GEODETIC ASPECTS OF ENGINEERING SURVEYS REQUIRING HIGH ACCURACY

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PREFACE

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GEODETIC ASPECTS OF ENGINEERING SURVEYS REQUIRING HIGH ACCURACY

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PREFACE

This report is an unaltered version of the author's masters thesis of the same title.

The thesis advisers under whom this work was carried out were Dr. Adam Chrzanowski and Dr. Klaus-Peter Schwarz. Acknowledgement of the assistance rendered by others is given in the Acknowledgements.

ABSTRACT

Many of today's engineering surveys require relative positional accuracies in the order of 1/100 000 or better. This means that positional observations must be very accurate, and that a rigorous geodetic approach must be followed.

This thesis is directed toward the geodetic aspect. Chapter 2 reviews the geodetic models and coordinate systems available. For an engineering survey requiring high relative positional accuracy a local plane coordinate system and a geodetic height system, both based on the classical geodetic model, is the appropriate choice. Chapter 3 reviews the well known geometric and gravimetric effects in a local coordinate system.

Special emphasis is placed on methods to determine deflections of the vertical in chapter 4. It was felt that a contribution could be made if a simple method could be developed to determine deflections, which describe variations in the gravity field. (Very often the effect of variations in the gravity field on survey observations are neglected only because they are difficult to determine.) Such a method was developed by the author by applying a difference method to the usual astrogeodetic deflection determination. The method is very simple and practical, and field test results indicate it is accurate

to 1" to 2". Extensive field work associated with the use of trigonometric levelling to determine local deflections led to inconclusive results because the effect of vertical refraction could not be isolated.

Chapter 5 shows the application of the material presented in the first four chapters, with emphasis on the effect of deflection of the vertical. The two problems considered show that often, even for engineering surveys requiring high accuracy, the effect of variations in the earth's gravity field can be safely neglected. This however can only be determined by analyzing each problem using accurate deflection components to estimate the effect in the horizontal and a small number of gravity values to estimate the effect on heights.

Being able to easily define the local gravity field a priori by the astrogeodetic difference method will probably have its best application in situations in which the local variations have their greatest effect, for example in the determination of heights in a three-dimensional coordinate system and in the determination of horizontal positions with inertial surveying systems.

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Often moral support was more urgently required than material support. When encouragement was needed, my fellow graduate students and my family and friends at home provided it.

1. INTRODUCTION

1.1 General

Throughout history there has been a need for engineering surveys. The accuracy requirements of most were such that no special effort or knowledge was required to execute them. There were some problems however that taxed the best minds of the day; the setting out of a long tunnel is the classic example. Today this same problem would challenge the abilities of any surveying engineer. Two other examples of modern day projects requiring engineering surveys of high accuracy are the setting out of nuclear accelerators and the alignment of a straight line in space for the positioning of the aerials of a radio telescope array.

Today's demand for engineering surveys of high accuracy has been matched by advances in survey instrumentation. With the advent of EDM, angles and distances can be measured with comparable accuracy; with careful use of routinely available equipment both can be measured with an accuracy greater than 1/100 000. Using proper methods of network design, measurement and adjustment, relative positional accuracies of the same order can be attained.

If relative positional accuracies of the order of 1/100 000 are actually to be attained, a rigorous geodetic approach must be followed. This means that for even small project areas the effect of the ellipsoidal shape of the earth and the effect of the earth's gravity field must be accounted for. Although these two effects are inextricably linked, they are most often dealt with separately. In this thesis the same approach will be followed.

Throughout this thesis reference to engineering surveys requiring high accuracy will imply relative positional accuracies in the order of 1/100 000 between locally stable points. Special engineering surveys concerned with the movements of points will not be considered.

1.2 Engineering Surveys and an Integrated Surveying System

Much work has been done in recent years to develop a workable survey control framework for position - related information at a regional or national level. This survey control framework and the position related information tied to it is generally referred to as an integrated survey system.

In order for integrated systems to operate for maximum benefit all surveying and mapping activities should be tied to them. For routine engineering surveys requiring relative positional accuracies in the order of 1/10 000, the survey control framework of integrated survey systems could be used directly as control. For engineering surveys requiring relative positional accuracies in the order of 1/100 000 or better, the survey control framework of integrated survey systems might not be adequate.

One reason that control points of integrated surveying systems might not have relative positional accuracies in the order of 1/100 000 is that the expense of attaining these accuracies is not justified for the vast majority of users of the system. Another possible reason is that integrated survey system networks may use a "higher order" network, for example the national geodetic network, as fixed and errorless in the adjustment of the integrated survey system network and thus be distorted.

Use of the first order national geodetic network directly may also not solve the problem. In North America readjustment of the national geodetic networks is currently underway. This readjustment will remove distortions in the network, but according to proposed specifications [Surveys and Mapping Branch (EMR), 1973] first order networks will have relative positional accuracies of about 1/50 000 in terms of the semi-major axes of the relative error ellipse at the 95% probability level. As stated by Linkwitz [1970], experience in Europe indicates that conventional geodetic networks may not be suitable engineering projects:

> "Most conventional geodetic networks have been designed measured and adjusted with overall homogeneity in mind. Often this quality makes them unsuitable for controlling engineering projects where high local precision is required".

An alternative then, for engineering surveys requiring high accuracy, is to adopt an appropriate geodetic model and local coordinate system. The local system could be tied to an integrated survey system if required but the observations used to make the tie would not be used for position determinations in the local system. A disadvantage of this approach is that coordinates (and their accuracies) from another coordinate system could not be utilized as additional information for position determinations in the local system without a transformation. The use of coordinates as observations (additional information) is discussed in Chrzanowski et al [1979] and Vanicek and Krakiwsky [in prep].

In reviewing the literature on engineering surveys requiring high accuracy it was found that only rarely, outside of Europe, is this approach followed. Often it seemed that only the mystique surrounding geodesy prevented those responsible for the survey control from attaining better accuracies.

2. A GEODETIC MODEL AND COORDINATE SYSTEM

2.1 Choices of Geodetic Models

A geodetic model of a set of points on the surface of the earth consists of a definition of a coordinate system and its location within the earth, and the coordinates of the points in this coordinate system.

Basically, there exist two different approaches to the problem of geodetic positioning. One approach regards the points on the surface of the earth as being perpetually in motion with respect to each other as well as with respect to the coordinate system. In this model the coordinates are therefore time varying and the model is four dimensional: three coordinates specifying position, and one coordinate specifying time. The other more conventional approach treats the positions of the points as permanent with respect to the coordinate system.

2.1.1 Time Varying Model

The ultimate goal in geodesy is to be able to provide instantaneous positions of ground points as they vary with time [Mather, 1974].

Ground, or more generally, surface movements can be due to three effects. These three effects are earth tides, sea tides loading, and aperiodic surface or crustal movements.

(i) The earth tides. This is a global phenomenon. It changes the shape of the earth so that a point on the surface of the earth can oscillate as much as -15 cm to +30 cm with respect to the center of mass of the earth.

There is also an annual distortion of the earth's surface caused partly by earth tides but mainly by atmospheric variations. Very little is known about the geodetic effects of this distortion.

(ii) Sea tides' loading. Sea tides are a more complex phenomenon than earth tides. Only at tidal stations can sea tides by directly measured, and in order to predict the loading at a point on the surface of the earth, the distribution of sea tides must be known over a large area. In addition to this the elastic properties of the earth's crust must be well known. For these reasons, the degree of reliability in predicting distoritons caused by sea tides' loading is low.

One sea tides loading prediciton gives a semi-diurnal (12 hour) loading effect in the immediate vicinity of the Bay of Fundy of several centimeters vertical. The associated grount tilt is about 0$''$.1. both of these effects fade waay inland. It should also be noted that the sea tides' loading effect in this area probably would be the largest experienced anywhere in the world since the Bay of Fundy experiences the world's highest tides.

(iii) Aperiodic surface or crustal movements. All other surface movements have been lumped together into this category only because they do not show a regular variation with time. These movements can be further divided into movements due to crustal loading (other than sea tides' loading), movements due to tectonic action, movements due to man's activities and movements due to other causes.

Movements due to crustal loading are predominantly vertical movements. The loading (or unloading) causing this movement may be due to a large water reservoir, a large city, sediments deposited or material eroded by a major river, post glacial isostatic rebound, or other factors. Tectonic action refers to movements of large plates of the earth's crust on the upper mantle material. These movements have recently become the subject of vigorous research, for example the movements associated with the San Andreas fault in California. Movements due to man's activities could be ground consolidation due to withdrawal of fluids such as oil or water, or ground swelling due to fluid waste disposal. Man's activities could also cause landslides,and subsidences following mining exploitation. Movements due to other causes would include thermo-elastic deformations of the earth, about which very little is known quantitatively, and regional anomalous uplifts or subsidences of no immediately explainable origin. An example of the latter is the vertical movement in the Lac St. Jean area of Quebec [Vanicek and Hamilton, 1972].

Before concluding this discussion of surface movements, the long term movement of mean sea level should be mentioned because mean sea level is commonly used as a height datum. Studies have shown a eustatic (world mean sea level) rise of the order of 10 cm per century [Holdahl, 1974].

2.1.2 Contemporary Three-Dimensional Model

In this model the coordinate system is three-dimensional and the positions of points are considered invariant with time. This approach is not new - it was first suggested by Bruns in 1878. The formulae used in contemporary three-dimensional geodesy are generally those contained in Wolf [1963], Hirvonen [1964] or Hotine [1969] and summarized in Heiskanen and Moritz [1967]. Many authors have refined these or similar formulae and applied them to simulated or actual networks. Examples are: Bacon [1966], Henderson [1968], Hradilek [1968; 1972], Fubara [1969], Stolz [1972], Vincenty [1973], Vincenty and Bowring [1978], Lehman [1979].

In recent years the three-dimensional approach has gained in popularity. There are several reasons for this. One reason is that surveying methods using photogrammetry, satellite receivers or inertial systems are inherently three-dimensional. Another reasons is that the computational requirements of simultaneously dealing with three coordinates are no longer a problem due to advances in computer technology. A third reason is that deflections of the vertical and geoid heights, which can be used as input into the three-dimensional model to obtain the most accurate values for the coordinates, can now be better determined. (Deflections of the vertical and changes in geoid heights express the variation of the earth's gravity field. They also affect the classical geodetic model, and this aspect is discussed in detail in chapter 3.)

In the three-dimensional model the position of a point on the terrain is given by the ellipsoidal coordinates (ϕ, λ, h) or by the geodetic cartesian coordinates (X_G, Y_G, Z_G). ϕ and λ are taken as positive to the north and east respectively. The geometric relationship between these coordinates is indicated in Figure 2-1.

The ellipsoid height h is obtained by adding together the orthometric height H and the geoid height N. A complete discussion of the relationship between ellipsoidal and cartesian coordinates, as well as other coordinate systems used in geodesy, is given in Krakiwsky and Wells [1971].

2.1.3 Classical Geodetic Model

In the classical geodetic model, the triplet of coordinates used to define the position of a point on the surface of the earth are separated into horizontal coordinates and a vertical coordinate. The horizontal coordinates may be geodetic latitude ϕ and geodetic longitude λ, or cartesian coordinates X and Y on a mapping plane. The vertical coordinate is a rigorous geodetic height such as dynamic height or orthometric height.

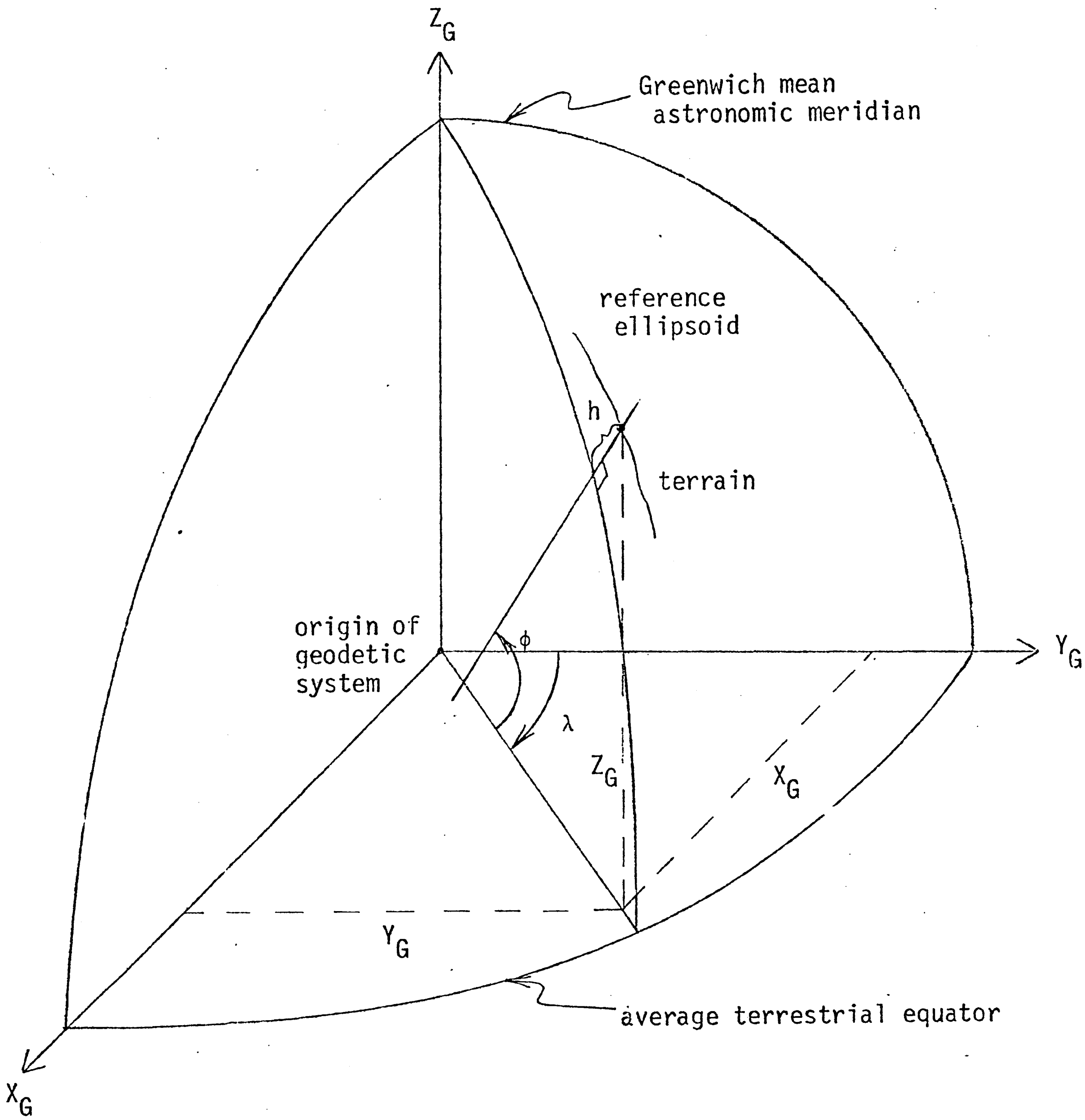


Figure 2-1

Ellipsoidal (ϕ, λ, h) and Geodetic Cartesian (X_G, Y_G, Z_G) Coordinates

The main reason for the separation of coordinates is purely practical. Horizontal geodetic stations, in order to be intervisible so that the traditional surveying measurements can be made, are generally located on hilltops. Precise geodetic levelling between these stations is usually very difficult and is very seldom performed [Krakiwsky and Thomson, 1974]. Vertical control points, on the other hand, are generally located along roads to that the levelling route is easily accessible.

2.1.4 The Choice of a Geodetic Model for an Engineering Survey

The time-varying model would only be an appropriate choice for special engineering surveys concerned with the movements of points. These types of surveys will not be considered in this thesis.

Movements due to earth-tides and sea tides' loading can cause movements with respect to the center of mass of the earth in the order of centimeters, but the relative changes in angles and distances in the area covered by an engineering project are very small - of the order of 10^{-8} [Melchior, 1966] and undetectable with present surveying instruments. Relative movements due to crustal loading are of the same order. Movements due to some of the remaining causes, for example tectonic action and man's activities, and anomalous movements could easily be large enough so that terrain points could not be used for local control purposes. In these cases terrain points would have to be carefully chosen so that they would be locally stable.

The choice of a geodetic model for an engineering survey has been reduced to either a three-dimensional model or the classical model. The choice between these two will take a little more consideration, for each model has a number of distinct advantages and disadvantages.

One of the advantages of a three-dimensional model is that observations do not have to be reduced to a reference surface. Only the usual atmospheric and instrumental corrections are made to the observations.

Another advantage of the three-dimensional model is that all three coordinates are determined for every point; however, for an engineering project this advantage might not be utilized. Just as design, measurement and adjustment of separate horizontal and vertical networks is a practical procedure, it is also a practical procedure to set out heights from vertical control and horizontal positions from horizontal control.

A third advantage of the three-dimensional model is that it can fully utilize satellite data. The three-dimensional model can also utilize the output of other systems that operate in three-dimensional space, for example photogrammetric systems [El Hakim, 1979] and inertial survey systems.

The fact that in a three-dimensional model the three coordinates are solved for simultaneously, leads to certain difficulties. The most obvious difficulty is that a larger system of equations must be solved. Another difficulty is that in a three-dimensional model the formulation of observation equations must be done in the ellipsoidal coordinate system since horizontal directions (or angles) and zenith angles cannot be expressed completely in cartesian coordinates [Hotine, 1969; Chovitz, 1974].

Neither of these difficulties however, should be considered as serious because of the computer facilities available today.

The most serious difficulty with a three-dimensional model is that a height coordinate is solved for at every point. This requires additional observations. Spatial distances, except where lines of sight are very steep, do little to accurately determine heights [Hradilek, 1968]. Measurement of the spatial distance between two points together with the zenith angles corrected for deflections of the vertical allow differences of ellipsoid heights to be calculated. These observations are essential for a three-dimensional geodetic model but, by themselves, are not sufficient to determine the height coordinates as accurately as the horizontal coordinates. Observations of astronomic latitude and astronomic longitude and observations of differences in spirit levelled heights are necessary to increase the accuracy of the height coordinates [Fubara, 1969; Vincenty, 1973]. (These observations provide information on the variation of the earth's gravity field between points in the network. The rôle of astronomic observations and observations of changes in height are discussed in detail in chapters 3 and 4). Even with these additional observations the height coordinates may not be of the same accuracy as the horizontal coordinates. This is because of the uncertainty associated with vertical refraction. For smaller scale three-dimensional networks, such as those used to determine ground movements, the uncertainty associated with vertical refraction still has a significant effect [Dodson, 1978].

Vertical refraction can be treated as an unknown parameter in a three-dimensional adjustment [Hradilek, 1972; Ramsayer, 1978] but only with special observing methods, for example simultaneous reciprocal zenith angles, or under special conditions, for example lines high above the ground, can vertical refraction and heights be well determined. An alternative to treating vertical refraction as an unknown parameter is to input it as a known quantity, but the difficulty of adequately modelling vertical refraction is well illustrated by the work of Angus-Leppan [1967; 1978], Brunner [1977] and others.

Another alternative to treating vertical refraction as an unknown parameter may be available in the future. Work is currently being carried out by Tengstrom [1977] at Uppsala University in Sweden and by Williams [1977] at the National Physical Laboratory in England on instruments to determine refraction directly by measuring the dispersion of two colours of light. (Vertical refraction is dealt with in more detail in chapter 4 in connection with determination of deflection of the vertical.)

The disadvantages of the classical geodetic model, especially when applied to an engineering survey, are minor.

One disadvantage is that observations have to be reduced from the terrain to the ellipsoid to the mapping plane. Since these reductions are performed together with the atmospheric and instrumental corrections, most likely within a computer program, it causes no real problem. The reductions due to gravity can be determined by a global geoid model or, with more accuracy, by any of the methods outlined in Chapter 4.

Another disadvantage of the classical geodetic model is that horizontal control points have accurate horizontal coordinates but only approximate heights, and vertical control points have accurate heights but only approximate horizontal coordinates. Again, this causes no real problem. Just as design, measurement and adjustment of separate horizontal and vertical networks is a practical procedure, it is also a practical procedure to set out heights from vertical control and horizontal positions from horizontal control.

A third disadvantage of the classical geodetic model is that it cannot fully utilize three-dimensional data such as satellite data. This is a very real disadvantage for national geodetic networks, but is not a major consideration for most engineering surveys requiring high accuracy. The reason for this is that in a small area, like that covering an engineering project, three-dimensional data (satellite, photogrammetric or inertial) is generally not sufficient to produce horizontal and vertical positional accuracies of 1/100 000; this is especially true for the heights. For small areas the traditional surveying measurements still provide the highest positional accuracies. An exception to this would be the positional accuracies of some satellite solutions, for example the short arc satellite solution in which accuracies of 0.25 m in all three coordinates are claimed [Brown, 1976]. If some of the control for an engineering survey were to be established by satellite methods, a datum shift would be required to make the satellite derived coordinates compatible with the coordinates derived from the traditional surveying methods. (The reasons for the datum shift is explained in section 2.2.1.) In this case the datum shift could be performed by a method outlined by Merry and Vanicek [1974]. A few special engineering surveying problems would be very difficult without satellite position determinations. Examples are the positioning and orientation of nuclear accelerators and radio telescopes in which "absolute" position and "absolute" orientation (position with respect to the center of mass of the earth and orientation with respect to the best-fitting geocentric ellipsoid - see section 2.2.1) are necessary.

The advantages of the classical geodetic model are due to practical considerations. The classical geodetic model minimizes the effect of atmospheric refraction by separating horizontal and vertical control. This enables positional accuracies of 1/100,000 to be attained. As has been discussed previously, it is also very practical to deal with horizontal and vertical control networks separately, and to set out from these networks separately

Another practical aspect of the classical geodetic model is it height component. In the classical geodetic model, heights whether dynamic or orhtometric, have a definite physical meaning. Points having the same dynamic height lie on the same equipotential surface. Points having the same orthometric height are the same height above the geoid. (Heights will be discussed in more detail in chapter 3.) In the three-dimensional geodetic model, the height component is either the local cartesian coordinate Z_L or the ellipsoid height h. Figure 2-2 shows the position and orientation of a local three-dimensional cartesian coordinate system. Given the parameters defining the shape and position of the reference ellipsoid, Z_L can be transformed to h and vice versa. Ellipsoid height h is related to orthometric height H by the formula $h = H + N$ (see section 2.1.2) but it is difficult to obtain an accurate value for N at a given point.

For engineering purposes neither Z_L or h is very useful. If heights on a given project were defined by Z_L's it would cause a great deal of confusion. By studying Figure 2-2, one can see that depending on the location of points with respect to the origin of the local three-dimensional cartesian coordinate system a point with a larger Z_L value than another point may or may not be higher than the other point! Use of ellipsoid heights would be better but still not satisfactory. Changes in ellipsoid heights approximate changes in dynamic or orthometric heights but for engineering surveys requiring high accuracy, especially in areas where variations in the gravity field are large, use of ellipsoid heights would not be satisfactory.

The classical geodetic model has been in use since man first began to investigate the size and shape of the earth. It is still in use today in all the national geodetic networks of the world. Despite advances in all areas of surveying-new equipment and methods, more dense coverage of data, the ability to rigorously adjust networks- the classical geodetic model remains the most practical and useful. For these reasons the classical geodetic model should be used in preference to the three-dimensional geodetic model for an engineering survey requiring high accuracy.

