COMPILATION OF A PRECISE REGIONAL GEOID

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PREFACE

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PREFACE

This unedited technical report is a reformatted version of a final contract report. That report was submitted to the Geodetic Survey Division, Geomatics Sector, Natural Resources Canada, Ottawa, on 12 June 1995.

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1. INTRODUCTION

In this contract, we were required to do the following:

1) Review the state of the art methodology for geoid height determination and error estimation;
2) Propose viable options to facilitate relative geoid height determination anywhere in Canada with errors not exceeding 10 cm;
3) Develop numerical procedures and computer software to calculate geoid heights in selected regions of Canada and compare results with independent determinations;
4) Implement procedures and software for geoid height error estimation and demonstrate their validity on practical examples;
5) Document work and recommend possible options in progress reports to be prepared for regular contract reviews;
6) Prepare full report on methodology and software at the completion of the contract.

During the course of our investigation, it became very clear that existing theories for geoid determination are not accurate enough to meet the contract requirements, i.e., to demonstrate that the geoid can be in fact determined with a decimetre accuracy. To compute the geoid to a decimetre accuracy, the theory has to hold to the one centimetre level; yet many of the approximations used in the existing theories, are likely to be good only to the one metre level, justifiable by the accuracy achievable at the time these theories were formulated. Consequently, we had to do much pioneering theoretical work, enjoyable but very time consuming, and were not able to complete the development of the theory and thus even the methodology for geoid computation. Problems yet to be solved or solutions tested include: the atmospheric attraction (condensation) effect, topographic density effect on the geoid, density effect on orthometric heights and, in turn, their effect on the geoid. Other problems of a more minor nature and possible alternative solutions to those opted for by us, are listed in section 9.

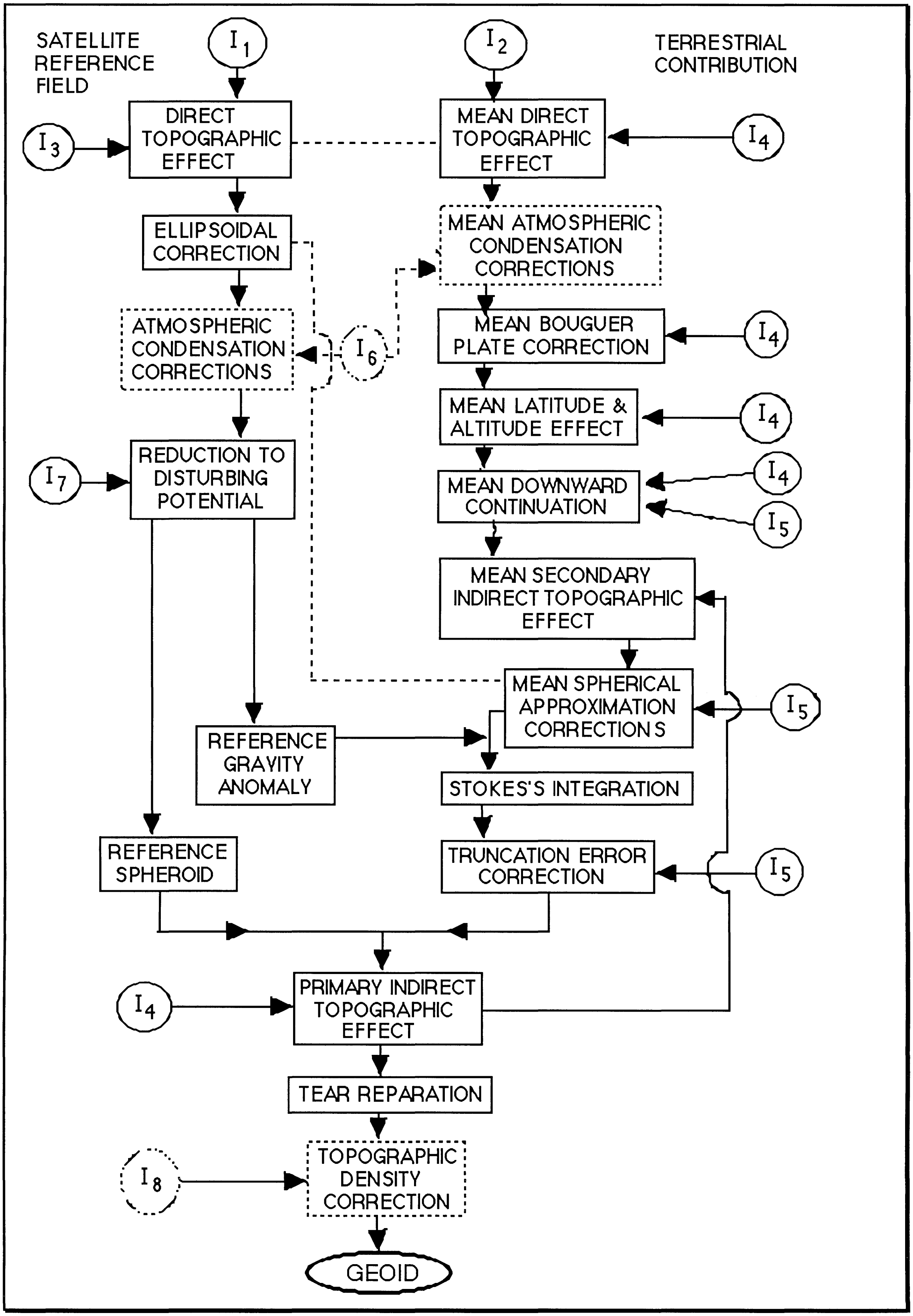
Most of the theoretical contributions described herein have already been published by us in the open literature, or manuscripts describing the contributions have been either accepted or submitted for publication - see section 10. We believe that this represents the best reviewing process for any research because the reviewing is done by an international group of peer referees. Thus, wherever appropriate, we refer to these papers, which make an external appendix to this report.

When formulating the theory for this report, we have continued along the lines of research embodied in our previous involvement with geoid work. What we report on here is basically a further development of our technique which we call the "Generalized Stokes's Technique" [Vaníček and Sjøberg, 1990; Vaníček et al., 1992], in combination with "Molodenskij's modification of the integration kernel" used by Vaníček et al. [1986] and the "Stokes-Helmert's scheme" investigated more recently by Vaníček and Martinec [1994]. There have been in the recent years many new ideas and developments proposed by different research teams from various countries. Thus a perfectly legitimate question may be asked: "Why did we not use any of these ideas and techniques in our approach?" The answer is: partly due to reasons described above, but mainly because most of the other teams are actually interested in the quasigeoid [Molodenskij et al., 1960], or "free-air geoid" (an equipotential surface of the external gravity field) [Vermeer, 1994]. The applicability of most of the developed methods to our goal, i.e., the determination of an equipotential surface of the internal gravity field, ranges from obscure to impossible. This statement should not be understood as a judgement on the merit of the alternative approaches.

Because, in spite of the time extension, we ran out of time (and of course out of funds) before we could solve all the theoretical problems, we have not attempted to compile the geoid over the whole of Canada. With some of the problems in the methodology still outstanding, it would not have made much sense. Instead, we have concentrated on a limited area 5 by 10 degrees (latitudes 49 to 54 degrees North, longitudes 236 to 246 degrees East) covering the south-eastern part of British Columbia and south-western part of Alberta. This area, specified by the contract Scientific Authority, Dr. A. Mainville, contains an important part of the Rocky Mountains and thus represents a challenging ground for testing the performance of the developed technique. The geoid in this area was computed on a 5' by 5' geographical grid. Thirteen GPS stations, whose orthometric heights were determined also by spirit levelling, were made available to us for comparisons.

2. GENERALIZED STOKES-HELMERT SCHEME

In this section we show the flow of the individual operations on both the satellite reference field and the terrestrial data and how these operations fit together. The following flowchart shows the whole methodology. Note that the boxes in dashed lines denote those operations that have not yet been implemented. We also point out, that operations relating to the various error estimation algorithms are not shown on the flowchart; the diagram would become too clattered if we tried to show these as well.


SATELLITE REFERENCE FIELD
TERRESTRIAL CONTRIBUTION
I_1
I_2
I_3
I_4
DIRECT TOPOGRAPHIC EFFECT
MEAN DIRECT TOPOGRAPHIC EFFECT
ELLIPSOIDAL CORRECTION
MEAN ATMOSPHERIC CONDENSATION CORRECTIONS
ATMOSPHERIC CONDENSATION CORRECTIONS
I_6
MEAN BOUGUER PLATE CORRECTION
MEAN LATITUDE & ALTITUDE EFFECT
REDUCTION TO DISTURBING POTENTIAL
I_7
MEAN DOWNWARD CONTINUATION
I_5
MEAN SECONDARY INDIRECT TOPOGRAPHIC EFFECT
MEAN SPHERICAL APPROXIMATION CORRECTIONS
REFERENCE GRAVITY ANOMALY
STOKES'S INTEGRATION
REFERENCE SPHEROID
TRUNCATION ERROR CORRECTION
PRIMARY INDIRECT TOPOGRAPHIC EFFECT
TEAR REPARATION
I_8
TOPOGRAPHIC DENSITY CORRECTION
GEOID

In this diagram, the circles stand for input. The individual input information is denoted thus:

I_1 - the first 20 by 20 potential coefficients of the satellite determined reference field;

I_2 - mean incomplete Bouguer anomalies for 5' by 5' geographical cells. We note that the production of mean 1° by 1° incomplete Bouguer anomalies is not shown on the flowchart. These, as well as the mean 1° by 1° corrections are evaluated simply by taking the averages of the 144 5' by 5' means;

I_3 - global topography in a spectral form (spherical harmonics);

I_4 - local detailed topography. The so called "1km by 1km topography" was used wherever available, the 5' by 5' topography was used everywhere else;

I_5 - global gravity field model to whatever degree and order (smaller than 360 by 360) is needed in the particular correction evaluation;

I_6 - global atmospheric density model - not used in our computations;

I_7 - normal gravity field and the corresponding reference ellipsoid to which the final geoid is to be referred;

I_8 - topographical density model - not used in our investigations.

The individual steps shown in the flowchart will now be discussed in detail in the following sections. It should be clear from the headings of these sections just what is described where.

3. REFERENCE GRAVITY FIELD AND REFERENCE SPHEROID

As explained by Vaníček and Sjøberg [1991], the "Generalized Stokes Technique" consists of taking a higher than second degree gravity field and the spheroid generated by its equipotential surface of a prescribed potential value, as the reference field and the reference surface. This is an obvious generalization of the classical Somigliana-Pizzetti's concept of normal field of second degree and the reference ellipsoid associated with it. We have shown [ibid] that practically all the relations used in the classical Stokes technique are valid even for this higher order reference field and reference spheroid, except for the Stokes function itself (cf. section 4).

The advantage of using a higher order reference field has been recognized by most people who work with the earth gravity field and with the geoid in particular. Some researchers opt for using a reference field of an order as high as possible. The price one has to pay for a higher than some 20 by 20 reference field is that such a (global) field is by necessity constructed using the same terrestrial gravity data that one wants to use in computing the geoid referred to this reference field. Thus the same data are used twice, often without a proper account being taken of so introduced correlations - see, e.g., [Vaníček and Sjøberg, 1991, eqns. (72) and (73)]. We thus prefer to use a reference field derived from independent data, namely satellite orbit analysis and have been doing it since the late 70's [John, 1980]. The additional advantage of a satellite-derived field is its better spatial homogeneity compared with a combined field.

Once the decision to use such a field is made, one cannot go too high with its degree because the pure satellite-derived field is reliably known only to a degree and order 20 by 20, except for resonant frequencies [Vaníček and Krakiwsky, 1986]. Thus our choice of using the purely satellite-determined reference field compels us to considering only relatively low degree and order fields and for the purpose of this investigation we decided to stay with our original choice of 20 by 20 [Vaníček et al., 1986]. We have also decided to use the new European global satellite model GRIM4-S4P [Schwintzer, 1993] up to degree and order 20. Its plot for Canada (after the "Helmertization" described in the next paragraph) is displayed in **Figure 3.1**; the values range between -47.60 and +41.94 metres. This field appears to have the smallest error (average error for Canada) of the new satellite fields that have become recently available, 11 cm compared to, for instance 30 cm for GEM-T3 [Lerch et al., 1992].

In the context of the Stokes-Helmert computation scheme used by us, it is necessary to "Helmertize" the (satellite-derived) reference field by subtracting from the real field the direct topographical effect V on potential, as explained in [Vaníček et al., 1994(a)]. The direct topographical effect on the reference spheroid [ibid, eqn. (20)] for the whole of Canada is shown in **Figure 3.2.** We note that the effect is relatively small; its range for the whole of Canada being between -9 and +25 centimetres. The direct and secondary indirect topographical effects on the (satellite-derived) reference gravity have to be also considered [ibid]. The former, for the territory of Canada, is shown in **Figure 3.3**, with the range being between -258 and +549 μGal. The latter effect is even smaller, ranging between -27 and +77 μGal for the whole of Canada and has not been considered in our computations. Its effect on the geoid would be of the order of a few millimetres in our area of interest.

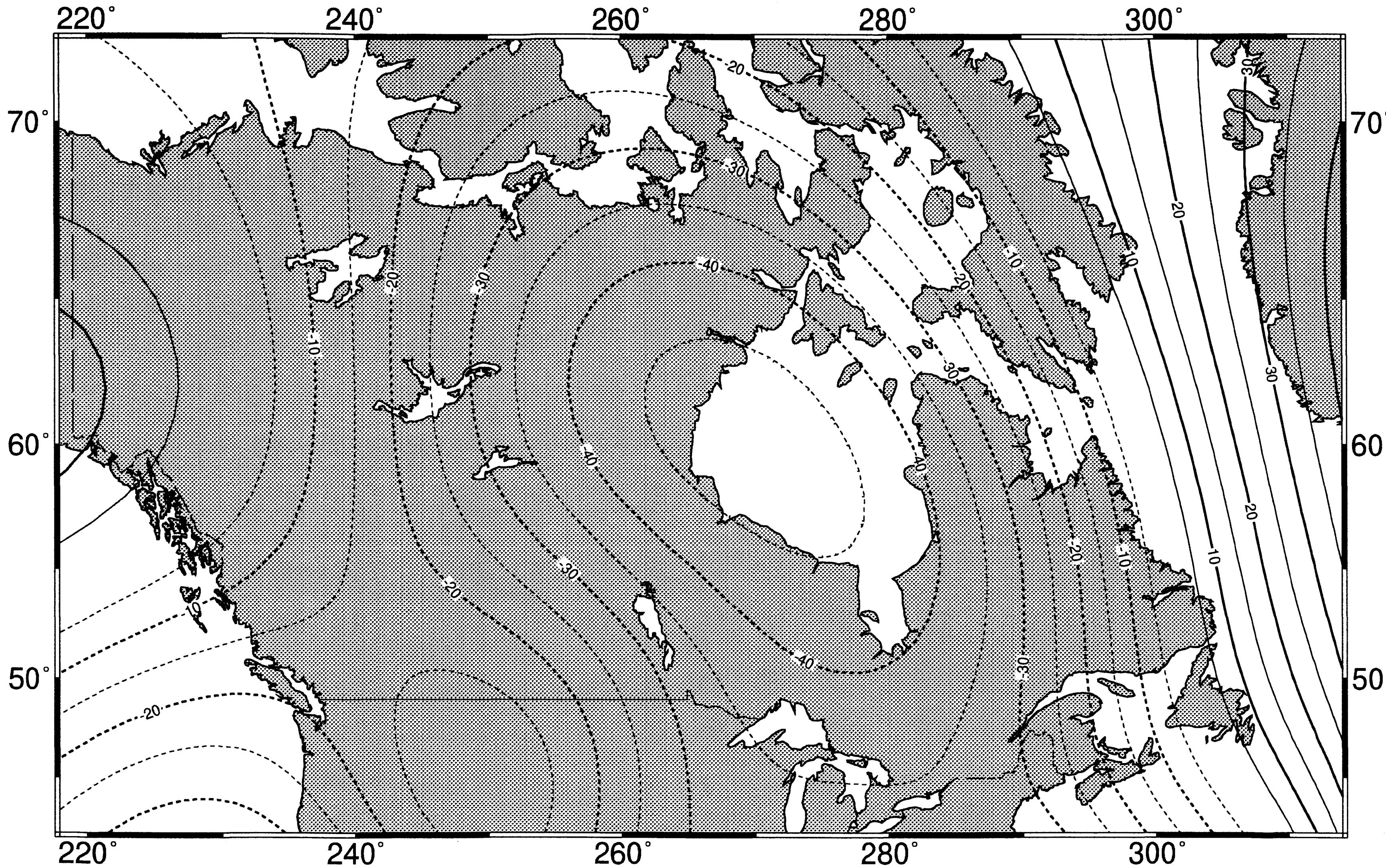


Figure 3.1: Helmert spheroid of degree and order 20 from GRIM4-S4P referred to GRS80 (in metres)
Contour interval = 5m

