OPTIMIZATION AND DESIGN OF DEFORMATION MONITORING SCHEMES

KUANG SHAN-LONG

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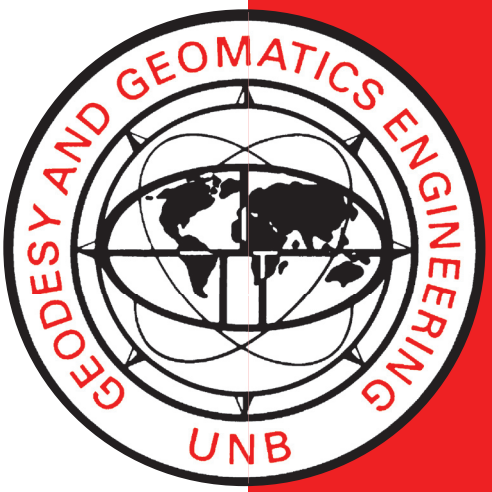

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PREFACE

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OPTIMIZATION AND DESIGN OF DEFORMATION MONITORING SCHEMES

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PREFACE

This technical report is a reproduction of a dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Surveying Engineering, July 1991. The research was supervised by Dr. Adam Chrzanowski, and funding was provided partially by the Natural Sciences and Engineering Research Council of Canada and by the University of New Brunswick.

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ABSTRACT

A methodology for the optimization and design of integrated deformation monitoring schemes has been developed in this research. Examples with real and simulated data are given which demonstrate the usefulness of the newly developed methodology. It is now possible to design integrated deformation monitoring schemes with any type of ***geodetic*** and ***geotechnical*** observations scattered in space and time to monitor any type of deformations. The methodology includes all the intentions of the conventional First Order, Second Order and Third Order Designs of geodetic networks by allowing for ***separate*** or ***simultaneous*** optimization of the geometrical configuration and weights of heterogeneous observables of a monitoring scheme ***analytically***. It allows for a simultaneous consideration of all the quality aspects, i.e. precision, internal and/or external reliability, sensitivity and economy of a monitoring scheme. The developed methodology can be used for the optimal design of either one-, two-, or three-dimensional monitoring schemes. An extension of its application to the optimal design of any geodetic networks for engineering purposes is quite straight forward.

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CHAPTER 1

INTRODUCTION

1.1 The Motive

Deformation refers to the changes a deformable body undergoes in its shape, dimension, and position. It can be said that any object, natural or man-made, undergoes changes in space and time. The determination and interpretation of the changes are the main goal of deformation surveys.

Deformation surveys are one of the most important activities in surveying, especially in engineering surveying. Their results are directly relevant to the safety of human life and engineering structures. Deformation surveys can provide not only the geometric status of the deformed object, but also information on its response to loading stress. This provides a better understanding of the mechanics of deformations and the checking of various theoretical hypotheses on the behavior of a deformable body. Examples of deformation surveys include the monitoring of ground deformations due to mining exploitation, withdrawal of oil or underground water, or construction of large reservoirs; the monitoring of accumulation of stress near active tectonic plate boundaries; and the checking of the stability of large or complex structures (e.g. hydro-electric dams).

As in conventional measurement, deformation measurements are undertaken in three phases (Chen et al., 1983):

(i) Design of the surveying scheme;

(ii) The field observation campaign, and

(iii) Post-analysis of the data.

In the last few years, a considerable effort has been made to develop new methodologies and new instrumentation for deformation monitoring, and for the post-analysis of the data. Very little effort, however, was paid to developing methodologies for the optimization and design of monitoring schemes with geodetic and non-geodetic observables, as reflected in Chen (1983):

> " ... A look at the research activities in the surveying community reveals that optimization and design of monitoring schemes with geodetic and non-geodetic observables require further research, especially in engineering surveys where the design problem is complicated ... "

The problem of the optimization and design of monitoring schemes has been recognized by and incorporated into the study program of the international "ad hoc" Committee (FIG Commission 6) on deformation analysis. The Department of Surveying Engineering at the University of New Brunswick (UNB), referred to as the "Fredericton Group", is a member of the FIG "ad hoc" Committee. Research Projects have been set up for the (Chrzanowski and Secord, 1983):

(i) Optimization and design of monitoring networks with geodetic and non-geodetic observables;

(ii) Evaluation of the observation data (including correlation of observations), detection of outliers, and systematic errors;

(iii) Geometrical analysis of deformations;

(iv) Physical interpretation of deformations.

For a number of years, the Fredericton Group at UNB has been involved in the development of new deformation surveying techniques and new methods for the post-analysis of deformation surveys. Since the ***"UNB Generalized Approach"*** for deformation analysis was developed (Chrzanowski et al., 1982; Chen 1983), it has been successfully applied in practice to a number of engineering and scientific projects.

Generally, this approach involves evaluation of the observation data; preliminary identification of the deformation models; estimation of the deformation parameters; and diagnostic checking of the models and the final selection of the "best" model. To ensure an efficient implementation of the ***"UNB Generalized Approach"*** , the optimization and design of the monitoring scheme should precede the field observation and analysis procedures. The optimization and design of a monitoring scheme are mainly comprised of:

(i) The determination of the required monitoring accuracy;

(ii) The selection of a monitoring methodology;

(iii) The determination of the optimal distribution of control and object points; and

(iv) The computation of the optimal distribution of required observational accuracies among heterogeneous observables.

An optimized monitoring scheme will ensure the most economic field campaign, and it will help in identifying, eliminating, or minimizing the effects of the gross and systematic errors existing in the observation data prior to the estimation of deformation parameters in order to avoid misinterpreting measuring errors as deformation phenomena. An optimized monitoring scheme will also ensure the detection of predicted deformations according to a selected tolerance criterion.

From the point of view of deformation mechanics, the state of a deformable body may be either static, or kinematic, or dynamic. The design of a monitoring scheme depends on the type, the magnitude, and the rate of the deformation. Compared with the design of geodetic positioning surveys, the design of an integrated monitoring scheme with geodetic and non-geodetic instrumentation is much more complicated. The accuracy criterion, and the plan of the "configuration", all require specialized treatment. Although optimization of geodetic positioning networks has been extensively discussed, no previous work has been done towards the optimization and design of integrated monitoring schemes with geodetic and non-geodetic observables. It is for these reasons that ***this research was intended***

to develop a methodology for the optimization and design of integrated monitoring schemes with geodetic and non-geodetic observables.

1.2 Identification of Problems and Scope of the Thesis

In the past years, an abundance of papers appeared concerning the optimal design of terrestrial geodetic networks, but very few have dealt specifically with deformation monitoring schemes. While the usual geodetic surveys are concerned, in a static sense, with determination of relative positions, a deformation monitoring scheme must be considered as being changeable, either dynamically or kinematically, in space and time. Thus the design of a monitoring scheme is concerned only with these changes e.g. displacements or certain deformation parameters, depending on the attributes of the specific problem. Niemeier (1981) proposed to optimize the configuration of levelling nets according to the precision criteria set for deformation parameters representing crust movements. Chen, et al. (1983) gave an example of optimizing the distribution of observation weights based on the desired accuracy of strain parameters.

In principle, the optimization and design of a monitoring scheme can be approached using basically the same philosophy as used in the optimal design of geodetic networks. In the following, a review of the presently available methods, including both the criterion set up and the solution methodology, is given first. Then the existing problems and the scope of the thesis are clarified.

Grafarend (1974) classifies the geodetic network design by order, i.e.

Zero Order(ZOD): Design of reference system

First Order(FOD): Design of the network configuration

Second Order(SOD): Selection of the Observation weights

Third Order(THOD): Addition of observations to improve the existing network

To the above design problems a fifth one can be added, called the Combined Design (COMD) (Vanicek and Krakiwsky, 1986) problem, where both the First- and Second-Order Design problems have to be optimally solved simultaneously with a preassigned covariance matrix of the parameters. A network should be designed in such a way that:

(1) The postulated precision of the network elements, and of arbitrary estimable quantities, can be realized;

(2) It is as sensitive as possible to statistical testing procedures, which allow for example the detection of outliers in the measurements;

(3) The marking of the points and the performance of the measurements are satisfying some cost criteria.

The starting point of analytical optimization techniques with regard to geodetic measurements was due to the dissertation of Helmert (1868), entitled "Studien über rationelle Vermessungen im Gebiet der höheren Geodäsie". Since that time several of the most exceptional geodesists have contributed to this subject, e.g. Schreiber(1882), Jung(1924), and Wolf (1961). They all attempted to minimize some objective function which describes the cost, precision, or reliability within a geodetic project by a scalar value. Baarda (1962) proposed a completely different concept which dealt with a so-called criterion matrix to be best approximated by the actual covariance matrix of the estimated parameters. These criterion matrices possess "ideal" structure (in a certain sense) which has to be specified in each case. Grafarend (1972) introduced the Taylor-Karman structured idealized variance-covariance matrix of Cartesian coordinates in two- and three-dimensional geodetic networks based on the theory of turbulence. In Grafarend and Schaffrin(1979), TK-structures were studied generally with variance matrices for azimuths, angles and distances derived from general and special dispersion matrices of Cartesian coordinates and coordinate differences. In a subsequent publication, Schaffrin and Grafarend (1981) dealt with the problem of allocated criterion matrices which were computed using generalized

inverses from the idealized variance-covariance matrix of azimuths, angles or distances constructed under the postulate of homogeneity and isotropy. Molenaar(1981) extended the two dimensional concept of Baarda into three dimensions by using quaternion algebra and spherical coordinates.

The problem with the Taylor-Karman structured criterion matrices is that the requirement for statistical homogeneity and isotropy is too strict for real networks; unrealistic requirements may lead to absurd design.

The solution strategies of the optimization problems in the different orders of the design are dependent both on the mathematical form to which the problem has been brought, and on the shape of the objective function which is representing the aim of the design. Unfortunately, purely analytical solutions are known mainly in the second order design, where the standard problems can be expressed in terms of linear equations and linear inequalities.

A primitive method which is suited for FOD, SOD, and THOD is the computer simulation, or "trial and error" method. In this method, a solution to the design problem is postulated and the design and cost criteria computed. Should either of these criteria not be fulfilled, a new solution is postulated (usually by slightly altering the original postulate) and the criteria are recomputed. The procedure is repeated until a satisfactory (unlikely to be the optimum) network is found. The process is summarized in more detail by the following steps (Cross,1985):

(i) specify precision and reliability criteria;

(ii) select an observation scheme (stations, observations, and weights);

(iii) Compute the covariance matrices of the desired least squares estimates and derive the values of the quantities specified as precision and reliability criteria;

(iv) if these values are close to those specified in (i) then go to the next stage; otherwise alter the observation scheme (by removing observations or

decreasing weights if the selected network is too good, or by adding observations or increasing weights if it is not good enough) and return to (iii);

(v) compute the cost of the network and consider the possibility of returning to (ii) and restarting the process with a completely different type of network (e.g. a traverse instead of triangulation). Stop when it is believed that the optimum (minimum cost) network has been found.

The method has been used for about twenty years now and is well established. Some descriptions of software include Mepham (1983), Cross(1981), and Frank and Misslin(1980). Recent research into the simulation method concentrates on the following:

(i) Increasing the computational efficiency of the process, e.g. by using sequential least squares as in Baran(1982), Mepham(1983), and Tang(1990);

(ii) The establishment of general rules to help designers decide quickly on suitable networks to select in stage(ii) of the simulation process;

(iii) The use of interactive graphics;

(iv) The automation of the alternative process (stage (iv) above) so that the computer rather than the designer chooses which observations to add or remove.

Two important interactive graphics systems were described in Nickerson (1979) and Conzett et al. (1980). With an interactive computer system with graphic terminals, the design can be criticized and directly improved in a dialogue mode.

The advantage of the simulation method is that arbitrary decision criteria can be used and compared together, in order to find the required design. There is no need to bring these criteria into a strong mathematical form which is indispensable if one uses purely analytical solutions with discrete risk functions. The obvious disadvantages of the method are that the optimum network may never be found and also a very large amount of work may be involved.

In contrast, the so-called " analytical" methods offer specific algorithms for the solution of particular design problems. Once set in motion, such an algorithm will automatically produce a network that will satisfy the user quality requirements and that will, in some mathematical sense, be optimum. So far, however, almost all of the advances in analytical methods have been in finding solutions only for the second order design problem. Although Koch (1982) tried to develop an analytical algorithm for the First Order Design, the derivatives needed in his mathematical modelling are also provided by numerical methods.

Firstly, the starting equations for SOD read

$$A^T P A = Q_{\hat{X}}^{-1} = P_X, \quad \text{(Grafarend, 1974)} \tag{1-1a}$$

$$(A^T \Theta A^T)\underline{p} = \text{vech}(P_x), \quad \text{(the diagonal SOD, Schaffrin, 1977)} \tag{1-1b}$$

where $\underline{p}$ is a vector containing the diagonal elements of P.

Bossler et al (1973) proposed a solution for Eq. (1-1a) using the Moore-Penrose inverse, i.e.,

$$P = (A^+)^T P_x A^+. \tag{1-2}$$

The solution results in a positive-definite weight matrix which is not at all realizable by practical measurements, and therefore useless for practical applications.

Schaffrin (1977) solves Eq. (1-1b) for a diagonal weight matrix P, i.e. for uncorrelated observations using the Khatri-Rao product and the Moore-Penrose inverse, i.e.,

$$\underline{p} = (A^T \Theta A^T)^+ \text{vech}\,(P_x), \tag{1-3}$$

Notice that in a design problem where m new stations are connected by n observations for two-dimensional networks, Eq.(1-1b) will be a set of m(2m+1) equations with n unknowns.

Equation (1-3) will produce a set of observational weights which, if later achieved, will yield a network whose covariance matrix best fits the criterion matrix in a least squares sense. Also, it will be optimum in the sense that $\underline{p}^T \underline{p}$ will be a minimum. A numerical problem with this solution arises because, even for quite small networks, ($A^T \Theta A^T$) is a large matrix, with size m(2m+1) by n, and the computation of its inverse is time consuming. The fact that it is sparse is not very helpful in practice and, of course, ($A^T \Theta A^T$) will always be a full matrix. Furthermore, the inversion involves poorly conditional matrices leading to numerical difficulties. Schaffrin et al. (1977), Schmitt(1977) suggested to rewrite Eq.(1-3) in its canonical form to improve the results; i. e.

$$\underline{p} = (Z^T \Theta Z^T)^+ \text{vech } (D) \tag{1-4}$$

where P_X has been decomposed by the similarity transformation

$$P_x = E D E^T \tag{1-5}$$

and

$$Z = A E. \tag{1-6}$$

Of course, Eq.(1-3) and Eq.(1-4) give identical solutions. Cross and Whiting (1981) have carried out a number of tests with the disappointing result that it regularly produced negative observation weights. These clearly have no physical meaning and are therefore difficult to interpret. One approach is to simply discard observations with negative weights but this leads to disconnected networks, i.e. networks split into several independent sections. Alternatively only the observation with the least negative weight can be discarded and the process repeated but tests have shown that the observation with the least negative weight is rarely the least valuable.

One way of avoiding negative weights is to use linear programming. Boedecker(1977) solved Eq.(1-1b) for the case of gravity networks by linear programming. Following his suggestion, Cross and Thapa(1979) attempted to find a solution for $\underline{p}$ in Eq.(1-1b) such that the resulting network would have a covariance matrix

that would, in some sense, be better than the criterion matrix. A network is bound to satisfy its design criteria if the variances in the covariance matrix are forced to be smaller than those in the criterion matrix and, conversely, the covariances larger.

Since Eq.(1-1b) involves an inversion of the criterion matrix, and since inversion is the matrix equivalent of a reciprocal, these inequalities were reversed and the linear programming constraint equations written as

$$(A^T \Theta A^T)\, \underline{p} \geq \text{vech}\,(P_x) \qquad \text{(diagonal elements)} \qquad (1\text{-}7)$$

$$(A^T \Theta A^T)\, \underline{p} \leq \text{vech}\,(P_x) \qquad \text{(off-diagonal elements)} \qquad (1\text{-}8)$$

$$p_i \geq 0 \qquad \text{for all } i \qquad (1\text{-}9)$$

The objective function is to minimize the sum of the weights.

Unfortunately, the method sometimes yields networks which do not satisfy the design criteria. The reason is that the simple reversal of inequality signs due to the inversion of the criterion matrix is not valid. It seems impossible to predict, for a given choice of method, the correct inequality signs for Eq.(1-7) and Eq.(1-8), and, therefore, it must be concluded that this linear programming technique can only be applied if the inversion of the criterion matrix is avoided.

Cross and Whiting (1980) have suggested that the inversion of the criterion matrix can be avoided by expanding the left hand side of Eq.(1-1a) using an unspecified generalized inverse

$$(A^T)^- P^- A^- = Q_X \qquad (1\text{-}10)$$

which, after application of the Khatri-Rao product, becomes

$$((A^-)^T \Theta (A^-)^T)\underline{w} = \text{vech}\,(Q_x) \qquad (1\text{-}11)$$

where $\underline{w}$ is a vector containing the reciprocals of required weights of the observations i.e. the diagonal elements of P. Eq.(1-11) can now be restated as a linear programming problem with the following constraint equations

$$((A^-)^T \Theta (A^-)^T)\underline{w} \leq \text{vech}\,(Q_x) \quad \text{(diagonal elements)} \tag{1-12}$$

$$((A^-)^T \Theta (A^-)^T)\underline{w} \geq \text{vech}\,(Q_x) \quad \text{(off-diagonal elements)} \tag{1-13}$$

$$w_i \geq 0 \quad \text{for all i} \tag{1-14}$$

The objective function for this set up must be maximized in order to reduce the total work.

Unfortunately, the method proved impractical as a suitable generalized inverse could not be found. Cross and Whiting (1980) have tried to use the Moore-Penrose inverse even though they showed that theoretically it was not valid. It resulted, in general, in designed networks being much more precise than required and hence too expensive.

Schaffrin (1980) suggested the use of the linear complimentary algorithm to overcome the negative weight problem. This involves determining a best-fit solution in the least squares sense to Eq.(1-1b), subject to a number of linear constraints which, as well as describing the required precision and cost of the network, also ensure that P is non-negative. The mathematical set up is essentially equivalent to a quadratic programming problem and can be written as

$$\text{Minimize } ((A^T \Theta A^T)\underline{p} - \text{vech}(P_x))^T((A^T \Theta A^T)\underline{p} - \text{vech}(P_x)) \tag{1-15}$$

$$\text{Subject to } (A^T \Theta A^T)\underline{p}\ (\geq; =; \leq)\ \text{vech}(P_x) \tag{1-16}$$

$$\underline{c}^T \underline{p} \leq d \tag{1-17}$$

$$p_i \geq 0 \quad \text{for all i} \tag{1-18}$$

where $\underline{c}$ is a vector of coefficients relating observation weight to cost and d the total allowable cost. Liew and Shim (1978) give details of a computer program suitable for the solution of this problem. Note that the difficulty regarding the inequality signs for use in Eq.(1-16) arises again but Schaffrin (1980) states that it may be avoided by reforming Eq.(1-15) and Eq.(1-16) using the canonical formulation and restricting Eq.(1-16) to the rows which correspond to the eigenvalues of P_x within vech D. Then Eq.(1-16) becomes

$$(Z^T \Theta Z^T)\underline{p} \geq \text{vech}\,(D)\,. \tag{1-19}$$

Schaffrin et al (1980) have successfully applied the method to a geodetic network with Taylor-Karman criterion matrices.

Wimmer (1982), and Cross and Fagir (1982) proposed a method by reforming the basic mathematical statement of the second order design as follows

$$P = P\,P^{-1}\,P. \tag{1-20}$$

Postmultiplying both sides of Eq.(1-20) by $A\,(A^T P A)^{-1}$ and premultiplying by $(A^T P A)^{-1}A^T$ yield

$$(A^T P A)^{-1} = ((A^T P A)^{-1}A^T P)P^{-1}(P A\,(A^T P A)^{-1}). \tag{1-21}$$

Denoting $$G = (P A\,(A^T P A)^{-1}), \tag{1-22}$$

and substituting Eq.(1-1) and Eq.(1-22) into Eq.(1-21) one obtains

$$Q_X = G^T\,P^{-1}\,G. \tag{1-23}$$

Applying the Khatri-Rao product to Eq.(1-23) and rearranging them yields

$$(G^T \Theta G^T)\underline{w} = \text{vech}\,(Q_x). \tag{1-24}$$

Putting $$H = (G^T \Theta G^T), \tag{1-25}$$

and substituting it into Eq.(1-24) yields

$$H\underline{w} = \text{vech}\,(Q_x) \tag{1-26}$$

where $\underline{w}$ contains the reciprocals of the diagonal elements of P.

This formulation is of a structure similar to Eq.(1-1b) but has the considerable advantage of being in terms of the criterion matrix itself rather than its inverse. All solutions to Eq.(1-24) must, of course, be iterative because, according to Eq.(1-22), the matrix G is itself in terms of P. Hence we must first assume a set of values for P, solve Eq.(1-24) for $\underline{w}$ (and hence P) and use this value to recompute G. The process is repeated until P ceases to change.

In summary, it can be said that in the realm of geodetic network design, except for the Zero Order Design, all the rest of the design orders were not fully solved (Schmitt, 1982) and the following conclusion may be drawn: (i) There is no fully analytical solution method for the First Order Design. The justification of the First Order Design has been

called in question for a time. In classical large scale networks there is no margin because of topographical realities. But it has its importance and qualification in the realm of deformation surveys. The only numerical method which leads, until now, to successful solutions is the computer simulation; (ii) The success of SOD depends on both the set up of the criterion matrix and proper formulation of the mathematical model. Up to now, most of the formulations for SOD are in terms of the inverse of the criterion matrix, not the criterion matrix itself. That makes it difficult, in some cases, to achieve the design criteria, since a good fitting of the inverse of the criterion matrix does not necessarily mean the good fitting of the criterion matrix itself. The problem of how to properly set up the criterion matrix was not solved. As discussed before, the Taylor-Karman structured criterion matrix is too strict for a real network. What is the empirical correlation behavior in real networks? How far is this behavior in correspondence to statistically ideal correlation situations such as the Taylor-Karman structure? How can we construct an allocated criterion matrix if the real input or the design problem is a criterion matrix of derived quantities? For example, in deformation measurements, the accuracy of deformation parameters as derived from the displacement field is the design target for a monitoring scheme. All these questions remain to be answered; (iii) As for the THOD and COMD, there exists no fully analytical solution. The mainly used way is by simulation and by "trial and error."

Having recognized these problems, a methodology for the optimization and design of integrated monitoring schemes with geodetic and non-geodetic observables has been developed by the author and successfully applied to a number of practical examples. This approach may be used for the First Order, Second Order, Third Order, or the Combined First Order and Second Order Design ***analytically***. It removes the need for the method of " Trial and Error". The aims of this thesis have been:

(i) To define the measures and optimality criteria for the quality of deformation monitoring schemes;

(ii) To formulate the mathematical models for optimization; and

(iii) To evaluate the mathematical models.

Deformation measurements are usually categorized as being of a local, regional, continental or global scale(Whitten,1982). This study concentrates on local and regional scales. The developed methodology is applicable mainly to the detection of local deformation, with a possible extension for regional applications. Usually, such a monitoring scheme consists of a geodetic network, plus some isolated non-geodetic observables which may not be geometrically connected with the geodetic network.

1.3 Organization of the Contents and Summary of Contributions

This study presents a systematic study of the optimization and design of deformation monitoring schemes. The remainder of this chapter gives an outline of the research work done and, at the same time, the contributions of the author are listed.

A good knowledge of the data acquisition and analysis techniques is a prerequisite for a successful design of deformation surveys. In Chapter 2, a brief review of the theory of deformation analysis, the geodetic and non-geodetic monitoring techniques coupled with their typical accuracies is given first. Then, the design problem involved is identified.

Chapter 3 is devoted to defining quality control measures and optimality criteria for monitoring schemes. Optimization of monitoring schemes means minimizing or maximizing an objective function which represents the criteria adopted to define the "quality of the scheme". Four general criteria are used to evaluate this quality: precision, reliability, sensitivity, and economy. Therefore, a quantification of these demands is the first step towards optimization. This is elaborated in Chapter 3, which begins with a survey of the present precision measures of conventional geodetic networks for positioning purposes. These measures are then modified to represent monitoring schemes established for the purpose of displacement detection or estimation of deformation parameters. Then the datum

problem for criterion matrix in designing deformation surveys is discussed. In Section 3.2, a general reliability criterion is proposed to search for minimum detectable gross errors and to minimize their effects on the solution of deformation parameters. The sensitivity criterion of monitoring schemes is developed in Section 3.3 to enable the detection of postulated deformation parameters of certain magnitude. Finally, the minimum cost criterion is discussed in Section 3.4.

The methodology for the optimization and design of deformation monitoring schemes is developed in Chapter 4. At first, the unknown parameters to be optimized in a monitoring scheme are identified in Sections 4.1 and 4.2, which include positions of both the geodetic and non-geodetic points and weights of both the geodetic and non-geodetic observables. Then in Section 4.3, the optimality criteria for precision, reliability, sensitivity and economy, as developed in Chapter 3, along with the physical environment in which the optimization is performed are transformed into constraints on the optimal solution of the unknown parameters in a three-dimensional space. The contributions made in this section include a new formulation of the precision criterion in terms of the criterion matrix itself, rather than its inverse; and all the criteria of precision, reliability, sensitivity, and economy are brought into a strong mathematical form. Sections 4.4, 4.5 and 4.6 lay down the mathematical foundation for optimization. After the five different possible mathematical models for optimization are developed in Section 4.4, the solution methods for these models are discussed in Sections 4.5 and 4.6, where a unified mathematical modelling i.e., the Multi-Objective Optimization Model (MOOM) is developed and suggested for practical appplication. This model aims at best approximating all the precision, reliability, sensitivity, and economy criteria "from both sides" or "from one side" by optimizing the monitoring configuration and weights of observations simultaneously under the given topography and instrumentation condition.

Chapter 5 elaborates on the full evaluation of the developed mathematical Model MOOM. At first, two simulation studies are performed. The simulation study No. 1

confirms the correctness of the developed mathematical model. The simulated example 2 illustrates the significance of applying relatively small position changes of netpoints for the optimal solution of observation weights, what has been underestimated by other authors. In the third example, the practical significance and advantages of the newly developed methodology over the conventional approaches are demonstrated. Finally, as a practical application, this model is applied to a geodetic monitoring network established to assist in deformation analysis of structures of the Mactaquac hydro-power Generating station in Canada, resulting in a saving of 20% field work while increasing the monitoring accuracy by a factor of two. Chapter 6 concludes this study.

CHAPTER 2

DEFORMATION MONITORING AND THE DESIGN PROBLEM

A good knowledge of data acquisition and analysis techniques is a prerequisite for a successful design of deformation surveys. In this Chapter a brief review of the basic deformation parameters and deformation models, the geodetic and non-geodetic monitoring techniques coupled with their typical accuracies, and the "***UNB Generalized Approach***" for deformation analysis is given first. Then the design problem involved in a deformation monitoring scheme is identified.

2.1 Basic deformation parameters and the Deformation Model

If acted upon by external forces (loads), any real material deforms, i.e., changes its dimensions, shape and position. According to Sokolnikoff (1956) the basic deformation parameters are rigid body translation, rigid body rotation (or relative translation and rotation of one "block" with respect to another), strain tensor and differential rotation components. If a time factor is involved, the derivative of the above quantities with respect to time is used instead. According to Chen(1983) and Chrzanowski et al (1983), the above deformation parameters in three-dimensional space can be obtained if the displacement field $\underline{d}(\underline{x},\underline{y},\underline{z};t-t_0)$ is known. The displacement field can be approximated by fitting a selected deformation model to displacements determined at discrete points:

$$\underline{d}(\underline{x}, \underline{y}, \underline{z}; t\text{-}t_0) = B(\underline{x}, \underline{y}, \underline{z}; t\text{-}t_0)\, \underline{e} \tag{2-1}$$

where $\underline{d}$ is the vector of displacement components of point (x_i, y_i, z_i) at time t with respect to t_0;

B is a matrix of base functional values; and

$\underline{e}$ is the vector of unknown deformation parameters.

The mathematical model Eq.(2-1) can be explicitly written as

$$\underline{d} = \begin{pmatrix} u(\underline{x}, \underline{y}, \underline{z}; t\text{-}t_0) \\ v(\underline{x}, \underline{y}, \underline{z}; t\text{-}t_0) \\ w(\underline{x}, \underline{y}, \underline{z}; t\text{-}t_0) \end{pmatrix} = \begin{pmatrix} B_u(\underline{x}, \underline{y}, \underline{z}; t\text{-}t_0)\, e_u \\ B_v(\underline{x}, \underline{y}, \underline{z}; t\text{-}t_0)\, e_v \\ B_w(\underline{x}, \underline{y}, \underline{z}; t\text{-}t_0)\, e_w \end{pmatrix} \tag{2-2}$$

where u, v, and w represent displacement components in the x, y, and z directions respectively, and they are functions of both position and time.

From Eq.(2-2), the non-translational deformation tensor can be calculated by

$$E = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{pmatrix} \tag{2-3}$$

and the normal strains, shear strains and the differential rotations around x, y, z axes are respectively

$$\varepsilon_x = \frac{\partial u}{\partial x},\ \varepsilon_y = \frac{\partial v}{\partial y},\ \varepsilon_z = \frac{\partial w}{\partial z}; \tag{2-4}$$

$$\varepsilon_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)/2,\ \varepsilon_{xz} = \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)/2,\ \varepsilon_{yz} = \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)/2; \tag{2-5}$$

$$\omega_x = \left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right)/2,\ \omega_y = \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)/2,\ \omega_z = \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)/2 \tag{2-6}$$

In addition, certain functions of these strain parameters, e.g., maximum strain (ε), dilatation (Δ), pure shear (r_1), simple shear (r_2), and total shear (r) may also be of interest and they are defined as (Frank,1966)

$$\varepsilon = \sqrt{\varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_z^2} \tag{2-7}$$

$$\Delta = \varepsilon_x + \varepsilon_y + \varepsilon_z \tag{2-8}$$

$$r_1 = (\varepsilon_x - \varepsilon_y) \tag{2-9}$$

$$r_2 = 2\ \varepsilon_{xy} \tag{2-10}$$

$$r = \sqrt{r_1^2 + r_2^2} \tag{2-11}$$

As for the selection of a deformation model, it depends on any a priori information that is available and, especially, from whatever trend or change is exhibited by the measurements or by the location of the stations. When using the "***UNB Generalized Approach***" in geometrical deformation analysis, the whole area covered by the deformation surveys is treated as a non-continuous deformable body consisting of separate continuous deformable blocks. Thus the blocks may undergo relative rigid body displacements and rotation, and each block may change its shape and dimensions. In the case of single point movement, the given point is treated as a separate block being displaced as a rigid body in relation to the undeformed block composed of the remaining points in the network. Examples of typical deformation models in two-dimensional space are given below (Chrzanowski et al., 1982; Chen,1983; Chrzanowski et al., 1986):

(1) Single point displacement or a rigid body displacement of a group of points, say, block B (Fig. 2.1a) with respect to block A. The deformation model is expressed as:

$$u_A = 0,\ v_A = 0;\ u_B = a_0 \text{ and } v_B = b_0, \tag{2-12}$$

where the subscripts represent all the points in the indicated blocks.

(2) Homogeneous strain in the whole body and differential rotation (Fig. 2.1b), the deformation model is

$$u = \varepsilon_x x + \varepsilon_{xy} y - \omega y$$
$$v = \varepsilon_{xy} x + \varepsilon_y y + \omega x \qquad (2\text{-}13)$$

where the physical meaning of the coefficients is defined in Eq.(2-4) to (2-6) with ω_z in Eq.(2-6) being replaced by ω.

(3) A deformable body with one discontinuity (Fig. 2.1c), say, between blocks A and B, and with different linear deformations in each block plus a rigid body displacement of B with respect to A. Then the deformation model is written as

$$u_A = \varepsilon_{xA} x + \varepsilon_{xyA} y - \omega_A y$$
$$v_A = \varepsilon_{xyA} x + \varepsilon_{yA} y + \omega_A x \qquad (2\text{-}14)$$

and

$$u_B = a_0 + \varepsilon_{xB}(x - x_0) + \varepsilon_{xyB}(y - y_0) - \omega_B (y - y_0)$$
$$v_B = b_0 + \varepsilon_{xyB}(x - x_0) + \varepsilon_{yB}(y - y_0) + \omega_B (x - x_0) \qquad (2\text{-}15)$$

where x_0, y_0 are the coordinates of any point in block B

The components Δu_i and Δv_i of a total relative dislocation at any point i located on the discontinuity line between blocks A and B can be calculated as

$$\Delta u_i = u_B(x_i, y_i) - u_A(x_i, y_i)$$
$$\Delta v_i = v_B(x_i, y_i) - v_A(x_i, y_i) \qquad (2\text{-}16)$$

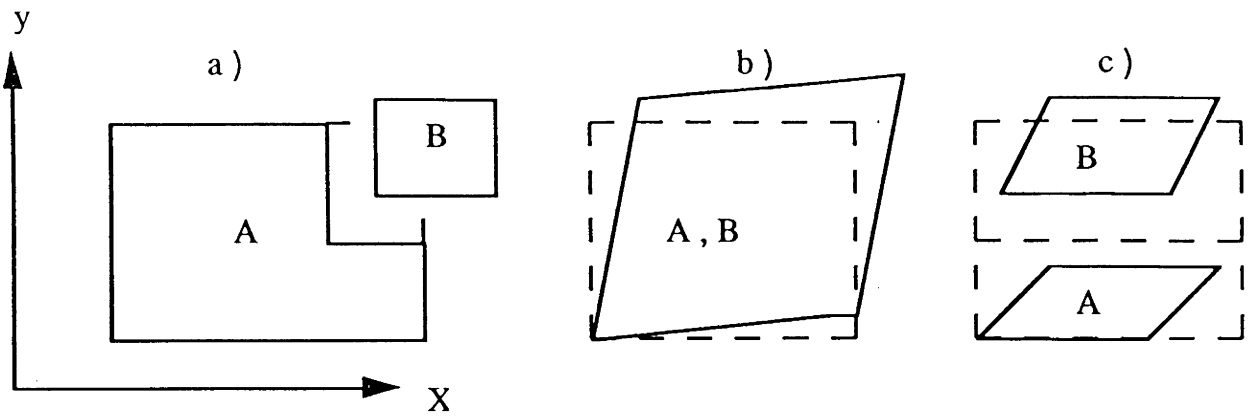

Fig. 2.1: Typical deformation models (after Chrzanowski et al., 1983)

Usually, the actual deformation model is a combination of the above simple models or, if more complicated, it is expressed by non-linear displacement functions which require the fitting of higher order polynomials or other suitable functions. If time dependent deformation parameters are sought, then the above deformation models will contain time variables. For instance, in the model of homogeneous strain, if a linear time dependence is assumed, the model becomes:

$$u(x , y , t) = \dot{\varepsilon}_x \, x \, t + \dot{\varepsilon}_{xy} \, y \, t - \dot{\omega} \, y \, t$$
$$v(x , y , t) = \dot{\varepsilon}_{xy} \, x \, t + \dot{\varepsilon}_y \, y \, t + \dot{\omega} \, x \, t \qquad (2\text{-}17)$$

2.2 Geodetic and Non-Geodetic Methods for Deformation Monitoring

Acquisition of deformation parameters is one of the main goals of deformation monitoring. Different methodologies and techniques have been used for this purpose. As compared with other types of surveys, deformation measurements have the following characteristics (Chen,1983):

(i) Higher accuracy requirement;

For example, in engineering projects, an accuracy of $\pm$ 1 mm or higher might be a typical requirement

(ii) Repeatability of observations;

The periods of resurveys range from seconds to years, depending on the rate of deformation

(iii) Integration of different types of observations;

Here not only **geodetic** methods should be considered but also **non-geodetic** instrumentation, e.g., pendula, tiltmeters, strainmeters, mechanical and laser alignment, hydrostatic levels and others in order to get more complete information

(iv) Network may be incomplete, scattered in space and time;

(v) Sophisticated analysis of the acquired data in order to avoid the misinterpretation of measuring errors as deformation and local phenomena as a global status.

Geodetic methods, which include terrestrial geodetic methods, photogrammetric methods, and space techniques, are used to monitor the magnitude and rate of horizontal and vertical deformations of structures, the ground surface, and accessible parts of subsurface instruments in a wide variety of construction situations. Frequently, these methods are entirely adequate for deformation monitoring. In **non-geodetic** methods, we have geotechnical and specialized monitoring devices. They are required only if greater accuracy is sought or if measuring points are inaccessible to **geodetic** methods. However, in general, whenever non-geodetic instruments are used to monitor deformation, geodetic methods are also used to relate measurements to a reference datum.

Up to now, there are hundreds of available models of various geodetic and non-geodetic instruments for deformation measurements. The decision on which instruments should be used and where they should be located leads to the need for a proper design and

optimization of a proposed measuring scheme which should be based on the best possible combination of all the available measuring instrumentation. In discussing geotechnical instrumentation for performance monitoring, Peck (in Dunnicliff, 1988) states that

> "... every instrument on a project should be selected and placed to assist answering a specific question, the wrong instruments in the wrong places provide information that may at best be confusing and at worst divert attention from telltale signs of trouble. Too much instrumentation is wasteful and may disillusion those who pay the bills, while too little, arising from a desire to save money, can be more than false economy: it can even be dangerous ..."

Therefore, the design of a monitoring scheme should satisfy not only the best geometrical strength of the network of the observation stations, as is the case in geodetic positioning surveys, but should primarily satisfy the needs of the subsequent physical interpretation of the monitoring results, i.e., should give optimal results when solving for the deformation parameters of the selected deformation model (Chrzanowski et al, 1986) and that is the main topic of this research.

The Geodetic Methods

According to Chrzanowski (1981), in deformation measurements by geodetic methods, whether they are performed for monitoring engineering structures or ground subsidence in mining areas or tectonic movements, the monitoring networks can be divided into relative networks and reference networks (see Fig. 2.2 and Fig. 2.3).

In relative networks, all the survey points are assumed to be located on the deformable body, the purpose in this case is to identify the deformation model, i.e., to distinguish, on the basis of repeated geodetic observations, between the deformations caused by the extension and shearing strains, by the relative rigid body displacements, and by the single point displacements. However, in reference networks, some of the points are, or are assumed to be, outside the deformable body (object) thus serving as reference points for the determination of absolute displacements of the object points.

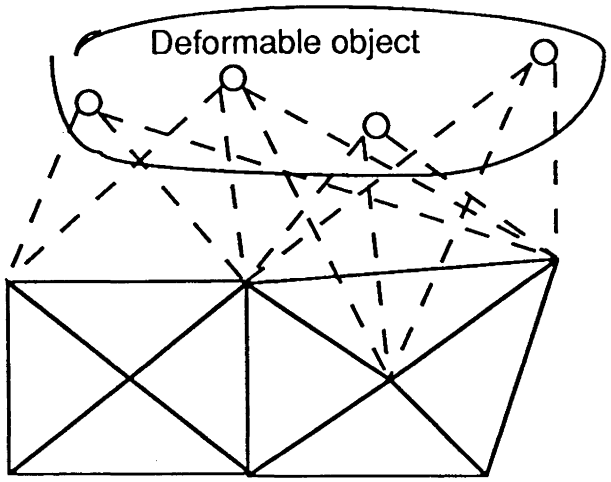

Fig. 2.2: A Reference Monitoring Network

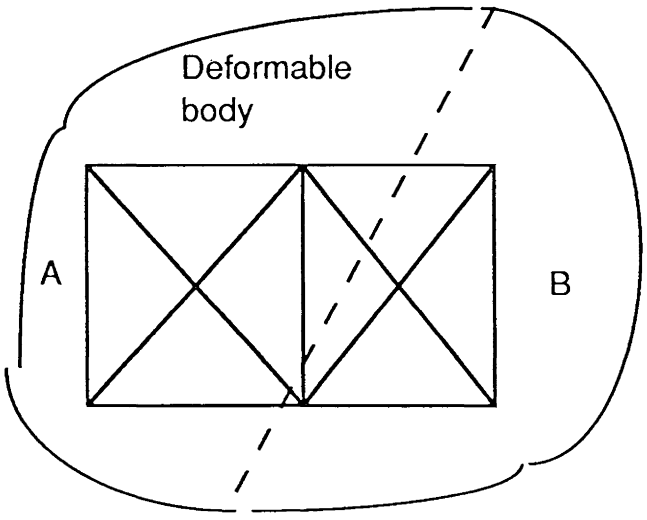

Fig. 2.3: A Relative Monitoring Network

Table 2.1: The geodetic methods for deformation monitoring

Methods	Achievable accuracy (σ)
i) Elevations by optical levelling	0.1 mm over a few tens of metres to about 1 mm over long distance
ii) Distance measurements with tapes or wires	0.1 mm over a few metres to about 2 ppm over a few hundred metres
iii) Offsets from a baseline using theodolite and scale	0.3 - 2 mm
iv) Traversing	1/30,000 - 1/150,000
v) Triangulation	1/30,000 - 1/1,000,000
vi) Electronic distance Measurement (EDM)	0.2 mm or 0.1 ppm to 5 ppm
vii) Trigonometric levelling	2 mm $\sqrt{km}$
viii) Photogrammetric methods	1/5000 - 1/100,000
(ix) Space techniques	
VLBI	0.01 ppm
SLR	0.01 ppm
GPS	0.1ppm - 2 ppm

Table 2.2: The Non-Geodetic Methods for Deformation Monitoring

Types of deformation	Methods and sample instruments	Typical accuracy
Extensions and strains	a) Wire and Tape extensometers	
	* ISETH Distometer	0.05 mm
	* CERN Distinvar	0.05 mm
	* Rock Spy	0.02 - 0.2 mm
	b) Rod and Tube extensometers	
	* Single-point extensometers	0.01 - 0.02 mm
	* Multi-point extensometers	0.01 - 0.02 mm
	* Torpedo type extensometers	0.1 mm
	c) Michelson type laser interferometers	
	* Laser strain meters	0.0004 ppm
Tilts and inclinations	a) Precision tiltmeters	
	* High precision mercury tiltmeter	0.0002"
	*Electrolevel	0.25"
	* Talyvel	0.5"
	b) Hydrostatic levelling	
	*Elwaag001	0.03 mm/40 m
	* Nivomatic telenivelling system	0.1 mm/ 24 m
	c) Suspended and Inverted Pendulum	0.1 mm
Alignment	a) Mechanical Methods	
	* Steel wire alignment	0.1 mm
	* nylon line alignment	0.035 mm - 0.070 mm
	b) Direct Optical alignment	1- 10 ppm
	c) Alignment with laser diffraction gratings	0.1 - 1 ppm

A list of the most commonly used geodetic methods for deformation monitoring and their associated approximate accuracies is given by Table 2.1. Over the last 20 years the availability of increasingly reliable and accurate EDM equipment has radically changed conventional surveying practices. EDM devices require fewer personnel than conventional optical instruments, are faster to use, and are more accurate. Therefore, horizontal triangulation monitoring networks have been gradually replaced by trilateration or triangulateration networks. Depending on the models, an EDM instrument can have a range of a few meters to several tens of kilometres. Recent progress in distance measurements includes the development of the multiple wavelength EDM to reduce the effect of tropospheric refraction internally, e.g., Terrameter (Hugget, 1982) and the EDM with high modulation frequencies, e.g. Kern ME 5000. The former can achieve an accuracy in the order of 0.1 ppm over distance of several kilometres while the latter can give 0.3 mm standard deviation over a few hundred meters.