2.2 The Classical Geodetic Model and a Local Coordinate System

In this section well known features of horizontal and vertical geodetic networks are reviewed. Definitions are kept to a minimum. No references are given to formulae as these are readily available from

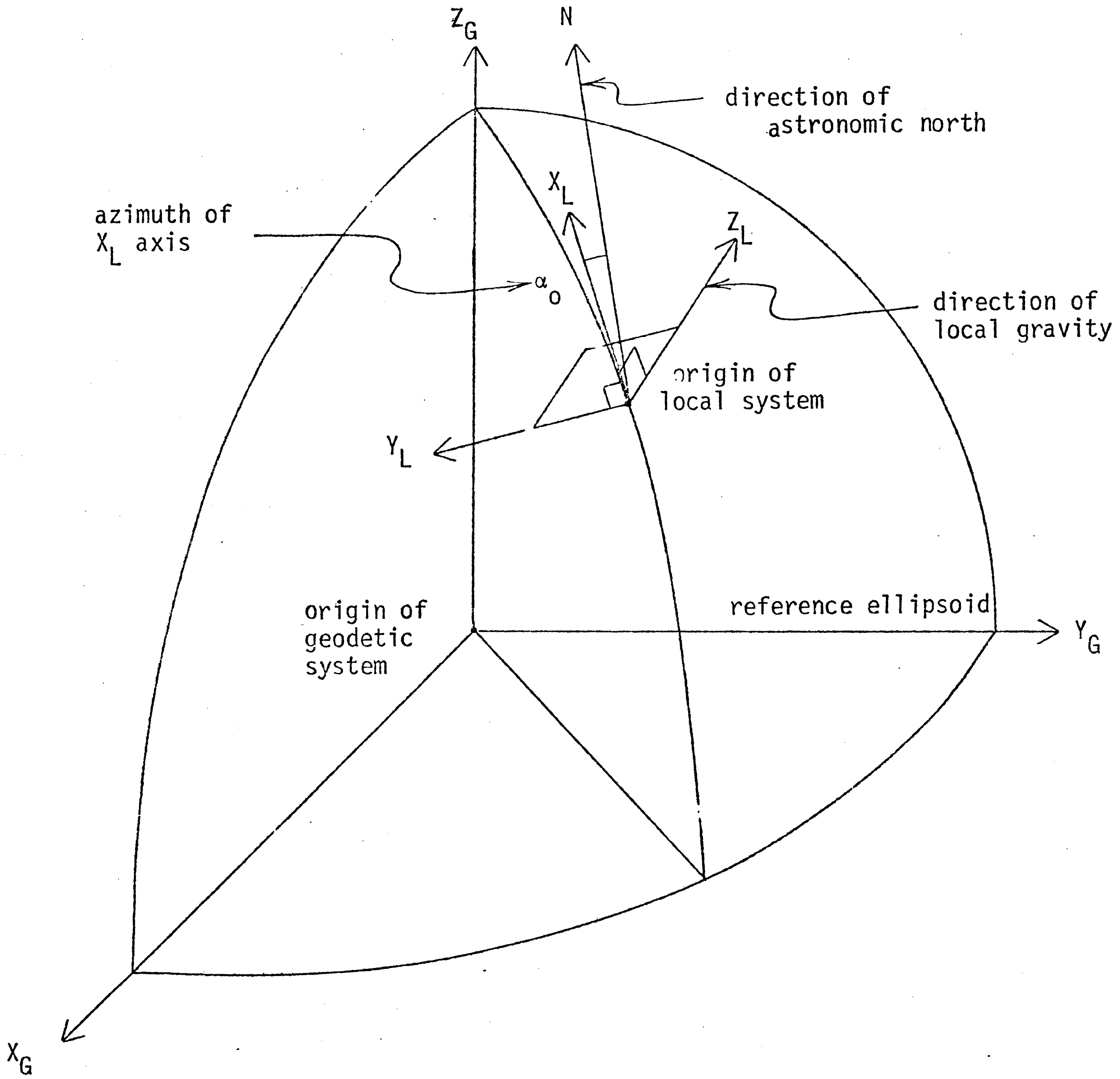


Figure 2-2

The Local Three-Dimensional Coordinate System any textbook on geodesy, for example Bomford [1975] and Vanicek and Krakiwsky [in prep].

In the classical geodetic model the horizontal position of a point is defined by geodetic latitude and longitude on the surface of a reference ellipsoid or by cartesian coordinates X and Y on a mapping plane. The vertical position in a classical geodetic model is defined by a rigorous geodetic height.

For local control purposes two-dimensional cartesian coordinates are preferable to geodetic coordinates. Cartesian coordinates are most easily obtained by reducing horizontal position observations to the mapping plane and then adjusting the reduced observations on the mapping plane. Before this can be done however, a horizontal geodetic datum must be established.

2.2.1 Establishment of a Horizontal Geodetic Datum

A horizontal geodetic datum is simply the surface of the reference ellipsoid. There are several ways in which a datum can be defined. The classical approach is to determine a set of parameters which define the datum, by making measurements on the surface of the earth. This is the approach that will be followed here.

A set of eight parameters which define a horizontal geodetic datum are: a, f, ϕ_0, λ_0, N_0, ξ_0, η_0, $\delta\alpha_0$.

a and f define the size and shape of the reference ellipsoid. a is the dimension of the semi-major axis of the reference ellipsoid. f is the flattening of the reference ellipsoid, and $f = \frac{a - b}{a}$ where b is the dimension of the semi-minor axis of the reference ellipsoid.

The values of a and f have been refined by satellite data. Typical values for a best-fitting geocentric ellipsoid for the entire earth have a=6378.135 km and $f = \frac{1}{298.26}$ [Seppelin, 1974].

Before going further, geocentric and best-fitting should be explained. Geocentric means that the center of the ellipsoid is located at the center of mass of the earth, and the semi-minor axis of the ellipsoid is coincident with the spin axis of the earth. Best-fitting for the entire earth means that the ellipsoid approximates the geoid to within $\pm$ 100 m everywhere. (The geoid or figure of the earth would coincide with the surface of the oceans if they were not subject to external influences such as tides, prevailing winds, currents, differences in density, etc. The departure of average sea level over a long period of time, or mean sea level, from the geoid is of the order of 1 m.) Figure 2-3 shows the relationship between a best fitting geocentric ellipsoid for the entire earth and the geoid. Later it will be shown that a non-geocentric reference ellipsoid, approximating the geoid (or some other equipotential surface closer to the terrain) in the region of use, is satisfactory.

Returning to the parameters which define a horizontal geodetic datum, the remaining six parameters (ϕ_0, λ_0, N_0, ξ_0, η_0, $\delta\alpha_0$) all refer to the initial point of the network. ϕ_0 and λ_0 are the geodetic latitude and geodetic longitude respectively of the initial point. N_0 is the geoid height or geoid-ellipsoid separation at the initial point. ξ_0 and η_0 are the components of the deflection of the vertical at the initial point. $\delta\alpha_0$ is the difference between the astronomic azimuth and the

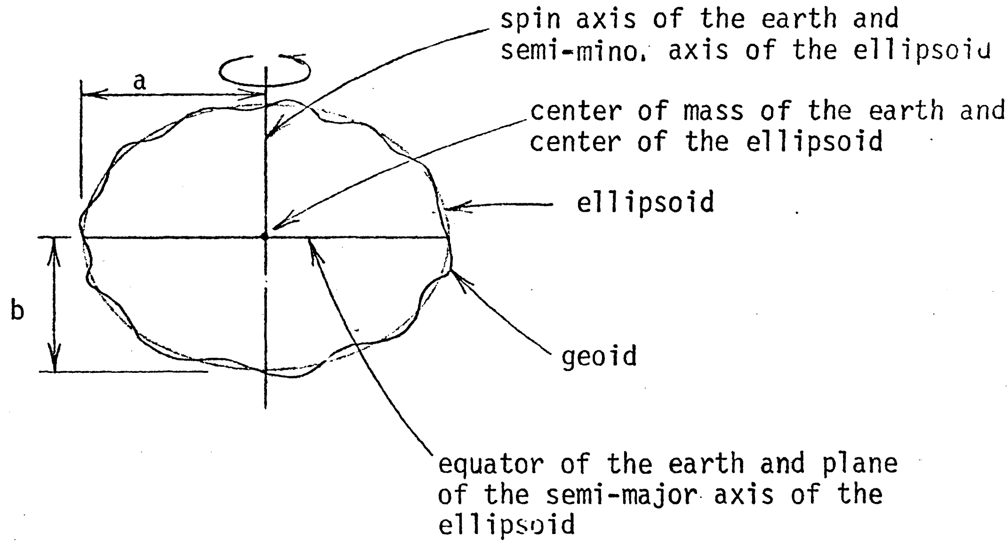


Figure 2-3

A Geocentric Reference Ellipsoid

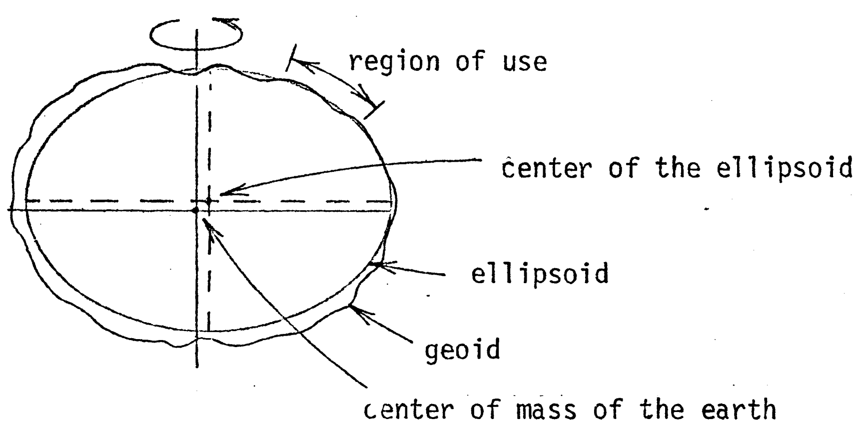


Figure 2-4

A Nongeocentric Reference Ellipsoid

geodetic azimuth between the initial point and another point. ϕ, λ and N have been defined previously; ξ, η and $\delta\alpha$ are defined in the following paragraphs.

Deflection of the vertical is the spatial angle between the plumbline and the normal to the reference ellipsoid. ξ and η are the orthogonal components of the deflection of the vertical. ξ is the north-south or meridian component; η is the east-west or prime vertical component. ξ and η are taken to be positive to the north and east respectively in order to correspond to the sign convention for ϕ and λ. Since the plumbline is a spatial curve, the value of deflection of the vertical will depend on where the angle is measured. Many tasks in geodesy require the deflection of the vertical at the geoid, others require the deflection of the vertical at the earth's surface; the latter are called surface deflections. Differences in deflection of the vertical between the terrain and the geoid have been computed to be as high as 3"/1000 m in the Alps [Kobold and Hunziker, 1962]. If the earth had no terrain, the geoid coincided with the reference ellipsoid and the density distribution within the earth were uniform, deflections of the vertical would be zero everywhere. Because of the earth's terrain, the position of the reference ellipsoid within the earth and density variations near the surface of the earth, deflections of the vertical of up to 01' can exist [Heiskanen and Vening Meinesz, 1958].

$\delta\alpha$ is expressed by the Laplace azimuth condition

$$\delta\alpha = A - \alpha = \eta \tan \phi + (\xi \sin \alpha - \eta \cos \alpha) \cot z \qquad (2\text{-}1)$$

where ϕ, ξ, η have been given previously

and A = astronomic azimuth

α = geodetic azimuth

z = zenith angle

The Laplace azimuth condition is one of the three parallelity condition equations. The other two equations are

$$\xi = \Phi - \phi \qquad (2\text{-}2)$$

$$\eta = (\Lambda - \lambda) \cos \phi \qquad (2\text{-}3)$$

where ϕ, λ, ξ and η were given previously

and Φ = astronomic latitide

Λ = astronomic longitude

Together the parallelity condition equations ensure that the semi-minor axis of the reference ellipsoid is parallel with the spin axis of the earth and the plane of the Greenwich meridian is parallel to the zero meridian of the ellipsoid.

With all of the parameters defining a horizontal geodetic datum explained, the problem of establishing a datum can be considered. Simply stated, the problem is to choose values for (a, f, ϕ_0, λ_0, N_0, ξ_0, η_0, $\delta\alpha_0$) such that the values of (ξ, η) or N elsewhere in the network are minimized. When this is done, it will result in a non-geocentric reference ellipsoid which approximates the geoid in the region of the network. (See figure 2-4).

The only practical problem of applying this to any network is to determine accurate (say $\pm$ 1") values for ξ and η. (In any network ξ and η are unlikely to vary by more than 20".) Traditionally ξ and η

have been determined by laborious 2nd order astronomic observations for Φ and Λ. More recently terrestrial gravity data has been used to improve the interpolation between astronomic stations, or satellite, astronomic and terrestrial data have been combined.

In chapter 4 a new very simple method to determine ξ and η is given. This new method, which was developed by the author, is based on astronomic difference observations for Φ and Λ. It was field tested and shown to be accurate to about $\pm$ 1".

The usual methods to determine deflections of the vertical and the new method are discussed in detail in chapter 4.

2.2.2 A Plane Coordinate System

A plane coordinate system can be obtained by the conformal mapping of the ellipsoid surface, along with coordinates of points on it, onto a flat two-dimensional plane. If this approach is used the observations must first be adjusted on the ellipsoid. An equivalent alternative approach is to reduce the observations to a conformal mapping plane, using reduction formulae derived from the particular conformal mapping function, and adjust the observations on the conformal mapping plane. The second approach is generally used when establishing a local horizontal control system since it is simpler: plane trigonometry is used as opposed to ellipsoidal geometry when working on the ellipsoid.

Mapping is a general term in mathematics. It means the transformation of information from one surface to another. For surveying purposes a conformal mapping is used because in this type of mapping angles are preserved and, as a result, linear scale is a function of position only.

By imposing different conditions various conformal map projections can be deduced. The more familiar conformal map projections are Mercator, Transverse Mercator, Stereographic and Lambert Conformal Conic. The Transverse Mercator map projection is probably the most commonly used map projection in surveying; for this reason the corrections to observations in reducing from ellipsoid to a Transverse Mercator mapping plane will be given in detail in chapter 3.

In a small area, such as that covering an engineering project, no conformal mapping projection has a distinct advantage; however, corrections to observations in all conformal mapping projections can be minimized by choosing a reasonable origin for the conformal mapping projection and a reasonable scale factor at the origin. This aspect will also be discussed in chapter 3, in reference to the Transverse Mercator map projection.

A complete coverage of Transverse Mercator and other map projections can be obtained in references such as Maling [1973], Richardus and Adler [1974] and Krakiwsky [1973].

2.2.3 A Geodetic Height Datum

A geodetic height datum is a surface to which heights are referred. In national geodetic networks it is common to use height above the geoid, as approximated by mean sea level, as the height datum. The problems with this approach were mentioned in sections 2.1.1 and 2.2.1. A more reasonable approach would be to use the equipotential surface passing through a stable point in the height network as the height datum. The height of this point would be arbitrarily assigned its approximate elevation above mean sea level.

In the classical geodetic model, heights are obtained from precise levelled height differences corrected for differences in gravity along the levelling route. Details of the geometric and gravimetric effects on heights are discussed in chapter 3.

3. GEOMETRIC AND GRAVIMETRIC EFFECTS IN A LOCAL COORDINATE SYSTEM

In this chapter the gravimetric and geometric effects on the traditional observations used to obtain accurate heights and horizontal positions in a local coordinate system are discussed, and the corrections for these effects are given. No references are given for the correction formulae since they are well known and available from textbooks on geodesy, for example Bomford [1975], Vanicek and Krakiwsky [in prep], or from other sources, for example; Department of the Army [1958], Krakiwsky [1973], Krakiwsky and Thomson [1974] and Thomson et al [1978].

It should also be noted that the gravimetric and geometric effects on traditional surveying observations is just one small aspect of the overall problem. If an accurate local coordinate system were to be established it would involve many other tasks - reconnaissance, preanalysis and design, performing field observations, and obtaining coordinates of control points together with their associated accuracies by adjusting the corrected observations. To discuss all these tasks is beyond the scope of this thesis; however, in chapter 5 preanalysis and adjustment are used to show the effect of neglecting the gravity field in a simulated tunnel survey.

3.1 Heights

Heights are obtained from measurements of height differences above or below the height datum. Using the well known procedures for precise spirit levelling, accuracies of height differences of the order of 2 mm/$\sqrt{km}$ of levelling route can be attained.

The most important procedure in precise spirit levelling is the equalizing of backsight and foresight distances. By this procedure the geometric effect due to the curvature of the earth is eliminated. This procedure also eliminates the error due to the horizontal collimation error of the instrument and the error due to the refraction of the line of sight on the backsight and foresight, assuming that the refraction is the same in the backsight and foresight directions.

If trigonometric levelling is carried out using equal backsight and foresight distances, and if heights of targets at beginning and end, zenith angles and slope distances are measured with a comparable high accuracy, the accuracy of the height difference may approach that of spirit levelling. This method should be considered where accurate heights are required in rough terrain. Because of the problems discussed in section 2.1.4, single observation trigonometric levelling or even reciprocal trigonometric levelling may be an order of magnitude less accurate than precise spirit levelling or trigonometric levelling with equal backsight and foresight distances.

The geometric effect on heights is removed by merely equalizing backsight and foresight distances. The gravimetric effect on heights however, cannot be eliminated, and can only be accounted for by making gravity observations along the levelling route. The gravity values obtained are used to make corrections to the measured height differences.

Gravity must be accounted for in levelling because equipotential, that is level, surfaces are not parallel. Since differences in level are actually differences in vertical distances between equipotential surfaces, and since equipotential surfaces are not parallel, a sum of differences in level will be path dependent. In order to uniquely define heights of points the effect of gravity must be included in levelling. Mathematically, $\oint dL \neq 0$ indicates that observed level differences are path dependent; $\oint gdL = 0$ indicates that the product of observed level differences and corresponding gravity values are not path dependent. ($\oint$ is the integration around a closed circuit.)

There are several height systems which include the effect of gravity so that heights of points can be defined uniquely. The system of geopotential numbers uses the property $\oint gdL = 0$ directly. Geopotential numbers are seldom used in engineering work because numerically they depart from measured heights by about 2% even when units are chosen in the most convenient way, that is gravity is expressed in kgals so that $g \doteq 1$.

The system of dynamic heights and the system of orthometric heights are two other height systems which include the effect of gravity by making a small correction to a measured height difference. In either system the difference in height between two points A and B is expressed as

$$\Delta H_{AB} = \Delta H^{M}_{AB} + \Delta_{AB} \tag{3-1}$$

where

ΔH_{AB} = difference in dynamic or orthometric height

ΔH^{M}_{AB} = measured difference in height

Δ_{AB} = dynamic or orthometric correction to the measured difference in height.

In the dynamic height system

$$\Delta H^{D}_{AB} = \Delta H^{M}_{AB} + \Delta^{D}_{AB} \tag{3-2}$$

and

$$\Delta^{D}_{AB} = \sum_{i} \frac{g_i - G}{G} \delta L_i \tag{3-3}$$

where

g_i = average value of gravity in a levelling section

G = reference gravity for the area

δL_i = measured difference in height in a levelling section.

In the orthometric height system

$$\Delta H^{O}_{AB} = \Delta H^{M}_{AB} + \Delta^{O}_{AB} \tag{3-4}$$

and

$$\Delta^{O}_{AB} = \Delta^{D}_{AB} + H^{M}_{A} \frac{\bar{g}_A' - G}{G} - H_B{}^M \frac{\bar{g}_B' - G}{G} \tag{3-5}$$

$$= \sum_{i} \frac{g_i - G}{G} \delta L_i + H_A{}^M \frac{g_A' - G}{G} - H_B{}^M \frac{g_B' - G}{G}$$

where

g_i, G, δL_i were given previously

$H_A{}^M$, $H_B{}^M$ = measured heights of points A and B

$\bar{g}_A'$, $\bar{g}_B'$ = average value of gravity on the plumbline between the terrain point and the geoid at point A and point B;

$\bar{g}_A' = g_{A_{obs}} + 0.0424 \times H_A^M$

(H_A^M in meters for $g_{A_{obs}}$ and $\bar{g}_A'$ in milligals);

similarly for $\bar{g}_B'$

The physical interpretation of dynamic and orthometric heights is slightly different. Dynamic heights are closely related to the concept of equipotential surfaces. One may say that they reflect the geometry of the physical space surrounding us. As was stated previously, the points lying on one equipotential surface have the same dynamic height. Orthometric heights may be considered "common sense heights". Points having the same orthometric height are the same vertical distance from the geoid but do not lie on the same equipotential surface.

The size of the corrections to measured heights in the dynamic and orthometric height systems depend on the differences in gravity values along the levelling route. Usually the corrections are smaller than the accuracy of the measured height differences. In many engineering surveys including some requiring high accuracy, gravity corrections are not made. This was the case, for example, in the Snowy Mountains Scheme in Australia which contained 90 miles of trans mountain tunnels and 80 miles of aqueducts [Wasserman, 1967]. In the Orange-Fish Tunnel in South Africa it was recommended that gravity corrections be made to measured height differences primarily due to a very large gravity anomaly in the area of the tunnel [Williams, 1969].

In the simulated tunnel problem discussed in chapter 5, the effect of neglecting the gravity corrections to measured height differences is shown.

The effect of gravity on heights is generally covered best by books on physical geodesy such as Heiskanen and Moritz [1967] and Vanicek [1976]. The subject is dealt with in detail in Krakiwsky [1966] and Nassar [1977].

3.2 Horizontal Positioning

Unlike heights, generally more than one type of measurement is necessary to obtain accurate horizontal positions. The traditional surveying measurements used to determine horizontal positions are azimuths, directions, angles and distances. The corrections to these observations in reducing from terrain to ellipsoid and ellipsoid to Transverse Mercator conformal mapping plane are given in sections 3.2.1 and 3.2.2.

It will be seen in the reduction formulae that corrective terms are very often functions of the horizontal position (either grid coordinates X and Y or geodetic coordinates ϕ and λ) of the point to which the observation is made. The easiest solution to this problem is to use the raw observations (except that all spatial distances are reduced to the horizontal and all azimuths are corrected for meridian convergence; approximate corrections for these are given in sections 3.2.1.1 and 3.2.2.2) to get an approximate graphical solution for all the grid coordinates of the horizontal network.

Grid coordinates are coordinates on a mapping plane determined with respect to the coordinates of one point in the network being given arbitrary X and Y values such that the X and Y values of all other points are positive. The positive Y axis is directed north and the positive X axis is directed east.

With the approximate geodetic coordinates (ϕ_i, λ_i) of one point in the network known, the approximate geodetic coordinates of all other points can be determined by the following formulae:

$$\phi_j = \phi_i + \frac{Y_j - Y_i}{(R + H_o)} \rho'' \quad (3\text{-}6)$$

$$\lambda_j = \lambda_i + \frac{X_j - X_i}{(R + H_o) \cos \phi_i} \rho'' \quad (3\text{-}7)$$

where

$(R + H_o)$ = mean radius of the reference ellipsoid (see section 3.2.1.1)

ρ'' = seconds of arc per radian = 206265"

Before the reductions from terrain to ellipsoid and ellipsoid to conformal mapping plane are given, the relative positions of the terrain, geoid or arbitrarily chosen equipotential surface, reference ellipsoid and conformal mapping plane should be shown. Figure 3-1 shows the usual situation for a national geodetic network. The reference ellipsoid approximates the geoid but may be separated from it (as shown here) to get the best fit to the geoid for the entire country. For a national geodetic network the geoid level is the most convenient

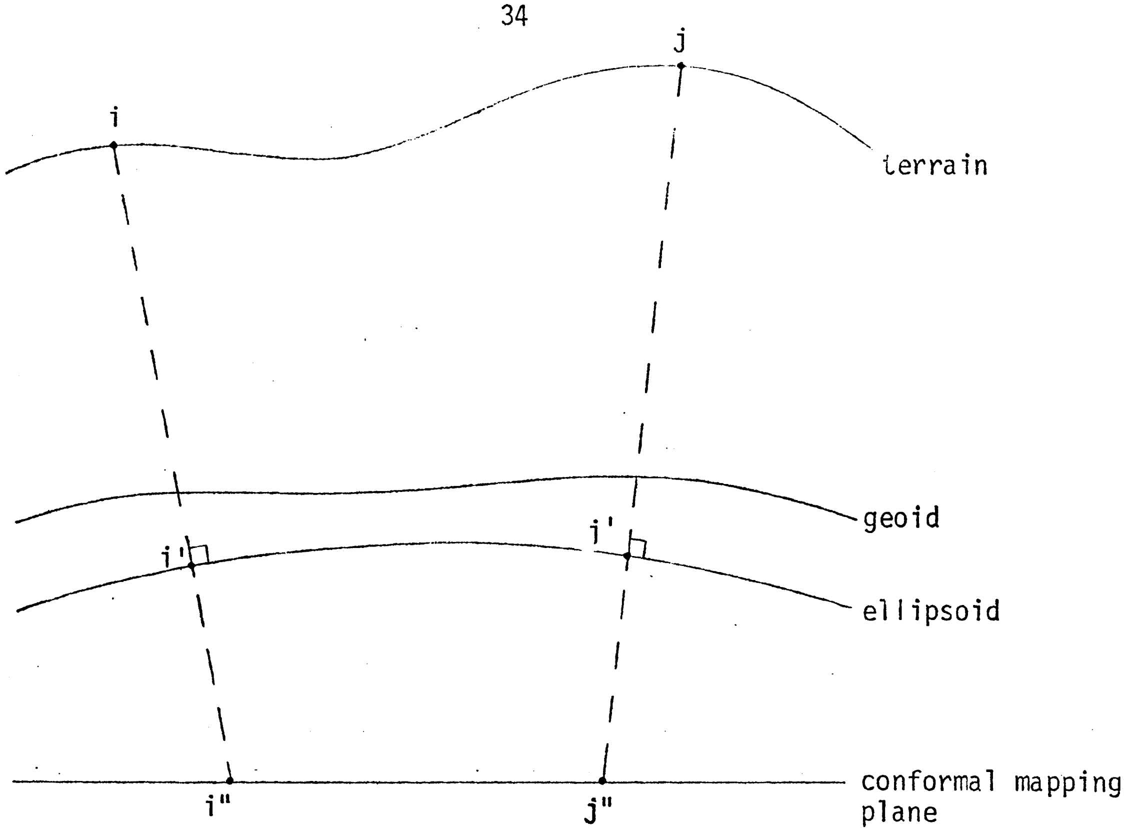


Figure 3-1

Ellipsoid and Conformal Mapping PLane for a
National Geodetic Network

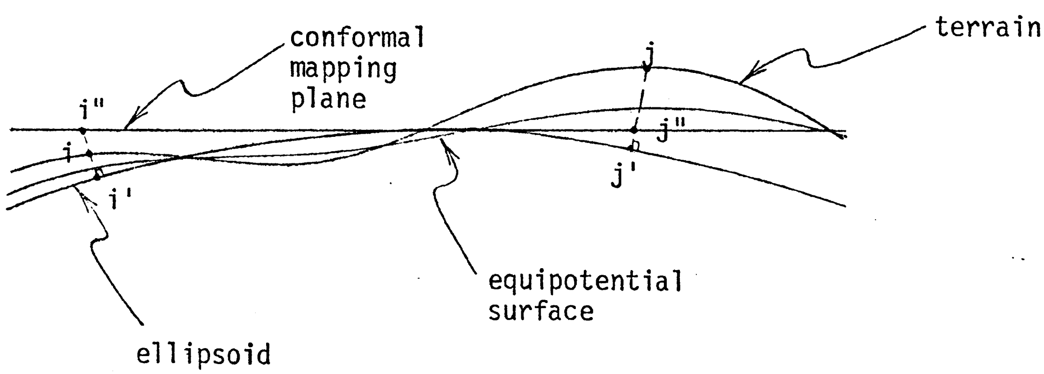


Figure 3-2

Ellipsoid and Conformal Mapping Plane for a
Local Geodetic Network

location for a reference ellipsoid. The conformal mapping plane is usually a secant plane to the reference ellipsoid in order to minimize the ellipsoid to conformal mapping plane linear scale distortion over the area of the conformal mapping plane.