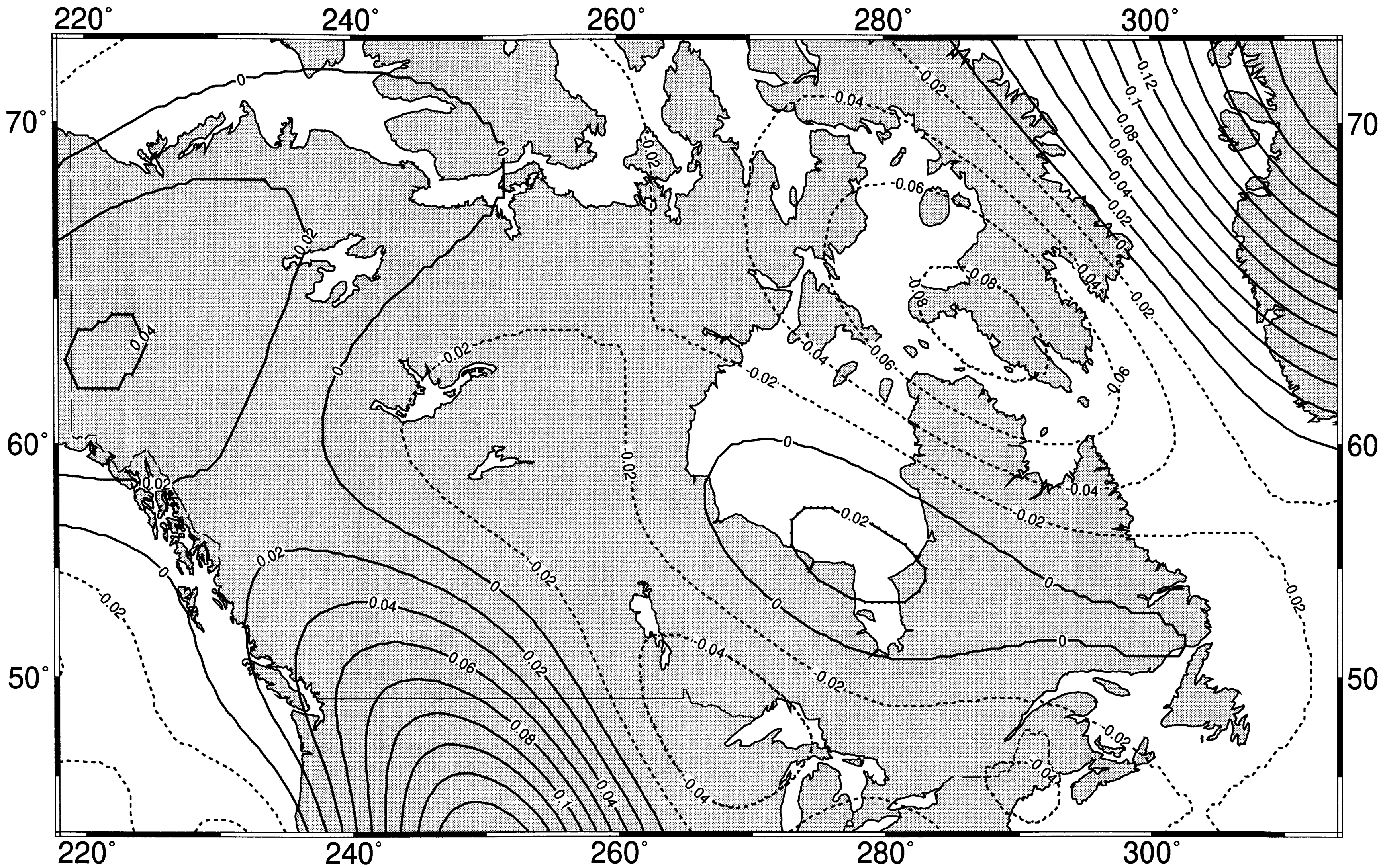


Figure 3.2: Direct topographical effect on the reference spheroid (in metres)
contour interval = 0.02 m.

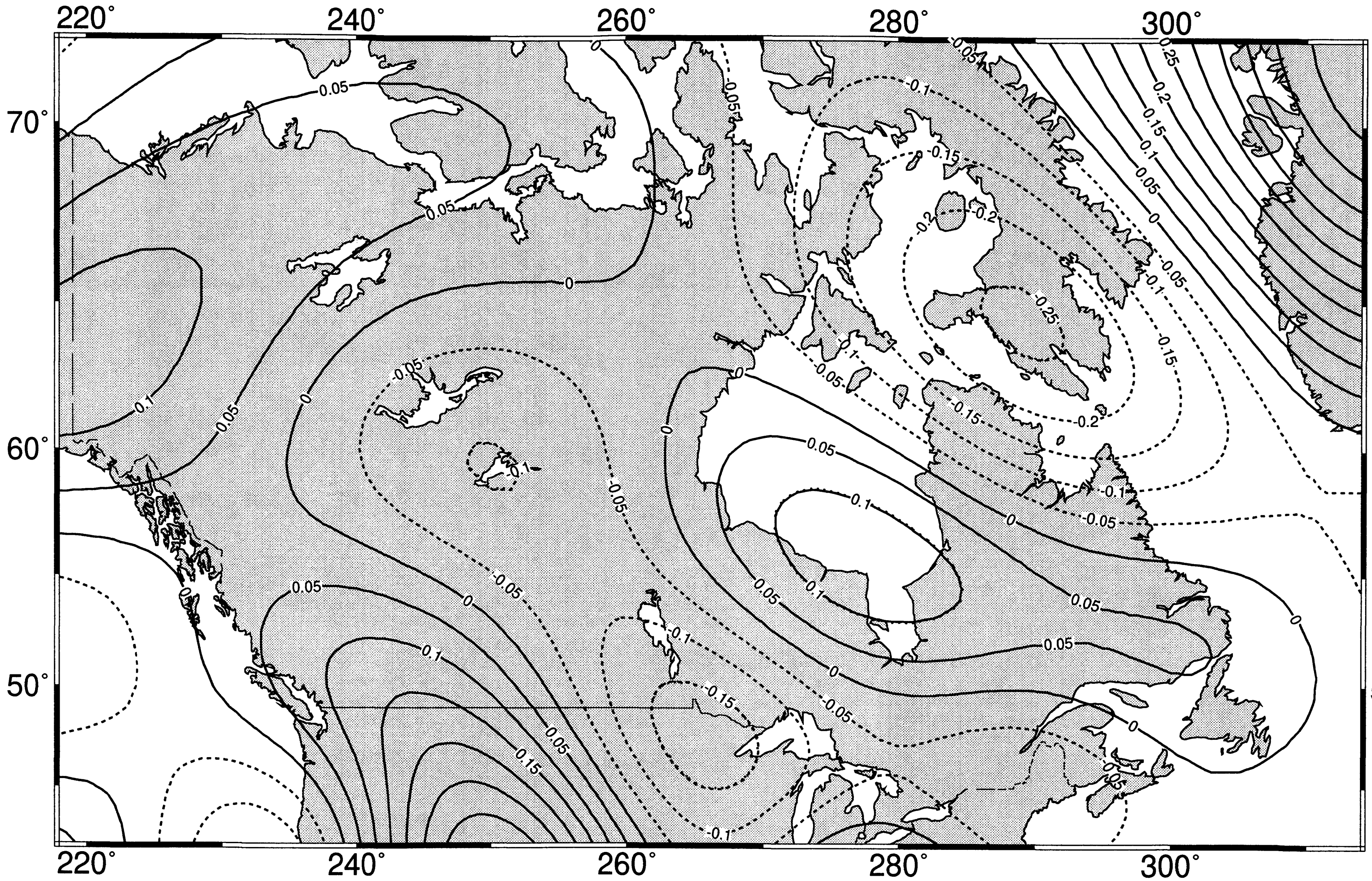


Figure 3.3: Direct topographical effect on the reference gravity (in mGals) contour interval = 0.05 mGal.

The Helmert reference potential W^h has to be converted into Helmert's disturbing potential T^h by subtracting from it the desired Somigliana-Pizzetti's ellipsoidal (2nd degree) normal field. The equations for this conversion are given in [Vaníček and Kleusberg, 1987, eqn. (22) to (25)] and the conversion is done to refer the estimated quantity (T^h) to a desired ellipsoidal (normal) reference field. Our choice here was the GRS 80 normal field and its reference ellipsoid - our results thus refer to GRS 80. The resulting expression for T^h in spectral form must then be reduced to the geoid by applying the ellipsoidal correction [Vaníček et al., 1994(a), eqn. (27)] that arises from the fact that the radial functions in the harmonic series must refer to the geoid rather than to a sphere. The amplitude of this correction is somewhat larger in our latitudes and for Canada it ranges between -88 and +65 centimetres. The correction values for Canada are plotted in **Figure 3.4**.

Turning now to errors associated with the reference field, it is the commission error that we are, of course, interested in. The commission error can be evaluated from the standard deviations of potential coefficients following the procedure described in [Vaníček et al., 1986, eqn. (2.36)]. From the standard deviations of GRIM4-S4P's [Schwintzer, 1993] first 20 by 20 potential coefficients, we obtain the estimated global mean commission error equal to 11 cm. From the standard deviations of the potential coefficients we can also compute the commission error (standard deviation) of the reference gravity as follows

$$\left(\Delta g_g^h\right)_{20} \approx \frac{1}{R}\sum_{n=2}^{20}(n-1)\sum_{m=0}^{n} T_{nm}^h Y_{nm} \ . \tag{3.1}$$

Applying the law of propagation of errors and assuming that there is no longitudinal variation in the potential coefficient standard deviations σ, i.e.,

$$\forall_{n,m}: \qquad \sigma_{n,m} = \sigma_n \ , \tag{3.2}$$

we get the following expression for the global mean value

$$\mathrm{mean}\left(\sigma^2_{\Delta g^{ref}}\right) \approx \frac{G^2 M^2}{R^4}\sum_{n=2}^{20}(2n+1)(n-1)^2\sigma_n^2 \ . \tag{3.3}$$

The value of the global mean for the GRIM4-S4P model is equal to 227 μGal. The mean value in Canada is 265 μGal.

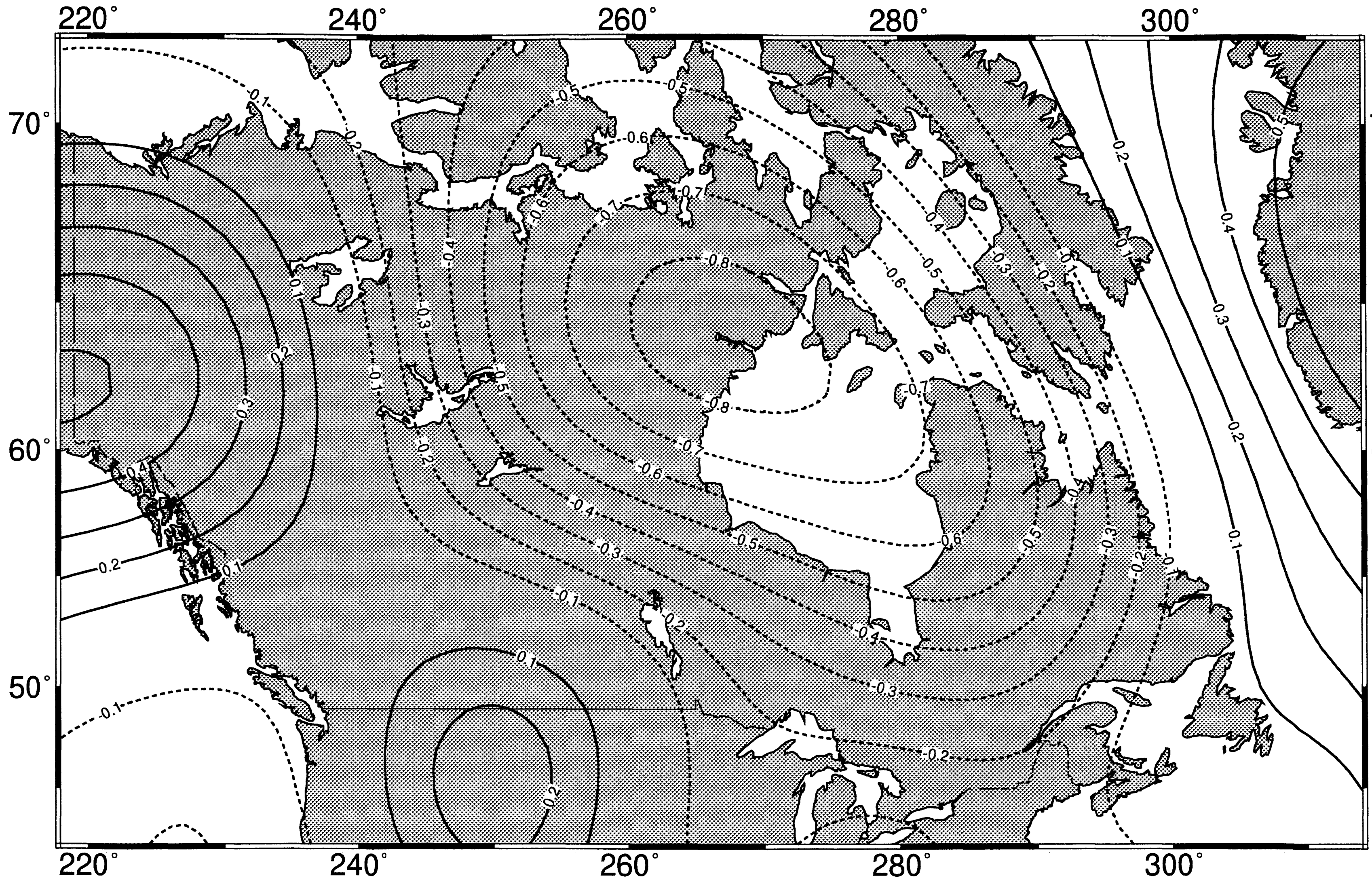


Figure 3.4: Ellipsoidal correction to the reference spheroid (in metres)
Contour interval = 0.1m

To conclude this section, let us mention that no attempt has been made to implement the atmospheric attraction correction to the reference field. This correction was investigated by Harrie [1993], but has not been implemented yet.

4. STOKES'S INTEGRATION

The numerical integration technique used here is essentially the same as that used in our 1986 and 1990 geoid compilation [Vaníček et al., 1986; Vaníček et al., 1990]. The notable difference is the treatment of the innermost zone integration. When looking into this numerical problem, we realized that only a few percent of computation points have enough point gravity anomalies in their innermost zone (10' by 10') to warrant the integration procedure that uses point values. Also, the (local) increase of accuracy gained by invoking this integration procedure is minimal in most of the cases. We have thus decided to eliminate this procedure systematically and by doing so, to eliminate the necessity of working with the point anomaly files at all. The 5' by 5' mean anomalies are now used even in the innermost zone integration, but the process is still kept different (more accurate) from the integration in the inner zone [ibid]. This leads to a substantial saving of computer processing time. The point anomaly procedure can be resurrected in the future when more point anomalies become available to make it worthwhile.

Another improvement of the numerical integration process as implemented in our GIN program concerns the "tears". In our numerical integration process, the batch of 5' by 5' mean anomalies needed in the inner and innermost zone integration, is replaced by a new batch whenever the border line between the 1° by 1° mean anomalies is crossed [Vaníček et al., 1986]. This discontinuity causes tears along the 1° boundaries in the inner and outer zone integration results. These tears in the geoid solution can and are now being repaired by distributing the perceived geoid height difference (between two adjacent points that belong to two adjacent regions where different batches of 5' by 5' mean anomalies are used) to 4 points along the latitude or longitude profiles on each side of the 1° break. The following algorithm has now been implemented:

i) denote geoidal height values on one side of the break by $N_i, N_{i-1}, N_{i-2}, \ldots$, on the other side of the break by $N_{i+1}, N_{i+2}, \ldots$, indicating that the break occurs between N_i and N_{i+1};

ii) compute the third difference Δ^3 as

$$\Delta^3 = (-N_{i-1} + 3N_i - 3N_{i+1} + N_{i+2}) / 2 = D \; ; \tag{4.1}$$

iii) test if D is larger than a selected threshold value, e.g., 5cm. If it is, then this is an indication that a tear had developed and 4 values before and 4 values after the break are corrected;

iv) correct the 4 values immediately following the break by adding to them +0.395D, +0.222D, +0.100D and +0.025D respectively. The 4 values immediately preceding the break get the same corrections, but with negative signs. These corrections follow a "quadratic bent".

