AN EXPLORATORY INVESTIGATION INTO SATELLITE POSITIONING TECHNIQUES TO BE EMPLOYED IN CANADIAN CONTINENTAL SHELF AND ARCTIC AREAS

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PREFACE

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AN EXPLORATORY INVESTIGATION INTO SATELLITE POSITIONING TECHNIQUES TO BE EMPLOYED IN CANADIAN CONTINENTAL SHELF AND ARCTIC AREAS

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ABSTRACT

The purpose of the investigation contained herein is to examine the possibilities of various simultaneous modes of satellite positioning from the error analysis point of view. That is, what accuracy can be expected in the determination of a sea position once given the network configuration and accuracy of the observables? The recommendations given are based in part on this exploratory study; They are intended to assist in giving partial direction to the present and future development of satellite positioning techniques in Canada.

PREFACE

The research conducted for this report is under the direction of Professor Edward J. Krakiwsky of The Department of Surveying Engineering, The Universtiy of New Brunswick. The contract agreement is with the Surveys and Mapping Branch of the Department of Energy, Mines and Resources.

The report was written by Professor Krakiwsky and Jan Kouba, with the latter performing the computer programming.

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1. INTRODUCTION

Background and Motivation

The background for use of satellites by the petroleum industry in Canada is given by the following quotation [Hittle, 1969, pp. 1-2]:

> "Permit holdings in excess of 300 miles from shore are not uncommon along the eastern coast line of Canada. At such ranges the present radio location systems are influenced by a number of disturbing factors normally attributed to the inponderables associated with radio waves. The effects of activities degrade positional accuracy beyond the limitations considered practical for geophysical operations."

The chief reasons for using satellites is stated to be:

(1) to provide lane identification for the Decca 12 F two range system,

(2) to confirm the positions of the offshore drilling vessels during rig locations.

It should be noted at this point that the satellite method of positioning employed to date on the east coast of Canada is the "orbital mode" of satellite geodesy. The method requires that the satellite positions be known at the time of observation; Further, the resultant sea position coordinates are "geocentric". The first requirement may cause positional errors in the order of tens of meters depending upon when and how accurately the orbital elements were computed. The transformation of geocentric coordinates to North American Datum (NAD) coordinates may cause an error of the same magnitude mentioned above, since the datum shift parameters are presently not accurately known or are not available to the public. The alternative to the orbital mode is the simultaneous mode of which the latter is not hampered by the two problems mentioned above; That is, the coordinates of the sea position can be determined in NAD coordinates without any precise knowledge of the orbital elements.

The motivation for the investigation contained herein originated in Banff, Alberta on October 15, 1969 during the Surveying and Mapping Colloquium for the Petroleum Industry. At that time it was suggested by the author [Canadian Petroleum Association, 1969, p. 161] that the simultaneous mode of satellite geodesy be used for geodetic positioning in the continental offshore areas. This approach utilizes simultaneous absolute satellite directions and/or ranges from several known ground stations and unknown sea stations to a satellite position. Several satellite positions are observed in this manner in order to strengthen the determination of the sea position coordinates

Another variation of the simultaneous mode is to employ the range difference from a given ground station to two satellite positions as an observed quantity. This basic concept is extended to include several ground and sea stations as well as many pairs of satellite positions.

In summary then, the various possible variations of the simultaneous mode are:

(1) satellite triangulation (directions only),

(2) satellite trilateration (ranges only),

(3) satellite total vector mode (simultaneous range and directions from each station),

(4) satellite range difference mode (range differences only).

The first two modes are treated rather extensively in the literature, thus less emphasis are placed on them. The total vector mode and range difference mode are treated in more detail.

2. MATHEMATICAL FORMULATION

The purpose of this Section is to outline the equations used in the simulation. First the adjustment scheme is treated. This is immediately followed by the specific mathematical models which make-up the various satellite modes, namely: total vector, triangulation, trilateration, and range difference. The last part of the Section deals with the normal equation system.

2.1 Adjustment Scheme

The mathematical model is of the following form:

$$F\ (X^a,\ Y^a,\ L^a) = 0 \quad , \tag{1}$$

where X^a is the desired adjusted group of ground (sea) stations; Y^a is a group of adjusted auxiliary parameters (e.g. satellite coordinates, frequency off-set, etc.) which are often called "nuisance parameters", and L^a represents the adjusted observations, namely, the topocentric right ascensions and declinations, topocentric ranges, and topocentric range differences.

The linearized form of the above mathematical model is

$$AX + CY + BV + W = 0 \quad , \tag{2}$$

where X, Y, V are the corrections to be applied respectively to the approximate values X^o, Y^o and the observations L^b, such that

$$X^a = X^o + X \ ,$$
$$Y^a = Y^o + Y \ , \qquad (3)$$
$$L^a = L^b + V \ .$$

Further

$$A = \frac{\partial F}{\partial X^a} \Big|^{*}_{X^a = X^o} \ ,$$

$$C = \frac{\partial F}{\partial Y^a} \Big|_{Y^a = Y^o} \ ,$$

$$B = \frac{\partial F}{\partial L^a} \Big|_{L^a = L^b} \ ,$$

are the design matrices, while

$$W = F\ (X^o, Y^o, L^b) .$$

2.2 Satellite Total Vector Mode and Satellite Triangulation Mode

The total vector mathematical model is [Krakiwsky, 1968, pp. 8-10] (Figure 1)

$$F_{ij} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \vec{X}_j - \vec{X}_i - \vec{X}_{ij} = 0 \ , \qquad (4)$$

where $\vec{X}_j$ is the radius vector to the j'th satellite position, $\vec{X}_i$ is the radius vector to the i'th ground station, and

$$\vec{X}_{ij} = R_2\ (-x)\ R_1(-y)\ R_3\ (GAST) \begin{bmatrix} r_{ij} \cos \delta_{ij} \cos \alpha_{ij} \\ r_{ij} \cos \delta_{ij} \sin \alpha_{ij} \\ r_{ij} \sin \delta_{ij} \end{bmatrix} \qquad (5)$$

is the ground station to satellite position vector obtained by rotating

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* indicates that the derivatives are evaluated at X^o, Y^o, L^b

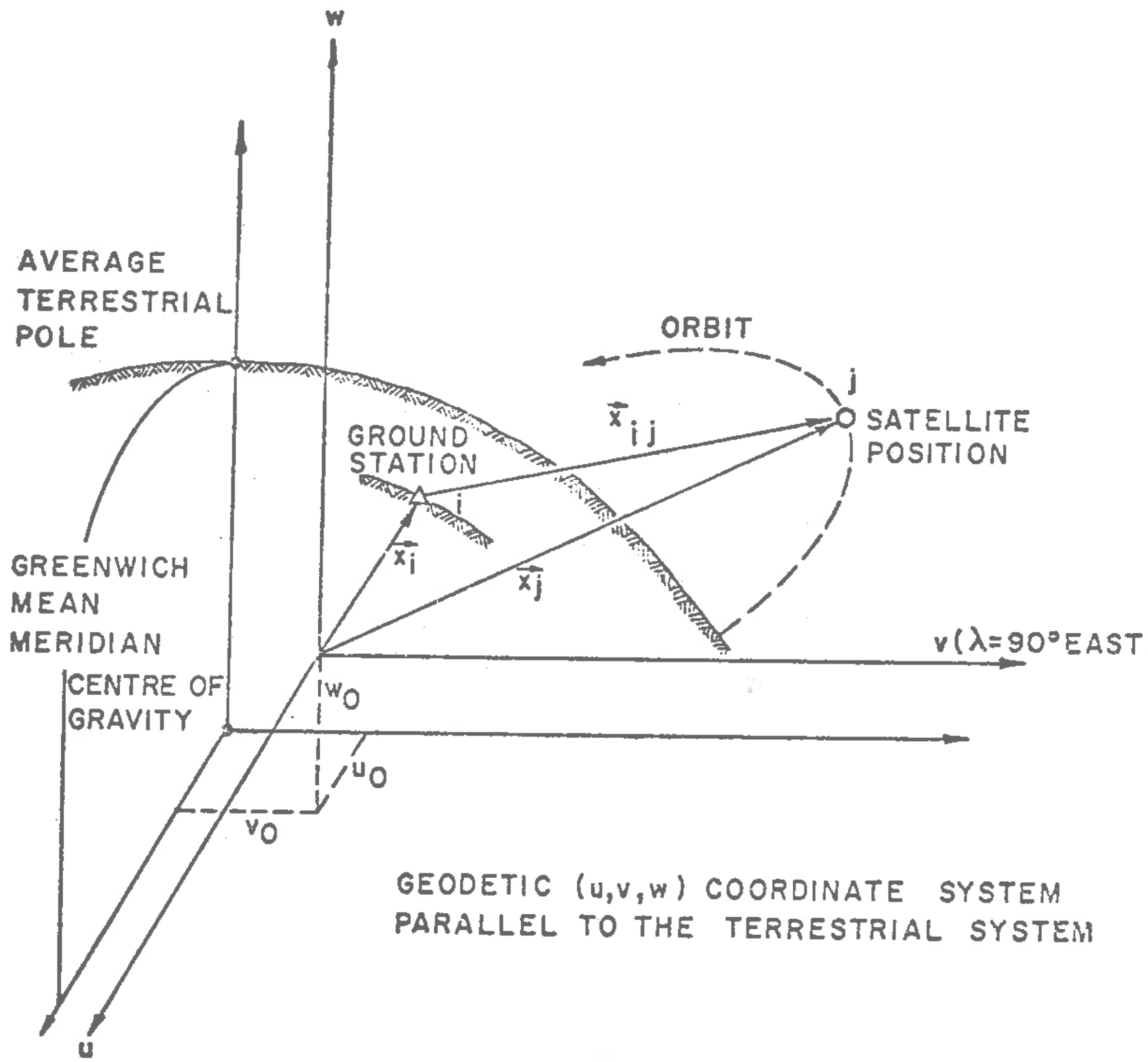

FIGURE 1. VECTORS TO GROUND STATION AND SATELLITE POSITION, AND TOPOCENTRIC VECTOR.

the topocentric vector from the true celestial system to the average terrestrial system* through the Greenwich apparent sidereal time (GAST) and two components of polar motion (x, y).

The design matrices A anc C are composed of three by three identity matrices (I or - I), namely

$$
\underset{r \times u}{A} =
\begin{array}{c}
\text{Ground Stations} \\
\begin{array}{cccccc} \vec{X}_1 & \vec{X}_2 & \cdots & \vec{X}_i & \cdots & \vec{X}_u \end{array} \\
\left[\begin{array}{cccccc}
-I & & & & & \\
 & -I & & & & \\
 & & \ddots & & & \\
 & & & -I & & \\
 & & & & & -I \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
-I & & & & & \\
 & -I & & & & \\
 & & & -I & & \\
 & & & & \ddots & \\
 & & & & & -I \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
-I & & & & & \\
 & -I & & & & \\
 & & \ddots & & & \\
 & & & -I & & \\
 & & & & \ddots & \\
 & & & & & -I
\end{array}\right]
\end{array}
, \quad
\underset{r x v}{C} =
\begin{array}{c}
\text{Satellite Stations} \\
\begin{array}{ccccc} \vec{X}_1 & \vec{X}_2 & \cdots & \vec{X}_j & \cdots \vec{X}_v \end{array} \\
\left[\begin{array}{ccccc}
I & & & & \\
I & & & & \\
\vdots & & & & \\
I & & & & \\
I & & & & \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
 & I & & & \\
 & I & & & \\
 & \vdots & & & \\
 & I & & & \\
 & \vdots & & & \\
 & I & & & \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
 & & & I & \\
 & & & I & \\
 & & & \vdots & \\
 & & & I & \\
 & & & \vdots & \\
 & & & I &
\end{array}\right]
\end{array}
$$

Note: r is the number of rows while u and v are the number of desired

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* See [Veis, 1963] for a detailed description of coordinate systems.

parameters and nuisance parameters, respectively. The B matirx of (equation 2) is composed of the three by three submatrices B_{ij} along the main diagonal, namely (this submatrix is obtained by differentiating equation 4 with respect to the observables, r, δ, α.)

$$B_{ij} = R_2(-X)\ R_1(-Y)\ R_3(GAST)*$$

$$* \begin{bmatrix} -r^b_{ij}\ \sin\delta^b_{ij}\cos\alpha^b_{ij} & ; & -r^b_{ij}\cos\delta^b_{ij}\sin\alpha^b_{ij} & ; & \cos\delta^b_{ij}\cos\alpha^b_{ij} \\ -r^b_{ij}\ \sin\delta^b_{ij}\sin\alpha^b_{ij} & ; & r^b_{ij}\cos\delta^b_{ij}\cos\alpha^b_{ij} & ; & \cos\delta^b_{ij}\sin\alpha^b_{ij} \\ \cos\delta^b_{ij} & ; & 0 & ; & \sin\delta^b_{ij} \end{bmatrix} . \qquad (5a)$$

The three rotation matrices are needed only when processing actual data, thus they are deleted from the simulation computation. The weight matrix is composed of three by three submatrices P_{ij} along main diagonal. The submatrix P_{ij} has the following form as expressed in terms of variances and covariances of the observed quantities:

$$P_{ij} = \begin{bmatrix} \sigma^2_\delta & \sigma_{\delta\alpha} & 0 \\ \sigma_{\alpha\delta} & \sigma^2_\alpha & 0 \\ 0 & 0 & \sigma^2_r \end{bmatrix}^{-1}$$

The satellite triangulation mode follows from the above equations by simply stipulating that the variance of the range is zero, i.e.

$$P_{ij} = \begin{bmatrix} \begin{bmatrix} \sigma^2_\delta & \sigma_{\delta\alpha} \\ \sigma_{\alpha\delta} & \sigma^2_\alpha \end{bmatrix}^{-1} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{bmatrix}$$

For a detailed discussion, concerning the satellite triangulation mode, see [Krakiwsky 1968, p- 15].

2.3 Satellite Trilateration Mode.

The mathematical model for ranges is [Krakiwsky, 1968, p. 22].

$$F_{ij} = [(x_j-x_i)^2 + (y_j-y_i)^2 + (z_j-z_i)^2]^{1/2} - r_{ij} = 0, \qquad (6)$$

where r_{ij} is the topocentric range from a ground station i to a satellite position j.

The design matrices A and C of equation (2) are composed of the three element vectors

$$a_{ij} = \left[\frac{x_j^o - x_i^o}{r_{ij}^o} \; ; \; \frac{y_j^o - y_i^o}{r_{ij}^o} \; ; \; \frac{z_j^o - z_i^o}{r_{ij}^o} \right] . \qquad (7)$$

Specifically, the matrices A and C have the form

$$\begin{array}{cc} \text{Ground Stations} & \text{Satellite positions} \\ \begin{array}{ccccc} \vec{X}_1 & \vec{X}_2 \cdots & \vec{X}_i & \cdots & \vec{X}_u \end{array} & \begin{array}{ccccc} \vec{X}_1 & \vec{X}_2 & \cdots & \vec{X}_j & \cdots \; \vec{X}_v \end{array} \end{array}$$

$$\underset{r \times u}{A} = \left[\begin{array}{ccccc} -a_{11} & & & & \\ & -a_{21} & & & \\ & & \ddots & & \\ & & & -a_{i1} & \\ & & & & \ddots \; -a_{u1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -a_{ij} & & & & \\ & -a_{2j} & & & \\ & & \ddots & -a_{ij} & \end{array} \right] , \quad \underset{r \times v}{C} = \left[\begin{array}{ccccc} a_{11} & & & & \\ a_{21} & & & & \\ \vdots & & & & \\ a_{i1} & & & & \\ \vdots & & & & \\ a_{u1} & & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & a_{ij} & \\ & & & a_{2j} & \\ & & & \vdots & \\ & & & a_{ij} & \end{array} \right] . \qquad (8)$$

The B matrix of equation (2) is a minus three by three identity matrix $-I$. The weight matrix is a diagonal matrix composed of inverse variances when the range observations are statistically independent.

2.4 Satellite Range Difference Mode

The observed quantity in this mode is the range difference Δr and is related to the ground (sea) stations and satellite positions (Figure 2). The mathematical model for this mode is

$$F_{jik} = -[(x_j-x_i)^2 + (y_j-y_i)^2 + (z_j-z_i)^2]^{1/2} + +[(x_k-x_i)^2 + (y_k-y_i)2 + (z_k-z_i)^2]^{1/2} - - \Delta r_{jik} = 0 \qquad (10)$$

This is rather a simplified mathematical model, however is sufficient for this preliminary investigation. The model could be expanded to include knowledge about the height of the sea position as well as the solution for unknown electronic constants of the system (See Section 6). Also see [Newton, 1966, p. 23] for a description of the relationship of the range difference to other parameters.

The design matrices in this case are

(Ground Stations)

$$\underset{r\times u}{A} = \begin{bmatrix} \vec{X}_1 & \vec{X}_2 & \cdots & \vec{X}_i \\ -a_{11}+a_{12} & -a_{21}+a_{22} & & \\ & & & -a_{i1}+a_{i2} \\ \vdots & \vdots & & \vdots \\ -a_{1j}+a_{1k} & -a_{2j}+a_{2k} & & \\ & & & -a_{ij}+a_{ik} \end{bmatrix}$$

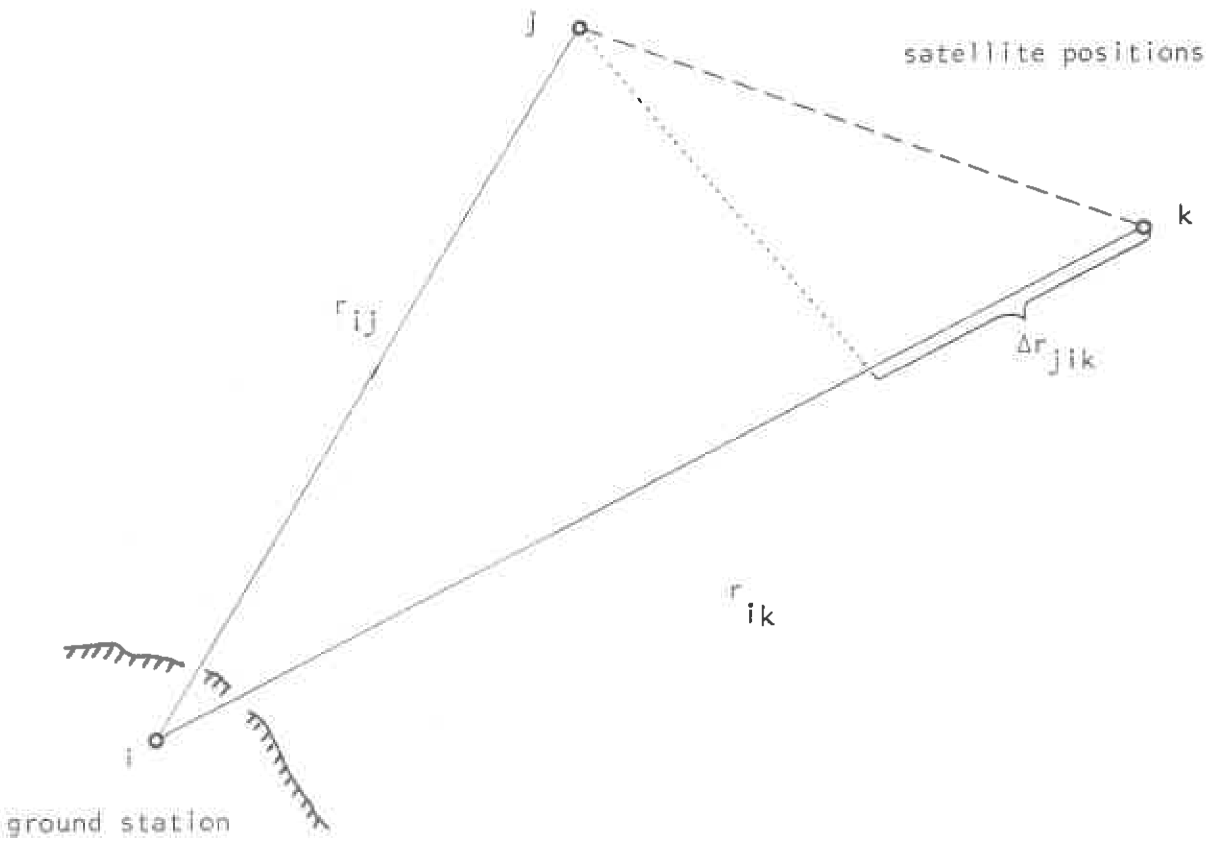

Figure 2. Satellite Range Difference

(Satellite Positions)

$$
\underset{r\times v}{C} = \begin{bmatrix}
\vec{X}_1 & \vec{X}_2 & & \vec{X}_j & \vec{X}_k \\
a_{11} & -a_{12} & & & \\
a_{21} & -a_{22} & & & \\
a_{i1} & -a_{i2} & & & \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
 & & & a_{1j} & -a_{1k} \\
 & & & a_{2j} & -a_{2k} \\
 & & & \vdots & \vdots \\
 & & & a_{ij} & -a_{ik}
\end{bmatrix}
$$

The matrix B is again a minus identity matrix which corresponds to a parametric type of adjustment.

2.5 Normal Equations.

The system of normal equations corresponding to the mathematical model of equation (1) is given in [Krakiwsky, 1968, equation 2.1 - 11], namely

$$N = A'M^{-1}A - A'M^{-1}C\ (C'M^{-1}C + P_y)^{-1}C'M^{-1}A + P_x, \quad (11)$$

where $M = BP^{-1}B'$. P_y and P_x are the weight-matrices pertaining to the approximate parameters Y^o and X^o, respectively. The solution for the unknown vector X is

$$X = - N^{-1}\ U \quad , \qquad (12)$$

where U is the absolute column; However for the simulation only the inverse N^{-1} is needed and is used in order to compute the variance - covariance matrix of the adjusted parameters, namely

$$\Sigma^{*}_{x}a = \sigma_{o}^{2} N^{-1} \quad , \tag{13}$$

where σ_o^2 is the apriori variance factor and in the simulation was set at 1 since the weight matrix was defined as the inverse of the variance-covariance matrix of the observables.