For local control purposes a more convenient positioning of the surfaces is shown in Figure 3-2. The reference ellipsoid approximates an equipotential surface at the average elevation of the area. The conformal mapping plane is a tangent plane to the reference ellipsoid near the center of the area. This positioning of the surfaces minimizes the reduction corrections.

3.2.1 Reduction of Observations from Terrain to Ellipsoid

3.2.1.1 Reduction of Spatial Distances

A terrain spatial distance is reduced to the ellipsoid by the following formula (see Figure 3-3):

$$S_{ij} = 2\,(R + H_o)\ \sin^{-1}\left\{ \frac{\left[\dfrac{r_{ij}^{2} - \Delta h^{2}}{\left(1 + \dfrac{h_i}{(R+H_o)}\right)\left(1 + \dfrac{h_j}{(R+H_o)}\right)} \right]^{1/2}}{2\,(R + H_o)} \right\} \qquad (3\text{-}8)$$

where

S_{ij} = ellipsoid distance between points i and j

r_{ij} = terrain spatial distance between points i and j, corrected for instrumental effects and atmospheric refraction

Δh = difference in ellipsoid height of points i, j

h_i, h_j = ellipsoid height of points i, j

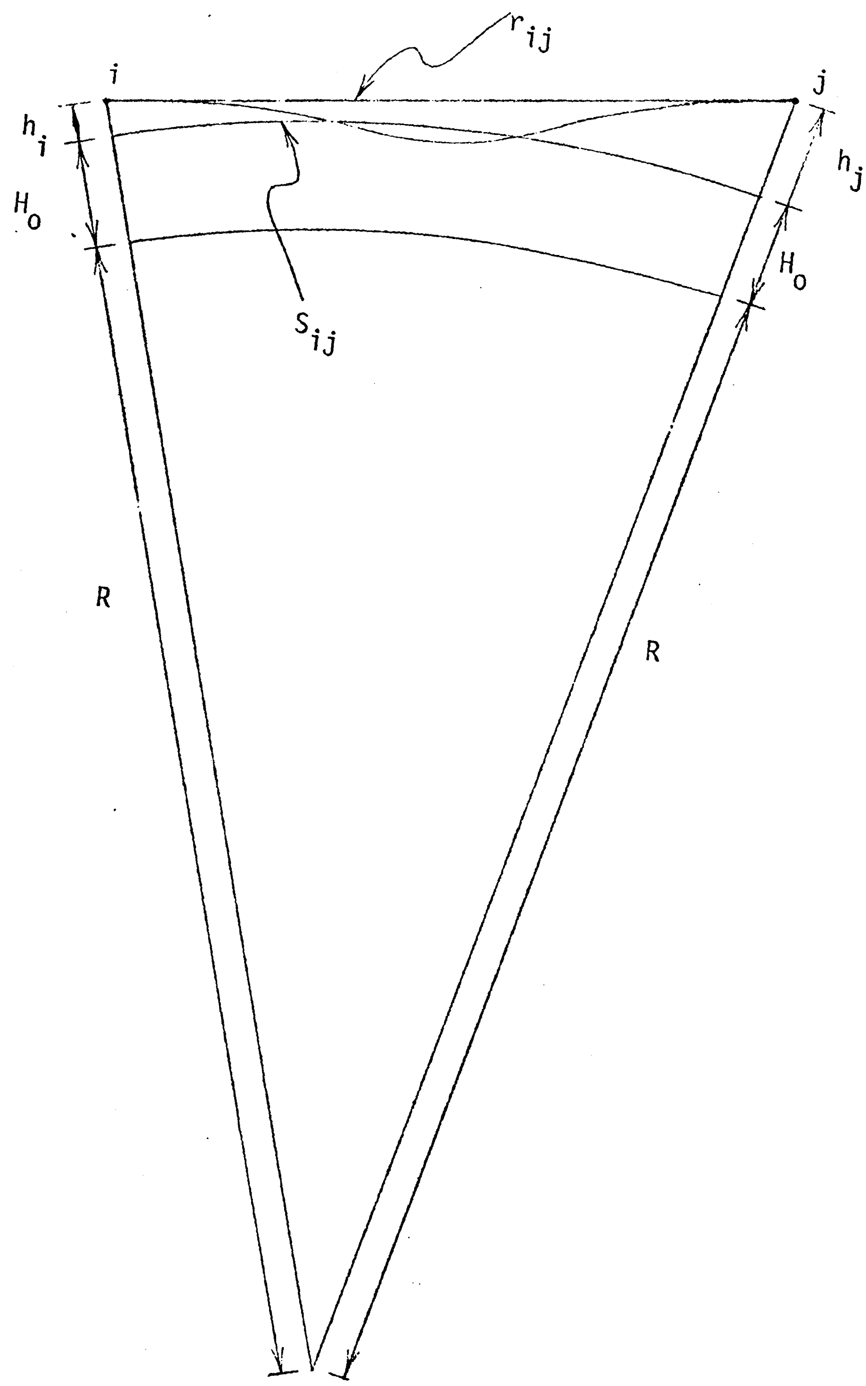


Figure 3-3

Spatial Distance Reduction

R = mean radius of the ellipsoid which best fits the geoid or mean sea level locally

$$= \frac{a(1 - 2f + f^2)^{1/2}}{1 - (2f - f^2)\sin^2\phi_m} \quad \text{and } \phi_m = \frac{\phi_i + \phi_j}{2};$$

from values of a = 6378.135 km and

$$f = \frac{1}{298.26} \text{ (see section 2.2.1)}$$

$R_{\phi = 0°}$ = 6335.438 km

$R_{\phi = 45°}$ = 6356.715 km

$R_{\phi = 90°}$ = 6378.135 km

H_o = approximate elevation of the reference ellipsoid above the ellipsoid of mean radius R; if H_o is chosen to position the reference ellipsoid at the average elevation of the area, h_i and h_j may be positive or negative.

For purposes of determining approximate coordinates on the mapping plane

$$\ell_{ij} \doteq S_{ij} \doteq r_{ij} \sin z_{ij} \qquad (3\text{-}9)$$

where

S_{ij} and r_{ij} were defined previously

ℓ_{ij} = distance on the mapping plane between points i and j

z_{ij} = zenith angle from point i to point j uncorrected for curvature of the earth and refraction.

This approximate reduction formula considers only the approximate slope correction which is generally much larger than the corrections included in the rigorous reduction formula.

3.2.1.2 Reduction of Astronomic Azimuths

An astronomic (observed) azimuth is reduced from the terrain to the ellipsoid by the following formula:

$$\alpha_{ij} = A_{ij} - \eta_i \tan \phi_i + c_1 + c_2 + c_3 \qquad (3\text{-}10)$$

where

α_{ij} = geodetic azimuth from point i to point j

A_{ij} = astronomic azimuth from point i to point j

c_1 = gravimetric correction; correction due to the deflection of the vertical at the instrument station;

$$c_1 = (-\xi_i \sin \alpha_{ij} + \eta_i \cos \alpha_{ij}) \cot z_{ij} \qquad (3\text{-}11)$$

in which ξ_i, η_i = components of the deflection of the vertical at point i

α_{ij} and z_{ij} were defined previously;

z_{ij} uncorrected for the effects of the deflection of the vertical is sufficient as a first approximation

c_2, c_3 = geometric corrections; corrections due to the positions of the instrument and target stations with respect to the reference ellipsoid; c_2 = skew normal or height-of-target correction; c_3 = normal section to geodesic correction

$$c_2 = \frac{h_j}{(R + H_o)} (2f - f^2) \sin \alpha_{ij} \cos \alpha_{ij} \cos^2 \phi_j \qquad (3\text{-}12)$$

and

$$c_3 = \frac{(f^2-2f)\, S_{ij}^2 \cos^2 \phi_m \sin 2\alpha_{ij}}{12\,(R + H_o)^2} \tag{3-13}$$

where all the terms were defined previously.

3.2.1.3 Reduction of Horizontal Directions

A horizontal direction is reduced from the terrain to the ellipsoid by the following formula:

$$d_{ij}^{e} = d_{ij}^{t} + c_1 + c_2 + c_3 \tag{3-14}$$

where

d_{ij}^{e} = horizontal direction on the ellipsoid from point i to point j.

d_{ij}^{t} = horizontal direction on the terrain from point i to point j

c_1, c_2, c_3 were defined previously.

3.2.1.4 Reduction of Horizontal Angles

A horizontal angle is reduced from the terrain to the ellipsoid by the following formula:

$$\beta_{ijk}^{e} = \beta_{ijk}^{t} + (c_1 + c_2 + c_3)_{ik} - (c_1 + c_2 + c_3)_{ij} \tag{3-15}$$

where

β_{ijk}^{e} = horizontal angle on the ellipsoid at point i from points j to k

β_{ijk}^{t} = horizontal angle on the terrain at point i from points j to k

c_1, c_2, c_3 were defined previously except that they now refer to the lines ik or ij

3.2.1.5 Magnitude of the Corrections

Figures 3-4 to 3-7 inclusive show the magnitude of corrections to observations in reducing the observations from the terrain to the ellipsoid.

Figure 3-4 shows the slope correction to a terrain spatial distance. This is generally the largest correction incorporated in the rigorous reduction formula. In this figure the approximate slope of the line of observation is given by its uncorrected zenith angle, whereas in the rigorous reduction formula the quantities r_{ij}, h_i, h_j and $(R + H_0)$ determine the slope of the line of observation.

Even without figure 3-4 it is obvious that a terrain spatial distance must be properly reduced to the ellipsoid. In the lowest order horizontal position computations the slope distance is "reduced to the horizontal", usually by formula (3-9).

Figures 3-5, 3-6 and 3-7 show the corrections c_1, c_2 and c_3 respectively.

c_1, the gravimetric correction, can easily reach a magnitude of several seconds in rugged terrain. As noted in chapter 2 and discussed in detail in chapter 4 there are many methods, including a new very simple method, to determine ξ and η so that the gravimetric correction can be applied. In chapter 5 the effects of neglecting the gravimetric correction in two different engineering surveying problems is shown.

c_2 and c_3, the geometric corrections, are very small. In most cases they will be opposite in sign and of approximately equal

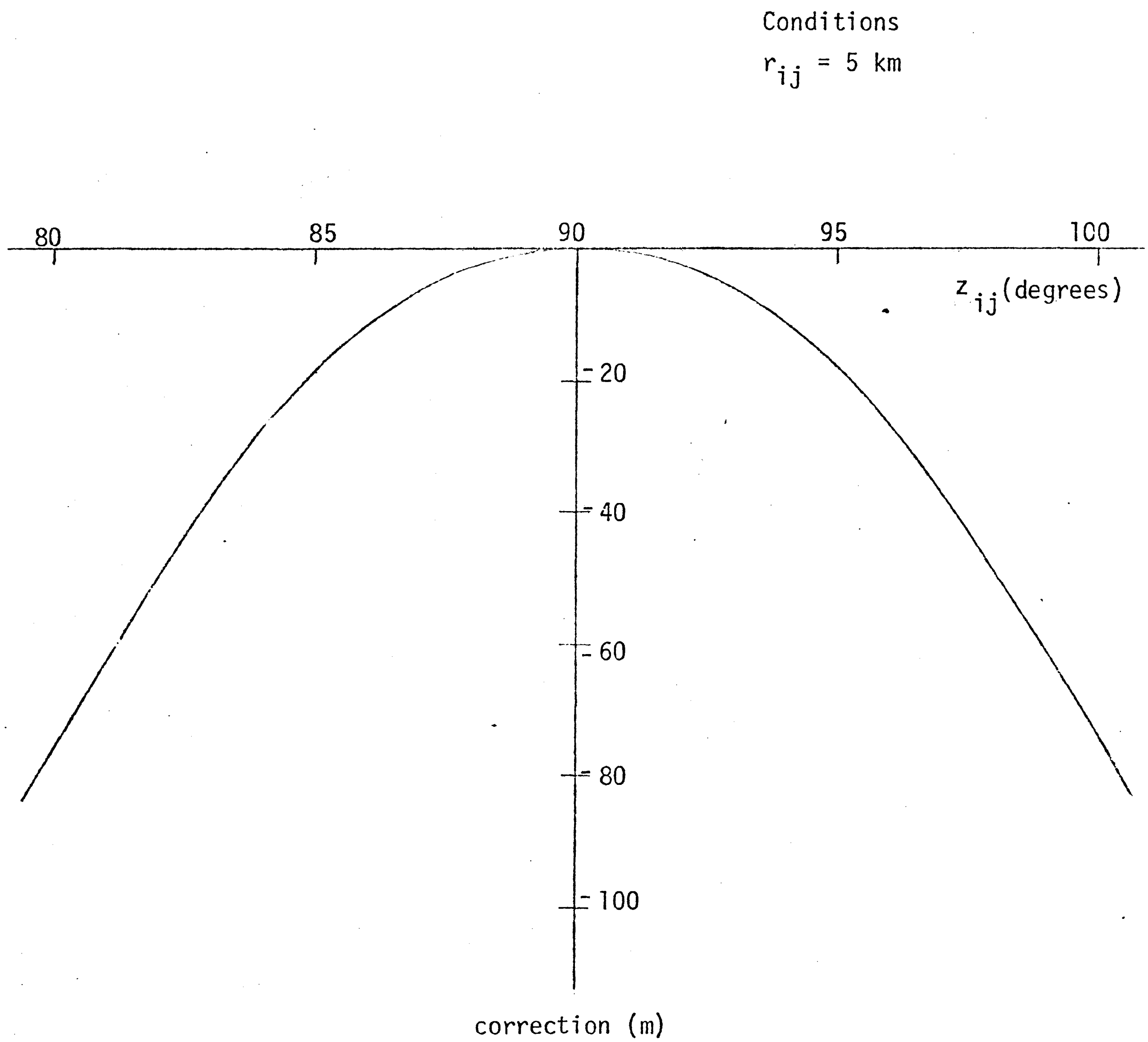


Figure 3-4

Slope Correction

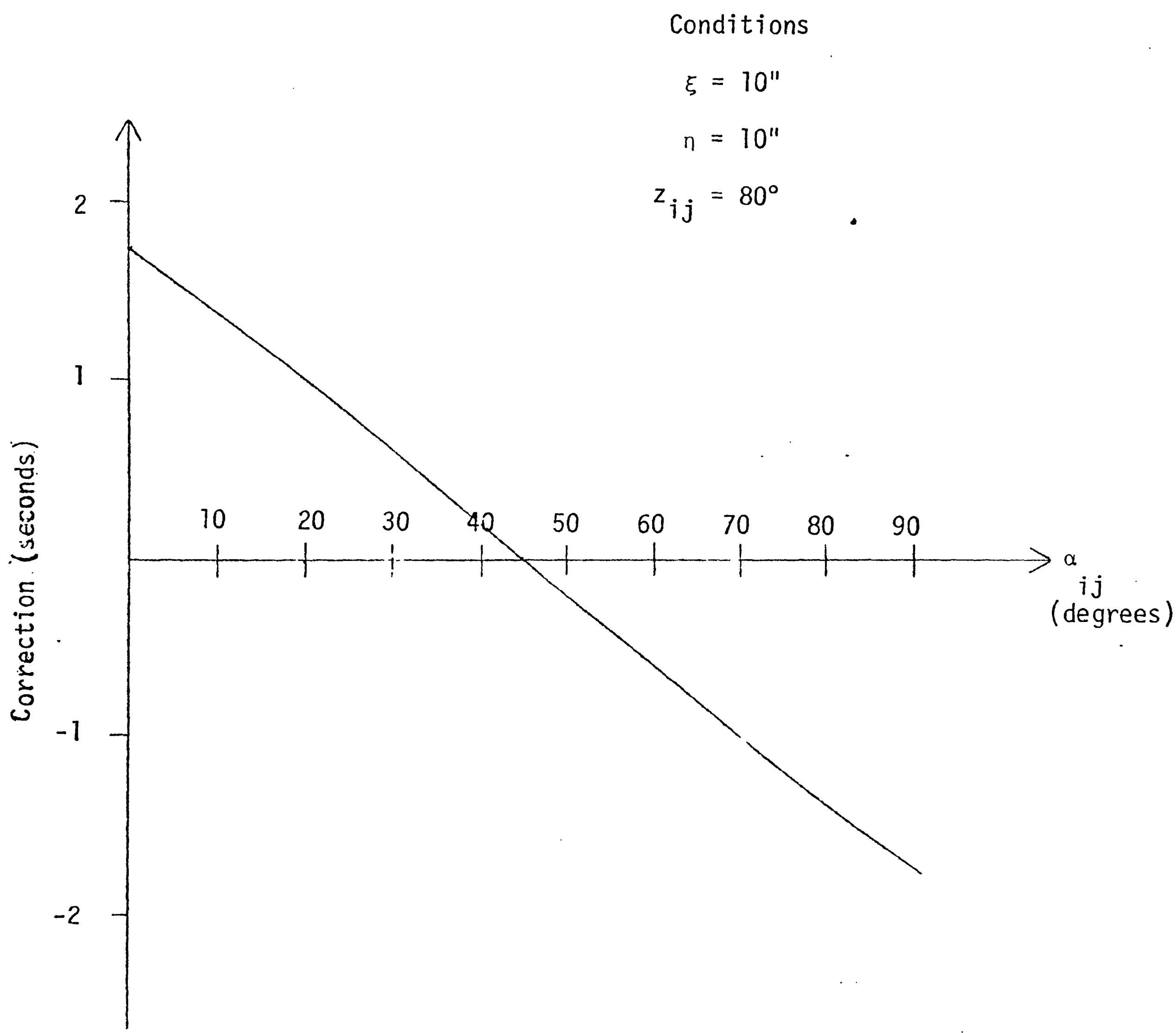


Figure 3-5

Gravimetric Correction

[Thomson et al, 1978]

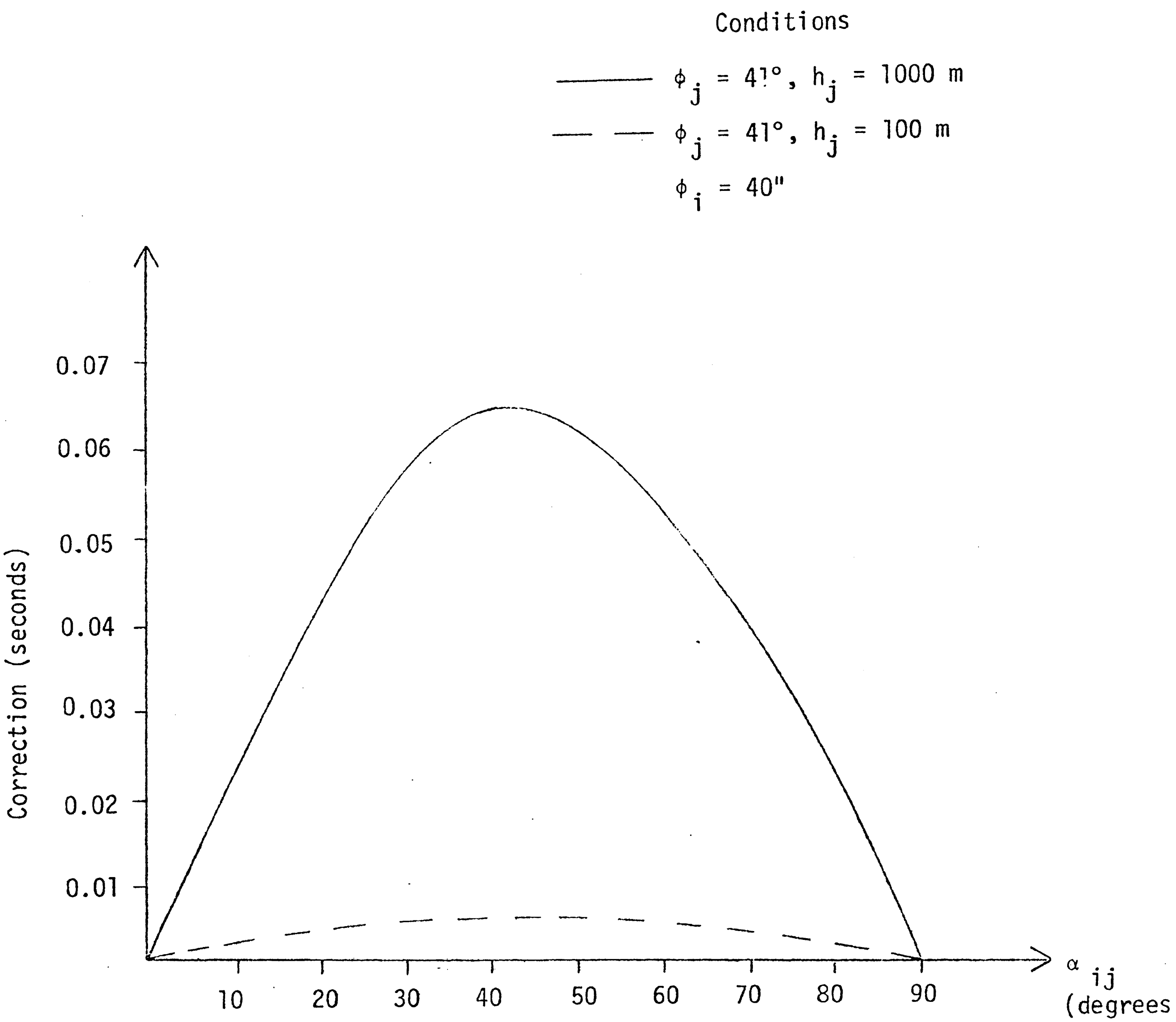


Figure 3-6

Skew Normal Correction

[Thomson et al, 1978]