We found out that setting the threshold value to 0 , i.e., smoothing the geoid indiscriminately across all the 1° lines, works the best.

Another modification to our GIN program that we have implemented is an added flexibility to select the area of integration at will. It is now possible to use the GIN program in a specified area and supply only the 5' by 5' and 1° by 1° mean anomalies pertaining to that area.

Following the work by Martinec [1993], we adopted the approach, whereby we no longer neglect the error caused by the Stokes integration truncated to a spherical cap of a specific radius, i.e., the <u>truncation error</u>. We now evaluate the truncation error from a global gravity model; the new combined European model GFZ93a [Gruber and Anzenhofer, 1993], complete to 360 by 360, is used in this investigation. It turns out that to compute the truncation error for the 6° spherical cap to 1cm accuracy, only the first 120 by 120 degrees and orders may be used. The range of the truncation error in Canada is between -24 and +36 cm (to an internally estimated accuracy of 3 mm!) and its plot, is shown in **Figure 4.1**.

We have elected to stay with the 6° integration cap, which we have used in all our computations till now, having had no compulsion to change it. Again, for reasons explained by Vaníček and Krakiwsky [1986], the spheroidal Stokes function is used and the truncation error minimized by Molodenskij's modification [Vaníček et al., 1986]. So modified a kernel is not "blind" to low frequencies in the integrated anomaly [Vaníček and Sjøberg, 1991, eqn. (43)] and care must be taken to make sure that the anomalies are the least possible contaminated in the low frequency domain - see below. (Interestingly, Martinec [1993] found that the truncation error of a Molodenskij-like modified spheroidal kernel contains only frequencies above the wave-number equal to the maximum wave-number of the reference field, while Vaníček and Sjøberg [1991, eqn. (42)] show presence of all frequencies.) There is generally still a room for improvement as far as the choice of integration kernel is concerned. The "strict frequency separation modification" discussed in [ibid], should be seriously considered.

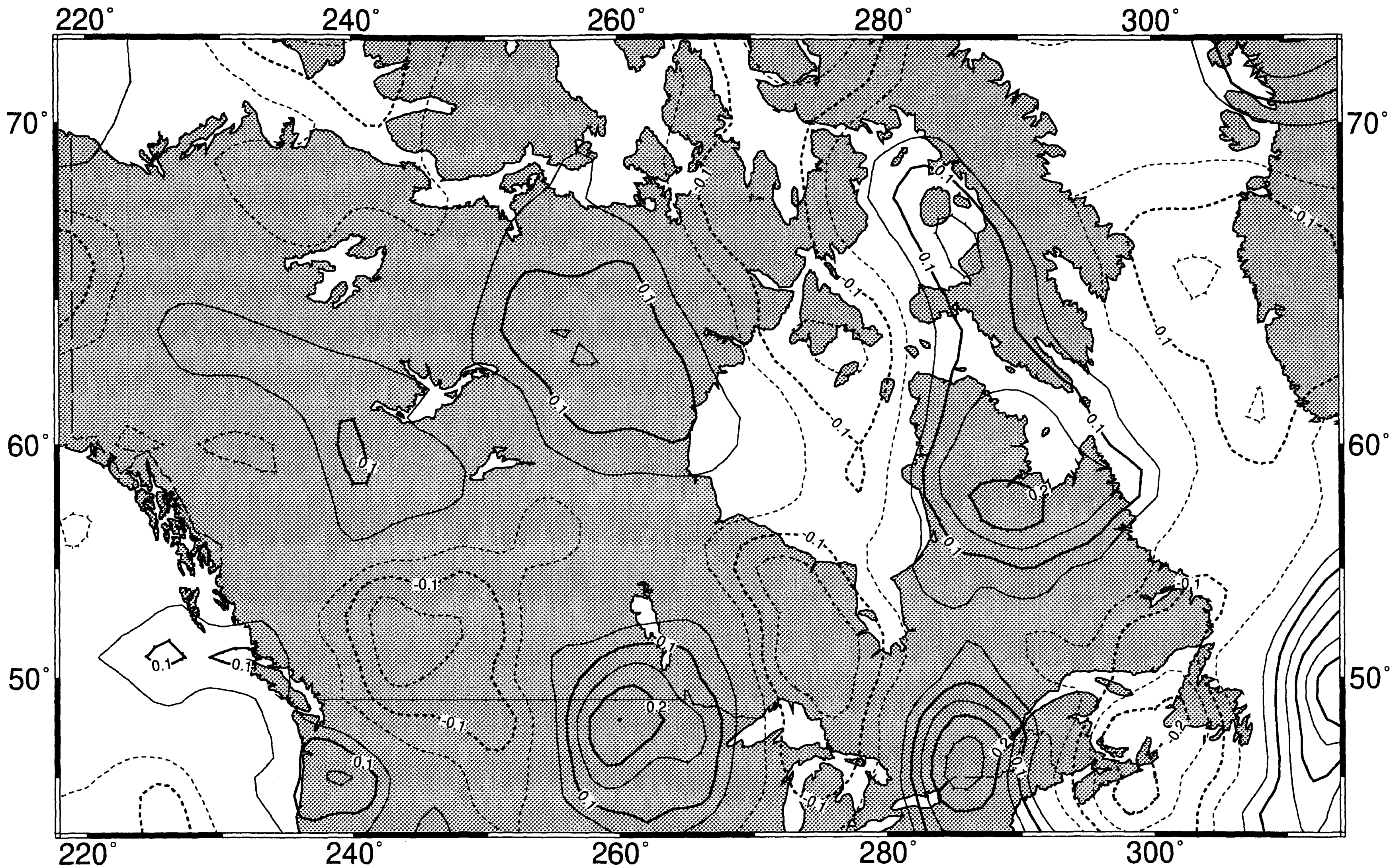


Figure 4.1: Truncation error of the modified Stokes's integral truncated at spherical cap of radius 6 degrees (in metres)
Contour interval = 0.05m

The question that comes to mind at this point is: "Why to minimize the truncation error when it can be evaluated?" The minimization must be employed to ensure that the available global models are accurate enough to use for the actual evaluation of the error, i.e., that they give essentially the same results within reasonable limits. As an illustration, we give here a plot of differences in metres - **Figure 4.2** - between the truncation error evaluated from the GFZ93a and OSU91a [Rapp et al., 1991] global models. Even with the minimization of truncation error implemented, the differences range between -5 and + 6 centimetres, large enough values to compete with the random noise in measurements. This error will tend to become less significant with an improvement of global potential models.

Do we have to subtract the 20 by 20 reference field from the terrestrial anomalies before using them in the Stokes integration? Yes! Since the modified spheroidal Stokes kernel is <u>not</u> blind to low frequencies a reasonable effort must be made to drive the amplitudes of the low frequency constituents to zero. As we shall see in the next section, the evaluation of the "residual" Helmert anomalies on the geoid is carried out in a rigorous way so that, in absence of measuring errors, the terrestrially determined anomalies on the geoid match the satellite determined anomalies in the low frequencies. But there is indeed a potential source of error here and in the next iteration of Canadian geoid compilation a different modification should be tested as stated above.

5. MEAN HELMERT ANOMALIES

Since the Stokes integration is done numerically, it is the <u>mean Helmert anomalies</u> that are needed for the Stokes integration. In the innermost and inner integration zones, 5' by 5' mean anomalies are used and it is these anomalies that we shall talk about here and call them only "mean anomalies". The 1° by 1° mean anomalies, used for the outer zone integration [Vaníček et al., 1986], are obtained simply by averaging over the 144 5' by 5' mean anomalies. Thus, in all our computations, we need only the 5' by 5' mean anomalies and all the corrections that have to be applied to the mean anomalies (supplied to us by the GSD personnel) must be corrections to mean anomalies, i.e., <u>mean corrections for the 5' by 5' cells</u>. This is advantageous in so far that the mean corrections are naturally smoother, but disadvantageous from the point of view of computation. In case the correction values vary widely within a cell, the mean correction has to be evaluated by actually averaging point corrections within the cell.

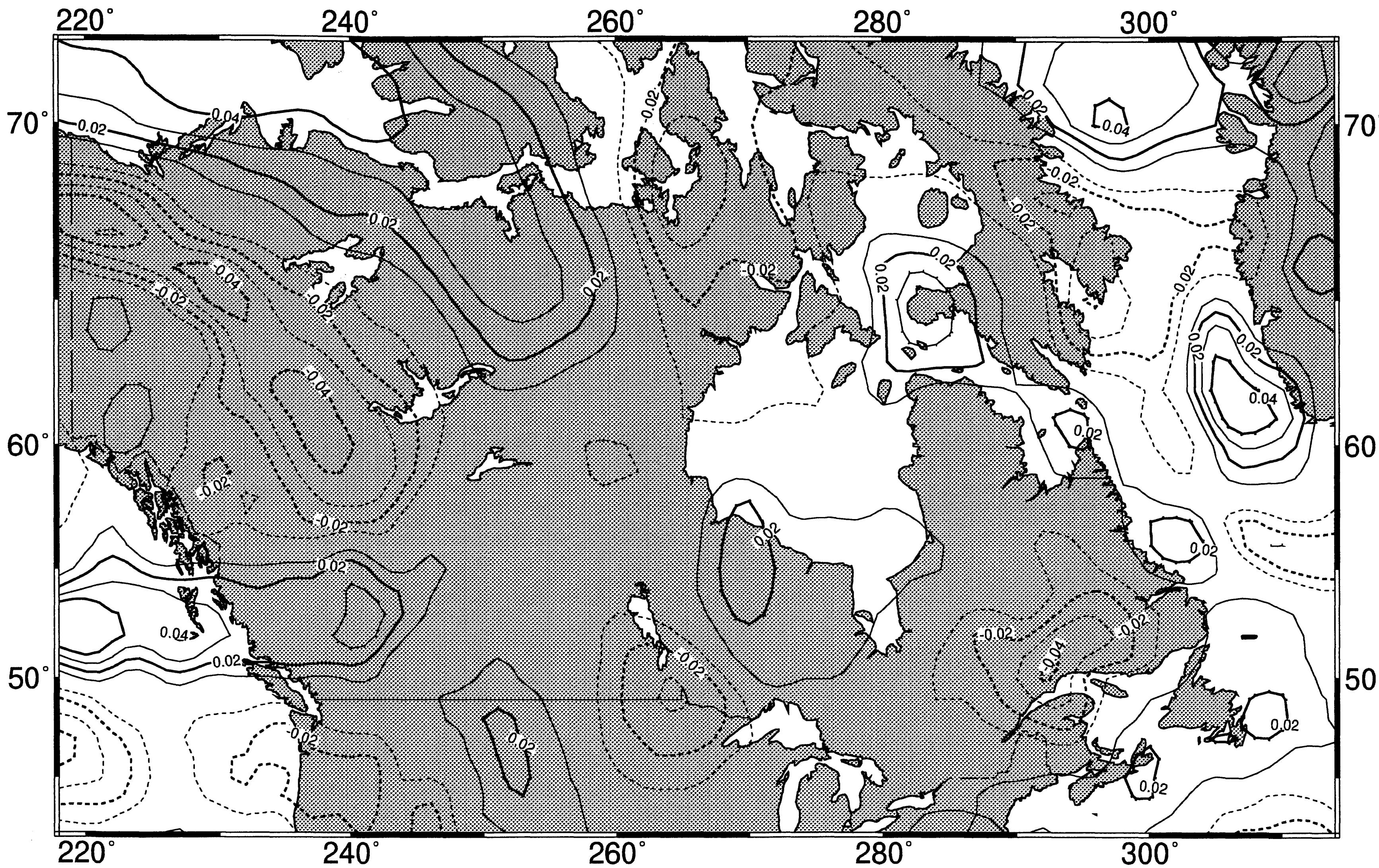


Figure 4.2: Difference in the truncation error estimates from GFZ93a and OSU91a (in metres)
Contour interval = 0.01m

It was agreed in March 1994 [Véronneau, 1994], that the mean anomalies prepared for us by the GSD personnel would be the mean incomplete Bouguer anomalies computed from the following formula

$$\mathrm{mean}(\Delta g_t^B) = \mathrm{mean}(g_t^* - 2\pi G\rho_0 H + 0.3086\mathrm{mGal/m}\ H - \gamma_0), \tag{5.1}$$

where g_t^* is the observed gravity value at the earth surface corrected for atmospheric attraction effect. Note, that no terrain correction or the curvature effect are applied. On the other hand, the mean Helmert anomaly we need, is given by [Vaníček and Martinec, 1994, eqn. 39]:

$$\mathrm{mean}\ (\Delta g_g^h) = \mathrm{mean}\ (g_t^* + \left[\frac{\partial V}{\partial r}\right]_t + |\mathrm{grad}(\gamma)|_0\ H + \left[2\frac{V}{R}\right]_g - Dg + D^s - \gamma_0), \tag{5.2}$$

where all the symbols are used in the same sense as in the cited paper: the second term on the right hand side is the direct topographical effect (DTE) on gravity at the earth surface, the fourth term is the secondary indirect topographical effect (SITE) at the geoid, Dg is the downward continuation of Helmert gravity disturbance (cf. section 6), D^s is the spherical approximation correction [ibid, eqn. 29] and the third term can be, to a sufficient accuracy, written as

$$|\mathrm{grad}(\gamma)|_0\ H \approx 0.3086\mathrm{mGal/m}\ H + Le + Ae. \tag{5.3}$$

Here, Le stands for the "latitude effect" on normal gravity gradient (described in [ibid], by eqn. 22, which contains both the first and the second terms on the right hand side of the above equation) and Ae stands for the "altitude effect" on normal gravity gradient [ibid, eqn. 37].

The transformation formula between the mean simple Bouguer anomaly supplied to us and the mean Helmert anomaly we need in our computations, is thus as follows

$$\mathrm{mean}\ (\Delta g_g^h) = \mathrm{mean}\ (\Delta g_t^B) + \mathrm{mean}\ (2\pi G\rho_0 H + \left[\frac{\partial V}{\partial r}\right]_t + Le + Ae + \left[2\frac{V}{R}\right]_g - Dg + D^s). \tag{5.4}$$

We note that both the DTE and SITE, depend on the kind of Helmert condensation we prescribe. For the purpose of this contract, we had decided to use the condensation that preserves the mass, for which the Helmert model earth has the same mass as the real earth. For the discussion of this point see Wichiencharoen [1982] or Vaníček et al. [1994(a)]. Let us just point out, that the expression for the DTE under this condensation prescription is given in [Martinec, 1993, eqn. (4.22)]. The equation may be understood as reflecting the roughness of the terrain.

Martinec et al. [1994(a)] have shown that, unfortunately, the usual isotropic and homogeneous integration kernel obtained through the Taylor development of the Newton integral and used by many geodesists for computing this roughness term, is not good enough when heights are densely sampled - as they must be if the geoid is to be computed to a 1 centimetre accuracy. In addition, the integration extends all over the world but, fortunately, the (new non-homogeneous and unisotropic) integration kernel tapers off rather rapidly so that the integration can be limited to a spherical cap of a manageable radius. From numerical experiments, we had established that a spherical cap of a radius of $2.5°$ gives a sufficient accuracy of a few tens of μGal. To speed up the computations, we use 2 integration zones: the inner zone, extending to a radius of $40'$ and the outer zone from $40'$ to $2.5°$. In the inner zone we use the heights on the 1 by 1 km grid, whenever these more densely sampled heights were available. In the outer zone, we use the heights given on the 5' by 5' grid.

The value of the integral depends strongly on the sampling step for heights as shown by Martinec et al. [1994(a)]. The grid step for heights used in this study is certainly not dense enough to ensure adequate accuracy in the DTE for the "1 centimetre geoid" in the mountains. The height sampling step in the mountains should be further reduced (to 100 metres? to 30 metres?) for the evaluation of the ultimate geoid in Canada.

The SITE is nothing else but a re-scaled primary indirect effect (PITE) on Helmert's co-geoid - see [Vaníček and Martinec, 1994, eqn. (40)]. Denoting the PITE on the Helmert co-geoid by V_g/γ, cf. section 7, then

$$\text{SITE} \approx 2\gamma/R \ \text{PITE} \ , \tag{5.5}$$

with a sufficient accuracy. It is thus advantageous, to compute the SITE simply from the PITE. For computing the PITE see section 7.

