THE COMPILATION OF A MAP OF RECENT VERTICAL CRUSTAL MOVEMENTS IN CANADA

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PREFACE

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Funding for the research discussed in this Technical Report was provided by an Energy, Mines and Resources Canada Research Contract.

The original contract report contained 14 appendices. As reprinting all these appendices would have been prohibitive, only Appendix N, the most meaningful appendix, is contained in this Technical Report. Anyone interested in obtaining the other appendices are advised to contact the authors.

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1 Introduction.

Ten years have elapsed since a first attempt was made to compile a map of vertical crustal movements in Canada (Vanícek and Nagy, 1980). A considerably larger database has been gathered since then. The sea level records used in this new study are not only ten years longer but we also also added 28 US tide gauge records in support of the southern portions of the map. This new study employs 39426 Canadian first order relevelled segments, six times as many as used in 1980, plus an additional 6231 first order relevelled segments distributed in the northern US. This study also adds much more lake level information in time and space than used previously. A total of 105 lake level tilt elements were determined over 17 lakes across Canada and the US, compared to 39 tilt elements used in 1980 over the Great Lakes exclusively. We were able to use 73 sea level trends now compared to the 48 available in 1980.

We have gained additional insight into the nature of the data going in the making of this new map. We were forced to devote an unexpectedly large amount of time verifying the integrity of all sea and lake water level records for the map. Our analyses included the study of the physical response and technical behaviour of Canadian tide gauges (Carrera and Vanicek, 1989), individual numerical analysis of hourly, daily and monthly record at various Canadian sites, and a search of historic records to expand the database and to verify the benchmark stability at all Canadian tide and lake level gauge sites. This process lead us to eliminate for a variety of reasons 28 Canadian and one US tide gauge records, as well as four records collected in two Canadian lakes.

Another new feature included in this study is the introduction of directional statistics on the relevelled segments which enables us to make an informed decission about the kind of regression surface that should be used with the data in each area. Our screening of the levelling data was based solely on a maximum amount of tilt allowed in each relevelled segment, i.e. one metre per 100 km per century. The application of this criterion lead us to delete 2867 Canadian segments from the database provided to us by the Geodetic Survey of Canada. More than 90% of these deleted segments were shorter than 1 km in length. This same rejection criterion was used to screen US levelling data from the levelling database produced in Cornell University (Coyne and Brown, 1988).

The contract awarded to the University of New Brunswick by the Department of Supplies and Services stipulates that a spatial prediction of vertical velocities, or uplift rates, should be carried out for Canada. We have attempted to do this by fitting a vertical velocity surface over all the sea level linear trends, lake level tilts, and levelling height difference differences data available today. This is the only technique capable of accomodating in one model these three kinds of information when the relevelled segments are scattered not only in space but also in time. We have obtained a single national solution plus several regional solutions that allow us to identify smaller features in the map.

The contract contemplated the possibility of comparing our results with those of the National Map of Radio-Carbon curves for Canada being assembled by Bernard Peltier in the Geological Survey of Canada. Unfortunately, this data was not available to us. Their compilation was not completed at the end of this project and the authors decided to withold all of its information until its future publication date. Our results in the east coast, however, confirm our earlier findings that radiocarbon curves and sea level linear trends tend to agree in sign and have a difference in magnitude of 10 cm per century.

Some geophysical features, such as postglacial rebound, are readily apparent in the map. However, we have concentrated our effort on the production of the database and the compilation of the map itself and thus no attempt has been made to interpret all the features in it. Recent reviews of the many sources of vertical crustal movements can be found elsewhere (Lambeck, 1988, chapter 10; Aubrey and Emery, 1990).

Although the advent of extra-terrestrial positioning techniques such as the Global Positioning System (GPS), Satellite Laser Ranging (SLR), and Very Long Baseline Interferometry (VLBI) makes it possible today to determine crustal movements more accurately than ever before (Mueller and Zerbini, 1989), anyone wishing to study the geodynamical phenomena that have taken place in Canada over the course of this century will be forced to use a database of levelling, sea and lake level trends such as the one produced here. We feel that the compilation of this much more improved database is at least as important as the compilation of the map itself.

2 Data and Error Analysis.

2.1 Sea Water Level Data.

The Canadian sea level information analyzed in this study includes monthly, and daily tide gauge values recorded at 73 sites and was provided to us by Mr. D. Spear, Marine Environmental Data Services (MEDS). Mr. D. Zilkoski, National Geodetic Survey (NGS), NOAA, provided us with 29 monthly tide gauge records from the northern US. From the total amount of 102 sites, 60 are located on the Atlantic Ocean, 13 on the Arctic and Hudson Bay, and 29 on the Pacific Ocean.

Our work started with the time series of monthly mean sea level values knowing well that the virtues of monthly mean as an effective filter of high frequency oceanic signals had been questioned, for example, by Godin (1972, p.56), Walters and Heston (1982) and Cartwright (1983). Our previous experiments had shown that the use of monthly means produces a "leakage" of frequency signals into the frequency spectrum of a sea level record but has very little effect on the linear trend of a ten year or longer time series. However, our problems with these monthly records had an origin of a different nature: outliers and systematic errors.

The first outlier detection procedure applied to the data was the smothness test suggested by Cartwright (1968). This test was applied to all records. A quintic polynomial was used to interpolate sea level values

$$S'_n = 0.75(S_{n+1} + S_{n-1}) + 0.30(S_{n+2} + S_{n-2}) + 0.05(S_{n+3} + S_{n-3}). \qquad (1)$$

The time series containing the differences between the observed S_n and interpolated S'_n sea level values

$$\delta S_n = S_n - S'_n, \qquad (2)$$

helps to detect potential isolated errors in the original data. An outlier in the original data produces a well defined sub-sequence of values. The local extreme of this sequence permits to find the location in time of such an isolated error. Outliers in this context may be produced either as the result of an error, human or mechanic, in the recording of the data or, alternatively, as consequences of severe storms at the time of recording.

A critical value for the detection of outliers in the residual time series must be postulated. Any difference given by the above equation greater in absolute value than this critical value is flagged as a possible outlier. Cartwright (1968) used a critical value of 0.092 m (0.3 ft), however, Zetler et al. (1979) found this value to be too stringent a requirement for tide gauge observations in Atlantic City, N.J. and used, instead, twice this amount. A critical value close to this later criteria was used in the present study (0.2 m). The set of values flagged as potential outliers were then examined going back to the daily and hourly time series used to derive them. Tide gauge records containing outliers were also differenced with neighbouring gauges and the difference time series was also tested for smoothness. Most of the outliers found in our study were found to be the result of blunders in the hourly, daily or monthly records or the result of weather storms, undocumented datum biases or artifacts of the averaging procedure employed by the Marine Environmental Data Services (MEDS) to arrive at monthly values.

A monthly mean value is published by MEDS in Canada if there are at least 21 continuous days of sea level values available in any one month. These monthly values were derived in the past from daily values and, more recently, from hourly values. This procedure introduced not only an occassional monthly bias due to the lack of data in some days but is further compounded because missing hourly values can introduce a bias in the daily means as well.

The test for smoothness described above to the sea level data above is designed to detect abrupt isolated errors, or outliers, but it does not provide any information on errors that could affect the data gradually in time. Marine growth and silting are two examples of these gradual processes. It has been shown that their presence induces a decrease in the cut-off frequency of the tide gauge (Carrera and Vanicek, 1989). If marine growth and silting reduce the inlet area of the tide gauge in excess of 75% the cut-off frequency may be decreased to reach the tidal frequency band. A gradual decrease in the cut-off frequency of a tide gauge results in a reduction in the amplitude of various tidal constituents and in an increase of their phases. We monitored the amplitude and phase variability of the most important diurnal (O1 and K1) and semi-diurnal (M2 and S2) tidal constituents in Halifax and Pointe au Père to detect any problems and found none in Halifax. But a portion of the record data after 1983 in Pointe au Père had to be eliminated after confirmation of silting problems with personnel of the Canadian Hydrographic Service in Quebec.

We found several undocumented datum changes in the monthly records: Halifax (29 cm, January 1987), Point Tupper (39 cm, January 1984), North Sydney (28 cm, January 1985), Riviere au Renard (12 cm, August 1975) and Baltimore in the US. More Puzzling was the bias found in the record of Boutilier Point (21 cm, January 1971): the data given to us by MEDS in a computer readable form do not agree with the values listed in their own publication (Fisheries and Oceans, 1984). We were forced after this discovery to verify visually the integrity and agreement of all computer generated records with their printed counterparts for all sites in Canada.

We also felt obliged to investigate the stability of most Canadian tide gauges by looking into the yearly levelling records that describe their ties with its neighbouring benchmarks. The instability of local datums in time either as the result of environmental problems such as the one posed by the presence of ice, unstable wharfs or human errors resulted in the introduction of monthly biases as well. The tide gauges located in Nain and West Saint Modeste were affected by ice, Savage and Point Sapin were located on unstable wharfs, and Grindstone Island was affected by human errors. The tide gauge located in Sandy Hook suffered from sinking in an area of beach erosion (McCann, 1981). All of these records had to be eliminated from the sea level database.

The database of sea level measurements in Arctic Canada is formed by 11 records. These records were measured at the same geographic locations but by means of different tide gauges each of which was replaced every year. It is uncertain whether observations carried out over consecutive years were referred to the same datum to an accuracy required by the present study (O'Reilly, pers. comm.). The only exception to this rule might be perhaps the record gathered in Resolute, N.W.T. where a permanent site was used for a number of years. We have concluded that the present state of the database in the Arctic renders it useless for any long term sea level study. The poor state of these records is explained by the fact that most of these observations were gathered in support of hydrographic surveys. The task of trying to rescue any long term information from these data by putting together the pieces of these records is further complicated by the fact, first, that no two contiguous yearly records overlap in time at any one location and, secondly, that these records are affected by unaccounted drift problems in some of the instruments themselves. Thus, any such attempt must be made on the basis of predictions of hourly values from contiguous years. This attempt, however, was considered beyond the scope of this study due to the time constraints imposed by the contract.

Additional insight was gained into the performance of individual tide gauges later in the project when their spatial coherence with neighbouring gauges was investigated through their correlation matrix. The spatial coherence of linear trends casted additional light on their performance.

Appendix A contains 22 figures displaying various real or suspected errors in the original hourly or monthly tide gauge database. Appendix B contains 84 plots of the Atlantic, Hudson Bay and Pacific monthly records for all sites investigated as potential candidates to be included in this study and corrected for errors whenever this was possible.

It appears from the error analysis performed in this study that past accuracy claims of 1 mm for monthly averages (Lennon, 1971; Lambeck, 1988, p. 109) obtained for even well maintained tide gauges are too optimistic.

2.2 Lake Water Level Data.

The Canadian lake level information analyzed in this study included monthly and daily water level values recorded at 75 sites on 19 major lakes and was provided to us by Mr. P.J. McCurry, Inland Waters Directorate (IWD). Mr. D. Zilkoski (NGS) provided us with monthly tide gauge records for 34 sites located on the US shores of the Great Lakes.

A description of the actual sites and number of instruments used to gather the records at each geographic location over the Great Lakes Region is given in the reports issued by the Coordinating Committee of the Great Lakes (1978, 1987). Following the recommendations issued by these reports all data recorded prior to 1916 were removed from the records. Analogue water level recording systems appear to have been introduced for the first time in that year. Many of the records obtained prior to that year were gathered by means of visual inspections of staffs a few times a day and there is no way to verify their integrity.

The smoothness test described in the previous section was also applied to differences of water level records from the same lake. Again any flagged outliers were investigated going back to daily mean records and any new differences were recomputed. An undocumented datum shift was found in the record for Toledo, Ohio in Lake Erie. These data were checked against printed records and corrected when possible or eliminated altogether.
The record obtained in Sault Ste Marie was eliminated from the analysis because water currents in the vicinity of the gauge affected its readings (Kite, 1972a).

A source of concern in the Great Lakes are coastal changes due to erosion (Birkemeier, 1981). We have verified the location and condition of most wharfs in this area and could not find any instability similar to the one affecting Sandy Hook in the Atlantic.

Similarly to sea level records, we also found monthly biases due to the method used to obtain monthly averages. This problem is more critical in a lake than at sea because the lake surface is less constrained to stay at the same level than sea level is. Our error analysis forced us to remove the portion recorded prior to 1925 in Port Colburne due to an undocumented datum bias. Two records obtained in the Lake Williston and other two in the Arrow Reservoir, B.C. were removed entirely from the database due to the presence of large outliers that could not be accounted for. Several records did not meet our requirement of being at least five years long and have been thus excluded from the final lake level database.

The underlying hypothesis that justifies the use of lake level information in the determination of crustal movements is that lake levels conform to a level of hydrostatic equilibrium, i.e.,

$$\frac{\partial p}{\partial z} = -\rho g. \tag{3}$$

Actual measurements of the water density distribution in the Great Lakes serves to determine the extent to which this assumption holds true (Csanady, 1984). Two further phenomena that can seriously question this assumption are river discharge and wind setup. We decided to eliminate from our study lake gauges clearly exposed to direct water currents from nearby rivers or reservoir outlets such as those in Sault St Marie and eliminated monthly means from all records at times of large spring streamflows.

Wind forcing over a shallow layer of fluid produces larger surface slopes than the same force acting over deep water. This can be easily illustrated by the solution for the free surface slope produced by a simple model with a uniform wind stress acting over a basin of constant depth H (Csanady, 1984, p. 26).

$$\zeta = \frac{u_*^2}{gH} y, \tag{4}$$

where ζ is water level, u_* is friction velocity, g is gravity, and y is oriented to ensure conservation of total water mass.

In shallow basins mean velocities are one order of magnitude weaker than wind driven transient flow. Csanady (1984, § 8.4) has pointed out the potential biasing effect that one or two wind episodes can have over monthly averaged quantities during winter months. Coastally trapped waves such as those in Lake Ontario with periods of 12-16 weeks further compound the problem.

Under summer conditions water level changes take place over time scales much too close to a month not to distort monthly averages. Only data from June, July, August and September when summer stratified conditions persist were included in our analysis (Csanady and Scott, 1980).

Appendix C contains 116 plots of the data provided to us originally for this study. From these only 105 records were actually used in the compilation of the map of vertical crustal movements for the reasons described above.

Additional insight in the behavior of individual differences is gained when their spatial coherence with other intra-lake differences is investigated in section 3.2 through the use of their correlation matrix. Similarly, spatial coherency of linear trends in any given lake can be used as tool to detect the abnormal behaviour of any pair of gauges.

2.3 Geodetic Re-levelled Segments.

The original Canadian 1st order levelling database provided to us by Dr. A. Mainville (GSC) contained 42293 relevelled segments (in a single 1600 bpi magnetic tape). The original US levelling data were supplied to us (in three 1600 bpi magnetic tapes!) in a format that did not allow us to learn the exact number of segments in it. This was due to the topology used originally to divide the data into regional blocks, i.e., a single relevelled segment that goes beyond a single block appears again in its entirety in all other blocks through which it runs. Special care was taken to avoid the presence in our database of the same segments coming from different blocks. The total number of relevelled segments extracted from the database in the northern US was 6231.

Although the Canadian relevelled segments included in this study are the result of first order levelling surveys, no refraction corrections, rod settlement corrections, magnetic corrections or tidal corrections have been applied to them to date. The accuracy or the nature of gravity field corrections is not likely to affect the data when the two differenced levellings follow close paths. The original contract envisaged that all these corrections would be available to us, but could not be accomplished prior to the deadline of this study. When these corrections become available we can either apply them directly to the data and recompute the map or "correction surfaces" can be generated and applied to the existing maps. The US relevelled segments extracted from the University of Cornell database are also the result of first order levelling surveys to which various kinds of gravity, refraction and magnetic effects corrections have been applied. Both sets of data were taken "as given" and no attempt was made on our part to investigate their accuracy or the quality of the corrections applied to them. These are probably the best sources of levelling data available in North America today.

Our only quality control criterion applied to all relevelled segments was implemented by means of a rejection criterion for height difference differences per distance in time, or tilt, greater than one metre per 100 km per century, $T = 1 \times 10^{-5}$, i.e.,

$$T = \frac{\Delta\delta h / S}{\Delta T} \tag{5}$$

where $\Delta\delta h$, is the height difference difference in m, S, is the distance in m along the levelling path, and, ΔT, is the period in years between the two levelling campaigns. The application of this criterion lead us to eliminate 2867 Canadian relevelled segments, 90% of which were shorter than 1 km in length. This same rejection criterion was used to screen US levelling data. The purpose of this rejection criterion was to eliminate locally disturbed benchmarks as well as potential blunders from our records. The final levelling database used to compile the map of vertical crustal movements was formed by 39426 Canadian and 6231 US segments.

The Canadian and northern US territory was divided into 184, 4 by 4 degree blocks spanning from latitudes 40 N to 72 N and from longitudes 52 W to 144 W to get a better overview of the spatial distribution of the levelling data. We proceeded to determine the average length, $\bar{L}$, between the beginning and end points (along a geodesic on the surface of a geodetic ellipsoid and not along the levelling path) of all the relevelled segments within each block. The rationale to determine the mean in this form is that although the distance along the levelling path is of value to us to weight the observations going into the mathematical model the distance over the ellipsoid sheds more light into the geometric strength of the data.

The directional statistics obtained for the relevelled segments within each block (Mardia, 1972; Watson, 1983) are the circular mean, $\bar{\alpha}$, and circular variance, S_o, where

$$L = \frac{1}{n}\sum_{i=1}^{n} L_i, \tag{6}$$

where

$$\bar{\alpha} = arc\tan\left(\frac{S}{C}\right), \tag{7}$$

where

$$C = \frac{1}{n}\sum_{i=1}^{n} \cos(\alpha_i), \tag{8}$$

$$S = \frac{1}{n}\sum_{i=1}^{n} \sin(\alpha_i), \tag{9}$$

are the trigonometric moments and

$$S_o = 1 - R. \tag{10}$$

for circular data $(0° \le \alpha < 360°)$, and

and

$$S_o = 1 - (1 - (1 - R)^{1/4}. \tag{11}$$

for directional data $(0° \le \alpha < 180°)$, where

$$R = \sqrt{C^2 + S^2}, \qquad (12)$$

is the mean resultant length that can be used for both circular and directional data provided that the angles in the second case are first doubled and the circular mean is halved (Mardia, 1972, p. 169).

The circular variance is a dispersion measure that takes on values between the closed interval [0,1]. A value of 0 represents no dispersion and 1 means a uniform distribution over the circle. Tables 1 and 2 list the values of the average geodesic length, circular mean and variance only for those blocks that contain more than one relevelled segment within the Canadian and US territories, respectively.

Appendix D contains 66 plots showing histograms to present the distance distribution of Canadian relevelled segments within each of the 4 by 4 degree blocks. Appendix D also contains 41 additional plots showing histograms to present the orientation distribution of Canadian relevelled segments within each block that contains more than one segment. Any one relevelled segment that crosses two or more blocks is listed only within the block in which the starting benchmark is located, i.e., multiple listings of the same segment in different blocks was avoided.

Appendix E contains 40 plots showing histograms of the distance distribution of US relevelled segments within each of the 4 by 4 degree blocks within the grid. Appendix E also contains other 40 additional plots showing histograms to present the orientation distribution of US relevelled segments within each block that contains more than one segment. These histograms of US data were obtained accessing the US database directly and it is quite possible that some segments might have been listed in more than one block (We made sure, however, that duplication of any relevelled segment in our database used to compile the map was totally avoided).