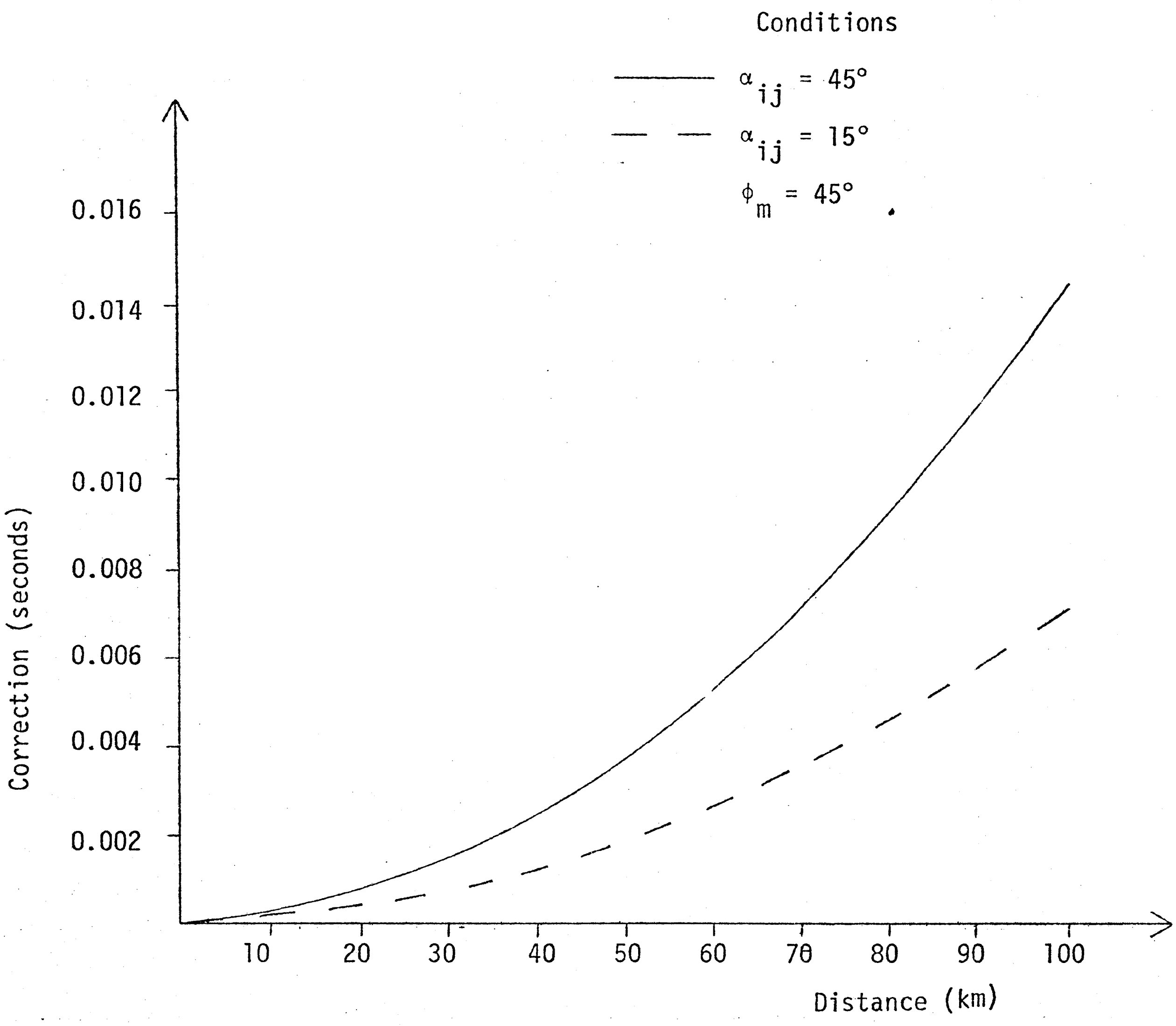


Figure 3-7

Normal Section to Geodesic Correction

magnitude so that neglecting to make these corrections would have an almost negligible effect on the accuracy of the horizontal position computations.

3.2.2 Reduction of Observations from Ellipsoid to Transverse Mercator Conformal Mapping Plane

Before giving the corrections to observations, the Transverse Mercator projection should be briefly described and the geometry of curves projected from the ellipsoid onto the mapping plane should be shown.

In the Transverse Mercator projection the scale is constant along an arbitrarily chosen central meridian. For local control purposes the central meridian should be chosen to pass near the center of the area so that reduction corrections are minimized. If the Transverse Mercator mapping plane is tangent to the reference ellipsoid along the central meridian, the scale is true at the central meridian and the scale factor at the central meridian, k_0, is equal to 1.

The origin of the Y-axis in the Transverse Mercator projection is at the equator. In order to avoid Y-coordinate values in the millions of metres, the origin can be arbitrarily shifted to the north or south as required. The origin of the X-axis in the Transverse Mercator projection is at the central meridian. In order to avoid negative X-coordinate values the origin can be arbitrarily shifted to the west; X_0 is then the x-coordinate of any point on the central meridian.

Figure 3-8, The Geometry of Projected Curves, illustrates the line scale factor $\overline{k}$, the meridian convergence γ and the (T-t) correction. The line scale factor $\overline{k}$ is the ratio of the length of the projected curve connecting two points to the length of the chord connecting the same two points. The meridian convergence γ at a point is the angle between the tangent to the projected meridian through the point, and the Y-axis. The (T-t) correction is the difference between the grid azimuth of the projected curve connecting two points and the grid azimuth of the chord connecting the same two points.

3.2.2.1 Reduction of Ellipsoid Distances

An ellipsoid distance is reduced to the Transverse Mercator conformal mapping plane by the following formula, which is accurate to 10^{-7} for lines up to 150 km in length and within 3° of the central meridian:

$$\ell_{ij} = \overline{k}_{ij} S_{ij} \tag{3-16}$$

where

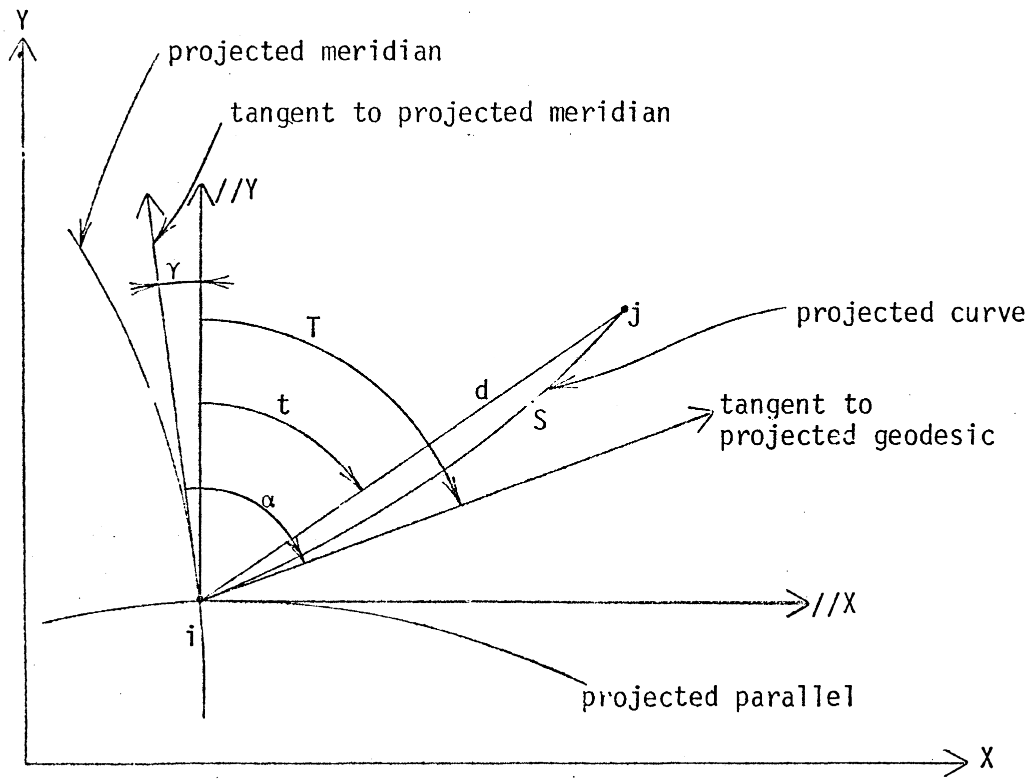
S_{ij} and ℓ_{ij} were defined previously

$\overline{k}_{ij}$ = line scale factor for the line between points i and j

and

$$\overline{k}_{ij} = k_o \left[1 + \frac{X_u^2}{6(R + H_o)^2} \left(1 + \frac{X_u^2}{36(r + H_o)^2}\right)\right] \tag{3-17}$$

in which $X_u^2 = (X_i - X_o)^2 + (X_i - X_o)(X_j - X_o) + (X_j - X_o)^2$ (3-18)



X, Y = grid coordinates
S = length of projected curve
d = chord length
$\bar{k}$ = line scale factor
α = geodetic azimuth
γ = meridian convergence
T = grid azimuth of projected curve
t = grid azimuth of chord

Figure 3-8

Geometry of Projected Curves

3.2.2.2 Reduction of Geodetic Azimuths

A geodetic azimuth is reduced to the conformal mapping plane by the following formula:

$$t_{ij} = \alpha_{ij} - \gamma_i - (T-t)_{ij} \tag{3-19}$$

where

α_{ij} was defined previously

γ_i = meridian convergence at point i

$(T-t)_{ij}$ = the (T-t) correction between points i and j

Meridian convergence for the Transverse Mercator projection is given by the following expression which is accurate to 0".01 within 3° of the central meridian:

$$\gamma_i = \Delta\lambda_i \sin\phi_i \, [1 + \frac{\Delta\lambda_i^{\,2} \cos^2\phi_i}{3(\rho")^2} (1 + 3\eta_i^{\,2} + 2\eta_i^{\,4}) + \frac{\Delta\lambda_i^{\,4} \cos^4\phi_i}{15(\rho")^4} (2 - \tan^2 \phi_i)] \tag{3-20}$$

where

$\Delta\lambda_i$ = change in longitude from the central meridian; positive east and negative west to conform to the usual sign convention for longitude

and

$$\eta_i^{\,2} = \frac{2f - f^2}{(1 - f)^2} \cos^2 \phi_i \tag{3-21}$$

The (T-t) correction for the Transverse Mercator projection is given by the following expression which is accurate to 0".02 for lines up to 100 km in length within 3° of the central meridian:

$$(T-t)_{ij} = \frac{(Y_j-Y_i)\ (X_j+2X_i-2X_o)}{6(R+H_o)^2}\ [1 - \frac{(Y_j+2X_i-2X_o)^2}{27\ (R+H_o)^2}]\ \rho'' \qquad (3\text{-}22)$$

where all the terms have been defined previously.

For purposes of determining approximate grid coordinates

$$t_{ij} \doteq A_{ij} \doteq \Delta\lambda_i \sin\phi_i \qquad (3\text{-}23)$$

where all the terms have been defined previously. This approximate reduction formula considers only the approximate meridian convergence which is generally much larger than the other corrections included in the complete reduction formula.

3.2.2.3 Reduction of Horizontal Directions

A horizontal direction is reduced from the ellipsoid to the Transverse Mercator conformal mapping plane by the following formula:

$$d_{ij}^{p} = d_{ij}^{e} - (T-t)_{ij} \qquad (3\text{-}24)$$

where

d_{ij}^{p} = horizontal direction on the mapping plane from point i to point j

d_{ij}^{e} and $(T-t)_{ij}$ were defined previously.

3.2.2.4 Reduction of Horizontal Angles

A horizontal angle is reduced from the ellipsoid to the Transverse Mercator conformal mapping plane by the following formula:

$$\beta_{ijk}^{p} = \beta_{ijk}^{e} + (T-t)_{ij} - (T-t)_{ik} \qquad (3\text{-}25)$$

where

β_{ijk}^{p} = horizontal angle on the mapping plane at point i from points j to k

β_{ijk}^{t} was defined previously

(T-t) was defined previously except that now one (T-t) term refers to line ij and the other to line ik

3.2.2.5 Magnitude of the Corrections

Figures 3-9 to 3-11 inclusive show the magnitude of corrections to observations in reducing from the ellipsoid to the Transverse Mercator conformal mapping plane.

Figure 3-9 shows the scale factor correction for the Transverse Mercator projection. The correction is proportional to the length of the line and increases approximately as the square of the distance of the line from the central meridian. At 10 km from the central meridian the correction is about 2ppm, at 50 km about 44ppm and at 100 km about 175ppm. Neglecting this correction in horizontal position computations on a plane would obviously only be possible in a very small area.

Figure 3-10 shows the meridian convergence correction for the Transverse Mercator projection. Like the slope correction to a spatial distance, this correction is so large that it is applied even in the lowest order horizontal position computations.

Figure 3-11 shows the (T-t) correction for the Transverse Mercator projection. This correction is larger than 1" only for very long lines having large north - south components.

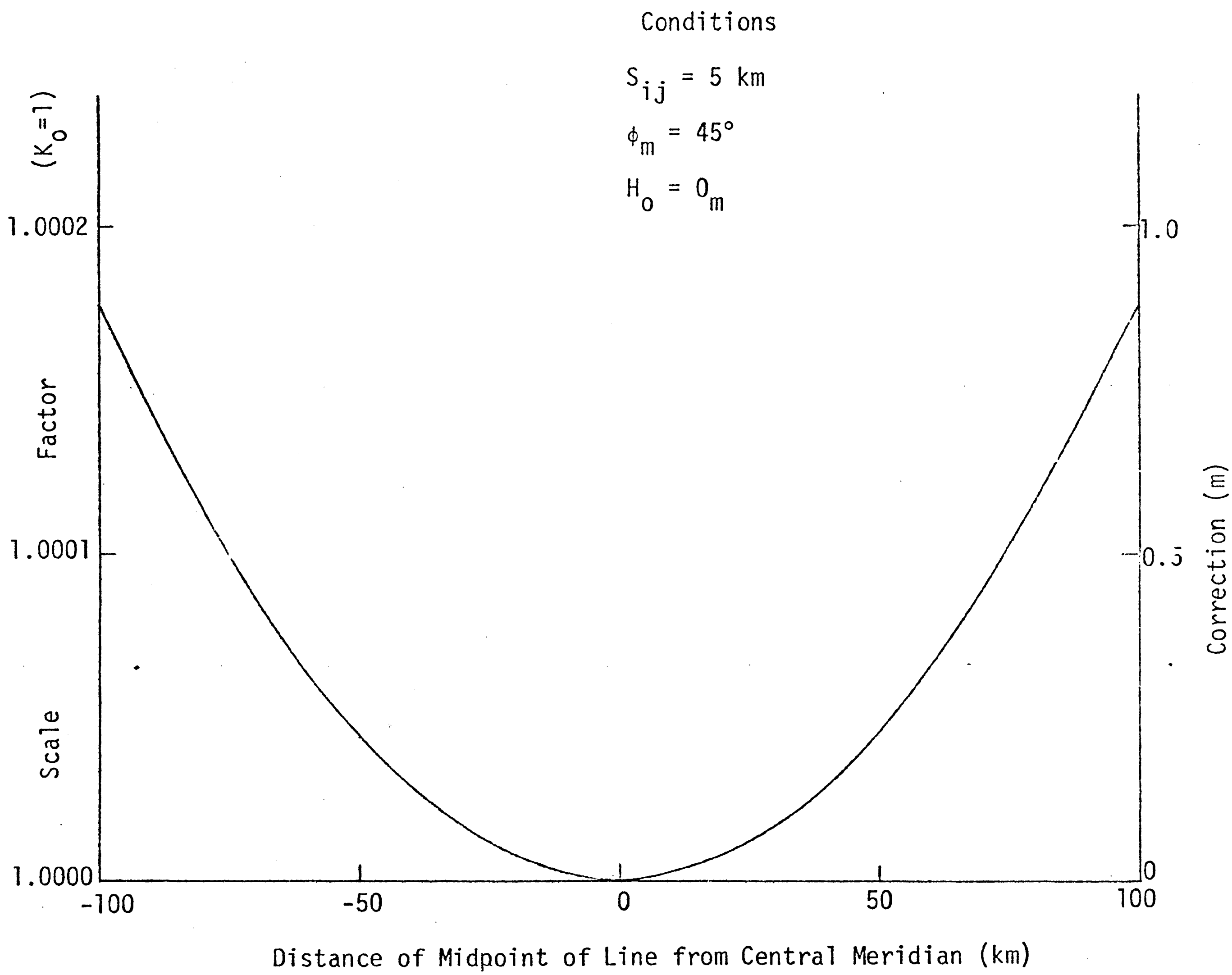


Figure 3-9

Scale Factor Correction (Transverse Mercator Projection)

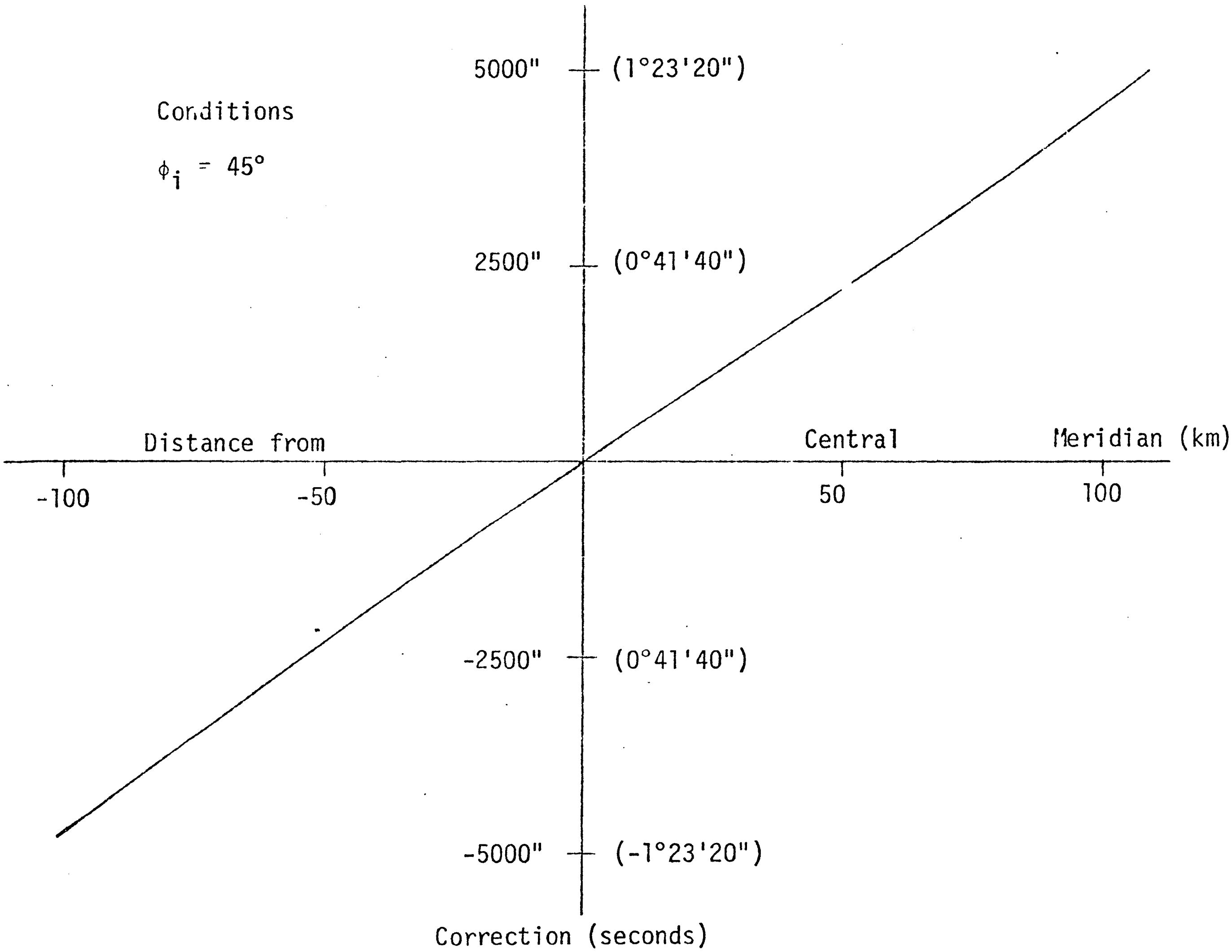


Figure 3-10

Meridian Convergence Correction (Transverse Mercator Projection)

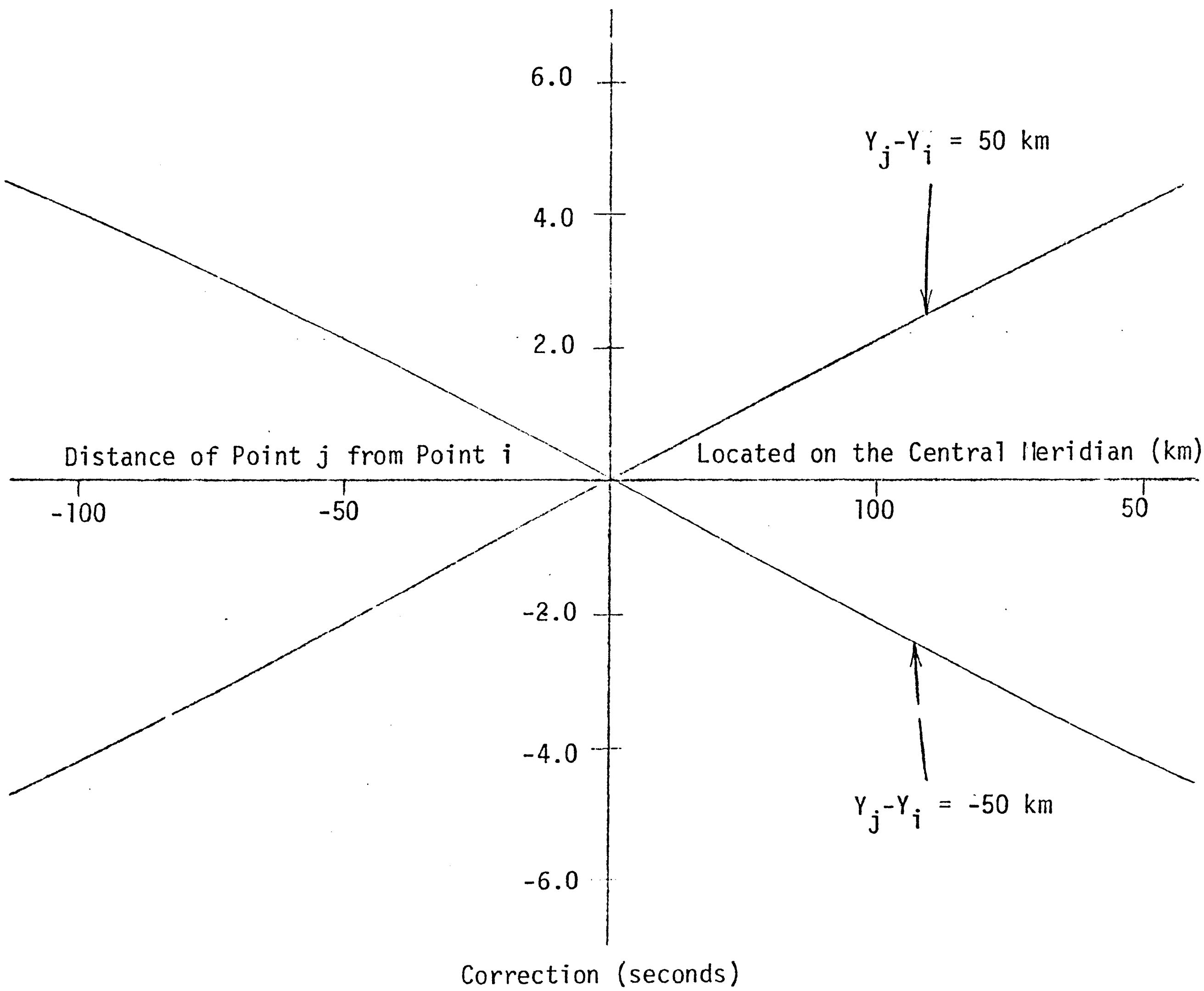


Figure 3-11

(T-t) Correction (Transverse Mercator Projection)

4. DETERMINATION OF DEFLECTIONS OF THE VERTICAL

In this chapter various methods of determining deflections of the vertical in the small area covered by an engineering survey will be considered. Unless noted otherwise, all deflections will be surface deflections rather than geoid deflections since it is the surface deflections that are required to make the gravimetric corrections to the traditional survey observations.