Trigonometric levelling is much more economical than conventional geodetic levelling when third-order accuracy is adequate and measuring points are physically inaccessible. According to Chrzanowski (1983), even the first order and second order accuracy can be achieved by a modified trigonometric levelling i.e. the leap-frog or reciprocal trigonometric levelling. This method is especially advantageous in mountainous areas.

Both terrestrial and aerial photogrammetry have been extensively used in the determination of deformations of large structures (e.g. Faig, 1978), and ground subsidence(Faig and Armenakis,1982). In terrestrial photogrammetry, an example is to use phototheodolites to take successive photographs from a fixed station along a fixed baseline. Movements are identified in a stereocomparator by stereoscopic advance or recession of pairs of photographic plates in relation to stable background elements. The procedure defines the components of movements taking place in the plane of the photograph. The

photogrammetric method has the advantage that hundreds of potential movements are recorded on a single stereo photographic pair, allowing an appraisal of the overall displacement pattern in a minimum time.

Geodetic space techniques, e.g. VLBI(Very Long Base Line Interferometry), SLR (Satellite Laser Ranging), and GPS (Global Positioning System) can provide deformation data of global extent, such as polar motion, variation of the earth's rotation, and relative motion between tectonic plates.

The Non-Geodetic Methods

The geodetic methods, through interconnections among the monitoring stations, can provide very useful information on the global deformation status of the monitored object and, in most cases, can also provide information on its rigid body translations and rotations with respect to reference points located outside the deformation area. However, as mentioned above, geodetic methods are limited only to open areas, they require intervisibility between the survey stations, or between the monitoring stations and the satellites, or between the object points and the cameras. The deformations inside the deformable body e.g. in foundations or foundation rocks of large engineering structures and relative movements of different layers of soil or rock formations in slope stability studies, etc, can only be approached by non-geodetic methods.

Non-geodetic methods include geotechnical instrumentation and other specialized monitoring devices. For instance, borehole inclinometers, extensometers, are examples of geotechnical instruments, while inverted pendula, hydrostatic levels, laser interferometers, and diffraction aligning equipment, are made for some specialized monitoring purposes. Geotechnical instrumentation does not require intervisibility between the stations and can be easily adapted for continuous and telemetric data acquisition with an instantaneous display of the deformations which is very advantageous in comparison with slow, labour intensive, geodetic surveys. Non-geodetic methods are usually used to measure three types of

deformations, i.e. extensions and strains; tilts and inclinations; and alignment. Typical examples of Non-Geodetic Instruments with their associated accuracies are listed in Table 2.2 (Chrzanowski, 1986).

One has to remember that the non-geodetic methods, despite their indisputable advantages, also have weak points: a) the measurements are very localized and they may be affected by local disturbances which do not represent the actual deformations; b) since the local observables are not geometrically connected with observables at other monitoring stations, and global trend analysis of the deformations is much more difficult than in the case of geodetic surveys unless the observing stations are very densely spaced (Chrzanowski,1986).

2.3 Estimation of Deformation Models and the Design Problem

2.3.1 The functional relationship between the deformation models and the observed quantities.

Any observation, geodetic or non-geodetic measurement made in deformation surveys, will contribute to the determination of deformation parameters and should be fully utilized in the analysis(Chrzanowski et al., 1986; Teskey, 1987). The functional relationships between different observable types and the selected deformation model are given below using a local geodetic coordinate system (Chen,1983)

(1) Observation of coordinates of point i, for instance, the coordinates derived from photogrammetric measurements or obtained using space techniques:

$$\begin{pmatrix} x_i(t) \\ y_i(t) \\ z_i(t) \end{pmatrix} = \begin{pmatrix} x_i(t_0) \\ y_i(t_0) \\ z_i(t_0) \end{pmatrix} + \begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \qquad (2\text{-}18)$$

(2) Observation of coordinate differences between points i and j, e.g., height difference (levelling) observation, pendulum (displacement) measurement, and alignment survey:

$$\begin{pmatrix} x_j(t) - x_i(t) \\ y_j(t) - y_i(t) \\ z_j(t) - z_i(t) \end{pmatrix} = \begin{pmatrix} x_j(t_0) - x_i(t_0) \\ y_j(t_0) - y_i(t_0) \\ z_j(t_0) - z_i(t_0) \end{pmatrix} + \begin{pmatrix} u_j - u_i \\ v_j - v_i \\ w_j - w_i \end{pmatrix} \qquad (2\text{-}19)$$

If the components of the displacement obtained from a pendulum observation do not coincide with the coordinate axes, a transformation to the common coordinate system has to be performed. Similarly, a coordinate transformation may be required in alignment surveys which provide a transverse displacement of a point with respect to a straight line defined by two base points.

(3) Observation of azimuth from point i to point j

$$\alpha_{ij}(t) = \alpha_{ij}(t_0) + \left(\left(-\cos\alpha_{ij}\right)/\left(s_{ij}\cos\beta_{ij}\right),\ \left(\sin\alpha_{ij}\right)/\left(s_{ij}\cos\beta_{ij}\right)\right) \begin{pmatrix} u_j - u_i \\ v_j - v_i \end{pmatrix} \qquad (2\text{-}20)$$

where β_{ij} and s_{ij} are the vertical angle and spatial distance from point i to point j, respectively. The observation of a horizontal angle is expressed as the difference of two azimuths.

(4) Observation of the distance between points i and j:

$$s_{ij}(t) = s_{ij}(t_0) + \left(\cos\beta_{ij}\ \sin\alpha_{ij},\ \cos\beta_{ij}\ \cos\alpha_{ij},\ \sin\beta_{ij}\right) \begin{pmatrix} u_j - u_i \\ v_j - v_i \\ w_j - w_i \end{pmatrix} \qquad (2\text{-}21)$$

(5) Observation of strain along the azimuth α and vertical angle β at point i:

$$\varepsilon(t) = \varepsilon(t_0) + c^T E\, c, \tag{2-22}$$

where

$$c^T = (\cos\beta \sin\alpha,\ \cos\beta \cos\alpha,\ \sin\beta)$$

$$E = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{pmatrix}$$

(6) Observation of a vertical angle at point i to point j

$$\beta_{ij}(t) = \beta_{ij}(t_0) + \left(\sin\beta_{ij}\ \sin\alpha_{ij}/s_{ij},\ -\sin\beta_{ij}\ \cos\alpha_{ij}/s_{ij},\ \cos\beta_{ij}/s_{ij}\right) \begin{pmatrix} u_j - u_i \\ v_j - v_i \\ w_j - w_i \end{pmatrix} \tag{2-23}$$

(7) Observation of a horizontal tiltmeter:

$$\tau(t) = \tau(t_0) + (\partial w/\partial x)\sin\alpha + (\partial w/\partial y)\cos\alpha \tag{2-24}$$

where α is the orientation of the tiltmeter.

In the above formulae, the quantities u, v, w and their derivative are replaced by the deformation model which is explicitly expressed in Eq.(2-2). Thus all the observations are functions of the unknown coefficients $\underline{e}$.

2.3.2 Estimation of deformation parameters and the design problem

Let $\underline{l}_i$ (i=0,1,2,...,k) be the n_i-vector of observations with weight matrix P_i in epoch i, which includes both geodetic and non-geodetic observables. The deformation model B $\underline{e}$ is related to the observables through the null hypothesis:

$$H_0: E(\underline{l}_i)=E(\underline{l}_0)+A_i B_i \underline{e} \tag{2-25}$$

where A_i is the configuration matrix which relates the observables to deformation model;

B_i is the coefficient matrix of deformation model, it is a function of position and time; and

$\underline{e}$ is the vector of deformation parameters.

The parameters $\underline{e}$ may be estimated from the following mathematical model

$$\begin{pmatrix} \underline{l}_0 \\ \underline{l}_1 \\ \vdots \\ \underline{l}_k \end{pmatrix} + \begin{pmatrix} \underline{v}_0 \\ \underline{v}_1 \\ \vdots \\ \underline{v}_k \end{pmatrix} = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{I} & A_1B_1 \\ \vdots & \vdots \\ \mathbf{I} & A_kB_k \end{pmatrix} \begin{pmatrix} \underline{\xi} \\ \underline{e} \end{pmatrix} \tag{2-26}$$

with $\underline{\xi}$ being a vector of nuisance parameters and the weight matrix (assuming there is no correlation between epochs)

$$P = \begin{pmatrix} P_0 & 0 & 0 & \cdots & 0 \\ 0 & P_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & P_{k-1} & 0 \\ 0 & \cdots & 0 & 0 & P_k \end{pmatrix} \tag{2-27}$$

Applying the least squares criterion to the above model and eliminating $\underline{\xi}$ allow the vector of deformation parameters $\underline{e}$ and its accuracy to be calculated from

$$\begin{aligned} \underline{e}= &(\sum_1^k B_i^T A_i^T P_i A_i B_i - \sum_1^k B_i^T A_i^T P_i (\sum_0^k P_i)^{-1} \sum_1^k P_i A_i B_i)^{-1} * \\ &*(\sum_1^k B_i^T A_i^T P_i \underline{l}_i - \sum_1^k B_i^T A_i^T P_i (\sum_0^k P_i)^{-1} \sum_0^k P_i \underline{l}_i) \end{aligned} \tag{2-28}$$

$$Q_{\underline{e}}=(\sum_1^k B_i^T A_i^T P_i A_i B_i - \sum_1^k B_i^T A_i^T P_i (\sum_0^k P_i)^{-1} \sum_1^k P_i A_i B_i)^{-1} \tag{2-29}$$

In the design phase it is justified to assume that the observation schemes are the same for all the epochs (i.e. A_i=A, P_i = P, for all i). And the expression for the estimation and accuracy of the estimated deformation parameters can be obtained by considering only two epoches. Therefore,

$$\underline{e} = (B^T A^T P A B)^{-1} B^T A^T P (\underline{l}_2 - \underline{l}_1) \tag{2-30}$$

$$Q_{\underline{e}} = 2(B^T A^T P A B)^{-1} \tag{2-31}$$

When B=**I** (identity matrix), then the deformation monitoring reduces to determine the displacements of object points. In this case, the cofactor matrix of the displacements can be expressed by

$$Q_{\underline{d}} = 2(A^T P A)^{-} \tag{2-32}$$

The design problem involves how to find the best configuration matrix (A) and the weight matrix (P) in order to attain the required accuracies of the deformation parameters or displacements in the most economical way.

CHAPTER 3

QUALITY CONTROL MEASURES AND OPTIMALITY CRITERIA FOR DEFORMATION MONITORING SCHEMES

Generally, the quality of a monitoring scheme may be characterized by precision, reliability, sensitivity and economy. Precision, as expressed by the a posteriori covariance matrix of the coordinates, displacements, or deformation parameters, etc, is the measure of the scheme's characteristics in propagating random errors; reliability describes the ability of the redundant observations to check observation errors; sensitivity describes the scheme's ability to detect postulated displacements or deformation parameters of certain magnitude; and finally, economy is expressed in terms of the observation program. Thus the optimization of a monitoring scheme may be said to design a precise-, reliable- and sensitive enough scheme which can also be realized in an economical way. But how precise, reliable, sensitive and cheap should the scheme be? A quantification of the demands is obviously indispensable. This chapter will discuss various measures and criteria for these different indicators of the quality of a monitoring scheme, and this serves as the foundation for the optimization.

3.1 Measures and Criteria for Precision

3.1.1 Scalar precision functions

A great deal of work has been carried out in the field of user precision requirements for geodetic networks. In the case of integrated monitoring schemes with geodetic and non-geodetic instruments, since the displacements or deformation parameters to be monitored can be derived from changes in point coordinates, thus a good knowledge of the ways to quantify the precision of geodetic networks is fundamental for us to properly establish precision measures and criteria for monitoring schemes.

The precision in geodetic networks is expressed in terms of the variance-covariance matrix of the point coordinates C_x or of estimable quantities derived from coordinates C_f. The purpose for which a network must serve is decisive for the determination of the precision required. In the case of multi-purpose networks (such as national control networks whose purpose may not be specified), it is difficult to establish a link between social values (related with the purpose) and geodetic numerical indicators of quality, while it is much easier for networks with limited and specific purposes. For instance, when defining a geodetic network for setting out an engineering structure, for controlling the breakthrough of tunnelling or for providing photogrammetric control, there may be quite special requirements e.g. the accuracy of certain functions pertaining to specific points or group of points. Thus to quantify the required precision is often quite straightforward. In the case of the general purpose networks, however, the precision requirements cannot be specified so easily. In such cases it is necessary to have some concepts of " ideal " networks. These ideal precision criteria can either be based on theoretic results, such as those of Grafarend(1972), on homogeneous and isotropic networks (Taylor-Karman Structure), or they can be derived from empirical studies with real networks.

One measure of precision takes the form of a scalar function of the elements of the covariance matrix of the coordinate variates. The purpose is to fill the need for an overall

representation of the precision of a network. A scalar function may be one of the following(Grafarend,1974)

i) **N-Optimality**

$$f = \| C_{\underline{x}} \| \rightarrow \min \tag{3-1}$$

with $\| \cdot \|$ denoting the norm of a matrix;

ii) **A-Optimality**

$$f = \text{Trace}(C_{\underline{x}}) = \lambda_1 + \lambda_2 + \cdots + \lambda_r \rightarrow \min \tag{3-2}$$

with $\lambda_1, \lambda_2, \ldots, \lambda_r$ the non-zero eigenvalues of the matrix $C_{\underline{x}}$;

iii) **E-Optimality**

$$f = \lambda_{max} \rightarrow \min \tag{3-3}$$

with λ_{max} the maximum eigenvalue of matrix $C_{\underline{x}}$;

iv) **S-Optimality**

$$f = (\lambda_{max} - \lambda_{min}) \rightarrow \min \tag{3-4}$$

with $(\lambda_{max} - \lambda_{min})$ the spectral width of matrix $C_{\underline{x}}$;

v) **D-Optimality**

$$f = \text{Det}(C_r) = \lambda_1 * \lambda_2 * \cdots * \lambda_r \rightarrow \min \tag{3-5}$$

where $C_r = \text{diag}(\lambda_1, \ldots, \lambda_r)$ with $\lambda_1, \ldots, \lambda_r$ the non-zero eigenvalues of the covariance matrix $C_{\underline{x}}$.

If we demand some certain function f of coordinates $\underline{x}$,

$$f = \underline{c}^T \underline{x}, \tag{3-6}$$

to have the highest precision, then a criterion may be written as

$$c_f = \underline{c}^T C_x \underline{c} \rightarrow \min \tag{3-7}$$

where $\underline{c}$ is a vector of constants.

There may be other scalar precision measures, depending on the purpose of the designed networks.

The advantage of trace, determinant, and eigenvalues of a covariance matrix is that they all are datum-independent quantities. However, the disadvantage is that an overall precision criterion does not control individual values. In addition, on account of correlation between the coordinate variates of different points, coordinate variances alone are not capable of representing variances of operational variates such as angles or distances. For instance, Mittermayer (1972) shows that the small coordinate variances produced by a minimum trace matrix can be deceptive. In his example of a trilateration network, the variances of adjusted distances will be the same whether they are computed from the minimum trace matrix or from a covariance matrix based on any other correct reference system. Scalar measures are rather coarse characteristics of the covariance matrix. They can be used as criteria for the comparison of different designs and for minimization, but they are difficult to be used as an absolute criterion, whose numerical value is, for instance, required not to surpass a certain pre-assigned value related to the purpose of the network. Therefore, the application of scalar precision measures in practice is limited.

3.1.2 Criterion matrices

A much more detailed control of precision is provided by a criterion matrix. A criterion matrix is an artificial variance-covariance matrix possessing an ideal structure, where "ideal" means that it represents the optimal accuracy situation in the planned network. If, instead of a scalar risk function, a criterion matrix is introduced in an optimal design procedure, the solution to the optimization problem must approximate it as closely as possible.

The structure of the criterion matrix for general purpose networks, such as the control networks for regional or national mapping, has been extensively studied by Grafarend (1972) and Baarda (1973). The results are in the form of the general Taylor-Karman structured criterion matrix or its chaotic structure.

3.1.2.1 The Taylor-Karman Structure

Grafarend (1972) introduced the Taylor-Karman structured idealized variance-covariance matrix of Cartesian coordinates in two- and three-dimensional geodetic networks based on the theory of turbulence. Suppose we are given a two-point tensor function $\Phi_{ij}(\underline{r}, \underline{r}')$, i, j ε {1,2,3}, where $\underline{r}$ is the position vector OP, $\underline{r}'$ the position vector OP', O the origin of the coordinate system. Let $\underline{e}_i(\underline{r})$ be a system of three orthonormal basis vectors at point P, $\underline{e}_j(\underline{r}')$ has the same meaning at point P'. Then the second-order form associated to the tensor function of second rank is written by

$$\Phi = \Phi_{ij}(\underline{r}, \underline{r}')\underline{e}_i(\underline{r})\underline{e}_j(\underline{r}') \tag{3-8}$$

The summation has to be carried out over two-by-two identical subscripts.

(a) Φ is called homogeneous if it is translation invariant: $\Phi_{ij}(\underline{r} + \underline{t}, \underline{r}' + \underline{t}) = \Phi_{ij}(\underline{r}, \underline{r}')$ with $\underline{t}$ a displacement vector. The tensor function $\Phi_{ij}(\underline{r}, \underline{r}')$ is strictly homogeneous if it is a function of the difference between the position vectors ($\underline{r} - \underline{r}'$);

(b) Φ is called isotropic if it is rotation invariant: $\Phi_{ij}(R\underline{r}, R\underline{r}') = R\Phi_{ij}(\underline{r}, \underline{r}')R^T$, where R is a rotation matrix. Provided that the second order form Φ is rotation invariant, the tensor function $\Phi_{ij}(\underline{r},\underline{r}')$ is written as $\Phi_{ij}(\underline{r}, \underline{r}') = \delta_{ij}\,\Phi(\underline{r}, \underline{r}')$, where $\Phi(\underline{r}, \underline{r}')$ is a scalar function (δ_{ij} is the Kronecker-identity-matrix);

(c) Φ is called homogeneous and isotropic if it is translation and rotation invariant. Provided that the second order form Φ is translation and rotation invariant, the tensor function $\Phi_{ij}(\underline{r}, \underline{r}')$ is written as

$$\Phi_{ij}(\underline{r}, \underline{r}') = \Phi_m(|\underline{r} - \underline{r}'|) + (\Phi_l(|\underline{r} - \underline{r}'|) - \Phi_m(|\underline{r} - \underline{r}'|))\Delta x_i \Delta x_j / |\underline{r} - \underline{r}'|^2 \tag{3-9}$$

where the characteristic functions Φ_l and Φ_m are called the longitudinal, and lateral functions, respectively. They are functions of the distance $|\underline{r} - \underline{r}'|$ between the points P and P'. (Δx_1 Δx_2 Δx_3) is the vector of the differences of Cartesian coordinates between the points P and P'.

Eq.(3-9) has been named after Taylor (1935) and Karman (1937) and was introduced to geodesy by Grafarend(1972). By applying the two- point tensor function on geodetic networks, for the special case of homogeneity and isotropy, an expression for the submatrix of cross-covariance between points P_i and P_j on the plane within the TK-structured criterion matrix equals to (Grafarend, 1972):

$$(C_{ij}) = \begin{pmatrix} \Phi_m(s) & 0 \\ 0 & \Phi_m(s) \end{pmatrix} + (\Phi_l(s)-\Phi_m(s)) \begin{pmatrix} (x_i-x_j)^2/s^2 & (x_i-x_j)(y_i-y_j)/s^2 \\ (x_i-x_j)(y_i-y_j)/s^2 & (y_i-y_j)^2/s^2 \end{pmatrix} \tag{3-10}$$

where $$\Phi_m(s) = \frac{4d^2}{s^2} - 2k_0(s/d) - \frac{4d}{s}k_1(s/d) \tag{3-11}$$

$$\Phi_l(s) = -\frac{4d^2}{s^2} + 2k_0(s/d) + \frac{4d}{s}k_1(s/d) + \frac{2s}{d}k_1(s/d) \tag{3-12}$$

$$s = |r_i - r_j| = \sqrt{(x_i-x_j)^2 + (y_i-y_j)^2} \tag{3-13}$$

in which k_0 and k_1 are the modified Bessel functions of the second degree and of zeroth and first order, d is the characteristic length. The magnitude of the characteristic distance d of a network still remains a problem. For example, according to Schmitt(1980), d should be chosen smaller than the minimum distance between any arbitrary two points of the network, while Wimmer(1982) recommends the maximum distance (or diameter) of the network as an upper bound for 10 d.

3.1.2.2 The Chaotic Structure

Eq.(3-10) leads to an error situation characterized by identical error circles at the netpoints. However, Baarda(1973) and Alberda (1974) proposed to use a special case of this structure called the "**chaotic structure**". In that case, the relative error ellipses are

also circles with a radius which depends on the distance between the points to which they refer. They adopted the following relation

$$\Phi_m(s) = \Phi_1(s) = d^2 - r^2 s,\ r^2 > 0 \tag{3-14}$$

for a suitable constant r^2. The submatrix of cross-covariance between points P_i and P_j on the plane is

$$(C_{ij}) = \begin{pmatrix} d^2 & 0 & d^2 - d_{ij}^2 & 0 \\ & d^2 & 0 & d^2 - d_{ij}^2 \\ \text{symmetric} & & d^2 & 0 \\ & & & d^2 \end{pmatrix} \tag{3-15}$$

where d^2 = constant; and

$d_{ij}^2 = d_{ji}^2 = f(l_{ij})$ is a positive monotonic non-decreasing function of the side length l_{ij}. The d^2 and d_{ij}^2 should be chosen so that C is positive definite. However, the variance-covariances of all kinds of coordinate differences do not have anything to do with d^2. For example, for $i, j \neq 1$:

$$\sigma^2_{(x_i x_1)} = 2\, d_{i1}^2 \tag{3-16}$$

$$\sigma^2_{(x_i x_1)(x_j x_1)} = d_{1i}^2 + d_{1j}^2 - d_{ij}^2 \tag{3-17}$$

From Eq. (3-16) and (3-17), we can see that by subtracting x_1 and y_1 from all other x_i and y_i we obtain new coordinate variates whose covariance matrix does not contain d^2 any more. So it plays no role in the formulation of the criterion matrix for precision, only the function d_{ij}^2 is important. It is called the "**Choice function**" because one can choose this function to set a criterion. This function should be defined so that the transformed criterion matrix referred to a S-base C_s is positive definite. Baarda (1973) proposed that

$$d_{ij}^2 = c_0 + c_1 l_{ij} \ , \ c_0 > 0, c_1 > 0 \qquad (3\text{-}18)$$

This function has been used in Netherlands for network reconnaissance. Alternative choices are (Karadaidis, 1984; Meissl, 1976)

$$d_{ij}^2 = c_1^2 c_2 \ln\{1 + \frac{l_{ij}}{c_2}\} + c_0^2 \qquad (3\text{-}19)$$

or

$$d_{ij}^2 = c_1^2 [1 - \exp(-c_2^2 l_{ij})] + c_0^2 \qquad (3\text{-}20)$$

Other choices are possible too. The actual choice of the function depends on the type of the network. Once a choice has been made, the parameters c_0, c_1 and c_2 can be used to set the actual criterion for precision. This may depend on the class of network e.g.: densification level, terrain type, etc.

3.1.2.3 Modification of the Present Covariance Matrix

For special purpose networks such as met in civil engineering and deformation measurements, the elements of the criterion matrix can be computed from user requirements, such as the shape of error ellipses or the accuracy of derived quantities. In this respect, an example was given by Koch (1982). He suggested to construct a criterion matrix by modifying the present covariance matrix of coordinates of geodetic netpoints in the following way:

Let $(\hat{x}_i, \hat{y}_i)$ and $(\hat{x}_j, \hat{y}_j)$ be the estimates of the projected coordinates of two points of a two-dimensional network and let their variance-covariances be given by

$$C\begin{pmatrix} \hat{x}_i \\ \hat{y}_i \\ \hat{x}_j \\ \hat{y}_j \end{pmatrix} = \begin{pmatrix} \sigma_{\hat{x}_i}^2 & \sigma_{\hat{x}_i\hat{y}_i} & \sigma_{\hat{x}_i\hat{x}_j} & \sigma_{\hat{x}_i\hat{y}_j} \\ & \sigma_{\hat{y}_i}^2 & \sigma_{\hat{y}_i\hat{x}_j} & \sigma_{\hat{y}_i\hat{y}_j} \\ \text{symmetric} & & \sigma_{\hat{x}_j}^2 & \sigma_{\hat{x}_j\hat{y}_j} \\ & & & \sigma_{\hat{y}_j}^2 \end{pmatrix} \qquad (3\text{-}21)$$

By the law of error propagation the covariance matrix of the coordinates differences $(\hat{x}_i, \hat{y}_i)$ and $(\hat{x}_j, \hat{y}_j)$ of the two points follow

$$C\begin{pmatrix} \hat{x}_j - \hat{x}_i \\ \hat{y}_j - \hat{y}_i \end{pmatrix} = \begin{pmatrix} \sigma^2_{x_i} + \sigma^2_{x_j} - 2\sigma_{x_i x_j} & \sigma_{x_i y_j} - \sigma_{x_i y_i} - \sigma_{x_j y_j} + \sigma_{x_j y_i} \\ \text{symmetric} & \sigma^2_{y_i} + \sigma^2_{y_j} - 2\sigma_{y_i y_j} \end{pmatrix} \qquad (3\text{-}22)$$

A confidence ellipse of circular shape follows for the first point from the equality

$$(\sigma^2_{x_i} - \sigma^2_{y_i})^2 + (2\sigma_{x_i y_i})^2 = 0$$

which is satisfied by

$$\sigma^2_{x_i} = \sigma^2_{y_i} \text{ and } \sigma_{x_i y_i} = 0 \qquad (3\text{-}23)$$

and for the second point by

$$\sigma^2_{x_j} = \sigma^2_{y_j} \text{ and } \sigma_{x_j y_j} = 0\,. \qquad (3\text{-}24)$$

If, in addition

$$\sigma_{x_i x_j} = \sigma_{y_i y_j} \text{ and } \sigma_{y_i x_j} = -\sigma_{x_i y_j}, \qquad (3\text{-}25)$$

then a relative confidence ellipse of circular shape is obtained for the coordinate differences of the two points.

Let $C_{\underline{x}}$ be the covariance matrix of the coordinates of the network to be optimized, the elements of the covariance matrix for the points to be optimized are then changed according to Eq.(3-23) - (3-25) so that smaller confidence ellipses with circular shapes for the coordinates and the coordinate differences of these points are obtained. This changed covariance matrix is then used as the criterion matrix for the optimization.

In deformation monitoring, the parameters of interest are usually displacements (d) or deformation parameters (e) derived from observations in the monitoring scheme as expressed by

$$\hat{\underline{d}} = \hat{\underline{x}}_2 - \hat{\underline{x}}_1 \tag{3-26}$$

$$\hat{\underline{e}} = (B^T C_{\hat{\underline{d}}}^{-1} B)^{-1} B^T C_{\hat{\underline{d}}}^{-1} \hat{\underline{d}} \tag{3-27}$$

with the corresponding covariance matrices for $\hat{\underline{d}}$, $\hat{\underline{e}}$ obtained from

$$\begin{aligned} C_{\hat{\underline{d}}} &= C_{\hat{\underline{x}}_1} + C_{\hat{\underline{x}}_2} \\ &= 2\, C_{\underline{x}} \qquad \text{(in the design phase)} \end{aligned} \tag{3-28}$$

$$\begin{aligned} C_{\hat{\underline{e}}} &= (B^T C_{\hat{\underline{d}}}^{-1} B)^{-1} \\ &= 2\, (B^T C_{\underline{x}}^{-1} B)^{-1} \qquad \text{(in the design phase)} \end{aligned} \tag{3-29}$$

where $\hat{\underline{x}}_1$, $\hat{\underline{x}}_2$ represents the estimates of the vector of coordinates in two different measurement epoches; $\hat{\underline{d}}$, and $\hat{\underline{e}}$ are the vector of displacements and deformation parameters respectively; and B is the coefficient matrix of the deformation model.

At first, the scalar precision measures of geodetic networks may be directly adopted for monitoring schemes by substituting $C_{\underline{d}}$ and $C_{\underline{e}}$ for $C_{\underline{x}}$ in Eq. (3-1) - Eq.(3-7), i.e.,

i) $f = \| C_{\underline{d}} \text{ or } C_{\underline{e}} \| \dashrightarrow \min;$ (3-30)

ii) $f = \text{Trace}(C_{\underline{d}} \text{ or } C_{\underline{e}}) \dashrightarrow \min;$ (3-31)

iii) $f = \lambda_{max} \dashrightarrow \min$ (3-32)

with λ_{max} being the maximum eigenvalue of matrix $C_{\underline{d}}$ or $C_{\underline{e}}$;

iv) $f = (\lambda_{max} - \lambda_{min}) \dashrightarrow \min$ (3-33)

with $(\lambda_{max} - \lambda_{min})$ being the spectral width of matrix $C_{\underline{d}}$ or $C_{\underline{e}}$;

v) $f = \text{Det}(C_r)$

$$= \lambda_1 * \lambda_2 * \dots * \lambda_r \dashrightarrow \min \tag{3-34}$$

where $C_r = \text{diag}(\lambda_1, \dots, \lambda_r)$ with $\lambda_1, \dots, \lambda_r$ the non-zero eigenvalues of the covariance matrix $C_{\underline{d}}$ or $C_{\underline{e}}$;

vi) if we demand that some certain function f of $\underline{d}$ or $\underline{e}$,

$$f = \underline{c}^T \underline{d} \text{ or } \underline{c}^T \underline{e}, \tag{3-35}$$

has the highest precision, then a criterion may be in the form:

$$c_f = \underline{c}^T C_d \underline{c} \text{ or } \underline{c}^T C_e \underline{c} \text{ --> min} \quad (3\text{-}36)$$

where $\underline{c}$ is a vector of constants

There may be many more other scalar precision measures, depending on one's purpose.

The criterion matrix for displacements and deformation parameters can be approached in different ways. At first, if the criterion matrix for point coordinates C_x^c is established, then from Eq.(3-28) and Eq.(3-29) we have the criterion matrix for displacements C_d^c and deformation parameters C_e^c as

$$C_d^c = 2\ C_{\underline{x}}^c \quad (3\text{-}37)$$

$$C_{\underline{e}}^c = 2\ (B^T (C_{\underline{x}}^c)^{-1} B)^{-1} \quad (3\text{-}38)$$

where C_x^c may take the form of homogeneous and isotropic precision or may be constructed by modifying the present covariance matrix of coordinates. The problem for this approach is that the precision of deformation parameters is not controlled, since good precision of coordinates may ensure good precision of displacements but may not ensure good precision of deformation parameters. Thus the criterion matrix for deformation parameters may also be obtained by modifying the present covariance matrix C_e^c of deformation parameters according to the required precision for each individual parameter e_i (i=1, ..., u). Once the criterion matrix for deformation parameters is properly defined, and since

$$C_{\underline{e}}^{-1} = B^T C_d^{-1} B \quad (3\text{-}39)$$

then a solution for the criterion matrix of displacements and coordinates can be (Schaffrin et al., 1977; Schaffrin and Grafarend, 1982):

$$C_{\underline{d}} = ((B^T)^+ C_{\underline{e}}^{-1} B^+)^-$$

$$= (B (B^T B)^{-1} C_{\underline{e}}^{-1} (B^T B)^{-1} B^T)^- \text{ (if } B^T B \text{ is non-singular)} \quad (3\text{-}40)$$

and from Eq. 3-37), we have

$$C_{\underline{x}} = \frac{1}{2} C_{\underline{d}} \tag{3-41}$$

Obviously this corresponds to a minimum norm solution of the system of equations (3-39).

Another approach to construct the criterion matrix for displacements was studied by Sprinsky(1978),Wimmer(1981), and Crosilla(1982;1983). They proposed to construct a criterion matrix for displacements to be monitored by contracting the eigenvalue spectrum of the covariance matrix of the adjusted network coordinates and/or by rotating the m-dimensional error ellipsoid to redistribute the allocated variance. A review of this technique is given below:

Referring to Eq.(3-26) and Eq.(3-28), an estimated (m-dimensional) displacement vector $\hat{\underline{d}}$ may be defined as the difference between two estimated coordinate vectors and relating to different measurement epoches:

$$\hat{\underline{d}} = \hat{\underline{x}}_2 - \hat{\underline{x}}_1$$

with covariance matrix

$$C_{\hat{d}} = C_{\hat{x}_1} + C_{\hat{x}_2}$$

In the hypothesis of a least-squares minimum norm solution for a free net

$$\text{rank}(C_{\hat{d}}) = h < m \tag{3-42}$$

The quadratic form

$$(\underline{d} - \hat{\underline{d}})^T (C_{\hat{\underline{d}}})^+ (\underline{d} - \hat{\underline{d}}) \le \xi_{\chi^2_{h,1-\alpha}} \quad \text{with } \underline{d} \text{ the expectation of } \hat{\underline{d}} \tag{3-43}$$

corresponds to the equation of an h-dimensional error ellipsoid, centered in $\hat{\underline{d}}$, in which the length of the semi-axes and their directions are given by the square root of the eigenvalues greater than zero, and the corresponding eigenvectors of $C_{\hat{d}}$ respectively:

$$C_{\hat{\underline{d}}} = \begin{pmatrix} V_1 & V_2 \end{pmatrix} \begin{pmatrix} \Lambda_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} = V_1 \Lambda_1 V_1^T \tag{3-44}$$

where $\Lambda_1 = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_h)$ (3-45)

with $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_h > 0$

V_1 and V_2 are the submatrices of the eigenvectors relating to the non-zero eigenvalues and zero eigenvalues respectively.

At the first stage, the contraction of the eigenvalue spectrum may be accomplished using a "contraction parameter t", with $0 \leq t \leq 1$, arbitrary but fixed in two different formulations

i) $\tilde{\lambda}_1 = \lambda_1 - t(\lambda_1 - \lambda_h)$ and (3-46)

$\tilde{\lambda}_i = \min(\tilde{\lambda}_i, \lambda_1)$ for i= 2, ... , h (Sprinsky,1978; Wimmer, 1982)

ii) $\tilde{\lambda}_i = \lambda_i - t(\lambda_i - \lambda_h)$ for i=1,2, ..., h (Crosilla,1983a) (3-47)

At the second stage, if a deformation is defined by means of a prediction method, the worst directions to detect point movements are those which correspond to the direction of the greater semi-axes of the error ellipsoid, that is, the eigenvectors relating to the greater eigenvalues of $C_{\hat{d}}$. According to the conceived deformation model, it might be useful to proceed with some rotations of the component pairs of the essential eigenvector in such a way that they would be oriented as close as possible along a direction orthogonal to that of the movement predicted by the deformation model. For a sound statistical study of the displacement vector $\underline{\hat{d}}$, based on the analysis of the principal components(Niemeier,1982) it is necessary to investigate a total variance percentage equal to 40 - 60% of $\text{tr}(C_{\hat{d}})$. Whenever λ_1 alone represents such a value it is sufficient to rotate only the component pairs of the first eigenvector. Otherwise, account must be taken of all the eigenvectors relating to the greatest eigenvalues for which $\sum \lambda_i$ is equal to 40 - 60% of $\text{tr}(C_{\hat{d}})$.

One rotation technique involves the rotation equal to an angle $\varphi_{x,y}$ of all the pairs of eigenvector components relating to the coordinates x_i, y_i of q netpoints ($1 \leq q \leq m/2$):

$$\begin{pmatrix} v^{o}_{x_{ii}} \\ v^{o}_{y_{ii}} \end{pmatrix} = \begin{pmatrix} \cos(\varphi_{xy_i}) & \sin(\varphi_{xy_i}) \\ -\sin(\varphi_{xy_i}) & \cos(\varphi_{xy_i}) \end{pmatrix} \begin{pmatrix} v_{x_{ii}} \\ v_{y_{ii}} \end{pmatrix} \quad (i=1, \ldots, h) \tag{3-48}$$

This follows from the fact that in order to ensure the orthogonality of all the h vectors V_1 of V it is necessary to perform the same rotation for all the pairs of elements of the h eigenvectors relating to the coordinates x_i, y_i of the netpoints.

Another rotation technique is the so-called procrustean transformation of the V_1 eigenvector matrix or of the factorial matrix $\Lambda^{1/2} V_1^T$. With the procrustean transformation each pair of h eigenvector components can be rotated independently. This is of particular importance in case in which the first eigenvalue λ_1 does not represent 40 - 60% of the total variance and it is, therefore, necessary to take more than one eigenvector into consideration in order to satisfy, from the statistical point of view, the condition of orthogonality between the pairs of essential eigenvector components and the predicted direction of the movement. From the independent rotations of each couple of eigenvector components, the resulting matrix - that is the target matrix V_0, is no longer constituted by orthogonal eigenvectors. The procrustean transformation (Schönemann,1966) is to find an orthogonal matrix T which satisfies the approximation

$$V_1 T \cong V_0 \tag{3-49}$$

from the least squares point of view.

Finally, the criterion matrix for the displacement vector $\hat{\underline{d}}$ resulting from the procrustean transformation technique and from the contraction of the eigenvalue spectrum can be written as

$$C_{\hat{d}} = V_1 \, T \, \tilde{\Lambda}_1 \, T^T V_1^T \tag{3-50}$$

where $\tilde{\Lambda}_1$ is the matrix of contracted eigenvalues

$T = X\,Y^T$ with X, Y from the results of the singular value decomposition of matrix $V_1^T V_0$ i.e.

$$V_1^T V_0 = X\,Z\,Y^T \tag{3-51}$$

3.1.3 The datum problem for criterion matrix

The Taylor-Karman structured criterion matrix Eq.(3-10) and Eq.(3-15) refer to a point field where all coordinates are stochastic. In principle, the criterion matrix should be created independently of any linear model connecting the sought parameters to certain observations. However that is not possible in real point fields where a network datum for coordinates should be defined. In order to make the criterion matrix comparable with a "real" covariance matrix which has a defined datum, the criterion matrix should be transformed with the same datum parameters. This transformation may be accomplished by executing a proper S-transformation (Baarda,1973; Schaffrin, 1985)

$$S = \mathbf{I} - H\,(D^T H)^{-1} D^T$$
$$C_{Trans.} = S\,C\,S^T \tag{3-52}$$

where C and $C_{Trans.}$ are the criterion matrices before and after transformation respectively; I is the identity matrix; H is a matrix satisfying A H =0 with A the configuration matrix of the network and D characterizes the datum of the network with datum equation $D^T \underline{x} = 0$ and rank(D)= datum defects of the network.

However, if a criterion matrix for datum-independent deformation parameters such as relative block movements, strain parameters and their derivative is defined in the optimization procedure, it can be directly used to fit the inverse of the normal equation matrix of the deformation parameters.

One should remember that although deformation strictly means change of shape and dimension, detection of scale changes, rotations and displacements must also be included.

In many cases, the deformation parameters are derived from changes in coordinates of the object points as a result of survey observations made at two or more epoches. The changes in coordinates (i.e., displacements) of points are the quantities that geodesists and surveyors like to work with because they allow for an easy identification of the deformation model for further geometrical analysis (Chrzanowski et al., 1982). However, the use of "coordinates of points" introduces problems associated with the way in which those coordinates are defined. i.e., the coordinates are datum-dependent. Although the estimation of the deformation parameters using the "***UNB Generalized Approach***" does not depend on the choice of datum, careful selection of a datum will provide a picture of the displacement field, which makes it easier to identify the deformation model in the space domain or to identify the suspected unstable points of a reference network.

At the design phase, the a priori knowledge of the expected deformation is of much help for the definition of an appropriate datum. Such information comes from a study of the relevant physical, geomechanic, and/or structural properties of the object. In the absence of any a priori reliable data about the likely behaviour of points on and around the object it is necessary in the first instance to assume that all points are subject to deformation, and some sophisticated techniques can be used to define an appropriate datum at the stage of post-processing of the data (Chrzanowski et al. 1983; Chen 1983; Chrzanowski et al. 1986).

3.2 Measures and Criteria for Reliability

Deformation analysis involves two types of errors: the errors of observations, and the errors in deformation models. In order to avoid a misinterpretation of systematic errors or outliers in the observations as deformation phenomena, screening of the observations for outliers or systematic errors should be done prior to the estimation of the deformation parameters. This is why in the "***UNB Generalized Approach***" for deformation

analysis, the adjustment of the geodetic monitoring network for the detection of outliers and systematic errors, whenever possible, occupies the first position.

The concept of reliability of geodetic networks originates from Baarda(1968). Generally, it refers to the ability of a network to resist against gross errors in the observations. In this respect, usually the "**internal reliability**" and "**external reliability**" are distinguished. The internal reliability of a network is its ability to allow the detection of systematic errors by tests of hypothesis made with a specific confidence level $(1-\alpha)$ and power $(1-\beta)$, while the external reliability of a network is related to the effect of undetected errors on the estimated parameters and on their functions.

The reliability of a network depends on the geometry of the network i.e. the configuration matrix and on the weight matrix of observations, not on the actual observations. Thus the problem of reliability should be considered at the design stage to ensure the detection of gross errors as small as possible and to minimize the effects of the undetected ones on the estimated parameters.

3.2.1 Gross errors and hypothesis testing

From the least squares parametric adjustment of a geodetic network with n_G observables one obtains the estimates of parameters and residuals as

$$\hat{\underline{x}} = (A^T P A)^{-1} A^T P \underline{l} \qquad (3\text{-}53)$$

$$\hat{\underline{v}} = (A(A^T P A)^{-1} A^T P - I)\underline{l} \qquad (3\text{-}54)$$

where A, P, and $\underline{l}$ represent the configuration matrix, weight matrix and the observations vector respectively.

Let us assume that the observational value l_i $(i=1, \ldots, n_G)$ is erroneous by ∇l_i, and introduce the vector, $\nabla_i \underline{l}^T = (0, 0, \ldots, \nabla l_i, 0, \ldots, 0)$, containing zeros except for ∇l_i at the i-th position. Thus the observation vector $\underline{l}$ is changed by $\nabla_i \underline{l}$. If the vector $\nabla_i \underline{l}$ is full it

describes a systematic effect on the observations. In this case there are two possible hypothesis:

null hypothesis: $H_0: E(\underline{l}|H_0) = A\,\underline{x}$

alternative hypotheses: $H_a: E(\underline{l}|H_a) = A\,\underline{x} + \underline{h}_i\,\nabla l_i$ (3-55)

with the vector $\underline{h}_i$ describing the effect of the error source on the observation. If we have a blunder ∇l_i in l_i then $\underline{h}_i$ is a unit vector $\underline{h}_i^T = (0, 0, \ldots, 1, 0, \ldots, 0)$. If the error source leads to a systematic effect, $\underline{h}_i$ is full in general and ∇l_i will be a single additional parameter, say s_j.