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* The inverse of the normal equation matrix is the weight coefficient matrix of the adjusted parameters even when the parameters themselves are weighted [Kouba and Krakiwsky, 1970].

3. DESCRIPTION OF SIMULATION

The purpose of a simulation is to compute the variance - covariance matrix given by equation (13) for the various situations.

3.1 Weighting of Observations

The following weighting scheme was adopted in the simulations:

(1) Total vector mode, directions $\sigma_\delta = \sigma_\alpha = 3$ **arcsec.**

ranges $\sigma_r = 10$ meters.

(2) Range mode, ranges $\sigma_r = 10$ meters.

(3) Range difference $\sigma_{\Delta r} = 1$ meter.

The equations have been arranged such that the above standard errors can be changed by any scale factor thereby changing the standard errors of the sea positions (indicated on figures) by the same factor. In some instances the above estimates are conservative, i.e. better results are certainly obtainable for at least the land based observing stations. More information indicating the accuracy of satellite observations is contained in [Mueller et al, 1970; Vonbun, 1969].

3.2 Weighting of Satellite Positions

In the simultaneous mode, the satellite positions can be treated as unknown parameters or as quasi - observables. The treatment as unknown parameters is feasible in the total vector and range modes;

while in the range difference mode significant redundancy is difficult to achieve when few stations (e.g. four) are involved in the simultaneous events. It is because of this that it is useful to weight the satellite positions. The following hypothetical values for the accuracy of the orbital parameters, from which the satellite positions are deduced, are claimed by NASA [Vonbun, 1969]:

(1) along the track, $\sigma = 38$ meters,

(2) across the track, $\sigma = 4$ meters,

(3) radial, $\sigma = 4$ meters.

It is cautioned that the weighting of the satellite positions must be in the same coordinate system as that of the fixed ground stations. If there are any datum shifts involved these must be accounted for, in fact, the latter can be weighted as well. Details on this matter can be found in [Krakiwsky, 1968, p. 157].

3.3 Simulation Arrangement

For the purpose of the simulation, four ground stations were selected to be fixed; these correspond to stations in the U.S. Coast and Geodetic Survey North American Satellite Triangulation Network, namely Halifax, St. Johns, Goose Bay, Greenland (Figure 3). In addition to these, several sea positions were chosen on the continental shelf which extend out to the limits of the shelf.

The satellite positions were simulated for two satellites; the first corresponding to a geodetic satellite of height 3,000 miles, and the second to the U.S. Navy Satellites of 600 miles (Figure 3).

All observations were generated from the above arrangements of ground, sea, and satellite positions by first converting their

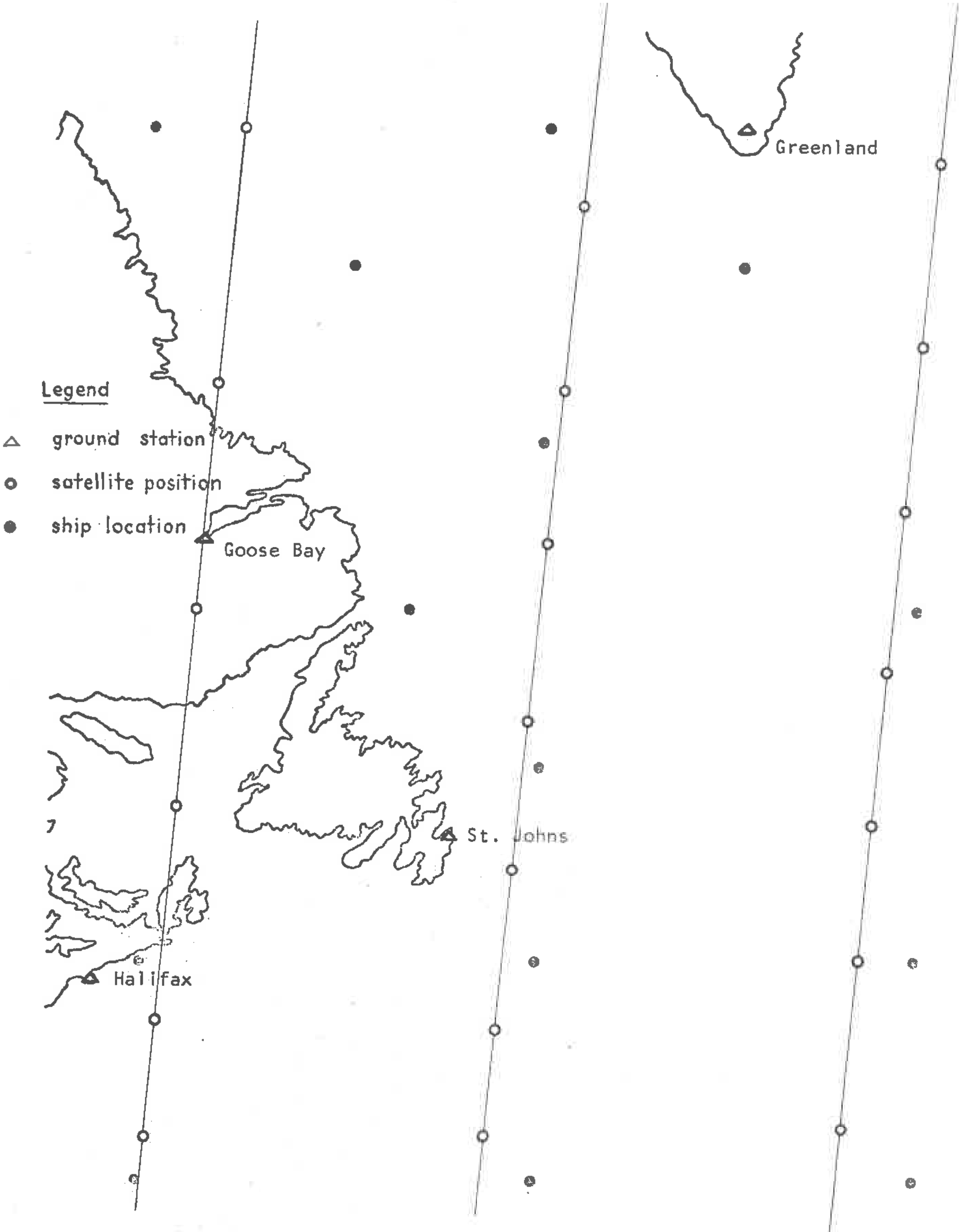


Figure 3. Simulation Scheme.

curvilinear geodetic coordinates to three-dimensional Cartesian coordinates. These Cartesian coordinates were then manipulated by the obvious formulae in order to produce for example the ranges, range differences, etc.

The satellite positions were chosen along "passes" at intervals of one to two minutes. This arrangement was found useful as numerous positions were made possible at a spacing which approximately corresponds to the "two minute doppler count interval" of the U.S. Navy Navigation Satellite System [Newton, 1966, p. 3].

3.4 Programming Considerations

All together four programmes were written in order to process the data (Figure 4).

The first FORTRAN IV programme was used to generate the data (observations) and output on tape. These observations served as input to another FORTRAN IV programme which computed the design matrices for the various modes. These design matrices were available on tape file and used as input to a MATLAN programme which computed the variance - covariance matrix of the sea positions. The last programme transformed the latter matrix from three-dimensional Cartesian coordinates into curvilinear geodetic coordinates according to

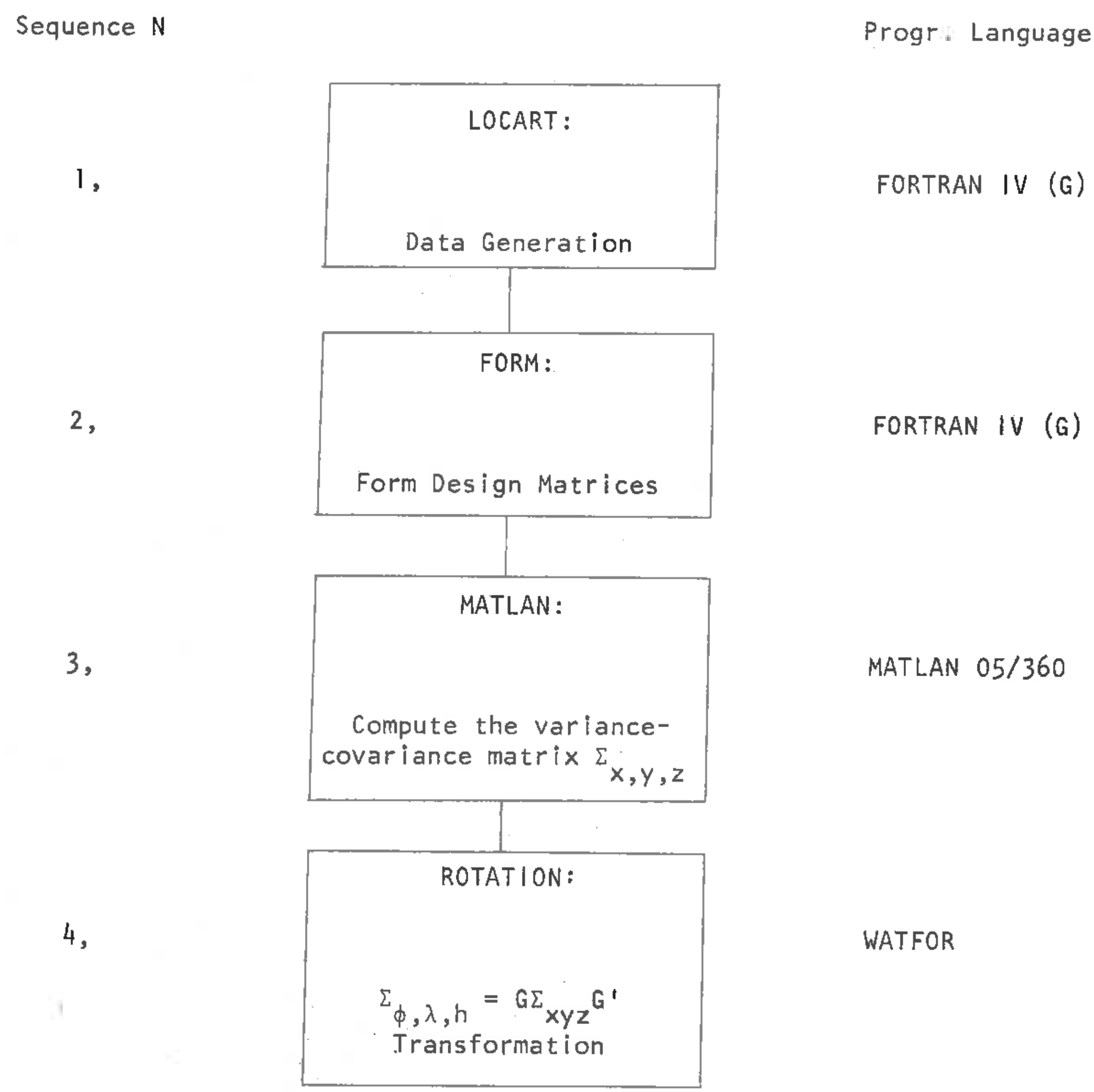
$$\Sigma_{\phi,\lambda,h} = G \, \Sigma_{x,y,z} \, G' \quad ,$$

where

$$G = \begin{bmatrix} -\sin\phi \cdot \cos\lambda & -\cos\phi \cdot \sin\lambda & \cos\phi \cdot \cos\lambda \\ -\sin\phi \cdot \sin\lambda & \cos\phi \cdot \cos\lambda & \cos\phi \cdot \sin\lambda \\ \cos\phi & 0 & \sin\phi \end{bmatrix} .$$

The description of a similar SCATRAN subroutine can be found in [Krakiwsky, et al, 1967, p. 23].

Figure 4, Programming Scheme



Note: Programs 3 and 4 could have been replaced by one MATLAN Program with a FORTRAN IV (G) subroutine. However, at the time the program was written, "peculiarities" of the MATLAN language did not permit this.

4. REPRESENTATION AND DISCUSSION OF RESULTS

The various satellite simulation computer runs are tabulated in Table 1 and the results of each is represented on a separate figure immediately following.

It is emphasised that the purpose of these simulations is not to form the basis for the deduction of "general laws", but to give an indication of what accuracy can be expected when given a certain mode, accuracy of observations, and net configuration. With this in mind, the following comments can be made with respect to the simulations.

4.1 Satellite Trilateration Mode

Runs 1 to 3 (for the high satellite) indicate that ranges to a single pass are not satisfactory; In fact several passes are needed as indicated by Run 4.

Runs 5 to 7 (for the low satellite) indicate again that a single pass is not adequate for an accurate "fix". The results are, however, better than for the high satellite.

Note that in all the figures a change in the accuracy of the observations changes the results by the same scale factor.

4.2 Satellite Range Difference Mode

This mode was computed for a low satellite only as its height

Table 1. List of Satellite Simulation Computer Runs

Run No.	Mode	Satellite Passes (height-miles)	No. Ground Sta.	No. Range Differences or Satellite Pos. per pass	Weighted Satellite Positions	Correlated Observ.	Page
1	Trilateration	1(3000)	4	6	no	no	23
2	Trilateration	1(3000)	4	7	no	no	24
3	Trilateration	1(3000)	4	7	no	no	25
4	Trilateration	3(3000)	4	6,7,7	no	no	26
5	Trilateration	1 (600)	4	6	no	no	27
6	Trilateration	1 (600)	4	7	no	no	28
7	Trilateration	1 (600)	4	7	no	no	29
8	Range Diff.	1 (600)	0	5	yes	no	30
9	Range Diff.	1 (600)	0	6	yes	no	31
10	Range Diff.	1 (600)	1	6	yes	yes	32
11	Range Diff.	1 (600)	2	6	yes	no	33
12	Range Diff.	1 (600)	3	6	yes	yes	34
13	Range Diff.	1 (600)	4	5	no	no	35
14	Range Diff.	1 (600)	4	6	no	no	36
15	Range Diff.	1 (600)	4	6	no	no	37
16	Range Diff.	3 (600)	4	5,6,6	no	no	38

Table I. (cont'd)

Run No.	Mode	Satellite Passes (height-miles)	No. Ground Sta.	No. Range Differences or Satellite Pos. per pass	Weighted Satellite Positions	Correlated Observ.	Page
17	Range Diff.	1 (600)	4	5	no	yes	39
18	Range Diff.	1 (600)	4	6	no	yes	40
19	Range Diff.	1 (600)	4	6	no	yes	41
20	Range Diff.	3 (600)	4	5,6,6	no	yes	42
21	Total Vector	1 (600)	3	6	no	no	43
22	Total Vector	1 (600)	3	7	no	no	44
23	Total Vector	1 (600)	3	7	no	no	45
24	Total Vector	3 (600)	3	6,7,7	no	no	46
25	Total Vector	1 (600)	3	6	no	no	47
26	Total Vector	1 (600)	3	7	no	no	48
27	Total Vector	1 (3000)	4	6	no	no	49
28	Total Vector	1 (3000)	4	7	no	no	50
29	Total Vector	1 (3000)	4	7	no	no	51
30	Total Vector	1 (600)	4	6	no	no	52
31	Total Vector	1 (600)	4	7	no	no	53
32	Total Vector	1 (600)	4	7	no	no	54
33	Total Vector	1 (600)	4	6,7,7	no	no	55

of 600 miles corresponds to that of the four existing doppler satellites of the U.S. Navy.

Runs 8 to 12 indicate the accuracy that can be expected when the number of ground stations involved in the simultaneous observations is small thereby necessitating the weighting of satellite positions in order to get redundancy or even a solution. Increasing the number of ground stations to one as in Run 10, or to two as in Run 11, or to three as in Run 12, significantly improves the accuracy (note that the satellite positions are still weighted).

Runs 13 to 16 indicate the accuracy that can be expected when employing the "pure simultaneous mode", that is, satellite positions along with a sea position are all taken as unknown; No orbital elements are required. For example in Run 13, the number of observations are five (range differences) times five (observing stations) equals 25; the number of unknowns are six (satellite positions) times three (X,Y,Z) equals 18, plus three (X,Y,Z for sea position) equals a total of 21; the redundancy is four. The one-pass runs indicate that the latitude is determined better than the longitude. Further, the longitude is better determined for those sea positions which are closest to the centroid of the ground station configuration and far from the line of the pass; however, the closeness to the centroid seems to dominate.

Run 16 gives the accuracy for three passes. Many more passes could be observed, especially when positioning an "anchored rig" as opposed to a "moving vessel". Again it is emphasised that changing the standard deviation of the range difference from 1 to 3 meters would simply increase the standard deviations of the latitude, longitude, and height of the sea positions by a factor of three.

Runs 17 to 20 indicate the increase in sea position accuracy when known covariance between adjacent range differences is encorporated.

Runs 16 and 20 indicate that those sea stations located closest to the centroid of the land station configuration are more accurately determined than that for far off sea stations (e.g. edge of continental shelf).

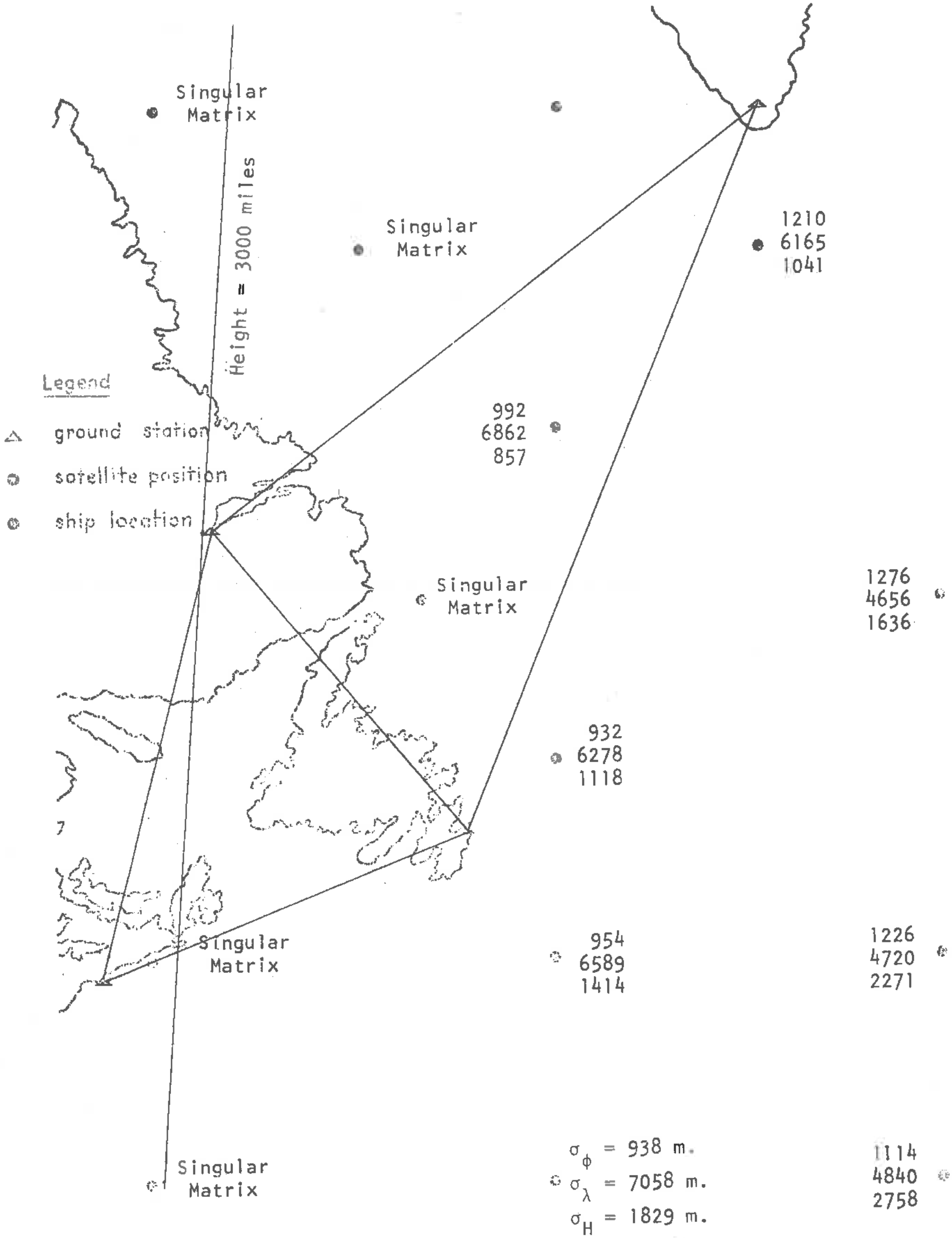
4.3 Satellite Total Vector Mode

The total vector mode has been computed for both high and low satellites. The number of ground based stations were varied from three to four.

