AUTOMATED TIDAL REDUCTION OF SOUNDINGS

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PREFACE

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AUTOMATED TIDAL REDUCTION OF SOUNDINGS

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ABSTRACT

In Hydrographic Surveying, soundings are reduced to a chart datum established at a reference gauge station from a long period of tidal observations. Unfortunately, due to the variations in tidal characteristics from place to place, soundings can only be reduced to the chart datum within the vicinity of the gauge station. As we move away from the gauge station, it becomes necessary to obtain new information on the tidal characteristics and apply necessary corrections to the chart datum to obtain an appropriate sounding datum for reducing the soundings.

To reduce soundings means to subtract the heights of tide, at the sounding locations and at the times of soundings, from the depths sounded to obtain the depths referenced to the chosen datum. Manual reduction of soundings is a tedious aspect of the field hydrographer's list of chores. There have been some attempts to automate the tidal reductions using digitized cotidal charts.

The objective of this work has been to develop alternative approaches to automated tidal reductions, namely, using analytical cotidal models. The range ratio and time lag fields have been approximated by surfaces described by two dimensional algebraic polynomials ($Pn(\phi,\lambda)$). The observed time series at a reference station has been approximated by one dimensional trigonometric polynomial

With the coefficients of these Polynomials stored in the computer, the range ratio and the time lag at any point (ϕ_i, λ_i) in the area can readily be predicted and the height of tide at the point and at time t can be predicted from the predicted height of tide at the reference station.

Test computations, using data from the 'Canadian Tides and Current Tables, 1978' for the Bay of Fundy have been done. It has been shown that the water level (h) at a location (ϕ_i, λ_i) can be predicted with a standard deviation (σ_{h_i}) of 0.5 m or better.

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I would also like to express my gratitude to the University of Nigeria, Nsukka for offering me the opportunity to undertake this research.

My thanks goes to Mrs. Rao for her excellent typing.

Finally, I wish to dedicate this work to my wife, Chinyere, in appreciation of her support and encouragement and for taking care of our baby.

I. INTRODUCTION

1.0 Chart Datum and Other Water Levels

The hydrographic surveyor must refer all his depth and height measurements to a reference datum. This reference datum, generally called chart datum, is a low water datum which by international agreement is so low that water level will seldom fall below it. The chart datum is, for purposes of integration and consistency, normally tied to the Geodetic datum which is usually defined by the mean sea level. For example, over a period of some years, tide gauges in Canada have been tied to the Geodetic Survey of Canada Datum (G.S.C.D.) [Atlantic Tidal Power Engineering and Management Committee Report, 1969]. This geodetic datum is based on the value of the mean sea level prior to 1910 as determined from a period of observations at tide gauge stations at Halifax and Yarmouth, Nova Scotia and Father Point, Quebec on the East Coast, and at Prince Rupert, Vancouver and Victoria on the Pacific. Mean Sea Level (M.S.L.), as its name implies, is the mean level taken up by the sea. It is determined at a tide gauge station from a long period of tide observations The geoid, which is supposed to be the datum for the heights, is defined as "that equipotential surface which on the average coincides with the mean sea level" [Thomson, 1974]. It therefore leaves the problem of mean sea level determination to be solved in order to define a height datum.

It is not easy to determine mean sea level since the actual level of the sea is continuously changing. Wemelsfelder [1970] , in his paper titled, 'Mean Sea Level as a Fact and as an Illusion', outlined two concepts of mean sea level: the Physical concept and the Emperical concept. The Physical concept according to him 'is that of a common parlance', it is the concept used in the verbal description, "the height of the mountains above sea level". This concept has the intent to overlook every motion of the sea, it intends to say, no waves, no tides, no storm surges, no wind influences, no seasonal changes, no density anomalies, no temperature anomalies. The mean sea level is rather conceptualized as, 'a physical object existing primarily in space, the way in which the ocean spans the earth.'

The emperical concept tries to quantify the mean sea level as the mean observed water levels at a tide gauge station over a period of time. This mean level even on the same sea varies from one tide gauge location to another and varies also with different time epochs. Wemelsfelder, [1970], enumerated 33 factors influencing the variations in the mean sea level and grouped them under global, regional, local and instrumental influences. Bomford, [1971], observed that apart from tidal forces whose mean effect over a long period should be zero, other forces cause the mean sea level to depart appreciably from an exact level (equipotential) surface. Thomson [1974], further noted that, 'the problem of determining the true physical surface of the oceans is analogous to that of using Stoke's formula for geoid determination - we would require an infinite number of tide gauges, atmospheric sensors, sea temperature and density determinations'. It appears then that mean sea level, thus the geoid, cannot be easily determined.

The various other water levels* that can be used as a datum, or that will be relevant to the subject matter of this work, will now be briefly defined and each is illustrated in Figure 1-1.

The average of recorded values of all the high and low waters over a period is called the Mean Tide Level (M.T.L.). It is obtained more easily than mean sea level and as such is sometimes used in calculations instead of the M.S.L.

The average throughout the year of heights of high waters during the spring tides is termed Mean High Water Springs (M.H.W.S.). The average throughout the year of the heights of low water during the spring tides is called Mean Low Water Springs (M.L.W.S.).

Mean High Water Neaps (M.H.W.N.) is the average

*see Appendix III for further details regarding definitions used in Canada.

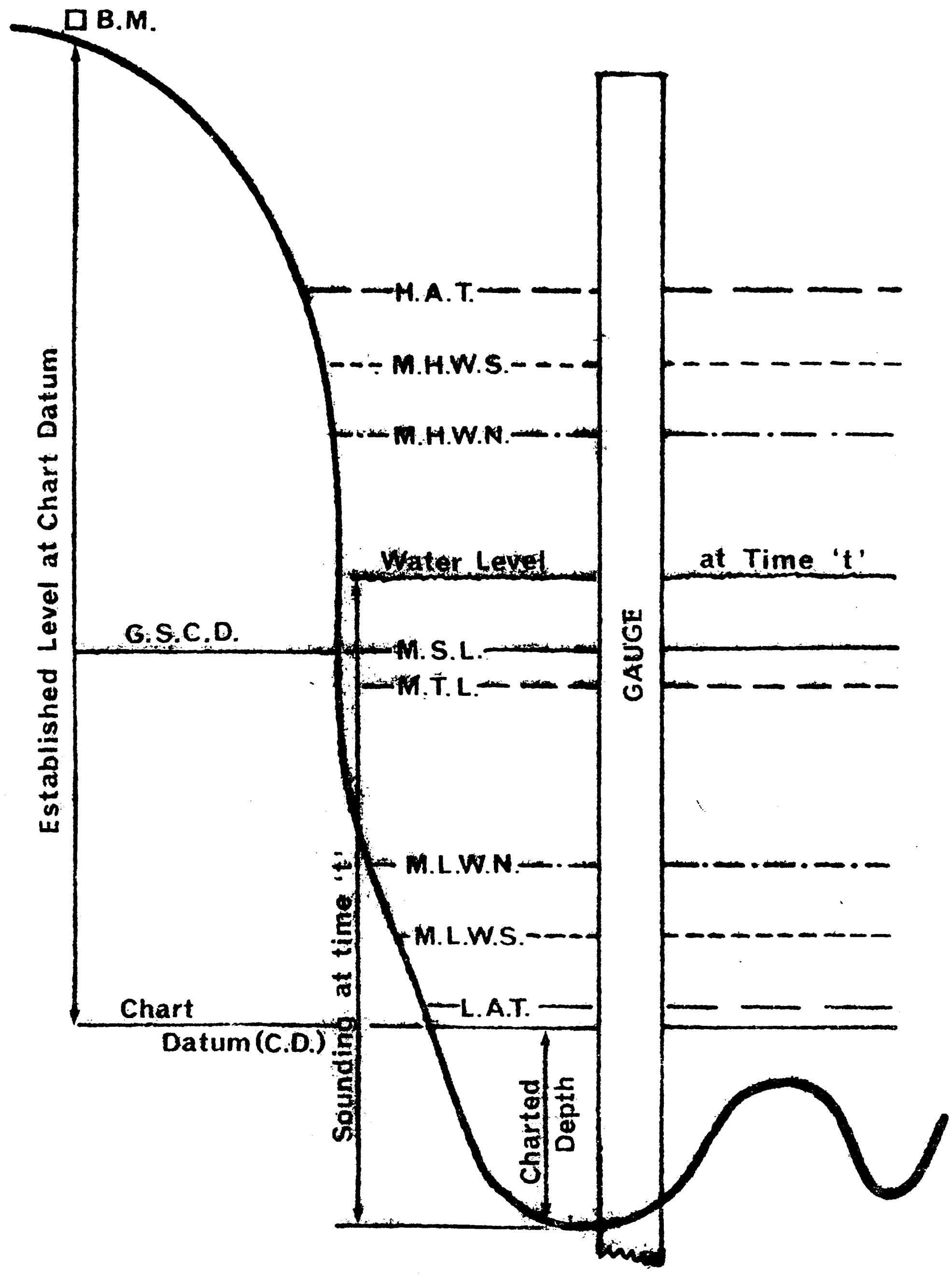

Figure 1-1

Relationship Between Various Water Levels

throughout the year of heights of high waters during the neap tides and the average throughout the year of heights of low water during the neap tides is called Mean Low Water Neaps (M.L.W.N.).

The highest tide which can be predicted to occur under average meterological conditions and under any combination of astronomical conditions is termed Highest Astronomical Tide (H.A.T.), while the lowest predictable tide is called the Lowest Astronomical Tide (L.A.T.).

Chart datum, as previously stated, is a low water level. It is the datum to which all soundings on published charts are reduced and to which tidal predictions and tide readings are referenced. Ideally, Lowest Astronomical Tide level should be taken as chart datum. But, since we cannot accurately define it, we choose chart datum arbitrarily as close to L.A.T. as possible such that, (i) tides will seldom fall below it, (ii) it is not so low as to give unduely shallow depths.

1.1 Sounding Datum

When a chart datum is chosen, it can only be used within the vicinity of the gauge location [Atlantic Tidal Power Engineering and Management Committee, 1969]. Depending on the variation of tidal characteristics, it is not advisable to reduce depth measurements to this chart datum if the reference tide gauge is more than 8 km away [Admiralty Manual of Hydrographic Surveying, 1969]. This leads to the necessity of establishing a local sounding datum. In the Admiralty Manual of Hydrographic Surveying, 1969 , the following rules are given as a guide to the choice of sounding datum:

(i) if possible, a sounding datum should agree with the chart datum.

(ii) changes in a sounding datum within the area of interest must be made whenever the nature and range of tides alter appreciably. It is difficult to lay down precise figures, but a difference in range of about one metre between two places would normally indicate the necessity for a change of datum somewhere between them.

(iii) the time difference between tides experienced at two places will not have any effect on the difference of sounding datum between two points. It may however have a considerable effect on the value of the reduction required to reduce soundings to datum. Therefore, it is important, even if the sounding datum does not alter, to obtain time differences between tidal stations so that time differences may be interpolated and applied to observed heights of tide used for the reduction of soundings.

(iv) If there is any doubt in the surveyor's mind concerning the behaviour of the tide, he should set up another tide gauge to find out what is happening.

Figures 1-2 and 1-3 show how the tidal ranges change along the southern and northern coasts of the Bay of Fundy. At Yarmouth, the range at the spring tides is about 4.9 metres (16 feet). The range increases to the east and at Burnt Coat Head, a distance of about 290 km away, the range reaches about 16.7 metres (55 feet). Along the northern coast, the range is about 8.5 mteres (28 feet) at Eastport, Me. and increases going eastward, and at Joggins Wharf, the range is about 12.2 metres (40 feet).

If a datum was established at Yarmouth or Eastport, Me. for the reduction of soundings, as the soundings progressed eastwards, the sounding datum should be altered. The ideal thing is to alter a sounding datum in a series of steps. Figures 1-2 and 1-3 depict the alteration of a sounding datum in steps of 0.6 m (2 feet). The correction to be

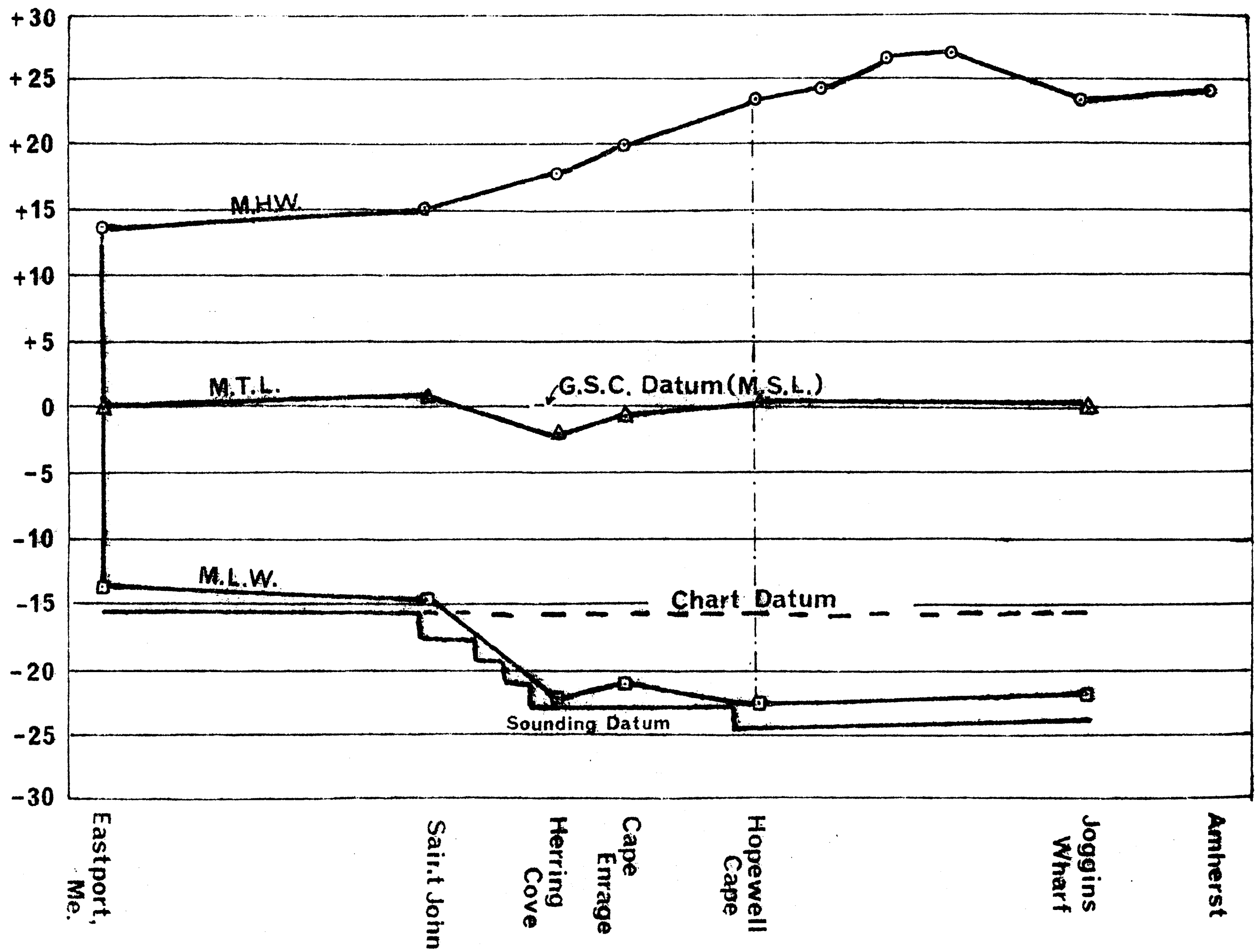

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Figure 1-2

Bay of Fundy; Variation of Tidal Ranges and Sounding Datum Along the Northern Coast

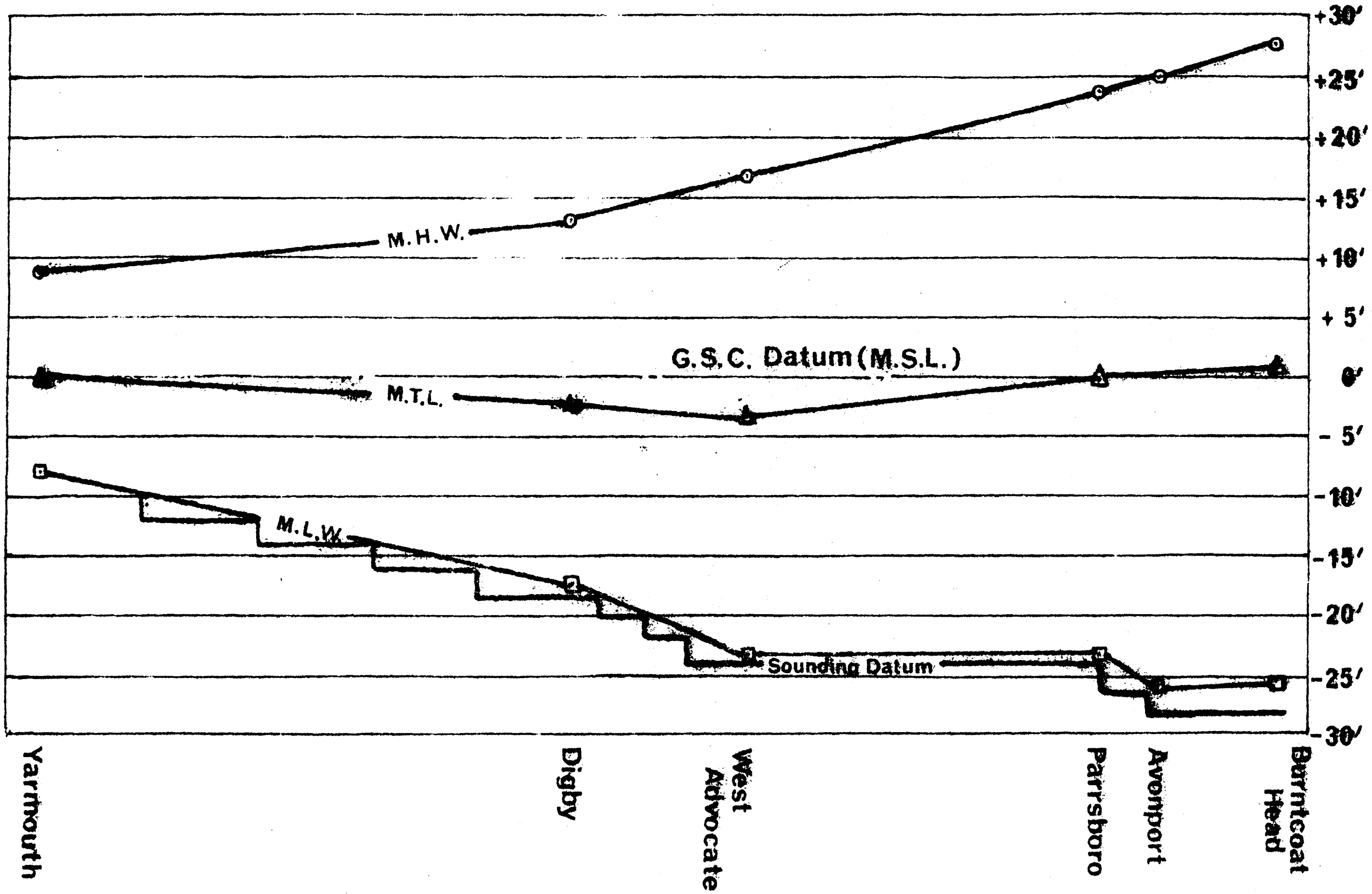


Figure 1-3

Bay of Fundy; Variation of Tidal Ranges and Sounding Datum Along the Southern Coast

applied to a chart datum (established datum at the reference station) to obtain the sounding datum is given by [Admiralty Manual of Hydrographic Surveying, 1969],

$$d = h - H\frac{r}{R} , \qquad (1.1)$$

where h is the height of the M.S.L. above the zero of the new reference gauge, H is the height of the M.S.L. above the established chart datum, r is the range of tide at the new reference station and R is the range of tide at the established reference station. It means that when $|d| \geq 0.6$ m (2 feet) the sounding datum is changed by 0.6 m (2 feet).

Figure 1-4 illustrates how a sounding datum could change in an estuary or a river. The configuration of the land and the slope of the sea bed will influence the tidal characteristics and hence the tidal ranges. The range of the tide increases at first proceeding up a river and then starts to decrease until it reaches zero at a point inland where the river ceases to be tidal.

It is not possible to establish one sounding datum for a hydrographic survey which covers a long stretch of coastline and where tidal conditions are unknown. Tidal information in the area must be built up and a sounding datum transferred gradually along the coast as the survey progresses. A hydrographic surveyor on a sounding mission could be met with any of the following situations regarding sounding datum:

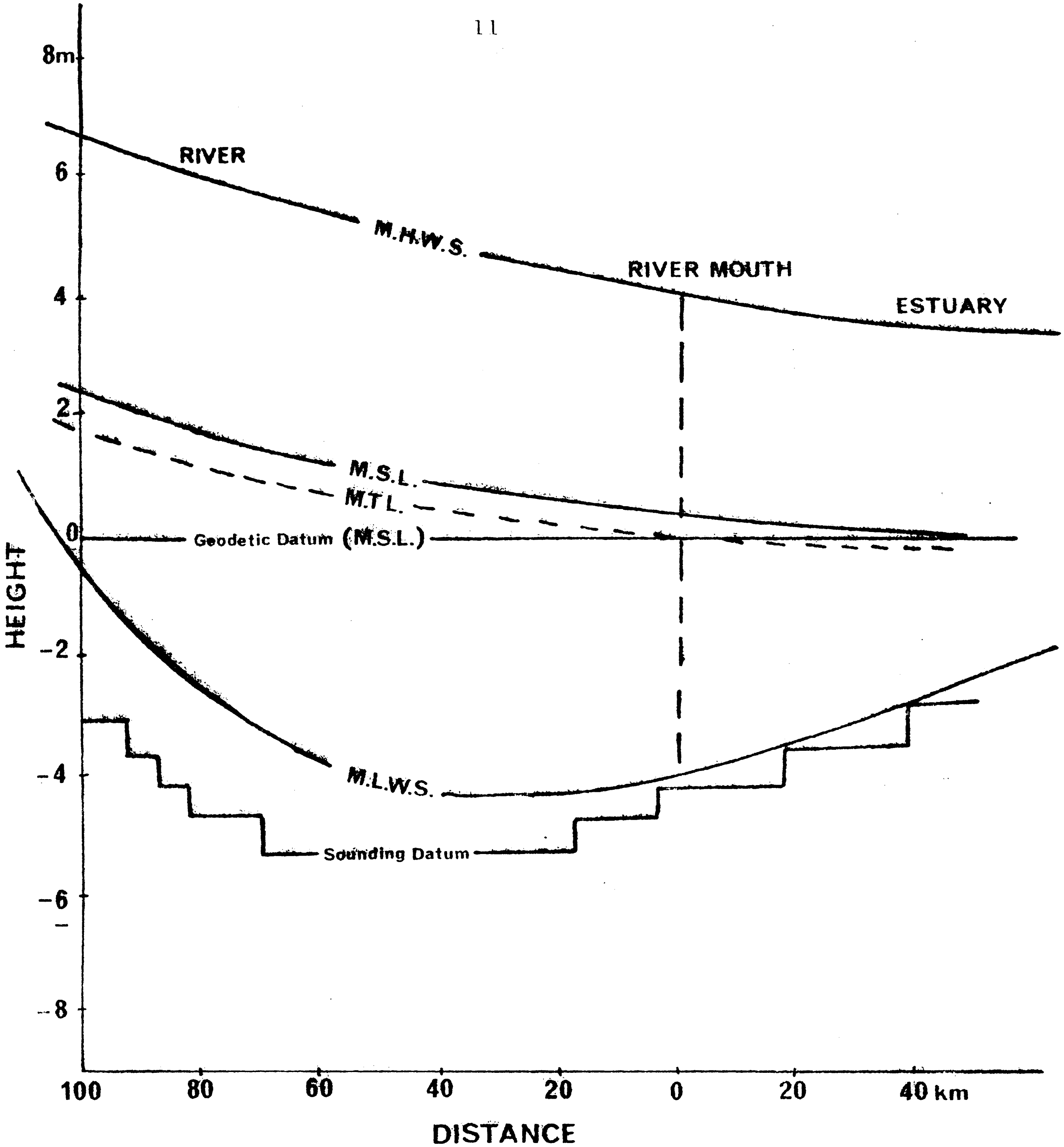


Figure 1-4

Variation of Sounding Datum in an Estuary or a River

(adapted from Admiralty Manual, 1969)

(i) a chart datum has already been established within the sounding area,

(ii) a chart datum has been established near the sounding area,

(iii) a chart datum has not previously been established anywhere nearby.

The actions corresponding to the above situations are:

(i) the surveyor should recover the established chart datum and use it,

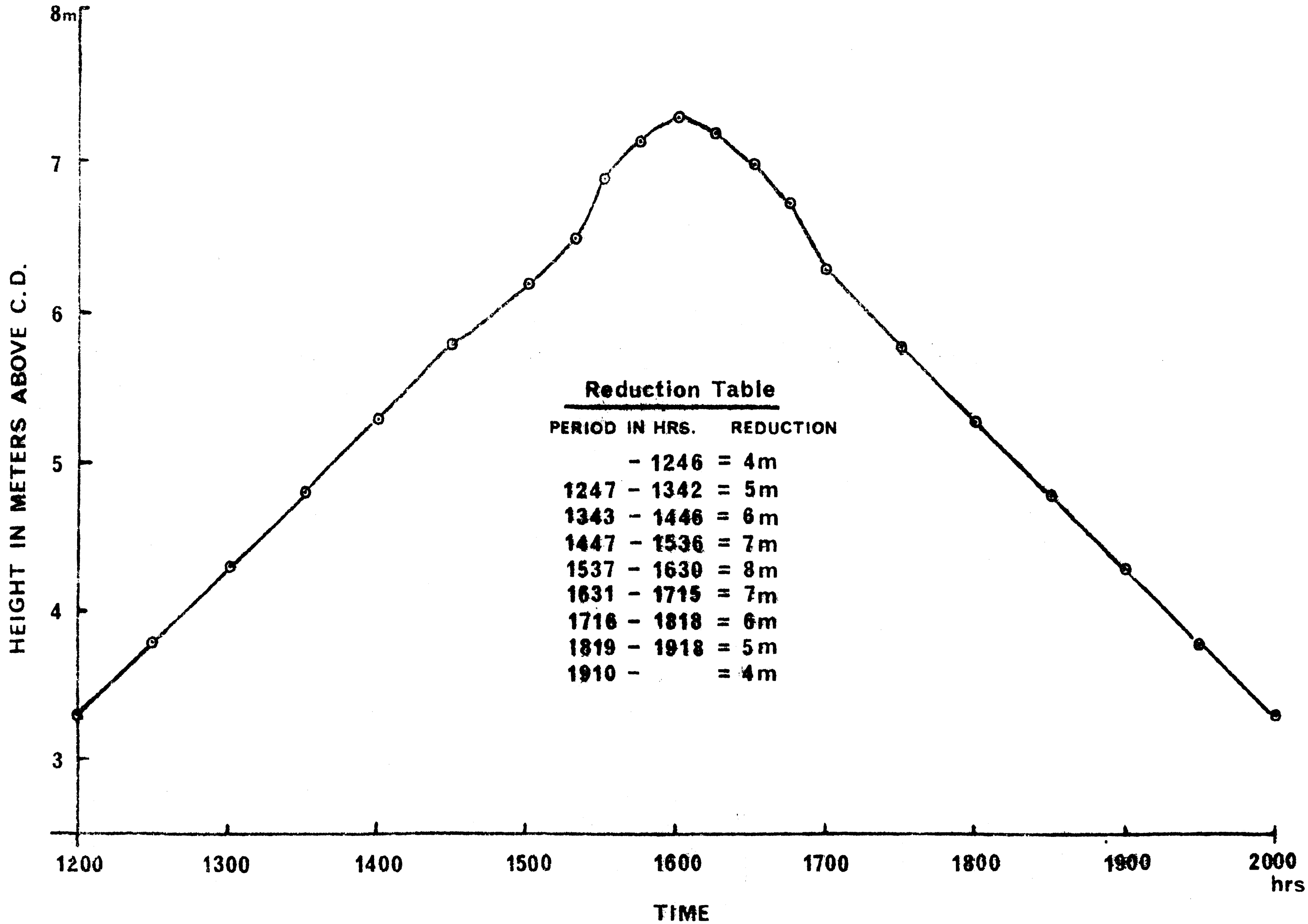
(ii) the surveyor should transfer the datum to the survey area; in other words, he should obtain a sounding datum for the area to be surveyed referenced to the established chart datum,

(iii) the surveyor should aim at establishing a chart datum.

1.2 Reduction of Soundings

Figure 1-1 illustrates the realtionship between a sounding at a time t and the chart datum. The height of tide at time t must be subtracted from the depth sounded to yield a reduced sounding. Manual reductions of soundings in tidal waters is a tedious aspect of the field hydrographer's tasks. It requires that a tide gauge be set up in the survey area and the rise and fall of tides observed while the sounding is performed. From the observed heights, it is possible to plot a curve showing the variations in the water levels and to reduce the soundings to a suitable reference plane. Figure 1-5 illustrates a typical reduction curve [Admiralty Manual of Hydrographic Surveying, 1969]. It has been drawn from the height observations at half hourly intervals with additional readings on either side of the high water. The reductions are scaled in steps of one metre and noted in the form of a table. For example, the reduction is 5 m from 1247 hrs to 1342 hrs, 6 m from 1343 to 1446 hrs.

For inshore surveys, it is usually convenient to set up a tide gauge and observe the tides while sounding is proceeding. If we are sounding offshore, the problem becomes complicated. It may be possible to use drying banks, islets or temporary structures such as drilling rigs as sites for tide gauges. Another possibility in the near future will be the use of automatic sea bed tide gauges [DeWolfe,1977].



Reduction Table

PERIOD IN HRS.	REDUCTION
- 1246	= 4m
1247 - 1342	= 5m
1343 - 1446	= 6m
1447 - 1536	= 7m
1537 - 1630	= 8m
1631 - 1715	= 7m
1716 - 1818	= 6m
1819 - 1918	= 5m
1910 -	= 4m

Figure 1-5

Tidal Reduction Curve

In the absence of the above alternatives, tidal observations could be made from an anchored survey vessel using an echo sounder.

If the cotidal charts for the area of interest are available or could be constructed, the necessary tidal information for the reduction of the sounding can be recovered from them. The objective of this report is to offer an automated analytical alternative to the manual task of tidal reduction of soundings through the use of tidal observations or cotidal chart information or a combination of the two.

Before describing the proposed scheme, an understanding of tidal theories and phenomena, analysis and prediction of tides, and the types and construction of cotidal charts are pertinent. Chapter II covers the theory of tide generation, harmonic analysis and prediction of tides. Chapter III is devoted to the types, construction and uses of cotidal charts.

II ANALYSIS AND PREDICTION OF TIDES

2.0 Introduction

When the water levels $h(t)$ have been observed at times t relative to a chosen datum at a tide gauge station, we have obtained a record distributed in time space (time series) and defined at the discrete time intervals. There is a trigonometric polynomial, Pn(t), of the form

$$\tilde{h}(t) = \sum_{i=0}^{n} (a_i \cos \omega_i t + b_i \sin \omega_i t), \qquad (2.1)$$

which can predict this time series at any time t in the interval. The analysis of this time series means the determination of the real numbers a_i, b_i, and ω_i. If we seek a least squares solution to this problem, we would have a system of normal equations that would be nonlinear. The presence of the non-linear trigonometric terms as unknowns leads to a serious problem which may or may not have a solution [Vaníček and Wells, 1972]. If, however, the frequencies ω_i are known, the coefficients a_i and b_i can be determined using least squares harmonic analysis.

The first and basic problem of harmonic tidal analysis, therefore, is the determination of the constituent frequencies ω_i. This is the first step in the complete decomposition of the observed time series into individual trigonometric terms. The first practical attempt at the determination of the constituent frequencies was made by

Darwin in 1886 using the orbital theories of the moon and the sun. In 1921, Doodson improved on the method by making a more complete expansion of the tidal potential using the modern luni-solar orbital theories.

The careful analysis of the tides at Honolulu and Newlyn by Munk and Cartwright [1966], indicated that the spectrum of a tidal record is a continuous function of frequency ω over the low frequency band, but that it approximates closely a line spectrum over the other frequencies - 'the constituent lines emerge from the noise background as trees from grass' [Godin, 1972]. As long as we do not work with the low frequency band, (as is generally the case in Hydrographic Surveying), it is reasonable to assume that to a good order of approximation the spectrum of a tidal record is a line spectrum. We can therefore treat the observed heights as a problem of spectral analysis of a time series. Letting

$$H_k = a_k^2 + b_k^2 ,$$

and

$$\alpha_k = \text{Arctan } (b_k/a_k) .$$

equation 2.1 can be rewritten as

$$\tilde{h}(t) = \sum_{k=0}^{\infty} H_k \cos(\omega_k t + \alpha_k) , \qquad (2.2)$$

where H_k is the amplitude of the constituent frequency ω_k, α_k is the phase of the constituent at time $t = 0$. If the function is defined on the finite set $M = \{0, \pm 1, \pm 2, \pm 3, \ldots \pm \ell\}$, the frequency ω_k is given by

$$\omega_k = \pi/\ell k \; . \tag{2.3}$$

H_k is obviously a non negative real number that describes the magnitude of the constituent frequency ω_k. By plotting the amplitude against integer frequencies, a visual interpretation of the contributions of the individual constituent frequencies (Figure 2-1) can be made. This represents the discrete transformation of the function from time space into frequency space [Vanicek and Wells, 1972].

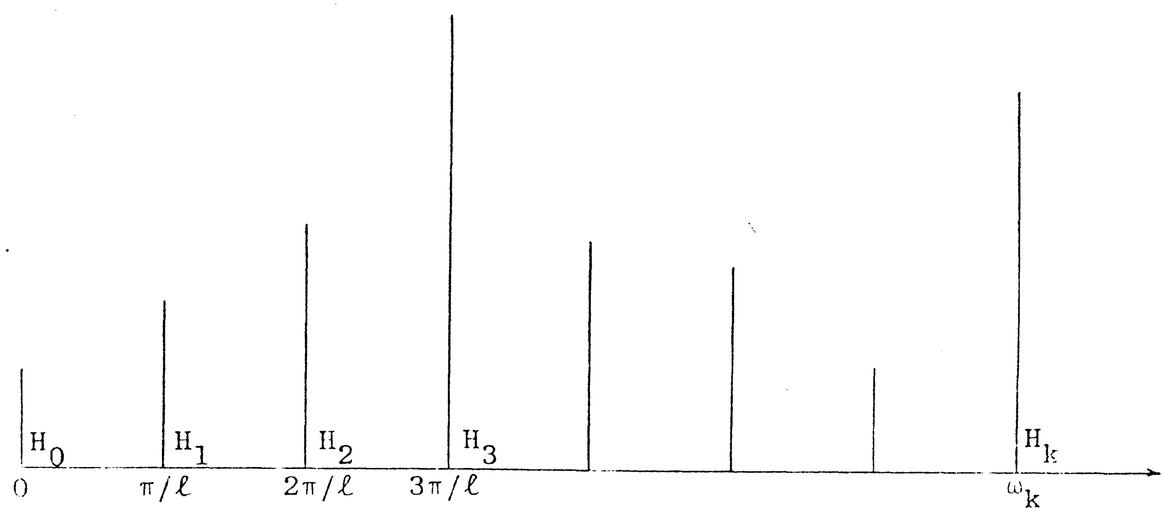


Figure 2-1

Line Spectrum of Function h(t)

Munk and Cartwright, [1966] introduced an entirely different method of tidal analysis which they called the response method. In this method, the potential is generated as a time series V(t) and an attempt is made at the prediction of height of the tide at a time t as the weighted sum of the past and present values of the potential

$$h(t) = \sum_{s} W(s)V(t - \tau_s). \qquad (2.4)$$

The weights W(s) are determined such that the prediction error $h(t) - \tilde{h}(t)$ is a minimum in the least square sense.

In this chapter, the theory of tidal generation and the traditional harmonic analysis and prediction of tides are described. The thinking behind the response analysis and prediction is briefly outlined.

