

# FESTSCHRIFT IN HONOUR OF ADAM CHRZANOWSKI

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**FESTSCHRIFT IN HONOUR OF  
ADAM CHRZANOWSKI**

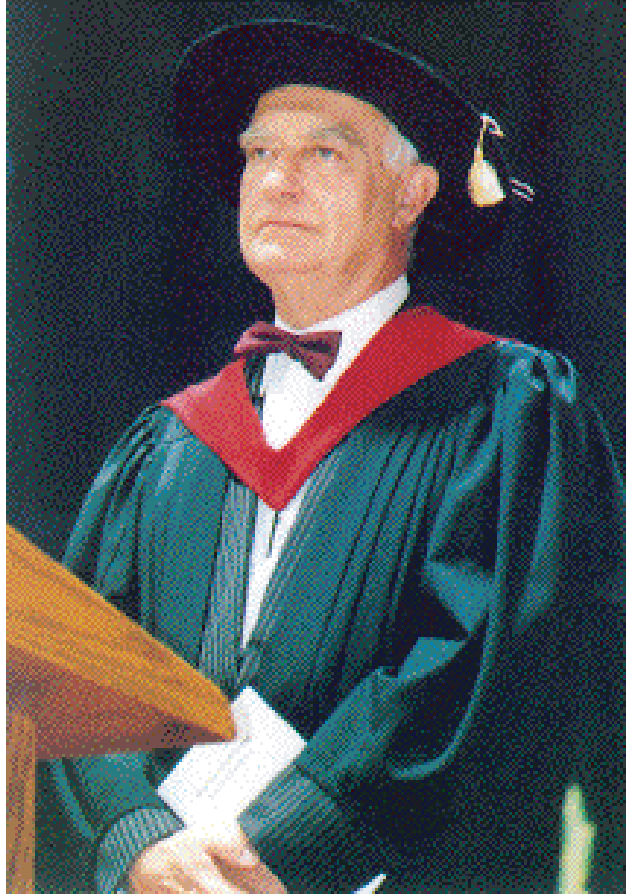
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## PREFACE

In order to make our extensive series of technical reports more readily available, we have scanned the old master copies and produced electronic versions in Portable Document Format. The quality of the images varies depending on the quality of the originals. The images have not been converted to searchable text.



Dr. Adam Chrzanowski at the ceremony of receiving the honoris causa title of Professor Emeritus at the University of New Brunswick, October 1998.

Adam Chrzanowski, born in Poland in 1932, received his B.Sc. (1954), M.Sc. (1956), and Ph.D. (1962) degrees in surveying engineering from the Technical University of Mining and Metallurgy in Krakow, Poland. He joined the Department of Surveying Engineering at the University of New Brunswick (UNB) in Canada in 1964, where he initiated a unique for North America specialisation in engineering and mining surveys at both undergraduate and graduate levels. In 1971, he became a full professor and served as the department chair of geodesy and geomatics engineering (formerly surveying engineering) between 1991 and 1995. About 50 graduate students have completed their studies and specialised in engineering surveys at the M.Sc.E. and Ph.D. levels under his supervision. Dr. Chrzanowski is an author of over 200 publications and co-author of several prestigious books on geodetic, engineering, urban, and mining surveying. In 1986, Dr. Chrzanowski was awarded the title of Honorary Professor at Wuhan Technical University of Surveying and Mapping in China in recognition of his contributions to the geodetic and engineering surveys. In 1995, he was awarded the Knight's Cross of the Order of Merit by Lech Walesa, the President of Poland, and in 1998, he was awarded the honoris causa title of Professor Emeritus at UNB. In 1999, he was awarded the title of Honorary Professor at the Technical University of Agriculture in Olsztyn, Poland. Dr. Chrzanowski officially retired from UNB in 1998, but he still carries on his research as Professor Emeritus and serves as a director of the Engineering and Mining Surveys Research Group.

## Table of Contents

Frontis page .....	ii
Table of Contents .....	iii
Introduction .....	iv
<i>It was not so easy 21 years ago</i> , A. Chrzanowski .....	1
<i>The story of “Urban Surveying and Mapping”</i> . T. J. Blachut .....	3
<i>Adam Chrzanowski: UNB’s Engineering and Mining</i>	
<i>Surveying Guru</i> , W. Faig .....	8
<i>For your retirement scrapbook</i> , M. and J. Miller .....	10
<i>Three professors, one engineer, and twelve lobsters</i> , J. Murria .....	11
<i>A good acquaintance</i> , S. Oszczak, A. Wasilewski, Z. Rzepecka .....	12
<i>Across Bulgaria with motor-scooters</i> , Z. Sitek .....	14
<i>Reminiscences of Adam</i> , W. F. Teskey .....	15
<i>Time series analysis for deformation monitoring</i> , W. Caspary .....	16
<i>Developments in deformation monitoring and analysis</i> ,	
Chen Yong-qi .....	24
<i>Design of a robotic monitoring system for the Eastside</i>	
<i>reservoir in California</i> , M. A. Duffy .....	33
<i>Photogrammetric techniques for deformation measurements</i> , W. Faig .....	49
<i>Bounding natural domains: The definition of geospatial objects</i>	
<i>from fields</i> , M. Kavouras .....	51
<i>Numerical modelling of heat and mass transfer processes using</i>	
<i>least squares method</i> , Z. Kolenda .....	65
<i>The footprint problem in deformation analysis</i> , H. B. Papo .....	81
<i>Modelling of subsidence in area of room and pillar salt mining using finite</i>	
<i>element method</i> , J. Pielok, K. Pietruszka, A. Szostak-Chrzanowski .....	93
<i>Geodetic deformation measurements of the Ball Mountain</i>	
<i>dam</i> , M. W. Rohde .....	100
<i>Effects of open pit mining on gravity changes</i> ,	
A. Szostak-Chrzanowski, E. Popiolek .....	111
<i>Implementation and methodologies for high-precision industrial</i>	
<i>surveys</i> , W. F. Teskey .....	123
<i>The detection of crustal movements by geodetic space</i>	
<i>techniques</i> , P. Vanicek .....	133
<i>Primary GPS control point network of Polish copper</i>	
<i>basin area</i> , A. Wasilewski, S. Oszczak, Z. Rzepecka, R. Kurpinski .....	139
<i>Juneau icefield, Alaska: Glacio-geodetic measurements</i> , W. M. Welsch .....	148
A lifetime of publications .....	163

# Introduction

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This “festschrift” is being published by the Department to mark Adam’s retirement (in 1998) and to commemorate his contributions over the 34 plus years that he has been here.

Having collaborated with Adam since the summer of 1981 (resulting in a B.Sc.E. “thesis”), I feel that I can speak on behalf of all of his graduate students since my M.Sc.E. and, eventually Ph.D., were also under his supervision. If it were necessary to sum up Adam’s “character” in a single word, such a daunting task would end with my suggesting “generous” - with his time, with his knowledge, and with his resources (financial and otherwise). Being such an active person, he always seems to have several projects on the go, but he always has had time to spend with his students, however trivial the matters. He is certainly very generous with his knowledge and is very quick to ensure that anyone else’s contribution is dutifully acknowledged - very often having co-authors when their contributions seem rather minor in comparison. His knowledge generosity is also apparent in his courses—a fact to which many a third-year geodetic surveying student would attest. And, Adam is certainly materially generous as well. Many of his graduate students have seen much of the world (e.g., Peru, Venezuela, Switzerland, and western Europe, Texas, California, Pakistan) through his assistance.

This publication is a rather modest collection of reminiscences and technical papers by contributors in countries other than just Canada - Germany, Greece, Peoples’ Republic of China, Poland, United States, and Venezuela. They span nearly four very full decades of activity.

One habit which I must admit to having cultivated with Adam is that of the untimely meeting of deadlines, especially for papers to conferences and publications. The original target for publishing this was for it to coincide with Adam’s official retirement in June 1998. The liberty that we have taken in meeting that is, I admit, a bit excessive even if we claim to be following a “tradition”. For this I apologize, especially when I realize how much I, and so many others, owe to Adam.

A collection of reminiscences in photographic form appears on the next 13 pages, highlighting Adam’s time spent at UNB.

## Reminiscences



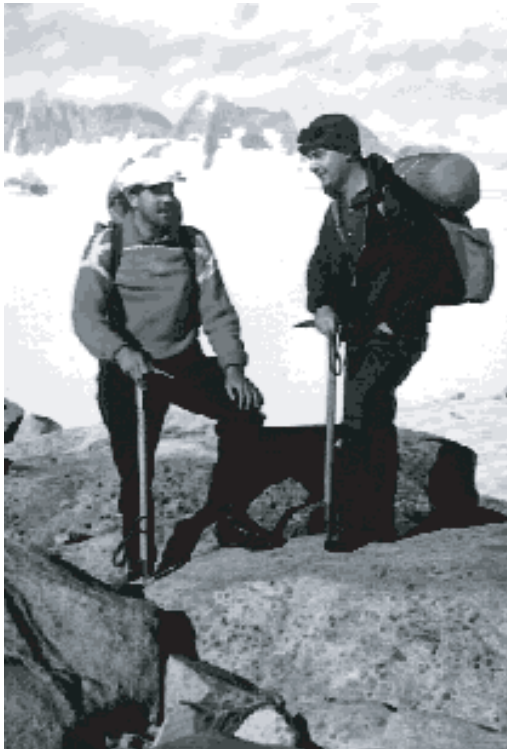
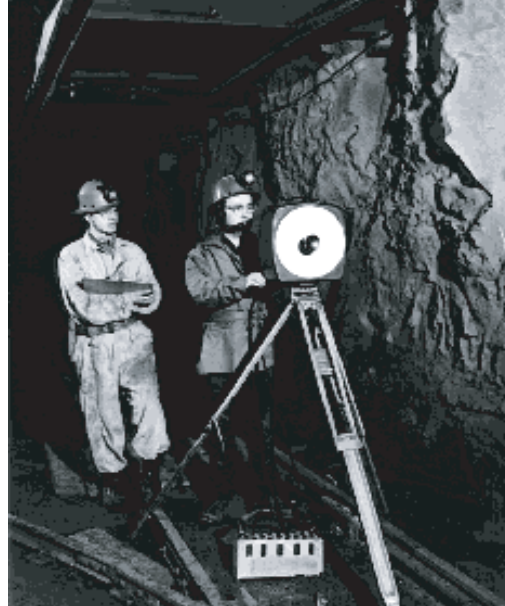
*1951: Adam (left) as a second-year student of surveying engineering at the Technical University of Mining and Metallurgy, Krakow, Poland.*



*1965: Adam, at the theodolite, establishes geodetic control at the border of Yukon and Alaska during the Mt. Kennedy expedition organized by the National Geographic Society and Boston Museum of Science. Senator Robert Kennedy participated in the expedition as the first man to put his foot on the top of the mountain named in honour of his assassinated brother. The expedition was plagued with snow blizzards. Below is the survey camp, called Misery, in which Adam and four other members of the expedition were supposed to spend only two days but, because of the storm, they were cut off from the base camp for 10 days.*



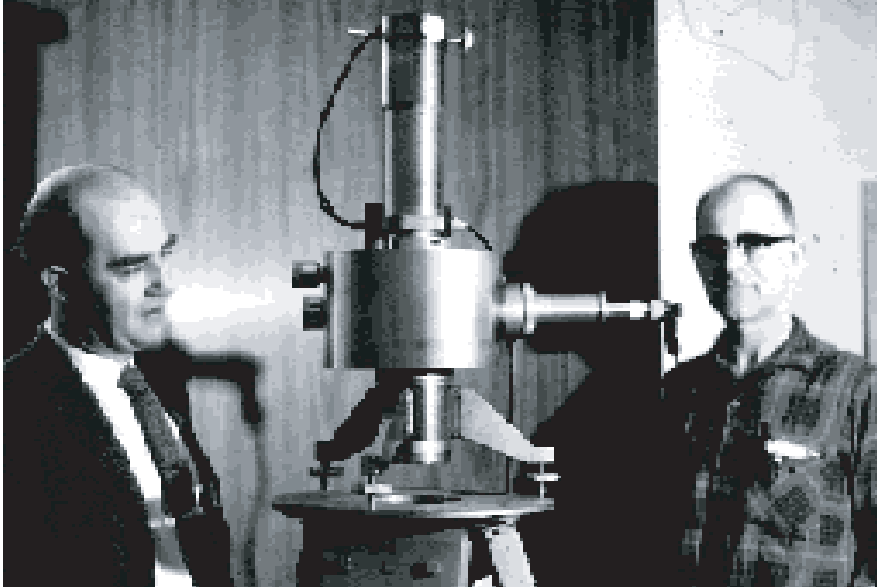
*1966: Adam (at the Tellurometer) evaluating tunnelling effects on microwave distance measurements at INCO mines in Sudbury, Ontario. Between 1965 and 1970, Adam and his associates pioneered the use of EDM instruments and gyrotheodolites in North American mines and developed prototypes of laser instruments for mine orientation surveys and for tunnel profiling.*



*1968: Adam (right) and Dr. Peter Angus-Leppan of Australia on their way to a glacier camp at Juneau Icefield in Alaska. Adam spent five summers (between 1965 and 1974) with the Glaciological and Arctic Sciences Institute in the Juneau Icefield Research Program as a visiting scientist to study glacier movements and to give lectures on geodetic surveys.*







*1968: Adam (left) and Ralph Carruthers, a precision machinist at UNB, at the prototype of a laser plummet developed together with Dr. Salem Masry. Between 1966 and 1975, Adam and his associates developed and constructed prototypes of various laser instruments for precision engineering surveys, including two types of laser plummet, a precision laser level, a self-centring laser target, and a laser system for precision alignment surveys using diffraction techniques. The research included an intensive study for the Defence Research Board on the propagation of lasers in a turbulent atmosphere.*



*1973: Adam and his family driving on a “highway” towards Tierra del Fuego at the southern tip of Argentina. The return trip of 60 000 km from Fredericton through all the Americas, took 15 months in 1972/73 with two major stops at the University of Brasilia (4 months) and the University of Tucuman in Argentina (4 months) where Adam worked as a visiting professor, organized the first Brazilian Symposium on Urban Surveys (1972), and gave courses on engineering surveys.*



*1979: Adam (right), Anna Szostak-Chrzanowski, his research associate, and Mark Dennler, a former UNB graduate student, at the site of a micro-geodetic monitoring network in the earthquake prone Peruvian Andes. Two networks at elevations of 4200 m and 5000 m were observed annually between 1975 and 1982 and analysed using the Generalized Method of Deformation Analysis developed by Adam, Dr. Chen Yongqi, and Dr. J. Secord at UNB. Below is the survey camp near Pitec at 4700 m elevation.*





*1980: Adam (right) and Dr. Yilmaz Fisekci of Canada Mineral and Energy Technology (CANMET) install a telemetric system for monitoring slope stability above an underground coal mine near Sparwood, B.C. This pioneering system, developed by Adam and his research associates for CANMET, allowed for a continuous monitoring of ground movements in the rugged terrain using an array of tiltmeters. The system was continuously operational for three years (1980-1983) at temperatures to  $-40^{\circ}\text{C}$  and under 3 m of snow during the winters.*





*1985: Adam and his associates developed for the Geodetic Survey of Canada a motorized system for trigonometric height traversing of high precision. At the instrument (above) is Mr. A. Kharagani, a former UNB graduate student. The development included intensive studies on effects of tropospheric refraction. Below, Dr. A. Jarzymowski, a visiting scientist from Poland, and Mr. S. Vamosi, of the Geodetic Survey of Canada, are at an instrument for temperature gradient recording.*





*1985: Adam (centre) with Julio Leal and Rick Wilkins, former UNB graduate students, on Lake Maracaibo at the site of a ground subsidence study in oil fields in Venezuela. The study was carried out between 1984 and 1992 and included the pioneering use of GPS (1986) in ground subsidence studies and in monitoring a 50 km long earth dyke.*





*1986 (above): Adam receives the title of Honorary Professor at the Wuhan Technical University of Surveying and Mapping, China.*

*1989 (below): Adam and a group of his Polish friends celebrate the 70th anniversary of the Technical University of Mining and Metallurgy in Krakow, Poland.*





*1991 (above): Adam (far right) takes over the chairmanship of the Department from John McLaughlin (far left) at a ceremony of the 30th anniversary of the Department at which a special monument was unveiled. Standing in the middle are Stefan Geneja and his wife, good friends of UNB.*

*1991 (below): Adam in his new office as Chair with Dr. A. Szostak-Chrzanowski and two visiting professors from Poland, Dr. J. Pielok and Dr. M. Milewski.*





1991: Adam and Dr. Anna Szostak-Chrzanowski, his research associate, work in their laboratory on the application of the iterative finite element analysis to modelling and predicting ground subsidence in mining areas. Their method, developed between 1985 and 1990 and known as the S-C Method, has been successfully applied in coal, copper, and potash mines in Canada, Poland, and the U.S.A. Recently, the method has been expanded to model gravity changes in mining areas and tectonic plate movements.

1992: Adam (left) with Trevor Greening and Greg Robinson, former UNB graduate students, at the construction site of the Superconducting Super Collider in Texas. Adam served as a consultant and scientific advisor on the design of geodetic control for the 87 km long collider tunnel. Out of a total of 20 geodetic engineers employed on this \$10 billion project, 16 were former students from UNB.







*1995: Attending the annual dinner of the Hong Kong Institute of Surveying. From left to right: Associate Professor Conrad Tang (former UNB graduate), Adam, Mr. Leung Shou Chun, president of the Institute, and Dr. Chen Yong-qi (former UNB graduate), head of the Department of Surveying at the Hong Kong Polytechnical University and Honorary Research Associate of UNB.*

*1996: Adam receiving the Knight's Cross of the Order of Merit at the Polish Embassy in Ottawa for his contributions to scientific exchange between Canada and Poland.*



*1996: Adam (right) at the Warsak hydro electric power station on the Kabul River in northern Pakistan. Adam and his graduate students designed and implemented an integrated survey scheme to monitor and analyse structural deformations and slope stability around the site. The project is to be completed in 2000.*



*1997: At the Eastside construction site of southern California's largest water reservoir. Three large earth dams will enclose a valley to create the reservoir. Several active GPS stations and 10 robotic total stations will continuously monitor the stability of the three dams and their surroundings. Cecilia Whitaker (left) and Mike Duffy (far right) of the Metropolitan Water District, are in charge of designing the monitoring system. Adam (third from left) and his UNB research group serve as advisors on the design and they have developed software for the system based on the UNB generalized method of deformation analysis. The monitoring system should be operational in early 2000.*



*1998: Two retirees, Adam and Dave Wells, cut their retirement cake, although neither of them takes his retirement seriously.*



*1999 (above): Adam receives an honorary professorship at Olsztyn University of Agriculture and Technology (ART) in Poland. Left to right: Prof. Dr. R. Gorecki, Vice-President of ART; Prof. Dr. A. Chrzanowski; Prof. Dr. W. Baran, of ART; and Prof. Dr. J. McLaughlin, Vice President of UNB.*

*1999 (right): Adam and Geoff Bastin, UNB research assistant, at the site of ground subsidence studies at the PCS potash mine in New Brunswick. Ground subsidence monitoring and finite element modelling of deformations, initiated by Adam and his associates in 1989, aid in the enhancement of mine planning and safety.*



# It Was Not so Easy 21 Years Ago\*

*Adam Chrzanowski*

It was a sunny morning on the 30<sup>th</sup> of September 1964. The train from Montreal was a little bit late and I looked impatiently through the window to catch my first glimpse of Fredericton. I did not know yet that in order to get to Fredericton I had to spend another hour on a bus from the railway hut in Fredericton Junction. Though sleepy and tired, I was happy. I was in Canada! The country of my childhood dreams about far north adventures; the country of Jack London's Grey Wolf, and the Family of Whiteoaks; the books which I had read in Polish when I was ten. On the day of my arrival, 21 years ago, I expected to spend only one year at UNB, as an NRC Postdoctoral Fellow, to work with Gottfried Konecny on integrated control surveys and to share with Canadian colleagues my experience in engineering and mining surveys. Coming from Krakow, the location of the second oldest university in Europe, a place full of traditions and academic respect, I was surprised to find UNB located nowhere — in the middle of the woods. From my correspondence with Professor Konecny, I expected to meet a shaky old man, the European image of a professor. So I was pleasantly surprised when I met two young looking men at the entrance to UNB and one turned out to be Gottfried; the second Peter Wilson. They missed the arrival of my bus from Fredericton Junction, but waited for me at the gates. Shortly I was introduced to the respectful looking Gerhard Gloss and Eugene Derenyi. It was a hard-working group of friends, a mixture of different nationalities bound together by one goal: to raise the status of surveying in Canada.

I was busy from the very start. Less than a month after my arrival we had the Second Canadian Geodetic Symposium, *The Impact of Electronic Computers on Geodetic Adjustment*, and I was asked to give a paper on Cracovian calculus. I thought that everybody had known the "Cracovians", so similar to matrix algebra and used by geodesists and astronomers in Poland for 30 years before anybody in surveying started using matrices. I was delighted to have free access to the marvelous IBM 1620 computer with which one could invert square matrices of up to dimension 20 by 20 in a matter of minutes! I remembered doing it back in Poland with a mechanical calculator: I needed two weeks to do the same job. I used the IBM 1620, together with Gottfried, to prepare examples for a paper on the comparison of triangulation, trilateration, and traversing which was published in *The Canadian Surveyor* in 1965. It looks trivial now, after 20 years, but at that time it was a pioneering introduction to the optimization of geodetic networks.

During the first year of my work at UNB, my main task was to introduce engineering and mining surveying into the curriculum. What a shock I had when I visited the Ontario Transportation, or Highway, Department and learned that it was the first year that they had started using coordinates in designing and laying out the highways. How was it possible to

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\* Originally published in the Annual Report of the Department of Surveying Engineering, 1984/85, pp. 3-4.

do it otherwise? I still have not figured that out. Then I discovered that there was not a single mining company in North America that would employ a surveying engineer as a chief surveyor. About 90% of the chief surveyors were technicians trained on the job. What an offence to somebody from central Europe! I realized how much work and how much of a challenge there was in Canada for newcomers with a surveying background.

Slowly, slowly, I had to adjust myself to the new situation and I adjusted to the extent that I decided to stay at UNB for good. New faculty members started joining our group, some permanent, some only for one or two years or as visiting professors who later on became our close friends: Rafael Sanchez from Argentina, Ludvik Hradilek from Czechoslovakia, Peter Angus-Leppan from Australia, Julya Alpar from Hungary, and several others. With Sam Masry we developed and patented the first laser plummet in 1968. Then, my first big success—the first Canadian Symposium on Mining Surveying, held at UNB in 1969. Then the first Ph.D. degree in engineering surveys (Fouad Ahmed from Egypt).

In 1968 and 1971 two Canadians joined our small international group, Ed Krakiwsky and Angus Hamilton. Our department was already well established. Our hard work paid off. We can be proud of what we have built from nothing. One by one new members joined our group, Wolfgang Faig, Petr Vaníček, John McLaughlin, Dave Wells, Richard Langley. Some of us have already lost our hair working at UNB, but we still work hard and our influence is very visible across Canada and beyond.

The engineering and mining surveys at UNB have gained a good reputation. Our graduates are in charge of important ongoing engineering projects, such as the survey of the 14 km CP tunnel through the Rogers Pass and monitoring of subsidence in oil fields in Venezuela. Our telemetry system for monitoring deformations works in oil sands in Alberta, our generalized approach to the analysis of deformations is used on power dams, dykes, in mining areas and in geotechnical projects in many parts of the world. The grants and contracts for research in engineering surveys at UNB reached almost \$250,000 in 1984/85.

I still have a heavy Polish accent when lecturing, but I do not call our secretaries “typewriters” anymore, as I did the first few weeks of my stay at UNB. If I were arriving now as a newcomer, I would perhaps call them “word processors.”

## **The Story of “Urban Surveying and Mapping” (the textbook co-authored by Prof. Adam Chrzanowski)**

*DR. T .J. Blachut  
RSC, PADI  
Ottawa*

As I was approached with the suggestion to write an article suitable to be a part of a commemorative publication dedicated to Prof. Adam Chrzanowski, I did not hesitate a second. It was a suitable occasion to bring to the attention of the scholarly community in the broad field of the geodetic disciplines Prof.. Chrzanowski's important contribution in the field with which he usually is not associated: his part in the creation of the textbook "Urban Surveying and Mapping". In the technical literary world the book occupies a very prestigious position. It is the only book on the subject in the Western technical literature! Originally it was published in New York (1979) but very soon has been translated and published in Spanish in Mexico (1980) and eventually in Chinese. I never learned how big the English (6000 - 7000 copies ?) and Spanish editions were, but the Chinese edition counted 11 500 copies! A university textbook published in the English, Spanish and Chinese languages speaks for itself. There are not many examples like this!

The thought of publishing a textbook on urban surveying and mapping originated within the Pan American Institute of 'Geography and History (PAIGH) about eight years before the book was published. At that time I was "president" of the Committee on Large Scale Surveying, mapping and Cadastre of the Institute. The Institute is a specialized agency of the Organization of American States, a typically inefficient, international organization, enjoying official status. Canada joined PAIGH in 1961 (but at present decided to discontinue the partnership, which is a pity and mistake) and, it is fair to say, contributed greatly in various fields covered by the PAIGH. Above all, Canada tried to fill the often empty, bombastic shell, with projects of practical values. This seemed to be very appreciated by the Latin American members, who needed and were seeking help in various fields. Canada not being a military power and not aiming at political domination of other countries, could offer invaluable competent help in various fields, without any strings attached. In "my" PAIGH Committee on Large Scale Surveying, Mapping and Cadastre, we concentrated on projects essential for a rational and efficient management of the lands. As the result, the Committee was very popular and enjoyed enthusiastic support of all members. The Committee organized symposia at which particularly burning issues were discussed, seeking ways how to speed up the progress of American countries.

One of the fields particularly neglected, and yet of basic importance from an economic and social point of view, was the development of urban areas. All neuralgic centers of countries' life are located in urban areas. They are not only the political power centers but also centers of knowledge and spiritual life. In spite of that, nowhere was a conscious effort visible to create more bearable living conditions in cities and particularly in monstrous megalopolises. A precondition for such a deliberate effort was a systematic gathering of relevant information in urban areas or the procurement of data as provided by modern surveying and mapping operations. In order to avoid costly and completely redundant operations, an appropriate cadastral surveying had to become an integral part of urban surveying and mapping, indeed its basis.

When I raised this problem at one of our symposia and introduced the subject, the decision was reached that of primary importance would be the availability of an appropriate text. As usual in such situations, the chairman (it meant me) was charged with the production of a suitable text.

I should mention that in North America, but especially in the USA, this field was (and probably still is) hopelessly neglected. Therefore, it was impossible to refer to any example. I counted therefore on the academic community in Canada to help in the production of the needed text. I invited various professors from the Laval and New Brunswick Universities (the only ones with Surveying Departments at that time in Canada). However, soon, when confronted with a concrete task, they resigned, with the exception of Prof. Chrzanowski. The task was, indeed, very demanding, which I fully realized.

I must say, that I felt comfortable with the project. As a young geodetic engineer I was in charge of several city surveying projects in Poland before the outbreak of World War II. The cities were small but the principles to follow were the same. Moreover, some of the cities in the very industrial part of Poland were exceptionally complex, with a variety of intricate infrastructures (surface, overhead and underground) to be surveyed. In addition, ten years spent in Switzerland (partly as an interned Polish soldier, but due to the magnanimity of Switzerland, employed as scientific researcher at the Technical University of Zurich and an engineer at the famous "Wild" factory) confirmed and completed my Polish experience.

It should be realized, that city surveying and mapping is a very complex subject. In addition, the revolutionary development in the surveying and mapping technology, required a very critical approach to the suggested new procedures and practices. When writing a longer technical text, consisting of several parts, the internal cohesiveness of the text must constantly be watched and the focus on the main goals should always be clearly present. This requires a lot of concentration and preferably the availability of free time to do the job properly. This was obviously not the case: both of us, Prof. Chrzanowski and myself we were already overtaxed with our normal duties and "extra mural" commitments. Obviously, some time could be found at nights, on weekends and holidays, which we did. In addition, in order to alleviate our task, I invited a classical geodesist, Jouko H. Saastamoinen, to join us. Initially, he joined the NRC Photogrammetric Research Section as a so called "postdoctoral fellow" from Finland. At the NRC he made various, internationally recognized contributions in the field of geodetic operations and decided to settle down in Canada, becoming a research scientist employed by the Section.

As a group of friends, we did not have any difficulty to subdivide among us the responsibilities. We agreed from the very beginning, that we would treat the text as our common good and responsibility. Consequently, we decided not to publicize clearly who did what. We also decided that we would split the eventual royalty equally among the three of us disregarding the size of the individual contributions. This was a very happy formula, indeed, which permitted to avoid the arguments about the value and size of the effort put in the preparation of individual chapters. It is also true (as it turned out) that the meager size of the final royalty was not worth to be argued about.

The text was growing somewhat erratically but consistently. Quite amusing incidents were not lacking. At a certain moment, being tired of my own writing of more theoretical chapters, I decided to start, in way of relaxation, the parallel work on a more descriptive chapter on urban cartography. However, after few weeks, I was interrupted by other duties and a longer, demanding trip outside of Canada. After returning home and after catching up with the day to day problems, I had to resume the writing of the text in the spare, "free" time. In further few weeks, being tired of the sustained, rigorous effort, I again came on

the idea of writing in parallel a more narrative chapter on urban cartography. As in the meantime I forgot completely my previous start on that subject, I started from the very beginning. Soon, I noticed that somehow I had the sentences and even whole sections of the text neatly formulated, ready in my memory. I suspected to have unwillingly copying the text which I previously read. I knew, however, that such text did not exist. As the result I felt very confused and puzzled until I discovered by accident several pages of my previous manuscript misplaced among other papers!

I often thought about this episode suspecting that Prof. Chrzanowski was writing under similar conditions, since in addition to demanding courses given at the university and the supervision of many magisterial and doctoral thesis's he conducted in various countries critical projects requiring exceptional attention and care (e.g. crustal movements, deformation of dams, construction of huge underground accelerators etc.).

The progress was correspondingly slow, but we did not compromise until the manuscript satisfied our own expectations. I did my best to integrate various chapters into uniform contents without losing the guiding philosophy of the book, which we considered of primary importance. We realized that particular techniques and procedures would eventually be superseded as the result of the general progress and development. However, the general message and the guiding philosophy of the book should remain valid into the future.

We were in quite comfortable conditions as far as the secretarial services were concerned. My highly motivated and always very skillful and competent secretaries did not have any difficulties in the production of a clean, integrated manuscript which was subsequently passed to A. Richens and Mrs. Mary Geroux, technical editor of the "Canadian Surveyor". Being in addition for many years the editor of various publication within the ministry of Mines and Technical Surveys, Mary Geroux was fully familiar with the terminology used in the text.

Eventually, the moment arrived (beginning of 1979) to have to look around for a publisher of the text. Probably it would be useless to try to locate a publisher before the manuscript was complete with illustrations. As the events have proven, I was right. We fully realized, that most of the countries constituted the potential market. My natural tendency was to have the book published in Canada, and I was told by knowledgeable people at the National Research Council that the Toronto University Press would be the best place. I was also convinced, that the undertaking should be financially attractive to the publisher. Therefore, I traveled in high spirit to Toronto. I had there a very friendly but at the same time very disappointing (for me) meeting. I was told that this kind of highly specialized technical text, with the entire world as destination, "could only be successfully published in the USA". Since the field under discussion was and probably still is horribly neglected in the USA, I wondered whether the recommendation was correct. However, I had no choice, but to follow the suggestion.

Looking for an acceptable solution, it occurred to me that the famous "Springer Verlag" from Germany, active in the general field of geodetic disciplines, had a branch in New York. Trying to play "safe" I made three copies of the master manuscript and sent one to Springer Verlag in New York and two other to the leading publishers of technical texts, also in New York.

After the disappointing experience with the Toronto University Press I awaited the answer with a certain apprehension. To my great surprise in few days I received a telephone call from Germany, from Dr. Springer himself, the president of Springer Verlag. He confirmed the receipt of the manuscript and after few a very flattering opening



sentences, he asked me not to contact any other publisher, since one of his editors in New York was already making preparation for a trip to Ottawa in order to discuss the publication of the text. He assured me that we would be satisfied with the work done in his house as the general field of surveying and mapping is well known to them. Assuming that there would be no unforeseen difficulties, the book would be in world-wide distribution within six months from the date of signing the contract obviously, the company would take proper care of publicity. Well, we could not expect much more, and the beginning was encouraging.

However, the two other copies of the manuscript were already sent and it was interesting to receive other offers. I patiently waited, even though I already signed the contract with the "Springer Verlag". Indeed, in about one month I received an answer from another publisher (nomina sunt odiosa!): they confirmed receipt of the manuscript and were looking for reviewers. However, should they decide to publish the book, all metric notations would have to be changed to the "foot system"(!). The publisher did not specify whether the English or American system was in mind. I thanked them politely for their effort. The third publisher never answered.

The "Springer Verlag" laid down a precise time schedule and the very complex volume (372 pages of technical text full of formulas, 153 figures and 2 color plates, a very complete alphabetic index, hard covers) was indeed in distribution in six months from the moment of signing the contract. I admired the meticulous care and competence with which the manuscript was prepared for the final printing.

Once the book was published we were looking for the opinions of reviewers in various technical and scientific journals. We received about forty (!) of them, from all continents and major countries, including such countries as India, Australia and South Africa. They were complimentary and some of them even flattering. The one from Germany was somewhat embarrassing for me. The obviously very competent author closed the technical review of the book by a flattering remark: in addition to the very valuable content, the book is written in an exceptionally clear and understandable English and it can serve as an example how such books should be written! I promptly passed a copy of this review to Mary Giroux with the wish, that the readers would refer the flattering remarks in the first place to her.

There were no problems with the Spanish edition. Due to personal efforts of a very active PAIGH member from Mexico, Eng. Alberto Villasana, the book was translated into Spanish and published for the Latin American countries. The copyrights for the Spanish edition were exempt from the contract with Springer Verlag. We were therefore able to donate them to those developing countries of Latin America.

The final surprise connected with the book awaited me on my first visit to China. In this to me entirely unknown territory I was everywhere greeted very friendly, even with signs of some reverence. At the airport of Canton I was told by the delegation from the surveying department responsible for the region, that contrary to the official program of my visit, they would retain me for three days. Apparently they had many technical questions which they would like to discuss with me. They assured me that everything was cleared (of course!) with the higher authority. They were visibly pleased when I did not show any opposition, but to the contrary I agreed quite enthusiastically. I must confess that I liked the idea and in addition I was totally disarmed by the unique friendliness of my hosts. At the same time I was somewhat flabbergasted by the warmth and a certain intimacy of their attitude. The situation was compounded by the language barrier. I do not know Chinese at all and only one person of the delegation that met me at the airport spoke some English.

When asked directly, they answered that they knew some of my publications, translated into Chinese.

Only at the University in Wuhan, which specializes exclusively in the field of geodetic disciplines (Prof. Chrzanowski became a honorary professor of this university, an enviable recognition indeed! ), I learned about the reason of the intimacy in the attitude of my Chinese hosts. When I made a discrete remark on this matter to the local professor who studied for several years at the Toronto University and spoke fluent English, he excused himself with visible amazement for a few moments. When he returned he was holding in his hand ... the Chinese edition of our textbook on Urban Surveying and Mapping! In turn I was completely perplexed, since I did not know anything about the Chinese translation and publication of the book. The friendly professor handing to me the book made the following remark: "In Chinese culture authors like you enjoy enormous prestige. They are considered to be our leading teachers. Meeting them is a rare privilege and it is expected that they should be treated with the maximum courtesy and friendliness!" In a few days I received a copy of the Chinese edition of the book as a souvenir of the visit to Wuhan.

Writing the text of the book required from the authors an enormous discipline. There was no example that could guide us, in a positive or negative sense. Moreover, the text had to be created in "spare" time when the authors were already exhausted by their ongoing, intense responsibilities. The authors did not enjoy the luxury of isolation provided to some medieval writers enclosed in their monasteries. Each of us had his specific characteristic and tried to cope with the additional demand in his particular way. I knew, however, that each of us, when working on the text, was feeling a kind of urgent "mission" that had to be fulfilled. This helped us to master the necessary endurance in order to overcome the often frustrating difficulties. Without the always good humored and exceptionally competent contribution by Prof. Chrzanowski this book could not have been written!

# **Adam Chrzanowski**

## **UNB's Engineering and Mining Surveying Guru**

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In the early years of surveying engineering at UNB, the division — later department — had to rely on outside help to deliver a well-rounded leading edge program. One of the first academic visitors was Adam, who came to UNB in 1964 as a National Research Council of Canada postdoctoral fellow. He brought with him a wealth of knowledge and experience gained at the University in Krakow, Poland.

Being one of a small group of surveying engineering graduate students within the larger civil engineering pool, I met Adam shortly after his arrival. Concentrating on finishing my thesis, I had little interaction with him at first, although we did discuss some of the surveying problems I had faced in the Arctic. I did, however, get an early appreciation of his teaching skills at the second Canadian Symposium on the Impact of Electronic Computers on Geodetic Adjustments. While I will never forget how well he managed to explain to us the mathematics of “Crakovians,” I have forgotten most of the details, but still remember that they were basically matrices with rows and columns reversed. I must say, I never heard of them again, and Adam quickly switched to matrix notation in his work.

I was looking forward to working with Adam on the Ward Hunt Ice Shelf that next summer — but unfortunately red tape prevented it. Coming from east of the Iron Curtain, he was not allowed north of the DEW-line in case he would walk or ski away (it is all down hill from the top of the world!). Instead, Adam went to Alaska as a key member in the Mount Kennedy expedition, and rumour has it he had quite a trip getting there and back.

Adam liked it so well at UNB that he accepted a faculty position in 1966, which marked the beginning of a strong engineering and mining surveying component at UNB. This quickly became the centre of expertise for North America and beyond.

When I returned to my alma mater, Adam was here as the senior professor, having developed and patented equipment, guided a successful and dedicated research group, and generated great interest in his courses.

When I taught his senior surveying course for him, while he was driving a VW bus through South America during his sabbatical, I was surprised, and a bit overwhelmed, to find out that half of it was hydrographic surveying. He quickly dropped that once Dave Wells came on the scene in 1980.

Over the years, our research interests overlapped with my attempts to utilize photogrammetry for industrial projects and for deformation studies. We collaborated on a variety of projects, most notably the mining subsidence studies near Sparwood, B. C. We spent some interesting days there on the slopes, Adam in a white coat, so that he stood out more from the coal seams, and whistling so that the bears would not be frightened and peacefully go away. The highlight of the day's work was the prime rib dinner at the miners' pub, with beer and entertainment.

Adam's reputation as engineering and deformation surveys expert quickly spread around the world, bringing him involvement in numerous interesting projects with colleagues on every continent, and also within international professional associations. He organized symposiums and workshops, wrote papers — usually about a month after the deadline — travelled, made presentations, and travelled again. Yet he found time to become department chair and efficiently and thoroughly perform his duties.

I admire his stamina, determination, and willingness to work hard and I count myself privileged to have a close and cooperative relationship with him, which — I am sure — will continue well into his retirements. With best personal wishes!

University of Idaho  
College of Mines and Earth Resources  
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Moscow, Idaho 83844-3022

May 30, 1999

Dr. Adam Chrzanowski

For Your Retirement Scrapbook:

What great pleasure and warm memories you have given to us in the JIRP family; and of course also to our esteemed Mt. Kennedy mapping crew. Your years on the Juneau Icefield (1965, 1966, 1967 and 1968) resulted in stellar contributions to our survey program and on that memorable 1965 Yukon expedition, the monumental map we produced was largely through your efforts. And you gave us significant and appreciated liaison with your distinguished surveying department at the University of New Brunswick. But more than those professional accomplishments you provided a joy of companionship, good cheer and Polish “Golumpki” (spell?) with meat rolled in cabbage and sprinkled with powdered TUMS. You have added zest and joy to our lives – and in the long run that is the most appreciated. Though the years have winnowed us all, you know we have extended an ongoing invitation to you to return to Camp 10 and 18 and the high heights of the Taku. Your mentoring of our exciting new cadre of students could continue to add zest to our operations. And when you come back bring your great story of how you used condoms for achievement in mine surveying.

May we have the renewed joy of a reunion soon. You are one of our greatest and fondest memories.

Always with affection

Maynard and Joan Miller  
And the JIRP and Mt. Kennedy Crews

# Three Professors, One Engineer, and Twelve Lobsters

*Juan Murria*  
*Member of the Board of Directors*  
*Venezuelan Foundation for Seismological Research (FUNVISI)*  
*Caracas, Venezuela*  
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Several years ago (it must have been in the late 1980s or early 1990s), I was in Fredericton at the invitation of Professor Chrzanowski to spend several weeks at the Surveying Engineering Department as “Distinguished Visiting Researcher” or something to that effect.

At that time, Professor Yong-qi Chen, now Chairman of the Surveying Department, Hong Kong Polytechnic Institute, but at that time at Wuhan University in China, was a “Visiting Research Professor” in Fredericton.

Professor Willy Caspari, of the Institute for Geodesy at Army University in Neubiberg, Germany, was also visiting Adam.

Adam, as was customary when he had visitors from out-of-town and particularly from abroad, had invited us over to his house for what he termed “a lobster boil.” We arrived at the beautiful Chrzanowski home in mid-afternoon of a beautiful Fredericton summer.

Well provided with a good selection of Canadian beers, for Adam, the delightful host that he was, knew my (or I should say) our taste for good beer, he proceeded to put on the stove three or four good sized pots full of water. And then the problems started: The professors and yours truly might have been the top of the heap in their respective careers as academics, researchers, consultants, and engineers but ... lobster boilers? I don't know! Opinions came and went as to how hot should the water be? how long should it boil before immersing the live lobsters? how long should we cook them? how many per pot? how much salt should be added? at what stage of the boiling process? Talk about a deformation measurements seminar!!! Theories, postulations, arguments went back and forth always accompanied by long satisfying quaffs of Moosehead and Labatt Blue.

Finally, some kind of a consensus was reached and one-by-one the twelve lobsters found their way into the pots, not without energetic movements on the part of the live crustaceans with the corresponding danger of first-degree burns for the three professors and the one engineer now in the role of lobster boilers as when the water started to boil again and the lobsters tried to jump out of the pots!

Fortunately, no damage to body or limb occurred. The lobsters lost their battles ... and their lives ... for the gastronomic satisfaction of Adam and his overseas guests.

# A Good Acquaintance

*Stanislaw Oszczak, Alojzy Wasilewski, Zofia Rzepecka*  
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Professor Dr. Eng. Adam Chrzanowski, born in Poland, studied at the Technical Mining Academy in Cracow. He began his work there, first as an assistant, then as a senior lecturer. In 1964, in the frame of scientific cooperation with the University in New Brunswick, he went to Canada taking the position there of associate professor. Then he was appointed full professor in the Department of Survey Engineering. In spite of an absence of 34 years from Poland, Professor Adam Chrzanowski has never broken his scientific nor social relations with his native country. His wide learning, friendliness, and favor have made him a host of friends in Poland.

Professor Adam Chrzanowski is a wonderful man. He takes everybody with his good humor, cheer, and wit.

Close relations between the Institute of Geodesy, Olsztyn University of Agriculture and Technology, and Adam date back to 1994. Two big international events that took place in August and September 1994, namely, the Perelmuter Workshop in Haifa and the first Symposium on Deformations in Istanbul, provided good occasions for our common meetings, discussions, and talks devoted to cooperation between our universities.

In 1995, Professor Chrzanowski accepted an invitation to come to our institute. He delivered a very interesting report on precise geodetic works he had taken part in during construction of a particle accelerator in Texas. Also in this year, representatives from our institute met him in Hong Kong, during the 8<sup>th</sup> FIG International Symposium on Deformation Measurements.

The next visit of Professor Chrzanowski, this time accompanied by his wife, Dr. Anna Szostak-Chrzanowski, took place in 1997. During this visit, research workers of our institute had an opportunity to hear two reports devoted to tension modelling in rock mass, using the application of the method of finite elements.

From our side, three visits to Canada took place. The first visit was in October 1996, by Professor Stanislaw Oszczak; the next, also in October but one year later, by the rector of our university; and the third visit in March 1999 by Professor Richard Gorecki, vice-president of our university, and Professor Vlodimir Baran, dean of our faculty of geodesy and land management. In 1997 a formal agreement concerning cooperation between the

Olsztyn University of Agriculture and Technology and the University of New Brunswick was signed by the both parties. In the frame of the document we agreed to a common research project concerning the study of deformations of the mass rock in the area of the Polish Copper Basin (LGOM). This project will be put forward to the Polish Committee of Scientific Research for approval and financial support.

In March 1999, our University unanimously approved for Professor Chrzanowski the title of honorary professor. Our institute is going to invite Professor Adam Chrzanowski to deliver a series of thematic lectures as the honorary professor.

In September 1999, our institute, in cooperation with Professor Adam Chrzanowski, organized the 9<sup>th</sup> FIG International Symposium on Deformation Measurements in Olsztyn.

We are impressed not only by scientific achievements of Professor Chrzanowski but also by some funny events that happened during our acquaintance with him. During his journey to Poland in 1995 his luggage was lost. Everybody can imagine that it is not very pleasant to have no clothes to change nor your own, let us say, handkerchief, but Professor seemed to be in good temper in spite of that. The weather changed, he did not have his summer blazer but still he was full of good humor. He suspected the luggage was lost at the Polish airport. When he learned it happened in very orderly Switzerland, he gave his apology by default to the Polish airport authorities.

Also during this same visit, Professor gave a lecture during a technical conference organized by the Polish Geodesists Association (SGP), in Gdansk. He showed there a new, brilliant way of geodetic monumentation — he simply produced a bottle of brandy from his bag, keeping it upside down!