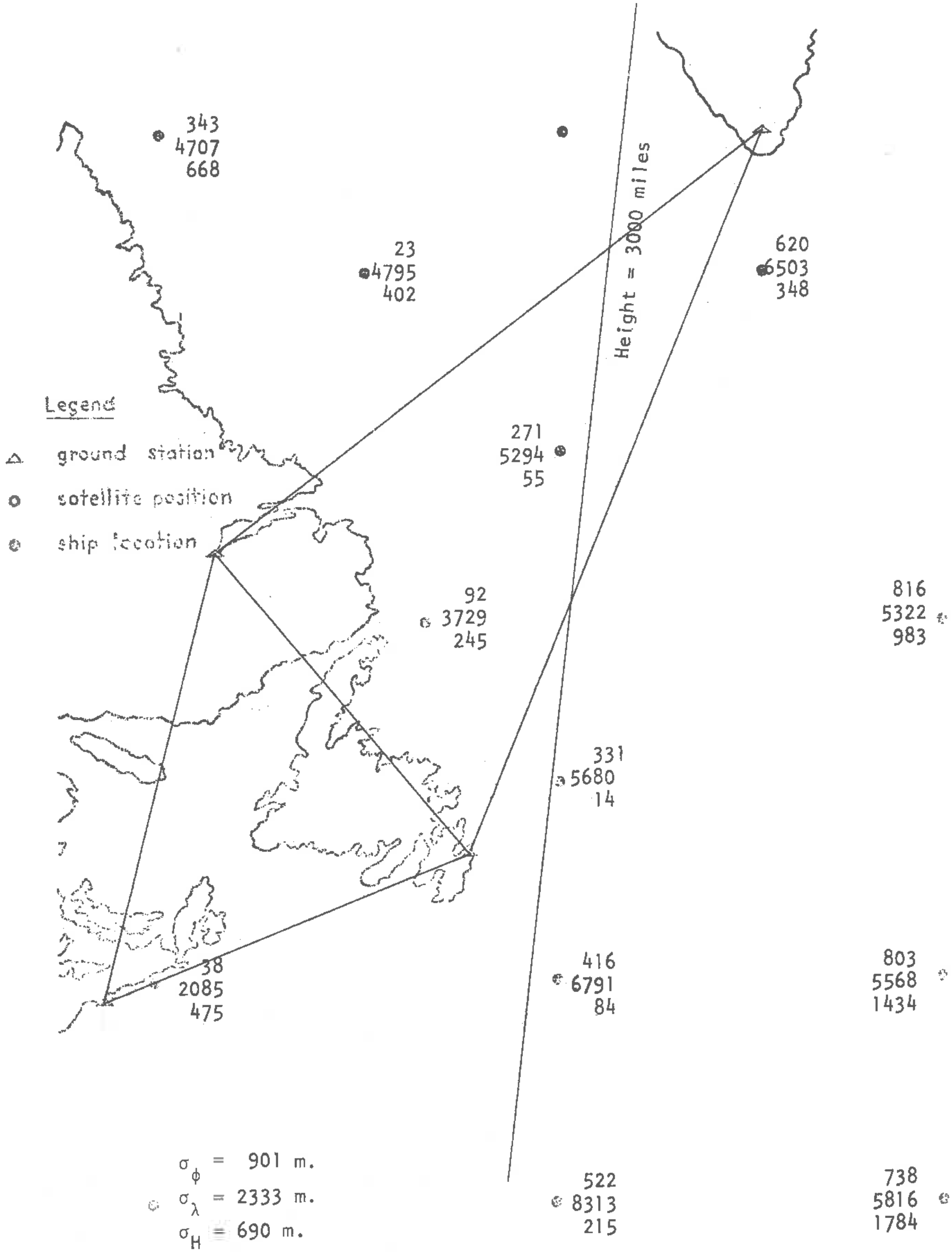
Runs 21 to 24 and Runs 25 to 26 indicate that a relatively high accuracy can be achieved in the sea position even with a conservative estimate of 3 arcsec. standard error for the satellite directions. The location of the sea position and three-ground station configuration, within the area studied, does not affect positional accuracy significantly. Run 24 indicates the high accuracy that can be obtained when several passes would be observed.

Runs 30 to 33 indicate the results obtainable when four ground based stations are involved in the simultaneous scheme; No significant increase in accuracy of the sea position is obtained due to the fourth station.

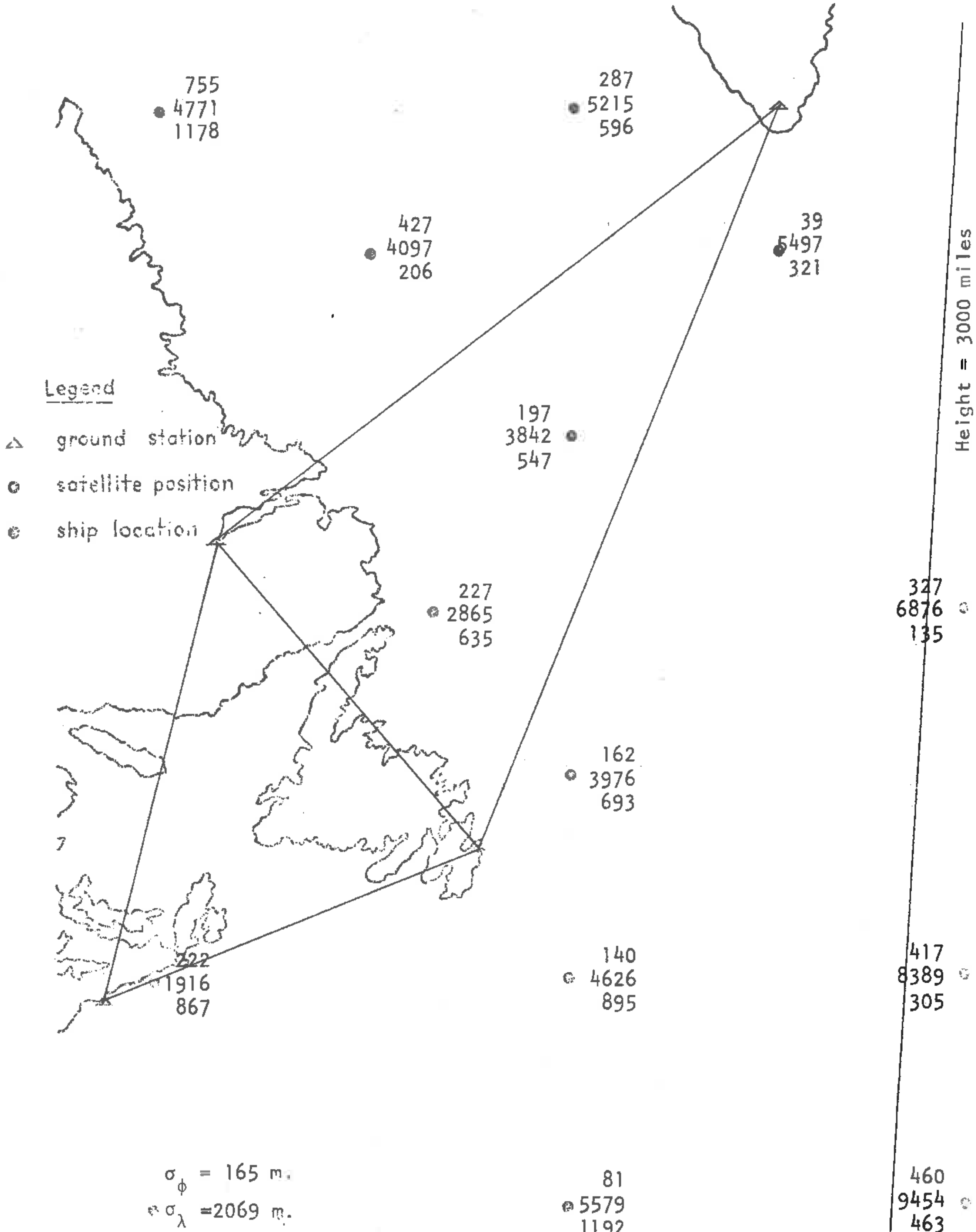
Runs 27 to 29 indicate a four ground station configuration scheme for a high satellite. On the average, the accuracy decreased two to three times due to the weaker geometry.



Satellite Trilateration Mode (σ_r = 10 m.)

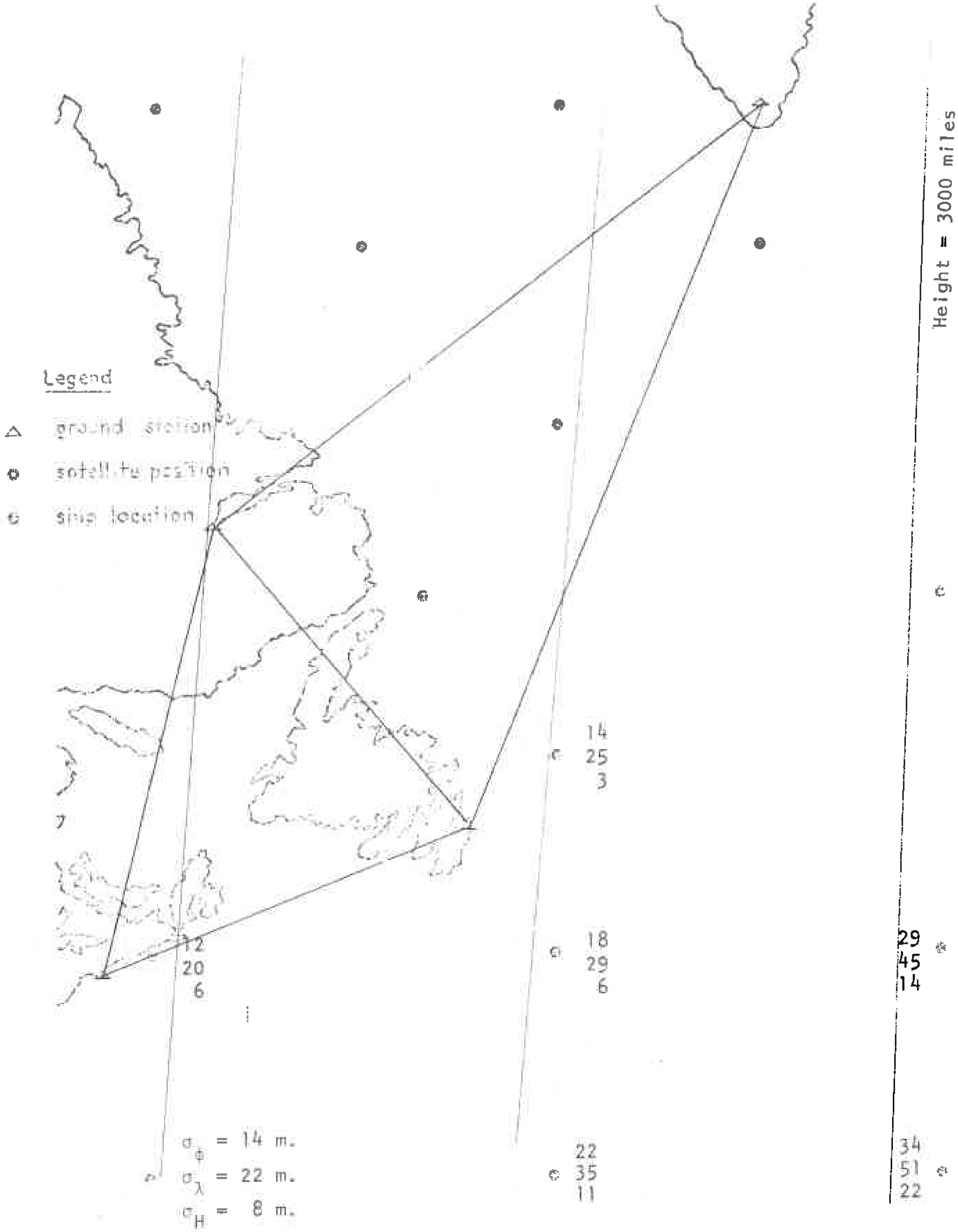


Satellite Trilateration Mode (σ_r = 10 m.)

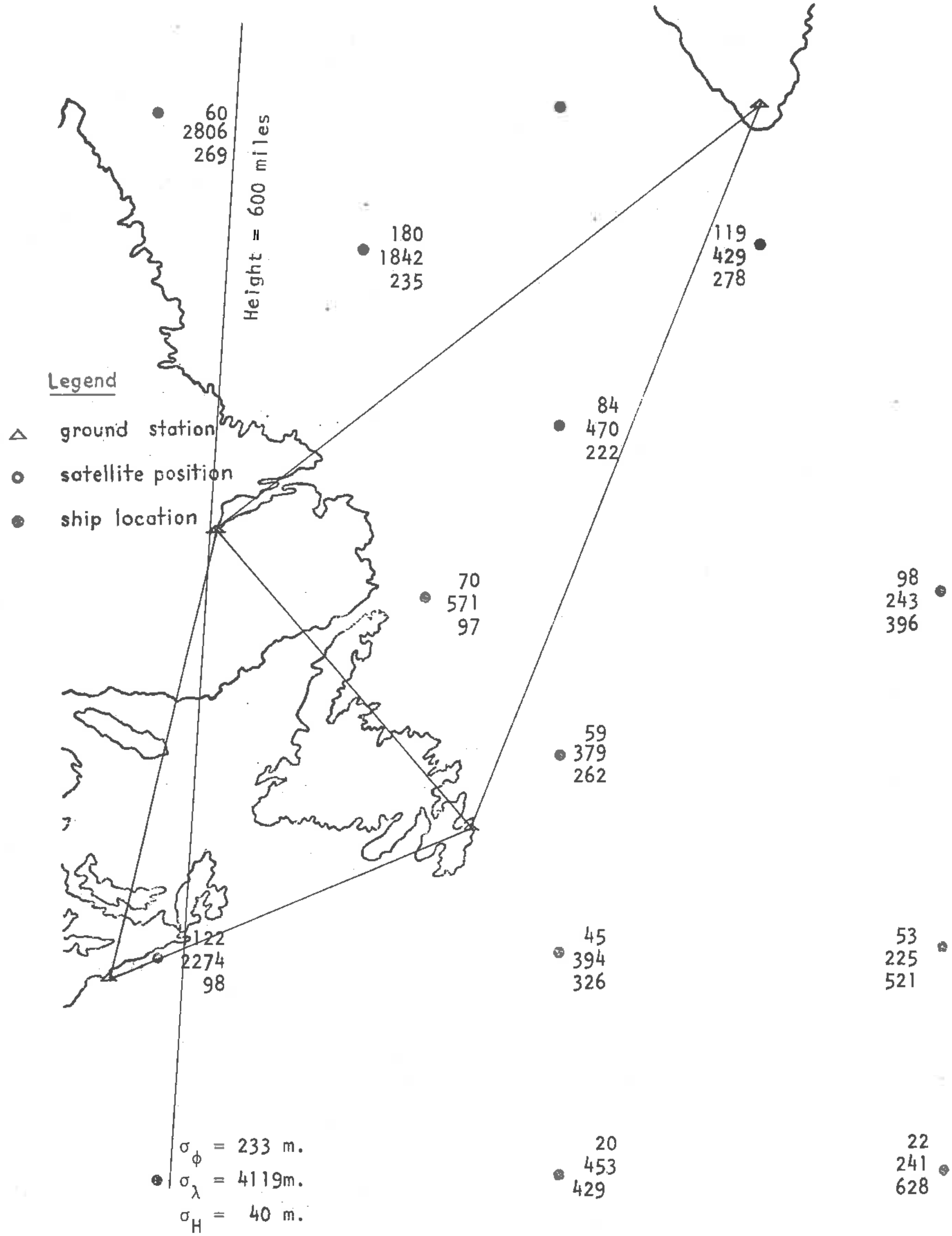


σ_ϕ = 165 m.
σ_λ =2069 m.
σ_H =1055 m.

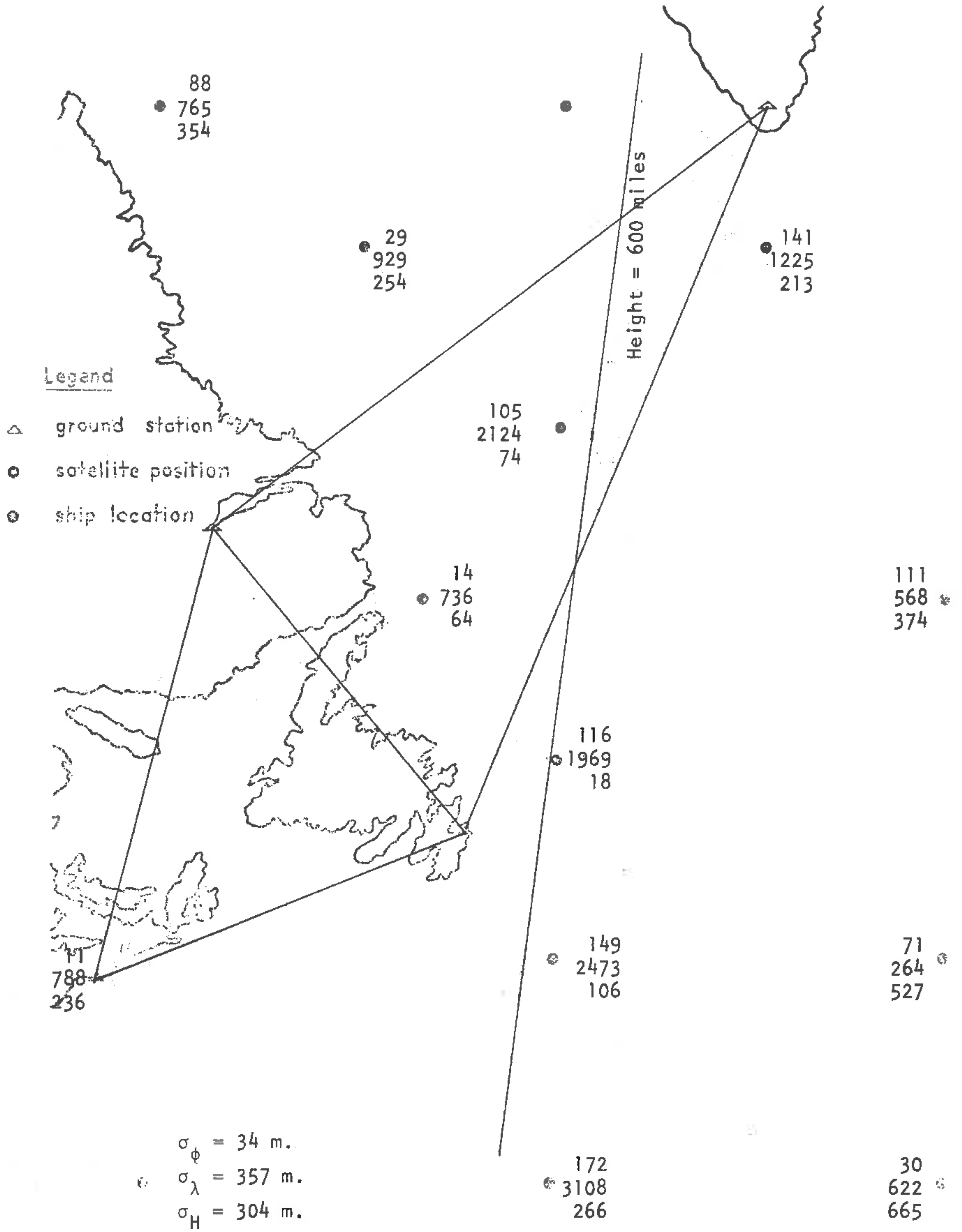
Satellite Trilateration Mode (σ_r=10 m)



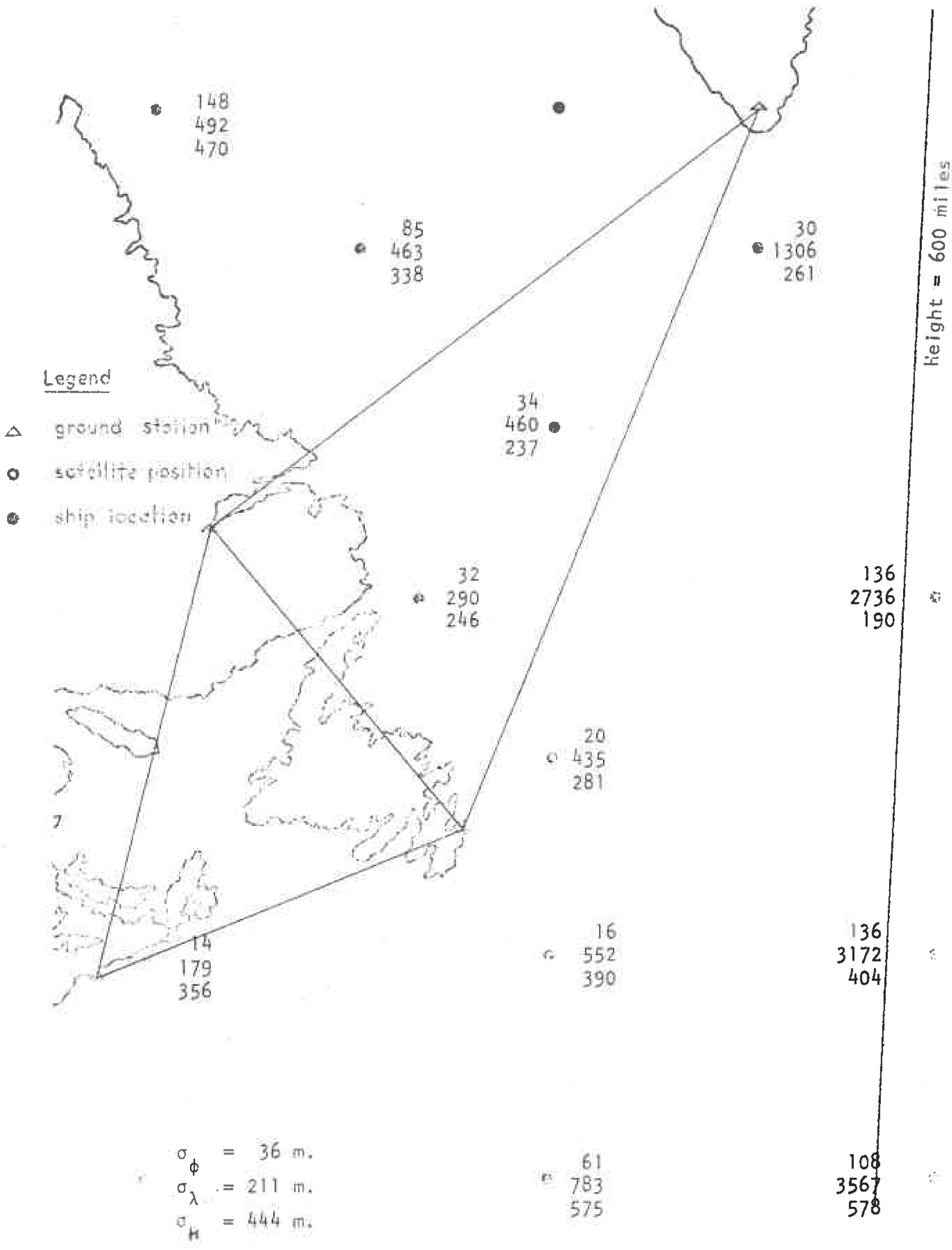
Satellite Trilateration Mode (σ_r = 10 m.)