The derivations above have been all done for the "total" mean Helmert anomaly. Yet, our approach is that of generalized Stokes variety, as noted above, where only the high frequency part of mean Helmert anomaly on the geoid, i.e., $\text{meam}[\delta\Delta g_g^h]$, is used. So how should this problem be dealt with? In fact, the reference field is subtracted from Δg_g^h, i.e., from the Helmert anomaly reckoned <u>on the geoid</u> and all we have to worry about is to produce the reference Helmert gravity anomaly on the geoid. This we have already done in section 3: the

DTE, SITE and spherical approximation correction, called the elliptical correction in the context of the reference field, have already been discussed.

The mean values of the above corrections for the 5' by 5' cells, called for by our formulation can be replaced by point values (for the centre of the cell) if the correction is sufficiently smooth (long wavelength). This is the case with the spherical approximation correction D^s , as can be seen on Figure 8.4. The mean values of the Bouguer plate correction ($2\pi G\rho_0 H$) and the Le and Ae corrections are obtained simply by evaluating these corrections for the mean height mean(H) of the cell. The DTE and SITE should be, of course, averaged from point values within the cell. This has not been done in this study for a lack of time and mainly for a lack of financial means. The production of both the meanDTE and the meanPITE , needed as the first step to produce the meanSITE , is very computer time intensive and would probably require the use of a supercomputer to accomplish successfully. The evaluation of the mean downward continuation correction Dg is treated in section 6 and we will not discuss it here.

Our software produces standard deviations of the computed point geoidal heights, through a simple error propagation of standard deviations of mean anomalies [Vaníček et al., 1986]. These latter standard deviations, computed from the expression developed by Marc Veronneau (and found by us to be correct) have been supplied to us by the GSD. We shall not discuss them here. We should mention however, that we feel the errors of all the applied corrections are significantly smaller than the error in the mean anomaly and can thus be neglected. This point though, may require further investigation. At present, we do not consider the contribution to the (high frequency) mean Helmert anomaly error due to the uncertainty of the (low frequency) reference field; it is very highly spatially correlated - as a matter of fact it is almost constant - and its introduction would require computations involving the correlation function of the reference field, which our software is not designed to handle.

6. DOWNWARD CONTINUATION OF MEAN HELMERT'S GRAVITY ANOMALIES

As has been experienced by various researchers, the downward continuation correction to gravity is a very difficult one to formulate - many have attempted and failed. It has been a very elusive quantity even in the Molodenskij concept, where downward continuation of external field is called for. In our previous geoid compilation, we assumed that this correction is equal to zero. This caused an exchange of opinions with Wang and Rapp [1990] and Sideris and Forsberg [1990] clarified finally in our paper Martinec et al., [1993] and acknowledged by Dr. Wang. The results presented here thus represent our attempt to do a better job this time around.

The theory of our approach to the problem and the numerical results for our area of interest are described in [Vaníček et al., 1994(b)]. It has turned out, we think, that the problem can be attacked more easily in the context of Stokes-Helmert model than in the context of Molodenskij model. Even though we have not proved the convergence of our formulation theoretically, it is encouraging to see that the numerical process converges rather nicely in both investigated norms, yielding reasonable values. It appears fairly certain that the averaging process involved in producing the mean 5' by 5' anomalies is a natural smoothing process which ensures the existence as well as the uniqueness of the solution even in very rugged terrain. In all probability, the same conclusion could be reached for mean anomalies computed for much smaller geographical cells. Since only mean anomalies are used in the solution of the boundary value problem of geodesy, it then becomes pointless to worry about possible non-existence of solution for point value anomalies and/or for anomalies given by a continuous prescription.

The differences between mean Helmert's anomalies on the earth surface and on the geoid - the downward continuation of mean Helmert's anomalies $D\Delta g^h$ - are surprisingly large, reaching well over 100 mGal in both negative and positive senses, and shown in Fig.8.5. Their character is, however, very short wavelength and after a convolution with (modified spheroidal) Stokes's function one can see that they contribute to the Helmert co-geoid but a few decimetres, at most 90 cm in the Canadian Rockies [Vaníček et al., 1994(b), Fig. 9], to be precise. Interestingly, the contribution due to the downward continuation is positive for all the points in the area. This is, of course, a natural consequence of the fact that the Helmert disturbing potential T^h is harmonic between the geoid and the earth surface [Vaníček and Martinec, 1994]; hence T^h must increase downward from the earth surface along every vertical.

The evaluation of the downward continuation is a very computationally demanding process. The main reason for this are the very large dimensions of the systems of equations one has to deal with. These dimensions depend on the size of the area one wants to compute the effect for. For future use, various schemes can be designed and tested, to cut down on the computational requirements.

7. TRANSFORMATION OF HELMERT'S CO-GEOID INTO GEOID

As the final step, the Helmert co-geoid must be transformed into the proper geoid by adding to it the primary indirect topographical effect (PITE). The expression for this effect was derived by Martinec and Vaníček, [1994(b), eqn.(50)] for the topographic column average condensation technique. For the condensation that preserves the mass, i.e., the condensation technique used in our investigations here, the expression changes only so far as to the "Bouguer term" is concerned; this term becomes negative instead of positive as shown by Vaníček and Martinec [1994, eqn. (48)]. The second, generally much smaller term, which can be called the "terrain roughness term", is not much affected by the condensation technique.

We note that the PITE represents a correction to point values of geoidal heights and is thus evaluated for the same locations as is the geoid, i.e., on a 5' by 5' mesh. No averaging is involved here. The main contribution to the PITE comes from the "Bouguer term", which is nothing else than just the topographical height squared and scaled. If the topographical heights used in the evaluation of this term are smoothed (by such a process as averaging), then the computed values will be systematically smaller than they should be. For a discussion of this point see [Martinec, 1993].

The "topographical roughness" term, cf. [Martinec and Vaníček, 1994(b), eqn.(50)] consists of an integral over a fairly complicated sub-integral function of density and height. The evaluation of this sub-integral function slows down the computation considerably. We have thus tried to simplify this function to expedite the computations. The simplified function we have derived reads as follows (we leave out the lengthy derivations that would only clutter this report):

$$G R^2 \rho_o \left\{ \ln \left[\left(H' + \sqrt{(l_o^2 + H'^2)}\right) / \left(H + \sqrt{(l_o^2 + H^2)}\right) \right] - (H' - H) / l_o \right\} . \qquad (7.1)$$

The accuracy of this approximation has been tested along two profiles across the Rockies and it was found that the error amounts to 4 cm or less - not good enough for the "one centimetre geoid" but adequate for the present study; we think that the accuracy of point heights used for the evaluation of the "Bouguer term" cannot guarantee a better geoid accuracy either and this shortcut has been taken to save computing time.

Our numerical tests had shown that the integration area can be reduced to a spherical cap of a radius of about $2.5°$, without affecting the centimetre level accuracy, see also [Martinec, 1993]. This is the cap that has been used in our study here.

8. NUMERICAL RESULTS

Since the "picture is worth a thousand words", we have decided to present the required numerical results in a graphical form. Herewith is a string of plots of the various quantities and corrections we have produced during this investigation. The actual numbers are contained in files described in the Appendix. As required, these files are being transferred to the Scientific Authority for inspection and testing. They are also available in the Department of Geodesy and Geomatics Engineering at UNB to anyone wishing to work with them.

The computer programs that have produced the results shown here are listed in the Appendix by names. They also have been transferred to the Scientific Authority for inspection and testing.

The mean direct topographical effect (DTE) is shown in **Figure 8.1**. This effect ranges from -54.26 mGal (latitude 50.62, longitude 243.42) to +79.46 mGal (latitude 51.96, longitude 242.62), with a mean value of +0.88 mGal. The effect is quite short wavelength and, as expected, highly correlated with topography. As discussed in [Vaníček et al., 1994(b)], the application of the DTE to free-air gravity anomalies, makes the latter smoother, making the Helmert anomaly a better choice for downward continuation. The application of the DTE to free-air anomalies in our area of interest has reduced the original range of (-143.62 mGal, +214.40 mGal) to (-134.17 mGal, +185.65 mGal), a reduction of 40 mGal.

The mean secondary indirect topographical effect (SITE) is plotted in **Figure 8.2**. It is 2 orders of magnitude smaller than the DTE, always negative, and ranges between -0.47 mGal (latitude 43.21, longitude 250.38) and 0 mGal , with a mean value of -0.04 mGal. Once again, the effect is short wavelength and as such contributes very little to the final geoidal heights. But the effect would be systematically negative and since it reaches more than 0.01 mGal (in absolute value), it must be taken into account if the 1cm accuracy is the aim - cf. [Vaníček and Martinec, 1994].

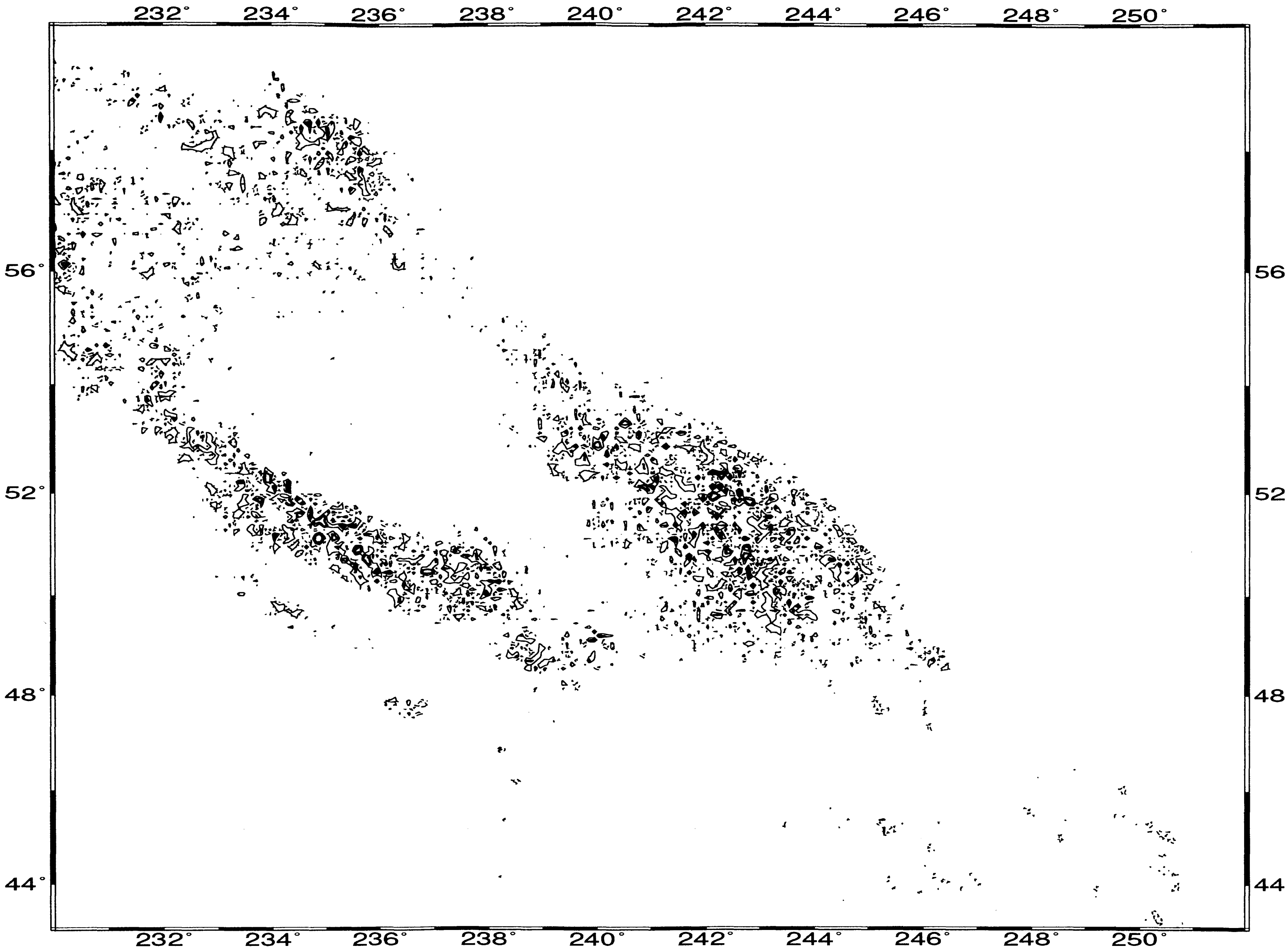


Figure 8.1: The mean direct topographical effect (mGal)
Contour interval = 20.0 mGals

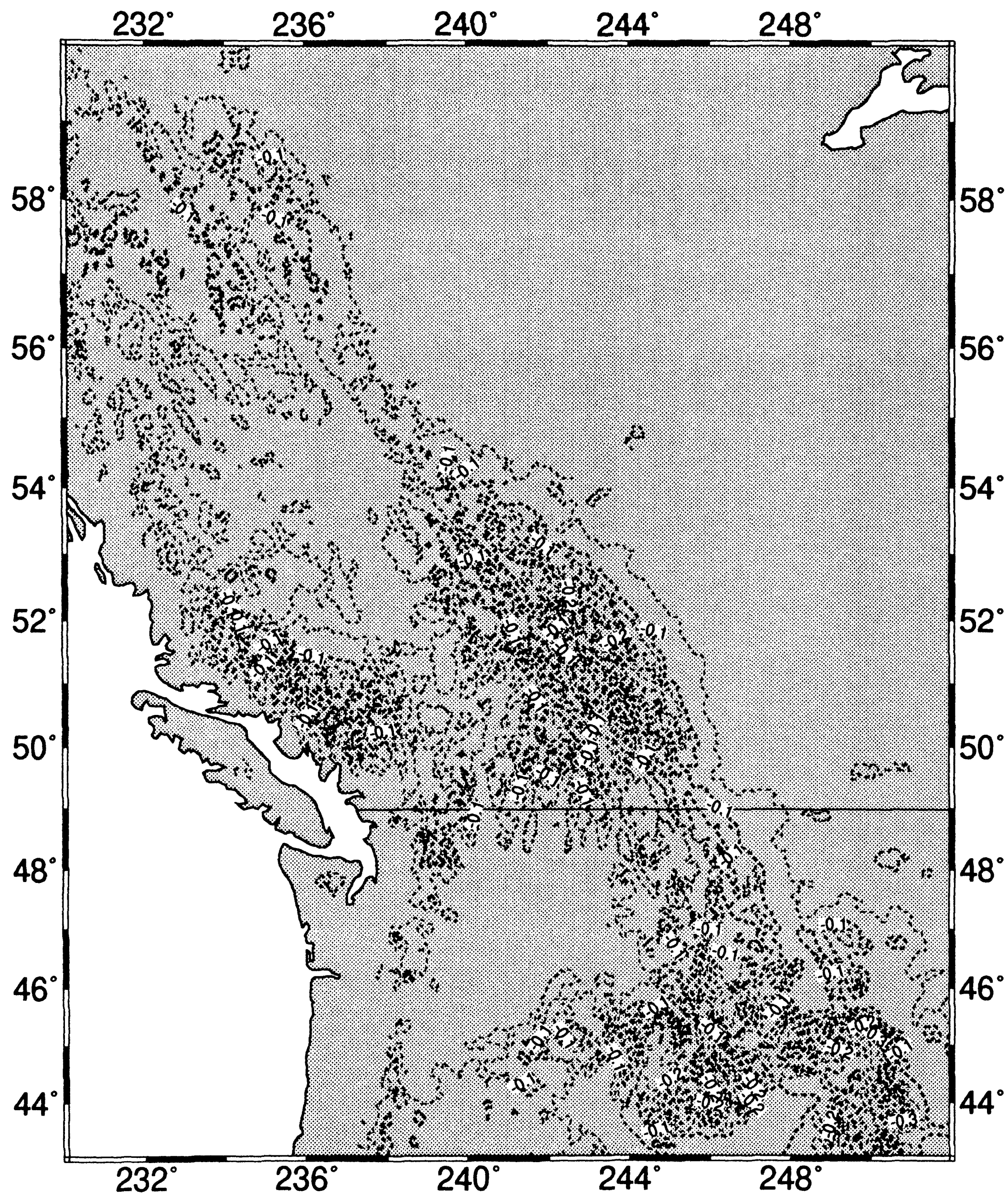


Figure 8.2: The mean secondary indirect topographical effect (mGal). Contour interval is 0.05 mGal.

The sum of mean latitude effect (Le) and the mean altitude effect (Ae) of the normal gravity gradient is plotted in **Figure 8.3**. It is always negative and ranges between -1.06 and 0 mGal, with a mean value of -0.16 mGal. It being short wavelength, once more, the contribution to the resulting geoid is small but systematically negative.