The effect of errors in the observations as described by the alternative hypothesis on the different estimates is:

$$\nabla_i\hat{\underline{x}} = (A^T P A)^{-1} A^T P \nabla_i \underline{l} \tag{3-56}$$

$$\nabla_i\hat{\underline{v}} = (A(A^T P A)^{-1} A^T P - \mathbf{I})\nabla_i \underline{l} \tag{3-57}$$

Since the expectation

$$E(\hat{\underline{v}}) = 0 \tag{3-58}$$

of $\hat{v}$ is known, in contrast to $E(\hat{\underline{x}})$, the effect $\nabla_i\hat{\underline{v}}$ of the error onto the residuals can be easily seen. By studying the effect of an error onto the residuals, we are able to obtain an estimate for the size of a gross error which then can be used to estimate its influence onto the unknown parameters.

By denoting $R = (\mathbf{I} - A(A^T P A)^{-1} A^T P)$, Eq.(3-57) reads

$$\nabla\hat{\underline{V}} = -R\,\nabla\underline{l} \tag{3-59}$$

The index is omitted here, as the equation holds for general $\nabla\underline{l}$. The matrix R, in general, is a full matrix. It contains the full information on the geometry of the design with respect to the influence of observational errors onto the residuals. The effect $\nabla_i\hat{v}_i$ of an error ∇l_i in observation l_i onto the corresponding residual is determined by the i-th diagonal element of R:

$$\nabla_i \hat{v}_i = -r_i \nabla l_i \qquad \text{with } r_i = (R)_{ii},\ 0 \le r_i \le 1,\ \sum r_i = r \tag{3-60}$$

where r is the total redundancy of the system. The diagonal elements of R are obviously decisive for our analysis. As they sum up to the total redundancy of the system, we call them " **redundancy numbers**", i.e., r_i can be seen as the contribution of the observation l_i to the total redundancy r of the system. And since r_i always is between 0 and 1, only a possibly small part of an error shows in the residuals. On the other hand if we have an indication that observation l_i is erroneous then from the residual $\hat{v}_i$ we can obtain an estimate $\widehat{\nabla l_i}$ for the size of the error (assuming $\hat{v}_i = \nabla_i \hat{v}_i$ in the presence of gross errors):

$$\widehat{\nabla l_i} = -\frac{\hat{v}_i}{r_i} \tag{3-61}$$

The standard deviation of $\hat{v}_i$ and $\widehat{\nabla l_i}$ are :

$$\sigma_{\hat{v}_i} = \sqrt{r_i}\ \sigma_{l_i} \tag{3-62}$$

$$\sigma_{\widehat{\nabla l_i}} = \sigma_{l_i} / \sqrt{r_i} \tag{3-63}$$

Statistical tests serve to determine whether or not anything has gone wrong with some basic postulate. In our context, a statistical test must tell whether the residuals give an indication that a certain prespecified observation is erroneous or not. The test is based on the test statistic w_i which is a function of the observed values $\underline{l}$. The probability density function, specifically the expectation and the variance of the test statistic, has to be known for the case where the null hypothesis H_0 is true. Good test statistics use all information, i.e., are sufficient and lead to the best distinction between the null and alternative hypothesis. According to Baarda(1968), the best test statistic for blunder detection is

$$w_i = \frac{-\underline{h}_i^T P \hat{\underline{v}}}{\sigma_0 \sqrt{\underline{h}_i^T P Q_{\hat{v}\hat{v}} P \underline{h}_i}} \tag{3-64}$$

which for uncorrelated observations reduces to the standardized residual

$$w_i = \frac{-\hat{v}_i}{\sqrt{r_i}\ \sigma_{l_i}} \tag{3-65}$$

The influence of an error ∇l_i in l_i onto the test statistic w_i is

$$\delta_i = \nabla w_i = - \frac{\nabla \hat{v}_i}{\sigma_{l_i} \sqrt{r_i}} = \frac{\nabla l_i \sqrt{r_i}}{\sigma_{l_i}} \tag{3-66}$$

and leads to a shift of the probability density function of w_i by δ_i. δ_i is called the non-centrality parameter. Assuming that the observational errors are normally distributed under the null hypothesis, then the probability function of w_i , under the null and alternative hypotheses H_0 and H_a , is central and non-central normal distribution respectively:

$$w_i \mid H_0 \quad \underset{\sim}{d} \quad n\,(0,1) \tag{3-67}$$

$$w_i \mid H_a \quad \underset{\sim}{d} \quad n\,(\delta_i,1)\,. \tag{3-68}$$

From Eq.(3-66) we can conclude that the weaker the geometry (i.e. the smaller r_i) the smaller is the influence of a blunder onto the test statistic w_i, thus the more difficult it will be to discern between H_0 and H_a .

Carrying out of the test is shown in Fig. 3.1. The boundary values $- w_{\alpha/2}$ and $+ w_{\alpha/2}$, divide the w-axis into two regions V and W with the following test procedure:

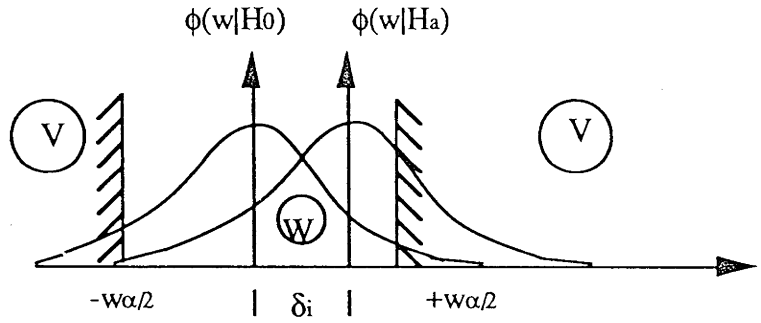

Fig. 3.1 Density functions of test statistic w_i

1) If $w_i \in W$ i.e. $|w_i| \leq w_{\alpha/2}$ then accept H_0 (3-69)

2) If $w_i \in V$ i.e. $|w_i| > w_{\alpha/2}$ then reject H_0 (3-70)

$$\lambda_0 = \max (\lambda_{i,0}) \tag{3-80}$$

can be considered as a measure of the reliability of the estimated coordinates.

Substituting σ_0^2 from Eq.(3-77) and using the vector $\underline{h}_i$, Eq.(3-79) becomes

$$\lambda_{i,0} = \delta_0^2 (\frac{1}{r_i} - 1) \tag{3-81}$$

Similarly, the influence of undetectable gross errors ∇l_i on the solution of deformation parameters can be defined by:

$$\nabla_{i,0} \hat{\underline{e}} = 2 (B^T A^T P A B)^{-1} B^T A^T P \nabla_{0,i} l, \tag{3-82}$$

$$\bar{\lambda}_{i,0} = \frac{1}{\sigma_0^2} (\nabla_{0,i} \hat{\underline{e}})^T Q_{\underline{e}}^{-1} (\nabla_{0,i} \hat{\underline{e}})$$

$$= 2 \delta_0^2 (\frac{(A B (B^T A^T P A B)^{-1} B^T A^T P)_{ii}}{r_i}), \tag{3-83}$$

and

$$\bar{\lambda}_0 = \max (\bar{\lambda}_{i,0}). \tag{3-84}$$

$\bar{\lambda}_0$ is used as a measure of the reliability of the estimated deformation parameters.

Replacing $\sigma_0^2 (B^T A^T P A B)^{-1}$ by one half of the criterion matrix of deformation parameters, i.e., $\frac{1}{2}\sigma_0^2 Q_{\underline{e}}^c$ and approximating r_i by an average value of $\bar{r} = \frac{r}{n_G}$ (Chen et al., 1983) with r being the total redundancy and n_G the number of geodetic observations in the geodetic network, we obtain

$$\bar{\lambda}_{0,i} = \frac{\delta_0^2 n_G}{r} (A B Q_{\underline{e}}^c B^T A^T P)_{ii} \tag{3-85}$$

3.2.3 The optimality criteria

Thus gross error larger than the boundary value can be detected. Otherwise, they are not detectable.

From Eq.(3-77) we can see that the boundary value $\nabla_0 l_i$ depends on the precision of the observation described by the standard deviation σ_{l_i}; the geometry of the design of the measuring procedure described by the redundancy number r_i; and the significance level α_0 and the required minimum power $(1-\beta_0)$ comprised in the lower bound δ_0 for the non-centrality parameter. The smaller the redundancy number r_i of the observation, the larger a gross error has to be in order to be detectable. The case of r_i=1 occurs only if the true value of the observation is known. If r_i=0 then a test of the observation is not possible: the boundary value is infinite.

3.2.2.2 External reliability

External reliability relates to the maximum effect of possibly undiscovered observational gross errors ∇l_i on the estimates of unknown parameters (e.g. coordinates, deformation parameters). The influence of ∇l_i on coordinates i.e. $\nabla_{0,i}\hat{\underline{x}}$ is given by

$$\nabla_{0,i}\hat{\underline{x}} = (A^T P A)^{-1} A^T P \nabla_{0,i}\underline{l} \tag{3-78}$$

Since $\nabla_{0,i}\hat{\underline{x}}$ is datum dependent, Baarda (1976) proposed a new standardized variate

$$\lambda_{i,0} = \frac{1}{\sigma_0^2} (\nabla_{0,i}\hat{\underline{x}})^T Q_{\underline{x}}^{-1} (\nabla_{0,i}\hat{\underline{x}}) \tag{3-79}$$

which is invariant with respect to the coordinates defined. However, one is interested mainly in the maximum value of this variate which is related to the minimum deviation from the null hypothesis that can be detected with a certain probability β_0. So the following variate (Baarda, 1976)

It is possible that this test leads to a wrong decision, which is called an error of Type I or Type II. The probabilities for these errors are given by

$$\text{Type I error: } \alpha = p\{ w_i \in V | H_0 \} = 2 (1 - \Phi(w_{\alpha/2}) \tag{3-71}$$

$$\text{Type II error: } \beta = p\{ w_i \in W | H_a \} = \Phi(w_{\alpha/2} - \delta_i) - \Phi(- w_{\alpha/2} - \delta_i) \tag{3-72}$$

where $\Phi(w)$ is the normal distribution function.

Physically, α is the probability that the observation is error free but one may conclude from $|w_i| > \xi_{N(0,1),\, 1-\alpha/2}$ that it contains an error, and is called the significance level. β is the probability that the observation is erroneous but one may conclude from $|w_i| \leq \xi_{N(0,1),\, 1-\alpha/2}$ that the observation is correct. $1-\beta$ is called the power of the test. Since the actual value ∇l_i of a gross error and thereby the non-centrality parameter δ_i is not known, the probability β can not be computed in practice. Baarda(1968) proposed to invert the relation between the power of the test and the non-centrality parameter and use

$$\delta = \delta(\beta, \alpha) \tag{3-73}$$

For a preset power of the test, Eq.(3-73) gives the distance between H_0 and H_a, i.e. the size of the error which can be detected with that power. If we now require that an alternative, i.e. an error has to be detected with a probability of $1-\beta$ which is larger than a prescribed bound $1-\beta_0$ then the distance δ between H_0 and H_a has to be larger than the bound δ_0 i.e.

$$1 - \beta > 1 - \beta_0 \quad \text{--> } \quad \delta > \delta_0,\ \delta_0 = \delta(\beta_0, \alpha_0) \tag{3-74}$$

As for the calculation of δ_0, from Eq.(3-72), the second term is usually very small if $\delta_i > 0$. If δ_i is larger than $w_{\alpha/2}$ then the term can be neglected and we obtain

$$\beta = \Phi (w_{\alpha/2} - \delta_i) \tag{3-75}$$

When α_0, β_0 are fixed, the noncentrality parameter can be computed from

$$\delta_0 = w_{\alpha_0/2} - \Phi^{-1} (\beta_0) \tag{3-76}$$

Because for $\beta < 0.5$, the function $\Phi^{-1}(\beta)$ is negative, the minimum bound δ_0 for the noncentrality parameter δ_i is always larger than the critical value $w_{\alpha/2}$ if one requires a power of at least 50%. But this also means that the distance δ between H_0 and H_a must be much larger than the critical value $w_{\alpha/2}$ if one with a high probability wants to correctly reject H_0 when H_a actually is true. Some values for δ_0 are listed in Table 3-1.

Table 3.1: Lower bounds δ_0 for noncentrality parameter in dependency of the significance number α_0 and the required minimum power $1-\beta_0$ of the test.

α_0 / $1-\beta_0$	0.01%	0.10%	1%	5%
50%	3.72	3.29	2.58	1.96
70%	4.41	3.82	3.1	2.48
80%	4.73	4.13	3.42	2.8
90%	5.17	4.57	3.86	3.24
95%	5.54	4.94	4.22	3.61
99%	6.22	5.62	4.9	4.29
99.90%	6.98	6.38	5.67	5.05

3.2.2 Reliability measures

3.2.2.1 Internal reliability

Internal reliability refers to the lower bounds for just detectable gross errors which can also be seen as a measure for the controlability of the observations. In the last section we have derived the lower bound δ_0 for the distance between H_0 and H_a which guarantees a required separability of the two hypotheses. The distance δ depends on the size of the gross error in the observation and can be derived using Eq.(3-66). Now the lower bound for just detectable gross errors $\nabla_0 l_i$ can be derived by substituting the actual influence δ_i of the gross error onto the test statistic by the lower bound δ_0:

$$\nabla_0 l_i = \sigma_{l_i} \frac{\delta_0}{\sqrt{r_i}} \tag{3-77}$$

At the design stage, a monitoring scheme should be designed such that

i) Gross errors should be detected and eliminated as completely as possible. An undetected gross error Δ_i in an observation l_i should be small in comparison with the standard deviation σ_{l_i} of l_i

ii) The effect of an undetected error Δ_i on the solution of deformation parameters should be as small as possible.

From Eq.(3-77), (3-81), and (3-83) we can see that the larger the redundancy number r_i the smaller is the size of the undetectable gross errors as well as its influence on the estimated coordinates or deformation parameters. Thus a general criterion for the internal- and external reliability can be stated respectively as

$$\| r \| \quad \text{--> maximum} \tag{3-86}$$

and

$$\| \bar{\lambda} \| \quad \text{--> minimum} \tag{3-87}$$

where $\underline{r} = (r_1 , r_2 , ..., r_{n_G})^T$;

$\underline{\bar{\lambda}} = (\bar{\lambda}_{0,1} , \bar{\lambda}_{0,2} , ..., \bar{\lambda}_{0,n_G})^T$; and

$\| \cdot \|$ represent the norm of a vector.

However, Baarda(1968) argues that it is desirable to have approximately a constant value for all r_i $(i=1, ..., n_G)$ so that the ability of detecting gross errors is the same in every part of the network. If certain observation variates have larger boundary values, then they are insufficiently checked and the network should be improved locally. With this in mind, a special reliability criterion can be of the type (Van Mierlo, 1981):

$$\min (r_i) = \max \tag{3-88}$$

or

$$\max (\bar{\lambda}_{0,i}) = \min \tag{3-89}$$

3.3 Measures and Criteria for Sensitivity

At the design stage of a monitoring scheme, postulated displacements or deformation parameters, based on the particular mechanism causing the deformation, can be made. The network should then be designed so that the postulated displacements or deformation parameters, if they occur, can be detected with specified probabilities α and β. This is called the sensitivity of the design.

One way to approach the sensitivity problem for displacement monitoring was described by Niemeier(1982). It is based on the following global congruency test (Pelzer,1971)

$$\omega = \frac{\hat{\underline{d}}^T Q_{\hat{\underline{d}}}^{+} \hat{\underline{d}}}{r_d \sigma_0^2} \sim F_{r_d, \infty} \tag{3-90}$$

where $\hat{\underline{d}} = \hat{\underline{x}}_2 - \hat{\underline{x}}_1$ is the estimated displacement vector;

$Q_{\hat{d}} = Q_{\hat{x}_1} + Q_{\hat{x}_2} = 2\, Q_{\hat{x}}$;

$r_d = \text{rank}(Q_{\hat{\underline{d}}})$; and

σ_0^2 = the a priori variance factor.

Under the null hypothesis

$$H_0 : E(\hat{\underline{d}}) = 0 \tag{3-91}$$

ω would follow the central F-distribution $F_{r_d, \infty}$. However, for an alternative hypothesis

$$H_i : E(\hat{\underline{d}}) = \underline{d}_i \neq 0 \tag{3-92}$$

ω would follow a non-central F-distribution $F_{r_d, \omega_i, \infty}$ with the non-centrality parameter

$$\omega_i = \frac{\underline{d}_i^T Q_{\hat{\underline{d}}}^{+} \underline{d}_i}{\sigma_0^2} \tag{3-93}$$

The power of a test is defined as the probability (1-β) that $\underline{d}_i$ will result in a rejection of the null hypothesis H_0 with a significance level α. Having fixed values of α and β, the critical value ω_0 for the non-centrality parameter is given by a complicated function

$$\omega_0 = f(r_d, \infty, \alpha_0, \beta_0) \tag{3-94}$$

The values of ω_0 can be found in tables given in Pearson and Hartley (1951) or Pelzer,1971). Thus all the alternative hypotheses, which have non-centrality parameters

$$\omega_i > \omega_0 \tag{3-95}$$

will lead to a rejection of the null hypothesis with the preset probabilities α_0 and β_0. We say, then the corresponding displacements $\underline{d}_i$ are detectable.

Let $\underline{d}_i = a_i \underline{k}_i$, where a_i is a scalar and $\underline{k}_i$ is a normalized factor. The vector $\underline{d}_i$ can be obtained if

$$\omega_i = \frac{a_i^2}{\sigma_0^2} \underline{k}_i^T Q_{\underline{d}}^{+} \underline{k}_i > \omega_0 \tag{3-96}$$

i.e. if

$$a_i > \sigma_0 \sqrt{\frac{\omega_0}{\underline{k}_i^T Q_{\underline{d}}^{+} \underline{k}_i}} \tag{3-97}$$

By this inequality we compute for each variant network the minimum (critical) values for a_i so that Eq.(3-97) is fulfilled. These scale factors allow for a comparison of different network designs.

Crosilla (1983) considered the sensitivity problem in the construction of the criterion matrix for displacements. Since the cofactor matrix of the displacements can be decomposed into

$$Q_{\underline{d}} = 2\, Q_{\underline{x}} = 2\, V\, \Lambda\, V^T = 2 \sum_{j=1}^{r_d} \lambda_j \underline{v}_j \underline{v}_j^T \tag{3-98}$$

where Λ and V correspond to the eigenvalues different from zeros and their eigenvectors of the matrix $Q_{\underline{x}}^{\wedge}$ respectively. The displacement directions which are most unfavourable from the point of view of getting real point movements are those which correspond to the greatest eigenvalues of $Q_{\underline{\varepsilon}}^{\wedge}$ and consequently of $Q_{\underline{x}}^{\wedge}$. The essential eigenvectors $\underline{v}_j$ (j=1, ..., $s < r_{\dot{c}}$) therefore have to be substituted for the form vector $\underline{k}$ to calculate the scalar value a_j using Eq.(3-97) and consequently to define, with $\tilde{\underline{d}}_j \geq a_j \underline{v}_j$, the value of vectors $\tilde{\underline{d}}_j$ which can be effectively detected along the directions $\underline{v}_j$ (j=1, ... , $s < r_{\dot{c}}$).

Substituting the s essential eigenvectors $\underline{v}_j$ for the form vector $\underline{k}$, the result is s displacement vector

$$\tilde{\underline{d}}_j = a_j \underline{v}_j \tag{3-99}$$

and since

$$\underline{k}_j^T Q_{\underline{d}}^{\wedge +} \underline{k}_j = \frac{1}{2} \underline{v}_j^T V \Lambda^{-1} V^T \underline{v}_j \tag{3-100}$$

under the condition of orthonormality of the matrix V, we obtain

$$\underline{k}_j^T Q_{\underline{d}}^{\wedge +} \underline{k}_j = \frac{1}{2\lambda_j} \tag{3-101}$$

with λ_j the j-th non-zero eigenvalue. Therefore, from Eq.(3-97)

$$\lambda_j \leq \frac{a_j^2}{2\,\sigma_0^2\,\omega_0} \tag{3-102}$$

Once the scale factor of the predicted movements has been ascertained or estimated and the value of the scale $a=a_j$ (j=1, ..., s) fixed, it may happen that for a certain value of σ_0^2 and ω_0, the above inequality is not fulfilled. If this is the case, it becomes necessary to

contract the value of λ_j —> $\tilde{\lambda}_j$ (j=1,...,s) so that the inequality is fulfilled and construct a criterion matrix according to the sensitivity analysis as

$$\tilde{C}_d = 2\,\sigma_0^2\, V\,\tilde{\Lambda}\,V^T \tag{3-103}$$

with $\tilde{\Lambda} = \text{diag}\,(\tilde{\lambda}_1,\ldots,\tilde{\lambda}_s\,\lambda_{s+1},\ldots,\lambda_{r_d})$

The disadvantage of the above methods is that they do not explicitly give any suggestions on the geometry of the network configuration and the observation plan which have to be satisfied in order to detect a displacement vector of certain magnitude. In the following, a new sensitivity criterion is derived by the author. Assume a displacement vector $\underline{d}$ to be monitored is postulated. From Eq.(3-95), the vector $\underline{d}$ is detectable if the following inequality holds:

$$\omega = \frac{\underline{d}^T Q_{\hat{\underline{d}}}^{+}\,\underline{d}}{\sigma_0^2} > \omega_0. \tag{3-104}$$

Replacing $Q_{\underline{d}}^{+}$ by $\frac{1}{2}Q_{\underline{x}}^{+} = \frac{1}{2}(A^T P A)$ reduces Eq. (3-104) to

$$\underline{d}^T A^T P A\ \underline{d} > 2\,\sigma_0^2\,\omega_0. \tag{3-105}$$

This inequality gives a requirement for the geometry of the network(i.e. the configuration matrix A) and the weight matrix P in order that the displacement vector $\underline{d}$ be detectable.

If only a scale factor of the predicted movements is postulated, then following the philosophy of Eq.(3-97), we have

$$\underline{k}_i^T A^T P A\ \underline{k}_i > \frac{2\,\omega_0\,\sigma_0^2}{a_i^2}. \tag{3-106}$$

The sensitivity analysis for the detection of deformation parameters can be approached in the same way. Since the significance of the estimated deformation parameters is tested by(Chen,1983)

$$\varpi = \frac{\hat{\underline{e}}^T Q_{\underline{e}}^{-1} \hat{\underline{e}}}{u\ \sigma_0^2} \sim F_{u,\infty} \tag{3-107}$$

where $\hat{\underline{e}}$ - the vector of estimated deformation parameters;

$Q_{\underline{e}}$ - the cofactor matrix of the estimated deformation parameters;

σ_0^2 - the a priori variance factor; and

u - the number of identifiable deformation parameters.

Testing of the null hypothesis

$$H_0 : E(\hat{\underline{e}}) = 0 \tag{3-108}$$

against the alternative hypothesis

$$H_a : E(\hat{\underline{e}}) = \underline{e} \neq 0 \tag{3-109}$$

gives the non-centrality parameter

$$\varpi_e = \frac{1}{\sigma_0^2} \underline{e}^T Q_{\underline{e}}^{-1}\ \underline{e} \tag{3-110}$$

After fixing error probabilities α and β, a critical value ϖ_0 of ϖ_e can be found. This is the value of ϖ_e at which the null hypothesis will just be rejected. Thus in order that an estimated scale of deformation parameters $\underline{e}$ be detectable, the following inequality should be satisfied

$$\frac{1}{\sigma_0^2} \underline{e}^T Q_{\underline{e}}^{-1}\ \underline{e} > \varpi_0 \tag{3-111}$$

Replacing $Q_{\underline{e}}^{-1}$ by $\frac{1}{2}(B^T A^T P A B)$, we obtain

$$\underline{e}^T (B^T A^T P A B)\underline{e} > 2\,\sigma_0^2\,\varpi_0 \qquad (3\text{-}112)$$

Thus an optimality criterion for sensitivity can be expressed in the form

$$\underline{e}^T (B^T A^T P A B)\underline{e} = \max \qquad (3\text{-}113)$$

3.4 Measures and Criteria for Economy

A monitoring scheme should be designed such that it satisfies the user precision, reliability and sensitivity requirements with the least cost. It is then essential to write down a general mathematical cost function involving the observations to be considered during the design process. For instance, Cross and Whiting (1981) proposed to evaluate the cost of a levelling network by

Cost = constant × length of levelling

thus minimizing cost amounts to minimizing the total length of levelling. However, cost functions for a two- or three-dimensional networks can be much more complex. Since most of the currently used analytical design techniques require the cost function to be primarily in terms of observation precision and this, in practice is not usually the case. Also the methods may require cost functions to be continuous, which again is unlikely to be true.

An approximate approach (Schmitt, 1982) to bring cost criteria into a mathematical form is to split the costs for the measurements into constant terms, e.g., cost of driving to the stations, cost of setting up of the instruments, and cost of signalling the points. The remaining free parameters are then the repetition numbers of the observations. Different instrumentations can be considered in this concept by the introduction of special efficiency numbers for these instruments or error formulas for a single measurement. By making the

assumption that the smaller the weight of an observation the less expensive it is to observe, a simplified formulation for minimum cost criteria can be (Schaffrin, 1985)

$$\| P \| = \text{minimum} \tag{3-114}$$

where $\| \cdot \|$ represents the norm of the weight matrix P.

It should be pointed out that in all the above derivations the author assumes the use of Gauβ-Markov Model only. Theoretically, if we allow "random effects" to enter our model, e.g., deformation parameters with known expected magnitude, then a Mixed Model (Schaffrin, 1985, 1986) should be used which reforms the basic matrix for any design problem, i.e., equation (2-31) to

$$Q_{\hat{e}} = (\frac{1}{2} B^T A^T P A B + P_{\hat{e}})^{-1}, \tag{3-115}$$

where $P_{\hat{e}}$ represents the a priori weight matrix associated with the expected values of the deformation parameters. However, from the practical point of view, in the design phase of a monitoring scheme, it is very difficult to evaluate the accuracy of the a priori information about the deformation parameters predicted from a study of the relevant physical properties, geophysical investigations, etc. Therefore, for the purpose of safety, it is justified to assume that the predicted expected values of the deformation parameters are not accurate enough to be given a proper weighting by setting

$$P_{\hat{e}} = 0. \tag{3-116}$$

In this case, the Mixed Model reduces to the standard Gauβ-Markov Model, and we are erroring in the safe side. Otherwise, the above derived external reliability criterion (equations (3-85; 87)) and sensitivity criterion (equations (3-112; 113)) should be modified accordingly. This research will stick to the use of the Gauβ-Markov Model only.

CHAPTER 4

OPTIMIZATION AND DESIGN OF DEFORMATION MONITORING SCHEMES

4.1 Design Orders of a Monitoring Scheme

Following the convention of design orders for geodetic networks by Grafarend(1974), one may consider the same classification of the desihgn orders for deformation monitoring schemes. There are, however, significant differences in the design problems in positioning networks versus monitoring schemes.

First of all, there is no Zero Order Design (ZOD) problem in monitoring schemes. For instance, in the case of a reference network, correct displacements can be obtained only by comparing coordinates at different epochs with respect to the same reference datum. Hence not only the reference datum at the initial epoch has to be defined, but also its motion at the subsequent epochs has to be identified. Therefore, the main problem here is not to define an optimal datum for the initial epoch but to confirm the stability of the reference frame at the initial epoch, and only the one which maintains the same position and orientation at the subsequent epochs can be considered as the "optimal" datum for it. Various techniques were developed for this purpose (Chrzanowski and Chen, 1986), including the aforementioned "***UNB Generalized Approach***" which provides datum independent deformation parameters. Thus the choice of the reference frame for the deformation analysis is of no practical importance.

In regard to the First Order Design (FOD), the shape of the network largely depends on the topography covered by the network or the shape of the structures to be monitored. Certain changes, however, of the positions of the points in the monitoring scheme are always possible, and they should be used to optimize the configuration of the network with respect to an optimality criterion. Generally, the location of the reference points should be outside of the deformable body and outside of the zone of the acting forces which produce the deformation. In contrast, the object points should be distributed over the area which is considered important from the safety point of view and in which the most obvious deformations (maximum values of the deformation parameters) are expected to occur. The existing prediction theories and methods such as finite element method can help in identifying the stable versus the most unstable areas. Since the parameters of interest in monitoring schemes are not the coordinates (positions) themselves but their variation (displacements) with time and other deformation parameters, all other isolated observations such as tiltmeter, strainmeter, pendula and repeated accurate measurements of baselines and angles not belonging to a geodetic network, must be integrated in order to obtain an optimally designed monitoring scheme. The geodetic network can be seen in this case not as a means of obtaining the absolute positions of the displaced points with the higher accuracy, but rather as a means of providing good spatial correlation between various observables and providing information on the global deformation trend. The location of geotechnical and other non-geodetic instrumentation should be selected where it represents the general trend of the deformation or where the maximum deformations are expected. Therefore in the FOD for monitoring schemes, the optimal positions for both geodetic and non-geodetic instrumentation should be solved for simultaneously.

The Second Order Design (SOD) problem for monitoring schemes is to find accuracies of both the geodetic and non-geodetic observables or weight matrix P which leads to accuracies of estimates of all unknown parameters as close as possible to some given idealized counterparts, e.g., the criterion matrix. The matrix P derived from the

solution of the SOD problem may give directions for the choice of instrumentation or observational procedures.

The Third Order Design (THOD) problem of improvement of existing networks might be very useful for monitoring schemes. When at some epoch it is realized that accuracies of displacements or deformation parameters are unsatisfactory or the observed deformations do not follow the expected trend, the monitoring scheme must be improved by additional observables or additional points in future epochs.

Finally, the Combined First Order and Second Order Design problem appears if neither the monitoring configuration nor the weights of observations are known and they have to be optimally solved for simultaneously.

4.2 Identification of Unknown Parameters to Be Optimized

Generally, as it is clear from the above section, in the optimization of a monitoring scheme, the free parameters are the configuration matrix A, characterized by the positions of both the geodetic and non-geodetic instrumentations, and the weight matrix P consisting of accuracies of both geodetic and non-geodetic observables. We say that a solution for these free parameters is optimal if it satisfies the optimality criteria adopted to define the " quality of the monitoring scheme" and it lies within the range defined by their physical properties, i.e., the parameters should be physically realizable.

Let us assume, for the convenience of mathematical modelling later on, that we have a monitoring scheme to be optimized as shown by Fig. 4.1. This scheme consists of a three-dimensional terrestrial geodetic network with m_G points plus some isolated non-geodetic instrumentations with m_{NG} points in order to estimate the deformation model $B\underline{c}$ with u deformation parameters. Denote the design matrix of the geodetic network by A_G with n_G all possible geodetic observables l_G, and the design matrix of non-geodetic

instruments as A_{NG} with all possible n_{NG} observables $\underline{l}_{NG}$. The weight matrix for geodetic and non-geodetic observables are designated as P_G and P_{NG} respectively.

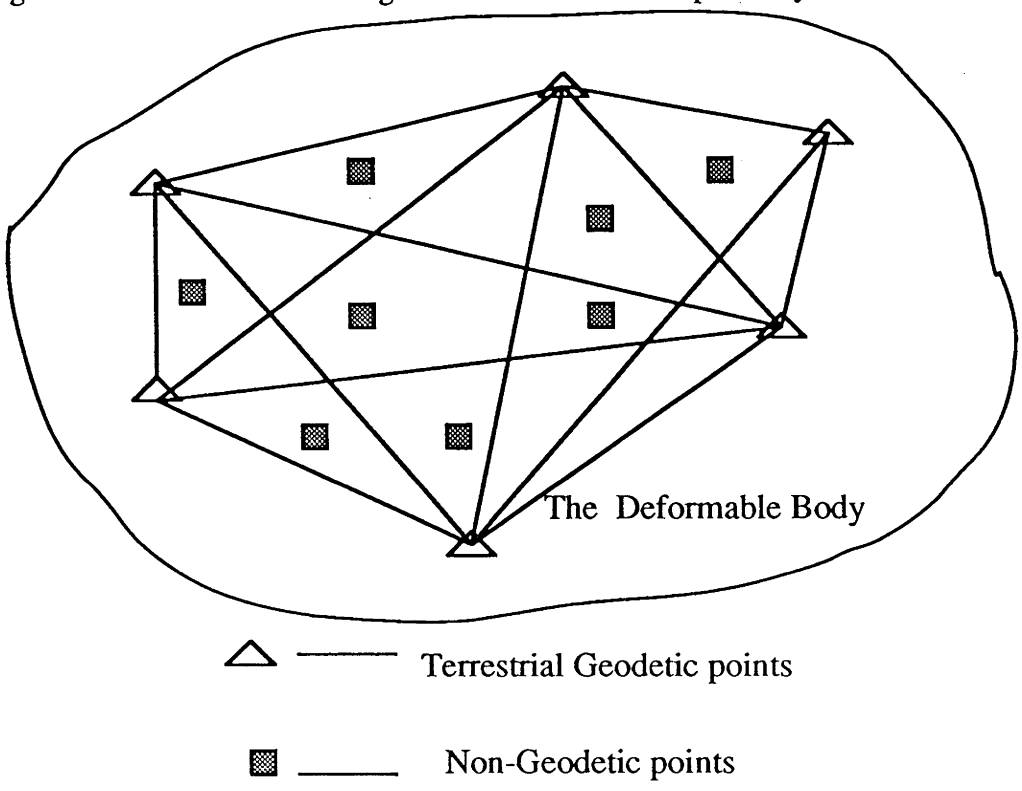

Fig. 4.1: A deformation monitoring scheme

Let

$$A = \begin{pmatrix} A_G \\ A_{NG} \end{pmatrix} \tag{4-1}$$

$$P = \begin{pmatrix} P_G & 0 \\ 0 & P_{NG} \end{pmatrix} \tag{4-2}$$

$$\underline{l} = \begin{pmatrix} \underline{l}_G \\ \underline{l}_{NG} \end{pmatrix} \tag{4-3}$$

$$m = m_G + m_{NG} \tag{4-4}$$

$$n = n_G + n_{NG} \tag{4-5}$$

Then the deformation parameters and their cofactor matrix in the design phase may be approximated by

$$\hat{\underline{e}} = (B^T A^T P A B)^{-1} B^T A^T P \underline{l}$$

$$= (B^T A_G^T P_G A_G B + B^T A_{NG}^T P_{NG} A_{NG} B)^{-1} (B^T A_G^T P_G l_G + B^T A_{NG}^T P_{NG} l_{NG}) \quad (4\text{-}6)$$

$$Q_{\hat{e}} = 2(B^T A^T P A B)^{-1}$$

$$= 2(B^T A_G^T P_G A_G B + B^T A_{NG}^T P_{NG} A_{NG} B)^{-1}. \quad (4\text{-}7)$$

In the design phase, the optimization procedure must give optimal matrices A, P in order to achieve required accuracies of deformation parameters. Note that the matrix A is a function of the positions of each netpoints, thus the above requirement is equivalent to solving for the optimal positions for all the geodetic and non-geodetic points characterized by coordinates (x_i , y_i , z_i , i= 1, ... , m) and optimal weights for all the geodetic and non-geodetic observables (p_i , i= 1, ... , n).

The word "**optimization**" has rather recently come into use in geodesy to indicate designing networks based on well-specified quantitative considerations and techniques; it suggests planning for the best solution, as brought out by the following quotation from Alberda (1974):

> " ... in particular with respect to terrain difficulties and the choice of methods of measurement, the planning of networks means in practice that one starts with a solution that is feasible under the given circumstances and available material means, and then introduces improvements until the plan is not too expensive and good enough. "

In the light of this quotation it is the **improvements** that will be identified as the unkown parameters to be optimally solved for at the design stage. As discussed in the above section, from the practical point of view, the specific positions of instruments in deformation monitoring are rather suggested by landform in the network area in connection with mutual observability conditions. That is, the general configuration of a monitoring scheme is usually fixed by topography of the surface of the earth covered by the deformable body, or by the shape of the buildings or the constructions, or suggested by the position where the maximum deformation for object points or minimum deformation for reference points may be expected based on the deterministic deformation analysis or other

external evidences. As aforementioned, certain changes of the position of netpoints, which may vary in magnitude from point to point, are always possible, and they should be used to optimize the configuration of the network. Assume that an approximate monitoring configuration with approximate coordinates (x_i^0 , y_i^0 , z_i^0, i=1, ... , m) was selected from a reconnaissance according to the topography and/or stability condition. The configuration of the monitoring scheme can be optimized by introducing improvements (Δx_i , Δy_i , Δz_i, i=1, ... , m) in the coordinates of each point. As for the solution of weights of observations, we can also start with a set of approximate weights p_i^0 (i=1, ... , n) which can be realized with the least efforts, then introduce improvements Δp_i (i=1, ... , n) in order to achieve the design target. Thus finally, we obtain the optimal positions and weights by

$$\begin{aligned} x_i &= x_i^0 + \Delta x_i \\ y_i &= y_i^0 + \Delta y_i \qquad i=1, ..., m \\ z_i &= z_i^0 + \Delta z_i \end{aligned} \tag{4-8}$$

and

$$p_i = p_i^0 + \Delta p_i \qquad i=1, ..., n \tag{4-9}$$

4.3 Basic Requirements for An Optimal Monitoring Scheme

As discussed above, the parameters to be optimized in the optimization of a monitoring scheme can be the improvements in positions and weights. We say that a solution for these parameters is optimal if it satisfies the optimality criteria adopted to define the quality of the monitoring scheme and they are physically realizable. According to the discussions made in Chapter 3, the quality of a monitoring scheme is represented by

precision, reliability, sensitivity, and cost. Following this line of thought, the basic requirements for the solution of the parameters can be formulated as follows.

4.3.1 Precision requirement

Refer to Eq. (4-7),

$$Q_{\hat{e}} = 2(B^T A^T P A B)^{-1} .$$

This is the starting equation for the optimal design of the configuration matrix (A) and the weight matrix (P) in order to attain the required accuracies of the deformation parameters.

Assuming $\sigma_0^2 Q_{\hat{e}}^c$ is the criterion matrix for deformation parameters, the design problem seeks an optimal configuration (A) and weights (P) such that it can be best approximated by $\sigma_0^2 Q_{\hat{e}}$. This basic relation can be expressed by

$$\sigma_0^2 (2(B^T A^T P A B)^{-1}) = \sigma_0^2 Q_{\hat{e}}^c \qquad (4\text{-}10)$$

or

$$B^T A^T P A B = 2(Q_{\hat{e}}^c)^{-1} . \qquad (4\text{-}11)$$

Using canonical formulation (Schaffrin,1981), Eq. (4-11) can be rewritten as

$$E^T B^T A^T P A B E = D \qquad (4\text{-}12)$$

where $2(Q_{\hat{e}}^c)^{-1}$ has been decomposed by similarity transformation as

$$2(Q_{\hat{e}}^c)^{-1} = E D E^T \qquad (4\text{-}13)$$

with E and D being the orthonormal matrix consisting of the normalized eigenvectors of $2(Q_{\hat{e}}^c)^{-1}$ and the eigenvalues of $2(Q_{\hat{e}}^c)^{-1}$ respectively.

Eq. (4-12) is usually inconsistent. In this case, one has to approximate the equation "from one side" in order to (Schaffrin, 1981):

(a) Assure that the solution of A, P yields a "better" variance-covariance matrix in some sense than the presupposed criterion matrix $\sigma_0^2 Q_{\hat{e}}^c$, i.e.,

$$\text{vecdiag}(E^T B^T A^T P A B E) \geq \text{vecdiag}(D) \tag{4-14}$$

and

(b) avoid that some of the unknown parameters becomes disproportionally "better" than the others, i.e.

$$\| E^T B^T A^T P A B E - D \| = \min_{A,\,P} \tag{4-15}$$

or alternatively

$$\| E^T B^T A^T P A B E - D \| \leq v_p \tag{4-16}$$

with $\| \cdot \|$ the norm of a matrix and v_p the measure for improvement.

Denote

$$N = E^T B^T A^T P A B E \tag{4-17}$$

Note that elements of matrix N are linear functions of weights (p_i,i=1,...,n), but non-linear functions of configuration (A) characterized by coordinates (x_i, y_i z_i , i = 1, ..., m) of netpoints. However, as discussed above, since the general shape of the configuration is fixed, the configuration is optimized by introducing small changes in positions. Therefore matrix N can be approximated by Taylor series restricted to linear term as

$$N = N^0 + \sum_1^m \left(\frac{\partial N}{\partial x_i}\right)\Delta x_i + \sum_1^m \left(\frac{\partial N}{\partial y_i}\right)\Delta y_i + \sum_1^m \left(\frac{\partial N}{\partial z_i}\right)\Delta z_i + \sum_1^n \left(\frac{\partial N}{\partial p_i}\right)\Delta p_i \tag{4-18}$$

where

$$N^0 = (E^T B^T A^T P A B E)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \tag{4-19}$$

$$\frac{\partial N}{\partial x_i} = \left(E^T B^T A^T P \left(\frac{\partial A}{\partial x_i} B + A \frac{\partial B}{\partial x_i}\right) E + E^T \left(B^T \left(\frac{\partial A}{\partial x_i}\right)^T + \left(\frac{\partial B}{\partial x_i}\right)^T A^T\right) P A B E \right)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \tag{4-20}$$

$$\frac{\partial N}{\partial y_i} = \left(E^T B^T A^T P \left(\frac{\partial A}{\partial y_i} B + A \frac{\partial B}{\partial y_i}\right) E + E^T \left(B^T \left(\frac{\partial A}{\partial y_i}\right)^T + \left(\frac{\partial B}{\partial y_i}\right)^T A^T\right) P A B E \right)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0}$$

(4-21)

$$\frac{\partial N}{\partial z_i} = (E^T B^T A^T P(\frac{\partial A}{\partial z_i} B + A \frac{\partial B}{\partial z_i}) E + E^T (B^T (\frac{\partial A}{\partial z_i})^T + (\frac{\partial B}{\partial z_i})^T A^T) P A B E) |_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0}$$

(4-22)

$$\frac{\partial N}{\partial pi} = (E^T B^T A^T (\frac{\partial P}{\partial pi}) A B E)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0}$$

(4-23)

m is the total number of netpoints

n is the total number of observables

$\underline{x}^0$, $\underline{y}^0$ $\underline{z}^0$ are vectors of initial coordinates of both geodetic and non-geodetic points selected in reconnaissance

p^0 consists of the approximate values of weights.