A chapter is devoted to this topic because of all the corrections applied to the traditional survey observations only the gravimetric correction to horizontal position observations, being a function of deflection of the vertical, is difficult to determine. All other corrections are functions of the observations themselves, the approximate positions of the ends of the lines of observation, or quantities such as the radius of the reference ellipsoid $(R + H_o)$, reference gravity G, etc.; all of which are readily available. These corrections can therefore be easily made if the accuracy of the survey requires it. The gravimetric corrections on the other hand are often neglected for no better reason than the fact that deflections of the vertical are difficult to determine. Section 4.2 describing a new simple method to determine deflection of the vertical will show that this no longer has to be the case.

Parameters describing the earth's gravity field, one of which is deflection of the vertical, are most often used to determine the general shape of the geoid over a large area. For purposes of national geodetic control this is sufficient. However, for purposes of local geodetic control, for example engineering surveys requiring high accuracy, local variations in the earth's gravity field may have to be considered. For this reason, in this chapter only methods having a resolution sufficient to determine a change in deflection of the vertical in the order of 2" in 5 km will be considered.

In the following sections seven methods to determine deflection of the vertical, which meet the resolution criteria, will be discussed. The existing methods will be discussed only briefly. The astrogeodetic difference method will be discussed in detail, and field test results from the trigonometric method and the astrogeodetic difference method will be presented and evaluated.

4.1 Review of Existing Methods

4.1.1 Trigonometric Method

This method through which change in deflection of the vertical is determined is well known, but it is seldom used because the uncertainty associated with vertical refraction makes it difficult to determine the accuracy of the result. Only in mountainous areas where atmospheric conditions are stable and lines of observation are high above the ground does the method appear to give satisfactory results [Hradilek, 1968].

When trigonometric levelling is used to determine change in deflection it is usually done within a three-dimensional adjustment in which ξ and η of each point are solved for as unknown parameters together with the three-dimensional coordinates of each point. The use of trigonometric levelling data in a three-dimensional adjustment is in fact the only new development associated with this method since problems with the method were outlined by Kobold [1956].

A unique solution for change in deflection of the vertical along the line connecting two points can be made if accurate trigonometric and spirit levelled height differences are available and if the coefficient of refraction is assumed to be the same at each end of the line.

From simultaneous reciprocal trigonometric levelling [Chrzanowski, 1978],

$$\Delta h_{AB} = S_s \sin \frac{z_B - z_A}{2} \tag{4-1}$$

where

Δh_{AB} = difference in trigonometric height between points A and B

S_s = slope distance between points A and B

z_A, z_B = simultaneous reciprocal zenith angles at points A and B corrected for deflections of the vertical.

If ΔH_{AB} is the spirit levelled height difference between points A and B, then $(S_s \sin \frac{z_B - z_A}{2} - \Delta H_{AB})$ is the separation, at point B, of the reference ellipsoid passing through point A and the equipotential surface passing through point A.

This separation is related to the change in deflection of the vertical along the line between the two points by [Vanicek and Krakiwsky, in prep]

$$\Delta\varepsilon = - \frac{2\Delta N}{S_o} \qquad (4\text{-}2)$$

where

$\Delta\varepsilon$ = change in deflection of the vertical

ΔN = difference in ellipsoid - equipotential surface separation

S_o = ellipsoid distance between the points.

Substituting for N, and expressing $\Delta\varepsilon$ in seconds,

$$\Delta\varepsilon_{AB} = \frac{2(\Delta H_{AB} - S_s \sin \frac{z_B - z_A}{2})}{S_o} \rho'' \qquad (4\text{-}3)$$

where $\Delta\varepsilon$ positive means that the change in deflection of the vertical is outward along the line joining the points. To determine $\Delta\varepsilon_{AB}$ from this formula iteration is required because $\Delta\varepsilon_{AB}$ appears on both sides of the equation: explicitly on the left hand side, and as a correction to observed zenith angles on the right hand side.

As was noted previously, when the difference in trigonometric and spirit levelled height differences is assumed to be due only to a change in the deflection it is necessary to assume that the coefficient or refraction is the same at each end of the line of observation. The coefficient of refraction k in this case is [Chrzanowski, 1978].

$$\frac{R}{S_o \rho''} [180° - (z_A + z_B)] + 1 \qquad (4\text{-}4)$$

where z_A, z_B, S_o and ρ'' were defined previously and

R = mean radius of the reference ellipsoid.

A completely different approach can be taken in which the difference in trigonometric and spirit levelled heights is assumed to be due to only a difference in coefficient of refraction at each end of the line of observation. The necessary assumption now is that there is no change in deflection from one point to another. The coefficient of refraction from point A to point B, k_{AB}, in this case is [Brunner, 1977]

$$\frac{2R\,(S_s \cos z_{AB} - \Delta H_{AB})}{S_s{}^s \sin^2 z_{AB}} + 1 \qquad (4\text{-}5).$$

A similar expression can be written for the coefficient of refraction from point B to point A, k_{BA}.

4.1.2. Astrogeodetic Method

This method was mentioned previously in the context of establishment of a horizontal geodetic datum. (See section 2.2.1.)

The formula for the components of the deflection of the vertical are

$$\xi = \Phi - \phi \qquad (4\text{-}6)$$

$$\eta = (\Lambda - \lambda) \cos \phi \qquad (4\text{-}7)$$

where

(ξ, η) = components of deflection of the vertical, north-south and east-west respectively

(Φ, Λ) = astronomic coordinates, latitude and longitude respectively

(ϕ, λ) = geodetic coordinates, latitude and longitude respectively.

This is the classical method of determining deflection of the vertical, and should provide the highest accuracy. (See Section 4.3.2.) The only disadvantage of this method is that laborious and costly 2nd order astronomic observations are required.

4.1.3 Gravimetric Method

This method uses gravity anomalies to determine deflections of the vertical. The conversion of gravity anomalies to deflection of the vertical is by the well known Vening-Meinesz formulae:

$$\begin{Bmatrix} \xi(\phi_A, \lambda_A) \\ \eta(\phi_A, \lambda_A) \end{Bmatrix} = \frac{1}{4\pi G} \iint_E \Delta g(\phi, \lambda) \begin{Bmatrix} \cos \alpha \\ \sin \alpha \end{Bmatrix} \frac{d\, S(\psi)}{d\psi} \, d\nu \qquad (4\text{-}8)$$

where

(ϕ_A, λ_A) = point of interest

(ϕ, λ) = running or dummy point

G = mean value of gravity on the surface of the reference ellipsoid

= 9.8 m/s^2

Δg = gravity anomaly

α = geodetic azimuth between (ϕ_A, λ_A) and (ϕ, λ)

$\frac{d\, S(\psi)}{d\psi}$ = Vening-Meinesz funtion; a known function of spherical distance

ν = solid angle between (ϕ_A, λ_A) and (ϕ, λ)

Often gravimetric deflections are only used to interpolate deflections between points at which astrogeodetic deflections have been determined. This approach is known as the astrogravimetric method.

Deflections from gravity anomalies are deflections at the geoid with respect to a geocentric reference ellipsoid. Small corrections would have to be made for curvature of the plumbline (the difference between geoid and surface deflection - see section 2.2.1), the datum shift and what is known as the indirect effect (see Vanicek and Krakiwsky [in prep]) to obtain surface deflections with respect to a nongeocentric reference ellipsoid. In addition to this, the theoretical requirement of the Vening-Meinesz formulae is that Δg must be given continuously over the entire earth. However, because the gravimetric deflection is often used only for interpolation, the Vening-Meinesz integration can be carried out over a small area (say 100 km radius) in the region of interest to obtain incomplete gravimetric deflections. These incomplete gravimetric deflections (which are also uncorrected for curvature of the plumbline, datum shift and the indirect effect) differ from the correct ones by an almost constant amount and thus are adequate to obtain accurate interpolated values of deflection between points of known astrogeodetic deflection .

The interpolation between points of known astrogeodetic deflection can be linear [Molodenskii, et al , 1962] but this results in a loss of information since deflections are inherently two dimensional. The loss of information is overcome by a method developed at the University of New Brunswick [Merry, 1975] which uses a two dimensional surface to interpolate between points of known astrogeodetic deflection.

One disadvantage of the gravimetric or astrogravimetric methods is that large amount of gravity data is required. A second disadvantage is the computational complexity of the method. Access to a large computer would be essential if these methods were to be used.

4.1.4 Topographic Method

Deflections, at a local level, are highly correlated with topography; as a result variations in deflections in rugged terrain are due almost entirely to the topography.

The formulae for the deflection components in a rectangular coordinate system expressed as a function of adjacent masses is given by Fischer [1974]:

$$\begin{Bmatrix} \xi \\ \eta \end{Bmatrix} = -\Sigma \frac{Km}{Gs^3} \begin{Bmatrix} Y_m - Y_A \\ X_m - X_A \end{Bmatrix} \tag{4-9}$$

where

K = universal gravitational constant = $6.81 \times 10^{-11}\ m^3 kg^{-1} s^{-2}$

G = mean value of gravity on the surface of the earth = $9.8\ m/s^2$

m = mass of an adjacent unit of mass

$(Y_m - Y_A)$, $(X_m - X_A)$ = horizontal distance from the centroid of the unit of mass to the point of interest.

Numerical integration of the formulae is required but it can be readily seen that the effect of distant masses decreases rapidly because of the s^3 term in the denominator. When used to determine deflections on an atoll in the Pacific Ocean [Fischer, 1974] an integration distance of several hundred kilometers proved to be adequate. The assumption of a flat earth, implicit in the formulae, was also adequate. If topographic deflections were used to interpolate between known astrogeodetic deflections, in the same way that incomplete gravimetric deflections are used in the astrogravimetric method, a much shorter integration distance would be satisfactory.

The serious disadvantage of this method is that the density distribution of the earth in the vicinity of the point of interest should be known. In very rugged terrain however, where the topography itself rather than the density distribution within the topography has the predominant effect, the method produces accurate results. On the Pacific atoll referred to previously a standard deviation of the difference in deflection (between astrogeodetic deflections and topographic deflections based on a simple density distribution model) of 1.″5 was obtained for

23 deflections ranging from about -30" to +30". In the horizontal control network for the 20 km long Simplon tunnel in Switzerland deflections were calculated by this method using only the visible mountain masses. [Richardus, 1974].

As an illustration of the general effect of topography on deflection consider the following example. Figure 4-1 shows hills having slopes of 5%, 10%, 50% and 100%; all 2 km long at the base.

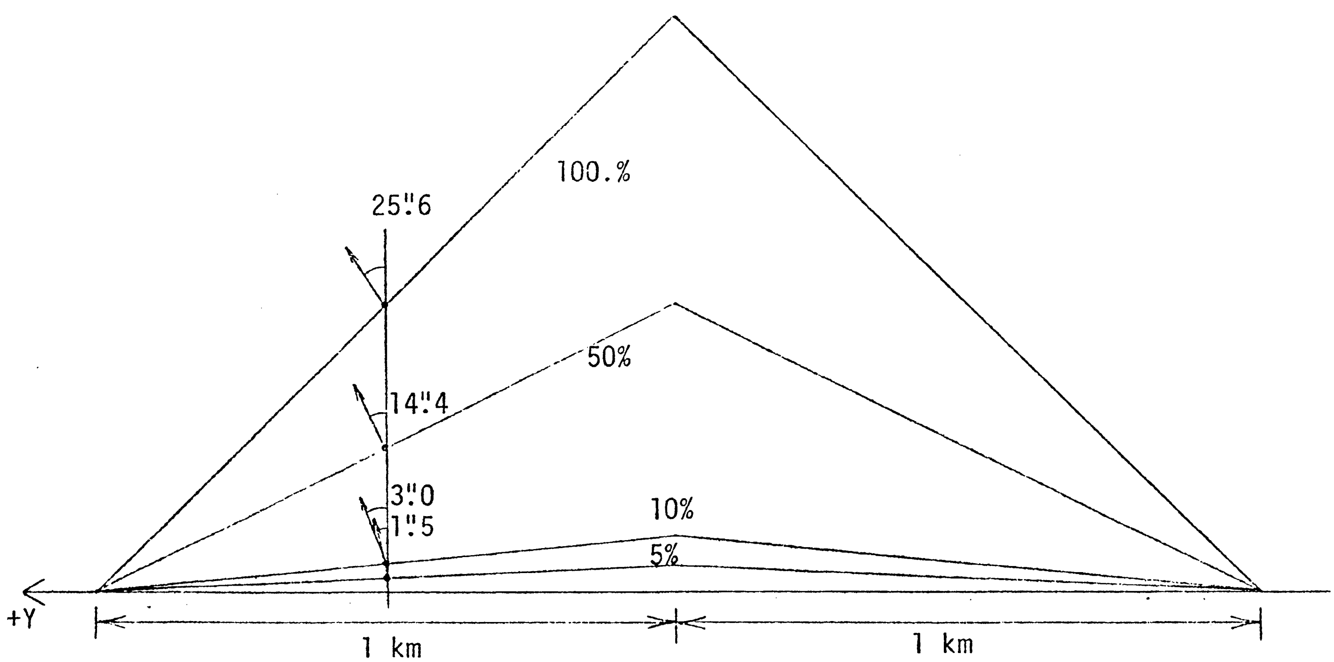


Figure 4-1 General Effect of Topography on Deflection of the Vertical

The hills are also 2 km long in the plane of the page and have the same cross section from one end to the other. The material in the hills has a uniform density of 2670 kg/m^3. The deflection of a point half way up to the north side of each hill with respect to zero deflection at a point not on the hill or a point at the top of the hills is 1".5 for the hill of 5% slope, 3".0 for the hill of 10% slope, 14".4 for the hill of 50% slope and 25".6 for the hill of 100% slope. All deflections are "downhill".

4.1.5 Combined Method (Least Squares Collocation Method)

A method of this type combines heterogenous data through least squares collocation to produce an optimal solution. By a combined method it is possible to compensate certain disadvantages in one type of data by advantages inherent in data of another type, and to interpolate numerically between discrete observations.

The method of Lachapelle [1975] which combines astrogeodetic deflections, gravity anomalies and low degree geopotential coefficients is a method of this type. The low degree geopotential coefficients provide the general features of the geoid and define a reference surface. The finer features are provided by astrogeodetic deflections which are accurate but widely spaced, and gravity anomalies which are usually abundant on land but sparse in the oceans. The solution for deflection at any point is given on the geoid with respect to a geocentric reference ellipsoid. The solution can be given with respect to any nongeocentric reference ellipsoid if its datum shift parameters are known. For a complete description of this method see Lachapelle [1975].

The disadvantages of this method are the same as those for the astrogravimetric method, that is data and computational requirements, only more so.

4.1.6 Inertial Method

An inertial survey system (ISS) has two types of main sensors, gyroscopes and accelerometers. The gyroscopes maintain the alignment of the ISS, and from the accelerometers a change in position can be determined by double integration of acceleration over time. This is the basic concept on an ISS; an actual ISS is a complex electromechanical device with many error sources. Adams [1979] describes the local level ISS and simulates position errors of the system caused by accelerometer bias and gyro drift.

The primary use of an ISS has been to determine horizontal positions between 2nd order points. Because of the sensors in an ISS, changes in deflection of the vertical can also be measured. Figure 4-2 shows the basic concept. In this figure, ε is the deflection, g is the gravity vector and γ is the vector normal to the reference ellipsoid. Accuracies of the order of 2" have been reported [Todd, 1978] but there are apparently systematic errors due to the filtering procedure [Schwarz, 1978].

ISS's are very complex and barely beyond the prototype stage but the speed and ease with which changes in deflection can be determined may offer advantages in certain applications. The method, if not

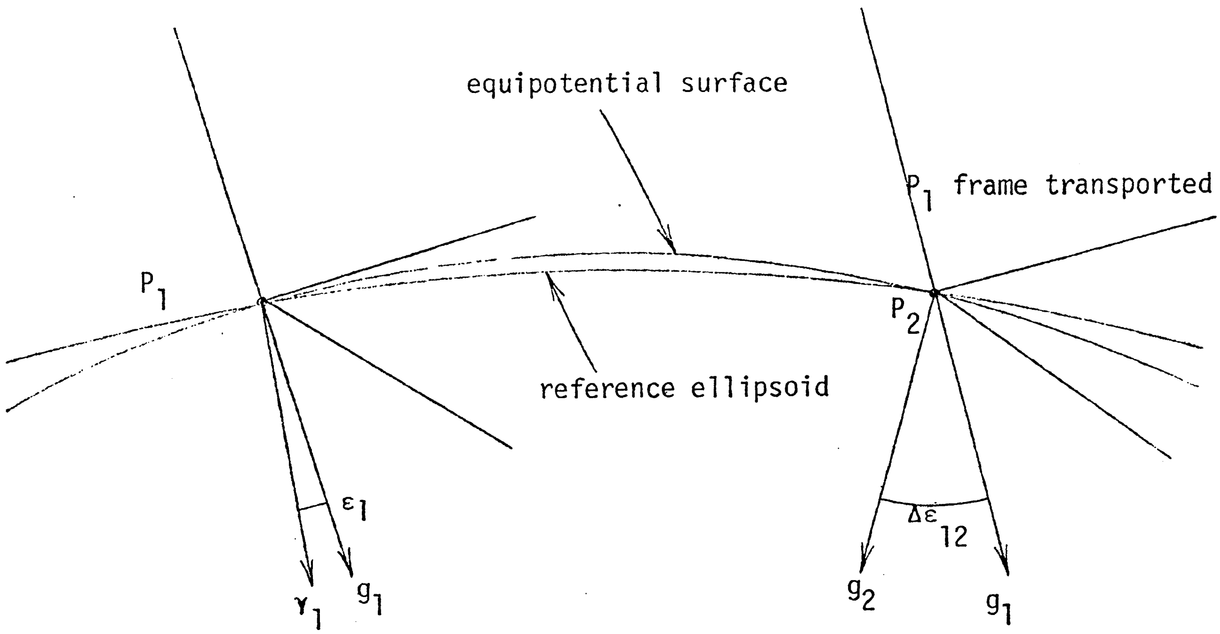


Figure 4-2

Basic Concept of Measurement of Change in Deflection by an ISS

already economically competitive with other methods to determine changes in deflection, will certainly be so in the future as ISS's become more generally available.

4.2 Astrogeodetic Difference Method

4.2.1 Description

This method to determine change in deflection of the vertical has been developed by the author, and to his best knowledge, it has never before been used.

The method is based on relative rather than absolute astronomic observations; that is it is a difference method. Accuracies of the order of 1" appear to be attainable using two 1" Theodolites with automatic vertical circle compensators and two-way radio communication so that a timer can give a simultaneous read signal to both observers. The reason that the high accuracy is attainable is that, as with other difference methods, a difference of two observables can be measured more accurately than either observable itself. By differencing, the effect of common systematic errors is eliminated.

$\xi = \Phi - \phi$ and $\eta = (\Lambda - \lambda) \cos \phi$ are the expressions for the meridian or north-south, and prime vertical or east-west deflections of the vertical respectively, where (Φ, Λ) are astronomic latitude and longitude and (ϕ, λ) are geodetic latitude and longitude. After differentiation, the expressions become

$$d\xi = d\Phi - d\phi \tag{4-10}$$

and

$$\begin{aligned} d\eta &= (d\Lambda - d\lambda) \cos \phi - (\Lambda - \lambda) \sin \phi \, d\phi \\ &\doteq (d\Lambda - d\lambda) \cos \phi \end{aligned} \tag{4-11}$$

For small changes in ξ and η, d can be replaced by Δ. It would also be permissible to drop the term $(\Lambda-\lambda) \sin \phi\Delta\phi$ for determinations of $\Delta\eta$ in which $\Delta\phi$ were small. For example, with $(\Lambda - \lambda) = 20"$, $\phi = 45°$, $\Delta\phi = 1000"$ (32 km$\pm$), the error in neglecting the second term would be only +0."1.

To determine the differences $\Delta\phi$ and $\Delta\lambda$ various methods can be used. For $\Delta\phi$ and $\Delta\lambda$ to be accurate to 0."1, ΔY and ΔX have only to be accurate to about 3 m and 2 m respectively at $\phi = 45°$. $\Delta\phi$ and $\Delta\lambda$ can therefore be determined from a large scale map, if it is available, or from any horizontal position determination giving a relative positional accuracy of about 2 m.

$\Delta\Phi$ and $\Delta\Lambda$ in the math models for $\Delta\xi$ and $\Delta\eta$ are determined by simultaneous astronomic zenith angles.

By applying spherical trigonometry to the astronomic triangle, or equivalently, by transforming between the horizon and hour angle celestial coordinate systems through the use of rotation matrices the following expression can be obtained [Mueller, 1969]:

$$\cos z = \sin \delta \sin \Phi + \cos \delta \cos h \cos \Phi \qquad (4\text{-}12)$$

where

z = zenith angle to the star

δ = declination of the star as tabulated by a star catalogue

Φ = astronomic latitude of the point of observation

h = hour angle of the star

After differentiation with respect to the observed quantities z, Φ and h, and simplification (see Appendix I), the expression becomes

$$\Delta\Phi = - \sec A \ \Delta z - \cos \Phi \tan A \ \Delta h \qquad (4\text{-}13)$$

This expression is used in Mueller [1969] to show the effect of small systemmatic errors in the measurements of zenith angle and hour angle on the determination of astronomic latitude. When the expression is used to determine $\Delta\Phi$ between two points at which simultaneous observations of zenith angle of the same star are made, then $\Delta\Phi = - \sec A \ \Delta z$ since $\Delta h = 0$. Further, if the star is chosen such that $A \doteq 0$, then $\Delta\Phi \doteq - \Delta z$.

The ideal choice of a star for the determination of $\Delta\Phi$ in the Northern hemisphere is obviously Polaris (α Ursae Minoris) since its azimuth at any time is very small. To illustrate this, consider the choice of Polaris for the determination of $\Delta\Phi$ at $\Phi = 45°$. The azimuth of a star is maximum at elongation, and [Mueller, 1969]

$$A_{elongation} = \sin^{-1} (\cos \delta \sec \Phi) \qquad (4\text{-}14)$$

For Polaris,

$$\delta \doteq 89° \, 10' \text{ (1979), and}$$

$$A_{elongation} = \pm 1°.2, \ \sec A = 1.0002$$

Therefore, if Polaris is chosen for the determination of $\Delta\Phi$ at $\Phi = 45°$, the math model $\Delta\Phi = -\Delta z$ can be used in place of $\Delta\Phi = -\sec A \, \Delta z$ with a maximum error of $-0''.2$ in $1000''$.

In order to determine $\Delta\Lambda$ from simultaneous astronomic zenith angles, the expression for zenith angle, $\cos z = \sin \delta \sin \Phi + \cos \delta \cos h \cos \Phi$, is again used except that astronomic longitude Λ is entered into the expression by making the substitution

$$h = \Lambda - \alpha + T \qquad (4\text{-}15)$$

where

h and Λ were defined previously

α = right ascension of the star as tabulated by a star catalogue

T = time

After differentiation with respect to the observed quantities z, Φ, Λ and T, and simplification (see Appendix I), the expression becomes

$$\Delta\Lambda = -\sec \Phi \cot A \, \Delta\Phi - \sec \Phi \operatorname{cosec} A \, \Delta z - \Delta T \qquad (4\text{-}16)$$

This expression is used in Mueller [1969] to show the effect of small systematic errors in the measurements of astronomic latitude, zenith angle and time on the determination of astronomic longitude. When the expression is used to determine $\Delta\Lambda$ between two points at which simultaneous observations of zenith angle of the same star are made, then $\Delta\Lambda = -\sec\Phi \cot A \;\; \Delta\Phi - \sec\Phi \operatorname{cosec} A \; \Delta z$ since $\Delta T = 0$.