The results shown in tables 1 and 2 were consulted every time that a decission had to be reached in regard to the powers in latitude and longitude of the base functions used in the regressions.

Table 1. Canadian Levelling Data Length and Directional Statistics.

Block Number	Average Length (km)	Circular Mean (deg)	Circular Variance *
1 3	2.409	125.3	0.063
2 3	3.937	80.9	0.086
2 4	6.726	77.2	0.002
2 5	1.134	98.8	0.059
3 1	22.148	140.6	0.040
3 2	13.477	4.2	0.081
3 3	8.569	134.3	0.065
3 4	5.579	72.2	0.047
3 5	34.322	133.0	0.099
3 6	4.263	142.2	0.130
3 7	11.507	148.2	0.173
3 8	5.607	72.4	0.122
4 4	29.069	164.1	0.102
4 5	24.691	125.7	0.061
4 6	3.601	82.7	0.144
4 7	9.461	176.0	0.298
4 9	5.614	133.3	0.122
4 11	1.141	12.7	0.011
4 13	25.516	2.3	0.034

4 19	0.802	158.3	0.000
5 4	10.775	179.8	0.268
5 5	21.194	116.2	0.035
5 6	12.228	141.9	0.257
5 7	5.365	122.8	0.503
5 8	4.179	107.0	0.228
5 9	9.128	174.9	0.250
5 10	7.719	16.7	0.422
5 11	8.147	81.1	0.228
5 12	2.249	36.1	0.246
5 13	0.839	178.6	0.104
5 17	8.375	76.2	0.054
5 20	3.309	1.3	0.111
6 5	6.038	131.7	0.195
6 6	4.764	140.7	0.345
6 7	6.425	6.2	0.239
6 8	4.765	149.8	0.308
6 9	5.913	83.2	0.080
6 10	5.399	174.3	0.271
6 11	8.049	155.8	0.352
6 12	4.797	103.8	0.316
6 13	14.460	96.7	0.444
6 14	9.813	8.3	0.323

6 15	13.086	27.8	0.069
6 16	2.542	112.3	0.092
6 17	4.591	96.6	0.353
6 18	3.767	55.3	0.315
6 19	3.869	20.1	0.218
6 20	7.906	156.9	0.122
6 22	12.064	85.4	0.050
6 23	7.017	99.7	0.086
7 15	1.808	123.0	0.155
7 16	2.114	111.6	0.202
7 17	3.025	82.6	0.468
7 18	1.111	59.5	0.432
7 20	6.089	90.4	0.205
7 21	4.568	86.4	0.274
7 22	8.918	150.7	0.192
7 19	3.492	33.2	0.252
7 23	4.175	12.8	0.536
8 16	2.018	146.0	0.414
8 17	1.561	53.6	0.444
8 20	7.649	163.3	0.301

* The circular variance has a dimensionless range within [0,1]

Table 2. US Levelling Data Length and Directional Statistics

Block Number	Average Length (km)	Circular Mean (deg)	Circular Variance *
3 1	9.585	130.1	0.064
4 3	22.483	27.6	0.000
6 5	60.056	160.5	0.041
6 6	46.109	166.9	0.194
6 7	33.149	140.6	0.204
6 8	74.795	137.9	0.244
6 9	67.965	89.4	0.062
6 10	88.593	92.1	0.095
6 11	27.850	100.1	0.259
6 12	34.626	132.6	0.234
6 13	32.862	92.0	0.305
7 6	22.496	112.5	0.363
7 7	29.406	63.5	0.497
7 8	60.737	114.1	0.309
7 9	67.057	70.8	0.381
7 10	43.217	40.8	0.386
7 11	41.745	149.6	0.308
7 12	38.933	105.7	0.175
7 13	18.943	113.6	0.285
7 14	75.261	154.4	0.226
7 15	35.422	44.9	0.166

7 16	13.383	143.9	0.010
7 17	93.488	79.9	0.019
7 18	31.253	4.5	0.105
7 19	18.787	59.7	0.285
7 20	28.911	42.7	0.203
8 5	79.168	2.3	0.010
8 6	91.827	8.6	0.050
8 7	82.030	141.8	0.032
8 8	74.994	9.9	0.081
8 9	198.216	164.0	0.017
8 11	203.735	134.5	0.007
8 13	105.054	48.9	0.011
8 14	75.137	177.2	0.129
8 15	49.470	161.2	0.248
8 16	34.230	22.7	0.405
8 17	15.254	71.8	0.500
8 18	21.674	176.7	0.329
8 19	15.690	28.1	0.103

* The circular variance has a dimensionless range within the closed interval [0,1].

3 Data Pre-processing.

3.1 Sea level linear trends and their correlation matrices.

The first problem that has to be addressed when trying to determine vertical crustal movements from sea level data is to decide what kind of regression model should be used to minimize oceanic signals and still leave untouched the signatures of geodynamic processes. Our first experiment was to fit simple linear trends to the records and determine the spectrums of the residual time series to identify individual oceanic or river discharge signals common to various gauges. Appendix F contains 2 plots showing the equivalent least squares spectrum and maximum entropy spectrum for Baltimore, Md., the only continuous tide gauge record in North America, as well as 10 other plots showing the least squares spectra of tide gauges in the estuary of the St Lawrence River. Not unexpectedly, we found the annual and semi-annual constituents to be the dominant frequencies in the residual time series and all the spectra to be "red", i.e., increasing in value towards the low frequencies. We proceeded then to perform new regressions removing as well these two periodic constituents from the time series and then compared the two sets of linear trends to investigate their sensitivity. The first 12 plots contained in Appendix G show the value of and the change in the linear trend induced by removing the annual and semi-annual constituents from tide gauges located on the Atlantic and Pacific Oceans. Not surprisingly, the linear trends determined from shorter records experience larger changes. The next 3 plots in Appendix G show the change in the correlation matrices of Pacific time series residuals obtained from regressions in which annual and semi-annual constituents were first ignored and later introduced. These results forced us to eliminate other tide gauges such as the one located in New Westminster, B.C. which is largely affected by river discharge. We also found that the three locations with the longest records in the Atlantic, Hudson Bay, and Pacific Canada, were Halifax, Churchill, and Victoria. Tables 3 to 5 show the results of Chi-square tests over the residuals obtained from regressions, including annual and semi-annual constituents, of the data gathered at these three sites.

These results show significamce levels greater than 0.01 for the regressions in Churchill and Halifax but also indicated a very low significance level for Victoria where in addition to the 6 expected residuals under a normal distribution, 9 monthly residuals went below -17 cm, some of which are directly associated with the times of occurrences of El Niño (cf. Appendix L).

The question still remains of how stable and how free from oceanic signals are the individual linear trends of all the records even after removing the dominant frequencies in the spectrum, i.e., are there very low frequency signals present in addition to tides that could bias the trend of short sea level records? Sturges (1987) has recently provided some information on this subject. He was able to identify very low frequency oceanic signals that could affect linear trends shorter than 40 years long. Other studies point in similar directions (e.g., Enfield and Allen, 1980; Chelton and Davis, 1982; Chelton and Enfield, 1986; Enfield 1989) due to phenomena such as El niño. It appears to us that only two approaches are available in a study such as this. We can either determine linear trends from a regression model which attempts to remove all oceanic sources of sea level variability at each site, be that in the time or frequency domain, or we can use differences of sea level values between neighbouring gauges hoping that common oceanic signals are cancelled out or attenuated in the process.

Prior to the determination of differences of sea level time series, we computed three regional correlation matrices of the sea level residuals time series from all regressions between tide gauges located in the regions of the Estuary of the St Lawrence River, and the Atlantic and Pacific Coasts. Appendix H contains 11 plots each of which represents a row of the correlation matrix for gauges located in the estuary of the St Lawrence River. Appendix I and Appendix J contain 53 and 29 plots each representing a row of the correlation matrices in the Atlantic and Pacific coasts, respectively.

Several of the time series included in the analysis contain gaps and also the individual series have different lengths. We had to decide between one of two ways to compute these correlation matrices. We either had to cut all the time series in each matrix to cover the maximum amount of observations common to all series, which would have produced a correlation matrix of small dimensions, i.e., including very few gauges, or a larger correlation matrix but constrained by the length of the shortest record in the entire database. Alternatively, the other option was to simply select all the pairs of sea level values common to any two time series and compute the Pearson correlation coefficient (,19) between them.

A justification for the use of this second approach is required. The underlying assumption for this approach is that the maximum of the correlation function between any two time series takes place at zero lag. This assumption is proved to be true as shown by the 4 plots of correlation functions shown in Appendix K. It should be noted that another implication of using this approach is that the probability level associated with each element of the correlation matrix is in general different.

The three correlation matrices allowed us to identify the anomalous behaviour of the following tide gauges on the Atlantic coast St Joseph, Riviere du Loup, Gros Cacouna, Nain, West St Modeste, Savage, Grindstone, Point Sapin, and Sandy Hook. On the Pacific coast, New Westminster was eliminated due to the large effect of the Fraser River, and other gauges that were eliminated as well from our database were Comox, Langara Point, Port Simpson, and Bella Coola.

It should be pointed out that correlation matrices such as the ones described above have also been used as the first step in the studies of the spatial coherence of linear trends by means of eigenanalysis (e.g., Barnett, 1978; Barnett, 1984; Aubrey and Emery, 1983; and Peltier and Tushingham, 1989) who have applied empirical orthogonal functions (eof) or principal component analysis to sea level data. For a review of the properties of eof the reader is adviced to look at the work of Backus and Preisendorfer (1978).

Table 3. Chi-square Test for the residuals in Halifax, N.S.

Lower Limit	Upper Limit	Observed Frequency	Expected Frequency	Chisquare
at or below	-10.3333	12	8.4	1.54774
-10.3333	-9.0000	11	9.5	.23111
-9.0000	-7.6667	15	17.4	.32541
-7.6667	-6.3333	22	29.0	1.69997
-6.3333	-5.0000	43	44.3	.04041
-5.0000	-3.6667	47	62.0	3.60928
-3.6667	-2.3333	86	79.2	.58747
-2.3333	-1.0000	102	92.6	.96266
-1.0000	.3333	110	99.0	1.22826
.3333	1.6667	102	96.8	.28018
1.6667	3.0000	104	86.6	3.50227
3.0000	4.3333	58	70.8	2.32932
4.3333	5.6667	47	53.0	.68377
5.6667	7.0000	26	36.3	2.92013
7.0000	8.3333	22	22.7	.02313
8.3333	9.6667	7	13.0	2.77940
9.6667	11.0000	7	6.8	.00491
above 11.0000		12	5.6	7.38045

Chi-square = 30.1359 with 15 degrees of freedom.

Significance level = 0.0114409

Table 4. Chi-square Test for the residuals in Churchill, Manitoba

Lower Limit	Upper Limit	Observed Frequency	Expected Frequency	Chisquare
at or below	-19.5714	5	6.2	.2231
-19.5714	-16.7143	9	7.7	.2135
-16.7143	-13.8571	17	14.5	.4299
-13.8571	-11.0000	26	24.5	.0956
-11.0000	-8.1429	38	37.1	.0235
-8.1429	-5.2857	59	50.4	1.4665
-5.2857	-2.4286	54	61.5	.9213
-2.4286	.4286	55	67.4	2.2924
0.4286	3.2857	65	66.3	.0271
3.2857	6.1429	56	58.6	.1148
6.1429	9.0000	44	46.5	.1302
9.0000	11.8571	42	33.1	2.4107
11.8571	14.7143	32	21.1	5.5877
14.7143	17.5714	11	12.1	.1041
above 17.5714		5	11.0	3.2602

Chi-square = 17.3007 with 12 degrees of freedom.

Significance level = 0.138633

Table 5. Chi-square Test for the residuals in Victoria, B.C.

Lower Limit	Upper Limit	Observed Frequency	Expected Frequency	Chisquare
at or below	-17.000	15	6.1	12.7415
-17.000	-14.333	9	11.0	.3684
-14.333	-11.667	23	24.7	.1141
-11.667	-9.000	35	47.6	3.3263
-9.000	-6.333	49	78.9	11.3603
-6.333	-3.667	115	112.7	.0456
-3.667	-1.000	168	138.5	6.2670
-1.000	1.667	170	146.5	3.7636
1.667	4.333	149	133.3	1.8367
4.333	7.000	96	104.5	.6838
7.000	9.667	55	70.4	3.3729
9.667	12.333	32	40.8	1.9161
12.333	15.000	17	20.4	.5643
above 15.000		16	13.4	.5066

Chisquare = 46.8671 with 11 degrees of freedom.

Significance level = 2.27053E-6

3.2 Linear trends of sea level differences and their correlation matrices.

In building the network, or tree diagram, of pairs of differenced tide gauges records on both coasts, we have combined the information provided by the correlation matrices described in the previous section together with the information on the length of all sea level records, i.e., we aimed at the optimization of the length of the records of sea level differences keeping in mind the correlation among time series to help us determine how far can we safely go in the search of pairs of gauges to effectively cancel out oceanic noise and improving the spatial coherence of the linear trends.

Once these rules are set in place, several tide gauges appear to play a crucial role in the compilation of the map. On the Atlantic coast the tide gauges located in Pointe au Père, Charlottetown, Halifax, Eastport and New York (the Battery) play the dominant role in determining the values assigned to every other tide gauge in the solution. On the Pacific coast the tide gauges located in Neah Bay, Victoria, Vancouver, Point Atkinson, and Bella Bella appear to play the most prominent roles. All of these gauges contain very long records with the exception of Bella Bella. As a matter of fact, the values derived for Prince Rupert, B.C. and Ketchikan, Alaska, are distorted by this only weak link in the chain. This situation, however, can not be helped because tide gauges located along the B.C. coast tend to be concentrated in two clusters, one in the Juan de Fuca and Georgia Straits and the other north of Queen Charlotte Island. The only tide gauge located between these two groups has been in operation for a relatively short period in Bella Bella. The only other record in this area, obtained for a brief period in Bella Coola, was replaced by Bella Bella and is not in operation anymore.

Once the tree diagram was designed, linear trends for the sea level differences are found solving for annual and semi-annual constituents at the same time. Addressing the zero order design problem of the solution comes next. We again have two options, either do not use any weighted constrained in the solution and compute only relative crustal movements referred to the centroid of the solution or, alternatively, use weighted constraints. We decided to use the second approach to be able to compare our results with previous studies. The tide gauge sites selected as weighted constraints in our solution were Halifax, N.S., the longest running record in the Atlantic, Churchill, Manitoba, the longest record in Hudson Bay, and Victoria, B.C. the longest record in the Canadian West coast. Appendix L contains 18 plots, 6 of which are devoted to the analysis of the regression residuals of each of the three tide gauge records. Linear trends can be propagated from these three original locations throughout the rest of the tree diagram, or network, along the same lines as it would be the case in a levelling or gravity network.

The selection of the weighted constraint approach, however, poses another problem. Namely, the introduction, or lack therof, of a eustatic water rise correction. This issue is further discussed below in section 5.2.

The last issue to be addressed in this section is the determination of whether the oceanic noise is effectively suppressed by the differencing approach. We found, indeed, this to be the case by computing the correlation matrices of the residual of the regressions over sea level differences. We found that tide gauge time series differences that do not share a common tide gauge show very little correlation. Even those time series differences that share a common tide gauge might not use the same data with their neighbours and this leads often to relatively small correlation values among themselves. Appendix M contains 48 plots each of which represents a row of the correlation matrix of the residuals of regressions over sea level differences in the Atlantic coast. Similar results were found for the West coast, but are not presented here.

3.3 Linear trends of lake water level differences and their correlation matrices.

The tree diagram of lake level differences was formed solely on the basis of maximizing the length of the lake level differences records. Similarly to the situation encountered along the Atlantic and Pacific coasts, some lake gauges appear to play a more prominent role than others in various lakes. In Lake Ontario, the most important gauges are located in Toronto and Oswego. In Lake Erie, the most important gauges are located in Port Stanley and Cleveland. In Lake Huron, the most important gauges are located in Goderich, Collingwood, Thessalon and Mackinaw. In Lake Michigan, the most important gauges are those located in Milwaukee and Sturgeon Bay. Finally, in Lake Superior, the most important gauges are located in Michipicoten, Marquette, and Thunder Bay. The importance of other gauge sites within other lakes is apparent from the tables listed in the next chapter.

In determining linear trends over the differences of lake levels we did not find an annual or semi-annual component left in the data. Local effects, such as channels and river discharge, did not appear on the spectra of the residuals either. This might be explained by the fact that we have selected to work in our analysis with data recorded only during the four summer months June, July, August, and September of every year, when river discharge is at a low.

Lake gauge records contain also gaps. We decided to deal with them in the same manner as we did with the sea level information, i.e., differencing monthly values without the introduction of any lag knowing that the maximum value of the correlation function for every pair of lake level monthly records takes place at zero lag.

We often find in the Great Lakes sites where two different water level gauges were used to obtain records over different periods but at the same site. We elected to form a single record from the data gathered by these two instruments accounting for an additional datum bias in the regression of the differences.

4 Comparison with other studies.

4.1 Comparison of results with other studies in the Atlantic Coast.

The following tables summarize the results of this study on the Atlantic coast. Tables 6 and 7 list the linear trends produced in this study for Canadian and US sites, respectively, and compare them whenever possible with other studies. The value for linear trends that was used in the map is the one listed under the heading "propagating differences". Table 8 lists the values of Atlantic tide gauge linear trends obtained for sites excluded from the map. Table 9 lists the values of the linear trends of the sea level differences determined along the Atlantic in this study. Very few studies have followed this approach and only values computed by Dohler and Ku (1970) could be used for comparison. We consider it very important to note that short records display linear trend values much closer to their longer counterparts when the method of propagation of differences is used. As a way of an example the linear trend in Rustico, P.E.I., shows a value of -0.18 m/century when analyzed as a point value but when differenced acquires a value of -0.32 very close to the value found for the gauge in Charlottetown (-0.35 m/century) only a few tens of km away. The reason for this result is the attenuation of oceanic noise when the differencing method is used. It is also interesting to note the good agreement between our results propagating differences and those published by the Department of Commerce (1988) for tide gauges located in the US Atlantic coast.

Table 6. Comparison of sea level linear trends and their standard deviations in m/century in Atlantic Canada (*).