The derivatives of matrices A, B and P with respect to x_i, y_i ,z_i(i=1, ..., m) and p_i (i=1,...,n) can be obtained analytically. Thus now Eq.(4-14) - Eq.(4-16) can be reformulated as

$$\text{vecdiag} (N^0 + \sum_1^m (\frac{\partial N}{\partial x_i}) \Delta x_i + \sum_1^m (\frac{\partial N}{\partial y_i}) \Delta y_i + \sum_1^m (\frac{\partial N}{\partial z_i}) \Delta z_i + \sum_1^n (\frac{\partial N}{\partial p_i}) \Delta p_i - D) \geq 0$$

(4-24)

$$\| (N^0 + \sum_1^m (\frac{\partial N}{\partial x_i}) \Delta x_i + \sum_1^m (\frac{\partial N}{\partial y_i}) \Delta y_i + \sum_1^m (\frac{\partial N}{\partial z_i}) \Delta z_i + \sum_1^n (\frac{\partial N}{\partial p_i}) \Delta p_i - D \| = \min$$

(4-25)

and

$$\| (N^0 + \sum_1^m (\frac{\partial N}{\partial x_i}) \Delta x_i + \sum_1^m (\frac{\partial N}{\partial y_i}) \Delta y_i + \sum_1^m (\frac{\partial N}{\partial z_i}) \Delta z_i + \sum_1^n (\frac{\partial N}{\partial p_i}) \Delta p_i - D \| \leq v_p$$

(4-26)

Denote

$$\underline{u} = \text{vec} (D) - \text{vec} (N^0)$$

(4-27)

$$H = (\mathrm{vec}(\frac{\partial N}{\partial x_1}) \; \mathrm{vec}(\frac{\partial N}{\partial y_1}) \; \mathrm{vec}(\frac{\partial N}{\partial z_1}) \; \ldots \; \mathrm{vec}(\frac{\partial N}{\partial x_m}) \; \mathrm{vec}(\frac{\partial N}{\partial y_m}) \; \mathrm{vec}(\frac{\partial N}{\partial z_m}) \; \mathrm{vec}(\frac{\partial N}{\partial p_1}) \ldots \mathrm{vec}(\frac{\partial N}{\partial p_n})) \tag{4-28}$$

$$\underline{w} = (\Delta x_1 \; \Delta y_1 \; \Delta z_1 \; \ldots \; \Delta x_m \; \Delta y_m \; \Delta z_m \; \Delta p_1 \; \ldots \; \Delta p_n) \tag{4-29}$$

where the operator " vec " is obtained by stacking the columns of a quadratic matrix one under another in a single column(Henderson/ Searle 1979).

Then Eq.(4-24) to Eq.(4-26) can be respectively written in matrix form as

$$H_1 \, \underline{w} - \underline{u}_1 \geq 0 \tag{4-30}$$

$$\| H \, \underline{w} - \underline{u} \| = \min \tag{4-31}$$

$$\| H \, \underline{w} - \underline{u} \| \leq v_p \tag{4-32}$$

where $H_1 = (\mathbf{I}_u \, \Theta \, \mathbf{I}_u)^T H$;

$\underline{u}_1 = (\mathbf{I}_u \, \Theta \, \mathbf{I}_u)^T \underline{u}$; and

$\mathbf{I}_u$ is the u by u unit matrix and Θ represents the Khatri-Rao product.

Unfortunately, as mentioned in Chapter 1 and from the author's experience, the above formulations may result in monitoring schemes which do not satisfy the set precision criteria, depending on the selection of the criterion matrix. The reason is that the simple reversal of inequality signs due to the inversion of criterion matrix is not valid. In this case, the above formulations must be modified to approximate the criterion matrix itself.

Starting from Eq.(4-10), we have

$$(B^T A^T P \, A \, B)^{-1} = \frac{1}{2} Q_{\underline{\ell}}^{c} \, . \tag{4-33}$$

Decomposing $\frac{1}{2}(Q_{\underline{\ell}}^{c})$ by similarity transformation, we obtain

$$\frac{1}{2} Q_{\underline{\ell}}^{c} = E_1 \, D_1 \, E_1^T \tag{4-34}$$

with E_1 and D_1 the orthonormal matrix consisting of the normalized eigenvectors of $\frac{1}{2}(Q_{\underline{e}}^{e})$ and the eigenvalues of $\frac{1}{2}(Q_{\underline{e}}^{e})$ respectively. And it can be shown that

$$E_1 = E \tag{4-35}$$

$$D_1 = D^{-1}. \tag{4-36}$$

In this case, Eq.(4-33) changes to

$$E_1^T (B^T A^T P\, A\, B)^{-1}\, E_1 = D_1. \tag{4-37}$$

Denoting $N_1 = B^T A^T\, P\, A\, B$, (4-38)

and approximating N_1 by Taylor series of linear form, we have

$$N_1 = N_0 + \sum_1^m \frac{\partial N_1}{\partial x_i}\Delta x_i + \sum_1^m \frac{\partial N_1}{\partial y_i}\Delta y_i + \sum_1^m \frac{\partial N_1}{\partial z_i}\Delta z_i + \sum_1^n \frac{\partial N_1}{\partial p_i}\Delta p_i \tag{4-39}$$

where $N_0 = (B^T A^T P\, A\, B)|_{\underline{x}^0,\, \underline{y}^0,\, \underline{z}^0,\, p^0}$ (4-40)

$$\frac{\partial N_1}{\partial x_i} = (B^T A^T P(\frac{\partial A}{\partial x_i}B + A\frac{\partial B}{\partial x_i}) + (B^T(\frac{\partial A}{\partial x_i})^T + (\frac{\partial B}{\partial x_i})^T A^T)P\, A\, B\,)\,|_{\underline{x}^0,\underline{y}^0,\, \underline{z}^0,\, p^0} \tag{4-41}$$

$$\frac{\partial N_1}{\partial y_i} = (B^T A^T P(\frac{\partial A}{\partial y_i}B + A\frac{\partial B}{\partial y_i}) + (B^T(\frac{\partial A}{\partial y_i})^T + (\frac{\partial B}{\partial y_i})^T A^T)P\, A\, B\,)\,|_{\underline{x}^0,\underline{y}^0,\, \underline{z}^0,\, p^0} \tag{4-42}$$

$$\frac{\partial N_1}{\partial z_i} = (B^T A^T P(\frac{\partial A}{\partial z_i}B + A\frac{\partial B}{\partial z_i}) + (B^T(\frac{\partial A}{\partial z_i})^T + (\frac{\partial B}{\partial z_i})^T A^T)P\, A\, B\,)\,|_{\underline{x}^0,\underline{y}^0,\, \underline{z}^0,\, p^0} \tag{4-43}$$

$$\frac{\partial N_1}{\partial p_i} = (B^T\, A^T\, (\frac{\partial P}{\partial p_i})A\, B)\,|_{\underline{x}^0,\underline{y}^0,\, \underline{z}^0,\, p^0} \tag{4-44}$$

m, n, $\underline{x}^0$, $\underline{y}^0$ $\underline{z}^0$, and p^0 mean the same as before.

The inverse of the normal equation matrix N_1 can be expressed as:

$$N_1^{-1} = \left[N_0 + \sum_1^m \frac{\partial N_1}{\partial x_i}\Delta x_i + \sum_1^m \frac{\partial N_1}{\partial y_i}\Delta y_i + \sum_1^m \frac{\partial N_1}{\partial z_i}\Delta z_i + \sum_1^n \frac{\partial N_1}{\partial p_i}\Delta p_i \right]^{-1}$$

$$= \left[\mathbf{I} + \sum_1^m N_0^{-1}\frac{\partial N_1}{\partial x_i}\Delta x_i + \sum_1^m N_0^{-1}\frac{\partial N_1}{\partial y_i}\Delta y_i + \sum_1^m N_0^{-1}\frac{\partial N_1}{\partial z_i}\Delta z_i + \sum_1^n N_0^{-1}\frac{\partial N_1}{\partial p_i}\Delta p_i \right]^{-1} N_0^{-1}$$

(4-45)

Here, the first term on the right hand side of the equation may again be approximated by applying the Neumann series restricted to linear term as:

$$\left[\mathbf{I} + \sum_1^m N_0^{-1}\frac{\partial N_1}{\partial x_i}\Delta x_i + \sum_1^m N_0^{-1}\frac{\partial N_1}{\partial y_i}\Delta y_i + \sum_1^m N_0^{-1}\frac{\partial N_1}{\partial z_i}\Delta z_i + \sum_1^n N_0^{-1}\frac{\partial N_1}{\partial p_i}\Delta p_i \right]^{-1} =$$

$$= \left[\mathbf{I} - \sum_1^m N_0^{-1}\frac{\partial N_1}{\partial x_i}\Delta x_i - \sum_1^m N_0^{-1}\frac{\partial N_1}{\partial y_i}\Delta y_i - \sum_1^m N_0^{-1}\frac{\partial N_1}{\partial z_i}\Delta z_i - \sum_1^n N_0^{-1}\frac{\partial N_1}{\partial p_i}\Delta p_i \right].$$

(4-46)

Finally, we get

$$N_1^{-1} = N_0^{-1} - \sum_1^m (N_0^{-1}\frac{\partial N_1}{\partial x_i}N_0^{-1})\Delta x_i - \sum_1^m (N_0^{-1}\frac{\partial N_1}{\partial y_i}N_0^{-1})\Delta y_i -$$

$$- \sum_1^m (N_0^{-1}\frac{\partial N_1}{\partial z_i}N_0^{-1})\Delta z_i - \sum_1^n (N_0^{-1}\frac{\partial N_1}{\partial p_i}N_0^{-1})\Delta p_i$$

(4-47)

And Eq.(4-37) leads to

$$(E_1^T N_0^{-1} E_1 - \sum_1^m (E_1^T N_0^{-1}\frac{\partial N_1}{\partial x_i}N_0^{-1}E_1)\Delta x_i - \sum_1^m (E_1^T N_0^{-1}\frac{\partial N_1}{\partial y_i}N_0^{-1}E_1)\Delta y_i -$$

$$- \sum_1^m (E_1^T N_0^{-1}\frac{\partial N_1}{\partial z_i}N_0^{-1}E_1)\Delta z_i - \sum_1^n (E_1^T N_0^{-1}\frac{\partial N_1}{\partial p_i}N_0^{-1}E_1)\Delta p_i) = D_1$$

(4-48)

Now, another set of formulations in regard to the precision criteria can be obtained as:

$$H_{11}\,\underline{w} - \underline{u}_{11} \le \underline{0} \tag{4-49}$$

$$\| H_1\,\underline{w} - \underline{u}_1 \| = \min \tag{4-50}$$

$$\| H_1\,\underline{w} - \underline{u}_1 \| \le v_p \tag{4-51}$$

where $\underline{u}_1 = \mathrm{vec}(D_1) - \mathrm{vec}(E_1^T(N_0^{-1})E_1$ (4-52)

$$\underline{u}_{11} = (\mathbf{I}_u \Theta \mathbf{I}_u)^T \underline{u}_1 \tag{4-53}$$

$$H_1 = -(\mathrm{vec}(N_{x1})\,\mathrm{vec}(N_{y1})\,\mathrm{vec}(N_{z1}) \cdots \mathrm{vec}(N_{xm})\,\mathrm{vec}(N_{ym})\,\mathrm{vec}(N_{zm})\,\mathrm{vec}(N_{p1}) \cdots \mathrm{vec}(N_{pn})) \tag{4-54}$$

$$H_{11} = (\mathbf{I}_u \Theta \mathbf{I}_u)^T H_1 \tag{4-55}$$

$$N_{xi} = E_1^T N_0^{-1} \frac{\partial N_1}{\partial x_i} N_0^{-1} E_1 \quad (i=1, \cdots, m) \tag{4-56}$$

$$N_{yi} = E_1^T N_0^{-1} \frac{\partial N_1}{\partial y_i} N_0^{-1} E_1 \quad (i=1, \cdots, m) \tag{4-57}$$

$$N_{zi} = E_1^T N_0^{-1} \frac{\partial N_1}{\partial z_i} N_0^{-1} E_1 \quad (i=1, \cdots, m) \tag{4-58}$$

$$N_{pi} = E_1^T N_0^{-1} \frac{\partial N_1}{\partial p_i} N_0^{-1} E_1 \quad (i=1, \cdots, n) \tag{4-59}$$

$\mathbf{I}_u$ is the u by u unit matrix and Θ represents the Khatri-Rao product.

4.3.2 Internal reliability requirement

Traditionally, the internal reliability criterion works for the geodetic monitoring network only. However, if in some cases the geotechnical instruments provide for geometric loop closures or other redundancies, a similar treatment can be made. From Chapter 3, a general internal reliability criterion for optimization can be

$$\| \underline{r} \| = \max \qquad \text{(optimal internal reliability)} \tag{4-60}$$

or

$$\| \underline{r} \| \geq r_l \qquad \text{(internal reliability control)} \tag{4-61}$$

where the redundancy numbers r_i are defined as

$$r_i = (\mathbf{I} - A_G(A_G^T P_G A_G)^{-1} A_G^T P_G)_{i\,i.} \qquad (i=1, \ldots, n_G) \tag{4-62}$$

Replacing $\sigma_0^2(A_G^T P_G A_G)^{-1}$ by the criterion matrix of coordinates $\sigma_0^2 Q_{\underline{x}}^c$, we have

$$r_i = (\mathbf{I} - A_G Q_{\underline{x}}^c A_G^T P_G)_{i\,i.} \tag{4-63}$$

Note that r_i are nonlinear functions of coordinates but linear functions of weights. The Taylor series expansion of r_i can be expressed as

$$r_i = (R^0 + \sum_1^{m_G} (\frac{\partial R}{\partial x_i})\Delta x_i + \sum_1^{m_G} (\frac{\partial R}{\partial y_i})\Delta y_i + \sum_1^{m_G} (\frac{\partial R}{\partial z_i})\Delta z_i + \sum_1^{n_G} (\frac{\partial R}{\partial p_i})\Delta p_i)_{i\,i} \tag{4-64}$$

where $R^0 = (\mathbf{I} - A_G Q_{\underline{x}}^c A_G^T P_G)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0}$ (4-65)

$$\frac{\partial R}{\partial x_i} = (- \frac{\partial A_G}{\partial x_i} Q_{\underline{x}}^c A_G^T P_G - A_G Q_{\underline{x}}^c (\frac{\partial A_G}{\partial x_i})^T P_G)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \tag{4-66}$$

$$\frac{\partial R}{\partial y_i} = (- \frac{\partial A_G}{\partial y_i} Q_{\underline{x}}^c A_G^T P_G - A_G Q_{\underline{x}}^c (\frac{\partial A_G}{\partial y_i})^T P_G)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \tag{4-67}$$

$$\frac{\partial R}{\partial z_i} = (- \frac{\partial A_G}{\partial z_i} Q_{\underline{x}}^c A_G^T P_G - A_G Q_{\underline{x}}^c (\frac{\partial A_G}{\partial z_i})^T P_G)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \tag{4-68}$$

$$\frac{\partial R}{\partial p_i} = (- A_G Q_{\underline{x}}^c A_G^T \frac{\partial P_G}{\partial p_i})|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \tag{4-69}$$

$Q_{\underline{x}}^c$ can be obtained by Eq.(3-40) and (3-41) if the criterion matrix of deformation parameters has been defined. Alternatively, it can also be approximated by

$Q_{\underline{x}}^c = (A_G^T P_G A_G)^{-1}$

$$= N_{20}^{-1} - N_{20}^{-1}\left(\sum_{1}^{m_G}\left(\frac{\partial N_2}{\partial x_i}\right)\Delta x_i + \sum_{1}^{m_G}\left(\frac{\partial N_2}{\partial y_i}\right)\Delta y_i + \sum_{1}^{m_G}\left(\frac{\partial N_2}{\partial z_i}\right)\Delta z_i + \sum_{1}^{n_G}\left(\frac{\partial N_2}{\partial p_i}\right)\Delta p_i\right)N_{20}^{-1} \tag{4-70}$$

where $N_{20} = (A_G^T P_G A_G)\,|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, p^0}$ (4-71)

$$\frac{\partial N_2}{\partial x_i} = \left(\left(\frac{\partial A_G}{\partial x_i}\right)^T P_G A_G + A_G^T P_G\left(\frac{\partial A_G}{\partial x_i}\right)\right)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, p^0} \tag{4-72}$$

$$\frac{\partial N_2}{\partial y_i} = \left(\left(\frac{\partial A_G}{\partial y_i}\right)^T P_G A_G + A_G^T P_G\left(\frac{\partial A_G}{\partial y_i}\right)\right)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, p^0} \tag{4-73}$$

$$\frac{\partial N_2}{\partial z_i} = \left(\left(\frac{\partial A_G}{\partial z_i}\right)^T P_G A_G + A_G^T P_G\left(\frac{\partial A_G}{\partial z_i}\right)\right)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, p^0} \tag{4-74}$$

$$\frac{\partial N_2}{\partial p_i} = \left(A^T{}_G \frac{\partial P_G}{\partial p_i} A_G\right)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, p^0} \tag{4-75}$$

In this case, the above derivatives Eq.(4-66) - Eq.(4-69) can be reformulated as

$$\frac{\partial R}{\partial x_i} = -\left(\frac{\partial A_G}{\partial x_i} R_{10} P_G + R_{01}\left(\frac{\partial A_G}{\partial x_i}\right)^T P_G - R_{01}\left(\frac{\partial N_2}{\partial x_i}\right) R_{10} P_G\right)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, p^0} \tag{4-76}$$

$$\frac{\partial R}{\partial y_i} = -\left(\frac{\partial A_G}{\partial y_i} R_{10} P_G + R_{01}\left(\frac{\partial A_G}{\partial y_i}\right)^T P_G - R_{01}\left(\frac{\partial N_2}{\partial y_i}\right) R_{10} P_G\right)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, p^0} \tag{4-77}$$

$$\frac{\partial R}{\partial z_i} = -\left(\frac{\partial A_G}{\partial z_i} R_{10} P_G + R_{01}\left(\frac{\partial A_G}{\partial z_i}\right)^T P_G - R_{01}\left(\frac{\partial N_2}{\partial z_i}\right) R_{10} P_G\right)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, p^0} \tag{4-78}$$

$$\frac{\partial R}{\partial p_i} = -\left(R_{01} A_G^T\left(\frac{\partial P_G}{\partial p_i}\right) - R_{01}\left(A_G^T \frac{\partial P_G}{\partial p_i} A_G\right) R_{10} P_G\right)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, p^0} \tag{4-79}$$

$$R_{10} = (A_G^T P_G A_G)^{-1} A_G \tag{4-80}$$

$$R_{01} = A_G (A_G^T P_G A_G)^{-1}. \tag{4-81}$$

Finally, if written in vector form, the vector $\underline{r} = (r_1 \quad r_2 \ \ldots \ r_{n_G})^T$ can be expressed as

$$\underline{r} = (\mathbf{I}_{n_G} \Theta \mathbf{I}_{n_G})^T (\underline{r}_0 + R_1 \underline{w}) \tag{4-82}$$

where $\mathbf{I}_{n_G}$ is the n_G by n_G unit matrix and Θ the Khatri-Rao product;

$$\underline{r}_0 = \text{vec}(R^0); \tag{4-83}$$

$$R_1 = \left(\text{vec}\left(\frac{\partial R}{\partial x_1}\right) \text{vec}\left(\frac{\partial R}{\partial y_1}\right) \text{vec}\left(\frac{\partial R}{\partial z_1}\right) \ldots \text{vec}\left(\frac{\partial R}{\partial x_{m_G}}\right) \text{vec}\left(\frac{\partial R}{\partial y_{m_G}}\right) \text{vec}\left(\frac{\partial R}{\partial z_{m_G}}\right) \mathbf{0}_1 \right.$$
$$\left. \text{vec}\left(\frac{\partial R}{\partial p_1}\right) \ldots \text{vec}\left(\frac{\partial R}{\partial p_{n_G}}\right) \mathbf{0}_2 \right); \tag{4-84}$$

$\underline{w}$ is the same as in Eq.(4-29); and

$\mathbf{0}_1, \mathbf{0}_2$ are sub-matrices of zeros corresponding to position improvements of non-geodetic points and weight improvements of non-geodetic observables respectively.

Finally, a general criteria for internal reliability can be

$$\| \underline{r} \| = \| (\mathbf{I}_{n_G} \Theta \mathbf{I}_{n_G})^T (\underline{r}_0 + R_1 \underline{w}) \| = \max \tag{4-85}$$

or

$$\| (\mathbf{I}_{n_G} \Theta \mathbf{I}_{n_G})^T (\underline{r}_0 + R_1 \underline{w}) \| \geq r_1 \tag{4-86}$$

where $\| \cdot \|$ represents the norm of the vector.

Denote $\underline{r}_{00} = (\mathbf{I}_{n_G} \Theta \mathbf{I}_{n_G})^T \underline{r}_0$ (4-87)

$$R_{11} = (\mathbf{I}_{n_G} \Theta \mathbf{I}_{n_G})^T R_1 . \tag{4-88}$$

Then Eq. (4-85) and Eq. (4-86) may be simplified as

$$\| (\underline{r}_{00} + R_{11} \underline{w}) \| = \max \tag{4-89}$$

$$\| (\underline{r}_{00} + R_{11} \underline{w}) \| \geq r_1 . \tag{4-90}$$

4.3.3 External reliability criteria

In the consideration of external reliability, only the influences of gross errors in geodetic observations on the solution of deformation parameters will be considered here. However, the formulations developed below can be easily extended to include also the influences of gross errors in non-geodetic observations. From Chapter 3, a general external reliability criteria for the estimation of deformation parameters can be

$$\| \bar{\underline{\lambda}} \| = \min \qquad \text{(optimal external reliability)} \tag{4-91}$$

or

$$\| \bar{\underline{\lambda}} \| \le \bar{\lambda}_m \qquad \text{(external reliability control)} \tag{4-92}$$

where $\bar{\lambda}_m$ is the given maximum allowable value and

$$\bar{\lambda}_{0,i} = \frac{\delta_0^2 n_G}{r} (A B Q_{\underline{e}}^c B^T A^T P)_{ii} \quad (i=1, \ldots, n_G). \tag{4-93}$$

Here again, $\bar{\lambda}_{0,i}$ (i=1, ..., n_G) are nonlinear functions of positions but linear functions of weights. Using Taylor series expansion, we have

$$\bar{\lambda}_{0,i} = (G^0 + \sum_1^m (\frac{\partial G}{\partial x_i}) \Delta x_i + \sum_1^m (\frac{\partial G}{\partial y_i}) \Delta y_i + \sum_1^m (\frac{\partial G}{\partial z_i}) \Delta z_i + \sum_1^n (\frac{\partial G}{\partial p_i}) \Delta p_i)_{ii} \tag{4-94}$$

where

$$G^0 = \frac{\delta_0^2 n_G}{r} (A B Q_{\underline{e}}^c B^T A^T P)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \tag{4-95}$$

$$\frac{\partial G}{\partial x_i} = \frac{\delta_0^2 n_G}{r} ((\frac{\partial A}{\partial x_i} B + A \frac{\partial B}{\partial x_i}) Q_{\underline{e}}^c B^T A^T P + AB Q_{\underline{e}}^c (B^T (\frac{\partial A}{\partial x_i})^T + (\frac{\partial B}{\partial x_i})^T A^T) P)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \tag{4-96}$$

$$\frac{\partial G}{\partial y_i} = \frac{\delta_0^2 n_G}{r} ((\frac{\partial A}{\partial y_i} B + A \frac{\partial B}{\partial y_i}) Q_{\underline{e}}^c B^T A^T P + AB Q_{\underline{e}}^c (B^T (\frac{\partial A}{\partial y_i})^T + (\frac{\partial B}{\partial y_i})^T A^T) P)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \tag{4-97}$$

$$\frac{\partial G}{\partial z_i} = \frac{\delta_0^2 n_G}{r} \left(\left(\frac{\partial A}{\partial z_i} B + A \frac{\partial B}{\partial z_i}\right) Q_{\underline{e}}^{c} B^T A^T P + AB\, Q_{\underline{e}}^{c} \left(B^T \left(\frac{\partial A}{\partial z_i}\right)^T + \left(\frac{\partial B}{\partial z_i}\right)^T A^T \right) P \right)\Big|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \tag{4-98}$$

$$\frac{\partial G}{\partial p_i} = \frac{\delta_0^2 n_G}{r} \left(ABQ_{\underline{e}}^{c} B^T A^T \frac{\partial P}{\partial p_i} \right)\Big|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \tag{4-99}$$

Again, if written in vector form, the vector $\underline{\lambda} = (\lambda_{0,1}\, \lambda_{0,2} \ldots \lambda_{0,n_G})^T$ can be expressed as

$$\underline{\lambda} = \frac{\delta_0^2 n_G}{r} (\mathbf{I}_{n_G} \Theta \mathbf{I}_{n_G})^T (g_0 + G_1 \underline{w}) \tag{4-100}$$

where $\mathbf{I}_{n_G}$ is the n_G by n_G unit matrix and Θ the Khatri-Rao product,

$$g_0 = \mathrm{vec}\,(G^0) \tag{4-101}$$

$$G_1 = \left(\mathrm{vec}\left(\frac{\partial G}{\partial x_1}\right) \mathrm{vec}\left(\frac{\partial G}{\partial y_1}\right) \mathrm{vec}\left(\frac{\partial G}{\partial z_1}\right) \ldots \mathrm{vec}\left(\frac{\partial G}{\partial x_m}\right) \mathrm{vec}\left(\frac{\partial G}{\partial y_m}\right) \mathrm{vec}\left(\frac{\partial G}{\partial z_m}\right) \mathrm{vec}\left(\frac{\partial G}{\partial p_1}\right) \ldots \mathrm{vec}\left(\frac{\partial G}{\partial p_n}\right) \right) \tag{4-102}$$

$\underline{w}$ is the same as in Eq.(4-29).

Thus, a general criteria for external reliability can be

$$\| (\mathbf{I}_{n_G} \Theta \mathbf{I}_{n_G})^T (g_0 + G_1 \underline{w}) \| = \min \tag{4-103}$$

or

$$\| (\mathbf{I}_{n_G} \Theta \mathbf{I}_{n_G})^T (g_0 + G_1 \underline{w}) \| \le \overline{\lambda_m} \tag{4-104}$$

where $\| \cdot \|$ represents the norm of the vector.

let $$g_{00} = (\mathbf{I}_{n_G} \Theta \mathbf{I}_{n_G})^T g_0 \tag{4-105}$$

$$G_{11} = (\mathbf{I}_{n_G} \Theta \mathbf{I}_{n_G})^T G_1 \,. \tag{4-106}$$

Then Eq. (4-103) to (4-104) may be written more compactly as

$$\| (g_{00} + G_{11} \underline{w}) \| = \min \tag{4-107}$$

and

$$\| (g_{00} + G_{11} \underline{w}) \| \le \overline{\lambda_m}. \tag{4-108}$$

4.3.4 Sensitivity requirement

From chapter 3, the sensitivity criteria for detection of deformation parameters in optimization can be stated as

$$\underline{e}^T (B^T A^T P A B) \underline{e} = \max \qquad \text{(maximum sensitivity)} \qquad (4\text{-}109)$$

or

$$\underline{e}^T (B^T A^T P A B) \underline{e} > 2 \sigma_0^2 \varpi_0. \qquad \text{(sensitivity control)} \qquad (4\text{-}110)$$

Using Taylor series expansion we have

$$\underline{e}^T (M^0 + \sum_1^m (\frac{\partial M}{\partial x_i}) \Delta x_i + \sum_1^m (\frac{\partial M}{\partial y_i}) \Delta y_i + \sum_1^m (\frac{\partial M}{\partial z_i}) \Delta z_i + \sum_1^n (\frac{\partial M}{\partial p_i}) \Delta p_i) \underline{e} = \max \qquad (4\text{-}111)$$

and

$$\underline{e}^T (M^0 + \sum_1^m (\frac{\partial M}{\partial x_i}) \Delta x_i + \sum_1^m (\frac{\partial M}{\partial y_i}) \Delta y_i + \sum_1^m (\frac{\partial M}{\partial z_i}) \Delta z_i + \sum_1^n (\frac{\partial M}{\partial p_i}) \Delta p_i) \underline{e} > 2 \sigma_0^2 \varpi_0 \qquad (4\text{-}112)$$

where $M^0 = (B^T A^T P A B)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0}$ (4-113)

$$\frac{\partial M}{\partial x_i} = (B^T A^T P (\frac{\partial A}{\partial x_i} B + A \frac{\partial B}{\partial x_i}) + (B^T (\frac{\partial A}{\partial x_i})^T + (\frac{\partial B}{\partial x_i})^T A^T) P A B) |_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \qquad (4\text{-}114)$$

$$\frac{\partial M}{\partial y_i} = (B^T A^T P (\frac{\partial A}{\partial y_i} B + A \frac{\partial B}{\partial y_i}) + (B^T (\frac{\partial A}{\partial y_i})^T + (\frac{\partial B}{\partial y_i})^T A^T) P A B) |_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \qquad (4\text{-}115)$$

$$\frac{\partial M}{\partial z_i} = (B^T A^T P (\frac{\partial A}{\partial z_i} B + A \frac{\partial B}{\partial z_i}) + (B^T (\frac{\partial A}{\partial z_i})^T + (\frac{\partial B}{\partial z_i})^T A^T) P A B) |_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \qquad (4\text{-}116)$$

$$\frac{\partial M}{\partial p_i} = (B^T A^T (\frac{\partial P}{\partial p_i}) A B)|_{\underline{x}^0, \underline{y}^0, \underline{z}^0, P^0} \qquad (4\text{-}117)$$

Let $s_0 = \underline{e}^T M^0 \underline{e}$ (4-118)

$$\underline{s}^T = \left(\underline{e}^T \frac{\partial M}{\partial x_1} \underline{e} \quad \underline{e}^T \frac{\partial M}{\partial y_1} \underline{e} \quad \underline{e}^T \frac{\partial M}{\partial z_1} \underline{e} \quad \cdots \quad \underline{e}^T \frac{\partial M}{\partial x_m} \underline{e} \quad \underline{e}^T \frac{\partial M}{\partial y_m} \underline{e} \quad \underline{e}^T \frac{\partial M}{\partial z_m} \underline{e} \right.$$

$$\left. \underline{e}^T \frac{\partial M}{\partial p_1} \underline{e} \quad \cdots \quad \underline{e}^T \frac{\partial M}{\partial p_n} \underline{e} \right) \qquad (4\text{-}119)$$

Then Eq.(4-111) to Eq.(4-112) reduce to

$$s_0 + \underline{s}^T \underline{w} = \max \qquad (4\text{-}120)$$

or

$$s_0 + \underline{s}^T \underline{w} > s_m \qquad (4\text{-}121)$$

where $\underline{w}$ is the same as in Eq.(4-29), and

$$s_m = 2 \sigma_0^2 \varpi_0. \qquad (4\text{-}122)$$

4.3.5 Cost requirement

A simplified cost criterion for optimization can be

$$\| P \| = \min \qquad \text{(minimum cost)} \qquad (4\text{-}123)$$

or

$$\| P \| \leq c_m \qquad \text{(cost control)} \qquad (4\text{-}124)$$

where P is the weight matrix of observations and $\| \cdot \|$ represents the norm of a matrix. When using Taylor series expansion, matrix P can be expressed as

$$P = P^0 + \sum_1^n \frac{\partial P}{\partial p_i} \Delta p_i \qquad (4\text{-}125)$$

where P^0 is the approximate weight matrix. Considering P is a diagonal matrix, we have

$$\| P \| = \| \text{vecdiag}(P) \|$$

$$= \| \ (I_n \Theta I_n)^T (\text{vec}(P^0) + \sum_1^n \text{vec} (\frac{\partial P}{\partial p_i}) \Delta p_i) \| \qquad (4\text{-}126)$$

Thus Eq.(4-123) to Eq.(4-124) can be reformulated as

$$\| \underline{c}_{00} + C_{11}\, \underline{w} \| = \min \tag{4-127}$$

or

$$\| \underline{c}_{00} + C_{11}\, \underline{w} \| \leq c_m \tag{4-128}$$

where

$$\underline{c}_{00} = (\mathbf{I}_n \Theta \mathbf{I}_n)^T \mathrm{vec}(P^0)\,; \tag{4-129}$$

$$C_{1\,1} = (\ \mathbf{0}\ \ (\mathbf{I}_n \Theta \mathbf{I}_n)^T \mathrm{vec}\left(\frac{\partial P}{\partial p_1}\right) \ \ldots\ \ (\mathbf{I}_n \Theta \mathbf{I}_n)^T \mathrm{vec}\left(\frac{\partial P}{\partial p_n}\right)\ )\,; \tag{4-130}$$

$\underline{w}$ is the same as in Eq.(4-29);

$\mathbf{I}_n$ is the n by n unit matrix and Θ the Khatri-Rao product; and

$\mathbf{0}$ is the sub-matrix of zeros corresponding to position improvements.

4.3.6 Physical constraints

(a) Datum consideration

Since a translation, rotation and scaling of the monitoring scheme does not change the shape of the scheme, the position improvements to be introduced for the netpoints should be constrained such that no translation, scalling and differential rotation with respect to the scheme to be optimized are introduced i.e.

$$D^T w = 0 \tag{4-131}$$

where $D^T = (D_c^T \ \ \mathbf{0}^T)$ with D_c the datum matrix used to calculate the coordinates of netpoints; $\mathbf{0}$ is a sub-matrix of zeros corresponding to weight improvements; and $\underline{w}$ is the same as in Eq.(4-29).

(b) Realizability

At first, the position improvements to be introduced should be bounded by the topography and/or other considerations, i.e.

$$\begin{aligned} a_{1\,i} \le \Delta x_i \le a_{2\,i} \\ b_{1\,i} \le \Delta y_i \le b_{2\,i} \qquad (i= 1, ..., m) \\ c_{1\,i} \le \Delta z_i \le c_{2\,i} \end{aligned} \tag{4-132}$$

where the boundary values $[a_{1\,i}, a_{2\,i}]$, $[b_{1\,i}, b_{2\,i}]$, and $[c_{1\,i}, c_{2\,i}]$ may be established in the field by reconnaissance.

As for the weight changes Δp_i (i=1, ..., n) to be introduced, the weights of observations should be non-negative and be bounded by the maximum achievable accuracy of the available instruments, i.e.,

$$0 \le p_i \le \frac{\sigma_0^2}{(\sigma_i)^2_{min}} \tag{4-133}$$

or

$$-p_i^0 \le \Delta p_i \le \frac{\sigma_0^2}{(\sigma_i)^2_{min}} - p_i^0 = (\Delta p_i)_{max} \qquad (i=1, ..., n) \tag{4-134}$$

where p_i^0 (i=1, ..., n) are the approximate values for weights p_i

σ_0^2 is the a priori variance factor; and

$(\sigma_i)^2_{min}$ (i=1, ..., n) are the minimum variance which can be achieved for each observable l_i (i=1, ..., n).

Combining Eq.(4-132) and Eq.(4-134) and written in matrix form we have

$$A_{00}\,\underline{w} \le \underline{b}_{00} \tag{4-135}$$

$$\text{where } A_{00} = \begin{pmatrix} \mathbf{I} \\ -\mathbf{I} \end{pmatrix} \tag{4-136}$$

with **I** the (3m+n) by (3m+n) unit matrix; and

$$\underline{b}_{00} = (a_{21}\ b_{21}\ c_{21}\ \ldots\ a_{2m}\ b_{2m}\ c_{2m}\ (\Delta p_1)_{max}\ \ldots\ (\Delta p_n)_{max}, \\ -a_{11}\ -b_{11}\ -c_{11}\ \ldots\ -a_{1m}\ -b_{1m}\ -c_{1m}\ p_1^0\ \ldots\ p_n^0\ )^T. \tag{4-137}$$

4.4 Formulation of Mathematical Models for Optimization

Mathematically, **optimization** means determining the maximum or minimum of a target function under a number of constraints (equalities or inequalities or both). For our purpose -- the optimization of a monitoring scheme, the target function will be one which represent the quality of the monitoring scheme. As discussed before, the four general measures used to evaluate this quality are precision, reliability, sensitivity and economy. A monitoring scheme should be designed in such a way that it can realize the required precision of the deformation parameters; can be resistant to gross errors in the observations and minimize the effects of undetected gross errors on deformation parameters; can allow testing of special hypothesis with higher significance; and can satisfy certain cost limit. Therefore, from the suggestion of Schaffrin (1985), our target function will be of the type:

$$\alpha_p \text{ (precision)} + \alpha_r \text{ (reliability)} + \alpha_s \text{ (sensitivity)} + \alpha_c \text{ (cost)}^{-1} = \max \tag{4-138}$$

for suitably chosen weight coefficients α_p, α_r, α_s, and α_c . This is a multi-objective optimization problem. If we let one of the coefficients go to infinity, we obtain some extreme cases of the target function (Schaffrin, 1985):

Case (1): $\alpha_c \dashrightarrow \infty$ ==> cost = min

precision $\geq$ constant

reliability $\geq$ constant

sensitivity $\geq$ constant

Here the cost is optimized while we have to control the precision, reliability, and sensitivity.

Case (2): $\alpha_p \dashrightarrow \infty$ ==> precision = max

reliability $\geq$ constant

sensitivity $\geq$ constant

cost $\leq$ constant

Here the precision is optimized while we have to control reliability, sensitivity, and cost.

Case (3): $\alpha_r \rightarrow \infty \Rightarrow$ reliability = max

precision $\geq$ constant

sensitivity $\geq$ constant

cost $\leq$ constant

Here the reliability is optimized while cost, precision, and sensitivity are controlled.

Case (4): $\alpha_s \rightarrow \infty \Rightarrow$ sensitivity = max

precision $\geq$ constant

reliability $\geq$ constant

cost $\leq$ constant

Here the sensitivity is optimized while precision, reliability, and cost are controlled.

Thus, depending on which alternative we choose, the mathematical models for the optimization of a monitoring scheme can be established by combining a number of the basic requirements out of the ones discussed in the above section.

Model I: Best approximation of the criterion matrix

$$\| H \underline{w} - \underline{u} \| = \min$$

Subject to:

$$H_1 \underline{w} - \underline{u}_1 \geq 0 \qquad \text{(I-1)}$$

$$\| (\underline{r}_{00} + R_{11} \underline{w}) \| \geq r_l \qquad \text{(I-2)}$$

$$\| (g_{00} + G_{11} \underline{w}) \| \leq \bar{\lambda}_m \qquad \text{(I-3)}$$

$$\| \underline{c}_{00} + C_{11} \underline{w} \| \leq c_m \qquad \text{(I-4)}$$

$$s_0 + \underline{s}^T \underline{w} > s_m \qquad \text{(I-5)}$$

$$D^T \underline{w} = 0 \qquad \text{(I-6)}$$

$$A_{00} \underline{w} \leq \underline{b}_{00} \qquad \text{(I-7)}$$

Model II: Maximizing the internal reliability

$$- \| (\underline{r}_{00} + R_{11}\,\underline{w}) \| = \min$$

Subject to:

$$H_1\,\underline{w} - \underline{u}_1 \geq 0 \qquad \text{(II-1)}$$

$$\| H\,\underline{w} - \underline{u} \| \leq v_p \qquad \text{(II-2)}$$

$$\| (g_{00} + G_{11}\,\underline{w}) \| \leq \bar{\lambda}_m \qquad \text{(II-3)}$$

$$\| \underline{c}_{00} + C_{11}\,\underline{w} \| \leq c_m \qquad \text{(II-4)}$$

$$s_0 + \underline{s}^T\,\underline{w} > s_m \qquad \text{(II-5)}$$

$$D^T\underline{w} = 0 \qquad \text{(II-6)}$$

$$A_{00}\,\underline{w} \leq \underline{b}_{00} \qquad \text{(II-7)}$$

Model III: Maximizing the external reliability

$$\| (g_{00} + G_{11}\,\underline{w}) \| = \min$$

Subject to:

$$H_1\,\underline{w} - \underline{u}_1 \geq 0 \qquad \text{(III-1)}$$

$$\| H\,\underline{w} - \underline{u} \| \leq v_p \qquad \text{(III-2)}$$

$$\| (\underline{r}_{00} + R_{11}\,\underline{w}) \| \geq r_1 \qquad \text{(III-3)}$$

$$\| \underline{c}_{00} + C_{11}\,\underline{w} \| \leq c_m \qquad \text{(III-4)}$$

$$s_0 + \underline{s}^T\,\underline{w} > s_m \qquad \text{(III-5)}$$

$$D^T\underline{w} = 0 \qquad \text{(III-6)}$$

$$A_{00}\,\underline{w} \leq \underline{b}_{00} \qquad \text{(III-7)}$$

Model IV: <u>Maximizing the sensitivity</u>

$$-(s_0 + \underline{s}^T \underline{w}) = \min$$

Subject to:

$$H_1 \underline{w} - \underline{u}_1 \geq 0 \qquad \text{(IV-1)}$$

$$\| H \underline{w} - \underline{u} \| \leq v_p \qquad \text{(IV-2)}$$

$$\| (\underline{r}_{00} + R_{11} \underline{w}) \| \geq r_l \qquad \text{(IV-3)}$$

$$\| (\underline{g}_{00} + G_{11} \underline{w}) \| \leq \bar{\lambda}_m \qquad \text{(IV-4)}$$

$$\| \underline{c}_{00} + C_{11} \underline{w} \| \leq c_m \qquad \text{(IV-5)}$$

$$D^T \underline{w} = 0 \qquad \text{(IV-6)}$$

$$A_{00} \underline{w} \leq \underline{b}_{00} \qquad \text{(IV-7)}$$

Model V: <u>Minimizing the cost of observations</u>

$$\| \underline{c}_{00} + C_{11} \underline{w} \| = \min$$

Subject to:

$$H_1 \underline{w} - \underline{u}_1 \geq 0 \qquad \text{(V-1)}$$

$$\| H \underline{w} - \underline{u} \| \leq v_p \qquad \text{(V-2)}$$

$$\| (\underline{r}_{00} + R_{11} \underline{w}) \| \geq r_l \qquad \text{(V-3)}$$

$$\| (\underline{g}_{00} + G_{11} \underline{w}) \| \leq \bar{\lambda}_m \qquad \text{(V-4)}$$

$$s_0 + \underline{s}^T \underline{w} > s_m \qquad \text{(V-5)}$$

$$D^T \underline{w} = 0 \qquad \text{(V-6)}$$

$$A_{00} \underline{w} \leq \underline{b}_{00} \qquad \text{(V-7)}$$

4.5 Solution of the Mathematical Models

4.5.1 The choice of norm

Up to now, the mathematical models for the optimization of a monitoring scheme have been established except that all the norms of matrices and vectors in the models were not defined. Depending on the choice of norm, the whole problem gets a different formulation and, as a consequence, we have to apply a different algorithm in order to find its solution. If, for instance, we allow for general norms within the models, we would obtain general mathematical programming problems with linear and non-linear constraints.

A mapping of a linear vector space $\mathbb{X}$ over $\mathbb{R}$ into $\mathbb{R}$ is called "norm", and its value for $\underline{x} \in \mathbb{X}$ is denoted by $\| \underline{x} \|$, iff it holds:

(1) $\| \underline{x} \| \geq 0$ for $\underline{x} \in \mathbb{X}$

(2) $\| \underline{x} + \underline{y} \| \leq \| \underline{x} \| + \| \underline{y} \|$ for $\underline{x}, \underline{y} \in \mathbb{X}$

(3) $\| \alpha \underline{x} \| = |\alpha| \, \| \underline{x} \|$ for $\underline{x} \in \mathbb{X}, \alpha \in \mathbb{R}$

(4) $\| \underline{x} \| = 0 \Longrightarrow \underline{x} = 0$.

A "matrix norm" can be defined in terms of a vector norm. Given a vector norm $\| \cdot \|$ and a matrix A, consider $\| A \underline{x} \|$ for all vectors such that $\| \underline{x} \| = 1$. The matrix norm induced by the vector norm is given by

$$\| A \| = \max_{\| \underline{x} \| = 1} \| A \underline{x} \| \tag{4-139}$$

Typical examples are the l^{ρ}- norms for finite dimensional vector spaces as defined with respect to some algebraic basis by

$$\| (x_1, x_2, \ldots, x_n) \|_{\rho} = \left(\sum_{1}^{n} | x_i |^{\rho} \right)^{1/\rho} \tag{4-140}$$

for any arbitrary chosen number ρ with $1 \leq \rho \leq \infty$. For $\rho = 1, 2,$ and ∞, the l^1-norm , l^2-norm , and l^{∞}-norm are defined respectively as

$$\| (x_1, x_2, \ldots , x_n) \|_1 = \sum_1^n | x_i | \tag{4-141}$$

$$\| (x_1, x_2, \ldots , x_n) \|_2 = (\sum_1^n x_i^2)^{1/2} \tag{4-142}$$

$$\| (x_1, x_2, \ldots , x_n) \|_\infty = \max \{ | x_i | , i=1, \ldots , n \} \tag{4-143}$$

The l^∞-norm is also called " uniform norm" or " Tschebycheff norm".