2.1 Theory of Tide Generation

2.1.1 The Movements of the Moon (Real) and the Sun (Apparent)

The moon and the sun are the principal tide generating agents. Other heavenly bodies are either too distant or have too little mass to exert any significant force on the earth's surface. Figure 2-2 shows the relationship between the orbit of the moon and the apparent orbit of the sun. The sun moves in an apparent path around the earth on a plane called the ecliptic once every 365.25 solar days. For our present purposes, this movement can be regarded as uniform and inclined at an angle of 23° 27' (obliquity of the ecliptic) to the celestial equator. The point where the ecliptic crosses the celestial equator from south to north (B in Figure 2-2) is called the Vernal equinox or the first point of Aries ♈.

The moon moves eastward around the earth in an orbit

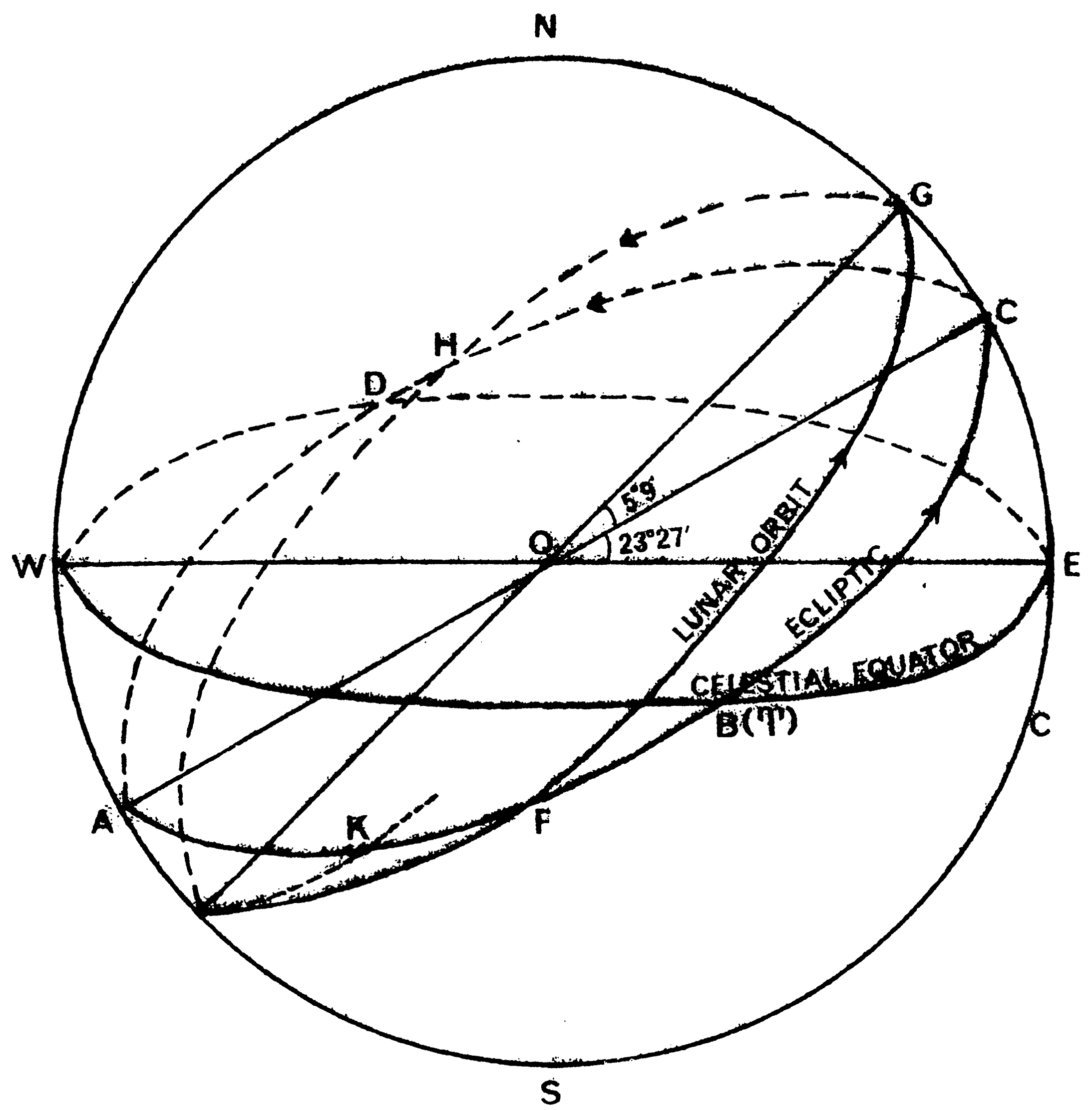


Figure 2-2

The Relationship Between the Orbital Motions of the Moon and the Sun inclined at about 5° 9' [Admiralty Manual of Hydrographic Surveying, 1969] to the ecliptic and crosses the ecliptic at the nodes. It takes approximately 27.2122 mean solar days for the moon to travel from the ascending node F to the ascending node K (Figure 2-2). As indicated in Figure 2-2, the lunar orbit does not cross the ecliptic at the same place consecutively. The nodes continually move westward along the ecliptic and this nodal movement or regression, as it is often called, has a period of 18.61 tropical years (one tropical year = 365.2422 mean solar days). Due to the nodal regression, the obliquity of the lunar orbit with respect to the celestial equator varies progressively between a maximum and a minimum, namely,

$$\text{Max.} = 23° \, 27' + 5° \, 9' = 28° \, 36' ,$$

$$\text{Min.} = 23° \, 27' - 5° \, 9' = 18° \, 18' .$$

2.1.2 The Tide Generating Forces and Potentials

To derive the mathematical expression for the tide generating forces of the moon and the sun, the principal factors to be taken into consideration are:

(i) the revolution of the moon around the earth in an orbit inclined to the equator,

(ii) the motion of the earth around the sun along the ecliptic which is also inclined to the equatorial plane,

(iii) the rotation of the earth around its axis.

The tide generating forces at the earth's surface result from a combination of two basic forces; (i) the force of gravitation exerted by the moon (and sun) upon the earth, and (ii) centrifugal forces produced by the revolutions of the earth and the moon (and the earth and the sun) around their common centre of mass known as the barycentre.

The magnitude of centrifugal force produced by the revolution of the earth-moon system around barycentre (which lies approximately 1709 km beneath the earth's surface on the side towards the moon and along the line connecting centres of mass of the earth and of the moon) is the same at any point on or beneath the earth's surface [National Ocean Survey, 1977]. Its magnitude is [Godin, 1972]

$$F_c = KM/\rho_0^2 ,$$

where ρ_0 is the distance between the centres of mass of the earth and of the moon (Figure 2-3), K is the universal gravitational constant, and M is the mass of the moon.* The gravitational force exerted by the moon is different at different positions on or beneath the earth's surface because the force of attraction

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*<u>Note</u>: The earth-moon system is used here to develop the equations for tidal potential. The same development resulting in similar equations can be used for the sun or any other heavenly body.

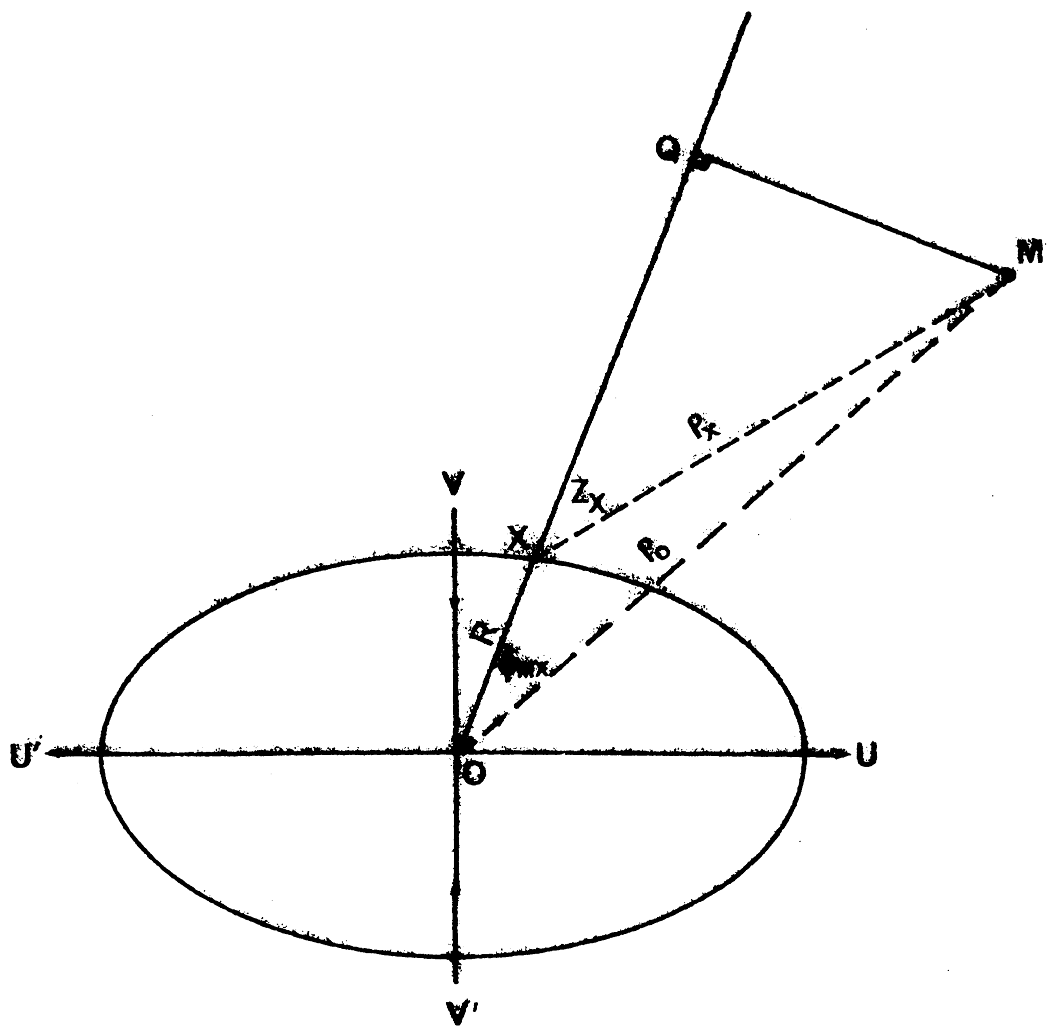


Figure 2-3

Effects of the Gravitational Attraction

Of a Heavenly Body M on the Earth

between two bodies is a function of the distance between them. This gravitational force at 0 (Figure 2-3) is

$$F_{g0} = KM/\rho_0^2 , \qquad (2.6)$$

and at X is

$$F_g = KM/\rho_x^2 , \qquad (2.7)$$

where ρ_x is the distance between the centre of mass of the moon and point X on the earth's surface. The tide generating force due to the moon M at point X (Figure 2-3) on the earth's surface is defined as the difference between the gravitational force at X and that at the resultant centre of mass of the earth-moon system where the gravitational and centrifugal forces are in equilibrium [Dronkers, 1972].

In terms of potentials, the attracting potential at X and at time t is

$$f_g = KM/\rho_x - KM/\rho_0 , \qquad (2.8)$$

and the potential of the constant vector field of the centrifugal force is

$$f_c = KM\ a \cos \Phi_{mx}/\rho_0^2 , \qquad (2.9)$$

where Φ_{mx} is the zenith distance as shown in Figure 2-3, and a is the mean radius of the earth. From equations 2.8 and 2.9 and making use of the definition of the tide generating force given above, the tide generating potential (V_m) due to the moon at X and at time t is [Dronkers, 1972].

$$V_m = KM[\frac{1}{\rho_x} - \frac{1}{\rho_0} - a \cos \Phi_{mx}/\rho_0^2] \ . \qquad (2.10)$$

Figure 2-4 shows the distribution on the earth of tide forces of lunar origin. At point A nearest to the moon, the force of attraction is greater than the centrifugal force. The resultant is the tidal force (F_t) towards the moon. At C, the centre of the earth, both centrifugal and the gravitational forces are equal. The tidal force at the centre consequently is zero. At B farthest from the moon where the centrifugal force is greater than the attractive force, the tidal force is directed away from the moon.

We can express ρ_x (equations 2.10) in terms of ρ_0 and Φ_{mx} using the cosine formula of plane trigonometry given by

$$\rho_x^2 = \rho_0^2 + a^2 - 2a\rho_0 \cos \Phi_{mx} \ . \qquad (2.11)$$

Equation 2.11 can be rewritten as

$$\frac{1}{\rho_x} = \frac{1}{\rho}[1 - 2\frac{a}{\rho_0} \cos \Phi_{mx} - (\frac{a}{\rho_0})^2] \ . \qquad (2.12)$$

When $\frac{1}{\rho_x}$ is expanded in powers of the parallax a/ρ_0 by means of a Taylor series, expansion in zonal harmonics is obtained and equation 2.10 is given as [Godin, 1972]

$$V_m = KM/\rho_0[P_0(\Phi_{mx}) + (a/\rho_0)P_1(\Phi_{mx}) + (a/\rho)^2$$

$$P_2(\Phi_{mx}) + (a/\rho_0)^3P_3(\Phi_{mx}) + \ldots\ldots]. \qquad (2.12a)$$

The first term of the expansion

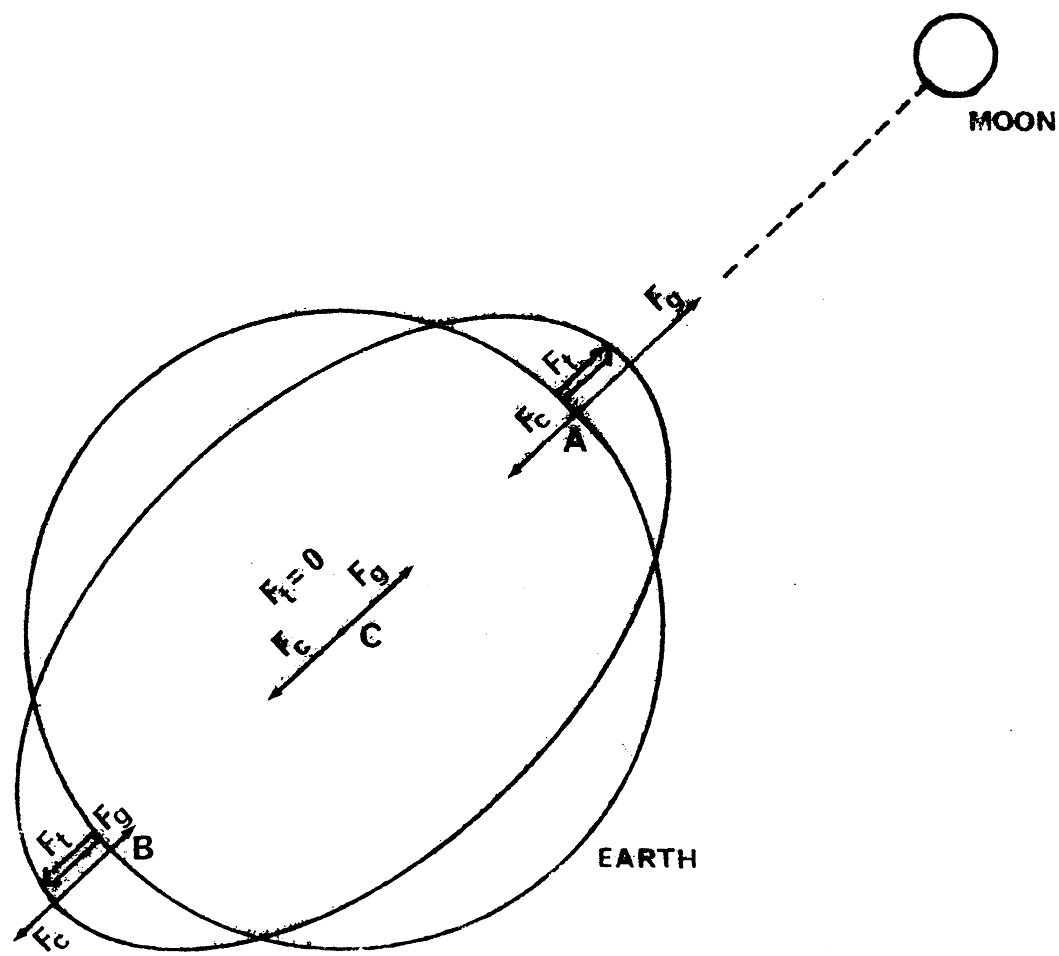


Figure 2-4

Distribution of Tidal Force

$$V_0 = KM/\rho_0 \ , \qquad (2.13a)$$

can be overlooked because it is a constant and hence has no physical significance.

The second term

$$V_1 = KM/\rho_0^2 \ a \cos \Phi_{mx} \ , \qquad (2.13b)$$

is the lunar gravitational force at the centre which is equivalent to the centrifugal force.

The third term is

$$V_2 = KM \ a^2/\rho_0 \ \tfrac{1}{2}(3 \cos^2\Phi_{mx} - 1) \ . \qquad (2.13c)$$

This is the significant term as far as tidal potential is concerned. The fourth term is

$$V_3 = KM \ a \ \frac{3}{\rho_0^4} \ \tfrac{1}{2}(5 \cos^3\Phi_{mx} - 3 \cos \Phi_{mx}) . \qquad (2.13d)$$

For practical purposes, the fourth term is of little significance. It must be considered when we are required to determine the potential with a higher degree of accuracy. Henceforth in this report, V_2 is the tidal potential. It is decomposed into constituent frequencies and this, as has been mentioned, is the first step in the harmonic analysis of tidal records.

We can rewrite equation 2.13 as

$$V_m = \tfrac{3}{2} \ KM \ a^2/\rho^3(\cos^2\Phi_{mx} - \tfrac{1}{3}) \ . \qquad (2.14)$$

The principal variable in the tide generating potential defined by equation 2.14 is the zenith distance Φ_{mx}. This quantity changes due to two effects [Dronkers, 1964], namely,

(i) the daily rotation of the earth about its axis (24 hours) combined with the motion of the moon in its orbit (50 minutes per day) giving a total periodicity of 24 hours, 50 minutes,

(ii) effects due to moon's motion in its orbit during a lunar month which results in a mean monthly periodicity of its declination δ of 27.3 mean solar days.

The other variable in the potential that must be accounted for is ρ_0, the mean distance of the moon to the earth which varies due to the irregular elliptical nature of lunar orbit.

The expression of the potential as a function of time dependant variables and as a function of position on the earth surface is achieved by transforming our Horizon co-ordinate system to the Hour Angle system using [Smart, 1971]

$$\cos \Phi_{mx} = \sin \phi \sin \delta + \cos \delta \cos \phi \cos t \ , \qquad (2.15)$$

where ϕ is the geodetic latitude, δ is the declination and t is the hour angle. We can evaluate $\cos^2\Phi_{mx}$ in terms of ϕ, δ and t which after some manipulation yields

$$V_m = G(a, \rho)\,[\cos^2\phi \cos^2\delta \cos 2t + \sin 2\phi \sin 2\delta \cos t + 3(\sin^2\phi - \tfrac{1}{3})(\sin^2\delta - \tfrac{1}{3})] , \qquad (2.16)$$

in which $G(a, \rho)$ is defined as the Doodson constant, namely $G(a, \rho) = \frac{3}{4} KM\, a^2/c^3$ (c is the mean semi-axis of the orbital ellipse of the moon).

Equation 2.16 contains the variables ρ, δ, t which are dependant on time. The first term of the equation containing cos 2t includes the semi-diurnal constituents with periods approximating half a lunar day. The second term containing cos t determines the diurnal constituents with periods approximating a lunar day. The third term is independent of t and hence contains the long period constituents. It is only subject to variations in declination δ and distance ρ of the celestial body. We have now been able to decompose the tidal potential into 3 frequency bands

0 - for long period constituents,

1 - for diurnal constituents,

2 - for semi-diurnal constituents.

This is only a step towards the complete decomposition of the tidal potential into the numerous periodic constituents. For the complete decomposition, the work of Darwin and Doodson are important. Darwin's decomposition provides readily the most important constituents and their relative importance while Doodson's method is more suitable for rigorous developments and provides a greater number of constituents.

2.1.3 Development According to Darwin

This development is based on deriving relations for $\sin \delta$ and $\cos \delta \cos t$, which occur in equation 2.16 in terms of

- t - the local solar time,
- s - the longitude of the moon referred to the equator,
- h - the mean ecliptic longitude of the sun.

Darwin used the old lunar theory and all quantities were given with respect to the moon's orbit projected onto the celestial equator. He considered

- P - the ecliptic longitude of the moon's perigee,
- n - the ecliptic longitude of the moon's nodes,
- Ps - the ecliptic longitude of the sun's perigee,

as constant over one year.

Referring to Figure 2-5, the relations are derived from right spherical triangles MAM' and MX'M' and the oblique triangle MAX'. A is a point of intersection of the lunar orbit and the equator, X' and M' are the projections of X and M onto the equator [Dronkers, 1964 Page 59]. From triangle MAM' and MX'M', the sine rule of spherical trigonometry yields

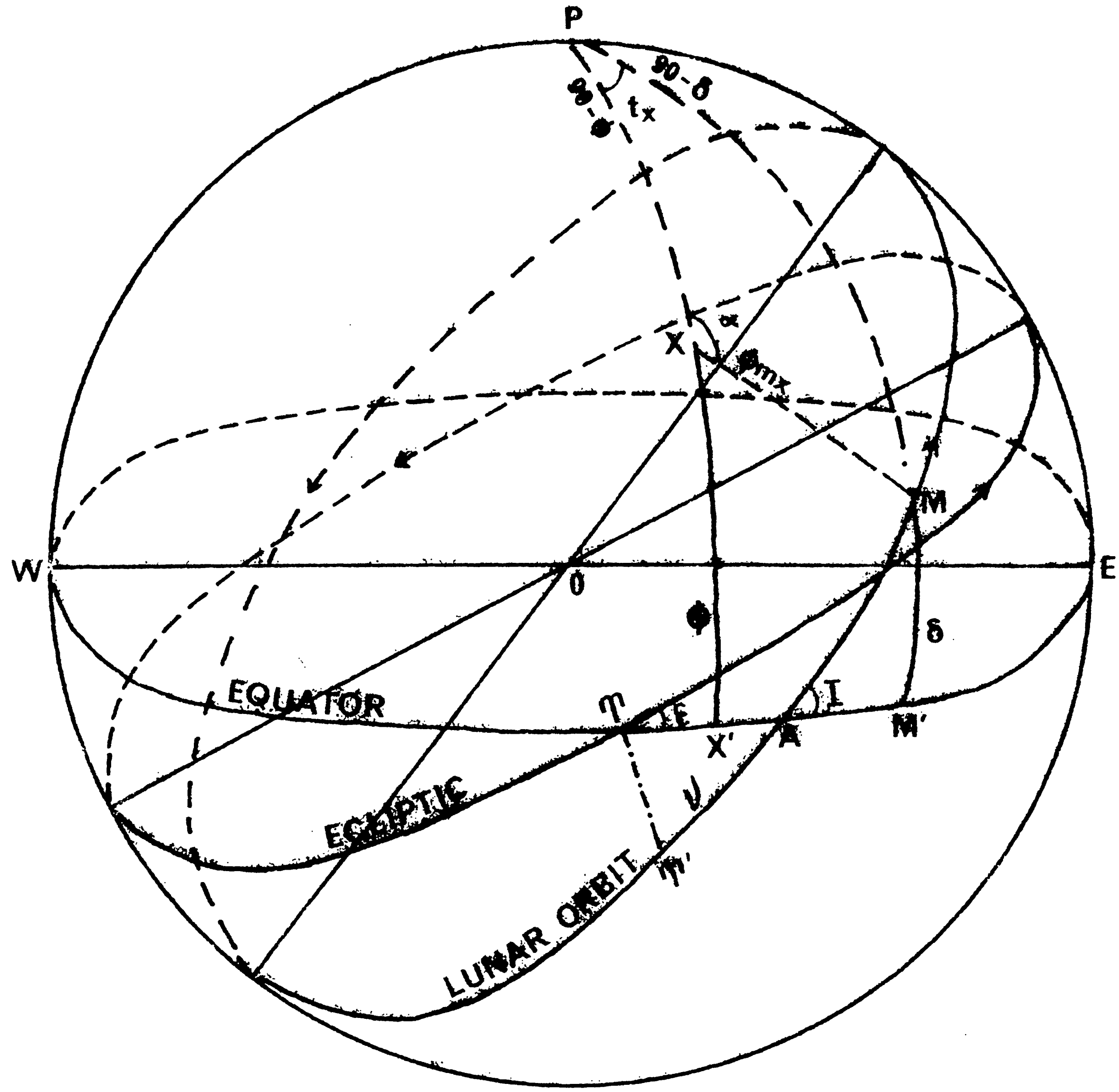


Figure 2-5

Orbital Parameters

$$\sin \delta = \sin I \sin(s - \nu + k), \qquad (2.17)$$

$$\cos \delta \cos t - \cos \chi, \qquad (2.18)$$

where I is the angle between the orbit of the moon and the celestial equator, s is the longitude of the mean moon on the equator, ν is the distance between the referred equinox γ' and the intersection of the lunar orbit with the equator at A, χ is the arc MX' and arc AM = $s - \nu + k$. k is the difference between the true longitude of the moon (s') measured from γ' (γ'M) and the longitude of the mean moon in the equator s. From oblique triangle MAX' and using the cosine formula we have that

$$\cos \chi = \cos(15°tx + h - v) \cos(s - \nu + k) + \sin(15°tx + h - v) \sin(s - \nu + k) \cos I \qquad (2.19)$$

in which h is the mean ecliptic longitude of the sun and v is the right ascension of A, 15° of arc is equal to one hour in time. The terms $\sin^2\delta$, $\sin 2\delta \cos t$ and $\cos^2\delta \cos 2t$ which are contained in the potential formula (equation 2.16), can be determined from equations 2.17, 2.18 and 2.19 in terms of the orbital elements tx, s, h and v. When these are substituted back into equation 2.16, we obtain a series of harmonic terms of which the arguments depend on the rotation of the earth (15°tx), the mean motion of the moon in its orbit (s) and the mean motion of the earth in orbit (h) namely,

$$V_m = G(a, \rho)\{\cos^2\phi[\cos^4 \frac{I}{2} \cos(30°tx - 2s - 2h - 2\nu - 2v - 2k)$$

$$+ \frac{1}{2} \sin^2 I \cos(30°tx + 2h - 2v)$$

$$+ \sin^4 \frac{I}{2} \cos(30°tx + 2s + 2h - 2\nu - 2v + 2k)]$$

$$+ \sin 2\phi[\sin I \cos^2 \frac{I}{2} \cos(15\ tx - 2s + h + 2\nu - v$$

$$- 2k - 90°) + \frac{1}{2} \sin 2I \cos(15\ tx + h - v + 90°)$$

$$+ \sin I \sin^2 \frac{I}{2} \cos(15°tx + 2s + h - 2\nu - v - 2k + 90°)]$$

$$+ (1 - 3 \sin^2\phi)[\frac{2}{3} - \sin^2 I + \sin^2 I \cos(s - \nu + k)]\}. \qquad (2.20)$$

In the development for solar constituents, the terms ν and v will vanish and angle I will change to ε.

2.1.4 Development According to Doodson

Doodson's method principally involves the use of a rigorous expansion of the ecliptic longitude and latitude of the moon. For the development of $\sin\delta$ and $\cos\delta \cos t$, he introduced the ecliptic longitude λ_m and latitude β_m of the moon and the local siderreal time θ of the point X (Figure 2-3) on the earth's surface. The equations are

$$\sin\delta = \sin\varepsilon \sin\lambda_m \cos\beta_m + \cos t \sin\beta_m , \qquad (2.21)$$

$$\cos\delta \cos t = \cos\beta_m \cos\lambda_m \cos\theta + (\cos\varepsilon \cos\beta_m \sin\lambda_m - - \sin\varepsilon \sin\beta_m)\sin\theta , \qquad (2.22)$$

where ε is the obliquity of the ecliptic.

Finally the potential V_m is developed as the sum of periodic functions of six variables, namely, tx, s, h, P, n and Ps.

Doodson obtained 400 periodic constituents from his development of which the principal ones are listed in Table 2-1 [Vaniček, 1973].

The constituent frequencies can be described in mathematical terms using Doodson numbers and the astronomical variables, namely

$$\omega_k = \overline{k}\overline{f} = k_1 f_1 + k_2 f_2 + k_3 f_3 + k_4 f_4 + k_5 f_5 + k_6 f_6 , \quad (k_X = 0 \pm 1 \pm 2). \tag{2.23}$$

$\overline{f}$ is a six dimensional vector whose components are the basic frequencies of the motions of the earth, the moon and the sun, namely

f_1^{-1} is the period of the earth's rotation τx (1 day),

f_2^{-1} is the period of moon's orbital motion $\dot{s}$ (1 month),

f_3^{-1} is the period of earth's orbital motion $\dot{h}$ (1 year),

f_4^{-1} is the period of lunar perigee $\dot{P}$ (8.85 years),

f_5^{-1} is the period of regression of lunar nodes $\dot{N}$ (18.61 years),

f_6^{-1} is the period of solar perigee $\dot{P}s$ (21000 years).

f_6 is usually omitted because it is insiginificant. k_X = 0, 1, 2 refers to the tidal species, 0 for long period, 1 for diurnal and 2 for semi-diurnal. (k_1, k_2) is called the group number. (k_1, k_2, k_3) is called the constituent number.

With the constituent frequencies determined, which are the same anywhere on the earth's surface, the first step in the harmonic analysis is now completed. In the next

Symbol	Velocity per hour	Amplitude 10^5	Origin (L, lunar; S, solar
		Long period components	
M_o	0°,000000	+ 50458	L constant flattening
S_o	0°,000000	+ 23411	S constant flattening
S_a	0°,041067	+ 1176	S elliptic wave
S_{sa}	0°,082137	+ 7287	S declinational wave
M_m	0°,544375	+ 8254	L elliptic wave
M_f	1°,098033	+ 15642	L declinational wave
		Diurnal components	
Q_1	13°,398661	+ 7216	L elliptic wave of O_1
O_1	13°,943036	+ 37689	L principal lunar wave
M_1	14°,496694	- 2964	L elliptic wave of ${}^mK_1$
π_1	14°,917865	+ 1029	S elliptic wave of P_1
P_1	14°,958931	+ 17554	S solar principal wave
S_1	15°,000002	- 423	S elliptic wave of ${}^sK_1$
${}^mK_1$	15°,041069	- 36233	L declinational wave
${}^sK_1$	15°,041069	- 16817	S declinational wave
ψ_1	15°,082135	- 423	S elliptic wave of ${}^sK_1$
ϕ_1	15°,123206	- 756	S declinational wave
J_1	15°,585443	- 2964	L elliptic wave of ${}^mK_1$
OO_1	16°,139102	- 1623	L declinational wave
		Semi-diurnal components	
$2N_2$	27°,895355	+ 2301	L elliptic wave of M_2

Table 2-1 Principal Tidal Constituents As Derived by Doodson.

Table 2-1 -continued .

Symbol	Velocity per hour	Amplitude 10^5	Origin (L, lunar; S, solar)
μ_2	27°,968208	+ 2777	L variation wave
N_2	28°,439730	+ 17387	L major elliptic wave of M_2
ν_2	28°,512583	+ 3303	L evection wave
M_2	28°,984104	+ 90812	L principal wave
λ_2	29°,455625	- 670	L evection wave
L_2	29°,528479	- 2567	L minor elliptic wave of M_2
T_2	29°,958933	+ 2479	S major elliptic wave of S_2
S_2	30°,000000	+ 42286	S principal wave
R_2	30°,041067	- 354	S minor elliptic wave of S_2
${}^mK_2$	30°,082137	+ 7858	L declinational wave
${}^sK_2$	30°,082137	+ 3648	S declinational wave
	Ter-diurnal component		
M_3	43°,476156	- 1188	L principal wave

section, the least squares harmonic analysis of observed tidal records, to determine the tidal constants H_k and g_k, where H_k is the amplitude of the constituent k and g_k the phase lag of the constituent k at the observed station, is described.

2.2 Least Squares Harmonic Analysis and Prediction of Tides

The height of tide h(t) at any place and at any time t can be expressed as the sum of harmonic terms [Dronkers, 1972]

$$h(t) = s_0 + \sum_{k=1}^{\infty} H_k \cos(\omega_k t + \alpha_k) \ , \tag{2.24}$$

where s_0 is the height of mean water level above the datum in use, ω_k is the constituent frequency, H_k is the amplitude of the constituent k and α_k is the initial phase of the constituent. The number of constituents included will depend on the accuracy required for prediction. For ordinary hydrographic works, the constituents M_2, S_2, N_2, O_1, K_1, P_1 are sufficient to yield an accuracy of 0.2 m in a prediction. α_k depends on the varying mean longitudes of the moon's perigee and sun's perigee with periods of approximately 8.61 and 21000 years respectively and the ecliptic longitude of the moon's ascending node with a period of 18.61 tropical years. To take these effects into account, f_5 and f_6 constituents are eliminated and a node factor f_k and a correction for equilibrium argument U_k are introduced.

Equation 2.24 is rewritten as

$$h(t) = s_0 + \sum_{k=1}^{N} f_k H_k \cos(\omega_k t + (V_k + U_k) - X_k), \tag{2.25}$$

in which $(V_k + U_k)$ is the value of the equilibrium argument of the constituent k when t = 0, generally called the astronomical argument, X_k is the phase lag of the tidal constituent behind the phase of the corresponding equilibrium constituent at Greenwich, N is the number of constituents in use.

All tide observations are made on local standard time, often referred to as zone time and denoted as ZT. Equation 2.25 therefore has to be modified so that allowance is made both for the zone time and the local longitude since the meridian of the observing station and the meridian defining zone time are usually not coincident (Figure 2-6).

If $(V_k - U_k)$ is the phase of the equilibrium constituent k at the Greenwich, P(= 0, 1, 2) is the tide species number, 0 for long period, 1 for diurnal and 2 for semi-diurnal and λ_x is the geodetic longitude of the point, say X_2 (Figure 2-6) west of the Greenwich, then $(V_k + U_k) - P\lambda_x$ is the phase expressed in Greenwich mean time of the equilibrium constituent k of the tide species P at the point X_2 west of Greenwich. This is now transformed into the zone time of the place. If the correction for zone time is ΔT (where ΔT is negative west of Greenwich and positive east of Greenwich) and the frequency of the constituent is ω_k, we must subtract $\omega_k . \Delta T$ from the phase of the equilibrium tide. Thus with respect to the point X_2 west of Greenwich, $V_k + U_k - P\lambda + \omega k . \Delta T$ is the phase of the equilibrium tide expressed in the local zone time.