Satellite Trilateration Mode (σ_r = 10 m.)

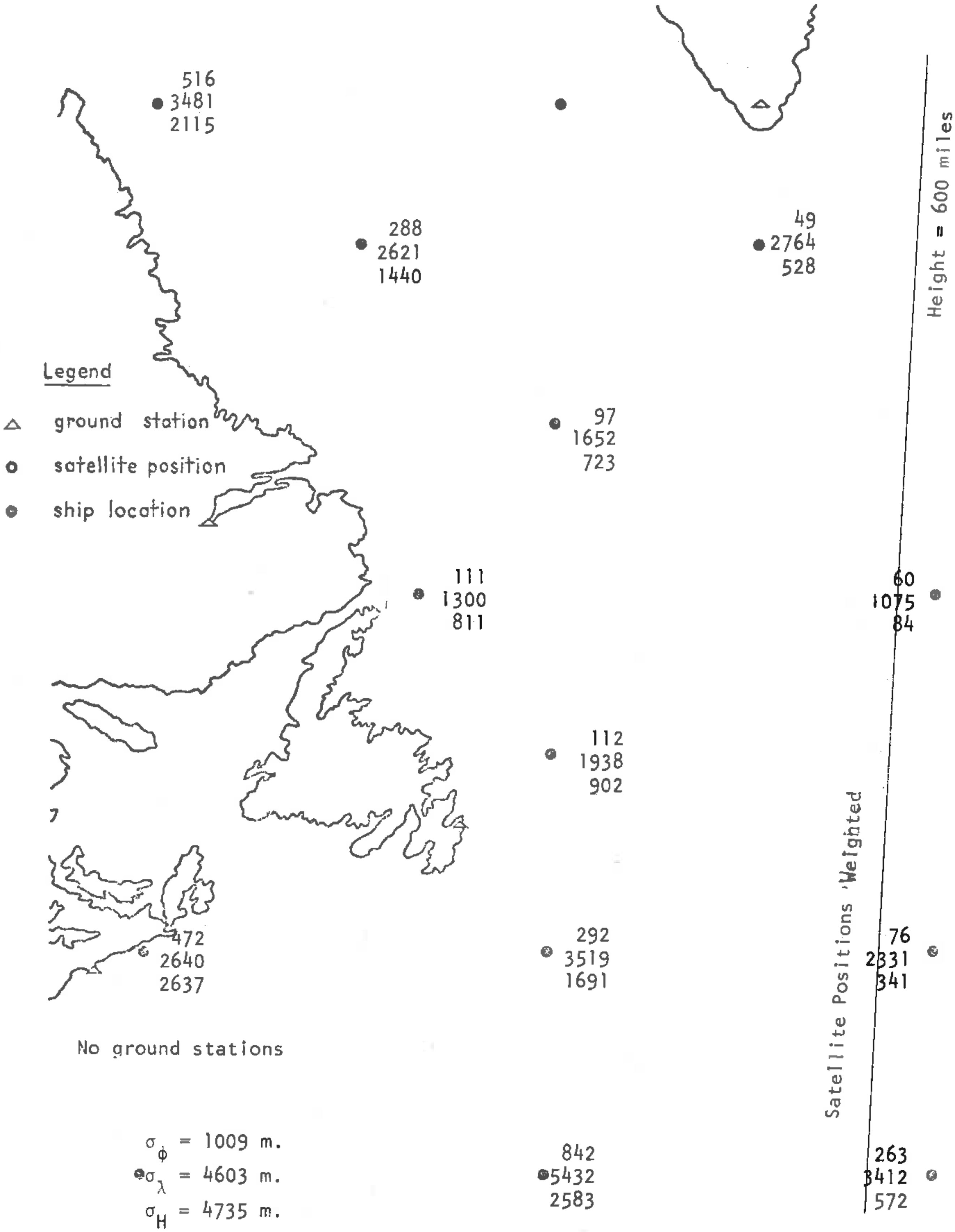


Satellite Trilateration Mode (σ_r = 10 m.)

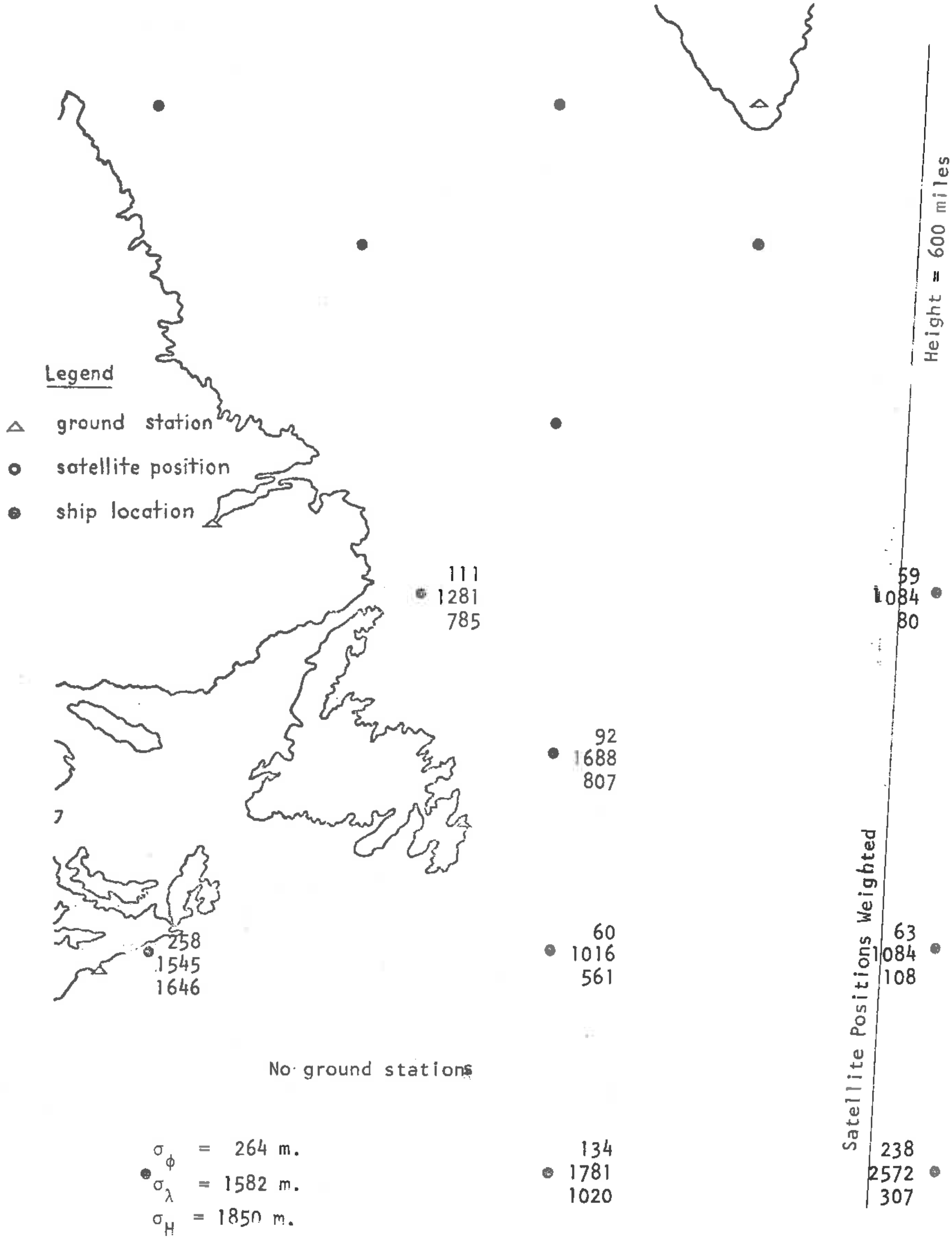


σ_ϕ = 36 m.
σ_λ = 211 m.
σ_H = 444 m.

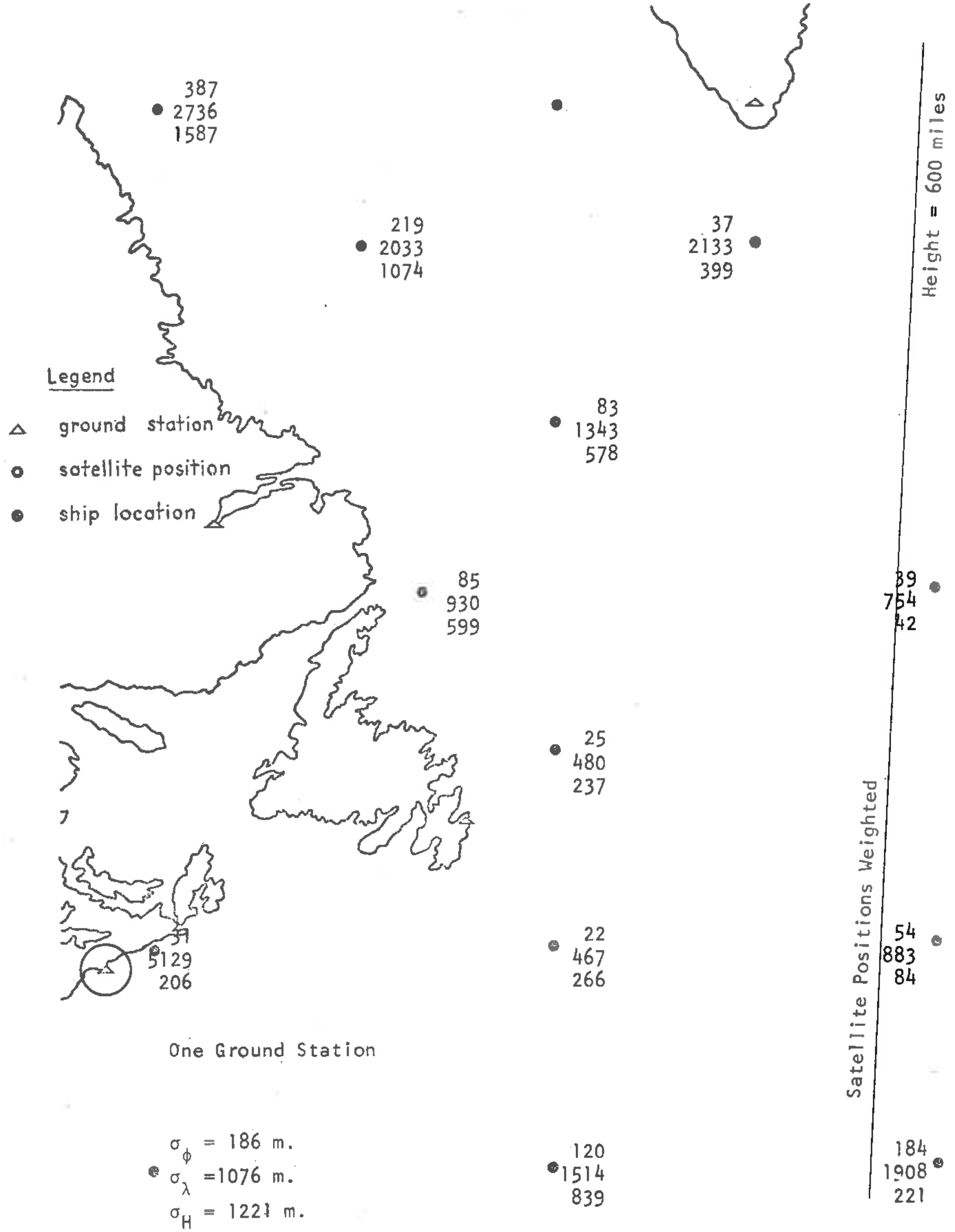
Satellite Trilateration Mode (σ_r = 10 m.)



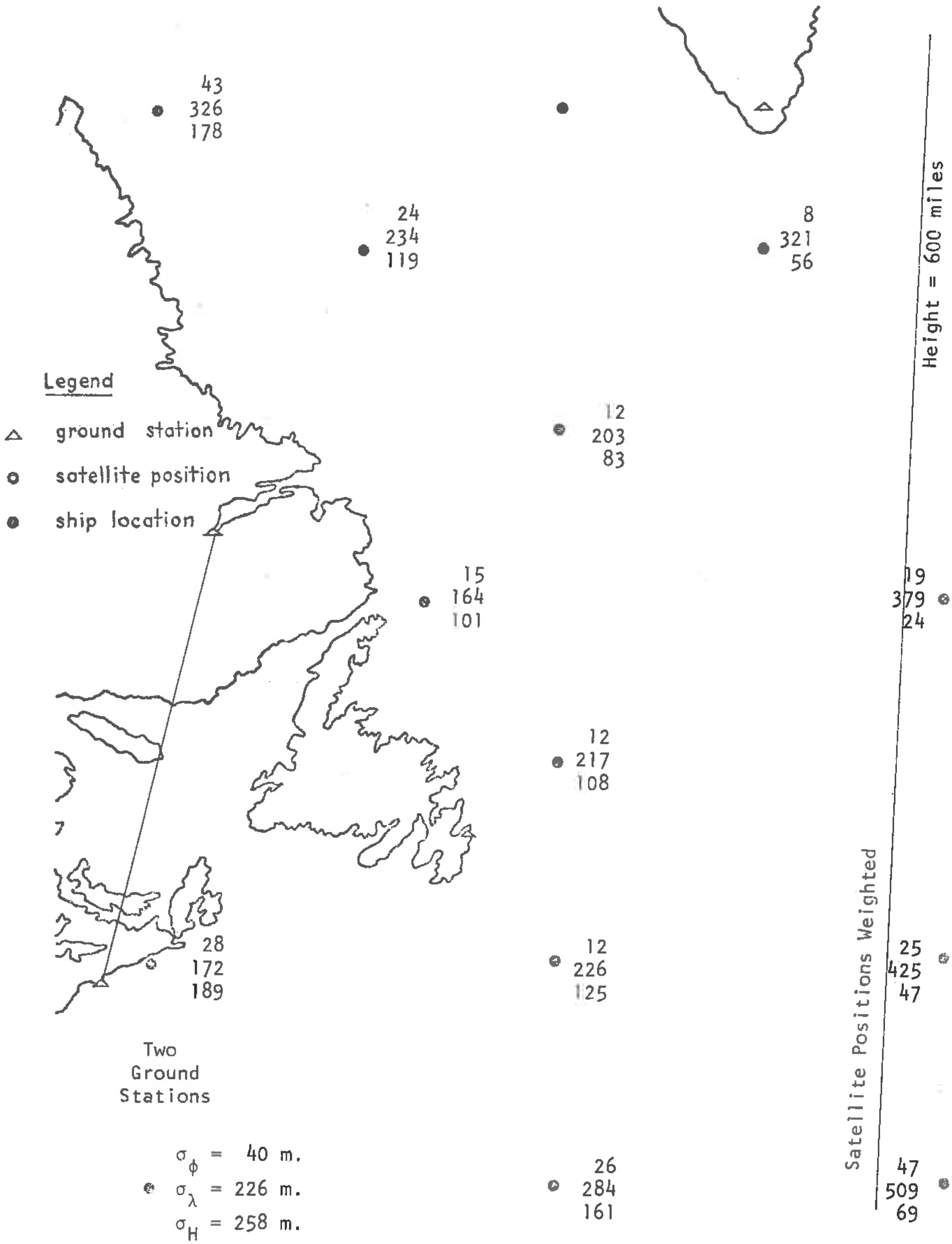
Satellite Range Difference Mode ($\sigma_{\Delta r}$ = 1 m)



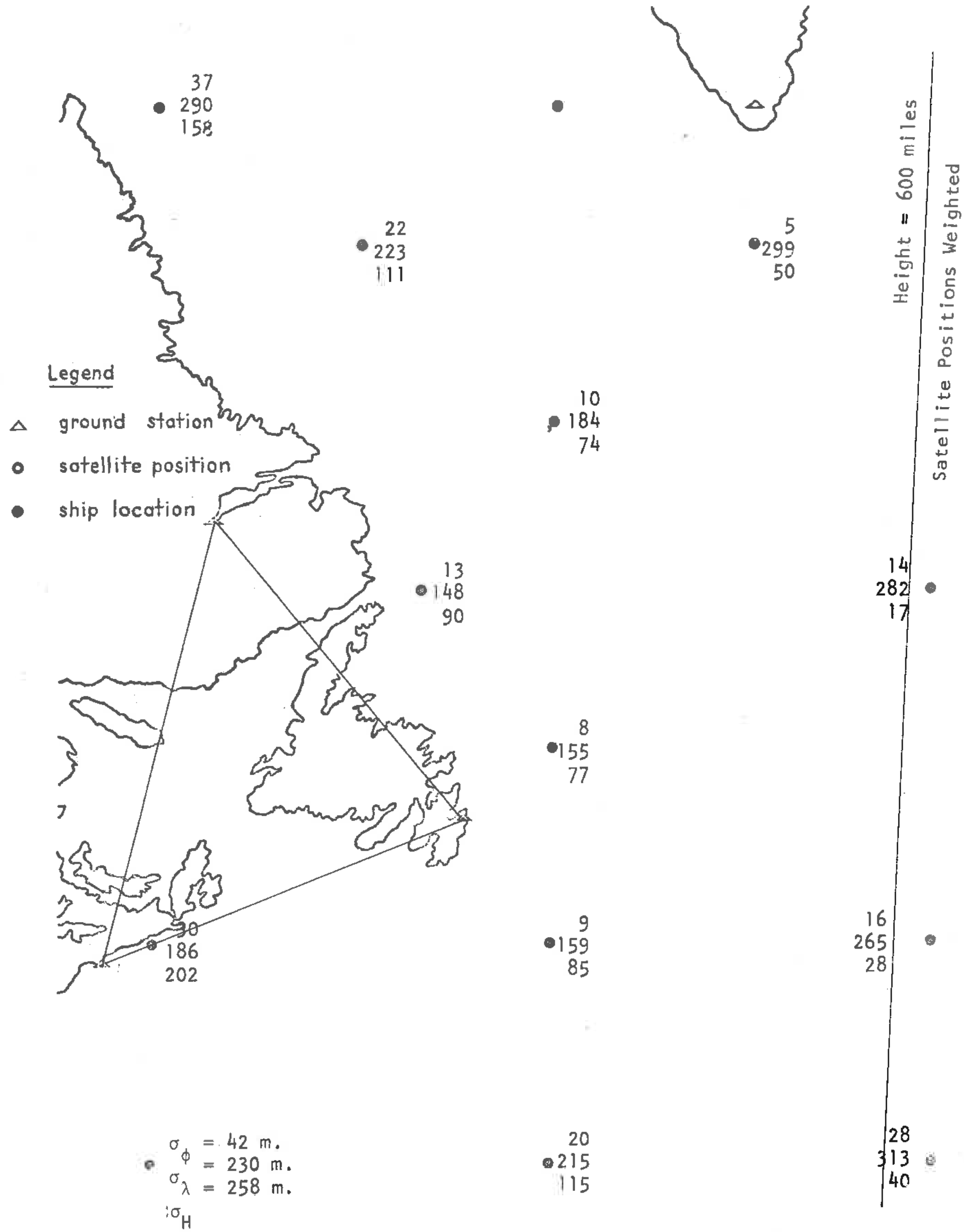
Satellite Range Difference Mode ($\sigma_{\Delta r}$ = 1 m.)