Being together in Hong Kong also made us sure of how cheerful Professor Chrzanowski is. We took part in an “ice-breaking” party, where there were some people from Poland. Professor attended our group, he sang Polish songs (very nicely and loudly), and he was the life and soul of the party. In this far country, Professor had also not so good an adventure. When we went together sight-seeing (and “shop-seeing”), Professor liked very much to stop at small restaurants, cafeterias, or even simply at places where poorer dealers sold their food that was eaten directly from the table! In one of these places, we ordered something very strange to eat (nobody knows what it was in English). Professor asked the owner how to eat it, since it was really a very strange dish. The dealer, full of good will, showed how to get through it – using his fingers, and dividing Professor’s portion into smaller pieces, with everything on the table, and without plates. Professor ate everything; looking at him the food seemed to be the most delicious in the world. We tried to be as brave as our friend from Canada was. But the next day, unfortunately, Professor did not feel well, he had to stay in his bed for some hours — it was horrible for such an active man as he!

Believe us, we look forward to meeting Professor Chrzanowski for the next time as quickly as possible!



# **Across Bulgaria with Motor Scooters**

*Zbigniew Sitek, Professor  
Technical University of Mining and Metallurgy  
Krakow, Poland*

Summary by Anna Szostak-Chrzanowski of a Polish text sent by the author.

My friendship with Adam Chrzanowski started 40 years ago, when both of us were assistant professors in the Faculty of Mining Surveying at the Technical University of Mining and Metallurgy in Krakow. At that time we were ranked by our colleagues as very dynamic researchers, but our dynamism and energy were not limited only to teaching and research. We liked hiking in the mountains, camping, and white water kayaking. One summer in the late fifties, we decided to visit Bulgaria and tour it using motor scooters as transportation. In the fifties it was not easy to travel from Poland to any foreign country even though Bulgaria belonged to the eastern block. It involved a lot of struggle with bureaucracy to obtain travel documents and permits to ship the motor scooters to Bulgaria. However, we managed to get all permits and after a long journey by train through Hungary and Rumania we landed in the coastal city of Varna located on the Black Sea. We started our tour along the Black Sea coast from Varna, through Zlatni Pjasaci, Nesebar, Pomorie to Brugas and Sosopol. The Black Sea coast is spectacular with gold sandy beaches and dramatic cliffs. After three weeks of travel, we turned towards Sofia and drove through rugged, hilly Bulgarian land. The roads were empty with little car and truck traffic. The journey was exhausting because of the heat, constant sun exposure, and weak motors of our scooters. We had to stop quite often to relax in the shade under trees. On the way we met some interesting people, for example, the son of a famous Bulgarian painter who gave us a tour through his art gallery.

After arriving in Sofia we were surprised to see that in the capital there was also limited car traffic. There were no lights at the main city intersections. We met a number of young people, whom we visited in private homes and went together to restaurants. We had good food and we tried good Bulgarian wines. We sang Bulgarian songs and till now we specially remember a song about an innkeeper: "Give us old and heavy wine so that we could forget the hardships of our life". The song became a symbol of our Bulgarian escapade and of that period of our life.

## Reminiscences of Adam

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One of my first recollections of Adam Chrzanowski was in an undergraduate lab. It was my first year at UNB where I was starting an M.Sc. program. Because my first degree was in Civil Engineering rather than Surveying Engineering (now Geomatics Engineering, Geodesy and Geomatics Engineering and other names depending on the institution to which you owe allegiance!), I was taking a number of senior undergraduate courses. In this particular lab, Adam spoke in great detail about EDM (electronic distance measurement) instruments. I'm sure that there was no advance preparation - he was just drawing on his extensive knowledge and experience. I remember being very impressed. I am still very impressed today. Adam is one of the few truly international-calibre teachers and researchers in engineering surveys. I have heard him speak at many international conferences. With his strong, authoritative voice and deep knowledge of the subject matter, his presentations are always among those you remember.

I completed my M.Sc. studies under Adam's supervision in 1979, and moved on to The University of Calgary where a new Surveying Engineering program was being set up. All of the first Faculty Members, including me, had a strong connection to UNB, and despite competition between "old" and "new" Surveying Engineering Departments, there was a lot of cooperation. In all research areas, we each promoted and participated in conferences organized by the other Department. I believe one of the best examples of UNB/UofC cooperation occurred at FIG Banff 1993 (abbreviated title of 7th International Symposium on Deformation Measurements together with 6th Canadian Symposium on Mining Surveying). This conference was particularly important to me because I was in the process of making the strongest case possible for my promotion from Associate to Full Professor. This conference was hosted and organized at UofC, but Adam and his research group had a very strong involvement. In addition to actively promoting the conference, Adam was directly responsible for a number of excellent papers relating to precise engineering survey work at the Superconducting Super Collider (then under construction in Texas) being presented. There is no question that these papers gave a significant boost to the technical level of FIG Banff 1993. Thanks Adam, and have a great retirement!

# Time Series Analysis for Deformation Monitoring

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This paper deals with a geodetic application of time series analysis. The observations forming the time series were taken at a power dam located in Canada. Prof. Chrzanowski, who has provided the data, has been conducting deformation measurements at this dam for many years. His analyses of these measurements have led to comprehensive deformation models and successful approaches to processing observational data which are exemplary and in use all over the world to monitor the behavior of comparable objects. Our modest contribution focuses on the special problem of deriving useful information from time series. As it turns out, the classical statistical methods fail, since the observations are contaminated by outliers. Only the introduction of robust estimation methods leads to some success. The result of our endeavor is only a very small contribution to the body of knowledge on deformations not at all comparable with the significant contributions of Prof. Chrzanowski.

## Deformation Monitoring of a large dam

Large dams suffer deformations from a number of causes like changing water levels, seasonal temperature shifts and the immense weight of the structure itself. Starting with the construction of a large dam, precision measurements are regularly carried out in order to obtain a stable reference frame and to establish a database from which the future behavior of the structure can be predicted. A large variety of equipment is used for this purpose including classical geodetic instruments like EDM and theodolites, geotechnical sensors, e.g. plumb lines, extensometers and jointmeters. Nevertheless, individual characteristics of each dam require special measurement setups and examination procedures. As an example, for the purpose of this paper, serves the well known Mactaquac power plant which was built at the Saint Johns River in the years 1964 to 1968. The main part of it consists of a concrete dam, 42 meters high with the power house situated in front of it.

Figure 1: Bird's view of the dam [Chrzanowski et al., 1989]

Since 1975 strong deformations of the dam and widening expansion joints at the power house were observed. The displacements with steady rates also took place in the succeeding years leading to enormous deformations of the spill building. Local

instabilities of the surrounding rocks, transfer of pressure through the stock pipes, tension within the structure itself and chemical processes were considered as causes and had to be rigorously investigated before exclusion. In the end it was found that the deformations were caused by an alkaline reaction of the used cement [Chrzanowski et al., 1991].

Figure 2: Cross section with placement of sensors [Chrzanowski et al., 1991].

Figure 2 shows a cross section of the dam and the powerhouse with the placement of the various sensors. At the expansion joints jointmeters are applied. The measurement principle of these gauges is rather simple. On one side of the gap three cones are arranged on a base plate, facing one single cone on the other side of the gap. By measuring all distances and applying simple geometric calculations the relative spatial movement can be determined. The accuracy is stated to be better than 0.1 mm [Wroblewicz/Solymar/Thompson, 1988]. Two of the thereby obtained time series are shown in figure 3.

Figure 3: Time series of jointmeter readings.

## Time Series Analysis of Jointmeter Readings

To prove the performance of a newly developed autocorrelation estimator [Sutor, 1997] the available data sets originating from the described jointmeter measurements were utilized. One time series was chosen to serve as an example which will be analyzed throughout this section. As the first step a trend function and an additional periodical component were adjusted to the observations. In figure 4 the estimated trends are displayed together with the residuals after adjusting a cyclic function to the time series. The respective parameters are given at the right margin.

Figure 4: Jointmeter readings with estimated trends.

The classical non robust least squares estimation leads to a quadratic trend as the best fit to the observations while the robust method yields a Gaussian trend function. The trends were chosen from six trial functions under the condition that both, the empiric variance and the number of required parameters attain a minimum. All estimated coefficients are significant as can be seen from the corresponding standard deviations. Figure 5 depicts the obtained residuals after the estimated trend was removed from the original time series.

Figure 5: Residuals after elimination of trend

The studentized residuals of the least squares fit (left graph of figure 5) contain obviously outliers, while the robust residuals (right graph of figure 5) do not. The iteratively computed weights of the robust adjustment correspond with individual empirical standard deviations which efficiently control the influence of outlying observations, so that the deterministic components of the time series could be successfully estimated and removed. The two resulting residual vectors, plotted in figure 5, can now be considered as realizations of a stationary stochastic processes and can, hence, be analyzed with methods based on the well known theory of linear time invariant systems. This has been done in two different ways. The residuals of the least squares

trend estimation have been treated further by non robust methods, while robust estimators have been applied to the robust residuals. In a first step the autocorrelation functions of the two series have been estimated. Figure 6 shows the result and gives a first impression of the overall stochastic behavior of the observations and enables a more detailed view of the micro structure.

Figure 6: Estimates of autocorrelation function

As already mentioned, a new robust estimator of the autocorrelation function has been developed by the authors. The main idea can be briefly described as follows. The influence of each individual observation on the autocorrelation estimate is analysed, then the original observations are appropriately reweighted in order to reduce the leverage effect of outlying values. As the right graph of figure 6 clearly shows, this robust estimation method uncovers periodicities in the time series which were not detected in the previous steps of the analysis. For the interpretation of these periodicities it would be desirable to have further information on the forces which cause the observed deformations, but since these are not available in this case, only conclusions from empirical modelling are possible. The dominant oscillation appearing in the right graph of figure 6 has a period around 25 observation intervals. Because the readings of the jointmeter were taken weekly, this period suggests a half year oscillation. The existence of additional, less strong oscillations can be conjectured from the close inspection of the robustly estimated autocorrelation function. But to get a more precise picture of the situation a (robust) estimation of the spectral density function is required. Less clear is the interpretation of the left graph of figure 6, originating from the non robust estimation of the autocorrelation function. It demonstrates the not at all satisfying properties of least squares estimation in the presence of outlying observations. An analysis of structural dependencies between the observations based on these estimates seems to be hardly possible and meaningful. Nevertheless, a certain systematic behavior can be observed for lags smaller than  $k=100$ . The interpretation will tend to assume random effects suggested by the overall plot.

The spectral density functions of the data sets plotted in figure 5 were estimated by the periodogram (non robust) and the so-called Blackman-Tukey estimator (robustified by the authors) [Caspary/Sutor, 1996]. Both methods are based on the estimates of the autocorrelation functions of figure 6. The periodogram uses the Wiener-Khintchin theorem to relate correlation function and density of the power spectrum, while the second method at first improves the estimate of the correlation function and then applies the fourier transformation. Both estimated spectral density functions are shown in figure 7.

Figure 7: Estimates of spectral density function

The left graph of figure 7 demonstrates the weakness of the non robust periodogram estimator. Since the observation vector contains a number of outliers the generally inferior statistical properties are amplified leading to an estimate of the spectral density function strongly suffering from distortions. In addition to this fault a so called line splitting appears rendering the interpretation even more difficult and risky. Especially in the low frequency range a localization of individual spectral lines is nearly impossible, and the micro structure of the observations which was by the track of robust methods

already detected in the autocorrelation estimate (see figure 6) is here completely hidden by noise. In contrast, the robustified Blackman-Tukey estimator clearly unveils a number of oscillations whose periods are either a fraction or a multiple of one year. To confirm this outcome a cross-check with a dynamic model of the dam based on forces and environmental parameters would be desirable.

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## Figures

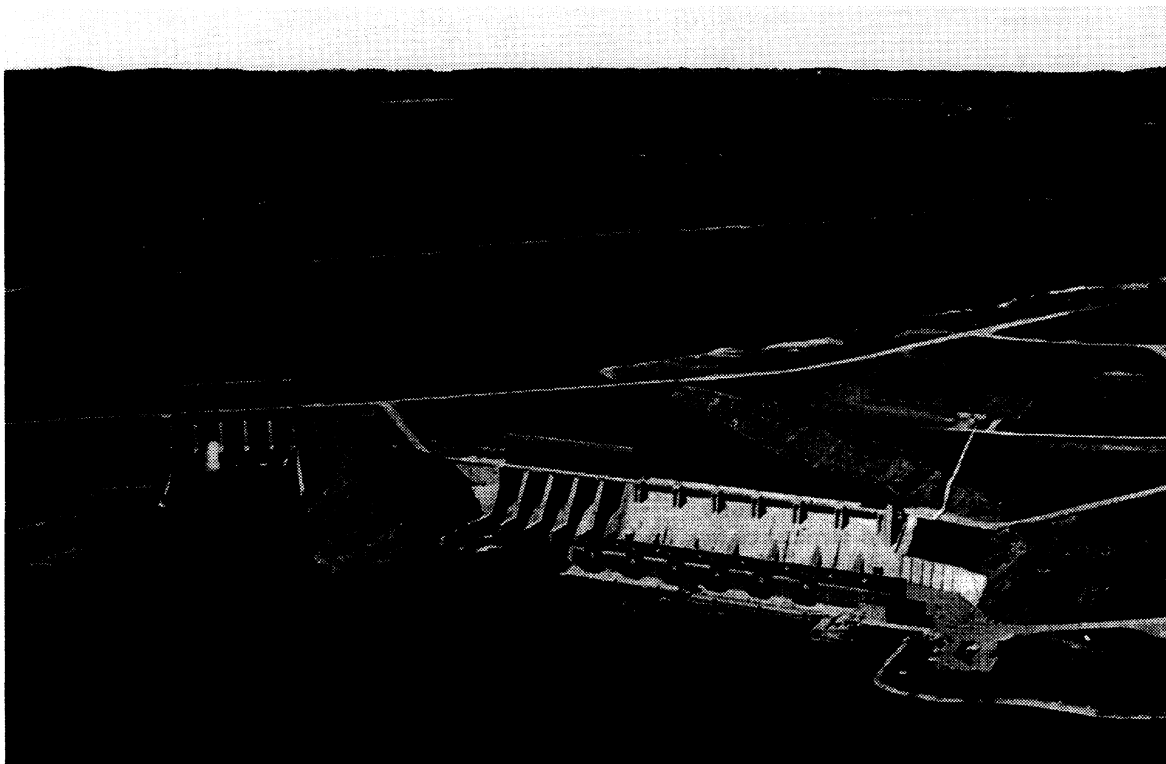


Figure 1: Bird's view of the dam [Chrzanowski et al., 1989]

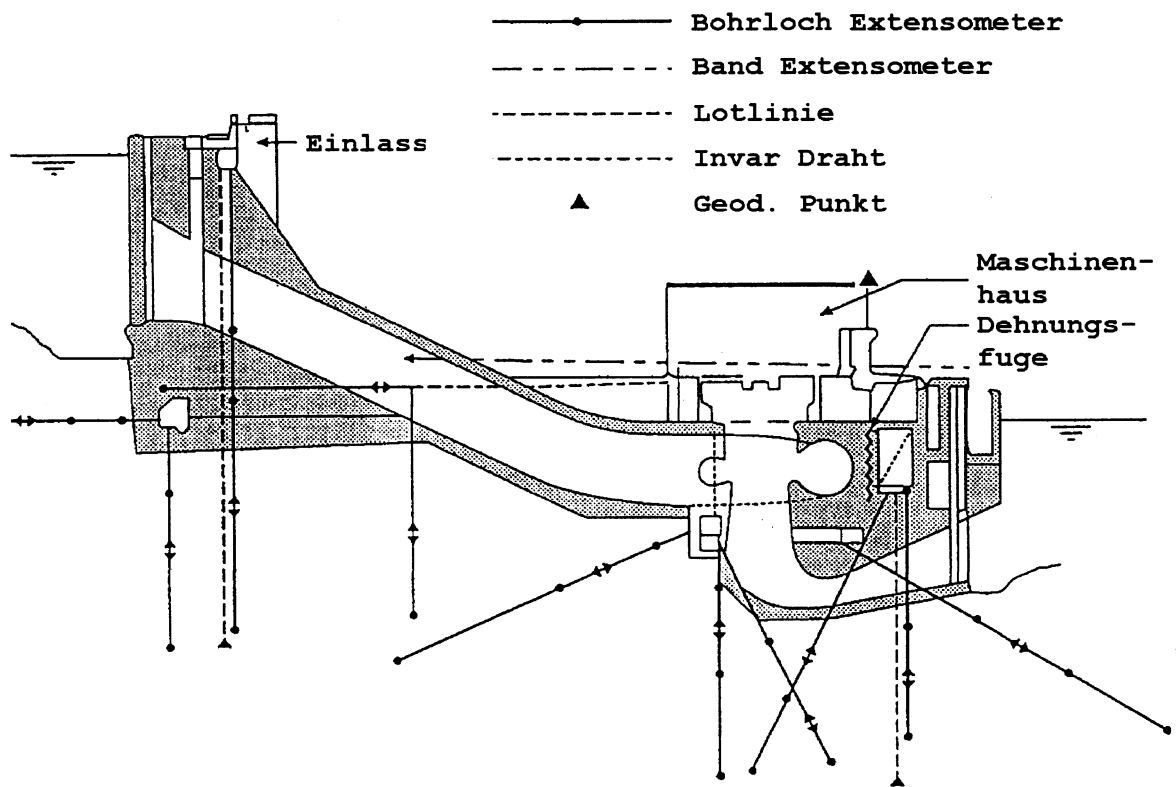


Figure 2: Cross section with placement of sensors [Chrzanowski et al., 1991].

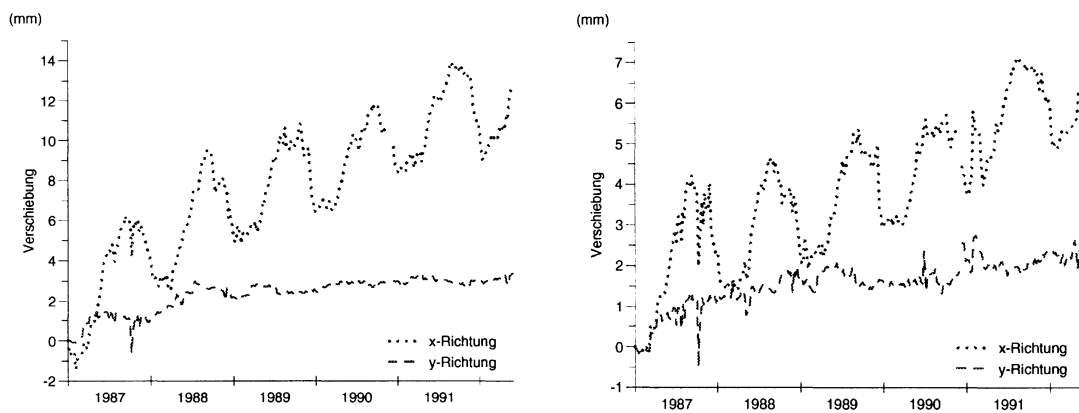
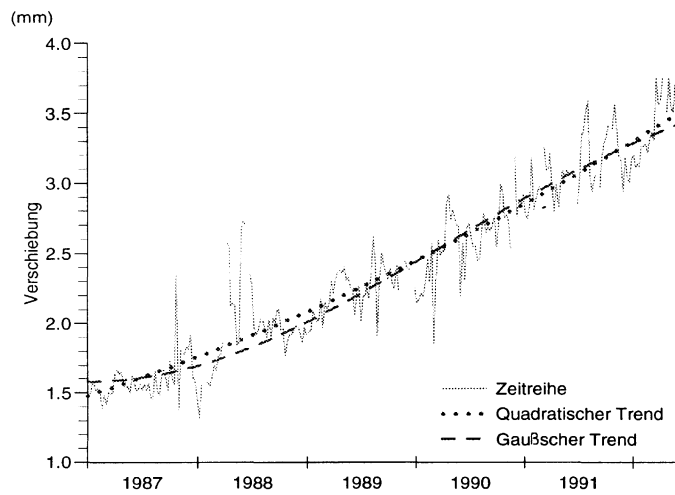


Figure 3: Time series of jointmeter readings





*Least squares adjustment*

Parameter	Value	Std. Dev.
a	8.E-6	1.0E-6
b	5.E-3	0.4E-3
K	1.47	0.02

*Quadratic Trend ( $s^2=0.020$ )*

*Hampel (bounded influence)*

Parameter	Value	Std. Dev.
a	18.E-6	2.0E-6
b	-2.4	0.11
K	3.98	0.11

*Gaussian Trend ( $s^2=0.019$ )*

Figure 4: Jointmeter readings with estimated trends

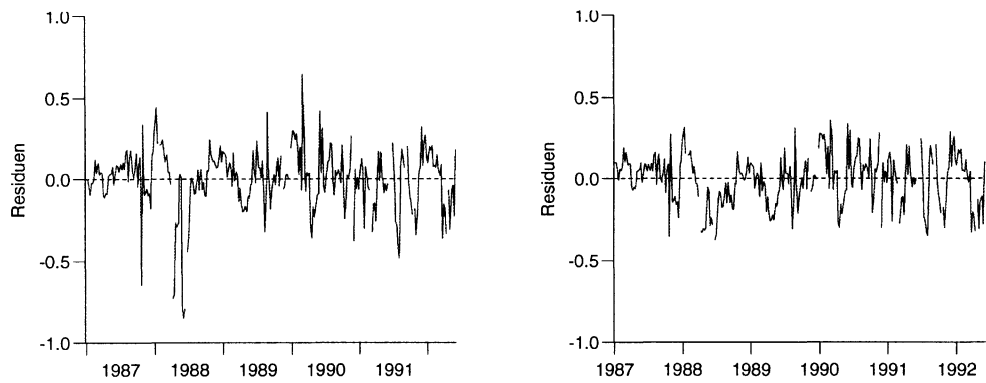


Figure 5: Residuals after elimination of trend

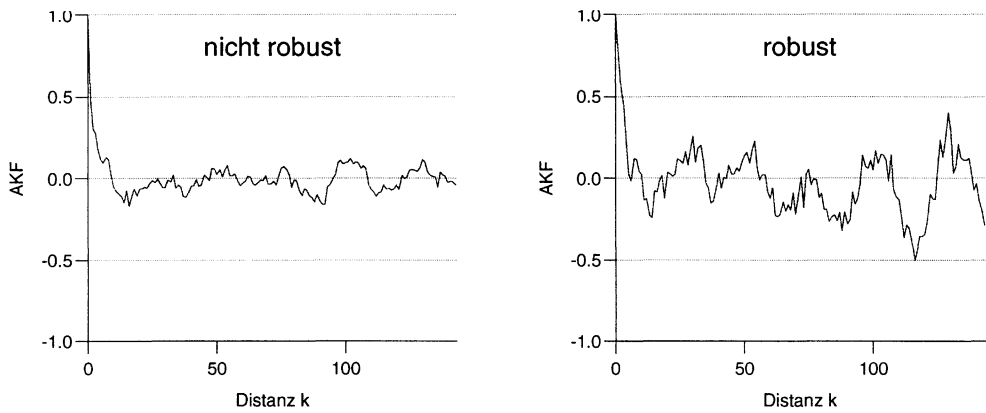


Figure 6: Estimates of autocorrelation function

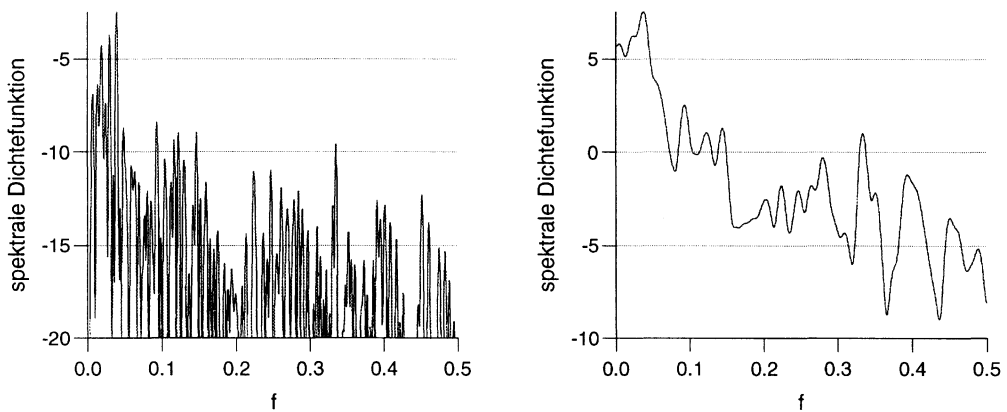


Figure 7: Estimates of spectral density function

# Developments in Deformation Monitoring and Analysis

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## Abstract

Deformation monitoring is one of the main professional activities of surveyors. FIG WG 6C with its current Chairman Professor Adam Chrzanowski has been active in the study of this area. Most of the recent developments within its activities are of a multi-disciplinary nature and are applicable to any deformation problems. This paper is aimed at highlighting the recent developments in the areas of monitoring techniques, analysis and interpretation of survey results, and design of monitoring schemes, to which Professor Adam Chrzanowski has made significant contribution.

## 1. Introduction

Safety control and understanding of deformation mechanism are the two objectives of deformation monitoring. Through deformation surveys the stability of an engineering structure or a geologically hazardous area can be examined; any disaster signals can be detected. A correct prediction of disasters will prevent or reduce property damage and life loss. On the other hand, the deformation mechanism of an object is usually not well defined. A number of hypotheses may be put forward. The deformation surveys can then be used to test and distinguish among those hypotheses. Moreover, with the deformation measurements the relationship between the loads (causative factors) to an object and its response (deformations) can be established, which can be used for the prediction of deformations.

Deformation survey is one of the main professional activities of engineering surveyors. FIG Commission 6 Working Group C under the current chairmanship of Professor Chrzanowski has been very active in the study of this area. Eight international symposia have been organized since 1975. In addition to the FIG efforts, several international societies deal with problems of deformable world, like International Society of Mining Surveying (ISM), International Association of Geodesy (IAG), International Society for Rock Mechanics (ISRM), and International Commission of Large Dams (ICOLD). But, the work of FIG WG 6C in this area is the most versatile and interdisciplinary because it is not biased by any particular applications. Most of the recent developments within its activities are of a multi-disciplinary nature and are applicable to any deformation problems, whether dealing with dam deformation monitoring, or ground subsidence studies, or tectonic plate movements (Chrzanowski, 1996).

On the occasion of the retirement of Professor Adam Chrzanowski, the author presents this paper in his honor. The paper highlights the recent developments in the areas of

monitoring techniques, analysis and interpretation of survey results, and design of monitoring schemes, to which Adam has made significant contribution.

## **2. Monitoring Techniques**

Monitoring techniques are classified into geodetic survey and geotechnical measurement (Chrzanowski, 1986). The former, including terrestrial, photogrammetric, and space positioning techniques, provides the displacements of points on an object, while the latter is usually used to determine the relative geometric changes between two points. With the advancement of technologies, especially in high precision directional borehole drilling, it is now possible to use geotechnical methods to determine absolute changes of geometric quantities (Chrzanowski, 1986). For instance, inverted plumb-lines and borehole extensometers anchored deep enough into the bedrock outside the deformation zone can determine the absolute displacements of object points in a particular direction to an accuracy comparable and even exceeding that of geodetic surveys. The geotechnical methods are traditionally employed by other professionals, e.g., civil and geotechnical engineers. Modern trend, however, is to integrate both classes of techniques into a monitoring scheme. The purpose of integration is to take full advantage of different measuring techniques and to provide independent check of the survey results.

### **2.1 GPS**

The use of GPS for deformation applications has received considerable interest in the last decade. Three operation modes are distinguished: periodic repeat surveys, permanent GPS arrays, and real time kinematic measurements. In the first mode GPS field observation campaigns are mounted to determine the relative position of groundmarks at a certain epoch. During each survey campaign the groundmarks are regarded stable (i.e., negligibly small movements). This is the most frequently used approach. However, special attention must be given to the systematic effects of GPS surveys and their modeling in deformation analysis (Chrzanowski and Chen, 1994). For some critical projects/areas (e.g., earthquake-active region, potential landslide areas, sensitive engineering structures) a number of GPS receivers are permanently installed, and the data are continuously collected and transmitted to a center for processing. This concept of continuous monitoring using GPS was first proposed by Ladd (1986), and has since been used in several projects, e.g., measurement of dam deformations (DeLoach, 1989), monitoring of the crustal movements (e.g., Bock, 1991), slope stability study (Shimizu et al., 1996). A few millimeters accuracy of displacement determination can be achieved for the projects with slow deformation rate. In these projects the GPS observations for a few or even tens minutes can be grouped into a survey epoch in data processing to reduce the effect of some error sources. In the study of the dynamic behaviors of engineering structures (like bridges and high-rise buildings) the real time kinematic positioning is required. Some example applications are in (Leach and Hyzak, 1994; Ashkenazi et al, 1996; Leroy, 1996). Recently, a methodology and software GPSMON for single epoch GPS deformation monitoring have been developed (Lutes, 1996; Chen and Lutes, 1998) by taking full advantage of the characteristics of deformation surveys, i.e., the position of a surveyed point changes relatively small between two consecutive survey epochs. The method can be used to process the GPS data from both slow deformation and rapid/dynamic deformation monitoring. In the application of GPS in deformation surveys, especially for engineering projects, special attention must be given to the multipath effect.

## 2.2 Photogrammetry

As a tool for deformation monitoring, photogrammetry displays a number of beneficial attributes such as network design flexibility and speed and non-contact nature of data acquisition phase. Example applications have ranged from ground subsidence study (Armenakis and Faig, 1982) to the monitoring of dams, retaining walls, landslides and industrial tooling (Fraser, 1993). The film-based photogrammetry also exhibits notable practical shortcomings. They include the time associated with image measurement, the expensive nature of the equipment involved and off-line, off-site nature of data processing phase. The introduction of digital cameras in photogrammetry has given rise to systems which facilitate both real time and near real time measurement. The main drawback of most digital cameras are their relatively low angular resolution compared with their counterpart. For a film based camera the image measurement resolution of 1-3  $\mu\text{m}$  is usually achievable. For a digital camera the measurement resolution is directly dependent on both the digital image processing algorithm used to determine image coordinates and the pixel size of the sensor. The typical pixel size of CCD cameras is about 10  $\mu\text{m}$ . The achievable measurement accuracy is 0.3 - 0.5 pixel employing either manual pointing to targets or automatic image matching, which translates to 3-5  $\mu\text{m}$ . The second factor causing the lower angular resolution is the shorter focal length of digital cameras, leading to smaller imaging scale. To offset the effect of small image scale superior image coordinate measurement must be ensured. Recently, some digital image measurement techniques have been developed for artificial targets, e.g., the centroiding (Shortis et al, 1994), the shape fitting (Cosandier and Chapman, 1992), which can offer accuracy to 2-10% of a pixel (Fraser, 1996, Lichti and Chapman, 1996). For a typical pixel size of 10  $\mu\text{m}$  this is equivalent to 0.2-1.0  $\mu\text{m}$  which well exceeds the precision of film measurement. With an average precision of 0.5  $\mu\text{m}$  the relative accuracy of object coordinate determination can achieve 1:50 000 for typical digital cameras.

## 2.3 Surveying robots

The terrestrial geodetic methods using EDM, theodolites and levels are still widely used in deformation surveys, because of their flexibility and suitability for different projects with different accuracy requirements and different environments. However, they are slow and labour intensive. Advent of georobots makes automatic data collection and real time processing possible. Surveying robots are specially-equipped motorized electronic theodolites which can carry out measuring tasks autonomously under the control of specialized software running on the external computer. Two types of automatic target detection are available: EDM signal scanning and video. The former systems can measure to reflectors in or near known positions and employ the return signal of the distance meter to confirm or improve pointing to these known reflector positions. The latter systems perform target detection with a CCD camera integrated in the telescope. Image processing hardware and software in a system controller conduct automatic target detection and measurement. Commercial georobot systems became available in the late 1980s, like the Geodimeter 140 SMS Slope Monitoring System appeared in about 1987 and Wild Automated Polar System (APS) in about 1990. Several models (also called on-man total station) are now in market. Their applications were reported in many projects, e.g., monitoring of bridges, landslides, dams and tunnels (Rueger, 1994; Katowski, 1995, 1996).

## 2.4 Geotechnical measurements

Geotechnical instrumentation may be divided into three groups: determination of physical properties of material; determination of acting forces (loads) using various load cells; and

determination of geometric changes (Chrzanowski, 1986; Dunncliff, 1993). The last group is of direct interest to surveyors. The major advantage of geotechnical instrumentation is easy adaptation for continuous and telemetric data acquisition. But its measurements are very localized and may be strongly affected by local disturbances.

*Measurement of changes in distance.* Extensometers are commonly used to measure change in a distance, for instance, development of a crack on rock, settlement or upheaval of soil, convergence of walls in a structure and underground excavations, movements of a structure with respect to the foundation rocks. They are made of various types of material (such as steel, invar, aluminum, or fiberglass) and in different form (tape, wire, or rod). Most of the extensometers have digital readout. Readings can be taken by site personnel, or stored in a data logger and transferred to computers afterwards. Some can be connected to telemetry system to log and check the readings in real time. Measurement ranges vary from a few centimeter (crack meters) upto hundred meters. Accuracy better than 0.1 mm can be obtained if the effect of error sources is correctly taken into account.

*Measurement of tilt and inclination.* Based on operation principle tiltmeters are divided into: liquid, vertical pendulum, and horizontal pendulum type. Their sensitivity ranges from 0.001" to tens of seconds. Tiltmeters have a wide range of applications. A series of tiltmeters if arranged along a terrain profile may replace geodetic leveling in the determination of ground subsidence (Chrzanowski and Fisekci, 1982). Another popular application is in the stability studies of slopes or earth dams. By scanning a borehole with a bi-axial inclinometer, a full profile of the borehole and its changes can then be determined through repeated surveys. In the cases of difficult access to a monitored area or a need for continuous data acquisition, the tiltmeters or borehole inclinometers with telemetry system are installed in the area (Chrzanowski, 1986). The tilt of ground or structure foundation can also be determined with hydrostatic level, which may be used in a form of network of permanently installed instruments. High accuracy can be achieved if the main error source of temperature change can be properly taken care of.

*Alignment measurement.* The alignment surveys measure the relative movements of points and cover a wide spectrum of applications from tooling industry through measurement of the amplitude of vibration of a building to monitoring of several kilometer long linear structures. Alignment can be carried out in the horizontal or in the vertical direction, and by mechanical method or optical method. The former includes tensioned wire alignment and suspended and inverted plumb lines. The latter includes direct laser alignment and alignment with diffraction gratings. The achievable accuracy varies from 10 ppm to 0.1 ppm.

### **3. Analysis of Deformation Surveys**

Under the action of external forces (loads) an object deforms. Depending on its purpose and the data available, analysis of deformation surveys is classified into geometric analysis and physical interpretation (or called dynamic analysis). The former is to provide the geometric status of an object in space and time, and the latter is to tell how an object responds to the loads or what is the relationship between the loads and deformations (Chen and Chrzanowski, 1986). In the system theory it is called system identification (Welsch, 1996).

### 3.1 Geometric analysis of deformations

Over the last fifteen years, considerable effort has been made by the FIG WG 6C Ad Hoc Committee on Deformation Analysis. Most problems of geometric analysis have been solved and the results were summarized in (Chrzanowski and Chen, 1986; Chrzanowski and Chen 1990). The only problem is the selection of proper models (functions). Recently, Chen and Chrzanowski (1996) have presented a methodology to handle the problem. As the deformation of an object is a function of position and time, it should be analyzed simultaneously in space and time domains using multi-epoch observations. In practice, however, this simultaneous estimation can be performed only the deformation models have been preliminarily identified. Otherwise, complication in the analysis may arise and some important deformation phenomena may be overlooked if only using the trial and error approach. Thus, if possible, one should try to identify deformation model in space and in time separately. Fortunately, in many practices the observations are grouped in distinct epochs. The survey results can then be analyzed by pairing epochs, which involves only deformation pattern in space. The deformation trend in time will be recognized through the analysis of successive pairs of epochs of observations. When the observations are scattered in time, one feasible approach is to analyze a time series of observations for each observable and obtain a set of time independent "observations", e.g., the rate of change for each observable. Then, the deformation pattern in space is identified using these pseudo-observations. Deformation model in space domain can be identified using a plot of the best displacement field or by a piecewise approximation approach. The former can be achieved by the weighted similarity transformation of a displacement field derived from a monitoring network (Chen, et al., 1990). It is a robust approach and generates the best displacement field. The latter is applicable to the case that deformation pattern is complicated or no displacements can be computed. The approach divides the area of study into elements and uses a linear model to approximate the deformation within each small element.

The established model can then be used to predict the deformations. However, the model represents only the deformation trend. To improve prediction accuracy the remaining signal can be extracted from the residuals of observations using the least squares collocation or the time series analysis techniques. Recently, attention has been given to the gray system theory (Chen and Tang, 1993). It operates on an accumulation-generated data set. With an accumulation generating process the weak signal in a data set becomes stronger and easier to be modeled. The gray model is generally a differential equation with some unknown parameters to be estimated from the data. The main advantage of the approach is a higher prediction accuracy is achievable with less sample data.

### 3.2 Physical interpretation

In the system theory the physical interpretation is called system identification, i.e., how a system transfers input signals to output. There are basically three approaches depending on how much one knows about the object. The statistic modeling refers to the case the object is treated as a *black box*. It establishes the functional relationship between the causative factors (loads) and deformations by analyzing the correlation between the inputs and outputs. Usually the regression technique is used. A good model can be established only if sufficient information on both input and output is available. This approach has been used by surveyors for quite some time. A typical example is to establish a model describing the deformation of a dam as function of time, water level in the reservoir, and temperature in air or concrete. The deterministic modeling utilizes information on the properties of the materials, the geometry of the object, and physical laws governing the stress-strain relationship, which is also called the white box model. The deformation behavior of an object is usually described by a differential equation, which is solved by a numerical approach, like the finite element method. The approach is mainly used by other specialists.

However, surveyors are getting interested in this method and working together with other specialists. In practices, however, a deformable object can hardly be treated as white box, but gray one. The third one, called hybrid modeling or the gray box model, has been developed. It uses the deterministic modeling to set up the functional relationship between input and output with some unknown parameters. These parameters are then estimated from the observed deformations using the least squares technique. This process is also called calibration of parameters.

A combination of geometric analysis with physical interpretation has been proposed to enhance deformation modeling (Chrzanowski et al., 1990). A typical example is integrated analysis of deformation surveys at Mactaquac Dam (Chrzanowski, et al., 1989).

### 3.3 Inverse analysis

This is a new area surveyors are exploring. Inverse analysis of deformation surveys (also called the back analysis) looks similar to the hybrid approach. The difference is in the purpose of analysis. The hybrid method aims at the establishment of a functional relationship between loads and deformations, while the inverse analysis aims at determination of some geometric and physical parameters of the object, and acting forces from the surveyed data. In contrast, the deterministic modeling should be called the direct analysis.

In the deterministic modeling the finite element method is commonly employed. Using the well known principle of the displacement approach, one can write

$$\mathbf{Kd} = \mathbf{f}$$

where  $\mathbf{d}$  is the vector of nodal displacements,  $\mathbf{K}$  is the global stiffness matrix of the material, which is a function of the material properties and the geometry of the object, and  $\mathbf{f}$  is the vector of external force. All the observed deformation quantities  $\mathbf{l}$ , including geometrical and physical ones, are related to the above displacement vector  $\mathbf{d}$  by a transformation matrix  $\mathbf{H}$  as

$$\mathbf{l} + \mathbf{v} = \mathbf{Hd} = \mathbf{HK}^{-1}\mathbf{f}$$

where  $\mathbf{v}$  is the vector of observation errors. The above is the basic equation for the inverse analysis and usually non-linear with respect to the unknown parameters, e.g., material properties, acting forces, and geometry of the body, expressing as

$$\mathbf{l} + \mathbf{v} = \mathbf{l}(\mathbf{p})$$

where  $\mathbf{p}$  is the vector of parameters to be estimated. The linear or non-linear least squares techniques are employed to solve the unknowns. If some information on the parameters is available, it can be incorporated into the least squares solution. For the detailed and demonstrating examples, the interested readers are referred to (Chen and Yang, 1996).

The concept of the inverse analysis widens the horizon for analysis of deformation surveys. The results not only enhance the direct analysis, improve the monitoring scheme, but also provide better understanding of the deformation mechanism.



## 4. Separability as a Design Criterion

Criteria for the optimum design of a geodetic control network include accuracy, reliability and cost. However, some differences between the design of geodetic networks and design of monitoring schemes exist. For geodetic networks, the accuracy of the coordinates of points are of major concern, while for monitoring schemes the accuracy of derived deformation parameters are of particular interest. Therefore, a sensitivity concept of monitoring networks was proposed and developed as a design criterion (Niemeier, 1982). In order to apply the criterion of sensitivity, deformation model must be known. This produces some practical difficulty at the design stage when the deformation pattern of the investigated object is not well defined. Several possible deformation models may have to be considered. Each of them represents a possible hypothesis on deformation mechanism. Thus a monitoring scheme must be so designed that several postulated deformation mechanisms can be correctly distinguished. The concept of separability was therefore developed (Chen and Chrzanowski, 1994).

Assume that there exist  $m$  possible deformation models, expressed as  $\mathbf{d} = \mathbf{B}_i \mathbf{c}_i$  with  $\mathbf{d}$  the vector of measured displacements,  $\mathbf{B}_i$  a deformation matrix whose elements are function of position and time, and  $\mathbf{c}_i$  the vector of coefficients to be determined, and that the smallest deformations to be correctly detected for each model is  $b_i$  ( $i=1, 2, \dots, m$ ). Then, a monitoring network must be so designed, i.e., design of configuration matrix  $\mathbf{A}$  and weight matrix  $\mathbf{P}$  of observations, that the following  $m(m-1)$  conditions are satisfied:

$$\lambda_{\min}\{M_{ij}\} \geq (\sigma_0^2 \delta_0)/b_i^2, \quad \text{for all } i,j$$

with

$$M_{ij} = \mathbf{B}_i^T \mathbf{P}_d \mathbf{B}_i - \mathbf{B}_i^T \mathbf{P}_d \mathbf{B}_j (\mathbf{B}_j^T \mathbf{P}_d \mathbf{B}_j)^{-1} \mathbf{B}_j^T \mathbf{P}_d \mathbf{B}_i, \quad \mathbf{P}_d = \mathbf{A}^T \mathbf{P} \mathbf{A} / 2$$

where  $\lambda_{\min}\{ \}$  represents the minimum eigenvalue of a matrix,  $\sigma_0^2$  variance factor and  $\delta_0$  the non-centrality parameters calculated based on Baadar (1968).

## Acknowledgement

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# **Design of a Robotic Monitoring System for the Eastside Reservoir in California**

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## **Abstract**

A new reservoir is being built by the Metropolitan Water District (MWD) of Southern California in Hemet, California. This project encompasses building 3 earthen dams of 2.9 km, 3.2 km and 0.8 km length to enclose a valley of approximately 7.2 km. long and 3.2 km. wide. At completion, this reservoir will hold 986.8 million cubic meters of water. There will be over 360 deformation monuments to be surveyed spread over 5 structures (3 dams, a forebay and a detention basin). These monuments will need to be monitored monthly starting from completion of structures through to the complete filling of the reservoir, estimated to take up to 5 years. A geodetic surveying system was designed to detect displacements of points larger than 10 mm at 95% confidence level using multiple, permanently installed, robotic total stations and an array of permanently mounted prisms to provide an economically feasible way to monitor such a large site. The total stations will be remotely operated and will automatically collect three dimensional data (X,Y and Z) on a set time schedule. These total stations will also be linked to an active GPS base station and real-time rover system which will be programmed to turn on the total stations whenever a preset tolerance level of deformation is exceeded due to, for instance, an earthquake or abrupt settlement.

We would like to gratefully acknowledge the expert guidance and design review that Dr. Adam Chrzanowski has provided us in the development of this system. We appreciate his willingness to share his many years of experience with us on this project and on several of our other deformation monitoring projects.

## **Introduction**

In 1996, the Metropolitan District of Southern California (MWD), started construction on a \$1.9 billion project of Southern California's largest water storage reservoir with a capacity of 986.8 million cubic meters (800,000 acre-feet or 260 billion gallons) of water. The reservoir, located near Hemet, California, about 160 km southeast of Los Angeles,

was designed to secure six months of emergency water supply (MWD 1997) to about 16 million inhabitants. It is being created by enclosing the Domenigoni/Diamond valleys at an elevation of about 500 m (1640 ft) by three earth/rock filled dams (Figure 1). The reservoir, about 7.2 km (4.5 mi) long and more than 3.2 km (2 mi) wide, will cover over 1800 hectares (4500 ac) of land. This earthfill dam project (which is the largest earthfill dam project in the US) will consist of:

- the West Dam which will be about 85 m (280 ft) high and 2.9 km (1.8 mi) long,
- the East Dam which will be 55 m (180 ft) high and 3.2 km (2.0 mi) long
- the Saddle Dam which will be 40 m (130 ft) high and 0.8 km (0.5 mi) long.

The project includes construction of a storage forebay at the West Dam, a detention basin at the east end and a pumping plant. The storage water will be supplied from the Colorado River (by a 387 km (240 mi) aqueduct) and from the California State Water Project (by a 710 km (441 mi) aqueduct) to Lake Silverwood Reservoir. From Lake Silverwood the water is transported by a 72 km (44.7 mi) long and 3.7 m (12 ft) diameter Inland Feeder Pipeline to the Eastside Reservoir. The construction work started in 1996 and it is planned to be finished by November 1999. The initial filling of the reservoir could take over 5 years.

The proposed method of deformation monitoring for the Eastside Reservoir Project will have ten permanently mounted robotic total stations that are part of an Automated Theodolite Measuring System (ATM). These Leica TCS1800 total stations will be linked to a remote computer system that will automatically control all functions. The precision of these total stations will allow all necessary field data to be collected remotely and still be within the accuracy requirements to detect displacements of >10 mm (0.03 ft) at the 95% confidence level for deformation surveys. The only data that will be necessary to be collected manually by field survey personnel will be on the upstream side of the three dams (after the reservoir is full, these monuments will be monitored only as the water level allows).

