE(6,6011) L,VBNAME(L)
 6011 FORMAT(1H1,T50,'RESIDUALS'/T50,9('='),/T90,'*..RESIDUAL.GT.CRITERI
     1A*STD. ERROR'//3X,'TIME',T108,'SOURCE',1X,'FF(HZ)',3X,'DLY(US)'//
     P ' DAYHR.MN',T50,'BASELINE NO.',I5,2X,A4,///)
      GO TO 61
   60 WRITE(6,6027) L,VBNAME(L)
 6027 FORMAT(1H1,T50,'RESIDUALS'/T50,9('='),/T90,'*..RESIDUAL.GT.CRITERI
     1A*STD. ERROR'//3X,'TIME',T108,'SOURCE',3X,'DLY(US)'//
     P' DAYHR.MN',T50,'BASELINE NO.',I5,2X,A4,///)
   61 WRITE(6,6012)(YSCALE(I),I=1,6)
 6012 FORMAT(9X,'FRINGE FREQUENCY PLOT SCALE SMALLER BY TEN'/4X,
     1 6(1PE10.3,10X)/9X,'+',10(9('-'),'+'))
      CALL TSPLOT(YFIRST,YINC,MTYPE,NY)
      DO 15 I=N1,IK
      READ(11) IDYOBS,UTH,UTM,UTC,JSORCE,IBASE,
     1   WF,SGMFF,IACOLF,((PF(J),NACOLF(J)),J=1,IACOLF),
     2   WD,SGMDLY,IACOL,((PD(J),NACOLD(J)),J=1,IACOL)
      DO 11 JJ=1,NY
      IOUT(JJ)=0
      IY(JJ)=0
      REJCT(JJ)=BLANK
   11 WR4(JJ)=0.
C TO USE TIME INTERVAL DIFFERENT FROM 1 MINUTE USE 60./(TIME INT)+1.5
C CHANGES REQUIRED IN TSPLOT TO HAVE 2 OR MORE OBS IN AN INTERVAL
      IX=((FLOAT(IDYOBS)-TOBS1)*24.+UTC-XFIRST)*60.+1.5
      IF(IX.LT.1) IX=1
      IF(MTYPE.EQ.2.OR.SGMFF.EQ.0.D0)GO TO 39
      NOB(1)=NOB(1)+1
      DO 79 J=1,IACOLF
```

```
 79 WF=WF+PF(J)*DELTA(NACOLF(J))
    IF(ISIGMA.EQ.1) WF=WF*SGMFF
    WW=WF*WF
    WR4(1)=SNGL(WF)
    SMUW(1)=SMUW(1)+WW
    II=(WR4(1)-YFIRST)/YINC+1.5
    IF(MTYPE.NE.2) II=(WR4(1)*10.-YFIRST)/YINC+1.5
    IF(II.LT.1) II=1
    IF(II.GT.101) II=101
    IY(1)=II
    IOUT(1)=1
    IF(ISIGMA.EQ.0)GO TO 38
    STD=WR4(1)/SNGL(SGMFF)
    IF(ABS(STD).GT.CRIT) REJCT(1)=STAR
    IF(STD.LT.-5.0) STD=-5.0
    IF(STD.GE.5.0) STD=4.999
    IHIST=(5.0+STD)/VAL+1.0
    NHIST(1,IHIST)=NHIST(1,IHIST)+1
    WF=WF/(SGMFF*SGMFF)
    WW=WW/(SGMFF*SGMFF)
 38 SUMW=SUMW+WF
    SSQR1=SSQR1+WW
 39 CONTINUE
    IF(MTYPE.EQ.1.OR.SGMDLY.EQ.0.D0) GO TO 14
    NOB(NY)=NOB(NY)+1
    DO 80 J=1,IACOL
 80 WD=WD+PD(J)*DELTA(NACOLD(J))
    IF(ISIGMA.EQ.1) WD=WD*SGMDLY
    WW=WD*WD
    WR4(NY)=SNGL(WD)
    SMUW(NY)=SMUW(NY)+WW
    COV=COV+DBLE(WR4(1)*WR4(2))
    II=(WR4(NY)-YFIRST)/YINC+1.5
    IF(II.LT.1) II=1
    IF(II.GT.101) II=101
    IY(NY)=II
    IOUT(NY)=1
    IF(ISIGMA.EQ.0)GO TO 40
    STD=WR4(NY)/SNGL(SGMDLY)
    IF(ABS(STD).GT.CRIT) REJCT(NY)=STAR
    IF(STD.LT.-5.0) STD=-5.0
    IF(STD.GE.5.0) STD=4.999
    IHIST=(5.0+STD)/VAL+1.0
    NHIST(NY,IHIST)=NHIST(NY,IHIST)+1
```