Location	Vanícek and Nagy 1980	Emery and Aubrey 1990	From Point Values	Propagating Differences
Quebec		-0.320 0.078	-0.028 0.023	-0.105 0.028
St Francois	-0.93 0.21	-0.253 0.080	-0.021 0.105	-0.170 0.067
St Jean Port Joli	-1.39 0.53		0.450 0.218	-0.088 0.165
Tadoussac		0.570 0.072	0.443 0.069	0.121 0.080
Pointe au Père	0.02 0.01	-0.051 0.100	0.012 0.010	-0.010 0.016
Baie Comeau	0.85 0.22		0.482 0.058	0.062 0.047
Ste Anne des Monts	0.61 0.23	0.217 0.096	0.324 0.050	-0.055 0.060
Sept Iles	-0.02 0.06		-0.215 0.034	-0.187 0.041
Riviere au Renard	1.06 0.23		0.320 0.055	-0.032 0.077
Harrington Harbour	0.05 0.02	0.033 0.100	0.060 0.014	-0.013 0.016
Lark Harbour	-0.68 0.13		-0.046 0.048	-0.297 0.040

Dalhousie			-0.419 0.101	-0.457 0.161
Lower Escuminac			-0.031 0.076	-0.212 0.048
Point du Chene			-0.126 0.082	-0.301 0.055
Charlottetown	-0.33 0.02	-0.307 0.100	-0.315 0.009	-0.355 0.011
Rustico	-0.01 0.05		-0.182 0.035	-0.328 0.047
Pictou	-0.31 0.12	-0.388 0.092	-0.221 0.045	-0.368 0.033
Port aux Basques	-0.72 0.03	0.179 0.097	-0.318 0.032	-0.375 0.026
Argentia			-0.062 0.082	0.073 0.073
St John's	-0.23 0.05	-0.138 0.100	-0.080 0.037	-0.193 0.036
North Sydney	0.79 0.47		-0.148 0.065	-0.387 0.046
Point Tupper	0.56 0.48		-0.255 0.073	-0.431 0.079
Halifax	-0.40 0.01	-0.370 0.100	-0.356 0.008	-0.356 0.008
Boutilier Point	0.08 0.26	0.173 0.090	-0.182 0.096	-0.397 0.049

Yarmouth	-0.28 0.03	0.053 0.088	-0.286 0.022	-0.475 0.035
St John	-0.36 0.02	-0.303 0.100	-0.279 0.012	-0.301 0.014

* The point values listed here were computed using all of the available data at each site. The next column lists the linear trends arrived at each site by propagating the linear trend from Halifax, N.S., using the linear trends of differences of sea level values among neighbouring tide gauges listed in Table 9.

Table 7. Comparison of sea level linear trends and their standard deviations in m/century in the Atlantic USA.

Location	Vaníček 1978	Department of Commerce 1988	Emery and Aubrey 1990	From Point Values	Propagating Differences
Eastport		-0.27 0.02	-0.367	-0.269 0.011	-0.271 0.011
Bar Harbor		-0.27 0.03	-0.278	-0.270 0.017	-0.296 0.020
Portland		-0.22 0.02	-0.217	-0.218 0.007	-0.183 0.024
Seavy Island	-0.119 0.024	-0.18 0.02	-0.179	-0.175 0.011	-0.105 0.025
Boston	-0.206 0.018	-0.29 0.02	-0.294	-0.225 0.009	-0.132 0.025
Woods Hole		-0.27 0.02	-0.274	-0.277 0.011	-0.241 0.026

Newport	-0.121 0.027	-0.27 0.02	-0.260	-0.262 0.010	-0.221 0.026
Providence	-0.213 0.024	-0.18 0.03	-0.180	-0.178 0.014	-0.169 0.027
New London		-0.21 0.03	-0.215	-0.199 0.014	-0.189 0.026
Bridgeport		-0.21 0.12	-0.127	-0.198 0.054	-0.290 0.030
Willets		-0.24 0.03	-0.244	-0.247 0.014	-0.205 0.034
Port Jefferson		-0.27 0.07	-0.267	-0.256 0.035	-0.270 0.037
Montauk		-0.19 0.05	-0.195	-0.199 0.021	-0.191 0.039
New York		-0.27 0.01	-0.308	-0.273 0.004	-0.241 0.034
Atlantic City		-0.39 0.02	-0.387	-0.383 0.010	-0.303 0.035
Philadelphia		-0.26 0.02	-0.273	-0.262 0.010	-0.388 0.045
Lewes		-0.31 0.04	-0.311	-0.309 0.015	-0.278 0.043
Kiptopeke		-0.31 0.05	-0.313	-0.317 0.029	-0.338 0.045
Baltimore		-0.32 0.01	-0.317	-0.314 0.007	-0.376 0.047

Annapolis		-0.36 0.02	-0.366	-0.369 0.013	-0.331 0.047
Solomons Island		-0.33 0.03	-0.326	-0.326 0.016	-0.306 0.047
Norfolk		-0.43 0.02	-0.430	-0.434 0.015	-0.426 0.046
Portsmouth		-0.37 0.03	-0.365	-0.364 0.018	-0.402 0.046

* The point values listed here were computed using all of the available data at each site. The next column lists the linear trends arrived at each site by propagating the linear trend from Halifax, N.S., using the linear trends of differences of sea level values among neighbouring tide gauges listed in Table 9.

Table 8. List tide gauge sites and their sea level linear trends in m/century in the Atlantic Coast that were eliminated from this study due to systematic errors.

Location	Vanicek and Nagy 1980	Emery and Aubrey 1990	From Point Values
St Joseph	-0.44		0.510
	0.65		0.166
Riv. du Loup -Gros Cacouna *	0.55		0.594
	0.25		0.123
West Saint Modeste	0.37		-0.017
	0.60		0.107
Savage	1.75		0.568
	0.46		0.162
Nain	0.42		0.364
	0.25		0.094
Grindstone Island			-0.850
			0.317
Point Sapin			1.440
			0.261
Parker's Cove			0.220
			0.105
Sandy Hook		-0.407	-0.405
			0.015

* These two records were obtained over different periods but nearly in the same location and were analyzed accordingly as a single record including a datum bias.

Table 9. Comparison of linear trends and their standard deviations in m/century of Atlantic sea level differences.

Location	- Location	Dohler and Ku, 1970	This Study
St Francois	- Quebec		-0.065 0.061
Quebec	- Pointe au Père	-0.094 0.043	-0.096 0.023
Tadoussac	- St Jean Port Joli		0.209 0.145
Pointe au Père	- Tadoussac		-0.131 0.078
Baie Comeau	- Pointe au Père		0.072 0.045
Ste Anne des Monts	- Pointe au Père		-0.045 0.058
Riviere au Renard	- Pointe au Père		-0.023 0.076
Dalhousie	- Pointe au Père		-0.447 0.160
Charlottetown	- Pointe au Père	-0.384 0.055	-0.345 0.012
Sept Iles	- Pointe au Père		-0.177 0.037
Charlottetown	- Harrington Harb.		-0.342 0.012

Lark Harbour - Harrington Harb.		-0.284 0.041
Charlottetown - Lower Escuminac		-0.143 0.047
Charlottetown - Pointe du Chene		-0.054 0.054
Rustico - Charlottetown		0.027 0.046
Pictou - Charlottetown		-0.013 0.031
Port aux Basques - Charlottetown		-0.020 0.024
North Sydney - Charlottetown		-0.032 0.045
Halifax - Charlottetown		-0.001 0.008
St John's - Argentia		-0.266 0.064
St John's - Harrington Harb.		-0.184 0.032
Point Tupper - North Sydney		-0.045 0.064
Boutilier Point - Halifax		-0.042 0.049
Yarmouth - Halifax		-0.119 0.034

St John - Halifax		0.055 0.011
Eastport - Halifax		0.085 0.009
Bar Harbor - Eastport		-0.024 0.016
Portland - Bar Harbor		0.113 0.014
Seavy I. - Portland		0.078 0.007
Boston - Seavy Island		-0.027 0.005
Woods Hole - Boston		-0.109 0.005
Newport - Woods Hole		0.020 0.004
Providence - Newport		0.052 0.006
New London - Newport		0.031 0.005
Bridgeport - New London		-0.101 0.014
Willets - Bridgeport		0.085 0.016
Port Jefferson - Willets		-0.065 0.015

New York - Willets		-0.036 0.005
Montauk - Port Jefferson		0.080 0.014
Atlantic City - New York		-0.062 0.005
Lewes - Atlantic City		0.026 0.010
Lewes - Philadelphia		-0.110 0.014
Kiptopeke - Lewes		-0.061 0.014
Solomon Island - Kiptopeke		0.032 0.013
Norfolk - Kiptopeke		-0.088 0.010
Annapolis - Baltimore		-0.045 0.004
Solomon Island - Annapolis		-0.024 0.005
Portsmouth - Norfolk		0.024 0.004

4.2 Comparison of results with other studies in all lakes.

Tables 10 to 26 summarize the lake level tilt results found in this study. With the exception of the Great Lakes, we could not find any other study to compare our results of other lakes against. Table 10 lists the results for Lac

St Jean. It appears that the lake level information available in this area has never been used to determine tilt. We could not find enough information in previous studies of this area to perform a comparison (cf. Frost and Lilly, 1966; Vanícek and Hamilton, 1972).

Studies in the Great Lakes region date back at least to the work of Gutenberg (1933, 1941, 1954). Some of the most extensive efforts to determine trends of water level differences in this region have been those made by Moore (1948), Kite (1972a,b), Coordinating Committee (1977), and Tait and Bolduc (1985). The two later studies were selected for comparison on the basis of being the ones produced most recently, prior to this study. Dohler and Ku (1970) determined a single linear trend of water level differences in the Lakes Ontario, Erie, Huron, and Superior where Canadian data is available.

Table 10. Linear trends and their standard deviations in m/century of lake level differences along Lake St Jean.

Location - Location	(*)	(*)	This Study
St Gedeon - Pointe Scott			0.143 0.148
St Henri - St Gedeon			-0.177 0.159

(*) No other study could be found as a reference for comparison

Table 11. Comparison of linear trends and their standard deviations in m/century of lake level differences along Lake Ontario.

Location - Location	Coordinating Committee 1977	Tait and Bolduc 1985	This Study
Toronto - Burlington			0.212 0.020
Toronto - Cobourg			-0.013 0.010
Toronto - Hamilton			0.008 0.064
Toronto - Port Dalhousie			0.013 0.006
Toronto - Port Weller			-0.006 0.007
Oswego - Toronto	0.094 0.009	0.088 0.007	0.085 0.005
Oswego - Kingston	-0.079 0.006	-0.076 0.004	-0.075 0.002
Oswego - Point Petr			0.330 0.105
Rochester - Oswego			-0.038 0.005
Olcott - Toronto			-0.066 0.015

Table 12. Comparison of linear trends and their standard deviations in m/century of lake level differences along Lake Erie.

Location - Location	Coordinating Committee 1977	Tait and Bolduc 1985	This Study
Kingsville - Bar Point			0.020 0.025
Toledo - Bar Point			0.027 0.027
Port Stanley - Erieau			0.055 0.020
Barcelona - Erieau			0.038 0.021
Pele Point W - Kingsville			0.040 0.228
Port Stanley - Port Colburne	0.061	0.046 0.012	0.038 0.006
Buffalo - Port Colburne	0.064 0.009		0.058 0.004
Sturgeon - Port Colburne			0.046 0.024
Port Stanley - Port Dover			-0.070 0.023
Barcelona - Port Stanley			0.046 0.023
Cleveland - Port Stanley	-0.055 0.015	-0.049 0.011	-0.041 0.005

Erie - Port Stanley			-0.011 0.021
Fairport - Cleveland			0.069 0.036
Marblehead - Cleveland			0.006 0.013
Toledo - Cleveland			0.021 0.007
Toledo - Fermi			0.016 0.013
Toledo - Monroe			0.066 0.043

Table 13. Linear trends and their standard deviations in m/century of lake level differences along Lake St Clair.

Location - Location	(*)	(*)	This Study
Tecumseh - Belle River			-0.076 0.021
Tecumseh - Gross Point			-0.035 0.006
Tecumseh - Port Lambton			-0.288 0.013

(*) No other study could be found as a reference for comparison.

Table 14. Comparison of linear trends and their standard deviations in m/century of lake level differences along Lake Huron.

Location - Location	Coordinating Committee 1977	Tait and Bolduc 1985	This Study
Goderich - Collingwood	-0.204 0.015	-0.206 0.016	-0.189 0.004
Parry Sound - Collingwood			0.120 0.012
Thessalon - Collingwood	0.0 0.009		0.018 0.006
Tobermory - Collingwood			0.042 0.018
Point Edward - Goderich			0.123 0.012
Harbour Beach - Goderich	0.015 0.012		0.023 0.004
Lakeport - Goderich			0.028 0.016
Thessalon - Little Current			-0.204 0.024
Detour * - Thessalon			-0.032 0.008
Mackinaw - Thessalon	-0.116 0.012		-0.092 0.005
Harbour Beach - Essexville *			0.017 0.019

Harrisville - Harbour Beach			0.047 0.012

* Two records were obtained over different periods using two different instruments but nearly in the same location and were analyzed accordingly as a single record accounting for a datum bias.

Table 15. Comparison of linear trends and their standard deviations in m/century of lake level differences along Lake Michigan.

Location - Location	Coordinating Committee 1977		This Study
Milwaukee * - Calumet	-0.128 0.012		-0.045 0.007
Sturgeon Bay - Green Bay *			-0.017 0.017
Milwaukee * - Holland			-0.147 0.013
Sturgeon Bay - Kewaunee			0.145 0.033
Milwaukee * - Ludington			-0.026 0.015
Sturgeon Bay - Ludington			0.116 0.007
Port Inland - Mackinaw			-0.068 0.018
Sturgeon Bay - Mackinaw			-0.139 0.006

* Two records were obtained over different periods using two different instruments but nearly in the same location and were analyzed accordingly as a single record accounting for a datum bias.

Table 16. Linear trends and their standard deviations in m/century of lake level differences along Lake Nippissing.

Location - Location	(*)	(*)	This Study
North Bay - French River			0.084 0.012

(*) No other study could be found as a reference for comparison.

Table 17. Comparison of linear trends and their standard deviations in m/century of lake level differences along Lake Superior.

Location - Location	Coordinating Committee 1977	Tait and Bolduc 1985	This Study
Point Iroquois - Grosse Cap			0.034 0.040
Marquette * - Michipicoten	-0.408 0.009	-0.394 0.008	-0.354 0.005
Point Iroquois - Michipicoten	-0.290 0.009		-0.254 0.005
Thunder Bay - Rossport			-0.274 0.032

Duluth - Thunder Bay	-0.290	-0.299	-0.308
	0.012	0.009	0.004
Grand Marais - Thunder Bay			-0.153
			0.022
Marquette * - Thunder Bay	-0.177	-0.175	-0.187
	0.012	0.011	0.005
Two Harbours - Duluth			0.059
			0.005
Ontonagon - Marquette *			-0.075
			0.021

* Two records were obtained over different periods using two different instruments but nearly in the same location and were analyzed accordingly as a single record accounting for a datum bias.

Table 18. Linear trends and their standard deviations in m/century of lake level differences along Lake of the Woods.

Location - Location	(*)	(*)	This Study
Warroad - Hanson Bay			-0.188
			0.021
Warroad - Keewatin			-0.093
			0.011
Keewatin - Clearwater Bay			-0.060
			0.040

(*) No other study could be found as a reference for comparison.

Table 19. Linear trends and their standard deviations in m/century of lake level differences along Lake Winnipeg.

Location - Location	(*)	(*)	This Study
Berens River - Anama Bay			-0.165 0.190
George I. - Berens River			-0.010 0.389
Matheson I. - Berens River			-0.077 0.027
Mission Point - Berens River			-0.029 0.033
Pine Dock - Berens River			-0.022 0.030
Victoria Beach - Berens River			-0.159 0.042
Winnipeg - Berens River			-0.050 0.043
Victoria Beach - Gimli			-0.189 0.039
Montreal Point - Mission Point			0.046 0.095

(*) No other study could be found as a reference for comparison.

Table 20. Linear trends and their standard deviations in m/century of lake level differences along Lake Manitoba.

Location - Location	(*)	(*)	This Study
Steep Rock - Meadow Portage			0.163 0.095
Steep Rock - The Narrows			-0.164 0.024
Steep Rock - Delta			0.084 0.034
Westbourne - The Narrows			0.011 0.056
Toutes Aides - Steep Rock			-0.097 0.056

(*) No other study could be found as a reference for comparison.

Table 21. Linear trends and their standard deviations in m/century of lake level differences along Lake Winnipegosis.

Location - Location	(*)	(*)	This Study
Winnipegosis - Dawson Bay			-0.621 0.050

(*) No other study could be found as a reference for comparison.

Table 22. Linear trends and their standard deviations in m/century of lake level differences along the Great Slave Lake.

Location - Location	(*)	(*)	This Study
Hay River - Fort Resolution			-0.632 0.178
Snowdrift - Fort Resolution			0.199 0.132
Yellowknife - Fort Resolution			0.002 0.026
Snowdrift - Hay River			0.400 0.163
Yellowknife - Hay River			0.304 0.034
Yellowknife - Snowdrift			-0.385 0.066

(*) No other study could be found as a reference for comparison.

Table 23. Linear trends and their standard deviations in m/century of lake level differences along Lake Athabasca.

Location - Location	(*)	(*)	This Study
Fort Chipewyan - Bustard Island			0.031 0.127
Fort Chipewyan - Crackingstone			-0.174 0.056

(*) No other study could be found as a reference for comparison.

Table 24. Linear trends and their standard deviations in m/century of lake level differences along Lake Kootenay.

Location - Location	(*)	(*)	This Study
Queen's Bay - Kuskonook			0.051 0.010
Queen's Bay - Procter			-0.017 0.118

(*) No other study could be found as a reference for comparison.

Table 25. Linear trend and its standard deviation in m/century of lake level differences along Lake Okanagan.

Location - Location	(*)	(*)	This Study
Penticton - Kelowna			-0.047 0.011

(*) No other study could be found as a reference for comparison.

Table 26. Linear trends and their standard deviations in m/century of lake level differences along Lake Shuswap.

Location - Location	(*)	(*)	This Study
Sicamous - Salmon Arm			0.262 0.058
Sorrento - Salmon Arm			0.298 0.038
Sorrento - Seymour Arm			0.175 0.045

(*) No other study could be found as a reference for comparison.

4.3 Comparison of results with other studies in Hudson Bay.

The record of tide gauge located in Churchill, Manitoba was the only information that could be used in our analysis. The other two tide gauges in this area Inoucdjouac and Fort Churchill were either affected by river runoff or too short for our analysis.

Our analysis of the linear trend in Churchill shows an accelerated uplift, i.e., it appears that there is a quadratic term in the record. This does not appear to be the result of a systematic error and it explains why our results are considerably higher than others.

Table 27. Comparison of sea level linear trends and their standard deviations in m/century in Hudson Bay.

Location	Barnett 1970	Dohler and Ku, 1970	Vanícek and Nagy, 1980	Emery and Aubrey 1990	This Study
Churchill	0.385	0.393	0.45	0.617	0.826
		0.067	0.04		0.028

4.4 Comparison of results with other studies in the Pacific Coast.

The following tables summarize the results found in this study for the Pacific coast. Tables 28 and 29 list the linear trends produced in this study for Canadian and US sites, respectively, and compare them whenever possible with other recent studies. The only study that appears to depart significantly from ours as well as all other studies in the region is that by Holdahl, et al. (1989). These discrepancies should be examined further. The value for linear trends that was used in the compilation of the map is the one listed under the heading "propagating differences". Table 30 lists the values of Pacific tide gauge linear trends for sites excluded from the map. Table 31 lists the value of the linear trends of the sea level differences used along the Pacific Coast in this study. Very few studies have followed this approach and only one value computed by Dohler and Ku (1970) could be used for comparison.

Table 28. Comparison of sea level linear trends and their standard deviations in m/century along Pacific Canada (*).