The mean spherical approximation correction (D^S) , evaluated from the global model GFZ93a, is shown in **Figure 8.4**. It is, for our area of interest, even smaller than the SITE; it ranges between -0.024 and +0.001 mGal, with a mean value of - 0.009 mGal. Its contribution towards the geoid is of the order of a few millimetres.

The mean downward continuation contribution (Dg) is shown in **Figure 8.5**. Note the very high frequency character of this term and its very large values, ranging between -126.408 mGal (latitude 52.29°, longitude 242.71°) to + 215.680 mGal (latitude 51.38°, longitude 234.79°), with the mean value of 0.387 mGal. Interestingly, when convolved with (modified spheroidal) Stokes's function, it gives a contribution to Helmert's co-geoid which is positive everywhere - for a detailed discussion see [ibid]. The truncation error correction to Poisson's integration has been evaluated from the global model GFZ93a.

Figure 8.6 shows the high frequency mean Helmert's anomaly, meam[$\delta\Delta g_g^h$] , referred to the GRIM4 -S4P global gravity model. For completeness sake, we give here the range (from -129.95 to +168.28 mGal) and the mean value of -5.08 mGal. The standard deviations associated with this quantity range between 0.51 and 19.27 mGal, with a mean value of 2.89 mGal. Their areal variations are shown in **Figure 8.7**

The primary indirect topographical effect PITE is shown in **Figure 8.8**. It is always negative - the geoid is everywhere lower than Helmert's co-geoid - and in our area of interest ranges between -104.5 centimetres (latitude 51.58, longitude 243.75) and 1 centimetre (at latitude 50.08 and longitude 236.25; note that the small positive number is an error due to the approximation of the integration kernel, it must theoretically be negative), with a mean of -23 cm. These are point values, computed by means of eqn. (7.1) using heights on the 1 by 1 km grid: the height value located the nearest to the geoid computation point is used as is, to avoid averaging, thus smoothing and making the geoid error systematic. Since these heights are, as we understand, somehow averaged, the real values of the PITE are probably somewhat larger (in absolute value) than those presented here.

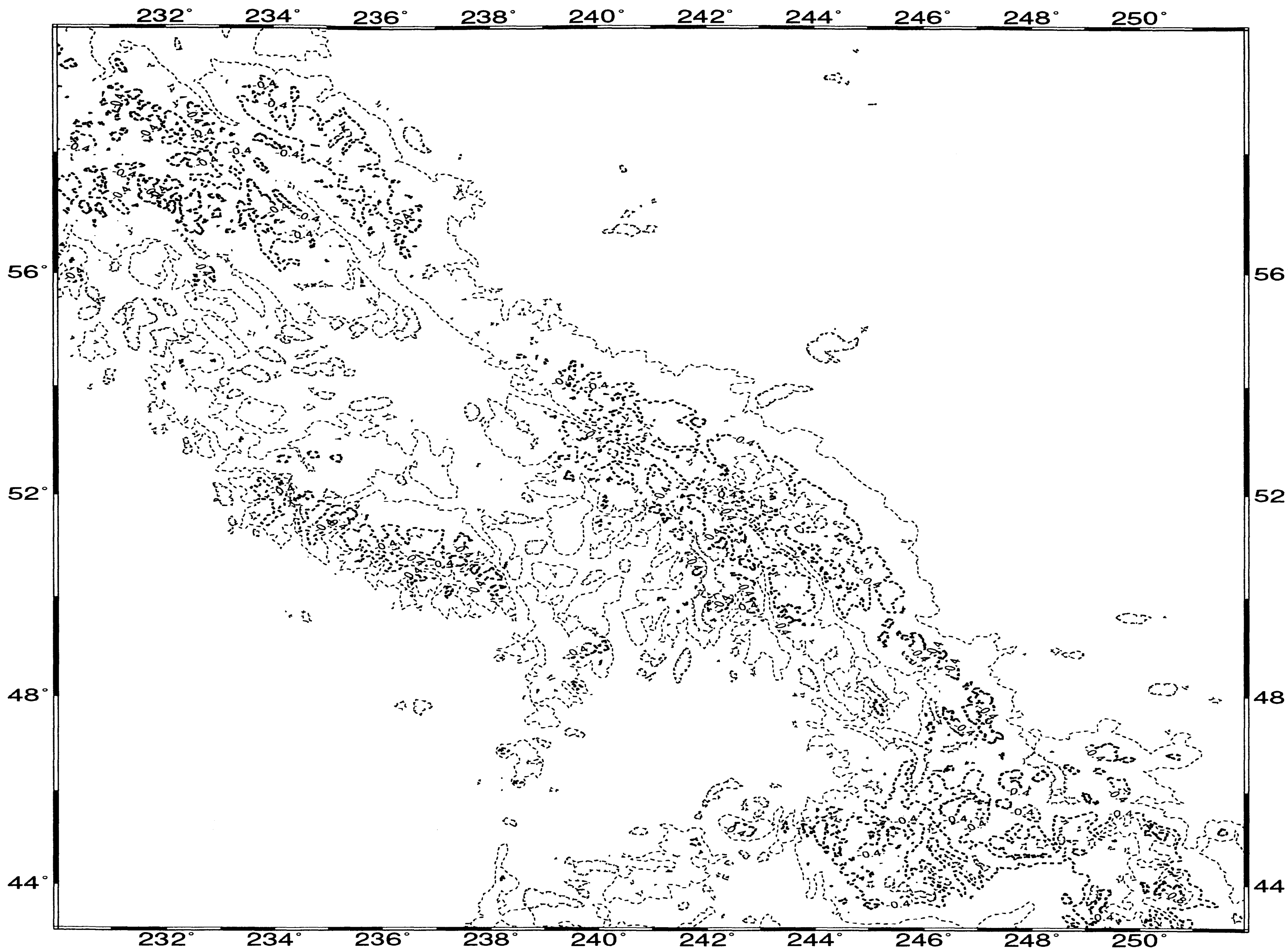


Figure 8.3: The sum of mean latitude and mean altitude effects (mGal)
Contour interval = 0.2 mGals

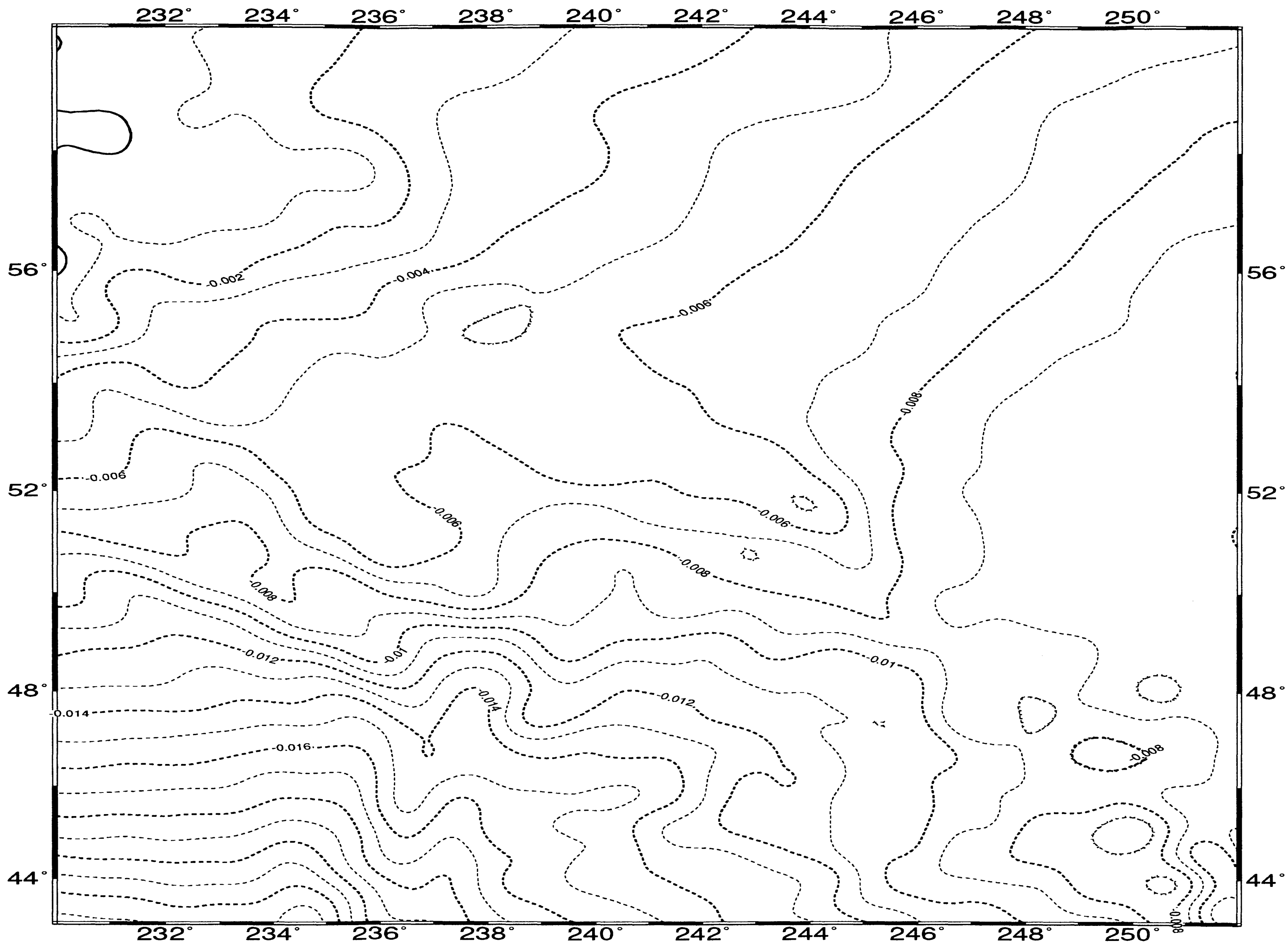


Figure 8.4: The mean spherical approximation correction (mGal)
Contour interval = 0.001 mGals

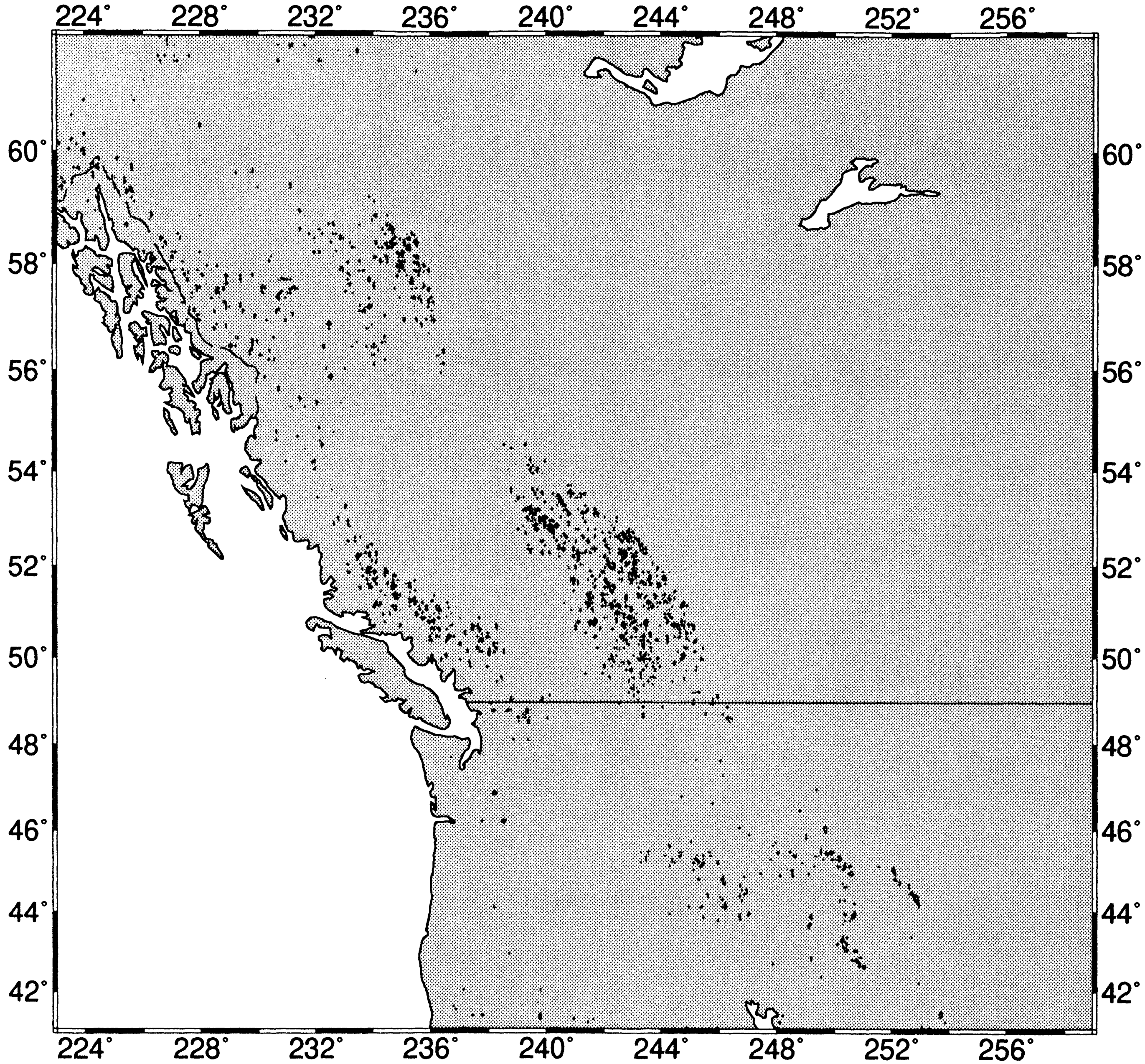


Figure 8.5: The difference between 5' by 5' mean Helmert anomalies on the topography and on the geoid, originated from the old Incomplete Bouguer anomalies file. Contour interval is 40 mGal.

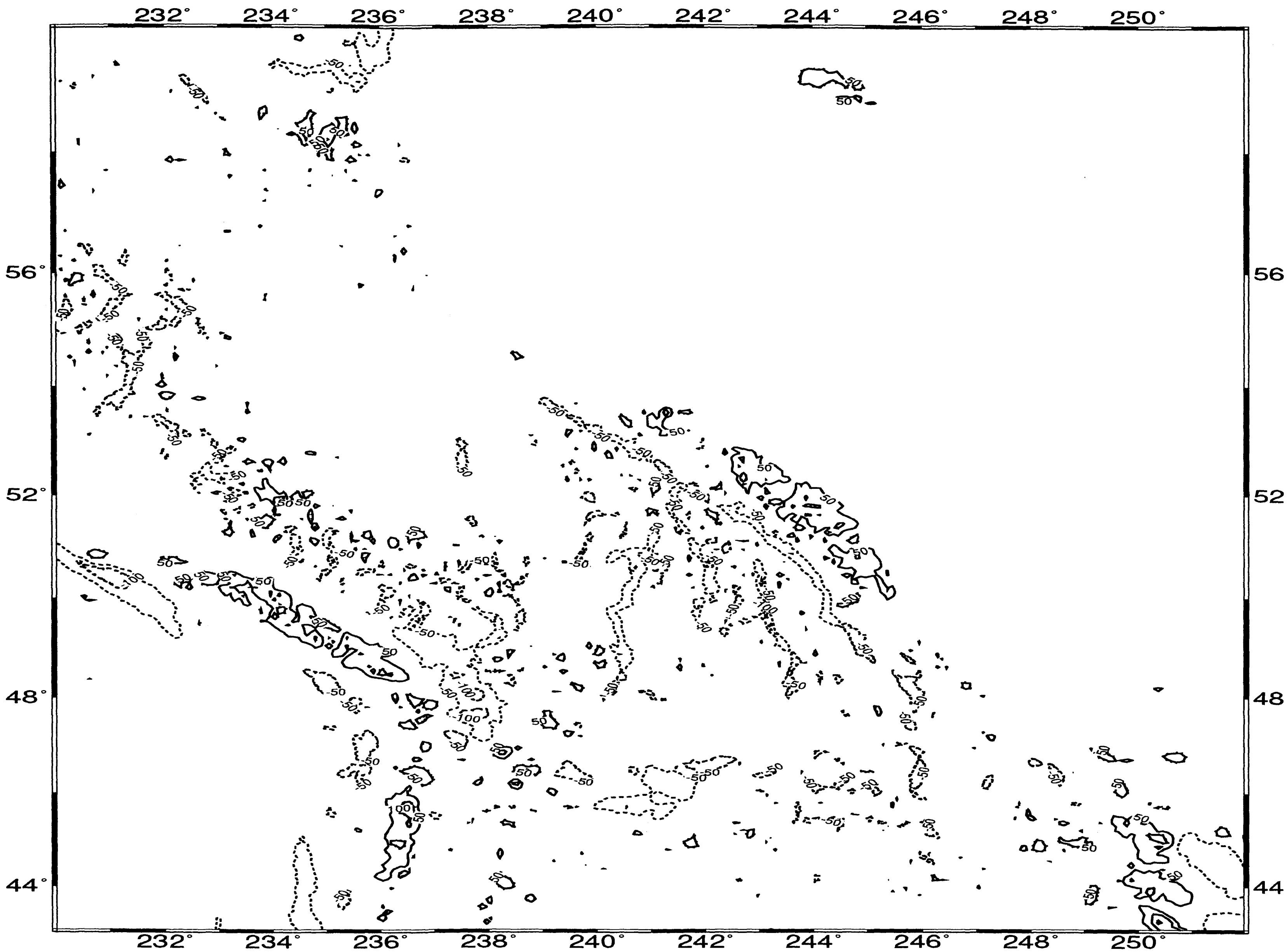


Figure 8.6: Mean Helmert's gravity anomalies referred to 20, 20 reference field on the geoid (mGal)
Contour interval = 50.0 mGals