```
      WD=WD/(SGMDLY*SGMDLY)
      WW=WW/(SGMDLY*SGMDLY)
   40 SUMW=SUMW+WD
      SSQR2=SSQR2+WW
   14 CONTINUE
      CALL RSPLOT(IX,IY,IOUT)
      WRITE(6,6015) IDYOBS,UTH,UTM,JSORCE,((WR4(K),REJCT(K)),K=1,NY)
 6015 FORMAT('+',I3,2F3.0,T111,I2,2(1PE9.2,A1))
   15 CONTINUE
      WRITE(6,6014)(YSCALE(I),I=1,6)
 6014 FORMAT(9X,'+',10(9('-'),'+')/5X,6(1PE10.3,10X)/9X,'FRINGE FREQUENC
     1Y PLOT SCALE SMALLER BY TEN')
   16 CONTINUE
      SSQWT=0.D0
      IF(NCONS.EQ.0) GO TO 49
      WRITE(6,6004)
 6004 FORMAT('0WEIGHTED CONSTRAINTS RESIDUALS')
      DO 48 I=1,NCONS
      W=0.D0
      NPARI=NCONP(I)
      SIGMSQ=SGMCON(I)**2
      DO 7 J=1,NPARI
      II=ICONS(I,J)
    7 W=W+DFLOAT(ISIGN(II))*X(IABS(II))
      W=W-ESTCON(I)
      WRITE(6,6005) W,(ICONS(I,J),J=1,NPARI)
 6005 FORMAT(T40,D16.6,T4,5I5)
      SUMW=SUMW+W/SIGMSQ
   48 SSQWT=SSQWT+W*W/SIGMSQ
C CHI-SQUARE TEST ON VARIANCE FACTOR. PLOTS HISTOGRAM OF RESIDUALS.
C CHI-SQUARE GOODNESS-OF -FIT TEST
   49 DENOM=DFLOAT(NOB(1)+NOB(2)+NCONS-NVARBL)
      SMACAP=(SSQR1+SSQR2+SSQWT)/DENOM
      WRITE(6,6000)
      WRITE(6,6007) SMACAP,DENOM
 6007 FORMAT('0ESTIMATED VARIANCE FACTOR: ',F17.7/' DEGREES OF FREEDOM:'
     &,F10.1)
      STD=SNGL(DENOM)
      P=1.0-ALPHA/2.0
      CALL MDCHI(P,STD,STAT1,IER)
      P=ALPHA/2.0
      CALL MDCHI(P,STD,STAT2,IER)
      STD=STD*SNGL(SMACAP)
      STAT1=STD/STAT1
```

```
      STAT2=STD/STAT2
      IF(STAT1.LT.1.0.AND.STAT2.GT.1.0) GO TO 62
      WRITE(6,6026) STAT1,STAT2
 6026 FORMAT('0CHI-SQUARE TEST ON VARIANCE FACTOR:'//'0',F16.6,' < 1.0 <
     & ',F16.6,'    FAILS'////)
      GO TO 63
   62 WRITE(6,6028) STAT1,STAT2
 6028 FORMAT('0CHI-SQUARE TEST ON VARIANCE FACTOR:'/'0',F16.6,' < 1.0 <
     &',F16.6,'    PASSES'////)
   63 DO 64 I=1,NY
   64 SMUW(I)=DSQRT(SMUW(I)/(DFLOAT(NOB(I)+NWTPRM+NCONS-NVARBL)))
      COV=COV/DFLOAT(NOB(1)+NWTPRM+NCONS-NVARBL)
      IF(MTYPE.EQ.2) GO TO 65
      WRITE(6,6029)
 6029 FORMAT(1H0,27X,'FRINGE FREQUENCY',30X,'DELAY')
      GO TO 66
   65 WRITE(6,6030)
 6030 FORMAT(1H0,33X,'DELAY')
   66 WRITE(6,6031)(NOB(I),I=1,2),(SMUW(J),J=1,2),COV
 6031 FORMAT(1H0,'NUMBER OF OBSERVATIONS',5X,I10,30X,I10//' STANDARD ERR
     &OR OF'/' UNWEIGHTED RESIDUALS',9X,D13.6,27X,D13.6,//' COVARIANCE',
     P 34X,D13.6)
      IF(ISIGMA.EQ.0.OR.NOBS.LE.11) GO TO 59
      CALL HISTGM(MTYPE,NY,NHIST,VAL,INTS,NOB,ISIGMA,ALPHA)
C TRANSFORMATION OF SUB-MATRIX OF COVARIANCE MATRIX FOR STATIONS INTO
C COVARIANCE MATRICES OF BASELINES
   59 L=0
      DO 21 I=1,3
      DO 21 J=1,I
   21 COVAR(I,J)=0.D0
      WRITE(6,6013)
 6013 FORMAT(1H1,43X,'BASELINE ERROR ANALYSIS'/44X,23('-'))
      DO 24 JJ=2,NSTNS
      IK=JJ-1
      DO 24 II=1,IK
      L=L+1
      DO 23 IAXIS=1,NAXIS
      IFROM(IAXIS)=II*3-(3-IAXIS)
      ITO(IAXIS)=JJ*3-(3-IAXIS)
   23 CONTINUE
      DO 72 J=1,NAXIS
      ITJ=ICOL(ITO(J))
      IFJ=ICOL(IFROM(J))
      DO 72 I=1,J
```