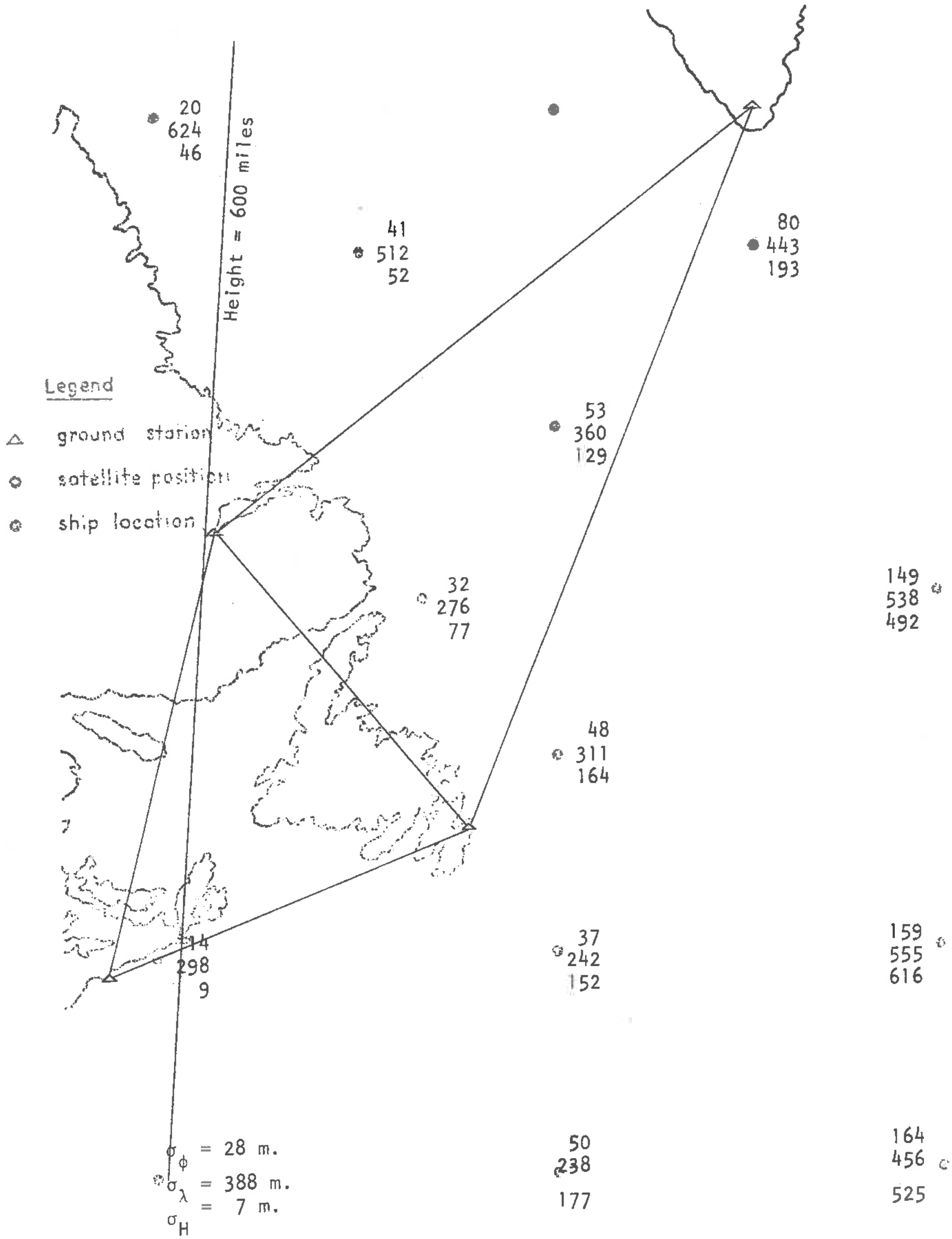
Satellite Range Difference Mode ($\sigma_{\Delta r}$ = 1 m.; covariance = 0.5 m^2)



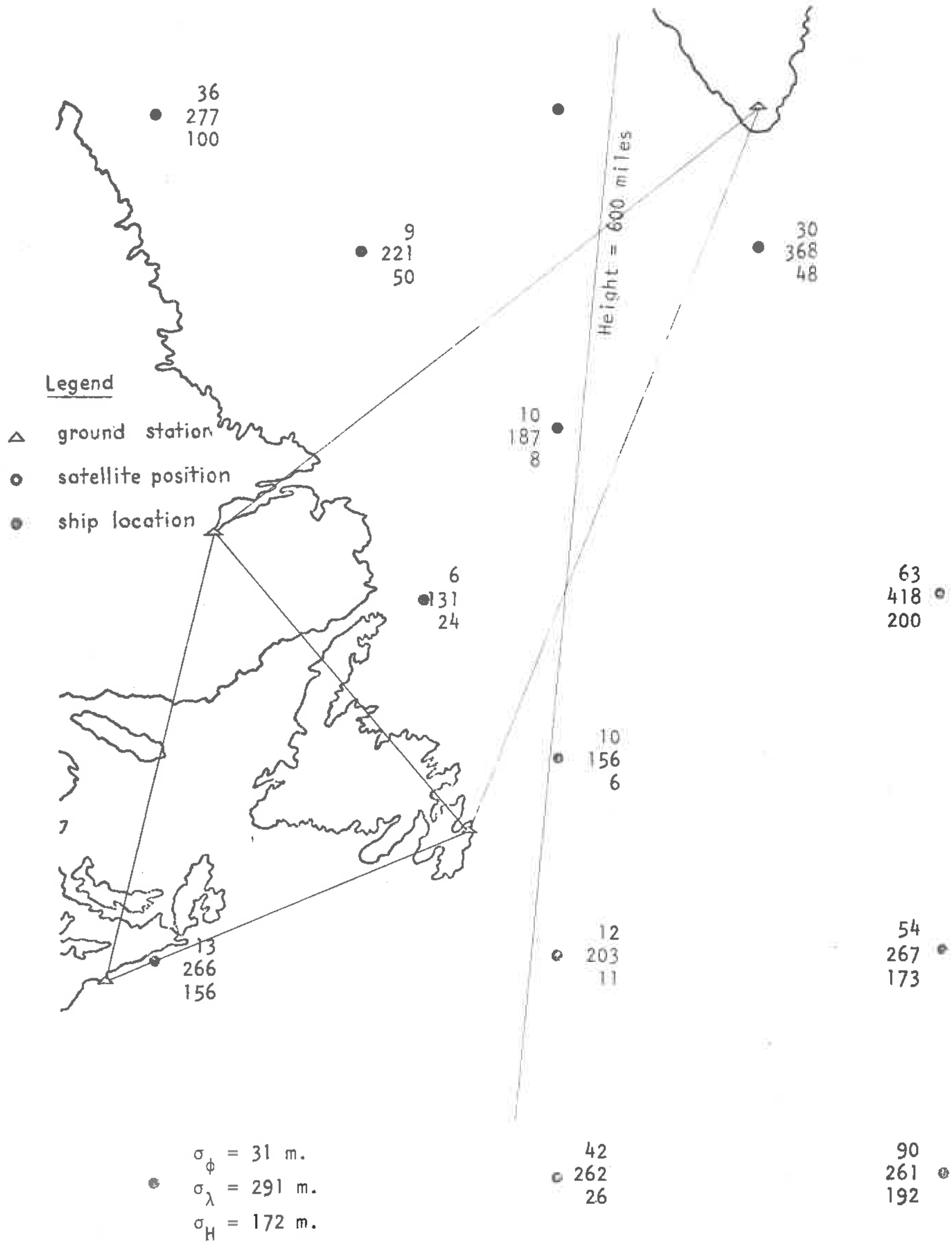
Satellite Range Difference Mode ($\sigma_{\Delta r}$ = 1 m.)



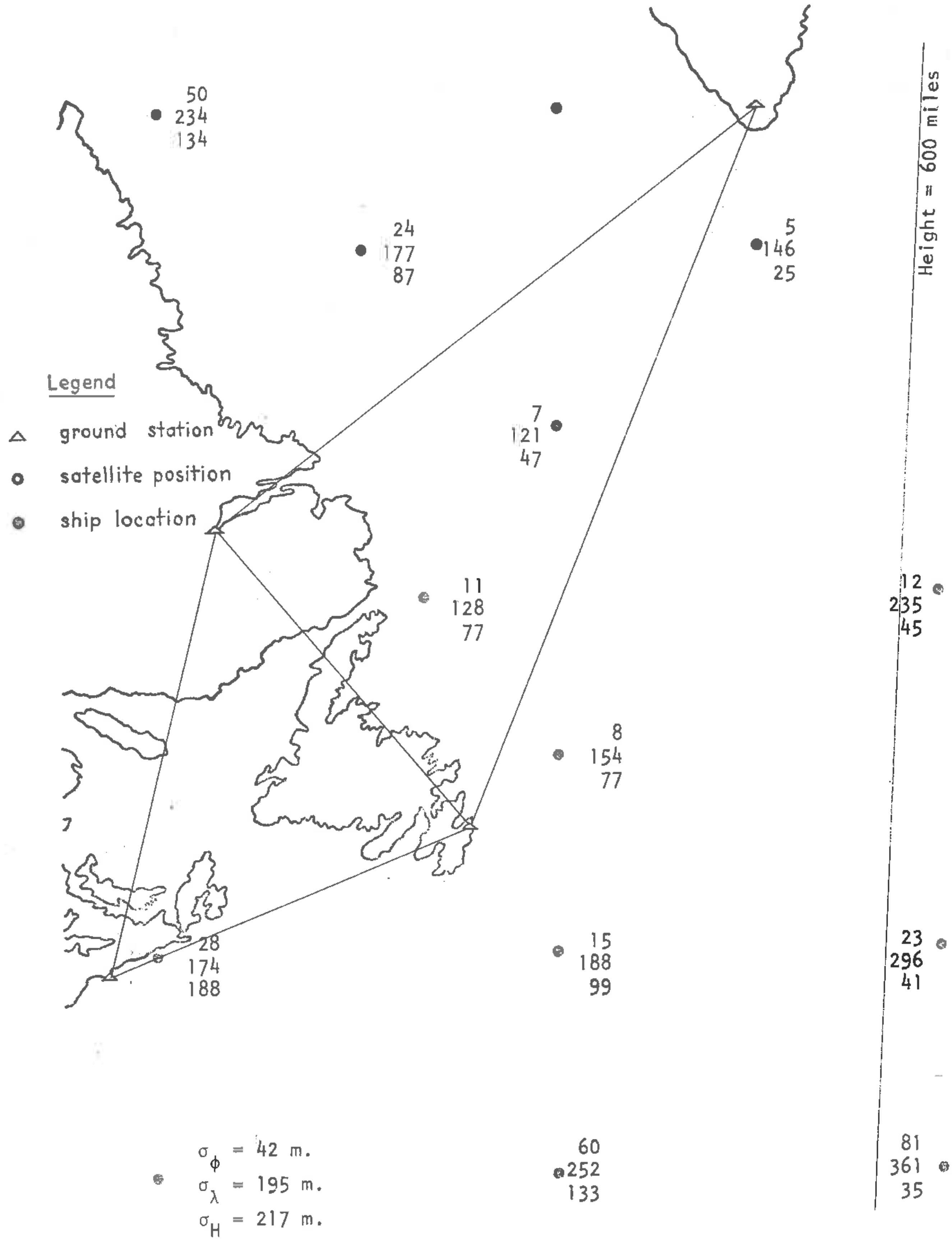
Satellite Range Difference Mode ($\sigma_{\Delta r}$ = 1 m; covariance = 0.5 m^2.)



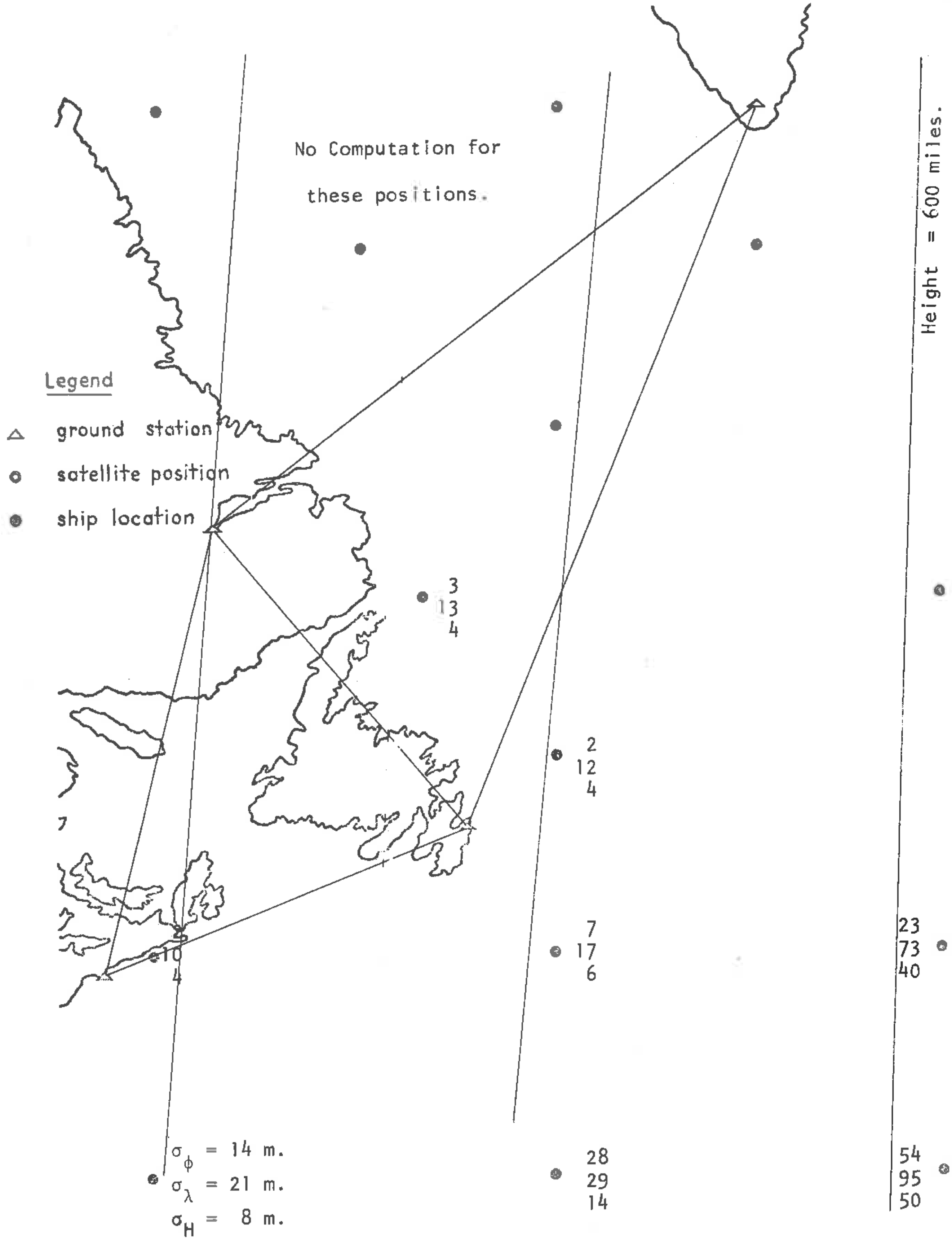
Satellite Range Difference Mode ($\sigma_{\Delta r}$ = 1 m.)



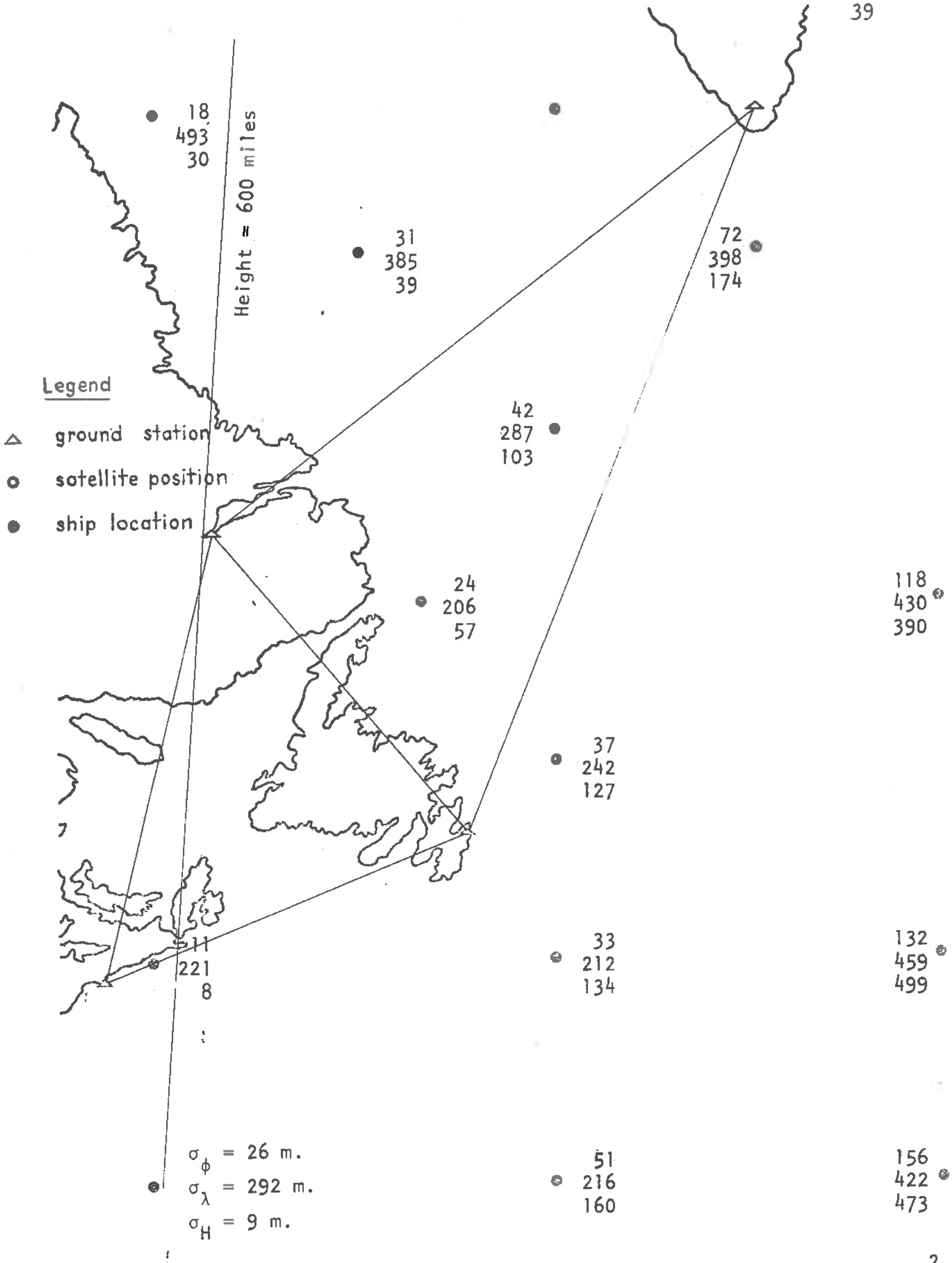
Satellite Range Difference Mode ($\sigma_{\Delta r}$ = 1 m.)



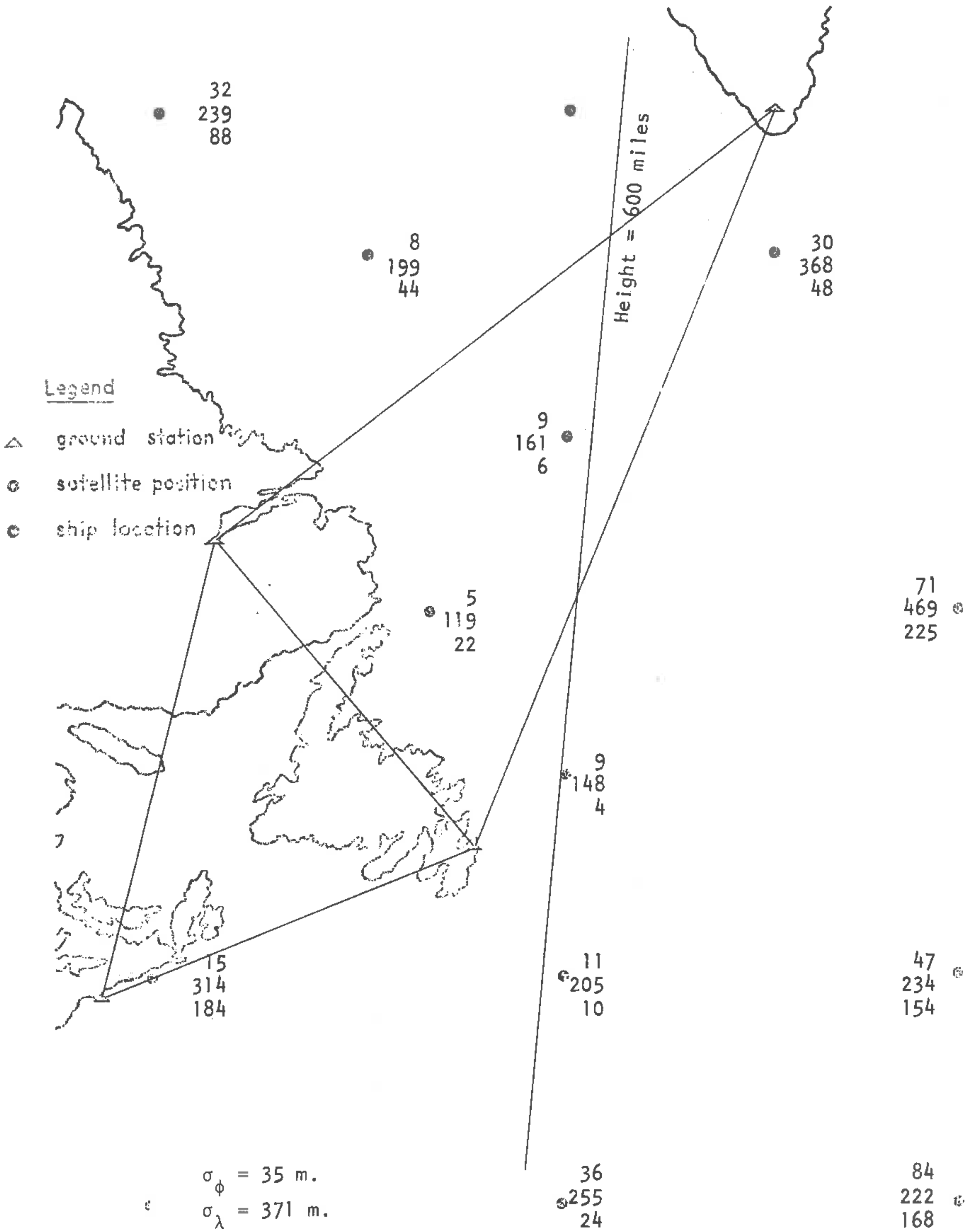
Satellite Range Difference Mode ($\sigma_{\Delta r}$ = 1m.)



Satellite Range Difference Mode ($\sigma_{\Delta r}$ = 1 m.)