On the other hand, for any space of matrices with arbitrary, but specified size we may obtain a matrix norm simply by identifying

$$\| A \| = \| \text{vec} (A) \| \tag{4-144}$$

for a certain vector norm, where the "vec" operator stacks one column of the matrix under the other.

As for the "choice of norm" for network optimization, it has been argued by Schaffrin (1981) that a choice of ρ with $1 \leq \rho < 2$ would be less suitable for fitting a criterion matrix because of the loss of smoothing power while for cost requirement ρ should not exceed 2. Van Mierlo (1981) suggested l^∞-norm for reliability. Therefore, as a typical case, we will use l^2-norm , l^1-norm , and l^∞-norm for the best fitting of the criterion matrix, for the minimum cost, and for the maximum reliability requirements respectively. And use l^1-norm for precision control Eq.(4-32). Under these considerations, the above proposed mathematical models (I) to (V) become:

Model I: Best approximation of the criterion matrix

Minimize $(\underline{w}^T H^T H \underline{w} - 2 \underline{u}^T H \underline{w} + \underline{u}^T \underline{u})$

Subject to:

$$H_1 \underline{w} - \underline{u}_1 \geq 0 \quad \text{(I-1)}$$

$$(\underline{r}_{00} + R_{11} \underline{w}) \geq \underline{r}_1 \quad \text{(I-2)}$$

$$(\underline{g}_{00} + G_{11} \underline{w}) \leq \underline{\bar{\lambda}}_m \quad \text{(I-3)}$$

$$\underline{\gamma}^T \underline{c}_{00} + \underline{\gamma}^T C_{11} \underline{w} \leq c_m \quad \text{(I-4)}$$

$$s_0 + \underline{s}^T \underline{w} > s_m \quad \text{(I-5)}$$

$$D^T \underline{w} = 0 \quad \text{(I-6)}$$

$$A_{00} \underline{w} \leq \underline{b}_{00} \quad \text{(I-7)}$$

where $\underline{\gamma}$ is a (3m+n) by 1 vector of constants;

$\underline{r}_1$ is the n_G by 1 vector with all elements being r_1;

$\underline{\bar{\lambda}}_m$ is the n_G by 1 vector with all elements being $\bar{\lambda}_m$.

Model II: Maximizing the internal reliability

Minimize $\max \{ - (\underline{r}_{00} + R_{11} \underline{w})_i \ , i = 1, \ldots , n_G \}$

Subject to:

$$H_1 \underline{w} - \underline{u}_1 \geq 0 \quad \text{(II-1)}$$

$$- \underline{v}_p \leq H \underline{w} - \underline{u} \leq \underline{v}_p \quad \text{(II-2)}$$

$$(\underline{g}_{00} + G_{11} \underline{w}) \leq \underline{\bar{\lambda}}_m \quad \text{(II-3)}$$

$$\underline{\gamma}^T \underline{c}_{00} + \underline{\gamma}^T C_{11} \underline{w} \leq c_m \quad \text{(II-4)}$$

$$s_0 + \underline{s}^T \underline{w} > s_m \quad \text{(II-5)}$$

$$D^T \underline{w} = 0 \quad \text{(II-6)}$$

$$A_{00} \underline{w} \leq \underline{b}_{00} \quad \text{(II-7)}$$

where $\underline{v}_p$ is a u^2 by 1 vector with all elements being v_p.

Model III: Maximizing the external reliability

Minimize $\max \{ (g_{00} + G_{11}\,\underline{w})_i \ , i = 1, \ldots , n_G \}$

Subject to:

$$H_1\,\underline{w} - \underline{u}_1 \geq 0 \qquad \text{(III-1)}$$

$$-\underline{v}_p \leq H\,\underline{w} - \underline{u} \leq \underline{v}_p \qquad \text{(III-2)}$$

$$(\underline{r}_{00} + R_{11}\,\underline{w}) \geq \underline{r}_1 \qquad \text{(III-3)}$$

$$\underline{\gamma}^T \underline{c}_{00} + \underline{\gamma}^T C_{11}\,\underline{w} \leq c_m \qquad \text{(III-4)}$$

$$s_0 + \underline{s}^T \underline{w} > s_m \qquad \text{(III-5)}$$

$$D^T \underline{w} = 0 \qquad \text{(III-6)}$$

$$A_{00}\,\underline{w} \leq \underline{b}_{00} \qquad \text{(III-7)}$$

Model IV: Maximizing the sensitivity

$$s_0 + \underline{s}^T \underline{w} = \max$$

Subject to:

$$H_1\,\underline{w} - \underline{u}_1 \geq 0 \qquad \text{(IV-1)}$$

$$-\underline{v}_p \leq H\,\underline{w} - \underline{u} \leq \underline{v}_p \qquad \text{(IV-2)}$$

$$(\underline{r}_{00} + R_{11}\,\underline{w}) \geq \underline{r}_1 \qquad \text{(IV-3)}$$

$$(g_{00} + G_{11}\,\underline{w}) \leq \underline{\bar{\lambda}}_m \qquad \text{(IV-4)}$$

$$\underline{\gamma}^T \underline{c}_{00} + \underline{\gamma}^T C_{11}\,\underline{w} \leq c_m \qquad \text{(IV-5)}$$

$$D^T \underline{w} = 0 \qquad \text{(IV-6)}$$

$$A_{00}\,\underline{w} \leq \underline{b}_{00} \qquad \text{(IV-7)}$$

Model V: Minimizing the cost of observations

Minimize $\underline{\gamma}^T \underline{c}_{00} + \underline{\gamma}^T C_{11} \underline{w}$

Subject to:

$$H_1 \underline{w} - \underline{u}_1 \geq 0 \qquad \text{(V-1)}$$

$$-\underline{v}_p \leq H \underline{w} - \underline{u} \leq \underline{v}_p \qquad \text{(V-2)}$$

$$(\underline{r}_{00} + R_{11} \underline{w}) \geq \underline{r}_1 \qquad \text{(V-3)}$$

$$(\underline{g}_{00} + G_{11} \underline{w}) \leq \underline{\lambda}_m \qquad \text{(V-4)}$$

$$s_0 + \underline{s}^T \underline{w} > s_m \qquad \text{(V-5)}$$

$$D^T \underline{w} = 0 \qquad \text{(V-6)}$$

$$A_{00} \underline{w} \leq \underline{b}_{00} \qquad \text{(V-7)}$$

4.5.2 The Solution methods

As is clear from the above discussion, obtaining the solutions of the above mathematical models can be summarized as solving one of the following standardized approximation problems with linear equality or inequality constraints:

(1) l^1 -approximation

Minimize $\| M \underline{w} - \underline{f} \|_1$

Subject to $E_{11} \underline{w} \leq \underline{f}_{11}$

(2) l^2-approximation

Minimize $\| M \underline{w} - \underline{f} \|_2$

Subject to $E_{22} \underline{w} \leq \underline{f}_{22}$

(3) l^∞-approximation

Minimize $\| M \underline{w} - \underline{f} \|_\infty$

Subject to $E_\infty \underline{w} \leq \underline{f}_\infty$

Model I is the l^2-approximation solvable by Quadratic Programming (QP); Model II and Model III are the l^∞-approximation solvable by Dual Linear Programming (DLP) or by QP;

and finally Model IV and Model V are the l^1 -approximation solvable by linear programming (LP). Standard software for Linear- or Quadratic Programming, if available, can be used to obtain the solutions for the mathematical models. Also, Schaffrin (1981) introduced the Linear Complementary Algorithm (LCA) or Parametric Linear Complementary Algorithm (PLCA) to solve the QP or LP problems, since the former may improve numerical stability and therefore are very efficient. The purpose of PLCA is to allow the infeasible constraints to be transformed into feasible ones by introducing additional parameters. Basic theories of QP, LP, and LCA will be discussed in appendix II.

4.6 Analysis of Solvability and the Multi-Objective Optimization Model (MOOM)

4.6.1 The Concept of Multi-Objective Optimization

The study of the multi-objective optimization is a well established branch in operations research. The mathematical theories involved are quite sophisticated. In the following a brief explanation of the problem statement and the solution concepts is given in order to apply this theory to solving our problems.

A Multi-Objective Optimization Problem (MOOP) is defined as:

$$\text{Minimize } \underline{f}(\underline{x}) = (f_1(\underline{x}), \ldots, f_p(\underline{x}))^T$$

$$\text{Subject to } \underline{x} \in \mathbb{X} = \{\underline{x}: \underline{g}(\underline{x}) = (g_1(\underline{x}), \ldots, g_m(\underline{x}))^T \leq 0 \}$$

where f_i (i=1, ..., p) are the individual objective functions and g_j (j=1, ..., m) the constraint functions. Here all the functions are assumed to be continuously differentiable. This kind of problem is also called a vector optimization.

Unlike traditional mathematical programming with a single objective function, an optimal solution that minimizes all the objective functions simultaneously does not necessarily exist. The final decision should be made by taking the total balance of

objectives into account. Thus the decision maker's value is usually represented by saying whether or not an alternative $\underline{x}$ is preferred to another alternative x', or equivalently whether or not f(x) is preferred to f(x'). For this purpose, a scalar-valued function $u(f_1, \cdots, f_p)$ representing the decision maker's preference called preference function has to be defined. Once we obtain such a preference function, our problem reduces to the traditional mathematical programming:

$$\text{Maximize } u(f_1, \cdots, f_p)$$
$$\text{Subject to } \underline{x} \in X = \{\underline{x}: \underline{g}(\underline{x}) = (g_1(\underline{x}), \ldots, g_m(\underline{x}))^T \leq 0\}.$$

Instead of strict optimality, the notion of efficiency is introduced in multi-objective optimization. A vector $\underline{f}(\hat{\underline{x}})$ is said to be efficient if there is no $\underline{f}(\underline{x})$ $(\underline{x} \in X)$ preferred to $\underline{f}(\hat{\underline{x}})$. Mathematically, the most fundamental kind of efficient solution is usually called a Pareto optimal solution or noninferior solution. A point $\hat{x} \in X$ is said to be a Pareto optimal solution(or noninferior solution) to the problem MOOP if there is no $\underline{x} \in X$ such that $f(\underline{x}) \leq f(\hat{\underline{x}})$. The final decision is usually made among the set of efficient solutions. There are a number of ways to characterize the efficiency of a solution. One characterization is taken as the best approximations to the ideal point. Consider the multi-objective optimization problem MOOP, an ideal point $\tilde{\underline{f}} = (\tilde{f_1}, \cdots, \tilde{f_p})^T$ is defined as:

$$\tilde{f_i} = \inf\{f_i(\underline{x}): \underline{x} \in X\} > -\infty, \quad i=1, \cdots, p. \tag{4-145}$$

However, we might take another ideal point $\bar{\underline{f}}$ as $\bar{\underline{f}} \leq \tilde{\underline{f}}$. A solution $\hat{\underline{x}}$ is efficient if

$$d = \| \underline{f}(\hat{\underline{x}}) - \bar{\underline{f}} \| = \inf\{\|\underline{f}(\underline{x}) - \bar{\underline{f}}\| \} \ \underline{x} \in X \tag{4-146}$$

where $\| \cdot \|$ represents the norm of a vector. It can be the l_ρ -norms as defined by

$$\| \underline{f} \|_\rho = \left[\sum_{i=1}^{p} | \mu_i f_i |^\rho \right]^{1/\rho}, \quad \rho \in [1, \infty) \tag{4-147}$$

or the one introduced by Dinkelbach and Isermann(1973)

$$\| \underline{f} \|_\rho = \| \underline{f} \|_\infty + \frac{1}{\rho} \left(\sum_{i=1}^{p} | \mu_i f_i | \right), \quad \rho \in [1, \infty) \tag{4-148}$$

where $\mu = (\mu_1, \mu_2, \cdots, \mu_p)^T$ is the weighting vector. Intuitively, if the ideal point is unattainable, $d = \| \underline{f}(\underline{x}) - \bar{f} \|$ represents the distance between $\underline{f}(\underline{x})$ and $\bar{f}$ and can be considered as a measure of regret resulted from unattainability of $\underline{f}(\underline{x})$ to $\bar{f}$. An efficient solution is to minimize this regret. In order to apply this idea in practice, the following problems have to be solved:

(i) How to choose the distance function?

(ii) How to decide the weighting vector? and

(iii) How to make a common scale for the different objective functions?

At first, Sawaragi et. al. (1985) suggests that it suffices to use the l_ρ -norms with $\rho \geq 1$ to evaluate the distance. The solution to problem (ii) is quite subjective, since that is closely related to the decision maker's preference attitudes to some specific objective functions. Thus, one used to adopt the weight based on his experience. If the same weight is selected for each objective function, then the resulting solution is the one that improves equally, in some sense, each criterion as much as possible. Dyer(1972) suggested a numerical method for deciding the weights under interaction with decision makers. The method is complicated and difficult to use in practice. Finally, if the l_ρ -norms distance function is used, it is important to make a common scale for the objective functions. For example, if the positive and negative values of the objective functions are mixed, or if their physical units and numerical orders are different from each other, then some of the objective functions are sensitive and others not. One approach to make a common scale for objective functions is to use the relative degree of the non-attainability of $f_i(x)$ to the ideal value $\bar{f}_i$ i.e. $(f_i(\underline{x}) - \bar{f}_i)/\bar{f}_i$ as new objective functions. This enables us to ignore the need to pay extra attention to the difference among the dimension and the numerical order of the different objective functions.

4.6.2 The Multi-Objective Optimization Model (MOOM) for Monitoring schemes

In the above sections, the mathematical models for the optimization of a monitoring scheme have been established. For practical applications, depending on the aspects of the problem we prefer, we may choose one of the formulations Models I to V to obtain the optimal configuration and observation plan of a monitoring scheme. Since the objective function in Model I is strictly convex for a positive-definite matrix H^TH, it has a unique minimizing solution if and only if the constraints in Eq.(I-1) to Eq.(I-7) are consistent. In Models II to V, the objective function are convex, they have unique minimizing solutions if and only if their respective constraints in Eq.(i-1) to Eq.(i-7) (i=II, III, IV, V) are consistent and no rank deficiency exists for the coefficient matrix of the whole set of constraints of each model. Therefore, the consistency of constraints is decisive for the solution of each of the mathematical models. The possibility of inconsistency of constraints in Eq.(I-1) was discussed by Schaffrin (1980). He proposed to approach the inconsistency by "parameterization" i.e. by adding a certain negative value at the right hand side in order to generate consistency and thus the existence of the desired solution. The problem with that approach is that through the "parameterization", the original constraints in Eq.(I-1) become in fact redundant.

An analysis of the constraints in Eq.(I-1) to Eq.(I-7) indicates that the physical constraints in Eq.(I-6) to Eq.(I-7) are compulsory. They represent the physical environment in which we can optimize our monitoring scheme. In most cases, the precision, reliability, sensitivity and cost constraints (Eq.(I-1) to Eq.(I-5)) may be contradictory, i.e., the best monitoring scheme simultaneously which would satisfy the objective function and all the constraints may not exist. That can be understood in this way: once the maximum precision of instrumentation, the total expenditure, and the topographic conditions are given, the maximum precision, reliability, and sensitivity attainable are limited. Thus if the criterion matrix of deformation parameters $\sigma_0^2 Q_{\xi}^{c}$, the criterion vector for internal and external reliability $\underline{\eta}$, $\underline{\lambda}_m$, and the criterion number for sensitivity s_m, are

not defined properly, i.e. if they exceed the reality, then the constraints in Eq.(I-1) to Eq.(I-5) will be inconsistent, and no solution exists. This analysis is applicable to all the other four Models II to V. Therefore, to ensure a universally applicable optimization procedure, a unified mathematical model, called the Multi-Objective Optimization Model (MOOM), is proposed, in which instead of approximating the constraints in Eq.(I-1) to Eq.(I-5) "from one side", an approximation "from both sides" is permitted. The point is to minimize the differences between the precision, reliability, sensitivity, cost and their respective ideal counterparts simultaneously, subject to the physical constraints. This model includes all the intentions of Models I to V, and can be described as:

$$\text{Minimize } \frac{\| H\,\underline{w} - \underline{u} \|}{\| \underline{u} \|} + \frac{\| R_{11}\,\underline{w} - (\underline{r}_1 - \underline{r}_{00}) \|}{\| \underline{r}_1 - \underline{r}_{00} \|} + \frac{\| G_{11}\,\underline{w} - (\bar{\lambda}_m - g_{00}) \|}{\| \bar{\lambda}_m - g_{00} \|} +$$

$$+ \frac{\| \underline{\gamma}^T C_{11}\,\underline{w} - (c_m - \underline{\gamma}^T \underline{c}_{00}) \|}{\| c_m - \underline{\gamma}^T \underline{c}_{00} \|} + \frac{\| \underline{s}^T \underline{w} - (s_m - s_0) \|}{\| s_m - s_0 \|} \qquad (4\text{-}149)$$

$$\text{Subject to: } D^T w = 0$$

$$A_{00}\,\underline{w} \le \underline{b}_{00}. \qquad (4\text{-}150)$$

By applying l^2 - norm from the point of view of computational benefit, the above model may be simplified as

$$\text{Minimize } (\underline{w}^T H_0^T H_0\,\underline{w} - 2\,\underline{u}_0^T H_0\,\underline{w} + \underline{u}_0^T \underline{u}_0) \qquad (4\text{-}151)$$

$$\text{Subject to: } D^T w = 0$$

$$A_{00}\,\underline{w} \le \underline{b}_{00} \qquad (4\text{-}152)$$

where

$$H_0 = \begin{pmatrix} H / \sqrt{\underline{u}^T \underline{u}} \\ R_{11} / \sqrt{\underline{u}_1^T \underline{u}_1} \\ G_{11} / \sqrt{\underline{u}_2^T \underline{u}_2} \\ \underline{\gamma}^T C_{11} / \sqrt{u_3 \cdot u_3} \\ \underline{s}^T / \sqrt{u_4 \cdot u_4} \end{pmatrix}, \quad \underline{u}_0 = \begin{pmatrix} \underline{u} / \sqrt{\underline{u}^T \underline{u}} \\ \underline{u}_1 / \sqrt{\underline{u}_1^T \underline{u}_1} \\ \underline{u}_2 / \sqrt{\underline{u}_2^T \underline{u}_2} \\ u_3 / \sqrt{u_3 \cdot u_3} \\ u_4 / \sqrt{u_4 \cdot u_4} \end{pmatrix} \qquad (4\text{-}153)$$

$$u_1 = \underline{r}_1 - \underline{r}_{00}$$

$$u_2 = \bar{\lambda}_m - g_{00}$$

$$u_3 = c_m - \gamma^T \underline{c}_{00}$$

$$u_4 = s_m - s_0.$$

In fact, this mathematical model corresponds to the solution of the multi-objective optimization problem(Eq.(4-138)) under the concept of "ideal point" with specified "ideal point" representing precision, internal and external reliability, sensitivity and cost being $\underline{u}_*$ $(\underline{r}_1 - \underline{r}_{00})$, $(\bar{\lambda}_m - g_{00})$, $(c_m - \gamma^T \underline{c}_{00})$, $(s_m - s_0)$ respectively. Interpreting this intuitively, this model tries to best approximate equally the precision, reliability, sensitivity, and cost criteria under the given geography and instrumentation condition in the sense of least squares. If an appropriate weighting vector $\underline{\alpha} = (\alpha_{pre.}, \alpha_{irelia.}, \alpha_{erelia.}\ \alpha_{sens.}, \alpha_{cos.})^T$ with elements for precision, internal reliability, external reliability, sensitivity, and cost respectively is selected, the selection of which is based on the specific problems to be solved and to a large extent subjective, then the decision maker desires to improve some of the criteria with larger weights more strongly than the others. This practical meaning encourages us to accept easily the obtained solution. This model has a unique minimizing solution if and only if the matrix $H_0^T H_0$ is positive definite. For practical applications, some of the quality criteria may be omitted in the objective function. In this case the model tries to best approximate the ones which appear in it. The rule which governs the solution does not change.

However, if there is a case where all the design criteria of precision, reliability, and sensitivity must be satisfied whatever the total expenditure of the project is, then the above mathematical model must be modified by approximating the design criteria "from one side", i.e.

$$\text{Minimize } (\underline{w}^T H_{00}^T H_{00} \underline{w} - 2\underline{u}_{00}^T H_{00} \underline{w} + \underline{u}_{00}^T \underline{u}_{00}) \tag{4-154}$$

$$\text{Subject to } A_{11}\underline{w} \le \underline{b}_{11}$$

$$D^T w = 0 \tag{4-155}$$

$$A_{00}\underline{w} \le \underline{b}_{00}$$

where

$$H_{00} = \begin{pmatrix} H_1/\sqrt{\underline{u}_1^T \underline{u}_1} \\ -R_{11}/\sqrt{\underline{u}_2^T \underline{u}_2} \\ G_{11}/\sqrt{\underline{u}_3^T \underline{u}_3} \\ \underline{\gamma}^T C_{11}/\sqrt{u_4 \cdot u_4} \\ -\underline{s}^T/\sqrt{u_5 \cdot u_5} \end{pmatrix}, \quad \underline{u}_{00} = \begin{pmatrix} \underline{u}_1/\sqrt{\underline{u}_1^T \underline{u}_1} \\ \underline{u}_2/\sqrt{\underline{u}_2^T \underline{u}_2} \\ \underline{u}_3/\sqrt{\underline{u}_3^T \underline{u}_3} \\ u_4/\sqrt{u_4 \cdot u_4} \\ u_5/\sqrt{u_5 \cdot u_5} \end{pmatrix} \tag{4-156}$$

$$A_{11} = \begin{pmatrix} H_{11} \\ -R_{11} \\ G_{11} \\ -\underline{s}^T \end{pmatrix}, \qquad \underline{b}_{11} = \begin{pmatrix} \underline{u}_{11} \\ \underline{u}_2 \\ \underline{u}_3 \\ u_5 \end{pmatrix} \tag{4-157}$$

H_1, $\underline{u}_1$, and H_{11}, $\underline{u}_{11}$ are defined in Eq. (4-49) and (4-50);

$\underline{u}_2 = \underline{r}_{00} - \underline{r}_1$;

$\underline{u}_3 = \bar{\underline{\lambda}}_m - \underline{g}_{00}$;

$u_4 = c_m - \underline{\gamma}^T \underline{c}_{00}$; and

$u_5 = s_0 - s_m$.

This model will result in a monitoring scheme in which the achievable precision, reliability, and sensitivity will be better or at least equal to the specified values. However, here the upper limits regarding the weights in vector $\underline{b}_{00}$ have to be put large enough so that all the constraints are feasible.

It should be noted that all the solutions to the above mathematical models should be iterative since Taylor series of linear form is used. A Fortran-77 computer program using Quadratic Programming to solve the above proposed mathematical models was completed by the author. The simulation study and its applications will be discussed in Chapter 5.

4.7 Summary of the Optimization Procedures

Step 1. Identification of the deformation parameters to be detected.

First of all, to establish a monitoring scheme, one has to clearly know the purpose it is to serve, i.e., what kind of deformation parameters is to be detected. At the design stage, the deformation model is usually not fully understood. Therefore, except for the case where the deformation model has been specified by the users or derived from previous observations, the construction of the deformation model may be based on a study of the relevant physical properties of the object and on the knowledge of the acting forces (deterministic modelling).

Step 2. Field reconnaissance

Once the purpose of the monitoring scheme is clarified, one has to perform the field reconnaissance. The purpose of the reconnaissance is to select locations for both geodetic and non-geodetic survey points; and to determine all possible types of both geodetic and non-geodetic instruments which would be suitable for the detection of the expected deformations. Then the approximate coordinates of the selected survey points are determined in an appropriate local coordinate system. The possible coordinate changes to be introduced for the initially selected survey points may also be determined from the topography and/or other conditions.

Step 3. Definition of the design criteria

The design criteria include precision, reliability, sensitivity, and economy. The methodology for the determination of these criteria is described in Chapter 3.

Step 4. Solution of the optimal configuration and observational weights of the monitoring scheme

The optimal configuration and observational weights of the monitoring scheme can now be solved for using the methodology developed in Chapter 4. The solution gives the optimal coordinate changes to be introduced to the initially selected survey points and optimal weight changes to be introduced to the initially adopted approximate weights. The selection of the initial weights may be arbitrary. Therefore, the optimal values of both coordinates and weights are obtained by adding the optimal changes, as obtained from the solution, to their corresponding approximate values.

Step 5. Finalization of the monitoring configuration and observation plan

The survey marks can now be monumented according to the solved optimal coordinates of all the initially selected points. As far as the observation scheme is concerned, all the observables which from the optimization in Step 4 obtained zero or close to zero weights can be eliminated from the final design of the monitoring scheme. Check the design criteria to make sure that all the criteria have been satisfied.

Figure 4.2 shows the computation flowchart.

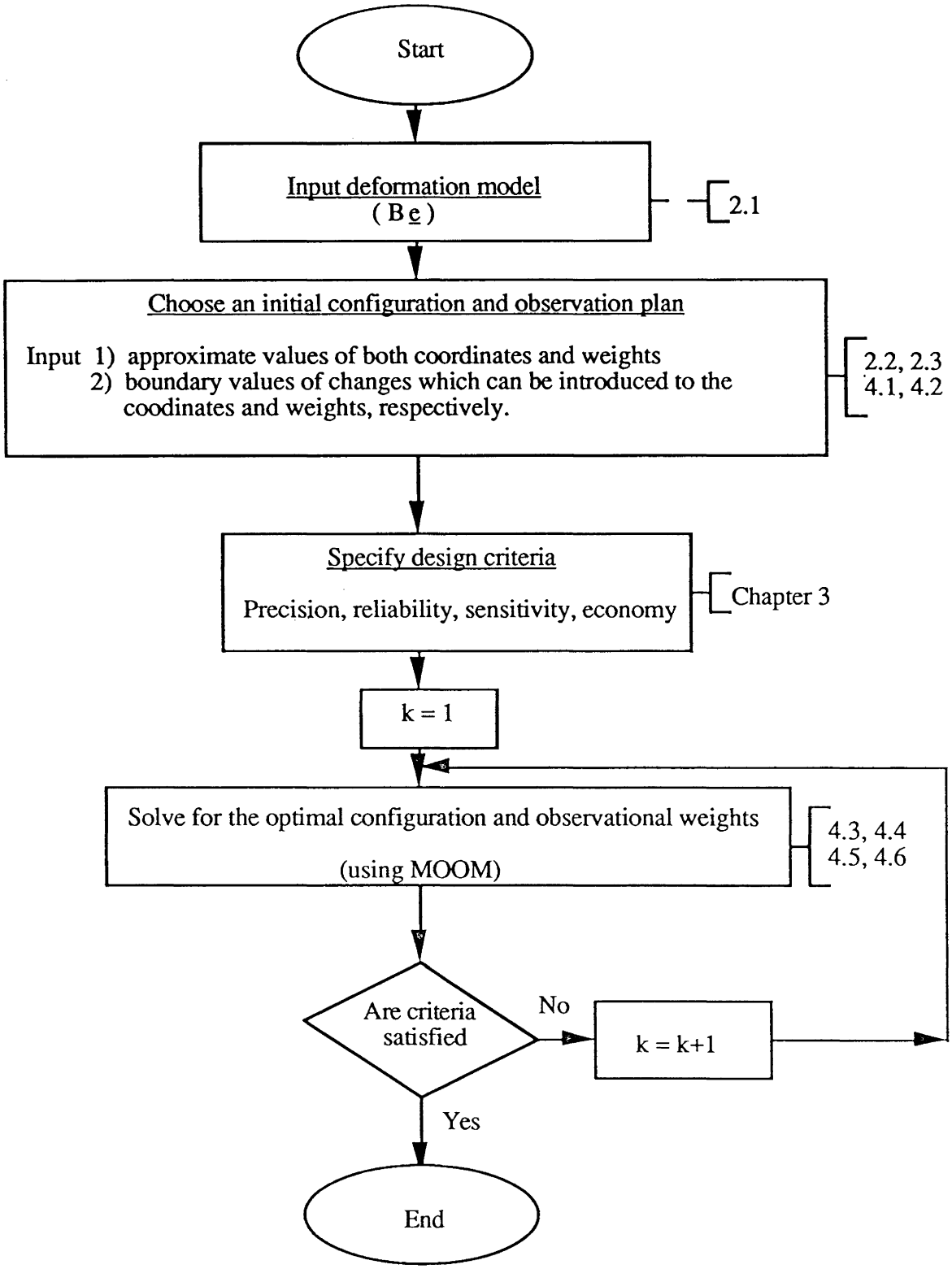

Fig. 4.2: Diagram of the Computation Procedure for the Optimization of Deformation Monitoring schemes

CHAPTER 5

SIMULATION STUDIES AND OPTIMIZATION EXAMPLES

5.1 Simulation Studies

In order to test the validity of the above developed methodology, simulation studies have been performed using two examples. The simulated example No. 1 confirms the correctness of the developed mathematical model MOOM. Simulation study No. 2 illustrates the significance of applying relatively small position changes of netpoints for the optimal solution of observation weights.

5.1.1 Simulation study No. 1: verification of the correctness of the developed mathematical models

Let us consider a simple monitoring scheme (Fig. 5.1) established for the purpose of detecting creep between two tectonic blocks along fault and strain accumulation. This scheme consists of a fault-crossing geodetic network plus some isolated strainmeters placed near each of the geodetic stations with the same orientations of 45 degrees. The desired coordinates of geodetic netpoints are listed in Table 5.1. The shortest and longest distances of the network are 2.00 km and 3.61 km respectively.

Assuming the deformation model includes a relative rigid body translation between Block II and Block I plus a homogeneous strain field over the whole area, it can be expressed as

Table 5.1: Desired coordinates for the geodetic netpoints

Point	x (m)	y (m)
1	500.00	600.00
2	500.00	2600.00
3	2500.00	600.00
4	2500.00	2600.00
5	5500.00	600.00
6	5500.00	2600.00

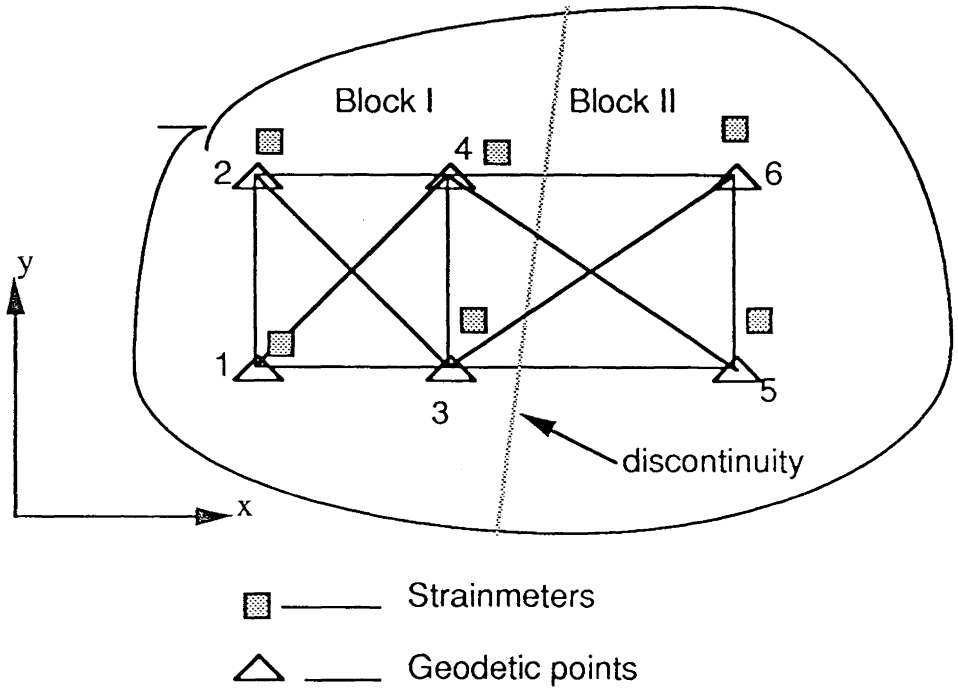

Fig. 5.1: The simulated monitoring scheme

$$u_I = \varepsilon_x X_I + \varepsilon_{xy} Y_I \qquad \text{(for Block I)}$$

$$v_I = \varepsilon_{xy} X_I + \varepsilon_y Y_I$$

$$u_{II} = a_0 + \varepsilon_x X_{II} + \varepsilon_{xy} Y_{II} \qquad \text{(for Block II)}$$

$$v_{II} = b_0 + \varepsilon_{xy} X_{II} + \varepsilon_y Y_{II}$$

The deformation parameters are a_0, b_0, ε_x, ε_y, ε_{xy} .

For the purpose of simulation study, the following procedure was followed:

Step 1: Assumption of an observation scheme

i) assume to use EDM ME 5000 to measure all the distances with nominal accuracy $\sigma_s^2 = (0.2 \text{ mm})^2 + (10^{-6} \cdot s)^2$. In this case, the four different standard deviations for the four different distance groups are:

$\sigma_{s_1} = 2.01$ mm for $s_1 = 2$ km

$\sigma_{s_2} = 2.84$ mm for $s_2 = 2.828$ km

$\sigma_{s_3} = 3.01$ mm for $s_3 = 3.0$ km

$\sigma_{s_4} = 3.61$ mm for $s_4 = 3.606$ km

ii) assume to use Kern E-2 electronic theodolites to measure all directions at each station with standard deviations as

$\sigma_{r_1} = 1''$ for station 1 and station 4;

$\sigma_{r_2} = 2''$ for station 2 and station 5;

$\sigma_{r_3} = 3''$ for station 3 and station 6.

iii) assume to use strainmeters to measure strains with standard deviation

$\sigma_{stra.} = 5$ ppm

Step 2: Computation of the covariance matrix for deformation parameters and point coordinates ; and the internal and external reliability measures

From the above assumed observation scheme and the desired coordinates of stations given in Table 5.1, the upper triangular part of the

covariance matrix of the deformation parameters C_e can be obtained from Eq.(4-7) as (unit : mm^2 for translations, and $\frac{mm^2}{m^2}$ for strains)

$$C_e = 2\cdot \begin{pmatrix} 6.59 & -0.30 & -1.19\cdot 10^{-3} & 5.51\cdot 10^{-5} & 5.70\cdot 10^{-5} \\ & 32.07 & 6.53\cdot 10^{-5} & -2.31\cdot 10^{-3} & 1.00\cdot 10^{-4} \\ & & 4.04\cdot 10^{-7} & -1.52\cdot 10^{-8} & -5.81\cdot 10^{-8} \\ \text{Symmetric} & & & 4.05\cdot 10^{-7} & -1.29\cdot 10^{-8} \\ & & & & 2.80\cdot 10^{-7} \end{pmatrix}$$

Without loss of generality, the covariance matrix for point coordinates of the geodetic monitoring network can be obtained by assuming point 1 and point 2 fixed as (unit: mm^2) :

$$C_x = \begin{pmatrix} 3.55 & 2.13 & -0.30 & 1.84 & 3.48 & 7.40 & -0.19 & 7.40 \\ & 10.08 & -1.79 & 8.32 & 2.40 & 15.39 & -1.68 & 15.44 \\ & & 3.58 & -2.15 & -0.21 & -7.60 & 3.57 & -7.58 \\ & & & 10.01 & 1.86 & 15.88 & -2.48 & 15.83 \\ & & & & 10.71 & 13.26 & -1.21 & 12.86 \\ & & & & & 65.91 & -13.28 & 63.89 \\ & \text{Symmetric} & & & & & 10.97 & -13.65 \\ & & & & & & & 65.76 \end{pmatrix}$$

These matrices C_e and C_x will be used as the criterion matrices for deformation parameters and point coordinates respectively in the following optimization procedure.

The vector $\underline{r}_1$ of redundancy numbers representing internal reliability and the vector $\underline{\bar{\lambda}}_m$ for external reliability under the assumption of $\alpha_0 = \beta_0 = 0.05$ (i.e. $\delta_0 = 3.61$) are listed in Table 5.6 and Table 5.7 respectively. Here it should be noted that, to avoid the introduction of station orientation parameters for direction measurements, all the directions at each station are replaced by appropriate correlated angles.

Step 3: Application of the mathematical model MOOM

To test the proposed mathematical model, we assume that the parameters to be optimized are the positions of geodetic points 4, 5 and 6 and the weights or standard deviations of all the above proposed observables. The positions of all the other geodetic netpoints and strainmeters are not supposed to change. The input approximate coordinates of the netpoints 4, 5 and 6 are listed in Table 5.2 which are obtained by purposely shifting certain values as given in Table 5.3 from the desired coordinates listed in Table 5.1; the input initial weights are $p_i = 1.0$ for all observables.

In order to use the mathematical model MOOM, C_e and C_x are used as the criterion matrices for the deformation parameters and the point coordinates, respectively; The vectors $\underline{r}_1$ and $\underline{\bar{\lambda}}_m$ are used as the boundary values for internal and external reliability respectively. Assume the vector of magnitude of deformation parameters (a_0, b_0, ε_x, ε_{xy}, and ε_y) to be detected is $\underline{e}^T$ = [16.00(mm) 40.0(mm) 4 ppm 4 ppm 4 ppm], then the boundary value for sensitivity (Eq. (4-110)) is s_m = 663.3721. The minimum cost criteria is not considered.

After using the proposed optimization procedure, a comparison of the simulated values and the optimization results is given in Table 5.3 - Table 5.7. From Table 5.3 we can see that the maximum difference between simulated position shifts and the optimized position correction is 0.57 m, and Table 5.4 - Table 5.7 indicate that the differences between the simulated weights, standard deviations, internal and external reliability measures and their corresponding optimized values are practically zero as expected. Also, the optimization procedure gives the same value for sensitivity criterion s_m as specified before.

To check the goodness of fitting of the criterion matrix, the difference matrix between the covariance matrix of deformation parameters as calculated from the optimized values and the criterion matrix is listed as follows:

$$\begin{pmatrix} 8.36\cdot 10^{-6} & -3.38\cdot 10^{-4} & -2.40\cdot 10^{-7} & 1.19\cdot 10^{-7} & 7.03\cdot 10^{-8} \\ & -7.76\cdot 10^{-6} & 2.18\cdot 10^{-7} & 1.49\cdot 10^{-7} & 4.05\cdot 10^{-8} \\ & & 8.64\cdot 10^{-11} & -3.39\cdot 10^{-11} & -1.43\cdot 10^{-11} \\ \text{Symmetric} & & & -2.05\cdot 10^{-11} & -1.24\cdot 10^{-12} \\ & & & & 1.52\cdot 10^{-12} \end{pmatrix}$$

An analysis of the above results indicates that by starting with a set of approximate coordinates of geodetic points and weights of observables, and given the optimality criteria for precision, reliability, and sensitivity, the developed optimization model MOOM can give us their corresponding optimal values which are practically the same as their true values. These results unbiguously confirm the correctness of the proposed mathematical model.

Table 5.2: The input approximate coordinates of geodetic netpoints

Point	x (m)	y (m)
1	500.00	600.00
2	500.00	2600.00
3	2500.00	600.00
4	2550.00	2690.00
5	5560.00	680.00
6	5570.00	2660.00

Table 5.3: Comparison between the simulated position shifts and the optimized position corrections

Point		Simulated position shifts $\Delta_{Simu.}$ (m)	Optimized position corrections $-\Delta_{Opti.}$ (m)	$\Delta_{Simu.}-(-\Delta_{Opti.})$
4	Δx	50.00	50.44	-0.44
	Δy	90.00	89.57	0.43
5	Δx	60.00	60.44	-0.44
	Δy	80.00	79.43	0.57
6	Δx	70.00	70.15	-0.15
	Δy	60.00	60.32	-0.32

Table 5.4: Comparison between the simulated weights and the optimized weights

Observables		Simulated weights ($p_{simu.}$)	Optimized weights ($p_{opti.}$)	($p_{simu.} - p_{opti.}$)
distances	1 - 2	0.24768	0.24770	-0.00002
	1 - 3	0.24768	0.24770	-0.00002
	1 - 4	0.12409	0.12410	-0.00001
	2 - 3	0.12409	0.12410	-0.00002
	2 - 4	0.24768	0.24770	-0.00002
	3 - 4	0.24768	0.24770	-0.00002
	3 - 5	0.11058	0.11060	-0.00002
	3 - 6	0.07672	0.07672	0.00000
	4 - 5	0.07672	0.07672	0.00000
	4 - 6	0.11058	0.11060	-0.00002
	5 - 6	0.24768	0.24770	-0.00002
directions	station 1	1.00000	1.00012	-0.00012
	station 2	0.25000	0.24999	0.00001
	station 3	0.11111	0.11105	0.00006
	station 4	1.00000	1.00012	-0.00012
	station 5	0.25000	0.24999	0.00001
	station 6	0.11111	0.11105	0.00006
strainmeters	strain. 1	4.00000	3.99995	0.00005
	strain. 2	4.00000	3.99995	0.00005
	strain. 3	4.00000	3.99995	0.00005
	strain. 4	4.00000	3.99995	0.00005
	strain. 5	4.00000	3.99995	0.00005
	strain. 6	4.00000	3.99995	0.00005

Table 5.5: Comparison between the simulated standard deviations and the optimized values

Observables		Simulated standard deviations ($\sigma_{Simu.}$)	Optimized standard deviations ($\sigma_{Opti.}$)	($\sigma_{Simu.} - \sigma_{Opti.}$)
distances	1 - 2	2.01(mm)	2.01(mm)	0.00
	1 - 3	2.01(mm)	2.01(mm)	0.00
	1 - 4	2.84(mm)	2.84(mm)	0.00
	2 - 3	2.84(mm)	2.84(mm)	0.00
	2 - 4	2.01(mm)	2.01(mm)	0.00
	3 - 4	2.01(mm)	2.01(mm)	0.00
	3 - 5	3.01(mm)	3.01(mm)	0.00
	3 - 6	3.61(mm)	3.61(mm)	0.00
	4 - 5	3.61(mm)	3.61(mm)	0.00
	4 - 6	3.01(mm)	3.01(mm)	0.00
	5 - 6	2.01(mm)	2.01(mm)	0.00
directions	station 1	1.00"	1.00"	0.00
	station 2	2.00"	2.00"	0.00
	station 3	3.00"	3.00"	0.00
	station 4	1.00"	1.00"	0.00
	station 5	2.00"	2.00"	0.00
	station 6	3.00"	3.00"	0.00
strainmeters	strain. 1	5 ppm	5 ppm	0.00
	strain. 2	5 ppm	5 ppm	0.00
	strain. 3	5 ppm	5 ppm	0.00
	strain. 4	5 ppm	5 ppm	0.00
	strain. 5	5 ppm	5 ppm	0.00
	strain. 6	5 ppm	5 ppm	0.00

Table 5.6: Goodness of fitting of the internal reliability

Observables		The simulated redundancy numbers ($\underline{r}$)	The optimized redundancy numbers ($\underline{r}'$)	($\underline{r} - \underline{r}'$)
distances	1 - 2	1.0000	1.0000	0.0000
	1 - 3	0.1208	0.1208	0.0000
	1 - 4	0.4238	0.4238	0.0000
	2 - 3	0.4184	0.4184	0.0000
	2 - 4	0.1127	0.1127	0.0000
	3 - 4	0.1482	0.1482	0.0000
	3 - 5	0.1930	0.1930	0.0000
	3 - 6	0.3689	0.3689	0.0000
	4 - 5	0.3688	0.3688	0.0000
	4 - 6	0.1815	0.1815	0.0000
	5 - 6	0.03931	0.03931	0.0000
Correlated angles	1 - 2 - 4	0.9862	0.9862	0.0000
	1 - 2 - 3	0.9465	0.9465	0.0000
	2 - 4 - 3	0.9967	0.9967	0.0000
	2 - 4 - 1	0.9866	0.9866	0.0000
	3 - 1 - 2	1.0000	1.0000	0.0000
	3 - 1 - 4	0.9962	0.9962	0.0000
	3 - 1 - 6	0.9903	0.9903	0.0000
	3 - 1 - 5	0.9861	0.9861	0.0000
	4 - 6 - 5	1.0000	1.0000	0.0000
	4 - 6 - 3	0.9520	0.9520	0.0000
	4 - 6 - 2	0.8859	0.8859	0.0000
	4 - 6 - 1	0.9309	0.9309	0.0000
	5 - 3 - 4	0.9989	0.9989	0.0000
	5 - 3 - 6	0.9779	0.9779	0.0000
	6 - 5 - 3	0.9967	0.9967	0.0000
	6 - 5 - 4	0.9931	0.9931	0.0000
Sum		19.0000	19.0000	0.0000

Table 5.7: Goodness of fitting of external reliability

Observables		The simulated exter. relia. $(\bar{\lambda}_m)$	The optimized exter. relia. $(\bar{\lambda}'_m)$	$(\bar{\lambda}_m - \bar{\lambda}'_m)$
distances	1 - 2	5.1813	5.1812	0.0001
	1 - 3	7.5408	7.5411	-0.0003
	1 - 4	10.8922	10.8915	0.0006
	2 - 3	10.9207	10.921	-0.0004
	2 - 4	7.5373	7.5376	-0.0003
	3 - 4	5.1837	5.1836	0.0001
	3 - 5	6.335	6.3351	-0.0001
	3 - 6	11.4933	11.4934	-0.0001
	4 - 5	11.5367	11.5361	0.0006
	4 - 6	6.335	6.3353	-0.0003
	5 - 6	5.1765	5.1763	0.0002
Correlated angles	1 - 2 - 4	0.1041	0.1041	0.0000
	1 - 2 - 3	0.7052	0.7053	-0.0001
	2 - 4 - 3	0.026	0.026	0.0000
	2 - 4 - 1	0.1763	0.1763	0.0000
	3 - 1 - 2	0.00138	0.00138	0.0000
	3 - 1 - 4	0.0496	0.0496	0.0000
	3 - 1 - 6	0.1008	0.1008	0.0000
	3 - 1 - 5	0.1614	0.1614	0.0000
	4 - 6 - 5	0.0667	0.0667	0.0000
	4 - 6 - 3	0.5833	0.5833	0.0000
	4 - 6 - 2	1.4738	1.4741	-0.0003
	4 - 6 - 1	0.8062	0.8062	0.0000
	5 - 3 - 4	0.00345	0.00346	0.0000
	5 - 3 - 6	0.2336	0.2337	-0.0001
	6 - 5 - 3	0.02547	0.02547	0.0000
	6 - 5 - 4	0.0802	0.0801	0.0001

5.1.2 Simulation study No. 2: Influence of the position changes on the solution of observation weights

The second simulation study deals with a simulated three-dimensional geodetic networks established for detecting displacements of some of the netpoints. The main purpose of this simulation is to investigate the influence of position changes of netpoints on the solution of the observation scheme. This problem is quite often underestimated by other authors (Mepham et. al. 1982).