```
      COV=0.D0
      ITI=ICOL(ITO(I))
      IFI=ICOL(IFROM(I))
      IF(ITI.EQ.0) GO TO 68
      IF(ITJ.EQ.0) GO TO 22
      IF(ITI.GT.ITJ) GO TO 76
      COV=COV+ANORM(ITI,ITJ)
      GO TO 22
   76 COV=COV+ANORM(ITJ,ITI)
   22 IF(IFJ.EQ.0) GO TO 68
      IF(ITI.GT.IFJ) GO TO 67
      COV=COV-ANORM(ITI,IFJ)
      GO TO 68
   67 COV=COV-ANORM(IFJ,ITI)
   68 IF(IFI.EQ.0) GO TO 72
      IF(ITJ.EQ.0) GO TO 70
      IF(IFI.GT.ITJ) GO TO 69
      COV=COV-ANORM(IFI,ITJ)
      GO TO 70
   69 COV=COV-ANORM(ITJ,IFI)
   70 IF(IFJ.EQ.0) GO TO 72
      IF(IFI.GT.IFJ) GO TO 71
      COV=COV+ANORM(IFI,IFJ)
      GO TO 72
   71 COV=COV+ANORM(IFJ,IFI)
   72 COVAR(I,J)=COV
      IF(MTYPE.NE.1) GO TO 74
      DO 73 I=1,2
   73 COVAR(I,3)=0.D0
   74 DO 75 I=1,NAXIS
   75 ANS(I)=DSQRT(COVAR(I,I))
      WRITE(6,6019)L,(((COVAR(I,J),J=1,3),ANS(I)),I=1,NAXIS)
 6019 FORMAT('-BASELINE   ',I6/' COVARIANCE MATRIX',17X,'X',19X,'Y',19X,
     P 'Z',15X,'STANDARD ERRORS'//28X,3(D15.6,5X),3X,'X:',F13.10//28X,
     Q 3(D15.6,5X),3X,'Y:',F13.10/28X,'SYMMETRIC MATRIX'/28X,3(D15.6,5X)
     R,3X,'Z:',F13.10)
      BASE1=DSQRT(XBASE(L)**2+YBASE(L)**2)
      SMLONG=( YBASE(L)**2*COVAR(1,1) + XBASE(L)**2*COVAR(2,2) - 2.D0 *
     P XBASE(L)*YBASE(L)*COVAR(1,2) )
      SMLONG=RADDEG*DSQRT(SMLONG)*3600.D0/BASE1**2
      SMEQL=DSQRT( ( XBASE(L)**2*COVAR(1,1) + YBASE(L)**2*COVAR(2,2) +
     P 2.D0*XBASE(L)*YBASE(L)*COVAR(1,2) )/BASE1**2 )
      WRITE(6,6020) SMLONG,SMEQL
 6020 FORMAT(1H0,21X,'LONGITUDE (SECONDS ARC)  EQUATORIAL LENGTH (KM)  D
```

```
&ECLINATION (SECONDS ARC)  TOTAL LENGTH (KM)'//' STANDARD ERRORS:',
P 4X,2(5X,F15.10,5X))
  IF(MTYPE.EQ.1) GO TO 24
  BSLNGT=DSQRT(XBASE(L)**2+YBASE(L)**2+ZBASE(L)**2)
  SMDEC=( (ZBASE(L)/BASE1)**2 * ( XBASE(L)**2*COVAR(1,1)+YBASE(L)**2
P *COVAR(2,2) ) + BASE1**2*COVAR(3,3) + 2.D0*ZBASE(L) * ( XBASE(L)*
Q YBASE(L)*ZBASE(L)*COVAR(1,2)/BASE1**2 - XBASE(L)*COVAR(1,3) -
R YBASE(L)*COVAR(2,3) ) )
  SMDEC=RADDEG*DSQRT(SMDEC)*3600.D0/BSLNGT**2
  SMTOTL=DSQRT( (XBASE(L)**2*COVAR(1,1) + YBASE(L)**2*COVAR(2,2) +
P ZBASE(L)**2*COVAR(3,3) + 2.D0*( XBASE(L)*YBASE(L)*COVAR(1,2) +
Q XBASE(L)*ZBASE(L)*COVAR(1,3) + YBASE(L)*ZBASE(L)*COVAR(2,3) ) )
R /BSLNGT**2 )
  WRITE(6,6021) SMDEC,SMTOTL
6021 FORMAT('+',65X,2(10X,F15.10)/)
 24  CONTINUE
  RETURN
  END

  SUBROUTINE RDWRT(X,CLPOLY,EPOCHS,ICOL,ISTAT,NPOLY,NPARAM,NPLNS,
1 IVARBL,DELTA,ATRW,ANS,ANORM,PACC,                    NVARBL,
2 IPARAM,
3 ICONS,NCONP,SGMCON,ESTCON,                          NCDIM,
1 MXEPOC,MD,NUSED,NFIXED,INTS,NUPDT)
C
C RDWRT READS AND WRITES ADJUSTMENT INFORMATION.  IT IS BASED ON DER
C WRITTEN BY R. B. LANGLEY.
C CALLED BY         MAIN
C CALLS  STNGEO     SOURCE     TAURE     MDNRIS     TIDES     LSQADJ
C        BLNOUT
C COMMON /LBIVAR/ USED IN     LSQADJ     FF$DLY     STNGEO
C COMMON /STNABB/ USED IN     LSQADJ
C COMMON /STATS/ USED IN      LSQADJ
C        D. A. DAVIDSON.
C
  IMPLICIT REAL*8(A-H,O-Z)
  COMPLEX*16 SESION
  REAL*4 XFIRST,YFIRST,YINC,AMAX1,AMAX2,AMAX,ALPHA,CRIT,P,ALPH,ADICT
  INTEGER*2 SABB1,SABB2,SABB
  COMMON /ADJUST/ FOFSET(10),OBSFRQ,XTRASM(10,2),
&SCNAME(10),VPNAME(10),
S XFIRST,YFIRST,YINC,
1 NOBSLN(10),NCONS,
```