This geodetic information is included as a part of a MWD Safety of Dams report to the California Division of Safety of Dams (DSOD) within the Department of Water Resources. The DSOD requires the monitoring of facilities under their jurisdiction. State jurisdiction is determined by structure height and impounded volume. There will be three State licenses required for the Eastside Reservoir Project - one for the reservoir, one for the forebay and one for the detention basin.

These structures need to be monitored monthly starting as soon as construction of the first berm is complete, throughout the rest of the construction, which lasts about 18 months, and then during the initial filling period of the reservoir, which could be over the course of many years. Monitoring will then be reduced to a quarterly basis for a period of 5 years or until the structures are stabilized. Monitoring will then be reduced, probably to twice yearly, in accordance with a State approved monitoring program for these facilities.

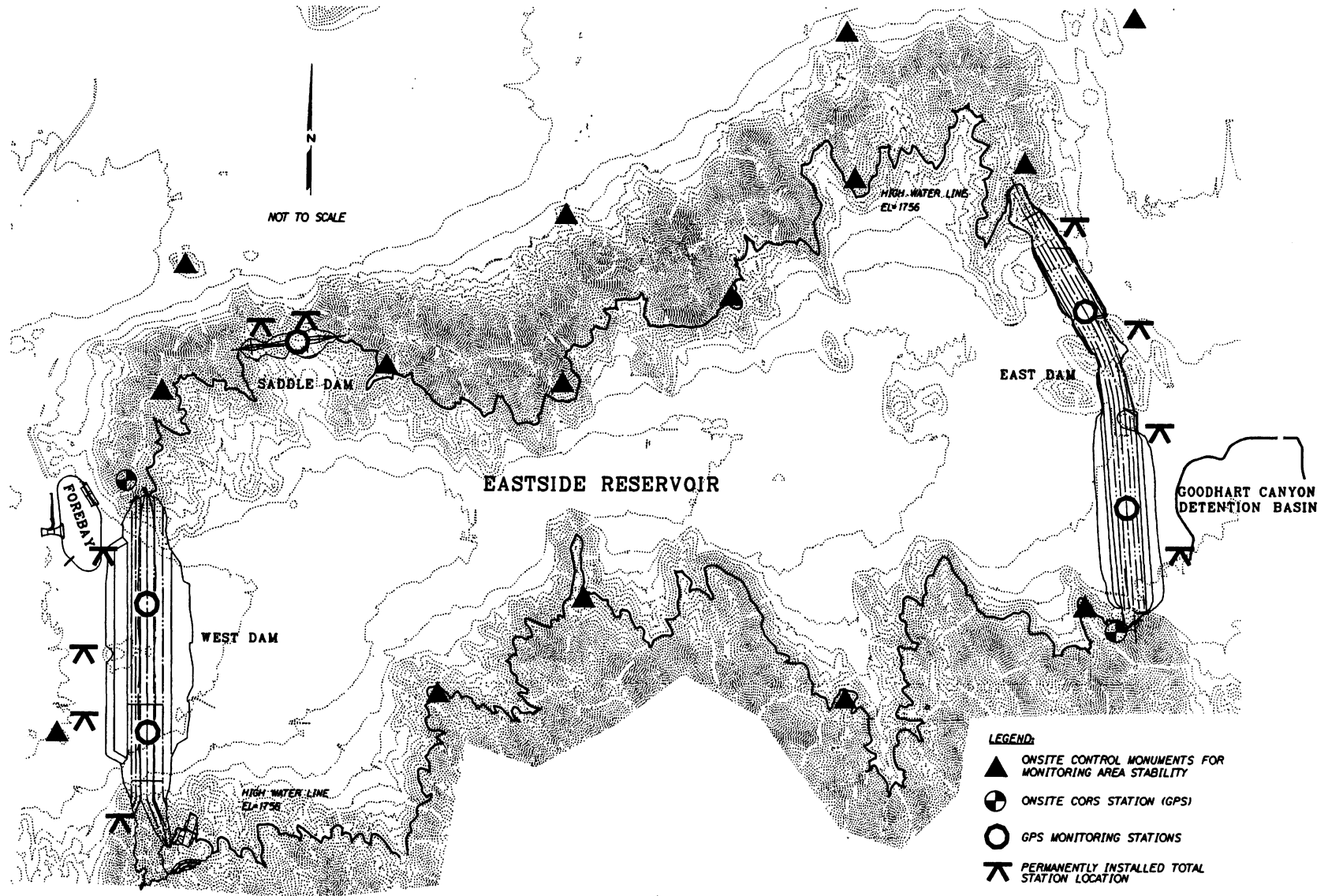


Figure No. 1

Geodetic Control Network Monuments

Geotechnical instrumentation has been designed independently of the geodetic portion of the monitoring plan. The geotechnical instrumentation will include 262 piezometers (various types), 18 strong motion accelerographs, 7 inclinometers, 74 settlement sensors, 6 fixed embankment extensometers and 14 weirs. The collected geotechnical information will be integrated with the geodetic information for structural analysis and physical interpretation of deformation using the methodology developed within the International Federation of Surveyors (FIG) activity of Working Group 6c (Chrzanowski et al. 1986; Chrzanowski et al. 1991).

This paper discusses the design of the geodetic monitoring surveys which are administered by the Geometronics Section of the Survey Engineering Branch at MWD.

## **Monitoring System Design**

The West Dam and East Dam will each have four permanently mounted robotic total stations. The Saddle Dam will have two permanently mounted robotic total stations (Figure No. 1). The Goodhart Canyon Detention Basin will have one “roving” robotic total station and will also use two of the East Dam total stations. The Forebay will be monitored using the northerly West Dam total station. All five areas will have prisms permanently mounted to the pillar type monuments (Figure 2). These pillars eliminate setting up tripods, object height errors and about 17 miles of walking. The monument spacing will be 152 m (500 ft) apart on each of the dam berms, at the toe of the dams, on the crest of the forebay and on the crest of the detention basin. The monuments will be 76 m (250 ft) apart on the crest of the dams. The total station array has been designed to keep all distance measurements at 500 m (1640 ft.) or less (Figure 3).

These total stations will be linked to a computer system that will automatically and remotely control all functions. This will allow for remote power-up (especially useful in emergencies), remote download, and real-time measurement as needed. The system will operate on a set time schedule that will automatically turn on at the correct time and start angle and distance measurement sets. The angle sets that each total station will collect is predetermined and programmed using the monitoring software.

The total stations will also be programmed to collect sets of angles to achieve a preset level of precision. When angle collection is complete, the system automatically shuts down and is basically “on-call” until the next scheduled power-up, or in the case of emergency, until someone remotely powers-up the system. The total stations will be monitoring horizontal angles, vertical angles and slope distance.

From this information, state plane coordinates and elevations for each point will be generated. The precision (0.8 arc seconds and 1mm+2ppm standard deviations) of these TCS1800 total stations (Leica, 1996) will allow all this information to be collected remotely and still be within the stated accuracy requirement. This eliminates the need for a separate procedure that revisits each site to obtain the vertical measurements as is required in procedures using GPS for horizontal measurement. This instrument is capable

of this type of angle measurements due to its unique automatic target recognition system which works with a standard prism. A video component within the instrument “sees” the reflected pixel configuration of the prism, calculates the center of mass of the reflector and then adds or subtracts this amount to the approximate angle to produce the precise angle measurement.

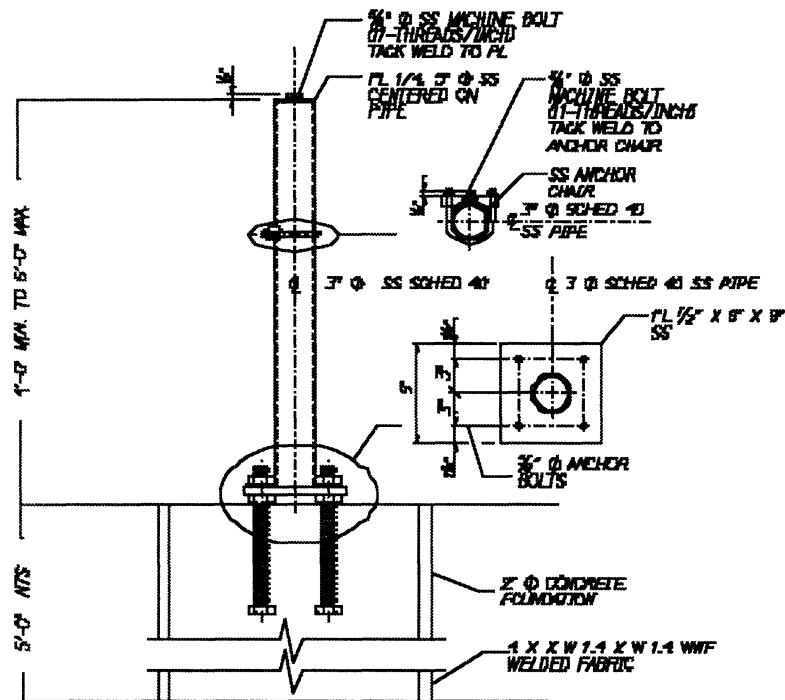


Figure 2 Pillar Survey Monument.

The only data that will be necessary to be collected manually by field survey personnel will be on the upstream side of the three dams. This data will be obtained using a total station and prisms mounted on fixed height rods. Three individuals equipped with these prism poles will walk each berm of the dam as the instrument person (setup on the middle berm) alternates taking shots between the three prisms. This will require four (4) days of total station work per survey to check the upstream side of all three dams. After the reservoir is full, these points will be monitored only as the water level allows.



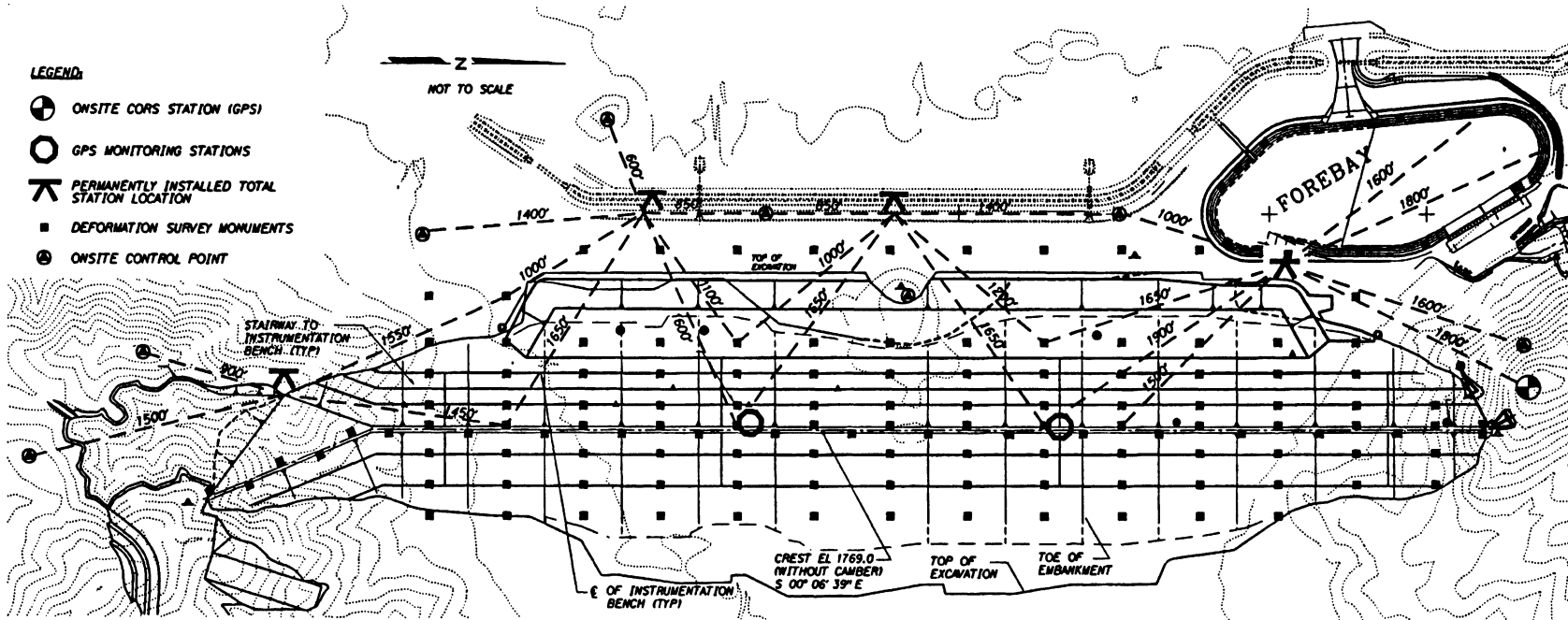


Figure No. 3 West Dam Deformation Monitoring Configuration

## **Procedures for Establishing and Monitoring Horizontal and Vertical Control Points**

The horizontal and vertical control network for the Eastside Reservoir includes several types of control which is critical when monitoring such a large site. The offsite control that will be utilized includes four GPS Continuously Operating Reference Stations (CORS) that are part of a growing system of 250 such stations that monitor crustal motion of the continental plates in southern California. These continuously operating GPS receivers are administered by a collaboration of scientists (Southern California Integrated GPS Network; SCIGN) studying crustal motion and earthquakes (Bock et al, 1997). They handle all data processing and analysis of the system and then post geodetic positions for all stations on the Internet. By utilizing this CORS system for the offsite control, which will then establish the horizontal positions for the onsite control, any crustal motion in the region will be determined. The onsite control that will be utilized consists of sixteen standard survey monuments situated on the two ridgelines surrounding the reservoir (Figure No. 1). These monuments have currently been tied to California's High Precision Geodetic Network (HPGN) horizontally and have National Geodetic Survey (NGS) first order North American Vertical Datum of 1988 (NAVD88) elevations vertically. These monuments will act both as onsite control for the total station network and as a monitor of the effects of increasing the load on the hills around the reservoir as the water level rises in the enclosed valley. The onsite control (HPGN monuments) will be used to establish and monitor the positions of the total stations and the backsight monuments. Utilizing these methods to monitor the control monuments will avoid contamination of the displacement measurements with non-structurally related settlement or movement such as regional slipping and creep.

### **Procedures for Horizontal Measurement of Control Monuments**

To establish horizontal positions on the onsite control (HPGN monuments), GPS receivers will be set up on each of the sixteen monuments (four at a time) for a static session of 30 - 45 minute duration. These static sessions will then be processed along with data downloaded for the four CORS stations. A least squares network adjustment will be done to obtain geodetic positions for the sixteen control monuments. These monuments will then be analyzed for any displacements between the two ridgelines.

Once this control network is established or verified, these monuments will be used inconjunction with the two onsite CORS stations to measure or verify a network of the total station pillars, the backsight pillars and the GPS "rover" stations on the crest of the dams. The reservoir will have three separate networks. The West Dam network will include the CORS station at its north end, three of the westerly onsite control (HPGN monuments), the four West Dam total station pillars, four backsight pillars, four secondary tie points and the two GPS "rover" stations on the West Dam crest. The Saddle Dam network will include the West Dam CORS station, three of the northwesterly onsite

control (HPGN monuments), the two Saddle Dam total station pillars, two backsight pillars, two secondary tie points and the GPS “rover” station on the Saddle Dam crest. The East Dam network will include the CORS station at its south end, three of the easterly onsite control (HPGN monuments), the four East Dam total station pillars, four backsight pillars, four secondary tie points and the two GPS “rover” stations on the East Dam crest. These networks will be remeasured every six months to verify the stability of our control monuments positions and the total station positions.

### **Procedures for Vertical Measurement of Control Monuments**

A determination of the true vertical elevation will need to be performed initially. This will be accomplished using an electronic level and bar code rods for greatest accuracy. This level run will be necessary only on the total stations, the backsights and the secondary tie points. The total station vertical position will be monitored by installing 3 brass cap monuments on the foundation of the observation shelter which is monolithic with the total station pillar. This survey will provide a check for any vertical settlement of the control points. If any significant change is determined, it would be corrected in the software to avoid misinterpretation as a shift in the deformation points. This stability check will need to be performed only once a year. The exceptional vertical accuracy of the total station during routine monitoring should alert us to any problems, at which time an intermediate check could be performed.

### **Deformation Monitoring**

After these control point stability checks are complete, the survey of deformation points would be accomplished by holding the value of the total station and the backsight established in these surveys and taking radial shots in direct and reverse mode to all the deformation points. Each total station with its subset of deformation points will be treated as a sub-network with independently determined displacements.

The positions established on each deformation point would not be determined by any type of adjustment; the coordinates would be generated based on the positions established on the total station and backsight and the averaged measurements for the deformation points (with any corrections for meteorological conditions taken into account). These values would then be compared to initial values and values from the previous survey to generate a table of delta values (change in northing, change in easting, change in elevation).

Dually determined points (those points measured from two different total stations) will be compared also to serve as a quality check of the survey. Other data analysis will include generating error ellipse plots of the accumulated displacements of the deformation points to show trends in the displacements.

All total station procedures will be able to be controlled from the office by a remote link. This will include scheduling the remote power-up and all measurements. The angles to be turned, the number of measurements and the acceptable standard deviation can all be controlled from the office.

The download and analysis of all data will also be done from the office by a remote link. When analyzing the data, if a problem shows up, the total stations can be turned on to re-measure the problem points. This will also be useful in the identification of prisms that may need maintenance.

## **Emergency Response Plan**

An emergency response plan (to be implemented after reservoir is operational) was developed to obtain verification of monument positions in a timely fashion after an event, with the most important measurements (those needed to verify structural integrity) obtained first.

Within the first hour after an event, all the total stations will be powered on and obtaining the subset of deformation measurements specific to each total station. This provides an immediate check on the status of the dams when viewed in conjunction with the real-time geotechnical data obtained. The total stations will be able to analyze its position in relation to at least two control points (one in bedrock) for a preliminary stabilization check.

This “power up” may occur in several ways. There will be four methods of alarming built into the operations of the geotechnical instrumentation of these dams. The instrumentation is all connected by an Automated Data Acquisition System (ADAS) that will have the capability to send an alarm if a preset tolerance for any of the designated instruments is exceeded. The instrument types integrated into the alarming system will be the GPS rover stations on the crests of the dams, strong motion accelerographs, seepage flow meters, and the facility operators system known as SCADA. The ATM system of total stations will be linked to this ADAS system and will be capable of receiving these alarm notifications and will be automatically powered on upon receipt of notification. The ATM system can also be remotely powered on by staff using any PC on the MWD computer network with pcANYWHERE™ remote accessing software.

Within the first day after an event, a GPS survey will be started to verify positions on the ten total stations and corresponding backsights. This survey will be completed utilizing the four CORS stations and manually collecting GPS data on the twenty onsite points. (Receivers will be hooked up to the antennas on the observation shelters and antennas will temporarily replace the prisms on the backsights.) Vectors measured from the total station to the backsight from the ATM system can also be added to the GPS network for analysis. GPS data from the rover stations on the dam crests will also be added to network for analysis.

The second day after an event, GPS measurements will be collected on the sixteen onsite control (HPGN monuments) along with the four CORS stations. By the second day after an event the CORS stations new geodetic coordinates (as determined and posted by SCIGN) will be available and analysis of the first day of GPS will be completed. At this point in time new positions of the total stations and backsights will be able to be compared to the existing positions to determine if there is any displacement in the ATM system control monuments. This information will then be used in analysis of the deformation monuments displacements.

By the third day after an event, analysis will be able to be completed that will verify the geodetic positions of all the total stations and backsights, the HPGN monuments and the GPS rover stations. This information will be compared to existing positions to determine if there is any displacement in any of these monuments. This information will then be used in analysis of the ridgeline monuments displacements and final analysis of the ATM system control points.

## **Design of Shelters and Data Communication System**

### **Computer Equipment**

Each total station requires a dedicated computer for the remote access capability and for data storage. This equipment would include 10 complete computer systems consisting of a computer with modem, network card, surge protection and remote monitoring and accessing software. The computers to run the theodolites will be 486/66 or better with a gigabyte hard drive, 16 MB memory and high speed modem. They need multiple serial ports for connecting the theodolite and the meteorological sensors. There will be a parallel port connection to allow a zip drive to be installed. The computers will be an “environmentally protected” type of computer - one that is meant to withstand dust, insects, outside use, etc.

### **Observation Shelters**

Each total station will be housed in an “observation shelter” to enclose and protect the total stations and computer equipment. The windows will be constructed of several large sheets of glass joined in a faceted and tilted arrangement, similar to a control tower window at an airport. The configuration of the windows must provide a view of all the monuments to be read from each total station. The various total stations each have different viewing requirements, however all shelters will be constructed with the greatest viewing capabilities on all sides to allow flexibility for future additional monitoring points (Figure 4). The following is a list of the structural requirements of the observation shelters.

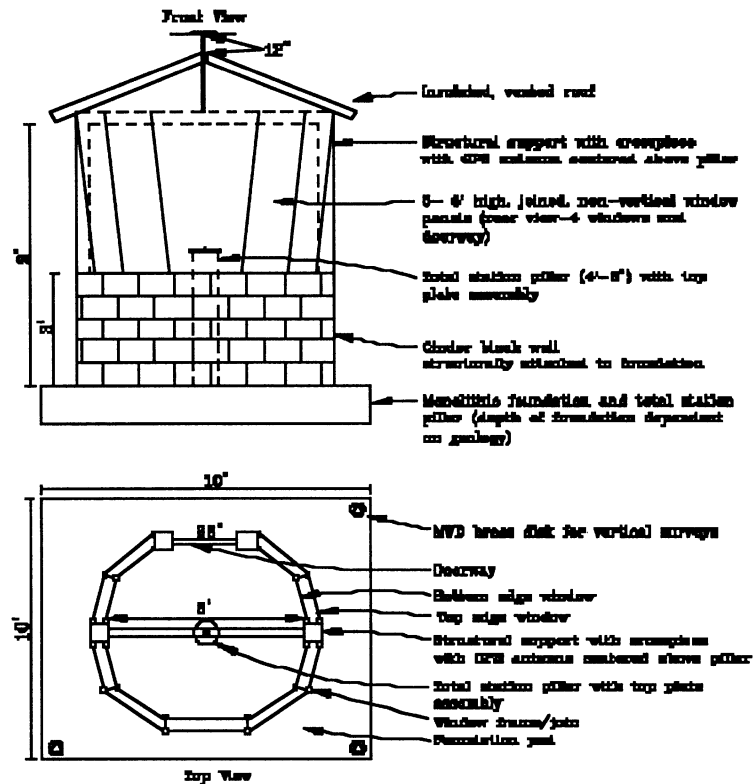


Figure 4 Total Station Observation Shelter.

1. These structures are required to be tall enough and wide enough to allow an individual to go inside to make adjustments to the theodolite and/or computer that will be housed inside.
2. The shelter will be climate controlled to protect the computer equipment from overheating (site is located in semiarid environment). Ideally the building should be well sealed to prevent dust, insects and rodents from entering and damaging the equipment as this is to be remotely operated equipment with a minimum of maintenance checks.
3. Windows should be approximately 6 feet high to allow for vertical clearance (line of sight) to points on top of dam. The windows should be installed at a slight angle. This is to prevent having the theodolite line of sight perpendicular to the window (vertical angle to monument equal to  $90^\circ$ ). This situation would create a mirror effect; instead of allowing the light beam from the electronic distance meter (EDM) to reflect off the prism on the monument, it reflects back off the window.
4. Once the system is set up and initial readings are taken, the "glass error" at each point will be relative to that point and will cancel out on subsequent measurements.
5. Window glass needs to be 6 mm thick sodalime float glass. This glass has a 91% transmission of light due to a low iron content. This is necessary to insure minimum light refraction for the EDM readings which are at a carrier wavelength frequency of  $0.850 \mu\text{m}$ .

6. The structures need to be connected to electricity (110 volt). Electric lines will be run to each of the ten total stations to provide reliable power during non-emergency situations which is the majority of the time. When field testing this ATM, battery failures occurred with both the total station and computer demonstrating the need for a dependable power source. The emergency backup system for power for the total stations and computer will be through internal batteries, however, these batteries will only run the equipment for up to eight hours reliably.
7. The foundation should be designed according to the geology of each location to provide as stable of a structure as possible. The pillar that the total station will be mounted on should be monolithic with the foundation of the structure.
8. A GPS antenna will be permanently mounted to a structural support on the roof of the shelter, directly above the center of the instrument, so that the stability of the foundation/pillar can be monitored by the GPS control surveys. The block walls and structural supports should be embedded in the foundation so that movement of the foundation/pillar will be translated to the GPS point.

### **Data Communication**

Data communication and remote access between the onsite PCs running the total stations and the survey office (located 120 km (75 mi) away) will be accomplished using a spread spectrum wireless LAN system. This system utilizes *Mavric™ Explorer* wireless data communication networking controllers which have been developed by Metric Systems Corporation (Brown, 1997). There will be one network controller at each monitoring (total stations) location, one on the crest of each dam and one at the “host PC” in the onsite pumping plant communication room. The three network controllers on the crest of the dams will act as control points in the data flow and will also provide redundant communication links between the dams and “host PC” in emergency situations (Figure No. 5). For data access, this host PC will be programmed (pcANYWHERE™ script file) to call up the ten total stations via the network controllers and have all data download to the host PC. The data on this PC will then be accessible from remote locations utilizing pcANYWHERE™ software. This host PC will be able to transmit the data by a connection on the MWD computer network and by modem. There will be no monthly charges for data communication using this system nor will there be any FCC licensing requirements. These network controllers are compatible with all standard protocols and are upgradeable as technology advances.

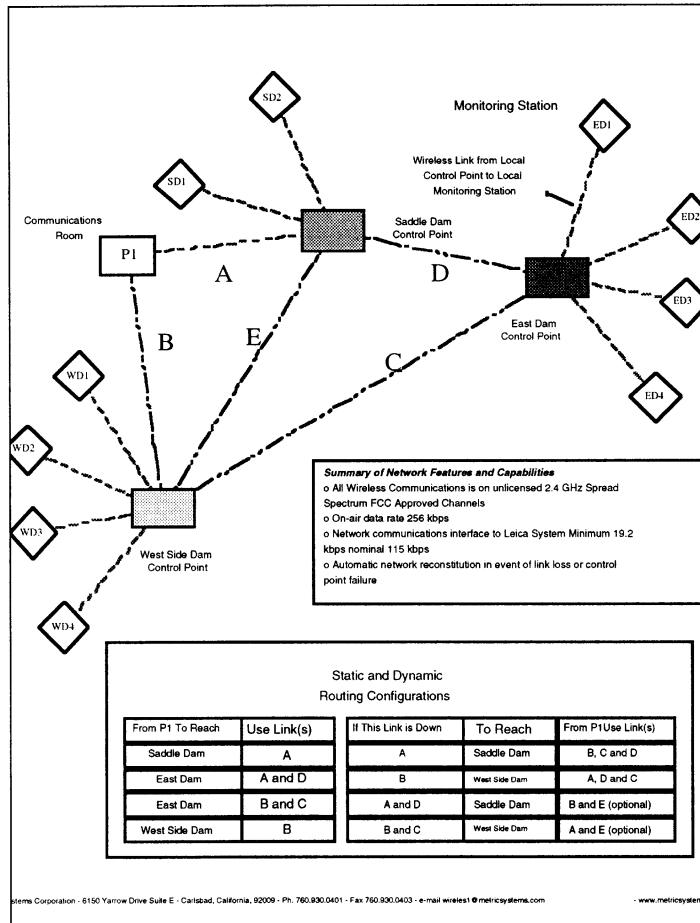


Figure No. 5 Overall Eastside Reservoir Communication Network Configuration.

## Other Methods of Monitoring Researched

Other methods of monitoring that were researched included using Real-time Kinematic (RTK) GPS, permanently installed static GPS and permanently installed straight line lasers.

The major disadvantage to all three systems included the need to run data communication cable to all the monuments to be monitored. The maintenance of this immense amount of cable from weather conditions, rodents and operational hazards made these options impractical.

Another disadvantage is the unacceptable accuracy for deformation purposes. Static GPS can determine vertical positions only to within  $\pm 2$  centimeter accuracy and this level of accuracy is only achieved with repeated sessions on different days. RTK GPS cannot detect displacements at the centimeter (0.03 ft) range horizontally or vertically at the 95% confidence level necessary for deformation surveys (Trimble Navigation, 1996).



Straight line lasers are typically used for measuring deformation of structures or instrumentation. The accuracy range for these instruments is suitable for deformation monitoring, but the distance range for measurement is restricted to 100 feet or less.

A manual variation of the proposed automatic method of monitoring could also be utilized. Instead of permanently installing the total stations, one or two robotic total stations would be moved manually from one instrument setup to another. A crew member would install the total station and laptop computer at an instrument setup position.

There are several disadvantages to this variation. The main disadvantage is that this system is no longer remotely operated. This takes away the capability of a 24 hour a day monitored system. This also eliminates the ability to turn the system on at will to monitor during or immediately after an earthquake or to take additional readings to check problem areas.

The safety factor for personnel is also eliminated in this variation. If an earthquake happens, crew members will be required to setup downstream of a possibly failed structure in order to check its position.

Another disadvantage is the need for personnel to take these measurements during the hours of 3:00 AM to 5:00 AM. In field testing this equipment (at another MWD site with similar conditions), the tests showed that the least amount of refraction due to heat waves occurred between these hours.

This is of great significance in obtaining accurate values (daylight operation reduces accuracy approximately by half) and a major advantage to implementing the proposed system. Personnel would have to be available to take measurements at these times to guarantee the repeatability of the surveys. To obtain measurements for all ten setups during this narrow window of time would increase the time to complete the survey from one hour with the ATM system to two or three days with manual setups.

Having employees working during night time hours has several disadvantages related to increased risk of liability for MWD, increased opportunity for injuries related to walking around the dam in the dark and driving at this time of night.

## **Advantages of Proposed Method**

### **Labor and Vehicle Cost Savings**

The advantages of this proposed ATM system becomes apparent when the extensive schedule of monitoring is viewed. The initial costs of the automatic system will be recovered easily within the first two years by reduced labor expenses. Cost reduction is also realized by reduced vehicle and travel costs and the decrease in liability due to

reduced travel. Cost savings are estimated to be \$1.7 million over the first five years of operation.

### **Procedural Advantages**

The main procedural advantage is the capability to measure at night (reduced atmospheric effects) and to take numerous readings at each point to greatly aid in achieving the desired accuracy standards, rapidly and safely.

Safety to personnel is enhanced because they would not be walking across rocks on the dam face to access monuments, reducing exposure to snakes and injuries, etc. Another procedural advantage is the elimination of two visits to each point - once for a horizontal measurement and then again for a vertical measurement, as in the current method. Both data components are being measured at once from a remote position.

### **Public Safety Advantages**

Another advantage to the system is the public safety factor. MWD will advertise Eastside Reservoir as a 24 hour a day monitored site. The GPS base station and rover system, along with the ATM total stations, will be linked to the Automatic Data Acquisition System (ADAS). If a movement over a certain tolerance is picked up by the GPS monitoring system, or if any of the geotechnical instruments monitored through the ADAS exceeded expected limits, the ATM system would be programmed to automatically turn on and start monitoring the dams. This eliminates the need to send our personnel into a dangerous situation after an emergency and provides near instantaneous response time.

### **Summary**

This continuous monitoring system was designed to provide adequate geodetic information over a large number of points for a long period of time with as little human intervention as possible. It also was designed to provide timely information after a major event, such as an earthquake, to help protect the public health and safety of the residents located near the reservoir. The system pays for itself in less than two years and provides all the geodetic information needed for reporting to the State of California in a timely fashion without requiring additional labor costs. It can analyze a good portion of the downstream surface area of all three dams in less than an hour, including analysis and reporting time.

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# Photogrammetric Techniques for Deformation Measurements

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A photograph represents a remoter, complete, and instantaneous record of an object, and a series of such instantaneous records taken at different points in time (epochs) provides an excellent measure of changes of the object (deformation).

Photography does not require physical contact with the object and thus causes no disturbance. By shifting the actual measuring task from the field to the office, the completeness of photographic records becomes very useful, as it is possible to go back to previous stages of deformation or to extend the measurements into areas previously assumed to be stable. This is impossible with other measuring approaches.

Photogrammetry covers the whole area, and point selection is therefore less critical. The actual measuring time has no influence on the resulting accuracy because measurements are taken on instantaneous records (“frozen” situation).

The accuracy of photogrammetric results is primarily a function of the photo scale, which means very high accuracies in close range applications and lower ones for smaller scales (e.g., for aerial photography). As a rule of thumb, an accuracy of about 10  $\mu\text{m}$  in photo scale can be obtained for each coordinate if a rigorous evaluation is used. This would mean millimetre accuracy or better in a laboratory, centimetre accuracy for photography with a 50 mm lens for up to 50 m object distance, or decimetre accuracy for low-altitude photography with a standard 15 cm aerial camera.

There is a requirement for expensive and specialized evaluation hardware and software (bundle adjustment with additional parameters). When using film based cameras, the rather slow turn around time makes photogrammetric methods unsuitable for real-time monitoring. With digital cameras, however, real-time photogrammetry has arrived, albeit the accuracy is not quite as high, and the area coverage of one frame is much smaller.

A photograph is a two-dimensional image of a three-dimensional object. Two photographs of the object taken from different stations are needed to recover the third

dimension (stereo-photogrammetry), provided that they are taken simultaneously or that the object remains stable and stationary between exposures. Simultaneous three-dimensional coverage at different epochs, provides the fourth dimension, namely, the change of the object during the interval between epochs. If the rate of change is small, consecutive exposures within a short time interval (e.g., during aerial photography) would not cause significant errors in an epoch.

Equivalent to the need of common points for relative orientation in stereophotogrammetry, there is a need for stable points between epochs, then only one epoch requires absolute object space control. If stable points are not available, object space control has to be provided for each epoch, and the resulting coordinate differences represent the deformation.

For measuring general deformations, it is essential that the same physical points are measured in each epoch. Targets are ideal but not always practical. In the latter case, a stereo cross-identification technique using photographs from different epochs can be applied.

If the deformation is acting in one direction only (e.g., vertical movement), mathematically defined points (i.e., grids, profiles) can be used, with photogrammetry basically providing elevation measurements.

It may also be possible to arrange the photogrammetric x-axis parallel to the direction of the movement. Then multiple photographs with unchanged exterior and interior orientation (same camera, same station, same set-up) provide “motion parallaxes,” and thus a “stereo image” of the movement. This approach has been successfully applied for glacier studies.

Analogue evaluation is only useful if the deformations are large. Photogrammetry and other types of remote sensing imagery can, of course, be used solely for interpretative purposes without a quantitative evaluation.

# Bounding Natural Domains: The Definition of Geospatial Objects from Fields\*

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## Abstract

Geoscientific applications face the problem of bounding natural domains by representing complex, highly irregular and fragmented natural objects with variable distribution of attributes. As with other GIS application domains (e.g., soil maps), geospatial objects by nature do not always exhibit a distinct boundary (as "exact objects" do), having thus a statistical rather than a deterministic character. In such "continuous fields" of data values, shapes are determined by the contents and various classification schemes. In other cases, this ill-definition of the boundary is the result of poor definitional data and of the uncertainty introduced in building an appropriate representation. This paper addresses both types of boundary uncertainty. The emphasis however is put on natural entity/object definition from continuous geoscientific fields, in the context of which a special classification and aggregation strategy is proposed.

**Key Words:** geoscientific modelling, continuous fields, discretisation, classification, aggregation.

## 1. Introduction

Geoscientific applications involve objects of great variety and complexity. Some of these have accurate spatial descriptions with sharp boundaries, and they are "well-behaved". Burrough [1996] and Frank [1996] call them also *exact entities*. Depending on the characteristics of an exact entity and the intended use, various geometric modelling techniques can be used to describe it at various degrees of detail. Any uncertainty in the boundary definition of "well-behaved" objects can be due to lack, loss, poor quality, inconsistency of definitional data, or limitations of the selected representation.

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\* A modified and extended version of Kavouras [1996], partly founded on Kavouras [1987]. Being an essential member of the advising committee, Professor Adam Chrzanowski has invaluablely contributed to this part.

In contrast to the exact and "well-behaved" entities (usually man-made features), other natural objects tend to be highly irregular, fragmented, fuzzier, with variable distribution of attribute values, and uneven sample data -- being thus very difficult to describe with distinct boundaries. Realisations that exhibit a statistical rather than a deterministic character, similar to that of the soil data in GIS applications [Burrough, 1991], are often called [Burrough and Frank, 1995; Burrough 1996; Frank, 1996; Worboys, 1995] *continuous fields*. In these cases, entities can be defined by the contents and various classification realisations.

*Field-based* models and *object-based* models had been an matter of fundamental debate in the GIS community in the early 1980s [Peuquet, 1984; Couclelis, 1992]. The definition of objects and boundaries from fields is a very complex issue, but of great importance to geoscientific applications. The topic of indeterminate boundaries also relates to data accuracy and quality issues which have been in the GIS research agenda for sometime [Goodchild and Gopal/Eds., 1989; Chrisman, 1991; Buttenfield/Ed., 1993]. Geoscientific but also environmental modelling have not experienced the same degree of awareness and development as 2-D GIS, for they address a smaller interest group with non-standard needs. They also present many and difficult generation, manipulation and visualisation problems. Three special landmark meetings -- a NATO Advanced Research Workshop at Santa Barbara USA in 1989 [Turner/Ed., 1992], a symposium at Freiburg im Breisgau Germany in 1990 [Pflug and Harbaugh/Eds., 1992], and a European Science Foundation meeting at Il Ciocco-Italy in 1992, and a number which followed afterwards, were held to address and promote research in the field of 3-D GIS and geoscientific applications. In this range of activities, one can mention applications in oil exploration [Youngmann, 1989; Sims, 1992; Lasseter, 1992], mining [Kavouras, 1987; Kavouras and Masry, 1987; Bak and Mill, 1989; Houlding, 1992; Kavouras, 1992], geological modelling [Denver and Phillips, 1990; Kelk, 1991; Hurni, 1993], sedimentology [Lee and Harbaugh, 1992], hydrogeology [Turner, 1989], environmental monitoring [Smith and Paradis, 1989], and meteorology [Slingerland and Keen, 1992]. For the new reader, a number of articles and textbooks containing a good selection of topics, are available by Raper [1989/Ed.], Raper and Kelk [1991], Bonham-Carter [1991], Turner [1992/Ed.], Pflug and Harbaugh [1992/Eds.], Breunig et al.[1994], and Scott [1998].

In the context of geoscientific data modelling, this paper focuses on the problem of entity/object definition from continuous fields (Sections 2 and 3). The approach being introduced can also find applicability in a number of domains. However, our primary application interest has been in geology and mining. Coping with the boundary uncertainty of exact entities is also a complex problem, yet much easier to handle, and it is only briefly introduced here (Section 4). The concluding Section (5) presents several issues that are open for further research.

## 2. Geoscientific Data Modelling

Some natural entities (geo-objects) are part of a continuum, being characterised by properties which vary continuously in space. The most common strategy for representing a continuous field is a 3-D (or n-D) domain-partitioning to regular or non-regular elements with variable attributes. A general and detailed description of fields, as they apply in geography, can be found in Tomlin [1990]. Entities can be spatially defined and subsequently modelled (i.e., represented or discretised) using a classification of this continuous field. Such entities are also known as '*definition limited*' [Raper and Kelk, 1991], as opposed to '*sampling limited*' -- a term used to describe exact geo-objects. The classification criteria are based on selected thematic values, and therefore control the spatial properties of the resulting entities. Different classification criteria result into different entity/object representations.

Before however, the necessary modelling strategy is presented (see Section 3), it is important to introduce some background concepts about the geoscientific modelling requirements. The common problem of *ore body definition* offers as a good base for discussion.

An ore body is conceptually defined as a mineral deposit that can be exploited at a profit, under certain market conditions. Thus, as economic conditions and technological capabilities change the ore body fluctuates (shrinks or expands) in a dynamic manner. The total extent of a mineral deposit and its overall grade are determined during exploration. Some deposits exhibit low variability about their overall grade. Most deposits however, have lower and higher grade zones. When a mine comes near production, it is of great importance to determine these variations with sufficient confidence.

Several techniques exist for estimating mineral inventories from sample data. Traditionally, geologists work on drill-holes or cross sections of the deposit and outline the ore-waste contact and zones of certain richness (ore grade). Block models are very suitable structures for both geological interpretation and mine design. The introduction of the theory of *geostatistics* by G. Matheron in the early 1960s, made it possible to formulate a rigorous block estimation as compared to the empirical techniques. The objective is not only to estimate the most likely values of the unknown parameters, but also to assess how accurate these estimates are, (their variability). The theory of geostatistics is concerned with variables which are distributed in space, known as *regionalized variables*. The distribution of grades of a particular mineralisation is an example of a regionalized variable [Journel and Huijbregts, 1978].

*Kriging* is a geostatistical procedure, in which the grade (or other) value at a given point (or block), is expressed as a linear combination of the N known (sample) values in its neighbourhood. The procedure requires the knowledge of the covariances between the known points and the point (to be estimated). The linear combination uses N weighting coefficients which depend on the structural characteristics of the mineralisation and the geometry of the sample data points and the point (or block) to be estimated.



The kriging procedure first computes the weighting coefficients and subsequently uses them to estimate the grade value at the point (or block) of interest. The estimation variance is computed as well. The procedure is repeated for all points.

The estimation of kriging is based on some assumptions of homogeneity and isotropy (local estimation). The presence of heterogeneous zones (significant trends), in the deposit, requires more complex treatment (global estimation) [Journel and Huijbregts, 1978]. Techniques such as *universal kriging* [Huijbregts and Matheron, 1970], and other non-linear estimates [Journel and Huijbregts, 1978] can be used in these cases.

### 3. Geospatial Object Definition From Continuous Fields

It was stated in the introduction, that in the modelling process, object definition can be based on a classification of continuous fields. It is argued here that this classification should not only be based on thematic attributes but also on some pre-defined confidence criteria. Consideration of adjacency, attributes and available accuracy measures results in better object definitions. This approach is explained in the rest of this Section. Details about the aggregation strategy of heterogeneous objects can also be found in Kavouras [1987].

An estimation uses a set of irregularly spaced sample points to estimate the "best" values (e.g., metal grades) for the block mesh in the continuous field, in form of a solution vector  $\mathbf{g}$  and the associated variances  $s_i^2$ . The estimation may also provide covariances  $s_{ij}$  between any two of the  $N$  blocks, in form of a covariance matrix  $\mathbf{S}$  of the thematic values, as follows:

$$\mathbf{g}^T = (g_1, g_2, \dots, g_N)$$

$$\mathbf{S} = \begin{pmatrix} s_1^2 & s_{12} & s_{13} & \dots & s_{1N} \\ & s_2^2 & s_{23} & \dots & s_{2N} \\ & & s_3^2 & \dots & s_{3N} \\ & & & \dots & \cdot \\ & & & & \cdot \\ & & & & \cdot \\ & & & & s_N^2 \end{pmatrix}$$

symmetric

A simple representation scheme can then be specified in the following four steps:

1. By defining a single generic primitive -- a cuboid (known as block, voxel, etc.) containing properties of interest.
2. By associating each block with a tuple  $vx(k_i, g_i, s_i)$ , where  $k_i$  is a locational key describing the spatial properties of the block,  $g_i$  is the block attribute, and  $s_i$  is the standard deviation of the estimated attribute  $g_i$ .

3. By defining the structuring rules needed to build structured representations and support the classification/aggregation scheme.
4. By defining some semantics to ensure that the representation is semantically correct (has desirable properties).

An object can be described as: "the set of blocks  $i$  whose attribute  $g_i$  is larger than or equal to a specified value, e.g., current cut-off grade  $g_t$ ". In order to attach some confidence to such classification, the covariance matrix information is to be used as well. If for each block  $i$ , a Normal probability distribution function is postulated, the blocks which satisfy the following inequality:

$$g_i - k*s_i \geq g_t, \text{ where } k \text{ is the coefficient based on a given confidence level -- e.g., 95\%.$$

Based on this inequality, blocks can be classified as being:

- clearly inside the object (*Rich* -- high grade),  $g_t + k*s_i \leq g_i$
- outside the object (*Poor* -- low grade below cut-off),  $g_i < g_t$ , or
- above cut-off grade, but no high confidence (*Uncertain*)  $g_t \leq g_i < g_t + k*s_i$

Figures 1 and 2 exemplify this classification using standardised and actual values (grades).

Aggregation of *uncertain* blocks generally results in larger more structured representations with higher reliability. Any data structuring procedure can be used in principle. The structuring rules can follow a simple hierarchical aggregation scheme, which builds larger blocks from smaller ones. When  $M = 2^m$  are aggregated ( $m > 0$ ), the resulting block will have a value  $s_a$  and a variance  $s_a^2$ , which, considering equal weights, are computed from:

$$g_a = (1/M) \sum_1^M g_i, \text{ and } S_a = a^T S_i a$$

where:  $a^T = (1/M)(1, 1, 1, \dots, 1)$  is the design matrix, and  
 $S_i =$  the part of the dispersion matrix  $S$  which refers to the  $M$  blocks.

Matrix  $S_a$  contains only one element -- the variance of the block after aggregation,  $s_a^2$ , which is a scalar of the form:

$$s_a^2 = (1/M^2) [ s_1^2 + s_2^2 + \dots + s_M^2 + 2 ( \begin{matrix} s_{12} + \dots & + s_{1M} \\ s_{23} + \dots & + s_{1M} \\ s_{34} + \dots & + s_{1M} \\ \dots & \\ s_{(M-2)(M-1)} + s_{(M-2)(M)} + s_{(M-1)(M)} \end{matrix} ) ]$$

The aggregation can be loosely viewed as a form of generalisation. While blocks of similar attributes (grades) are to be grouped, some restrictions are also to be imposed so

that there is no significant loss of local detail due to overgeneralization. Thus, certain *cut-off sizes* are selected, in order to specify at what level aggregation should start and where it should end, even if blocks may still be classified as *Uncertain*.