Location	Dohler and Ku, 1970	Holdahl, et al., 1989	Vanícek and Nagy, 1980	From Point Values	Propagating Differences
Tofino		0.19		0.137	0.104
		0.08		0.014	0.070
Port Alberni			0.19	0.065	-0.307
			0.08	0.049	0.060
Bamfield				0.007	-0.105
				0.172	0.046
Port Renfrew				0.065	-0.024
				0.096	0.056
Sooke			0.18	-0.094	0.030
			0.21	0.092	0.039
Victoria	-0.055	0.02	-0.06	-0.074	-0.074
	0.018	0.08	0.02	0.010	0.010
Patricia Bay				0.151	-0.066
				0.232	0.048
Fulford Harbour		0.08	0.00	-0.029	-0.020
		0.09	0.08	0.031	0.013
Steveston			0.21	-0.152	-0.175
			0.59	0.093	0.038
Vancouver	0.018	0.07	-0.01	-0.018	-0.024
	0.018	0.08	0.02	0.011	0.010
Point Atkinson		-0.02	-0.12	-0.103	-0.093
		0.08	0.02	0.013	0.011

Campbell River		0.23 0.11	0.31 0.28	0.151 0.069	-0.009 0.023
Alert Bay		0.28 0.13	0.11 0.07	0.152 0.042	0.106 0.022
Port Hardy		0.33 0.11	0.25 0.27	0.109 0.057	-0.056 0.038
Bella Bella			0.01 0.20	-0.094 0.053	-0.192 0.046
Queen Charlotte			0.19 0.14	0.060 0.043	-0.128 0.054
Prince Rupert	0.049 0.024		-0.07 0.02	-0.099 0.012	-0.332 0.053

(*) The point values listed here were computed using all of the available data at each site. The next column lists the linear trends arrived at each site by propagating the linear trend from Victoria, B.C., using the linear trends of differences of sea level values among neighbouring tide gauges listed in Table 31.

Table 29. Comparison of sea level linear trends and their standard deviations in m/century along the Pacific U.S.A. *

Location	Vanicek 1978	Holdahl, et al., 1989	Department of Commerce, 1988	From Point Values	Propagating Differences
Crescent City			-0.06 0.03	-0.062 0.017	0.128 0.027
Astoria		0.17 0.10	-0.03 0.04	-0.025 0.020	0.097 0.020
Neah Bay	-0.136 0.027	0.25 0.08	-0.11 0.03	-0.124 0.021	0.189 0.012
Seattle	0.213 0.017	-0.14 0.08	0.20 0.01	0.204 0.008	-0.198 0.013
Friday Harbor	0.023 0.019	0.04 0.08	0.14 0.03	0.126 0.018	-0.063 0.012
Ketchikan			-0.01 0.03	-0.009 0.014	-0.170 0.054

(*) The point values listed here were computed using all of the available data at each site. The next column lists the linear trends arrived at each site by propagating the linear trend from Victoria, B.C., using the linear trends of differences of sea level values among neighbouring tide gauges listed in Table 31.

Table 30. List of tide gauge sites and their sea level linear trends in m/century along the Pacific Coast that were eliminated from this study due the presence of systematic errors (*).

Location	(**)	(**)	Vanicek and Nagy, 1980	From Point Values
Tsawwassen			-0.28 0.34	-0.204 0.168
New Westminster			0.68 2.54	0.848 0.229
Comox				-3.240 0.153
Bella Coola				-0.532 0.204
Port Simpson				0.725 0.359
Langara Point				-1.170 0.326

(*) The point values listed here were computed using all of the available data at each site.

(**) No other study could be found as a reference for comparison.

Table 31. Comparison of linear trends and their standard deviations in m/century of Pacific sea level differences.

Location - Location	Dohler and Ku, 1970	(*) Wigen and Stephenson 1980	This Study
Astoria - Crescent City			0.031 0.018
Neah Bay - Astoria			-0.092 0.016
Bamfield - Neah Bay			0.290 0.045
Port Renfrew - Neah Bay			0.210 0.055
Victoria - Neah Bay			0.260 0.008
Friday Harbor - Seattle			-0.130 0.006
Victoria - Friday Harbor			0.011 0.006
Bamfield - Tofino		-0.361 0.173	0.210 0.052
Bamfield - Port Alberni			-0.200 0.037
Victoria - Sooke		-0.084 0.046	0.100 0.038
Patricia Bay - Victoria			-0.007 0.048

Fulford Harbour - Victoria		-0.002 0.018	-0.054 0.010
Vancouver - Victoria	-0.067 0.006	-0.037 0.011	-0.050 0.004
Vancouver - Steveston			-0.150 0.037
Point Atkinson - Vancouver		0.018 0.014	0.069 0.005
Campbell River - Point Atkinson		0.077 0.099	-0.085 0.020
Alert Bay - Point Atkinson			-0.200 0.019
Port Hardy - Campbell River			0.048 0.030
Bella Bella - Port Hardy			0.140 0.027
Queen Charlotte - Bella Bella			-0.064 0.028
Prince Rupert - Bella Bella		-0.076 0.091	0.140 0.026
Ketchikan - Prince Rupert			-0.160 0.009
Vancouver - Tsawwassen			-0.670 0.053

(*) These authors used yearly mean sea level values instead of monthly means.

5 Compilation of the Map.

5.1 Mathematical model and regressions per region.

The contract awarded to us by GSC stipulates that a spatial prediction of vertical velocities, or uplift rates, should be carried out for Canada. We have attempted to do this by fitting a vertical velocity surface over the sea level linear trends, lake level tilts, and levelling height difference differences data reviewed in previous chapters. This is the only technique capable of accommodating in one model these three kinds of information when the relevelled segments are scattered not only in space but also in time. The assumptions underlying this approach are that the uplift rates are linear in time and that they vary smoothly with location.

We have obtained a velocity surface of the form

$$v(x,y,t)=\sum_{k=1}^{n_t} c_{ok}T_k(t)+\sum_{i=1}^{n_x}\sum_{j=1}^{n_y}\sum_{k=1}^{n_t} c_{ijk}x^i y^j T_k(t), \qquad (13)$$

where n_x and n_y, are the powers of the generalized polynomial and, n_t, is the sum of number of times, or episodes, for which a solution is to be found plus the maximum power of the generalized polynomial in time, x and y are easting and northing in a Sinusoidal or Sanson-Flamsteed map projection (Snyder, 1982. p. 219),

$$x=R(\lambda-\lambda_o)\cos\phi$$

$$y=R(\phi-\phi_o)$$

where R is the Gauss radius of curvature, ϕ and λ are geodetic latitude and longitude, and ϕ_o and λ_o are the geodetic coordinates of the origin of the grid. We selected to set $n_t=1$ in all solutions given that two main levelling surveys are the source of our information.

The system of normal equations reads

$$[\Delta\Phi\,|\,\Phi]P[\Delta\Phi\,|\,\Phi]^T \mathrm{c}=[\Delta\Phi\,|\,\Phi]P[\delta v\,|\,v]^T, \qquad (14)$$

where Φ and $\Delta\Phi$ are base functions and differences of base functions, P is the weight matrix of the input velocities and velocity differences δv and δv and c is the vector of coefficients. The variance of the predicted surface is given by

$$\sigma_v^2(x,y) = \Phi^T C_c \Phi, \tag{15}$$

where C_c, is the covariance matrix of the estimated coefficients.

The weight matrix, P, is formed using the inverse of the covariance matrix of the observations, C_l. The diagonal elements of C_l are the variances of linear trends coming from the regressions of lake level differences and sea level data whereas the variances of the relevelled segments where derived from

$$\sigma^2 = (\sigma_{1.0}^2 + \sigma_{2.0}^2)\frac{S}{\Delta T^2}, \tag{16}$$

where σ_1,0 and $\sigma_{2.0}$ are the standard deviations in cm of a 1 km first order levelling line for the first and second surveys and S is the distance in km of the relevelled segment. We used in all our solutions the same value postulated in the compilation of the 1980 map, i.e., $sqrt\sigma_{1.0}^2 + \sigma_{2.0}^2 = 3.79 \times 10^{-1}$.

The off-diagonal elements of C_l were computed for lake level tilts sharing the same water gauge using the expression

$$\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j, \tag{17}$$

where σ_i and σ_j are the standard deviations of the tilt segments i and j and ρ_{ij} is their correlation coefficient given by

$$\rho_{ij} = \frac{\sqrt{n_i n_j}}{2n_{ij}}. \tag{18}$$

where n_i and n_j are the number of monthly values of one lake gauge used in the determination of the tilt segments i and j, and n_{ij} is the number of observations common to both segments from the same gauge. Clearly, the maximum value that ρ_{ij} can take is 0.5.

A statistical filtering of the coefficients is always done by means of an orthogonalization of the base functions at a prescribed confidence level. We have selected 2 standard deviations as our confidence level in all solutions. Only the significant coefficients at this level are de-orthogonalized back into the original space. This results in a smoother yet statistically significant solution.

The mathematical model described above was implemented in the program VCM4D. The version of the program used was the one ported to the Cyber computer in EMR in 1979 which was able to accommodate only less than 8000 relevelled segments. We modified the code extensively by removing unnecessary portions from it and improved its quality. The computer program was ported to the IBM PC and Vax computers. The IBM PC version is limited to 2,500 relevelled segments and 30 coefficients in the regression and the Vax version is limited to 30,000 relevelled segments and 100 coefficients. The program imposes some of the largest demands of CPU power for medium size programs. A single 6 by 3 national solution containing close to 30,000 segments and 73 tide gauge linear trends took over 24 hours to complete when it worked as a single task on a Vax 750 computer. The amount of time required by the problem to find a solution increases geometrically with the addition of new parameters to the model. An 8 by 4 solution was interrupted before completion after six continuous days of execution in the same Vax 750. The total amount of CPU time (not execution time) required to obtain the solutions listed in this section was in excess of 150 hours. We failed to port the program to the Cyber under the NOS operating system. The lack of virtual memory in this operating system made it impossible to accommodate all the 30,000 relevelled segments (input data) and the executable code at once.

We started by obtaining a national solution by fitting a 6 by 3 generalized polynomial to the data. This solution demonstrates the strengths and weakneses of the map. Wherever there was tide gauge or lake level tilts the solution shows the expected behaviour of postglacial rebound as well as the uplift of Vancouver Island relative to the mainland. On the down side, the solution is the weakest in southern Alberta and Saskatchewan and in northern Montana. The subsidence in this area appears to be a result of the combination of lack of US levelling data and the fact that the lake tilt data in the neighbouring Provinces, i.e., B.C. and Manitoba point towards the existence of a low in that region. No lake tilts are available in this area and the Canadian relevelled database carries considerably less weight in the map due to the shorter time intervals between campaigns compared to the lake derived tilts. Our consensus is that although the existence of a low in this region appears to be real, the data presently available do not allow us to determine its magnitude accurately.

We then proceeded to determine regional solutions over areas where we found the densest coverage by tide gauge and lake tilts. The Atlantic solutions show similar patterns to the ones encountered in the previous map, with the exception of the zero contour line which has shifted north due to two reasons: first, the linear trends at Rustico, Point du Chene, Lark Harbour, and Pictou show larger negative values through the use of the differential technique than encountered before, and secondly because a correction for eustatic water rise was not applied in this new map but was applied to the 1980 map in the amount of 10 cm/century.

Our solution for the Great lakes includes several sea level tide gauges located in the vicinity of New York and Long Island. Thus the contours shown are referred to sea level and explain the standard deviation map associated with it which shows an increase in value away from the Atlantic.

Our solution in the Pacific indicates a systematic uplift of Vancouver Island relative to the mainland. This is not unexpected given our present knowledge of the tectonic behaviour in this region (e.g., Riddihough, 1982). Once more, it is important to point out the difference between these findings against those found by Holdahl et al., (1989) for the same region.

In order to determine the amount of information lost due to the use of smooth approximating functions, we proceeded to perform an interpolation over sea level linear trends in the Atlantic and Pacific coasts using a universal kriging technique (Goldberger, 1962; Matheron, 1969). These results reinforce our conviction that the polynomial technique provides smoother but consistent trends with those obtained by the universal kriging technique.

Appendix N contains figures showing all regressions and their standard deviations as well as 2D and 3D representations of universal kriging in Atlantic and Pacific Canada.

Appendix O, an external Appendix in a computer tape, contains 12 computer printouts including input data, residuals, and the values of the coefficients and their variance-covariance matrices.

Table 32. Compilation of national and regional maps of vertical crustal movements.

Region	Number of coefficients in x (nx)	Number of coefficients in y (ny)	Parameters (nx+1)(ny+1)	Number of rejected coefficients
National	6	3	28	0
Atlantic	5	4	30	17
Atlantic	3	6	28	8
Great Lakes	5	4	30	1
Great Lakes	6	4	35	1
Winnipeg	3	3	16	2
Winnipeg	4	4	25	0
Kootenay	2	2	9	0
Kootenay	3	3	16	0
Southern BC	5	3	24	0
Southern Vancouver I.	2	2	9	5
Southern Vancouver I.	3	2	12	8

5.2 Eustatic corrections.

Few scientific issues are the subject of such a debate as the existence and magnitude of eustatic water level rise. A review in favour of it is given, for example, by Etkins and Epstein (1982). This phenomenon, if real, would introduce a systematic bias in all crustal movements derived from sea level information. Although our own research appears to justify a eustatic correction of 10 cm/century, we decided not to incorporate any correction into the data or the results of the map. This is not due to a position negating its existence but because we wish to let others to apply the corrections that they seem to find most appropriate in light of their own research.

Among the techniques used to try to determine this eustatic rise one finds average of linear trends, area averages of linear trends and of corrected linear trends, empiric orthogonal functions of linear trends and of corrected linear trends for eustatic rebound. Table 33 lists some of the most comprehensive attempts to determine its magnitude.

Pirazzoli (1986), on the other hand, analyzed 229 sea level series and concluded that any eustatic rise value would be biased by their geographic distribution and this obstacle could not be overcome to solve the problem.

Table 33. Global linear trend averages suggested to represent eustatic signatures of sea level.

Study	Number of tide gauge records used worldwide	Number of regions used worldwide	Global mean (m/century)	Note
Gutenberg (1941)	69	21	0.11	*
Lisitzin (1958)	6	-	0.11	**
Fairbridge and Krebs (1962)	23?	3	0.12	*
Emery (1980)	247	-	0.30	**
Gornitz et al. (1982)	193	14	0.12	*
Barnett (1984)	82	6	0.14	*
Peltier and Tushingham(1989)	81	7	0.24	***

* Area average of linear trends from all regressions

** Average of the linear trends from all regressions

*** Linear regression and eigenanalysis of adjusted sea level

6 Conclusions and Recommendations.

The most dramatic change that has been experienced since the compilation of the last map of vertical crustal movements 10 years ago is the tremendous increase in data assembled for this map. We have at our disposal nearly 7 times as much levelling data, 3 times as many lake tilts, and twice as many sea level linear trends.

Our research into the quality of sea level data lead us to the discovery of a few problems. For example, large portions of older data, such as that from 1894 to 1914 from St John, N.B. and from 1912 to 1938 from Charlottetown, P.E.I., are not included in various sea level series provided to us by MEDS. These data, however, appear to exist in the archives of the Permanent Service for Mean Sea Level (PSMSL) in the U.K. Similarly, entire records from various locations, such as St Paul Island, N.S. and Forteau Bay, Labrador (cf. Dawson, 1902), are missing from MEDS computer archives. At least some of this information is available from MEDS but in formats that are not computer readable (Wigen and Stephenson, 1980; Bolduc, pers. comm. 1989).

Another severe problem was the discovery of several undocumented datum shifts in some records. A few of these shifts can be found in Halifax (29 cm, January 1987), Point Tupper (39 cm, January 1984), North Sydney (28 cm, January 1985), Riviere au Renard (12 cm, August 1975), among others. More Puzzling is the shift in the record of Boutilier Point (21 cm, January 1971) due to the fact that the data given to us by MEDS in a computer readable format does not agree with their own printed publication. This discovery lead us to inspect visually all of our time series.

We had aimed to present an assessment of vertical crustal movements in the Canadian Arctic but this was impossible to achieve. The present status of the tide gauge records in this area renders them useless for any long term sea level study. These records would be extremely valuable not only in studies of crustal movements but also in those of global sea level behaviour.

We found that the time consuming effort put into the verification of the stability (or lack therof) of benchmarks and tide gauges installations was well worth its investment. We corrected or discarded data affected by local instabilities that otherwise would have gone into the compilation of the map (as it did in 1980).

We were supossed to have received Canadian relevelled segments corrected for several systematic errors, such as refraction, but this was not the case. These systematic effects can be corrected in two ways in the future. The straightforward method would be to apply the corrections to the data and recompute the map. The second method could be the generation of correction surfaces and apply them to the map. However, any of these options should be investigated carefully. The only quality control check that we have applied to the levelling data was a rejection criterion based on the maximum tilt allowed within a relevelled segment. This action was taken to remove outliers and disturbed benchmarks. However, it also possible that this correction might have had a smoothing effect, i.e., it migh have acted as a spatial filter on the map by possibly removing seismic-related displacements from short relevelled segments.

We were able to compute for the first time a map of vertical crustal movements for Canada computed in a single solution (nx=6,ny=3). This approach frees the new map of several assumptions made during the compilation of the 1980 map. We feel that the dominant physical feature in this map, the rapid rise in the Hudson Bay area and the location of the zero velocity contour line, is the effect of postglacial rebound. No attempt, however, has been made to perform a geophysical interpretation of the features described by the 11 regional solutions computed here.

Although the sea level linear trends used in 1980 have been included in previous chapters for comparison, a full comparison between this and the older map is somewhat unfair not only because we had nearly 7 times more data this time but because the two maps followed quite different routes during their compilation. Vanicek and Nagy (1980) selected to compile the map putting together a "mosaic" of solutions, whereas, we have decided (and had enough information) to compute a solution at one time. The only noticeable difference that we wish to point out is the shift to the North of the zero contour line in this new map. This is a result of the combined effect of not applying a eustatic correction and the new sea level trend values being computed through the differencing approach. These are important factors to be considered if someone chooses to perform a full comparison.

The comparisons presented in the previous chapters between this map and other regional studies show that our results improve upon the work that has preceded them. A reason for this, of course, is the larger size in space and time of our database, but another reason that reinforces this notion is the increase in the spatial coherency of the sea level linear trends computed through the differencing approach. The only discrepancy with our study is the one produced by Holdahl et al. (1989) in the West coast. We believe that this discrepancy should be investigated in greater detail.

We wanted to compare our results with an independent source of information on vertical crustal movements such as the one given by gradients of radiocarbon curves. We attempted to obtain a copy of the National Map of Radio-Carbon curves for Canada being assembled by Bernard Peltier in the Geological Survey of Canada. Unfortunately, these data were not available to us. Their compilation was not completed at the end of this project and the authors decided to withold all of its information until its future publication date. Our results in the east coast, however, confirm our earlier findings that radiocarbon curves and sea level linear trends tend to agree in sign and have a difference in magnitude of 10 cm per century (Carrera and Vanicek, 1988). Although comparisons with gradients of radiocarbon curves can be performed with several regional results, i.e., the Great Lakes (Lewis, 1970), the Arctic (Blake, 1975) and Pacific (Mathews et al., 1970, Clague, 1975, 1981, Clague, et al. 1982), we decided to postpone such comparison until this comprehensive compilation in the Geological Survey is completed and becomes available.