```
      2 MTYPE,ISIGMA,NY,NSTNS,NSKIP,ICORR
      3,SABB(40)
       COMMON /LBIVAR/
      1RA(10),DEC(10),XBASE(10),YBASE(10),ZBASE(10),OFFSET(5),HEIGHT(5),
      2UTPOLY(3),XPOLE,YPOLE,OMEGA,TOBS1,
      3NCP(10),JDJANO,NBASE,NSORCE,ITIDE,ISAME,NOBS,IMAX
       COMMON /STNABB/SABB1,SABB2
       COMMON /STATS/ CRIT,ALPHA
       DIMENSION RLAT(5),RLONG(5),
      1 X(NPARAM),CLPOLY(5,NPLNS),EPOCHS(10,MXEPOC),NPOLY(NPLNS),
      2 ICOL(NPARAM),ISTAT(NPARAM),
      3 ANORM(NVARBL,NVARBL),ATRW(NVARBL),DELTA(NVARBL),PACC(NVARBL),
      6 ANS(NVARBL),IVARBL(NVARBL),IPARAM(NUSED),
      7 NCOMP(NCDIM),ICONS(NCDIM,5),SGMCON(NCDIM),ESTCON(NCDIM)
       DATA AMAX/'MAX '/,ADICT/'DICT'/
       WRITE(6,6000)
 6000  FORMAT('1',55X,'CANADIAN LBI PROGRAMME'/' ',55X,22('-'),/,'0',49X,
      1 'FRINGE FREQUENCY AND DELAY ANALYSIS'/50X,35('-')/'0',54X,
      2 'LEAST SQUARES ADJUSTMENT'/55X,24('-')/'0',55X,
      3 'VERSION: U.N.B. MAY 1980'/56X,24('-'))
       DO 1 K=1,NPARAM
       ISTAT(K)=0
    1  ICOL(K)=0
       READ(5,5001)((IPARAM(K),ISTAT(IPARAM(K))),K=1,NUSED)
 5001  FORMAT(13(I4,I2))
C THIS LOOP DERIVES "COMPRESSED A MATRIX" COLUMN NUMBERS OF ALL
C VARIABLE PARAMETERS
       I=0
       DO 2 K=1,NUSED
       L=IPARAM(K)
       IF(ISTAT(L).LT.2)GO TO 2
       I=I+1
       ICOL(L)=I
       IVARBL(I)=L
    2  CONTINUE
       IF(I.NE.NVARBL)GO TO 998
       IF((NFIXED+NVARBL).NE.NUSED) GO TO 999
       READ(5,5002) MTYPE,ITIDE,ISIGMA,IMAX,ISAME,ICORR
 5002  FORMAT(3(I1,1X),I2,1X,I1,1X,I1)
       NY=1+MTYPE/3
       IF(NY.EQ.1) ISAME=0
C THIS INPUT SET SHOULD BE AFTER NBASE COMPUTED, THEN LOOP TO NBASE(8)
       IF(ISIGMA.EQ.1)READ(5,5003)(XTRASM(K,1),K=1,8)
       IF(MTYPE.EQ.3.AND.ISIGMA.EQ.1)READ(5,5003)(XTRASM(K,2),K=1,8)
```

```
 5003 FORMAT(8F10.5)
      READ(5,5004) SESION
 5004 FORMAT(2A8)
      READ(5,5005) OBSFRQ, JDJANO
 5005 FORMAT(F10.5,I15,I10)
      READ(5,5006) TOBS1
 5006 FORMAT(F15.5)
      READ(5,5007)XPOLE,YPOLE,OMEGA,(UTPOLY(K),K=1,3)
 5007 FORMAT(3D20.5/3D20.5)
      WRITE(6,6001)SESION,OBSFRQ,XPOLE,YPOLE,OMEGA,(UTPOLY(K),K=1,3)
 6001 FORMAT('-','OBSERVING SESSION: ',2A8,/'-','OBSERVING FREQUENCY (MH
     1Z): ',F10.3/'-','COORDINATES OF POLE'/'0',' X("): ',F6.3/'0',' Y("
     2): ',F6.3/'-','ROTATION RATE (RADIANS PER UT SECOND): ',1PD25.15/,
     3'-','UT1-UTC POLYNOMIAL COEFFICIENTS: ',1P3D25.16)
      IF(NCONS.EQ.0) GO TO 10
      WRITE(6,6016)
 6016 FORMAT(1H0,'PARAMETER CONSTRAINTS USED:')
      IF(NCDIM.LT.NCONS) WRITE(6,6030) NCDIM,NCONS
 6030 FORMAT(' *WARNING* DIMENSION (',I5,') LESS THAN NUMBER (',I5,')')
      READ(5,5000)(NCONP(I),I=1,NCONS)
 5000 FORMAT(26I3)
      DO 9 I=1,NCONS
      K=NCONP(I)
      READ(5,5011) ESTCON(I),SGMCON(I),(ICONS(I,J),J=1,K)
    9 WRITE(6,5011) ESTCON(I),SGMCON(I),(ICONS(I,J),J=1,K)
 5011 FORMAT(D25.16,D10.3,9I5)
   10 CONTINUE
      OBSFRQ=OBSFRQ*1.D6
      CALL STNGEO(X,NPARAM,NSTNS,RLAT,RLONG,SA3B,VBNAME)
      DO 3 L=1,NBASE
    3 READ(5,5008) FOFSET(L)
 5008 FORMAT(F10.4)
      DO 4 L=1,NBASE
      READ(5,5009) NCP(L)
 5009 FORMAT(I1)
      K=NCP(L)
      IF(K.GT.1)READ(5,5010)(EPOCHS(L,J),J=2,K)
 5010 FORMAT(4D20.10)
    4 EPOCHS(L,1)=TOBS1
      READ(5,5000)(NPOLY(L),L=1,NPLNS)
      DO 5 L=1,NPLNS
      J=NPOLY(L)+1
      DO 5 II=1,J
    5 CLPOLY(II,L)=0.D0
```