The appropriate choice of a star for the determination of $\Delta\Lambda$ is one for which $A \doteq \pm 90°$, that is a star near east or west prime vertical crossing, since with $A = \pm 90°$, $\cot A = 0$, $\operatorname{cosec} A = 1$ and $\Delta\Lambda = -\sec\Phi \; \Delta z$. The last expression shows only the predominant term in the determination of $\Delta\Lambda$; to avoid large errors the complete expression must be used in which $\Delta\Phi$ was determined previously, Φ is the approximate astronomic latitude and [Mueller, 1969]

$$A = \cos^{-1} \left(\frac{\sin\delta - \cos z \sin\Phi}{\sin z \cos\Phi}\right) \qquad (4\text{-}17)$$

where

δ and Φ were defined previously

$$z = \frac{z_1 + z_2}{2} + \text{refraction correction}$$

In the actual determination of $\Delta\Lambda$, the star chosen is one near east or west prime vertical crossing at the time of observation. To positively identify the star, the approximate azimuth, zenith angle and time are used to compute the approximate declination δ and right ascension α of the star. From [Mueller, 1969]

$$\delta = \sin^{-1} (\cos A \sin z \cos \Phi + \cos z \sin \Phi) \qquad (4\text{-}18)$$

$$\alpha = \Lambda - h + UT + R; \quad h = \cos^{-1} \left(\frac{\cos z - \sin \delta \sin \Phi}{\cos \delta \cos \Phi}\right) \qquad (4\text{-}19)$$

where

δ, A, z, Φ, α, Λ, and h were defined previously

UT = universal time

$R \doteq \alpha_m - 12^h$

in which α_m = right ascension of the mean sun.

R or α_m are tabulated in star catalogues. With the star identified, the tabulated value of δ is used in the formula for azimuth.

The description of the determination of $\Delta\Phi$ and $\Delta\Lambda$ is complete except for the application of the differential refraction correction. In order to make this correction pressure and temperature are measured at both points before and after each set of observations. The differential refraction correction is then applied to Δz by using refraction tables, for example those provided in Mueller [1969].

Summarizing the method of determining change in deflection of the vertical by the astrogeodetic difference method;

$$\Delta\xi = \Delta\Phi - \Delta\phi \qquad (4\text{-}10 \text{ repeated})$$

where

$\Delta\phi$ is determined approximately (to 0".1) by any suitable method,

$\Delta\Phi$ (for Polaris) = $- \Delta z$ + diff. refr. corr.; (4-20)

$$\Delta\eta = (\Delta\Lambda - \Delta\lambda)\cos\phi \qquad \text{(4-11 repeated)}$$

where

$\Delta\lambda$ is determined approximately (to 0".1) by any suitable method,

$$\Delta\Lambda = -\sec\Phi\cot A\,\Delta\Phi - \sec\Phi\,\mathrm{cosec}\,A\,(\Delta z + \text{diff. refr. corr.}), \qquad (4\text{-}21)$$

in which $\Delta\Phi$ was determined previously and A is calculated from z, Φ, and δ of the known star near prime vertical crossing.

An HP-29C program for the determination of $\Delta\Lambda$ from Δz is given in Appendix II.

4.2.2 A Priori Error Analysis

Applying the law of propagation of errors to the math model for $\Delta\xi$,

$$\sigma^2_{\Delta\xi} = \sigma^2_{\Delta\phi} + \sigma^2_{\Delta\Phi} \qquad (4\text{-}22)$$

If the error in $\Delta\phi$ is assumed to be small in comparison to the error in $\Delta\Phi$, then

$$\sigma^2_{\Delta\xi} = \sigma^2_{\Delta\Phi} \qquad (4\text{-}23)$$

Applying the law of propagation of errors to $\Delta\Phi = -\Delta z$ + differential refraction correction,

$$\sigma^2_{\Delta\Phi} = \sigma^2_{\Delta z} + \sigma^2_{\Delta r} \qquad (4\text{-}24)$$

where

Δr = differential refraction correction

and

$$\sigma^2_{\Delta\xi} = \sigma^2_{\Delta z} + \sigma^2_{\Delta r} \qquad (4\text{-}25)$$

In the measurement of zenith angles there are three errors; the error in reading the zenith angle in the readout system, the error in pointing the vertical crosshairs of the instrument at the target and the error in levelling of the vertical circle index.

The reading, pointing and levelling errors (σ_r, σ_p and σ_ℓ respectively) are determined by the types of instruments used to make the simultaneous zenith angle measurements. For example, if two - 28X, 1" theodolites with automatic vertical circle compensators and artificial light for the readout system were used, the errors would be:

σ_r = 1" (from previous experience of the author)

σ_p = 0".5 (from Chrzanowski [1977])

σ_ℓ = 0".3 (from Cooper [1971])

The error in the differential refraction correction ($\sigma_{\Delta r}$) is estimated to be of the same order as the correction itself, and a value of 0".5 is used.

For each determination of Δz, four measurements of zenith angle are taken (one measurement on each face of each instrument). If the previous values of σ_r, σ_p, σ_ℓ and $\sigma_{\Delta r}$ are used, the standard deviation for each determination of $\Delta\xi$ would be

$$\sigma_{\Delta\xi} = (\sigma^2_{\Delta z} + \sigma^2_{\Delta r})^{1/2}$$

$$= [4\ (\sigma_r{}^2 + \sigma_p{}^2 + \sigma_\ell{}^2) + \sigma_{\Delta r}{}^2]^{1/2}$$

$$= [4\ (1^2 + 0.5^2 + 0.3^2) + 0.5^2]^{1/2}$$

$$= 2''.4$$

σ_r, σ_p, σ_ℓ but not $\sigma_{\Delta r}$ can be reduced by $\sqrt{n}$ sets, therefore if, for example, 12 sets are taken

$$\sigma_{\Delta\xi} = [\frac{4\ (1^2 + 0.5^2 + 0.3^2)}{12} + 0.5^2]^{1/2} = 0''.8$$

Applying the law of propagation of errors to the math model for $\Delta\eta$,

$$\sigma^2_{\Delta\eta} = \cos^2\phi\ \sigma^2_{\Delta\Lambda} + \cos^2\phi\ \sigma^2_{\Delta\lambda} \qquad (4\text{-}26)$$

If the error in $\Delta\lambda$ is assumed to be small in comparison to the error in $\Delta\Lambda$, then

$$\sigma^2_{\Delta\eta} = \cos^2\phi\ \sigma^2_{\Delta\Lambda} \qquad (4\text{-}27)$$

Assuming that $\Delta\Lambda$ is determined when the star is near prime vertical crossing,

$$\Delta\Lambda \doteq \sec\Phi\ (\Delta z + \Delta r) \qquad (4\text{-}28)$$

Applying the law of propagation of errors to this expression,

$$\sigma^2_{\Delta\Lambda} = \sec^2\Phi\ (\sigma^2_{\Delta z} + \sigma^2_{\Delta r}) \qquad (4\text{-}29)$$

Substituting into the expression above for $\sigma^2_{\Delta\eta}$,

$$\sigma^2_{\Delta\eta} = \sigma^2_{\Delta z} + \sigma^2_{\Delta r} \qquad (4\text{-}30)$$

which is the same as the expression for $\sigma^2_{\Delta\xi}$.

In the determination of Δz for $\Delta\eta$ however, there is a simultaneous timing error in addition to the other errors. This error is negligible in the determination of $\Delta\Phi$ because for $\Delta\Phi$ a star near the celestial pole is observed. The apparent motion of this star is very slow. For $\Delta\Lambda$ a star near prime vertical crossing is chosen. The apparent motion of a star, along its track, at prime vertical crossing equals the rotation rate of the earth, that is 360° per 24 hours or

15" arc per 1^s time, making the star more difficult to track and to point to at an instant of time. The vertical component of the apparent motion of a star at prime vertical crossing is 15" cos ϕ per 1^s. The simultaneous timing error ($\sigma_{\Delta t}$) of two observers reacting to a read signal given by a third person is estimated to be $0\overset{s}{.}1$. At $\phi = 45°$ this corresponds to an error in zenith angle of $1\overset{''}{.}0$. If the previous values of σ_r, σ_p, σ_ℓ and $\sigma_{\Delta t}$ are used, the total standard deviation for each determination of $\Delta\eta$ is

$$\sigma_{\Delta\eta} = (\sigma_{\Delta z}^2 + \sigma_{\Delta r}^2)^{1/2}$$

$$= [4\ (\sigma_r^2 + \sigma_p^2 + \sigma_\ell^2 + \sigma_{\Delta t}^2) + \sigma_{\Delta r}^2]^{1/2}$$

$$= [4\ (1^2 + 0.5^2 + 0.3^2 + 1.0^2) + 0.5^2]^{1/2}$$

$$= 3\overset{''}{.}1$$

σ_r, σ_p, σ_ℓ, $\sigma_{\Delta t}$ but not $\sigma_{\Delta r}$ can be reduced by $\sqrt{n}$ sets, therefore if, for example, 12 sets are taken

$$\sigma_{\Delta\eta} = [\frac{4(1^2 + 0.5^2 + 0.3^2 + 1.0^2)}{12} + 0.5^2]^{1/2} = 1\overset{''}{.}0$$

This error analysis for $\Delta\xi$ and $\Delta\eta$ assumes that there is no systematic shift between the two observers and instruments making the simultaneous observations. To check for this possibility, a set of simultaneous zenith angles on the same star near prime vertical crossing should be determined for the two observers and instruments side by side. This procedure should be performed at the beginning and at the end of a night of observations.

In the field work which was done (see section 4.3) some practice time was required to eliminate this shift. It was noticed during these practice sessions that shifts as high as 10" could occur when one or more of the following conditions existed:

1. parallax between the instrument crosshairs and the star,
2. vibration of the vertical circle compensator causing vibration in the readout system because of an unstable instrument setup,
3. poor lighting of the crosshairs and readout system.

During the observations care must be taken to eliminate these conditions: crosshairs and star must be properly focussed, a stable instrument setup must be made and batteries for the lighting system must be changed as soon as the light begins to dim.

4.3 Field Tests

In order to determine whether the astrogeodetic difference method was actually practical and produced results more or less consistent with other independent determinations, field tests were carried out in the Fredericton and Fundy Park areas.

In the Fredericton area results from the astrogeodetic difference method were compared with results from the trigonometric method and results provided by Dr. Lachapelle using his combined method. Dr. Lachapelle also provided results for the Fundy Park area and these were compared with results from the astrogeodetic method and the astrogeodetic difference method.

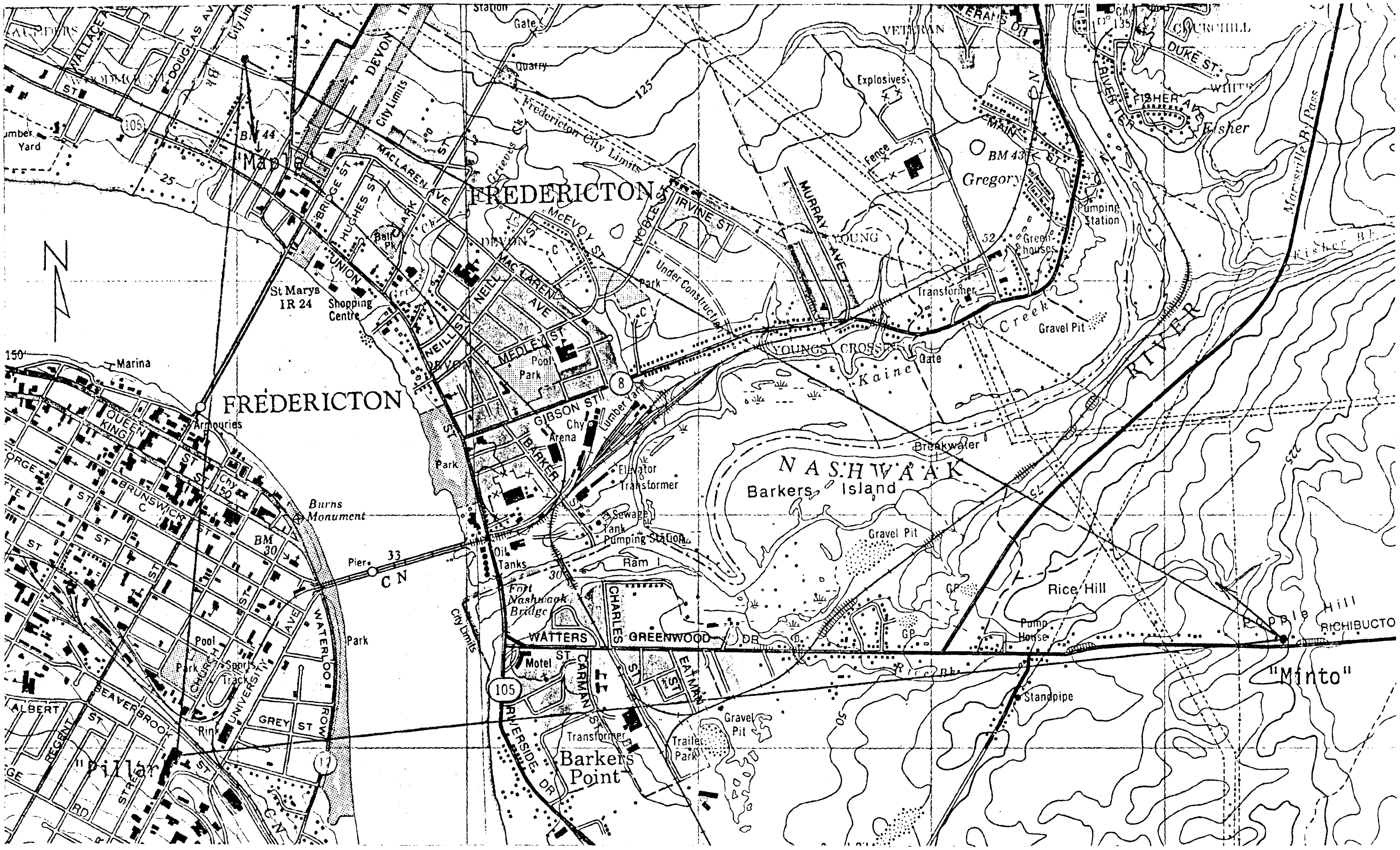
The astrogeodetic difference and trigonometric methods produce surface deflections of the vertical. The results which Dr. Lachapelle provided are geoid deflections. Section 2.2.1 gives the maximum curvature of the plumbline between the geoid and the terrain as 3"/1000 m height above the terrain; therefore for both the Fredericton area (H ≐ 10 m) and Fundy Park area (H ≐ 350m) the geoid deflections provided by Dr. Lachapelle are essentially the same as the surface deflections.

Dr. Lachapelle's results were determined from the Goddard Earth Model (GEM) 10B potential coefficients and adjacent gravity anomalies. No astrogeodetic deflection data was used in either the Fredericton or Fundy Park areas.

4.3.1 Fredericton Area

Figure 4-3 shows the location of three points in the Fredericton area ("Pillar", "Maple" and "Minto") between which changes in deflection of the vertical were determined.

The Fredericton area was chosen for convenience and, in order to utilize the terrain to make the changes in deflection as large as possible, the three points were located on opposite sides of the St. John and Nashwaak River valleys. "Pillar" is the East astro pillar located on the roof of the Engineering Building, University of New Brunswick. "Maple" is marked by an orange colored wooden stake on a slight rise in an open field. "Minto" is marked by a brass marker set in a rock outcrop. The line between "Pillar" and "Minto" is denoted as line 1, the line



Scale 1:25 000

All elevations in feet

0 2 4 6 8 10

Scale for Deflections (seconds)

Figure 4-3

Deflections of the Vertical in the Fredericton Area

by the Astrogeodetic Difference Method

between "Pillar" and "Maple" as line 2 and the line between "Maple" and "Minto" as line 3.

4.3.1.1 Use of the Trigonometric Method

The trigonometric method was used only on line 1.

The slope distance (S_s) of line 1 was measured independently by four groups of students using microwave EDM equipment, and is believed to be accurate to $\pm$ 0.1 m.

The difference in height (ΔH) between "Pillar" and "Minto" (each end of line 1) was measured by two groups of students under the supervision of the writer. Precise levelling equipment was used and precise levelling procedures were rigorously followed. The levelling route was divided into short sections so that blunders could be isolated. The misclosure in each section and for the entire levelling route was smaller than that required for special order levelling which has an allowable misclosure of $\pm$ 3 mm/$\sqrt{}$km of levelling route [Surveys and Mapping Branch (EMR), 1973]. The total misclosure for the entire route was 0.00527 m. Based on 200 sightings and a standard deviation of 0.4 mm/sighting [Chrzanowski, 1978], the standard devaition of the height difference was estimated to be $\pm$ 0.006 m. Because the forward and backward levellings were performed on essentially the same route, the gravity correction was negligible.

The standard deviations given for $\Delta\varepsilon$, k, k_{AB} and k_{BA} were determined by applying the law of propagation of errors to the expressions for each of these terms. In each case, the contribution of the standard deviation of ΔH , S_s and S_o was negligible in comparison to the contribution to the standard deviation of z:

$$\sigma^2_{\Delta\varepsilon_{AB}} \doteq \sigma^2_{z_B} + \sigma^2_{z_A} \qquad (4\text{-}31)$$

$$\sigma^2_{k_{AB}} \doteq \left(\frac{2R}{S_s \rho''}\right) \sigma^2_{z_A} \qquad (4\text{-}32)$$

$$\sigma^2_{k} \doteq \left(\frac{R}{S_o \rho''}\right)^2 \left(\sigma^2_{z_A} + \sigma^2_{z_B}\right) \qquad (4\text{-}33)$$

where

σ^2 = (standard deviation $)^2$ or variance

The standard deviation of each zenith angle was determined from a set of 12 zenith angles. Each reciprocal zenith angle was measured simultaneously (by means of two-way radio communication)and in rapid succession.

Line 1 is far from an ideal choice of a line on which to determine an accurate trigonometric height difference. According to Bomford [1975] there are several unfavorable situations for the determination of accurate trigonometric height differences:

1. great width of river crossing,
2. low ground or water clearance,
3. asymmetry of terrain profile,
4. aymmmetry of ground and water,
5. clear skies,
6. great heat or cold,
7. absense of wind.

Conditions 1, 3, and 4 exist for line 1, and obviously nothing could be done to reduce their effect. Meterological effects (5, 6 and 7) were minimized by making the observations on afternoons of windy, cloudy days.

In an attempt to reduce the uncertainty of the determination of change in deflection from trigonometric levelling, the vertical temperature gradient was measured at "Minto" and at "Pillar" for the zenith angle observations made on June 24. The vertical temperature gradient at "Minto" was measured just before the zenith angles were observed, and the vertical temperature gradient at "Pillar" just after. The temperature gradient was measured by mounting three thermisters l m apart on a levelling rod. The lowest thermister was mounted 0.1 m below the telescope. The readout system gave difference in temperature between any two of the three thermisters. The results obtained are shown in the following table.

Location	Vertical A to B	Temperature B to C	Gradient (°c/m) A to C
Pillar	-0.306	-0.222	-0.264
Minto	-0.639	-0.133	-0.389

A 0.1 m below telescope

B 0.9 m above telescope

C 1.9 m above telescope

Table 4-1

Vertical Temperature Gradients on line 1

To minimize the effect of heat from the Engineering Building, zenith angles and vertical temperature gradients for "Pillar" were measured close to the edge of the building.

The results were not what had been expected. If the average value of vertical temperature gradient (A to C) is used to determine coefficient of refraction,

from

$$k = 502 \frac{P}{T^2} (0.0341 + \frac{dT}{dh}) \text{ [Angus-Leppan, 1967]} \qquad (4\text{-}34)$$

where

P = atmospheric pressure in millibars

T = temperature in °K

$\frac{dT}{dh}$ = temperature gradient in °c/m

$k_{Pillar} = -1.37$

$k_{Minto} = -2.13$

An average value of vertical temperature gradient of -0.009 °C/m had been expected, which would have produced k values of about 0.150. The conclusion that can be drawn is that vertical temperature gradients in the immediate vicinity of the instrument are not representative of the average vertical temperature gradient at one end of a line of observation, probably because of the rapid change in $\frac{dT}{dh}$ and k near the ground.

A solution to this problem might have been to measure the temperature gradient further away from the instrument but still along the line of observation. For line 1 this was not feasible: at the

"Pillar" end the instrument was already near the edge of a high building, and at the "Minto" end the slope dropped off rapidly along the line of observation.

Table 4-2 shows the results of a difference in trigonometric and spirit levelled height differences interpreted as a change in deflection of the vertical (Approach 1) and as a difference in coefficient of refraction at each end of the line of observation (Approach 2). For this particular line the difference in heights may have been due just as much to one as the other, but the relative contribution of each could not be determined because of the unsuccessful attempt to make an independent determination of the coefficient of refraction at each end of the line of observation.

4.3.1.2 Use of the Astrogeodetic Difference Method

The astrogeodetic difference method was used on lines 1, 2 and 3 so that the misclosure in $\Delta\Phi$ and $\Delta\Lambda$ could be calculated to check for a possible systematic error in either of these quantities.

"Pillar" was arbitrarily assigned a zero deflection of the vertical. The values of $\Lambda\phi$ and $\Delta\lambda$ were determined by measuring azimuths accurate to about $\pm$ 10" and EDM distances accurate to about 0.1 m, and applying Puissant's formula. Astronomic coordinates accurate to about 0".1 had been previously determined for "Pillar" and these were used in the computations, but approximate astronomic coordinates would have been just as good.

In all the determinations of $\Delta\xi$ and $\Delta\eta$, values for δ, α, and

Observers (Zenith Angle)	Date	Time	Weather	$*S_s$(m)	$*\Delta H$(m)	Approach 1		Approach 2	
						$\Delta\xi('')$	k	k_{AB}	k_{BA}
Leal and Teskey	May 17	3:30 pm	30% cloud cover, light wind	4833.0 ± 0.1	41.739 ± 0.006	-1.4 ± 1.0	0.136 ± 0.005	0.163 ± 0.011	0.108 ± 0.006
Leal and Teskey	May 20	12:30 pm	100% cloud cover, light wind	4833.8 ± 0.1	41.732 ± 0.006	-0.6 ± 0.6	0.139 ± 0.005	0.150 ± 0.005	0.128 ± 0.004
Leal and Teskey	June 24	4 pm	100% cloud cover, light wind	4832.5 ± 0.1	41.695 ± 0.006	-0.8 ± 0.7	0.147 ± 0.004	0.162 ± 0.007	0.131 ± 0.004

Approach 1: Same k at each end of the line of observation assumed.

Approach 2: No change in deflection of the vertical assumed.

$*S_s$ and ΔH different for each set of observations because instrument and target setups different.

Table 4-2

Trigonometric Levelling on line 1

R for the stars observed were taken from "The Star Almanac for Land Surveyors for the Year 1979" (SALS, 1979) [Her Majesty's Nautical Almanac Office, 1978]. 12 sets of observations were made. Values of $\Delta\xi$ and $\Delta\eta$ were determined twice for line 1 to check that time of observation and star observed did not have a significant effect.

The check for the systematic shift between the two observers and instruments was performed before and after the observations for $\Delta\xi$ and $\Delta\eta$ on line 1. The check was performed only after the observations for $\Delta\xi$ and $\Delta\eta$ on line 2 because the results from line 1 had shown no significant systematic shift. (The systematic shifts for both determinations of $\Delta\xi$ and $\Delta\eta$ on line 1 were less than 1" and the standard deviations were about 2".) The check was not performed after the observations for $\Delta\xi$ and $\Delta\eta$ on line 3 because no stars were visible at the time due to heavy cloud cover.

One pair of observers (Leal and Teskey) performed the observations on lines 1 and 2. A different pair of observers (Sujanani and Teskey) performed the observations on line 3. This was done only because Leal was not available for the observations on line 3.

Table 4-3 summarizes the determinations of $\Delta\phi$, $\Delta\lambda$, $\Delta\Phi$ and $\Delta\Lambda$ in the Fredericton area. When $\Delta\Phi$ and $\Delta\Lambda$ were not corrected for differential refraction, the misclosures (about 1" for $\Delta\Phi$ and 2".5 for $\Delta\Lambda$) were about the same magnitude as the standard deviations of the misclosures. When $\Delta\Phi$ and $\Delta\Lambda$ were corrected for differential refraction, the misclosures (about 0".5 for $\Delta\Phi$ and 1".0 for $\Delta\Lambda$) were about one-half the magnitude of the standard deviations of the misclosures. These

Observers	Date	Line	From	To	Δφ(")	Δλ(")	ΔΦ(") (uncorrected)	ΔΦ(") (corrected)	ΔΛ(") (uncorrected)	ΔΛ(") (corrected)
Leal and Teskey	June 27	2	Pillar	Maple	97.2	16.2	94.9 ± 1.1	94.4 + 1.1	17.3 ± 1.5	16.6 ± 1.5
Sujanani and Teskey	July 7 & 8	3	Maple	Minto	-85.9	208.5	-80.9 ± 0.8	-80.4 ± 0.8	205.6 ± 2.1	205.5 ± 2.1
Leal and Teskey	{May 22, 23 May 17, 18}	1	Minto	Pillar	-11.3	-224.7	{-13.0 ± 0.8 -13.1 ± 0.7}	{-13.3 ± 0.8 -13.6 ± 0.7}	{-220.8 ± 1.9 -220.3 ± 1.3}	{-221.5 ± 1.9 -221.3 ± 1.3}
			Sum		0.0	0.0	{1.0 ± 1.6 0.9 ± 1.5}	{0.7 ± 1.5 0.4 ± 1.5}	{2.1 ± 3.2 2.6 ± 2.9}	{0.6 ± 3.2 0.9 ± 2.9}

Table 4-3

Differences in Astronomic and Geodetic Coordinates in the Fredericton Area results indicate that a small systematic error may exist in the results and that the differential refraction correction may reduce the systematic error. Table 4-3 also shows that the time of observation and the star observed probably do not have a significant effect on the determination of $\Delta\Phi$ and $\Delta\Lambda$ since the two determinations of $\Delta\Phi$ and $\Delta\Lambda$ on line 1 are the same at the 1σ level.