If g_k is the phase lag X_k corrected for longitude and

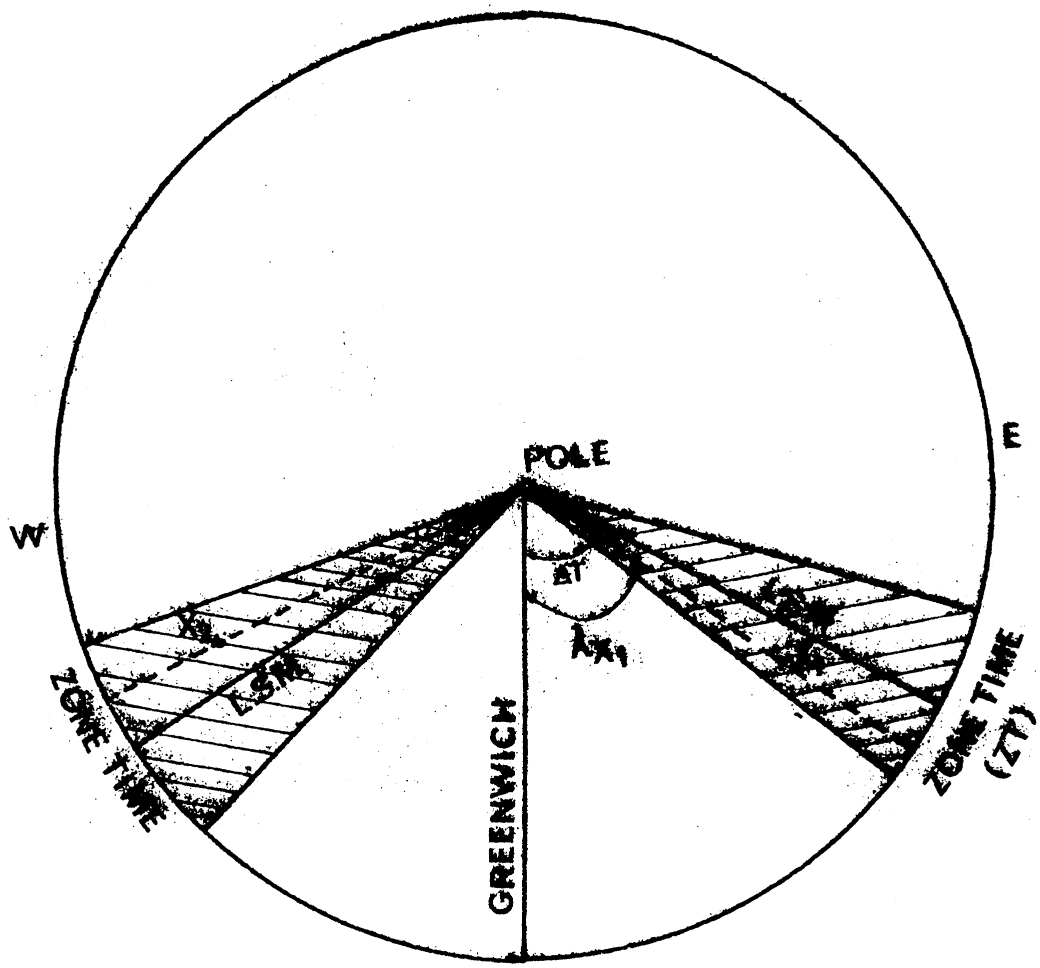


Figure 2-6

Time Relationships

zone time, then we have that

$$V_k + U_k - g_k = V_k + U_k - P\lambda - \omega_k . \Delta T - X_k ,$$

$$g_k = X_k + P\lambda + \omega_k . \Delta T . \qquad (2.26)$$

The determination of H_k in equation 2.25 and g_k in equation 2.26 are the objectives in the harmonic analysis of tides. They are determined from a series of observed tides at a tide gauge station and are called the harmonic constants for that station. The estimation of these constants for a station is improved when more observations are available.

From equation 2.25, using trigonometric relations for compound angles

$$f_k H_k \cos[\omega_k . t + (V_k + U_k) - X_k)] \equiv f_k H_k \cos((V_k + U_k) - X_k) \cos(\omega_k . t) + (f_k H_k \sin((V_k + U_k) - X_k) \sin(\omega_k . t) . \qquad (2.27)$$

If we let

$$f_k H_k \cos((V_k + U_k) - X_k) = A_k , \qquad (2.28)$$

$$f_k H_k \sin((V_k + U_k) - X_k) = B_k , \qquad (2.29)$$

equation 2.25 is rewritten as

$$h(t) = S_0 + \sum_{k=1}^{N} A_k \cos(\omega_k . t) + \sum_{k=1}^{N} B_k \sin(\omega_k . t) . \qquad (2.30)$$

Equation 2.30 is a trigonometric polynomial that can predict the observed time series h(t) at time t in the given interval of time. Least squares approximation methodology [Vanicek and Wells, 1972; Moritz, 1977; Appendix I] can be used to determine the coefficients S_0, A_k, B_k

(k = 1, 2, 3, ... N). The number of coefficients to be solved is

$$U = 2N + 1, \tag{2.31}$$

where N is the number of constituent frequencies used. We can choose our base functions as

$$\psi \equiv \{1, \cos \omega_1 t, \sin \omega_1 t \; \dots \; \dots \; \cos \omega_N t, \sin \omega_n t\}. \tag{2.32}$$

The Vandermonde's design matrix A is

$$\underset{MxU}{A} = \begin{bmatrix} 1, & \cos \omega_1 t_1, & \sin \omega_1 t_1, & \dots & \cos \omega_N t_1, & \sin \omega_N t_1 \\ 1, & \cos \omega_1 t_2 & \sin \omega_1 t_2, & \dots & \cos \omega_N t_2, & \sin \omega_N t_2 \\ \vdots & & & & & \\ 1, & \cos \omega_1 t_m, & \sin \omega_1 t_m & \dots & \cos \omega_N t_m, & \sin \omega_N t_m \end{bmatrix}, \tag{2.33}$$

in which m equals the number of measurements h(t) that have been made. For weights, we can consider each observation as having been made independently with equal amount of reliability. The error in observations (σ_{X_L}), can be taken to be equal to the resolution of the tide gauge used so that

$$\underset{mxm}{\Sigma_L} = \text{diag}\left(\sigma^2_{L_1}, \sigma^2_{L_2} \; \dots \; \sigma^2_{L_m}\right),$$

and the corresponding weight matrix is

$$\underset{mxm}{P} = \Sigma_L^{-1} = \text{diag}\left(\frac{1}{\sigma^2_{L_1}}, \frac{1}{\sigma^2_{L_2}}, \dots \frac{1}{\sigma^2_{L_m}}\right), \tag{2.34}$$

in which σ_0^2 (the a priori variance factor) is taken as unity.

The solution for the vector of coefficients is given as

$$\hat{C} = (A^T P A)^{-1} A^T P F, \tag{2.35}$$

in which $\hat{C} = [S_0, A_1, B_1, A_2, B_2 \ldots A_k, B_k]^T$.

The solution for the residual vector is

$$\hat{V} = A\hat{C} - F \quad , \tag{2.36}$$

where F is a vector of observed heights.

The associated variance covariance matrix of the vector of coefficients is

$$\Sigma_{\hat{c}} = \hat{\sigma}_0^2 [A^T P A]^{-1} \quad , \tag{2.37}$$

where $\hat{\sigma}_0^2$ is the estimated variance factor given by

$$\hat{\sigma}_0^2 = \frac{\hat{V}^T P \hat{V}}{df} \quad , \tag{2.38}$$

df represents the degree of freedom given in this case by the number of observations minus the number of coefficients (df = m - u).

With the coefficients S_0, A_k, B_k determined, equations 2.26, 2.28 and 2.29 yield the harmonic constants H_k and g_k. Note that if however it is not intended to predict the tides in the past or in the future, the constants need not be computed. The tide at any time t in the time interval can be predicted using the polynomial.

From 2.28 and 2.29

$$\frac{f_k H_k \sin((V_k - U_k) - X_k)}{f_k H_k \cos((V_k + U_k) - X_k)} = \tan((V_k + U_k) - X_k) = \frac{B_k}{A_k} \; ,$$

or

$$((V_k + U_k) - X_k) = \tan^{-1}(B_k/A_k) , \tag{2.39}$$

and

$$f_k H_k \cos((V_k + U_k) - X_k) = A_k \quad ,$$

$$H_k = A_k / f_k \cos((V_k + U_k) - X_k) \quad , \qquad (2.40)$$

or

$$f_k H_k \sin((V_k + U_k) - X_k) = B_k \quad ,$$

$$H_k = B_k / f_k \sin((V_k + U_k) - X_k) \quad . \qquad (2.40a)$$

To completely solve our problem, we have to determine the astronomical argument ($V_k + U_k$) and the nodal factor (f_k). The values are usually tabulated in tide tables (eg. Admiralty Tide Tables), or they can be computed.

The astronomical argument is given as [Godin, 1972; pp. 171-178]

$$V_k(t) = k_1\hat{\tau} + k_2\hat{S} + k_3\hat{h} + k_4\hat{P} + k_5\hat{N} + k_6\hat{Ps} \quad . \qquad (2.41)$$

where $\hat{\tau}$, $\hat{S}$, $\hat{h}$, $\hat{P}$, $\hat{N}$ and $\hat{Ps}$ are the values of the astronomical variables at the instant of time t from the origin of time and are given as

$$\hat{S} = S_0 + \Delta t\ \dot{S} ,$$

$$\hat{h} = h_0 + \Delta t\ \dot{h} ,$$

$$\hat{P} = P_0 + \Delta t\ \dot{P} ,$$

$$\hat{N} = N_0 + \Delta t\ \dot{N} ,$$

$$\hat{Ps} = Ps_0 + \Delta t\ Ps ,$$

$$\hat{\tau} = 0.0416\ (hh\ mm) + \hat{h} - \hat{S} \ .$$

S_0, h_0, P_0, N_0 and Ps_0 are the values of the astronomical variables at the time $t = 0$, hh mm represents the hours and minutes of the day, $\dot{S}$, $\dot{h}$, $\dot{P}$, $\dot{N}$, $\dot{Ps}$ are the rates of change of the astronomical variables in cycles per mean lunar day. U_k is the phase of the astronomical argument (V_k) at time t = 0.

The nodal (modulation) factor is given by [Godin, 1972]

$$f_k = 1 + \sum_{j=1}^{n_0} |r_{kj}| \exp[2\pi\ i(\Delta k_4(j)\hat{P} + \Delta k_5(j)\hat{N} + k_6(j)\hat{P}_s)], \tag{2.42}$$

in which r_{kj} is a complex number which depends on Δk_4, Δk_5 and Δk_6. The j's inside the differences in Doodson numbers indicate that they depend on a specific constituent within a cluster.

It is important to note that in the discussion so far, there was no mention of removing the noise part of the observed series before the analysis is made. The harmonic constants obtained are therefore likely to include other effects beside those of the astronomic forces and are consequently in a certain measure variable. The harmonic analysis should be based on a series of very selective filterings so as to permit isolation of an oscillation having a maximum tide/noise ratio. Godin [1972] has given several filters that could be used to eliminate the noise part or suppress certain frequencies.

Vaníček [1970] pointed out that there is an obvious danger in removing the noise part of a series when the magnitudes are not known. On the other hand, it is usually equally deterimental to leave these constituents unattended because they may distort the spectral image of the series to a considerable degree. He described a method of least squares spectral analysis that could be used to analyse a time series and locate the frequencies accurately without first removing the noise part.

Mosetti and Manca [1972] described a number of methods for separating a certain number of tidal constituents by means of successive approximations and thus to completely extract astronomic tide from the tidal records. The frequency interval in which the tidal constituents occur are divided into a number of wave groups, the periods within each group being very close to each other but sufficiently distinct from the periods of constituents in all other groups. By drawing the graph of oscillations in each group, it is easy to see that the modulations are perturbed to some extent due to interference phenomena from waves within the group. If we are dealing with series extending over a fairly long period, it is possible to evaluate the intervals on the record that are least perturbed and where the amplitudes vary with regularity dictated by astronomic laws. The harmonic constants can then by computed for those intervals.

2.3 Tidal Analysis and Prediction by Response Method

2.3.1 General

Munk and Cartwright [1966] presented an entirely different method of tidal analysis and prediction. They applied the theory of time series to the tidal observations at a gauge station to determine certain coefficients which replaced the amplitudes H_k and the phase lags g_k of the tidal constituents as in the harmonic analysis. Even though the theory of this method is more involved than the harmonic method, the authors claim that the response method gives a simpler and physically more meaningful representation of tides than the harmonic method. Unlike the traditional harmonic method which attempts to express the tides as the sum of harmonic functions of time, the response method expresses tide as the weighted sum of the past, present and future values of a relatively small number of time varying input functions.

Dronkers [1972] described the method as a more empirical modification of the equilibrium tide based on the theory of time series. He added that the principal advantage of the response method is that the total number of coefficients is less than the number of constituents used for the harmonic prediction of comparable accuracy. In the response method we deal with complete potential instead of a set of discrete frequencies as in the harmonic method.

Lambert [1974] noted that the principal advantage of response method over the harmonic method lies in the fact that separate admittance functions (Fourier transform of response weights) can be calculated for sufficiently distinct uncorrelated inputs, thus making the method adaptable for earth tide analysis.

The response method of tide analysis and prediction as developed by Munk and Cartwright [1966] is applied to various observed series to obtain frequency dependent admittances that describe the tidal characteristics in a similar sense to what can be deduced from the traditional harmonic constants. To bridge the gap between the response and harmonic methods, Zetler, Cartwright and Munk [1969] have described procedures for deriving harmonic constants from the response admittances. They showed that the harmonic constants H_k and g_k of a tide constituent k can be determined for a place using response analysis and the result is compatible with the conventional harmonic analysis.

2.3.2 Brief Outline of the Theory of Response Method

The tidal potential can be generated as a time series V(t) and an attempt can be made at predicting the height of tide for a time t as the weighted sum of the past and present values of the potential,

$$\tilde{h}(t) = \sum W(s)\, V(t - \tau s). \tag{2.43}$$

The weights W(s) are determined such that the prediction error $h(t) - \tilde{h}(t)$ is a minimum in the least squares sense, τs is the time lag used in the argument of the potential.

The weights represent the sea level response at the place of interest to a unit impulse

$$V(t) = \delta(t).$$

In the response approach of Munk and Cartwright, V(t) is expressed in spherical harmonics as

$$V(\phi,\, \lambda,\, t) = g \sum_{n=0}^{n} \sum_{m=0}^{n} [a_n^m(t) U_n^m(\phi,\, \lambda) + i b_n^m(t) V_n^m(\phi,\, \lambda)]. \tag{2.44}$$

Here $U_n^m + i\ V_n^m$ are a set of complex spherical harmonics of order m and degree n, a(t), b(t) are the amplitudes of the real and imaginary parts of the spherical harmonics and can be computed for any desired time interval for any location.

The prediction formalism becomes [Munk and Cartweight, 1966]

$$\tilde{h}(t) = \sum_{mn} \sum_{s} [U_n^m(s)\ a_n^m(t - \tau s) + iV_n^m(s)\ b_n^m(t - \tau s)]. \tag{2.45}$$

Letting

$$W_n^m(s) = U_n^m(s) + i\ V_n^m(s)\ .$$

and

$$C_n^m(t - \ s) = a_n^m(t - \tau s) - i\ b_n^m(t - \tau s),$$

equation 2.45 is rewritten as

$$\tilde{h}(t) = \sum_{mn} \sum_{s} W_n^m(s)\ C_n^m(t - \tau s). \tag{2.46}$$

The weights $W_n^m(s)$ define the relation between the linear part of the tide and the equilibrium tide, thus the determination of $W_n^m(s)$ is the essential point in the response method.

III COTIDAL CHARTS AND THEIR USES

3.0 Introduction

In Chapter II we have seen how the tidal constituent frequencies are obtained from the decomposition of tidal potentials and how the tidal characteristic for a location, that is, the tidal constants (amplitude H_k and phase lag g_k for any constituent k) for major constituents can be determined using the harmonic or response methods of tidal analysis. In this chapter, the types and methods of constructing cotidal charts and their uses, are discussed.

3.1 Types of Cotidal Charts

3.1.1 Range/Time Cotidal Charts

Most often, a range/time cotidal chart is constructed by graphical means. On it, two sets of curves connect points having equal range differences (or range ratios) and points having simultaneous high and low waters [Admiralty Manual of Hydrographic Surveying, 1969]. All cotidal curves indicate a relationship to the tides at the reference gauge station. Figure 3-1 illustrates a typical range/time cotidal chart. The range curves (shown by pecked lines) indicate the range ratios of the tide at the reference station A. At B for example, the tidal range is 0.65 times the range at A. The time curves (shown by full lines) indicate time lags or corrections which must be applied to the times of high or low waters at the reference gauge station to obtain the times of high or

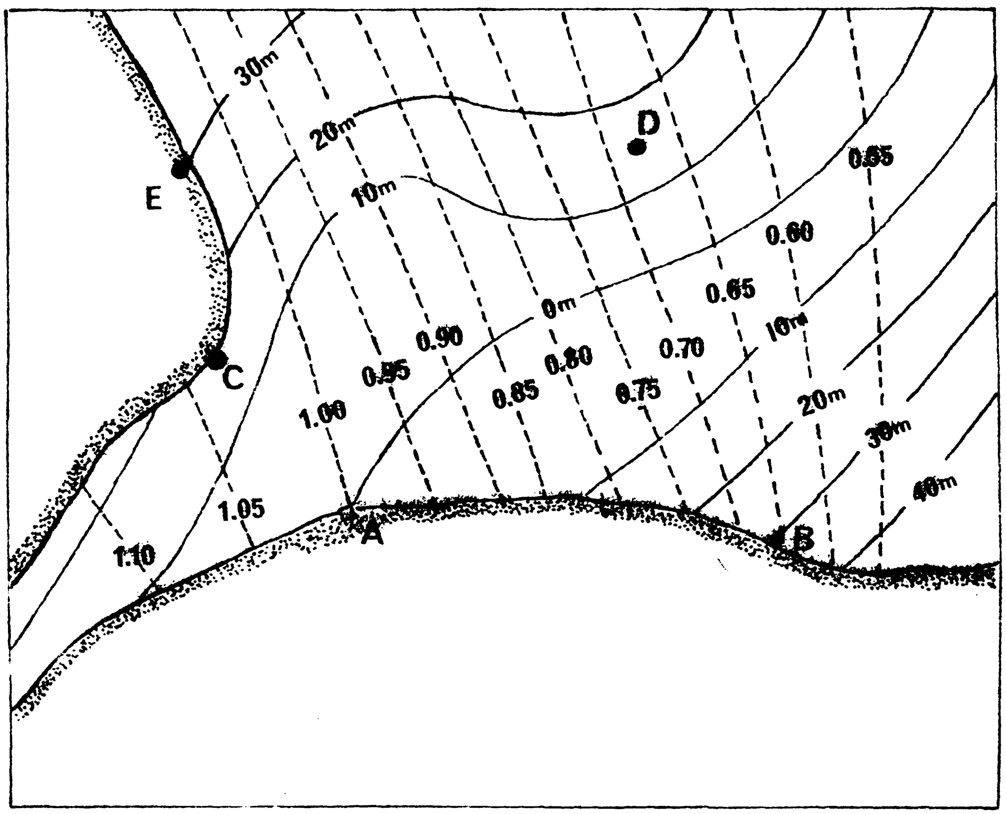


Figure 3-1

Range/Time Co-Tidal Chart

low waters at a place of interest.

To construct this type of cotidal chart, simultaneous tide observations are made at the reference station and at other well distributed temporary tide stations such as at points B, C, D and E in Figure 3-1. From mean high waters and mean low waters, the mean range is obtained for each station. The range ratios are determined from the relation: mean range at a gauge station/mean range at the reference station. The mean time lag for each station is determined by finding the mean time differences between the occurrence of high and low waters at the reference station and at other gauge stations. Both sets of cotidal curves are interpolated in between stations as contours are interpolated in between spot heights for a topographic map.

3.1.2 Amplitude/Phase Cotidal Charts

This type of cotidal chart is referred to as being semi-graphical. It is more difficult to produce and more complicated to use than a range/time cotidal chart but, could be more reliable and more versatile. The number of such charts needed for an area would be equal to the number of constituent frequencies being taken into account for our tidal predictions. For ordinary practical purposes in hydrographic surveying, four major constituents are considered, namely M_2, S_2, K_1 and O_1 [Admiralty Manual of Hydrographic Surveying, 1969]. This means that four cotidal charts would be needed each containing two sets of curves. Figure 3-2 illustrates one such cotidal chart of an area for the M_2 constituent. The full lines connect points having equal values of phase lag g_m in degrees and the pecked lines connect points having equal amplitudes H_m.

To produce the amplitude/phase cotidal charts, tide gauges are set up at well distributed locations in the area such that tidal characteristics should as much as possible vary linearly from one gauge station to another. This means that there should be no major physical features or structures which may influence the propagation of tidal waves between any two tide stations. (For example, Larsen [1977] in his study of the tides in the Pacific Ocean near the Hawaiian Islands, observed that the phase lag of the M_2 semi-diurnal tide differs by 46° between the nearby tide stations at Mokuoloe and Honolulu that are on the opposite sides of the Hawaiian ridge but differs by only 15° between Mokuoloe and a distant station at Hilo that are on the same side of the ridge. Also for the K_1 diurnal tide, the differences are found to be 8° and 3° respectively). Tides are observed at the stations for a minimum period of 29 days. The tidal records are then analysed using the harmonic or the response method to determine the harmonic constants H_k and g_k for each constituent frequency at each gauge station. The amplitude and phase lag curves are then interpolated as contours are interpolated for a topographic map.

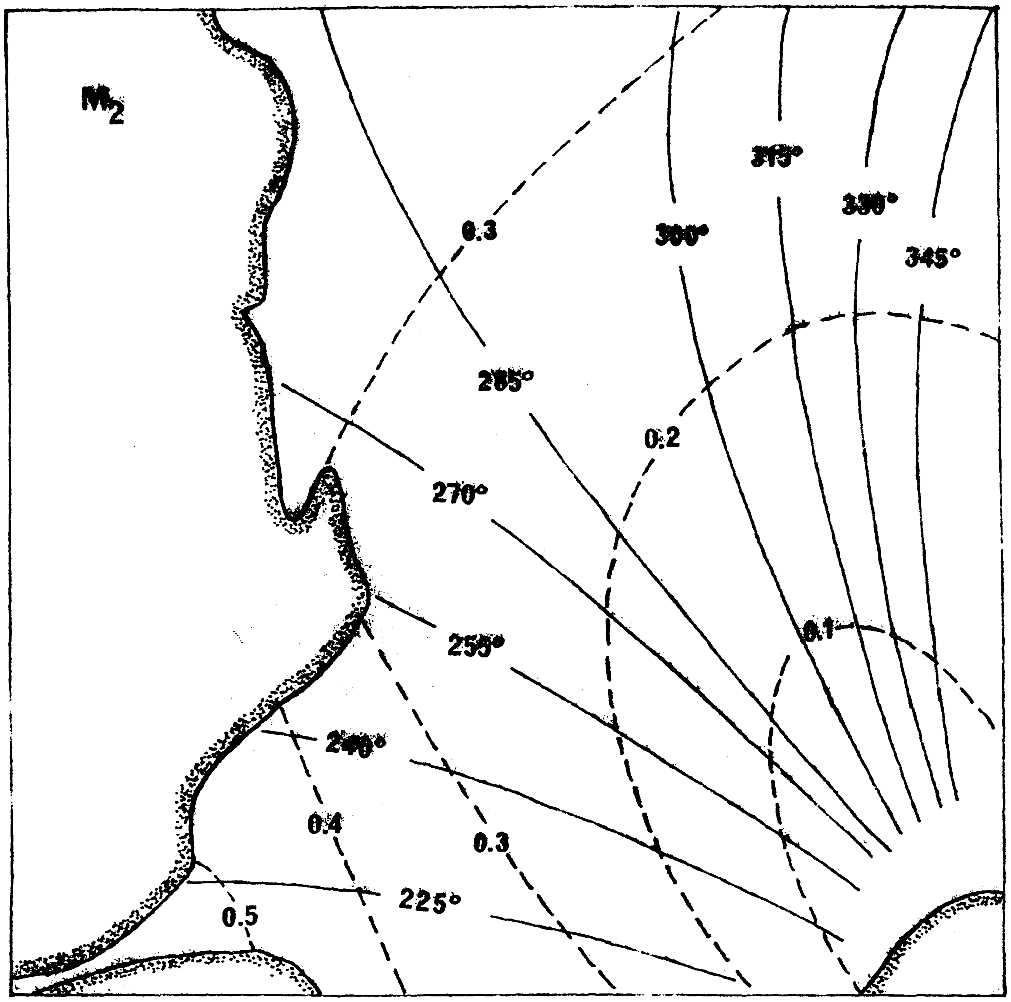


Figure 3-2

Amplitude/Phase Co-Tidal Chart for M_2

The amplitude/phase cotidal chart cannot be used to directly convert tide readings made at the reference station to those observable at any other place as is the case with the range/time cotidal charts. With it however, tide at any point of interest in the area covered by the chart can be predicted at any time t using equation 2.25.

Interpolating between gauge stations has been the classical method of producing amplitude/phase cotidal charts. Presently a more meaningful method of producing this type of cotidal chart is through the solution of numerical schemes. Luther and Wunsh [1974] however used 350 sets of constants, obtained partly from the publications of the International Hydrographic Bureau (IHB) and partly from other investigators, to produce the cotidal charts for the central Pacific ocean which they claim are comparable with the numerical charts of Pekeris and Accad [1969] and Hendershott [1972].

3.2 Numerical Schemes

The various numerical schemes for the production of cotidal charts stem from various solutions of the Laplace tidal equations [Bye and Heath, 1975; Hendershott and Munk, 1970]

$$\frac{\partial u}{\partial t} - fv = \frac{g}{a \cos \phi} \cdot \frac{\partial(\xi - \bar{\xi})}{\partial \lambda} \quad . \tag{3.1}$$

$$\frac{\partial v}{\partial t} + fu = \frac{-g}{a} \cdot \frac{\partial(\xi - \bar{\xi})}{\partial \phi} \quad , \tag{3.2}$$

$$\frac{\partial \xi}{\partial t} + \frac{1}{a \cos \phi}\left(\frac{\partial u_Q}{\partial \lambda} + \frac{\partial v_Q}{\partial \phi} \cos \phi\right) = 0 \quad , \tag{3.3}$$

where ϕ, λ are the geodetic latitude and longitude respectively,

u, v are the latitudinal and longitudinal components of the fluid velocity,

a is the earth mean radius,

$f(= 2\Omega \sin \phi)$ is the Coriolis parameter in which Ω is the angular velocity of the earth,

Q is the undisturbed depth of the ocean,

ξ is the elevation of the sea surface above the undisturbed level, and

$\bar{\xi}(= V/g)$ is the equilibrium tide.

The Laplace tidal equations representing equations of motion, though they look simplified, are difficult to solve even in the case of uniform depth covering the globe. The early solutions were given by Lord Kelvin in 1845 and Hough in 1897 who replaced the Laplace power series in sine with an expansion in spherical harmonics thus regarding the earth's rotation as very small. In 1898, Lord Kelvin introduced the concept of f-plane approximation in which he considered the oscillations of the horizontal sheet of fluid of uniform depth rotating about its normal and this reduces the Laplace tidal equations to [Hendershott and Munk, 1970]

$$\frac{\partial u}{\partial t} - fv = - g.\frac{\partial(\xi - \bar{\xi})}{\partial x} , \tag{3.4}$$

$$\frac{\partial v}{\partial t} + fu = - g.\frac{\partial(\xi - \bar{\xi})}{\partial y} . \tag{3.5}$$

$$\frac{\partial \xi}{\partial t} + Q(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0 \ . \qquad (3.6)$$

in which x, y are the Cartesian coordinates in the plane of the fluid. Larsen [1977] used the f-plane solution to produce the cotidal charts for the Pacific ocean near the Hawaiian Islands. He approximated the Island as an elliptically shaped cylinder with the plane ocean taken to be tangent to the earth at the coordinates ϕ_0 = 20.7°N and λ_0 = 156.8°W which corresponds to the coordinates of the centre of the elliptically shaped Island. On the plane ocean, he took the rectangular coordinate system with the X-axis eastwards and normal to the axis of the ridge formed by the island and the Y-axis northwards and parallel to the ridge axis and with the origin at the tangent point (ϕ_0, λ_0).

The boundary condition assumes that the velocity normal to the coast vanishes and free tide solutions are added in order to fit the observed tide at the boundary. The cotidal charts for the various constituents are constructed by mapping the amplitude and phase of the total tide, that is the resultant of the equilibrium tide, forced tide and free tide, as a function of the elliptic coordinates. The author evaluated the accuracy of the cotidal chart by comparing the observed tides at some locations with the values of tides predicted by the model. He observed that the plane wave model of the tides connect the tidal observations together in a simple way and thus allows the tide to be interpolated between gauge stations and extrapolated into the ocean beyond the tidal sites.

Rossby in 1939 introduced the beta-plane approximation. In this, the Laplace tidal equations are written as in f-plane approximation but with the coriolis parameter made a linear function of y, namely

$$f = f_0 + \beta y \qquad (3.7)$$

The variation of f with y corresponds to an expansion of the coriolis parameter about the latitude ϕ_0

$$2\Omega \sin \phi = 2\Omega \sin \phi_0 + \left(\frac{2\Omega}{a}\right) a(\phi - \phi_0)\cos \phi_0, \qquad (3.8)$$

in which β is of the order $\frac{2\Omega}{a}$.

When $\beta = 0$, we then have f-plane approximation.

With the advent of large computers, the application of the method of finite differences to the tidal problems become popular. Freeman and Murty [1976] studied the cooscillating and independent tides in Hudson Bay and James Bay by applying the finite differences to solve the Laplace tidal equations. They linearised the equations of motion in spherical polar coordinates and vertically integrated retaining variable coriolis, pressure gradient, bottom stress and direct tidal potential terms. The equations thus solved in the model are

$$\frac{\partial u}{\partial t} = 2\Omega\ v \sin \phi - \frac{gh}{a \cos \phi} \cdot \frac{\partial \eta}{\partial \lambda} - \frac{T_{B\lambda}}{\rho} + \bar{F}_\lambda , \qquad (3.8)$$

$$\frac{\partial v}{\partial t} = -\ 2\Omega\ u \sin \phi - \frac{gh}{a} \cdot \frac{\partial \eta}{\partial \phi} - \frac{T_{B\phi}}{\rho} + \bar{F}_\phi , \qquad (3.9)$$

$$\frac{\partial \eta}{\partial t} = \frac{1}{a \cos \phi} \left(\frac{\partial u}{\partial \lambda} + \frac{\partial v}{\partial \phi} \cos \phi \right), \tag{3.10}$$

where T_β is the bottom stress, $\bar{F}_\lambda$, $\bar{F}_\phi$ are the horizontal components of the tide generating force, η is the deviation of the water level from the mean tide level, h is the water depth and ρ is the density of water.

The cooscillating tide is modeled by setting the tide generating force terms to zero and specifying the free surface elevation across the mouth of the Hudson Bay by

$$\eta_k^1(\phi, \lambda) = H_k(\phi, \lambda) \cos(\omega_k t - g_k(\phi, \lambda)), \tag{3.11}$$

where η_k^1 belongs to the constituent k at the open mouth boundary location and is referred to the mean tide level.

The independent tide is modeled by setting the normal velocity on the open mouth boundary to zero and specifying the tide generating force. For example, for M_2

$$\bar{F}_2\lambda = \frac{-48.8}{a}.gh \cos \phi \sin(\omega_m t + 2\lambda + \omega_m T), \tag{3.12}$$

$$\bar{F}_2\phi = \frac{-48.8}{a}.gh \cos \phi \sin \phi \cos(\omega_m t + 2\lambda + \omega_m T), \tag{3.13}$$

and for K_1

$$\bar{F}_1\lambda = \frac{-28.5}{a}.gh \sin \phi \sin(\omega_k t + \lambda + \omega_k T), \tag{3.14}$$

$$\bar{F}_1\phi = \frac{-28.5}{a}.gh(\sin^2\phi - \cos^2\phi)\cos(\omega_k t + \lambda + \omega_k T). \tag{3.15}$$

Here T is the number of hours from the Greenwich mean time to the local zone time. The linear form of bottom friction due to Heaps is used and is given as

$$T\beta_\lambda = \frac{PR}{h} U, \quad T\beta_\phi = \frac{PR}{h} V. \qquad (3.16)$$

The authors used a rectangular grid of 15 and 10 minutes of arc in longitudinal and latitudinal directions respectively. The grids are drawn so that the fluid velocity components (U, V) are defined on the closed boundary locations and the water levels (η) at the open boundary at the mouth of the Bay. In the formulation of the numerical scheme, central finite differences are used in both space and time. Using a leap-frog scheme, water levels (η) are computed at even time steps (i.e. i = 2, 4, 6, 8..) and the horizontal flow components (U, V) computed at odd time steps (i.e. i = 1, 3, 5, 7 ...).

The numerical scheme is thus given by [Freeman and Murth, 1976]

$$\frac{U^{i+1}_{kj} - U^{i-1}_{k,j}}{2\Delta t} = 2\Omega\, V^{i}_{kj} \sin\phi_j - \frac{gh_{kj}}{a\cos\phi_j}\left(\eta^{i}_{k+1,j} - \eta^{i}_{k-1,j}\right) - \frac{1}{\rho} T\beta_{\lambda_{k,j}}^{i-1} + \bar{F}^{i}_{\lambda k,j}, \qquad (3.17)$$

$$\frac{V^{i+1}_{k,j} - V^{i-1}_{k,j}}{2\Delta t} = -\,2\Omega\, U^{i}_{k,j} \sin\phi_j - \frac{gh_{k,j}}{a}\left(\eta^{i}_{k,j+1} - \eta^{i}_{k,j-1}\right) - \frac{1}{\rho} T\beta^{i}_{\phi i,j} + \bar{F}^{i}_{\phi_{k,j}}, \qquad (3.18)$$

$$\eta^{i+1}_{k,j} - \eta^{i-1}_{k,j} = -\frac{1}{a\cos\phi_j}\left(\frac{U^{i}_{k+1,j} - U^{i}_{k-1,j}}{2\Delta\lambda} + \frac{V^{i}_{k,j+1}\cos\phi_{j+} - V^{i}_{k,j-1}\cos\phi_{j-1}}{2\Delta\phi}\right), \qquad (3.19)$$

and the output of the computations are U, V and η as functions time. From these parameters, the current ellipses and the co-phase and co-amplitude lines are constructed.

In numerical schemes, the problem generally posed is to solve the Laplace tidal equations in their primitive form or after elimination of one or two dependent variables with prescribed boundary conditions. For example

(i) Vanishing normal velocity at coast lines [Pekeries and Accad, 1967],

(ii) Specified or observed values of the constituents at the coastal stations only [Hendershott, 1966],

(iii) Specified or observed values of the constituents at selected coastal and island stations plus vanishing normal velocity at the remaining coastal boundary points [Larsen, 1977].

3.3 Uses of Cotidal Charts

Cotidal charts are found useful in many situations.

They are useful in the study of the impact of large engineering structures on the tidal regime, for example, the proposed tidal power project on the Bay of Fundy in Eastern Canada [Atlantic Tidal Power Engineering and Management Committee Report, 1969; Garrett and Greenberg, 1976].

They are indispensable in navigation especially when deep draught ships have to navigate through a complex estuary where drying sand banks alternate with deeps such as that obtained in the port of London [White, 1971]. Here deep draught tankers navigate to Thameshaven and Coryton to evacuate oil from the principal oil refineries. In such a situation the pilot and the captain of the vessel would want the information on

(i) the critical depths in the channel at chart datum,

(ii) the points along the track where these critical depths occur,

(iii) the times the tidal heights at these points would be sufficient for safe passage of a vessel with a particular draught,

(iv) the latest times along the route that the passage depths are available.

If the underkeel clearance is not so critical, this information can easily be obtained using cotidal charts and appropriate up to date navigation charts and tide tables. If the underkeel clearance is critical, the use of cotidal chartsis supplimented by several radio linked tide gauges.

The application of prime concern here is the use of cotidal charts for the reduction of sounding data. As was shown previously, all depth measurements are reduced to the chart datum; therefore the height of tide at time t must be subtracted from the depth sounded at the time t. This implies that we should observe tides at the same time we take our soundings. If we are working on the coast or on the inland tidal waters, it is possible to establish tide gauges close to the sounding area and observe tides at the same time. If we are involved with extensive sounding offshore, the possibilities of observing tides close to the sounding area are remote. It becomes more feasible to do the tidal reductions using predicted tides, and when this is the case, the use of cotidal charts become convenient.

Range/time cotidal charts can be used in which case we only need to observe or predict tides at the reference station and then obtain the equivalent at the desired locations, or, we can use amplitude/phase cotidal charts and predict the tides at the desired locations independent of a reference station. Finally, a combination of the two approaches can be used.

The Canadian Hydrographic Service has done some automated tidal reductions using digitized range/time cotidal charts of the Hudson Bay and the Lower St. Lawrence River [Tinney, 1977]. In these schemes, the cotidal charts were digitized by breaking the survey area into equal size blocks based on lines of latitude and longitude and approximating the boundaries of the cotidal zones with the edges of those blocks. Those digitizations were coded and stored in the computer. To locate a particular block and retrieve the cotidal values, the geodetic coordinates (ϕ, λ) of the position of the sounding were used.