This three-dimensional network consists of the same horizontal geodetic network as described above plus the third dimension. The desired coordinates of the network are listed in Table 5.8.

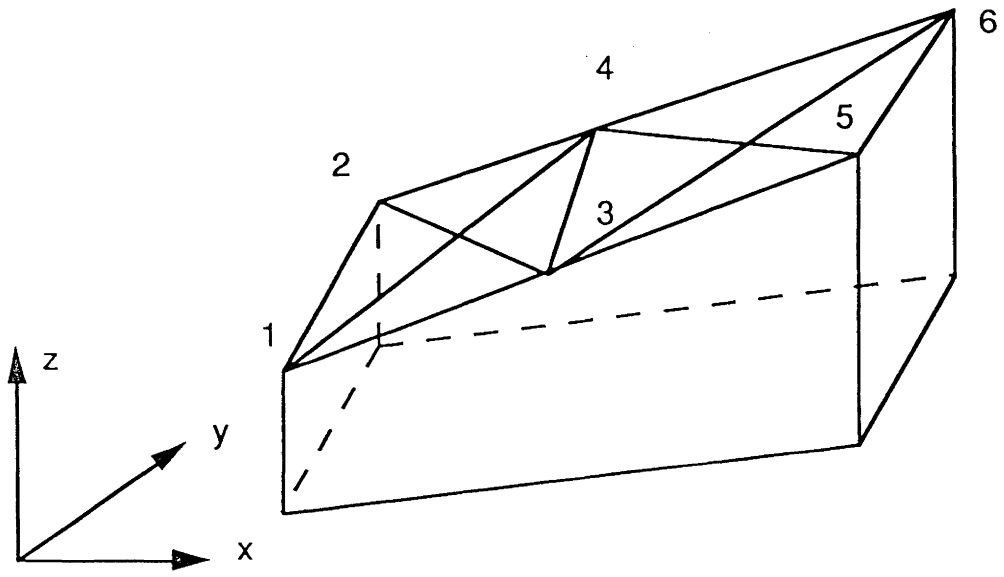

Fig. 5.2 The simulated 3-D geodetic network

The simulated observation scheme includes distances, azimuths, directions and vertical angles, the weights of which are shown in Table 5.11. For the simulation purpose,

the procedure adopted in the simulation study No. 1 is followed except that only the precision requirement is enforced. The covariance matrix of the coordinate variates as calculated from the simulated coordinates listed in Table 5.8 and the simulated weights of observations under the assumption that point 1 is fixed can be expressed as (unit: mm^2)

$$C_x = \begin{pmatrix}
51.18 & 0.00 & -0.34 & 1.46 & -49.07 & -0.21 & 49.70 & -49.04 & -0.22 & 4.15 & -117.84 & -0.28 & 46.98 & -117.83 & -0.18 \\
 & 3.44 & 0.00 & -0.28 & 1.86 & -0.01 & -0.29 & 1.70 & -0.01 & -0.27 & 1.79 & -0.01 & -0.28 & 1.79 & -0.01 \\
 & & 37.00 & -0.46 & -0.39 & 21.76 & -0.10 & -0.39 & 24.35 & -0.45 & -0.92 & 23.14 & -0.11 & -0.92 & 23.25 \\
 & & & 3.37 & 0.48 & -0.69 & 1.34 & 0.25 & -0.54 & 3.10 & 2.57 & -0.61 & 1.64 & 2.57 & -0.61 \\
 & & & & 56.89 & -0.24 & -49.58 & 55.18 & -0.25 & -1.93 & 128.38 & -0.18 & -46.88 & 128.39 & -0.28 \\
 & & & & & 31.52 & -0.30 & -0.25 & 24.95 & -0.58 & -0.57 & 28.00 & -0.38 & -0.57 & 27.74 \\
 & & & & & & 51.61 & -49.80 & -0.42 & 4.32 & -120.72 & -0.43 & 48.63 & -120.71 & -0.32 \\
 & & & & & & & 56.55 & -0.26 & -2.37 & 128.32 & -0.20 & -47.35 & 128.29 & -0.31 \\
 & & & & & & & & 32.22 & -0.61 & -0.62 & 28.84 & -0.38 & -0.62 & 29.14 \\
 & & & & & & & & & 8.82 & -0.29 & -1.00 & 4.67 & -0.61 & -0.89 \\
 & & & & & & & & & & 322.69 & -0.45 & -117.49 & 320.76 & -0.71 \\
 & & & & & & & & & & & 45.08 & -0.72 & -0.46 & 38.64 \\
 & & & & & & & & & & & & 51.71 & -117.83 & -0.71 \\
 & & & & & & & & & & & & & 322.57 & -0.72 \\
 & & & & & & & & & & & & & & 44.31
\end{pmatrix}$$

Symmetric

This matrix will be used as the criterion matrix for coordinate variates in the following optimization procedure.

For the simulation purpose, the input approximate coordinates of the netpoints are listed in Table 5.9, which are obtained by shifting all coordinate components of points 2 to 6 by 100 meters(point 1 is assumed as fixed). The input approximate weights of observables are all equal to unity, i.e. $p_i = 1$. Thus now, the parameters to be optimized are the positions of points 2 to 6 and weights of all observables. After using the proposed optimization procedure, a comparison between the simulated values and optimized values is shown in Table 5.10 and Table 5.11. The goodness of fitting of the criterion matrix is given by:

$$C_x - C_x^s = \begin{pmatrix} -4.94 & 0.30 & 0.11 & 0.51 & 6.25 & 0.17 & -5.36 & 6.14 & -0.21 & 0.43 & 13.07 & 0.26 & -4.55 & 12.47 & -0.13 \\ & -0.04 & -0.03 & -0.01 & -0.30 & -0.05 & 0.22 & -0.26 & -0.05 & -0.07 & -0.67 & -0.07 & 0.18 & -0.64 & -0.06 \\ & & 0.07 & 0.04 & -0.13 & 0.32 & 0.19 & -0.13 & 0.28 & 0.08 & -0.35 & 0.28 & 0.24 & -0.54 & 0.36 \\ & & & -0.08 & -0.58 & 0.02 & 0.51 & -0.56 & 0.14 & 0.00 & -1.37 & -0.01 & 0.51 & -1.34 & -0.02 \\ & & & & -7.71 & -0.20 & 6.71 & -7.53 & 0.20 & -0.45 & -16.38 & -0.31 & 5.76 & -15.74 & 0.09 \\ & & & & & 0.40 & 0.30 & -0.27 & 0.25 & 0.08 & -0.71 & 0.00 & 0.34 & -0.94 & 0.07 \\ & & & & & & -5.61 & 6.29 & -0.14 & 0.38 & 13.50 & 0.30 & -4.76 & 12.89 & -0.09 \\ & & & & & & & -7.28 & 0.19 & -0.39 & -15.84 & -0.29 & 5.58 & -15.17 & 0.10 \\ & & & & & & & & 0.32 & 0.02 & 0.28 & 0.00 & -0.03 & 0.08 & 0.04 \\ & & & & & & & & & 0.12 & -1.29 & -0.06 & 0.60 & -1.28 & -0.04 \\ & & & & & & & & & & -32.40 & -0.87 & 11.19 & -30.13 & 0.15 \\ & & & & & & & & & & & -0.43 & 0.46 & -1.48 & -0.46 \\ & & & & & & & & & & & & -3.69 & 10.27 & -0.04 \\ & & & & & & & & & & & & & -28.42 & -0.11 \\ & & & & & & & & & & & & & & -0.52 \end{pmatrix} \cdot 10^{-13}$$

Symmetric

From the above results we can see that the fitting of the criterion matrix is perfect, i.e. the covariance matrix of the optimized network is in fact the same as that of the simulated network. From Table 5.11, all the optimized weights of observables are practically the same as the simulated weights. Table 5.10 shows, however, that except in the z-direction, some of the position shifts in x- and y-direction are not correctly back shifted. It means that the precision of the network is mostly affected by changes in positions in the z-direction while position changes of certain values in some of the positive x- and/or y-direction of some points do not contribute to the improvement of the precision. This finding obviously reflects the importance of optimization of the geometric configuration of a network. However, some authors may advocate that small changes in relative positions of stations have no significant effect on the accuracy and thus there is no need for the optimization of the configuration. This point of view is not pertinent. For instance, if in this example we neglect all the simulated position shifts in x-, y-, and z-directions and if we would apply the Second Order Design instead of the simultaneous First Order and Second Order Designs, then a comparison between the simulated weights and the optimized weights is shown in

Table 5.12. The results show that by neglecting changes of positions we would obtain an observation scheme which would be much different from the simulated observation scheme, and the fitting of the criterion matrix would be very bad, as reflected by the following matrix (the variance-covariance matrix of coordinates minus the criterion matrix):

$$C_x - C_x^* = \begin{pmatrix} 12.30 & -2.93 & -0.75 & 3.25 & -12.06 & -0.91 & 12.01 & -12.02 & -0.73 & 3.90 & -24.30 & -0.83 & 11.36 & -24.30 & -0.80 \\ & 0.23 & -1.72 & -0.22 & 2.92 & -1.07 & -2.88 & 2.91 & -1.14 & -0.38 & 6.69 & -1.10 & -2.72 & 6.69 & -1.11 \\ & & 6.77 & -1.12 & -1.32 & 6.55 & -0.61 & -1.25 & 4.42 & -1.04 & -1.85 & 5.39 & -0.68 & -1.85 & 5.31 \\ & & & 0.27 & -3.50 & -1.57 & 3.25 & -3.44 & -1.19 & 0.42 & -7.77 & -1.39 & 3.09 & -7.77 & -1.37 \\ & & & & 11.46 & -1.06 & -11.72 & 11.45 & -1.10 & -4.14 & 22.85 & -1.07 & -11.07 & 22.86 & -1.09 \\ & & & & & 6.79 & -0.81 & -0.99 & 4.37 & -1.46 & -1.85 & 5.83 & -0.93 & -1.85 & 5.73 \\ & & & & & & 11.73 & -11.68 & -0.65 & 3.89 & -23.55 & -0.74 & 11.09 & -23.55 & -0.71 \\ & & & & & & & 11.44 & -1.07 & -4.08 & 22.86 & -1.02 & -11.04 & 22.87 & -1.05 \\ & & & & & & & & 2.24 & -1.11 & -1.71 & 3.04 & -0.71 & -1.71 & 2.96 \\ & & & & & & & & & 0.65 & -9.02 & -1.31 & 3.68 & -9.03 & -1.28 \\ & & & & & & & & & & 44.93 & -1.74 & -22.29 & 44.94 & -1.78 \\ & & & \text{Symmetric} & & & & & & & & 5.03 & -0.85 & -1.74 & 4.76 \\ & & & & & & & & & & & & 10.52 & -22.29 & -0.82 \\ & & & & & & & & & & & & & 44.96 & -1.78 \\ & & & & & & & & & & & & & & 4.52 \end{pmatrix}$$

Here we can see that the maximum difference between the elements of the criterion matrix and that of the covariance matrix calculated from the optimization results reaches to 44.96 mm^2. Therefore, this example shows that the influence of even 4% changes in relative positions of stations on the improvement of network precision can be very significant. Generally, different configurations may give quite different optimal solutions for weights of observations. In this example, the constraints of Eq.(4-30) are not applied. A good fitting of the criterion matrix of coordinate variates can be achieved only by significantly increasing the weights of observations if changes of the configuration are neglected. This point of view will be further elaborated in the following optimization example.

Table 5.8: Desired coordinates for the three-D. geodetic network

Point	x (m)	y (m)	z(m)
1	500.00	600.00	550.00
2	500.00	2600.00	650.00
3	2500.00	600.00	700.00
4	2500.00	2600.00	700.00
5	5500.00	600.00	800.00
6	5500.00	2600.00	800.00

Table 5.9: The input approximate coordinates for the three-D. geodetic network

Point	x (m)	y (m)	z(m)
1	500.00	600.00	550.00
2	600.00	2700.00	750.00
3	2600.00	700.00	800.00
4	2600.00	2700.00	800.00
5	5600.00	700.00	900.00
6	5600.00	2700.00	900.00

Table 5.10: Comparison between the simulated position shifts and the optimized position corrections

Point		Simulated position shifts $\Delta_{Simu.}$ [m]	Optimized position corrections $-\Delta_{Opti.}$ [m]	$\Delta_{Simu.}-(-\Delta_{Opti.})$ [m]
2	Δx	100.00	100.00	0.00
	Δy	100.00	60.00	40.00
	Δz	100.00	100.00	0.00
3	Δx	100.00	60.00	40.00
	Δy	100.00	100.00	0.00
	Δz	100.00	99.01	0.99
4	Δx	100.00	60.27	39.73
	Δy	100.00	60.27	39.73
	Δz	100.00	99.01	0.99
5	Δx	100.00	0.68	99.32
	Δy	100.00	100.00	0.00
	Δz	100.00	97.02	2.98
6	Δx	100.00	0.68	99.32
	Δy	100.00	60.27	39.73
	Δz	100.00	97.02	2,98

Table 5.11: Comparison between the simulated weights and the optimized weights

Observables		Simulated weights ($p_{Simu.}$)	Optimized weights ($p_{Opti.}$)	($p_{Simu.} - p_{Opti.}$)
distances	1 - 2	0.24768	0.24768	0.00000
	1 - 3	0.24768	0.24768	0.00000
	1 - 4	0.12409	0.12409	0.00000
	2 - 3	0.12409	0.12409	0.00000
	2 - 4	0.24768	0.24768	0.00000
	3 - 4	0.24768	0.24768	0.00000
	3 - 5	0.11058	0.11058	0.00000
	3 - 6	0.07672	0.07672	0.00000
	4 - 5	0.07672	0.07672	0.00000
	4 - 6	0.11058	0.11058	0.00000
	5 - 6	0.24768	0.24768	0.00000
azimuth	1 - 2	1.00000	1.04012	0.04012
	5 - 6	1.00000	1.04012	0.04012
vertical angles	1 - 2	1.00000	1.04012	-0.04012
	1 - 3	1.50000	1.56019	-0.06019
	1 - 4	2.00000	2.08025	-0.08025
	2 - 3	2.50000	2.60031	-0.10031
	2 - 4	3.00000	3.12037	-0.12037
	3 - 4	3.50000	3.64043	-0.14043
	3 - 5	4.00000	4.1605	-0.1605
	3 - 6	4.50000	4.68056	-0.18056
	4 - 5	5.00000	5.20062	-0.20062
	4 - 6	5.50000	5.72068	-0.22068
	5 - 6	6.00000	6.24075	-0.24075
directions	station 1	1.00000	1.04012	-0.04012
	station 2	1.50000	1.56019	-0.06019
	station 3	2.00000	2.08025	-0.08025
	station 4	2.50000	2.60031	-0.10031
	station 5	3.00000	3.12037	-0.12037
	station 6	3.50000	3.64043	-0.14043

Table 5.12: Comparison between the simulated weights and the optimized weights (SOD only)

Observables		Simulated weights ($p_{Simu.}$)	Optimized weights ($p_{Opti.}$)	($p_{Simu.}$ - $p_{Opti.}$)
distances	1 - 2	0.24768	0.24073	0.00695
	1 - 3	0.24768	0.25406	-0.00638
	1 - 4	0.12409	0.13198	-0.00789
	2 - 3	0.12409	0.12392	0.00017
	2 - 4	0.24768	0.24628	0.00140
	3 - 4	0.24768	0.24903	-0.00135
	3 - 5	0.11058	0.10535	0.00523
	3 - 6	0.07672	0.07455	0.00217
	4 - 5	0.07672	0.07659	0.00013
	4 - 6	0.11058	0.10869	0.00189
	5 - 6	0.24768	0.24866	-0.00098
azimuth	1 - 2	1.00000	0.64888	0.35112
	5 - 6	1.00000	1.46813	-0.46813
vertical angles	1 - 2	1.00000	0.46274	0.53726
	1 - 3	1.50000	0.02951	1.47049
	1 - 4	2.00000	4.50279	-2.50279
	2 - 3	2.50000	3.61813	-1.11813
	2 - 4	3.00000	2.90386	0.09614
	3 - 4	3.50000	3.28663	0.21337
	3 - 5	4.00000	4.88663	-0.88663
	3 - 6	4.50000	7.15309	-2.65309
	4 - 5	5.00000	2.61994	2.38006
	4 - 6	5.50000	5.08790	0.41210
	5 - 6	6.00000	5.64983	0.35017
directions	station 1	1.00000	0.69075	0.30925
	station 2	1.50000	2.34522	-0.84522
	station 3	2.00000	1.92279	0.07721
	station 4	2.50000	2.18637	0.31363
	station 5	3.00000	2.82130	0.17870
	station 6	3.50000	3.55782	-0.05782

5.2 Optimization Examples

In the previous sections, the simulated study No. 1 show the validity of the developed mathematical models. Study No. 2 illustrated the significance of applying position changes of netpoints to the optimal solution of observation weights. The latter point is further elaborated in the following optimization example No. 1, where the practical significance of the newly developed methodology is illuminated by approaching a practical design problem. Through a comparison of the results obtained by different methods, the advantages of the new approach over the others are easily identified.

The application of the developed optimization algorithm goes to a practical geodetic network established to assist in deformation monitoring and analysis of the structures of a hydro-power generation station in Mactaquac, N.B., Canada.

5.2.1 Example No. 1: Optimal design of a monitoring network - Comparison of different approaches.

Suppose we are given a monitoring network to be optimally designed as shown by Fig. 5.3. The approximate coordinates of the netpoints are listed in Table 5.13. The minimum and maximum side lengths of the network are approximately 1.7 km and 5.9 km, respectively. Assume that the deformation model to be detected includes a homogeneous strain field over the whole area plus single point movements of points 3, 4 and 5. That is, the vector of deformation parameters to be detected can be expressed as:

$$\underline{e} = (dx_3 \;\; dy_3 \;\; dx_4 \;\; dy_4 \;\; dx_5 \;\; dy_5 \;\; \varepsilon_x \;\; \varepsilon_{xy} \;\; \varepsilon_y)^T$$

where dx_i, dy_i (i=3, 4, 5) represent the displacements of points 3, 4 and 5 in x- and y-directions respectively, and ε_x, ε_y and ε_{xy} the normal strain and shear strain parameters respectively. The deformation model can be expressed as:

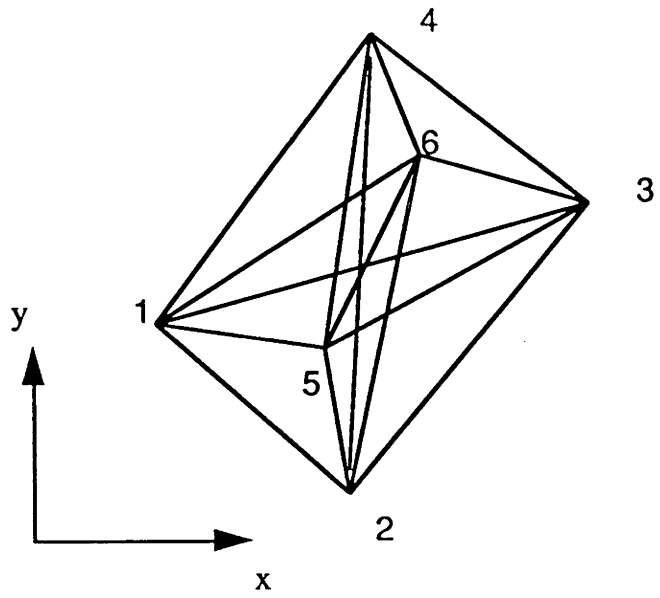

Fig. 5.3: The monitoring network

Table 5.13: The approximate coordinates of the monitoring network

Point	x(m)	y(m)
1	1125.00	1625.00
2	4625.00	375.00
3	6250.00	4625.00
4	3250.00	5875.00
5	3375.00	1500.00
6	4375.00	4625.00

$$u_i = \varepsilon_x x_i + \varepsilon_{xy} y_i$$
$$v_i = \varepsilon_{xy} x_i + \varepsilon_y y_i \qquad \text{for i=1, 2, 6; and}$$

$$u_j = dx_j + \varepsilon_x x_j + \varepsilon_{xy} y_j$$
$$v_j = dy_j + \varepsilon_{xy} x_j + \varepsilon_y y_j \qquad \text{for j=3, 4, 5.}$$

To simplify the computation, only the precision criterion is considered here. A diagonal matrix:

$$C_{\underline{e}} = 2 \cdot \text{Diag}\{(0.5\text{ mm})^2 \cdots (0.5\text{ mm})^2 \ (0.1\text{ppm})^2 \cdots (0.1\text{ppm})^2\}$$

is adopted as the criterion matrix of the deformation parameters for the optimization. That is, the standard deviations to be achieved for the detection of the displacements are respectively 0.71 mm, and those for strain parameters are 0.14 ppm.

To achieve the above set precision criteria, the optimal configuration of the network and optimal weights of observations have to be solved for. Assume that all the distances will be measured with EDM instruments with achievable accuracies ranging from $\sigma_s^2 = (1\text{mm})^2 + (1\text{ ppm s})^2$ to $\sigma_s^2 = (0.1\text{ ppm s})^2$, and assume that the positions of points 1 and 2 cannot be changed, while the possible changes of positions for points 3, 4, 5 and 6 range from -200 m to +200 m in both x- and y-directions respectively. The input approximate weights for distances are calculated from the initial accuracy of instruments i.e. $\sigma_s^2 = (1\text{mm})^2 + (1\text{ ppm s})^2$ and are listed in Table 5.15. Therefore, the unknown parameters to be optimally solved for include the position changes to be introduced to points 3, 4, 5, and 6, and changes to the weights of all the distances.

To approach the above optimal design problem, the following different possible methods may be applied:

Method I -- Analytical Second Order Design (SOD) only;

Method II -- The conventional approach: To perform the First Order Design (FOD) by trial and error first, and then apply the analytical Second Order Design;

Method III -- The modified conventional approach: To perform analytical First Order Design first, and then do analytical Second Order Design;

Method IV -- The newly developed approach: To perform the simultaneous First Order and Second Order Design analytically.

At first, Method I is used since one used to underestimate the significance of relatively small position changes of netpoints for the optimal design problem. Therefore, for most of the real world design problems of engineering or monitoring networks, only SOD is carried out. Method II is hard to follow. Since no fully analytical FOD algorithm exists, one used to optimize the configuration of a network by trial and error, and, therefore, to some extent subjective. The decision about how to change the configuration is made through one's intuitive evaluation of the geometry of the network or by comparison of the resulting variance-covariances matrix of the unknown parameters interactively with a computer. Nevertheless, the exact numerical values of the position changes to be introduced for the netpoints are very difficult to decide. The above difficulties may be the main reason why the FOD did not deserve proper attention before. Although Koch (1982) tried to develop an analytical algorithm for FOD, the derivatives needed in his mathematical modelling are also provided by numerical method. In this case, the obtained derivatives will be influenced by the selection of the step lengths needed. Therefore, instead of the conventional approach, the modified conventional approach i.e., Method III is applied for this example. Here, the developed fully analytical modelling in this research is used for the FOD with the weights of observations fixed as the input approximate values, then the desired weights of observations are solved for by fixing the resulting configuration of the network. Finally, Method IV is the simultaneous FOD and SOD as developed in this

research. Here the all the unknown position changes of netpoints and observation weights are optimally solved simultaneously.

The results of applying Method I, Method III, and Method IV to the same design problem are listed in Tables 5.14 to 5.16.

Table 5.14 shows that all the three methods have satisfied the set precision criteria. However, from Table 5.15, we can see that to achieve the same objective, Method IV requires the minimum weights. The total weight required from Method I is 15% larger than that from Method III, and is 25% larger than that from Method IV. By identifying weights as an indicator of cost, the observation scheme as obtained from Method IV requires the minimum cost. This shows once again that introducing even relatively small position changes of netpoints has significance for the optimal design of a network. They should not be neglected. Table 5.16 shows the possible position shifts of netpoints and the actual shifts from method III and method IV.

There is a serious problem with the modified conventional approach i.e., the separate analytical FOD and SOD. Since at the FOD stage the weights of observations have to be fixed, the solution for position changes depends on the initially adopted approximate values of the weights. This leads to different solution for the desired weights of observations at the SOD stage for different approximate values of the weights adopted at the FOD stage. This problem is overcome when the simultaneous FOD and SOD is performed. Whatever initial approximate weights are adopted, they do not influence the obtained results. This is illustrated by the numerical example. Tables 5.17 to 5.19 show the results obtained by the separate FOD and SOD approach, and the Simultaneous FOD and SOD approach when the maximum achievable weights are used as the approximate weights. From Table 5.17, Table 5.18, and Table 5.19, one can see that to achieve the same objective, after changing the input approximate weights of observations, method IV gives practically the same results as those obtained before. However, the results from

method III are quite different from those obtained before. The total weight amounts to 99.35, which is nearly twice as much as that given by method IV. Obviously this method is not recommendable.

From the above discussion and numerical example, the following conclusions are easily drawn:

1) If both the configuration of a network and weights of observations are to be optimized, it is not recommended to perform SOD alone. By introducing position changes of netpoints, a stronger geometry of the network can be established. That will lead to saving of effort in the field observation campaign,.i.e., saving of money. In this example, if only 200 metres position changes are considered, which amount to a 5% of the relative positions of the network (the average side length of the network is 3.8 km), then such relatively small changes in configuration have reduced the total weight by 25% as compared to the case when these position changes are not introduced (i.e. SOD only). Furthermore, if we allow the maximum possible coordinate changes for each point up to $\pm$ 400 meters, i.e. around 10% of the relative positions of the network, then one can see from Table 5.20 that the required total weight as obtained by the approach of simultaneous FOD and SOD is 47.43, which is only 68% of that obtained by SOD only.
2) Because of the awkward procedure of the trial and error method, the conventional separate FOD and SOD approach should be avoided.
3) By applying the fully analytical separate FOD and SOD approach, it is found that the results of this approach are dependent on the input approximate weights of observations. Different input approximate weights adopted at the FOD stage will lead to different position changes to be introduced for the netpoints. Therefore, they lead to different desired weights solved at the SOD stage. Theoretically, this approach should lead to stable results only by performing a

number of FODs and SODs iteratively. However, this cumbersome procedure can now be avoided by the newly developed simultaneous FOD and SOD approach, the results of which are independent of the selection of the input approximate weights.

4) From the above discussion, one may conclude that the approach of simultaneous FOD and SOD is the optimal approach to the optimal design problem. Although this approach is induced by the concept of "introducing relatively small position changes", large changes can actually be accommodated by increasing the number of iterations during the solution procedure.

Table 5.14: Goodness of fitting of the precision criteria

Parameters	Required precision σ(Re.)	σ(I)	σ(III)	σ(IV)
dx_3	0.71 mm	0.52 mm	0.55 mm	0.58 mm
dy_3	0.71 mm	0.71 mm	0.71 mm	0.71 mm
dx_4	0.71 mm	0.66 mm	0.64 mm	0.71 mm
dy_4	0.71 mm	0.59 mm	0.64 mm	0.66 mm
dx_5	0.71 mm	0.68 mm	0.68 mm	0.68 mm
dy_5	0.71 mm	0.65 mm	0.65 mm	0.65 mm
ε_x	0.14 ppm	0.14 ppm	0.14 ppm	0.14 ppm
ε_{xy}	0.14 ppm	0.10 ppm	0.10 ppm	0.10 ppm
ε_y	0.14ppm	0.11 ppm	0.11ppm	0.11 ppm

where σ(Re.) - the required precision;

σ(I), σ(III), and σ(IV) - the obtained precisions by method I, III, and IV respectively.

Table 5.15: The desired weights of the observations obtained by different approaches

Distances	p(Approx.)	p(I)	p(III)	p(IV)	p(Max.)
1 - 2	0.06751	5.01637	4.84161	4.80926	7.23982
1 - 3	0.02757	2.83562	2.83562	2.83562	2.83562
1 - 4	0.04241	4.42907	4.01781	4.35522	4.42907
1 - 5	0.16452	2.46406	2.20997	1.97162	19.69231
1 - 6	0.04863	5.11182	5.11182	5.11182	5.11182
2 - 3	0.04608	4.83019	4.83019	4.83019	4.83019
2 - 4	0.03017	3.11133	3.11133	3.11133	3.11133
2 - 5	0.26122	1.70221	1.84226	1.84898	35.35911
2 - 6	0.05229	5.51724	5.51724	5.51724	5.51724
3 - 4	0.08649	9.46745	7.46287	3.33546	9.46745
3 - 5	0.05255	3.13335	3.02009	2.55709	5.54593
3 - 6	0.22145	7.63964	4.5133	4.11555	28.44444
4 - 5	0.04961	1.21321	1.02701	1.43396	5.22023
4 - 6	0.26122	10.94819	6.96372	4.13256	35.35911
5 - 6	0.08499	2.50626	2.79949	2.93194	9.28882
Sum	1.49671	69.926	60.10433	52.89784	181.45249

where p(Approx.) - the input approximate weights;

p(Max) - the maximum achievable weights of observations;

p(I), p(III), and p(IV) - the solved weights by method I, III, and IV respectively.

Table 5.16: Possible coordinate shifts of netpoints and actual coordinate shifts resulting from the optimization

Point		3	4	5	6
Possible shifts	Δx(m)	±200.00	±200.00	±200.00	±200.00
	Δy(m)	±200.00	±200.00	±200.00	±200.00
Actual shifts	Δx(III) (m)	-200.00	200.00	-80.80	200.00
	Δy(III) (m)	200.00	-171.55	-200.00	200.00
	Δx(IV) (m)	-200.00	-200.00	-161.94	200.00
	Δy(IV) (m)	-200.00	-200.00	-200.00	200.00

where Δx, Δy - the possible coordinate shifts;

Δx(III), Δy(III) and Δx(IV), Δy(IV) are the actual shifts from method III and IV respectively.

Table 5.17: Goodness of fitting of the precision criteria with the input approximate weights of observations changed

Parameters	Required precision σ(Re.)	σ(III)	σ(IV)
dx_3	0.71 mm	0.48 mm	0.59 mm
dy_3	0.71 mm	0.72 mm	0.71 mm
dx_4	0.71 mm	0.69 mm	0.71 mm
dy_4	0.71 mm	0.54 mm	0.66 mm
dx_5	0.71 mm	0.68 mm	0.69 mm
dy_5	0.71 mm	0.65 mm	0.65 mm
ε_x	0.14 ppm	0.14 ppm	0.14 ppm
ε_{xy}	0.14 ppm	0.11 ppm	0.10 ppm
ε_y	0.14 ppm	0.14 ppm	0.11 ppm

where σ(Re.) - The required precision;

σ(III) and σ(IV) - The obtained precisions by method III and IV respectively.

Table 5.18: The desired weights of observations with the input approximate weights of observations changed

Distances	p(Approx.)	p(III)	p(IV)	p(Max.)
1 - 2	7.23982	5.35699	4.80878	7.23982
1 - 3	2.83562	2.83562	2.83562	2.83562
1 - 4	4.42907	4.42907	4.33216	4.42907
1 - 5	19.69231	3.72786	1.92265	19.69231
1 - 6	5.11182	5.11182	5.11182	5.11182
2 - 3	4.83019	4.83019	4.83019	4.83019
2 - 4	3.11133	0.00000	3.11133	3.11133
2 - 5	35.35911	0.80252	1.88681	35.35911
2 - 6	5.51724	3.59146	5.51724	5.51724
3 - 4	9.46745	9.46745	3.37011	9.46745
3 - 5	5.54593	2.61856	2.5296	5.54593
3 - 6	28.44444	17.07175	4.17594	28.44444
4 - 5	5.22023	1.54274	1.44492	5.22023
4 - 6	35.35911	35.35911	4.1002	35.35911
5 - 6	9.28882	2.6063	2.9281	9.28882
Sum	181.45249	99.35145	52.90548	181.45249

where p(Approx.) - the input approximate weights;
p(Max) - the maximum achievable weights of observations;
p(III), and p(IV) - the solved weights by method III and IV respectively.

Table 5.19: Possible coordinate shifts and actual coordinate shifts with the input approximate weights of observations changed

Point		3	4	5	6
Possible shifts	Δx(m)	±200	±200	±200	±200
	Δy(m)	±200	±200	±200	±200
Actual shifts	Δx(III) (m)	200.00	-200.00	200.00	-200.00
	Δy(III) (m)	200.00	200.00	-200.00	-200.00
	Δx(IV) (m)	-200.00	-200.00	-172.49	200.00
	Δy(IV) (m)	-200.00	-200.00	-200.00	200.00

where Δx, Δy - the possible coordinate shifts;

Δx(III), Δy(III) and Δx(IV), Δy(IV) are the actual shifts from method III and IV respectively.

Table 5.20: Results of Method IV when the maximum possible position shifts can be up to ± 400 meters

The obtained precision		Position shifts		The optimized weights	
Parameters	σ(IV)	point	Δx (IV) Δy (IV)	distances	p(IV)
dx_3	0.69 mm			1 - 2	7.23982
			-400.00	1 - 3	2.83562
dy_3	0.71 mm	3		1 - 4	3.41254
			-400.00	1 - 5	1.88894
dx_4	0.71 mm			1 - 6	5.11182
		4	-400.00	2 - 3	2.90403
dy_4	0.69 mm			2 - 4	3.11133
			-400.00	2 - 5	1.90803
dx_5	0.69 mm			2 - 6	5.51724
		5	-394.80	3 - 4	0.42232
dy_5	0.71 mm			3 - 5	1.25491
			-400.00	3 - 6	3.42277
ε_x	0.13 ppm			4 - 5	1.45609
			400.00	4 - 6	3.9237
ε_{xy}	0.10 ppm	6		5 - 6	3.01779
ε_y	0.11 ppm		400.00	Sum	47.42695

where σ(IV)- the obtained precision by method IV;

Δx(IV), Δy(IV) - the actual position shifts from method IV; and

p(IV) - the obtained weights by method IV

5.2.2 Example No. 2: Optimization of the Mactaquac Monitoring Network

The Mactaquac generating station was constructed on the St.John River near Fredericton, New Brunswick, between 1964 and 1968. It is comprised of a rock-fill dam, a sluice-way, a 42 m high concrete gravity dam(intake and spillway), and a power house with six power generating units connected to the intake structure via six penstocks of steel plate encased in concrete. An illustration of the layout of the Mactaquac generating station is shown in Fig. 5.4. A few years ago, some irregular deformations of the intake and power house were noticed in addition to predicted seasonal expansions and contractions of the structure. In order to find the source of the deformations a geodetic horizontal monitoring network was established as part of the deformation monitoring scheme. It is a trilateration network (Fig. 5.5) of 27 stations measured annually since 1983 using precision electronic distance meters, i.e. Kern Mekometer ME3000 and Tellurometer MA200. Five object points of the network, PR-1, PR-2, PR-3, TR-1, and TR-2, are located on the power house, two, TK-1, and TK-7, on the top deck of the intake, and one point, M-1, is located on the top of the spillway. The approximate coordinates of both the reference and object points are listed in Table 5.21. The UNB Generalized Method was applied to determine absolute displacements of the geodetic stations from various combinations of pairs of survey campaigns. A report by Chrzanowski et al.(1988) has indicated systematic downstream movement of points TR-1 and TR-2 at the average rate of 3 mm/year, while the roof points PR-1, PR-2, and PR-3 at a slower rate of about 2 mm/year over the period between 1985 and 1987. Most of the other stations exhibit significant, though random, movements. This information has played an important role in the overall trend analysis of the deformations.

In the 1989 campaign, an inverted pendulum, called INVP-B, near the object point TR-1 on the power house was also included in the geodetic network. Thus the main geodetic network now has 28 stations. The problem with the original observation scheme

is that accuracies of displacement detection for object points TR-1, TR-2, INVP-B and some reference points such as C-200, C-555, etc. are not satisfactory to detect displacements in the order of 2 - 3 mm/year. Also too many redundancies of distance observations have been made. The author's approach has been applied to optimize the observation scheme to enable the detection of movements of 2 mm/year of object points with the available instrumentations and with the minimum effort. The configuration of the monitoring network was not supposed to change.

Using the above developed optimization algorithm, and a diagonal matrix with all diagonal elements being 0.36 mm^2 (i.e., the standard deviations of all coordinates components being 0.6 mm and the semimajor axises of the 95% point error ellipses being 2 mm) adopted as the criterion matrix, the following procedure was followed.

a) The datum of the monitoring network was selected as:

Fixed point C-400

Fixed Azimuth C-400--> I-3

b) The optimization started with the original trilateration scheme (use EDM ME3000, input accuracy $\sigma_s^2 = (0.3\ mm)^2 + (2\ ppm \cdot s\ (mm))^2$) which consisted of measuring 176 distances. These distances are shown in Fig. 5.5. The station 95% error ellipses for this observation scheme are shown in Fig. 5.10.

c) After applying the optimization algorithm, 31 distances could be deleted because they obtained zero values for their weights, i.e., they did not contribute to the improvement of the network accuracy. The deleted distances are graphically shown in Fig. 5.6. The station 95% error ellipses after deleting these distances are shown in Fig. 5.11. From the comparison between Table 5.22 and Table 5.23, one can see that deleting these 31 distances causes the semi-major axis of station 95% error ellipse of reference point C-200 to increase only by 0.3 mm, and the semi-major axes of station 95% error ellipses of all the object points are effected by 0.1 mm at the maximum. This result shows that measuring the

31 distances could not add to the improvement of accuracies of displacement detection for both the reference and object points. The optimized trilateration observation scheme is shown in Fig. 5.7. The weights for all these distances are adopted as the ones achievable using EDM ME3000.

d) From the original and optimized trilateration scheme the accuracies of points C-100, C-200, C-555, REF-201, REF-202, TR-1, TR-2, INVP-B have not been satisfactory, with the maximum semi-major axes of station 95% error ellipses being 6.1 mm and 5.4 mm for reference and object points, respectively. Thus roughly, in the worst case, displacements less than 7.6 mm and 8.5 mm for object and reference points respectively could not be detected. To improve the accuracies, the optimization algorithm was applied again to optimize the monitoring network by adding to it direction measurements.

Assuming the use of Kern E-2 electronic theodolite with minimum achievable standard deviation of direction observation being 0.7", we have input all 28 possible stations with total 352 directions as shown in Fig. 5.8. The optimization procedure has shown that only 14 stations with a total of 88 directions would be good for the improvement of the accuracies of the above mentioned points. The standard deviations of direction observations given by the optimization algorithm for these stations are all 0.7". These directions are graphically represented in Fig. 5.9. The station 95% error ellipses after adding these directions are shown in Fig. 5.12. From Table 5.24, one can see that the maximum semi-major axes corresponding to this optimized triangulateration observation scheme are 2.9 mm and 2.2 mm (at 95%) for reference and object points, respectively. Displacements larger than 4.1 mm and 3.1 mm for reference and object points respectively would be detectable.

e) The optimization algorithm has also been used assuming that one's aim would be to improve only the accuracies of object points TR-1, TR-2 and INVP-B. The optimization procedure has indicated that only 5 stations with a total of 36 directions have to be

measured. These directions are graphically shown in Fig. 5.13. Table 5.25 shows that the maximum semi-major axis of object points corresponding to this observation scheme is also 2.2 mm (at 95%). Thus Displacements larger than 3.1 mm are possible to be detected. Fig. 5.14. shows a picture of station 95% error ellipses corresponding to this observation scheme.