We conclude by recommending the following actions:

- that corrections to the levelling data for systematic errors be computed and be applied either individually or be treated in the form of correction surfaces.
- to obtain additional regional and national solutions of vertical crustal movements from our database for regions of geodynamic activity (e.g., Bower, 1989).
- to investigate the effect on the levelling network of a new temporal homogenization of the levelling network for the NAVD adjustment (Carrera and Vanicek, 1985).
- to monitor sea level and lake level site movements through extra-terrestrial positioning methods (Carter et al., 1989).

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8 References.

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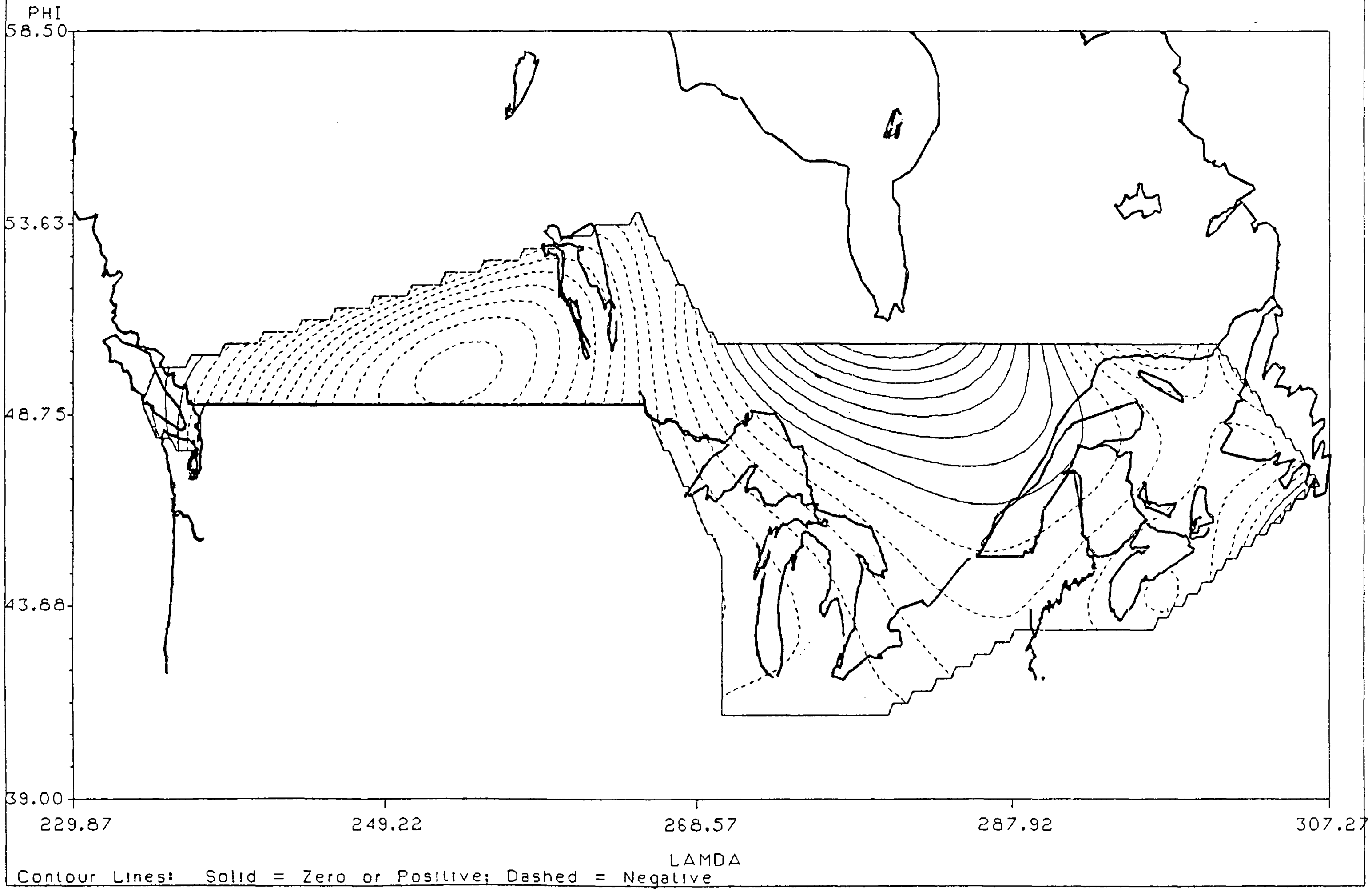
APPENDIX N



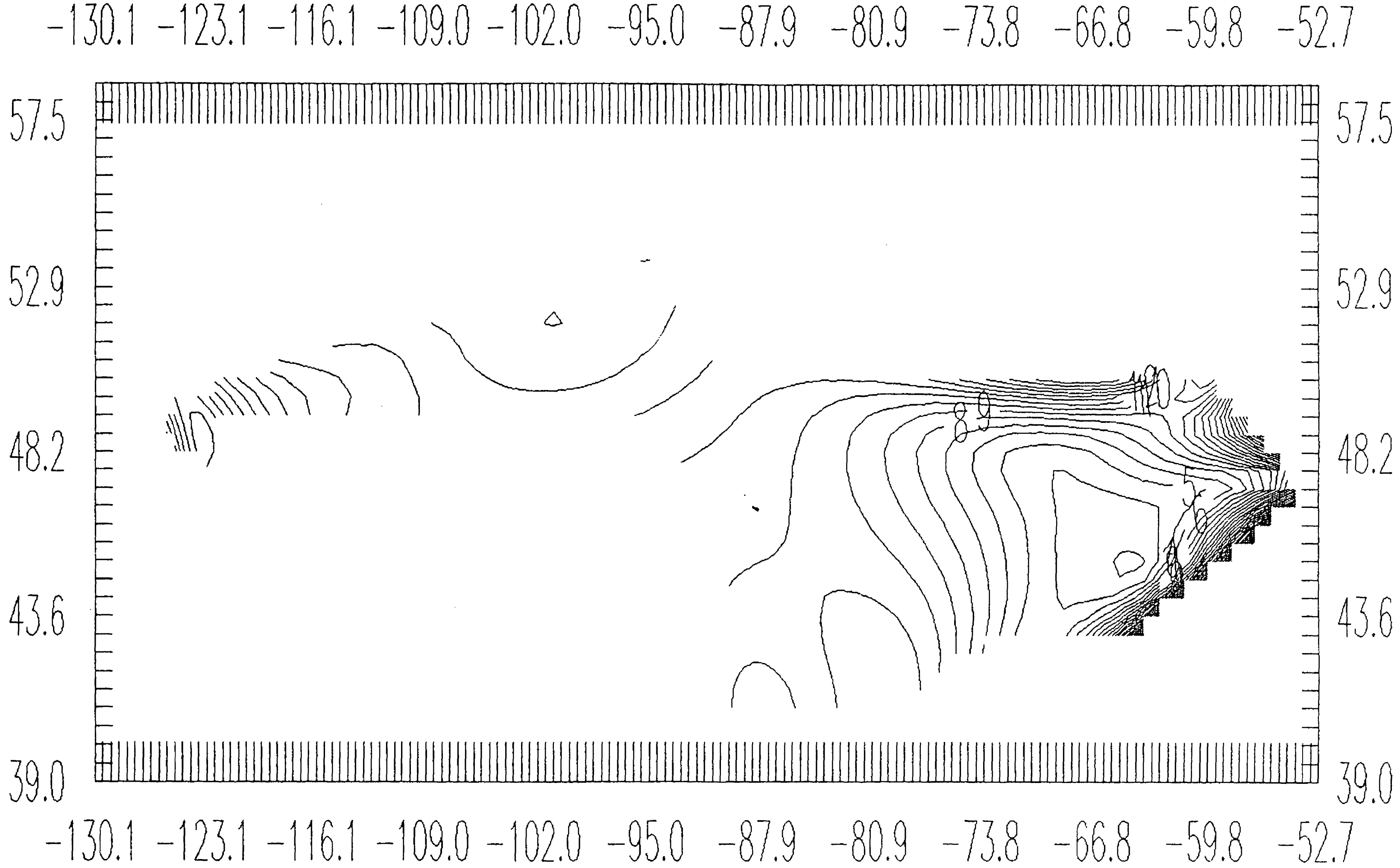
Map of Vertical Crustal Movements 1990

6 by 3 National Solution

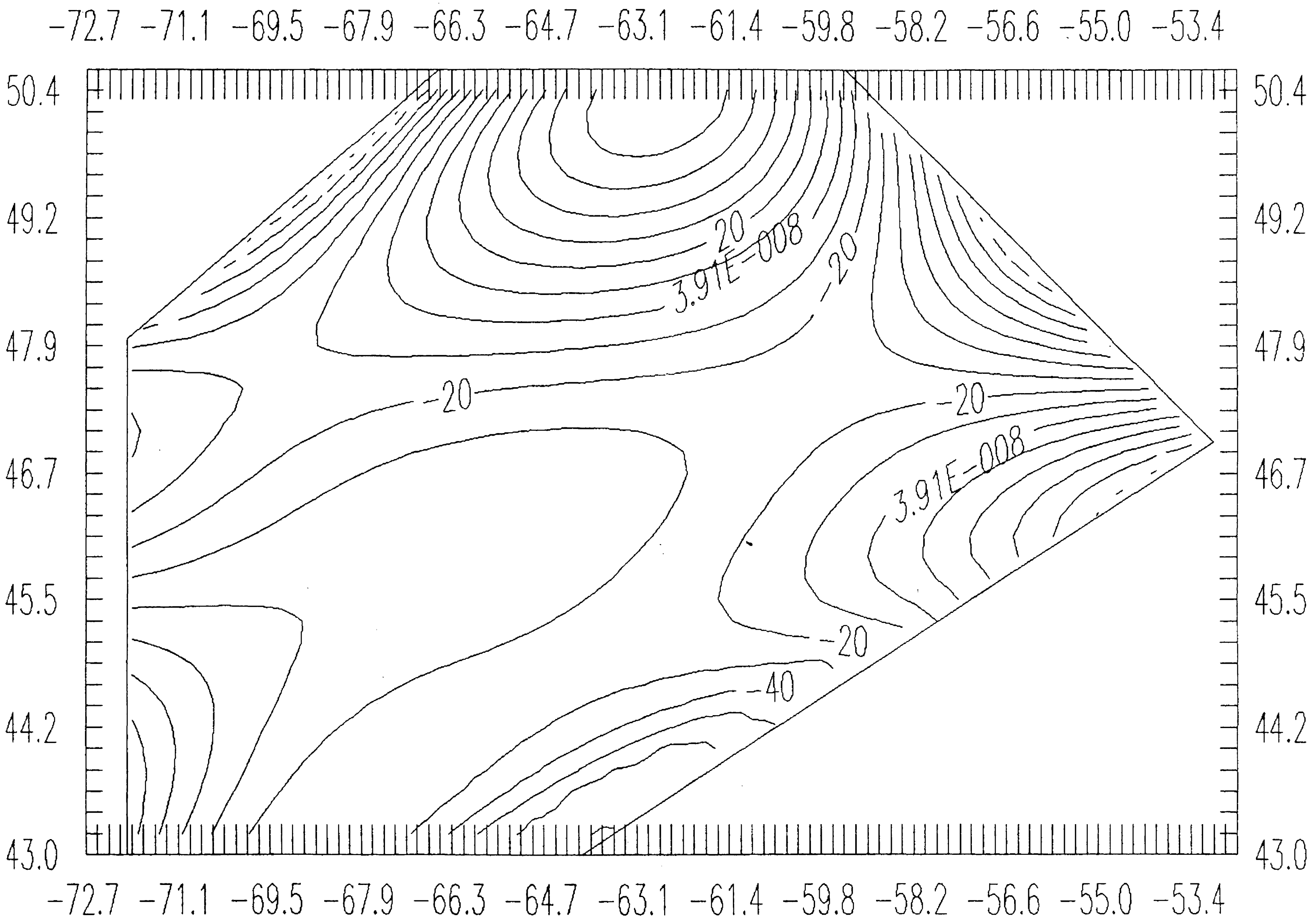
Contour Interval: 10 cm/century

Contour Lines: Solid = Zero or Positive; Dashed = Negative

Standard Deviation for the Uplift Solution for Canada (6x3)



Uplift in Atlantic Canada and New England (5x4)



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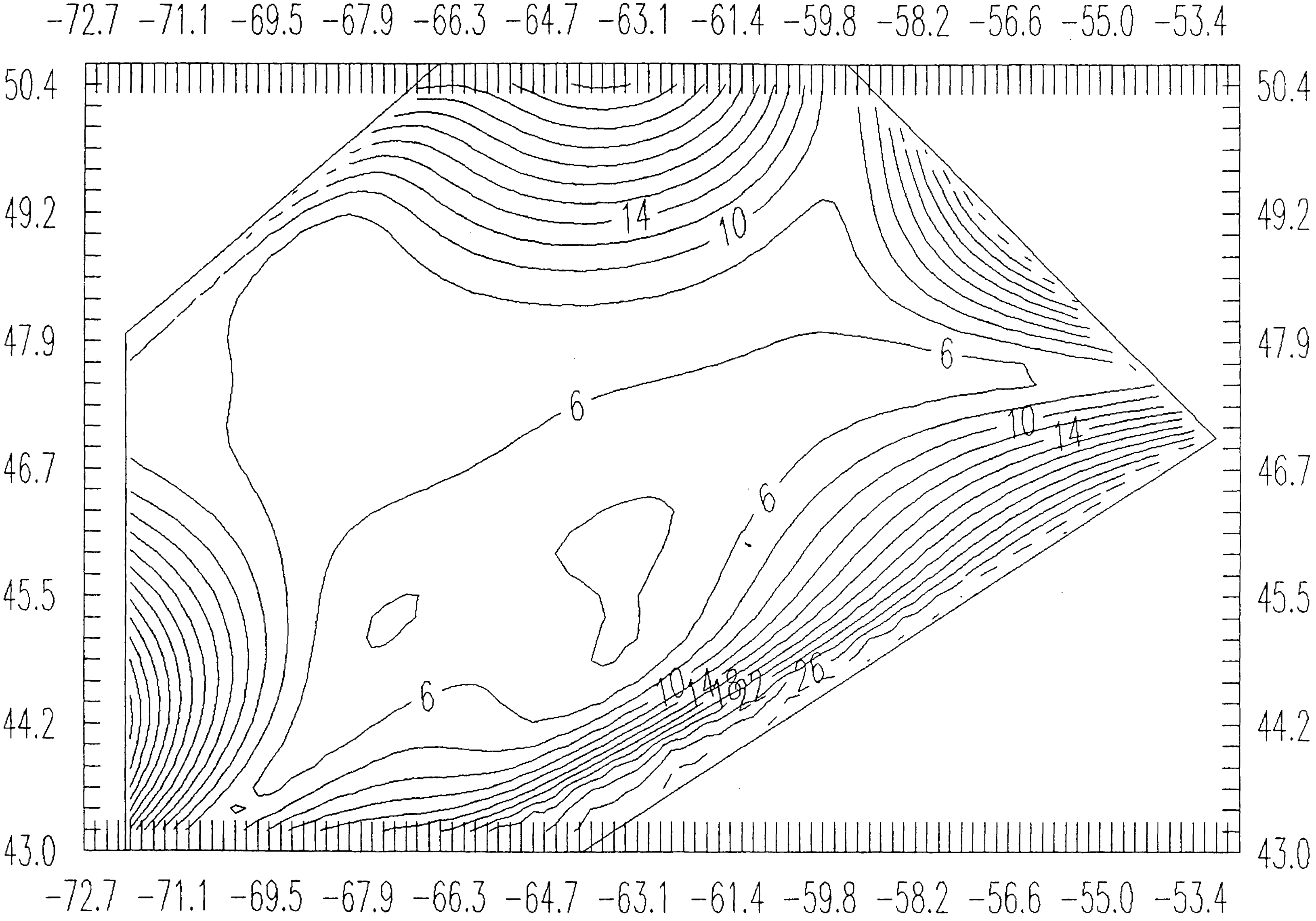
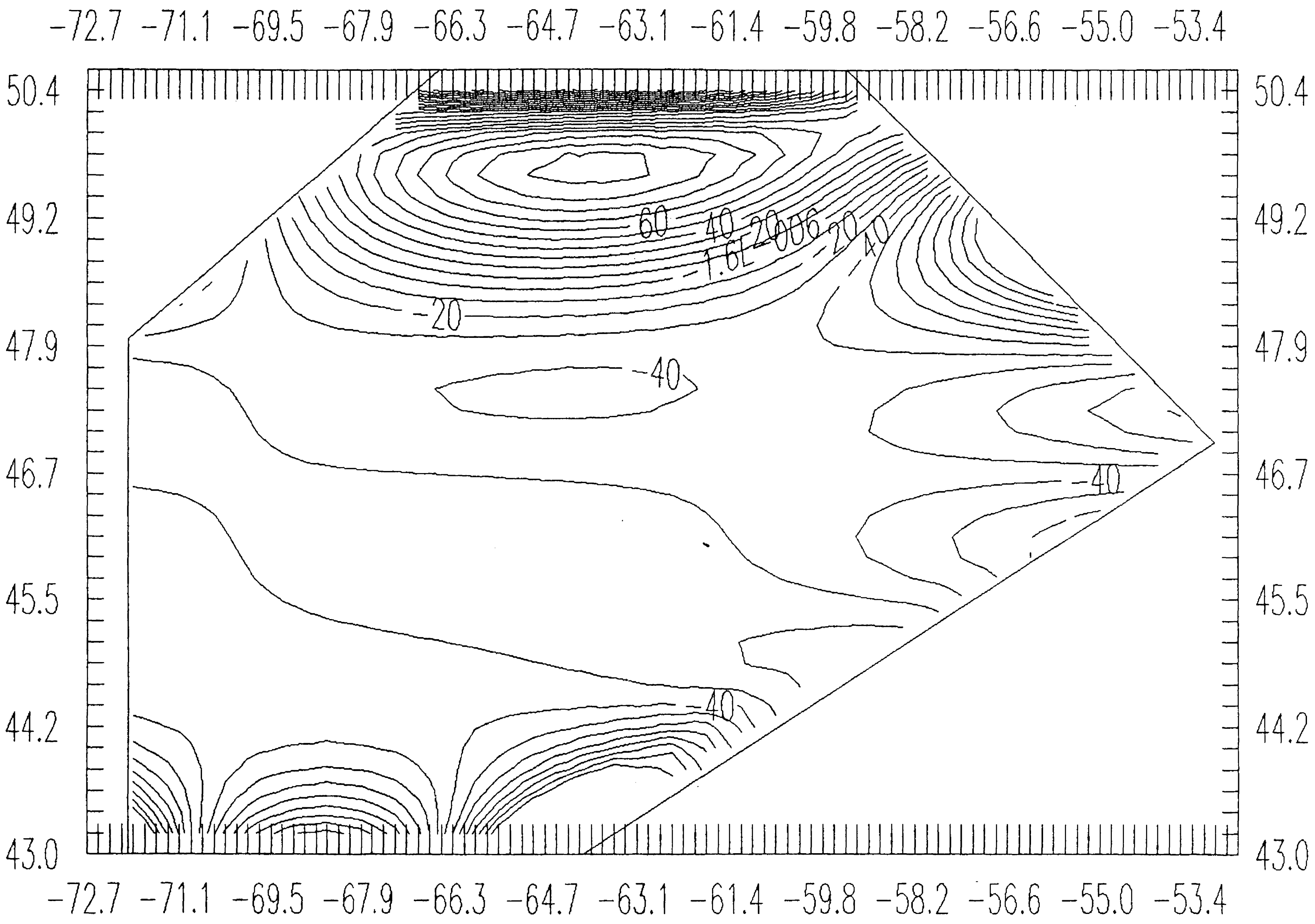