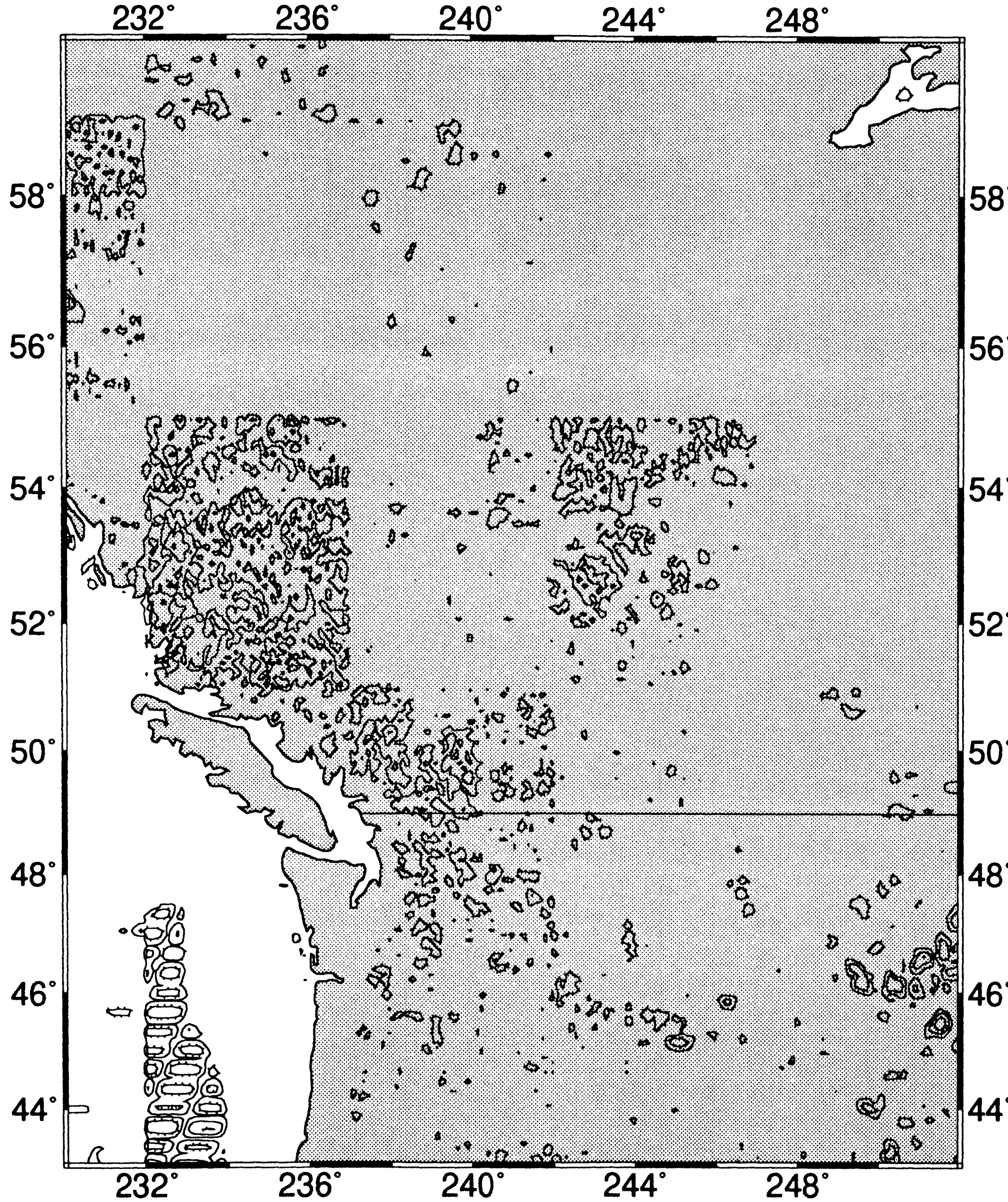


Figure 8.7: The standard deviation of the anomalies displayed in figure 8.6 (mGal) Contour interval is 5 mGal.

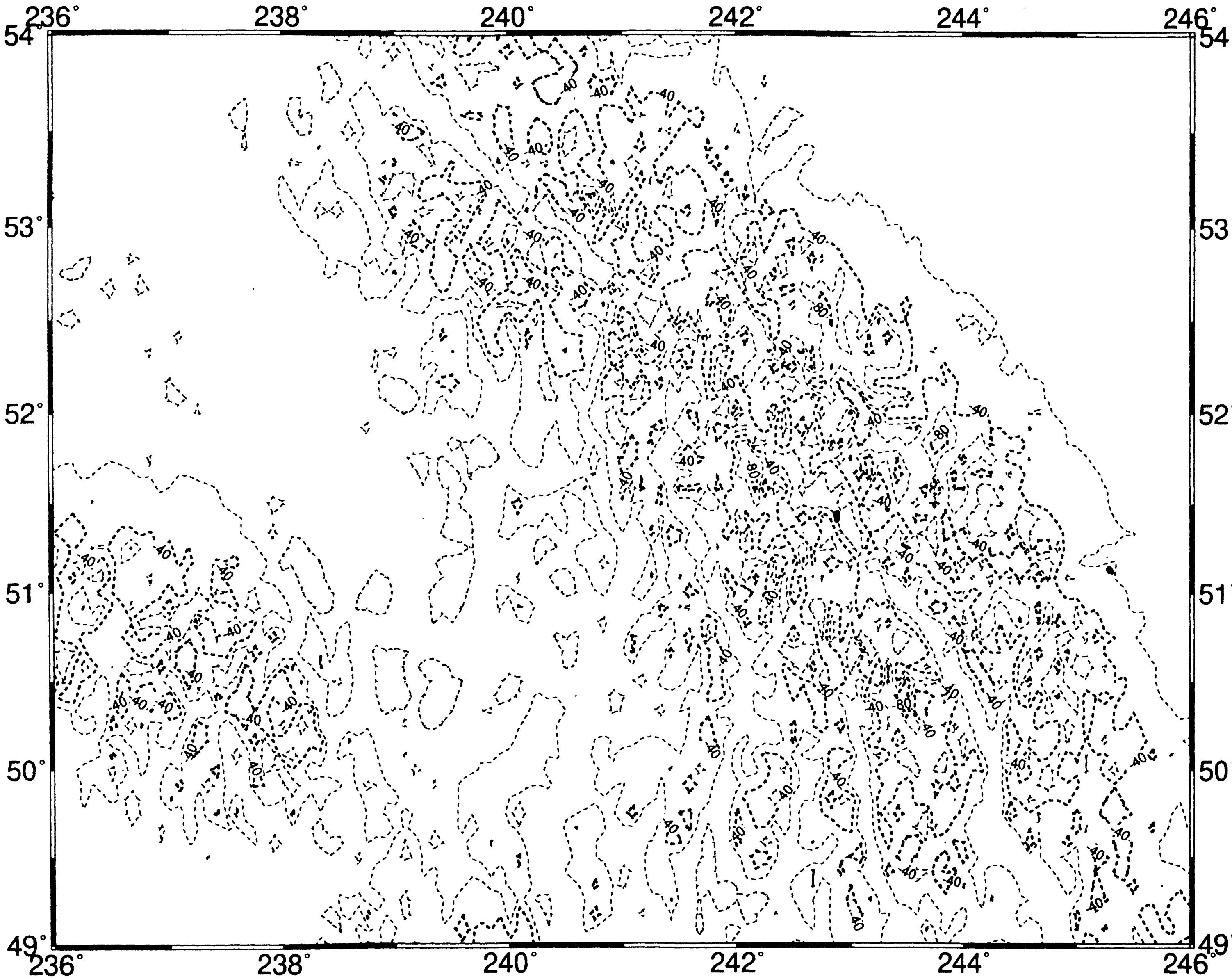


Figure 8.8: The primary indirect topographical effect (cm)

Contour interval = 20 cm

Figure 8.9 shows the plot of the geoid produced under this contract for the required area, called here the UNB 94 model. In the area of interest, it ranges between -23.86 and -14.52 metres with a mean of -18.34 metres. The computed standard deviations associated with this solution are plotted in **Figure 8.10**. They range between 16 and 40 cm, with the mean value being 26 cm. These are relatively large values and they reflect the fact that collected gravity data are relatively sparse and uncertainties in heights are high. Corresponding errors in other parts of Canada would be somewhat smaller. It should be borne in mind, that the standard deviations presented here are quite highly correlated, particularly for short distances; treating them as independent would result in distortions of the error information contained in these deviations.

9. COMPARISON WITH GPS/LEVELLING RESULTS AND THE UNB 95 SOLUTION

GPS determined positions of thirteen points have been given to us together with their orthometric heights, as external test data. The orthometric heights are those given with respect to the North American Vertical Datum of 1988 (NAVD 88) and not the officially published heights (which are still given with respect to the Canadian Vertical Geodetic Datum of 1928 - CVGD 28). These heights were selected by the GSD because they had been corrected for observed gravity and they differ from the official heights by approximately 1.65 m [Mainville, 1995]. These data are recapitulated in **Table 9.1**. Also shown in this table are the geoidal heights from our solution (UNB 94) and the differences between GPS/levelling and UNB 94 values.

Station name	h (m)	H (m)	h-H (m)	UNB94 (m)	(h-H) -UNB94
50C9501	-14.4723	5.633	-20.105	-22.305	2.200
19713	-16.2893	4.908	-21.197	-23.008	1.811
77C048	-0.6864	19.110	-19.796	-22.311	2.515
83C174	19.8064	37.980	-18.174	-21.080	2.906
61C028	1048.5298	1065.294	-16.764	-19.306	2.542
59C037	638.8256	655.908	-17.082	-19.260	2.178
58C144	723.8253	740.974	-17.149	-19.310	2.161
60C004	336.5755	354.009	-17.434	-19.522	2.088
887006	541.7580	559.603	-17.845	-19.856	2.011
68C026	404.4994	422.300	-17.801	-20.004	2.203
68C047	339.7462	357.014	-17.268	-19.558	2.290
68C129	787.5272	802.817	-15.290	-17.981	2.691
68A050	1395.3611	1411.067	-15.706	-17.302	1.596

Table 9.1 - UNB 94 geoid versus GPS and orthometric height comparison.

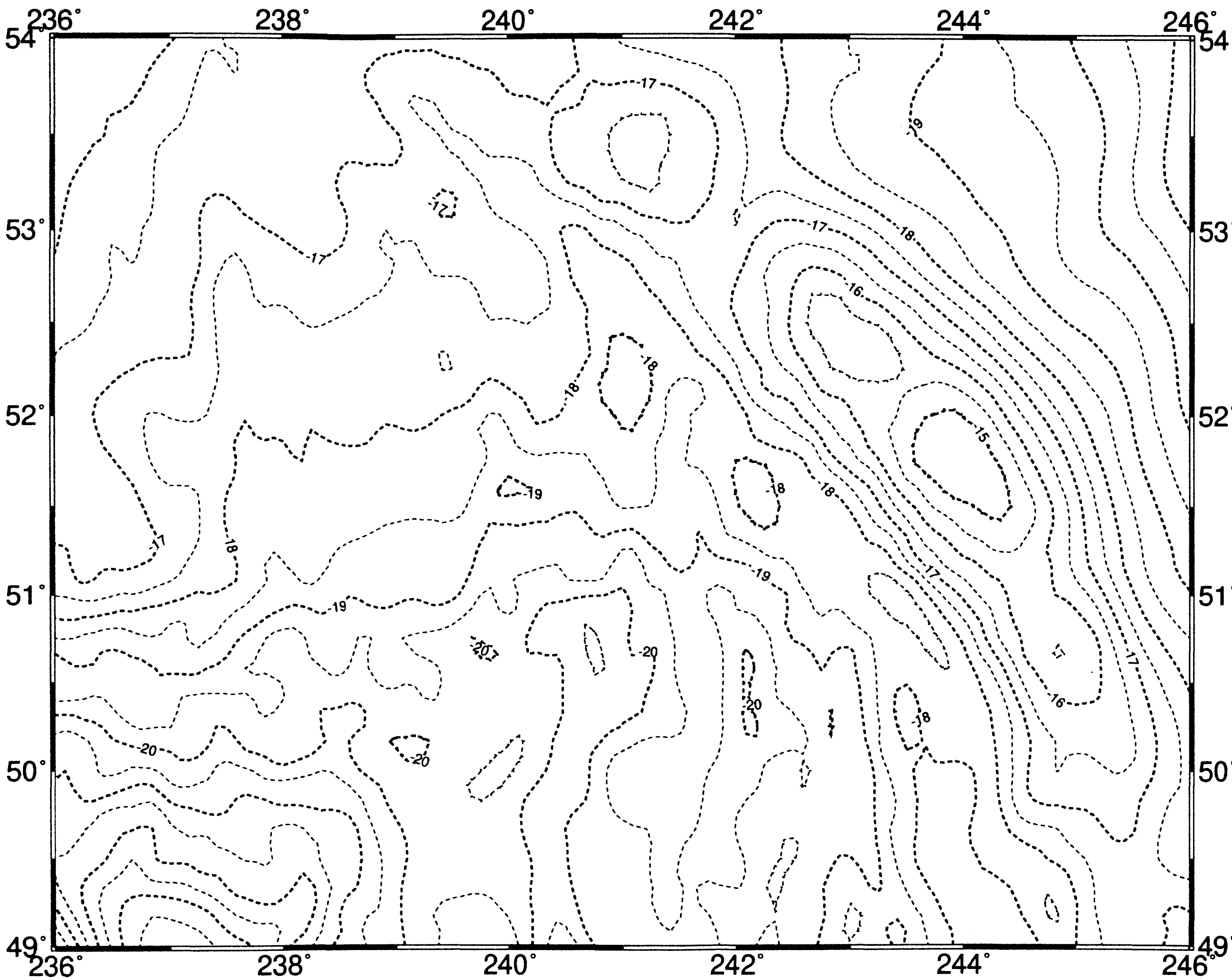


Figure 8.9: UNB94 geoid model (m)