A detailed description of the aggregation procedure can be found in Kavouras [1987]. Here, and without loss of generality, the procedure is best illustrated by a two-dimensional example. The sequential nature of the procedure makes extension to the 3-D case straightforward. The input data of 64 equal size blocks with their estimated grades and associated standard deviations (Figure 3) are from Clark [1979]. The additional parameters that have to be specified for the aggregation procedure are:

- a. The cut-off grade  $g_t = 33$  (user specified)
- b. The largest sizes allowed by the aggregation, to prevent overgeneralization (cut-off sizes).
- c. The level of statistical confidence (e.g., 68 %).

Blocks are classified as *Poor* if their grade  $g_i$  is less than  $g_t=33$ , as *Rich* if  $33+s_i \leq g_i$ , and as *Uncertain* if  $33 \leq g_i < 33+s_i$ . The first classification is graphically illustrated in Figure 4. Figure 5 shows the final result of the aggregation, if a quadtree structuring representation is employed.

The proposed aggregation technique exploits the spatial coherence of nature, also known as spatial homogeneity [Journel and Huijbregts, 1978], which normally characterises regionalised variables. It is usually accepted [ibid.], that in a limited zone of a homogeneous mineralisation the correlation that exists between two data values  $z(\mathbf{x}_1)$  and  $z(\mathbf{x}_1+\mathbf{h})$  does not depend on their particular positions within the zone, but rather on the distance  $\mathbf{h}$  which separates them. In order to formulate this spatial structure and make inferences, the range of homogeneity  $\bullet$  has to be established. Outside this range,  $\mathbf{h}$  is considered to be large and  $z(\mathbf{x}_1)$  and  $z(\mathbf{x}_1+\mathbf{h})$  are considered as uncorrelated. A covariance function  $C_v(\mathbf{h})$  or a semi-variogram  $\bullet(\mathbf{h})$  is often used to model this spatial structure [Journel and Huijbregts, 1978; Clark, 1979].

When block grades are estimated from sample points, the *natural correlation* between the blocks shows in their attribute values (grades). In a homogeneous zone it is expected that neighbouring blocks will have similar grades. The aggregation of blocks in poor and rich areas exploited this spatial coherence of nature.

The covariances between block grades can be stored in a covariance matrix of size  $(N \times N)$  where  $N$  is the total number of blocks. This covariance matrix is formulated as the matrix  $\mathbf{S}$  shown above. Its sparseness pattern depends on the ordering of the blocks. If the blocks are ordered in a way which approximately preserves adjacency in space (like e.g., the Morton sequence), then we would expect a band of non-zero elements along the main diagonal of the covariance matrix. The bandwidth is primarily a function of the selected range  $\bullet$ . Since the simple covariance model in a homogeneous zone is based only on the distance between points, a *distance matrix* would be equivalent to the covariance matrix.

The above *natural correlation* between points, which shows in the attribute values (grades), should not be confused with the covariances shown in the covariance matrix **S**. The latter express the *model correlation*, i.e., how the estimation of block i relates to the estimation of block j. If, for example, the block size is large and the distribution of sample data points is dense and homogeneous, the number of data points lying inside each block may be sufficient for the estimation of its metal grade, without having to consider sample points outside the block but still within range  $\bullet$ . Such estimation would produce only variances in the covariance matrix **S** of the estimated block grades. Of course, the absence of model correlation should not deceive us to claim that metal grades of adjacent blocks are not naturally correlated, and vice versa.

If covariances are available from the geostatistical estimation (i.e., the full covariance matrix **S** is available), then the variance of the aggregated block ( $s_a^2$ ) can be accurately computed as described above. If however, covariances are not available, then the user has two alternatives:

To ignore the absence of the off-diagonal elements of matrix **S**, i.e., to ignore the term:

$$(2/M^2) (s_{12} + s_{12} + \dots + s_{(M-1)(M)}) \geq 0$$

If this happens, the variance of the aggregated block  $s_a^2$  will be smaller than the true variance. Since the richness of each aggregated block is tested from the inequality:

$g_t + k*s_a \leq g_a$ , smaller  $s_a$  implies that more blocks will be classified as *Rich* while they are actually *Uncertain*.

To design a covariance model based on an assumed drill-hole distribution, and spatial structure of the regionalised variable. Such a solution however, is not very practical or realistic, and furthermore expensive to implement with very little gain (of a hopefully more accurate estimation of the variance of the aggregated block). It is rather preferable to resort to the initial sample data for a new estimation, if this data are available of course.

The problem of spatial correlation was only touched on here, for it relates to the aggregation computations. The topic is much wider itself and has received much attention by many researchers [Matérn, 1960; Cole and King, 1968; Cliff et al., 1975; Cliff and Ord, 1981].

#### 4. Uncertainty in Object-Based Approaches

In geoscientific applications, exact entities, i.e., those which have a known and accurate spatial description, can be well-surveyed excavations and sample point locations. Even certain mineralisations, which are quite homogeneous in quality and have sharp distinctive boundaries (e.g., salt domes or perched aquifers), can be accurately located and represented. In modelling (descretising) this reality, entities are described from selective observations, being therefore termed '*sampling limited*' [Raper and Kelk, 1991]. In both cases, any uncertainty in the boundary definition of the entities can be due to a number of reasons, such as:

- lack or loss of definitional data,
- inaccurate surveys -- data of poor quality,
- inconsistency of data coming from multiple sources ,
- data processing errors,
- uncontrollable generalisation operations, and
- limitations of the selected representation scheme.

Currently, there are no systems which estimate, model, propagate and represent uncertainty and quality of geoscientific entities in a systematic and integrated manner, although a significant amount of research has been conducted already [Goodchild and Gopal/Eds., 1989; Buttenfield/Ed., 1993]. What has been recognized for current 2-D GIS systems [Burrough and Frank, 1995], is even more valid for geoscientific information systems.

Modelling uncertainty requires some external knowledge and some assumptions about the spatial structure. Such an uncertainty model can build confidence regions (bands or smooth distributions) around the entities, the shape and size of which depends on the accuracy of definitional data and some statistical assumptions about the entities involved. The visualisation issue of the data quality problem has been addressed by McGranaghan [1993] and Fisher [1993]. Obviously, building such confidence regions increases the dimensionality and therefore the embedded space complexity of the spatial entities. Point uncertainty can be thus expressed by ellipsoids. Space curves are expressed as sweep representations (volumes swept by 2-D or 3-D shapes as they are moved along a trajectory in space). Solids are expressed as hyper-solids.

Drill-holes offer a good example for understanding the uncertainty issue of "well-behaved" geoscientific objects. Drill-holes can be geometrically represented as straight lines. In real life however, drill-holes can be quite curved; in that case, it would be more appropriate to represent them by space curves. Sometimes, due to instrumentation malfunctioning, subsurface drill-hole directional data are lost. This means that the actual location of the drill-hole underground is unknown. Of course, one can assume that the drilling was done vertically and so use the attribute information, or ignore this drill-hole data completely. There are cases however, when underground activities should not interfere with the drill holes. In this case, it is important to at least estimate the region in which the drill-hole may lie. In order to tackle this problem, the locational uncertainty of the vector representation of the drill-hole object is represented by a solid model built with information about the surrounding rock properties, the directional deviation of neighbouring drill-holes, and some confidence levels set by the user. Furthermore, this solid can be spatially enumerated with a block model in order to enable attachment of individual properties and uncertainty measures in each point in space.

In addition to the *uncertainty problem in object representations*, one further issue -- almost completely neglected, is that of the *uncertainty of topological relationships*. In general, topological relationships are critical to understand, describe, and maintain. Not all relationships carry however the same reliability or significance.

Uncertainties in boundary definition can be also due to limitations of the selected representation. Hierarchical data structures for example, have been proven very efficient in

modelling irregular geo-solids [Kavouras, 1987/1991; Bak and Mill, 1989], but pose certain limitations (e.g., they are not invariant under certain transformations) which create topological inconsistencies. Currently, there is research being done towards the development of more robust topological models [Egenhofer et al., 1990; Egenhofer and Herring, 1991; Hadzilakos and Tryfona, 1992].

## 6. Closing Remarks

In this paper, an attempt was made to solve the problem of geoscientific modelling of spatial entities and objects from continuous fields, and a classification/aggregation strategy has been proposed. Exact entities are also complex, yet much easier to handle.

Many issues related to quality and uncertainty in geoscientific modelling are currently in the GIS research agenda, some of which are:

- Formalisation of uncertainty and fuzziness of spatial entities and relations.
- Formal models of transition zones.
- Estimation of uncertainty and definition of order of topological relationships.
- Quality measures and error propagation during generic operations.
- Visualisation of quality and uncertainty.
- Definition of a set of generic dynamic modelling functions.

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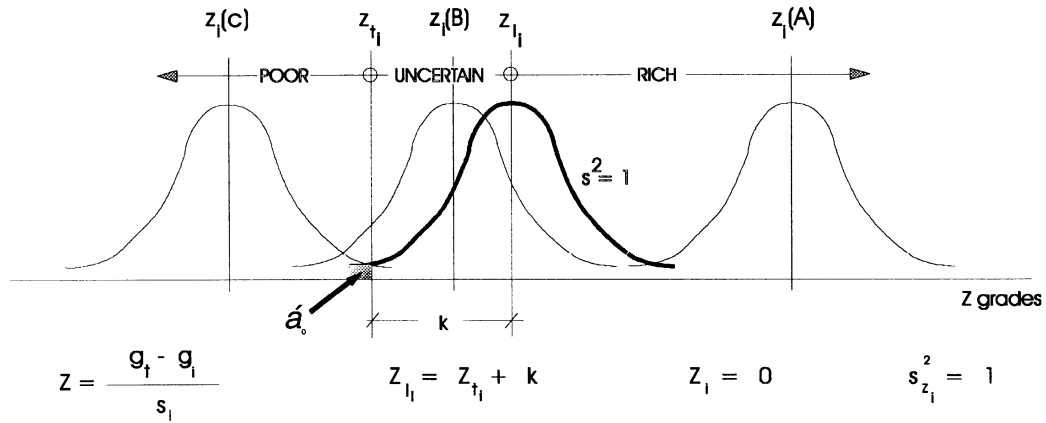


Figure 1: Classification intervals R(rich), P(poor), U(uncertain), for the standardized grade  $Z_i$  of block  $i$ .

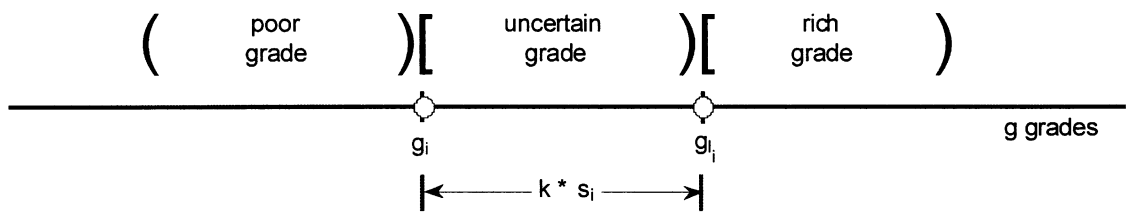


Figure 2: Classification intervals R(rich), P(poor), U(uncertain), for the actual grades  $g$ .

27.5 1.9	31.7 1.4	36.9 3.0	38.0 2.8	37.9 4.4	37.9 4.5	40.1 4.1	40.5 3.0
29.0 4.0	31.7 3.7	35.9 3.8	36.5 2.8	35.2 3.6	37.3 3.3	41.4 1.7	39.8 1.9
30.5 2.9	31.7 2.9	33.8 4.1	33.6 2.7	32.1 2.2	35.2 1.7	40.0 1.9	38.3 2.7
32.4 3.0	32.0 3.6	34.1 3.5	37.9 2.4	34.9 2.2	36.8 1.9	37.6 1.1	33.5 1.1
31.7 1.6	31.3 2.8	33.6 1.3	37.1 1.4	32.7 2.0	33.0 2.5	31.6 1.0	29.4 1.5
30.3 1.1	31.1 2.7	31.0 2.3	31.8 2.7	31.2 2.6	34.4 1.4	34.5 1.2	30.8 2.5
33.3 2.2	33.0 3.4	34.9 3.4	33.6 2.3	29.0 1.7	29.1 1.6	35.2 2.5	32.5 11.5
33.4 2.0	33.7 2.6	39.4 1.6	38.8 1.5	33.8 1.6	30.2 3.1	32.9 4.0	32.2 3.5

Figure 3. A 64-block simulated deposit with estimated grades and standard deviations (from Clark, 1979, page 112).



# Numerical Modelling of Heat and Mass Transfer Processes Using Least Squares Method

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*Dear Adam,*

*First of all I do wish to thank you for all you have done for me. Thank you for your warm friendliness, continuous inspiration, encouragement, hope and optimism during awful time of martial law in Poland. With this paper I join your colleagues and friends the world over in wishing you many more years of fruitful investigations, activities, long life, health and happiness.*

## MOTTO:

*"Observations are useless until they have been interpreted."..."The analysis of experimental data forms a critical stage in every scientific inquire - a stage which has been responsible for most of the failures and fallacies of the past."*

(E. B. Wilson "An Introduction to Scientific Research")

## **Abstract**

Mathematical models of heat and mass transfer processes are usually studied using formulation that is unique in the mathematical sense. An extension of such an approach by introducing the supplementary data concerning the process has been proposed here as a method for verifying the accuracy of the model equations. This means that the mathematical model consist of more equations than unknowns, which leads in consequence to a finite set of solutions. A criterion for choosing the most probable and unique solution has been proposed using total least squares method. Theoretical considerations have been illustrated by numerical solutions of simple mathematical models describing heat and mass transfer processes.

## **Introduction**

Conventional methods of mathematical modelling of heat and mass transfer processes are commonly based on schemes leading to unique solutions. For example, solving initial-boundary value problems, the mathematical model consists of the governing equations and initial and boundary conditions. The uniqueness of the solution can be proved mathematically for different kinds of boundary conditions and the problems are well

posed. A validation of the model is usually based on a direct comparison of experimental data and calculation results (Fig.1).

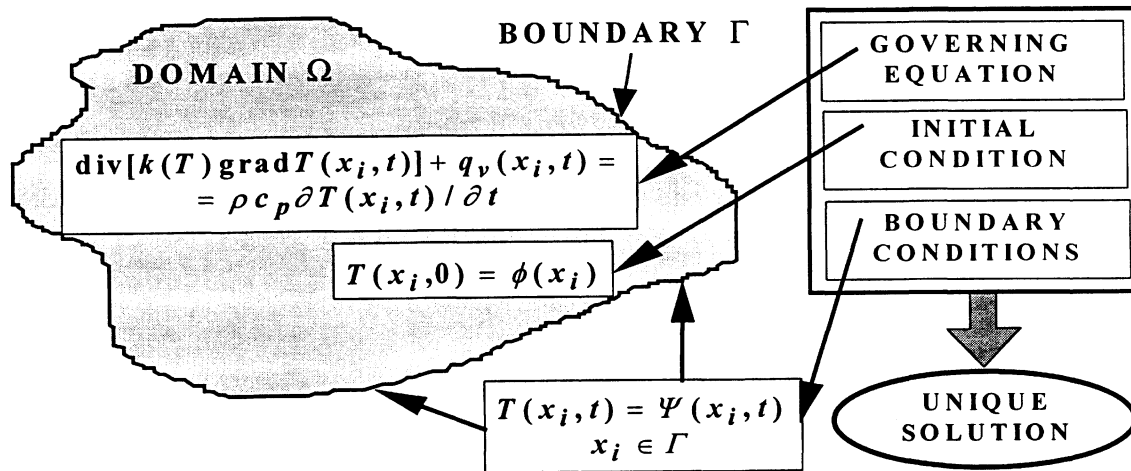


Fig.1. Conventional model.

The statements like: - the analytical results *agree satisfactorily* with the corresponding experimental data, the experimental results agree well with the theoretical results or comparison with the data shows *reasonable agreement* between the present theory and experimental results can often be encountered as the final conclusions of many research papers, without explanation of the meaning of *agree satisfactorily*, *agree well* and *reasonable agreement*. In our analysis a method based on the generalised least squares principle is proposed as validation procedure. The method can only be used when the number of directly measured variables is greater than the minimum number of distinct variables sufficient for unique solution. Additional data that are commonly used for validation purposes are proposed to be directly included into mathematical model as new (supplementary) variables not necessary for unique solution (Fig.2.).

The system of model equations will now contain more equations than unknowns and, because of the measurement errors, they will be internally contradicted. The problem that arises is how to correct the values of the directly measured variables (observations) to obtain the most probable solution of the mathematical model. The answer can be obtained with a *generalised least squares method* (Fig.3.).

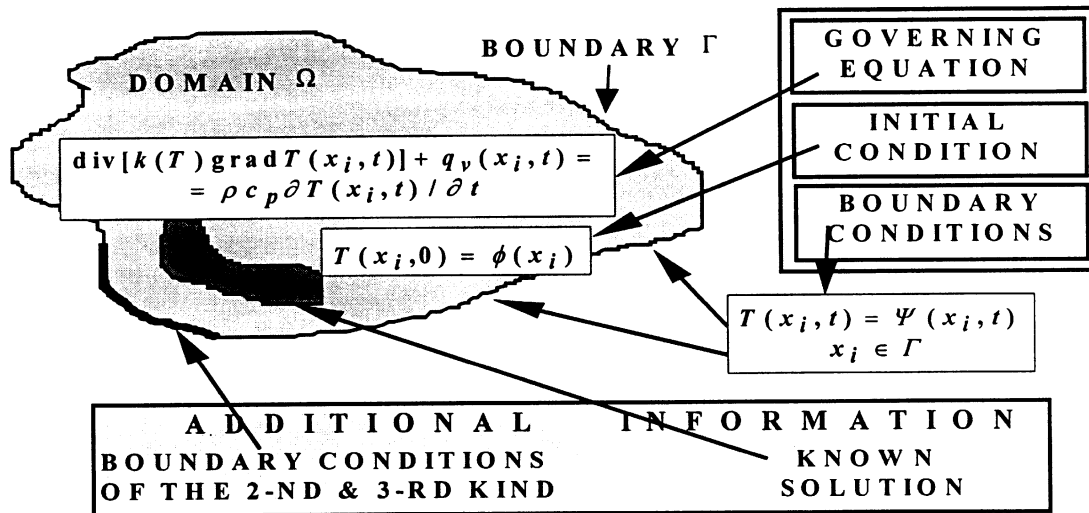


Fig.2. Model with supplementary information.

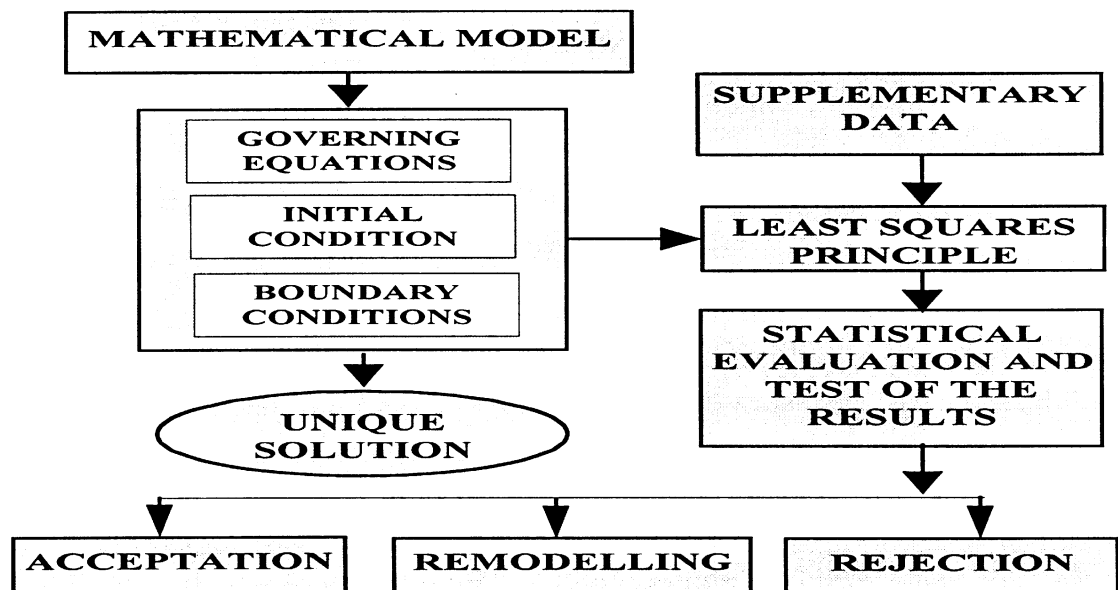


Fig.3. Least squares procedure.

The methods presented in this paper have been successfully applied to a mathematical modelling of heat transfer processes and in the theory of material, energy and exergy balances of chemical processes [Bocardo,1994], [Holda,1995], [Kolenda and Allman, 1974], [Kolenda, 1980], [Kolenda and Trela, 1981], [Kolenda et al., 1983], [Kolenda et al., 1995], [Norwicz, 1972], [Styrylska, 1986], [Styrylska and Pietraszak, 1992], [Szargut

and Ryszka, 1952], [Szargut and Kolenda, 1968], [Szargut et al., 1981], [Szmyd et al., 1992].

## Theoretical Development

Let  $n_0$  denote a minimum number of independent variables necessary for unique solution of the mathematical model and let  $n$  be a number of given functionally independent observations. When  $n$  is greater than  $n_0$ , the redundancy or number of statistical degrees of freedom defined as  $r = n - n_0$  is said to exist, and an adjustment becomes necessary in order to obtain a unique solution of the mathematical model. Because of the statistical properties of the experimental results, redundant observations are not compatible with the model and any arbitrary chosen subset of experimental results can be used to satisfy the model equations, [Mikhail and Ackermann, 1976].

Let  $\mathbf{l}$  denote vector of all experimental results and let  $\mathbf{l}^*$  be a vector of estimates that satisfies the model equations. In general the values of  $\mathbf{l}^*$  are different from  $\mathbf{l}$  and a difference vector

$$\mathbf{v} = \mathbf{l}^* - \mathbf{l}$$

which has been termed as either *corrections* or *residuals*, plays an important role in calculations. Due to the redundancy the number of estimates for  $\mathbf{l}^*$  or  $\mathbf{v}$  is infinite. To calculate the most probable solution, consistent with the model, the least squares principle is commonly used as an additional criterion. The least squares principle requires the condition

$$f(\mathbf{v}) = \mathbf{v}^T \mathbf{M}^{-2} \mathbf{v} = \sum_{i=1}^n \left( \frac{v_i}{\mu_i} \right)^2 \rightarrow \text{minimum}$$

to be satisfied simultaneously with the model equations where  $\mathbf{M}^{-2}$  is the weight matrix of the observations (experimental results). The weight  $\mathbf{M}^{-2}$  matrix is square and diagonal with  $\mu_i^{-2}$  and of order equal to the number of observations.

## Calculation procedure - classical approach

Mathematical model equations, usually in a differential form, describing any physical or chemical process are always divided into two groups - governing equations and conditions for unique solution. Their mathematical form is usually non-linear and an analytical solution can be obtained for very simple problems only. To obtain a solution, numerical methods are widely used. It means that the system of differential model equations is transformed into a system of non-linear algebraic equations and our further discussion will be focused on such an approach.

Let us assume that the mathematical model can be performed by the following system of algebraic non-linear equations

$$f_i(\mathbf{l}^*, \mathbf{x}^*) = 0 \quad (i=1, \dots, J) \tag{1}$$

where vector matrices  $\mathbf{l}^*$  and  $\mathbf{x}^*$  represent a set of variables the values of which are estimated a priori by experimental results ( $\mathbf{l}$ ) and a set of unknowns, ( $\mathbf{x}$ ), calculated from model equations (1).

Introducing experimental results  $\mathbf{l}$  and approximations for unknowns  $\mathbf{x}$  a system of model equations (1) is replaced by

$$f_i(\mathbf{l}, \mathbf{x}) = w_i \quad (2)$$

where  $\mathbf{l} = (l_1, \dots, l_k)$ ,  $\mathbf{x} = (x_1, \dots, x_m)$  and  $w_i$  represent the residua of the original system of non-linear model equations (1) and their values are evaluated using experimental results and approximations of unknowns.

To solve the problem numerically a linearization procedure is applied using the zero and first-order terms of the Taylor expansion. Defining the estimates (most probable values) as:

$$\mathbf{l}^* = \mathbf{l} + \mathbf{V}$$

and

$$\mathbf{x}^* = \mathbf{x} + \mathbf{Y}$$

where  $\mathbf{V}$  represents unknown corrections to the experimental results  $\mathbf{l}$ , and  $\mathbf{Y}$  corrections to the approximations of non-measured variables  $\mathbf{x}$ , the system (1) can be written in the form :

$$f_i(\mathbf{V}, \mathbf{Y}) = w_i$$

and after linearization, in the matrix form

$$\mathbf{A} \mathbf{V} + \mathbf{B} \mathbf{Y} = \mathbf{W} \quad (3)$$

where

$\mathbf{A} = \partial \mathbf{f} / \partial \mathbf{l}$  is  $J \times k$  Jacobi matrix of a rank equal to  $J$ ,

$\mathbf{B} = \partial \mathbf{f} / \partial \mathbf{x}$  is  $J \times m$  Jacobi matrix of a rank equal to  $m$ ,

$\mathbf{f} = \{f_1, \dots, f_J\}$  and  $\mathbf{W} = \{w_1, \dots, w_J\}$ .

The least squares procedure can now be formulated as follows:

$$\text{minimize } \Phi(\mathbf{V}) = \mathbf{V}^T \mathbf{M}^{-2} \mathbf{V}$$

*subject to model equations*

$$\mathbf{A} \mathbf{V} + \mathbf{B} \mathbf{Y} = \mathbf{W}$$

Variables  $(\mathbf{V}, \mathbf{l}^*, \mathbf{l}, \mathbf{Y}, \mathbf{x}^*, \mathbf{x}) \in E^n$  where  $E^n$  denotes an  $n$  - dimensional Euclidean space ( $n=m+k$ ). To solve the problem effectively, the Lagrange multipliers method can be used which leads to the system of additional linear equations [Mikhail and Ackermann, 1976]

$$\mathbf{A}^T \mathbf{K} = \mathbf{M}^{-2} \mathbf{V}$$

and



$$\mathbf{B}^T \mathbf{K} = \mathbf{0}$$

where  $\mathbf{K}$  is the column matrix of Lagrange multipliers. A routine calculation gives finally:

$$\mathbf{Y} = \mathbf{G}^{-1} \mathbf{B}^T \mathbf{F}^{-1} \mathbf{W} \quad (4)$$

$$\mathbf{V} = \mathbf{M}^2 \mathbf{A}^T \mathbf{F}^{-1} (\mathbf{W} - \mathbf{B}\mathbf{Y}) \quad (5)$$

where

$$\mathbf{F} = \mathbf{A} \mathbf{M}^2 \mathbf{A}^T$$

and

$$\mathbf{G} = \mathbf{B}^T \mathbf{F}^{-1} \mathbf{B}$$

If the accuracy of the solution of a linearized problem is not sufficient the iterative procedure must be applied. In such a case to get the solution of an original non-linear problem the values of elements of the Jacobi matrices  $\mathbf{A}$  and  $\mathbf{B}$  are continuously corrected at each iteration step. The solutions (4) and (5) are now used to calculate a posteriori errors of the directly measured variables, unknowns and any function containing model variables. Using the law of error propagation, the expressions for covariance matrices can be derived in the forms [Brandt, 1970], [Mikhail and Ackermann, 1976],

$$\mathbf{M}_l^2 = \mathbf{M}^2 - \mathbf{C} \mathbf{A} \mathbf{M}^2$$

and

$$\mathbf{M}_x^2 = [\mathbf{B}^T \mathbf{F}^{-1} \mathbf{B}]^{-1}$$

where

$$\mathbf{C} = \mathbf{M}^2 \mathbf{A}^T \mathbf{F}^{-1} [\mathbf{E} - \mathbf{B} \mathbf{G}^{-1} \mathbf{B}^T \mathbf{F}^{-1}]$$

and  $\mathbf{E}$  is the unit diagonal matrix.

A condition for the model adequacy can be formulated in different ways but the Lipschitz condition in the form [Chrzanowski et al., 1982]

$$|\mathbf{I}^* - \mathbf{I}| < \kappa \mu_j \quad (6)$$

or

$$|v_j| < \kappa \mu_j \quad (7)$$

(where  $\kappa$  is the Lipschitz constant) seems to be most effective from numerical point of view. The mathematical model is then accepted if the Lipschitz condition (6 or 7) is satisfied for all experimental results. In our calculations  $\kappa=2$  will be chosen (confidence level = 95% ).

## Unified approach

In many practical problems the necessary, from mathematical point of view, condition the rank of the Jacobi matrix  $\mathbf{A}$  to be equal to the number of model equations  $J$  is not satisfied. Such a situation is observed when the number of directly measured variables is less than the number of equations. The solutions (4) and (5) can not be used in such a case as an inverse matrix  $\mathbf{F}^{-1}=(\mathbf{A}\mathbf{M}^2\mathbf{A}^T)^{-1}$  does not exist ( $\det(\mathbf{A}\mathbf{M}^2\mathbf{A}^T)=0$ ) and another method of solution must be sought. One of the possibilities is an unified least squares method proposed by Mikhail and Ackermann [1976] for surveying problems. This method will be adapted here for the steady state and transient heat and mass transfer mathematical models. The basic principle and the most important assumption in the unified approach is that all variables in the mathematical model are observations (results of direct measurement). It means that unknown variables (solution of the model and model parameters) are treated numerically in the same manner as directly measured variables but with sufficiently large error values in comparison with a priori errors  $\mu_j$ .

The model equation (3) can now be written in a simplified form

$$\overline{\mathbf{A}} \overline{\mathbf{V}} = \overline{\mathbf{W}} \quad (8)$$

where

$$\overline{\mathbf{A}} = [\mathbf{A}, \mathbf{B}] \quad \text{and} \quad \overline{\mathbf{V}} = [\mathbf{V}^T, \mathbf{Y}^T]$$

and the corresponding a priori weight matrix is

$$\overline{\mathbf{M}}^{-2} = \begin{vmatrix} \mathbf{M}^{-2} & 0 \\ 0 & \mathbf{M}_y^{-2} \end{vmatrix} \quad (9)$$

where no correlation is assumed between the two vectors  $\mathbf{l}$  and  $\mathbf{x}$ .

A new weight matrix  $\mathbf{M}_y^{-2}$  is the inverse of a priori covariance (diagonal) matrix for unknowns (zero matrix in classical approach). Using, as previously, the Lagrange multipliers method the final solution becomes in the form

$$\overline{\mathbf{V}} = \overline{\mathbf{M}}^2 \overline{\mathbf{A}}^T \overline{\mathbf{F}}^{-1} \overline{\mathbf{W}} \quad (10)$$

where

$$\overline{\mathbf{F}} = \overline{\mathbf{A}} \overline{\mathbf{M}}^2 \overline{\mathbf{A}}^T$$

with the covariance matrix for all variables

$$\overline{\mathbf{M}}^2 = \overline{\mathbf{M}}^2 - \overline{\mathbf{M}}^2 \overline{\mathbf{A}}^T \overline{\mathbf{F}}^{-1} \overline{\mathbf{A}} \overline{\mathbf{M}}^2 \quad (11)$$

The partition of the calculated variables into *measured* and *unknowns* is not necessary in the calculations.

## Transient problems

The methods described above can be effectively used when transient problems are considered and an unified approach is especially recommended for numerical calculations. In such cases the solutions for each time level can be obtained with equation (10) using a posteriori covariance matrices  $\overline{\mathbf{M}}^2$ , Eq.(11), instead of a priori covariance matrix  $\mathbf{M}^2$ , Eq.(9). The values of residues  $\mathbf{W}$  must be modified at each next time step using solutions from the previous time level. For transient problems the system of model Eqs. (8) is written for each time level ( $n=1,2,\dots,N$ ) separately to give [Kolenda et al., 1995], [Styrylska, 1986]

$$\begin{array}{ll} \mathbf{A}_1 \mathbf{V} = \mathbf{W}_1 & \text{for } n=1 \\ \mathbf{A}_2 \mathbf{V} = \mathbf{W}_2 & \text{for } n=2 \\ \vdots & \vdots \\ \mathbf{A}_N \mathbf{V} = \mathbf{W}_N & \text{for } n=N \end{array}$$

The procedure is as follows: at the first step the least squares principle is used to adjust a subset of model equations  $\mathbf{A}_1 \mathbf{V} = \mathbf{W}_1$  ( $n=1$ ) with a diagonal a priori covariance matrix  $\mathbf{M}^2$ . The solution is given by Eq. (10) with a posteriori matrix  $\overline{\mathbf{M}}^2$  given by Eq. (11). At the second step the next subset of model equations,  $\mathbf{A}_2 \mathbf{V} = \mathbf{W}_2$ , is adjusted with new covariance matrix  $\overline{\mathbf{M}}^2$  which is now treated as the a priori covariance matrix for the time level  $n=2$ . To obtain the global solution the procedure as above is repeated  $n=N$  times. The partial solution for any  $n$ -time level is

$$\mathbf{V}_n = \overline{\mathbf{M}}_{n-1}^2 \mathbf{A}_n^T \mathbf{F}_{n,n-1}^{-1} \mathbf{W}_n$$

where

$$\mathbf{F}_{n,n-1} = \mathbf{A}_n \overline{\mathbf{M}}_{n-1}^2 \mathbf{A}_n^T$$

and

$$\overline{\mathbf{M}}_n^2 = \overline{\mathbf{M}}_{n-1}^2 - \overline{\mathbf{M}}_{n-1}^2 \mathbf{A}_n^T \mathbf{F}_{n,n-1}^{-1} \mathbf{A}_n \overline{\mathbf{M}}_{n-1}^2$$

The final solution is given by the sum

$$\mathbf{V} = \sum_{i=1}^n \mathbf{V}_i$$

If new independent variables appear in the consecutive subset of model equations the a priori covariance matrix must be enlarged by the non-zero elements at the main diagonal only. The procedure presented above is very effective from computational point of view. It can be easily simplified further to avoid calculations of the inverse matrices  $\mathbf{F}_{n,n-1}^{-1}$  at each step. It is achieved if the only single equation, instead of the subset of equations, is added at the next time step.

The matrix

$$\mathbf{F}_{n,n-1} = \mathbf{A}_n \overline{\mathbf{M}}_{n-1}^2 \mathbf{A}_n^T$$

(1,1)      (1,k) (k,k) (k,1)

where  $k$  is the number of actual variables, becomes a scalar and all calculations are limited to the matrix multiplications. If the computer program is properly designed the computational procedure is very simple and effective. It also allows to avoid excessive round-off errors to obtain very exact final solution.

The procedure described above has been used in calculations.

## Numerical examples

### Conduction of heat in solid

Consider 1-D steady state flow of heat along adiabatically isolated rod of finite length. Temperature distribution was measured along the rod at six points (Fig. 4)

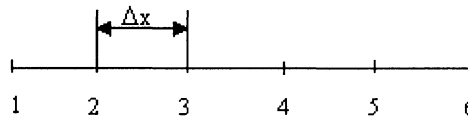


Fig.4. 1-D heat conduction.

Assuming for simplicity that  $(k/\Delta x)=1$ , governing equations take the form

$$T_{i-1} - 2T_i + T_{i+1} = 0 \text{ for } i=1,\dots,4 \quad (12)$$

introducing additionally entropy generation rate equations written for each nodal point as

$$\begin{aligned} \sigma_1 &= \left( \frac{T_1 - T_2}{T_1} \right)^2 \\ \sigma_i &= \left( \frac{T_{i-1} - T_{i+1}}{T_i} \right)^2 \quad \text{for } i=2,\dots,5 \\ \sigma_6 &= \left( \frac{T_5 - T_6}{T_6} \right)^2 \end{aligned} \quad (13)$$

the system of model equations contains ten equations with six directly measured temperatures  $T_i$  ( $i=1,\dots,6$ ) and only six unknowns  $\sigma_j$  ( $j=1,\dots,6$ ) representing entropy generation rates. Eqs. (12) and (13) correspond to general model Eqs. (2). Unified least squares method has been applied to calculate the most probable values of  $T_i$  and  $\sigma_j$ . Standard deviation for  $\sigma_j$  have been evaluated as 10 times  $\sigma_j^0$  where  $\sigma_j^0$  are the estimates of generation rates calculated using temperature measurement results. Measurement and calculation results are shown in Table 1.

Table 1. Calculation results.

Node	T <sub>i</sub> [K] measurement results	Standard deviation of T <sub>i</sub>	Approximation of σ <sub>i</sub>	Results of calculations	
				T* <sub>i</sub> [K]	σ <sub>i</sub> * [W/m <sup>2</sup> K]
1	970.	± 30.	0.024	955.±22	0.0114±0.001
2	830.	± 30.	0.06	853±16	0.0570±0.007
3	760.	± 30.	0.072	752±12	0.0734±0.010
4	635.	± 30.	0.078	650 ±12	0.0983±0.007
5	575.	± 30.	0.14	548±16	0.1380±0.026
6	435.	± 30.	0.078	446±22	0.0521±0.012

### Radiative transfer

Consider steady state heat transfer process by radiation between two concentric spherical surfaces or two coaxial cylindrical surfaces separated by non-absorbing and non-emitting gas. Heat flux absorbed by the surface A<sub>1</sub> is:

$$Q = -A_1 \epsilon_{1-2} \sigma (T_1^4 - T_2^4) \quad (14)$$

where ε<sub>1-2</sub> is total emissivity coefficient, σ Stefan-Boltzman constant, T<sub>1</sub> and T<sub>2</sub> represent temperatures of A<sub>1</sub> and A<sub>2</sub> respectively. Let us assume that Q, ε<sub>1-2</sub>, T<sub>1</sub> and T<sub>2</sub> were directly measured and the measurement results and associated errors were

$$\begin{aligned} Q &= -38000 \pm 6000 \text{ [W]} \\ T_1 &= 1150 \pm 70 \text{ [K]} \\ T_2 &= 850 \pm 70 \text{ [K]} \\ \epsilon_{1-2} &= 0.83 \pm 0.15 \\ A_1 &= 1.0 \text{ [m}^2\text{]} \text{ (exact value)} \end{aligned}$$

Introducing measurement results into Eq. (14) the value of residue is

$$\begin{aligned} w &= Q + A_1 \epsilon_{1-2} \sigma (T_1^4 - T_2^4) \\ w &= -38000 + 1 \times 0.83 \times 5.67 \cdot 10^{-8} \times (1150^4 - 850^4) \\ w &= 19713 \text{ [W]} \end{aligned}$$

Applying classical least squares method, (Eq. 10), (Eq.(14) does not contain unknown variable) the results of calculation are

$$\begin{aligned} Q^* &= -39300 \pm 5700 \text{ [W]} \\ T^*_1 &= 1100 \pm 43 \text{ [K]} \\ T^*_2 &= 870 \pm 64 \text{ [K]} \end{aligned}$$

$$\varepsilon^*_{1-2} = 0.77 \pm 0.14$$

If entropy production equations are involved into calculation the system of condition equations (2) becomes

$$Q = -A_1 \varepsilon_{1-2} \sigma (T_1^4 - T_2^4)$$

$$\sigma_1 = Q/T_1 + (4/3) A_1 \varepsilon_{1-2} \sigma (T_1^3 - T_2^3) \quad (15)$$

$$\sigma_2 = Q/T_2 + (4/3) A_1 \varepsilon_{1-2} \sigma (T_2^3 - T_1^3) \quad (16)$$

Eqs. (15) and (16) have been derived by Szargut, 1986 . Evaluation of  $\sigma_1$  and  $\sigma_2$  gives  $\sigma_1 = 3.30$  [W/K] and  $\sigma_2 = 4.40$  [W/K] . Applying unified least squares methods for three equations (14), (15) and (16), and using the same measurement results as above the solution is:

$$Q^* = -38700 \pm 5400 \text{ [W]}$$

$$T^*_1 = 1101 \pm 42 \text{ [K]}$$

$$T^*_2 = 879 \pm 58 \text{ [K]}$$

$$\varepsilon^*_{1-2} = 0.78 \pm 0.14$$

$$\sigma^*_1 = 3.62 \pm 1.0 \text{ [W/K]}$$

$$\sigma^*_2 = 5.27 \pm 1.9 \text{ [W/K]}$$

On the contrary a posteriori errors of entropy generation rates calculated from classical law of errors propagation is as high as  $\mu_{\sigma_1} = \pm 20.6$  [W/K].

It means that value of  $\sigma_1$  could be evaluated as  $\sigma_1 = 3.30 \pm 20.6$  [W/K] which is not acceptable from physical point of view. The use of least squares method allows to decrease this error significantly.

## Convection

The example to be discussed is a simple convective heat transfer problem of the steady fully developed laminar flow of air inside circular tube. Assuming constant properties of air, the governing equations in the radial co-ordinate system  $r, z$  are

- momentum equation

$$\frac{\mu}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) = \frac{dp}{dz} \quad (17)$$

- energy equation

$$\frac{k}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = \rho c_p u \frac{\partial T}{\partial z} \quad (18)$$



Table 2. Results of measurements and calculations.

Node	Measurement result	Approximate value	Correction	Result of calculation
T10	15.10±0.1	-	-0.067	15.033±0.08
T9	15.00±0.1	-	0.02	15.020±0.08
T1	15.00±0.1	-	0.003	15.003±0.08
T2	13.70±0.1	-	-0.006	13.694±0.08
T3	12.30±0.1	-	0.008	12.308±0.07
T4	11.10±0.1	-	-0.021	11.079±0.08
T5	9.90±0.1	-	0.026	9.926±0.07
T6	9.10±0.1	-	0.005	9.104±0.06
T7	8.60±0.1	-	-0.02	8.580±0.05
T8	8.40±0.1	-	0.005	8.405±0.09
r9	0.08±0.001	-	-	-
r2	0.06±0.001	-	0.00009	0.06009±0.0006
r3	0.05±0.001	-	-0.00012	0.04988±0.0006
r4	0.04±0.001	-	0.00025	0.04025±0.0006
r5	0.03±0.001	-	-0.00024	0.02976±0.0008
r6	0.02±0.001	-	-0.00002	0.01998±0.0009
r7	0.01±0.001	-	0.00004	0.01004±0.0010
r8	0.00±0.001	-	0.00001	0.00001±0.0010
m	7.00±0.5	-	-0.00073	6.00027±0.4995
u2	-	0.05	0.0022	0.0522±0.005
u3	-	0.1	-0.0023	0.0977±0.007
u4	-	0.13	0.0032	0.1332±0.010
u5	-	0.16	0.0032	0.1632±0.012
u6	-	0.18	0.0033	0.1833±0.013
u7	-	0.19	0.0057	0.1957±0.014
q	3.1±0.1	-	0	3.1±0.07

The system of Eqs. (19) together with Fourier equation for the wall corresponds to Eq. (2) and does not contain unknown variables.

### **Radiative transfer inside laboratory combustion chamber [Kolenda et al., 1983]**

From mathematical point of view the problem is highly non-linear. Water emulsion of oil (10-30%) mixed with pulverised coal (0.024 to 2.0%) was used in experiments.

The gas temperature distribution along the axis, the local concentration of combustion gas, the temperature of the furnace walls, and the distribution of heat flux to the furnace bottom wall were used as the supplementary data in the calculations. On the basis of measurement data, standard deviation of the measured gas temperature was estimated to be ±5% of  $T_g$ , and those of the wall temperature, soot concentration, and concentration of gas components to be ±30 K, ±0.01g/m<sup>3</sup>, and ±0.1% respectively. The Hotell zone method was used as the mathematical model. To facilitate this method, the combustion



chamber was divided into 10 streamwise segments as elementary zones. The equations used in calculations are

- energy balance for  $i$ -th gaseous zone

$$\sum_j \overline{g_i g_j} E_{gj} + \sum_j \overline{s_j g_i} W_{gj} - 4 K_i V_i E_{gj} + H_{in} - H_{out} + \sum h A_k (T_{s,k} - T_{g,i}) = 0 \quad (20)$$

where convective inflow and outflow of enthalpy  $H_{in}$  and  $H_{out}$  were calculated from the total mass flow rate, concentration of gas species and gas temperature. The convective heat transfer coefficient,  $h$ , between the furnace wall and the gas was estimated separately. In (20),  $W$  represents radiosity,  $T_s$  and  $T_g$  are temperatures of wall and gas,  $K$  gas absorption coefficient,  $V$  gas volume,  $A$  surface area.

- energy balance for the wall zone

$$Q_i = A_i [(\epsilon_i / \rho_i)(W_i - E_i) + h(T_{g,k} - T_{s,i})] \quad (21)$$

where  $\epsilon$  and  $\rho$  are emissivity and reflectivity of the wall surface, and  $E$  is black body emissive power.

- radiation balance on surface zone

$$\sum_j \left[ \overline{s_i s_j} + \delta_{ij} (A_i / \rho_i) \right] W_j + A_j (\epsilon_i / \rho_i) E_{s,i} + \sum_j \overline{s_i s_j} E_{gj} = 0. \quad (22)$$

Emissivity  $\epsilon$  and transmissivity  $\tau$  of the gas were computed separately for each gaseous zone using the formula [Hottel and Sarofim, 1967]

$$\epsilon = 1 - \exp\{-(k_g p_g L + B T_g)\}$$

$$\tau = 1 - [e^{(T_s p L T_g)}] (T_g / T_s)^{0.65}$$

where  $k_g$  represents linear absorption coefficient,  $p_g$  is gas pressure,  $L$  mean length,  $T_s$  and  $T_g$  are wall and gas temperatures respectively.  $B$  is a constant. The evaluation of the effective exchange areas  $\overline{gg}$ ,  $\overline{gs}$ ,  $\overline{ss}$  is described in [Hottel and Sarofim, 1967]. Figure 6 shows the comparison of the measured and calculated values of gas temperature along flame axis.

The difference at a point of 1 m downstream from the burner does not exceed 50 K and is around 20K over the rest of furnace. Figure 7 shows the result for the furnace wall temperature. The difference between the measured and calculated values does not exceed 20K all along the furnace length. Eqs. (20), (21) and (22) correspond to the system of condition Eqs. (2).