Uplift in Atlantic Canada and New England (3x6)



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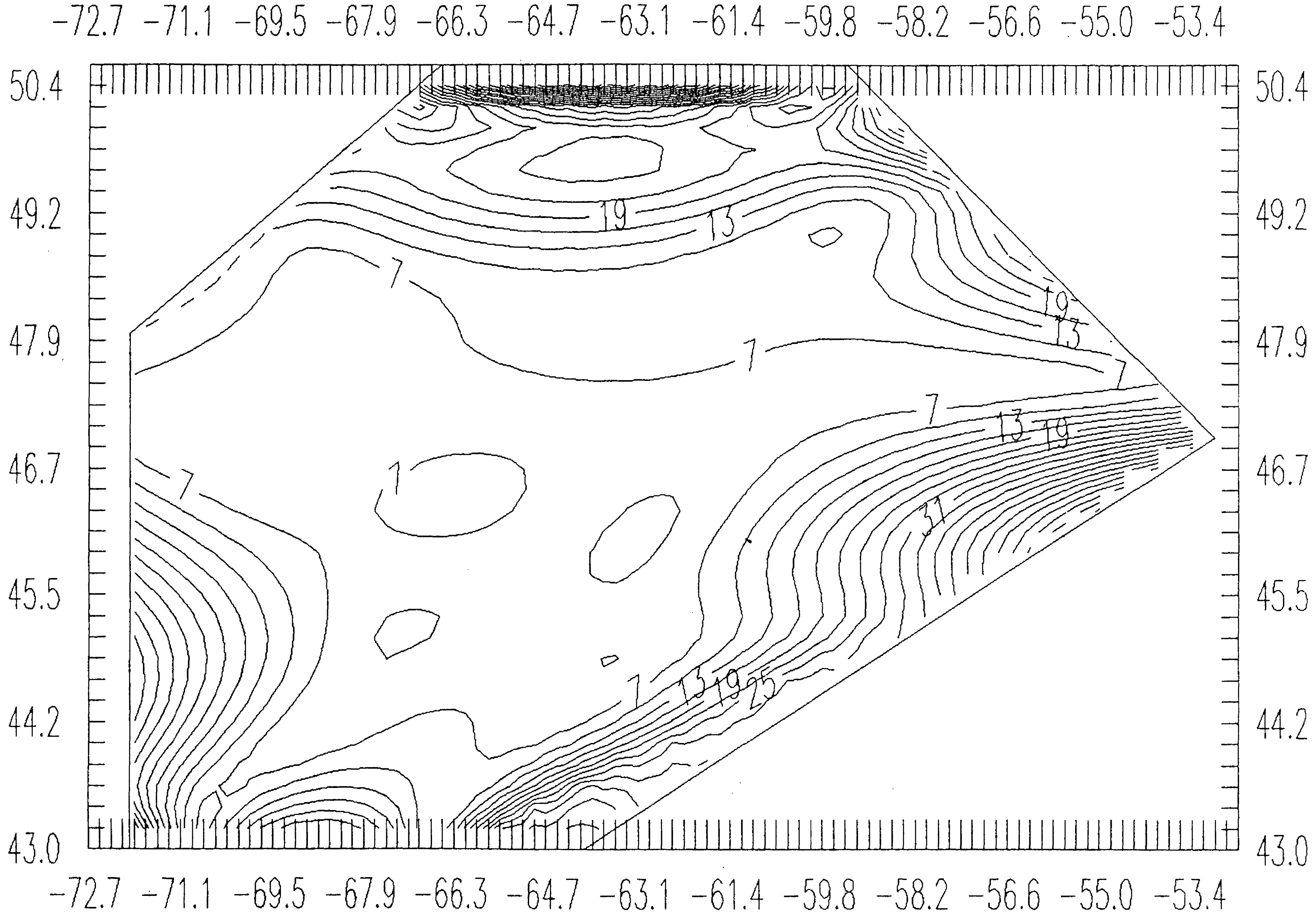

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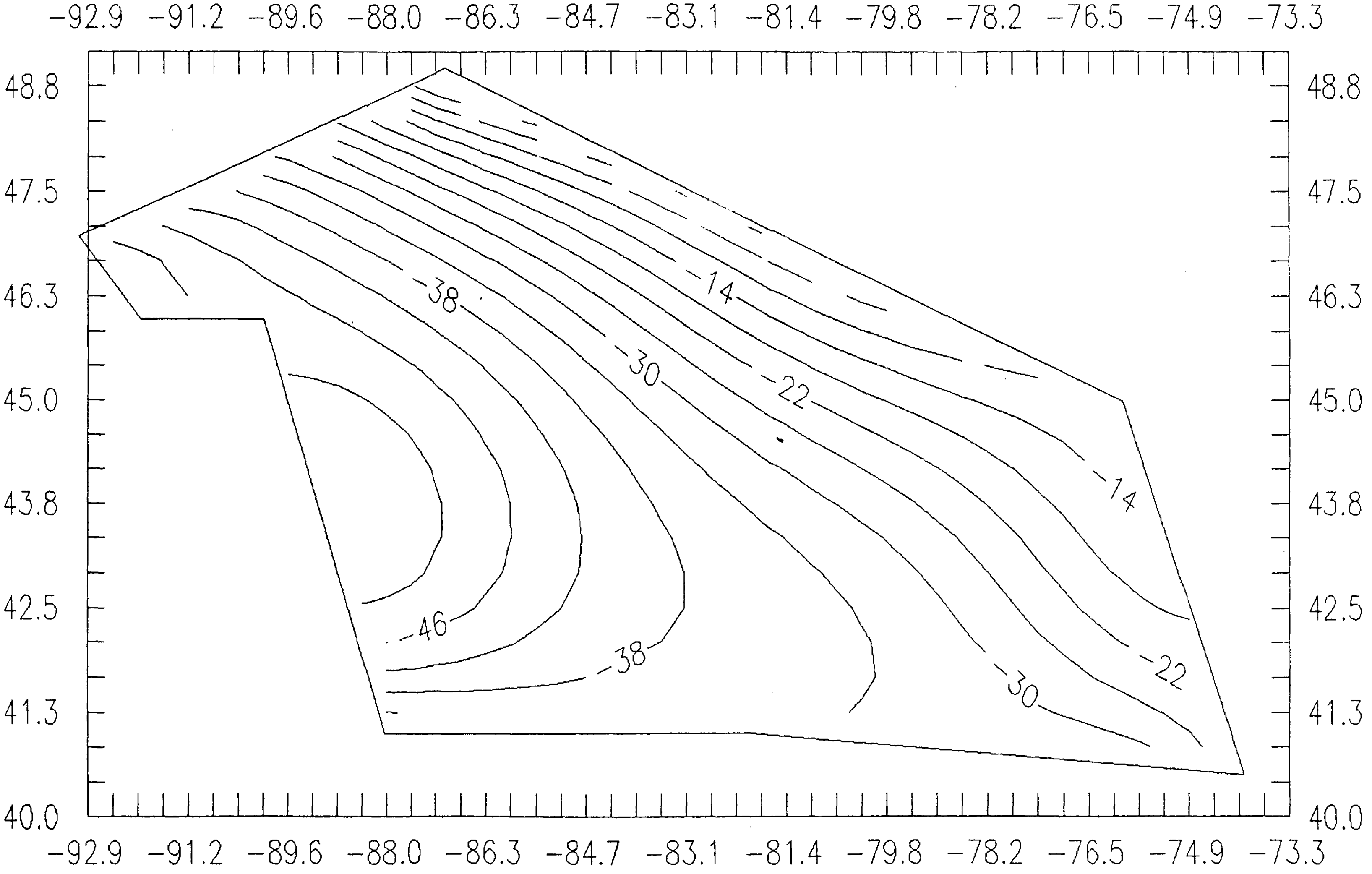
Uplift in the Great Lakes Region (5x4)



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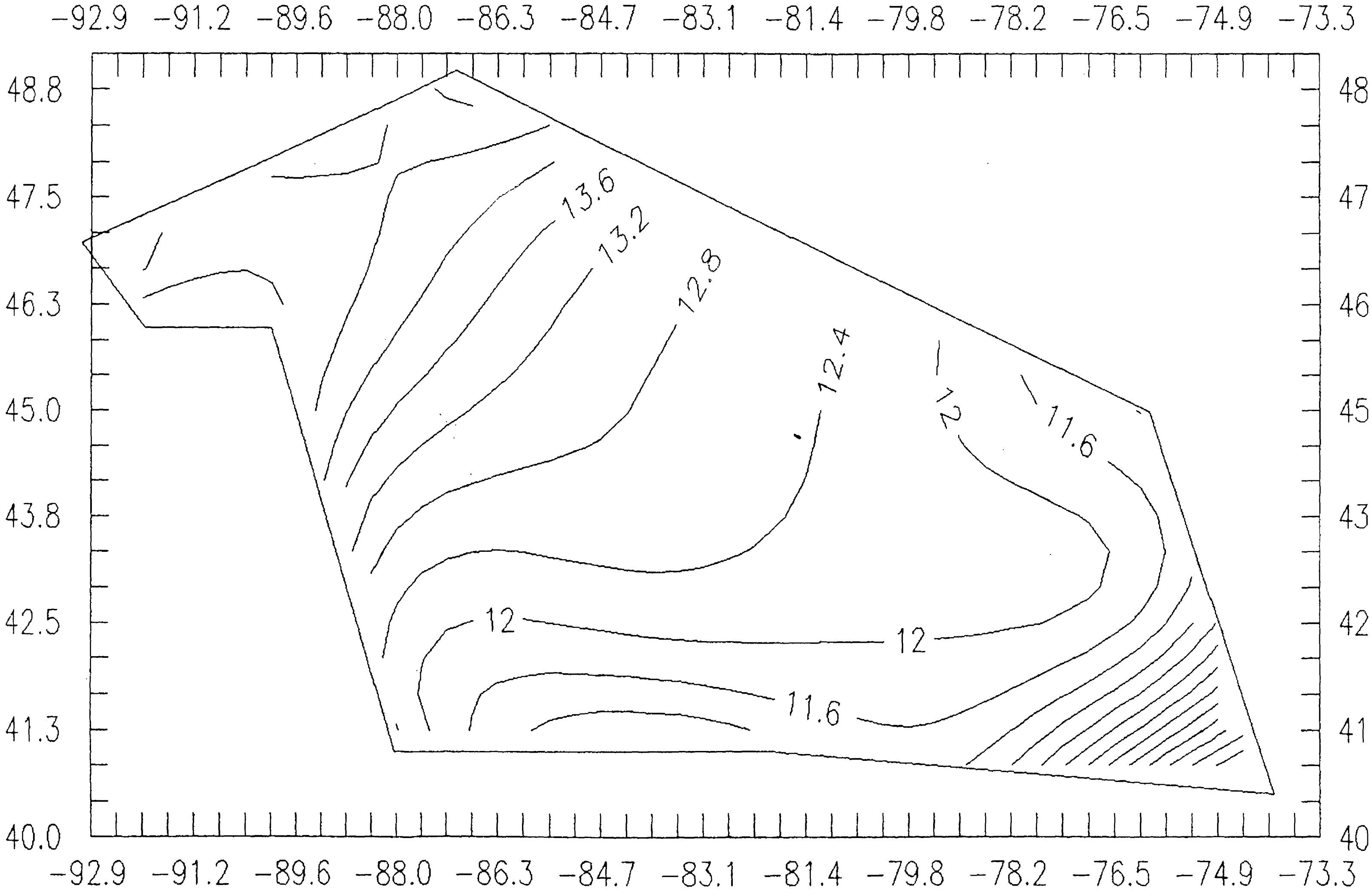

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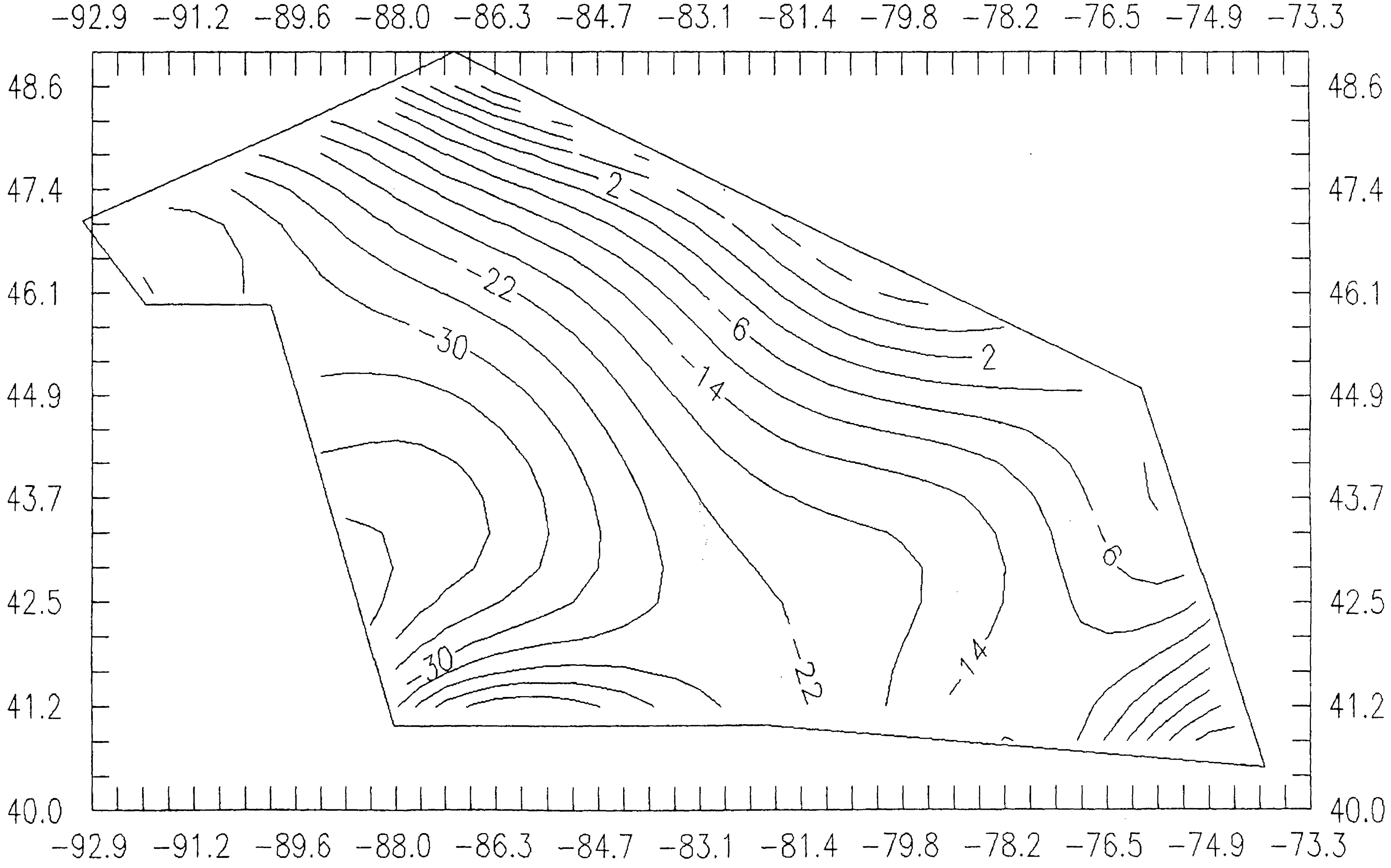