The choice of the size of the blocks would obviously

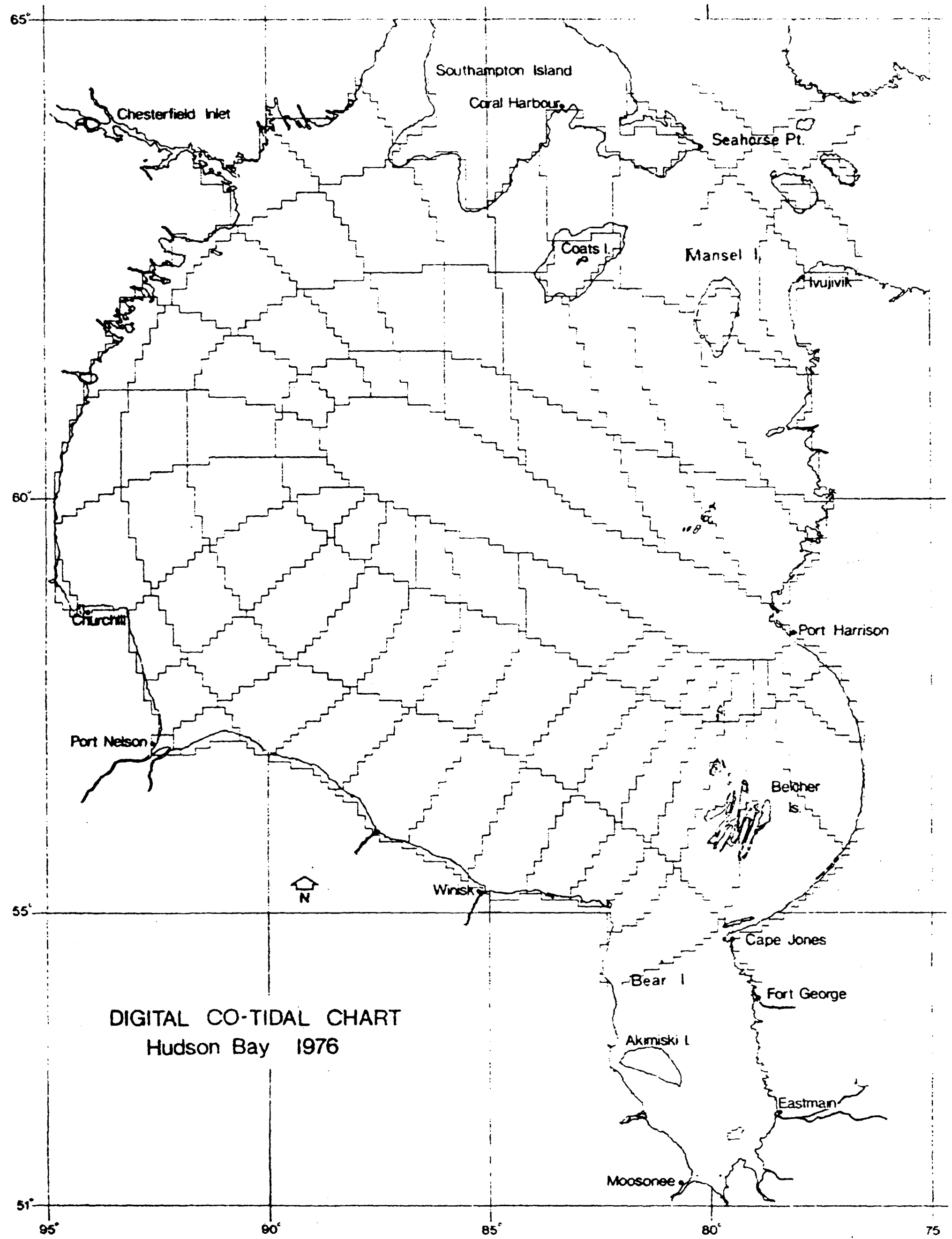

65°
60°
55°
51°
95°
90°
85°
80°
75
Southampton Island
Coral Harbour
Chesterfield Inlet
Seahorse Pt.
Coats I.
Mansel I.
Ivujivik
Churchill
Port Harrison
Port Nelson
Belcher Is.
Winisk
N
Cape Jones
Bear I
Fort George
DIGITAL CO-TIDAL CHART
Hudson Bay 1976
Akimiski I.
Eastmain
Moosonee

Figure 3-3

depend on the amount of computer space available and the accuracy requirements. With smaller size blocks, the zone boundaries would be better approximated but more computer space would be required. Figure 3-3 shows the digital breakdown of the cotidal chart used for the Hudson Bay. Block sizes of 5' latitude and 10' longitude were used giving a total of 13,986 blocks dividing the Bay into 93 reduction zones. The tide station at Churchill served as the reference station for the cotidal chart and during the survey, the predicted heights from the reference station were used instead of the observed heights. However, in the survey of the Lower St. Lawrence River with Pointe-an-Pēre as the reference station, observed tides were used.

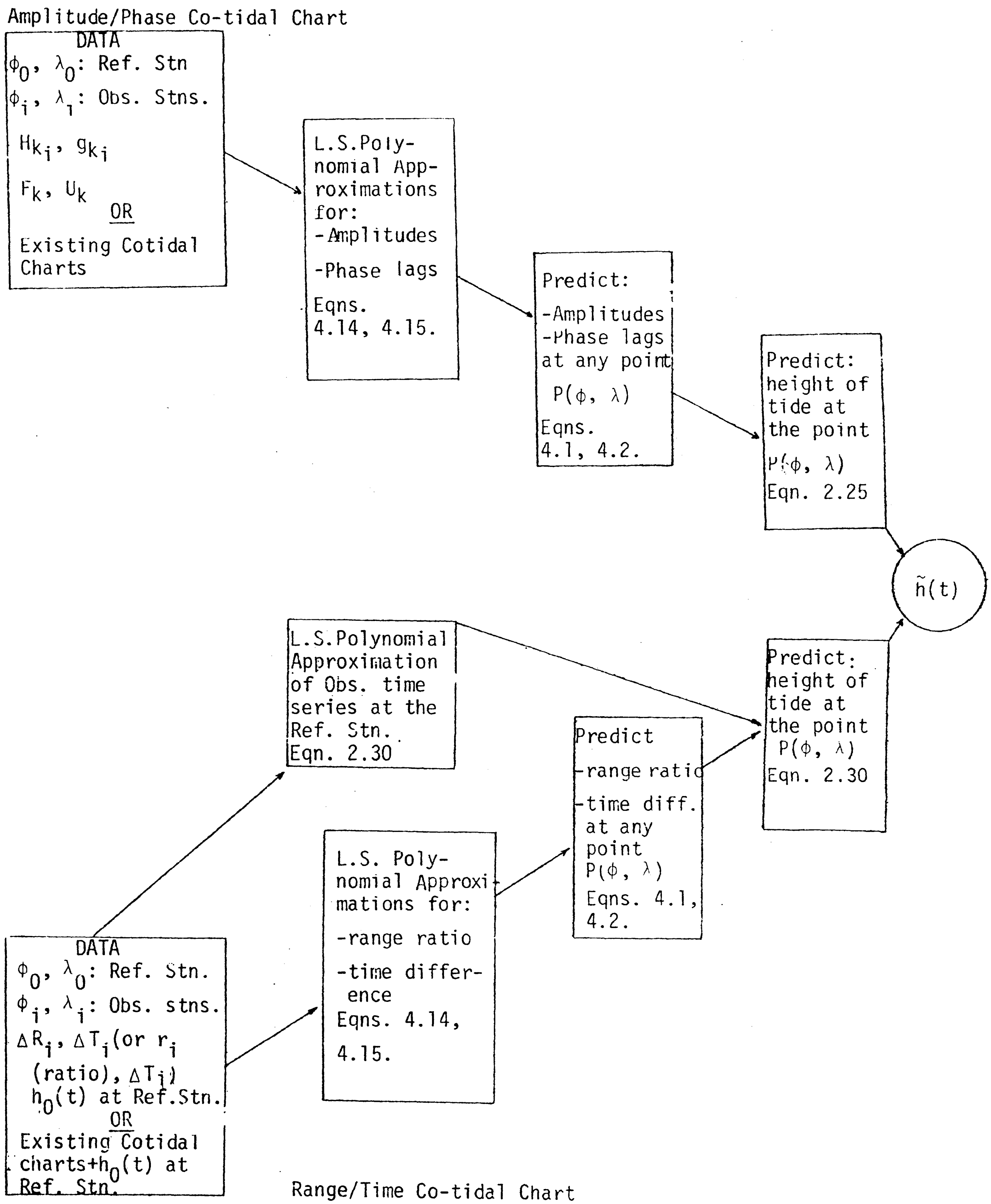
IV THE PROPOSED ANALYTICAL SCHEME

4.0 Introduction

The proposed analytical scheme is aimed at achieving automated tidal reductions using little computer space and time and with advantageous accuracy and flexibility. Figure 4-1 illustrates the proposed scheme in a flow-chart. It shows that we can work with amplitude/phase cotidal model or range/time cotidal model. The same objective is achieved using either model but it does not necessarily mean that the same degree of accuracy and flexibility is attained. Basically the data requirements for either are the same except that with amplitude/phase cotidal model, the amplitude H_k and the phase lag g_k for each constituent k we wish to take into account and at each observation station are required. With the range/time cotidal model, we require the mean range ratios and the mean differences of the times of occurrence of high and low waters between each tide gauge station to be considered and a reference gauge station. With the range/time cotidal model, we have the option of carrying out the tidal reduction based on the observed tides or on the predicted tides at the reference station.

In each case, the aim is to produce an analytical cotidal model using observed data or existing cotidal charts. The analytical model could then be stored conveniently in a computer so that when observed sounding

Figure 4-1 The Proposed Scheme - Flow Chart



data are input, the output would be reduced soundings. The theory and mathematical models for the two approaches are basically the same. In Section 4.1 of this chapter, the mathematical models are discussed, and in Section 4.2 the data requirements are explained explicitly.

4.1 Models

Earlier, it was shown that the tides are functions of time and position on the surface of the earth and that the tidal characteristics, that is, the amplitude H_k and the phase lag g_k for the constituent k are constant for a place. These constants can be estimated by performing harmonic or response analysis of a long period tidal records. Knowing the estimated tidal constants for a place, the tide at the place can be predicted at any time t.

Now suppose we consider a section of a body of tidal water, not so extensive in area and where the constants H_k and g_k are defined at a reference station whose geodetic coordinates are (ϕ_0, λ_0), and at several other points $P_j(\phi_j, \lambda_j)$ within the area. We can define mathematically surfaces that can describe the distribution of those constants with reference to the primary station. The aim is to approximate, in the Least Squares sense, the amplitude and phase lag fields by surfaces described by two dimensional algebraic polynomials. The coefficients of these polynomials are determined in such a way as to fit the observed data in the Least Squares sense. Using this technique, the amplitudes H_k and the phase lag g_k can be predicted at any point of interest $P_i(\phi_i, \lambda_i)$ within the area by the polynomials

$$\tilde{\Delta}H_k(x_i, y_i) = \sum_{j=0}^{\ell} C_j^H \psi_j(x_i, y_i), \tag{4.1}$$

$$\tilde{\Delta}g_k(x_i, y_i) = \sum_{j=0}^{\ell} C_j^g \psi_j(x_i, y_i), \tag{4.2}$$

where $\tilde{\Delta}H_k(x_i, y_i)$ and $\tilde{\Delta}g_k(x_i, y_i)$ are the predicted differences in amplitude and phase lag respectively for the constituent k between the reference station and the point i, C_j^H and C_j^g are the coefficients of the polynomials, $\psi(x_i, y_i)$ are base functions (two dimensional) of the approximating polynomials, and ℓ is the number of base functions. The selection of the prescribed functions ψ can be, from the theoretical point of view, purely arbitrary. The sufficient and necessary condition for the prescribed functions $\psi \equiv \{\psi_1, \psi_2 \ldots \psi_\ell\}$ to create a base is that they are linearly independent on the functional space (G_m). If and only if ψ is a base can the coefficients of the best fitting polynomial be uniquely determined [Vaníček and Wells, 1972].

Even though the position of a point may be expressed in terms of geodetic coordinates (ϕ_i, λ_i), it is more convenient to work with local orthogonal coordinates (x_i, y_i). The relationship between the two systems is defined as

$$x_i = R_0(\phi_i - \phi_0), \tag{4.3}$$

$$y_i = R_0 \cos \phi_0(\lambda_i - \lambda_0) . \tag{4.4}$$

where R_0 is the mean radius of curvature of the earth computed at the reference station and is given by [Krakiwsky and Wells, 1971]

$$R_0 = \sqrt{M_0 N_0} \quad (4.5)$$

in which

$$M_0 = a(1 - e^2)/(1 - e^2 \sin^2 \phi_0)^{3/2}, \quad (4.6)$$

and

$$N_0 = a/(1 - e^2 \sin^2 \phi_0)^{1/2}. \quad (4.7)$$

The first eccentricity squared is

$$e^2 = (a^2 - b^2)/a^2, \quad (4.7b)$$

and for the Clarke 1866 ellipsoid, the semi-major axis $a = 6378.2064$ km, while the semi-minor axis $b = 6356.5838$ km.

Regarding the choice of base functions, we can use mixed algebraic functions which are particularly simple to deal with [Nassar and Vaníček, 1975], namely,

$$\psi = \{x^\ell y^j\}, \ (\ell, j = 0, 1, 2 \dots n) \quad (4.8)$$

where n is the degree of the polynomial. Equation 4.1 and 4.2 can now be rewritten as

$$\tilde{\Delta} H_k(x_i, y_i) = \sum_{\ell, j=0}^{n} C^H{}_j \ x_i^\ell \ y_i^j , \quad (4.9)$$

$$\tilde{\Delta} g_k(x_i, y_i) = \sum_{\ell, j=0}^{n} C^g{}_j \ x_i^\ell \ y_i^j . \quad (4.10)$$

The problem is to solve for the coefficients $C^H{}_j$ and $C^g{}_j$ of the polynomials. The number of coefficients U to be solved for is determined from the relation

$$U = (n + 1)d. \quad (4.11)$$

where n is the degree of the polynomial and d is the dimensionality of the base functions.

4.2.1 Least Squares Solution of the Models

To determine the unknown coefficients C^H_j and C^g_j of the models represented by equations 4.9 and 4.10, observation equations can be written for each data point i where the amplitude difference ΔH_k and the phase lag difference Δg_k referred to a reference station are known. The equations are

$$\tilde{\Delta} H_k(x_i, y_i) + V_{H_{ki}} = \Delta H_k(x_i, y_i), \qquad (4.12)$$

$$\tilde{\Delta} g_k(x_i, y_i) + V_{g_{ki}} = \Delta g_k(x_i, y_i), \qquad (4.13)$$

where $\tilde{\Delta} H_k$ and $\tilde{\Delta} g_k$ are the predicted values, $V_{H_{ki}}$ and $V_{g_{ki}}$ are the residuals of observations, and the terms on the right hand side (ΔH_k and Δg_k) are the known or observed values. Substituting equations 4.9 and 4.10 into equations 4.12 and 4.13 yields

$$\sum_{\ell, j=0}^{n} C^H_{\ell j} \; x_i^{\ell} y_i^{j} + V_{H_{ki}} = \Delta H_k(x_i, y_i), \qquad (4.14)$$

$$\sum_{\ell, j=0}^{n} C^g_{\ell j} \; x_i^{\ell} y_i^{j} + V_{g_{ki}} = \Delta g(x_i, y_i). \qquad (4.15)$$

Putting equations 4.14 and 4.15 in matrix form we have

$$\underset{m \times u}{A} \; \underset{u \times 1}{C^H} + \underset{m \times 1}{V_H} = \underset{m \times 1}{L_H}, \qquad (4.16)$$

$$\underset{m \times u}{A} \; \underset{u \times 1}{C^g} + \underset{m \times 1}{V_g} = \underset{m \times 1}{L_g}. \qquad (4.17)$$

It is pertinent to note here that equations 4.16 and 4.17 are the same as the observation equations for a parametric case in the least squares adjustments. The parametric least squares adjustment differs only in purpose and notations from the least squares approximation of a function (F) defined on a discrete or compact domain (M) [Vaniček and Wells, 1972]. The purpose of the least squares approximation is to find an approximating polynomial (P_n) for a given function or for a given set of functional values. The purpose of the least squares adjustment is to find the least squares statistical estimates of unknown parameters which are related to the observed values by linear (or linearized) mathematical models.

The matrix A is known as Vandermonde's design matrix and is given by

$$\underset{m \times u}{A} \equiv \begin{bmatrix} \psi_0(x_1^0 y_1^0), & \psi_1(x_1^0 y_1^1), & \psi_2(x_1^0 y_1^2), & \ldots & \psi_u(x_1^n y_1^n) \\ \psi_0(x_2^0 y_2^0), & \psi_1(x_2^0 y_2^1), & \psi_2(x^0 y^2), & \ldots & \psi_u(x_2^n y_2^n) \\ \\ \psi_0(x_m^0 y_m^0), & \psi_1(x_m^0 y_m^1), & \psi_2(x_m y_m), & \ldots & \psi_u(x_m^n y_m^n) \end{bmatrix} . \qquad (4.18)$$

C^H and C^g are the vectors of coefficients. V_H and V_g are the vectors of residuals of the observations L_H and L_g. L_H and L_g are the vectors of observed values (or the functional values) at the discrete points i. The solution of the system of equations given by 4.16 and 4.17 for the coefficients, using least squares approximation methodology [Vaniček and Wells, 1972; Christodoulidis, 1973; Balogun, 1977; Appendix I] is given by

$$\hat{C} = N^{-1}U \; , \tag{4.19}$$

where N is the Gram's matrix defined by

$$\underset{u\times u}{N} \equiv [A^TPA] \equiv \begin{bmatrix} \langle\psi_0\psi_0\rangle, \langle\psi_0\psi_1\rangle & \cdots & \langle\psi_0\psi_u\rangle \\ \langle\psi_1\psi_0\rangle, \langle\psi_1\psi_1\rangle & \cdots & \langle\psi_1\psi_u\rangle \\ \langle\psi_u\psi_0 \;, \langle\psi_u\psi_1\rangle & \cdots & \langle\psi_u\psi_u\rangle \end{bmatrix} \; , \tag{4.20}$$

and

$$\underset{u\times 1}{U} \equiv A^TPL \equiv \langle L, \; \psi_i\rangle \; . \tag{4.21}$$

The sign $\langle \;\; \rangle$ indicates a scalar product [Appendix I]. Since our prescribed functions form a base, the Gram's determinant must be different from zero and must have an inverse.

The solution for the residual vector is given by

$$\hat{V} = A\hat{C} - L \; . \tag{4.22}$$

The associated variance covariance matrix for the coefficients is given by

$$\textstyle\sum_{\hat{C}} = \hat{\sigma}^2 N^{-1} \; , \tag{4.23}$$

where $\hat{\sigma}^2$ is the a posteriori variance factor given by

$$\hat{\sigma}^2 = \hat{V}^T P\hat{V}/df \; , \tag{4.24}$$

in which df represents the degree of freedom given by

$$df = m - u \; . \tag{4.25}$$

P is the weight matrix

$$P = \textstyle\sum_L^{-1} = \mathrm{Diag}[\frac{1}{\sigma_{L_1}{}^2} \; , \; \frac{1}{\sigma_{L_2}{}^2} \; \cdots \; \frac{1}{\sigma_{L_m}{}^2}] \tag{4.26}$$

where σ_L is the standard error of the observables. The weight matrix is diagonal when we are dealing with statistically independent observables, that is, the observations are assumed uncorrelated.

For statistical reasons, we may wish to work with orthogonal bases, and usually the base ψ is not an orthogonal one. Schmidt's orthogonalization process [Appendix I] may be applied to obtain an orthogonal base ψ^*. Using an orthogonal base, the normal equation is

$$A^{*T} P A^* \hat{C}^* = A^{*T} PL. \qquad (4.27)$$

Again setting

$$A^{*T} P A^* = N^* ,$$

and

$$A^{*T} PL = U^* ,$$

we have that

$$\hat{C}^* = N^{*-1} U^* . \qquad (4.28)$$

A^* is the Vandermonde's design matrix obtained using the orthogonal base. $\hat{C}^*$ is a vector of Fourier coefficients, N^* is the Gram's matrix, this time diagonal because we are dealing with orthogonal base functions and is given by

$$\underset{u \times u}{N^*} \equiv \begin{bmatrix} \langle \psi_0^* \; \psi_0^* \rangle & 0 & 0 \\ 0 & \langle \psi_1^* \; \psi_1^* \rangle & 0 \\ & & \\ 0 & 0 & \langle \psi_u^* \; \psi_u^* \rangle \end{bmatrix} , \qquad (4.29)$$

and U^* is given by

$$\underset{u\times 1}{U^*} \equiv \begin{bmatrix} \langle\psi_0^* & L\rangle \\ \langle\psi_1^* & L\rangle \\ \vdots & \\ \langle\psi_u^* & L\rangle \end{bmatrix} . \qquad (4.30)$$

The associated variances are given by

$$\Sigma_{\hat{C}^*} = \hat{\sigma}^2{}_* \; N^{*-1} , \qquad (4.31)$$

where

$$\hat{\sigma}^2{}_* = \hat{V}^{*T} P \hat{V}^* / df . \qquad (4.32)$$

The solution of normal equation becomes trivial as the normal equation matrix N* (Grams matrix) is diagonal and each Fourier coefficient can be solved for independently.

We subject the Fourier coefficients to statistical screening by comparing each coefficient against j times its standard error. [Christodoulidis, 1973], that is, if

$$|\hat{C}_i^*| < j \, \sigma_{\hat{C}_i^*} , \qquad (4.33)$$

then $\hat{C}_i^*$ is statistically insignificant at that level and is discarded. j takes the values 1, 2 or 3 depending upon what level of significance of their standard deviations we wish to test the coefficients. The discarded Fourier coefficients are set equal to zero. Once the appropriate Fourier coefficients are discarded, the residuals, the variance factor and the variances are recomputed using only the accepted coefficients. The residuals are given by

$$\hat{V}^{*1} = A^* \hat{C}^* - L . \qquad (4.34)$$

The a posteriori variance factor is recomputed by

$$\hat{\sigma}^{2*1} = \hat{V}^{*1T} P \hat{V}^{*1} / df^{1}, \tag{4.35}$$

where

$$df^{1} = m - u + d ,$$

in which d represents the number of Fourier coefficients discarded. The new variances are

$$\Sigma_{\hat{C}^{*1}} = \hat{\sigma}^{2*1} N^{*-1} . \tag{4.36}$$

Using the transformation matrix (see Appendix I)

$$\underset{u \times u}{B} \equiv \begin{bmatrix} 1 & \beta_{12} & \beta_{13} & \cdots & \beta_{1u} \\ 0 & 1 & \beta_{23} & \cdots & \beta_{2u} \\ & & & & \\ 0 & 0 & 0 & & 1 \end{bmatrix} \tag{4.37}$$

and the remaining statistically significant Fourier coefficients, the original coefficients are computed by

$$\hat{C} = B\hat{C}^{*} . \tag{4.38}$$

The correct number of original coefficients are obtained even though we are solving for them using fewer number of Fourier coefficients. If, however, the last Fourier coefficients are the ones discarded, a fewer number of original coefficients will be recovered. The variance-covariance matrix of the original coefficients can be computed using the variance-covariance law, namely,

$$\Sigma_{\hat{C}} = B \, \Sigma_{\hat{C}^{*1}} \, B^{T}, \tag{4.39}$$

where $\Sigma_{\hat{C}^{*1}}$ is given by equation 4.36.

Once we have computed the coefficients of the original polynomials and their variance-covariance matrix from the statistically significant Fourier coefficients, statistically significant surfaces which describe the distributions of amplitudes and phase lags (or range ratios and time differences) in the area of interest have been obtained. Analytical cotidal models for amplitudes and phase lags (or range ratios and time differences) have thus been obtained. With the analytical models, the values of the amplitude and phase lags (or range ratio and time lags) can be predicted for any point $Pi(\phi_i \lambda_i)$ in the area using equations 4.1 and 4.2. The prediction variance covariance matrix is given by

$$\Sigma_{\tilde{H}} = J \; \Sigma_{\hat{C}} \; J^T \qquad (4.40)$$

in which J is Jacobian of transformation defined by A matrix.

4.3 Data Requirements and Reduction Algorithms

4.3.1 Amplitude/Phase Cotidal Model

As previously noted, to produce cotidal models for amplitudes and phase lags, we need to define the amplitudes and the phase lags of each constituent frequency at a reference station and at several other observation stations adequately distributed in an area of interest. Working with four major constituents, eight analytical models are needed to describe the tidal characteristics of the area. For a fair estimate of the amplitudes and the phase lags, the tidal analysis must be made from

369 days of tidal records, and for a barely acceptable estimate, observation should cover a period of 29 days. The more observations added in the analysis, the better will be the estimate of the harmonic constants.

It may not be easy to adequately distribute observing stations and obtain sufficient data to enable the production of a desired analytical model. An alternative is to use cotidal charts, produced from the numerical schemes such as those described in III, Section 3.2, as a source of data. The cotidal charts are digitized as mentioned in Section 3.3 and the digitized values are used in the least squares polynomial approximations to produce the analytical cotidal models. As a check on the compatibility of the analytical models and the original chart, the area is grided at close intervals and the amplitudes and phase lags predicted at the grid intersections using equations 4.1 and 4.2. The co-amplitude and co-phase curves can then be easily drawn in.

If the amplitude/phase cotidal models are being used for the reduction of soundings, the reduction algorithms can be summarized in steps as follows:

(i) At each sounding location i, the depth (D_i), the time (t) and the geodetic coordinates (ϕ_i, λ_i) are observed.

(ii) With the observed geodetic coordinates (ϕ_i, λ_i), the amplitudes and phase lags of the constituents being used can be predicted using the analytical models.

(iii) Using the tide prediction approach as described in II Section 2.2 and the predicted amplitudes and phase lags from (ii) above, the height of tide $h_i(t)$ at the sounding location above chart datum are predicted.

(iv) The sounding reduced to the chart datum is $d_i = D_i - h_i(t)$ (4.41)

4.3.2 Range/Time Cotidal Model

Some assumptions must be made at the outset for this model. Considering a body of water of relatively small extent, such that one can safely assume that the meteorological variables in the area are not remarkably different from place to place, it can be further assumed that given any two points $A(\phi, \lambda)$ and $B(\phi, \lambda)$ in the area, the tides at A bear constant relationships with the tides at B. Those relationships will change when there are marked topographical changes due, for example, to errosion, engineering structures, which tend to change the pattern of the propagation of tidal waves. If we establish the relationship existing between a reference station and any other point, it is possible to predict with some degree of certainty the tides at that other point from the observed (or predicted) tides at the reference station.

The relationships between the tides at any two stations can be established from the ratio of their ranges and the difference in the times of occurrence of high and low waters. In other words, it is assumed that the unwanted noise has perturbed observations equally so that when the range ratios and time differences are determined, the unwanted noise is eliminated.

To produce range ratios and time lags cotidal models, we require

(i) the mean range R_{m0} at the reference station and the mean ranges R_{mj} at discrete points (ϕ_j, λ_j); the range ratios are then given as

$$r_j = R_{mj}/R_{m0} \ . \qquad (4.42)$$

(ii) the mean time differences between the times of high and low waters at the reference station and at the discrete points given in minutes of time.

If the sounding reduction is to be done with range/time analytical cotidal model, the reduction aligorithms can be summarized in the following steps:

(i) The tide is observed at the reference station to cover the time interval M (the soundings are also performed within the same interval of time). A least squares approximation of the observed series at the reference station is done so that at any time t in the interval, the height of tide can be predicted.

(ii) At each sounding location i, the depth (D_i), the time (t) and the geodetic coordinates (ϕ_i, λ_i) are observed.

(iii) With the observed geodetic coordinates (ϕ_i, λ_i), the range ratio (r_i) and time difference (correction to time) are predicted using the analytical models.

(iv) Using the corrected time at the reference station and the approximating polynomial from step (i) above, the height of tide ($h_0(t)$) at the reference station is predicted.

(v) The height of tide at the observed location i is computed from the relation

$$h_i(t) = h_0(t) \times r_i \quad . \tag{4.43}$$

(vi) The reduced sounding is

$$d_i = D_i - h_i(t) \quad . \tag{4.44}$$

It is more convenient and simple to work with range/time cotidal models because (i) unlike the amplitude/phase models where 2 x NCON (NCON is the number of constituents used) analytical models are needed to describe the tides, only two models are needed to completely describe the tides, (ii) working with range/time cotidal models allows us to use the observed tides at the reference station to reduce soundings instead of the predicted tides .

V TEST COMPUTATIONS AND THE RESULTS

5.0 Data

To test the proposed analytical scheme, there was unfortunately no adequate data immediately available. However, the tidal information for secondary ports on the Bay of Fundy, published in the Canadian Tides and Current Tables, 1978 by the Canadian Hydrographic Service was minimally adequate for testing the analytical range/time cotidal models. This tidal information is given with reference to the Port of Saint John. In Table 5-1, the data as extracted are tabulated for 35 secondary stations (Figure 5-1).

The predicted tides for the Port of Saint John from January 1-15, 1978, were extracted from the same Canadian Tides and Current Tables, 1978 and treated as observed tides in the computations. The zero hour of the day the observation started is taken as the origin of time and times are given in hours from the origin of time. The observations are treated such that the period of the sounding exercise is covered, in other words, it is assumed that the tides were observed at Saint John throughout the period of the sounding. In Table 5-2, the tides as supposedly observed are tabulated and from Table 2-1 the following 7 major constituent frequencies are used.

Symbol	Frequency (deg./hr)
M_2	28.984104
S_2	30.00000

Index No.	Location Name	Zone Time(ZT)	Latitude °	'	Longitude °	'	Mean Range	Range Ratio(r)	Mean Time Diff.(min)	Remark
0065	Saint John	+4	45	16	-66	04	25.10	1.0	0.0	Ref. St.
0001	Outer Wood Isl.	+4	44	36	-66	48	16.60	0.6614	-28.5	*
0015	Welshpool	+4	44	53	-66	57	16.90	0.6733	+ 5.0	*
0040	St. Andrews	+4	45	04	-67	03	22.60	0.9004	+15.5	*
0060	Partridge Isl.	+4	45	14	-66	03	25.00	0.9960	-10.0	*
0129	St. Martins	+4	45	21	-65	32	30.15	1.2012	+ 9.0	*
0140	Herring Cove	+4	45	34	-64	58	33.25	1.3247	+19.0	*
0150	Cape Enrage	+4	45	36	-64	47	35.40	1.4104	+17.0	*
0160	Grindstone Isl.	+4	45	44	-64	37	38.30	1.5359	+20.0	
0170	Hopewell Cape	+4	45	51	-64	35	39.90	1.5896	+19.0	
0190	Pecks Point	+4	45	45	-64	29	38.70	1.5418	+19.0	
0215	Joggins Wharf	+4	45	41	-64	28	38.15	1.5199	+18.5	
0225	Cape Capstan	+4	45	28	-64	51	33.05	1.3167	+11.0	*
0235	West Advocate	+4	45	21	-64	49	32.90	1.3107	- 1.0	*
0240	Cape D'or	+4	45	18	-64	47	36.55	1.4562	+16.5	*
0245	Port Greville	+4	45	40	-64	56	36.70	1.4622	+30.0	
0247	Diggent River	+4	45	24	-64	27	39.50	1.5737	+33.0	
0250	Cape Sharp	+4	45	22	-64	23	37.95	1.5120	+48.5	
0260	Five Isl.	+4	45	23	-64	08	43.05	1.7151	+56.0	
0270	Burnstooat Head	+4	45	18	-63	48	44.30	1.7649	+67.0	
0285	Avon Port	+4	45	06	-63	13	45.05	1.7948	+32.5	
0290	Cape Blomidon	+4	45	16	-64	21	29.80	1.1873	+46.0	
0300	Scots Bay	+4	45	19	-64	26	37.10	1.4781	+14.5	

Table 5-1 Bay of Fundy - Tidal Information on Secondary Port.

Index No.	Location Name	Zone Time (ZT)	Latitude °	Latitude '	Longitude °	Longitude '	Mean Range	Range Ratio (r)	Mean Time Diff.(min)	Remark
0305	Baxter Harbour	+4	45	14	-64	31	37.4	1.4900	+12.0	*
0312	Ile Haute	+4	45	15	-65	00	34.15	1.3606	0.0	*
0315	Margaretsville	+4	45	03	-65	04	31.75	1.2649	-12.0	*
0320	Parkers Cove	+4	44	48	-65	32	26.60	1.0598	-14.0	*
0325	Digby	+4	44	38	-65	45	25.25	1.0060	- 9.0	
0330	Deep Cove	+4	44	24	-65	50	24.00	0.9562	-15.5	*
0335	Sand Cove	+4	44	30	-66	06	21.15	0.8426	-18.0	*
0336	East Sandy Narro.	+4	44	29	-66	05	19.10	0.7610	-37.0	*
0337	Tiverton	+4	44	23	-66	13	17.45	0.6952	-45.0	
0340	West Port	+4	44	16	-66	21	18.10	0.7211	-34.0	*
0345	Lighthouse Cove	+4	44	15	-66	24	17.90	0.7131	-34.0	*
0353	Church Point	+4	44	20	-66	07	18.10	0.7211	+18.0	*
0355	Meteghan	+4	44	12	-66	10	16.90	0.6733	+18.0	*

* Data used in test computations

Table 5-1 (cont'd).

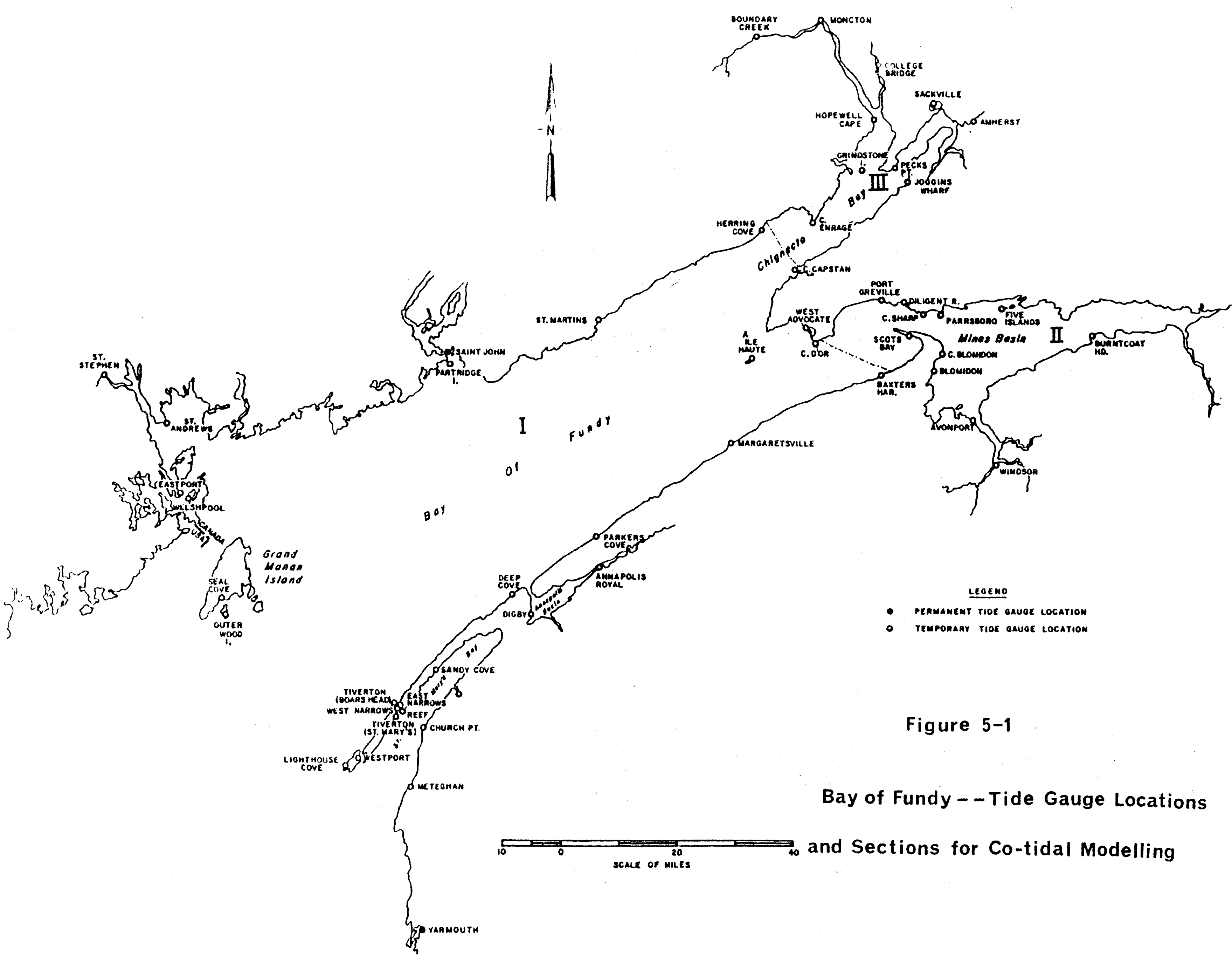


Figure 5-1

Bay of Fundy--Tide Gauge Locations and Sections for Co-tidal Modelling

Station: Saint John   Time Zone: + 4

Coords.: Lat. = 45 16'N
Long. = 66 04'W   Date: Jan. 1-15, 1978

Time (Hrs)	Height(m)	Time (Hrs)	Height(m)	Time (Hrs)	Height(m)
03.833	7.4	128.500	8.1	253.167	8.4
10.250	1.3	134.833	0.6	259.333	0.2
16.250	7.3	141.083	7.7	265.500	8.0
22.350	1.2	147.333	0.8	271.917	0.6
28.667	7.4	153.417	8.3	278.000	8.1
35.00	1.2	159.833	0.3	284.333	0.4
41.250	7.3	166.083	7.8	290.500	7.8
47.250	1.2	172.167	0.6	296.667	0.8
53.500	7.6	178.25	8.4	302.833	7.8
59.917	1.2	184.917	0.2	309.083	0.7
66.083	7.3	190.917	8.0	315.333	7.6
72.417	1.1	197.00	0.5	321.500	1.0
78.583	7.7	203.333	8.5	327.750	7.4
84.833	1.0	209.667	0.1	334.167	1.0
91.083	7.4	215.750	8.1	340.250	7.4
97.417	1.1	222.083	0.4	346.500	1.3
103.583	7.9	228.167	8.5	353.000	7.1
109.915	0.8	234.667	0.1	359.167	1.3
116.167	7.5	240.750	8.0		
122.333	0.9	246.915	0.5		

Table 5-2 Tide Observations.