The final results, ξ and η of "Maple" and "Minto" with respect to $\xi = \eta = 0$ at the pillar, are shown on Figure 4-3. The standard deviations of the results compare well with those calculated from an a priori error analysis. The ξ and η values for "Maple" are those determined from the astronomic observations at "Pillar" and "Maple", and the ξ and η values for "Minto" are those determined from the astronomic observations at "Pillar" and "Minto", that is neither set of deflection components were adjusted for the small misclosure around the triangle.

Figure 4-3 also shows the deflection of the vertical as a vector. When shown this way it appears that the deflection is affected by the topography in the general area of the station. This may be by chance considering the magnitude of the deflection components and their standard deviations, or it may be due to a small systematic shift between observers and instruments during the observations. It may also be that the deflection components are substantially correct, and that they exist because of the high correlation between topography and deflection of the vertical. This correlation between topography and deflection was discussed in section 4.1.4.

Point	[1]Deflection Component	Astrogeodetic Difference	Trigonometric	[2]Combined
Maple	$\xi('')$	-2.8 ± 1.1		-0.77 ± 1.4
	$\eta('')$	$+0.4 \pm 1.1$		-0.02 ± 1.3
Minto	$\xi('')$	$\left\{\begin{matrix}+2.3 \pm 0.7 \\ +1.6 \pm 0.8\end{matrix}\right\}$		-0.05 ± 1.4
	$\eta('')$	$\left\{\begin{matrix}-2.2 \pm 1.3 \\ -2.4 \pm 0.9\end{matrix}\right\}$	[3] $\left\{\begin{matrix}-1.4 \pm 1.0 \\ -0.6 \pm 0.6 \\ -0.8 \pm 0.7\end{matrix}\right\}$	$+0.34 \pm 1.3$

Notes:

1. Determined with respect to $\xi = \eta = 0$ at "Pillar".
2. Provided by Dr. Lachapelle using GEM10B potential ceofficients and adjacent gravity anomalies only.
3. Determined only along line of observation which is predominantly east and west.

Table 4-4

Deflections of the Vertical in the Fredericton Area

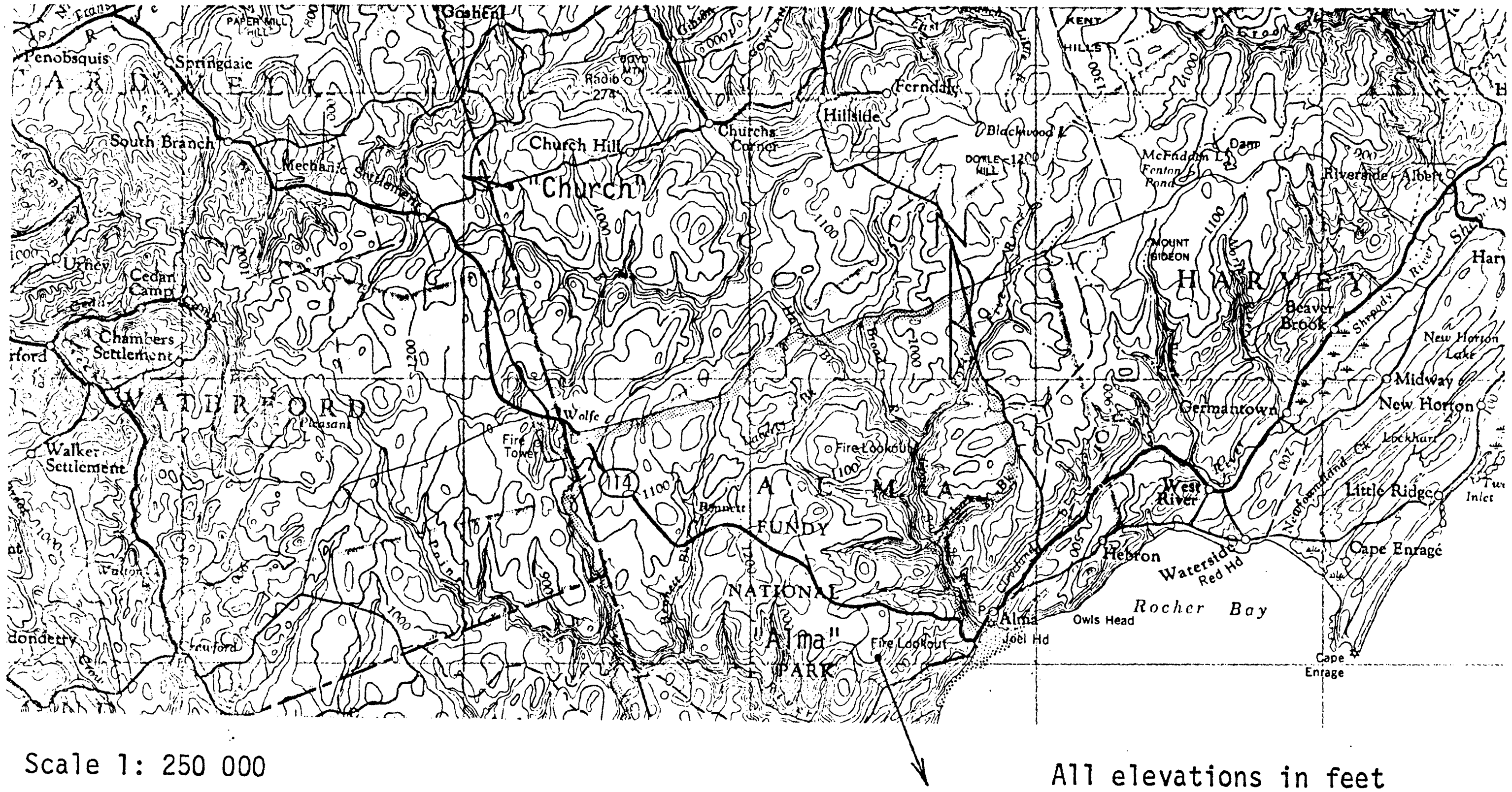
4.3.1.3 Comparison of the Results

Table 4-4 shows a comparison of deflection components in the Fredericton area.

Because the deflection components determined by each independent method are roughly the same magnitude as their standard deviations, no conclusions can be drawn regarding the correctness of one method versus another. Certain observations concerning the methods however, can be made. The trigonometric determination is probably the weakest due to the effect of vertical refraction on a line of observation near the ground. The astrogeodetic difference determinations could easily be affected by a small systematic shift between instruments and observers during the observations. The resolution of the combined determinations may not be as high as the astronomic or trigonometric determinations because point gravity anomalies used are an average distance of 10 km apart.

4.3.2 Fundy Park Area

Figure 4-4 shows the location of two 1st order geodetic control points in the Fundy National Park area. Also shown on Figure 4-4 are the deflections of the vertical (with respect to NAD 27) that were determined for these points by the observation of astronomic latitude and astronomic longitude. At "Alma" (station No. 14103) astronomic coordinates were observed in 1914, 1929 and 1964. At "Church" (station No. 641007) astronomic coordinates were observed in 1964. The standard deviations of the deflection components are estimated to



Scale 1: 250 000

All elevations in feet

0 10 20 30

Scale for Deflections (seconds)

Figure 4-4

Deflections of the Vertical in the Fundy Park Area

1:250,000

SYMBOLS BLACK (UNLESS OTHERWISE STATED)

SYMBOLES NOIRS (SAUF INDICATION CONTRAIRE)

MISCELLANEOUS CULTURE	OUVRAGES D'ART	
Building	Bâtiment	
Church	Église	
School	École	
Built-up area	Agglomération	Screened red / Trame rouge
Town	Ville	
Village or settlement	Village ou hameau	

be $\pm$ 0''.3 for both north-south components and $\pm$ 0''.5 for both east-west components [Robbins, 1977], since the standard deviation of a north-south component is about the same as the standard deviation of astronomic latitude and the standard deviation of an east-west component is about the same as the standard deviation of (astronomic longitude) x cos (astronomic latitude). $\Delta\xi$ by this determination is 12''.1 with $\sigma_{\Delta\xi} = 0''.3 \times \sqrt{2} = 0''.4$ because each deflection component was determined independently. $\Delta\eta$ by the same determination is 8''.3 with $\sigma_{\Delta\xi} = 0''.5 \times \sqrt{2} = 0''.7$, again because each deflection component was determined independently.

In addition to the astrogeodetic determination of the deflection components, a combined determination was provided by Dr. Lachapelle. $\Delta\xi$ by this determination is 11''.3 $\pm$ 1''.4, and $\Delta\eta$ is 4''.8 $\pm$ 1''.4.

Because both of these determinations of deflection components are independent of one another, it is almost certain that the differences in deflection components, which are large in comparison with their standard deviations, do exist between "Alma" and "Church". The reason for the large differences in deflection components is due mostly to the fact that "Alma" is located at a point where the highlands of the Caledonia Hills begin to slope steeply toward the Bay of Fundy. Large differences in deflection components could have been found in other coastal areas or in the Rocky Mountains but the close proximity of the Fundy Park area to Fredericton made it the obvious choice.

The field procedures followed in the Fundy Park area were essentially the same as those followed in the Fredericton area. Geodetic coordinates of "Alma" and "Church" were provided by the Geodetic Survey of Canada. $\Delta\Phi$ and $\Delta\Lambda$ were each determined by 12 sets of observations. Davidson and Teskey performed the observations. Because the two-way radios used in the Fredericton area did not have sufficient range, previous arrangements had been made for the Fundy Park two-way radio system to be used.

After the observations had been completed, the check for a systematic shift was performed. The magnitude of the shift ($0''.8$) was again smaller than its standard deviation ($\pm$ $1''.5$), and no correction was applied to the observations. Small eccentric corrections were applied to $\Delta\Phi$ and $\Delta\Lambda$ because both instruments had to be set up a short distance away from the stations in order to get a reasonable field of view.

Table 4-5 shows a comparison of the deflection components determined by the astrogeodetic, combined and astrogeodetic difference methods. The agreement of the results of the three methods is remarkably good. The results of the astrogeodetic difference method agree more closely with those of the astrogeodetic method than the combined method possibly because the astronomic methods are conceptually the same while the combined method is based on a completely different concept.

These field results, together with those obtained in the Fredericton area, indicate with some certainty that changes in

Point	Deflection Component (NAD 27)	Astrogeodetic	*Combined	Astrogeodetic Difference Without Δr	Astrogeodetic Difference With Δr	Difference (Astro - Astro Diff) Without Δr	Difference (Astro - Astro Diff) With Δr
Church	$\xi('')$	$+0.9 \pm 0.3$	1.9 ± 1.4	-		-	
	$\eta('')$	-4.2 ± 0.5	-3.4 ± 1.4	-		-	
	$\Delta\xi('')$	-12.1 ± 0.4	-11.4 ± 2.0	-11.0 ± 1.3	-12.7 ± 1.3	- 1.1	+ 0.6
	$\Delta\eta('')$	$+8.4 \pm 0.7$	$+4.8 \pm 2.0$	$+8.3 \pm 1.8$	$+10.6 \pm 1.8$	+ 0.1	- 2.2
Alma	$\xi('')$	-11.2 ± 0.3	-9.5 ± 1.4	-		-	
	$\eta('')$	$+4.2 \pm 0.5$	$+1.4 \pm 1.4$	-		-	

* provided by Dr. Lachapelle using GEM 10B potential coefficients and adjacent gravity anomalies only

Table 4-5

Deflections of the Vertical in the Fundy Park Area deflections of the vertical can be determined by the astrogeodetic difference method with an accuracy of 1" to 2". The weakness of the method is the possibility of an undetected systematic shift between observers and instruments during the observations.

5 APPLICATION TO ENGINEERING SURVEYS

In this chapter the application of the material presented in chapters 1 to 4 will be shown. Two different engineering surveying problems will be considered: a simulated tunnel survey and alignment of a straight line in space. The emphasis in both problems will be on the effect of the gravity field.

5.1 A Simulated Tunnel Survey

A tunnel survey is an excellent problem to investigate when considering high accuracy requirements in an engineering survey. The critical problem in a tunnel survey is to minimize the breakthrough error of headings driven from opposite ends of the tunnel. This is difficult to do since the lateral breakthrough is determined by an open traverse and the vertical breakthrough by an open levelling line.

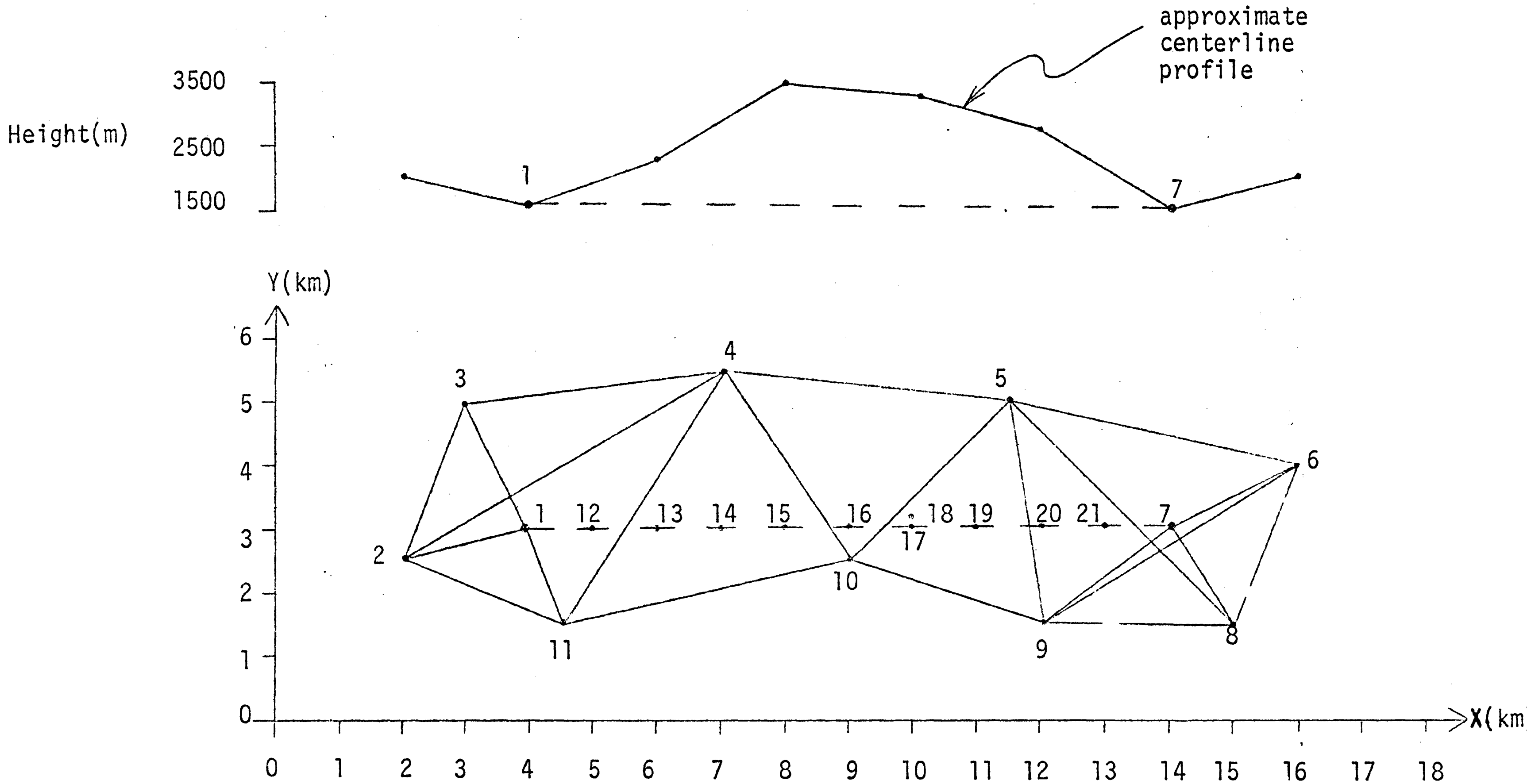
For short tunnels the portals are commonly connected on the surface by a traverse; for longer tunnels the portals are connected by a trigonometric network [Wassermann, 1967]. In either case the only alternative for horizontal control in the tunnel itself is a traverse. For vertical control precise spirit levelling is used. The portals may be connected by a single precise levelling line or by a network of such lines [Wassermann, 1967]. Vertical control in the tunnel itself is extended by a precise levelling line.

The problem that will be considered is the error which will occur at the breakthrough of a tunnel in a mountainous area if the effect of the gravity field is neglected. It will be assumed that all systematic errors (other than those due to neglecting the gravity field) have been eliminated by proper survey procedures, and that the only errors which remain are random errors. The lateral and vertical breakthrough errors will be considered separately.

5.1.1 Lateral Breakthrough Error

For horizontal control various network and traverse configurations have been considered. The configurations are shown on Figure 5-1. Table 5-1 show approximate plane coordinates, heights and deflection components of each point in a local coordinate system. For sake of convenience the X-axis has been aligned along the tunnel.

Heights of the points were estimated assuming that the tunnel passes beneath a high mountain in a range which runs generally north and south. Deflections were assumed to be dependent largely on topography and thus all deflections are "downhill". Differences in deflection were estimated to be larger across the mountain range, that is east-west, than along it, that is north-south. The maximum difference in east-west deflection was estimated to be 20"; the maximum difference in north-south deflection, 10". Both the magnitude of the deflection



Configuration	Observations between Portals
1	all distances
2	all directions with 2 distances
3	all distances and directions
4	South side traverse (1-11-10-9-7)
5	North side traverse (1-3-4-5-6-7)
6	South and North side traverses
7	Zig-zag traverse (1-11-4-10-5-9-7)

Figure 5-1

Horizontal Control for a Simulated Tunnel Survey

Point	X(m)	Y(m)	Ht (m)	ξ(")	η(")
1	4000	3000	1500	0	0
2	2000	2500	2000	-5	+5
3	3000	5000	2500	+5	+5
4	7000	5500	3000	+5	-10
5	11500	5500	3250	+5	+10
6	16000	4500	2000	+5	-5
7	14000	4000	1500	0	0
8	15000	3000	2250	-5	-5
9	12000	1500	2500	-5	10
10	9000	1500	3500	-5	0
11	4500	2500	2500	-5	-5
12	5000	1500	1500	0	-2.5
13	6000	3000	1500	0	-5.0
14	7000	3000	1500	0	-2.5
15	8000	3000	1500	0	0
16	9000	3000	1500	0	0
17	10000	3000	1500	0	+2.5
18	10000	3000	1500	0	+2.5
19	11000	3000	1500	0	+5.0
20	12000	3000	1500	0	+5.0
21	13000	3000	1500	0	+2.5

Table 5-1

Horizontal Control Data for Simulated Tunnel Survey differences and the direction of the deflections is in general agreement with those reported for mountainous terrain in Wassermann [1967], Richardus [1974] and Maclean [1977]. (In a real problem deflections could be determined by any of the methods discussed in chapter 4. Probably the easiest and most convenient method would be the astro-geodetic difference method.)

Directions, angles and distances were used to determine horizontal positions. The estimated standard deviation of each of these observations was obtained by referring to Chrzanowski [1977], considering that directions and angles were measured with a 1" theodolite and distances were measured with commonly available electro-optical EDM equipment. The estimated standard deviations were:

directions: $\pm$ 1."5

angles: $\pm$ 2."0

distances: $\pm$ (5 mm + 4 ppm)

No azimuths were included since the standard deviation of an azimuth determined with a 1" theodolite is 5" to 10" [Robbins, 1976] and this would do very little to improve the horizontal position determinations in a small area considering the much smaller standard deviations of directions, angles and distances. (In a real problem, a low order azimuth would be required to approximately orient the Y-axis to north so that geometric and gravimetric corrections could be applied to the observations.)

The random lateral breakthrough error for each configuration was determined by the method proposed by Chrzanowski [1978]. In this method the random lateral break-through error (at the 1 σ level) is the error in the direction perpendicular to the tunnel centerline of the error (pedal) curve described on the standard relative error ellipse between two points very close to the breakthrough point. The breakthrough point must be considered as two separate points in this method so that a relative error ellipse can be obtained.

The relative error ellipse is usually interpreted as depicting a region which defines the relative positional accuracy of two points whose positions were determined from observations having only random errors. The parameters defining a relative error ellipse are a, the dimension of its semi-major axis; b, the dimension of its semi-minor axis; and ϕ, the azimuth of the semi-major axis. These parameters are determined from the estimated variances of the set of observations for a particular configuration of points, by parametric least squares preanalysis. Parametric least squares preanalysis for horizontal geodetic networks is described in many references, for example Krakiwsky and Thomson [1978] and Chrzanowski [1977].

The error in any direction between two points is obtained from the relative error ellipse between the two points by [Chrzanowski, 1977]:

$$\sigma_\theta = (a^2 \cos^2 \theta + b^2 \sin^2 \theta)^{1/2} \qquad (5\text{-}1)$$

where θ = clockwise angle from the semi-major axis to the desired direction

For this problem

$$\theta = (180° - \phi) \qquad (5\text{-}2)$$

therefore $$\sigma_\theta = (a^2 \cos^2 \phi + b^2 \sin^2 \phi)^{1/2} \qquad (5\text{-}3)$$

The systematic lateral breakthrough error for each configuration was determined by calculating errorless observations and then adjusting these by the corrections which should have been applied for deflections of the vertical. Using the errorless observations adjusted for the deflections, the misclosure in the Y direction at the breakthrough is the systematic lateral error.

The corrections applied to the directions and angles were the c_1 corrections described in section 3.2.1. The corrections for directions varied from -3".0 to +4".0, and for angles from -4".2 to +6".2.

The corrections applied to the distances were obtained by differentiating, with respect to the zenith angle, the expression for the reduction of slope distance to the horizontal:

$$\ell_{ij} = r_{ij} \sin z_{ij} \qquad (5\text{-}4)$$

and

$$d\ell_{ij} = r_{ij} \cos z_{ij} \, dz_{ij} \qquad (5\text{-}5)$$

The correction for a zenith angle is [Thomson et al, 1978]

$$\Delta z_{ij} = (\xi_i \cos \alpha_{ij} + \eta_i \sin \alpha_{ij}) \qquad (5\text{-}6)$$

It was assumed that to obtain the highest accuracy in the reduction of spatial distances to the mapping plane simultaneous reciprocal zenith angles were observed, therefore Δz_{ij} for a given distance would be the sum of Δz_{ij}'s at each end of the line. The corrections for distances varied from -0.033m to +0.090 m.

Several things can be noticed about the corrections to the observations:

1. There are no corrections to the observations for the underground traverse since it is horizontal.
2. The corrections to directions and angles are proportional to the component of the deflection perpendicular to the line(s) and the slope of the line(s); the corrections to distances are proportional to the sum of the components along the line at each end of the line, and the slope of the line.
3. Because of the orientation of the deflection components, the corrections for distances are largest for steep lines running generally east and west or north and south; the corrections for directions are largest for lines running generally northeast and southwest or northwest and southeast.
4. The corrections in most cases are within the 2σ level and appear to be random.

Calculation of the misclosures at the breakthrough was done by parametric least squares adjustment which is just an extention of parametric least squares preanalysis. In preanalysis of horizontal geodetic networks variances of observations yield variances and covariances of coordinates. In adjustment of horizontal geodetic networks observations with variances yield adjusted coordinates with variances and covariances. Any reference which describes preanalysis of horizontal geodetic networks, such as those referred to previously, would also describe adjustment.