Satellite Range Difference Mode ($\sigma_{\Delta r}$ = 1 m.; covariance = 0.5 $m.^2$)

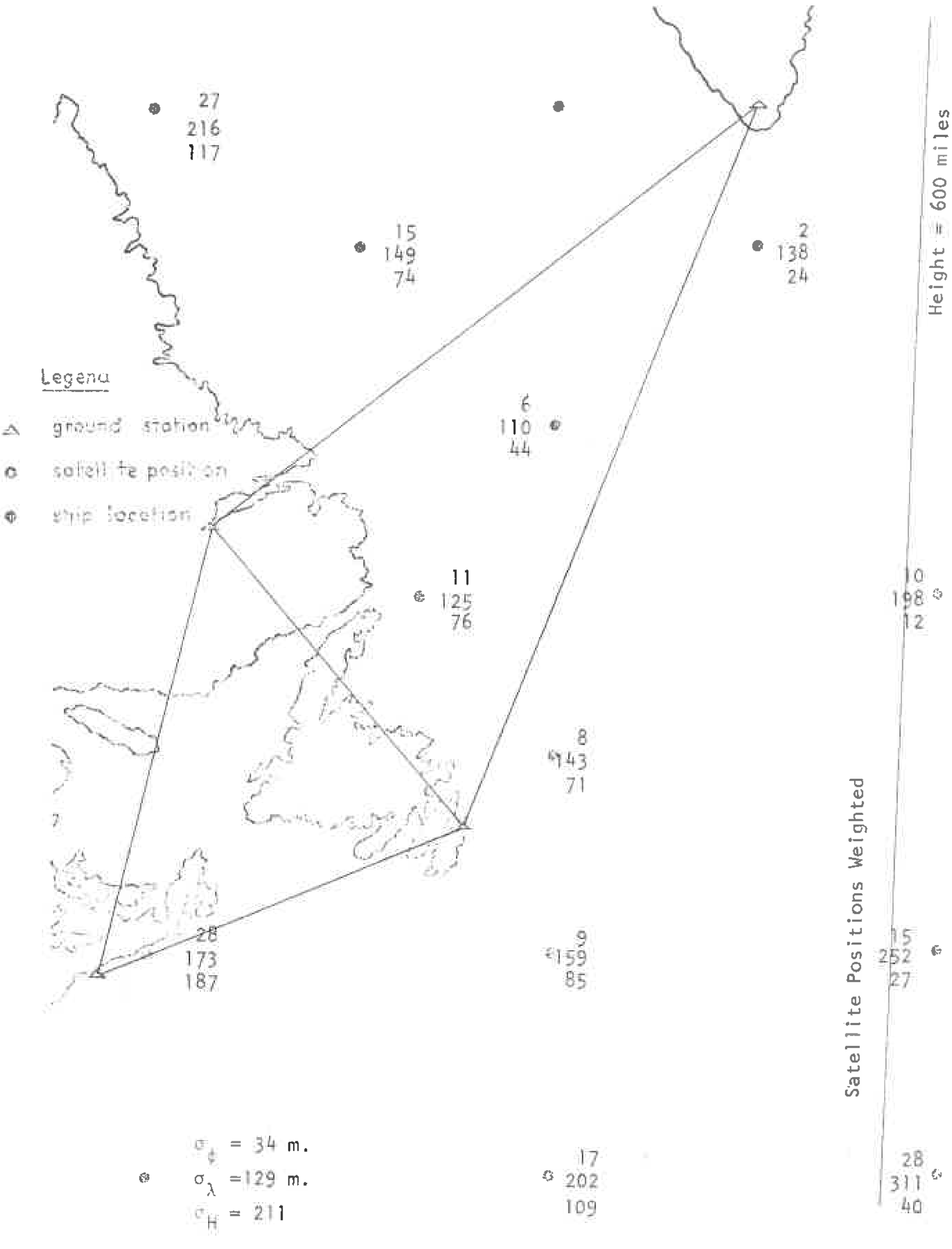


σ_ϕ = 35 m.

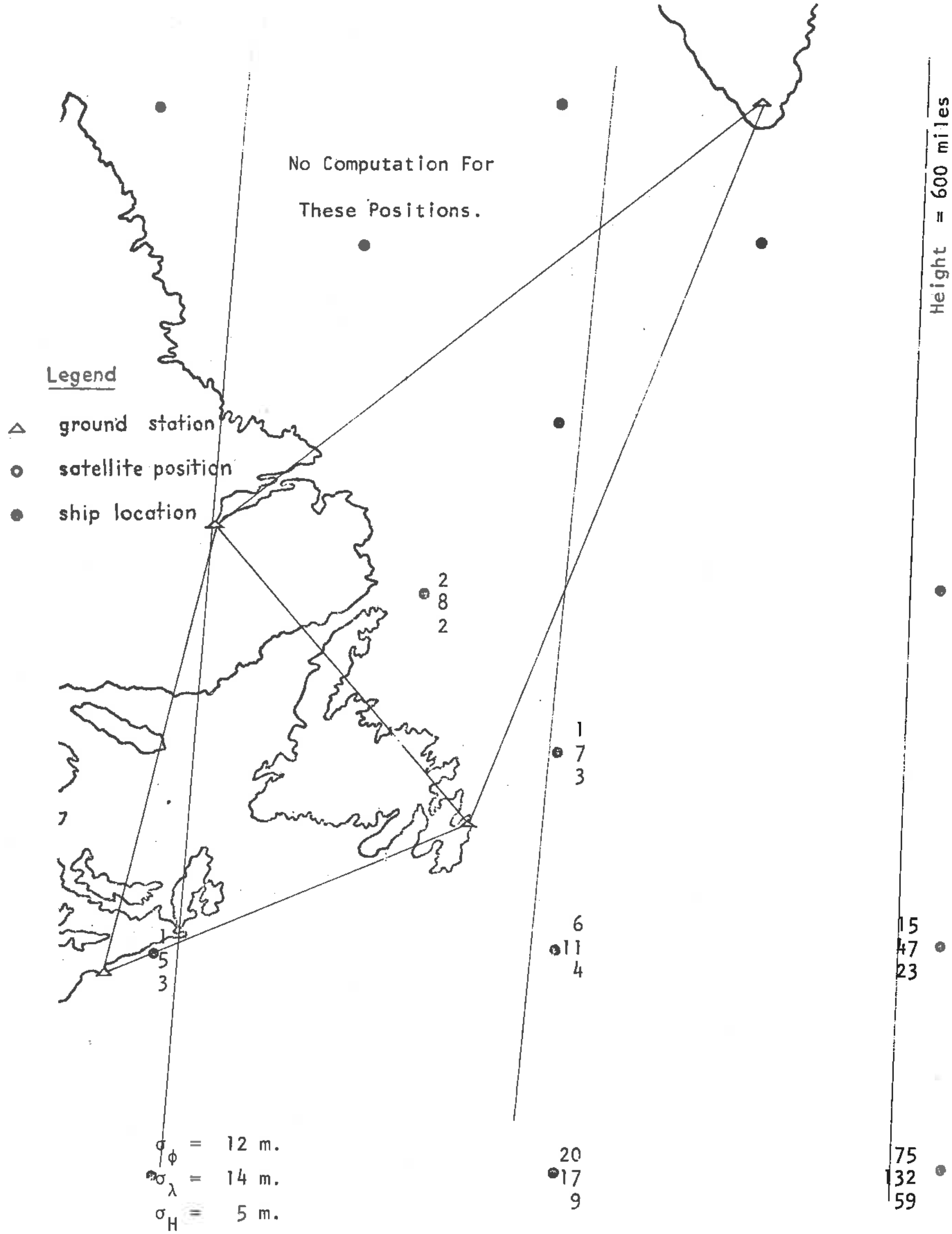
σ_λ = 371 m.

σ_H = 216 m.

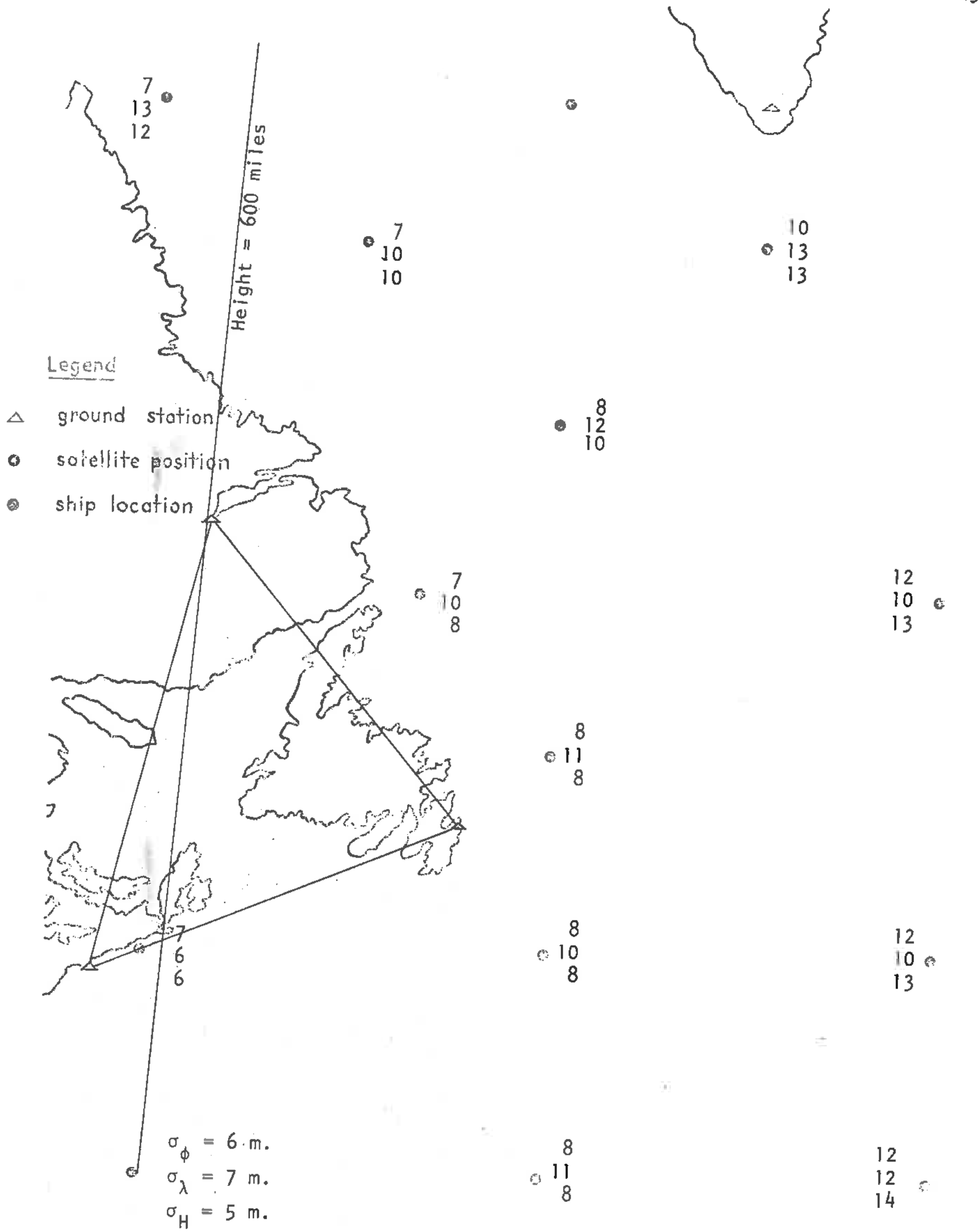
Satellite Range Difference Mode ($\sigma_{\Delta r}$ = 1 m.; covariance = 0.5 m.^{2})



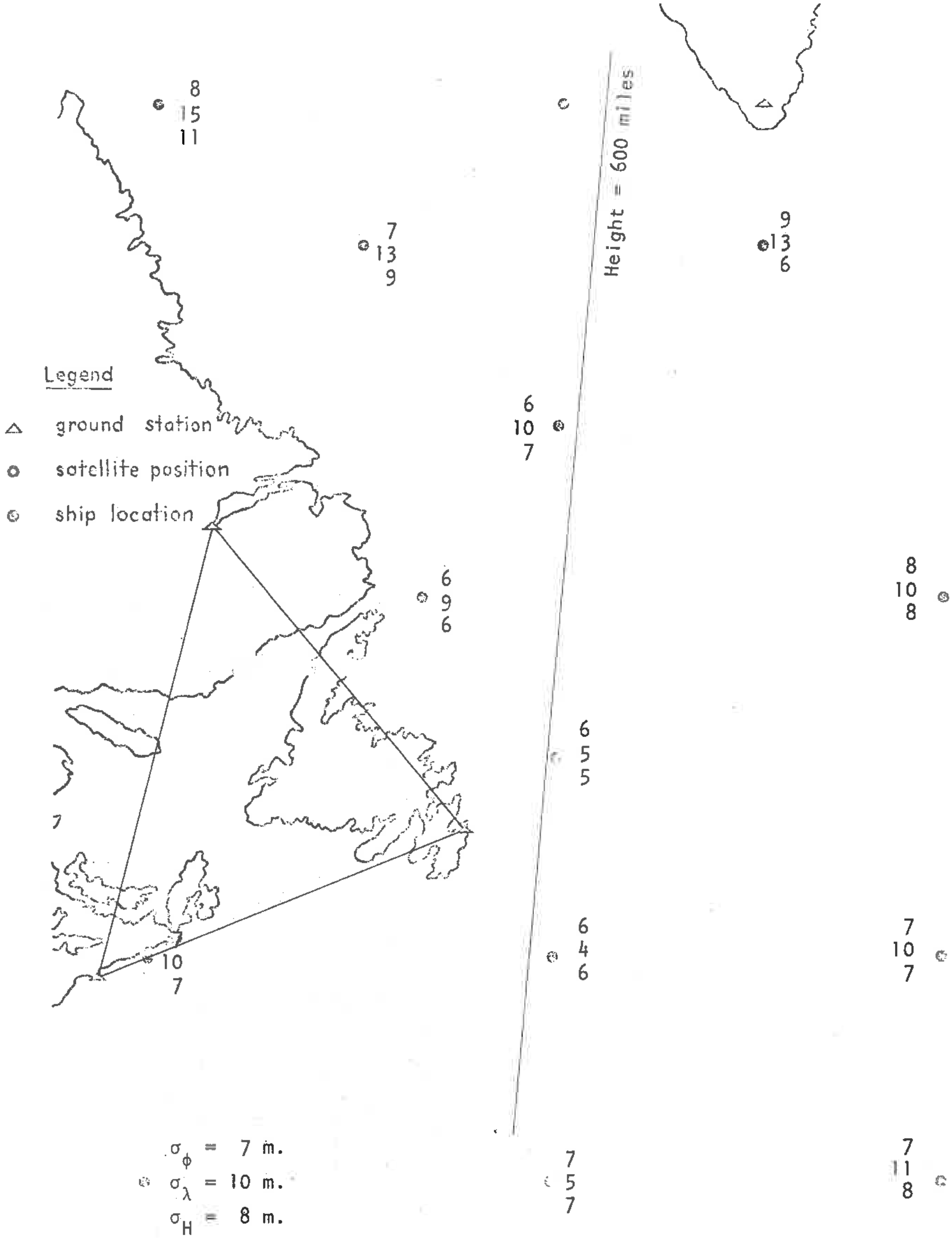
Satellite Range Difference Mode ($\sigma_{\Delta r}$ = 1m; covariance = 0.5 m^2.)



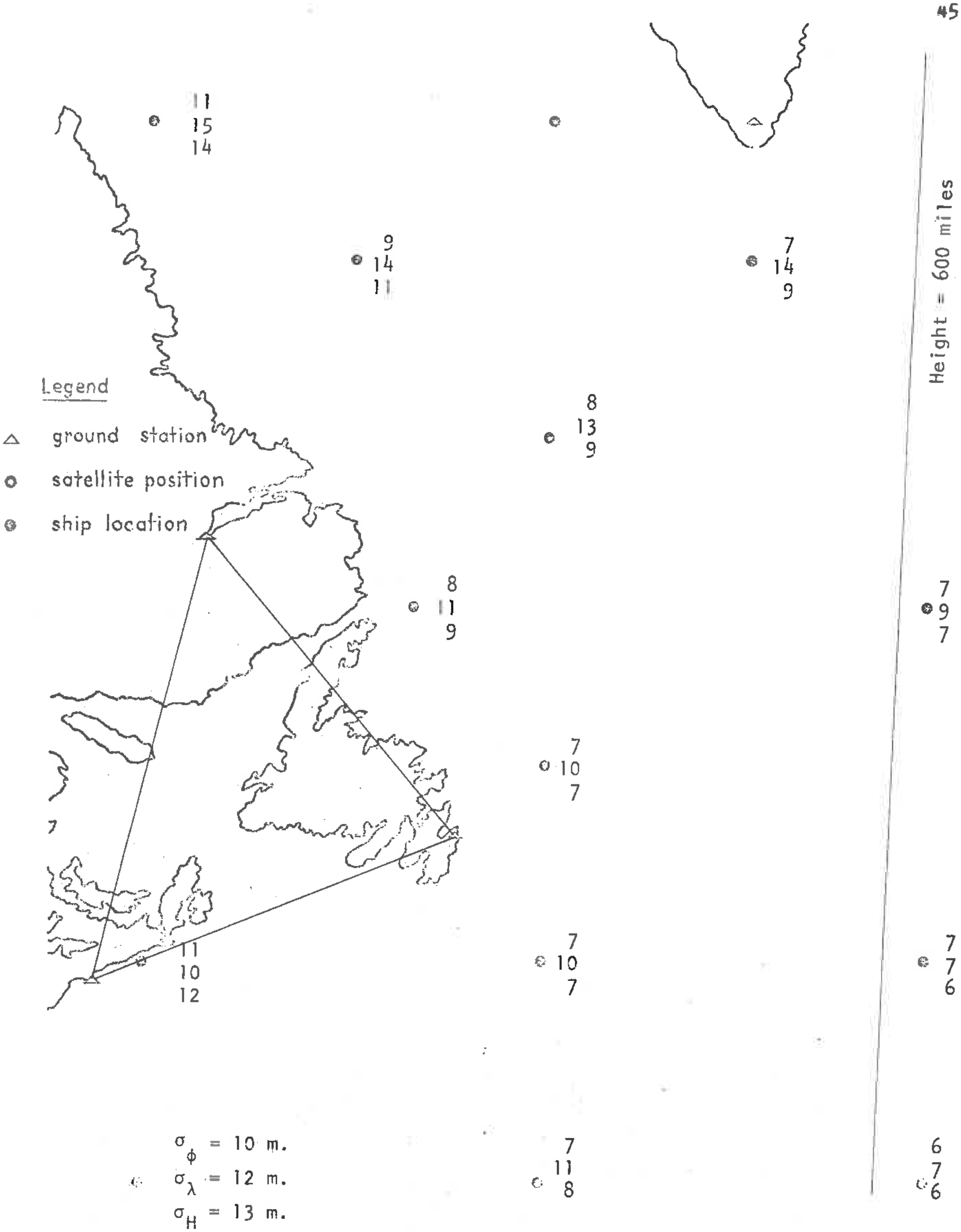
Satellite Range Difference Mode ($\sigma_{\Delta r}$ =1 m.; covariance = 0.5 m^2.)



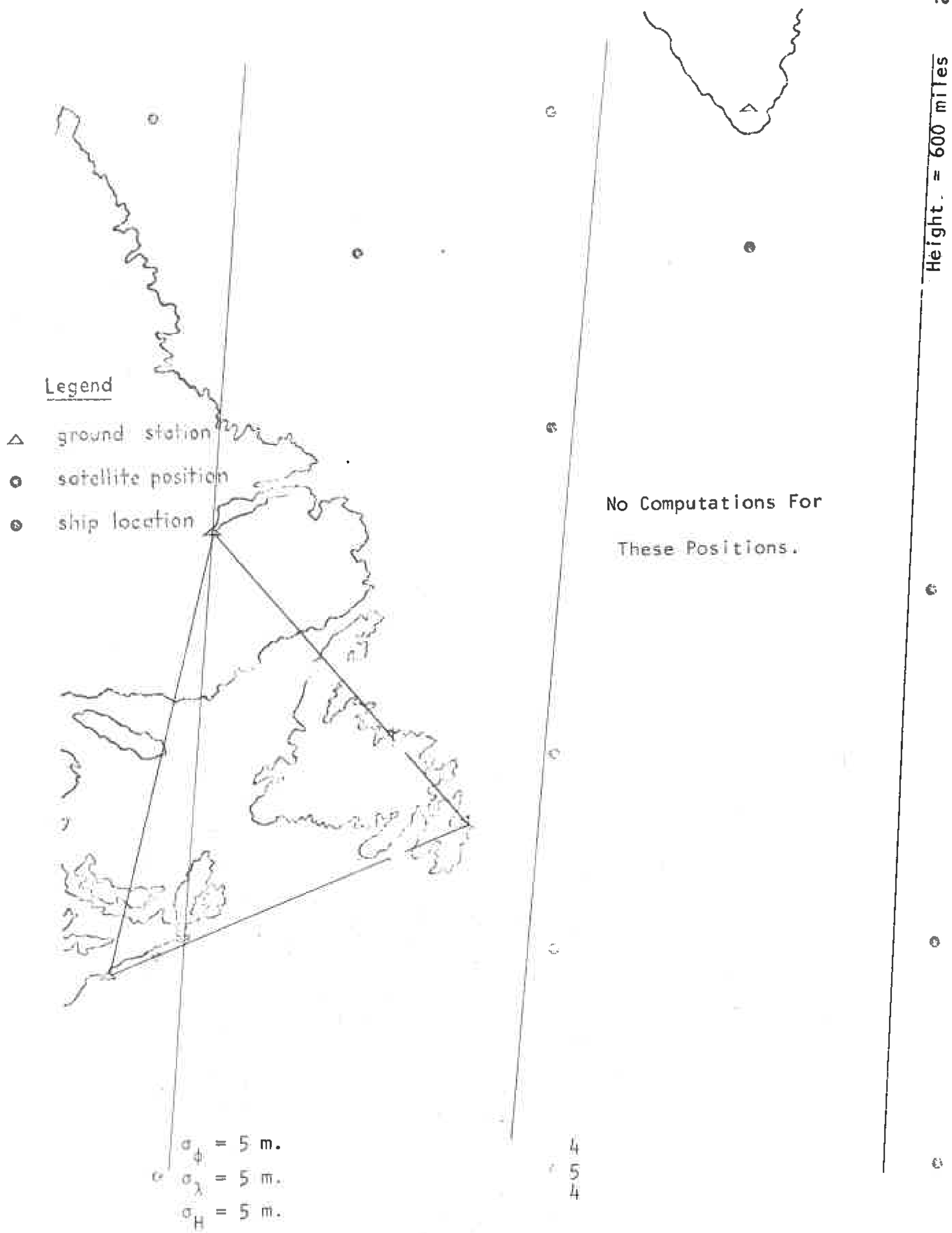
Satellite Total Vector Mode ($\sigma_\delta = \sigma_\alpha = 3''$; σ_r = 10 m.)