```
      CALL SOURCE(NPARAM,SCNAME,RA,DEC,X,NSORCE)
      WRITE(6,6002)
 6002 FORMAT('1')
      I=0
      DO 6 KK=1,NY
      IF(MTYPE.NE.2.AND.KK.EQ.1)WRITE(6,6003)
 6003 FORMAT('-',52X,'FRINGE FREQUENCY')
      IF((MTYPE.EQ.2.AND.KK.EQ.1).OR.(MTYPE.EQ.3.AND.KK.EQ.2))WRITE(6,60
     104)
 6004 FORMAT('-',58X,'DELAY')
      WRITE(6,6005)
 6005 FORMAT(' ',52X,'CLOCK POLYNOMIALS'/'0',30X,'BASELINE',10X,'POLYNOM
     1IAL',10X,'EPOCH',10X,'AVAILABLE'/84X,'PARAMETERS'/)
      J=51
      IF(KK.EQ.2.AND.ISAME.EQ.1) J=51+5*NPLNS/2
      DO 6 K=1,NBASE
      LL=NCP(K)
      WRITE(6,6006)VBNAME(K)
 6006 FORMAT(33X,A4)
      DO 6 L=1,LL
      I=I+1
      II=NPOLY(I)
      JJ=J+II
      WRITE(6,6007)L,EPOCHS(K,L),J,JJ
 6007 FORMAT('+',52X,I2,10X,F13.9,5X,I4,' - ',I4/1X)
    6 J=JJ+5-II
      WRITE(6,6008)(IPARAM(K),K=1,NUSED)
 6008 FORMAT('1','PARAMETERS USED IN ANALYSIS'//(' ',20I5))
      IF(MTYPE.EQ.1)WRITE(6,6009)
      IF(MTYPE.EQ.2)WRITE(6,6010)
      IF(MTYPE.EQ.3)WRITE(6,6011)
 6009 FORMAT('-','FRINGE FREQUENCY DATA ONLY')
 6010 FORMAT('-','DELAY DATA ONLY')
 6011 FORMAT('-','FRINGE FREQUENCY AND DELAY DATA')
      IF(ISIGMA.NE.1)GO TO 7
      WRITE(6,6012)
 6012 FORMAT('-','WEIGHTED DATA')
      IF(MTYPE.EQ.1.OR.MTYPE.EQ.3)WRITE(6,6013)(VBNAME(K),XTRASM(K,1),K=
     &1,NBASE)
 6013 FORMAT('0'/(1X,'INPUT FRINGE FREQUENCY VARIANCES FOR BASELINE ',
     1A4,' INCREASED BY (',D15.7,')**2'))
      J=MTYPE-1
      IF(MTYPE.EQ.2.OR.MTYPE.EQ.3)WRITE(6,6014)(VBNAME(K),XTRASM(K,J),K=
     &1,NBASE)
```

```
 6014 FORMAT('0'/(1X,'INPUT DELAY VARIANCES FOR BASELINE ',A4,' INCREASE
     &D BY (',D15.7,')**2'))
    7 CONTINUE
      IF(ITIDE.EQ.1)WRITE(6,6015)
 6015 FORMAT('0','EARTH TIDE CORRECTION INCLUDED IN MODEL')
      READ(5,5911)NSKIP,NOBS,(NOBSLN(K),K=1,NBASE)
 5911 FORMAT(8I10)
      IF(NUPDT.EQ.0) GO TO 28
      WRITE(6,6028)
 6028 FORMAT('0UPDATED PARAMETERS:'/' PARAMETER',11X,'VALUE')
      DO 27 J=1,NUPDT
      READ(5,5016) K,X(K)
      WRITE(6,5016) K,X(K)
 5016 FORMAT(I5,5X,D25.16)
      IF(K.LE.30.OR.K.GE.51) GO TO 27
      JJ=(K-30)/2
      IF(K/2*2.NE.K) GO TO 26
      DEC(JJ)=X(K)
      GO TO 27
   26 RA(JJ+1)=X(K)
   27 CONTINUE
C COMPUTES OUTLYING RESIDUAL CRITERIA...CRIT
   28 READ(5,5015)AMAX1,AMAX2,ALPHA,XFIRST,YFIRST,YINC,CRIT
 5015 FORMAT(2A4,F7.4,6F10.6)
      IF(AMAX1.EQ.ADICT) GO TO 25
      K=NOBS-NVARBL+NWTPRM+NCONS
      IF(AMAX1.EQ.AMAX) GO TO 23
      IF(ISIGMA.NE.1) GO TO 22
      P=1.0-ALPHA/2.0
      CALL MDNRIS(P,CRIT,IER)
      GO TO 25
   22 DALPHA=DBLE(ALPHA)
      CALL TAURE(1,K,DALPHA,DCRIT)
      CRIT=SNGL(DCRIT)
      GO TO 25
   23 ALPH=ALPHA/FLOAT(NOBS)
      IF(ISIGMA.NE.1) GO TO 24
      P=1.0-ALPH/2.0
      CALL MDNRIS(P,CRIT,IER)
      GO TO 25
   24 DALPHA=DBLE(ALPH)
      CALL TAURE(1,K,DALPHA,DCRIT)
      CRIT=SNGL(DCRIT)
   25 WRITE(6,6027) ALPHA,AMAX1,AMAX2,CRIT
```