Table 5.21: The approximate coordinates of the Mactaquac monitoring network

Stations	x(m)	y(m)
C-400	864.511	576.27
I-3	1009.078	1180.25
REF-200	1152.051	1068.975
REF-100	738.534	1069.058
C-600	481.324	576.131
C-500	739.652	591.885
C-200	1584.227	1431.552
C-301	951.443	576.76
C-100	1143.233	558.283
REF-201	1273.949	1138.799
REF-202	1204.666	1200.884
C-555	958.596	1433.629
S-252A	360.764	813.886
S-251A	388.256	900.093
S-250A	432.046	979.683
DS-1	532.996	1078.807
DS-2	604.105	1138.284
I-1	887.909	1282.613
I-2	941.425	1231.489
PR-1	1043.291	986.334
PR-2	967.139	986.386
PR-3	903.871	986.275
M-1	816.993	1069.183
TK-7	914.345	1066.457
TK-1	1061.602	1066.493
TR-1	1055.058	981.482
TR-2	923.8	981.494
INVP-B	1031.826	982.221

Table 5.22: Station 95.000 % confidence ellipses for the original observation scheme (Factor used for obtaining these ellipses from standard error ellipses =2.4484)

Stations	Semi-major axis (m)	Semi-minor axis (m)	Azimuth of Semi-major axis (o ' ")
I-3	0.0010	0.0001	13 27 39
REF-200	0.0015	0.0008	340 58 37
REF-100	0.0012	0.0008	1 27 40
C-600	0.0014	0.0009	335 46 17
C-500	0.0010	0.0008	354 30 19
C-200	0.0058	0.0016	326 59 42
C-301	0.0009	0.0008	1 46 58
C-100	0.0033	0.0011	69 36 12
REF-201	0.0031	0.0012	324 24 19
REF-202	0.0031	0.0014	294 47 55
C-555	0.0042	0.0013	303 25 29
S-252A	0.0016	0.0012	349 34 11
S-251A	0.0016	0.0011	351 5 57
S-250A	0.0016	0.0011	346 11 3
DS-1	0.0014	0.0010	354 5 16
DS-2	0.0013	0.0010	9 39 33
I-1	0.0012	0.0010	56 47 6
I-2	0.0011	0.0008	33 9 49
PR-1	0.0011	0.0009	67 22 58
PR-2	0.0010	0.0010	284 20 9
PR-3	0.0011	0.0011	296 16 25
M-1	0.0012	0.0008	3 47 29
TK-7	0.0010	0.0008	10 18 2
TK-1	0.0009	0.0008	21 20 32
TR-1	0.0034	0.0013	290 57 28
TR-2	0.0030	0.0013	277 25 14
INVP-B	0.0053	0.0014	295 50 55

Total area of station ellipses = 0.19571D-03

Table 5.23: Station 95.000 % confidence ellipses after deleting 31 distances (Factor used for obtaining these ellipses from standard error ellipses =2.4484)

Stations	Semi-major axis (m)	Semi-minor axis (m)	Azimuth of Semi-major axis (° ′ ″)
I-3	0.0011	0.0001	13 27 39
REF-200	0.0017	0.0009	340 47 19
REF-100	0.0012	0.0009	358 50 38
C-600	0.0016	0.0013	322 39 03
C-500	0.0011	0.0008	357 16 49
C-200	0.0061	0.0018	329 11 28
C-301	0.0010	0.0008	4 15 31
C-100	0.0035	0.0012	69 44 41
REF-201	0.0032	0.0012	324 21 18
REF-202	0.0032	0.0015	295 17 48
C-555	0.0043	0.0014	303 40 36
S-252A	0.0017	0.0012	349 13 35
S-251A	0.0016	0.0012	351 38 14
S-250A	0.0016	0.0011	347 31 52
DS-1	0.0015	0.0011	336 49 40
DS-2	0.0013	0.0011	10 12 46
I-1	0.0013	0.0011	52 9 20
I-2	0.0012	0.0008	24 7 52
PR-1	0.0012	0.0010	71 18 34
PR-2	0.0011	0.0010	283 30 14
PR-3	0.0012	0.0011	293 4 49
M-1	0.0013	0.0010	16 8 13
TK-7	0.0011	0.0009	18 28 23
TK-1	0.0010	0.0009	19 33 34
TR-1	0.0035	0.0013	291 18 26
TR-2	0.0031	0.0013	277 37 57
INVP-B	0.0054	0.0014	295 49 51

Total area of station ellipses = 0.21965D-03

Table 5.24: Station 95 % confidence ellipses for the optimized observation scheme (Factor used for obtaining these ellipses from standard error ellipses =2.4484)

Stations	Semi-major axis (m)	Semi-minor axis (m)	Azimuth of Semi-major axis (° ' ")
I-3	0.0010	0.0001	13 27 39
REF-200	0.0010	0.0008	345 32 38
REF-100	0.0011	0.0008	357 19 27
C-600	0.0014	0.0012	319 23 54
C-500	0.0008	0.0007	318 50 48
C-200	0.0029	0.0015	328 49 19
C-301	0.0009	0.0007	7 19 19
C-100	0.0019	0.0010	67 7 16
REF-201	0.0014	0.0010	323 48 7
REF-202	0.0013	0.0010	309 33 27
C-555	0.0029	0.0013	301 18 57
S-252A	0.0015	0.0011	343 3 9
S-251A	0.0015	0.0011	345 45 44
S-250A	0.0015	0.0011	341 47 28
DS-1	0.0014	0.0011	329 0 52
DS-2	0.0012	0.0010	4 54 26
I-1	0.0012	0.0011	70 19 51
I-2	0.0011	0.0008	26 40 34
PR-1	0.0011	0.0009	77 58 51
PR-2	0.0010	0.0009	287 54 56
PR-3	0.0011	0.0010	298 39 51
M-1	0.0012	0.0010	18 7 44
TK-7	0.0010	0.0008	17 33 53
TK-1	0.0009	0.0008	7 52 58
TR-1	0.0018	0.0012	292 59 42
TR-2	0.0017	0.0012	277 5 11
INVP-B	0.0022	0.0013	295 28 31

Total area of station ellipses = 0.12698D-03

Table 5.25: Station 95.000 % confidence ellipses for the optimized observation scheme (aim at improving accuracies of object points only, factor used for obtaining these ellipses from standard error ellipses =2.4484)

Stations	Semi-major axis (m)	Semi-minor axis (m)	Azimuth of Semi-major axis (o ' ")
I-3	0.0011	0.0001	13 27 39
REF-200	0.0016	0.0009	339 49 33
REF-100	0.0012	0.0009	356 43 54
C-600	0.0015	0.0012	318 22 33
C-500	0.0009	0.0008	316 7 47
C-200	0.0046	0.0017	329 17 51
C-301	0.0009	0.0008	5 51 18
C-100	0.0031	0.0011	70 58 37
REF-201	0.0023	0.0011	326 3 27
REF-202	0.0024	0.0014	295 4 9
C-555	0.0037	0.0013	303 1 42
S-252A	0.0016	0.0012	344 12 3
S-251A	0.0015	0.0011	347 6 30
S-250A	0.0015	0.0011	344 5 44
DS-1	0.0015	0.0011	331 1 26
DS-2	0.0012	0.0011	8 34 39
I-1	0.0012	0.0011	63 58 11
I-2	0.0012	0.0008	25 3 56
PR-1	0.0012	0.0009	74 2 36
PR-2	0.0011	0.0009	281 35 45
PR-3	0.0012	0.0010	290 4 18
M-1	0.0012	0.0010	18 12 23
TK-7	0.0011	0.0009	21 46 7
TK-1	0.0010	0.0009	21 0 15
TR-1	0.0020	0.0013	292 10 38
TR-2	0.0018	0.0012	276 40 53
INVP-B	0.0022	0.0013	295 16 52

Total area of station ellipses = 0.16422D-03

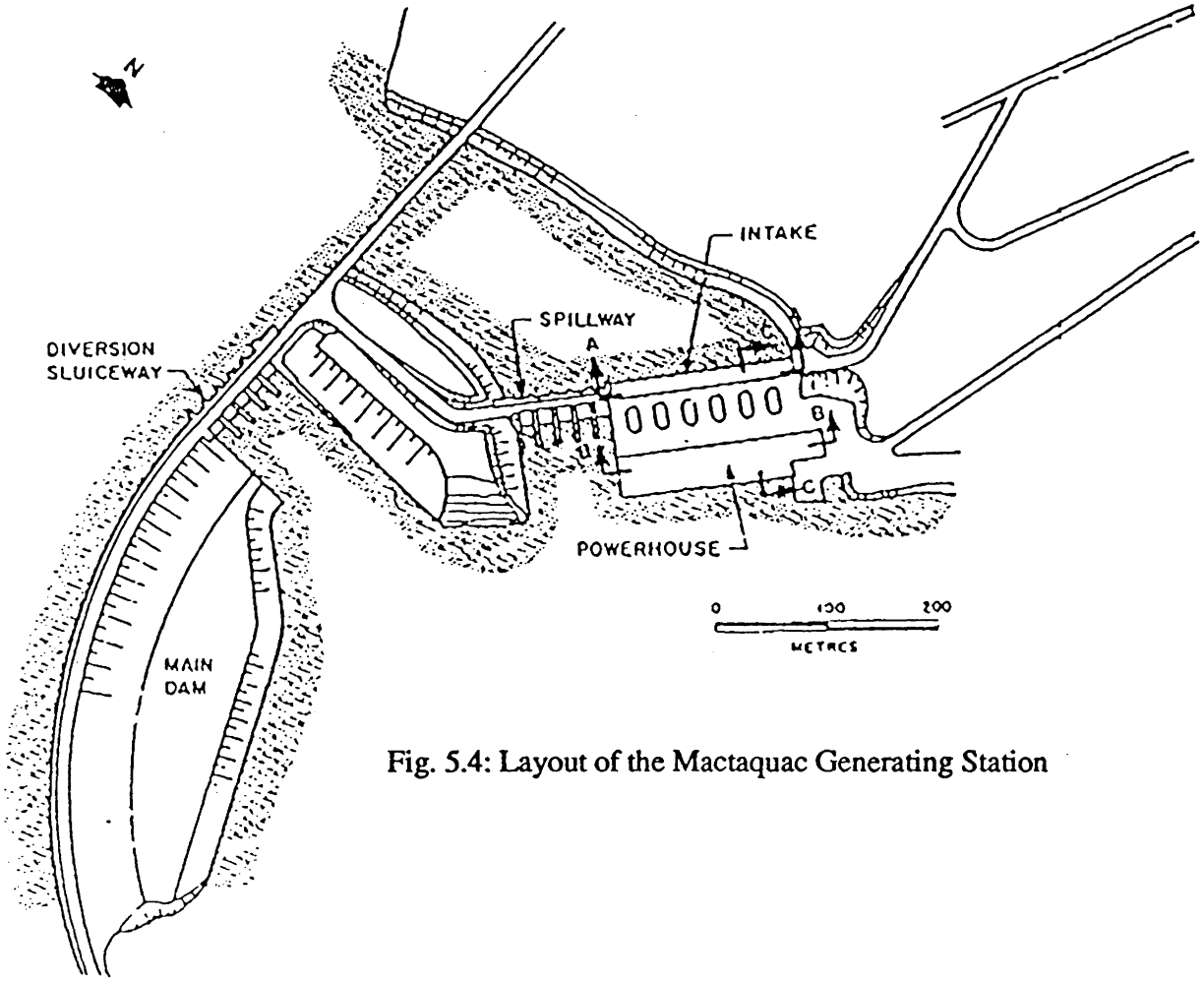

Fig. 5.4: Layout of the Mactaquac Generating Station

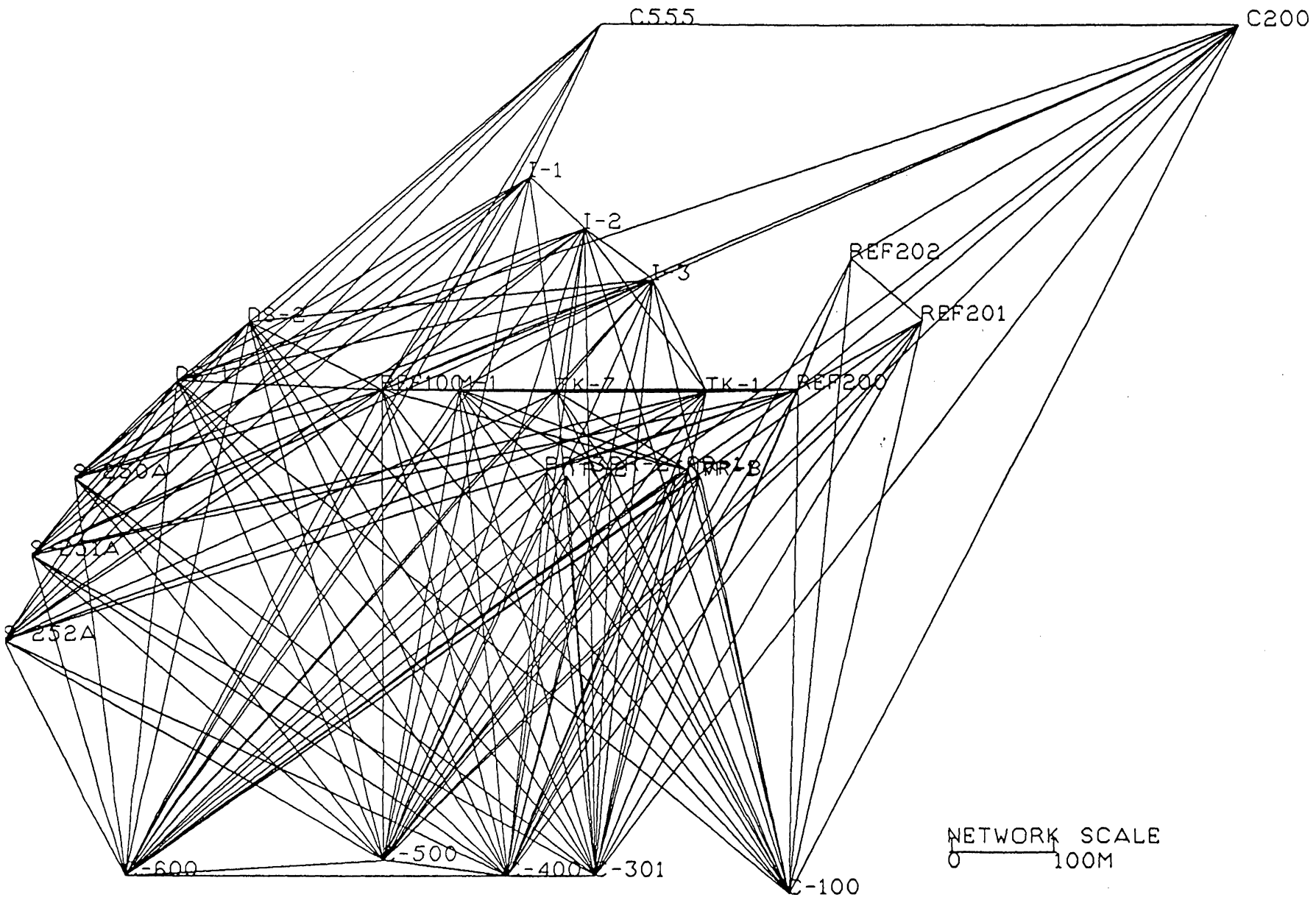

FIG.5.5:MACTAQUAC MONITORING NET
-- THE OBSERVABLE DISTANCES
(TOTAL:176 DISTANCES)

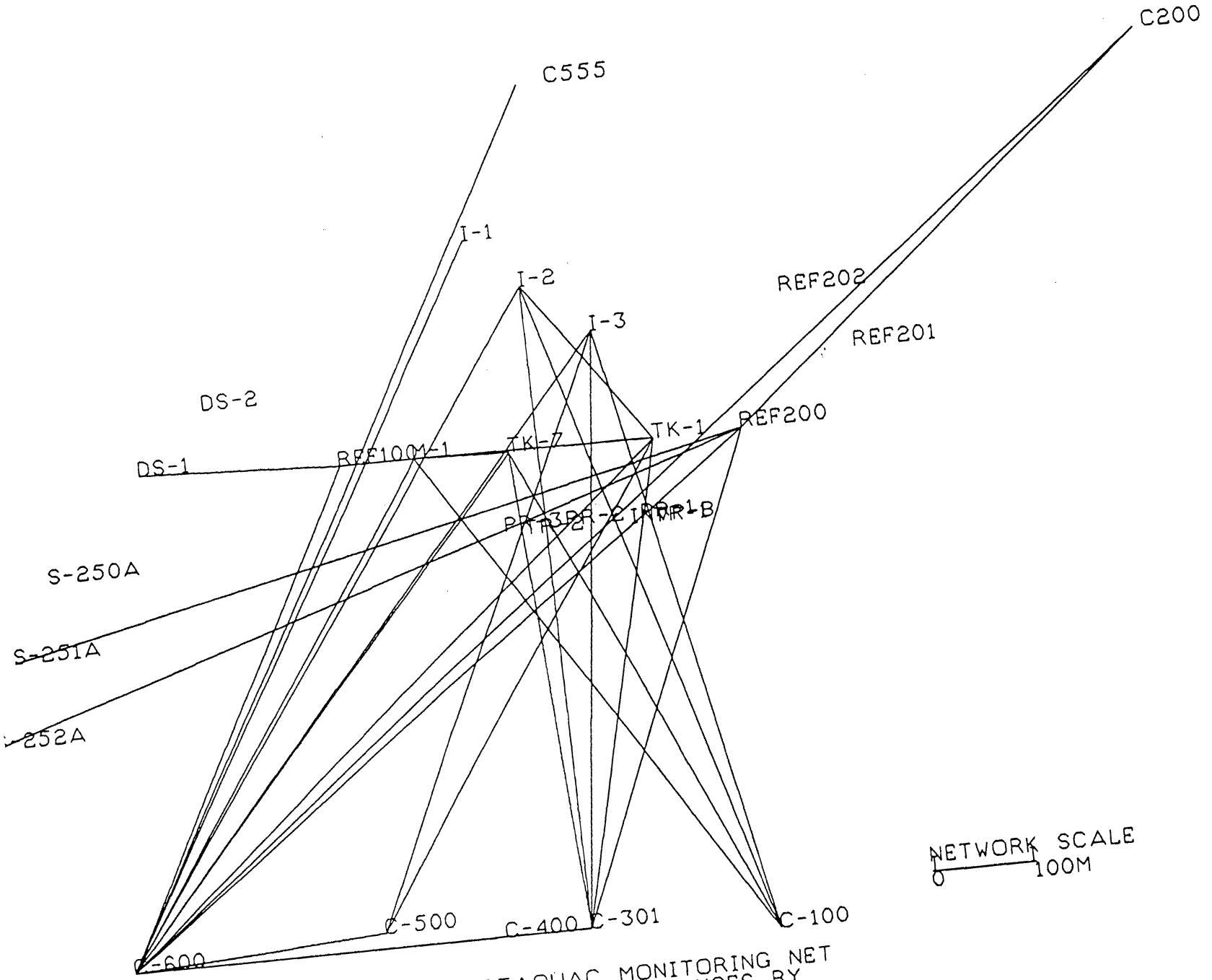

FIG.5.6:MACTAQUAC MONITORING NET
-- THE DELETED DISTANCES BY
OPTIMIZATION(TOTAL:31)

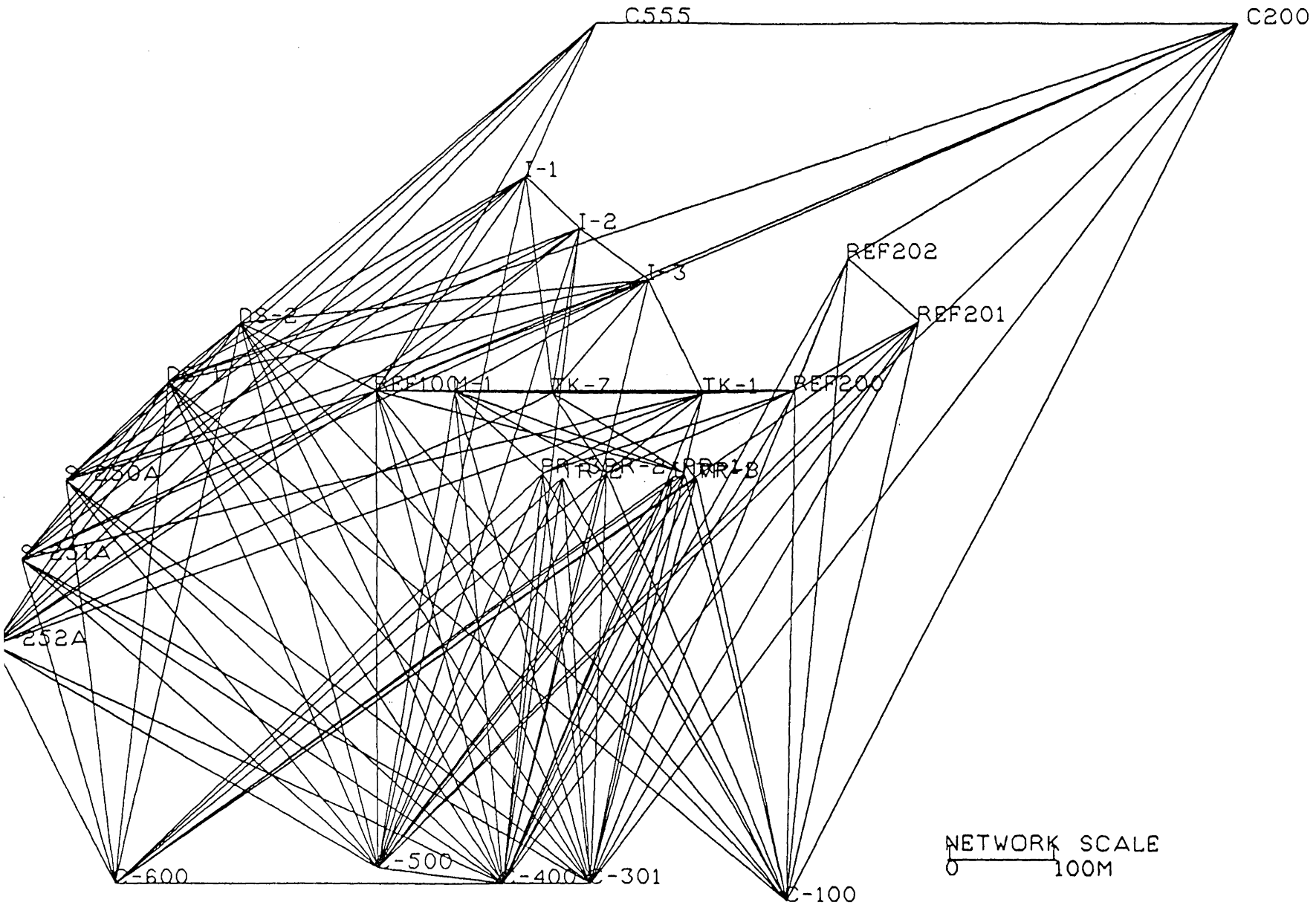

FIG.5.7:MACTAQUAC MONITORING NET
-- THE OPTIMIZED TRILATERATION

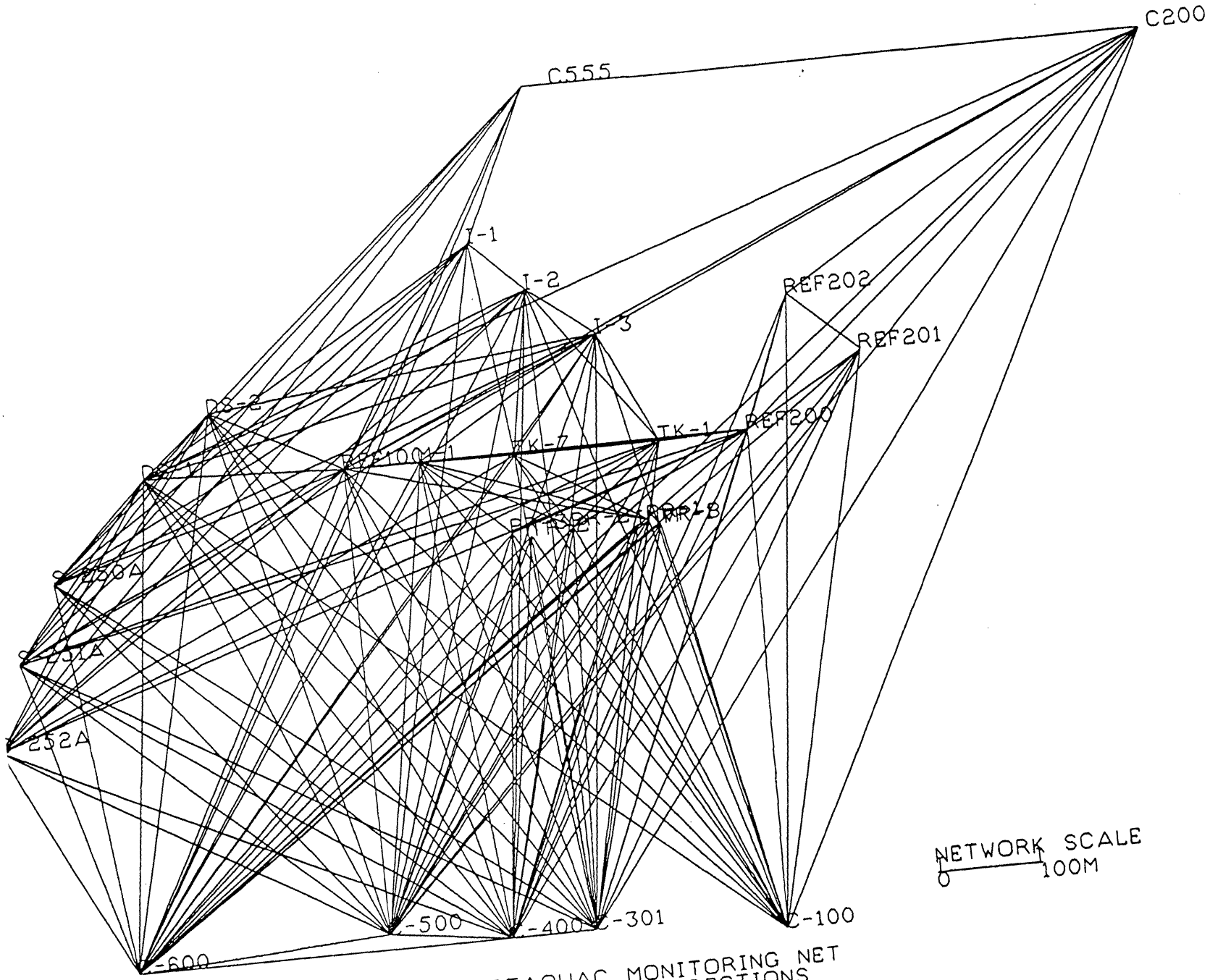

FIG.5.8:MACTAQUAC MONITORING NET
-- THE OBSERVABLE DIRECTIONS
TOTAL: 352 DIRECTIONS

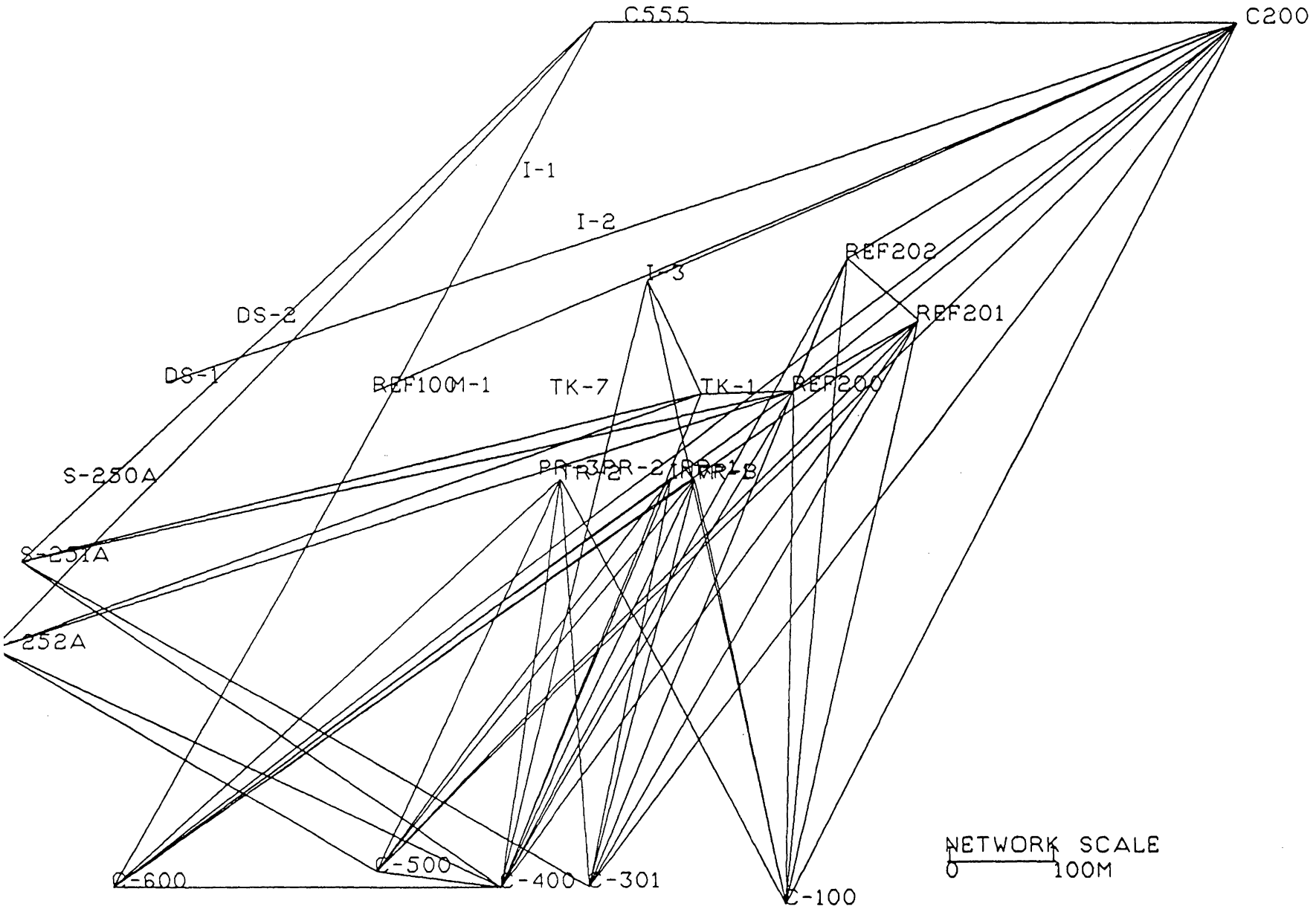

FIG.5.9:MACTAQUAC MONITORING NET
--THE PROPOSED DIRECTIONS BY
OPTIMIZATION(TOTAL:88)

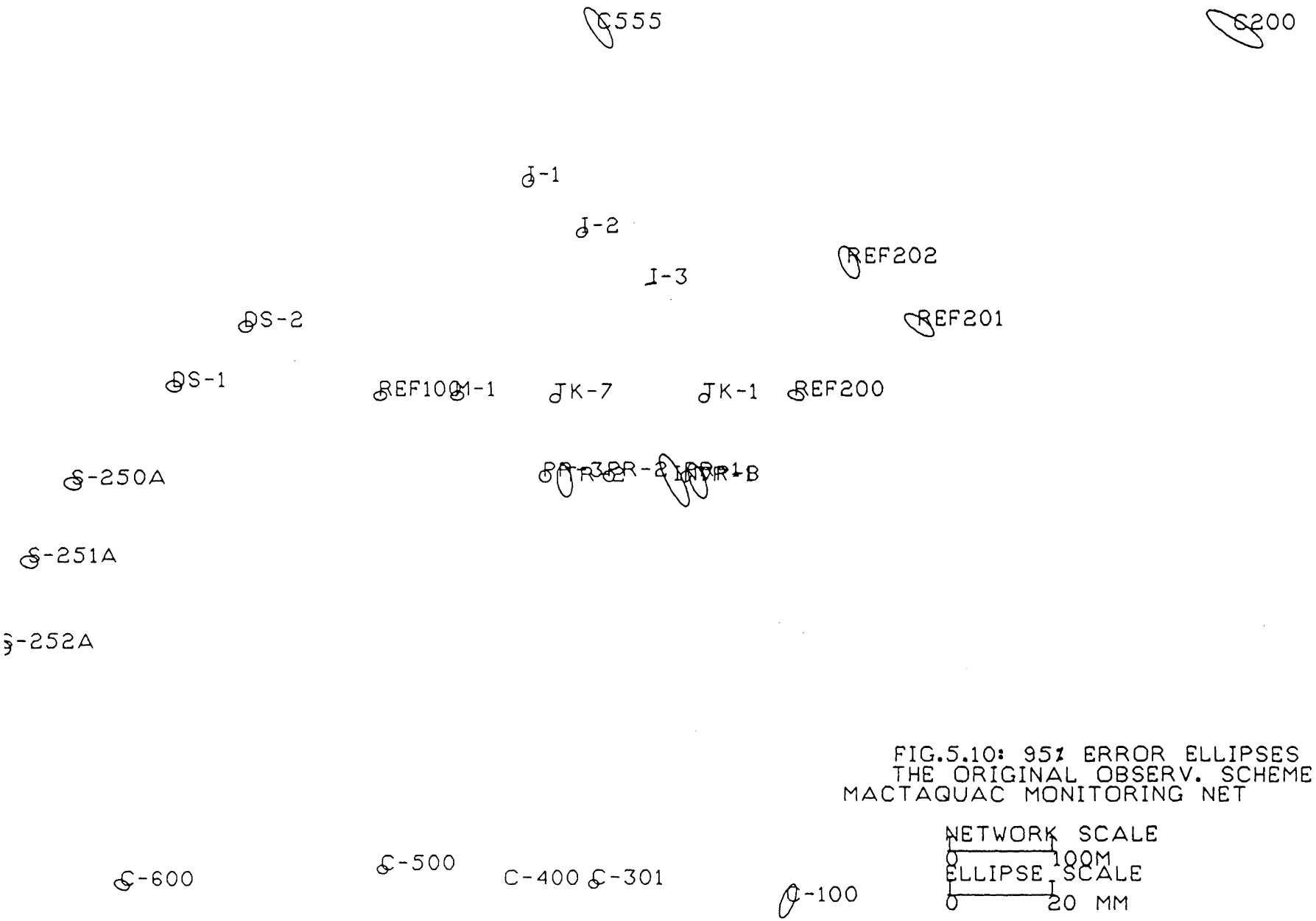

FIG.5.10: 95% ERROR ELLIPSES
THE ORIGINAL OBSERV. SCHEME
MACTAQUAC MONITORING NET

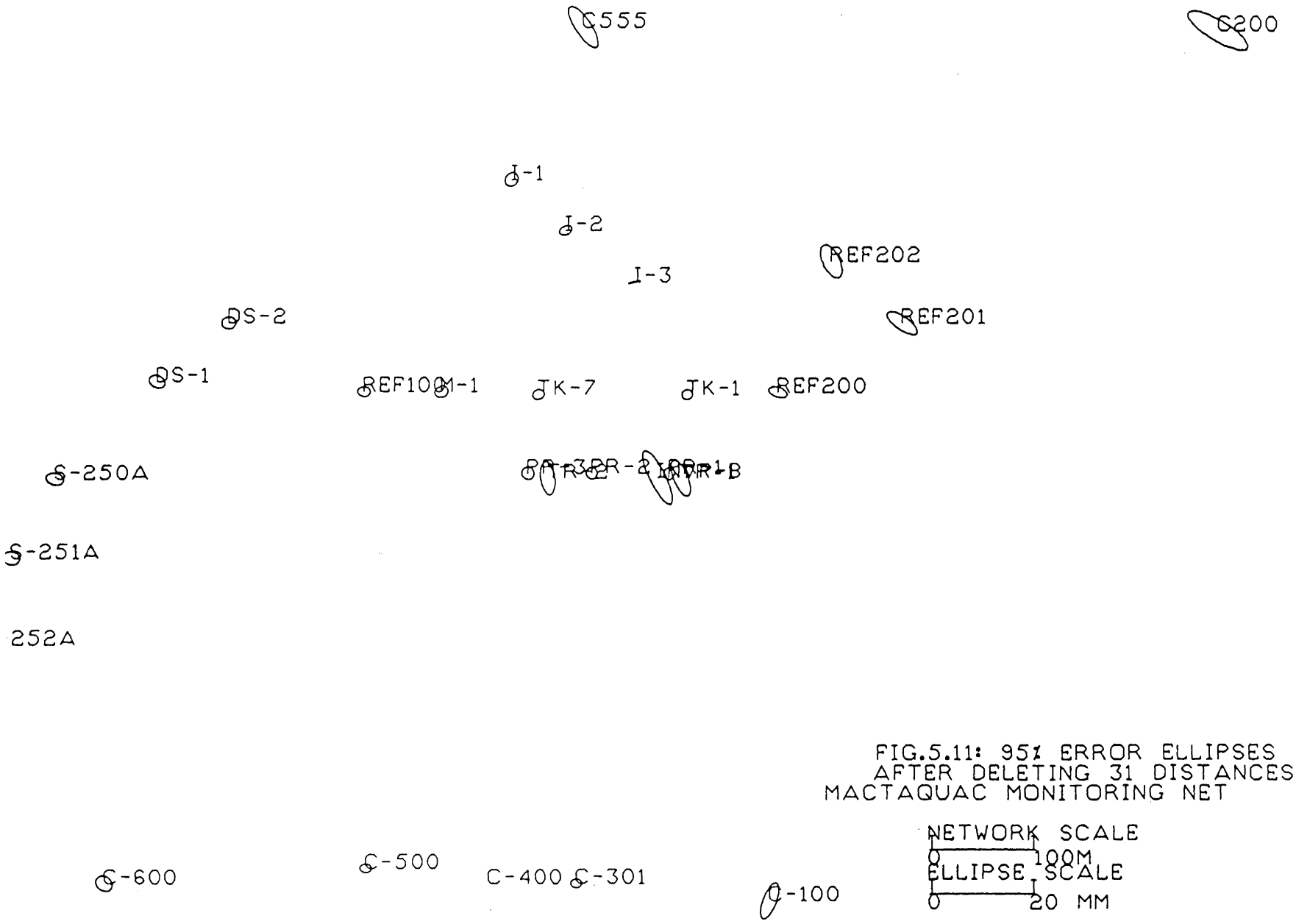

FIG.5.11: 95% ERROR ELLIPSES
AFTER DELETING 31 DISTANCES
MACTAQUAC MONITORING NET

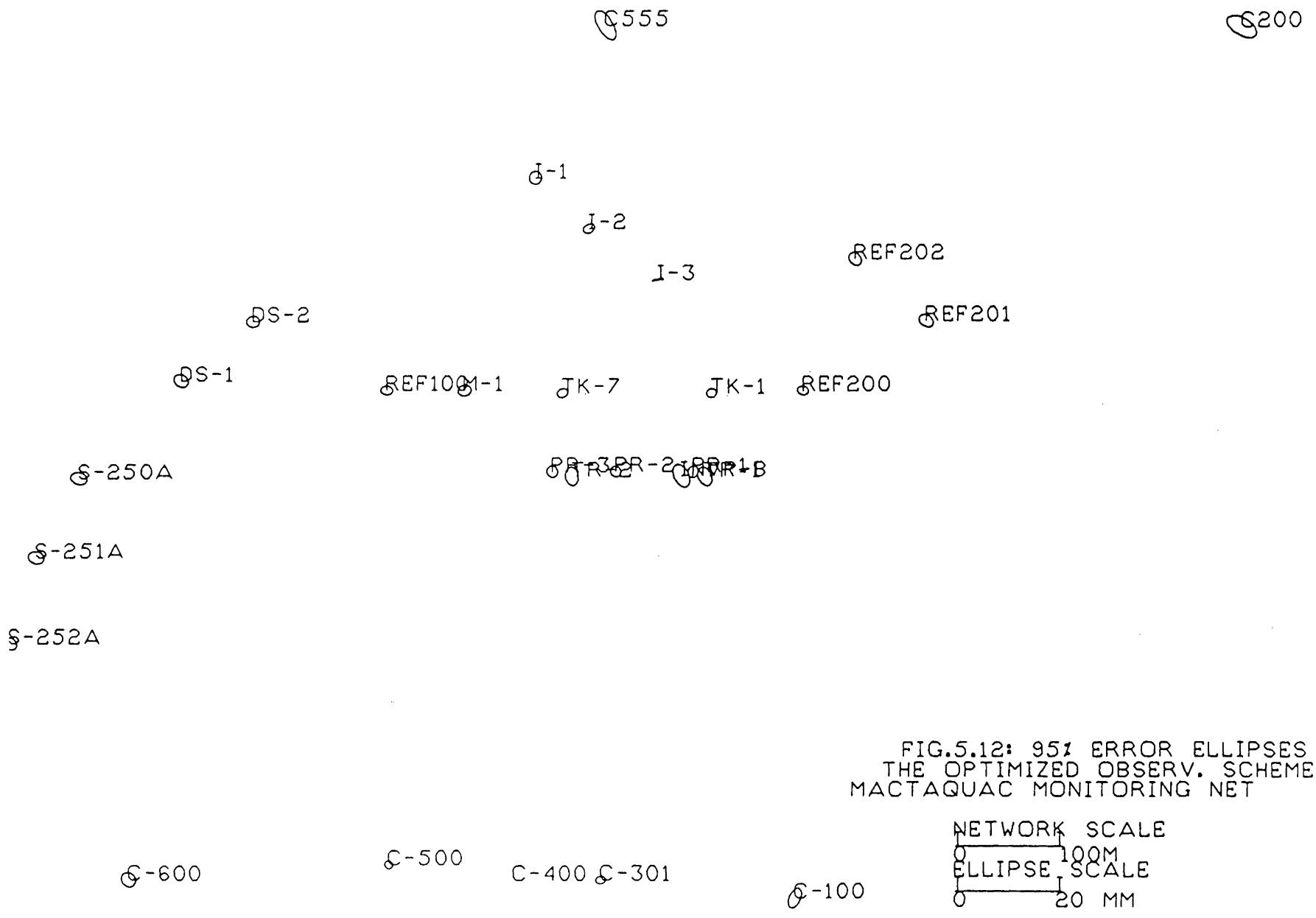

FIG.5.12: 95% ERROR ELLIPSES
THE OPTIMIZED OBSERV. SCHEME
MACTAQUAC MONITORING NET

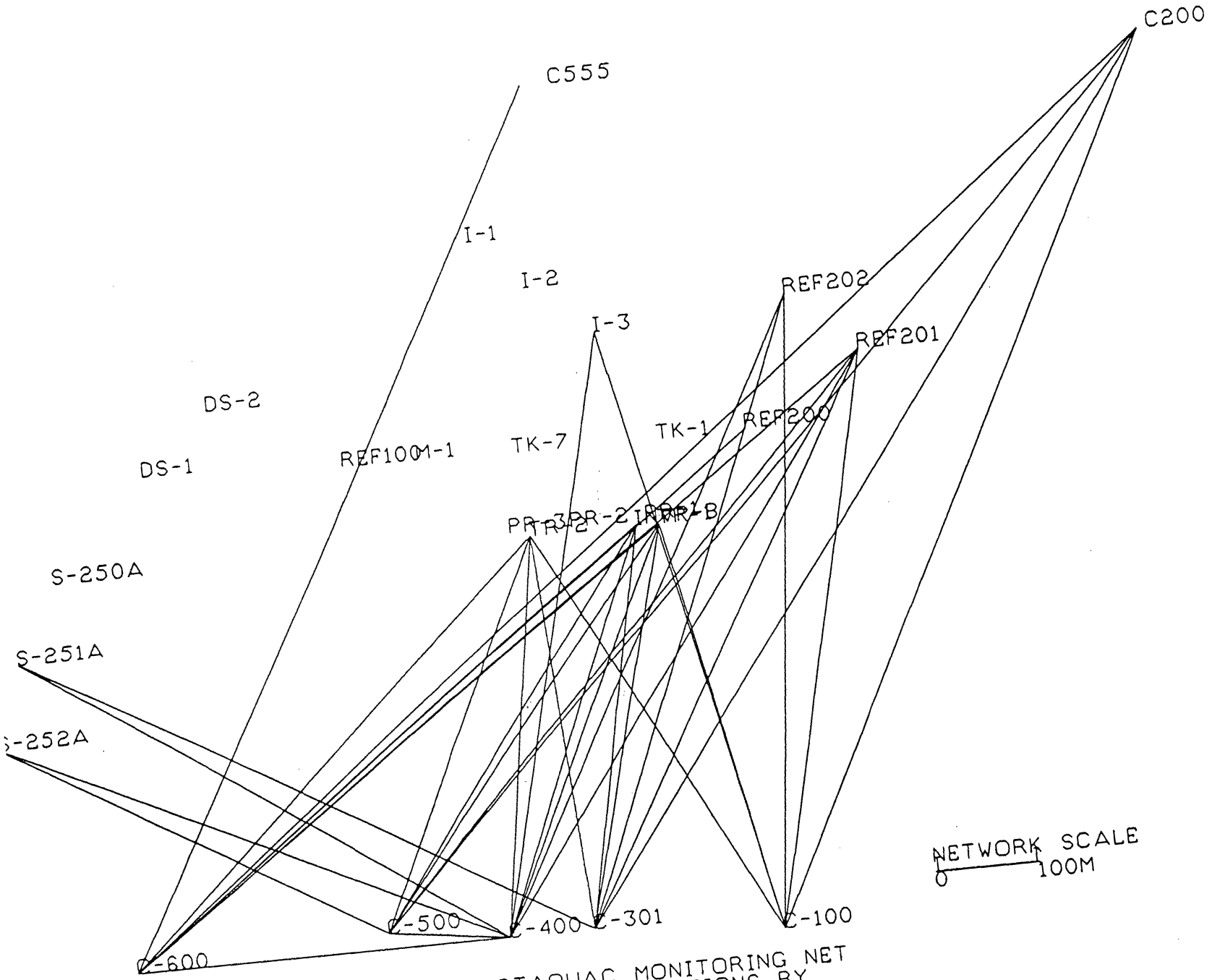

FIG.5.13:MACTAQUAC MONITORING NET --THE PROPOSED DIRECTIONS BY OPTIMIZATION(TOTAL:36)