Symbol	Frequency (deg./hr)
O_1	13.943036
K_1	15.041069
P_1	14.958931
K_2	30.082137
N_2	28.439730

5.1 Computations and the Results

The computations have been completed in three steps. First, least squares approximations were done to determine the coefficients of the polynomials that will predict the range ratio (r_i) and the time difference (correction to time) at a point P_i (ϕ_i, λ_i). Second, a least squares polynomial approximation of the observed time series at the reference station (Table 5-2) was completed to determine the coefficients of the polynomial that will predict the height of tide $\bar{h}_0(t)$ at the reference station at any time $t \varepsilon M$. Finally, using the results of the first two steps, the observed geodetic coordinates at a point $P_i(\phi_i, \lambda_i)$ and the observed time at the location, the height of tide at the ship was computed for the determination of the reduced depth.

5.1.1 Determination of the Coefficients of the Approximating Polynomials

Of the 35 secondary gauge stations spread around the Bay of Fundy, 21 of them that are located around the main body ofthe Bay were used. Because of the intervening peninsula which bifurcates the Bay at about longitude 64° 55', the tidal wave propagation have been greatly affected along the two branches. A single analytical cotidal model for the entire area could not therefore be produced. The Bay has been divided into three sections numbered I, II and III in Figure 5-1. We have used the 21 secondary stations to model section I (those stations marked with * in Table 5-1 under remarks column). It should be noted that the origin of the local Cartesian coordinate system is approximately at the centre of the area being modelled (ϕ_0 = 45° 05' 00N, λ_0 = 65° 35' 00W). The data at the reference station (Saint John) was not fixed giving a total of 22 data points for the approximation.

Using equation 4.11, it was deduced that the highest degree of polynomial possible with 22 data points is 3, giving a total of 16 coefficients and 6 degrees of freedom. This does not however mean that the polynomial of degree 3 will give a better approximation than polynomials of degree 1 or 2. In Table 5-3 the degrees of the polynomials and their associated a posterori variance factors are tabulated. Two of the functions (Range ratio and Function A) have their variance factors reach a minimum at degree 2, while the variance factor of the third function (Function B) varies more slowly at degree 2. The conclusion is that the polynomial of degree 2 will give the best approximation with this data.

The approximation for time lag required some extra

n	Degree of Freedom (df)	Std. Dev. of Obs. σ_L (m)	Range Ratio $\hat{\sigma}_0^2$	Function A = R cos(v) $\hat{\sigma}_0^2$	Function B = R sin(v) $\hat{\sigma}_0^2$
1	18	0.1	0.94497	14.02610	20.60978
2	13	0.1	0.84280	12.40109	13.66546
3	6	0.1	1.14585	17.50086	13.43903

Table 5-3 A Posterori Variance Factor for Various Degrees of the Polynomials.

data manipulation. First, the time differences given in minutes were converted to angular measure using the relation

$$12 \text{ hrs} = 360°,$$
$$1 \text{ hr} = 30°,$$
$$1 \text{ min} = 0.5°,$$

(the Bay of Fundy tide is mainly semi-diurnal). Attempts to approximate the time lag converted to angular measure yielded large variance factors which of course decreased with increase in the degree of the polynomial. Unfortunately the highest degree of polynomial with the data available is 3. The conclusion reached was that the time lag distribution is not simple enough to be approximated by lower degrees of the polynomials.

From Chapter II

$$h(t) = \frac{1}{2} R \cos(\omega_k t + \alpha_k) ,$$

$$= \frac{1}{2} R \cos \alpha_k \cos \omega_k t + \frac{1}{2} R \sin \alpha_k \sin \omega_k t,$$

$$= A \cos \omega_k t + B \sin \omega_k t, \qquad (5.1)$$

where $$A = \frac{1}{2} R \cos \alpha_k . \qquad (5.2)$$

$$B = \frac{1}{2} R \sin \alpha_k . \qquad (5.3)$$

α_k is time lag (or phase lag), and R is the mean tide range at the station.

Also

$$\alpha_k = \text{Arctan } (B/A) , \qquad (5.4)$$

$$\frac{1}{2}R = (A^2 + B^2)^{1/2} \quad . \tag{5.5}$$

A and B can therefore be evaluated at each station using equations 5.2 and 5.3 respectively. We can now seek for the polynomials that can predict A and B at any point P_i (ϕ_i, λ_i). Once A and B are predicted, the predicted time lag (phase lag) can be obtained using equation 5.4. The associated variance (assuming no correlation between A and B eg. $\sigma_{AB} = 0$) is given by

$$\sigma_\alpha^2 = \left(\frac{\partial\alpha}{\partial A}\right)^2 \sigma_A^2 + \left(\frac{\partial\alpha}{\partial B}\right)^2 \sigma_B^2 \ , \tag{5.6}$$

where

$$\frac{\partial\alpha}{\partial A} = \frac{1}{1 + (B/A)^2} \times (- B/A^2) = \frac{B}{A^2(1 + (B/A)^2)} \ , \tag{5.7}$$

$$\frac{\partial\alpha}{\partial B} = \frac{1}{1 + (B/A)^2} \times \frac{1}{A} = \frac{1}{A(1 + (B/A)^2)} . \tag{5.8}$$

σ_A^2 and σ_B^2 are prediction variances of A and B respectively from the least squares approximations. For weighting, it was assumed that all the stations have been observed independently with equal amount of care. The standard error of the observed range was set at 0.1m.

If observed data is used, it is pertinent to note the following:

(i) The standard error of the observed mean range should be computed from the observed data using the relation

$$\sigma_R = \sqrt{\sum (R_{mD} - \bar{R}_m)^2 / n} \ . \tag{5.9}$$

where R_{mD} is the daily mean range, $\bar{R}_m$ is the mean of mean ranges, n is the number of observations.

(ii) A and B should be computed from equation 5.1 in the least squares sense using observed heights and the dominant constituent frequency in the semi-diurnal or diurnal band, depending on the type of tide.

In Table 5-4, the Fourier coefficients and their associated standard deviations for the range ratio and time lag are tabulated. The last four Fourier coefficients in the range ratio and function A have been eliminated, and in function B, two of the Fourier coefficients have been eliminated in the middle. In Table 5-5, the original coefficients of the polynomials are tabulated. Because of the discarding of the last four Fourier coefficients in the range ratio and function A, only five original coefficients can be recovered. In function B where the Fourier coefficients discarded are not the last ones, all the 9 original coefficients were recovered. (Note, each Fourier coefficient was tested against its standard deviation)

To compare the analytical cotidal model with other cotidal charts, the area was divided into a rectangular grid of 10' latitude and 10' longitude (Figure 5-2), and the values of range ratios (r) and time lags have been

RANGE RATIO		TIME LAG			
Coeff.(C_r)	σ_{C_r}	Coeff.(C_A)	σ_{C_A}	Coeff.(C_B)	σ_{C_B}
1.049	0.01957	3.970	0.07508	0	0
0.4623E-5	0.330E-6	0.1783E-4	0.1265E-5	0.4906E-5	0.1237E-5
0.1940E-10	.6191E-11	0.7554E-10	0.2375E-10	0.7091E-10	0.2321E-10
0.2126E-5	0.5570E-6	0.8618E-5	0.2138E-5	0.3977E-5	0.2089E-5
-0.2648E-10	0.1350E-10	-0.1140E-9	0.5178E-10	0	0
0	0	0	0	0.1728E-14	0.1004E-14
0	0	0	0	0.2869E-9	0.1042E-9
0	0	0	0	0.2024E-14	0.1582E-14
0	0	0	0	-0.5218E-19	0.3350E-19

Table 5-4 Fourier Coefficients After Discarding those of them greater than their Standard Deviations.

RANGE RATIO		TIME LAG			
Coeff.(C_r)	σ_{C_r}	Coeff.(C_A)	σ_{C_A}	Coeff.(C_B)	σ_{C_B}
1.120	0.03537	4.264	0.1357	-0.5691	0.16978
0.4041E-5	0.5680E-6	0.1550E-4	0.2179E-5	0.8730E-5	0.3614E-5
0.1364E-10	0.7644E-11	0.5374E-10	0.2932E-10	0.1484E-9	0.4318E-10
0.1247E-5	0.7150E-6	0.4835E-5	0.2742E-5	0.1695E-4	0.5526E-5
-0.2648E-10	0.1350E-10	-0.1140E-9	0.5177E-10	-0.2617E-9	0.1938E-9
0	0	0	0	-0.3887E-14	0.2208E-14
0	0	0	0	0.4269E-9	0.1391E-9
0	0	0	0	0.5786E-15	0.1834E-14
0	0	0	0	-0.5218E-19	0.3349E-19

Table 5-5 The Original Coefficient of the Polynomials and their Associated Standard Deviations.

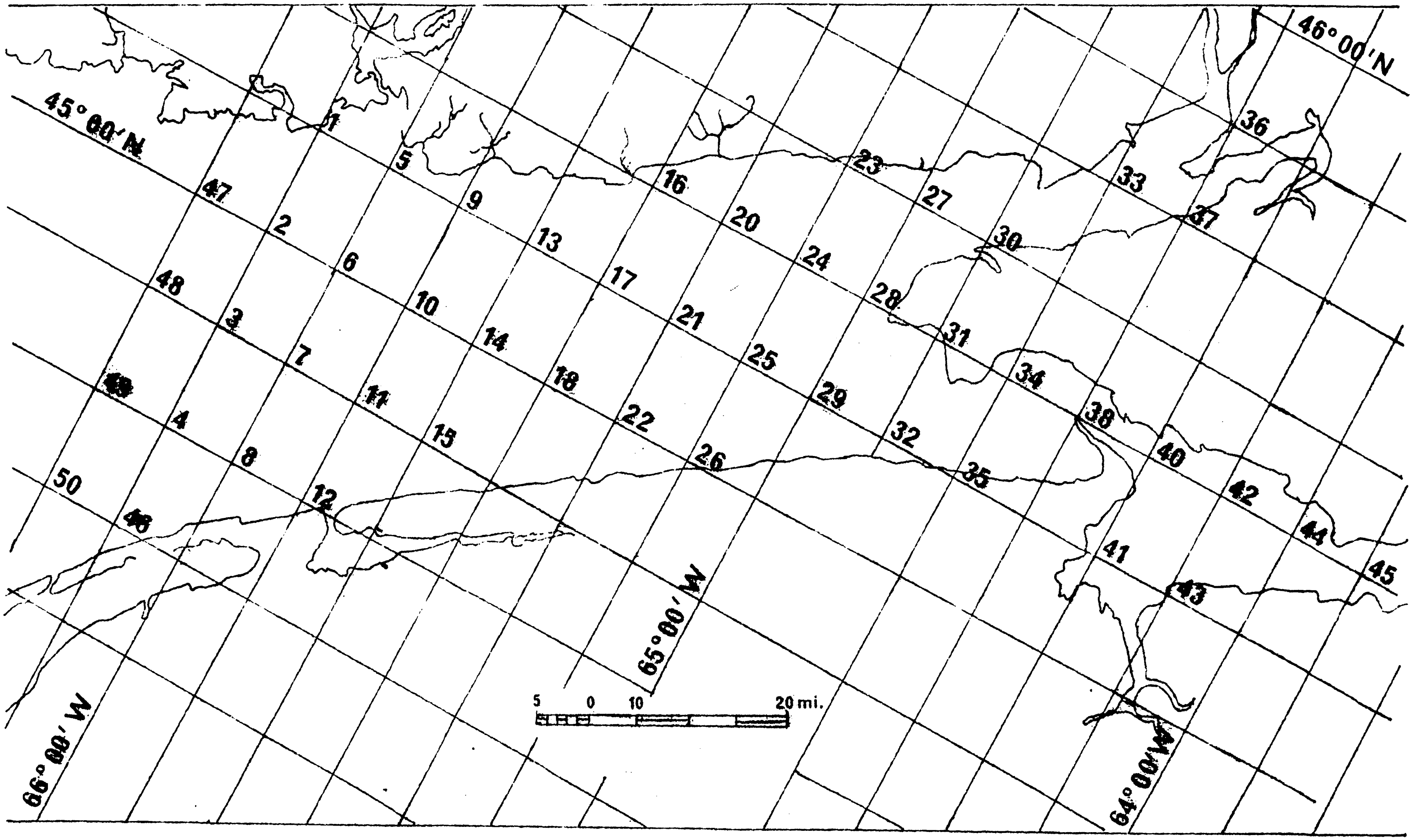


Figure 5-2

Grid Numbering

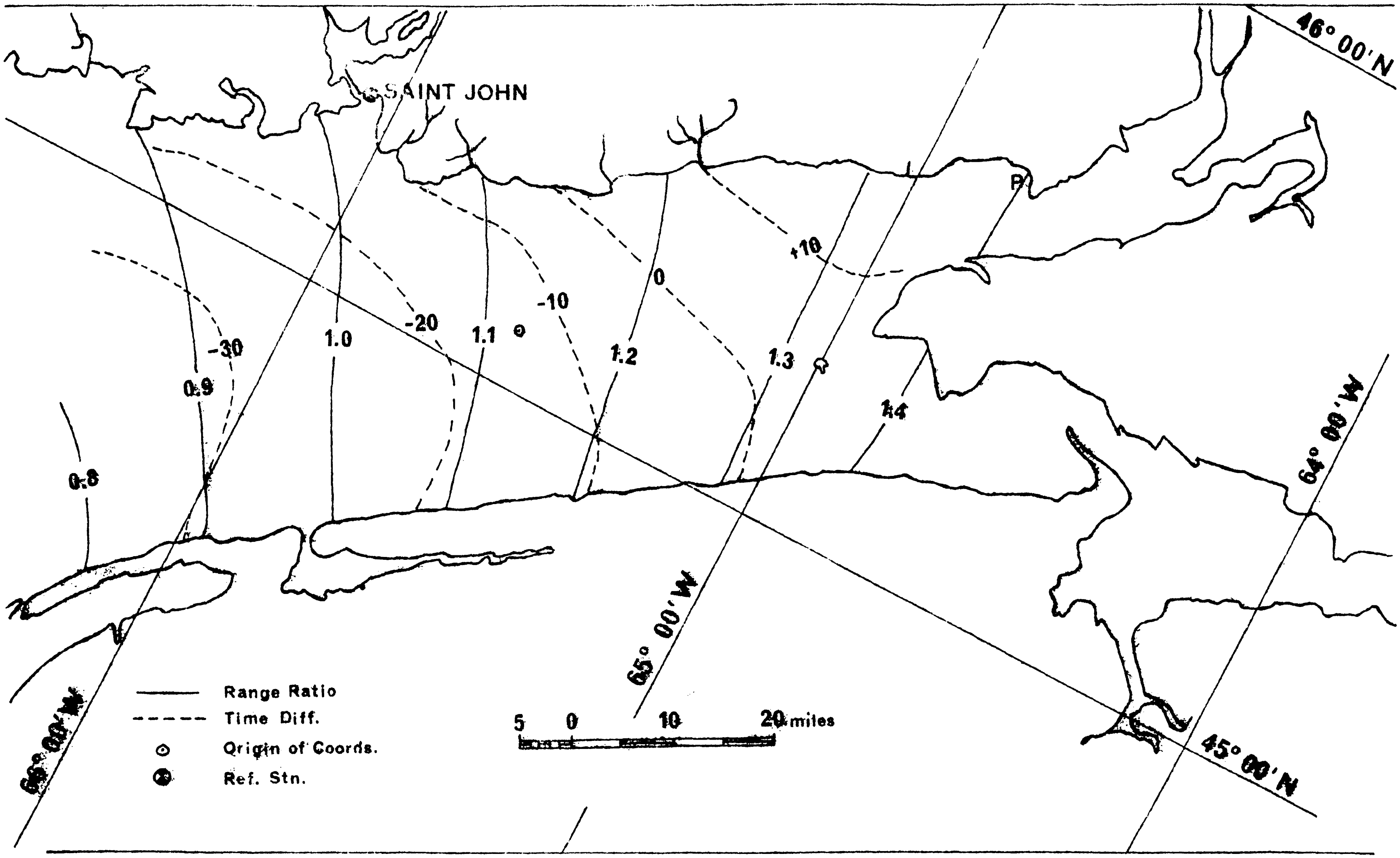


Figure 5-3

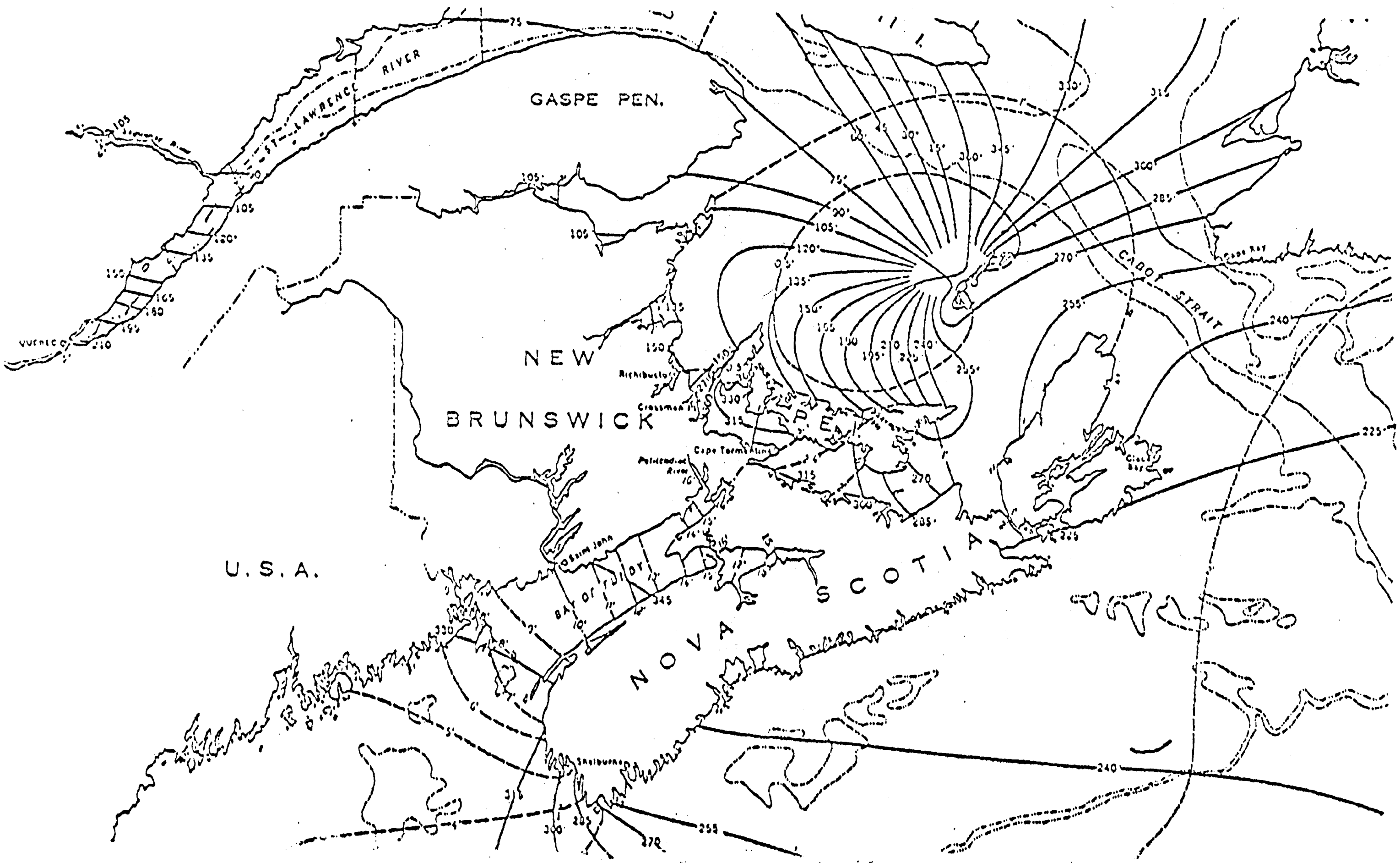
Bay of Fundy -- Range/Time Co-Tidal Curve from the Analytical Co-Tidal Models

PREDICTED RANGE RATIOS AND TIME LAGS AT THE GRID POINTS

NO	LATITUDE	LONGITUDE	RANGE RATIO	SIGMA	TIME LAG	SIGMA
1	45.166670	-66.166670	0.986543	0.4405D-01	-13.338520	0.5822D 01
2	45.000000	-66.166670	0.940871	0.3616D-01	-26.008451	0.7681D 01
3	44.833330	-66.166670	0.895199	0.3015D-01	-33.234233	0.9196D 01
4	44.666670	-66.166670	0.849530	0.2727D-01	-34.578871	0.9258D 01
5	45.166670	-66.000000	1.022218	0.4303D-01	-13.691588	0.5319D 01
6	45.000000	-66.000000	0.982984	0.3624D-01	-26.041995	0.7262D 01
7	44.833330	-66.000000	0.943749	0.3145D-01	-31.769547	0.8312D 01
8	44.666670	-66.000000	0.904517	0.2964D-01	-30.424472	0.7825D 01
9	45.166670	-65.833330	1.062573	0.4136D-01	-12.923846	0.4379D 01
10	45.000000	-65.833330	1.029776	0.3590D-01	-24.231832	0.6462D 01
11	44.833330	-65.833330	0.996980	0.3323D-01	-28.197832	0.6968D 01
12	44.666670	-65.833330	0.964185	0.3401D-01	-24.349502	0.5079D 01
13	45.166660	-65.666670	1.107604	0.3872D-01	-11.163316	0.4395D 01
14	45.000000	-65.666670	1.081246	0.3519D-01	-20.811055	0.5423D 01
15	44.833330	-65.666670	1.054887	0.3578D-01	-22.808907	0.5851D 01
16	45.333330	-65.500000	1.177242	0.4110D-01	6.276973	0.4590D 01
17	45.166670	-65.500000	1.157322	0.3625D-01	-8.537109	0.5231D 01
18	45.000000	-65.500000	1.137400	0.3475D-01	-15.998606	0.4251D 01
19	44.833330	-65.500000	1.117477	0.3980D-01	-15.874801	0.5117D 01
20	45.333330	-65.333330	1.225201	0.3593D-01	6.841768	0.4446D 01

21	45.166670	-65.333330	1.211718	0.3169D-01	-5.178430	0.3227D 01
22	45.000000	-65.333330	1.198233	0.3581D-01	-10.012239	0.3628D 01
23	45.500000	-65.166670	1.284885	0.4210D-01	21.128669	0.3956D 01
24	45.333330	-65.166670	1.277837	0.3087D-01	6.951205	0.3693D 01
25	45.166670	-65.166670	1.270790	0.2987D-01	-1.217189	0.2733D 01
26	45.000000	-65.166670	1.263743	0.3989D-01	-3.080484	0.4198D 01
27	45.500000	-65.000000	1.335766	0.3932D-01	16.463540	0.3503D 01
28	45.333330	-65.000000	1.335157	0.2831D-01	7.208467	0.2940D 01
29	45.166670	-65.000000	1.334547	0.3247D-01	3.216478	0.2630D 01
30	45.000000	-64.833330	1.391329	0.4012D-01	10.978109	0.2634D 01
31	45.333330	-64.833330	1.397156	0.3170D-01	7.417273	0.2776D 01
32	45.166670	-64.833330	1.402984	0.4097D-01	8.003851	0.3130D 01
46	44.500000	-66.166670	0.803858	0.2851D-01	-29.008920	0.7601D 01
47	45.000000	-66.333330	0.903441	0.3618D-01	-23.809221	0.7707D 01
48	44.833330	-66.333330	0.851332	0.2965D-01	-32.240976	0.9492D 01
49	44.666670	-66.333330	0.799226	0.2732D-01	-36.402650	0.9779D 01
50	44.500000	-66.333330	0.747117	0.3019D-01	-35.009593	0.8116D 01

Table 5-6 Predicted Values at Grid Intersections



Amplitude of the Main Semi-Diurnal Tide in Feet - - - 2' - - - -

Phase Lag of the Main Semi-Diurnal Tide - —150°————

(30°≈ Approx. 1 hr. in time)

Figure 5-4

The Average Progression of Semi-Diurnal Tide in the Bay of Fundy (Dohler, 1966).

predicted at each grid intersection. The co-range curves and the co-time curves were plotted as shown in Figure 5-3. Figure 5-2 shows the grid numbering, and in Table 5-6, the predicted values at each grid intersection and their associated standard deviations are tabulated. The cotidal curves from the proposed analytical models compared favourably with the cotidal curves (Figure 5-4) taken from 'Tides in Canadian Waters' [Dohler, 1966] showing the progression of semi-diurnal tides in the Bay of Fundy.

5.1.2 Least Squares Polynomial Approximation of Observed Time Series at the Reference Station

In this case, the heights of the tide defined at discrete times (t_i) in the time interval M are given and it is required to determine the coefficients of the polynomial that will best predict the height of tide h(t) at any other time $t \in M$. A one dimensional trignometric polynomial (Eqn. 2.30, Chapter II, Section 2.2) and the 7 constituent frequencies listed on page 82 have been used. The number of coefficients is given by

$$U = 2\ \mathrm{Ncon} + 1 \quad , \tag{5.10}$$

where Ncon is the number of constituent frequencies being used. For weighting, it was assumed that each height was observed independently, with equal amount of care and precision, and $\sigma_{h(t)} = 0.05$ m. The weight matrix is therefore

$$\underset{m\times m}{P} = \mathrm{Diag}\left(\frac{1}{\sigma_{h_1}^{2}} \quad \frac{1}{\sigma_{h_2}^{2}} \quad \cdots \quad \frac{1}{\sigma_{h_m}^{2}}\right) \quad . \tag{5.11}$$

In Table 5-7 , the Fourier coefficients and the recovered coefficients of the approximating polynomial of the observed time series, and their associated standard deviations are tabulated. Two Fourier coefficients were discarded in the middle of the series thus all the 15 original coefficients were recovered.

5.1.3 Tidal Reduction

For this set of computations, simulated sounding observations (corresponding in location to the 22 data points and with all observations made within the time interval M) were used to illustrate a proposed reduction algorithm. At each sounding location i, the depth (D_i), the time (t) and the geodetic coordinates (ϕ_i, λ_i) or the local Cartesian coordinates (x_i, y_i) are observed.

The arguments of the approximating polynomials for range ratios and time lags are the local Cartesian coordinates (x, y) and the argument of the approximating polynomial for the heights of tide at the reference station is the time (t). With the polynomial coefficients and their associated standard deviations stored in the computer, only the arguments (x_i, y_i) are needed to predict the range ratio (r_i) and the time lag (tc_i). The time lag is, in a sense, the correction to be applied to the observed time at the ship (sounding location i) to get the equivalent time at the reference station. With the equivalent time at the reference station computed, the height of the tide at the reference station is predicted

FOURIER COEFFS. AFTER TEST AGAINST THEIR STD. DEVS.		COEFFICIENTS OF THE ORIGINAL POLYNOMIAL	
Coeff.(F_c)	Std. Dev. σ_c	Coeff.(C)	Std. Dev. σ_c
4.279	0.5861E-2	4.279	0.6227E-2
-5.339	0.9072E-2	-1.951	0.1423
2.335	0.02048	2.226	0.1081
0.4590	0.9760E-2	-0.1756	0.2113
-0.0496	0.8784E-2	0.2592	0.3972
0.0360	0.8436E-2	0.0678	0.0100
-0.0909	0.8295E-2	-0.1031	0.8507E-2
-0.1338	0.8309E-2	-0.2305	0.0319
0.1584	0.8360E-2	0.0335	0.0322
0	0	0.1392	0.0382
0	0	0.0832	0.0229
-0.4193	0.0954	0.4132	0.1602
0.9868	0.1508	-0.0302	0.3586
0.4479	0.0840	0.7741	0.1272
0.4300	0.1259	0.4300	0.1259

Table 5-7 Coefficients of the Polynomial for the Observed Time Series at the Reference Station.

using the known time as the argument of the predicting polynomial. The height of tide at the ship, which is the required reduction, is obtained using equation 4.43. The reduced sounding is computed using equation 4.44. Applying the law of propagation of errors, the standard deviation of reduced sounding is given by

$$\sigma_{d_i} = \left\{ \left(\frac{\partial d_i}{\partial D_i} \right)^2 \sigma^2_{D_i} + \left(\frac{\partial d_i}{\partial h_i(t)} \right)^2 \sigma^2_{h_i(t)} \right\}^{1/2} , \quad (5.12)$$

where σ_{D_i} is the standard deviation of the depth sounded, $\sigma_{h_i(t)}$ is the standard deviation of the predicted height at the ship, $\frac{\partial d_i}{\partial D_i} = 1$ and $\frac{\partial d_i}{\partial h_i(t)} = 1$. The heights of the tide at the ship, required to reduce the soundings, are tabulated in Table 5-8 along with their estimated standard deviations. The predicted time lags and range ratios are compared with the original data set (observed values) as shown in Table 5-9. At this stage it is important to mention that normality in the distribution of residuals was assumed and chi square tests on the variance factor performed at 95% confidence level. The test passed for the range ratio and observed time series approximations but failed for the time lag. There are several possible reasons for the failure of this test and as such a definite conclusion cannot be made without performing several other statistical tests [Vaníček and Krakiwsky, Chapter 13, 1978]. However, it can be concluded from our earlier discussions (Section 5.1) that eithe the time lag is not simple enough to be approximated by the lower order polynomial or the information available is not sufficient to approximate the time lag. In the present circumstance it may be safer to assume σ_0^2 known and equal to 1, so that

$$\Sigma_{\hat{c}} = \sigma_0^2 N^{-1}. \tag{5.13}$$

TIDAL REDUCTIONS

NUM	LATITUDE	LONGITUDE	OBS. DEPTH	TIME AT SHIP	TIME AT REF	TIDE AT REF	PR. RATIO	TIDE AT SHIP	STDEV
1	45.350000	-64.816667	12.000	2.750	2.624	6.855	1.403	9.616	0.8976D 00
2	45.300000	-64.783333	13.620	3.250	3.124	7.181	1.418	10.183	0.9208D 00
3	45.233333	-64.516667	14.250	4.500	4.274	7.218	1.538	11.105	0.9573D 00
4	45.250000	-65.000000	13.720	4.833	4.758	6.935	1.335	9.255	0.6678D 00
5	45.030000	-65.066667	12.820	5.117	5.403	6.314	1.306	8.246	0.5426D 00
6	44.800000	-65.533333	10.600	6.500	6.775	4.394	1.100	4.834	0.4540D 00
7	44.400000	-65.833333	13.980	25.333	25.312	4.035	0.911	3.677	0.4929D 00
8	44.500000	-66.100000	10.130	28.533	28.965	7.391	0.828	6.116	0.6320D 00
9	44.483333	-66.083333	12.080	29.250	29.647	7.026	0.829	5.828	0.5863D 00
10	44.266667	-66.350000	9.880	30.300	30.737	5.819	0.668	3.865	0.7069D 00
11	44.250000	-66.400000	9.500	31.500	32.020	3.871	0.643	2.491	0.5744D 00
12	44.333333	-66.116667	8.520	32.250	32.425	3.253	0.778	2.530	0.5165D 00
13	44.200000	-66.166667	7.080	32.833	32.755	2.762	0.721	2.007	0.4990D 00
14	44.600000	-66.800000	8.750	33.750	34.285	1.315	0.655	0.861	0.3824D 00
15	44.683333	-66.950000	8.350	35.167	35.111	1.205	0.752	0.906	0.3172D 00
16	45.066667	-67.050000	12.330	45.333	45.090	2.862	0.827	2.363	0.5901D 00
17	45.233333	-66.050000	13.100	46.583	46.700	1.294	1.027	1.329	0.4388D 00
18	45.590000	-65.533333	14.230	145.583	145.448	2.118	1.170	2.478	0.5612D 00
19	45.566667	-64.966667	15.330	149.300	146.966	2.017	1.346	2.714	0.5388D 00
20	45.600000	-64.783333	16.200	152.300	152.099	7.340	1.404	10.305	0.9844D 00
21	45.466667	-64.650000	15.890	155.917	155.744	6.132	1.386	8.501	0.6340D 00
22	45.266667	-66.066667	13.690	156.250	156.300	5.093	1.032	5.257	0.4241D 00

Table 5.8 Tidal Reductions

Station Index No*	Obs. Time Lag	Predicted Time Lag	Diff.	Obs. Range Ratio	Predicted Range Ratio	Diff.
0065(22)	0	-3	+3	1.0	1.032	-0.032
0001(14)	-28	-32	+4	0.661	0.655	+0.006
0015(15)	+5	+3	+2	0.673	0.752	-0.079
0040(16)	+16	+16	0	0.900	0.827	+0.073
0060(17)	-10	-7	-3	0.996	1.027	-0.031
0129(18)	+9	+8	+1	1.201	1.170	+0.031
0140(19)	+19	+20	-1	1.325	1.346	-0.021
0150(20)	+17	+12	+5	1.410	1.404	0.006
0225(21)	+11	+10	+1	1.317	1.386	-0.069
0235(1)	-1.0	+8	-9	1.311	1.403	-0.092
0240(2)	+16	+8	+8	1.456	1.418	+0.038
0305(3)	+12	+14	-2	1.490	1.538	-0.048
0312(4)	0	+5	-5	1.361	1.335	+0.026
0315(5)	-12	+1	-13	1.265	1.306	-0.041
0320(6)	-14	-16	+2	1.060	1.100	-0.040
0330(7)	-16	+1	-17	0.956	0.911	+0.045
0335(8)	-18	-26	+8	0.843	0.828	+0.015
0336(9)	-37	-24	-13	0.761	0.829	-0.068
0340(10)	-34	-26	-8	0.721	0.668	+0.053
0345(11)	-34	-31	-3	0.713	0.643	+0.070
0353(12)	+18	-10	+28	0.721	0.778	-0.057
0355(13)	18	+5	+13	0.673	0.721	-0.048

<u>Summary:</u>

Time Lag : $0 < \text{Diff.} < |28|$

Range Ratio : $|0.005| < \text{Diff.} < |0.1|$

Time Lag, RMS of the Diff. (observed-predicted) = 9.44 min

Range Ratio, RMS of the Diff. (observed-Predicted) = 0.0505

Table 5-9 Difference Between Predicted and Observed Values.

* Numbers in brackets corresponds to the serial numbers in Table 5-8.

VI CONCLUSIONS AND RECOMMENDATIONS

The objective of this work has been to produce analytical cotidal models, using observed data or existing cotidal charts, which could be stored conveniently in a computer so that when observed sounding data are input, the output would be reduced soundings. The principal advantages of the proposed analytical scheme are the following.

(i) The analytical models can be obtained and updated using the observed data in addition to that of already produced charts. This allows up-dating the model when more observations are available.

(ii) This scheme does not require large computer storage space. For example, instead of storing many digitized numbers, the digitized values are used to determine a few coefficients of the best approximating polynomials.