```
 6027 FORMAT('OALL STATISTICS BASED ON PROBABILITY OF A TYPE I ERROR(ALP
     &HA): ',F10.5/' RESIDUAL OUTLIERS DETECTED USING ',2A4,' CRITERIA.
     & FACTOR: ',F16.6)
      IF(ITIDE.EQ.0)GO TO 17
      TDI=DFLOAT(JDJAN0)+TOBS1-0.5D0
      READ(5,5002,END=16) JUNK
   16 CALL TIDES(TDI,NSTNS,RLAT,RLONG)
   17 CONTINUE
      IF(NOBS.LE.50) INTS=INTS/2*2
      IF(INTS.GT.40) INTS=40
      WRITE(6,6029) MD
 6029 FORMAT(1H1,'LEAST SQUARES PARAMETRIC ADJUSTMENT'//' MODEL NUMBER:'
     &,I10)
      CALL LSQADJ(X,ICOL,ISTAT,                          NPARAM,
     1 ANORM,ATRW,DELTA,RACC,ANS,IVARBL,                 NVARBL,
     2 IPARAM,                                           NUSED,
     3 ICONS,NCONF,SGMCON,ESTCON,                        NCDIM,
     5 CLPOLY,NPOLY,          NPLNS,          EPOCHS,    MXEPOC,
     6 INTS                                                    )
C PRINTS PARAMETER RESULTS AND STANDARD ERRORS                  C
      WRITE(6,6017)
 6017 FORMAT('1RESULTS'/' ',7('-')/' PARAMETER',10X,'ESTIMATE',17X,
     1 'STANDARD ERROR',/)
      DO 20 II=1,NUSED
      L=IPARAM(II)
      IF(L.GT.30) GO TO 18
      WRITE(6,6018)L,X(L)
 6018 FORMAT(1H0,I5,5X,F15.6,' KM')
      IF(ISTAT(L).LT.2)GO TO 20
      STDER=DSQRT(ANORM(ICOL(L),ICOL(L)))
      WRITE(6,6022) STDER
 6022 FORMAT('+',T46,F10.6,'  KM')
      GO TO 20
   18 IF(L.GT.50)GO TO 19
      IF(L/2*2.EQ.L)GO TO 21
      HRS=X(L)/15.D0
      IHR=HRS
      RMINS=DABS(HRS-DFLOAT(IHR))*60.D0
      IMINS=RMINS
      RSEX=(RMINS-DFLOAT(IMINS))*60.D0
      WRITE(6,6021)L,IHR,IMINS,RSEX
 6021 FORMAT(1H0,I5,5X,I5,' HR ',I5,' MIN',F8.4,' SEC')
      IF(ISTAT(L).LT.2)GO TO 20
      STDER=DSQRT(ANORM(ICOL(L),ICOL(L)))/15.D0*3600.D0
```

```
      WRITE(6,6023) STDER
 6023 FORMAT('+',T46,F10.6,' SEC')
      GO TO 20
 21   CONTINUE
      IDEG=X(L)
      RMINS=DABS(X(L)-DFLOAT(IDEG))*60.D0
      IMINS=RMINS
      RSEX=(RMINS-DFLOAT(IMINS))*60.D0
      WRITE(6,6019)L,IDEG,IMINS,RSEX
 6019 FORMAT(1H0,I5,5X,I5,' DEG',I5,' MIN',F7.3,' SEC')
      IF(ISTAT(L).LT.2)GO TO 20
      STDER=DSQRT(ANORM(ICOL(L),ICOL(L)))*3600.D0
      WRITE(6,6024) STDER
 6024 FORMAT('+',T45,F10.5,'   SEC')
      GO TO 20
   19 WRITE(6,6020) L,X(L)
 6020 FORMAT(1H0,I5,5X,D17.7)
      IF(ISTAT(L).LT.2)GO TO 20
      STDER=DSQRT(ANORM(ICOL(L),ICOL(L)))
      WRITE(6,6025) STDER
 6025 FORMAT('+',T46,D10.3)
 20   CONTINUE
      CALL BLNOUT
      RETURN
  998 WRITE(6,6992)
 6992 FORMAT('-','PARAMETERS GIVEN AS VARIABLE STATUS DO NOT SUM TO NVAR
     &PL')
      GO TO 9999
  999 WRITE(6,6991)
 6991 FORMAT('-','NFIXED+NVARBL DOES NOT EQUAL NUSED')
 9999 CONTINUE
      RETURN
      END

      SUBROUTINE TAURE( NT,NU,ALPH,CRTAU )
C    COMPUTES THE REJECTION LEVEL FOR NORMALISED RESIDUALS FOR A GIVEN NU
C    OBSERVATIONS , DEGREES OF FREEDOM AND DESIRED LEVEL OF TYPE I ERROR
C      PARAMETERS
C        NT - NUMBER OF OBSERVATIONS
C        NU - DEGREES OF FREEDOM
C        ALPH - DESIRED PROBABILITY OF TYPE I ERROR
C        CRTAU - CRITICAL VALUE ( MAX-TAU ) PRODUCED BY THE SUBROUTINE
C    REFERENCE : A.J. POPE (1976) - "THE STATISTICS OF RESIDUALS AND THE
```