Contour interval = 0.5 m

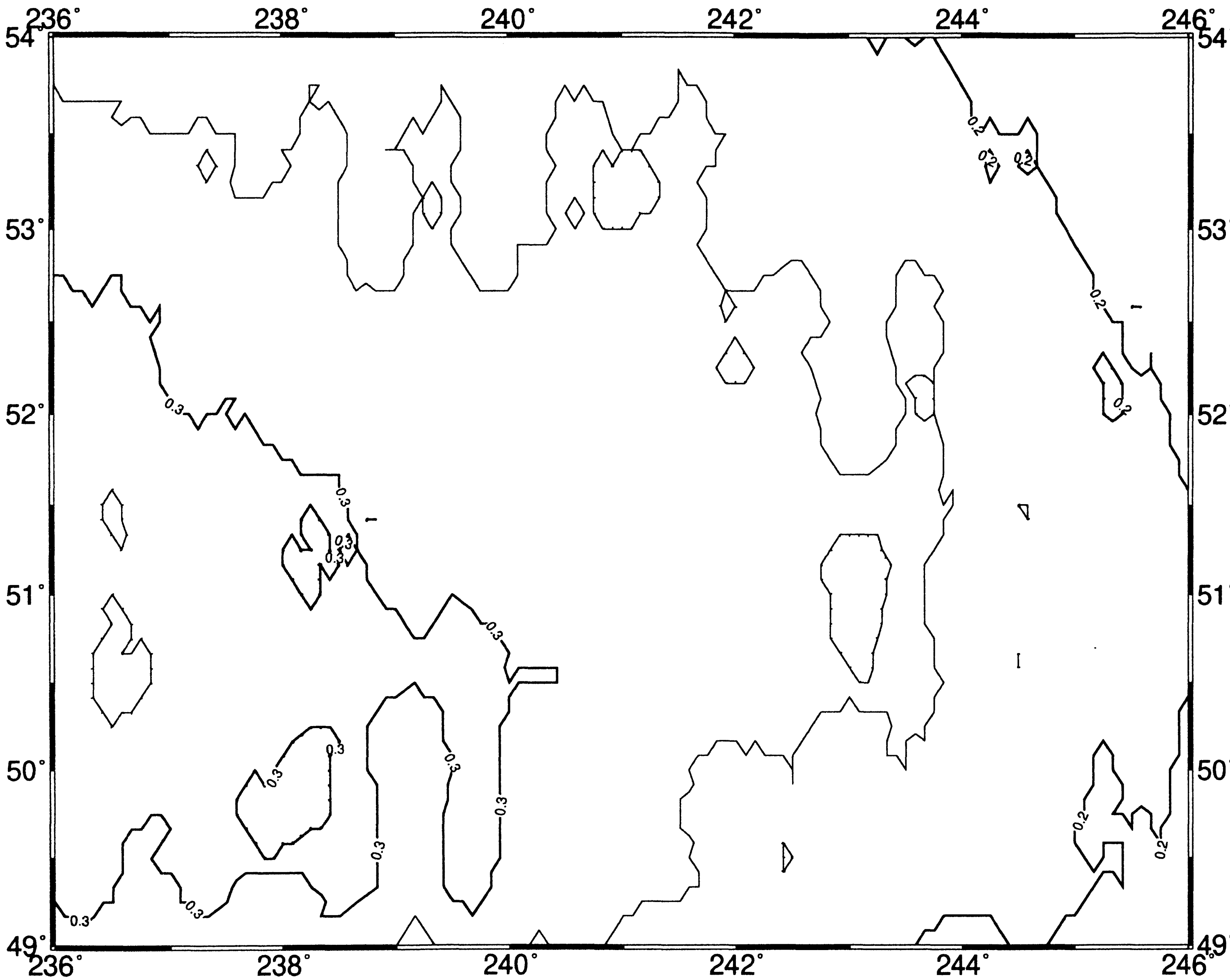


Figure 8.10: The standard deviation of UNB94 displayed in figure 8.9 (m)

Contour interval = 0.05 m

At this point we have realized that there was something drastically wrong with this solution and started to check our procedures. First we looked into the reference field GRIM-S4P to see if it could possibly explain the 4 metre bias of the UNB 94 geoid. Comparison of this field with GEM-T3 taken to degree 20 and properly referred to GRS 80, shown in **Figure 9.1** for the whole of Canada, convinced us that the reference field could not be responsible for the distortion. The differences between the two fields in our area of interest are at most of the order of 30 cm. Moreover, this experiment shows just how good the satellite derived fields have become.

The totality of terrain related corrections, the DTE, PITE, the downward continuation and the SITE, contribute between 60 and 180 cm to geoidal heights at the GPS points. We thus did not suspect that the main problem was with these corrections. We tested them nevertheless and found error in neither the formulation, nor the code, nor the results. We have also checked all the other corrections and found no fault with any of them.

We then turned our attention to the Stokes integration. We completely re-wrote the GIN program, which is now completely flexible. It now allows to vary the size of the innermost and inner zones, the computation of geoidal heights on a grid, on a string of points (a profile), or on individual points, and a much more efficient handling of the 5' by 5' mean anomaly files. This re-write resulted in a much faster running program, which we call "GIN 95". We have tested this new integrator on data generated from a global model taken to degree and order 360, 360, producing geoidal heights both directly from the potential coefficients as well as by integrating over similarly generated anomalies (and correcting for the truncation error). The two solutions agree to a few centimetres and we can thus conclude that the new integrator works as well as can be possibly expected.

As a by-product of this testing, we have learnt that the discretisation error in the integration does not exceed 6 centimetres in either positive or negative sense. We have also discovered that somewhat more accurate results can be obtained when the inner zone is extended from the 2° in latitude by 2° in longitude to 4° by 4°. We have also enlarged the innermost zone to 10' in latitude by 15' (or larger in higher latitudes) in longitude. In spite of all these changes, the solutions we were getting showed only slight differences from the original, apparently drastically wrong solution (cf. Table 9.1).

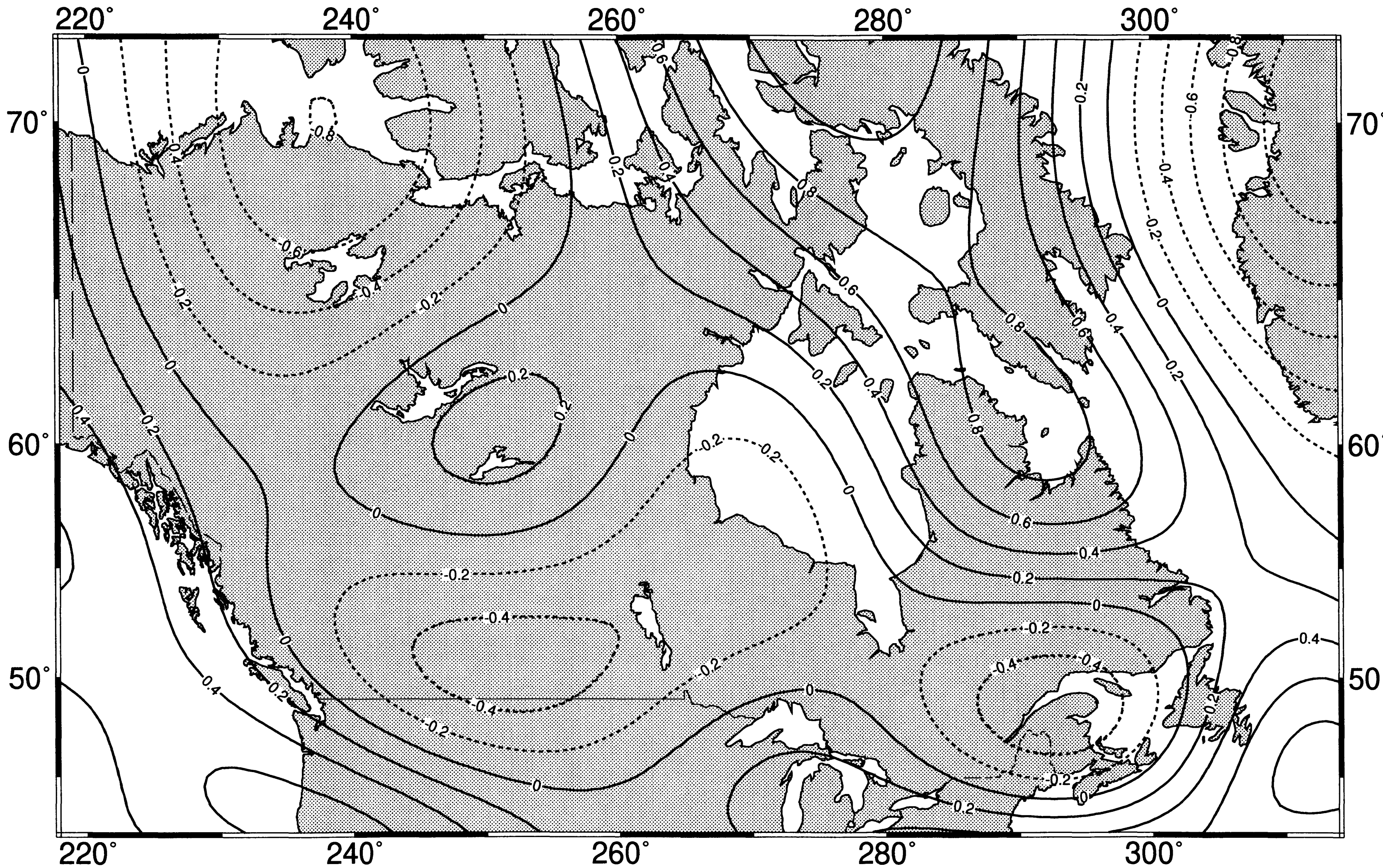


Figure 9.1: The difference between GRIM4-S4P and GEM-T3 satellite solutions (taken only to degree and order 20) for Canada. Contour interval is 0.2 m.

The only remaining explanation was that the mean simple Bouguer anomalies we were using were in error. To check this last possible explanation, we asked the Scientific Authority for the permission to use the "mean Helmert anomalies" used by the GSD personnel in their compilation of the GSD geoid, which shows a much better agreement with the GPS/levelling derived geoidal heights on the 13 test points than our UNB 94 geoid does. Since in the compilation of these Helmert anomalies the DTE had somehow been already included, we have not used our own DTE. We have, however added all the other corrections as described in this report, including a recomputed downward continuation shown in **Figure 9.2**. This figure should be compared with Fig. 8.5. Note again the very high frequency character of this term and its very large values, ranging between -133.035 mGal (latitude 43.96°, longitude 250.62°) to $+234.090$ mGal (latitude 46.88°, longitude 238.21°), with the mean value of 0.636 mGal.

The use of these GSD Helmert anomalies, plus all our corrections, resulted in the UNB 95 geoid shown in **Figure 9.3.** The estimated standard deviations of this geoid could not be plotted because the "Helmert anomaly" file given to us did not contain the requisite standard deviations.

Comparison of the UNB 95 geoid with the GPS/levelling derived geoidal heights for the 13 test points is shown in **Table 9.2**. For completeness, the GSD geoidal height are also listed. From this Table we can see that the UNB 95 fits much better to the external standard. The difference between the UNB94 and UNB95 geoids on the 13 GPS points reaches about 4 metres, a difference caused solely by using a different set of mean anomalies. In addition, our results appear to be somewhat closer to the external standard, than the GSD results. This should not be immediately interpreted as a proof that our solution is better than the GSD solution; it merely shows that our technique seems to work as designed. Let us remark here, that a positive difference between the geoid and the GPS/levelling results is to be expected. The orthometric heights in western Canada are probably too large by more than a metre due to systematic errors in levelling [Zilkoski et al., 1992; Mainville, 1994].

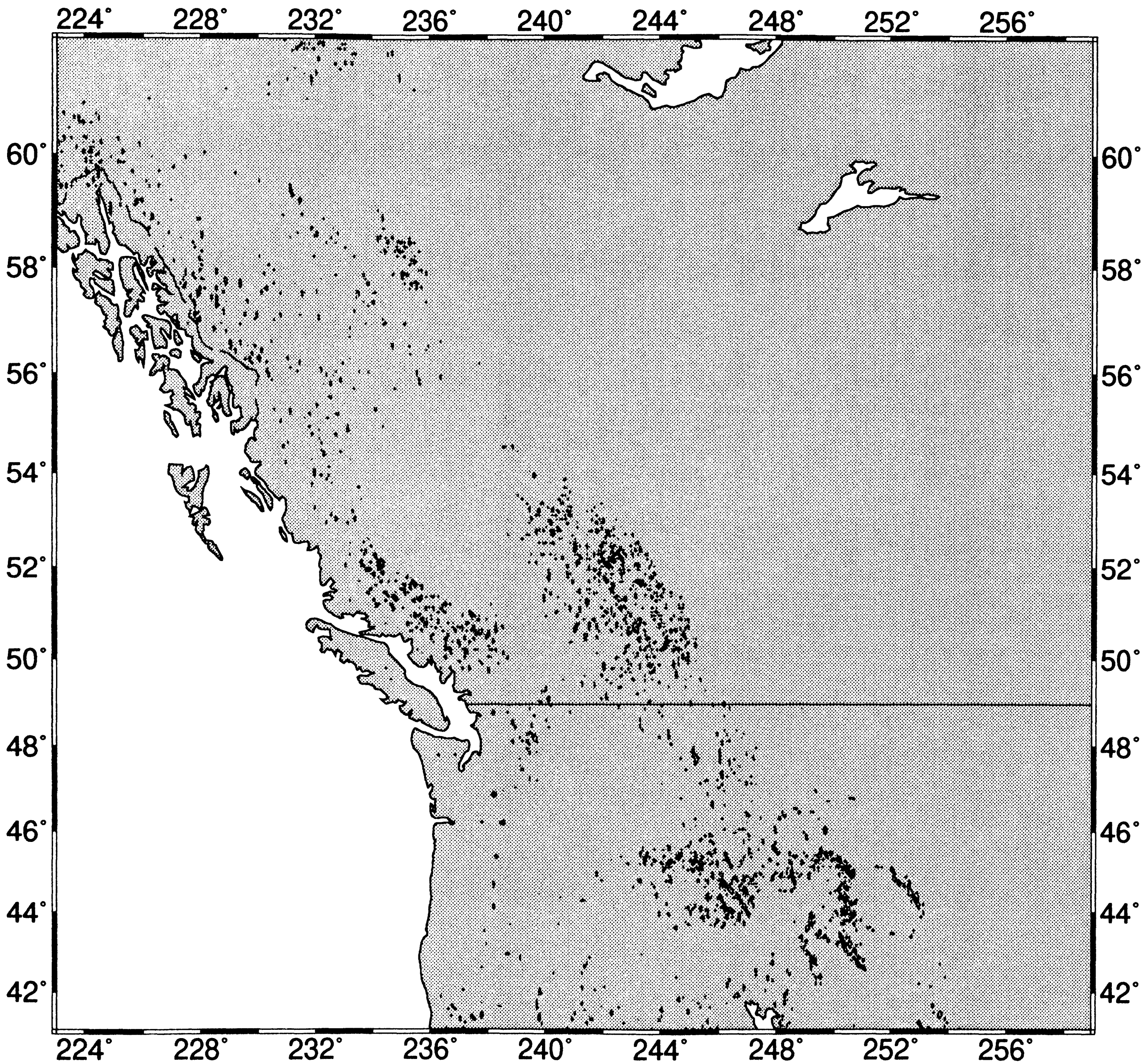


Figure 9.2: The difference between 5' by 5' mean Helmert anomalies on the topography and on the geoid, computed from the new "Helmert anomaly file", Contour interval is 40 mGal.

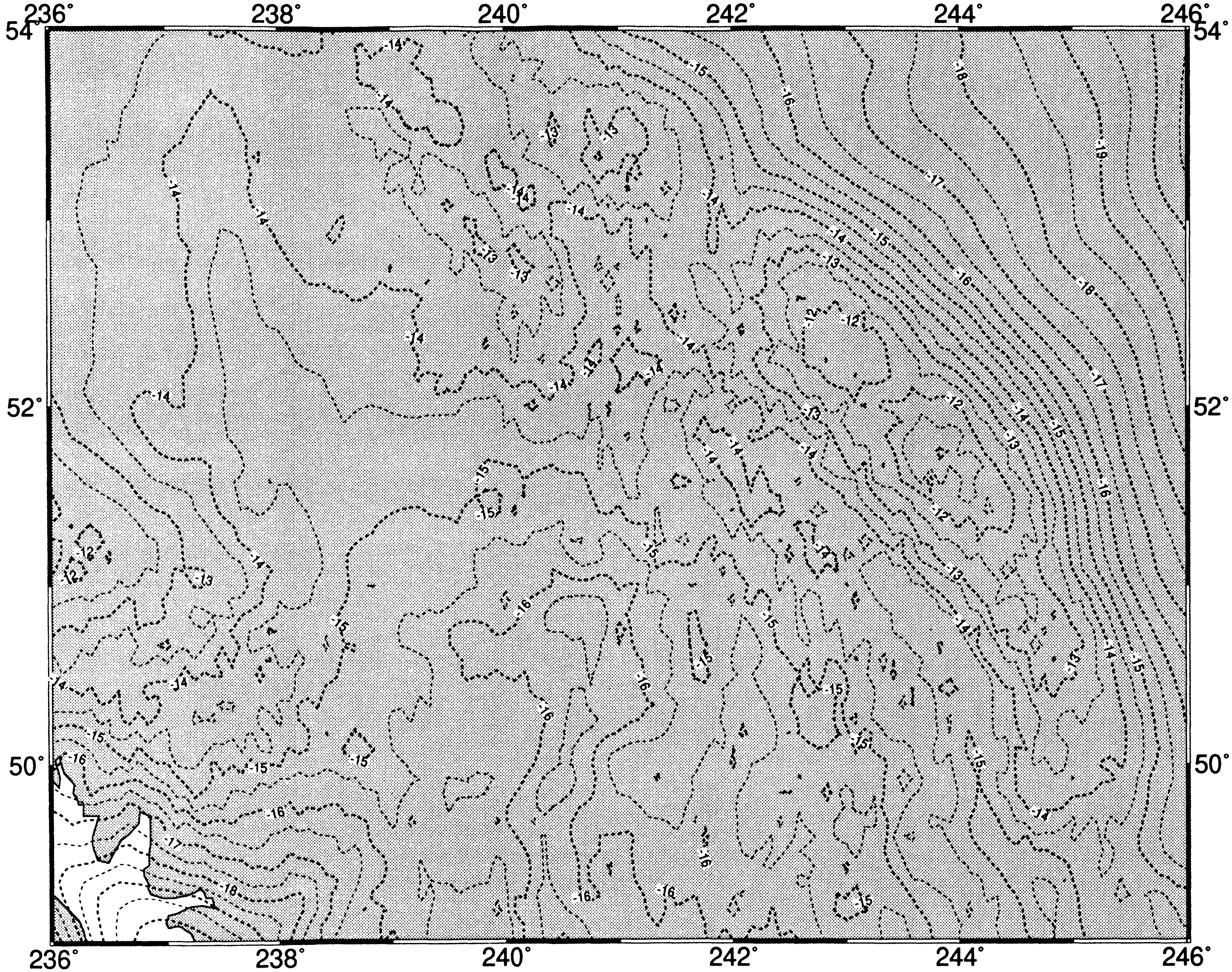


Figure 9.3: UNB 95 geoid model (m). Contour interval=0.5 m.