(iii) These models allow for the rigorous propagation of errors. With associated estimated standard deviations, the reliability of the final result can be easily obtained.

(iv) A degree of flexibility is offered. It is convenient to use data from existing cotidal charts, observations or a combination of the two.

Least squares polynomial approximation is applied to either

(i) recover a function F(x) from a known set of its values, or

(ii) to replace the known function in further computations by a more trackable polynomial.

The problem of least squares polynomial approximation as applied in this work is that defined by (i) above. It would be interesting to view the problem as in (ii) above and apply it to the Laplace tidal equations to obtain the necessary polynomials.

From the test computations using the data on the Bay of Fundy, the computer effective run time is 28.46 secs and the storage space is 336,136 Bytes for Least squares polynomial approximation for range ratios and time lags. For the polynomial approximation for time series at the reference station the effective run time is 23.03 secs and the storage space is 299,288 Bytes. For the Tidal Reduction therefore we have a total of 42 coefficients and their associated standard deviations to store in the computer. In the program to execute this for 22 sounding stations, the time of execution was 0.76 sec and the storage space used was 14,480 Bytes. The result also shows that the water level at a location (ϕ_i, λ_i) can be predicted with a standard deviation (σ_{h_i}) of 0.5 m or better.

It is recommended that the prediction of tides at the reference station with the polynomial should be done strictly within the time interval M used in the least squares approximation of the observed series. When extrapolation is required, it is advisable to use the amplitude/ phase analytical cotidal models and carry out the prediction using the procedure described in Chapter II, Section 2.2.

Finally, since the data immediately available was not adequate to fully test the proposed analytical schemes, it is suggested that proper data be obtained to facilitate complete testing.

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A P P E N D I C E S

I OUTLINE OF THE LEAST SQUARES APPROXIMATION THEORY

Least squares polynomial approximation is applied to either

(i) recover a function $F(x)$ from a known set of its values, or

(ii) replace the known function $F(x)$ in further computations by a more trackable polynomial.

The problem of least squares polynomial approximation as used in this report is that defined by (i) above. A brief outline of the least squares approximation theory due mainly to Vaníček and Wells, [1972] is here given.

Given:

(i) a function F defined on a finite set M

$M \equiv \{X_1 \; X_2 \; \ldots \; X_m\}$, M discrete

$M \equiv [a, b]$, M compact

(ii) a base $\Psi = \psi_1, \psi_2 \ldots \psi_u$, a set of u linearly independent prescribed functions from the functional space Gm,

(iii) a weight function W, defined and non-negative on M,

then the problem of least squares approximation is to determine the vector of coefficient $(C_1, C_2 \ldots C_u)$ of a generalized polynomial Pn which minimizes the weighted distance $P(F, Pn)$ defined as

$$P(F,\ Pn) \equiv \left(\sum_{X \varepsilon M} W(x)(F(x) - Pn(x))^2 \right)^{1/2} , \quad M \text{ discrete} \qquad (I.1)$$

$$P(F,\ Pn) \equiv \left(\int_M W(x)(F(x) - Pn(x))^2 \right)^{1/2} . \quad M \text{ compact} \qquad (I.2)$$

The approximating polynomial is given by

$$Pn = \sum_{i=1}^{n} C_i \psi_i \ . \qquad (I.3)$$

The scalar product of two functions G, H ε Gm is defined as

$$\langle G,\ H \rangle = \begin{cases} \sum_{X \varepsilon M} W(x).G(x).H(x), & M \text{ discrete} \\ \int_M W(x).G(x).H(x). & M \text{ compact} \end{cases} \qquad (I.4)$$

If the product of two functions G, H ε Gm is zero, then the functions are orthogonal. If the base functions are orthogonal,

$$\langle \psi_i \psi_j \rangle = \begin{cases} k_i \neq 0 & i = j \\ 0 . & i \neq j \end{cases} \qquad (I.5)$$

If i = j, it means

$$k_i = \langle \psi_i\ \psi_i \rangle = ||\psi_i||^2,\ \varepsilon\ E^+$$

or

$$\langle \psi_i\ \psi_i \rangle = ||\psi_i||^2\ \delta_{ij} ,$$

where δ_{ij} is known as Kronecker delta and is defined as

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j . \end{cases} \qquad (I.6)$$

Returning to the problem of least squares approximation we are seeking for the coefficients $C_1, C_2 \ldots C_u$ of the polynomial Pn that would make the distance $||Pn - F||$ the minimum. This means minimizing the Eucleidean distance

$$\sum_{X\varepsilon M} W(x)(F(x) - Pn(x))^2$$

with respect to $C_1, C_2 \ldots C_u$. The condition is written as

$$\underset{C_1,\, C_2,\, \ldots\, C_u \,\varepsilon}{\text{Min}} \rho^2(P, F)$$

$$= \underset{C_1,\, C_2,\, \ldots\, C_u \,\varepsilon\, E}{\text{Min}} \sum_{X\varepsilon M} W(x)(Pn(x) - F(x))^2$$

$$= \underset{C_1,\, C_2,\, \ldots\, C_u \,\varepsilon\, E}{\text{Min}} \sum_{X\varepsilon M} W(x) \sum_{i=1}^{u} (C_i\psi_i(x) - F(x))^2. \quad (I.7)$$

When the partial derivatives of the above w.r.t individual C's are equated to zero, the minimum distance is obtained.

Minimizing we have

$$\frac{\partial \sum_{X\varepsilon M} [W(x)(\sum_{j=1}^{u} C_j\psi_j(x) - F(x))^2]}{\partial C_i}$$

$$= 2\sum_{X\varepsilon M} W(x) \sum_{j=1}^{u} (C_j\psi_j(x) - F(x)) \frac{\partial \sum_j C_j\psi_j(x)}{\partial C_i}$$

$$= 2\sum_{X\varepsilon M} W(x) \sum_{j=1} C_j\psi_j(x) - F(x)\ \psi_i(x) \quad ,$$

$$= 2\sum_{X} W(x) \sum_{j}^{u} C_j\psi_j(x)\psi_i(x) - 2\sum_{X} W(x)F(x)\psi_i(x) \quad ,$$

$$= 0 \ . \quad (I.8)$$

From the definition of the scalar product, the above can be written as

$$\sum_{j=1}^{u} \langle\psi_i, \psi_j\rangle C_j = \langle F, \psi_i\rangle. \qquad (I.9)$$

Equation I.9 gives the system of normal equations which can be solved to obtain the coefficients C_1, C_2, ... C_u. Putting I.9 in matrix form we have

$$[\langle\psi_i \ \psi_j\rangle]C = [\langle F, \psi_i\rangle]. \qquad (I.10)$$

Letting

$$N = [\langle\psi_i, \psi_j\rangle], \qquad (I.11)$$

and

$$U = [\langle F, \psi_i\rangle], \qquad (I.12)$$

the solution of normal equation is given by

$$C = N^{-1} U . \qquad (I.13)$$

N is the Gram's matrix and Gram's determinant $\det(N) \neq 0$ because we are dealing with linearly independent base functions Ψ. Equation I.13 therefore has a unique solution.

If we are dealing with orthogonal system of base functions Ψ^*, then

$$N^* = \mathrm{Diag}[\langle\psi_i^* \ \psi_i^*\rangle] = \mathrm{Diag}(||\psi_i^*||^2) .$$

The solution of the normal equation becomes trivial and is given by

$$C^* = \langle F, \psi_i^*\rangle/||\psi_i^*||^2, \quad i = 1, 2, \ldots u. \qquad (I.14)$$

Each Fourier coefficient C* can be solved independently.

The system of base functions Ψ often encountered are not usually orthogonal. The system can however be orthogonalised using Schmidt's orthogonalization process. The procss works as follows:

i) choose

$$\psi_1^* = \psi_1 \qquad X \varepsilon M \tag{I.15}$$

ii) define

$$\psi_2^* = \psi_2 + \beta_{2,1}\psi_1^*, \quad X \varepsilon M, \ \beta_{2,1} \varepsilon E. \tag{I.16}$$

Multiplying the above equation I.16 by $W\psi_1^*$ and summing up all the equations for all the X yields

$$<\psi_2^*, \psi_1^*> = <\psi_2 \ \psi_1^*> + \beta_{2,1}<\psi_1^*, \psi_1^*> . \tag{I.17}$$

To make the system orthogonal, $<\psi_2^*, \psi_1^*>$ must be zero. The unknown coefficient $\beta_{2,1}$ can be determined from

$$\beta_{2,1} = <\psi_2, \psi_1^*>/<\psi_1^*, \psi_1^*> . \tag{I.18}$$

iii) Define next

$$\psi_3^* = \psi_3 + \beta_{3,2}\psi_2^* + \beta_{3,1}\psi_1^*, \ X \varepsilon M, \ \beta_{3,2}, \beta_{3,1} \varepsilon \ . \tag{I.19}$$

Multiplying by $W\psi_1^*$ and $W\psi_2^*$ yield respectively

$$<\psi_3^*, \psi_1^*> = <\psi_3, \psi_1^*> + \beta_{3,2}<\psi_2^*, \psi_1^*> + \beta_{3,1}<\psi_1^*, \psi_1^*>,$$

$$<\psi_3^*, \psi_2^*> = <\psi_3, \psi_2^*> + \beta_{3,2}<\psi_2^*, \psi_2^*> + \beta_{3,1}<\psi_1^*, \psi_2^*>.$$

By reason of orthogonality,

$$<\psi_3^* \ \psi_1^*> = <\psi_2^*, \psi_1^*> = <\psi_3^*, \psi_2^*> = <\psi_1^*, \psi_2^*> = 0.$$

We therefore have that

$$<\psi_3, \psi_1^*> + \beta_{3,1}<\psi_1^*, \psi_1^*> = 0 ,$$

$$<\psi_3\ \psi_2^*> + \beta_{3,2}<\psi_2^*,\ \psi_2^*> = 0\ ,$$

that is

$$\beta_{3,1} = <\psi_3,\ \psi_1^*> / <\psi_1^*,\ \psi_1^*>\ . \tag{I.20}$$

$$\beta_{3,2} = <\psi_3\ \ \psi_2^*>/<\psi_2^*,\ \psi_2^*> \tag{I.21}$$

The process can be generalized for any coefficient β_{ji} thus

$$\beta_{ji} = <\psi_j\ \psi_i^*>/<\psi_i^*\ \psi_i^*>\ . \tag{I.22}$$

Expressing the original system in terms of the orthogonal system we have

$$\psi_1 = \psi_1^*\ ,$$

$$\psi_2 = -\ \beta_{2,1}\ \psi_1^* + \psi_2^*\ ,$$

$$\psi_3 = -\ \beta_{3,1}\ \psi_1^* - \beta_{3,2}\ \psi_2^* + \psi_3^*\ ,$$

$$\vdots$$

$$\psi_u = -\ \beta_{u,1}\ \omega_1^* - \beta_{u,2}\ \psi_2^*\ \cdots\ -\ \beta_{u,u-1}\ \psi_{u-1}^* + \psi_u^*\ .$$

Putting it in matrix form we have

$$\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \\ \psi_u \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -\beta_{2,1} & 1 & 0 & \cdots & 0 \\ -\beta_{3,1}, & \beta_{3,2}, & 1 & \cdots & 0 \\ \\ -\beta_{u,1}, & -\beta_{u,2} & \beta_{u,3} & \cdots & 1 \end{bmatrix} \begin{bmatrix} \psi_1^* \\ \psi_2^* \\ \psi_3^* \\ \\ \psi_u^* \end{bmatrix}\ . \tag{I.23}$$

β_{ji} is defined by equation I.22.

Letting

$$\underset{u \times u}{B} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -\beta_{2,1} & 1 & 0 & \cdots & 0 \\ -\beta_{3,1} & -\beta_{3,2} & 1 & \cdots & 0 \\ -\beta_{u,1} & -\beta_{u,2} & -\beta_{u,3} & \cdots & 1 \end{bmatrix},$$

equation I.23 is written as

$$\psi = B\psi^{*}. \qquad (I.24)$$

B is the transformation matrix that transforms non orthogonal system to orthogonal system. It is an u x u triangular matrix and the determinant $\det(B) \neq 0$.

If we have that

$$\Psi^{T}C = \Psi^{*T}C^{*} \quad ,$$

using equation I.24, we can transform the Fourier coefficients into the coefficients of the original base functions, thus

$$(B\Psi^{*})^{T}C = \Psi^{*T}C^{*}$$

$$C = (B^{T})^{-1}C^{*} \quad . \qquad (I.25)$$

II BRIEF DESCRIPTION OF THE COMPUTER PROGRAMS USED

The computer programs used in the computations are in three parts, namely

(i) Least squares polynomial approximation for cotidal curves,

(ii) Least squares polynomial approximation of observed time series at the reference station,

(iii) Tidal reductions.

II.1 Least Square Polynomial Approximation for the Cotidal Curves

Figure A-1 is the flow chart describing the program.

INPUTS

1st card, FORMAT(5X, 7I4)

ID - The dimension of the polynomial

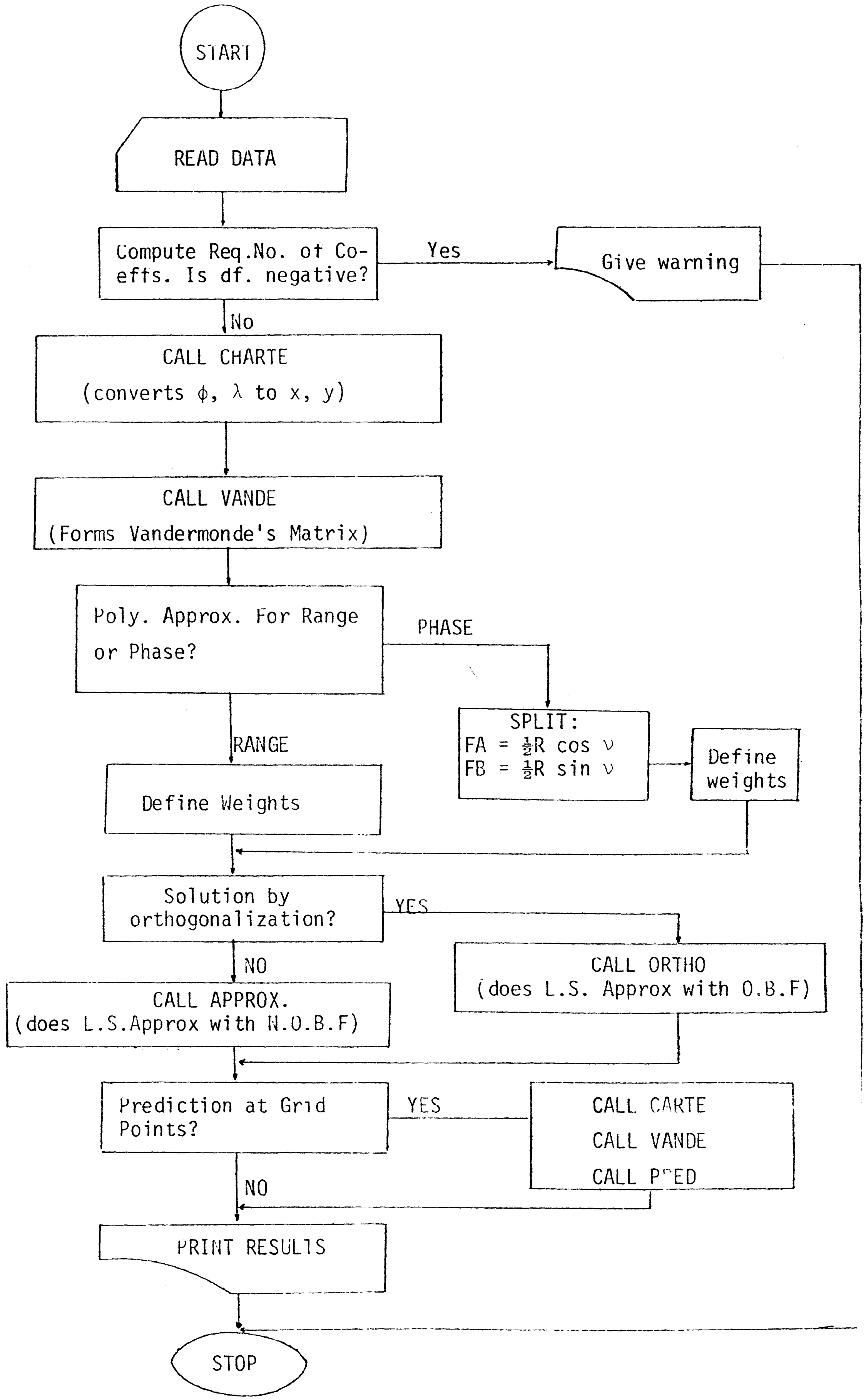
N - The degree of the polynomial

M - Number of data points for the approximation

NPP - Number of grid points for prediction. If there is no prediction NPP = 0

INDEX - Code for the type of function to be approximated. If index = 1, the polynomial approximation for range ratio (amplitude) is performed. If index = 2, the polynomial approximation for time lag (phase lag) is performed.

Figure II-1 Polynomial Approximation of Cotidal Curves - Flow Chart



ID - Code for orthogonal or non orthogonal solution, 1 - for orthogonal solution
2 - for non orthogonal solution.

ITEST - Code for testing Fourier coefficients
0 - for no test
1 - against its Standard Deviation
2 - against 2 times its Standard Deviation
3 - against 3 times its Standard Deviation.

2nd Card: FORMAT(5X, I5, 5X, F10.6, 5X, F10.6
5X, F10.6, 5X, F10.6)

This card contains the identification number, latitude ϕ and longitude λ of the reference station, the range ratio (amplitude) and time lag (phase lag) at the reference station.

n cards: Format as in the second card. Each card contains, station identification number, latitude (ϕ), longitude (λ) of the data points, the range ratio (amplitude diff.) and the time lag (phase lag diff.) at each data.

NPP cards: FORMAT(5X, I5. 5X, F10.6, 5X, F10.6)

If there are no prediction at the grid points, these cards will be omitted. Each card contains the grid point number, the geodetic coordinates of the grid points (ϕ, λ).

SUBROUTINES:

SUBROUTINE CARTE; computes the local cartesian coordinates (x_i, y_i) given the geodetic coordinates of the points (ϕ_i, λ_i), the geodetic coordinates of the origin of the local system (ϕ_0, λ_0) and the dimensions of the ellipsoid.

SUBROUTINE VANDE - computes the prediction matrix given the geodetic coordinates (ϕ, λ) of the prediction points, the number of prediction points, the dimension of the polynomial and the number of coefficients.

SUBROUTINE APPROX. - does the Least Square approximation of the function given the number of coefficients, the number of data point, the Vandermonde's matrix, the weight matrix and the functional values.

SUBROUTINE ORTHO - orthogonalizes the Vandermonde's matrix using Gram Schmidt method, computes the Fourier Coefficients of the orthogonalized matrix, derives the coefficients of the Vandermonde's matrix, computes the variances of the Fourier Coefficients and the variance-covariance matrix of the original coefficients.