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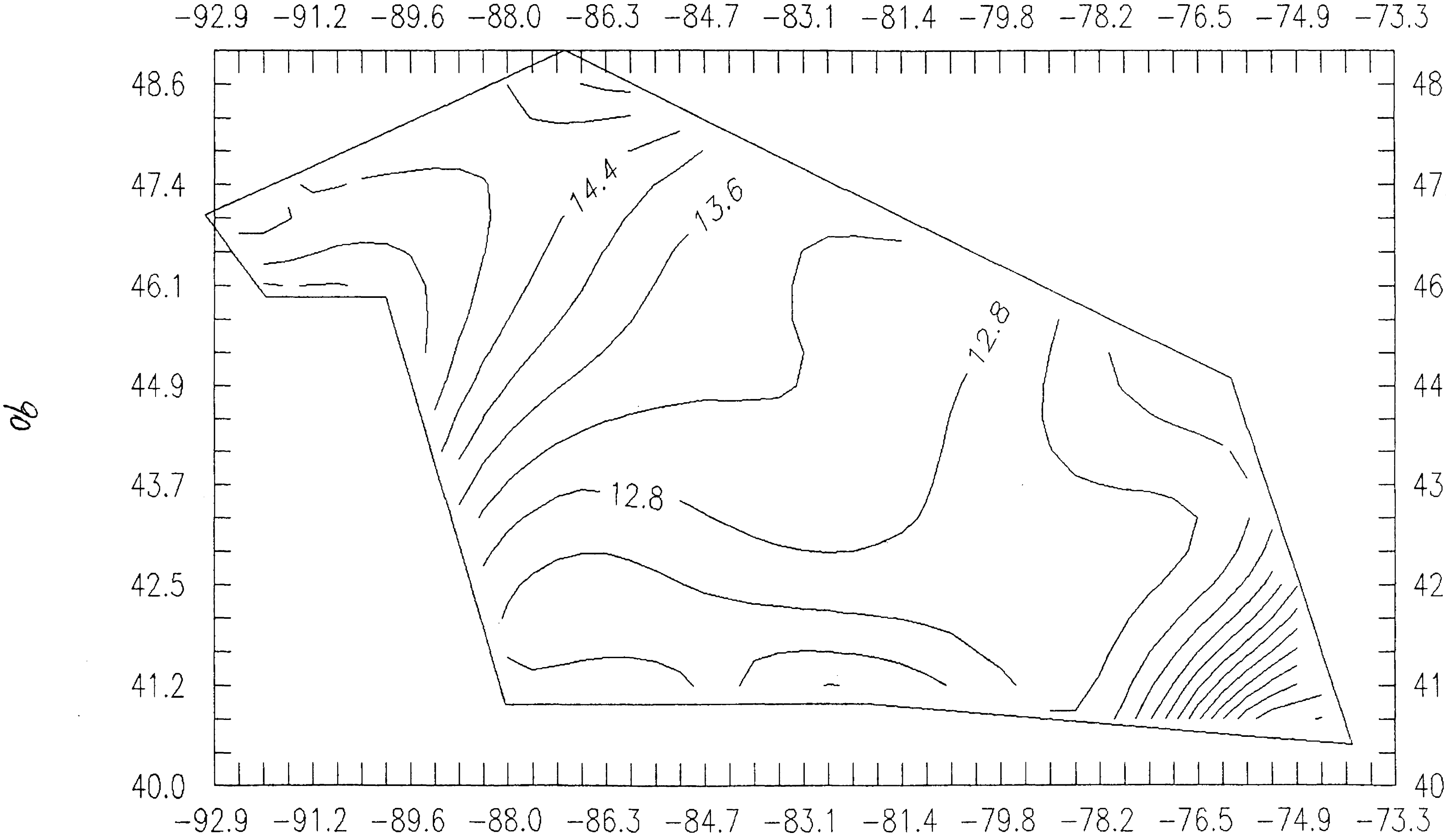
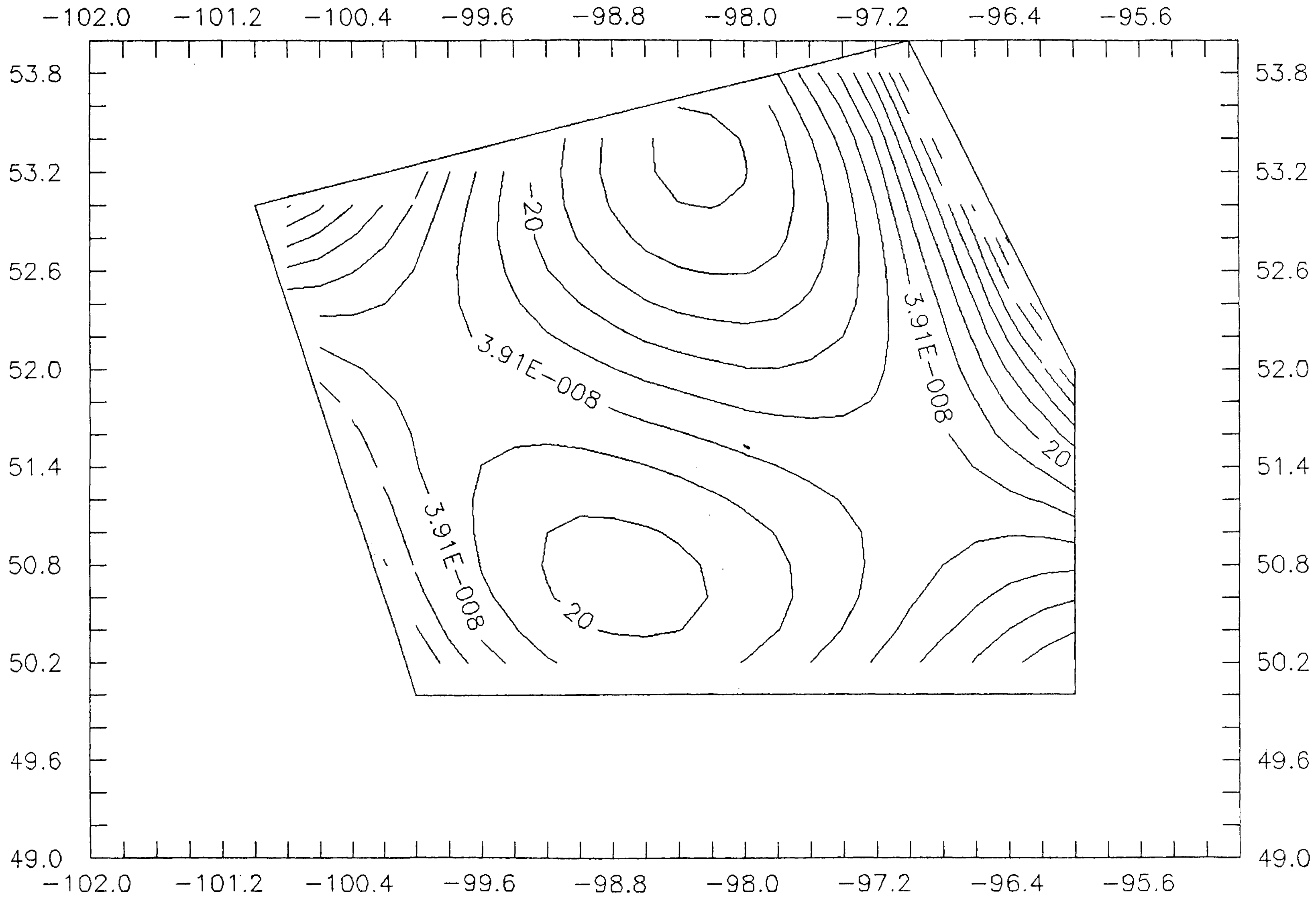


Uplift in the Winnipeg, Winnipegosis and Manitoba Lakes Region



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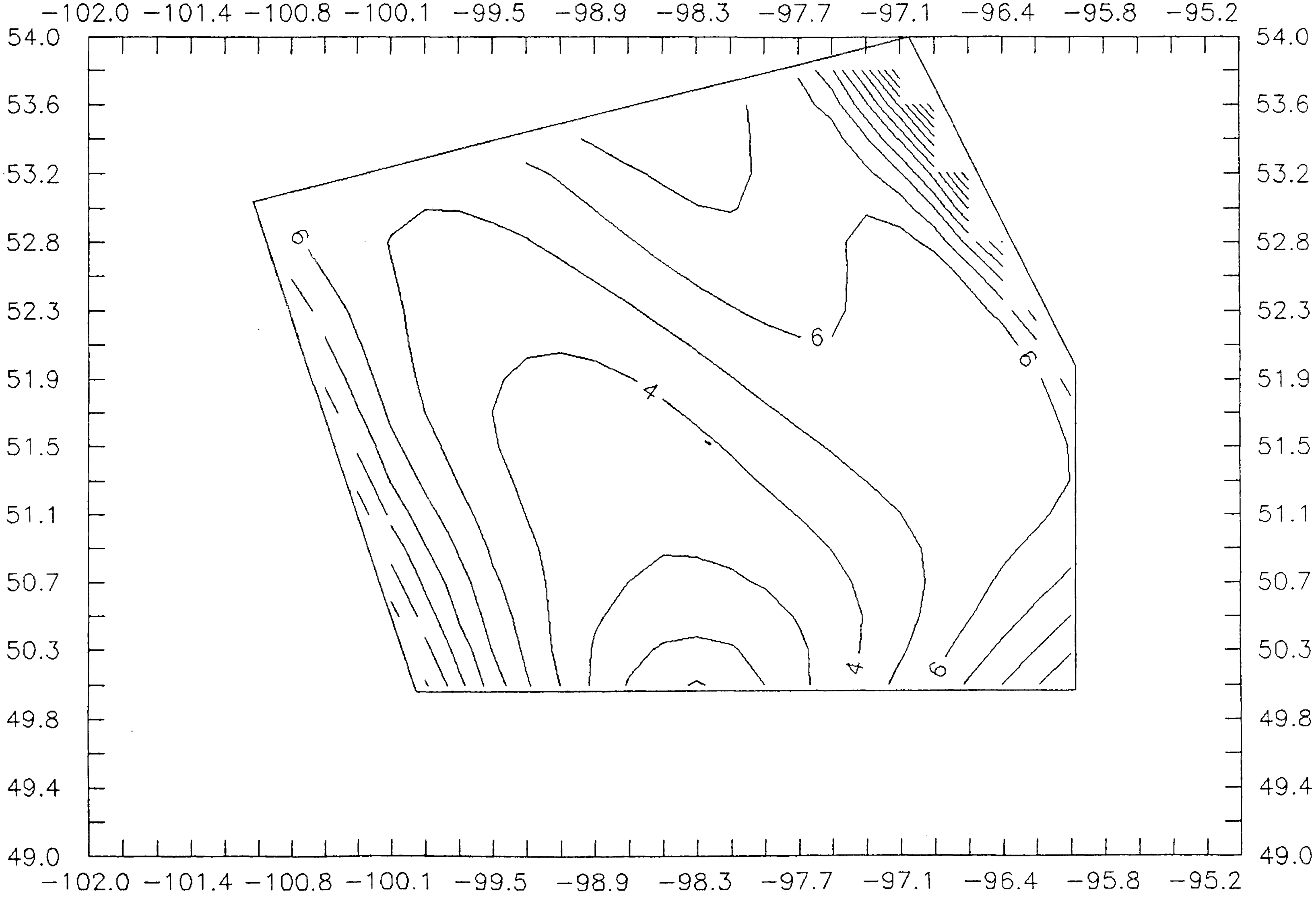
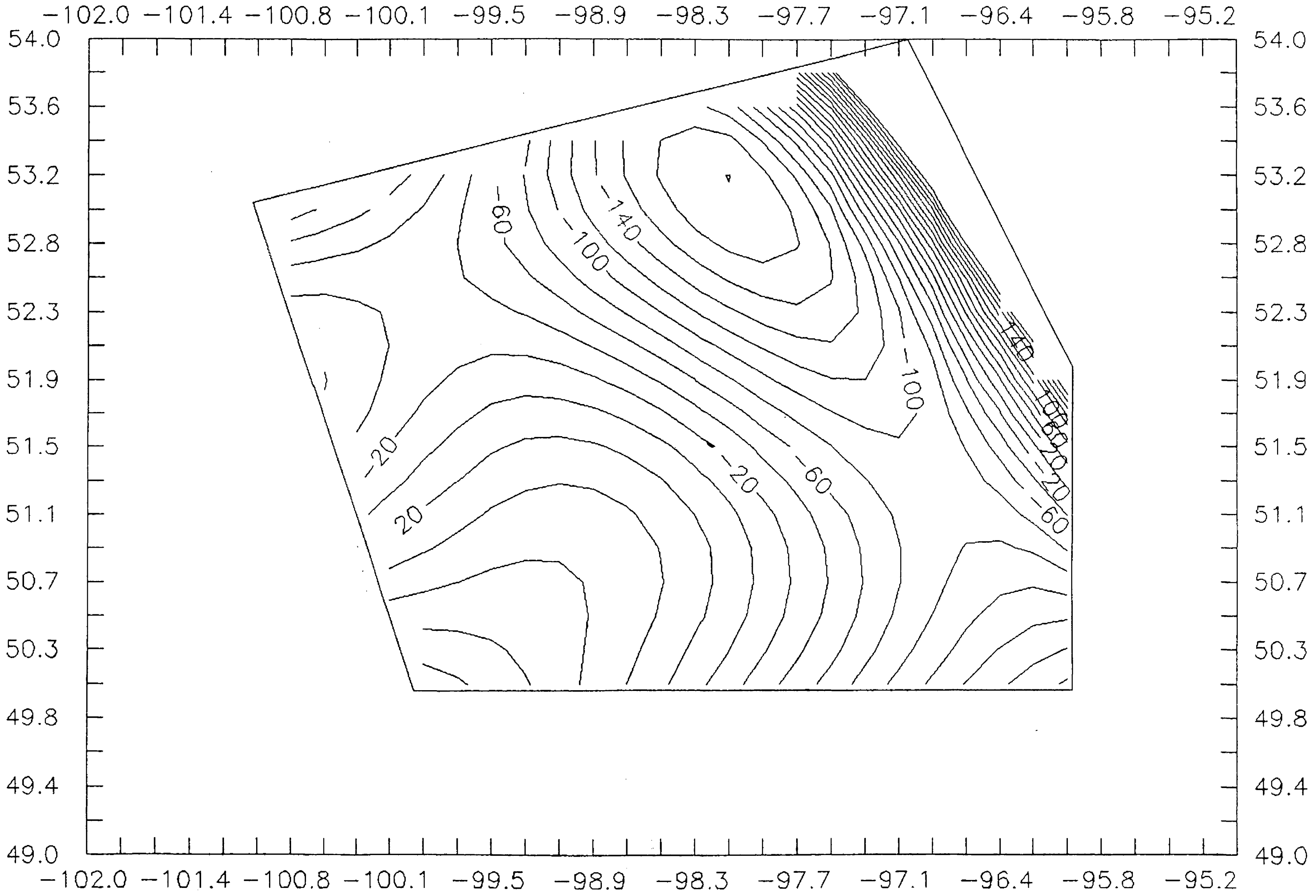
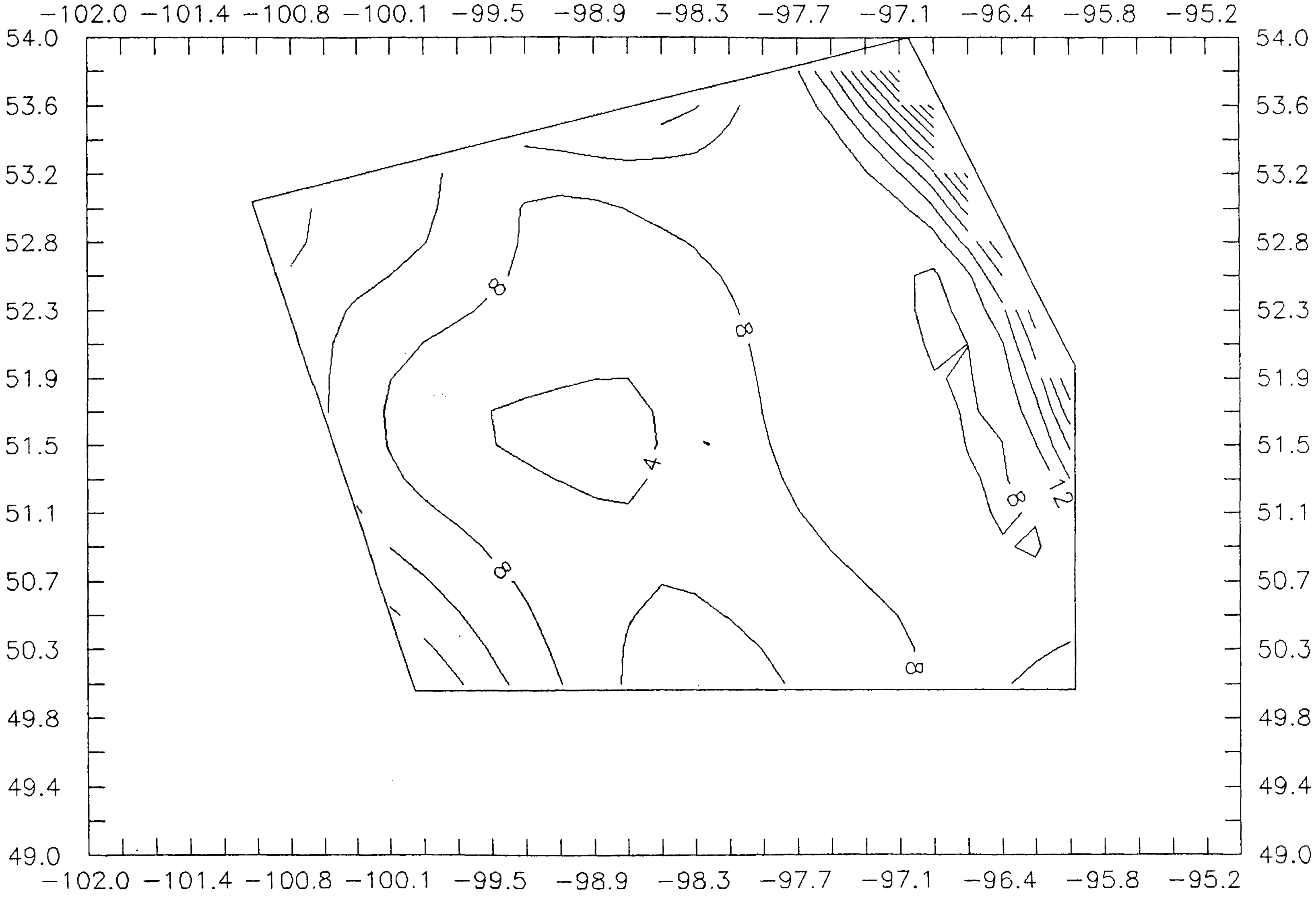


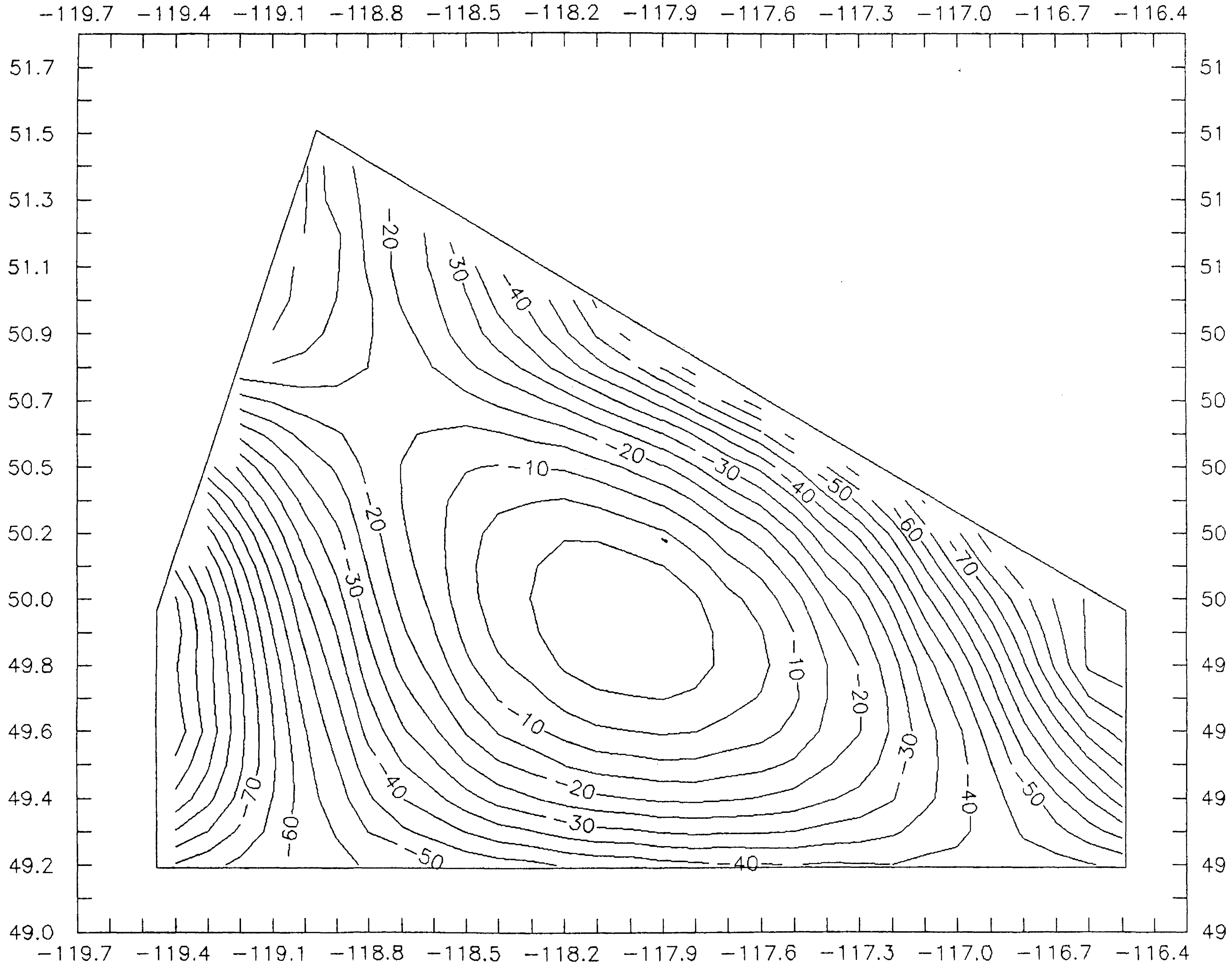
Uplift in the Winnipeg, Winnipegosis and Manitoba Lakes Region (4x4)