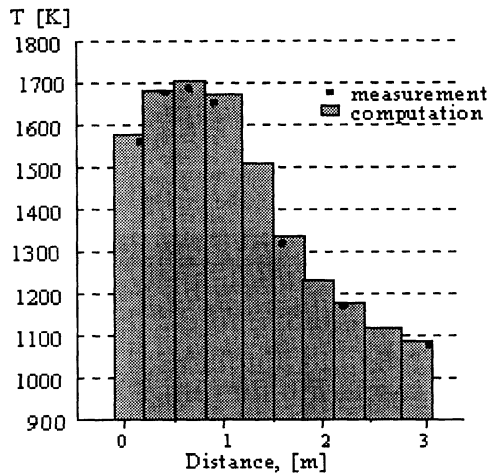


Fig. 6. Gas temperature distribution along the furnace.

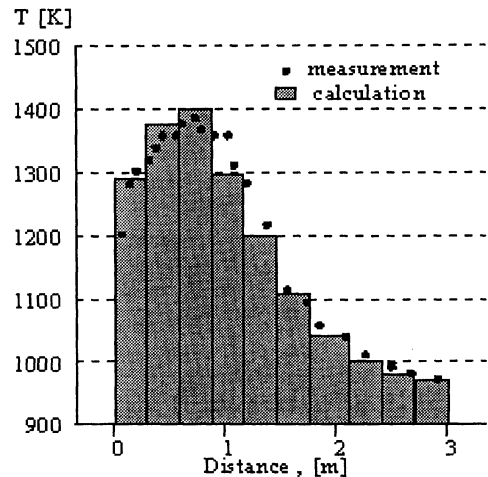


Fig. 7. Wall temperature distribution along the furnace.

## Conclusions

A new approach to the mathematical modelling of heat and mass transfer processes using supplementary data has been proposed. Calculation results show that the method works effectively giving new possibilities of more rigorous analysis. Supplementary information allows to obtain the estimates (most probable values) of calculation results and experimental data. The method makes also possible a sensitive analysis of the model equations. Additionally, a posteriori errors calculated using the law of errors propagation, when supplementary data are included into numerical model, give important information about the accuracy of model solutions. The values of corrections  $v_i$  allow to formulate an objective criterion for the verification of the mathematical model. The method proposed in this paper can easily be used in solution of many inverse problems.

The method presented here has been successfully applied to the much more advanced problems of transient heat and mass transfer processes during solidification of binary alloys [Escobedo Bocado, 1994] (macro and micro models) and mass transfer of autocatalytic dissolution of metallic copper in oxygen-containing ammonia solutions using the rotating disc technique [Kolenda et al., 1995].

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# The Footprint Problem in Deformation Analysis

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## Abstract

Control networks constructed to monitor deformations of the earth crust that have been surveyed by GPS, SLR or by VLBI are characterized by an extremely high inner precision. Accuracies of one part per million could be obtained formerly only at the cost of enormous investment in work and instrumentation. Today such accuracies are achieved effortlessly and as a routine outcome of relatively short surveys. Higher accuracies of the order of  $10^{-7}$  and even  $10^{-8}$  are also possible today. They require longer measurement sessions and rigorous processing of the data by means of sophisticated software. Opposed to the above impressive progress in measurement technologies, the state of the art in the area of monumentation of control points has not changed much since the beginning of this century. In deformation monitoring networks the above contrast in quality between monumentation of the points and the subsequent measurements is entirely unacceptable. The above contrast in quality casts a shadow of doubt over the impressive results of the analysis and could place a question mark on many of its conclusions. To the monumentation we have to add uncertainties or inadequacies in our knowledge of the structure and recent behavior of the rock on which the control points are monumented. As a possible solution to the above problems we propose the creation of a “footprint” around each monitoring (control) point. The footprint is a small cluster (network) of secondary points (footprint points) which are constructed around the primary control point and together with it constitute its footprint. The measurements within each footprint are processed independently following principles of free network adjustment. The monumentation of additional points, the extra measurements and the subsequent processing within the domain of each footprint are indispensable for preserving the extremely high relative accuracies which are obtained in deformation monitoring and analysis. There are also “side benefits” which should not be overlooked. The points of the footprint network can serve as safety (back up) points against the possibility of destruction (or movement) of the principal monitoring point. The paper presents the

mathematical models for analysis of the measurements. It ends up with a simulated numerical example intended to serve as an illustration of the algorithm.

## **Definition of the footprint problem**

During the second half of this century geodesy and surveying which rank among the oldest professions have been blessed by a sequence of phenomenal technological innovations. EDM, VLBI, SLR and lately GPS have placed at our disposal a formidable arsenal of means for achieving accuracies that were unthinkable only 50 years ago. Relative accuracies of the order of 1 ppm are regarded today as normal and “standard”. Higher accuracies ( $10^{-7}$ ,  $10^{-8}$ ) are also possible at the cost of longer measurement periods, better equipment and more sophisticated data processing algorithms.

In sharp contrast to the euphoria which prevails in measurement technologies we have to admit that in some related fields we continue to use methods and technologies which have changed very little in the past 50 and even 100 years. Monumentation of control points, setting up (3-D relative positioning with respect to the monumented mark) of sensors (prisms, antennas), determination of the sensor's focal point, checking focal point stability, those and many more "elementary" activities are still being carried out by rather primitive methods and with an accuracy which is definitely inferior as compared to the accuracy of the measuring instruments. As we all know “the strength of the wall is determined by the weakest of its bricks”. The above situation is not new and all those engaged in measurements and surveying of high precision are well aware of the painstaking care that has to be employed in order to achieve the desired accuracy. In GPS measurements, for example, where it appears that the role of the surveyor and his expertise have been reduced to a minimum, we are still expected to take good care of quite a few trivial details. With the consistent rise in measurement accuracy and the respective fall of the level of standard deviations of the errors, certain "rocks" begin to "show their ugly heads" above the surface of the sea of measurements. We may have known or suspected their existence although so far we did not have the means for their detection or removal. One of those "sea-bottom-rock" problems which has been revealed by the consistent fall of the measurements error-level is the representation problem of control points or as we did rename it in this paper - the footprint problem.

A geodetic control network designed to monitor deformations of the earth crust consists of a relatively small number of well monumented points which are distributed uniformly over the area under investigation. Distribution density (typical distance between network points) is usually a compromise between two contradictory considerations:

- (a) Insure proper representation to a maximum number of areas that may have different patterns of motion and deformation. Ideally each control point represents to the accuracy limits of the measurements only the adjacent area in its immediate neighborhood.

- (b) Each additional point in the network rises the cost of monumentation as well as the cost of the measurements that have to be carried out subsequently.

It so happens that in many cases control points are at distances of tens or hundreds and even thousands of kilometers. In such cases to the problem of proper representation an additional problem of a practical nature has to be taken care of. Appropriate measures have to be taken to preserve the extremely high accuracy that is obtained in relative positioning between the control points against the possibility of their inadvertent motion or - even - their destruction. A handy solution to this problem is a small set of safety points that are monumented in the vicinity of each control point. In the case of control point destruction, the safety points are used as a basis for their reconstruction with a minimum loss in accuracy. In the following sections of this paper we will discuss systematically the geometry and kinematics of such a network of "safety points" which has been constructed around the primary control point and forms together with it its footprint.

Surveying a high precision network involves an enormous investment in time and in money. The additional investment associated with the construction and surveying of footprints should be regarded as an insurance against the possibility of a "total loss" of the primary investment. By its very nature, deformation monitoring is an endeavor that depends on consistency and continuity over extended periods of time. Thus the stability of each control point with respect to its vicinity is of cardinal importance and its timely examination and control should receive the highest priority and should become an integral part of the surveying operations.

## **Geometry and kinematics of the footprint**

Let us consider a deformation monitoring network "D network" composed of a discrete number of points  $D_i$  ( $i=1,2, \dots$ ) . Around each  $D_i$  point we define an area of an irregular shape and of a variable size (from one point to another) and denote it as its footprint domain. Within each domain a number of secondary points  $F_{ij}$  ( $j=1,2, \dots$ ) are marked, thus forming together with the primary point  $D_i$  its footprint. For the sake of simplicity, without affecting the generality of the results, we will limit discussions in this paper to 2-D space, i.e., to a horizontal reference surface where horizontal distances are the only measurements. Initial positions of the  $D_i$  and  $F_{ij}$  points (at an arbitrary initial epoch  $t_0$ ) are given in the  $X_D$  and  $X_F$  vectors, respectively.

The respective velocities are given in  $\dot{X}_D$  and  $\dot{X}_F$  .

$$\mathbf{X}_D^T = (x_1, y_1, x_2, y_2, \dots, x_i, y_i \dots)_D$$

$$\mathbf{X}_F^T = (\mathbf{X}_{F1}^T, \mathbf{X}_{F2}^T, \dots, \mathbf{X}_{Fi}^T, \dots)$$

$$\mathbf{X}_{Fi}^T = (x_1, y_1, x_2, y_2, \dots, x_j, y_j, \dots)_{Fi}$$

$$\dot{\mathbf{X}}_F = d\mathbf{X}_F/dt \quad ; \quad \dot{\mathbf{X}}_D = d\mathbf{X}_D/dt$$

The coordinates (at epoch  $t_0$ ) and the velocities (assumed to be uniform) are absolute in the sense of being defined with respect to an inertial (x,y) coordinate system. At a sequence of epochs ( $t_1, t_2, \dots, t_k$ ) a discrete number of independent sets of measurements are made which involve only the  $D_i$  points. We denote those sets by  $L_{Dk}^b$  ( $k=1,2, \dots$ ).

The mathematical model of those measurements is given by the following equations:

$$L_{Dk} + V_{Dk} = \begin{bmatrix} A_{Dk} & \dot{A}_{Dk} \end{bmatrix} \begin{bmatrix} X_D \\ \dot{X}_D \end{bmatrix} \quad (1)$$

$$L_{Dk} = L_{Dk}^b - L_{Dk}^o \quad ; \quad \dot{A}_{Dk} = A_{Dk} \cdot (t_k - t_0)$$

Adjustment of the measurements which were collected over the epochs  $t_1, t_2, \dots, t_k$  produces an estimate of the velocity field of the D network. The sequence of computational steps needed to transform the  $L_{Dk}$  measurements into the  $\dot{\mathbf{X}}_D$  vector are shown in Appendix A. We should remember the well-known defect which is inherent in the above scheme and is typical to deformation analysis. While the velocities  $\dot{\mathbf{X}}_D$  are defined, indeed, with respect to the inertial (x,y) coordinate system, the measurements themselves being relative entities do not contain any information which refers to the inertial (x,y) system.

As a viable alternative for the inertial (x,y) system and on the basis of the measurements only we can define a conventional (subjective) reference (x,y) system. As a result of the above compromise in deformation analysis we end up with a velocity field that is defined with respect to a subjective (not necessarily inertial) conventional (x,y) system (see Appendix A).

Points in the primary deformation monitoring D network are split into two subsets: I and II. Points in subset II are distinguished from the rest by being a congruent set, i.e., a set that maintains an invariant geometry (invariant relative positions between its points). Due to their congruence, the points in subset II can be used as a basis for defining a conventional (x,y) coordinate system:  $(x,y)_{DII}$ . The  $(x,y)_{DII}$  system can serve as a substitute

for the hypothetical inertial  $(x,y)$  system. It can be shown that in the  $(x,y)_{D_{II}}$  system the velocities of points belonging to the II subset are identically zero.

$$\dot{X}_{D_{II}} = 0 \quad (2)$$

The points in the II subset form a so-called “basis for datum definition of the D network”.

So far we have described in short the “velocity approach” in deformation analysis where the inherent “footprint problem” of each of the  $D_i$  control points has been intentionally ignored.

Let us take now a closer look at a typical  $D_i$  point and at the area in its immediate vicinity - at its footprint domain. Let us assume that independently of the  $L_D^b$  measurements that involved only  $D_i$  points (the primary deformation monitoring network) we perform measurement sets confined to points that belong to the footprint of each  $D_i$  point.

The linearized model of measurements made at epoch  $t_n$  within the footprint domain will be:

$$L_{in} + V_{in} = [A_{in}, \dot{A}_{in}] \begin{bmatrix} X_{F_i} \\ X_{D_{ii}} \\ \dot{X}_{F_i} \\ \dot{X}_{D_{ii}} \end{bmatrix} \quad (3)$$

$$L_{in} = L_{in}^b - L_{in}^o ; \dot{A}_{in} = A_{in} (t_n - t_o)$$

where  $\dot{X}_{D_{ii}}$  is the velocity vector of point  $D_i$  with respect to the conventional coordinate system of footprint  $F_i$ . Here (within the footprint  $F_i$  domain) we identify a congruent subset  $F_{iii}$  in order to use it as a basis for defining its datum, i.e., to define a conventional coordinate system specific to footprint  $F_i$ . Note that point  $D_i$  does not belong to the  $F_{iii}$  subset. Analysis of such  $L_{in}$  measurements made at two or more different epochs  $t_n$  would produce an estimate of the  $\dot{X}_{F_i}$  and  $\dot{X}_{D_{ii}}$  velocities. As it was stated above (referring to subset II of the  $D_i$  points – see equation (2)), here also the velocities of points in the  $F_{iii}$  subset are identically zero:

$$\dot{X}_{F_{iii}} = 0 \quad (4)$$

It is important to note that the velocity of  $D_i$  with respect to this  $(x,y)$  reference system (denoted by  $\dot{X}_{D_{ii}}$ ) could be entirely different from  $\dot{X}_{D_i}$ . The  $\dot{X}_{D_i}$  velocity which was



obtained from analysis of the  $L_D$  measurements, refers to an entirely different datum as follows:

$$\begin{aligned}\dot{X}_{D_{ii}} &- \text{refers to datum } F_{ii} \\ \dot{X}_{D_i} &- \text{refers to datum } D_{ii}\end{aligned}$$

Let us consider the case where the  $F_{ii}$  points span the major part of the respective footprint domain and the velocity of  $D_i$  with respect to datum  $F_{ii}$  is significantly different from zero. We could state that point  $D_i$  is an outlier within its own domain or, in other words, that it does not represent properly the kinematics of its footprint. If this is the case and in accordance with a quasi “democratic” rationale we should identify the motion of the footprint  $F_i$  with the motion of its  $F_{ii}$  subset and not with the point  $D_i$  itself.

The velocity of the footprint  $F_i$  with respect to the  $(x,y)_{D_{ii}}$  reference system is obtained by the following difference:

$$\dot{X}_i = \dot{X}_{D_i} - \dot{X}_{D_{ii}} \quad (5a)$$

In case  $\dot{X}_{D_{ii}} = 0$  it would mean that point  $D_i$  is moving consistently with its footprint and consequently that

$$\dot{X}_i \equiv \dot{X}_{D_i} \quad (5b)$$

We would like to go back and to discuss the method of selecting the  $D_{ii}$  subset. Before making a distinction between the motion of  $D_i$  and the motion of its footprint  $F_i$  we applied linear constraints to the  $\dot{X}_D$  velocity vector where matrix  $P_x$  was defined so as to identify the  $D_{ii}$  subset as shown in the following

$$\dot{X}_D^T \cdot P_x \cdot \dot{X}_D = \dot{X}_{D_{ii}}^T \cdot \dot{X}_{D_{ii}} = \min \quad (6a)$$

$$P_x \dot{X}_D = \dot{X}_{D_{ii}} = 0$$

Those linear constraints together with the  $L_D$  measurements define the conventional reference system  $(x,y)_{D_{ii}}$ . All we have to do now in order to be ready for possible inconsistencies between the  $D_i$  points and their respective footprints would be to impose the constraints (equation (6)) on  $\dot{X}$  instead of  $\dot{X}_D$  (where  $\dot{X}$  is the vector of footprint velocities).

$$\begin{aligned}\dot{X}^T P_x \dot{X} &= \dot{X}_{ii}^T \cdot \dot{X}_{ii} = \min \\ P_x \dot{X} &= \dot{X}_{ii} = 0\end{aligned} \quad (6b)$$

The meaning of the above “shift of constraints” is that a deformation monitoring network is represented by the footprints of its points rather than by the points ( $D_i$ ) themselves.

We conclude this theoretical section by a remark of a rather practical nature. The footprints in a control network do not have to be uniform in terms of size and geometry, number of points, frequency and type of measurements. Common sense should be applied in addition to input from other disciplines such as geology, geophysics and soil-mechanics in order to make a choice from a wide range of possible footprints. The range can be anywhere between a large and extensive footprint down to a "no-footprint" at all. As we emphasized earlier, measurements of the primary network are entirely independent of the measurements that are made within each footprint. This should be regarded as an important convenience. Thus the D net could be surveyed by a national or international organization (as the case may be) while surveying the various footprints would be the responsibility of local or regional authorities.

### Numerical example of a footprint

To illustrate the principles of the proposed solution of the footprint problem we simulated a small monitoring network in the Upper Galilee region in Northern Israel spanning both sides (east and west) of the Jordan river. The network consists of 6  $D_i$  points where points  $i=1, 2, 3$  are west of and points  $i=4, 5, 6$  are east of a simulated left transform fault which runs in a general north south direction. The footprint of point  $D_4$  consists of four points  $F_{4j}$  ( $j=1, 2, 3, 4$ ). Table 1 shows the absolute positions (with respect to a hypothetical inertial system) at epoch  $t_0$  and the respective velocities of all 10 points.

**Table 1: Absolute  $t_0$  positions and velocities of points  $D_i$  and  $F_{4j}$**

	x [km]	y [km]	$\dot{x}$ [mm/y]	$\dot{y}$ [mm/y]
$D_1$	787	251	15	5
$D_2$	777	257	10	0
$D_3$	766	254	10	5
$D_4$	769	267	10	10
$D_5$	779	268	25	0
$D_6$	788	263	25	-5
$F_{41}$	770	266	20	-5
$F_{42}$	770	267.5	20	0
$F_{43}$	768	267	25	0
$F_{44}$	768	265.5	20	-5

The D net was surveyed at two epochs with an interval of 3 years in between. The first epoch was chosen (declared) as  $t_0$ . The footprint of  $F_4$  was surveyed 3 times at intervals of 2 years. Here too the epoch of the first survey was declared as  $t_0$ . Note that

the two initial epochs  $t_0$  do not coincide. Datum of the  $D_i$  net was based on points 1, 2, 3 (western subset) and so the (12 by 12)  $P_x$  was defined as

$$P_x = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$$

where  $I$  and  $0$  are (6 by 6) identity and zero. Datum of the  $F_4$  footprint net was based on all four  $F_{4j}$  points. Note that point  $D_4$  itself does not participate in the footprint datum definition although it is an integral part of the footprint net.

The standard deviation of the measurements within the  $D$  net was assumed to be

$$\sigma_i = 0.7 * 10^{-6} * \ell$$

while the footprint net  $F_4$  measurements were of a significantly lower quality:

$$\sigma_4 = 5 * 10^{-6} * \ell$$

All the measurement sets were simulated as being complete, i.e., that each set contains all possible distances:

- in the  $D$  net - 15 distances
- in the  $F_4$  footprint - 10 distances

Results of the adjustment (after elimination of the position unknowns) are shown in table 2.

Referring to equation (5), the results shown in table 2 indicate that the footprint  $F_4$  is moving with respect to the datum of the primary  $D$  network by:

$$\dot{x}_4 = +10.64 \text{ mm/year}$$

$$\dot{y}_4 = -5.33 \text{ mm/year}$$

**Table 2: Results of the adjustment of the  $D$  primary and the  $F_4$  footprint networks**

point	$\dot{x}$	$\dot{y}$	$\sigma_{\dot{x}}$	$\sigma_{\dot{y}}$
$D_4$	-11.58	12.42	1.53	2.46
$F_{41}$	-.92	-1.92	1.61	1.41
$F_{42}$	-1.91	3.08	1.43	1.46
$F_{43}$	3.42	1.76	1.45	1.48
$F_{44}$	-.59	-2.91	1.49	1.48
$D_1$	3.17	1.09	2.71	1.33
$D_2$	-1.50	-3.35	1.73	2.81
$D_3$	-1.67	2.26	2.53	1.65
$D_4$	-.94	7.09	4.16	3.61
$D_5$	14.12	-3.46	4.28	3.31
$D_6$	13.84	-8.97	3.67	3.70

Note: The values in table 2 are given in mm/year.

It is easy to see that in this simulation the velocity of the  $F_4$  footprint conforms to the velocities of points  $D_5$  and  $D_6$  much better than the velocity of point  $D_4$  itself. As expected, the velocities obtained by our analysis are entirely different from the absolute velocities that were shown in table 1. This is explained by the apparent (but non-estimable) velocity of the  $D_1, D_2, D_3$  subset with respect to the hypothetical inertial  $(x,y)$  system.

## Summary

In this paper we presented a simplistic solution of the footprint problem which is applicable in high precision geodetic control networks. The proposed solution is general in the sense that size, number of points, geometry, and measurement accuracy may differ from one footprint to another. Local geological and logistic constraints will usually dictate a variety of footprint designs. Additional flexibility is gained by the complete independence in choosing the epochs of measurement of the primary vs. the footprint networks. Yet in spite of all the above variability the mathematical model and the over-all algorithm of data processing are simple and uniform so that design and construction of an appropriate software package should be a straightforward matter. The main (and only) drawback of the proposed solution is its price. Monumentation of footprint points and in particular those points which belong to the  $F_{III}$  subset (points which define the footprint datum) has to be of a quality which is nearly equivalent to the quality of the  $D_i$  point itself. Any compromise following a pseudo-economical attitude in this matter may take us back to the original problem and may eradicate completely the advantages gained by adopting the proposed solution.

## Appendix A

### Velocity field in 2-D space from measured horizontal distances

Let us have a horizontal kinematic control network composed of  $m$  points. At two or more epochs ( $t_1, t_2 \dots t_k$ ) “complete” sets of distances ( $L_1, L_2 \dots L_k$ ) have been measured between the  $m$  points. A set of measurements is considered “complete” if network geometry is completely defined and apart of the need for datum definition, coordinates of all the  $m$  points can be estimated. The condition of completeness is a desirable property, particularly in the first two sets, although it is not indispensable. The datum defect in such a net is of size 3 which means that three linear constraints have to be imposed in order to define origin and orientation of the conventional  $(x, y)$  coordinate system. As we are dealing with two sets of independent unknowns, i.e., positions at  $t_0$  and uniform velocities of all  $m$  points we need two independent sets of constraints - one for the positions and a second one for the velocities.

The linearized mathematical model of the  $L_k$  set of measurements is

$$L_k^b + V_k = L_k^o + A_k X + \dot{A}_k \dot{X} \quad ; \quad \dot{A}_k = A_k (t_k - t_0) \quad (A1)$$

where  $A_k$  is a matrix of partial derivatives.

After two or more ( $k$ ) sets of measurements have been performed we have:

$$\begin{aligned} L + V &= AX + \dot{A}\dot{X} \quad ; \quad L = L^b - L^o \\ A^T &= [A_1^T, A_2^T, \dots, A_k^T] \\ \dot{A}^T &= [A_1^T(t_1 - t_0), A_2^T(t_2 - t_0), \dots, A_k^T(t_k - t_0)] \\ L^T &= [L_1^T, L_2^T, \dots, L_k^T] \\ V^T &= [V_1^T, V_2^T, \dots, V_k^T] \end{aligned} \quad (A2)$$

The conventional coordinate system for positions and velocities is defined by the following equations of constraints:

$$\begin{aligned} \bar{C}^T \cdot X &= 0 \quad \leftarrow \quad X^T P_x X = \min \\ \bar{C}^T \cdot \dot{X} &= 0 \quad \leftarrow \quad \dot{X}^T P_x \dot{X} = \min \end{aligned} \quad (A3)$$

where

$$\bar{C}^T = C^T P_x$$

and  $C$  is the well known (2m by 3) Helmert's matrix.

$$C^T = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & \dots \\ 0 & 1 & 0 & 1 & 0 & 1 & \dots \\ -y_1 & x_1 - y_2 & x_2 - y_3 & \dots & & & \end{bmatrix}$$

Note that  $x_i, y_i$  in the above  $C$  matrix are the approximate (nominal) coordinates of the points in the conventional ( $x, y$ ) system.  $P_x$  is a diagonal matrix with diagonal elements consisting of zeros and ones. Points marked by "1" in  $P_x$  form a subset that is distinguished by remaining congruent in time, i.e., maintaining an invariant geometry. The "bootstrapping" property of this method of datum definition is such that the velocities of points in the selected subset are identically zero. Note that the above property (zero velocities) is objective in the sense that it is supported by the measurements.

Application of the above linear constraints together with minimization of the  $V^T P V$  scalar function and following the elimination of the  $X$  unknowns (see Appendix B) results in the following normal equations:

$$\begin{bmatrix} \bar{N} & \bar{C} \\ \bar{C}^T & O \end{bmatrix} \begin{bmatrix} \dot{X} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} \bar{U} \\ 0 \end{bmatrix} \quad (\text{A4})$$

where  $\dot{\lambda}$  is the vector of Lagrange multipliers.

Each successive set of measurements results in a new update of the  $\dot{X}$  estimate. With each new set of measurements the congruence of the “datum” subset is verified statistically. If a point is suspected of “misbehavior”, i.e., an apparent inconsistent motion relative to the datum subset, then the respective elements in the diagonal of the  $P_x$  matrix are changed from “1” to “0”. The  $\dot{X}$  vector is then S-transformed to a different (but more stable) datum which is independent of the suspected point.

## Appendix B

### Elimination of the X unknowns ( $t_0$ positions)

Let us have the following linear model:

$$V = AX + \dot{A}\dot{X} - L \quad ; \quad \dot{A} = A \cdot (t - t_0) \quad (\text{B1})$$

We impose two sets of linear constraints:

$$\begin{cases} 0 = \bar{C}^T X \\ 0 = \bar{C}^T \dot{X} \end{cases} \quad ; \quad \bar{C} = P_x C \quad (\text{B2})$$

C is a basis of the null space of A and satisfies the following equation:  $AC = 0$

We seek to minimize the following  $\phi$  function:

$$\begin{aligned} \phi &= V^T P V + 2\lambda^T C^T X + 2\dot{\lambda}^T \bar{C}^T \dot{X} = \min \\ \frac{\partial \phi}{\partial X} &= 2V^T P \frac{\partial V}{\partial X} + 2\lambda^T \bar{C}^T = 2V^T P A + 2\lambda^T \bar{C}^T = 0 \\ \frac{\partial \phi}{\partial \dot{X}} &= 2V^T P \frac{\partial V}{\partial \dot{X}} + 2\dot{\lambda}^T \bar{C}^T = 2V^T P \dot{A} + 2\dot{\lambda}^T \bar{C}^T = 0 \end{aligned} \quad (\text{B3})$$

A trivial matrix manipulation brings us to the following form of the normal equations:

$$\begin{bmatrix} \mathbf{N} & \bar{\mathbf{C}} & \dot{\mathbf{N}} & 0 \\ \bar{\mathbf{C}}^T & 0 & 0 & 0 \\ \dot{\mathbf{N}} & 0 & \ddot{\mathbf{N}} & \bar{\mathbf{C}} \\ 0 & 0 & \bar{\mathbf{C}}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \lambda \\ \dot{\mathbf{X}} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{U} \\ 0 \\ \dot{\mathbf{U}} \\ 0 \end{bmatrix} \quad (\text{B4})$$

where

$$\mathbf{N} = \mathbf{A}^T \mathbf{P} \mathbf{A} \quad ; \quad \dot{\mathbf{N}} = \mathbf{A}^T \dot{\mathbf{P}} \mathbf{A} = \dot{\mathbf{A}}^T \mathbf{P} \mathbf{A} \quad ; \quad \ddot{\mathbf{N}} = \dot{\mathbf{A}}^T \dot{\mathbf{P}} \mathbf{A}$$

$$\mathbf{U} = \mathbf{A}^T \mathbf{P} \mathbf{L} \quad ; \quad \dot{\mathbf{U}} = \dot{\mathbf{A}}^T \mathbf{P} \mathbf{L}$$

The following sub-matrix is regular and has a regular inverse; we are interested in Q only:

$$\begin{bmatrix} \mathbf{N} & \bar{\mathbf{C}} \\ \bar{\mathbf{C}}^T & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{Q} & \mathbf{B} \\ \mathbf{B}^T & 0 \end{bmatrix} \quad (\text{B5})$$

Thus X can be expressed as a function of  $\dot{\mathbf{X}}$  as follows:

$$\mathbf{X} = \mathbf{Q} \mathbf{U} - \mathbf{Q} \dot{\mathbf{N}} \dot{\mathbf{X}} \quad (\text{B6})$$

Substitution in the third row of the normal equations results in:

$$\dot{\mathbf{N}} \mathbf{Q} \mathbf{U} - \dot{\mathbf{N}} \mathbf{Q} \dot{\mathbf{N}} \dot{\mathbf{X}} + \dot{\mathbf{N}} \dot{\mathbf{X}} + \bar{\mathbf{C}} \lambda = \dot{\mathbf{U}} \quad (\text{B7})$$

to be followed by:

$$\begin{bmatrix} (\ddot{\mathbf{N}} - \dot{\mathbf{N}} \mathbf{Q} \dot{\mathbf{N}}) & \bar{\mathbf{C}} \\ \bar{\mathbf{C}}^T & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{X}} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} (\dot{\mathbf{U}} - \dot{\mathbf{N}} \mathbf{Q} \mathbf{U}) \\ 0 \end{bmatrix} \quad (\text{B8})$$

which is the final form of the normal equations as shown at the end of Appendix A:

$$\begin{bmatrix} \bar{\mathbf{N}} & \bar{\mathbf{C}} \\ \bar{\mathbf{C}}^T & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{X}} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{U}} \\ 0 \end{bmatrix}$$

# **Modelling of Subsidence in Area of Room and Pillar Salt Mining Using Finite Element Method**

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*NOTE: the authors dedicate this paper to Dr. Adam Chrzanowski who has spent many years of his scientific career on the problems related to ground subsidence in mining areas. He has co-authored the development of the method of modelling ground subsidence that has been applied in the case study presented in this paper.*

## **Introduction**

The investigated salt mine is located near the city of Inowroclaw in Poland. The horizontal projection of the salt deposit has an elliptic shape with dimensions 2.5 km x 1.0 km, with the center located under the city. An upper boundary of the deposit is at a depth of 125 m and the lowest at 4000 m. The deposit is covered by the gypsum cap rock with thickness of 100 m. The cap rock was created by diluting of the top layers of salt deposit by surface water. The first mining activities started in 1920. At that time, inadequate design of pillars allowed for intrusion of water to the mined areas and led to a change of the ground water level that resulted in a large ground subsidence and creation of caved-in zones in the city.

After two decades, a new mining method was designed in order to carry on mining activities in the area. The main goal of the design was to limit influence of the mining exploitation on the deformation of the cap rock and on the surface. The new mining area was located below the earlier workings and was separated from them by 200 m of unmined rock. Also the dimensions of boarder pillars were increased to 250 m.

The study of ground subsidence over the mine area using finite element method is presented. Because of the unusual history of mining, complicated geometry of mining sequence, and limited knowledge of the rock properties, only approximate modelling was possible. The first attempt to model the surface subsidence and deformation in the mine was performed at the University of New Brunswick in 1993 and the second at the



Technical University of Mining in Krakow in 1997. The results of the analyses are presented below.

## **Modelling of Mining Subsidence**

Modelling of mining subsidence can be performed using either empirical (statistical) or deterministic methods. Most of the empirical theories for predicting subsidence have been developed in Central Europe and the United Kingdom, where systematic and accurate monitoring surveys have been conducted in mining areas for several decades. The empirical models have been obtained through a correlation of the observed deformations with the causative factors that are usually expressed only in terms of the geometry and depth of the mined deposit and, in some theories, with qualitative information on geology and the strength of rocks. Since other parameters, such as mechanical properties of the rock and tectonic stresses, are not taken into account, the prediction theories are applicable only to the areas where the mining, geological, and tectonic conditions are the same or very similar to the area where the empirical data for the theory was collected. It also should be stressed that the empirical theories are generally not reliable in cases of complicated geometry of mined deposits, in the presence of faulting, and in areas of previous extensive mining operations. The deterministic methods are more universal than the empirical theories because they can be applied in any geological and mining conditions, and they provide information not only about the surface subsidence but also on deformations within the rock masses and within the mine workings. The deterministic methods, however, require reliable information on the in-situ properties of rocks, initial stresses, and tectonics of the area. With the development of computer technology numerical methods and particularly finite element method became very popular.

## **Geometry and Mining Sequence of Salt Mine Inowroclaw**

After the flooding in 1920s of the upper levels of the salt mining operation and stabilization of water inflow, the mining activities started again in 1941 at the depth of 470 m to 632 m. The mining method was a room-and-pillar with hydraulic salt removal. Dimensions of rooms were: length 100 m, initial width 4 m and final width 25 m, and height of 9 m (Jasinski and Laskowski, 1970; Pielok, 1985). The exploitation was progressing simultaneously at seven levels with a total volume of rooms being 380 000 m<sup>3</sup>. After the salt removal, the rooms were backfilled to decrease the surface subsidence. A geological cross-section of the mine is shown in Fig.1.

In 1959, a leveling survey of points along selected lines on the surface was initiated. In 1978, the surveys were repeated every 2 years. At the same time, a survey of vertical deformations at the first working level at the depth 470 m was also conducted. Between 1959-78, the maximum subsidence of the surface and the level I of the mine reached 121

mm and 295 mm respectively. The measured subsidence profiles on the surface and on level I are shown in Fig.2 and Fig.3 respectively.

## **Modelling of Ground Subsidence in Salt Mine Inowroclaw**

Ground subsidence is a time dependent process. Generally, in the salt mines, the maximum (final) subsidence reaches the surface after several decades (Kratzsch, 1983). There are cases when the effects of subsidence may be temporarily stopped by overlain strong rocks and re-activated after a prolonged period of time.

Modelling of the behavior of salt rocks, and, particularly, modelling of the time dependent response of the ground surface in potash and salt mines is very complex. The choice of the reological model and its parameters is important. The parameters may be determined on the bases of long time laboratory test of the salt samples or using the field observations. To develop a reliable method for either empirical or numerical modelling of ground subsidence, one has to utilize long sets (many years) of field observation data. In case of the Inowroclaw mine, there is lack of observation data of the surface subsidence and deformation at level I at the early stage of mining. The observations were performed only for limited time.

### **Modelling of Ground Subsidence using Software FEMMA**

A method for numerical modelling of ground subsidence has been developed at the University of New Brunswick (Szostak-Chrzanowski, 1988; Szostak-Chrzanowski and Chrzanowski, 1991a, Chrzanowski et al, 1997) within the activity of the Engineering and Mining Surveys Research Group led by Dr. Adam Chrzanowski. The method is based on a combination of sequential finite element solutions with empirical knowledge on the behavior of rock masses. The method considers the salt rock as a non-Newtonian liquid and the overburden brittle rocks as a non-tension material (Zienkiewicz et al., 1968). The method is supported by software FEMMA (Finite Element Method for Multipurpose Applications) for 2-D and 3-D analyses developed at the UNB (Szostak-Chrzanowski and Chrzanowski, 1991b).

Modelling of ground subsidence in salt mine Inowroclaw was performed in two following phases:

- (1) modelling of the salt rock response to the underground openings and calculation of the expected subsidence at the top of the salt formation (at the cap rock); and
- (2) modelling of the response of the rock overlying the salt deposit, which is treated as brittle and non-tension material.

Salt rock is considered as a non-Newtonian liquid with high and not constant viscosity (Mraz et al., 1987). As a liquid, the intact salt rock deposits are under isotropic lithostatic stress conditions that are characterized by all stress components being equal to each other and equal to the overburden stress. Therefore, the shear stress in intact salt

rock is equal to zero. Development of shearing stresses due to mining activity causes the flow of the salt mass into the excavated areas in order to achieve a new equilibrium state of stresses. The 'flow-in' zone is determined by the FEM elements in which maximum shearing stresses are developed by introduction of the mining opening.

In modelling the final maximum subsidence at the top of the salt formation, the original value of E in the 'flow' zone is decreased to give the same volume of the subsidence basin (under the cap rock) as is the volume of mining openings (minus the volume of the backfill). The initial stress in the intact salt rock is assumed to be isotropic lithostatic.

Once the equivalent subsidence trough at the top of the salt is determined, the response of the brittle rock is analysed. Similar to the 'flow-in zone' in salt, a 'weak zone' is determined in the overburden brittle rocks. The 'weak' zone is delineated by the FEM elements in which the maximum shearing stresses develop at the boundary between the zone of rocks subjected to tensional stresses above the underground opening and the surrounding rocks subjected to compressive stresses (Chrzanowski et al, 1996a; Chrzanowski et al, 1996b).

In case of "Inowroclaw" mine, an attempt was made to model only the surface subsidence process which would correspond to the monitoring time interval. In calculations of the subsidence, the volume of the flown in salt rock was calculated on the basis of 1959-1978 measurements. The modeled subsidence is shown in fig.2. In the two dimensional analysis, a "weighting method" (Pielok, 1985) was introduced to represent the three dimensional mining workings. The "weights" are shown in Fig.1 under the extracted area.

### **Modelling of Mining Subsidence using Software ABAQUS**

Recently, an attempt to model the subsidence in the area of the salt mine Inowroclaw was performed at the Technical University of Mining in Krakow using finite element software ABAQUS. In modelling the salt rock, an initial relaxation model was used. The following formula was used in the two dimensional model:

$$\dot{\epsilon}_i^e = A \sigma_i^n t^m$$

where  $\dot{\epsilon}$  is a velocity of relaxation process,  $\sigma$  is stress,  $t$  is time, and  $A$ ,  $n$ ,  $m$  are selected constants. The coefficients  $A$ ,  $n$ , and  $m$  were determined by calibration of the computed results against geodetic measurements. The following calibrated values of the coefficients have been obtained:

$$A = -8.8 \times 10^{-11}, n = 2, \text{ and } m = -0.5.$$

Various viscosity models are being used in modelling ground subsidence. The research is in progress.

## Conclusions

The finite element modelling of subsidence in area of the salt mine Inowroclaw has a good potential in describing the time dependent process of subsidence. In order to better understand the process of subsidence, surface leveling measurements and measurements of closures of level I should be repeated to see whether the subsidence has already reached its maximum value.

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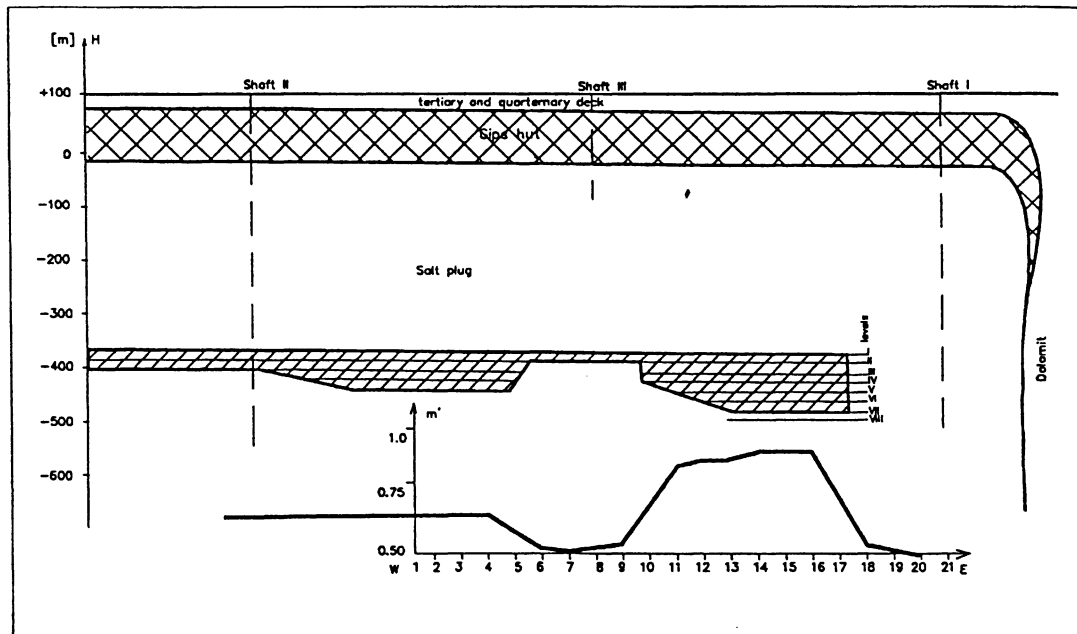


Fig. 1. Geological cross-section salt mine Inowroclaw.

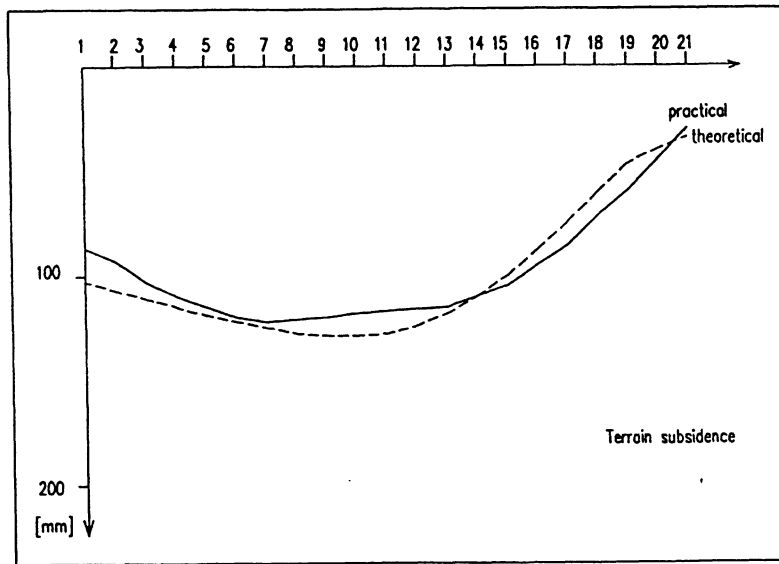


Fig. 2. Surface Subsidence -Measured and calculated using FEMMA

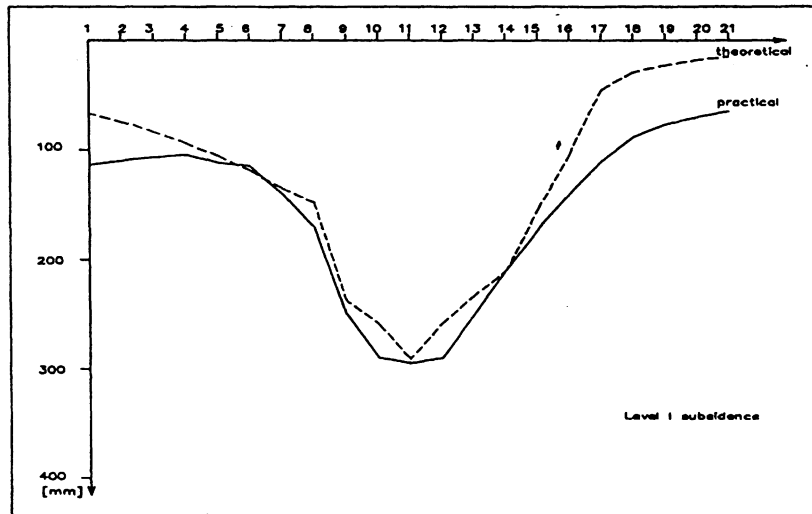


Fig. 3 Vertical displacement at level I measured and calculated using FEMMA.

# GEODETIC DEFORMATION MEASUREMENTS OF THE BALL MOUNTAIN DAM

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## Introduction

Ball Mountain Dam is owned and operated by the Army Corps of Engineers (ACOE) in Waltham, Massachusetts. The project is located on the West River near the small town of Jamaica, Vermont. Construction of the dam began in 1957 and was completed in November 1961. The dam is part of the system of reservoirs and local protection works for the control of floodwaters in the Connecticut River basin. The impoundment area is approximately 800 acres and supports a drainage area of 172 square miles. The capacity of the reservoir with the water level at the spillway crest elevation is 54,000 acre feet.

The flood control system at the Ball Mountain Dam is comprised of an embankment dam, spillway, reservoir, and outlet works. The dam is a rolled earth and rockfill embankment. The structure rises 265 ft above the riverbed with a crest length of 915 ft and a 20 ft crest width. An emergency spillway consisting of an uncontrolled concrete ogee weir is located in the right abutment adjacent to the dam and is separated from the embankment by a concrete retaining wall. The spillway crest is 35 ft below the crest of the dam.

The outlet works are located in the right abutment and consist of an inlet channel, an intake tower, a 13.5 ft diameter tunnel under the dam, and an outlet channel. The dam is a zoned structure consisting of a central rolled glacial till impervious core. This is protected by an upstream and downstream rockfill shell. Gravel filter layers are provided between the impervious core and the more porous rockfill shells to prevent the loss of the fine-grained core material from normal water seepage. Additional gravel filter layers are provided between the downstream rockfill shell and the underlying foundation soils.

Seepage beneath the dam is reduced by a compacted glacial till cutoff under the impervious core extending to glacial till in the valley and to bedrock in the abutments. A grout curtain in the bedrock of the abutments provides additional seepage control. The upstream slope varies from 2H:1V to 2.5H:1V, and the downstream slope is 1.75H:1V. The axis of the dam is arched approximately 50 ft toward the upstream side.

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<sup>1</sup> Note from the Editor: Between 1985 and 1987, Mr. Rohde was involved in the original design and implementation of the geodetic monitoring scheme at Ball Mountain Dam as a graduate student working towards his M.Sc.E. degree at UNB under the supervision of Dr. A. Chrzanowski.

A bulge in the downstream slope was noticed in 1962, less than one year after construction. Surveys of the downstream slope were performed in 1963 and showed that the bulge represented a deviation of about 2 ft from the design slope line of the dam. One possible explanation given for the bulge was that a survey error had occurred during the construction of the dam. Crest settlement was observed and estimated to be approximately 0.5 ft (0.15 m) in 1962 and 1 ft (0.30 m) in 1963. Three survey monuments were then installed along the crest centerline in 1963.