```
C                 DETECTION OF OUTLIERS" , U.S. DEPT. OF COMMERCE , NOAA T
C                 REPORT NOS 65 NGS 1.
      IMPLICIT REAL*8(A-H,O-Z)
      DATA PI/ 3.1415926535898 /
      RD = 2. /PI
      S = 1.
      WNU = NU
      U = WNU -1.
      IF( U.EQ.0. ) GO TO 72
      IF ( ALPH.EQ.0. ) GO TO 72
      IF ( ALPH.LT.1. ) GO TO 10
      CRTAU = 0.
C
      RETURN
C
   10 Q = NT
      IF ( ALPH.GT.0.5 ) GO TO 19
      AA = ALPH / Q
      DELT = AA
      DO 18 I = 1,100
      XI = I
      DELT = DELT * ALPH * (( XI*Q - 1.)/(( XI+1.)*Q))
      IF ( DELT.LT.1.D-14 ) GO TO 20
   18 AA = AA + DELT
   19 AA = 1. - (1.-ALPH)**(1./Q)
   20 P = 1. - AA
      IF(U.EQ.1. ) GO TO 71
      F = 1.3862943611199 - 2.*DLOG(AA)
      G = DSQRT(F)
      X = G - (2.515517 + 0.802853*G + 0.010328*F)
     $  /  (1. + 1.432788*G + F*(0.189269 + 0.001308*G))
      Y = X*X
      A = X*(1. + Y)/4.
      B = X*(3. + Y*(16. + 5.*Y))/96.
      C = X*(-15. + Y*(17. + Y*(19. + 3.*Y)))/384.
      E = X*(-945. + Y*(-1920. + Y*(1482. + Y*(776. + 79.*Y))))/92160.
      V = 1./U
      T = X + V*(A + V*(B + V*(C + E*V)))
      S = T/DSQRT(U + T*T)
      UM = U - 1.
      DELL = 1.
      DO 70 M = 1,50
      H = 1. - S*S
      R = 0.0
```

```
      IF ( DMOD(U,2.D0).EQ.0.0 ) GO TO 49
      DD = DSQRT(H)
      N = 0.5*UM
      DO 45 I = 1,N
      Z = 2*I
      R = R + DD
      D = DD
   45 DD = DD * H * (Z/(Z+1.))
      R = PD*(R*S + DARSIN(S))
      D = PD*D*UM
      GO TO 61
   49 DD = 1.
      N = 0.5*U
      DO 55 I = 1,N
      Z = 2*I
      R = R + DD
      D = DD
   55 DD = DD*H*((Z-1.)/Z)
      R = R*S
      D = D*UM
   61 CONTINUE
      DEL = (P-R)/D
      IF( DABS( DEL/DELL ) .GT.0.5) GO TO 72
      DELL = DEL
      S = S + DEL
      IF( DABS(DEL) .LT. 1.D-8 ) GO TO 72
   70 CONTINUE
      GO TO 72
   71 S =DSIN(P/PD)
   72 CRTAU = S*DSQRT(WNU)
      RETURN
      END

      SUBROUTINE TSPLOT(YFIRST,YINC,MTYPE,NY)
C
C TSPLOT PLOTS DLY AND FF RESIDUALS AGAINST TIME FOR EACH BASELINE.
C TIME SCALE IS DOWN THE PAGE;RESIDUAL SCALE ACROSS THE PAGE.
C IT IS BASED ON TSPLOT BY R.B.LANGLEY 19 NOVEMBER 1979.
C CALLED IN LSQADJ -TSPLOT INITIALISES FOR A BASELINE.
C                  -ENTRY RSPLOT PLOTS RESIDUALS FOR EACH OBSN. TIME
C INPUT PARAMETERS
C  YFIRST  EXTREME NEGATIVE VALUE OF RESIDUALS TO BE PLOTTED
C  YINC    INCREMENT PER. PRINTER SPACE ACROSS PAGE
```

```
C   MTYPE     TYPE OF OBSERVATIONS 1=FF , 2=DLY , 3=FF AND DLY
C   NY        NUMBER OF OBSERVATION TYPES IE. EITHER 1 OR 2
C   IX        LINE NUMBER FROM START TIME OF PLOT
C   IY        POSITION ACROSS PAGE FOR EACH RESIDUAL
C   IOUT      DENOTES PRESENCE OF DLY AND FF RESIDUALS FOR TIME POINT
C             D. A. DAVIDSON      MAY 1980
C
      REAL*4 YFIRST,YINC
      LOGICAL*1 STRING(101)
      LOGICAL*1 LINE/'|'/,BLANK/' '/
      LOGICAL*1 SYMBOL(3)/'F','D','B'/
      INTEGER IOUT(2),IY(2),IXOLD,IYZERO,IX
      IXOLD=1
      IYZERO=1.5-YFIRST/YINC
      IF(MTYPE.EQ.2) SYMBOL(1)=SYMBOL(2)
      RETURN
      ENTRY RSPLOT(IX,IY,IOUT)
      IF(IX.LE.IXOLD) GO TO 2
      IF(IX-IXOLD.LT.20) GO TO 5
      IXOLD=IX-3
      WRITE(6,6003)
 6003 FORMAT(/////)
    5 IXM1=IX-1
      DO 1 K=IXOLD,IXM1
    1 WRITE(6,6001) LINE
 6001 FORMAT(T10,A1)
    2 CONTINUE
      STRING(1)=LINE
      STRING(101)=LINE
      STRING(IYZERO)=LINE
      DO 3 K=1,NY
      IF(IOUT(K).EQ.0) GO TO 3
      STRING(IY(K))=SYMBOL(K)
    3 CONTINUE
      IF(IOUT(1)*IOUT(2).EQ.1.AND.IY(1).EQ.IY(2)) STRING(IY(1))=SYMBOL(3
     &)
      WRITE(6,6002)(STRING(K),K=1,101)
 6002 FORMAT(T10,101A1)
      DO 4 K=1,NY
    4 STRING(IY(K))=BLANK
      IXOLD=IX+1
      RETURN
      END
```