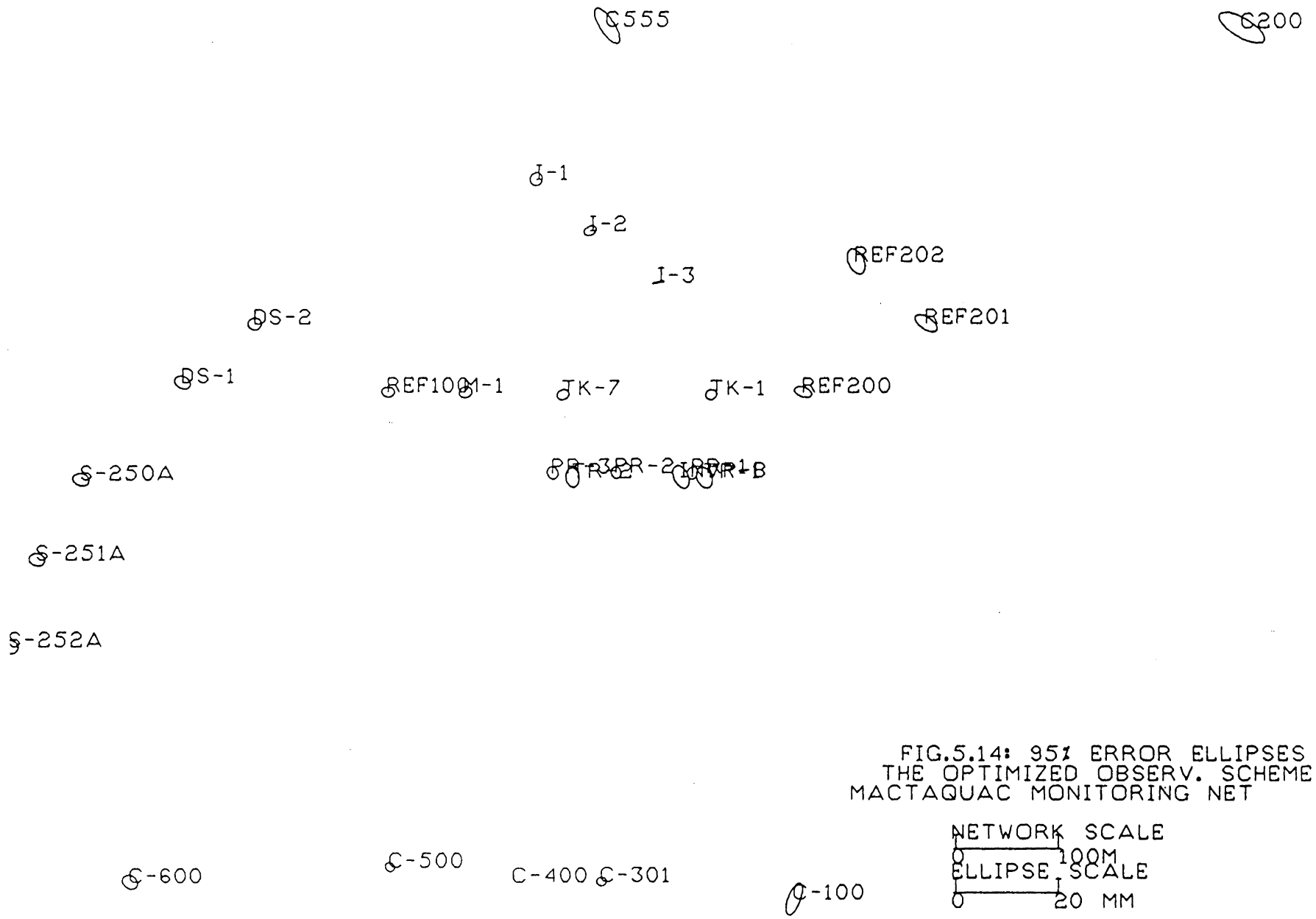

FIG.5.14: 95% ERROR ELLIPSES
THE OPTIMIZED OBSERV. SCHEME
MACTAQUAC MONITORING NET

CHAPTER 6

SUMMARY OF RESULTS AND CONCLUSIONS

The objective of this study has been achieved. As a result of this research, some conclusions and recommendations can be drawn.

The *necessity* of detecting, monitoring, and interpreting deformations is mainly dictated by safety and scientific reasons. It is obvious that continuous and rigorous monitoring of the natural or man-made environment is vital in predicting and preventing a future disaster. The precise determination of variations of a body can support and aid in improving new design theories by providing information about the load-deformation relationship. The body knowledge on deformation mechanisms can also be enhanced, which leads to the development of more sophisticated and refined procedures.

The *aim* of a monitoring scheme is to determine a set of deformation parameters which characterize the changes of a deformable body in its dimensions, shape, and position. Different methodologies and techniques, which may be classified as geodetic and non-geodetic, can be used for this purpose. The selection of monitoring techniques depends mainly on the type, magnitude, and rate of the expected deformation. Since each method has advantages and disadvantages, the integration of different systems is highly recommended to provide a more complete and more accurate picture of the deformable body.

Before any deformation measurement campaign is started, the geodesists should know about the result of their work according to the set objectives. This leads to the need for the optimization and design of deformation monitoring schemes. Essentially, the

purpose for the optimization and design of monitoring schemes is to prevent the deformation measurement campaigns from failing. It enables us to make decisions on which instruments should be selected from the hundreds of available models of various geodetic and non-geodetic instruments and where they should be located in order to estimate the unknown parameters and achieve the desired criteria derived from and determined by the purpose of the monitoring scheme. It is for this reason that a methodology for the optimization and design of deformation monitoring schemes has been developed in this research.

In general, the *unknown parameters* to be optimized for a monitoring scheme include the optimal positions for each geodetic and non-geodetic point characterized by coordinates and optimal weights for each geodetic and non-geodetic observable. However, from the *practical* point of view, since the specific positions of instruments in deformation monitoring are rather suggested by landform in the monitoring area in connection with mutual observability conditions and/or stability considerations, the unknown parameters to be optimized for a monitoring scheme can be the "improvements" to the approximate coordinates of both geodetic and non-geodetic points which are obtained by reconnaissance in the field and the "improvements" to the approximate weights of the observations which can be realized with the least effort. The optimal positions and weights are obtained then by adding the solved "improvements" to their respective approximate values. The criteria of optimization for monitoring schemes include precision, reliability, sensitivity, and economy. The optimization of a monitoring scheme means to design a precise-, reliable-, and sensitive enough scheme which can also be realized in an economical way. Precision is a measure of the variance and covariance of the estimated parameters; reliability refers to the measure to detect, localize and eliminate outlying observations; sensitivity refers to the detection of minimum magnitude of parameters; and finally economy is a measure of cost and benefit of the project. These criteria must be fulfilled for the different design problems of a monitoring scheme. For the purpose of optimization, all the optimality criteria of

precision, reliability, sensitivity, and economy have to be expressed in terms of the "improvements" to the approximate positions and weights. In addition, the "improvements" to be optimized are also subject to physical constraints constructed according to the topography and/or stability conditions as well as the maximum achievable accuracies of instruments. The proposed multi-objective optimization mathematical model can be applied to solve for the optimal values of the "improvements" to be introduced.

As compared with the existing approaches for network optimization, the newly developed methodology for the optimization and design of deformation monitoring schemes has the following features:

(1) Any type of geodetic and non-geodetic observables can be considered to construct an integrated deformation monitoring scheme. These may be the coordinate observables from geodetic space techniques and photogrammetry, individual terrestrial geodetic observations such as distances, directions, azimuths, horizontal and vertical angles, levelling, etc, or any physical-mechanical measurements of tilts, strains, alignment observations, etc.

(2) The methodology can be used to design a monitoring scheme which will give optimal results when solving for any type of deformation parameters. This is reflected by the unspecified deformation model B $\underline{e}$. Following the concept of the "***UNB Generalized Approach***" for deformation analysis, the whole area covered by the deformation survey is treated as a non-continuous deformable body consisting of separate continuous deformable blocks. Thus, the blocks may undergo relative rigid body displacements and rotations, and each block may change its shape and dimensions. In the case of single point movement, the given point is treated as a separate block to be displaced as a rigid body. Therefore, the methodology can be used to design a monitoring scheme aiming at either displacement detection of unstable area or detection of rigid body movements between blocks and the strain components. However, if

the deformation model B $\underline{\ell}$ to be detected consists of general polynomials, then a monitoring scheme can be designed for the optimal solution of the unknown coefficients in the polynomials. In the design phase, the a priori knowledge about what deformation will take place may be used for the selection of an appropriate deformation model. Such information will come from a study of the relevant physical properties of the object; geomechanics for crustal movement and structural analysis for deformation of dams, bridges and other large structures, etc.

(3) Essentially, an *optimization* procedure tries to maximize or minimize some kind of target function under a number of constraints(equalities or inequalities or both). Formerly, it was very difficult, if not impossible, to construct such a target function that contained parameters expressing precision, reliability, sensitivity, and cost of observations all in the same utility unit. This difficulty can now be overcome by introducing the theory of multi-objective optimization. Rather than the "trial and error" method, all the criteria of precision, reliability, sensitivity, and economy can now be expressed analytically in terms of the unknown parameters to be optimized. Based on the theory of multi-objective optimization, a suitable target function that includes all the criteria can be formulated under a common scale and the multi-objective optimization mathematical model (**MOOM**) for the optimization and design of monitoring schemes is established. Thus it is now possible to consider all the quality aspects i.e. precision, reliability, sensitivity and economy of a monitoring scheme simultaneously.

(4) By identifying the unknown parameters to be optimized for a monitoring scheme as the optimal positions for each geodetic and non-geodetic point characterized by coordinates and optimal weights for each geodetic and non-geodetic observable, the developed methodology can analytically perform the

Combined First Order and Second Order Design (Vanicek and Krakiwsky, 1986) i.e. the simultaneous optimization of the geometrical configuration and weights of heterogeneous observables in a monitoring scheme. For practical applications, after appropriate optimality criteria for deformation parameters or displacements are established, the optimization model **MOOM** gives us the desired weights or standard deviations for each observable and it can also give the desired position shifts of the initially selected netpoints in reconnaissance in order to obtain the best configuration. From the given simulation studies and practical examples one can see that even relatively small changes in configuration may significantly contribute to the solution of an optimal design problem. They can be used to establish a monitoring configuration with stronger geometry, which may lead to significant savings of effort in the field observation campaign. Conventionally, for positioning or monitoring networks, one used to ignore small changes in relative positions of stations assuming that they would have no significant contribution to the improvement of the accuracy. This point of view has now been proved to be impertinent. Although this methodology is induced by the concept of "introducing relatively small position changes", relatively larger changes can actually be accommodated by increasing the number of iterations during the solution procedure.

If the positions of the netpoints selected in reconnaissance can not be changed, the developed optimization model **MOOM** reduces to the Second Order Design. On the other hand, if the measurement instrumentation and accuracy for the given job are fixed, then the model reduces to the First Order Design. Finally, if positions of some points and weights of some observables have to be optimized, this model reduces to the Third Order Design. Therefore, one can see that the conventional different Orders of Design Problem are

embeded in the proposed mathematical model. The author believes that the optimization procedure developed in this research is the most practical and flexible one for the optimal design of monitoring schemes or geodetic networks established for engineering purposes. All the solutions are automatic thus removing the need for the method of "trial and error".

(5) The developed methodology can be used for the optimal design of either one-, two-, or three-dimensional monitoring schemes. The developed mathematical models can also be easily modifed to treat the optimization problems with the Mixed Models, and the application of the methodology to the optimal design of any geodetic networks for engineering purposes is quite straight forward.

The examples with simulated and real data have shown that the developed methodology is correct and works well. Further research is suggested to continue and expand the practical applications of the developed methodology.

REFERENCES

Abbreviations

AVN :Algemeine Vermessungs-Nachrichten

Bull. Geod. :Bulletin Geodesique

CIS : The Canadian Institute of Surveying

DGK : Deutsche Geodatische Kommission

JASA : Journal of American Statistics Association

ZFV : Zeitschrift fur Vermessungswesen

ALBERTA, J.E. (1974). "Planning and optimization of networks: Some general considerations." Boll. Geod. Sci. Aff. No. 33, pp. 209-240.

ALBERTA, J.E. (1980). A review of analysis techniques for engineering survey control schemes. Paper presented at the Industrial and Engineering Survey Conferences, London, UK.

BAARDA, W. (1962). "A generalization of the concept Strength of Figure." Publ. of the Computing Center of the Geodetic Inst., Delft.

BAARDA, W. (1967). "Statistical concepts in geodesy." Neth. Geod. Com., publ. on Geodesy, New Series 2, No. 4, Delft, Netherlands.

BAARDA, W. (1968). "A testing procedure for use in geodetic networks." Neth. Geod. Com., publ. on Geodesy, New Series 2, No. 5, Delft, Netherlands.

BAARDA, W. (1973). "S- Transformations and Criterion matrices." Neth. Geod. Com., publ. on Geodesy, New Series 5, No. 1, Delft, Netherlands.

BAARDA, W. (1976). Reliability and precision of networks. VII International Course for Engineering Surveys of High Precision, Darmstadt, pp. 17-27.

BARAN, W. (1982). Some new procedures of sequential adjustment. Int. symp. on geodetic networks and computations (1981), DGK, Munchen, VIII: 69-79.

BOOT, J.C.G.(1964). "Quadratic Programming", North-Holland, Amsterdam, 1964.

BOEDECKER, G. (1977). The development of an observation scheme from the variance covariance matrix of the unknowns. Proc. Int. Symposium on optimization of design and computation of control networks, Sopron.

BOSSLER, J., E. GRAFAREND and R. KELM (1973). "Optimal design of geodetic nets 2." Journal of Geophysical Research, vol. 78, no. 26.

CHEN, Y.Q. (1983). Analysis of Deformation Surveys - A generalized method. Technical Report No. 94, Dept. of Surveying Engineering, UNB, Canada.

CHEN, Y.Q., KAVOURAS M. and SECORD J.M. (1983). Design considerations in deformation monitoring. FIG XVII International Congress, Sofia, Bulgaria, Paper No. 608.2

CHEN, Y.Q., A. CHRZANOWSKI (1986). An overview of the physical interpretation of deformation measurements. at the Deformation Measurements Workshop, M.I.T., Cambridge, Mass. 31 October-1 November 1986

CHRZANOWSKI, A. (1981). A comparison of different approaches into the analysis of deformation measurements. Proc. of FIG XVI International Congress, Montreux, Paper No. 602.3, August 9-18.

CHRZANOWSKI, A. (1983). Economization of vertical control surveys in hilly areas by using modified trigonometric levelling. Presented at the 1983 ACSM-ASP Convention, Washington, D.C., March.

CHRZANOWSKI, A., Y.Q. CHEN, J.M. SECORD (1983). "On the strain analysis of tectonic movements using fault crossing geodetic surveys." Tectonophysics, 97, pp. 295-315.

CHRZANOWSKI, A., and J.M. SECORD (1983). Report of the 'ad hoc' Committee on the Analysis of Deformation Surveys. Presented at the FIG-17th International Congress, Paper 605.2, Sofia, June 19-28.

CHRZANOWSKI, A. (1986). Geotechnical and other non-geodetic methods in deformation measurements. at the Deformation Measurements Workshop, M.I.T., Cambridge, Mass. 31 October-1 November 1986

CHRZANOWSKI, A., Y.Q. CHEN, P. ROMERO, J.M. SECORD (1986). "Integration of geodetic and geotechnical deformation surveys in geoscience." Tectonophysics, 130, pp. 369-383.

CHRZANOWSKI, A., Y.Q. CHEN, J.M. SECORD, G.A. THOMPSON and Z. WROBLEWICZ (1988). Integration of geotechnical and geodetic observations in the geometrical analysis of deformations at the Mactaquac Generating Station. Conference 1988 on deformation surveys, Fredericton, Canada.

CONZETT, R., FRANK A. and MISSLIN C. (1980). Interactive triangulation. VIII Int. Kurs für Ingenieurermessung, Zürich.

COTTLE, R.W. AND G.B. DANTZIG (1968). "Complementary pivot theory of mathematical programming." Linear Algebra Application No. 1 (1968), 103-125.

CROSILLA, F. (1982). "Geodetic network optimization for the detection of crustal movements using a mekometer." Bollettino di Geodesia e Sc. Affini.

CROSILLA, F. (1983a). "A criterion matrix for the second order design of control networks." Bull. Geod. 57:226-239.

CROSILLA, F. (1983b). "Procrustean transformation as a tool for the construction of a criterion matrix for control networks." Manuscripta Geodetica 8: 343-370.

CROSILLA, F. (1985). "A criterion matrix for deforming networks by multifactorial analysis" in Optimization and design of geodetic networks edited by Grafarend and Sanso. Springer: Berlin etc. pp. 429-435.

CROSS, P.A. and B.M. WHITING (1980). The design of national vertical control networks. Proc. 2nd Int. Symposium On Problems Related to the Redefinition of North American Vertical Geodetic Networks, Ottawa, Canada.

CROSS, P.A. and K. THAPA (1979). "The optimal design of levelling networks." Survey Review, vol. XXV, No.192, pp.68-79.

CROSS, P.A. and B.M. WHITING (1981a). On the design of vertical geodetic control networks by iterative methods. VI International Symposium On Geodetic Computations, Munich.

CROSS, P.A. (1981b). The geodetic suite. Working paper 4, North East London Polytechnic, Department of Land Surveying, 92 pp.

CROSS, P. and A. FAGIR (1982). Procedures for the First and Second Order Design of vertical control networks. Proc. Survey Control Networks, Heft 7, Munchen.

CROSS, P.A. (1985). "Numerical methods in network design." in Optimization and design of geodetic networks edited by Grafarend and Sanso. Springer: Berlin etc. pp. 429-435.

DANTZIG, G.B.(1963). Linear programming and extensions. Princeton University Press, Princeton, N.J., 1963.

DERMANIS, A. (1985). "Optimization problems in geodetic networks with signals." in Optimization and design of geodetic networks edited by Grafarend and Sanso. Springer: Berlin etc. pp. 221-256.

DINKELBACH, W., H. ISERMANN (1973). "On decision making under multiple criteria and under incomplete information." in Multiple Criteria Decision Making, J.L. Cochrane and M. Zeleny, (eds.), Univ. of South Carolina Press, Columbia, South Carolina.

DUNNICLIFF, JOHN (1988). Geotechnical instrumentation for monitoring field performance. John Wiley & Sons, New York/ Chichester/ Brisbane/ Toronto/ Singapore/

DYER, J.S. (1972). "Interactive goal programming." Management Science, No. 19, pp. 62-70.

FAIG, W.(1978). The utilization of photogrammetry of deformations of structural parts in the shiping building industry. Proc. of the II international Symposium of deformation measurements by geodetic methods, Bonn, West Germany, Konrad Wittwer, stuttgart.

FAIG, W. and C. ARMENAKIS (1982). Subsidence monitoring by photogrammetry. Proceedings of the Fourth Canadian Symposium On Mining surveying and deformation measurements, Bannf, Alberta, June, CIS 197-208.

FRANK, C.F. (1966). "Deduction of earth strain from survey data." Bulletin Seismological Society of America, 56 (1):35-42.

FRANK, A., MISSLIN C. (1980) INTRA: a programme for interative design and adjustment of geodetic networks. Institute für Geodasie und Photogrammetrie, Eidgenössische Technische Hochschule Zürich, Bericht 43, 11 pp.

GRAFAREND, E. (1972). "Genauigkeitsmasse geodatischer Netze." DGK-publ. A-73, Munchen

GRAFAREND, E.W. (1974). "Optimization of geodetic networks." Bolletino di Geodesia a Science Affini, Vol. 33, No. 4, pp.351-406.

GRAFAREND, E.W. and B. SCHAFFRIN (1979). "Kriterion-matrizen I-zweidimensional homogene und isotrope geodatische netze." ZFV, No. 4, pp.133-149.

GRAFAREND, E.W., HEISTER. H., KELM R., KROPFF H. and SCHAFFRIN B. (1979). Optimierung geodetischer messoperationen. Karlsruhe

GRAFAREND, E.W. (1985). "Criterion matrices for deforming networks." in Optimization and design of geodetic networks edited by Grafarend and Sanso. Springer: Berlin etc. pp. 363-428.

GRAFAREND, E.W., F. KRUMM and B. SCHAFFRIN (1986). "Kriterion-matrizen III -zweidimensional homogene und isotrope geodatische netze." ZFV, No. 5, pp.197-207.

HELMERT, F.R. (1868): "Studien über rationelle Vermessungen im Gebiet der höheren Geodäsie." Z. Math. Phys. Schlömilch vol. 13 (1868), pp. 73-120, 163-168.

HENDERSON, H.V, S.R. SEARLE (1979). "Vec and Vech operators for matrices with some uses in Jacobians and multivariate statistics." Canadian Journal of statistics 7,65-81

HUGGET, G.R. (1982). A Terrameter for deformation measurements. Proceedings of the Fourth Canadian Symposium on Mining Surveying and Deformation measurements, Banff, Alberta, June 7-9, CIS, 179.

ILLNER, M. (1986). "Anlage und Optimierung von Verdichtungsnetzen." DGK, C-317, Munich.

ILLNER, M. (1988). "Different models for the sequential optimization of geodetic networks." Manuscripta Geodaetica 13 (1988), 306-315.

JÄGER, R. (1988). "Analyse und Optimierung geodatischer Netze nach spectralen Kriterien und mechanische Analogien." DGK, C-342, Munich, 1988.

JÄGER, R. (1989). "Optimum positions for GPS-points and supporting fixed points in geodetic networks." Proc. IAG General Meeting (Edinburgh, 1989), Springer Lect. Notes, 1990.

JÄGER, R., H. KALTENBACH (1990). "Spectral analysis and optimization of geodetic networks based on eigenvalues and eigenfunctions." Manuscripta Geodaetica 15 (1990), 302-311.

JUNG, I. (1924). "Uber die gunstigste Gewischitsverteilung in Basisnetzen." Akadem. Abh. Uppsala 1924

KARADAIDIS, D. (1984). "A substitute matrix for photogrammetrically determined point fields." ITC-Journal, 1984-3.

KARMAN, T. (1937). "On the statistical theory of turbulence." Proc. Natl. Acad. Sci. USA, 23.

KOCH, K.R. (1982a). "Optimization of the configuration of the geodetic networks." DGK, B, 258/III, 82-89, Munchen, 1982.

KOCH, K.R. (1985)."First Order Design: Optimization of the configuration of a network by introducing small position changes". in Optimization and design of geodetic networks edited by Grafarend and Sanso. Springer: Berlin etc. pp. 56-73.

KOCH, K.R. and D. FRITSCH (1981). "Multivariate hypothesis tests for detection of crustal movements." Tectonophysics, 71, pp. 301-313.

KUANG S.L. (1986). Study on the Post-Analysis of Positioning Data for Marine Geophysical Exploration. M.Sc.E thesis, Wuhan Technical University of Surveying and Mapping(WTUSM), April, 1986.

KUANG S.L. (1988). "Study of Outlier Detection and Position Estimation for Point Positioning in Marine Surveying." Journal of Wuhan Technical University of Surveying and Mapping(WTUSM), pp. 85-97, Vol. 12, No.4, 1988.

KUANG S.L. (1989) "Using Minimax Principle to Estimate Surveying Parameters and Perform Accuracy Analysis." Journal of Hohai University, pp. 16-23, Vol.17, No.3, 1989.

LIEW, C.K. and J.K. SHIM (1978). "Computer program for inequality constrained least squares estimation." Econometrica, vol. 46, pp 237.

MEISSL, P. (1976). " Strength analysis of two-dimensional Angular Anblock networks." Manuscripta Geodetica, Vol. 1, No. 4.

MEPHAM, M.P.A., MACKENZIE A.P., KRAKIWSKY E.J. (1982). Design engineering and mining surveys using Interactive Computer Graphics. Presented at the 4th Canadian Symposium on Mining Surveying and Deformation Measurements, Banff, Canada.

MEPHAM, M.P.A., KRAKIWSKY E.J. (1983) Description of CANDSN: a computer aided survey network design and adjustment system. Presented at the 76th Annual meeting of the Canadian Institute of Surveying, Victoria, Canada.

MIERLO, J.VAN (1981). "Second Order Design: precision and reliability aspects." AVN, pp.95-101.

MITTERMAYER, E. (1972). "Adjustment of free networks." ZFV, No. 11, pp. 481-489.

MOLENAAR, M. (1981). "A further inquiry into the theory of s-transformations and criterion matrices." Netherlands Geodetic commission, Publications on geodesy, New series, vol. 7, No. 1, Delft 1981

MÜLLER, H. (1986). Zur Berucksichtigung der Zuverlassigkeit bei der Gewichtsoptimierung geodatischer Netze." ZFV 111 (1986), 157-169

NICKERSON, B.G. (1979). Horizontal network design using computer graphics. M.sc. Eng. Thesis, Dept. of Surveying Engineering, UNB, Fredericton, Canada.

NIEMEIER, W. and G. ROHDE (1981). On the optimization of levelling nets with respect to the determination of crust movements. IAG Symposium on Geodetic networks and computations, Munich.

NIEMEIER, W. (1982). Principal component analysis and geodetic networks-some basic considerations. Proc. of the meeting of FIG study Group 5B.

PEARSON, E.S., and H.O. HARTLEY (1951). "Charts of the power Function of the analysis of variance tests, derived from the non-central F-distribution." Biometrika, vol. 38, pp.112-130.

PELZER, H. (1971). Zur Analyse Geodatischer Deformationsmessungen, DGK, C 164, Munchen

RAMSAY, J.G. (1967). Folding and Fracturing of Rocks. McGraw-Hill, New York, 568 pp.

RAO, C.R. (1973). Linear statistical Inference and its applications. John Wiley & Sons, New York.

SAWARAGI, Y., H. NAKAYAMA AND T. TANINO (1985): Theory of multi-objective optimization. Academic Press, Inc. Orlando San Diego New York London Toronto Montreal Sydney Tokyo.

SCHAFFRIN, B. (1977). A study of the Second Order Design problem in geodetic networks. Proceedings of the International Symposium on Optimization of Design and Computation of control Networks, Sopron, 1977, pp. 175-177.

SCHAFFRIN, B., E.W. GRAFAREND, G. Schmitt (1977). "Kanonisches Design geodatischer Netze I." Manuscripta Geodaetica 2 (1977) 263-306.

SCHAFFRIN, B., F. KRUMM and D. FRITSCH (1980). Positive diagonale genauigkeitsoptimierung von realnetzen uber den komplementaritats-algorithms. VIII International Kurs Fur Ingenieurvermessung, Zurich

SCHAFFRIN B.(1981). "Some proposals concerning the diagonal Second Order Design of geodetic networks." Manuscripta Geodetica, vol. 6, No. 3, p.303-326

SCHAFFRIN, B. and GRAFAREND, E.W. (1982). "Kriterion-Matrizen II-Zweidimensionale homogene und isotrope geodatische Netze." ZFV, Tei IIa, No. 5, pp. 183 - 194, Tei IIb, No. 11, pp. 485 - 493, 1982.

SCHAFFRIN, B. (1983). "On some recent modifications regarding the optimal design of geodetic networks." Geodezja (Krakow) 79 (1983), 43-56.

SCHAFFRIN, B. (1985a). "On design problems in geodesy using models with prior information." Statistics and Decision, Suppl. Issue 2 (1985), 443-453

SCHAFFRIN, B. (1985b). "Aspects of network design." in Optimization and design of geodetic networks edited by Grafarend and Sanso. Springer: Berlin etc. pp. 548-597.

SCHAFFRIN, B. (1986). "New estimation/prediction techniques for the determination of crustal deformations in the presence of geophysical prior information." Tectonophysics 130 (1986), 361-367.

SCHAFFRIN, B. (1988). Tests for random effects based on homogeneously linear predictors, in H. Lauter (ed.), Proc. of the Intl. Workshop on "Theory and Practice in Data Analysis", Acad. of Sci. of GDR, Report No. R-MATH-01/89, Berlin (East), pp. 209-227.

SCHMITT, G. (1977). Experience with the second order design of geodetic networks. Proc. International Symposium of Design and Computation of Control Networks, Sopron, Hungary.

SCHMITT G. (1978). Numerical problems concerning the second order design of geodetic networks. Second International Symposium Related to the Redefinition of North American Geodetic Networks, Washington.

SCHMITT G. (1980). "Second Order Design of free distance networks considering different types of criterion matrices." Bull. Geod. 54 pp. 531-543.

SCHMITT G. (1982). "Optimal design of geodetic networks." DGK, B, 258/III, 82-89, Munchen, 1982.

SCHMITT G. (1985). "A review of network designs, criteria, risk functions and design ordering." in Optimization and design of geodetic networks edited by Grafarend and Sanso. Springer: Berlin etc. pp. 6-10.

SCHMITT G. (1985). "Second Order Design". in Optimization and design of geodetic networks edited by Grafarend and Sanso. Springer: Berlin etc. pp. 74-121.

SCHMITT G. (1985). "Third Order Design". in Optimization and design of geodetic networks edited by Grafarend and Sanso. Springer: Berlin etc. pp. 122-131.

SCHÖNEMANN, P.H. (1966). "The generalized solution of the orthogonal procrustes problem." Psychometrica 31:1-16.

SCHREIBER O. (1882). "Anordnung der winkelbeobachtung im Gottinger Basisnetz." ZFV 11 (1882), 129-161

SECORD, J. M. (1985). Implementation of a generalized method for the analysis of deformation surveys. Technical Report No. 117, Dept. of Surveying Engineering, University of New Brunswick, Fredericton, N.B., Canada.

SECORD, J. M. (1986). Terrestrial survey methods for precision deformation measurements. at the Deformation Measurements Workshop, M.I.T., Cambridge, Mass. 31 October-1 November 1986

SOKOLNIKOFF, I.S. (1956). "Mathematical Theory of Elasticity." McGraw-Hill, New York.

SPRINSKY, W.H., (1978). "Improvement of parameter accuracy by choice and quality of observation." Bulletin Geodesique 52, 269-279.

TARLOR, G.I. (1935). "Statistical theory of turbulence." Proc. R. Soc. London, Ser. A, 151-421.

TANG, H.W.C. (1990). "Optimum design of geodetic network using integer linear programming and rank one updating procedure." M.Eng. thesis. Dept. of Surveying Engineering, Fredericton, UNB, Canada.

TESKEY, W.F. (1987). Integrated analysis of geodetic, geotechnical, and physical model data to describe the actual deformation behaviour of earthfill dams under static loading. Institute für Anwendungen der geodäsie im Bauwesen. Universität Stuttgart, May, 1987.

TEUNISSEN, P.J.G. (1985)."Zero Order Design: Generalized inverses, Adjustment, the Datum problem and S-Transformations " in Optimization and design of geodetic networks edited by Grafarend and Sanso. Springer: Berlin etc. pp. 11-55.

TEUNISSEN, P.J.G. (1985)."Quality control in geodetic networks." in Optimization and design of geodetic networks edited by Grafarend and Sanso. Springer: Berlin etc. pp. 526-547.

VANICEK, P., E.J. KRAKIWSKY (1986). Geodesy: The Concepts. North-Holland publishing company.

VANICEK, P., A. KLEUSBERG, D. WELLS and R. LANGLEY (1986). "GPS survey design." in Guide to GPS positioning, Chapter 14. Canadian GPS associates. Printed in Fredericton, N.B., Canada by UNB Graphic Services.

WELLS, D.E., and E.J. KRAKIWSKY (1971). The method of least squares. Dept. of Surveying Engineering, University of New Brunswick, Fredericton, N.B., Canada.

WEMPNER, G.(1973). Mechanics of Solids with applications to thin bodies. McGraw-Hill, New York, 623 pp.

WHITTEN, A. (1982). Monitoring Crustal Movement, Lecture Notes used in P.R. China.

WHITTLE, P. (1954). "On stationary processes in the plane." Biometrika 41 (1954), pp. 434-449.

WIMMER, H. (1981). Second order design of geodetic networks by an iterative approximation of a given criterion matrix. VI International Symposium on geodetic computations, Munich.

WIMMER, H. (1982a). "Ein Beitrag Zur Gewichtsoptimierung geodatischer Netze." Pul. DGK c -269, Munchen.

WIMMER, H. (1982b). "Second Order Design of geodetic networks by an iterative approximation of a given criterion matrix." DGK, Reihe B, Heft 258/III: 112-127.

WOLF, H. (1961). "Zur Kritik von Schreibers Satz uber die Gewichtsverteilung in Basisnetzen." ZFV 86 (1961), 177-179

APPENDIX I

NOMENCLATURE

I.1 General Conventions

1. Vectors are lowercase letters underscored, e.g., $\underline{a}$.
2. Matrices are uppercase letters, e.g., A, or letters in parentheses, e.g., (a_{ij}); or diag $\{a_1, a_2 \cdots\}$ in the case of diagonal matrix with diagonal elements being $a_1, a_2 \cdots$, or diag $\{A_1, A_2, \cdots\}$ in the case of block diagonal matrix with A_i being a submatrix.
3. Terminologies are consistent with

   i) Wells and Krakiwsky (1971) for the least squares adjustment;

   ii) Baarda (1968) and Rao (1973) for statistics;

   iii) Chrzanowski (1981a) and Chen(1983) for deformation analysis;

   iv) Grafarend (1974) for the optimization of networks.

I.2. Symbol Definition

ε	is an element of
:=	equal by definition
I	identity matrix
Θ	Khatri-Rao product
vec	an operator obtained by stacking the columns of a quadratic matrix one under another in a single column

vech	an operator obtained by stacking the columns of a quadratic matrix one under another in a single column starting each column at its diagonal element

I.3 Notations and Operations on Matrices A and B

A^T	transpose of A
$\|A\|$	determinant of a square matrix
$A^\perp$	a matrix of maximum rank such that $A^T A^\perp = 0$
A^-	any generalized inverse of A (g-inverse) such that $AA^-A = A$
A^+	pseudo-inverse or Moore-Penrose inverse such that $AA^+A = A,\ A^+AA^+ = A^+,\ (AA^+)^T = AA^+,\ (A^+A)^T = A^+A$
$r\{A\}$	rank of A
$S(A)$	linear vector space generated by the columns of A
$rd\{A\}$	rank defect of A
$Tr\{A\}$	trace of A, equal to $\sum_i a_{ii}$
$P_{A/B}$	projection operator onto S(A) along S(B)

I.4 Notations and Operations on vectors and Random Variables

$\|\| \underline{x} \|\|$	the norm of, or the length of $\underline{x}$
$\langle \underline{x}, \underline{y} \rangle$	inner product of $\underline{x}$ and $\underline{y}$
$\underline{x} \sim d$	$\underline{x}$ is distributed as
$E\{\underline{x}\}$	expected value of $\underline{x}$
$D\{\underline{x}\}$	dispersion matrix of $\underline{x}$
$V\{\underline{x}\}$	variance of random variable $\underline{x}$
$pr\{\xi > \xi_\alpha\}$	probability when $\xi > \xi_\alpha$

APPENDIX II

The Problems of QP, LP, LCP

This appendix describes the problems of Quadratic programming(QP), Linear Programming(LP), and the Linear Complementary Problem(LCP). For the details of the solution methods, some references are suggested.

1) The Problem QP

Generally, the standardized form of the problem of Quadratic Programming can be stated as:

$$\text{Minimize } \frac{1}{2}\underline{x}^T H \underline{x} + \underline{c}^T \underline{x} \tag{II-1}$$

$$\text{Subject to } A\underline{x} \le \underline{b} \tag{II-2a}$$

$$\underline{x} \ge \underline{0} \tag{II-2b}$$

Where $\underline{x}$, $\underline{c}$ are n by 1 vectors;

H is a symmetric n by n matrix;

$\underline{b}$ is a m by 1 vector; and

A is a m by n matrix

When introducing a n+m by 1 vector $\underline{v} = (v_1^2, v_2^2, \cdots, v_{m+n}^2)^T \ge \underline{0}$ of slack variables as well as the vector of Lagrangian multipliers $\underline{\lambda}$, the Lagrangian function can be set up as:

$$L_1(\underline{x}, \underline{v}, \underline{\lambda}) = \frac{1}{2}\underline{x}^T H \underline{x} + \underline{c}^T \underline{x} + \underline{\lambda}^T\left(\begin{bmatrix} A \\ -\mathbf{I} \end{bmatrix}\underline{x} + \underline{v} - \begin{bmatrix} \underline{b} \\ 0 \end{bmatrix}\right) \tag{II-3}$$

With **I** the n by n identity matrix. The minimum value of $L_1(\underline{x}, \underline{v}, \underline{\lambda})$ can be obtained by setting the partial derivatives to zeros

$$\frac{\partial L_1}{\partial \underline{x}} = H\,\underline{x} + \underline{c} + [A^T \; -I]\,\underline{\lambda} = \underline{0} \qquad \text{(II-4)}$$

$$\frac{\partial L_1}{\partial \underline{\lambda}} = \begin{bmatrix} A \\ -I \end{bmatrix}\underline{x} + \underline{v} - \begin{bmatrix} \underline{b} \\ \underline{0} \end{bmatrix} = \underline{0} \qquad \text{(II-5)}$$

$$\frac{\partial L_1}{\partial v_i} = 2\,\lambda_i\, v_i \quad \text{for } i=1, \cdots, m+n \qquad \text{(II-6)}$$

Eq.(II-6) may be reformulated as

$$\underline{\lambda}^T \underline{v} = \underline{0} \;\text{ with }\; \underline{\lambda} \geq \underline{0} \text{ and } \underline{v} \geq \underline{0} \qquad \text{(II-7)}$$

Eq. (II-4) - Eq.(II-6) are the famous Kuhn-Tucker conditions for Quadratic programming, which are necessary and efficient to get feasible solutions x for the problem QP. The problem QP has a unique minimizing solution when H is positive definite and the constraints (II-2) are feasible. Algorithm for the solution of a Quadratic Programming problem can be obtained from Boot (1964). Alternatively, the above set of Kuhn-Tucker conditions may be rewritten as

$$\underline{v} = M\,\underline{\lambda} + \underline{k} \qquad \text{(II-8a)}$$

$$\underline{\lambda}^T \underline{v} = \underline{0} \;\text{ with }\; \underline{\lambda} \geq \underline{0} \text{ and } \underline{v} \geq \underline{0} \qquad \text{(II-8b)}$$

where $M = \begin{pmatrix} A\,H^{-1}\,A^T & -A\,H^{-1} \\ -H^{-1}\,A^T & H^{-1} \end{pmatrix}$

$$\underline{k} = \begin{pmatrix} A\,H^{-1}\,\underline{c} + \underline{b} \\ -H^{-1}\,\underline{c} \end{pmatrix}$$

This is called the Linear Complementarity Problem (LCP), which will be discussed later.

2) The Problem LP

The standardized form of the problem of Linear Programming is written as

Minimize $\underline{c}^T \underline{x}$ (II-9)

Subject to $A_1\,\underline{x} = \underline{b}_1$ (II-10a)

$$A_2 \underline{x} \leq \underline{b}_2 \qquad \text{(II-10b)}$$

$$\underline{x} \geq \underline{0} \qquad \text{(II-10c)}$$

Where $\underline{x}$, $\underline{c}$ are the n by 1 vectors;

A_1, A_2 are the m_1 by n and m_2 by n matrices respectively; and

$\underline{b}_1$, $\underline{b}_2$ are the m_1 by 1 and m_2 by 1 vectors respectively.

There are different ways to approach the solution of a linear programming problem, in which the Simplex Method due to G. Dantzig is firstly recommended. Details of the approach refers to G. Dantzig(1963).

3) The Problem LCP

The problem of LCP is defined as

Find $\quad \underline{v}, \underline{\lambda} \geq \underline{0}$

Such that $\quad \underline{v} = M\underline{\lambda} + \underline{k} \qquad \text{(II-11a)}$

$$\underline{v}^T \underline{\lambda} = 0 \qquad \text{(II-11b)}$$

From Eq.(II-8), we can see that every Quadratic Programming can be transformed into an equivalent LCP. In order to solve LCP, the Complementary Pivot Theory is applied. Details of the method can be obtained from Cottle and Dantzig(1968), Lemke(1968). Liew and Shim(1978) created a computer program suitable for the solution of this problem.

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"Study of Outlier Detection and Position Estimation for Point Positioning in Marine Surveying", Journal of Wuhan Technical University of Surveying and Mapping(WTUSM), pp. 85-97, Vol. 12, No.4, 1988.

"Using Minimax Principle to Estimate Surveying Parameters and Perform Accuracy Analysis", Journal of Hohai University, pp. 16-23, Vol.17, No.3, 1989.