The ACOE performed periodic inspections of the dam between 1962 and 1985. These irregularities have been noted in periodic inspection reports by the ACOE during the past twenty years (1963, 1975, 1978, and 1982). In September 1984 the ACOE, based on surveys made of the crest monuments installed in 1963, concluded that the crest centerline had settled approximately 10 in. (0.25 m) since 1963 and about 2 in. (0.05 m) between 1978 and 1982. It was also estimated (based on visual inspection) that the downstream edge of the crest had settled by as much as 2 ft (0.61 m) since 1963.

In November 1984, an inspection of the dam was performed by representatives from the Corps of Engineers Office of the Chief of Engineers (OCE), Washington, DC in an attempt to resolve the magnitude and scope of movement of the downstream slope. In the report prepared after the inspection, it was concluded that the bulge in the downstream slope extended over the entire length of the embankment and approximately 18 - 24 in. (0.46 to 0.61 m) of crest settlement had occurred. There were also indications of a shallow slide in the downstream slope. Definite conclusions as to the magnitude, rate, and cause of the embankment movement could not be made, at that time, due to the lack of standardization in the measurement and survey techniques employed.

## **Geodetic Deformation Monitoring Program**

Evidence of crest settlement and the presence of the downstream slope bulge at Ball Mountain Dam prompted the ACOE to investigate the deformations using geodetic and geotechnical survey methods in 1985. Historical deformation data available on the Ball Mountain Dam consists of both geodetic and geotechnical measurements. The ACOE awarded a contract to Coler & Colantonio, Inc. in 1995 to continue the investigation of the deformations of the Ball Mountain Dam for the geodetic monitoring program.

The geotechnical data is produced from 6 slope inclinometer casings which were installed in the dam during the period of October 1985 to January 1986. The geotechnical survey design and installation of the casings, as well as performing monthly readings of the inclinometer installations began in January 1986 by an independent consulting firm. The original geodetic surveys were performed by another consulting firm. A total of 6 bimonthly survey campaigns were performed over a 15 month time span, beginning July 1986 and ending October 1987. Deformation survey number 7 was completed by Coler & Colantonio, Inc. in October 1995. Table 1 summarizes the dates of all the surveys.

The shaded areas in Table 1 indicate that the first two campaigns were considered to be unreliable due to non-rigorous handling of the influence of the inclination of the vertical axis of the theodolite. Heights of instruments and targets were not properly observed or maintained. The zenith angle observation procedure also did not contribute to the specialized adjustment method developed for modeling the effects of refraction.



## **Project Accuracy Requirements**

The existing survey program requires equipment, field procedures, and data analysis methods capable of detecting horizontal and vertical displacements of  $\pm 3.0$  mm at the 95.0% confidence level. The theory of error propagation requires that individual surveys (campaigns) performed at different times (epochs) will require a higher accuracy in order to achieve the detection of  $\pm 3.0$  mm displacements. A requirement often overlooked during the design phase of most deformation monitoring programs.

Conventional variance-covariance analysis was used to design a network capable of detecting the minimum displacement of  $\pm 3.0$  mm at the 95.0% confidence level. Assuming no correlation between a pair of measurement epochs and for the detection of a  $\pm 3.0$  mm horizontal displacement at the 95.0% confidence level, the design value for the semi-major axis of the standard confidence ellipse is  $\pm 0.87$  mm for the horizontal positional accuracy in a single deformation survey campaign. For the detection of a  $\pm 3.0$  mm vertical displacement at the 95.0% confidence level, the design value for the semi-major axis of the standard confidence interval is  $\pm 1.08$  mm for the vertical positional accuracy in a single deformation survey campaign.

## **Deformation Monitoring Network Design**

The strategy taken in the design of the survey network at Ball Mountain Dam was to consider the horizontal and vertical movements separately. The justification for this decision was based on the uncertainty of the effects of atmospheric refraction on the vertical observations. However, during the analysis procedure the goal was to combine both the horizontal and vertical components as a three-dimensional deformation network.

The deformation network diagram showing the location of the object points as established for the project is shown in Figure 1. A total of 27 object points are to be monitored on the downstream slope and crest of the dam. Nine of the object points are located on the crest of the dam. The intake tower is also included as an object point in order to monitor its movement.

Each of the 18 slope monuments consists of 0.20 ft diameter by 5 ft long galvanized steel pipe placed in a 5 ft deep, circular cored hole, into the rockfill shell to a depth of approximately 2 ft. The pipes were filled with concrete as well as the annulus between the pipe and the sides of the cored hole. A Wild GRT10 centering stem is permanently installed in the top of each pipe for mounting precision targets and reflector assemblies. Custom designed locking access covers protect the centering stems from damage.

The 9 crest monuments are similar to the slope monuments. These monuments are set flush with the surface of the access road across the crest. Locking access covers protect the recessed Wild GRT10 centering stems. Extension rods are used to elevate the precision targets during the surveys.

The use of an absolute reference network was selected for the Ball Mountain Dam project to monitor the deformations. This involves determining the relative accuracy of proposed object points located on the dam with respect to a network of stable reference points. A computer simulation and pre-analysis was performed to design the geometric configuration of the absolute reference network and to select surveying instrumentation capable of meeting the required accuracy specifications. Figure 2 on the previous page illustrates the final deformation monitoring network design.

The mode of network observations is essentially triangulation with the measurement of several interpillar distances. All elevations are determined by non-reciprocal trigonometric heights using measured zenith angles only (no slope distances). Daily water levels of the reservoir were also available. During the first 6 epochs, instrumentation consisted of the Wild T2000 and Kern E2 precision electronic theodolites and the Wild DI4L and Kern DM502 electronic distance measuring instruments (EDMI).

The absolute reference network points consist five, 18 in. diameter concrete pillars. Four of the pillars were installed directly into competent bedrock approximately 2 ft below grade. The remaining pillar (P3) was installed in a sand, glacial till deposit extending to a depth of approximately 5 ft below grade. Wild ball and socket type centering systems, utilizing the older K&E standard, were installed in the top center of each concrete pillar.

### **Epoch 7 Deformation Survey Campaign**

No major changes were made to the observation scheme originally proposed for the deformation monitoring program of the dam in 1985. The geodetic deformation monitoring program designed and implemented for the dam continues to provide the ACOE with a system capable of detecting movements of  $\pm 3.0$  mm at the 95% confidence level in both the horizontal and vertical directions.

The measurements for the detection of horizontal movements were based on the network design and pre-analysis results necessary to satisfy the project accuracy requirements as discussed above. Horizontal directions measured with a standard deviation of  $\pm 0.5$  arcseconds and distances with a standard deviation of  $\pm 3$  mm + 4 ppm or better are required. A Kern E2 electronic theodolite ( $\pm 0.5''$  accuracy) was used during the epoch 7 deformation survey.

A computer-based automated data collection system was available during the epoch 7 deformation survey. Hewlett-Packard HP-71B and a Husky FS/2 handheld computers were linked to the electronic total stations to allow for automated data acquisition during the field surveys. The units are also used during precise differential leveling surveys with data manually keyed into the computer. The data collection software features a graphical user interface using pull-down menus allowing the data to be downloaded to a notebook computer for further processing.

Once the field observations were completed, initial screening of the input data was performed and the data further processed by applying corrections to the observations for atmospheric conditions, applicable calibration parameters, and by performing geometrical reductions to the project coordinate system. The data was then prepared in a format for the subsequent least squares adjustment and statistical analysis.

During the epoch 7 deformation survey, the Leica DI2002 precision EDM was used to measure distances between the points established for the deformation reference network. The manufacturer's stated accuracy for this instrument is  $\pm 1$  mm + 1 ppm which exceeds the deformation monitoring program design specifications.

The Leica DI2002 EDM was calibrated for zero error before and after the epoch 7 deformation survey. Significant zero errors, if left undetected, will introduce systematic errors in the results of the separate epoch adjustments and distort the subsequent least squares and deformation analyses. Taking the average of the two determinations

produced a zero error of only  $-0.00017 \text{ m} \pm 0.00038$  metres. Thus the average zero error is statistically insignificant, therefore no corrections were required.

### **Network Adjustment Results**

The survey campaigns were subjected to a single epoch least squares adjustment and statistical analysis. The purpose of the single epoch adjustment was to: (1) screen the data for gross errors; (2) account for systematic errors either through the adjustment mathematical model or during the preprocessing stage; and (3) estimate the *a priori* accuracy of the observations. The quality of the data obtained in deformation surveys must be known in order to avoid a misinterpretation of systematic errors or outliers in the observations as a deformation.

Two-dimensional separate epoch adjustments for the horizontal deformation reference network were performed for two primary reasons: (1) to screen the data for outliers, and (2) to provide the data necessary (adjusted coordinates and their variance-covariance matrix) for the reduction of the zenith angles to their mark-to-mark equivalent. All field observations must be reduced to their mark-to-mark equivalents before the three-dimensional least squares adjustment can be performed.

The mark-to-mark reduction accounts for the differences in target and EDM reflector heights used during the various field surveys and reduces the observations to a common datum prior to the adjustment. For the reference pillars, the mark-to-mark reference point is defined as the top center (in 3-D space) of the Wild centering socket installed in the concrete pillar. The mark-to-mark reference point for the object points are defined by the center (in 3-D space) of the Wild GZT1 targets which are placed on the Wild GRT10 centering stems embedded in the crest and slope monuments during the surveys.

Results from the separate epoch least squares adjustment were then obtained for the three-dimensional network using a minimally constrained adjustment. Statistical tests carried out on the results of the separate epoch adjustment passed at the 95% confidence level. Horizontal and vertical positional accuracies for the reference and object points met or exceeded the accuracy specifications required to satisfy the detection of a  $\pm 3.0$  mm displacement at the 95% confidence level in both the horizontal and vertical directions.

### **Stable Point Analysis and Deformation Modeling**

The deformation analysis methodology utilized in the deformation studies of the dam are based on a generalized method for the analysis of deformation surveys as outlined in Figure 3. This method was developed by the Department of Surveying Engineering at the University of New Brunswick (UNB) in Canada.

The generalized method is based on a least squares fitting of selected deformation models to the displacement field obtained from repeated observations of deformations at discrete points on a deformable body. The choice of the best model is based on the statistical significance of its estimated parameters, the number of parameters and simplicity of the model, and the model's physical appropriateness.

## Reference Point Stability Analysis

When performing a stable point analysis, the initial assumption is made that all the points in the reference network must be treated as potentially unstable. In the stable point analysis only the reference points are considered. Generally the object points are not included since the assumption is that they are unstable anyway, reflecting the movement of the deformable object. The analysis is based on a comparison of two epochs (deformation surveys). A two epoch comparison was performed for the dam using epoch 6 vs. 3 and epoch 7 vs. 6. The Iterative Weighted Similarity Transformation (IWST) technique, developed by the Department of Surveying Engineering at UNB, was used for the stable point analysis.

The comparison of epoch 6 vs. 3 indicates that only pillars P3 and P4 were stable. The comparison of epoch 7 vs. 6 appears to indicate that none of the reference pillars remained stable. Pillar P4 was reconstructed before the epoch 7 deformation survey and thus has been moved from its original position. The remaining pillars continue to exhibit instability in the vertical direction, as was evidenced by the 1986 - 1987 deformation survey data. This would appear to indicate that dry shrinkage of the concrete continued, especially in the case of pillars P1, P2, and P5 which are anchored directly into bedrock. Pillar P3 was placed in a glacial deposit, extending down to a depth of approximately 5 ft below the ground surface. The vertical instability of P3, however, was not as pronounced as that of the remaining pillars. The horizontal and vertical movements are typically less than 10 millimetres.

Reasons behind the instabilities of any of the reference pillars are unknown at this time. The movement may be due to localized changes in the subsurface soil column caused by varying soil water content in the case of pillar P3. Terrain movements caused by changes in the water level of the reservoir could be one explanation. Reference points may experience random horizontal or vertical movements after they are initially constructed and even after many years of use. The vertical movements are most likely due to dry shrinkage of the concrete. Due to the long (8 year) separation between epoch 7 and 6, it is difficult to tell if the dry shrinkage has decreased. Deformation surveys performed on a more regular basis could help to explain the pillar movements.

The results from the stable point analysis of the reference pillars clearly demonstrates the importance of performing such an analysis. Without such an analysis, the casual user may have blindly accepted the reference points as fixed, thus providing a distorted picture of the actual movement of the structure. At this point it was not necessary to define the datum with respect to the reference points since so few of the points remained stable between epochs. The next step was to proceed with the modeling of the deformations.

## Deformation Modeling

The deformation modeling was accomplished by performing a least squares fitting of selected deformation models to the displacements of discrete deformations of the dam. *A priori* knowledge on the expected deformations was provided by the deformation modeling results obtained during the 1986 -1987 deformation monitoring program. Expected movements to consider were settlement or uplift, and downstream or upstream movement of the dam.

Deformation models can be expressed in terms of single-point displacements, rigid body displacements, homogeneous strain, or as a combination of these types of deformations. It was decided to consider the dam as a non-continuous deformable body with only single-point displacements of the object points. This presumption would

appear to represent the structural nature of a rockfill embankment dam rather well. Similar to the reference point stability analysis, deformation models are considered based on a two epoch analysis of observed deformation survey campaigns, namely epoch 6 vs. 3 and epoch 7 vs. 6.

The goal was to perform a three-dimensional (3-D) modeling of the dam deformations by generating a geometrical description of the deformations, combining the vertical (1-D) and the horizontal (2-D) components simultaneously. The approach is based on considering any stable reference points as a stable block or blocks. All the object points were treated as separate blocks or zones undergoing rigid body motion with respect to any block or blocks of stable points in the network.

A displacement function  $d(x, y, z)$  was given for each point describing the horizontal deformations of the whole block, and thus all the points contained in the block. Following the generalized method for the analysis of deformations, the coefficients of the deformation models were estimated through the least squares fitting of the polynomial approximations of the displacements of each point or group of points. A statistical assessment of the appropriateness of the model and significance of the estimated model coefficients was then performed. Approximately 12 deformation models were analyzed for each epoch pair comparison.

The geometrical analysis of the object points located along the downstream slope of the dam indicates an overall trend of movement in the vertical downslope and horizontal downstream direction. The displacement vectors clearly delineate the movements of the downstream edge of the crest and the surface of the downstream rockfill shell.

Comparing the epoch 7 (1995) deformation survey with the previous epoch 6 (1987) survey, average rates of change for the downstream and settlement components were calculated. Crest monuments (except A3, B3 and C3) showed an average settlement annual rate of change of 2.5 mm/yr. ( $\pm 0.6$  mm/yr.). Crest monuments (A3, B3 and C3) and all downstream slope monuments displayed a downstream average annual rate of change of 4.1 mm/yr. ( $\pm 1.8$  mm/yr.) and a settlement average annual rate of change of 5.3 mm/yr. ( $\pm 2.1$  mm/yr.).

The values in parentheses are the standard deviations of the rates of change, which in this case are low, indicating that the rate of movement is fairly constant along each row of object points. Between epoch 7 and epoch 6, the crest and downstream slope monuments have downstream displacements ranging from 1.1 mm (0.004 ft) to 59.8 mm (0.20 ft) and settlement from 14.2 mm (0.05 ft) to 68.6 mm (0.23 ft). The three-dimensional displacement vectors for all the object points on the dam ranged from 16.3 mm (0.05 ft) to 95.0 mm (0.31 ft), primarily in the downstream/downslope direction.

It is important to mention that as robust as statistical methods are, they are used as a guide to assist the professional analyst in establishing the most realistic deformation model. The choice of the best model is based on the statistical significance of its estimated parameters, the number of parameters and simplicity of the model, and the model's physical appropriateness.

Several plots were produced to graphically display the displacements showing trends, magnitudes, and rates of deformation. Cross section plots of the dam along each row of object points clearly show movements of the downstream edge of the crest and the rockfill shell. Displacements of the object points located along row B are shown in Figure 4 and are representative of displacements for rows A and C plotted on a cross-section of the dam crest and downstream slope.

## Conclusions

A geodetic deformation monitoring program has been designed and successfully implemented to determine the deformations of the Ball Mountain Dam near Jamaica, Vermont. Coler & Colantonio, Inc. performed the epoch 7 deformation survey, evaluated the deformation survey field observation data, performed a stable point analysis on the reference network points, and performed a geometrical analysis on the deformations of the dam.

It has been demonstrated that the geodetic surveys, and the subsequent deformation analysis performed for the Ball Mountain Dam, continue to clearly identify the spatial and temporal distribution of the object and reference point displacements. Implementation of the three-dimensional capability of the UNB Generalized Method of deformation analysis has reinforced the importance of performing a stable point analysis for the reference points, and has realized the goal of identifying and localizing movements exceeding  $\pm 3.0$  mm, at the 95% confidence level, in the horizontal and vertical directions.

Table 1 Dates and duration of deformation survey campaigns.

<i>Epoch</i>	<i>Deformation Survey Dates</i>
1	1986 07 14 - 1986 07 20
2	1986 09 24 - 1986 09 28
3	1986 11 18 - 1986 11 24
4	1987 05 05 - 1987 05 08
5	1987 07 21 - 1987 07 23
6	1987 09 29 - 1987 10 02
7	1995 10 04 - 1995 10 08

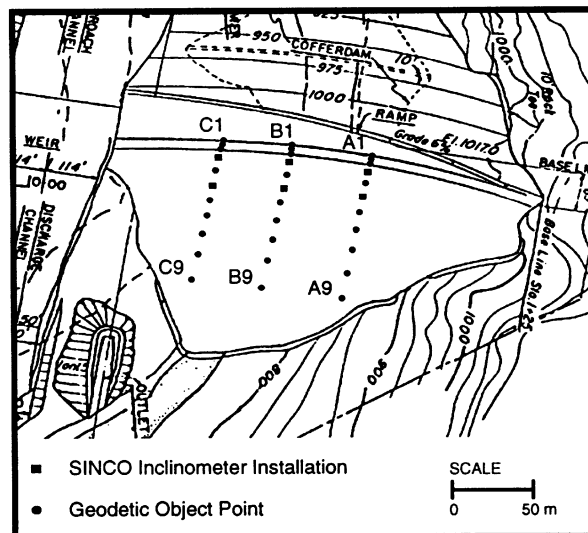


Figure 1 Inclinerometer and object (monitoring) point locations on downstream slope.

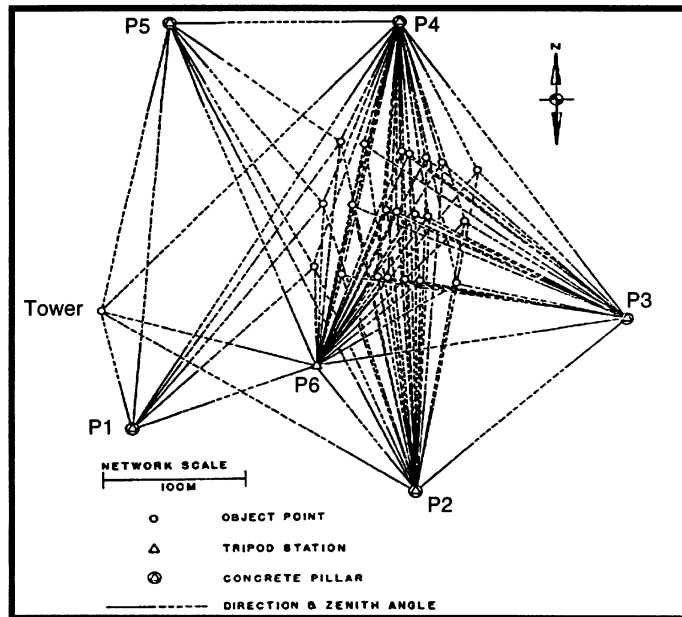


Figure 2 Ball Mountain Dam deformation monitoring network design.

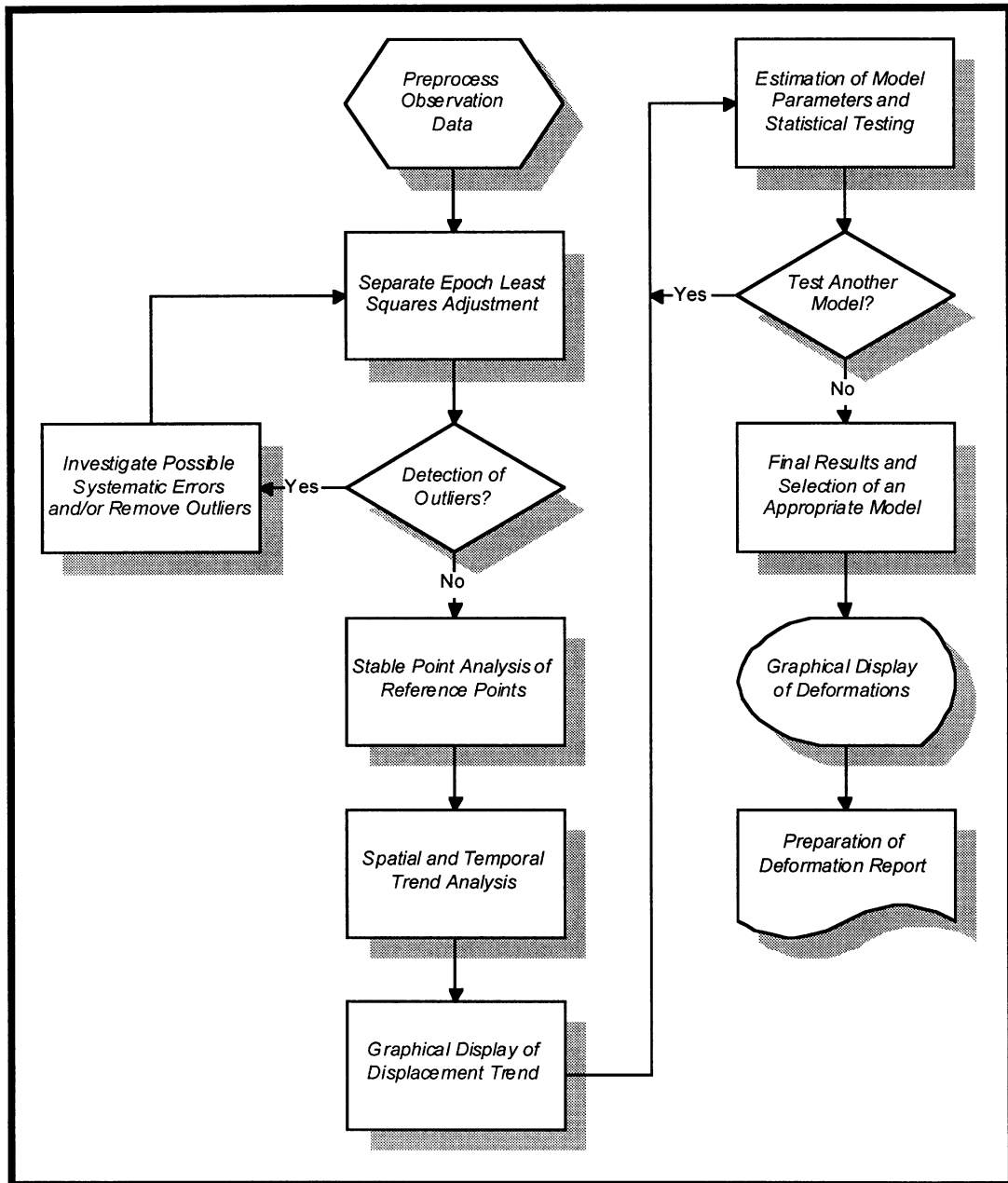


Figure 3 Flowchart of the generalized method for the analysis of deformations.



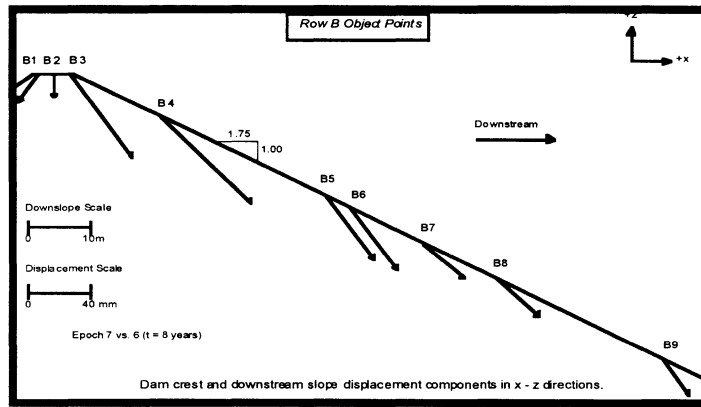


Figure 4 Typical modeled displacements from epoch 7 versus 6 comparison.

# Effects of Open Pit Mining on Gravity Changes

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## Abstract

Relocation of rock masses and resulting rock strata deformations in mining areas may produce significant local changes of the earth's gravity field. Finite Element Method (FEM), supported by FEMMA software, has been applied in a simultaneous determination of changes of gravity and deflections of the vertical caused by the relocation of mass, changes of density, and displacements of observation points. The method was used in modelling gravity changes in a large open pit mine in Poland. Gravity changes up to 18 mGal and changes of the vertical up to 4" (sec.of arc) can be expected in the mining area. The knowledge on gravity changes is important for correcting geodetic surveys of high precision and in identifying any abnormal behaviour of rock strata.

## Introduction

Relocation of rock masses due to mining activities may produce significant local changes to the gravity field in the vicinity of the excavated areas, stockpiled ore, and dumps of waste rocks. Besides the influence of the relocation of rock masses, the mining operation usually triggers rock strata deformations resulting in additional gravity changes due to density changes of the rock masses and displacements of observation points. The gravity changes result in local tilts of the level surface (equipotential surface of gravity) and, consequently, changes of the direction of the plumb lines to which the majority of geodetic and some geotechnical measurements are referenced. Thus, if any geodetic measurements of high precision are required during the mining operation, for example, for the purpose of monitoring the stability of surface structures, they should be corrected for changes of gravity and deflection of the vertical as a function of time. Otherwise, differences in the repeated observations (angles, distances, gyroazimuths, height differences, tilts, etc.) caused by the gravity changes could be misinterpreted as deformations of the observed object. In addition, by comparing the expected (modelled) gravity changes with the values observed with precision gravimeters, one may gain information on the behaviour of the rock masses disturbed by mining activity. The latter may give useful information on the existence of discontinuities in the rock masses and a possible presence of old mining openings. Various aspects of the mining 'microgravimetry' are discussed in Fajklewicz (1980).

The calculation of gravity changes involves solving complex integrals (summation of influences of point masses). The theory of gravity and various numerical methods used in

the gravity calculations, are given in textbooks on geodesy, for example Vanicek and Krakiwsky (1986); Torge (1991); Heiskanen and Moritz (1967).

The Engineering and Mining Surveying Research Group at the University of New Brunswick, led by Dr. A. Chrzanowski, has developed, within a broader research program on modelling and prediction of deformations in engineering and geosciences, a method for numerical modelling of gravity changes caused by the simultaneous effects of mass relocation and rock deformations. The method utilizes the finite element analysis supported by FEMMA (Finite Element Method for Multipurpose Applications) software developed at UNB (Szostak-Chrzanowski, 1988). The method has been used, among other uses, in modelling regional deformations and gravity changes of tectonic origin (Szostak-Chrzanowski et al., 1993) and, recently, has been applied to modelling expected gravity changes in a large open pit mine, Belchatow, in Poland. Results of the latter application are given in this paper preceded by a brief review of the basic principle of gravity calculations and finite element modelling of gravity changes.

## Gravitational Forces and Gravity Acceleration

According to Newton's law of gravitation, two points with masses  $m$  and  $m_p$  attract each other with a gravitational force

$$f=(Gm/R^2)m_p \quad (1)$$

where  $G$  is Newton's gravitational constant given as  $G= 6.67259 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$  and  $R$  is the distance between the two points. Introducing a Cartesian coordinate system  $X,Y,Z$ , and denoting the coordinates of the attracting mass  $m$  by  $x', y', z'$  and the coordinates of the attracted point  $P$  by  $x,y,z$ , one can calculate the components of the gravitational force vector  $\mathbf{F}$  (Fig.1) of magnitude  $f$  from

$$\begin{aligned} f_x &= - f \cos\alpha = - [G m (x-x') / R^3] m_p \\ f_y &= - f \cos\beta = - [G m (y-y') / R^3] m_p \\ f_z &= - f \cos\gamma = - [G m (z-z') / R^3] m_p \end{aligned} \quad (2)$$

where

$$R=[(x-x')^2+(y-y')^2 +(z-z')^2 ]^{1/2} \quad (3)$$

Since force is a product of mass and acceleration (Newton's second law), the terms in brackets of eqs.(2) are components of the gravitational acceleration or, briefly, gravitation.

In the case of a solid body with volume  $v$  and density  $\rho$ , the total gravitational force acting on point  $P$  (Fig. 2) with unit mass ( $m_p = 1$ ) will be calculated as a sum of influences of individual point masses  $dm$  of volume  $dv$  over the entire volume  $v$ . Assuming that the point masses are distributed continuously over the volume  $v$  with density

$$\rho=dm/dv \quad (4)$$

the components of the gravitational acceleration at point  $P$  will be calculated from

$$a_x = -G \rho_1 \int_V \frac{(x - x')}{R^3} dv ,$$

$$a_y = -G \rho_1 \int_V \frac{(y - y')}{R^3} dv ,$$

and

$$a_z = -G \rho_1 \int_V \frac{(z - z')}{R^3} dv ,$$

(5)

where  $dv = dx' dy' dz'$ .

A point of unit mass rotating with the earth is acted upon by *gravity*,  $g$ , which is the resultant of the gravitational forces of the earth and of other celestial bodies, as well as the centrifugal force due to the earth's rotation. The gravity (called also gravity acceleration) is expressed in *Gals* ( $1 \text{ Gal} = 1 \text{ cmsec}^{-2}$ ). The average value of  $g$  on the earth surface is approximately 981 Gals. With the present technology, the gravity changes can be measured with an accuracy of  $50 \mu\text{Gal}$  or better, using portable gravimeters.

## Calculation of the Local Changes of the Earth's Gravity

### Gravity Change due to Mass Relocations

The aforementioned relocation of rock masses and ground deformations due to mining activities change the earth's gravitational force, while other components of the earth's gravity can be considered as constant. Thus, one may determine the local gravity change,  $\Delta g$ , caused by the relocation of rock masses, as equal to the gravitational acceleration produced by the removed (excavated) or brought-in (stockpiled) rock masses of the given density and given geometry. Thus, for example, the  $x$  component of the gravity change,  $\Delta g_x$ , can be obtained from

$$\Delta g_x = a_x \tag{6}$$

where  $a_x$  is calculated using eqs.(5).

For a convenient interpretation of the gravity changes, the axis  $z$  of the local coordinate system is oriented in the vertical direction (direction of the plumbline) at the point at which the magnitude of gravity is calculated.

### Gravity Change due to Density Change

The change of density may be caused by a change in the state of stress, resulting, for example, from the deformation of the rock strata due to mining activity.

Let us assume that due to a change in the stress field in individual elements of the rock masses, the mass of the rocks remains constant but volumes of the elements change. The assumption of constant mass is expressed by

$$\rho_1 v_1 = \rho_2 v_2 \quad (7)$$

where  $\rho_1$  and  $\rho_2$  are the densities and  $v_1$  and  $v_2$  are volumes before and after the deformation, respectively. The new density may be expressed as

$$\rho_2 = \rho_1 \frac{v_1}{v_2} \quad (8)$$

Using eqs. (5), (6), and (8), one may calculate the gravity changes. For example, the z component of the gravity change,  $\Delta g_z$ , is

$$\Delta g_z = -G \rho_1 \frac{v_1}{v_2} \int_{v_2} \frac{(z - z')}{R^3} dv. \quad (9)$$

### Free Air and Topographic Corrections

Ground deformation caused by the mining activity may produce subsidence or uplift of the physical point at which the local gravity changes are to be determined.

A vertical displacement of the observed point changes its distance to the earth's centre of mass which changes the vertical component of gravity at the point. The increment of gravity due to the change of height is calculated approximately from (Torge, 1991; Vanicek and Krakiwsky, 1986):

$$dg_A = -b dH \quad (10)$$

where  $b$  is a gravity gradient equal to  $-0.3086$  mGal/m (here, one should not confuse  $m$  for metre with  $m$  for milliGal), and  $dH$  is the height change in metres along the plumbline. The correction (10) is called the *free air correction*.

In addition to the free air effect, the uplift or subsidence of the point produces a relative relocation of the topographic masses and, consequently, a change in the value of gravity. Therefore, eqs. (5) and (6) could be used for calculations of the gravity changes by taking a topographic area of a sufficiently large diameter around the investigated point for the integration of the effects. The topographic correction is expected, however, to be very small in comparison with the effects of the mass relocations due to mining extraction. Therefore, the topographic correction can be approximated to the first order by the so-called Bouguer plate reduction  $dg_B$ , which corresponds to the gravitation of a horizontal plate with infinite extension and the change of height  $dH$ . The Bouguer plate reduction  $dg_B$  is calculated from (Torge, 1991):

$$dg_B = 2 \pi G \rho_o dH \quad (11)$$

where  $\rho_0$  is the average density of the rocks surrounding the point of calculations. The total change of gravity is calculated from the sum of gravity changes and the free air and Bouger plate corrections.

### **Calculations of Deflection of Vertical**

Changes in the horizontal components,  $\Delta g_x$  and  $\Delta g_y$ , of gravity, produce a change in the direction of the plumbline at the point of interest. This can be calculated (in radians) from

$$\begin{aligned}d\alpha_x &= \Delta g_x / g_0 \\d\alpha_y &= \Delta g_y / g_0\end{aligned}\tag{12}$$

where  $g_0$  is an appropriate value of absolute gravity at the point. For all practical purposes, the average value of  $g_0 = 9.81 \text{ m s}^{-2} = 981 \text{ Gal}$  can be taken for the calculations.

## **Numerical Modelling of Gravity Changes**

### **Use of the Finite Element Method**

Two calculation problems are involved in the analysis of gravity changes: one is the solution of the integrals in eqs. (5) in the case of complicated geometry of the relocated mass; and the other is in the determination of the change of geometry (deformation) and resulting change in the density of the attracting object and displacements of the attracted point masses (points of observation). Although several numerical methods have been suggested by other authors for solving the integrals involved in eqs. (5) and (9), no method has been put forward addressing the simultaneous solution for the effects of the deformation. Use of the finite element method (FEM) permits the simultaneous solution by adapting the numerical solutions of integrals inherent in the FEM (involved in the calculation of the stiffness matrix of the investigated material) and by performing the load-deformation analysis on the basis of a known stress-strain relationship and geomechanical properties of the investigated material (Szostak-Chrzanowski and Popielek, 1995, 1996; Szostak-Chrzanowski et al., 1995).

The total gravity change caused by a change of density is calculated as the sum of effects of density changes in each element of the finite element model using eqs. (9). The volumes  $v_1$  and  $v_2$  of each element are calculated from the initial and after-the-deformation (obtained from the stress-strain finite element analysis) coordinates of the nodal points.

### **FEMMA Software**

FEMMA, Finite Element Method for Multipurpose Applications software, can be applied to either two- or three-dimensional problems in the analysis of elastic, linear-elastic, and visco-elastic deformations (Szostak-Chrzanowski and Chrzanowski, 1991), heat transfer, and rigorous propagation of random errors of the input data (Szostak-Chrzanowski et al.,

1994). The analysis of gravity changes has been added as an optional 3-D version in which the Gaussian quadrature rule is applied to solve the integrals of eqs. (5) and (9).

FEMMA uses brick elements with 8 nodal points in a 3-D analyses. The subprogram MESHGEN generates the mesh.

### Gaussian Quadrature Rule

To calculate the integral in eqs. (5), the Gaussian quadrature rule is used (Zienkiewicz and Taylor, 1989).

Let the function under the integral from eqn. (5) be represented by  $F(x,y,z)$ . Then, for example, for the  $z$  component of the gravitation, we have

$$\int_v F(x,y,z) dv = \int_v \frac{(z-z')}{R^3} dv. \quad (13)$$

By introducing local coordinates  $\xi$ ,  $\eta$ , and  $\zeta$  in each element, one can write:

$$\begin{aligned} & \iiint F(x,y,z) dx dy dz = \\ & = \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 G(\xi,\eta,\zeta) \det[J] d\xi d\eta d\zeta \end{aligned} \quad (14)$$

where  $\det[J]$  is the determinant of Jacobian matrix. The local coordinates are chosen as

$$-1 \leq \xi \leq 1, \quad -1 \leq \eta \leq 1, \quad \text{and} \quad -1 \leq \zeta \leq 1.$$

The determinant of matrix  $J$  is a magnification factor (scale factor) that yields:

volume =  $dx dy dz = \text{Det}J d\xi d\eta d\zeta$ . Using the Gaussian quadrature rule, one can write:

$$\begin{aligned} & \iiint_{-1}^1 G(\xi,\eta,\zeta) \det[J] d\xi d\eta d\zeta = \\ & = \sum_{m=1}^n \sum_{j=1}^n \sum_{i=1}^n W_i W_j W_m \det[J] G(\xi_i, \eta_j, \zeta_m) \end{aligned} \quad (15)$$

where  $W_{i,j,m}$  are weights. In order to evaluate matrix  $G$ , a transformation of the coordinates is necessary. The coordinate transformation is established using standard shape

functions given in terms of local coordinates as explained, for example, in Zienkiewicz and Tylor (1989).

## Example of Gravity Changes in an Open Pit Mine

### Belchatow Open Pit Brown Coal Mine

The presented method and FEMMA software have been used in calculating expected gravity and deflections of vertical changes at Belchatow, an open pit mine in Poland. The purpose of the calculations was to learn about the approximate magnitude of gravity changes which could be expected in a large mining operation and to compare the results with an earlier analysis performed by Sas (1993).

Belchatow is one of the largest open pit mines in the world producing brown coal. It started mining production in 1975. Figure 3 shows the approximate geometry and dimensions of the excavated area and of the waste rock (overburden) dump which are 2.5 x 2.6 km with the depth of 0.4 km and 2.7 x 2.7 km with a depth of 0.17 km, respectively. Table 1 shows the geomechanical parameters of the main components of the rock strata. Both the dimensions of the mine and the geomechanical data have been taken from Sas (1993) who calculated the gravity changes caused by the mining operation. In his calculations, Sas used a conventional method based on subdividing the excavated area and dump into simple geometrical blocks to facilitate analytical solutions of the integrals. His method could consider only a simplified geometry of the mine and only the influence of the mass relocation, neglecting the deformation effects.

**Table 1.** Geomechanical parameters of the rock mass.

Rock strata	Young Modulus [GPa]	Poisson ratio	Density [gcm <sup>-3</sup> ]
brown coal	1.0	0.3	1.39
crushed rock & sand	0.9	0.3	1.60

In order to compare the FEMMA results with the results obtained by Sas, the same simplified geometry of the mine was used in the finite element analysis. Two analyses were performed. The first, preliminary analysis, was to estimate the approximate magnitude of influences of the mass relocation versus influences of the rock deformations (density changes and displacements of surface points). The second, the final analysis, was to perform calculations with a refined FEM mesh and using only those affecting factors which proved, in the preliminary analysis, to be dominant.

In the preliminary analysis, the whole area of the investigated rock mass was modelled as a block with dimensions 12 500 m x 12 000 m and 970 m high divided into 332



elements with 519 nodal points. The finite element analysis of the gravity changes included effects of:

- 1) change (relocation) of mass,
- 2) change of density, and
- 3) change of the heights of the surface points.

Table 2 shows a sample of results at surface points 2, 3, and 18 (Fig. 3) for the various effects. As one can see, in this particular example, the relocation of mass is the dominating effect accounting for over 90% of the calculated gravity changes. Therefore, the final analysis was performed considering only the effect of the mass relocation. This provided also a better comparison with the results obtained by Sas.

**Table 2.** Results of the preliminary analysis of various effects.

Point No.	Gravity change [mGal]		
	caused by mass relocation	caused by mass reloc.+ density ch.+ free air red.	caused by mass reloc.+ density ch. + free air + plate
18	- 0.64	- 0.62	- 0.60
2	- 15.05	- 14.99	- 14.86
3	- 13.71	- 13.64	- 13.52

In the final analysis, the gravity change was calculated using eqs. (5). In the FEM model, the open pit was divided into 164 elements and the dump of the overburden rocks into 153 elements. The gravity changes were calculated on the surface for an area of 12500 m x 12 000 m. The model had a total of 5385 nodal points. Figures 4 and 5 summarize results of calculations of the total and vertical ( $dg_z$  component) gravity changes, respectively, along cross section A-A (Fig.3). Figure 5 shows also a comparison with the gravity changes calculated by Sas. A very good agreement has been obtained between the two methods of calculations. The total gravity changes, caused by the relocation of rock masses reached - 18 mGal .

The calculated components  $\Delta g_x$  and  $\Delta g_y$  of gravity changes have been used to calculate components of the deflection of vertical using equations (12). Figure 6 shows the  $d\alpha_y$  components of the changes of the deflection of vertical (in seconds of arc) along the cross-section A-A as calculated with FEMMA and by Sas (1993). Again, the agreement is very good.

## Conclusions

The presented method of modelling gravity changes allows for a simultaneous determination of the changes caused by relocation of masses and by deformation of the rock strata. The method is applicable to any geometry of the investigated effects (attracting masses). A good agreement with conventional calculations of gravity changes, for a simple geometry of the relocated masses, has provided a verification of the results obtained with the FEMMA software.

The presented example of effects of a large open pit mine indicates that mining activities may produce very significant gravity changes (several milliGals) and changes of the deflection of the vertical in the order of several seconds of arc.