To perform the calculations for the random and systematic lateral breakthrough errors the program GEOPAN (Geodetic Plane Adjustment and Analysis) [Steeves, 1978] was used. With the program used in the adjustment mode only one run was required for each particular configuration and set of observations since the adjusted coordinates of the two points at the breakthrough provide the systematic error, and the relative error ellipse between the two points at the breakthrough, calculated from the variances of and the covariances between these points, provides the random error.

Table 5-2 shows the random and systematic lateral breakthrough errors associated with each configuration connecting the portals. The random error is about 0.120 m (1 σ) for all configurations, and is little affected by the degrees of freedom. In fact, the lowest random error is a combination of two traverses having only three degrees of freedom. The systematic error due to neglecting the gravity field, on the other hand, can be reduced to less than the 1 σ random error by only a few degrees of freedom. With zero degrees of freedom or a unique determination (all the single traverses) the systematic error can be larger than the 2σ random error - unquestionably large enough that it should be eliminated.

5.1.2 Vertical Breakthrough Error

The random vertical breakthrough error and systematic

Configuration	Observations between Portals	Minimum Constraints Point	Minimum Constraints Azimuth	Degrees of Freedom	Lateral Error (m) Random	Lateral Error (m) *Systematic
1	all distances	5	5-4	3	± 0.126	-0.079
2	all directions with 2 distances	5	5-4	16	± 0.125	+0.059
3	all distances and directions	5	5-4	36	± 0.114	+0.001
4	south side traverse	10	10-11	0	± 0.122	-0.093
5	north side traverse	5	5-4	0	± 0.144	+0.210
6	south and North side traverses	5	5-4	3	± 0.107	+0.076
7	zig-zag traverse	10	10-4	0	± 0.129	-0.293

* + indicates heading from east is to the north of heading from West at breakthrough

Table 5-2

Lateral Breakthrough Errors of Simulated Tunnel Survey vertical breakthrough error will be discussed separately. For both the random and systematic errors it will be assumed that a single precise levelling line is used.

The estimate for the random vertical breakthrough error, based on a standard deviation of 0.4 mm/sighting [Chrzanowski, 1973] and an average sighting distance of 25 m, is ± 0.00283 m/√km of levelling route. Considering the rugged terrain, the length of the levelling route would be several times longer than the straight line distance. With the levelling route 50 km long the random vertical breakthrough error would be ± 0.020 m.

The systematic vertical breakthrough error was estimated by using estimated values of gravity along the route 17-1-3-4-5-6-7-17 shown in Figure 5-1. Heights of these points are shown in Table 5-1 although in an actual levelling line the height of the 1-3-4-5-6-7 portion of the line would be at a slightly lower elevation. The length of each levelling section with a gravity correction varied from about 2 km to 6 km, since according to Ramsayer [Heiskanen and Moritz, 1967] gravity values 5 km apart are sufficient in mountainous areas.

Values of gravity along the 1-3-4-5-6-7 portion of the line were estimated using the Bouguer vertical gradient of gravity (≐ -0.2 mgal/m height) since this gradient is the best estimate for the surface of the earth [Vanicek and Krakiwsky, in prep]. Along this same portion of the line the values of gravity were also adjusted for the effect of the mountainous terrain. This effect was assumed to vary uniformly from - 50 mgal at H = 1500 m to -100 mgal at H = 3500 m. This estimate of the effect of the terrain was obtained by referring to Vanicek and Krakiwsky [in prep.]. In the tunnel (sections 17-1 and 7-17) values of gravity were estimated using the Poincare-Prey vertical gradient of gravity (≐ -0.1 mgal/m height) from the surface downward to the level of the tunnel. The Poincare-Prey vertical gradient of gravity is the average value of the gradient in the surface layer of the earth [Vanicek and Krakiwsky, in prep]. As a starting point for the calculation of gravity values along the route, the gravity values at the points 1 and 7 were estimated to be 981 000 mgal.

A second set of gravity values was also calculated assuming a gradient of the refined Bouguer anomaly (the Bouguer anomaly corrected for the terrain effect) of about 20 mgal/10 km distance caused by a mass anomaly. This value of the gradient of the refined Bouguer anomaly is believed to be a reasonable estimate of its maximum value. The maximum value of the gradient was assumed to be parallel to the tunnel centerline so that is had its largest effect.

For sake of simplicity the gravity corrections for a dynamic height system are calculated. In this system the gravity correction to a height difference is

$$\Sigma_i \frac{g_i - G}{G} \delta L_i \tag{5-7}$$

where

g_i = average value of gravity in a levelling section

G = arbitrarily chosen reference gravity for an area

δL_i = change in height in a levelling section

Since the levelling line 17-1-3-4-5-6-7-17 is a closed loop the gravity corrections represent the systematic vertical breakthrough error.

Table 5-3 summarizes the vertical control data for this simulated tunnel survey. Using this data and formula (5-7) with G = 980 500 mgal the systematic vertical breakthrough error is -0.00382 m without the mass anomaly and +0.0629 m with the mass anomaly. The positive sign indicates that the heading from the east is below the heading from the west at the breakthrough.

The very small systematic vertical breakthrough error for no mass anomaly is to be expected since the gravity corrections up the mountain will almost completely cancel those down the mountain. There is no gravity correction in the tunnel since $\delta L_i = 0$. The systematic vertical breakthrough error with the mass anomaly is several times the random vertical breakthrough error but it may be unrealistically high because the value used for the gradient of the Bouguer anomaly is unrealistically high.

5.1.3 The Effect of Neglecting the Gravity Field

Based on the results of this simulated tunnel survey and the results of actual tunnel surveys in rugged terrain [Wassermann, 1967; Richardus,1974; Maclean, 1977] the following conclusions have been reached regarding the effect of neglecting the gravity field:

1. Because changes in deflection may be largely determined by topography, these changes will generally be largest parallel to rather than

Section	δL_i(m)	g_{avg} without anomaly (mgal)	g_{avg} with anomaly (mgal)
17-1	0	980810	980816
1-3	1000	980835	980833
3-4	500	980665	980867
4-5	250	980580	980590
5-6	-1250	980695	980715
6-7	-500	980895	980917
7-17	0	980805	980820

Table 5-3

Vertical Control Data for Simulated Tunnel Survey perpendicular to the tunnel. These large deflection differences will most affect distances parallel to the tunnel and directions perpendicular to the tunnel, both of which have little effect on the lateral breakthrough error. Thus, if changes in deflection are determined mainly by topography, by the nature of direction and distance observations for horizontal control of a tunnel, the effect of the gravity field is minimized. If however, changes in deflection are determined by mass anomalies near the surface of the earth, this argument is no longer valid.

2. For the simulated tunnel survey which had deflections estimated on the basis of topography only, the systematic lateral error due to neglecting the gravity field was reduced to less than the 1σ random error by simply increasing the degrees of freedom. Further simulations would have to be carried out to determine if the same would be true for deflections due to a mass anomaly near the surface of the earth.

3. The effect of the gravity field on the lateral breakthrough error might be safely neglected but this can only be determined by analyzing the specific problem. Good estimates of deflection components at main points in the horizontal control network would be necessary for this analysis.

4. The effect of the gravity field on the vertical breakthrough error might also be safely neglected. Again, this can only be determined by analyzing the specific problem. For this analysis gravity values at a spacing of about 5 km would be adequate. These could be estimated from a detailed gravity anomaly map of the area, if it is available,

or obtained by actual gravity measurements in the field. Field gravity measurements are easily made with a portable gravimeter.

5.2 Alignment of a Straight Line in Space

This is a very specialized engineering surveying problem which illustrates well several points:

1. the advantages of using a local coordinate system,
2. the advantage of using the astrogeodetic difference method rather than the astrogeodetic method to determine deflections of the vertical,
3. the significant effect of deflection of the vertical.

Discussion of this problem will be based on Preiss [1971] which describes the alignment in space of a 5 km radio telescope aerial array for the Cavendish Laboratory, Cambridge. An accurate alignment was required because the performance of the telescope is dependent on how accurately the intersection of the polar and declination axes of eight dish aerials fits a straight line in space. The end result of the alignment survey were three orthogonal corrections to a preliminary reference line defined by stable ground marks.

The corrections to the ground marks in the direction parallel to the line of ground marks were determined by measurements with a Mekometer, the most accurate short range EDM instrument available. The corrections to ground marks in the horizontal direction perpendicular to line of ground marks were determined by the usual optical alignment method except that the best available equipment was used (a 100X Fennel alignment telescope mounted on a massive concrete pillar and targets specially designed for the long distances), and a large number of observations were made with experienced personnel. The corrections to the ground marks in the vertical direction were made by combining the results of precise spirit levelling along the line of ground marks and determinations of deflection of the vertical, by the astrogeodetic method, at four of the ground marks.

The alignment parallel to the line of ground marks and the alignment in the horizontal direction perpendicular to the line of ground marks will not be discussed further. Only the alignment in the vertical direction will be examined in more detail since the alignment in this direction is most affected by the shape of the earth and the variations in its gravity field. The method used for the Cambridge radio telescope will be briefly described and suggestions will be given as to how the same or better accuracy might have been attained in a much shorter time.

Basically, the problem of determining corrections to the ground marks in the vertical direction consists of locating, along the line of ground marks, the equipotential surface passing through an arbitrarily chosen ground mark by precise spirit levelling; and then determining the shape of the equipotential surface, with respect to the ellipsoid surface, along this line by deflections of the vertical. When this has been done corrections can easily be calculated.

Figure 5-2 shows the correction applied along the vertical at one ground mark to obtain a straight line in space. The corrections along the vertical at all other ground marks would be obtained in a similar manner. The straight line in space in this example is the straight line, in the tangent plane to the equipotential surface at A, along the line of ground marks. The terms in Figure 5-2 are defined as follows:

ΔH_{AB} = precise levelled height difference between points A and B

Δh_{AB} = height difference between tangent plane to equipotential surface at point A, and at point B, measured along the vertical at point B

$$= \frac{S^2_{AB}}{2R} \qquad (5\text{-}8)$$

ΔN_{AB} = change in separation between ellipsoid and equipotential surface from point A to point B

$$= \frac{\varepsilon_{AB}\, S_{AB}}{2} \qquad (5\text{-}9)$$

Consider a typical example:

$$\Delta\, H_{AB} = 0.3129 \text{ m}$$

$$\Delta\, h_{AB} = \frac{S^2_{AB}}{2R} = \frac{2000^3}{2 \times 6375 \times 10^3} = 0.3146 \text{ m}$$

$$\Delta N = \frac{\Delta\varepsilon_{AB}\, S_{AB}}{2}$$

$$= \frac{0.2 \times 2000}{2 \times 206265}$$ ($\Delta\varepsilon_{AB}$ at Cambridge was about 0".2 in 2 km)

$$= 0.0010 \text{ m}$$

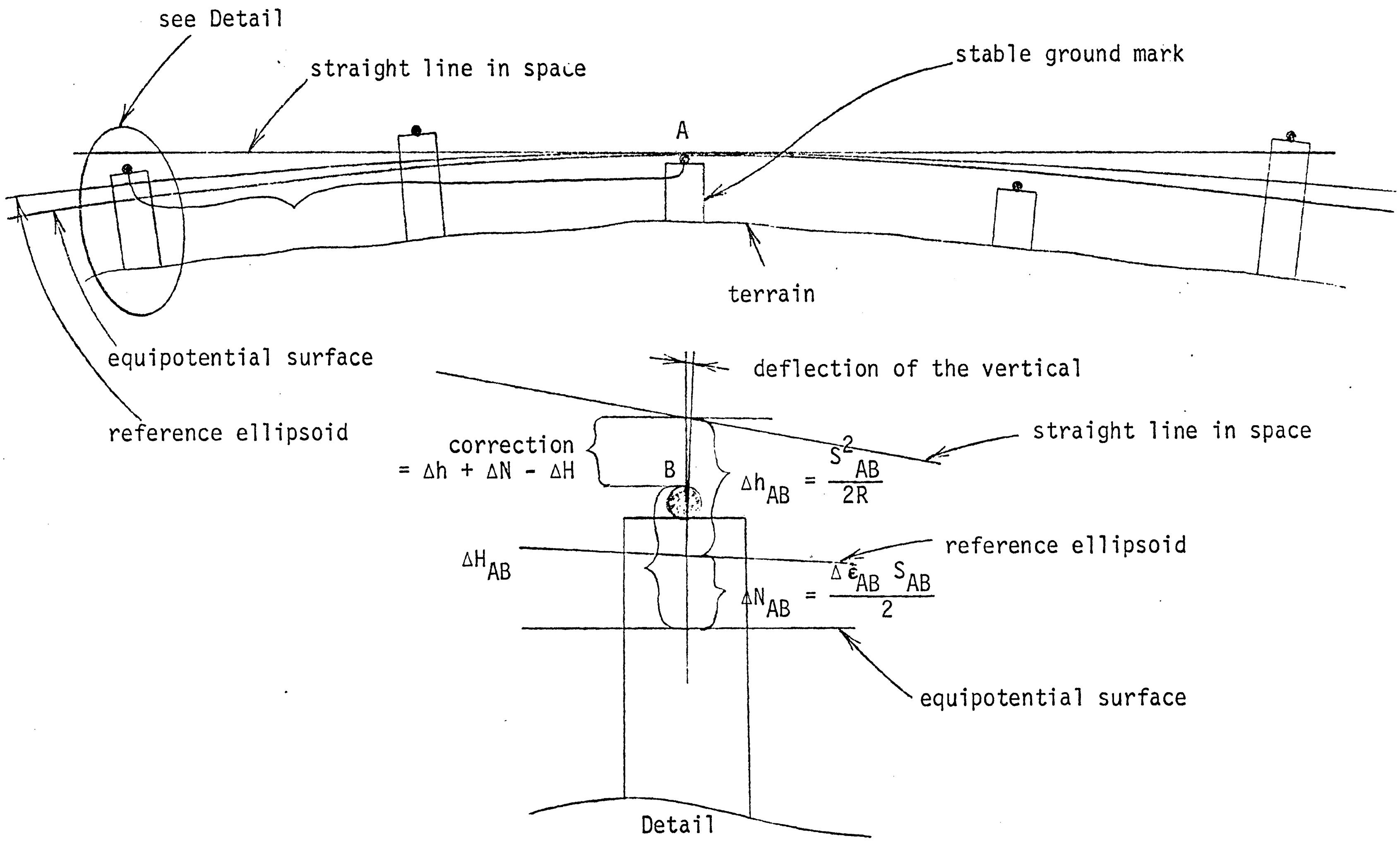


Figure 5-2

The Vertical Component of Alignment of a Straight Line in Space and correction

$= \Delta h + \Delta N - \Delta H$

$= + 0.0027$ m

To determine corrections to the ground marks in the vertical direction for the Cambridge radio telescope geodetic levelling was first performed along the line of ground marks. The national geodetic network in the area of the project was then reobserved, with connections to the ground marks, and readjusted to obtain geodetic coordinates for the ground marks. 1st order astronomic observations for latitude and longitude were observed at four of the ground marks. Combining the results of the astronomic and geodetic position determinations, deflections of the vertical were obtained.

Much of this work was not necessary. If a local geodetic coordinate system such as that described in chapter 4 for the determination of changes of deflection in the Fredericton area had been used, all of the work associated with the determination of geodetic coordinates could have been reduced to a single determination of second order astronomic azimuth ($\sigma \doteq 5''$) and a few distances accurate to about 0.1 m.(A 1st order azimuth of the line was required however for other purposes.)

Field and computation work could have been further reduced by using the astrogeodetic difference method rather than the usual astrogeodetic method to determine deflections. Standard deviations of deflection changes for the astrogeodetic difference method quoted in chapter 4 could be substantially reduced by using two 1st order instruments (for example two- Wild T-4's), each with an impersonal micrometer and a chronocord. The tracking record from each instrument with simultaneous timing marks would provide a very accurate measure of difference in zenith angle from which a difference in deflection is calculated. To ensure the highest accuracy a correction for the systematic shift between the two observers and instruments would be applied. Standard deviations of about 0".1 were quoted for the astronomic latitude and longitude determinations for the Cambridge radio telescope. If the same equipment and observers were used to determine $\Delta\Phi$ and $\Delta\Lambda$, the standard deviation could be $0''.1 \times \sqrt{2} = 0''.14$ or better with much less observation time.

The alignment of a straight line in space for the Cambridge radio telescope was performed by considering the alignment in the horizontal and vertical separately. For this particular problem it might have been more appropriate to use a three-dimensional geodetic model. A three-dimensional geodetic model was used for a similar alignment of a series of baselines connected to the National Aeronautics and Space Administration/Jet Propulsion Laboratory (NASA/JPL) MARS Deep Sapce Station located at the Goldstone Deep Space Communication Complex in California [Carter and Pettey, 1978].

6. CONCLUSIONS AND RECOMMENDATIONS

Many of today's engineering surveys require relative positional accuracies in the order of 1/100 000 or better. Advances in survey instrumentation and development of a methodology to eliminate systematic errors from the survey observations themselves has generally kept pace with the demand for higher and higher accuracies. However, in order to obtain the full benefit of these higher measuring accuracies and actually attain the high relative positional accuracies, a rigorous geodetic approach has to be followed.

Against this background the following recommendations are made for engineering surveys requiring high accuracy:

1. A local coordinate system should be used to avoid propagating errors from other coordinate systems. The local system could be tied to an integrated survey system if required but the observations used to make the tie would not be used for position determinations in the local system. A disadvantage of this approach is that coordinates (and their accuracies) from another coordinate system could not be utilized without a transformation. The advantages of a local coordinate system are well illustrated by the two problems considered in chapter 5, especially by the method used for the vertical component of the alignment of a

straight line in space.

2. The local coordinate system should be based on the classical geodetic model. This involves the separation of horizontal and vertical positioning but allows relative positional accuracy in the height component to be determined with about the same accuracy as relative positional accuracy of the horizontal components. Because of variations in the gravity field and the uncertainty associated with vertical refraction the relative positional accuracy in the height component of a three-dimensional coordinate system determined only by trigonometric measurements is at least one order of magnitude greater than the relative positional accuracy of the horizontal components.

3. Gravity corrections may have to be applied to precise levelling lines in engineering surveys requiring high accuracy. The error due to not making this correction can be estimated by using a detailed gravity anomaly map or by making small number of gravity measurements. If gravity corrections are required, gravity values can easily be measured with a portable gravimeter.

4. Horizontal position observations should be rigorously reduced to a mapping plane. In certain applications some of the reduction corrections can be omitted without adversely affecting the relative horizontal positional accuracy, but this should first be shown by analysis of the particular problem. This is especially true of the gravimetric reduction correction which is often omitted only because it is difficult to determine.

In this thesis special emphasis was placed on methods to determine deflection of the vertical. It was felt that a contribution could be made if a simple method could be developed to determine deflections in the small area covered by an engineering survey. Application of a difference method to the usual astrogeodetic deflection determination proved to be completely successful. Deflection changes accurate to 1" to 2" were determined on five different nights and in two different locations using only two 1" theodolites and two-way radio communication. Attempts to use trigonometric levelling to determine deflection changes led to inconclusive results because of the uncertainty associated with vertical refraction.

Concerning the astrogeodetic difference method to determine deflections, the following recommendations are made:

1. The method should be tested with more sets of observers to learn more about the systematic shift between instruments and observers, since this is the only real weakness of the method.
2. The method should be tested with 1st order astronomic equipment to determine the saving in time over the astrogeodetic method. This could be an important application since geodetic astronomy still provides the most accurate deflection determinations [Robbins, 1977].
3. In a three-dimensional coordinate system unknown deflection components have a large effect on height determinations. The astrogeodetic difference method could be used to provide these components (in the form of a priori astronomic latitudes and longitudes) and reduce the standard deviations of the heights.
4. Variations in the local gravity field have a large effect on ISS

position determinations. This is an area in which more research is required [Adams, 1979]. A possible application of the astrogeodetic difference method is the definition of the local gravity field so that this information can be provided a priori, although an interpolation method such as the astrogravimetric method would probably be more useful since by this method the gravity field can be determined at any point.

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APPENDIX I

Derivation of $\Delta\Phi = -\sec A\Delta z - \cos\Phi \tan A\Delta h$

All the terms given in the deviation are defined in section 4.7.

From the astronomic triangle, or equivalently, by transforming between the horizon and hour angle celestial coordinate systems

$$\cos z = \sin\delta \sin\Phi + \cos\delta \cos h \cos\Phi \qquad \text{(I-1)}$$

Differentiating with respect to the observed quantities z, Φ and h

$$-\sin z dz = (\sin\delta \cos\Phi - \cos\delta \cos h \sin\Phi)\, d\Phi$$

$$- \cos\Phi \cos\delta \sin h\, dh$$

or after replacing d with Δ and rearranging the terms

$$\Delta\Phi = -\frac{\sin z}{\sin\delta \cos\Phi - \cos\delta \cos h \sin\Phi}\Delta z$$

$$+\frac{\cos\Phi \cos\delta \sin h}{\sin\delta \cos\Phi - \cos\delta \cos h \sin\Phi}\Delta h \qquad \text{(I-2)}$$

Applying the five consecutive parts rule to the astronomic triangle

$$\cos A \sin z = \cos(90°-\delta)\sin(90°-\Phi) - \cos(90°-\Phi)\sin(90°-\delta)\cos(360°-h)$$

$$= \sin\delta \cos\Phi - \sin\Phi \cos\delta \cos h \qquad \text{(I-3)}$$

Applying the sine law to the astronomic triangle

$$\frac{\sin h}{\sin z} = \frac{-\sin A}{\sin(90° - \delta)} = \frac{-\sin A}{\cos\delta} \qquad \text{(I-4)}$$

Substituting cos A sin z for (sin δ cos Φ - sin Φ cos δ cos h) from (I-3) and $-\frac{\sin A \sin z}{\cos \delta}$ for sin h from (I-4) into (I-2), and simplifying yields

$$\Delta\Phi = -\sec A \,\Delta z - \cos \Phi \tan A \,\Delta h \qquad \text{(I-5)}$$

<u>Derivation of $\Delta\Lambda = -\sec \Phi \cot A \,\Delta\Phi - \sec \Phi \operatorname{cosec} A \,\Delta z + \Delta T$</u>

All the terms given in the derivation are defined in Section 4.7.

From the astronomic triangle, or equivalently, by transforming between the horizon and hour angle coordinate systems

$$\Lambda = \alpha + h - T \qquad \text{(I-6)}$$

Differentiating with respect to the observed quantities Δ, h and T

$$d\Lambda = dh - dT \qquad \text{(I-7)}$$

Replacing d with Δ and substituting $(\Delta\Lambda + \Delta T)$ for h from (I-7) into (I-5) and simplifying yields

$$\Delta\Lambda = -\sec \Phi \cot A \,\Delta\Phi - \sec \Phi \operatorname{cosec} A \,\Delta z + \Delta T \qquad \text{(I-8)}$$

HP-29C Program for $\Delta\Lambda$

The appropriate formula, in whch all terms are defined in section 4.7, is

$$\Delta\Lambda = -\sec\Phi\cot A\,\Delta\Phi - \sec\Phi\csc A\,\Delta z$$

where

$$A = \cos^{-1}\left(\frac{\sin\delta - \cos z\sin\Phi}{\sin z\cos\Phi}\right)$$

and

$$z = \frac{z_1 + z_2}{2} + \text{refraction correction}$$

Φ (° ' "), δ(° ' "), $\Delta\Phi$(") and the refraction correction (") are stored as shown in the program listing.

The step by step program listing follows.

←enter z("), z_1(° ' "), z_2(° ' ")

```
[1]  LBL1
[2]  →H
[3]  STO5
[4]  R↓
[5]  →H
[6]  RCL5
[7]  +
[8]  2
[9]  ÷
[10] RCL9  (refr. corr.)
[11] +
[12] STO5
[13] R↓
[14] STO6
[15] RCL0 (Φ)
[16] sin
[17] RCL5
[18] cos
[19] x
[20] CHS
[21] RCL4 (δ)
[22] sin
[23] +
[24] RCL0
[25] cos
[26] RCL5
[27] sin
[28] x
[29] ÷
[30] cos⁻¹
[31] CHS  )
[32] 360  } for WPVC only
[33] +    )
[34] STO8
[35] sin
[36] 1/x
[37] RCL0
[38] cos
[39] 1/x
[40] x
[41] RCL6
[42] x
[43] CHS
[44] RCL7 (ΔΦ)
[45] RCL8
[46] tan
[47] 1/x
[48] x
[49] RCL0
[50] cos
[51] 1/x
[52] x
[53] CHS
[54] +
[55] RTN → displays ΔΛ
```