Station name	h (m)	H (m)	h-H (m)	GSD geoid	UNB95 (m)	(h-H) -UNB95
50C9501	-14.4723	5.633	-20.105	-18.383	-18.909	-1.196
19713	-16.2893	4.908	-21.197	-19.314	-19.928	-1.269
77C048	-0.6864	19.110	-19.796	-17.850	-18.529	-1.267
83C174	19.8064	37.980	-18.174	-16.482	-17.075	-1.099
61C028	1048.530	1065.294	-16.764	-15.044	-15.608	-1.156
59C037	638.8256	655.908	-17.082	-15.296	-15.776	-1.306
58C144	723.8253	740.974	-17.149	-15.452	-16.001	-1.148
60C004	336.5755	354.009	-17.434	-15.674	-16.255	-1.179
887006	541.7580	559.603	-17.845	-15.836	-16.483	-1.362
68C026	404.4994	422.300	-17.801	-16.216	-16.787	-1.014
68C047	339.7462	357.014	-17.268	-15.572	-16.170	-1.098
68C129	787.5272	802.817	-15.290	-13.655	-14.449	-0.841
68A050	1395.361	1411.067	-15.706	-14.145	-15.390	-0.316

Table 9.2 - UNB 95 geoid versus GPS and orthometric height comparison.

10. CONCLUSIONS AND RECOMMENDATIONS

One of the main conclusions of this report must be that we must take a closer look at the procedures used for the evaluation of mean gravity anomalies. The original set of mean 5' by 5' simple Bouguer anomalies was apparently affected by a large and systematic effect. The reason for this effect is unclear; possibly, the simple Bouguer anomalies are not smooth enough. It seems that the application of the terrain correction, i.e., use of complete, rather than simple Bouguer anomalies (in the set of mean anomalies used for compiling the UNB 95 geoid) results in much smoother field, in which the averaging works much better. In addition, we understand that about 75% of the "mean anomalies" had to be actually predicted from surrounding values (due to the very low density of point gravity observations), rather than evaluated through averaging [Mainville and Véronneau, 1989]. This fact argues even more strongly for working with the smoothest possible field, i.e., with complete Bouguer anomalies.

In producing the UNB 95 geoid, we really worked with "topographically corrected free-air anomalies", rather than Helmert anomalies. We have taken the mean complete Bouguer anomalies supplied to us and converted these to mean free-air anomalies; we have not applied the DTE. We have not had the time either to do it, or even to inquire into this problem and cannot thus offer any estimates as far as the potential effect such incongruity may have. Clearly, some mean topographical corrections should have been subtracted from the mean anomaly values and mean DTE should have been added to them. The effect of using the topographical correction instead of the DTE might even be attenuated by the downward continuation term, which, for the UNB 95 geoid, was of course also computed from the (topographically corrected) mean free-air anomalies. Judging from the apparently reasonable fit of the UNB 95 geoid to the auxiliary geoidal heights, however, the effect is probably fairly small and smooth. Only when this problem is solved will we be able to assess properly the performance of the methodology we had developed under this contract.

Assuming that the problem with mean anomalies can be sorted out, there are other problems to be sorted out. Nowhere in this report, other than in section 8 which describes the actual results, do we speak either of the accuracy of Canadian gravity data, or their spatial distribution. We have considered the gravity data distribution and accuracy to be beyond the scope of our investigation for the following reason: we feel that in order to pass any judgement on our gravity data we must first make sure that our theory is accurate enough to handle the data adequately. We think we have now almost reached this point and one of the main goals for the not-too-distant a future should be to look seriously at the accuracy limits imposed on us by the existing gravity data set. Our conviction is that some 5cm geoid accuracy is possibly the best we can expect. Any recommendations for the improvement of gravity data accuracy and/or distribution must await the results of such an investigation.

If the "1cm Canadian geoid" (actually even a "decimetre geoid"!) is to be ever compiled, the topographical density correction discussed in [Martinec, 1993; Martinec et al., 1994(b)] must be considered in the final computation. (We note in passing that, of course, the determination of quasigeoid does not require any knowledge of topographical density.) A geologist/geophysicist should be recruited to help with the density data acquisition and we propose that the Canadian Geoid Committee become involved in organizing this effort.

Atmospheric (Helmert) condensation - similar to topographic condensation - must be properly formulated and implemented in the final computation. Only the mean anomalies given to us by GSD had been corrected for atmospheric attraction. The total effect of atmosphere on the geoid amounts to a few decimetres and as such must be considered. We have made a first attempt in [Vaníček and Martinec, 1994] and [Harrie, 1993] but more work is required to convert these attempts to meaningful algorithms. In this contract, we simply ran out of time and funds, to do so.

Mean values of the DTE (and perhaps the SITE, but this would not be crucial) must be used in the geoid computation. Their evaluation is computationally very intensive and may require a supercomputer to accomplish. Point values have been used by us in producing the UNB 94 geoid, because of lack of funds and time and this may have resulted in errors in mean Helmert's anomalies of several mGal and errors in the resulting geoid of several centimetres, even decimetres. As stated above, the DTE has not been used in the compilation of the UNB 95 geoid at all.

Which brings us to heights. A denser than 1 by 1 km height sampling must become available in the mountainous areas of Canada for a more accurate evaluation of topographical effects that depend on either point or mean heights. We think that the use of the 1 by 1 km sampling grid may have introduced errors of several decimetres in the primary indirect effect and thus in the resulting geoid. The existing height file for the 30 metre grid (with somewhat restricted availability) would be adequate, if it were not for the large errors associated with these heights. The construction of such a topographical file is, we feel, another issue with which the Canadian Geoid Committee should get involved. There is certainly a lot of room for improvement in this "department".

Corrections to orthometric heights due to topographic density variations should be evaluated, once the downward continuation is well understood and topographical density variations estimated. This will have also a second order effect on the computed geoid. The problem can be formulated as follows: the Poincaré-Pray gravity gradient (0.0848 mGal per metre) is used in the definition of Helmert's orthometric heights. This gradient value is derived from the exact Bruns formula [Vaníček, and Krakiwsky, 1986 (eqn. 21.26)] , by adopting the simplified assumption that crustal density ρ is constant and equals to 2.67 g cm^{-3}. The denominator in the defining equation for orthometric height H is given as [ibid ,eqn.(16.97)]:

$$\text{mean } g' = \left(0.3086 \text{ mGal/m} - 4\pi G\rho\right)\frac{H}{2}, \tag{10.1}$$

and the change of orthometric height with density is then

$$\frac{\partial H}{\partial \rho} \approx 0.114 * 10^{-6} m^{-1} \frac{H^2}{\rho}. \tag{10.2}$$

It is easily seen that even modest changes in topographical density cause centimetre and decimetre errors in orthometric heights.

Although a lot of effort went into a better estimation of errors in mean anomalies, a more thorough error analysis to accompany the developed methodology is called for. How should the uncertainty in the reference field be accounted for? Are all the corrections really determined so much better than the mean anomalies themselves? How are the estimated (random) errors in geoidal heights correlated? Such an analysis represents a substantial investigation and a substantial time and financial investment.

Different Stokes's kernel modification schemes should be investigated and tested. The Molodenskij modification employed by us has worked quite well but, for reasons described in section 4, it may not be the optimal technique to use. Our suggestion is to make some experiments with the "strict frequency separation kernel" as discussed earlier.

The primary indirect topographical effect may be recomputed using a more accurate integration kernel. This again is a computationally very intensive proposition and may require the use of a supercomputer. It would be essential, however, for producing the ultimate "1 centimetre geoid".

11. ACKNOWLEDGMENTS

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The graphics included in this report were produced using the GMT package [Wessel and Smith, 1991].

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13. APPENDIX

1) Program name: **general5.for**

This program computes the roughness term of the direct topographical effect ($\delta V^R(r, \Omega)$) to the gravity. The equation used here is shown in [Martinec and Vaníček, 1994(a), eqn.(42)].

Input data: The 5min. and the 30" by 60" DEM files whose names are introduced into an optional file of the program called **optgeneral5.inp.**

Output data: $\delta V^R(r, \Omega)$ in mGal computed and stored into a new file the name of which is also specified in the option file.

2) Program name: **bougdte5min.for**

This program computes the "Bouguer" term of the direct topographical effect ($\delta V^B(r, \Omega)$) to the gravity. The equation used here is shown in [Martinec and Vaníček, 1994(a), eqn.(41)].

Input data: The coordinates and heights of computation points as an input file whose name is given into an option file called **optbougdte5min.inp.**

Output data: $\delta V^B(r, \Omega)$ in mGal computed and stored into a new file specified into the option file.

3) Program name: **pvker_1.for**

This program computes the roughness term of the primary indirect topographical effect ($\delta V^R(R, \Omega)$) to the geoid. The equation used here is shown in [Martinec and Vaníček, 1994(b), eqn.(50)]

Input data: The 30" by 60" DEM file and a file containing coordinates and heights of computation points whose names are given into an option file called **optpvker_1.inp.**

Output data: $\delta V^R(R, \Omega)$ in metres computed and stored into a new file specified in the option file.

4) Program name: **ptbougpite.for**

This program computes the "Bouguer" term of the primary indirect topographical effect ($\delta V^B(R, \Omega)$) to the geoid. The formula used here is shown in equation (48) [Vaníček and Martinec, 1994, eqn.(48)].

Input data: A file containing coordinates and heights of computation points. The file name is given in an option file called **optptbougpite.inp.**

Output data: $\delta V^B(R, \Omega)$ in centimetres computed and stored into a new file specified by the option file.

5) Program name: **sphelm.f**

This program computes a Helmert reference spheroid of degree L (e.g., 20), [Vaníček et al., 1994(a), eqn. (2)], the direct topographical effect to the reference spheroid, and the direct topographical effect to the reference gravity anomalies [ibid, eqn. (18)], and the reference SITE.

Input data: Global satellite potential coefficients (L, L) and the height-squared coefficients (derived from TUG87 (90, 90)) files called **GRIM4.s4p** and **TUG87.hsq**.

Output data: Spheroid of degree L in metres, the direct topographical effect to the reference spheroid in metres, and the direct topographical effect to the reference gravity anomalies in mGal, and the reference SITE in mGal computed and stored into files called **sphelm.mape, sphelm.dtes, sphelm.dteg, and sphelm.site** respectively.

6) Program name: **hgrvan.f**

This program computes reference gravity anomaly of degree L (20), Helmert reference gravity anomaly of degree L, employing ellipsoidal approximation, and vertical gradient of the reference gravity anomaly. The equation used here is shown in [Vaníček and Krakiwsky, 1986, eqn. (23.60)].

Input data: Global satellite potential coefficients (20, 20) and the height squared coefficients (derived from TUG87 (90, 90)) files called **GRIM4.s4p** and **TUG87.hsq**.

Output data: Reference gravity anomalies, Helmert reference gravity anomalies in mGal, direct topographical effect to the reference gravity anomalies in mGal, and vertical gradient of the reference gravity anomalies in mGal/m computed and stored into the files called **grvanm.map, hgrvan.map, hgrvan.dte, and hgrvan.grd** respectively.

7) Program name: **dsterm.f**

This program computes the spherical approximation effect The equation used here is shown in [Vaníček and Martinec, 1994, eqn.(29)].

Input data: Global potential coefficients (360, 360 field) called **GFZ93a**.

Output data: D^s gravity anomalies in mGal computed and stored into the file called **dsterm.map**.

8) Program name: **trnerr.f**

This program computes the truncation error The equation used here is shown in [Martinec, 1993, eqn.(6.28)].

Input data: Global satellite potential coefficients (360, 360 field) file called **GFZ93a**.

Output data: Truncation errors (metres) computed and stored into the file called **trnerr.map**.

9) Program name: **hdelgtr.f**

This program computes some of the quantities in eqn 5.4: the Bouguer term, Latitude effect, altitude effect and combines them with other quantities: the simple Bouguer anomalies and the vertical gradient of the residual topographical potential evaluated on the topography, i.e., except the last three terms. After subtracting the Helmert reference gravity anomaly, computed on the topography, the program builds up the mean residual Helmert gravity anomaly on the topography, mean $[\delta (\Delta g_t^h)]$, to be ready for the downward continuation.

Input data: Mean incomplete Bouguer anomalies, mean direct topographical effect on gravity, reference gravity anomalies, and gradient of the reference gravity anomalies stored into old files whose names should be given by the option file called **hdelgtr.opt.**

Output data: The mean $[\delta(\Delta g_t^h)]$ in mGal computed and stored in a new file specified by the option file.

10) Program name: **GIN95.f**

This program evaluates numerically the modified spheroidal Stokes's convolution integral as described in this report and the standard deviations.

Input data: mean $[\delta(\Delta g_g^h)]$ on a 5' by 5' grid covering the required area and the mean 1° by 1°. gravity anomalies and their standard deviations stored in files whose names are introduced into an input option file called **GIN95.opt**.

Output data: Partial geoidal height and the corresponding standard deviations for the computation area computed and stored into a new file called **GIN95.map**.

11) Program name: **GINsmth.f**

This program smooths the tears in the geoid solution along the 1° boundaries in the inner and outer zone integration. The formula is that coded in equation (4.1).

Input data: The partial Stokes's solution file. Any arbitrary name (not exceeding 35 characters) of this file, name of the output new file, and the boundaries of the area covered by the solution should be introduced into an option file called **GINsmth.opt**.

Output data: The smoothed partial Stokes's solution stored into the new file prescribed into the option file.

The following suites of programs are needed to compute the downward continuation of Helmert's gravity anomalies for the area of interest to this contract.

12) Program name: **ktable.for**

This program computes the table of the k coefficients as described in [Vaníček et al., 1994(b)].

Input data: Modification coefficients of the Poisson kernel given in [Vaníček et al., 1994(b)].

Output data: The k-table in file **ktable.dat**.

13) Program name: **kreform.for**

This program reformats the k-table so that one can calculate the A-matrix as described in [Vaníček et al., 1994(b)].

Input data: The k-table in file **ktable.dat.**

Output data: The reformatted k-table in file **kreformd.dat.**

14) Program name: **amatrix.for**

This program computes the A matrix (filter) for downward continuation of gravity anomalies or disturbances described in [Vaníček et al., 1994(b)]. Note: this program should be run on the IBM mainframe (TSO).

Input data: The reformatted k-table in file **kreformd.dat** and height data in the area of interest in file **height.dat**.

Output data: The A matrix in file **amatrix.dat**.

15) Program name: **dgt.for**

This program computes the truncation error of Poisson integration, using a global potential model [Vaníček et al., 1994(b)].

Input data: Global potential coefficients, modified Poisson kernel and the height data in the area of interest.

Output data: the truncation error in file **dgt.dat.**

16) Program name: **dcont.for**

This program evaluates the downward continuation of the input data by iterations as described in [Vaníček et al., 1994(b)]. Note: because of the large memory requirements this program should be run on IBM mainframe (TSO) .

Input data: The initial input vector for iteration in file **ini.dat** (here the high frequency Helmert's gravity anomaly on topography minus the truncation error) and the A matrix in file **amatrix.dat.**

Output data: Downward continuation of Helmert's gravity anomaly in file **delggh.dat.**