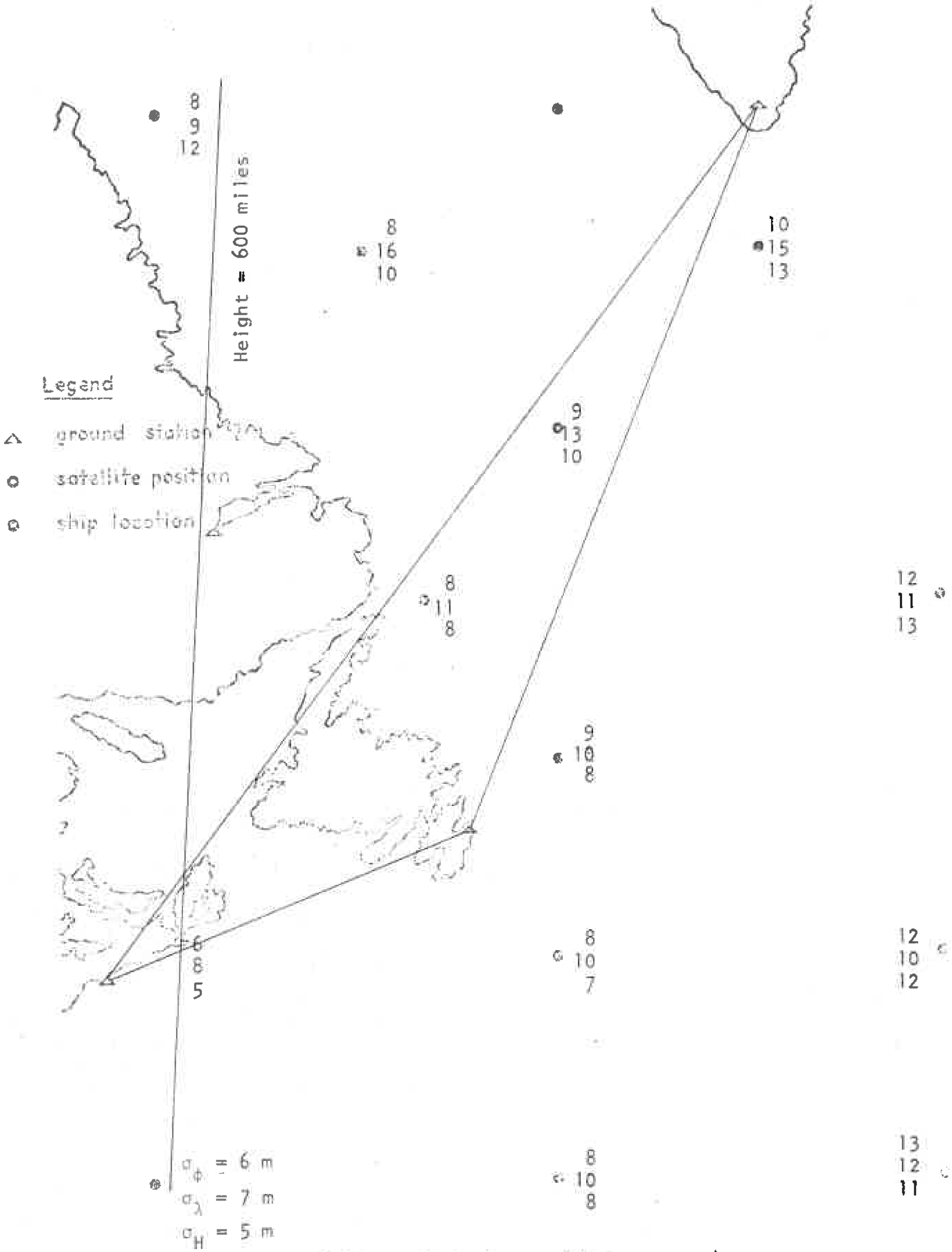
Satellite Total Vector Mode ($\sigma_\delta = \sigma_\alpha = 3''$; $\sigma_r = 10$ m.)



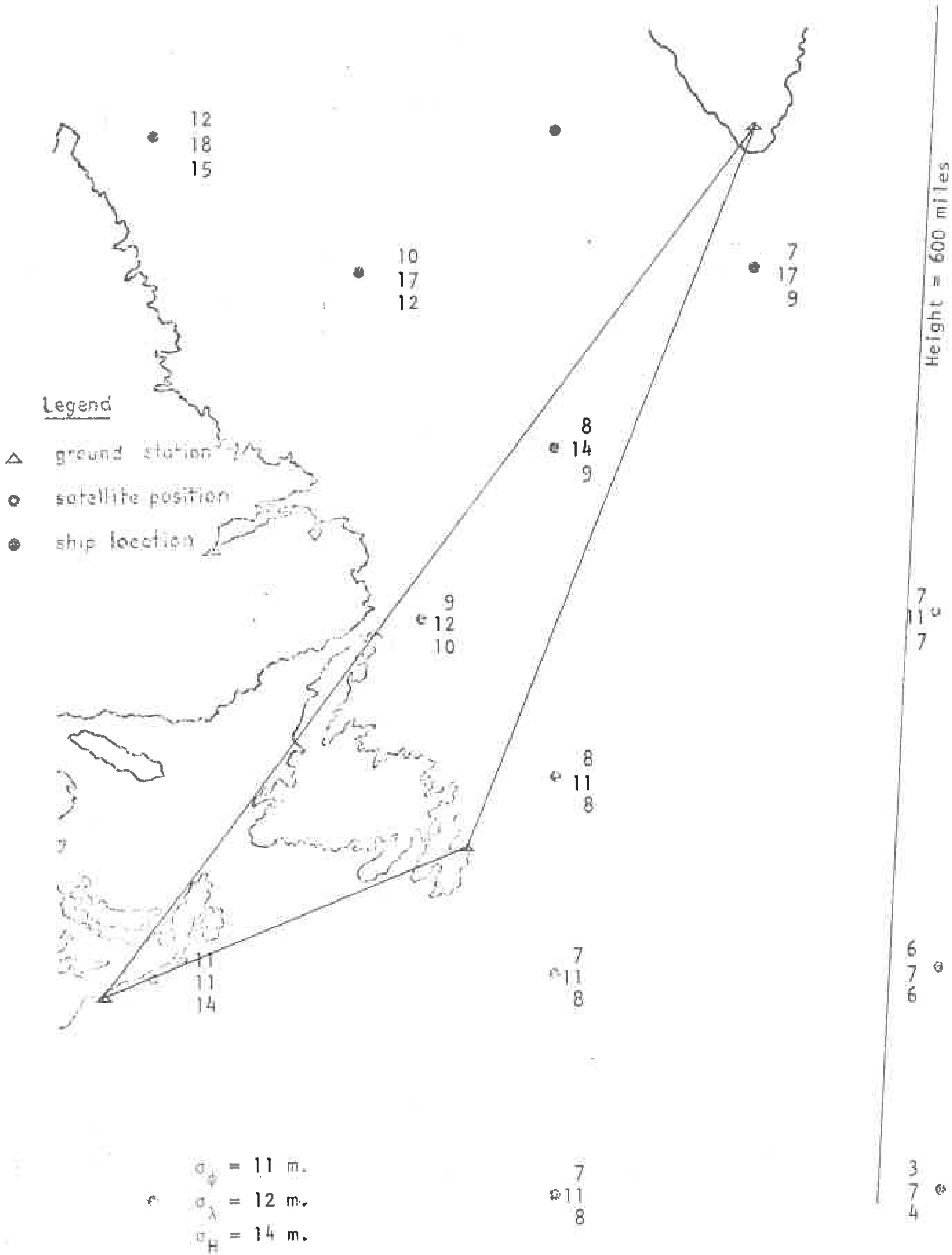
Satellite Total Vector Mode ($\sigma_\delta = \sigma_\alpha = 3''$; $\sigma_r = 10$ m.)



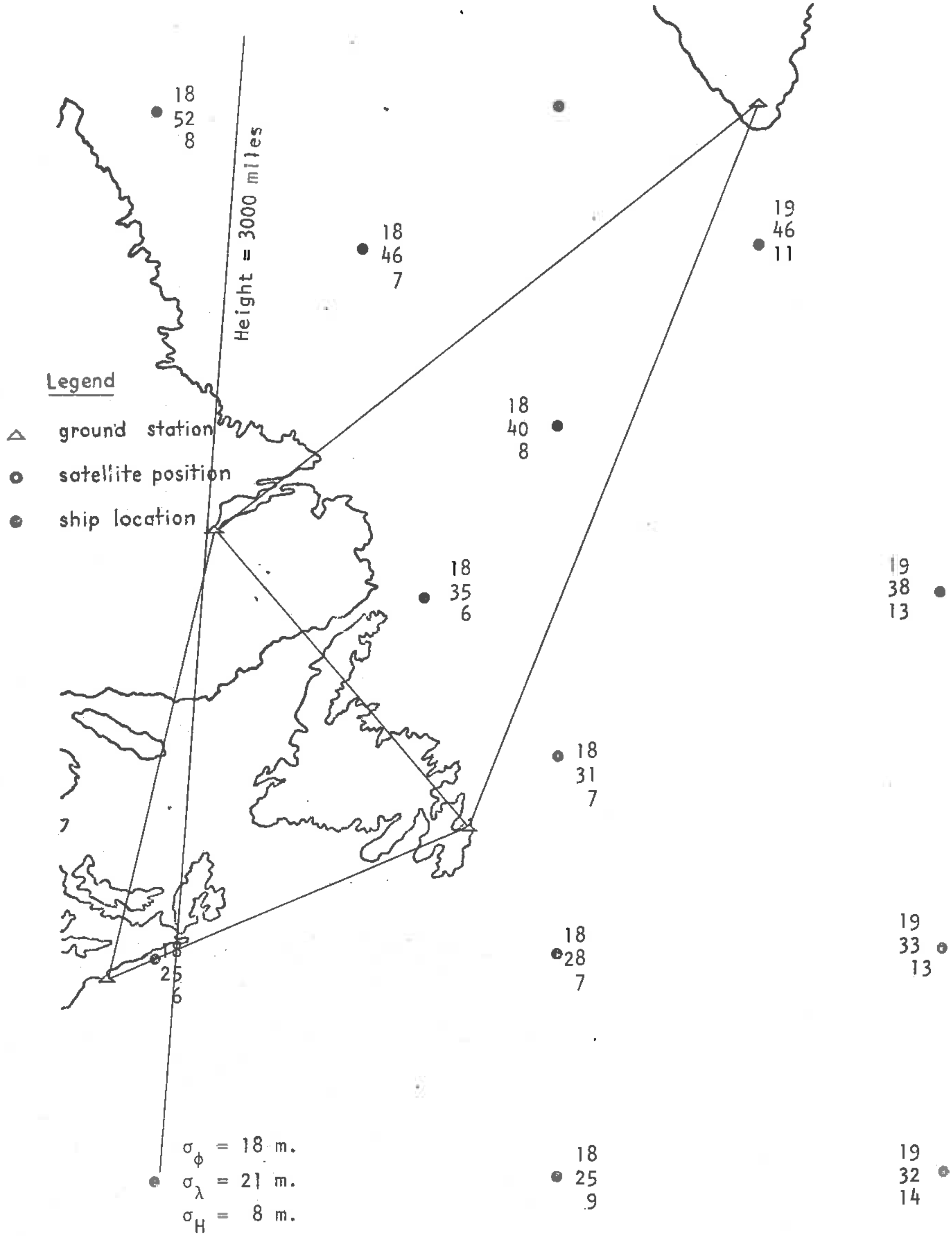
Satellite Total Vector Mode ($\sigma_\delta = \sigma_\alpha = 3''$; σ_r = 10 m.)



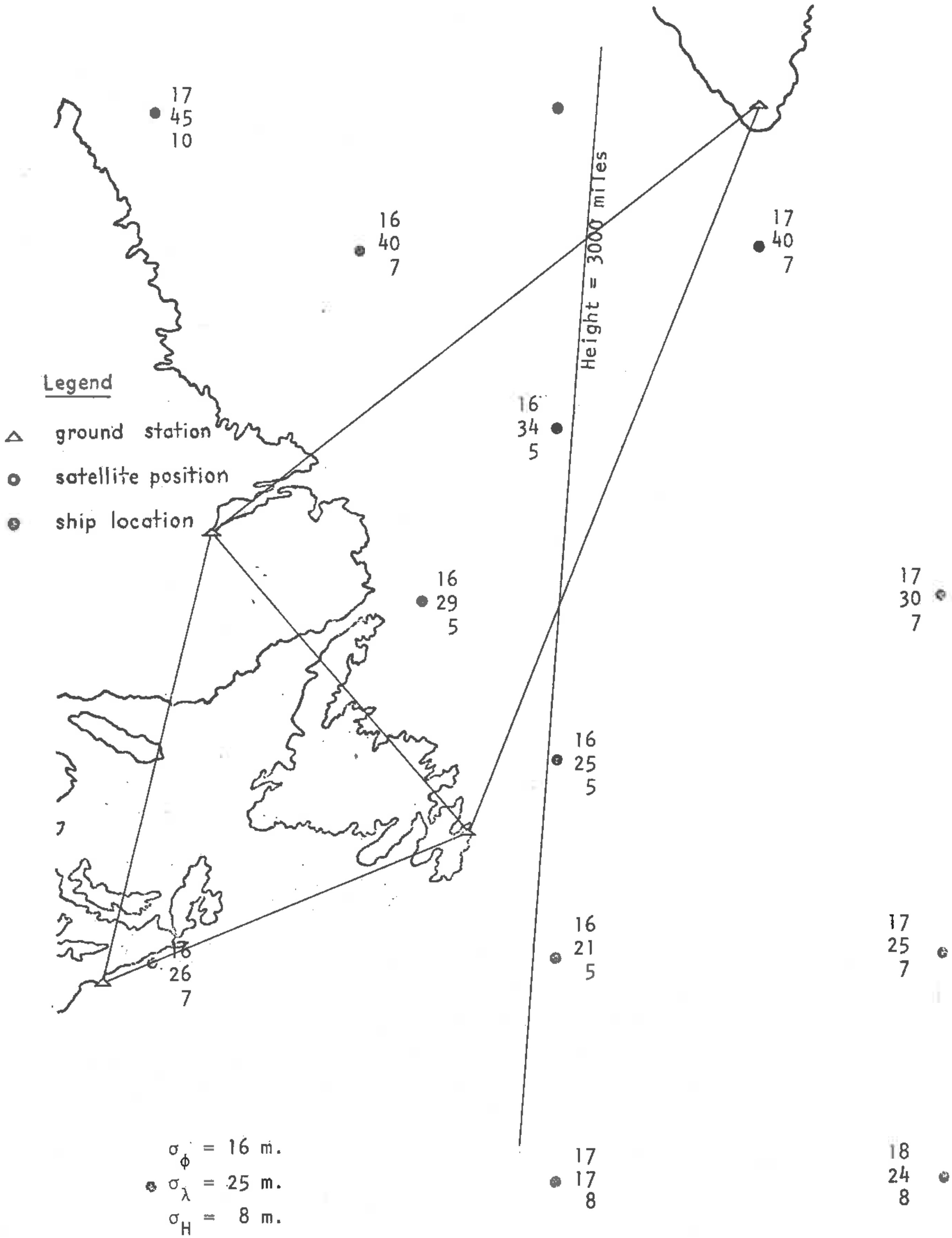
Satellite Total Vector Mode ($\sigma_\delta = \sigma_\alpha = 3''$; σ_r = 10 m)



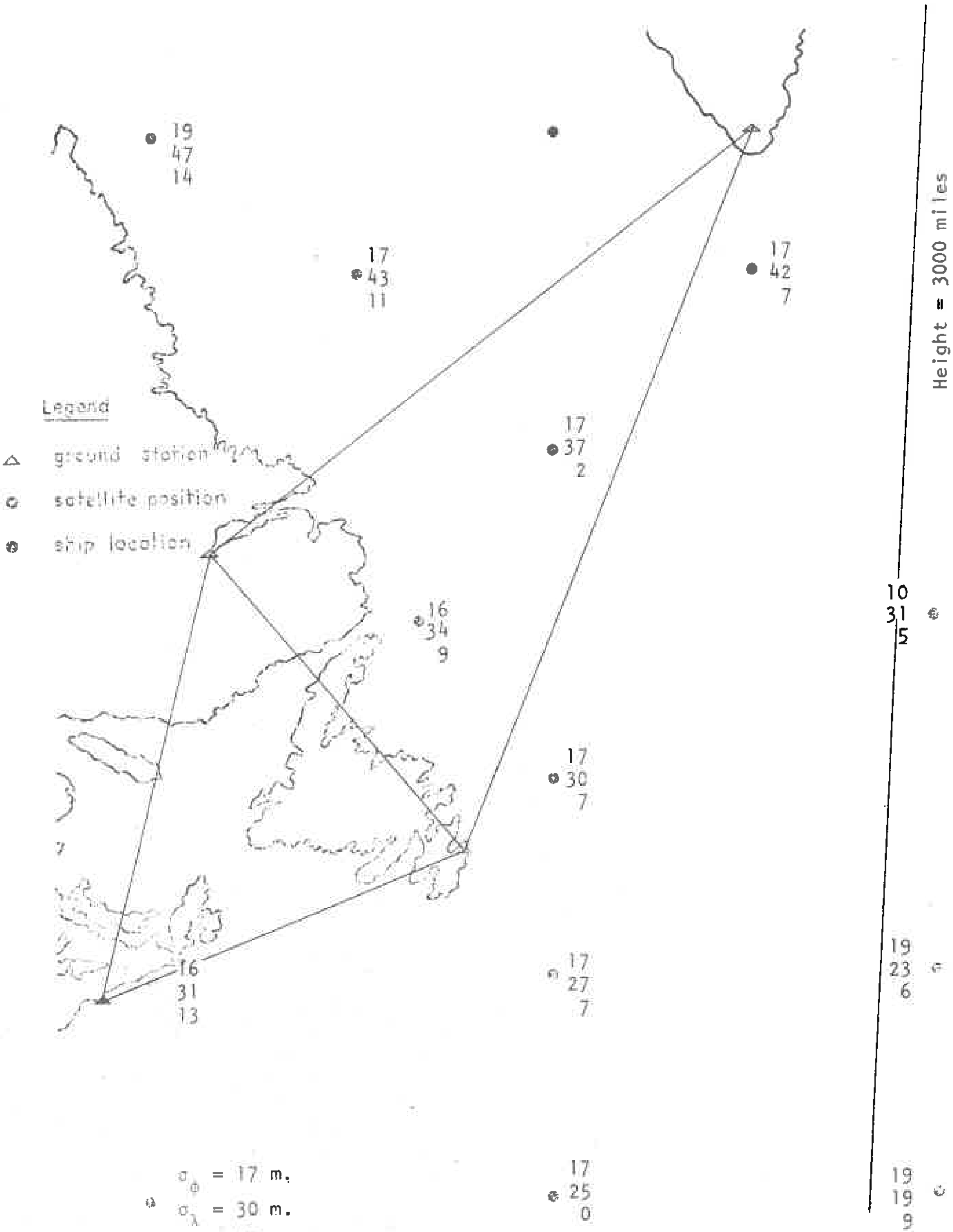
Satellite Total Vector Mode ($\sigma_\delta = \sigma_\alpha = 3''$; σ_r = 10 m.)



Satellite Total Vector Mode ($\sigma_\delta = \sigma_\alpha = 3''$; $\sigma_r = 10$ m.)

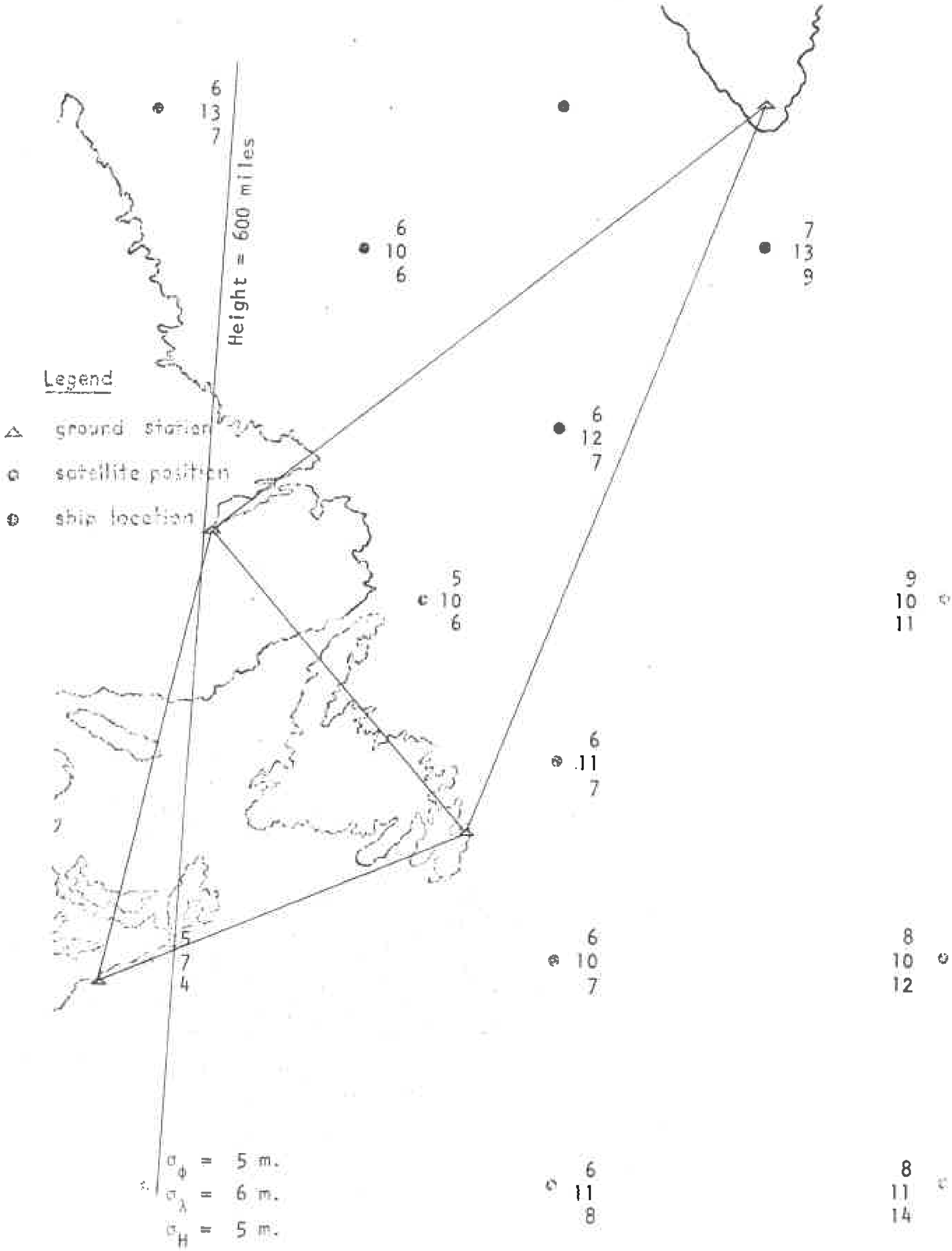


Satellite Total Vector Mode ($\sigma_\delta = \sigma_\alpha = 3''$; σ_r = 10 m.)