```
      SUBROUTINE XSLINV(T,B,N,NDIM,NCODE,D,DET,IDEXP,X)
C
C CHOLESKY INVERSION AND SOLUTION.
C BASED ON VARIOUS INVERSION ROUTINES OF R. R. STEEVES
C GIVES SOLUTION TO AX=B, WHILE LSQADJ REQUIRES SOLUTION TO AX=-B
C THEREFORE NEGATIVE VALUE OF B VECTOR ENTERED INTO ROUTINE
C INPUT PARAMETERS
C   T          MATRIX TO BE INVERTED OR SOLVED
C   B          CONSTANT VECTOR
C   N          SIZE OF T , NPARAMETERS TO BE SOLVED
C   NDIM       DIMENSIONED SIZE OF T,B,D,X.     NDIM >= N
C   NCODE      =1 GIVES INVERSE OF T
C              =2 GIVES SOLUTION VECTOR(X)
C              =3 GIVES SOLUTION AND INVERSE
C   D          WORK VECTOR
C OUTPUT PARAMETERS
C   T        INVERSE MATRIX IF NCODE=1 OR 3
C   X          SOLUTION VECTOR
C   DET,IDEXP DETERMINANT OF INPUT MATRIX T.OUTPUT AS (F10.5,'D',I6)
C ENTRY  INVERS    INVERSE ONLY REQUIRED
C        INVERT    GIVES INVERTED T IF XSLINV PREVIOUSLY GAVE X
C        D. A. DAVIDSON       MAY 1980
C
      IMPLICIT REAL*8(A-H,O-Z)
      INTEGER NDIM
      DIMENSION T(NDIM,NDIM),D(NDIM),B(NDIM),X(NDIM)
      ENTRY INVERS(T,N,NDIM,NCODE,DET,IDEXP)
C  FIND SQUARE ROOT
      DET=0.D0
      DO 4 J=1,N
      GOOG=T(J,J)
      DO 4 I=1,J
      IF(I.EQ.1) GO TO 2
      M=I-1
      SUM=0.0D0
      DO 1 K=1,M
1     SUM=SUM+T(K,I)*T(K,J)
      T(I,J)=T(I,J)-SUM
2     IF(I.EQ.J) GO TO 3
      T(I,J)=T(I,J)/T(I,I)
      GO TO 4
3     CONTINUE
      GOOG=T(I,I)/GOOG
```

```
      IF(GOOG.LT.0.1D-3) WRITE(6,6000) I,GOOG
 6000 FORMAT(' *WARNING*  GOOGE NUMBER FOR PARAMETER',I6,' SIGNIFIES SIN
     &GULARITY: ',D16.6)
      DET=DET+DLOG10(T(I,I))
      T(I,I)=DSQRT(T(I,I))
4     CONTINUE
      IDEXP=DET
      RPART=DET-IDEXP
      APART=DABS(RPART)
      IF(APART.GE.1.D-20)GO TO 9
      DET=1.D0
      GO TO 21
9     DET=10.D0**RPART
21    CONTINUE
      IF(NCODE.EQ.1) GO TO 10
C    FOWARD SUBSTITUTION...
      D(1)=B(1)/T(1,1)
      DO 6 I=2,N
      SUM=0.0D0
      K=I-1
      DO 5 J=1,K
5     SUM=SUM+T(J,I)*D(J)
6     D(I)=(B(I)-SUM)/T(I,I)
C    BACKWARD SUBSTITUTION...
      X(N)=D(N)/T(N,N)
      M=N-1
      DO 8 I=1,M
      SUM=0.0D0
      J=N-I+1
      L=N-I
      DO 7 K=J,N
7     SUM=SUM+T(L,K)*X(K)
8     X(L)=(D(L)-SUM)/T(L,L)
      IF(NCODE.EQ.2)GO TO 20
      ENTRY INVERT(T)
10    DO 17 J=1,N
      DO 17 I=1,J
      IF(I.LT.J) GO TO 15
      T(J,J)=1.0D0/T(J,J)
      GO TO 17
15    SUM=0.0D0
      M=J-1
      DO 16 K=I,M
16    SUM=SUM-T(I,K)*T(K,J)
```

```
      T(I,J)=SUM/T(J,J)                                                 XSLI  88
 17   CONTINUE                                                          XSLI  89
      DO 19 J=1,N                                                       XSLI  90
      DO 19 I=1,J                                                       XSLI  91
      SUM=0.0D0                                                         XSLI  92
      DO 18 K=J,N                                                       XSLI  93
 18   SUM=SUM+T(I,K)*T(J,K)                                             XSLI  94
      T(I,J)=SUM                                                        XSLI  95
 19   CONTINUE                                                          XSLI  96
 20   RETURN                                                            XSLI  97
      END                                                               XSLI  98
```