SUBROUTINE PRED. - predicts the function values at the grid points and computes the variance-covariance matrix of the prediction.

```
$JOB CKENWA/G
C     ************************************************************************
C     *  PROGRAME TO CONSTRUCT ANALYTICAL COTIDAL CHARTS USING ANALYSED *
C     *   TIDAL CONSTANT OR RANGE RATIOS AND TIME DIFF. REF.TO A STANDAR*
C     *   D STATION  OR DIGITIZED VALUES FROM EXISTING COTIDAL CHART     *
C     *                                                                  *
C     *  THIS USES THE LAT. AND LONG. OF DISCRETE POINTS,THE AMPL.AND    *
C     *  PHASE LAGS  OF RANGE RATIOS AND TIME DIFFS. AT THE POINTS AS    *
C     *  THE INPUTS TO COMPUTE THE COEFFICIENTS OF THE BEST PREDICTING   *
C     *  POLYNOMIALS AT ANY OTHER POINT.                                 *
C     *                                                                  *
C     *  GUIDE TO SOME NOTATIONS USED IN THE PROGRAM                     *
C     *  INPUT:                                                          *
C     *        PHI - LATITUDE OF STN.I                                   *
C     *        ALON - LONGITUDE OF STN. I
C     *        FH,FG - AMPLITUDE AND PHASE LAG OR RANGE RATIO AND TIME   *
C     *                  DIFF.IN MINUTES OF TIME.                        *
C     *        ID - DIMENSION OF THE APPROX.                             *
C     *        N - DEGREE OF THE POLYNOMIAL                              *
C     *        M - NUMBER OF OBSERVATIONS                                *
C     *        IO - CODE FOR ORTHOGONAL OR NON ORTHOGONAL SOLUTION.      *
C     *               1 - FOR ORTHOGONAL SOL.                            *
C     *               2 - FOR NON ORTHOGONAL SOL.                        *
C     *        ITEST :    CODE FOR TESTING FOURIER COEFFS.
C     *                   0  -  NO TEST                                  *
C     *                   1  -  AGAINST ITS STD DEV.                     *
C     *                   2  -  AGAINST 2 TIMES ITS STD. DEV.            *
C     *                   3  -  AGAINST 3 TIMES ITS STD. DEV.            *
C     *        NPP: - NO. OF GRID POINTS FOR PREDICTION.IF THERE IS      *
C     *                NO PREDICTION AT GRID POINTS NPP=0                *
C     *                                                                  *
C     *  OUTPUT:                                                         *
C     *         L - NO. OF COEFFICIENTS REQUIRED. (WHEN THE NO. OF COEFF*
C     *              EXCEED THE NO. OF OBS THE PROGRAM IS ABORTED)
C     *         C,CA,CB - VECTORS OF COEFFICIENTS.                       *
C     *                                                                  *
C     *                                                                  *
C     *  SEE SUBROUTINE ORTHO FOR MORE EXPLANATIONS OF OUTPUT NOTATIONS *
C     *                                                                  *
C     *                                                                  *
C     ************************************************************************
C         *          MAIN PROGRAME
C
          IMPLICIT REAL*8(A-H,O-Z)
          DIMENSION PHI(50),ALON(50),FH(50),FG(50),A(50,50),P(50,50)
     *   ,COVAR(50,50),VAR(50,50),C(50),EN(50,50),X(50),Y(50),V(50),
     *    U(50),NUM(50),NGRID(50),XP(50),YP(50),PMT(50,50),PMCO(50,50)
     *    ,AC(50),VARP(50)
          DIMENSION ALAT(50),ALONG(50),PM(50,50),PFH(50),PFG(50)
          DIMENSION ALPHA(50,50),W(50),FC(50),SUMFC(50),SGN(50),STDP(50),
     * D(50),SIGMAF(50),SIGMAR(50)
          DIMENSION FA(50),FB(50),FA2(50),FB2(50),WA(50),WB(50),CA(50),
     *    CB(50),CCA(50,50),COB(50,50),PFA(50),PFB(50),VARPA(50,50),
     *    VARPB(50,50),B(2,2),AR(50,50),FR(50)
          IRDA=IDA=IDB=IDC=50
          RHO=206265.0D0
          PI=3.14159265D0
```

```
            IW=6
            SIGMA=1.0D0
C
C        READ IN PROGRAME SPECIFICATION
C
            READ(5,146)IC,N,M,NPP,INDEX,IO,ITEST
 146        FORMAT(5X,7I4)
C
C           COMPLTE NO. OF COEFFICIENTS AND DEGREE OF POLY AND DEGREE OF
C            OF FREEDOM.  IF DF IS ZERO OR NEGATIVE GIVE WARNING AND STOP
C
                L=(N+1)**ID
                IDF=M-L
            IF(IDF.LE.0)THEN DO
                PRINT,'PROGRAME SPECIFICATION INADEQUATE'
                STOP
                ELSE DO
                END IF
         CONTINUE
C
C           READ IN DATA ON THE STANDARD OR REF. STATION
C
            READ(5,200)NUMO,PHI0,ALON0,FH0,FG0
 200        FORMAT(5X,I5,5X,F10.6,5X,F10.6,5X,F10.6,5X,F10.6)
 100        FORMAT(5X,I5,5X,F10.6,5X,F10.6)
C
C            READ IN DATA FROM OTHER STATIONS
C
            DO 1 I=1,M
            READ(5,200)NUM(I),PHI(I),ALON(I),FH(I),FG(I)
   1        CONTINUE
C
C           PRINT ALL THE INPUT DATA
C
            PRINT101
 101         FORMAT(//,4X,'NO',8X,'LATITUDE',10X,'LONGITUDE',10X,'RANGE RA
     *TIO',10X,'TIME LAG')
 102        FORMAT(5X,I4,7X,F10.6,9X,F10.6,9X,F10.6,9X,F10.6)
            PRINT102,NUMO,PHI0,ALON0,FH0,FG0
            DO 3 I=1,M
            PRINT102,NUM(I),PHI(I),ALON(I),FH(I),FG(I)
   3         CONTINUE
C
            PRINT131
 131        FORMAT(//,5X,'DEG.OF POLY.',5X,'NO.OF OBS.',5X,'NO.OF COEFF.',
     *      5X,'DEG.OF FREEDOM')
            PRINT132,N,M,L,IDF
 132        FORMAT(//,8X,I2,12X,I3,15X,I3,20X,I3)
            PRINT130
 130        FORMAT(//,5X,'CARTESIAN COORD. OF THE GIVEN STATIONS')
            PRINT127
 127        FORMAT(//,5X,'STN. NO.',5X,'X-COORD.',15X,'Y-COORD.')
            CALL CARTE(M,PHI,ALON,PHI0,ALON0,X,Y)
                DO 4 I=1,M
            PRINT111,NUM(I),X(I),Y(I)
 111        FORMAT(/,6X,I4,4X,F15.6,7X,F15.6)
   4            CONTINUE
C
C       COMPLTE FOR A  MATRIX
```

```
C
            DO 5 I=1,M
            DO 5 J=1,L
            A(I,J)=0.D0
    5       CONTINUE
            IDP=N+1
            DO 7 IRDP=1,M
            ICDP=0
            DO 13 I=1,IDP
            IA=I-1
            DO 14 J=1,IDP
            JA=J-1
            ICDP=ICDP+1
            A(IRDP,ICDP)=X(IRDP)**IA*Y(IRDP)**JA
   14        CONTINUE
   13      CONTINUE
    7       CONTINUE
            PRINT128
  128       FORMAT(//,5X,'VENDEMOND MATRIX')
            PRINT,'          '
            CALL MOUTD(A,IDA,M,L)
C
C     DETERMINE THE VALUES AT THE DISCRETE POINTS
C
            IF(NPP.EQ.0) GO TO 97
            DO 44 I=1,NPP
            READ(5,100)NGRID(I),ALAT(I),ALONG(I)
   44        CONTINUE
                 CALL CARTE(NPP,ALAT,ALONG,PHIO,ALONO,XP,YP)
            CALL VANDE(NPP,L,IDP,ALAT,ALONG,PHIO,ALONO,XP,YP,PM)
   97         CONTINUE
                 DO 15 ISIG=1,INDEX
            IF(ISIG.EQ.1)THEN DO
                 PRINT,'++  POLYNOMIAL APPROXIMATION FOR RANGE RATIO'
                 PRINT103
  103       FORMAT(4X,'******************************************' )
C
C     IF AMPLITUDE OF RANGE RATIO IS TO APPROXIMATED PROCEED,
C     OTHERWISE GO TO STATEMENT NO 46
C
            IF(IO.EQ.1)GO TO 46
C           FORMATION OF WEIGHT MATRIX
            DO 8 I=1,M
            DO 8 J=1,M
            P(I,J)=0.D0
    8       CONTINUE
            DO 9 I=1,M
            J=I
            P(I,J)=100.D0
    9       CONTINUE
C
C     PERFORM THE LEAST SQUARES APPROX BY CALLING THE SUBROUTINE APPROX
C
            CALL APPROX(L,M,A,P,PH,EN,C,V,AC,U,COVAR,APVF)
            PRINT105
  105       FORMAT(10X,'VECTOR OF COEFFICIENTS' )
            CALL MOUTD(C,IDA,L,1)
          PRINT106
  106       FORMAT(10X,'RESIDUAL',5X,'VECTOR' )
```

```
            CALL MOUTD(V,IDA,M,1)
            PRINT107,APVF
  107       FORMAT(10X,'A POSTERIORI VARIANCE FACTOR =',F10.6)
            PRINT108


  108       FORMAT(10X,'VARIANCE COVARIANCE MATRIX OF THE COEFFICIENTS')
            CALL MOUTD(COVAR,IDA,L,L)
            PRINT110
  110       FORMAT(//,5X,'POLYNOMIAL APPROX OF THE GIVEN FUNCTIONS')
            CALL MOUTD(AC,IDA,M,1)
            IF(IC.EQ.2)GO TO 51
   46       CONTINUE
            DO 57 I=1,M
            W(I)=100.0D0
   57       CONTINUE
C
C    PERFORM LEAST SQUARES APPROX USING ORTHOGONAL BASE FUNCTIONS
C     BY CALLING THE SUBROUTINE ORTHO
C
            CALL ORTHO(M,L,SIGMA,A,IDA,SIGMFC,VFC,NPC,ITEST,V,COVAR,FH,
     *      W,C,ALPHA,FC,SUMFC,SQN,STDP,IW)
            PRINT133
            DO 47 I=1,L
            PRINT134,FC(I),SUMFC(I)
   47       CONTINUE
            PRINT135
            DO 48 I=1,L
              PRINT136,C(I)
   48       CONTINUE
            PRINT,'NO. OF COEFF.OF ORIGN.POLY.AFTER TEST =',NPC
            PRINT137
            CALL MOUTD(COVAR,IDA,L,L)
            PRINT138
            DO 49 I=1,M
            PRINT136,V(I)
   49       CONTINUE
            PRINT,'    A POSTEORI VARIANCE FACTOR=   ',VFC
   51       CONTINUE
C
C     IF PREDICTION IS REQUIRED CALL THE SUBROUTINE PRED
C
            IF(NPP.EQ.0) GO TO 98
                CALL PRED(NPP,L,PM,C,COVAR,PMT,PFH,PMCO,VAR)
                ELSE DO
C
C
C     IF APPROX IS FOR TIME LAG (PHASE LAG)PROCEED
C
            PRINT,'++  POLYNOMIAL PROXIMATION FOR TIME LAG'
            PRINT103
            CK=0.5*PI/180.
            RO=25.10/3.2808
            ER=0.125*PI/180.
            ERR=0.10
            DO 60 I=1,M
            PH=FC(I)*CK
            R=RO*FH(I)/2.
            FA(I)=R*DCOS(PH)
            FB(I)=R*DSIN(PH)
            WA(I)=1./ERR**2
```

```
            WB(I)=1./ERR**2
C
C     FOR ORTHOGONAL BASE FUNCTIONS STAEMENT 52 IS EXCUTED
C
   60       CONTINUE


C
            IF(IC.EQ.1)GO TO 52
                DO 17 I=1,M
                DO 17 J=1,M
                P(I,J)=0.D0
   17           CONTINUE
               DO 18 I=1,M
                J=I
            P(I,J)=WA(I)
   18           CONTINUE
            PRINT140
            CALL APPROX(L,M,A,P,FA,EN,CA,V,AC,U,COA,APVF)
            PRINT105
            CALL MOUTD(CA,IDA,L,1)
            PRINT106
            CALL MOUTD(V,IDA,M,1)
            PRINT107,APVF
            DO 16 I=1,M
            DO 16 J=1,M
            P(I,J)=0.0D0
   16       CONTINUE
            DO 19 I=1,M
            J=I
            P(I,J)=WB(I)
   19        CONTINUE
            PRINT139
            CALL APPROX(L,M,A,P,FB,EN,CB,V,AC,U,COB,APVF)
            PRINT105
            CALL MOUTD(CB,IDA,L,1)
            PRINT106
            CALL MOUTD(V,IDA,M,1)
            PRINT107,APVF
            IF(IC.EQ.2)GO TO 56
   52        CONTINUE
C     FIND POLYNOMIAL APPROX. FOR FUNCTIONS FA,FB
C
            PRINT140
  140       FORMAT(//,5X,'POLYNOMIAL APPROXIMATION FOR FUNCTION  FA')
            CALL ORTHO(M,L,SIGMA,A,IDA,SIGMFC,VFC,NPC,ITEST,V,COA,FA,WA,CA,
     *      ALPHA,FC,SUMFC,SGN,STDP,IW)
            PRINT133
             DO 53 I=1,L
            PRINT134,FC(I),SUMFC(I)
   53       CONTINUE
             PRINT135
            DO 54 I=1,L
            PRINT136,CA(I)
   54       CONTINUE
            PRINT,'NO. OF COEFF.OF ORIGN.POLY.AFTER TEST =',NPC
            PRINT137
            CALL MOUTD(COA,IDA,L,L)
            PRINT138
            DO 55 I=1,M
            PRINT136,V(I)
```

```
   55      CONTINUE
           PRINT,'A POSTERORI VARIANCE FACTOR=',VFC
C
           PRINT139
  139      FORMAT(//,5X,'POLYNOMIAL APPROXIMATION FOR FUNCTION  FB')
C


           CALL LRTFO(M,L,SIGMA,A,IDA,SIGMFC,VFC,NPC,ITEST,V,COB,FB,WB,CB,
     *     ALPHA,FC,SUMFC,SCN,STOP,IW)
           PRINT133
           DO 61 I=1,L
           PRINT134,FC(I),SUMFC(I)
   61      CONTINUE
           PRINT135
           DO 62 I=1,L
           PRINT136,CB(I)
   62      CONTINUE
           PRINT,'NO OF POLY.AFTER TEST=',NPC
           PRINT137
           CALL MOUTD(CCB,IDA,L,L)
           PRINT138
           DO 63 I=1,M
           PRINT136,V(I)
   63      CONTINUE
           PRINT,'A POSTERORI VARIANCE FACTOR=',VFC
C
   56      CONTINUE
C
C     IF THERE IS NO PREDICTION STATEMENT NO 98 IS EXCUTED
C
           IF(NPP.EQ.0) GO TO 98
C     PREDICTION OF FUNCTIONS FA,FB AT GRID POINTS
           CALL PRED(NPP,L,FM,CA,CCA,FMT,PFA,PMCC,VARPA)
C
           CALL PRED(NPP,L,FM,CB,CCB,FMT,PFB,PMCC,VARPB)
C
C      COMPUTE THE PREDICTED TIME LAG AND ASSOCIATED VARIANCES
           DO 64 I=1,NPP
           PFG(I)=DATAN(PFB(I)/PFA(I))/CK
           B(1,1)=(1./(1.+(PFB(I)/PFA(I))**2))*(-PFB(I)/PFA(I)**2)
           B(1,2)=(1./(1.+(PFB(I)/PFA(I))**2))*(1./PFA(I))
C     COMPUTE VARIANCES
           VARP(I)=B(1,1)**2*VARPA(I,I)+B(1,2)**2*VARPB(I,I)
           SIGMAF(I)=DSQRT(VARP(I))/CK
   64      CONTINUE
            END IF
   98        CONTINUE
  15       CONTINUE
           PRINT,'*** PREDICTION      MATRIX  ****'
           CALL MOUTD(FM,IDA,NPP,L)
           PRINT126
           PRINT127
  126      FORMAT(//,5X,'CARTESIAN COORDS. OF GRID POINTS')
           DO 50 I=1,NPP
           PRINT111,NGRID(I),XP(I),YP(I)
   50       CONTINUE
             PRINT125
  125      FORMAT('1',//,5X,'PREDICTED RANGE RATIOS AND TIME LAGS AT THE G
     *RID POINTS')
           PRINT141
```

```
                DO 45 I=1,NPP
            ALAT(I)=ALAT(I)*180./PI
            ALONG(I)=ALONG(I)*180./PI
            SIGMAR(I)=DSQRT(VAR(I,I))
            PRINT142,NGRID(I),ALAT(I),ALONG(I),PRH(I),SIGMAR(I),PRG(I),
     *      SIGMAP(I)
   45           CONTINUE

            DO 66 I=1,L
            J=I
            WRITE(7,145)C(I),COVAR(I,J)
   66        CONTINUE
            DO 67 I=1,L
            J=I
            WRITE(7,145)A(I),COA(I,J)
   67        CONTINUE
            DO 68 I=1,L
            J=I
            WRITE(7,145)B(I),COB(I,J)
   68        CONTINUE
  133       FORMAT(//,5X,'FOURIER COEFFICIENT',10X,'VARIANCES')
  134       FORMAT(/,8X,E11.4,10X,E11.4)
  135       FORMAT(//,5X,'VECTOR OF ORIGINAL COEFFICIENTS')
  136       FORMAT(/,5X,E11.4)
  137       FORMAT(//,5X,'VARIANCE-COVARIANCE MATRIX OF THE COEFF.')
  138       FORMAT(//,5X,'VECTOR OF RESIDUALS')
  141       FORMAT(//,4X,'NO',8X,'LATITUDE',8X,'LONGITUDE',10X,'RANGE RATIO
     *      ',4X,'SIGMA',7X,'TIME LAG',5X,'SIGMA')
  142       FORMAT(/,3X,I4,5X,F10.6,6X,F10.6,6X,F10.6,6X,E11.4,6X,F10.6,
     *      6X,E11.4)
  145       FORMAT(5X,2E11.4)
            STOP
            END
C
            SUBROUTINE CARTE(M,ALAT,ALON,PHIO,ALONO,X,Y)
C
C     THIS SUBROUTINE COMPUTES THE CARTESIAN COORDS. X,Y, FROM GEOGRAPHICAL
C   COORDS.,LATITUDE AND LONGITUDE,.
C
C
            IMPLICIT REAL*8(A-H,O-Z)
            DIMENSION ALAT(50),ALON(50),X(50),Y(50)
            RA=6378206.4D0
            RB=6356583.8D0
            PI=3.14159265D0
            EC=(RA**2-RB**2)/RA**2
                PHIR=PHIO*PI/180.
                ALONR=ALONO*PI/180.
            XM=((1.-EC)*RA)/(DSQRT(1.-EC*(DSIN(PHIR)**2))**3)
            XN=RA/DSQRT(1.-EC*(DSIN(PHIR)**2))
            R=DSQRT(XM*XN)
             DO 4 I=1,M
            ALAT(I)=ALAT(I)*PI/180.
            ALON(I)=ALON(I)*PI/180.
                X(I)=R*(ALAT(I)-PHIR)
                Y(I)=R*DCOS(PHIR)*(ALON(I)-ALONR)
    4       CONTINUE
              RETURN
              END
```

```
C
      SUBROUTINE VANDL(NPP,L,IDP,ALAT,ALONG,PHIO,ALONO,XP,YP,PM)
C   THIS SUBROUTINE COMPUTES THE PREDICTION MATRIX PM.
C
CC
         IMPLICIT REAL*8(A-H,O-Z)
         DIMENSION PM(50,50),ALAT(50),ALONG(50),XP(50),YP(50)

         DO 40 I=1,NPP
         DO 40 J=1,L
         PM(I,J)=0.0D0
 40      CONTINUE
         DO 41 I=1,NPP
         ICDP=0
         DO 42 K=1,IDP
         KA=K-1
         DO 43 J=1,IDP
         JA=J-1
         ICDP=ICDP+1
         PM(I,ICDP)=XP(I)**KA*YP(I)**JA
  43      CONTINUE
  42      CONTINUE
  41       CONTINUE
          RETURN
            END
C
         SUBROUTINE PRED(NPP,L,PM,C,COVAR,PMT,PF,PMCO,VAR)
C . THIS SUBROUTINE PREDICTS THE FUNCTION VALUES AT THE GRID POINTS
C   AND COMPUTES THE PREDICTION VARIANCE COVARIANCE MATRIX.
C
C
      IMPLICIT REAL*8(A-H,O-Z)
         DIMENSION PM(50,50),C(50),PF(50),PMCO(50,50),COVAR(50,50)
   *  ,VAR(50,50),PMT(50,50)
         IRDA=IDA=IDB=IDC=50
         CALL MMULD(PF,IDC,PM,IDA,C,IDB,NPP,L,1)
         CALL TRNSD(PMT,IDE,PM,IDA,NPP,L)
         CALL MMULD(PMCO,IDC,PM,IDA,COVAR,IDB,NPP,L,L)
         CALL MMULD(VAR,IDC,PMCO,IDA,PMT,IDB,NPP,L,NPP)
      RETURN
      END

      SUBROUTINE APPROX(L,M,A,P,F,EN,C,V,AC,U,COVAR,APVF)
C     ***************************************************************
C     *     THIS SUBROUTINE DOES THE LEAST SQUARES APPROXIMATION    *
C     *        OF THE GIVEN FUNCTIONS AND RETURNS THE VECTOR OF COEF *
C     *   FICIENTS TOGETHER WITH THE VAR. COVAE MATRIX              *
C     ***************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(50,50),P(50,50),EN(50,50),AC(50),C(50),F(50),
   *  COVAR(50,50),ATP(50,50),AT(50,50),VTP(1,50),VF(1),U(50),
   *  IW1(50),IW2(50),VT(1,50),V(50,1)
        IRDA=IDA=IDB=IDC=50
      CALL TRNSD(AT,IDE,A,IDA,M,L)
      CALL MMULD(ATP,IDC,AT,IDA,P,IDB,L,M,M)
      CALL MMULD(EN,IDC,ATP,IDA,A,IDB,L,M,L)
```

```
            DO 5  I=1,L
            DO 5  J=1,L
            EN(I,J)=EN(I,J)*1.0D-20
  5         CONTINUE
      CALL  MINVD(EN,IRDA,L,DETA,IW1,IW2)
            DO 6  I=1,L
            DO 6  J=1,L
            EN(I,J)=EN(I,J)*1.0D-20
  6         CONTINUE

      CALL  MMULD(C,IDC,EN,IDA,U,IDB,L,L,1)
C     COMPUTE RESIDUALS
      CALL  MMULD(AC,IDC,A,IDA,C,IDC,M,L,1)
      CALL  MSUBD(V,IDC,AC,IDA,F,IDC,M,1)
C   COMPUTE A POSTERIORI VARIANCE FACTOR
      CALL  TRNSD(VT,1,V,IDA,M,1)
      CALL  MMULD(VTP,1,VT,1,P,IDB,1,M,M)
      CALL  MMULD(VP,1,VTP,1,V,IDB,1,M,1)
      IDF=M-L
      APVF=VP(1)/IDF
C     COMPUTE VARIANCE COVARIANCE MATRIX OF COEFFICIENT
      DO 10 I=1,L
      DO 10 J=1,L
      COVAR(I,J)=APVF*EN(I,J)
 10   CONTINUE
 16   RETURN
      END
C

      SUBROUTINE CHOLD(A,IRDA,NA,DETA,*)
C
C     THE USE OF THIS SUBROUTINE IS OPTIONAL
C MATRIX INVERSION USING CHOLESKI DECOMPOSITION
C
C INPUT ARGUMENTS
C A = ARRAY CONTAINING POSITIVE DEFINITE SYMMETRIC INPUT MATRIX
C IRDA = ROW DIMENSION OF ARRAY CONTAINING INPUT MATRIX
C NA = SIZE OF INPUT MATRIX
C OUTPUT ARGUMENTS
C DETA = DETERMINANT OF INPUT MATRIX
C A = CONTAINS INVERSE OF INPUT MATRIX (INPUT DESTROYED)
C * = ERROR RETURN (TAKEN IF NA .LT. 1 OR IF DETA .LT. SING)
C
      DOUBLE PRECISION A,DETA,SUM,SQRT,DSQRT,ABS,DABS,SING
      DIMENSION A(IRDA,NA)
      SQRT(SUM)=DSQRT(SUM)
      ABS(DETA) = DABS(DETA)
      DATA SING/1D-50/
C CHOLESKI DECOMPOSITION OF INPUT MATRIX INTO TRIANGULAR MATRIX
      IF(NA .LT. 1) GO TO 18
      DETA = A(1,1)
      A(1,1) = SQRT(A(1,1))
      IF(NA .EQ. 1) GO TO 6
      DO 1 I = 2,NA
    1   A(I,1) = A(I,1) / A(1,1)
      DO 5 J = 2,NA
        SUM = 0.
        J1 = J - 1
        DO 2 K = 1,J1
    2     SUM = SUM + A(J,K) ** 2
```

```
          A(J,J) = SQRT(A(J,J) - SUM)                                   00007570
          DETA = DETA * (A(J,J) - SUM)                                  00007560
          IF(J .EQ. NA) GO TO 5                                         00007580
          J2 = J + 1                                                    00007590
          DO 4 I = J2,NA                                                00007600
            SUM = 0.                                                    00007610
            DO 3 K = 1,J1                                               00007620
    3         SUM = SUM + A(I,K) * A(J,K)                               00007630
    4       A(I,J) = (A(I,J) - SUM) / A(J,J)                            00007640
    5     CONTINUE                                                      00007650

    6 IF(ABS(DETA) .LT. SING) GO TO 16                                  00007660
C INVERSION OF LOWER TRIANGULAR MATRIX                                  00007670
      DO 7 I = 1,NA                                                     00007680
        A(I,I) = 1. / A(I,I)                                            00007690
    7 CONTINUE
      IF(NA .EQ. 1) GO TO 10                                            00007700
      N1 = NA - 1                                                       00007710
      DO 9 J = 1,N1                                                     00007720
        J2 = J + 1                                                      00007730
        DO 9 I = J2,NA                                                  00007740
          SUM = 0.                                                      00007750
          I1 = I - 1                                                    00007760
          DO 8 K = J,I1                                                 00007770
    8       SUM = SUM + A(I,K) * A(K,J)                                 00007780
    9     A(I,J) = - A(I,I) * SUM                                       00007790
C CONSTRUCTION OF INVERSE OF INPUT MATRIX                               00007800
   10 DO 15 J = 1,NA                                                    00007810
        IF(J .EQ. 1) GO TO 12                                           00007820
        J1 = J - 1                                                      00007830
        DO 11 I = 1,J1                                                  00007840
   11     A(I,J) = A(J,I)                                               00007850
   12   DO 14 I = J,NA                                                  00007860
          SUM = 0.                                                      00007870
          DO 13 K = I,NA                                                00007880
   13       SUM = SUM + A(K,I) * A(K,J)                                 00007890
   14     A(I,J) = SUM                                                  00007900
   15   CONTINUE                                                        00007910
      RETURN                                                            00007920
   16 WRITE(6,17) DETA                                                  00007930
   17 FORMAT(10X, 'SINGULAR MATRIX IN CHOLD. DET =',E20.5)              00007940
      RETURN 1                                                          00007950
   18 WRITE(6,19)                                                       00007960
   19 FORMAT(10X,'MATRIX OF DIMENSION ZERO IN CHOLD')                   00007970
      RETURN 1                                                          00007980
      END                                                               00007990

      SUBROUTINE ORTHO(N,M,SIGMA,PHI,MRO,SIGMAF,VFC,NPC,INDEX,V,SUMD,F,W00007370
     &, ,ALPHA,      C,SUMC,SC2,STOP,IW)                                00007380
C THIS SUBROUTINE ORTHOGONALIZES THE MATRIX PHI USING THE GRAM-SCHMIDT  00007390
C METHOD, COMPUTES THE FOURIER COEFFICIENTS OF THE ORTHOGONALIZED MATRIX00007400
C DERIVES THE COEFFICIENTS OF PHI,COMPUTES THE VARIANCES OF THE FOURIER 00007410
C COEFFICIENTS AND THE VARIANCE-COVARIANCE MATRIX OF THE COEFFICIENTS   00007420
C INPUTS :                                                              00007430
C    1. PHI(OPTIONAL - COULD BE FUNCTION SUBPROGRAM INSTEAD) - AN N BY M 00007440
C       CONTAINING THE BASE FUNCTIONS EVALUATED FOR EACH OBSERVATION    00007450
C    2. N - THE NUMBER OF OBSERVATIONS                                  00007460
C    3. M - THE NUMBER OF BASE FUNCTIONS (EQUAL OR GREATER THAN 2)      00007470
C    4. W - A VECTOR OF LENGTH N CONTAINING THE COMPUTED WEIGHT FUNCTIONS00007480
C    5. F - FUNCTIONAL VALUES                                           00007490
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C     6. SIGMA - THE A PRIORI VARIANCE FACTOR                            00007500
C     7. NRO - THE MAXIMUM ROW DIMENSION OF PHI                          00007510
C     8. MRO - THE MAXIMUM COLUMN DIMENSION OF PHI                       00007520
C     9. INDEX - PERMITS OPTIONAL TEST FOR STATISTICAL SIGNIFICANCE      00007530
C                OF FOURIER COEFFICIENTS....                             00007540
C                IF 0,STATISTICAL TEST FOR FOURIER COEFFICIENTS ABANDONED 00007550
C                IF 1,TESTS AGAINST ONE TIME ITS STANDARD DEVIATION      00007560
C                IF 2,TESTS AGAINST TWICE ITS ST.DEVIATION               00007570
C                IF 3,TESTS AGAINST THREE TIMES ITS ST.DEVIATION         00007580
      IMPLICIT REAL*8(A-H,O-Z)                                           00007750
C    10. IW - WRITE CODE OF THE COMPUTER                                 00007590


C OUTPUTS :                                                              00007600
C     1. ALPHA - AN MRC BY M MATRIX CONTAINING THE ALPHA'S USED IN COMPUTI00007610
C        THE ORTHOGONALIZED MATRIX AND IN COMPUTING THE COEFFICIENTS OF PH00007620
C     2. C - THE M FOURIER COEFFICIENTS OF THE ORTHOGONALIZED MATRIX     00007630
C     3. D - THE M COEFFICIENTS OF THE INPUT MATRIX PHI                  00007640
C     4. SUMC - THE VARIANCES OF THE FOURIER COEFFICIENTS                00007650
C     5. SUMD - THE VARIANCE-COVARIANCE MATRIX OF THE COEFFICIENTS       00007660
C     6. SC2 - THE SQUARES OF THE NORMS OF THE ORTHOGONALIZED MATRIX     00007670
C     7. SIGMAF - THE FOURIER POLYNOMIAL A POSTEORI VARIANCE FACTOR      00007680
C     8. V - THE N RESIDUALS                                             00007690
C     9. VFC - THE ORIGINAL POLYNOMIAL A POSTEORI VARIANCE FACTOR        00007700
C    10. NPC - NUMBER OF THE COEFFICIENTS OF THE ORIGINAL POLYNOMIAL     00007710
C              AFTER THE STATISTICAL TEST IS PERFORMED                   00007720
C    11. STDP - VECTOR AGAINST WHICH THE ABSOLUTE VALUES OF FOURIER      00007730
C               COEFFICIENTS ARE TESTED                                  00007740
         DIMENSION ALPHA(50,50),W(50),F(50),C(50),D(50)
         DIMENSION SUMD(50,50),SUMC(50),SC2(M),V(50),STDP(50),
     *   PHI(50,50)
C TEST FOR NEGATIVE DEGREES OF FREEDOM                                   00007800
      IF (N.LT.M) GO TO 100                                              00007810
      K=1                                                                00007820
      ALPHA(M,M)=1.D0                                                    00007830
C DETERMINE THE ALPHA'S FOR COMPUTATION OF ORTHOGONALIZED MATRIX         00007840
   10 DO 3 J=K,M                                                         00007850
      IF(J.NE.K) GO TO 6                                                 00007860
      ALPHA(K,K)=1.D0                                                    00007870
      GO TO 3                                                            00007880
    6 SC1=0.D0                                                           00007890
      SC2(K)=0.D0                                                        00007900
      SC3=0.D0                                                           00007910
      DO 2 I=1,N                                                         00007920
      P=PHI(I,K)                                                         00007940
      IF(K.EQ.1) GO TO 4                                                 00007950
      K1=K-1                                                             00007960
      DO 5 J1=1,K1                                                       00007970
    5 P=P+ALPHA(J1,K)*PHI(I,J1)                                          00007980
    4 SC1=SC1+W(I)*PHI(I,J)*P                                            00007990
      SC3=SC3+F(I)*W(I)*P                                                00008000
    2 SC2(K)=SC2(K)+W(I)*P**2                                            00008010
      ALPHA(J,K)=-SC1/SC2(K)                                             00008020
      ALPHA(K,J)=ALPHA(J,K)                                              00008030
    3 CONTINUE                                                           00008040
C DETERMINE THE FOURIER COEFFICIENTS FOR THE ORTHOGONALIZED MATRIX       00008050
      C(K)=SC3/SC2(K)                                                    00008060
      K=K+1                                                              00008070
      IF(M.EQ.2) GO TO 34                                                00008080
      IF(K.LT.3) GO TO 10                                                00008090
C DETERMINE THE ALPHA'S USED IN COMPUTING THE COEFFICIENTS OF PHI        00008100
```

```
      JK=K-1                                                            00008110
    9 JL=K                                                              00008120
      JK=JK-1                                                           00008130
      JJ=K-JK-1                                                         00008140
      DO 8 LM=1,JJ                                                      00008150
      JL=JL-1                                                           00008160
    8 ALPHA(JK,K)=ALPHA(JK,K)+ALPHA(JK,JL)*ALPHA(K,JL)                   00008170
      IF(JK.NE.1) GO TO 9                                               00008180
      IF(K.LT.M) GO TO 10                                               00008190
C DETERMINE THE LAST FOURIER COEFFICIENT                                00008200
   34 SC2(K)=0.D0                                                       00008210
      SC3=0.D0                                                          00008220

      DO 7 I=1,N                                                        00008230
      P=PHI(I,K)                                                        00008250
      K1=K-1                                                            00008260
      DO 1 J=1,K1                                                       00008270
    1 P=P+ALPHA(J,K)*PHI(I,J)                                           00008280
      SC2(K)=SC2(K)+W(I)*P**2                                           00008290
    7 SC3=SC3+F(I)*W(I)*P                                               00008300
      C(K)=SC3/SC2(K)                                                   00008310
C DETERMINE THE COEFFICIENTS OF PHI                                     00008320
      IDEKT=1                                                           00008330
      ICOUNT=0                                                          00008340
 1000 CONTINUE                                                          00008350
      DO 13 I=1,M                                                       00008360
      D(I)=C(I)                                                         00008370
      IF(I.EQ.M) GO TO 13                                               00008380
      II=I+1                                                            00008390
      DO 14 J=II,M                                                      00008400
   14 D(I)=D(I)+ALPHA(I,J)*C(J)                                         00008410
   13 CONTINUE                                                          00008420
C COMPUTE THE VARIANCE OF THE FOURIER COEFFICIENTS AND THE VARIANCE-COVA00008430
C MATRIX OF THE COEFFICIENTS                                            00008440
      DO 15 I=1,M                                                       00008450
      DO 15 J=1,M                                                       00008460
   15 SUMD(I,J)=0.D0                                                    00008470
      SC4=0.D0                                                          00008480
      DO 22 I=1,N                                                       00008490
      FN=0.D0                                                           00008510
      DO 21 J=1,M                                                       00008520
   21 FN=FN+D(J)*PHI(I,J)                                               00008530
      V(I)=F(I)-FN                                                      00008540
      V2=V(I)**2                                                        00008550
   22 SC4=SC4+V2*W(I)                                                   00008560
      SIGMAF=SC4/(N-M+ICOUNT)*SIGMA                                     00008570
      VFC=SIGMAF                                                        00008580
      IF(IDEKT.EQ.2) VFC=SC4/(N-NPC)*SIGMA                              00008590
      IF(INDEX.EQ.0) NPC=M                                              00008600
      DO 28 I=1,M                                                       00008610
      SUMC(I)=SIGMAF/SC2(I)
      IF(IDEKT.EQ.1) GO TO 28                                           00008630
      IF(C(I).EQ.0D0) SUMC(I)=0D0                                       00008640
   28 CONTINUE                                                          00008650
      DO 23 I=1,M                                                       00008660
      DO 23 J=1,I                                                       00008670
      DO 23 K=J,I                                                       00008680
   23 SUMD(J,K)=SUMD(J,K)+ALPHA(J,I)*ALPHA(K,I)*SUMC(I)                 00008690
      DO 24 I=1,M                                                       00008700
      II=I+1                                                            00008710
```

```
      IF(II.GT.M) GO TO 30
      DO 24 J=II,M
   24 SUMD(J,I)=SUMC(I,J)
   30 CONTINUE
C OPTIONAL CHECK FOR STATISTICALLY SIGNIFICANT FOURIER COEFFICIENTS
      IF(INDEX.EQ.0) GO TO 40
      IF(IDEKT.EQ.2) GO TO 40
      PINDEX=DFLOAT(INDEX)
      DO 31 I=1,M
      STDP(I)=FINDEX*DSQRT(SUMC(I))
      IF(DABS(C(I)).LT.STDP(I)) GO TO 32
      GO TO 31
   32 C(I)=CDC

      ICOUNT=ICOUNT+1
      SUMC(I)=CDC
   31 CONTINUE
      NFC=0
      DO 33 I=1,M
      IF(C(I).NE.CDO) NFC=I
   33 CONTINUE
      IDEKT=2
      GO TO 1000
   40 RETURN
 1000 WRITE(IW,102)
  102 FORMAT('0','*ERROR*  NEGATIVE DEGREES OF FREEDOM')
      RETURN
      END
```

$GO

NO	LATITUDE	LONGITUDE	RANGE RATIO	TIME LAG
1000	45.083330	-65.583330	0.000000	0.000000
65	45.266670	-66.066670	1.000000	0.000000
1	44.600000	-66.800000	0.661400	-28.500000
15	44.883330	-66.950000	0.673300	5.000000
40	45.066670	-67.050000	0.900400	15.500000
60	45.233330	-66.050000	0.996000	-10.000000
120	45.350000	-65.533330	1.201200	9.000000
140	45.566670	-64.966670	1.324700	19.000000
150	45.600000	-64.783330	1.410400	17.000000
225	45.466670	-64.850000	1.316700	11.000000
235	45.350000	-64.816670	1.310700	-1.000000
240	45.300000	-64.783330	1.456200	16.500000
305	45.233330	-64.516670	1.490000	12.000000
312	45.250000	-65.000000	1.630600	0.000000
315	45.050000	-65.066670	1.264900	12.000000
320	44.[illegible]0000	-65.533330	1.059800	-14.000000
330	44.[illegible]000	-65.833330	0.956200	-15.500000
335	44.500000	-66.100000	0.842600	-18.000000
336	44.[illegible]83330	-66.0[illegible]3330	0.761000	-37.000000
340	44.266670	-66.3[illegible]0000	0.721100	-34.000000
345	44.250000	-66.400000	0.713100	-34.000000
353	44.333330	-66.116670	0.721100	18.000000
355	44.200000	-66.166670	0.673300	18.000000

DEG.OF POLY. NO.OF OBS. NO.OF COEFF. DEG.OF FREEDOM

II.2 Least Squares Polynomial Approximation of Observed Time Series

Figure A-2 is the flow chart describing the program. The program uses any number of constituent frequencies - ICON and the required number of coefficients is computed from

$$U = 2*ICON + 1 .$$

INPUTS

1st card: FORMAT Free, contains the following:

M - number of observations

ICON - number of constituent frequencies

ITEST - code for testing Fourier Coefficients

0 - for no test

1 - test against its Standard deviation

2 - test against 2 times its Standard deviation

3 - test against 3 times its standard deviation

ICON cards: FORMAT(10X, F15.6)

Each card contains one constituent frequency.

ICON cards: FORMAT(5X, F10.6, 5X, F10.6)

Each card contains the nodal (modulation) factor and the astronomical arguments required if harmonic constants are to be computed. If harmonic constants are not being computed, these cards should be omitted.

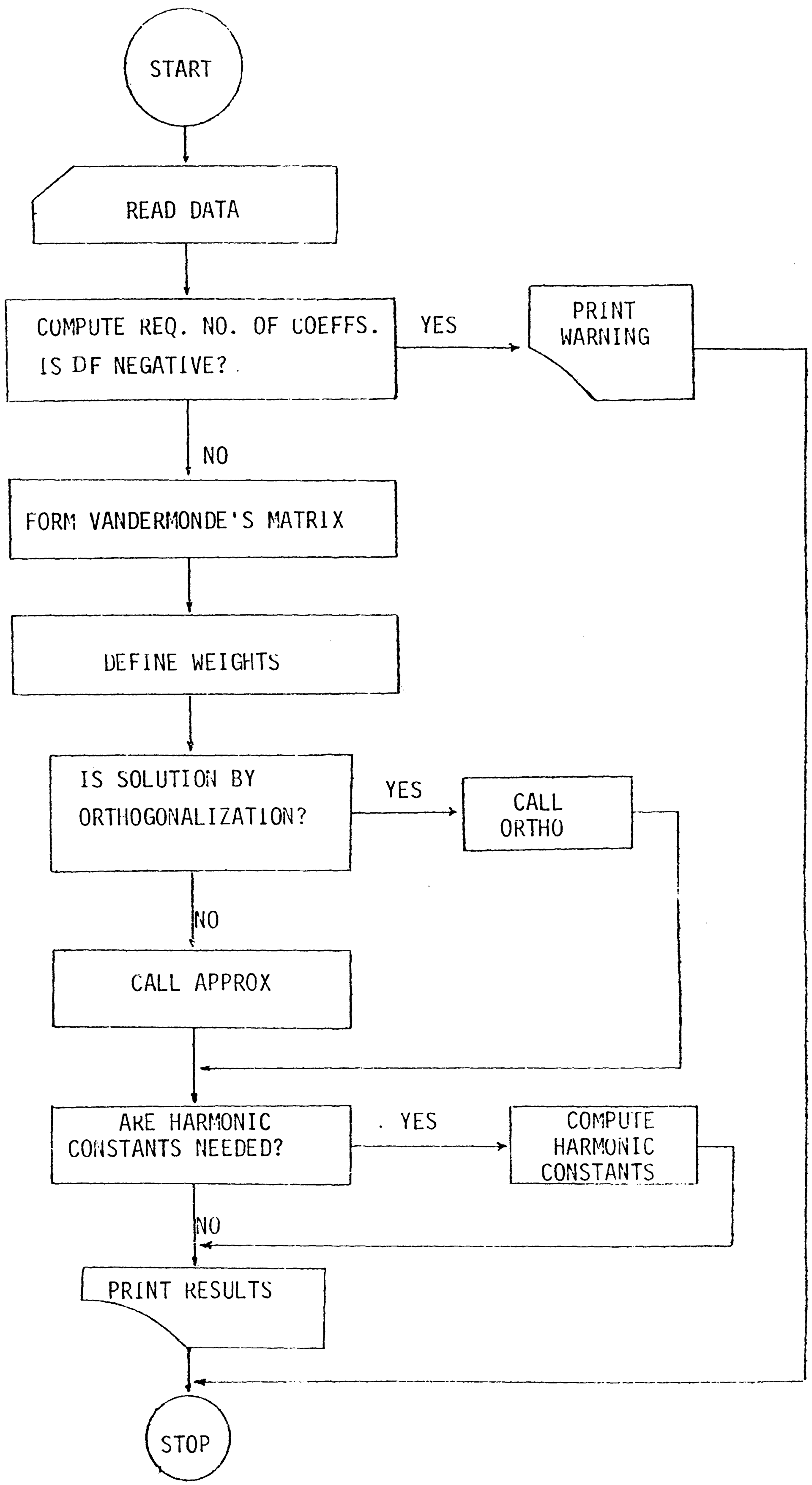
M cards: FORMAT(5X, F10.3, 5X, F10.3),

each card contains the observed height and the time of observation.

SUBROUTINES:

The Subroutines used are APPROX and ORTHO as in the previous case.