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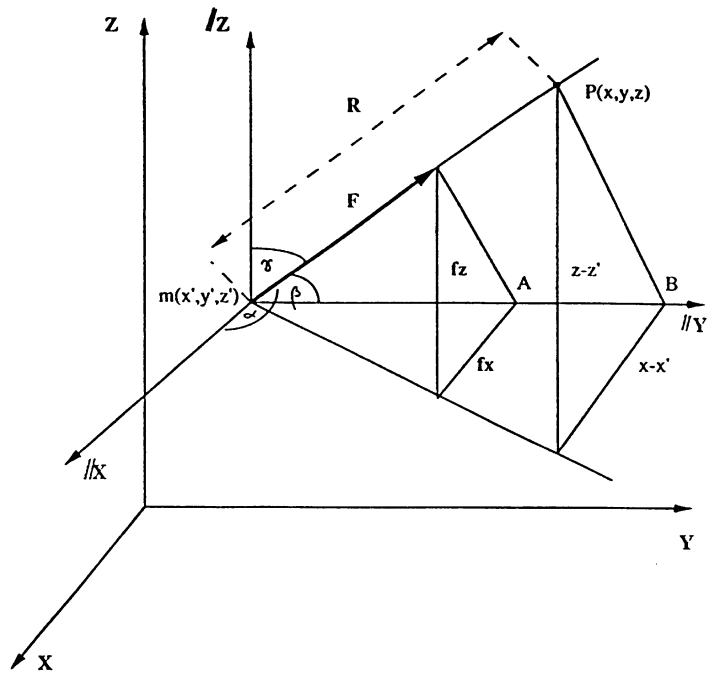


Fig. 1. The components of the gravitational force [after (Heiskanen and Moritz, 1967)]

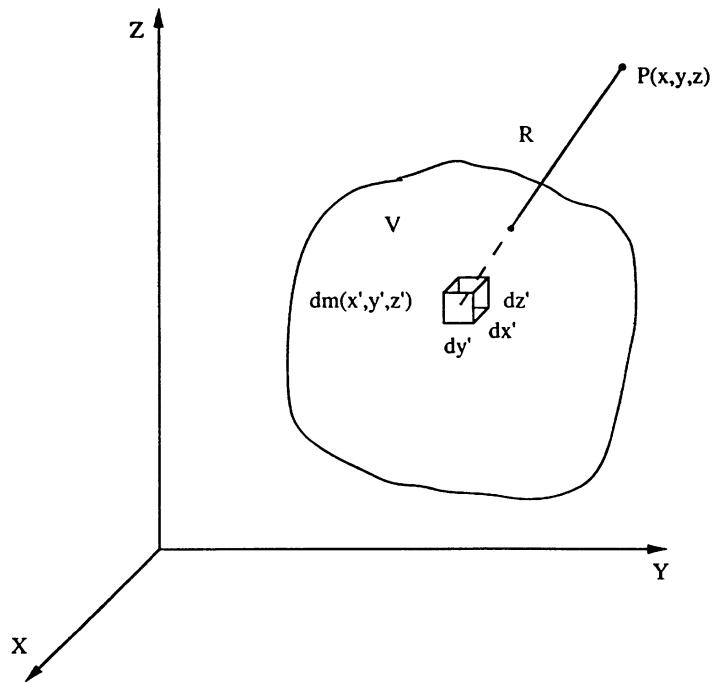
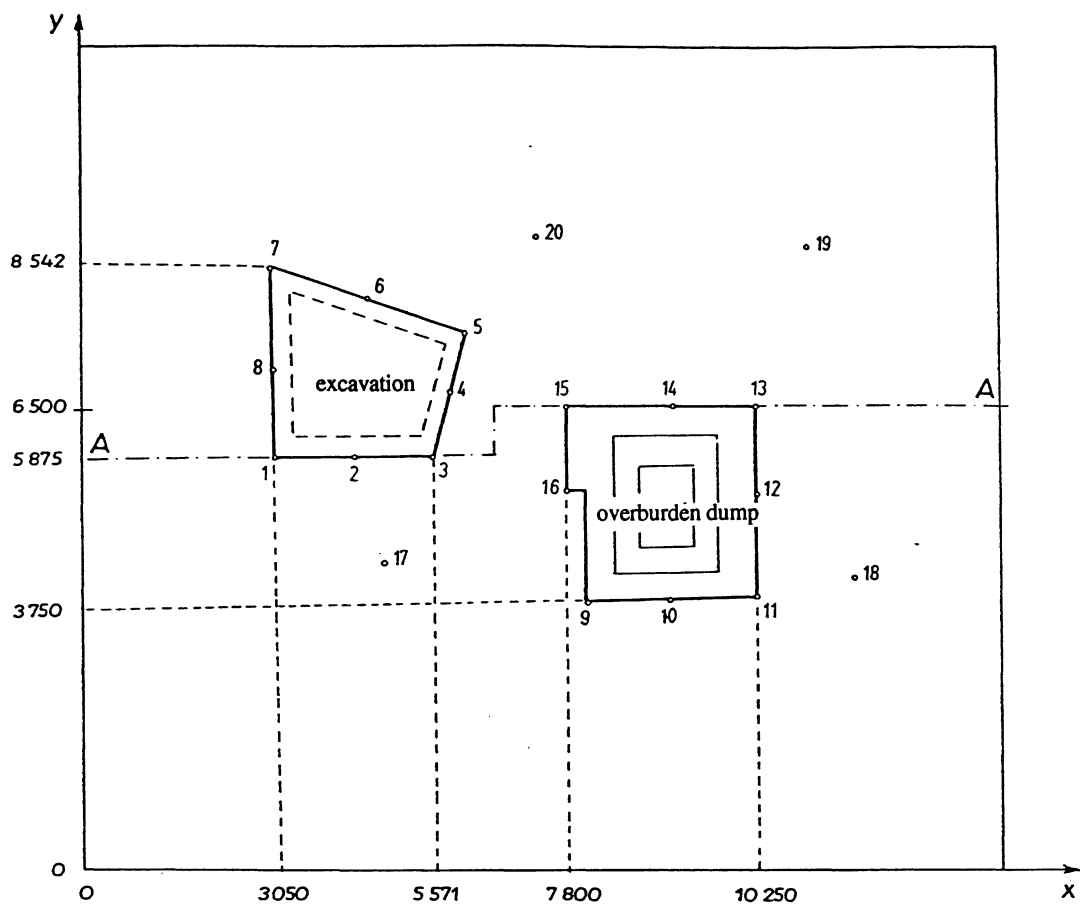


Fig. 2. Attraction of point P of unit mass by a point mass  $dm$  of a solid body [after (Heiskanen and Moritz, 1967)]



Vertical Cross-section

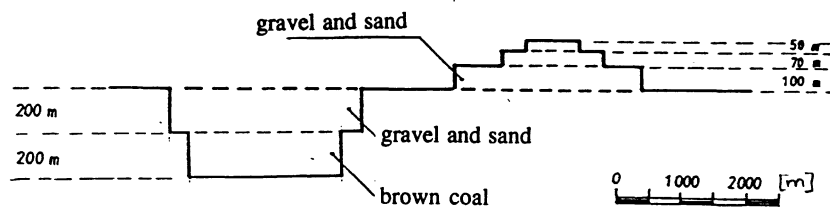


Fig. 3. Belchatow open pit mine (simplified geometry)

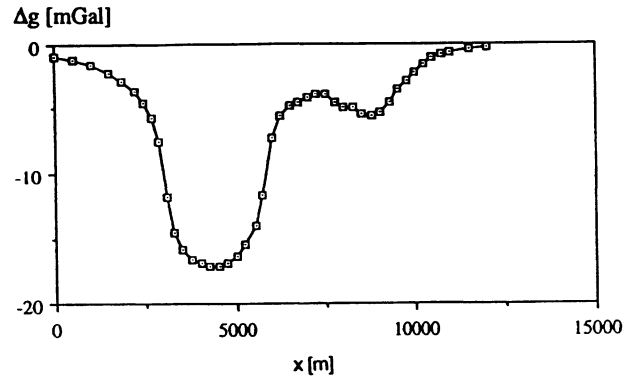


Fig. 4. Total change of gravity on the surface in A-A cross-section

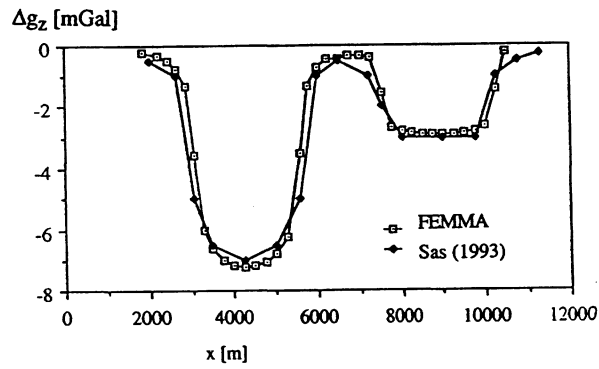


Fig.5. Vertical component of the gravity change on the surface in A-A cross-section

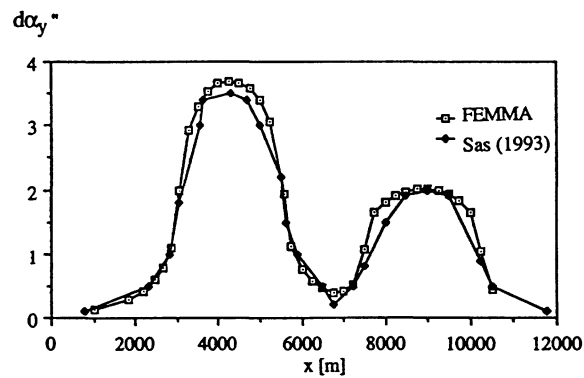


Fig. 6. Change of the deflection of vertical (y component) on the surface in A-A cross-section

# Instrumentation and Methodologies for High-Precision Industrial Surveys

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## Abstract

High-precision industrial surveys are required for a number of tasks including precise setting out, deformation monitoring, and setting and monitoring alignment changes in operating machinery. For those tasks which require three-dimensional coordinates or coordinate changes (movements), the theodolite intersection method, the total station method, or the single-theodolite triangulation method can be used. For tasks that require three-dimensional orientations or rotations, the autoreflexion method can be used. These methods are described, along with related systematic errors and how these errors can be controlled.

## Introduction

High-precision industrial surveys are those in which the requirement for relative precision among adjacent one-, two-, or three-dimensional survey points is normally in the order of a few tens of microns to a few millimetres. Some tasks that require high-precision industrial surveys are:

- (a) *Precise setting out for prebuilt components.* Examples include manufacture and assembly of aircraft and motor vehicles, construction of bridges, and construction of buildings.
- (b) *Precise setting out for long tunnels.* Examples of recently constructed long tunnels include the Channel Tunnel between England and France and the Superconducting Supercollider Tunnel in Texas (Greening et al. 1993).
- (c) *Deformation monitoring.* Examples include monitoring of slide areas, tectonically active areas, dams, open pit mines, bridges (Teskey and Obidowski 1998), buildings (Teskey 1998; and Teskey and Biacs 1991), and pipelines (Simmonds et al. 1993; Teskey et al. 1992).
- (d) *Setting alignment and monitoring alignment changes in operating machinery.* The alignment can either be coaxial (e.g. alignment of rotating shafts of two coupled

machines) or parallel (e.g. alignment of rolls in a paper mill). Alignment changes in operating machines are due to temperature changes (thermal growth) and changes in operating forces (Teskey et al. 1996).

In the following sections, methods of achieving the tens of microns to millimetres levels of precision are described. Some important systematic errors are also described, along with how they can be controlled.

## **Three-Dimensional Coordinates and Coordinate Changes**

### **Theodolite Intersection Method**

The standard method of obtaining three-dimensional coordinates or coordinate changes (movements) in high-precision industrial surveys is by theodolite intersection in which horizontal circle and vertical circle readings are observed. In this method, one, two or more theodolites can be used; a typical setup is shown in Figure 1.

The simplest solution approach is to compute the (x, y) coordinates of the free stations by double resection (see, for example, Sebert 1996), then the (x, y) coordinates of the points on the object of interest (M1, M2, M3, M4; usually marked by flat paste-on targets) by intersection. The height coordinates (z-coordinates) of (M1, M2, M3, M4) can be computed from each free station. These computed values, for each point, will not be the same but can be used as an indication of the quality of the survey. For typical lengths of lines of observation (5m or less), one should expect differences to be less than 0.2 mm, and the average computed values can be adopted.

Other solution approaches include intersection of three-dimensional vectors from two “fixed” theodolite positions (Allan 1996), and three-dimensional least squares adjustment. In the first approach, average (x, y, z) values at the point of closest approach of the two vectors pointing to each target are adopted, as long as a point of closest approach tolerance (e.g. less than 0.2 mm) is satisfied. There is no real advantage of this method other than perhaps computational efficiency. The second approach can be justified when target points, which must be in the same coordinate system, cannot be observed from only two theodolite stations. In these situations, three-dimensional least squares adjustment is the most general and flexible solution approach (Wilkins et al. 1988; Bayly and Teskey 1992).

The advantages of theodolite intersection are that only relatively low-cost, readily available equipment is required (any 1 arc second theodolite, e.g. a Wild T2, is adequate), and high-precision results can be obtained (standard deviations of (x, y, z) coordinates in the order of 0.1 mm). The disadvantages of theodolite intersection are that it is time-consuming, and the level of precision decreases rapidly for longer standoff distances (greater than 5 m). One must also have good pointing between the two free stations shown in Figure 1. (This is not a requirement, however, for the least squares adjustment approach.) This can be accomplished by mutual pointing of two theodolite telescopes (aside from the requirement for two theodolites, it is difficult to achieve (Kissam 1962)), by use of a reversible target in a properly adjusted forced centering device, or by the use of a reference object to place the two free stations and the reference object on a straight line in the horizontal plane.

It is also important to note that for deformation surveys in which coordinate changes (movements) are of interest, the same scale must be reproduced each time the target points

are reobserved. This is accomplished through the use of a calibrated scale bar (the length of which is temperature corrected if necessary) to define the relative positions of points R1 and R2, or the reobservation of stable points, for example, points on the foundation supporting the object of interest (Teskey et al. 1995a).

### **Total Station Method**

In this method, only one total station (theodolite plus calibrated electronic distance measurement instrument (EDMI)) is required; a typical set up is shown in Figure 2. Note that reference points are not necessary since reproducible scale is provided by the EDM component of the total station. Reference points may be added if one is interested in the movement of points (M1, M2, M3, M4) relative to the reference points, but usually only relative movements among points (M1, M2, M3, M4) are of interest.

The solution for three-dimensional coordinates is straight forward: compute (x, y, z) coordinates of (M1, M2, M3, M4) in the total station system, then rotate the (x, y) coordinate system so that it has a convenient alignment, for example, the alignment with the line M1-M4 parallel to the x-axis. Three-dimensional least squares adjustment could also be used as a solution approach, but even with more complex observational scenarios its use may not be justified; consider two typical situations:

- (a) *Second free station setup from which (M1, M2, M3, M4) can be observed.* In this case, use the two independent computations of (x, y, z) coordinates to compute average (x, y, z) coordinates, as long as each coordinate difference (dx, dy and dz for each point) is less than a given tolerance, for example, 0.3 mm.
- (b) *Additional free station setups along an object of interest.* In this case, carry forward the (x, y, z) coordinate system by observing two optimally located common points (these common points could be temporary points set on tripods); complete a loop closure back to the original two common points; compute equal shift adjusted (x, y, z) coordinates, as long as (dx, dy, dz) values for the original two common points are less than a given tolerance, for example,  $0.3 \text{ mm} \times (\text{no. of setups} - 1)^{1/2}$ .

In the total station method, a target/reflector is required at the target point. There are two possibilities: separate target and reflector, or combined target and reflector. The second possibility is preferred because it speeds up the measurement task. One type of combined target and reflector is shown in Figure 3. The cutaway steel ball bearing (typically 1 inch (25.4 mm) diameter) is set in a magnetic mount (this allows the target and reflector to be pointed toward the total station); the magnetic mount is then screwed into a permanently installed bushing. One cutaway ball bearing and magnetic mount is moved from point to point.

Either a noncoaxial or coaxial total station can be used; however, the latter is strongly preferred in order to avoid errors that could be introduced in the geometric reduction of the EDM distance to the theodolite line of observation. The total station should have a 1 arc second or 0.1 arc second resolution for the horizontal circle and vertical circle measurements, and a 0.1 mm resolution for the EDM measurements; one instrument that satisfies these requirements is the Leica TC2002.

The advantages of the total station method versus the theodolite intersection method are that the total station method is very fast (including computation of results on-the-fly), and scale and relative precision among target points are both homogeneous. The disadvantages of the total station method versus the theodolite intersection method are that the



instrumentation is relatively expensive, and slightly less precise results are produced (standard deviations of  $(x, y, z)$  coordinates in the order of 0.2 mm with standoff distances of 5 m or less). In many applications, the advantages outweigh the disadvantages; as a result, the total station method has become the preferred method for high-precision industrial surveys (Greening et al. 1993; Teskey et al. 1995b; Teskey et al. 1997).

### **Single-Theodolite Triangulation Method**

In this method, only one theodolite is required. Scale information is provided by a scale bar moved from point to point. The advantages of this method are the same as those for the total station method, but at a much lower cost since only a theodolite rather than a total station is required. The disadvantage of this method is that precision decreases rapidly (especially for the  $(x, y)$  coordinates) at standoff distances greater than 10 m. An example of the application of this method is deformation monitoring of the New Brunswick-Prince Edward Island Confederation Bridge (Teskey and Obidowski 1998).

### **Three-Dimensional Orientations and Rotations**

Movements of discrete points on a deforming object can be completely described by three-dimensional translations  $(dx, dy, dz)$  in the  $(x, y, z)$  directions respectively (see Figures 1 for orientation of axes), and three rotations  $(rotx, roty, rotz)$  about the  $(x, y, z)$  axes respectively. Methods of measuring three-dimensional translations have been described in the previous sections. The measurement of three-dimensional rotations (changes of three-dimensional orientations from one measurement epoch to another) can be conveniently carried out by autoreflexion.

In autoreflexion, the condition which exists when a line of sight is normal to a reflecting surface (typically a mirror) is shown in Figure 4. The procedure used to achieve this condition with a theodolite can be summarized as follows: focus on the mirror first, then focus on the reflection of the autoreflexion ring around the telescope (the reflected image will be at twice the distance to the mirror; see Figure 4); translate and rotate the theodolite to achieve the autoreflexion condition.

With a levelled theodolite having a line of sight normal to a mirror, the zenith angle is a direct measure of the orientation of the mirror about an axis in the horizontal plane. A second mirror orthogonal to the first allows one to measure the orientation about an orthogonal axis in the horizontal plane. Further, if one resects the position of the theodolite from control points when the autoreflexion condition has been achieved to either mirror, the orientation about the  $z$ -axis can be measured. Finally, if all measurements are made in a second measurement epoch, rotations can be calculated by differencing the orientations from each epoch. If a permanently installed mirror is used, its three-dimensional orientation in a given epoch can be determined to a precision of a few arc seconds.

Further information on autoreflexion and a similar technique called autocollimation is given in Kissam (1962). Autoreflexion is used instead of autocollimation in industrial applications because the latter requires specialized equipment (an autocollimation eyepiece), cannot be achieved under normal to bright lighting conditions, and is very sensitive to vibrations (the blurred image cannot be seen).

Applications of autoreflexion or related techniques by the writer and his research colleagues include the following: measurement of rotations of buttressed columns in the Olympic Speedskating Oval (Teskey 1988), measurement of rotational deformations in

buried pipelines (Teskey et al. 1992), and measurement of alignment changes in operating rolls (Radovanovic 1997).

## **Systematic Errors and Their Control**

In the following sections it is assumed that the theodolite or total station is levelled (preferably with fine levelling by dual axis compensation), observations are made in both positions of the telescope and mean values are used in computations, the total station and target/reflector combination is calibrated, and meteorological corrections are applied to EDM distances.

### **Scale Errors**

If scale is determined by a scale bar, one should use a scale bar fabricated from a material with a very low coefficient of linear expansion, for example, invar which has a coefficient of linear expansion of less than  $10^{-6}/^{\circ}\text{C}$ . If a scale bar with a significant coefficient of linear expansion is used or if scale is determined by two reference points, one should estimate the scale effect caused by temperature change within or between measurement epochs, and apply a scale correction if the correction is significant.

As an example, consider two reference points separated by 3 m on a reinforced concrete foundation. The temperature change is  $10^{\circ}\text{C}$ . The coefficient of linear expansion of the steel reinforcement and the concrete are both equal to approximately  $10^{-5}/^{\circ}\text{C}$ . This corresponds to a scale change of  $10^{-5}/^{\circ}\text{C}/\text{m}$  or 10 ppm (parts per million)/ $^{\circ}\text{C}/\text{m}$ . For a temperature change of  $10^{\circ}\text{C}$ , the scale change is 100 ppm or 0.3 mm in 3 m.

### **Errors Caused by Instrument Temperature Changes**

Temperature changes in an instrument, e.g. the EDM component of a total station, can produce scale errors because the internal dimensions of the instrument change. The alignment of components will also change if temperature change is not uniform.

The best way of dealing with these effects is to not capture data with an instrument until after a “warm up” period. The manufacturer’s instructions will indicate the length of this warm up period. In some applications, it may also be necessary to calibrate the instrument at different temperatures.

### **Errors Caused by Atmospheric Refraction**

Refraction of all electromagnetic radiation is described mathematically by the differential form of Snell’s Law:

$$\frac{1}{R} = \sin \theta \frac{1}{n} \frac{dn}{dh}$$

with R ... radius of curvature of the path

$\theta$  ... angle between path and refractive index gradient

$n$  ... refractive index

$\frac{dn}{dh}$  ... refractive index gradient

Consider the following example:

$\theta = 90^\circ$  (maximum effect)

$$\frac{1}{n} \doteq 1$$

$$\frac{dn}{dh} \doteq 1 \times 10^{-5} \text{ m}^{-1} \text{ (temperature gradient of } 10^\circ\text{C/m)}$$

$$\Rightarrow R = 1 \times 10^5 \text{ m}$$

If this is applied to horizontal refraction on a 10 m line of observation,

$$\alpha = \frac{s}{R} = \left( \frac{10\text{m}}{1 \times 10^5 \text{m}} \right) \text{ radians} \times 206265 \text{ arc seconds/radian}$$
$$\doteq 20 \text{ arc seconds}$$

The angular deflection at one end of the 10 m line, i.e. the angular error caused by atmospheric refraction, is given by:

$$\frac{\alpha}{2} = 10 \text{ arc seconds}$$

To control lateral (horizontal) and vertical refraction effects when observing horizontal circle and vertical circle readings with a theodolite or total station one can do the following:

- (a) Avoid grazing lines, especially those close to the instrument.
- (b) Mix the air in the vicinity of the instrument with an industrial blower. This produces a neutral atmospheric condition ( $\frac{dn}{dh}$  is minimized) versus stable (horizontally layered air masses) or unstable (warm air masses rising) atmospheric conditions.

### **Errors Caused by Obstruction in the Field of View**

If the field of view from the telescope to the target point is obstructed by an object not symmetrical about the line of observation, an error is produced due to the fact that the centre of concentration of rays from the target is shifted off the optical centreline of the telescope (Kissam 1962).

This error may be significant for horizontal circle and vertical circle readings with a theodolite or total station. As an example, the writer has experienced a 0.5 mm height error with a standoff distance of 5 m and a zenith angle of 90°. This corresponds to an angular error of approximately 20 arc seconds. The error was caused by a bipod leg very close to the line of observation. An obstruction near the line of observation may also produce an EDM distance error because maximum outgoing and return signal strength is not attained.

The way in which these errors can be controlled is obvious: ensure that there are no obstructions in the field of view.

### **Errors Caused by Machinery Vibrations**

If the object of interest and the observing instrument are in fixed positions (e.g. operating machine bolted to concrete machine foundation, theodolite or total station on tripod set in tripod base which is duct taped to concrete floor), the machinery vibration will only result in a slight increase in the angular and distance random errors. This is because the peak-to-peak amplitude of machinery vibrations is usually less than 0.1 mm. A slight blurring of the image may result, but the observer's eye will estimate the correct target centroid position. Random distance errors are slightly larger than they would be with no machinery vibrations because of the relative motion between instrument and target.

Vibration dampening within the instrument itself varies from instrument to instrument. In some applications, vibration dampening at the instrument may be improved by introducing a dampening material (e.g. a neoprene pad) between the instrument and the top of the tripod. For very critical projects, it is also possible to go even further and tune the vibration dampening to the specific application.

### **Conclusion**

The theodolite intersection method, the total station method, or the single-theodolite triangulation method can be used in high-precision industrial surveys in which three-dimensional coordinates or coordinates changes (movements) are required. In some high-precision industrial surveys, three-dimensional orientations and rotations are also required. These can be conveniently measured with a theodolite using the autoreflexion technique. With systematic errors controlled, precisions of three-dimensional coordinates and coordinate changes can be at the sub-millimetre level, and precisions of three-dimensional orientations and rotations can be at the level of a few arc seconds.

### **Acknowledgements**

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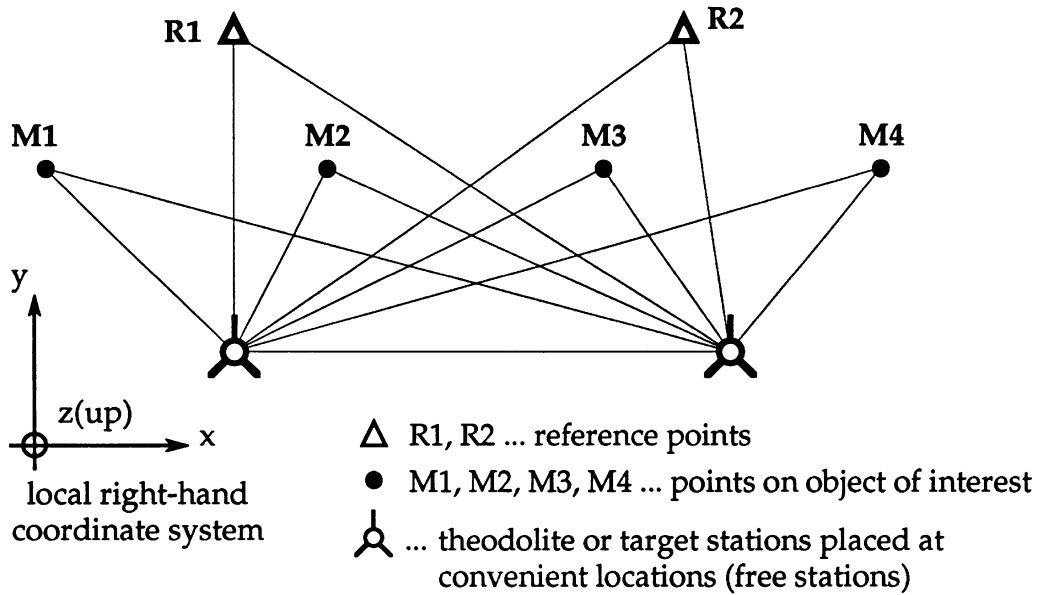
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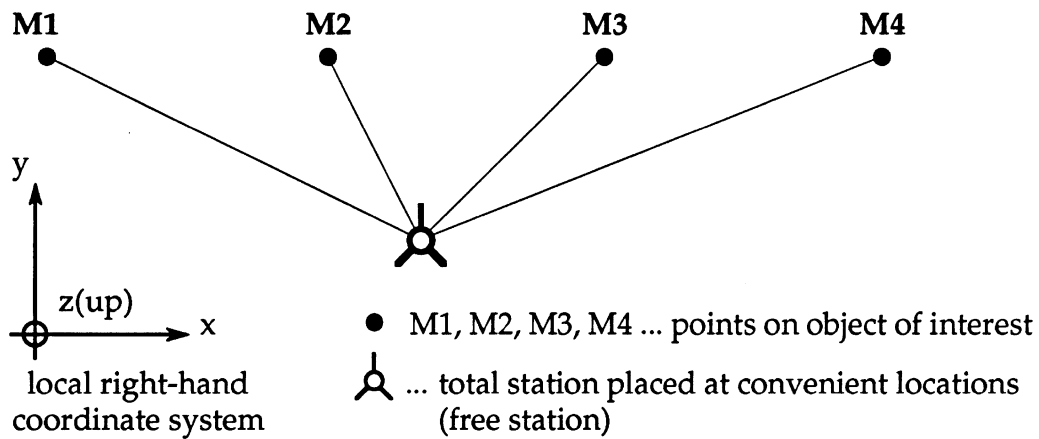
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$$R1(x,y,z) = (10.00000m, 10.00000m, 10.00000m)$$

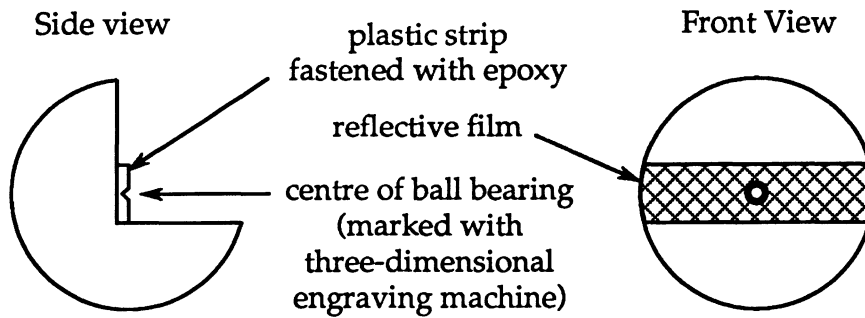
$$R2(x,y,z) = (10.00000+X.XXXXXm, 10.00000m, XX.XXXXXm)$$



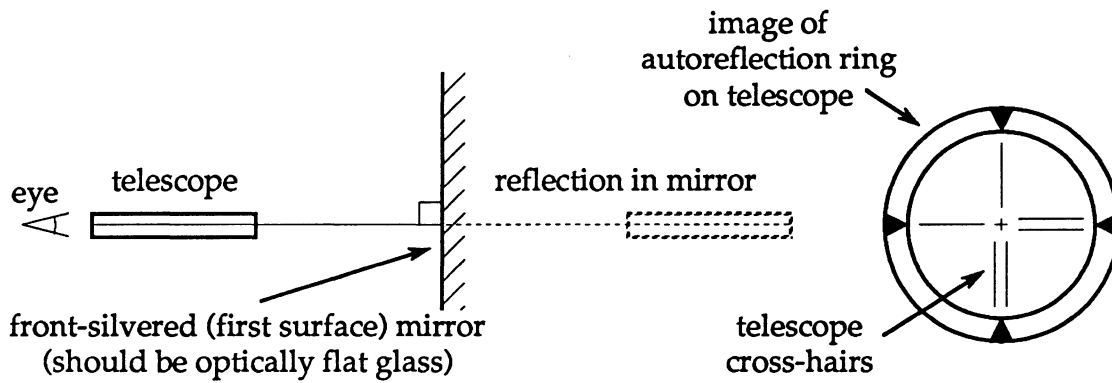
**Figure 1 - Theodolite Intersection Method (Typical Setup)**



**Figure 2 - Total Station Method (Typical Setup)**



**Figure 3 - Combined Target and Reflector for Total Station**



**Figure 4 - Autoreflexion**

# The Detection of Crustal Movements by Geodetic Space Techniques

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## The Deforming Earth

The earth's surface is continuously deforming! The most conspicuous causes of these deformations, arranged from the best understood (the most accurately predictable) to the least understood (the least accurately predictable) are as follows:

### 1) Tides

- body tides: the “breathing of the earth” causes global deformations in the one metre range;
- sea tides: the sea covers over 72% of the earth's surface, hence sea surface deformations should be considered alongside the solid earth deformations; the largest range is 16.3 metres;
- sea-tide loading: small, in the centimetre range.

### 2) Crustal loading

- ice: the weight of existing Antarctic ice is estimated at  $2.7 \times 10^{19}$  kg; the Laurentide region was depressed during the last glaciation by about 500 m;
- ice melt (water): after the last ice age, an estimated  $3 \times 10^{19}$  kg of water was released from the melted ice; consequently, the sea had risen by about 80 m and the sea bottom subsided by metres under the load of this water;
- river deposits: the Mississippi river deposits up to  $8 \times 10^{11}$  kg of silt annually close to its estuary; this is an example of an evolution of a sedimentary basin, where accumulated subsidence can be measured in tens of metres;
- tidal water: (see above);
- man made loads (reservoirs, cities, etc.): Lake Kariba weighs about  $1.5 \times 10^{14}$  kg; weight causes subsidence in decimetres;



## and unloading -

- ice melting: the Laurentide region rebounded by 350 metres and is still rising by up to one centimetre per year;
- evaporation of lakes: Great Salt Lake lost some  $8.2 \times 10^{15}$  kg of water and consequently rebounded by up to 7 metres;
- erosion: comparable to evaporation, no reliable data are available.

### 3) Tectonic deformations

- spreading boundaries: average rate 2 centimetres per year, rifting much faster, up to 15 cm/year; volcanic activity, earthquakes;
- converging boundaries: trench evolution, orogeny, the most destructive earthquakes; volcanic activity;
- intra-plate phenomena: geosyncline, faulting, graben, and escarpment evolution; earthquakes.

### 4) Man-made and other

- ground compaction: is caused by fluid withdrawal and is of local or regional character; many metres of subsidence have been observed;
- caving in: is caused by underground mining and by a collapse of natural cavities; subsidence by several metres has been observed;
- landslides: a self-evident change of the shape of the earth's surface;
- sea-level variations: eustatic rise (2 mm/year), waves, tides (see above), tsunami, seiches, wind-driven pile-ups, etc.

## The Problem of Prediction

Many of the above mentioned deformations are inconsequential for everyday human activities. Take for instance the body tide which is too global for us to sense it, or the evolution of escarpments or sedimentary basins which is too gradual for us to worry about it, or sea-tide loading which is too small to affect us in any way. These phenomena we do not have to even know about. Or, we may be living far away from where some of these things are happening. When we live in Japan, we do not have to worry about post-glacial rebound, or the lake evaporation rebound. When we live in eastern North America, we do not have to worry about volcanic eruptions.

Some of these deformations, such as sizable earthquakes, tsunami, or volcanic eruptions, affect many people very directly in different parts of the world, often with disastrous consequences. It is these phenomena, that we would like to be able to predict, to minimize the potential death toll and damage.

How do we go about predicting the phenomena of interest? As in any other field, prediction is done by models. These models are nothing but mathematical equations that describe the behaviour of the earth in time. To put together these equations, i.e., the model, we have to first understand the physics of the phenomenon, i.e., the forces that cause the deformation and the way the earth responds to these forces. This understanding

comes only from studying the ongoing processes. And this is where geodesy comes on the scene.

## **The Role of Geodesy**

What is geodesy? Geodesy is the science that concerns itself with the geometry of the earth, the earth's gravity field, and their temporal variations. Clearly, the evolution of the geometry of surface deformations, or crustal motions, as we often refer to them as, is of a paramount interest to geodesy. However, the focus of professional attention for a geodesist is not the design of a realistic model for a deformation phenomenon; rather, it is the impact of the crustal motion on positions (coordinates) of geodetic control points. Positions of points at the earth's surface are needed whenever one deals with spatial arrangement of objects, from the traditional maps to computerised geographical information systems.

Whatever their interest may be, the fact is, that geodesists are the people best equipped to measure and calculate repeatedly the most accurate positions of points on the earth's surface. Such repeated positions that geodesists determine, can then be used to describe the evolution of earth's surface deformation geometry. When a more spatially detailed description is needed, more densely distributed points have to be repeatedly surveyed. When a more refined picture of the evolution in time is desired, the time intervals between repeated surveys have to be shortened.

In the past, the positions of geodetic points on the earth's surface were determined in an incremental fashion from terrestrial measurements of angles, distances, height differences, astronomical azimuths, astronomical latitudes, and longitudes. With the advent of artificial satellites and radio-astronomy, and their progress in the past 30 years, geodesists now have new and generally more accurate tools for positioning. With these new "space tools," it is now possible to determine accurately even positions of isolated points, as well as accurate relative positions of points located from a few metres to many thousands of kilometres apart.

To study local and regional crustal motions, the classical terrestrial positioning techniques can be and still are used. For studying the global crustal motions, when changes in positions of points located very far apart are needed, space techniques must be used. Some space techniques, however, are used even to position points in the regional, or even local, context.

## **Geodetic Space Techniques**

What are these space techniques all about? They are all based on the idea of taking measurements to some extra-terrestrial objects. The best results, i.e., the most accurate positions or position differences, are obtained by using either the closest available objects, artificial satellites, or the most distant objects, quasars (quasi-stellar radio sources). For determining global crustal motions to a sufficiently high accuracy, we thus use, on the one hand, satellite positioning techniques and, on the other hand, radio-astronomy. Other existing space techniques do not have the required accuracy to be really useful in crustal motion applications.

The measurements to these space objects are, in the ascending order of positioning accuracy: directions, distances, and distance differences. Again, for meaningful global

crustal motion determination, only distances and distance differences can be measured accurately enough. These are measured by means of precise timing of the propagation of electromagnetic waves of very high frequencies.

In addition to the measurements, the positions of the satellites, or quasars vis-a-vis the earth at the instants of measurements, must be known as accurately as possible. Since quasars are believed to be millions to billions of light years away from the earth, their distances from the earth are considered practically infinite and only directions to quasars measured from the earth are required.

The three most popular positioning systems that are accurate enough, and thus usable for global crustal motion monitoring, are: the Satellite Laser Ranging (SLR), Global Positioning System (GPS), and Very Long Baseline Interferometry (VLBI). These systems may be briefly characterised as follows:

### 1) SLR

SLR is a loosely organised and very accurate system consisting of dedicated satellites and some permanent as well as mobile tracking stations. At present, there are 6 dedicated “laser satellites” (LAGEOS I & II, AJISAI, STARLETTE, ETALON I & II) available for ranging. These satellites are completely passive, in a shape of dense balls of up to one metre in diameter, covered with retro-reflectors. They are in orbits of inclinations ranging between 50 and 110 degrees, at altitudes ranging between 810 and 19,100 km. Because of their simple spherical shape, their orbits, i.e., strings of positions in space, can be computed very accurately, more accurately than orbits of any other satellites.

The ground stations designated to track these satellites with lasers, must be equipped with good astronomical telescopes designed for fast tracking, and with high quality clocks. The laser must be powerful enough to emit light pulses capable of traveling for several thousands of kilometres and passing through the whole thickness of atmosphere back and forth. The total travel time of the laser pulse from the transmitting telescope to the satellite retro-reflector and back to the receiving telescope is timed precisely. This time interval is then converted to distance. The position of the laser is then computed from these distances and the post-fitted orbits of the tracked satellites, at some later date. The errors in so determined positions are of the order of one centimetre.

### 2) GPS

GPS is an all weather system consisting of 24 identical satellites orbiting the earth at an altitude of about 20,000 km, revolving around the earth twice a day, at an inclination of 55 degrees, and a set of ground control stations. Each satellite continuously transmits its own particular signal at two frequencies of about 1200 and 1500 MHz. The signals of all the satellites are precisely synchronised and the frequencies kept as constant as possible by three very accurate atomic and one crystal clocks that operate (not simultaneously though) on board each satellite. Each satellite also continuously transmits information about its own position in space.

The antenna of the receiver, the instrument with which we make the measurements, receives signals from the satellites that are “visible” to the observer. These instruments, commercially available, relatively cheap, and in geodetic applications employed always in pairs, then measure very accurately the continuous changes of ranges to all the simultaneously tracked satellites. This is done by means of another precise clock built into each receiver. From these changes, and from the received satellite positions,

receivers can determine very accurate spatial differences in the positions of their respective antennas, i.e., the spatial “baselines” between the two antennas. These baselines may be of an arbitrary length and orientation and can be readily determined to an accuracy of a few parts in  $10^{-7}$  (a few decimetres in 1000 kilometres) and even better with a sophisticated analysis.

### 3) VLBI

VLBI is a radio-astronomical technique designed to measure very long baselines, typically several thousands of kilometres long. It uses quasars as sources of the signal tracked by the receivers. Quasars transmit signals that contain a broad band of frequencies; from these only narrow bands around 2 and 8 GHz are actually used for tracking. Quasars are so far away that the directions to them appear stationary in space.

Since the signals coming from quasars are very faint, very large radio-telescopes and powerful amplifiers are needed to track them. These radio-telescopes, called “dishes” in the technical slang, are employed in pairs, simultaneously to track quasar signals, one at a time. The selected bands of the signal, corresponding to the wavelengths of a few centimetres, are then recorded on tapes, together with very precise time marks issued periodically by the observatories’ dedicated hydrogen masers, the most stable atomic oscillators available. After each observing session, lasting normally for one to a few days, the recordings are brought together and processed in a “correlator,” a device that converts the recorded signals into range differences. From these continuously varying range differences and the known directions to quasars, very accurate baselines can be computed. The errors in so derived baselines may be as small as a few centimetres.

There exist other spatial systems that can and are used for global crustal motion monitoring, such as, for example, the French DORIS system, but these are less popular and we shall not describe them here.

## Examples of Some Ongoing Projects

The best known and the longest running (since 1979) global project is the **Crustal Dynamics Project**, renamed in 1992 to **Dynamics of the Solid Earth (DOSE)**, conceived and managed by NASA. Under the auspices of this project, NASA and scores of its external collaborators periodically employ many of the world’s radio-telescopes and lasers to survey and re-survey positions of strategically selected points all around the world. From so determined position changes, the pattern of motion of tectonic plates can be discerned. The results obtained from about 15 years of observations seem to confirm, to a good accuracy of a few centimetres per year, the estimated plate velocities derived from other evidence.

The recently (1991) established **International GPS Geodynamics Service**, a cooperative venture of some 150 different institutions around the world under the auspices of the International Association of Geodesy, continuously determines positions of some 120 stations and operates 6 centres, which independently produce very accurate post-fitted orbits for all GPS satellites. It became fully operational at the beginning of 1994 and started providing not only the information on position changes of its 120 permanent (fiducial) stations but also precise GPS orbits for independent researchers who wish to use the GPS for crustal motion monitoring.

Many regional crustal motion projects have been and are being conducted. Most of these projects employ the GPS as their basic tool, because it is the cheapest and the easiest to use. The longest running of these projects is probably the cooperative project in California, which monitors the ongoing deformations along the Saint Andreas system of tectonic faults.

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# Primary GPS Control Point Network of Polish Copper Basin Area

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## Abstract

The copper basin area, named LGOM, is located in the south-western part of Poland. Area of the basin is about 800 km<sup>2</sup>. This terrain undergoes considerable deformations because of intensive mine activities that take place there. A special horizontal control network was established on this terrain in 1973 for deformation monitoring purposes. It was classically measured (angles and distances) in 1975, 1985, and 1988. From 1988 until 1993 the control measurements were not performed there because of continuously progressing destruction of the wooden triangulation towers (signals of the network points in a forestry terrain). To make further measurements possible a new GPS network was designed and established there. It consists of new eccentric points suitably situated for satellite GPS measurements and moreover for tying them to the classical marks of the existing primary horizontal network of LGOM.

In this paper an application of a GPS technique to modernize the LGOM horizontal control network is presented. Results of satellite observations and computations are given. The accuracy of determination of current horizontal coordinates of the primary network points is estimated to be in the order of about 10 mm.

## 1. Introduction

The Polish Copper Basin (LGOM) is located in south-western part of Poland, between two towns: Lubin and Glogow. It covers an area of about 800 km<sup>2</sup>. Since the very beginning of mining activity in this area (i.e., since 1960) investigations and measurements to discover the factors shaping the deformation of rock mass and terrain surface processes have been carried out there. On the basis of such a long observation period two kinds of influences of the mining exploitation on the surface as well as on the rock mass there can be distinguished:

- direct influences, caused by displacement of the rock towards the post-exploitation free space (filling it with broken-down roof rock),
- indirect influences, caused by water escape (draining effects) from the rock mass that results in displacements and deformations of the ground, changes in water conditions in the region, changes of surface properties, etc.

Detailed analysis of levelling measurement results from the period of 1960-71 proved that there exists, in higher degree than was suspected, the indirect influence of the copper ore exploitation on the terrain surface caused by rock mass drainage. The vertical movements, which followed the drainage process, began to extend and cover bigger area, considerably overrunning the area of the direct influences. Currently, all mining plants are within these influences, and even areas beyond LGOM are also affected by the influences. The greatest subsidence, caused by direct influences, is located along Tertiary outcrops. The centres of troughs formed in these regions reach the maximum subsidence of 2000 to 2500 mm (the state for December 1994). Subsidence caused by indirect influences alone (east part of LGOM) ranges between 30 and 450 mm. Interpretation of the subsidence caused by direct and indirect influences was carried out mainly on the basis of precise levelling measurements of a vertical network of the second class established on this terrain specially for these purposes.

It is planned to use GPS results to elaborate a model of deformation on the area of LGOM, taking advantage of the Finite Element Method (FEM) approach [Szostak-Chrzanowski and Chrzanowski, 1994]. Using GPS makes it possible to measure the displacement of the ground surface. The FEM provides a powerful technique to study the deformational behaviour inside the mass rock.

## **2. Vertical Control Network of Second Class in the Copper Basin Area**

The vertical network covers an area of about 400 km<sup>2</sup>. Distances between bench-marks of the network average 1.5 to 2 km. The first measurements from the early 1960s were devoted to engineering surveying of artificial objects. The next measurements performed in 1967, 1971, 1975 and further on every 2 to 3 years discovered that deformations resulting from mining activities took place on the terrain. After the deformations were discovered the time span between successive measurements was shortened to 1 to 1.5 month.

The results of detailed analysis of measurements from 1971 and later proved that the copper ore deposit exploitation indirectly influenced the terrain surface, through the rock mass drainage. The vertical movements that followed the drainage began to extend and cover a bigger area, considerably overrunning the area of the direct influences. It caused the necessity of expanding the network. Currently, the levelling network covers an area of about 2300 km<sup>2</sup>, it consists of 1241 km of levelling polygons, 149 lines and 78 points. Measurement accuracy of the network amounts to 0.75 to 1.2 mm per 1 km.

## **3. Horizontal Control Classical Network**

The horizontal control classical network on the area of LGOM was established in 1973 and then extended in 1975. It consists of 38 points. Also in 1975 a local system of coordinates named "Pieszkowice" was introduced on this terrain. Mostly, points of the network were over-built with wooden triangulation towers. The first measurement of the network was performed in 1973, and then repeated in 1975. The classical measurements were performed with the accuracies given in Table 1.

Table 1. Accuracy estimation of the classical horizontal control network after adjustment.

	1973	1975
Mean error of angles after adjustment $m_a$ [cc]	3.0	2.0
Mean error of distances after adjustment $m_d$ [mm]	$5.6 \text{ mm} + 0.8 \times 10^{-9} d$	6 - 13
Mean position error $m_p$ [mm]	53	20

Such accuracy of point position determinations made this network suitable for applications in rock mass deformation studies as well as in multi-point tying of specific engineering networks to the control points. The next classical measurements campaigns were performed in 1985 and 1988. On the basis of these observations analysis of points stability was performed. Stability of 18 points in the period of 1975-85 was proven. Similar analysis was carried out for the period of 1985-88 and then only 13 points could be regarded as stable. On the basis of all three measuring campaigns a common reference basis of only 7 stable points was distinguished. It should be mentioned here that it was not possible to measure coordinates of all the points of the control network because of continuous destruction of the wooden triangulation towers, constituting signals of the network points in a forestry terrain. Maintaining of the towers in good technical condition was very expensive.

The situation in LGOM before 1992 can then be summarised as follows:

- the terrain there undergoes considerable displacements that must be monitored,
- the monitoring became more and more difficult (sometimes even impossible) because of very bad condition of the wooden towers (signals of geodetic points).

Looking for a solution to this problem, it was decided that GPS should be applied in this region. In 1992 a team from Institute of Geodesy, Olsztyn University of Agriculture and Technology, in cooperation with Polish Copper Mining Inc. and the Geodetic Office from Wroclaw, performed a GPS experiment aiming at checking the possibility of applying GPS to the study of this regional stability. The results confirmed the possibility, but they showed the necessity of transferring the tower table observation points to the ground. The GPS technique can be successfully applied to modernisation of the horizontal control classical network of LGOM, to the study of point stability, to establishment of special purposes networks, and to monitoring of the terrain surface displacements in the area of mining activity.

#### **4. Concept of GPS-Aided Modernisation of LGOM Horizontal Network**

On account of the progressing destruction of wooden towers built over points of the control horizontal network of LGOM, it seems that the only way to make further measurements possible is completion of the network with new points, meeting requirements for GPS surveys. GPS observations would be performed at these points, and then, on the basis of



classical tying measurements, coordinates of basic points of the network would be determined. Two examples of tying the GPS observations with classical ones is given in Fig. 1.

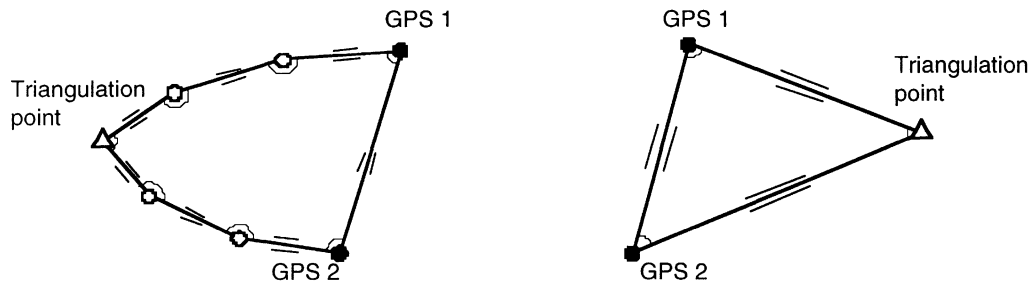


Fig. 1. Example of tying of GPS and classical observations

GPS measurements of the whole control network of LGOM allowed for:

- resignation from signal towers over points of the network,
- tying the new GPS points to existing points of the control network using classical measurements,
- increase of network accuracy,
- location of reference points outside of the area of mining activity influences,
- application of GPS points for studies of displacements of the ground,
- shortening of time needed for observations and elaboration of results, and
- using GPS points for other geodetic purposes.