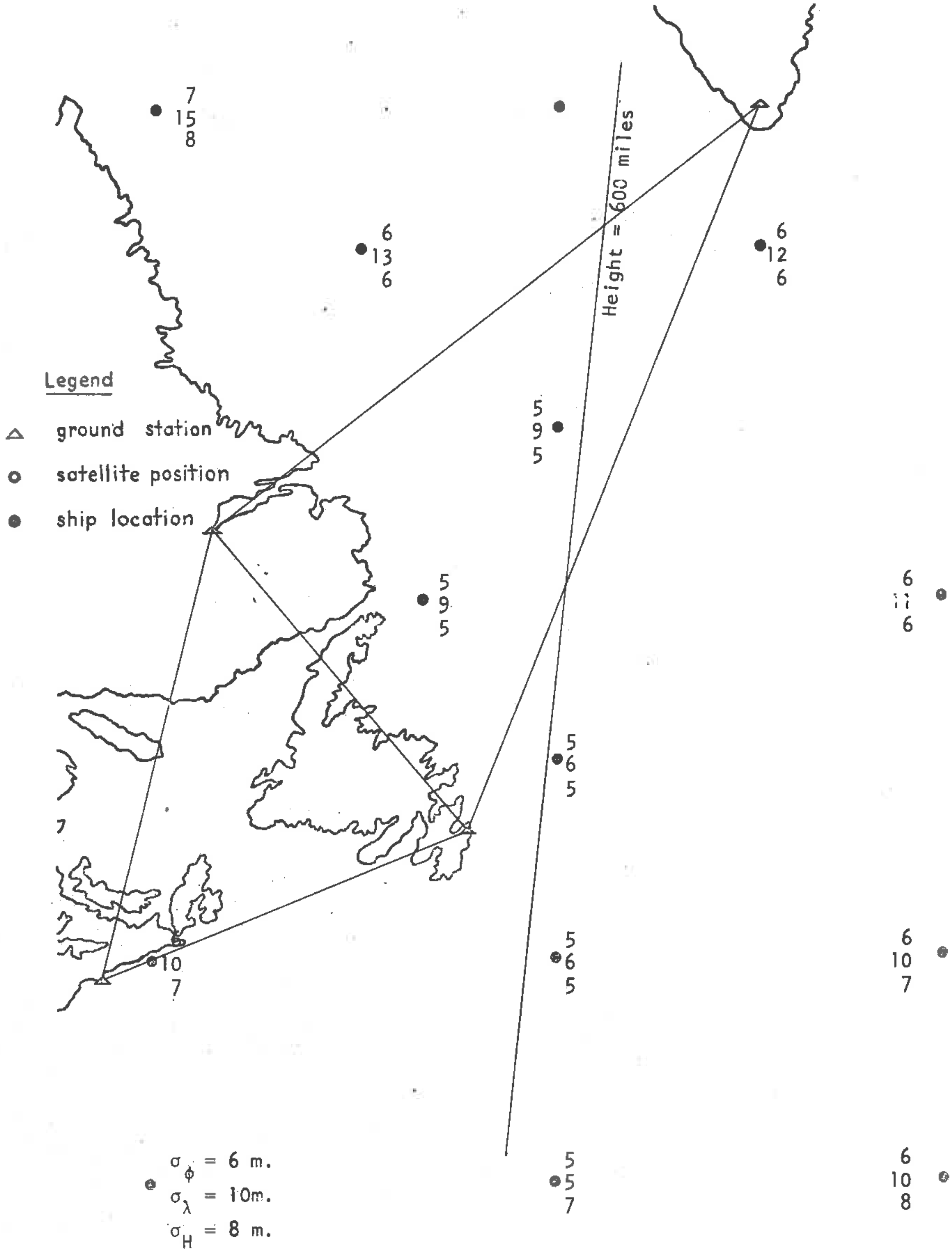


σ_ϕ = 17 m.
σ_λ = 30 m.
σ_H = 13 m.

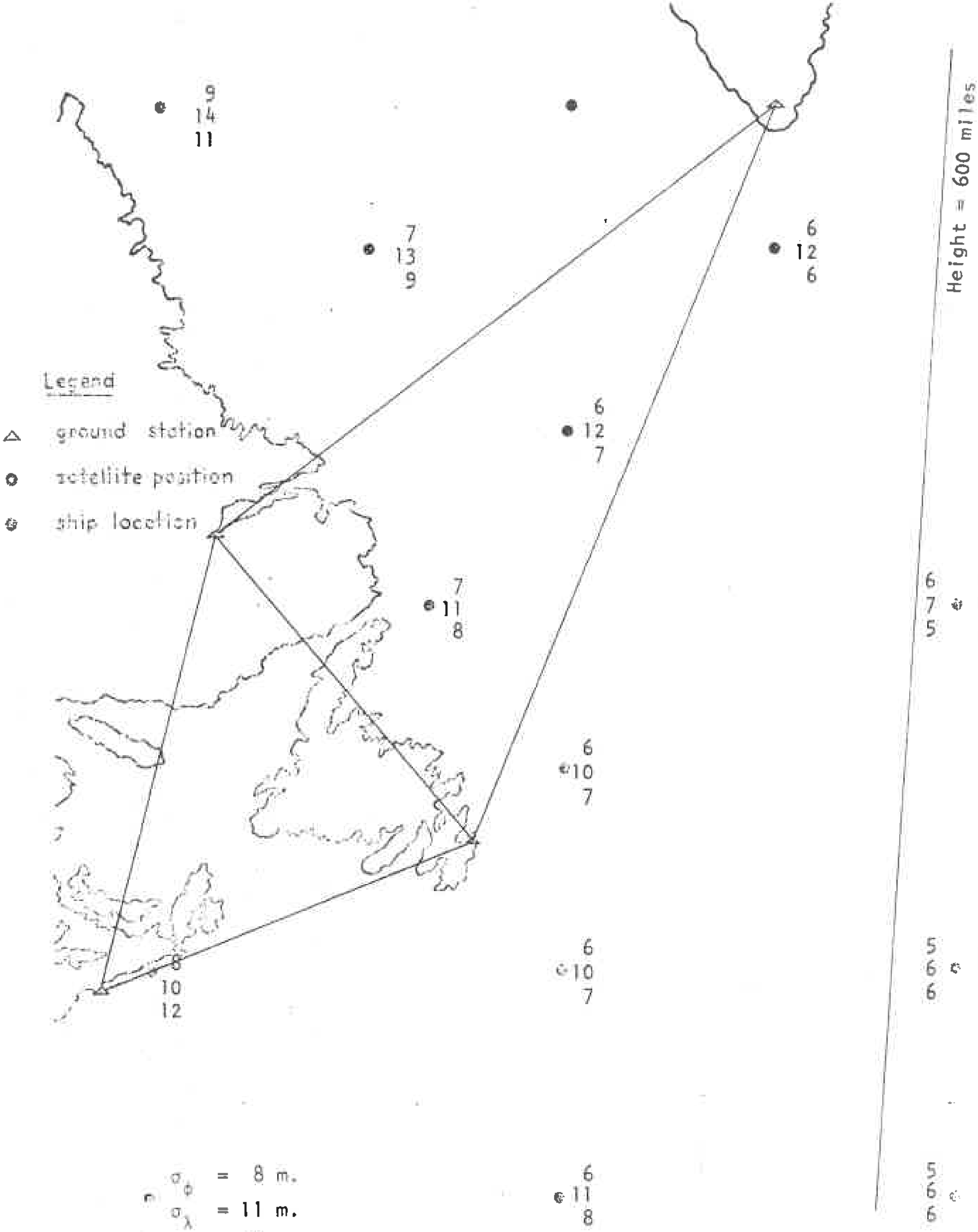
Satellite Total Vector Mode ($\sigma_\delta=\sigma_\alpha=3''$; $\sigma_r=10$ m)



Satellite Total Vector Mode ($\sigma_\delta = \sigma_\alpha = 3''$; σ_r = 10m)

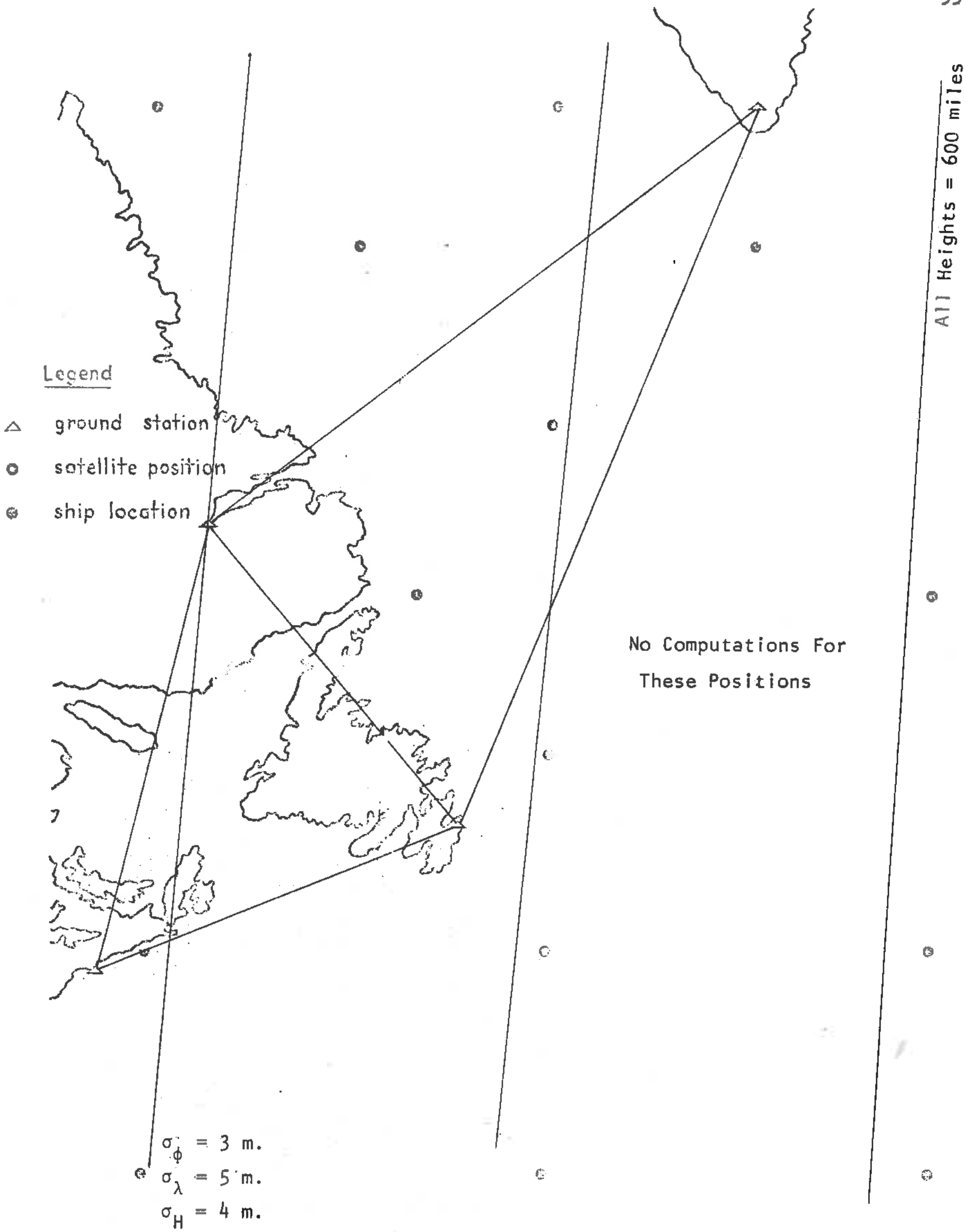


Satellite Total Vector Mode ($\sigma_\delta = \sigma_\alpha = 3''$; $\sigma_r = 10$m.)



σ_ϕ = 8 m.
σ_λ = 11 m.
σ_H = 13 m.

Satellite Total Vector Mode ($\sigma_\delta = \sigma_\alpha = 3''$; σ_r = 10 m.)



Satellite Total Vector Mode ($\sigma_\delta = \sigma_\alpha = 3''$; σ_r = 10 m.)

5. SUMMARY

5.1 Expected Accuracy of the Various Modes of Satellite Positioning

Satellite Triangulation. This mode is reported to give a positional accuracy ranging from σ = 5 m. to 25 m. depending upon the station configuration (separation 1,000 km. to 3,000 km.); height of satellite (400 to 6,000 km.); and accuracy of the directions (σ = 1 to 2 arcsec.). Details on various adjustments can be found in [Krakiwsky, 1968, pp. 197-239; Mueller et al, 1970, pp. 8-12]. The problem of photographing a satellite from a "dynamic platform" is discussed in the context of "Required Instrumentation" in Section 5.2.

Satellite Trilateration. The attainable accuracy for this mode has been exemplified by the various simulations (Runs 1 to 7). A positional standard deviation comparable to that of the Satellite Triangulation mode can be theoretically achieved. In fact an adjustment using actual SECOR range data yielded a station position accuracy of less than σ = 5 m. [Krakiwsky, 1969]; it should be noted that in this case extensive preprocessing of the data was required in order to remove ionospheric refraction effects [Peat, 1967]. General laws have been formulated for this mode of positioning [Rinner, 1966], such that accuracy estimates may be deduced for station configurations other than those computed in this report.

Satellite Total Vector Mode. The various simulations (Runs 21 to 33) computed for this mode indicate a strong solution for as few ground based stations as three; Even simulations involving only one pass yeilded good results. This approach undoubtedly gives the best positional accuracies e.g. $\sigma < 10$ m. in most cases. Again the main problem is the photographing of the satellite from a dynamic platform (See Section 5.2).

Satellite Range Difference Mode. The simulation (Runs 8 to 20) performed for this mode of positioning are satsifactory when at least four ground based stations are involved in a simultaneous observational scheme with the sea position. The geometry is rather weak for a single pass, however when three passes are observed - Runs 16 and 20 (not impractical), the positional accuracy is $\sigma = 20$ m. on the average.

The required accuracy for "lane checks" is less than one-half the average width of a lane; For the DECCA 12 F system this factor is approximately 400 feet. The total vector mode with one pass meets this requirement. The other modes meet this requirement only if several passes would be observed. For a stationary sea position such as a drilling vessel, all methods (considering several passes) are indicated to be satisfactory. The positional standard errors vary from a few meters to a few tens of meters (For Runs 4, 16, 20, 24, Table 1, p. 19).

5.2 Required Instrumentation for the Various Satellite Positioning Modes

5.21 Satellite Triangulation Instrumentation

Table 2 [Moorat, 1969] lists most of the satellite cameras in use to date. The standard error of directions attained by the various cameras varies from several arcsec. to less than 1 arcsec.,

Table 2. Magnitude Limit of Western European Subcommission Cameras [Moorat, 1969]

Focal Length mm.		250	300			500		620		1040		1206
f Number		2.5	2.5	3.0	4.5	1.0	3.0	1.0	2.0	2.6	5.0	4.0
Camera Type		NAFA 30/25	K37	BC4	IGN	BAKER NUNN	BC4	HEWITT	VAISALA SWEDEN	VAISALA SWISS	P.C1000	DELFT
Velocity 0/Sec.	0.9	3.6	3.7	3.3	2.5	6.3	3.8	6.5	5.2	5.0	3.6	4.2
	0.6	3.9	4.1	3.7	2.9	6.8	4.2	7.0	5.6	5.4	3.9	4.6
	0.3	4.7	4.9	4.5	3.8	7.5	5.0	7.7	6.4	6.3	4.7	5.4
	0.2	5.1	5.4	4.9	4.2	7.9	5.4	8.2	6.8	6.6	5.1	5.8
	0.1	5.8	6.1	5.6	5.0	8.6	6.1	9.0	7.5	7.4	5.9	6.5
	0.05	6.6	6.9	6.4	5.8	9.4	6.9	9.8	8.2	8.1	6.6	7.2

e.g. σ = 2 arcsec. for the PC-1000 camera [Mueller et al, 1970, p. 8].

The cost of the cameras also varies greatly and is approximately $10,000 for the IGN, while the cost of the BC4 (if it can be bought) would exceed this cost by several times.

A solution for the problem of photographing satellites in the background of the stars from a dynamic platform is proposed in [Jury, 1966]. This system is called POSE (Photogrammetric Ocean Survey Equipment) and consists of "a gyrostabilized, stellar-oriented camera, with associated timing equipment, mounted aboard a ship". This ship camera is involved in simultaneous observations with land based cameras with known coordinates. Tests have been made with this equipment and it is reported in the above reference to give results which differ by 57 feet in latitude and 192 feet in longitude from a position determined independently by LORAC- B equipment. The equipment is said to be able to produce results better than σ = 100 ft. in position. Better results should be attainable from observations from "drilling rig platforms" whose motions are damped.

5.22 Satellite Trilateration Instrumentation

One satellite ranging system in existence to date is SECOR (Sequential Collation of Range); It was developed for the U.S. Army Map Service [Cubic Corporation, 1964]. The availability of these receivers for civilian use is not known.

The acquisition of ranges by laser techniques have been employed [Veis, 1969; Lefebvre, 1969]. The instrumentation is at the prototype stage and is said to cost well over $100,000 each.

5.23 Satellite Total Vector Mode Instrumentation

This mode simply combines the instrumentation of the previous two modes in a colocation (same site) situation, therefore the previous remarks hold here as well.

5.24 Satellite Range Difference Mode Instrumentation

Range difference measurements obtained via doppler measurements has been made possible through research in the United States [Newton, 1966] and France [Lefebvre, 1969]. Doppler receivers are available for civilian use from U.S. manufacturers (I.T.T. Federal, Magnavox, and Honeywell) at a cost of approximately $35,000; Auxiliary equipment including a computer is an additional $40,000. Information on the French equipment is not available.

5.3 Availability of Satellites

An extensive study of the existing satellites of potential geodetic value has been made by Geonautics Inc. for NASA [Mancini, 1970]. A detailed analysis of the 102 satellites is necessary in order to determine their applicibility in a given area of the globe for a specific purpose. For example such an analysis has shown that seven satellites are useful for camera observations from the western European satellite triangulation stations [Moorat, 1969].

The following satellites [Lefebvre, 1969, p. 1] furnish the possibility for both camera and laser observations since they are equipped with retroreflectors: D1.C, D1.D, BEB, BEC, GEOS A, GEOS B.

Four U.S. doppler satellites are continuously available for range difference observations. Two French doppler satellites, DIAPASON and DIADEME, are in (exclusive?) use by the French.

6. RECOMMENDATIONS

The recommendations to follow pertain to offshore positioning on Canada's eastern continental shelf and arctic regions; The methods under consideration are the various simultaneous modes of satellite geodesy (i.e. satellite triangulation, trilateration, total vector mode, and range difference mode). The following factors were considered:

(1) the potential accuracy of the various modes of positioning, as compared to the required accuracy,

(2) availability and cost of equipment,

(3) availability of satellites,

(4) ease with which the mode of positioning can be put into a working system.

<u>Recommendations for the Immediate Future</u>

(1) To perform additional simulations with an expanded mathematical model for the range difference mode, i.e. Equation (10) be expanded to include one frequency offset* unknown per satellite pass as well as an equation containing information about the height of the sea station above the NAD ellipsoid. (Schedule - by December 31, 1970).

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* This unknown is peculiar to the U.S. Navy doppler satellites.

(2) To acquire simultaneous range difference data from at least four ground based stations and one sea position. These five stations should accumulate observations to at least 100 satellite passes during different times of the day. Presently, there are ten doppler receivers in Canada, thus such a scheme should be possible. The details of such a scheme should be worked out by those interested parties in government, private industry, and the universities. (Schedule by December 31, 1970; but after (1)).

(3) The processing of the raw data should be processed with the present U.S. Navy software (orbital mode) as well as with the simultaneous mode equations. Datum shift and ionospheric studies should be integral parts of the analysis. (Schedule - first results by June 30, 1971).

Recommendations for the Future

Some sort of additional joint government - industry - university activity should be commenced in the area of camera observations, range differencing, and ranging to satellites and/or to other elevated targets (e.g. aircraft, balloons).

For instance, range difference measurements, as obtained from the doppler technique cited in this study, could have application in geodetic control establishment as well as for controlled photography. The expanded form of the mathematical model expressed by equation (10) can be used as the basis.

Also, studies concerning the monitoring of the position of Canada's communication satellite to be, should be made in order to keep abreast with developments at Telesat Canada.

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