Figure II-2 Polynomial Approximation of Observed Time Series - Flow Chart



```
**************************************************************************************************
$JOB CKLNWA/G
      C    *****************************************************************************
      C    *                                                                           *
      C    *   PROGRAM TO APPROX. THE OBSERVED TIME SERIES (HEIGHTS OF TIDE) BY        *
      C    *   TRIGONOMETRIC POLYNOMIAL. THE PROGRAME USES ANY NUMBER OF               *
      C    *   CONSTITUENT FREQUENCIES-ICON-   THE NUMBER OF COEFFICIENT -L   IS       *
      C    *   COMPUTED FROM THE NUMBER OF CONSTITUENTS.                               *
      C    *  INPUTS:                                                                  *
      C    *     PROGRAM SPECIFICATION,
      C    *        M = NUMBER OF OBSERVED HEIGHTS                                     *
      C    *        ICON = NUMBER OF CONSTITUENT FREQUENCIES BEING USED                *
      C    *        ITEST = CODE FOR TESTING FOURIER COEFFICIENTS                      *
      C    *           0 _ FOR NO TEST                                                 *
      C    *           1 _ FOR TEST AGAINST STANDARD DEVIATION                         *
      C    *           2 _ TEST AGAINST 2 TIMES THE STANDARD DEVIATION                 *
      C    *           3 _ TEST AGAINST 3 TIMES THE STANDARD DEVIATION                 *
      C    *        IC = CODE FOR METHOD OF APPROXMATION                               *
      C    *           1 _ FOR APPROXMATION USING ORTHOGONALISED BAES FUNCTIONS        *
      C    *           2 _ FOR APPROXIMATION USING LEAST SQUARES METHOD WITH NON       *
      C    *                ORTHOGONAL BASE FUNCTONS                                   *
      C    *        NHC = CODE FOR THE COMPUTATION OF HARMONIC CONSTANTS               *
      C    *           0 _ NO HARMONIC CONSTANT NEED BE COMPUTED                       *
      C    *           1 _ COMPUTE THE HARMONIC CONSTANTS                              *
      C    *     DATA,                                                                 *
      C    *        W = CONSTITUENT FREQUENCY                                          *
      C    *        FH = OBSERVED HEIGHT IN METRES                                     *
      C    *        T = OBSERVED TIME IN HOURS                                         *
      C    *        FK = NODAL  FACTOR                                                 *
      C    *        VK = ASTRONOMICAL ARGUMENT                                         *
      C    *                                                                           *
      C    *                                                                           *
      C    *****************************************************************************
      C
  1           IMPLICIT REAL*8(A-H,O-Z)
  2           DIMENSION FH(60),T(60),A(60,60),W(60),P(60,60),EN(60,60)
           *  ,C(60),V(60),AC(60),COVAR(60,60),VAR(60,60),VT(60),VTP(60),
           *  IW1(60),IW2(60),AT(60,60),ATP(60,60),U(60)
  3           DIMENSION PHASE(20),HK(20),VK(20),VKR(20),XKAPA(20),FK(20)
           *  ,SIGMA(20),E(20),PHI(30)
  4           DIMENSION ALPHA(60,60),WT(60),SUMFC(60),STDP(60),SC2(60),
           *  FC(60),BP(2,2),BA(2,2),SIGMAA(60),SIGMAF(60)
  5           IRDA=IDA=IDE=IDC=60
  6           PI=3.141592650D0
      C
      C     READ IN COMPUTATION SPCIFICATION
      C
  7           READ,M,ICON,ITEST,IC,NHC
      C
      C     COMPUTE THE NUMBER OF COEFFICIENT AND THE DEGREE OF FREEDOM
      C
  8           L=2*ICON+1
  9           IDF=M-L
 10           APW=1.0
 11           IW=0
      C
      C     READ IN THE CONSTITUENT FREQUENCIES
      C
```

```
                DO 20 I=1,ICON
                READ(5,112)W(I)
      112       FORMAT(10X,F15.6)
       20       CONTINUE
                DO 32 I=1,ICON
                READ(5,121)FK(I),VK(I)
      121   FORMAT(5X,F10.6,5X,F10.6)
                VKR(I)=VK(I)*PI/180.
       32       CONTINUE
C
C       READ IN THE OBSERVED TIME AND HEIGHT. NOTE THE ORIGIN FOR TIME
C       CAN BE TAKEN AS ZERRO HOUR OF THEFIRST DAY OF OBSERVATION--
C       THE TIME IS IN HOURS.
C
                DO 21 I=1,M
                READ(5,113)HH(I),T(I)
      113   FORMAT(5X,F10.4,5X,F10.3)
       21       CONTINUE
            ERROR=0.05
C
C       PRINT OUT THE INPUT DATA
C
            PRINT,'***  CONSTITUENT  FRECUENCIES ******'
            DO 22 I=1,ICON
            PRINT112,W(I)
       22   CONTINUE
            PRINT114
      114   FORMAT(//,5X,'HEIGHT OBSERVED',5X,'TIME OF OBS.')
            DO 23 I=1,M
            PRINT124,HH(I),T(I)
      124   FORMAT(5X,F10.4,10X,F10.3)
       23   CONTINUE
C
C       CONVERT FREQUENCIES TO RADIAN MEASURE
            DO 29 K=1,ICON
            W(K)=W(K)*PI/180.
       29   CONTINUE
C
C       FORMATION OF VANDEMOND MATRIX
C
                DO 24 I=1,M
                DO 24 J=1,L
                A(I,J)=0.0D0
       24       CONTINUE
                DO 25 I=1,M
                J=1
                A(I,J)=1.0D0
                K=0
                DO 27 J=2,L,2
                K=K+1
                A(I,J)=DCOS(W(K)*T(I))
       27        CONTINUE
                K=0
                DO 28 J=3,L,2
                K=K+1
                A(I,J)=DSIN(W(K)*T(I))
       28        CONTINUE
       25       CONTINUE
C
```

```
C   PRINT THE MATRIX  A
C
            PRINT,'***  VANDEMOND  MATRIX ***'

            CALL MOUTD(A,IDA,M,L)
C
               IF(IO.EQ.1)GO TO 35
C
C       FORMATION OF WEIGHT MATRIX
C
            DO 30 I=1,M
            DO 30 J=1,M
            P(I,J)=0.0D0
   30       CONTINUE
            DO 31 I=1,M
            J=I
          P(I,J)=1./ERROR**2
   31       CONTINUE
C
C       PERFORM THE LEAST SQUARES APPROX. BY CALLING THE    SUBROUTINE
C       APPROX
C
          CALL APPROX(L,M,A,P,PH,EN,C,V,AC,U,COVAR,APVF)
            PRINT116
  116       FORMAT(//,10X,'VECTOR OF COEFFICIENTS')
            CALL MOUTD(C,IDA,L,1)
            PRINT117
  117       FORMAT(//,10X,'RESIDUAL',10X,'VECTOR')
            CALL MOUTD(V,IDA,M,1)
            PRINT,'NUMBER OF DEGREES OF FREEDOM=',IDF
            PRINT118,APVF
  118       FORMAT(//,10X,'A POSTERIORI VARIANCE FACTOR =',F10.6)
            PRINT119
  119       FORMAT(//,10X,'VARIANCE COVARIANCE MATRIX OF THE COEFF. ')
            CALL MOUTD(COVAR,IDA,L,L)
               IF(IO.EQ.2)GO TO 40
   35          CONTINUE
               DO 36 I=1,M
          WT(I)=1./ERROR**2
   36          CONTINUE
          CALL CRTFC(M,L,APW,A,IDA,SIGM,VFC,NPC,ITEST,V,COVAR,PH,
     *    WT,C,ALPHA,FC,SUMFC,SC2,STDP,IW)
          PRINT125
          DO 37 I=1,L
          PRINT126,FC(I),SUMFC(I)
   37     CONTINUE
          PRINT,'FOURIER COEFF. -A POSTEORI V.F.= ',SIGM
          PRINT,'NO OF ORIGINAL COEFF. AFTER TEST= ',NPC
          PRINT127
               DO 38 I=1,L
               PRINT128,C(I)
   38          CONTINUE
          PRINT129
          CALL MOUTD(COVAR,IDA,L,L)
          PRINT130
               DO 39 I=1,M
               PRINT128,V(I)
   39          CONTINUE
          PRINT,' A POSTEORI V.F. ORIGINAL POLYNOMIAL =',VFC
```

```
C     PREDICTION OF HEIGHTS USING THE POLYNOMIAL FOR SOME TIME INTERVALS
C
      WRITE(6,132)
      WRITE(6,133)
       DO 45 I=1,50
      TP=I
      PHI(1)=1.0
      DO 46 K=1,ICON
      JC=2*K
      JS=2*K+1
      PHI(JC)=DCOS(W(K)*TP)
      PHI(JS)=DSIN(W(K)*TP)
   46 CONTINUE
      SUM=0
      SUMVA=0
      DO 47 IP=1,L
      PH=C(IP)*PHI(IP)
      PV=PHI(IP)**2*COVAR(IP,IP)
      SUM=SUM+PH
      SUMVA=SUMVA+PV
   47 CONTINUE
      STD=DSQRT(SUMVA)
      WRITE(6,131)TP,SUM,STD
   45 CONTINUE
      DO 48 I=1,L
      WRITE(7,42)C(I),COVAR(I,I)
   48 CONTINUE
   42 FORMAT(5X,E11.4,5X,E11.4)
  132 FORMAT('1',15X,'HOURLY PREDICTIONS IN THE TIME INTERVAL')
  133 FORMAT(//,5X,'TIME IN HOURS',5X,'PR. HEIGHTS',5X,'STD. ERR. OF
     *PR. HEIGHTS')
  131 FORMAT(/,8X,F6.2,9X,F6.3,12X,F10.5)
  125 FORMAT(//,5X,'FOURIER COEFFS.',10X,'VARIANCES')
  126 FORMAT(/,5X,E11.4,10X,E11.4)
  127 FORMAT(//,5X,'VECTOR OF ORIGINAL COEFFICIENTS')
  128 FORMAT(/,5X,E11.4)
  129 FORMAT(//,5X,'VARIANCE COVARIANCE MATRIX OF ORIGINAL COEFF.')
  130 FORMAT(//,5X,'VECTOR OF RESIDUALS')
   40 IF(NHC.EQ.0)GO TO 41
            DO 33 I=2,L,2
            J=I+1
            IA=I/2
            PHASE(IA)=DATAN2(C(J),C(I))
            E(IA)=1./(FK(IA)*DCOS(PHASE(IA)))
            FK(IA)=C(I)*B(IA)
            VKR(IA)=2.*PI-VKR(IA)
            XKAPA(IA)=VKR(IA)-PHASE(IA)
      BP(1,1)=(1./(1.+(C(J)/C(I))**2))*(-1*C(J)/C(I)**2)
      BP(1,2)=(1./(1.+(C(J)/C(I))**2))*1./C(I)
      SIGMAF(IA)=BP(1,1)**2*COVAR(I,I)+BP(1,2)**2*COVAR(J,J)
      EA(1,1)=1./FK(IA)*DCOS(PHASE(IA))
      EA(1,2)=C(I)*DSIN(PHASE(IA))/(FK(IA)*DCOS(PHASE(IA))**2)
      SIGMAA(IA)=EA(1,1)**2*COVAR(I,I)+EA(1,2)**2*SIGMAF(IA)
   33       CONTINUE
            PRINT122
  122 FORMAT(//,5X,'CONSTITUENT',5X,'AMPLITUDE',5X,'SIGMA AMPL.',10X,
     *'PHASE LAG',5X,'SIGMA PHASE')
            DO 34 K=1,ICON
```

```
                  XKAPA(K)=XKAPA(K)*180./PI
              SIGMAF(K)=DSQRT(SIGMAF(K))*180./PI
              SIGMAA(K)=DSQRT(SIGMAA(K))
              PRINT123,W(K),FK(K),SIGMAA(K),XKAPA(K),SIGMAF(K)
  123         FORMAT(//,5X,F12.6,3X,F10.6,5X,E11.4,10X,F12.6,5X,E11.4)

   34            CONTINUE
   41         CONTINUE
                  STOP
                  END
C
              SUBROUTINE APPROX(L,M,A,P,F,EN,C,V,AC,U,COVAR,APVF)
C
C             ***************************************************************
C             *      THIS SUBROUTINE DOES THE LEAST SQUARES APPROXIMATION     *
C             *         OF THE GIVEN FUNCTIONS AND RETURNS THE VECTOR OF COEF *
C             *   FICIENTS TOGETHER WITH THE VAR. COVAR MATRIX                *
C             ***************************************************************
C
              IMPLICIT REAL*8(A-H,O-Z)
              DIMENSION A(60,60),P(60,60),EN(60,60),AC(60),C(60),F(60),
     *        COVAR(60,60),ATP(60,60),AT(60,60),VTP(1,60),VF(1),U(60),
     *        IW1(60),IW2(60),VT(1,60),V(60,1)
              IRDA=IDA=IDB=IDC=60
              CALL TRNSD(AT,IDB,A,IDA,M,L)
              CALL MMULD(ATP,IDC,AT,IDA,P,IDB,L,M,M)
              CALL MMULD(EN,IDC,ATP,IDA,A,IDB,L,M,L)
              CALL MMULD(U,IDC,ATP,IDA,F,IDB,L,M,1)
              CALL MINVD(EN,IRDA,L,DETA,IW1,IW2)
              CALL MMULD(C,IDC,EN,IDA,U,IDB,L,L,1)
C        COMPUTE RESIDUALS
              CALL MMULD(AC,IDC,A,IDA,C,IDC,M,L,1)
              CALL MSUBD(V,IDC,AC,IDA,F,IDC,M,1)
C   COMPUTE A POSTERIORI VARIANCE FACTOR
              CALL TRNSD(VT,1,V,IDA,M,1)
              CALL MMULD(VTP,1,VT,1,P,IDB,1,M,M)
              CALL MMULD(VF,1,VTP,1,V,IDB,1,M,1)
              IDF=M-L
              APVF=VF(1)/IDF
C        COMPUTE VARIANCE COVARIANCE MATRIX OF COEFFICIENT
              DO 10 I=1,L
              DO 10 J=1,L
              COVAR(I,J)=APVF*EN(I,J)
   10         CONTINUE
   10         RETURN
              END
C
          SUBROUTINE ORTHO(N,M,SIGMA,PHI,MRO,SIGMAF,VFC,NPC,INDEX,V,SUMD,F,W00007370
         &,D,ALPHA,     C,SUMC,SC2,STDF,IW)                                  00007380
C
C THIS SUBROUTINE ORTHOGONALIZES THE MATRIX PHI USING THE GRAM-SCHMIDT  00007390
C METHOD, COMPUTES THE FOURIER COEFFICIENTS OF THE ORTHOGONALIZED MATRIX00007400
C DERIVES THE COEFFICIENTS OF PHI,COMPUTES THE VARIANCES OF THE FOURIER 00007410
C COEFFICIENTS AND THE VARIANCE-COVARIANCE MATRIX OF THE COEFFICIENTS   00007420
C INPUTS :                                                              00007430
C    1. PHI(OPTIONAL - COULD BE FUNCTION SUBPROGRAM INSTEAD) - AN N BY M 00007440
C        CONTAINING THE BASE FUNCTIONS EVALUATED FOR EACH OBSERVATION   00007450
C    2. N - THE NUMBER OF OBSERVATIONS                                  00007460
```

```
C     3. M - THE NUMBER OF BASE FUNCTIONS (EQUAL OR GREATER THAN 2)        00007470
C     4. W - A VECTOR OF LENGTH N CONTAINING THE COMPUTED WEIGHT FUNCTIONS00007480
C     5. F - FUNCTIONAL VALUES                                             00007490
C     6. SIGMA - THE A PRIORI VARIANCE FACTOR                              00007500
C     7. NRD - THE MAXIMUM ROW DIMENSION OF PHI                            00007510
C     8. MRD - THE MAXIMUM COLUMN DIMENSION OF PHI                         00007520

C     9. INDEX - PERMITS OPTIONAL TEST FOR STATISTICAL SIGNIFICANCE        00007530
C                OF FOURIER COEFFICIENTS....                               00007540
C                IF 0,STATISTICAL TEST FOR FOURIER COEFFICIENTS ABANDONED 00007550
C                IF 1,TESTS AGAINST ONE TIME ITS STANDARD DEVIATION        00007560
C                IF 2,TESTS AGAINST TWICE ITS ST.DEVIATION                 00007570
C                IF 3,TESTS AGAINST THREE TIMES ITS ST.DEVIATION           00007580
      IMPLICIT REAL*8(A-H,O-Z)                                             00007750
C    10. IW - WRITE CODE OF THE COMPUTER                                   00007590
C OUTPUTS :                                                                00007600
C     1. ALPHA - AN MRD BY M MATRIX CONTAINING THE ALPHA'S USED IN COMPUTI00007610
C        THE ORTHOGONALIZED MATRIX AND IN COMPUTING THE COEFFICIENTS OF PH00007620
C     2. C - THE M FOURIER COEFFICIENTS OF THE ORTHOGONALIZED MATRIX      00007630
C     3. D - THE M COEFFICIENTS OF THE INPUT MATRIX PHI                    00007640
C     4. SUMC - THE VARIANCES OF THE FOURIER COEFFICIENTS                  00007650
C     5. SUMD - THE VARIANCE-COVARIANCE MATRIX OF THE COEFFICIENTS         00007660
C     6. SC2 - THE SQUARES OF THE NORMS OF THE ORTHOGONALIZED MATRIX      00007670
C     7. SIGMAF - THE FOURIER POLYNOMIAL A POSTERORI VARIANCE FACTOR       00007680
C     8. V - THE N RESIDUALS                                               00007690
C     9. VPC - THE ORIGINAL POLYNOMIAL A POSTEORI VARIANCE FACTOR          00007700
C    10. NPC - NUMBER OF THE COEFFICIENTS OF THE ORIGINAL POLYNOMIAL       00007710
C              AFTER THE STATISTICAL TEST IS PERFORMED                     00007720
C    11. STDP - VECTOR AGAINST WHICH THE ABSOLUTE VALUES OF FOURIER        00007730
C               COEFFICIENTS ARE TESTED                                    00007740
        DIMENSION ALPHA(60,60),W(60),F(60),C(60),D(60),SUMC(60),
     *  SUMD(60,60),SC2(60),V(60),STDP(60),PHI(60,60)
C TEST FOR NEGATIVE DEGREES OF FREEDOM                                     00007800
      IF (N.LT.M) GO TO 100                                                00007810
      K=1                                                                  00007820
      ALPHA(M,M)=1.D0                                                      00007830
C DETERMINE THE ALPHA'S FOR COMPUTATION OF ORTHOGONALIZED MATRIX           00007840
   10 DO 3 J=K,M                                                           00007850
      IF(J.NE.K) GO TO 6                                                   00007860
      ALPHA(K,K)=1.D0                                                      00007870
      GO TO 3                                                              00007880
    6 SC1=0.D0                                                             00007890
      SC2(K)=0.D0                                                          00007900
      SC3=0.D0                                                             00007910
      DO 2 I=1,N                                                           00007920
      P=PHI(I,K)                                                           00007940
      IF(K.EQ.1) GO TO 4                                                   00007950
      K1=K-1                                                               00007960
      DO 5 J1=1,K1                                                         00007970
    5 P=P+ALPHA(J1,K)*PHI(I,J1)                                            00007980
    4 SC1=SC1+W(I)*PHI(I,J)*P                                              00007990
      SC3=SC3+F(I)*W(I)*P                                                  00008000
    2 SC2(K)=SC2(K)+W(I)*P**2                                              00008010
      ALPHA(J,K)=-SC1/SC2(K)                                               00008020
      ALPHA(K,J)=ALPHA(J,K)                                                00008030
    3 CONTINUE                                                             00008040
C DETERMINE THE FOURIER COEFFICIENTS FOR THE ORTHOGONALIZED MATRIX         00008050
      C(K)=SC3/SC2(K)                                                      00008060
      K=K+1                                                                00008070
      IF(M.EQ.2) GO TO 34                                                  00008080
```

```
      IF(K.LT.3) GO TO 10                                               00008090
C DETERMINE THE ALPHA'S USED IN COMPUTING THE COEFFICIENTS OF PHI       00008100
      JK=K-1                                                            00008110
    9 JL=K                                                              00008120
      JK=JK-1                                                           00008130
      JJ=K-JK-1                                                         00008140
      DO 8 LN=1,JJ                                                      00008150
      JL=JL-1                                                           00008160
    8 ALPHA(JK,K)=ALPHA(JK,K)+ALPHA(JK,JL)*ALPHA(K,JL)                   00008170
      IF(JK.NE.1) GO TO 9                                               00008180
      IF(K.LT.M) GO TO 10                                               00008190
C DETERMINE THE LAST FOURIER COEFFICIENT                                00008200
   34 SC2(K)=0.D0                                                       00008210
      SC3=0.D0                                                          00008220
      DO 7 I=1,N                                                        00008230
      P=PHI(I,K)                                                        00008250
      K1=K-1                                                            00008260
      DO 1 J=1,K1                                                       00008270
    1 P=P+ALPHA(J,K)*PHI(I,J)                                           00008280
      SC2(K)=SC2(K)+W(I)*P**2                                           00008290
    7 SC3=SC3+F(I)*W(I)*P                                               00008300
      C(K)=SC3/SC2(K)                                                   00008310
C DETERMINE THE COEFFICIENTS OF PHI                                     00008320
      IDEKT=1                                                           00008330
      ICOUNT=0                                                          00008340
 1000 CONTINUE                                                          00008350
      DO 13 I=1,M                                                       00008360
      D(I)=C(I)                                                         00008370
      IF(I.EQ.M) GO TO 13                                               00008380
      II=I+1                                                            00008390
      DO 14 J=II,M                                                      00008400
   14 D(I)=D(I)+ALPHA(I,J)*C(J)                                         00008410
   13 CONTINUE                                                          00008420
C COMPUTE THE VARIANCE OF THE FOURIER COEFFICIENTS AND THE VARIANCE-COVA00008430
C MATRIX OF THE COEFFICIENTS                                            00008440
      DO 15 I=1,M                                                       00008450
      DO 15 J=1,M                                                       00008460
   15 SUMD(I,J)=0.D0                                                    00008470
      SC4=0.D0                                                          00008480
      DO 22 I=1,N                                                       00008490
      FN=0.D0                                                           00008510
      DO 21 J=1,M                                                       00008520
   21 FN=FN+D(J)*PHI(I,J)                                               00008530
      V(I)=F(I)-FN                                                      00008540
      V2=V(I)**2                                                        00008550
   22 SC4=SC4+V2*W(I)                                                   00008560
      SIGMAF=SC4/(N-M+ICOUNT)*SIGMA                                     00008570
      VFC=SIGMAF                                                        00008580
      IF(IDEKT.EQ.2) VFC=SC4/(N-NFC)*SIGMA                              00008590
      IF(INDEX.EQ.0) NFC=M                                              00008600
      DO 28 I=1,M                                                       00008610
      SUMC(I)=SIGMAF/SC2(I)
      IF(IDEKT.EQ.1) GO TO 28                                           00008630
      IF(C(I).EQ.0D0) SUMC(I)=0D0                                       00008640
   28 CONTINUE                                                          00008650
      DO 23 I=1,M                                                       00008660
      DO 23 J=1,I                                                       00008670
      DO 23 K=J,I                                                       00008680
   23 SUMD(J,K)=SUMD(J,K)+ALPHA(J,I)*ALPHA(K,I)*SUMC(I)                 00008690
```

```
      DO 24 I=1,M                                                       00008700
      IT=I+1                                                            00008710
      IF(IT.GT.M) GO TO 30                                              00008720
      DO 24 J=IT,M                                                      00008730
   24 SUMD(J,I)=SUMD(I,J)                                               00008740
   30 CONTINUE                                                          00008760
C OPTIONAL CHECK FOR STATISTICALLY SIGNIFICANT FOURIER COEFFICIENTS      00008750
      IF(INDEX.EQ.0) GO TO 40                                           00008770

      IF(IDEKT.EQ.2) GO TO 40                                           00008780
      PINDEX=DFLOAT(INDEX)                                              00008790
      DO 31 I=1,M                                                       00008800
      STDP(I)=PINDEX*DSQRT(SUMD(I))                                     00008810
      IF(DABS(C(I)).LT.STDP(I)) GO TO 32                                00008820
      GO TO 31                                                          00008830
   32 C(I)=0D0                                                          00008840
      ICOUNT=ICOUNT+1                                                   00008850
      SUMD(I)=0D0                                                       00008860
   31 CONTINUE                                                          00008870
      NPC=0                                                             00008880
      DO 33 I=1,M                                                       00008890
      IF(C(I).NE.0D0) NPC=I                                             00008900
   33 CONTINUE                                                          00008910
      IDEKT=2                                                           00008920
      GO TO 1000                                                        00008930
   40 RETURN                                                            00008940
  100 WRITE(IW,102)                                                     00008950
  102 FORMAT('0','*ERROR*  NEGATIVE DEGREES OF FREEDOM')                00008960
      RETURN                                                            00008970
      END                                                               00008980
```

```
$GO
***  CONSTITUENT  FREQUENCIES ******
             28.984104
             30.000000
             13.943036
             15.041069
             14.958931
             30.082137
             28.439730

HEIGHT OBSERVED      TIME OF OBS.
      7.4000             3.833
      1.3000            10.250
      7.3000            16.250
      1.2000            22.350
      7.4000            28.667
      1.2000            35.000
      7.3000            41.250
      1.2000            47.250
      7.6000            53.500
      1.2000            59.917
      7.3000            66.083
      1.1000            72.417
      7.7000            78.583
      1.0000            84.833
      7.4000            91.083
      1.1000            97.417
```

II.3 Tidal Reductions

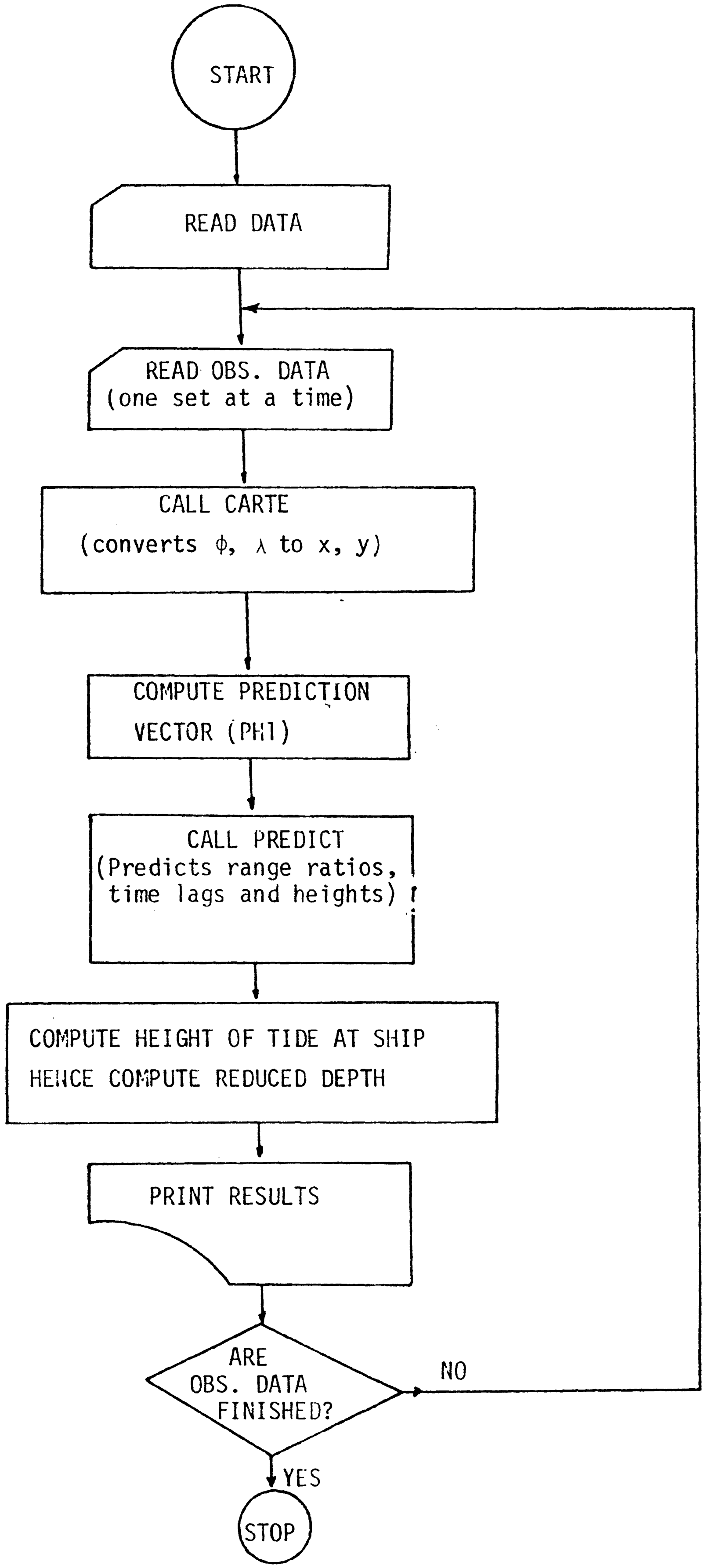
Figure A-3 is the flow chart describing the tidal reduction computations.

The program uses as input the following:

- the results of computations 1 and 2, that is; the coefficients of the approximating polynomials C_r, C_A and C_B and their associated standard deviations.
- the observed data at each sounding namely: the depth sounded (D), time of sounding (t) and the geodetic coordinates (ϕ, λ) or the local Cartesian coordinates (x, y). The observed data at each sounding are punched in one card and read into the computer one card at a time.

The SUBROUTINE PREDICT used in this program is different from the SUBROUTINE PRED. The subroutine predict uses prediction vector and predicts for one point at a time while the Subroutine Pred uses prediction matrix and predicts for all the points at once.

Figure II-3 Tidal Reductions - Flow Chart



```
**************************************************************************************************FABL
$JOB CKENWA/G,TIME=8,PAGES=10
C     @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
C
C     PROGRAM TO IMPLIMENT TIDAL REDUCTIONS USING ANALYTICAL COTIDAL
C     CHART IN REAL TIME.
C                               N O T A T I O N S
C         N=DEGREE OF POLY. FOR RANGE RATIO
C         L=NUM.OF COEFFS, IN THE POLY.
C         LT=NUM.OF COEFFS.IN THE POLY.OF THE TIME SERIES AT THE REF. STATION
C         NCON= NUM OF CONSTITUENT FREQ USED IN REAL TIME APPROX AT REF. STN.
C         ITYPE= THE TYPE OF INFORMATION
C                  1- FOR LAT.   LONG.GIVEN
C                   2- FOR X,Y COORDS.GIVEN
C         ISPLIT = CODE INDICATING WETHER THE TIME LAG IS SPLIT INTO
C         FUNCTION A AND B
C              1 _ FOR NO SPLIT
C              2 _ FOR SPLIT
C         CT= POLY. COEFFS.FOR REAL TIME AT THE REF STN.
C         CR=POLY. COEFFS.FOR RANGE RATIO
C         VACT,VACR=  ASSOCIATED VARIANCES FOR CT,CR RESPECTIVELY
C         CA,CB=COEFFS.OF POLY. FOR TIME LAG
C           VACA,VACB=ASSOCIATED VARIANCES FOR CA CB RESPECTIVELY
C         DEGLAO,XLAMO SECLAO = LATITUDE OF REF, STN. IN DEGREE,MIN,
C          AND SECONDS
C         DEGLOO,XLOMO,SECLOO = LONGITUDE OF REF. STN.IN DEGREES MIN.
C           AND SECONDS
C         T,XMIN = OBSERVED TIME IN HOURS AND MINS.
C         DEPTHO = OBSERVED DEPTH IN METRES
C         DEGLA,XLAMIN,SECLA = OBSERVED LATITUDE IN DEGREES,MINS,AND SECS.
C         DEGLO,XLOMIN,SECLO = OBSERVED LONGITUDE IN DEGREES.MINS.AND SECS.
C
C       @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
C
C
          IMPLICIT REAL*8(A-H,O-Z)
          DIMENSION CT(30),CR(30),PHIT(30),PHIR(30),CA(30),CB(30),
     *    VACT(30),VACR(30),VACA(30),VACB(30),H(300),STE(300),PHI(300),
     *    ALONG(300),W(10),TL(300)
          PI=3.14159265
          CK=0.5*PI/180.
          READ(5,40)L,LT,N,NCON,ITYPE,ISPLIT
          DO 1 K=1,NCON
          READ(5,50)W(K)
          W(K)=W(K)*PI/180.
    1     CONTINUE
          DO 6 I=1,LT
          READ(5,45)CT(I),VACT(I)
    6     CONTINUE
          DO 7 I=1,L
          READ(5,45)CR(I),VACR(I)
    7     CONTINUE
          DO 8 I=1,L
          READ(5,45)CA(I),VACA(I)
    8     CONTINUE
          IF(ISPLIT.EQ.1)GO TO 10
          DO 9 I=1,L
          READ(5,45)CB(I),VACB(I)
    9     CONTINUE
   10     CONTINUE
```

```
          READ(5,44)DEGLAO,XLAMO,SECLAO,DEGLOO,XLOMO,SECLOO
          CALL DEGRAD(DEGLAO,XLAMO,SECLAO,PHIO)
          CALL DEGRAD(DEGLOO,XLOMO,SECLOO,ALON)
          WRITE(6,57)
          WRITE(6,56)
          KOUNT=0
    2     CONTINUE
          IF(ITYPE.EQ.1)THEN DO
          READ(5,55)T,XMIN,DEPTHC,DEGLA,XLAMIN,SECLA,DEGLO,XLOMIN,SECLO
          IF(T.LT.0.0)GO TO 39
          KOUNT=KOUNT+1
          CALL DEGRAD(DEGLA,XLAMIN,SECLA,XLAT)
          CALL DEGRAD(DEGLO,XLOMIN,SECLO,XLONG)
          CALL CARTE(XLAT,XLONG,PHIO,ALON,X,Y)
          ELSE DO
          READ(5,60)T,XMIN,DEPTHC,X,Y
          IF(T.LT.0.0)GO TO 39
          END IF
C
C   COMPUTE THE VECTOR PHI FOR PREDICTION
C
          IDP=N+1
          I=0
          DO 4 K=1,IDP
          KA=K-1
          DO 5 J=1,IDP
          JA=J-1
          I=I+1
          PHIR(I)=X**KA*Y**JA
    5     CONTINUE
    4     CONTINUE
          CALL PRDICT(L,PHIR,CR,VACR,R,STDR)
          CALL PRDICT(L,PHIR,CA,VACA,A,STDA)
          IF(ISPLIT.EQ.1)GO TO 11
          CALL PRDICT(L,PHIR,CB,VACB,B,STDB)
          TC=DATAN(B/A)/CK
          B1=(1./(1.+(B/A)**2))*(-B/A**2)
          B2=(1./(1.+(B/A)**2))*(1./A)
          VATC=B1**2*STDA+B2**2*STDB
   11     IF(ISPLIT.EQ.2)GO TO 12
          TC=A
          VATC=STDA
   12      CONTINUE
          TL(KOUNT)=TC
          TCH=TC/60.
          TOH=T+XMIN/60.
          TAR=TOH-TCH
C
C  COMPUTE PHI FOR THE PREDICTION OF HEIGHT AT THE REF. STATION
C
          PHIT(1)=1.0
          K=0
          DO 3 I=2,LT,2
          IA=I+1
          K=K+1
          PHIT(I)=DCOS(W(K)*TAR)
          PHIT(IA)=DSIN(W(K)*TAR)
    3     CONTINUE
          CALL PRDICT(LT,PHIT,CT,VACT,HTC,STDH)
          HTIDE=HTC*R
```

```
      STDEV=DSQRT(R**2*STDH+HTO**2*STDR)
      DEPTH=DEPTHO-HTIDE
      STDER=DSQRT(.01**2+STDEV**2)
      H(KOUNT)=HTIDE
     STE(KOUNT)=STDEV
      PHI(KOUNT)=XLAT
      ALONG(KOUNT)=XLONG
      XLAT=XLAT*180./PI
      XLONG=XLONG*180./PI
      WRITE(6,61)KOUNT,XLAT,XLONG,DEPTHO,TOH,TAR,HTO,R,HTIDE,STDEV
      GO TO 2
   39 CONTINUE
      NOBS=KOUNT
      WRITE(6,42)
      DO 13 KOUNT=1,NOBS
      WRITE(6,41)KOUNT,TL(KOUNT)
   13 CONTINUE
   42 FORMAT('1',5X,'PREDICTED   TIME   LAGS')
   41 FORMAT(/,3X,I3,5X,F6.2)
   40 FORMAT(5X,6I3)
   50 FORMAT(5X,F10.6)
   45 FORMAT(5X,2E11.4)
   55 FORMAT(5X,9F6.2)
   60 FORMAT(5X,3F6.2,2F12.6)
   44 FORMAT(5X,6F6.2)
   57 FORMAT('1',//,5X,'TIDAL',5X,'REDUCTIONS')
   56 FORMAT(//,3X,'NUM',4X,'LATITUDE',6X,'LONGITUDE',4X,'OBS. DEPTH'
     *,3X,'TIME AT SHIP',2X,'TIME AT REF',2X,'TIDE AT REF',2X,'PR. RATIO
     *',2X,'TIDE AT SHIP',3X,'STDEV')
   61  FORMAT(/,3X,I3,2X,F10.6,3X,F12.6,5X,F7.3,7X,F7.3,6X,F7.3,7X,F7
     *.3,4X,F7.3,6X,F7.3,2X,E11.4)
      STOP
      END
C

      SUBROUTINE CARTE(ALAT,ALON,PHIO,ALONO,X,Y)
C
C     THIS SUBROUTINE COMPUTES THE CARTESIAN COORDS. FROM THE LAT.
C    AND LONGITUDE
C   ********************************************************************
C
      IMPLICIT REAL*8(A-H,O-Z)
      RA=6378206.4D0
      RB=6356583.8D0
      PI=3.14159265D0
      EC=(RA**2-RB**2)/RA**2
      XM=((1.-EC)*RA)/(DSQRT(1.-EC*(DSIN(PHIO)**2))**3)
      XN=RA/DSQRT(1.-EC*(DSIN(PHIO)**2))
      R=DSQRT(XM*XN)
      X=R*(ALAT-PHIO)
      Y=R*DCOS(PHIO)*(ALON-ALONO)
      RETURN
      END

      SUBROUTINE PREDICT(L,PHI,C,VAR,PV,STDEV)
C
C    THIS SUBROUTINE PREDICTS VALUES OF THE FUNCTION USING THE COEFF.
C    OF THE POLYNOMIAL AND THE BASE FUNCTIONS.
C   ********************************************************************
C
```

```
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION PHI(30),C(30),VAR(30)
      SUM=0.0
      SUMVA=0.0
      DO 10 I=1,L
      FUN=PHI(I)*C(I)
      VA=PHI(I)**2*VAR(I)
      SUM=SUM+FUN
      SUMVA=SUMVA+VA
   10 CONTINUE
      PV=SUM
      STDEV=SUMVA
      RETURN
      END
C
      SUBROUTINE DEGRAD(A,B,C,OUT)
C
C CONVERTS DEG, MIN, SEC TO RADIANS
C
      IMPLICIT REAL*8(A-H,O-Z)
      OUT=(A+B/60.0D0+C/3600.0D0)*DARSIN(1.0D0)/90.0D0
      RETURN
      END

$GO
```

III CANADIAN DEFINITIONS OF CHART AND SOUNDING DATUMS

This appendix has been added to supplement the information given in Sections 1.0 and 1.1 of this report. The information given here has been taken directly from the Hydrographic Tidal Manual 1970 [Energy, Mines, and Resources Canada]. The descriptions and definitions presented concern tidal waters; for similar information regarding non-tidal waters, the reader is referred to the above mentioned reference.

Chart datum is the datum plane adopted for a published chart. It is a low water datum which by international agreement is so low that the water level will seldom fall below it. It is the level above which tidal predictions and water level records are based. The datum is only used within a gauge location and differs from place to place depending on the range of tide or water level.

For tidal waters, the Canadian Hydrographic Service has adopted the level of Lower Low Water Large Tides (see Figure III-1) as its reference for chart datum, and Higher High Water Large Tides as a reference for elevations.

A sounding datum is the reference surface to which soundings are reduced during the course of a hydrographic survey. It is the datum used when compiling a "field sheet" for a survey. It may or may not be the same as chart datum.

When selecting a datum, the following must be considered:

(i) the datum should be sufficiently low so that under normal weather conditions there will always be at least the charted depth of water,

(ii) the datum should not be so low that it gives an unduly pessimistic impression of the least depth of water likely to be found,

(iii) the datum should be in close agreement with those of neighbouring surveys.

The following are the definitions of various reference surfaces (datum planes) and water level variations in tidal waters used by the Canadian Hydrographic Service.

Graphical representations of several of these are given in Figure III-1.

(i) Higher High Water Large Tides (H.H.W.L.T.) is the highest predictable tide from the available tidal constituents, with the astronomical (nodal) factor f_k close to unity.

(ii) Higher High Water Mean Tides (H.H.W.M.T.) is the mean of the predicted heights of the higher high waters of each day.

(iii) Lower Low Water Mean Tides (L.L.W.M.T.) is the mean of the predicted heights of the lower low waters of each day.

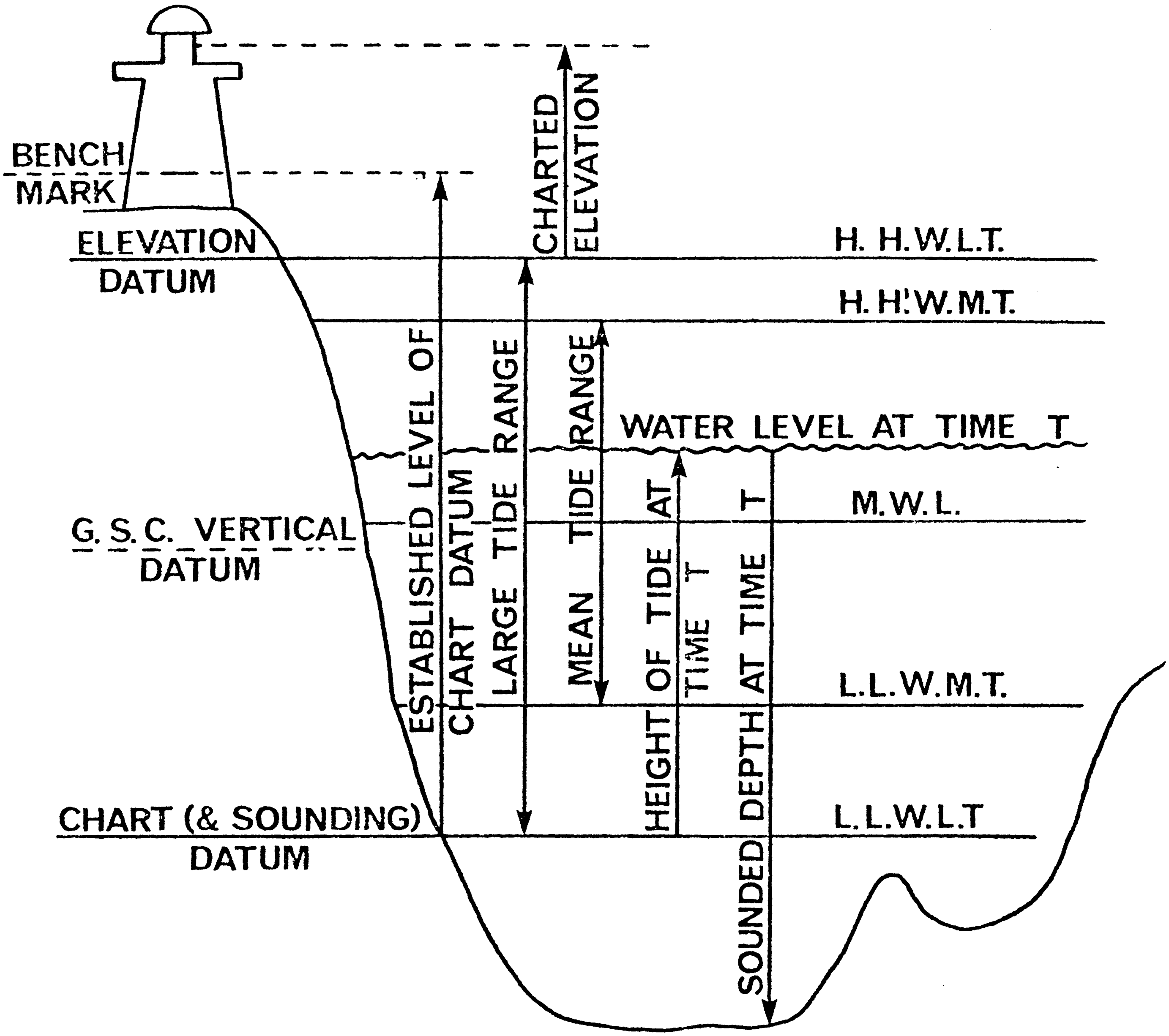
(iv) Lower Low Water Large Tides (L.L.W.L.T.) or Lowest Normal Tides (L.N.T.) is the lowest predictable tide from the available tidal constituents, with the astronomical (nodal) factor f_k close to unity.

(v) Mean Water Level (M.W.L.) is the mean of hourly water levels for a period of observations.

(vi) Mean Tide Level (M.T.L.) is the mean of all high and low water heights over a period of observation.

(vii) Charted Elevation is the vertical distances between an object and the reference surface of Higher High Water Large Tides.

(viii) Charted Depth is the vertical distance from the chart datum to the sea floor.



REFERENCE SURFACES AND WATER LEVEL VARIATIONS