## 5. Application of GPS to the Determination of Coordinates of LGOM Horizontal Control Network Points

As has already been mentioned, the classical control network of LGOM was established in 1973, then it was measured in 1975, 1985 and 1988. Horizontal coordinates of the network points were determined in each observational epoch on the basis of classical observations (angles and distances) with 20 mm accuracy. Since 1988 the measurements have not been performed because of continuous destruction of the wooden towers built as signals over points of the network. It was concluded that the only way is to complete this classical network with new points that meet the requirements of GPS observations.

Putting the ideas given in part 4 of this paper into practice, a network of additional points (eccentric) was designed to be convenient for GPS and simultaneously to enable tying them to the main points of the control network with classical observations. Accordingly, 89 eccentric points with new, durable monumentation, were measured with GPS. To make tying of the measured network to the Polish National System of Coordinates possible, also 6 points of the National Triangulation Network of the I class were measured during the same campaign. On the other hand, the new network has also been tied to the EUREF (European Reference Network) network through including one point belonging to the EUREF network (zero-order) and 3 points of the POLREF network (first-order) [Baran, 1995]. The POLREF network had been measured on the basis of 11 points of the EUREF network located in Poland and having their coordinates in the European Terrestrial Reference Frame from 1989 (ETRF'89). This enabled determination of LGOM point coordinates in the ETRF'89 system.

The GPS measurements of the network of eccentric points were performed in September 1995, May 1996 and May 1997, using 6 Ashtech MD-XII GPS receivers. The duration of observation sessions ranged between 90 to 120 minutes. Elaboration of results was carried out with Ashtech GPPS software (computation of vectors) and with GeoLab by BitWise Ideas Inc. being used for the adjustment. The analysis of accuracy showed that in both cases the semiaxes of confidence ellipses, at 95% confidence level, did not exceed 12 mm (major) and 9 mm (minor), while mean errors of height determinations did not exceed 14 mm. For more details see Table 2.

Table 2. Accuracies obtained from GPS measurements after adjustment - satellite system of coordinates

Confidence ellipses of point positions (at 95 %)					
Years					
Semi - axis	1993	1994	1995	1996	1997
major	5.1 - 9.4 mm	1.6 - 7.7 mm	2.1 - 7.7 mm	4 - 12 mm	2.1 - 6.8 mm
minor	3.9 - 7.3 mm	1.4 - 4.2 mm	1.8 - 5.4 mm	2.5 - 8.4 mm	1.5 - 4.7 mm

Confidence ellipses of relative positions in mm (at 95%)					
Semi - axis	1993	1994	1995	1996	1997
major	2.6 - 8.3	1.6 - 7.8	1.5 - 5.2	4 - 12	3.0 - 6.8
minor	2.2 - 6.0	1.3 - 4.4	1.1 - 4.6	2.5 - 8.4	2.5 - 4.8

Mean vertical errors in mm (at 95%)				
1993	1994	1995	1996	1997
8 - 15	3.3 - 9.2	5.2 - 12.1	6 - 14	5 - 11

Mean relative vertical errors in mm (at 95%)				
1993	1994	1995	1996	1997
4.3 - 13.5	2.9 - 9.2	2.6 - 10.4	6 - 14	5 - 11

The obtained coordinates were transformed to the Polish National System of coordinates on the basis of the 6 triangulation points of I class. The next step to be performed was computation of coordinates of the control network points in the Polish National System using GPS coordinates and classical observations. The measurements of angles were carried out with accuracy  $m = 10^c$  and those of distances had accuracy  $m_d = 0.005$  m.

On the basis of adjustment results it can be stated that the triangulation points

determined on the basis of GPS eccentric points were positioned with mean error  $m_p < 10$  mm. The applied method enabled analysis of the behaviour of the network during 20 years of its existence on the area of intensive mining activity. Analysis of LGOM control horizontal network points stability in the period of 1975 - 1995 was performed [Wasilewski et al., 1996].

## **6. Accuracy Analysis of GPS and Triangulation Points Positions Determination**

### **6.1. Accuracy of the Primary Horizontal Control Network**

As has been already mentioned, the coordinates of the triangulation points of LGOM network were determined on the basis of classical measurements tying the GPS points (eccentric points) to the main (triangulation) points. For each triangulation point the classical observations were performed, forming a small network to be adjusted (see Fig. 1). The adjustments were performed in two variants:

- each “object” independently, assuming the GPS fixed points as errorless, giving them coordinates obtained from adjustment of GPS network alone (after transformation of the coordinates into the Polish National System);
- common adjustment of all the observations (GPS together with classical ones for every “object” treated as one network).

For the adjustment computations special computer programs, written at our Institute by prof. I. Gajderowicz, were used [Gajderowicz, 1996].

In both the variants we admitted:

- mean errors of angle measurements  $m = 10''$ ,
- mean error of distance measurements  $m_d = 0.005$  m.

After analysis of the mean errors of positions obtained from both adjustments it can be stated that adjustment of GPS and classical observations independently causes too optimistic estimation of positioning accuracy  $m_p$ . In some cases, in the first as well as in the second variant of adjustment, the mean positioning errors are considerably bigger than average values. These triangulation points were determined on the basis of poor geometric constructions followed by difficult terrain conditions (thick woods).

The GPS points after transformation into the Polish National System of Coordinates obtained the mean position errors  $m_p$  that do not exceed the value of about 7 mm (see Table 3). It can be seen clearly that the GPS points are determined more accurately than the triangulation points. Corrections to X and Y coordinates in the transformation are given in Table 4.

Table 3. Polish National System of Coordinates “65”

Components of mean position errors					
Component	1993 [mm]	1994 [mm]	1995 [mm]	1996 [mm]	1997 [mm]
$m_x$	13 - 22	5 - 8	4 - 6	3 - 4	3 - 6
$m_y$	13 - 22	5 - 8	4 - 6	3 - 4	3 - 6

Table 4. Corrections to tying points coordinates in the system „65”  
(transformation from the satellite system to „65”)

Point No	1993 [mm]		1994 [mm]		1995 [mm]		1996 [mm]		1997 [mm]	
	$v_x$	$v_y$	$v_x$	$v_y$	$v_x$	$v_y$	$v_x$	$v_y$	$v_x$	$v_y$
22/20 Grabik (G022)	4	16	5	-1	2	6	0	-3	4	4
20/160 Pogorzelska (0020)	-15	20	-16	-2	-12	-7	9	-4	8	-8
15/94 Proszyce (0015)	-20	-25	-3	-4	-2	-6	-6	0	-10	3
16/110 ORSK (0016)	3	14	5	25	7	18	-7	3	-8	-2
17/135 Buszkowice (0017)	49	28	0	-13	-3	-9	5	-3	16	-5
22/183 Miłoradzice (0022)	-21	-50	10	-4	13	0	-	-	-	-
022A Miłoradzice	-	-	-	-	-5	0	-1	8	-	-
0001 Raszowa Duża	-	-	-	-	-	-	-	-	-9	8
	$m_0=32.2$		$m_0=12.6$		$m_0=9.7$		$m_0=4$		$m_0=9$	

## 6.2. Accuracy of the Special Horizontal Control Network

To assess real accuracy and reliability of the GPS results, the obtained coordinates were compared to those obtained on the basis of classical measurements performed for selected points of the network. The latter were carried out by a team from the Mining-Technical Academy in Cracow, in 1993. The comparison comprised points located in the region of the post-flotation reservoir “Zelazny Most,” having numbers 100, 111, 201, 219, 411 and 500. GPS measurements at these points were performed in the years 1993, 1994, 1995 and 1996. The coordinates of these points were transformed to the local reference system

“Zelazny Most.” The transformation was performed in two stages, according to the project given by the Institute of Mining Geodesy of the Mining-Technical Academy in Cracow. Also 3 tying points were included to the transformation as references, these had the following numbers and names: 15- Proszyce, G1- Gawrony and W1- Wadroze. The two transformation stages were as follows:

- Transformation from the WGS 84 to the Transverse Mercator system with the central meridian coinciding with point 500, the scale at the central meridian was set to 1.0
- Transformation from the Transverse Mercator system to the local reference system “Zelazny Most.”

The transformation results are given in Table 5.

Table 5. The WGS 84 to “Zelazny Most” transformation results for the years 1993-96.

	1992 [mm]	1993 [mm]	1994 [mm]	1995 [mm]	1996 [mm]
$V_{x\hat{r}}$	2	1.8	2.8	1.6	2
$V_{y\hat{r}}$	5	2.8	2.8	2.2	5

## 7. Conclusions

1. On the basis of the performed analysis of accuracy, it can be stated that semi-axes of triangulation point position error ellipses, determined using GPS, at 95% confidence level, do not exceed the value of 9 mm. Most of the points obtained the value of this parameter at the level of 3 mm. Components of the positioning errors of the GPS points after the transformation into the Polish National System of coordinates are on average 3 - 4 mm for horizontal coordinates and 4 - 5 mm for the vertical coordinate.
2. Application of GPS in combination with total station observations made it possible to connect observations from the past (1973-88) with the results obtained in 1995 and 96. Thanks to that, it was also possible to determine values of displacements of the network points. The common adjustment of GPS and classical observations made it possible to perform a uniform analysis of accuracy for all the points together (GPS and triangulation).
3. GPS enables choosing the location of stable points, used as references in displacement studies, far outside of the area of mining activities influences. Thanks to the satellite technique, it is not necessary to establish intermediate points.
4. Taking advantage of GPS considerably shortens the time needed for the observations to be performed. Observations of the whole network can be performed in not longer than 2-3 days. It provides the possibility of real (current) displacements monitoring of the terrain.
5. Taking into consideration the area of the Copper Basin that is affected by mining activity (800 km<sup>2</sup> of direct and 2300 km<sup>2</sup> of indirect influences) as well as the above

listed advantages of GPS, the satellite technique seems to be optimal for the displacements study in LGOM as well as for other geodetic purposes.

6. The obtained accuracy of GPS results makes it possible to study tensions inside the mass rock. It is planned to apply the FEM approach for this purpose.

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# **Juneau Icefield, Alaska: Glacio-Geodetic Measurements**

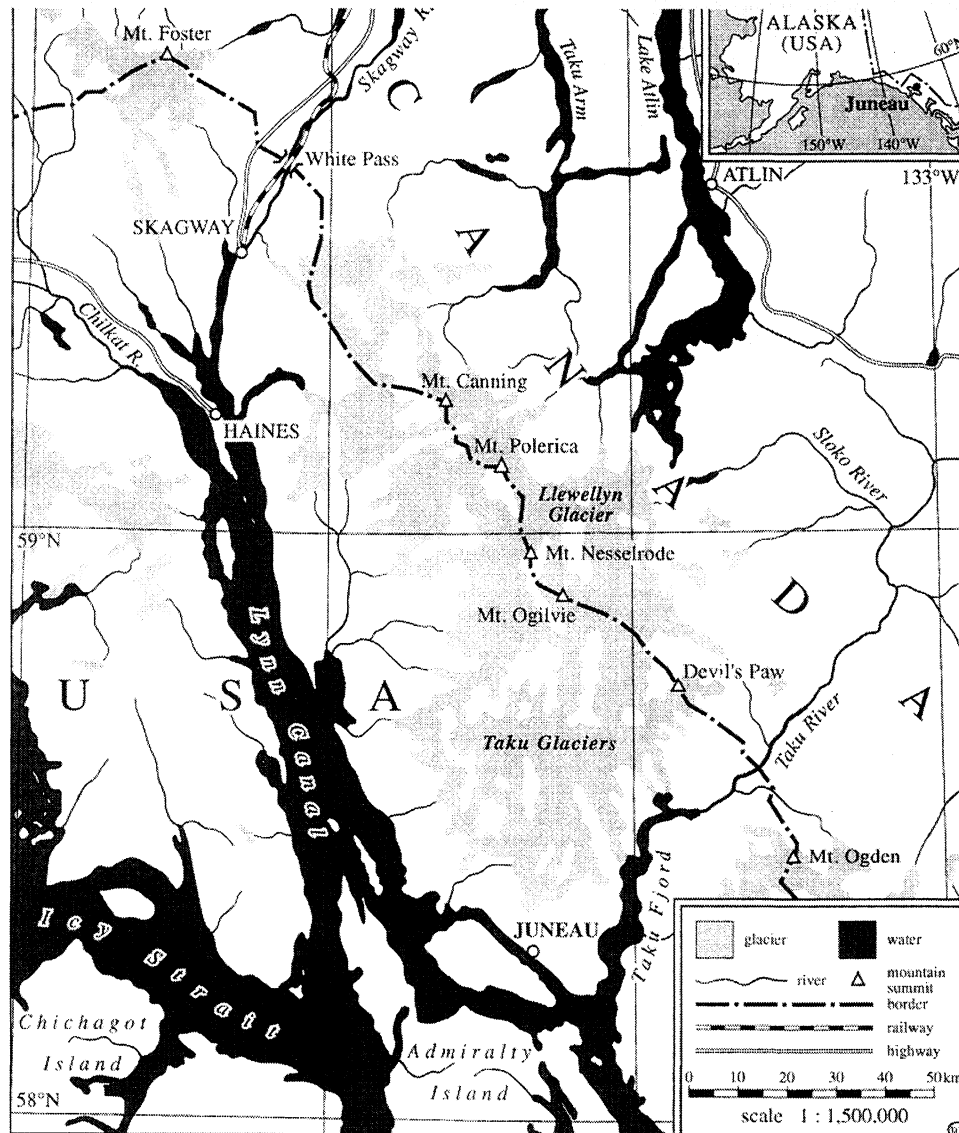
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Prof. Dr. Adam Chrzanowski spent four years in a row (1965-1968) with the Juneau Icefield Research Project as part of the University of New Brunswick's cooperation with Prof. Dr. Maynard M. Miller of the University of Idaho, U.S.A., who has been running this long-term glaciological program since 1946. Maynard's wife Joan tells about Adam on the Icefield: "Adam was great fun, loved the women, and they all loved him. I do remember that he sent the equipment list and arrived with all new gear, red down-filled parka, etc., as we all wore in those days. I believe his first year was the best weather we have EVER had on the Icefield. Adam told everyone from then on that they did not need heavy equipment. Then followed years of bad weather, but Adam never brought his heavy gear back again. — We miss him and can't believe he is retiring."

The author of the following contribution has been acquainted with Adam since he was a Visiting Professor at the Bundeswehr University in Munich, Germany, in 1976. Since then the cooperation with Adam in joint projects and scientific committees has been close. The article below — dedicated to Adam — is a summary of the results achieved by the author and his work-fellows since he took over the glacio-geodetic work on the Juneau Icefield in 1981.

## **1. The Juneau Icefield and Glacio-Geodetic Observation Techniques**

Since the foundation of the Juneau Icefield Research Program (JIRP) the survey work was mainly focused on the determination of surface velocities on various profiles of the Taku Glacier system. Later strain rate determination and analysis became another main topic of the JIRP surveying program. Surface velocities and strain rates are two of the key parameters to describe the behavior of a glacier. Fig. 1 represents the Juneau Icefield, Alaska.



**Fig. 1: The Juneau Icefield, Alaska.**

Wooden stakes with flagging material attached and set in a line across a glacier usually serve as point markers for the determination of glacial surface velocities. A spacing of 250 to 500 m depending on the width and the slope of the glacier turned out to be sufficient. The movements of these flags represent the main flow characteristics of the glacier.

Up to the early nineties all profiles were surveyed using theodolite and electronic distance measurements. Thus the point coordinates were determined by either intersection or polar survey. After an appropriate period of time the measurements were repeated. The movements of the flags are calculated from the coordinate differences between the measurement epochs.

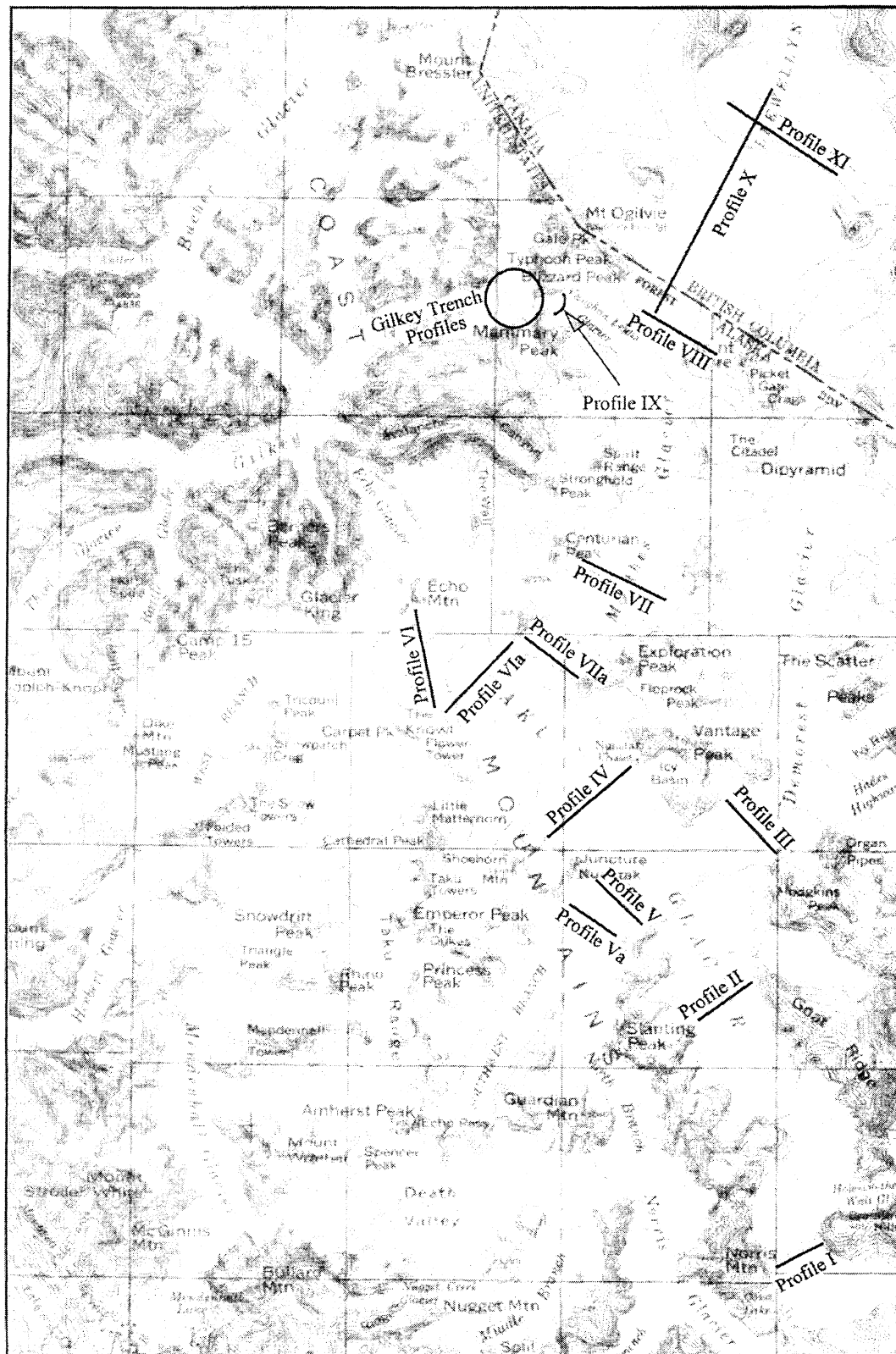


Since 1992 the use of GPS has consequently been intensified. More profiles per season and profiles in remote areas open up better research possibilities on the Icefield. GPS measurement techniques offer numerous advantages:

- greater time efficiency: using real-time GPS and motorized transport, a typical profile can be measured in two or three hours only;
- higher and distance independent accuracy: using differential GPS, a position can be derived with a standard deviation of about 1cm for baseline lengths shorter than 5 km;
- the positions of the flags of former years can be re-established very accurately and in no time using real-time GPS: a more consistent comparison of movements and heights over the years is enabled;
- the measurements are weather independent: in principle surveys can be carried out even under white-out conditions.

## **2. Movement Profiles**

The profiles shown in Fig. 2 represent the locations of movement measurements of the Taku Glacier system over the period 1986 - 1996. Small variations of the profile locations (up to 500 m) and the numbers of points per profile show no remarkable effects on the movement patterns and on the individual movement vectors.



**Fig. 2: The Movement Profiles.**

## 2.1 Ablation Area of the Taku Glacier

Profile I is located between Norris Mountain and Brassiere Hills on the lower Taku Glacier about 5 km above its present terminus. The out-radiating movement pattern reflects the fact that the profile is situated on the last narrow cross-section above the terminus. The zone of maximum movement starts 800 m east of the Norris Mountain glacier border and continues for some 1.3 km ending 400 m in front of the bedrock of the Brassiere Hills. The velocity of the maximum movement zone is fairly consistent with an average of 85 cm/day and a maximum of 91 cm/day close to the western end indicating an asymmetrical channel flow. On the western end of the profile the movement vectors are orientated towards the margin of the glacier. This fact may be caused by two factors. First, the steep valley sidewalls are responsible for an increasing ablation rate towards the margin so that the glacier intends to compensate for the loss. Second, the still strong positive mass balance [Peltó and Miller, 1990] of the Taku Glacier leads to a convex height profile—this is typical for advancing glaciers. The movement pattern is in surprisingly good accordance with the one observed in the early fifties [Miller, 1953]. The mean movement rate of Profile I is 65 cm/day.

Profile II has a location which varies from year to year. The location corresponds to the annual Equilibrium Line Altitude (ELA). In 1993 the profile was located between Slanting Peak and the northern end of Goat Ridge. Higher accumulation in the following winter required a shift of the profile some 1.5 km down-glacier a year later.

The highest velocity of any profile on the Taku Glacier can be expected in Profile II for three reasons: all tributaries enter the main Taku above this line, the ELA characterizes the area of highest mass transportation, and the combination of a narrow glacier bed and a steep slope of about  $2.5^\circ$  acts as a nozzle.

The 1993 profile shows a highly uniform velocity in the central part of the glacier (80-85 cm/day). The 1994 profile, situated on a narrower section of the lower Taku Glacier reveals the highest velocity (93 cm/day). The variations on the central section of the profile are comparable to the 1993 profile (83–93 cm/day). The mean velocity (71 cm/day) of Profile II as found in 1994 is the highest of all Taku Glacier profiles. The mean velocity in 1993 (66 cm/day) equals the one of Profile I.

Heavily crevassed and well defined shear zones can be seen on the margins of both profiles. In combination with the strong increase of the velocity in these marginal zones and the uniform velocity in the central part, a Block-Schollen movement has to be considered for Profile II. A counter-evidence remains: the velocity distribution in the marginal zones, which stretch over 700 m on both sides, cannot be measured because these areas are heavily crevassed. Thus the typical velocity profile of a Block-Schollen flow across the whole glacier cannot be verified. In addition, best fit polynoms of fourth order reveal rather a sort of parabolic flow. As a consequence, the flow in the area of both profiles can be characterized as a transition form of parabolic to Block-Schollen flow.

## **2.2 Main Taku Glacier and its Lower Branches (Demorest Glacier, SW-Branch)**

Profile III between Taku A and Peak 4785 (a summit in the west of Hodgkins Peak) allows to monitor the eastern accumulation area of the Taku Glacier mainly formed by the Demorest Glacier and its tributaries. In most of the recent summers this area was above the ELA. Movement surveys were conducted in 1987 and from 1991 to 1996.

The velocity pattern over the years is quite stable. The mean velocity of 20 cm/day was derived from a best fit fourth order polynom. It shows clearly a parabolic flow pattern.

Profile IV stretches from Camp 10 to the northeast ridge of Shoehorn Mountain. The significance of the profile is given by the fact that yearly movement surveys have been carried out here since the late forties creating one of the longest continuous velocity records of an individual glacier. Since 1993 the profile consists of two parallel lines approximately 300 m apart, the lower line representing the original location. This setup allows to gather a lot more parameters like strain-rates, glacier gradients and simplified mass balances [McGee, 1997]. The movement pattern on both lines is nearly identical, slight differences occur in the marginal zones reflecting topographic peculiarities. A parabolic flow with a broadened central zone spreads more than two kilometers showing a mean velocity of 50 to 60 cm/day. During the timespan from 1986 to 1996 the variations are negligible. A fourth order polynom fits all the data except those of 1987 revealing a mean velocity of 38 cm/day (Fig. 3). In 1949, 1950 and 1964 maximum velocities of 90 cm/day and higher were reported [Miller, 1953; Havas, 1965]; the reason for the significantly lower velocity in our days is not clear.

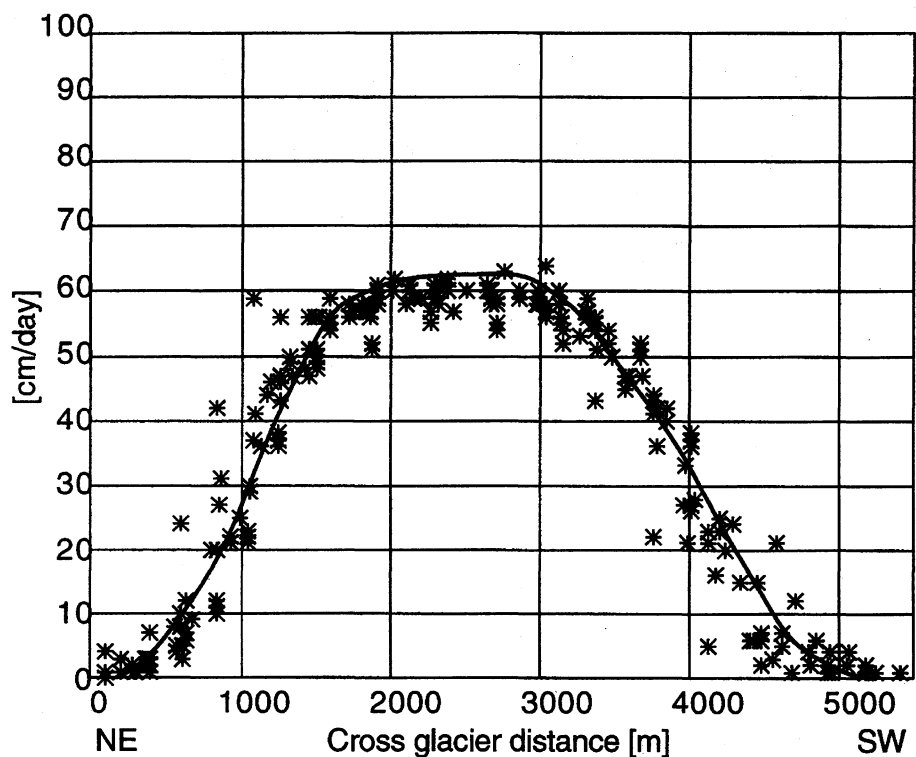


Fig. 3: Profile IV: measured (\*) and adjusted (-) velocities.

Profile V is located between Juncture Peak and Peak 4066 some 800 m above the confluence of the SW Branch and the Taku Glacier. It has been observed yearly since 1986. With an area of about 40 km<sup>2</sup> the SW Branch is the smallest tributary within the Taku Glacier system. The flow pattern is perfectly parabolic and stable over the period observed. The mean velocity is 6 cm/day; the maximum velocity is 12 cm/day.

Approximately 1.5 km up-glacier Profile Va was established only in 1989. The flow of the western part of the profile is obviously affected by a rocky ridge stretching from Shoehorn Peak southeast. The eastern part of the profile shows a parabolic flow up to 8 cm/day.

### 2.3 Convergence Area of the Matthes and Taku Glaciers

The NW-Branch of the Taku Glacier and the Matthes Glacier are similar in their areas, both covering about 150 km<sup>2</sup>. Although the exact depth profiles of both glaciers are not very well known, the latest results of seismic investigations [Sprenke, 1996] suggest a similarity so that the volumes of both glaciers can be regarded as equal. The NW-Branch of the Taku Glacier represents the maritime side of the Icefield whereas the Matthes Glacier is situated in the transition zone to the drier continental side of the Icefield. Besides topographic peculiarities different movement patterns may reflect this fact.

Profile VI extends from NW Taku Point situated northeast of the Flower Tower to Echo Mountain. After the first measurements in the early fifties the movement vectors were determined in the last decade only rarely (1986, 1989 and 1992). Profile VIa stretches from NW Taku Point to Taku D. In addition to the ice masses passing through Profile VI, the ice developing in the basin formed by Echo Mountain, The Wall, Centurian Peak and Taku D has to pass through Profile VIa. This profile has been measured in 1990 and 1991 and every year since 1993.

Both profiles depict an asymmetric parabolic flow. This is due to the rocky ridges continuing from Echo Mountain and Taku D. The greater volume passing through the lower Profile VIa is reflected by a 10% increase of the maximum velocity (32 cm/day vs. 29 cm/day at Profile VI) and of the mean velocity (18 cm/day vs. 16 cm/day). Movement variations over the years were not detected.

Profile VII extends from Camp 9 towards Centurian Peak. It was up to the early nineties the only profile on the lower Matthes Glacier. Data were gathered 1986, 1987, 1989, 1990, 1994 and 1996. Close to the confluence zone of Matthes and Taku Glacier, Profile VIIa between Taku C and Taku D was established in 1993 and yearly monitored.

The flow direction of the easterly part of Profile VII reflects the outflow of the small basin below Camp 9 towards the Matthes Glacier. Apart from this irregularity the flow is perfectly parabolic. Over the last decade the movement pattern has been very stable. The maximum velocity reaches 38 cm/day and the mean velocity is 26 cm/day. Compared to the results of earlier JIRP seasons the flow has not changed for the last fifty years. This is in contrast to other profiles.

Towards the confluence zone the Matthes Glacier valley narrows from 4 km at Profile VII to 3 km at Profile VIIa. Since between both profiles no mass is added to the Matthes Glacier, the result is a gain in velocity. Its maximum increases to 45 cm/day and the mean to 29 cm/day. A polynomial fit reveals again a parabolic flow pattern, which is asymmetric with higher velocities towards Taku C. The movement variations are very small and can be assigned to the small differences of the positions of the profile over the last three years.

Although the distances to the confluence zone are comparable, the lower Matthes Glacier (Profile VIIa) moves about 50% faster than the NW-Branch of the Taku Glacier (Profile VIa). This fact should give rise to further investigations of depth profiles in various locations of both glaciers.

#### **2.4. Nevé Area of the Matthes, Vaughan Lewis, and Llewellyn Glaciers**

The so-called High Plateau at an elevation of 1800-1900 m is the source of the two main glaciers of the Juneau Icefield: the Llewellyn Glacier representing the continental part of the Icefield flowing north into Canada and being the ultimate headwaters of the Yukon,

and the Matthes Glacier flowing south into the Taku Glacier. The Matthes Glacier is one of the main tributaries of the Taku representing 20% of the Taku Glacier system.

The area of the High Plateau is characterized by extensive ice surfaces with only a few nunataks. Long periods of poor visibility due to haze and fog were limiting factors for terrestrial surveying methods. In addition to the classical profiles VIII (Upper Matthes Glacier) and IX (Upper Vaughan Lewis Glacier) new profiles were established only when GPS became available. The new profiles on the Llewellyn Glacier and in the area of the Divide give a first glance of the movement patterns of the highest part of the Taku-Llewellyn system.

Profile VIII is located between Blizzard Point and Camp 8. The orientation of the movement vectors (up to 8 cm/day) at the eastern and western end of the line reflects the influence of the ice movements down from Blizzard Peak and Mount Moore resp. Nevertheless the flow pattern is parabolic. The maximum velocity ranges from 16 to 19 cm/day and the mean movement is 9 cm/day. With respect to the location of the profile close to the Divide and an average gradient of  $0.5^\circ$  these values appear very high. The reason could be a substantial depth of the glacier which provides the big masses necessary for high velocities.

In 1995 Profile VIIIa was set 3 km down-glacier of Profile VIII. The purpose was to demarcate the Matthes from the Vaughan Lewis Glacier. The results were not satisfying, because the profile was set too far down the Matthes Glacier. The movement pattern is as expected with a maximum velocity of 22 cm/day. It has to be considered that the profile covers only one third of the glacier's width. The survey of Profile VIIIa was not repeated in the following years.

Profile IX is located about 1 km above the top of the Vaughan Lewis Icefall forming a semicircle between the ridge of Camp 18 and Mammary Peak. The area is usually heavily crevassed. In some years it has been too dangerous to set the flags or to perform EDM or GPS measurements. Besides, due to the difficult accessibility the location of the profile varied over the years. Therefore the individual maximum velocities (26 to 35 cm/day) reflect rather the distance to the Icefall than the change of the movement pattern. The orientation of the vectors, however, is consistent throughout the years: radiating towards the icefall and perpendicular to the contour lines.

Profile X was established in 1995 and prolonged a year later to find out where the divide between the Matthes and the Llewellyn Glacier may be located. The profile extends some 8 km starting in the center of Matthes Glacier at Profile VIII and ends about 5.5 km beyond the Canadian border. The Divide is in the area of flag 15 and 16, where the orientation of the movement vectors changes towards the Llewellyn Glacier.

Profile XI is located 8 km beyond the Canadian border and some 3 km before the Llewellyn Glacier enters the narrow channel between F10 Peak and Sloko Ridge. Extending 4 km, it covers half the width of the glacier. The eastern end shows an erratic but not significant pattern of movement mainly directed towards the center of the

Llewellyn Glacier. Further to the west the velocity of the movement increases up to 15 cm/day; the orientation is northwest. The plateau around Mount Nesselrode and Mount Bressler contributes masses to the Llewellyn Glacier which are not monitored. An even higher velocity in continuation of the western end of the profile can be assumed.

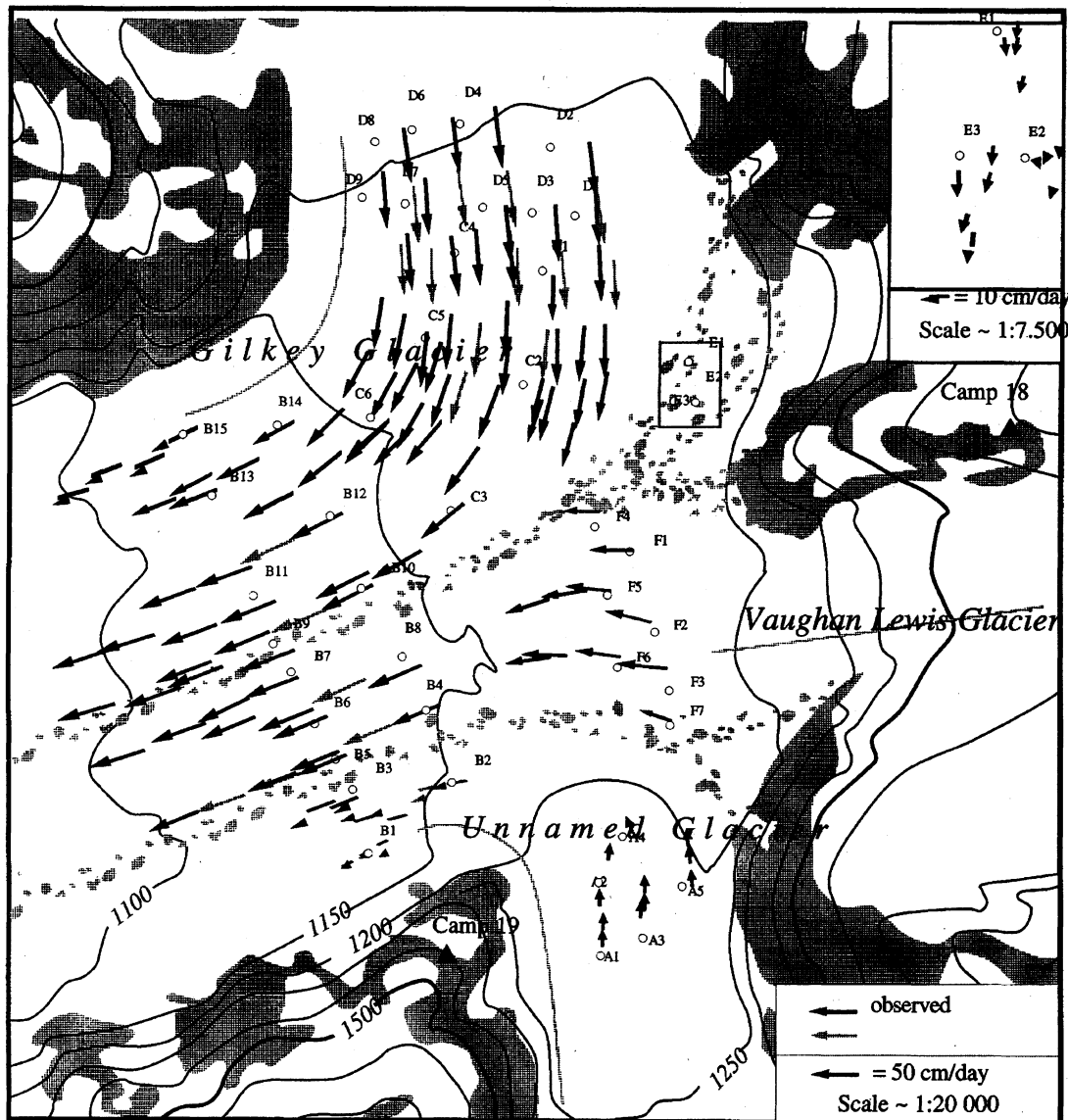
It is proposed to relocate Profile XI between F10 Peak and Sloko Ridge, where all the masses originating from the High Plateau have to pass through.

## **2.5 Gilkey Trench**

In 1990 six profiles with a total of 45 flags were distributed in the confluence area of the Gilkey Trench (Fig. 4). Here Gilkey Glacier is forced to change its flow direction from south to west within some 1.5 km, Unnamed Glacier changes its flow direction in a similar way from north to west in less than 1 km. The Vaughan Lewis Glacier is pushing in between after cascading down the icefall and forming wavebulges with amplitudes of about 25 m.

All profiles except for Profile E were set in double lines. Thus slope gradients, movement vectors and strain rates can be derived. Profile A was set on the Unnamed Glacier before its sharp turn, Profile B traverses the Gilkey Trench directly below the confluence of the three glaciers. Profile C consists of two longitudinal lines in the area of the bend of Gilkey Glacier. Profile D was placed on a transect above the Gilkey Glacier turn, Profile E is a single triangle below the icefall of the Little Vaughan Lewis Glacier, and Profile F was originally placed on the crests of the first and second wavebulges below the Vaughan Lewis Icefall. A significant number of flags could be found again year after year until 1995, permitting an almost complete and conclusive long-term movement record of the confluence area.





**Fig. 4: Movement pattern in the Gilkey Trench.**

Comparing the short-term “summer” velocities derived in August 1990 and the long-term yearly velocities, the differences are in general negligible. Therefore a uniform movement throughout the year can be taken as a conclusion.

The central part of Gilkey Glacier (Profile B flags 9-12, Profile C and D) is moving with an average velocity of about 50 cm/day. Since the profiles do not cover the margins of the three glaciers of the trench, definite conclusions about the decrease in velocity on both sides cannot be drawn. However, the numerous crevasses indicate a sudden decrease in velocity, so that a transition from parabolic to Block-Schollen flow can be assumed.

The Unnamed Glacier moves with an average velocity of about 12 cm/day in the area up-glacier the sharp turn. The equivalents of the points A1, A3 and A5 are the points B1, B2 and B3 (see Fig. 4). Remarkable is the increase of the velocity of these points: while the increase from A1 to B1 is only 4 cm/day, the increase from A3 to B2 is 16 cm/day and the acceleration from A5 to B3 is even 25 cm/day. At the same time the width of the Unnamed Glacier halves. The reason for the unequal flow velocities is obviously the higher velocity of the mighty neighbouring glaciers which carry along the small Unnamed Glacier.

The Vaughan Lewis Glacier shows the greatest velocity variations of all glaciers on the Icefield. Above the icefall maximum velocities of 30 cm/day are reported (see Profile IX in section 2.4), whereas in the icefall dramatic 5.7 m/day were observed [Rentsch et al., 1990]. In the lower section the icefall slows down to less than 1 m/day [Lang, 1993]. Points F1, F2 and F3 located on the first wavebuldge move with 40 cm/day, the points F4 to F7 on the second wavebuldge are slower by 15%. Points B4 and B8 on wavebulge No. 8 and points B5 to B7 on bulge No. 10 of the Vaughan Lewis Glacier have an average velocity of 50 cm/day. The width of the Vaughan Lewis Glacier diminishes from 750 m at the base of the icefall to 500 m at Profile B which may explain the increase in velocity.

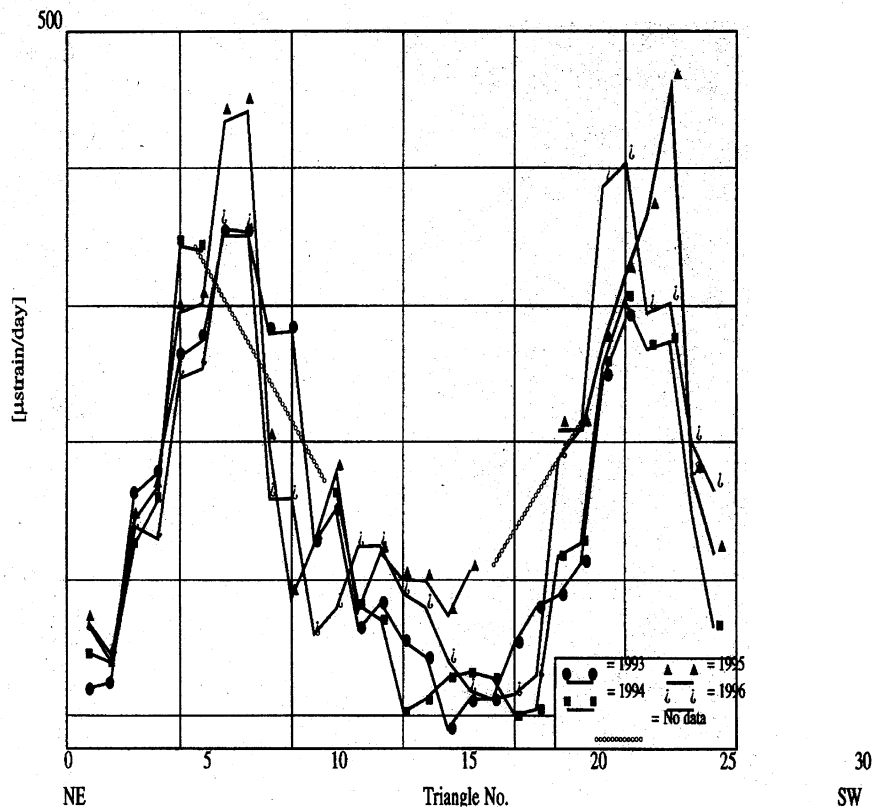
All the variations of the velocities do not provide a clue to the bed topography of the three glaciers. They are rather a result of the complex interaction of the three glaciers. Further conclusions could be drawn if the bed topography of the Gilkey Trench was reliably investigated by seismic or other means.

### **3. Strain Rates**

Strain rates can be derived from repeated observations of geometrical figures like triangles [Welsch, 1981]. Strain rate measurements are the geodetic contribution to stress and strain relations which are the basis for further glacier studies applying methods of continuum mechanics. A simple example is given in Daellenbach and Welsch [1990]. In the following section the results of strain rate measurements on the main Taku Glacier and in the Gilkey Trench are presented.

#### **3.1 Main Taku Glacier - Profile IV**

In 1994 the western end of Profile IV was extended so that 29 triangles instead of 25 can now be used for strain rate analysis. The distribution of the strain rates across the main Taku Glacier is consistent over the years. In Fig. 5 the maximum principle strain rates ( $\epsilon^1$ ) are shown.



**Fig. 5: Profile IV: maximum principal strain rates ( $e^1$ ), 1993 – 1996.**

The strain rates of the first two triangles at the eastern end are very small, indicating only little stress within the first 500 m. This is in accordance with the small increase of velocity in this area. Within the next 1 km the velocity is increasing with the consequence of maximum principle strain rates which are 10 times higher than before. This is made obvious by numerous shear crevasses. In the central area of Profile IV a more or less homogeneous movement is dominant indicated by strain rates which are as low as at the eastern end. Towards the western end the velocity slows down causing again a peak of maximum principle strain. The velocity and the strain rates at the western end show the disturbing influence of the ice flow from the basin between Shoehorn Mountain and Juncture Peak.

### 3.2 Gilkey Trench

The movement of Profile A on the Unnamed Glacier is very consistent. Thus the relatively small strain rates observed are to be expected.

The results of Profile B traversing the Gilkey Trench completely, reveal some peculiarities. The strains of the Unnamed and the Vaughan Lewis Glaciers are rather homogeneous but decreasing towards the Gilkey Glacier. Here compressional flow is

dominant extending far across the glacier. The strain of the last triangle of the profile is extensional. The crevasses beyond the profile may indicate that the extensional strain increases towards the bank of the glacier.

The triangles of Profile C cover the area where Gilkey Glacier has its 90° turn. Apart from the triangle which includes point C6, the strain values and the orientations of the strain ellipses are consistent with the uniform speed in this area.

The strain values of Profile D are astonishingly high and variable. Beginning from the eastern end there is a built-up of extensional flow, a sudden decrease in the center, and towards the corner again an increase of extension by more than a magnitude. Finally at the western end compressional flow dominates. So far there is no obvious explanation of this phenomenon.

The result found with Profile E is as expected: point E2 as the closest to the glacier's margin is the slowest, so that increasing extensional strain towards the faster moving center is regular.

Extensional strain in the flow direction of the Vaughan Lewis Glacier could be assumed for all the triangles formed by the points of Profile F. But it turns out to be true only for some triangles. Especially in the central part, where the curvature of the wavebulges is minimal, compressional flow is found.

The strain ellipses correspond reasonably to the crevasse pattern [Rentsch, 1997].

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# Adam Chrzanowski

## A Lifetime of Publications

(to 30 Nov.,1999)

### Papers Published in Refereed Journals

- Chrzanowski, A., C. Monahan, B. Roulston, and A. Szostak-Chrzanowski (1997). "Integrated monitoring and modelling of ground subsidence in potash mines." *Int. J. Rock Mech. Min. Sci.*, Vol. 34, No. 3/4, p. 621 (abstract) and CD Rom (full text).
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