Std Dev in the Winnipeg, Winnipegosis and Manitoba Lakes Region (4x4)



Uplift in the Kootenay, Okanagan and Shuswap Lakes Region (2x2)



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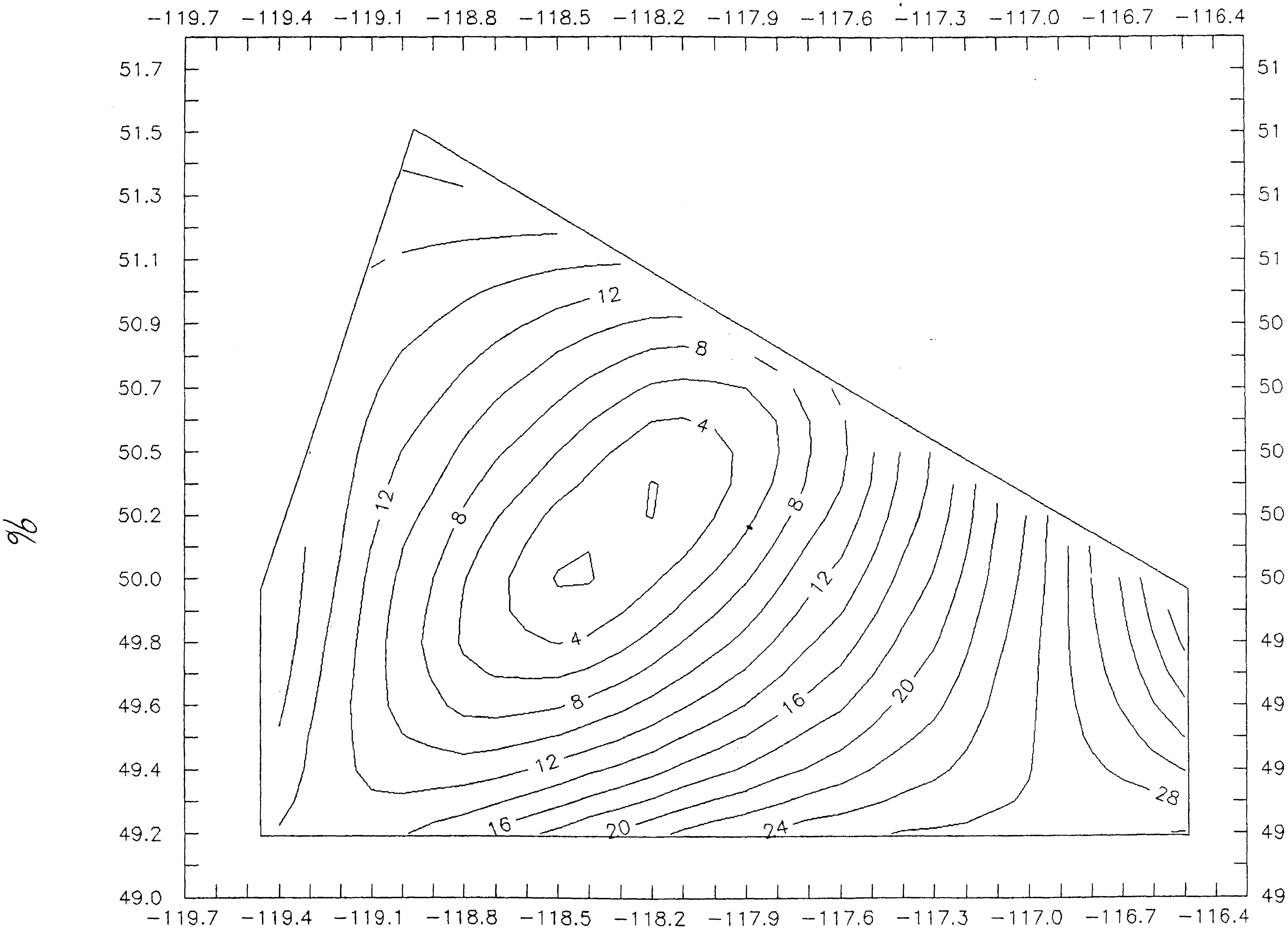
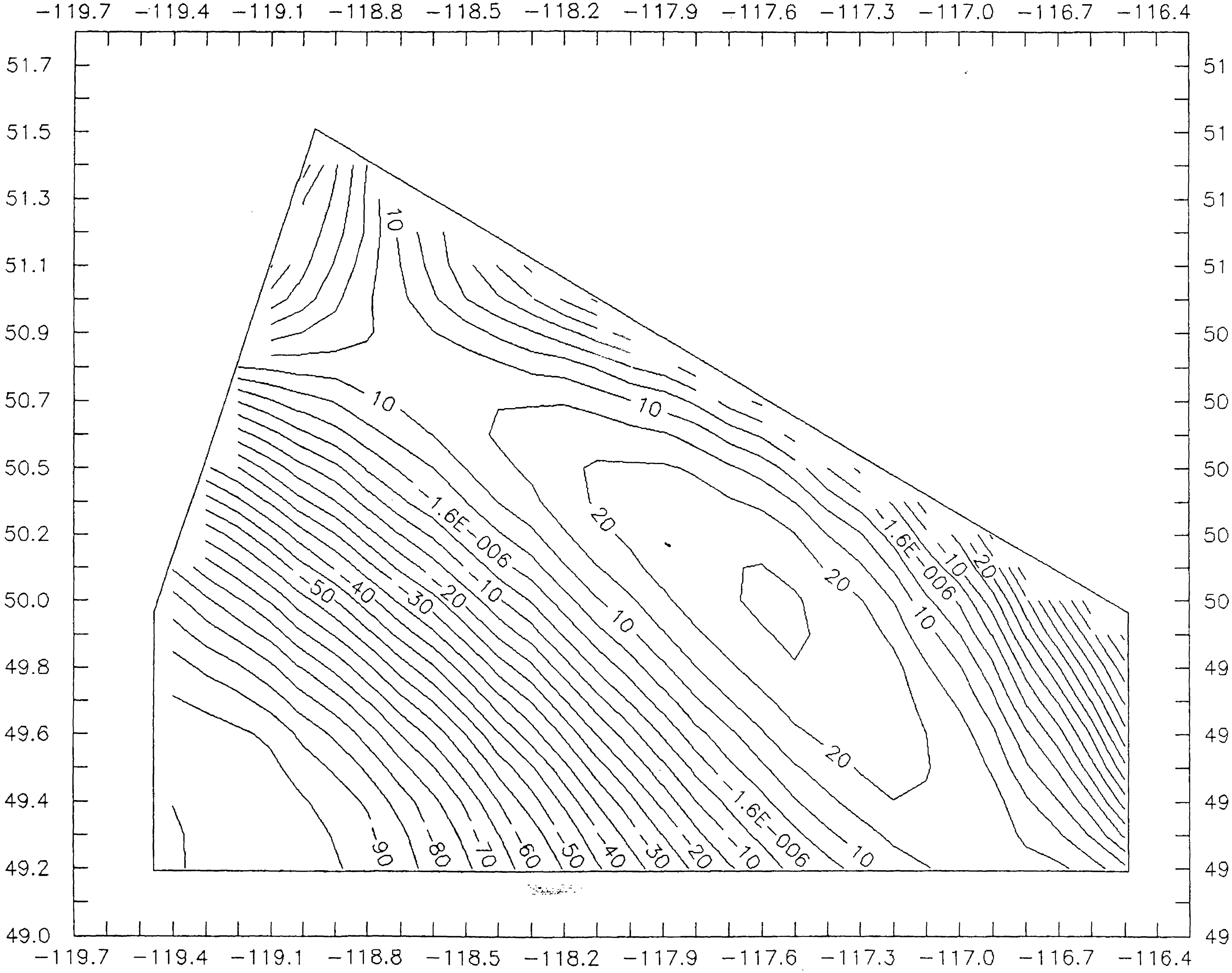
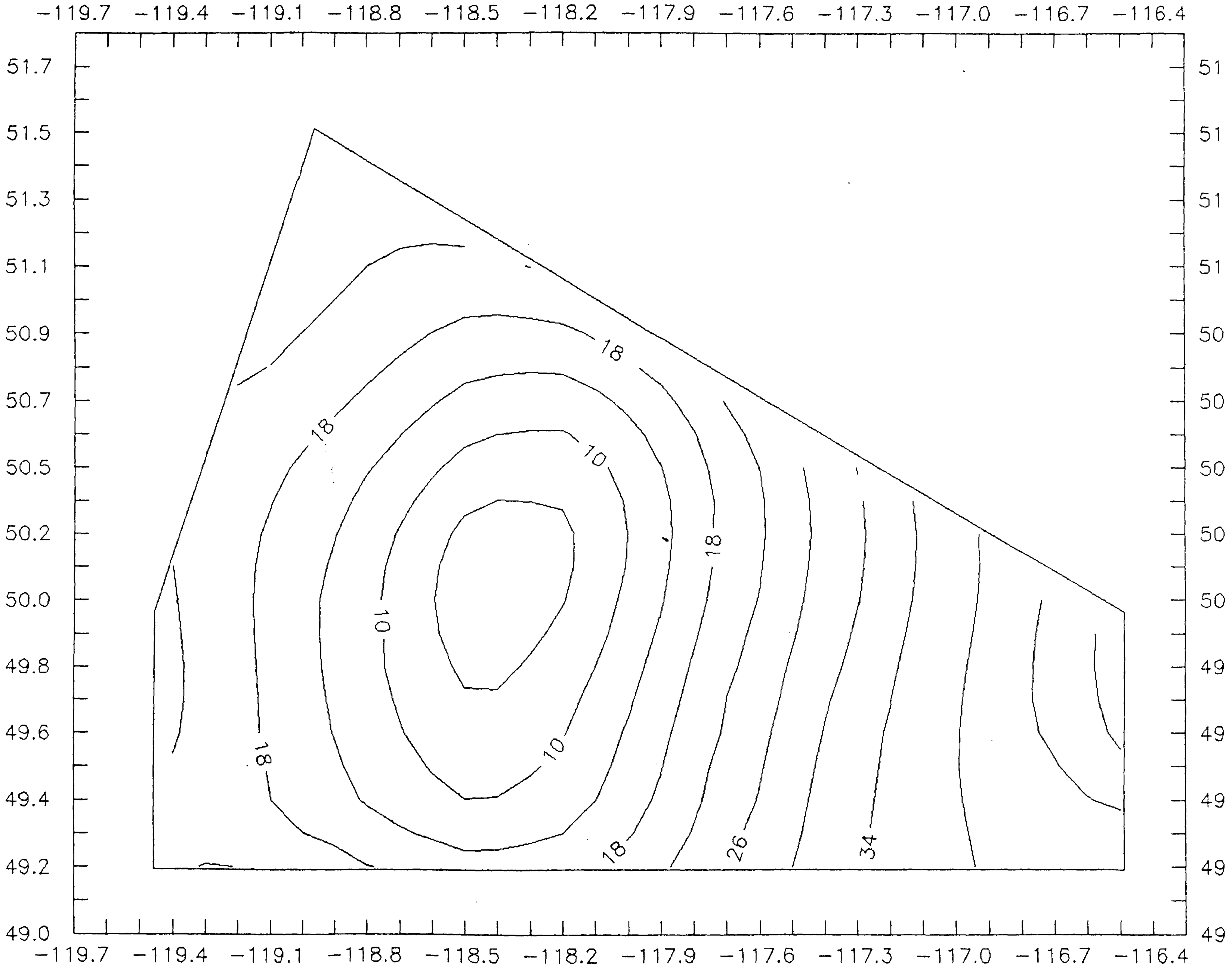
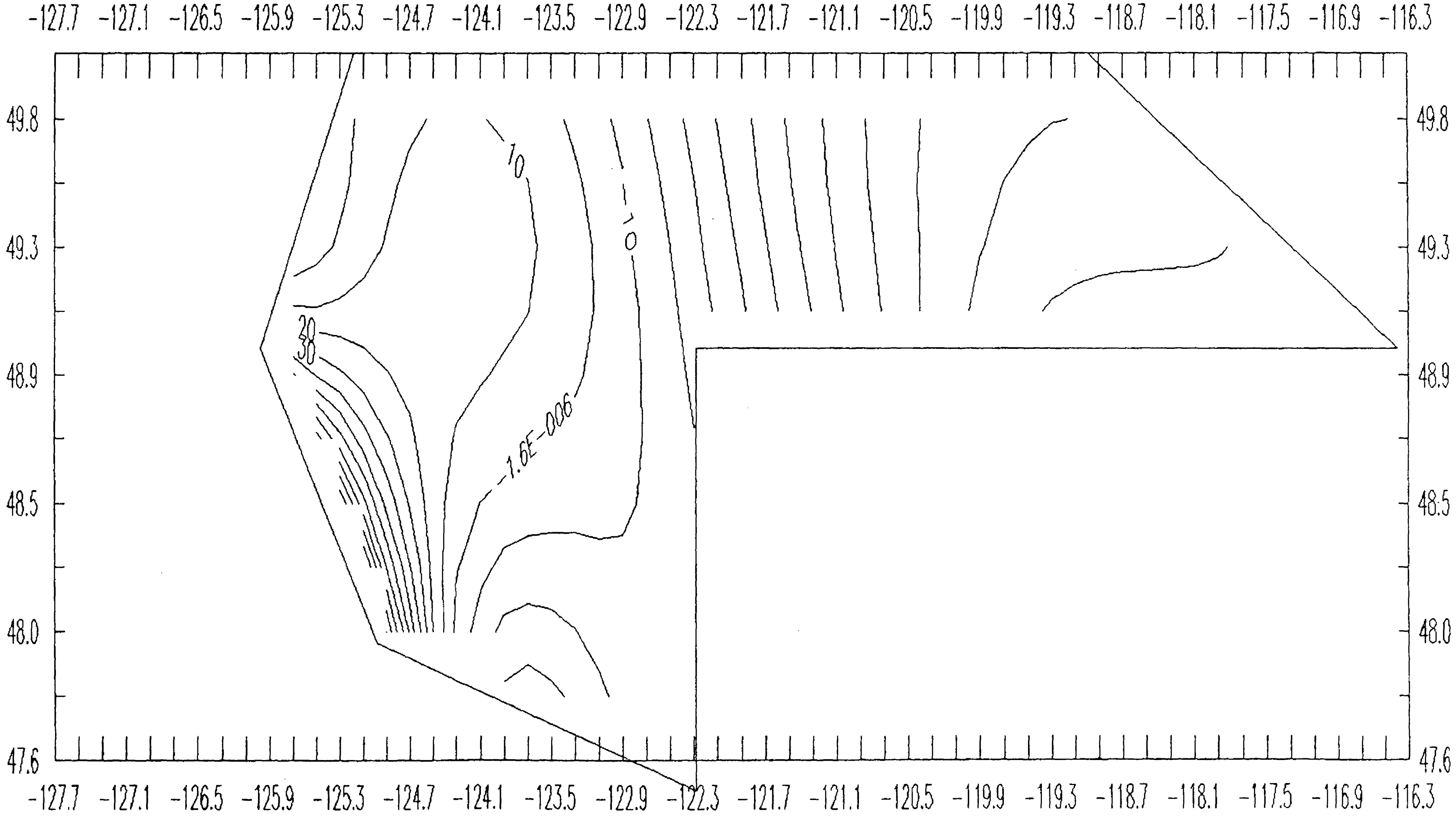


Uplift in the Kootenay, Okanagan and Shuswap Lakes Region (3x3)



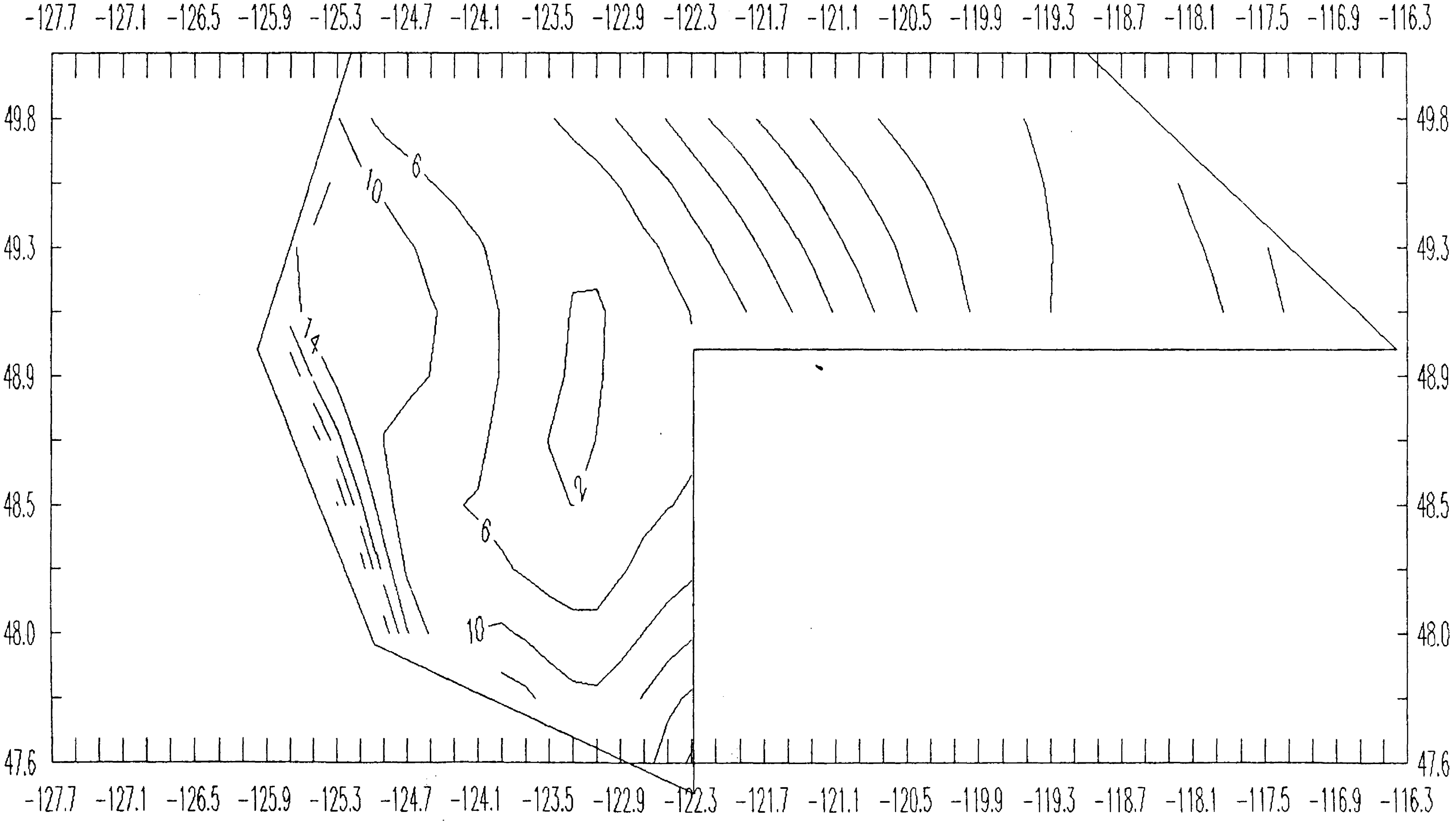
Std Dev in the Kootenay, Okanagan and Shuswap Lakes Region (3x3)



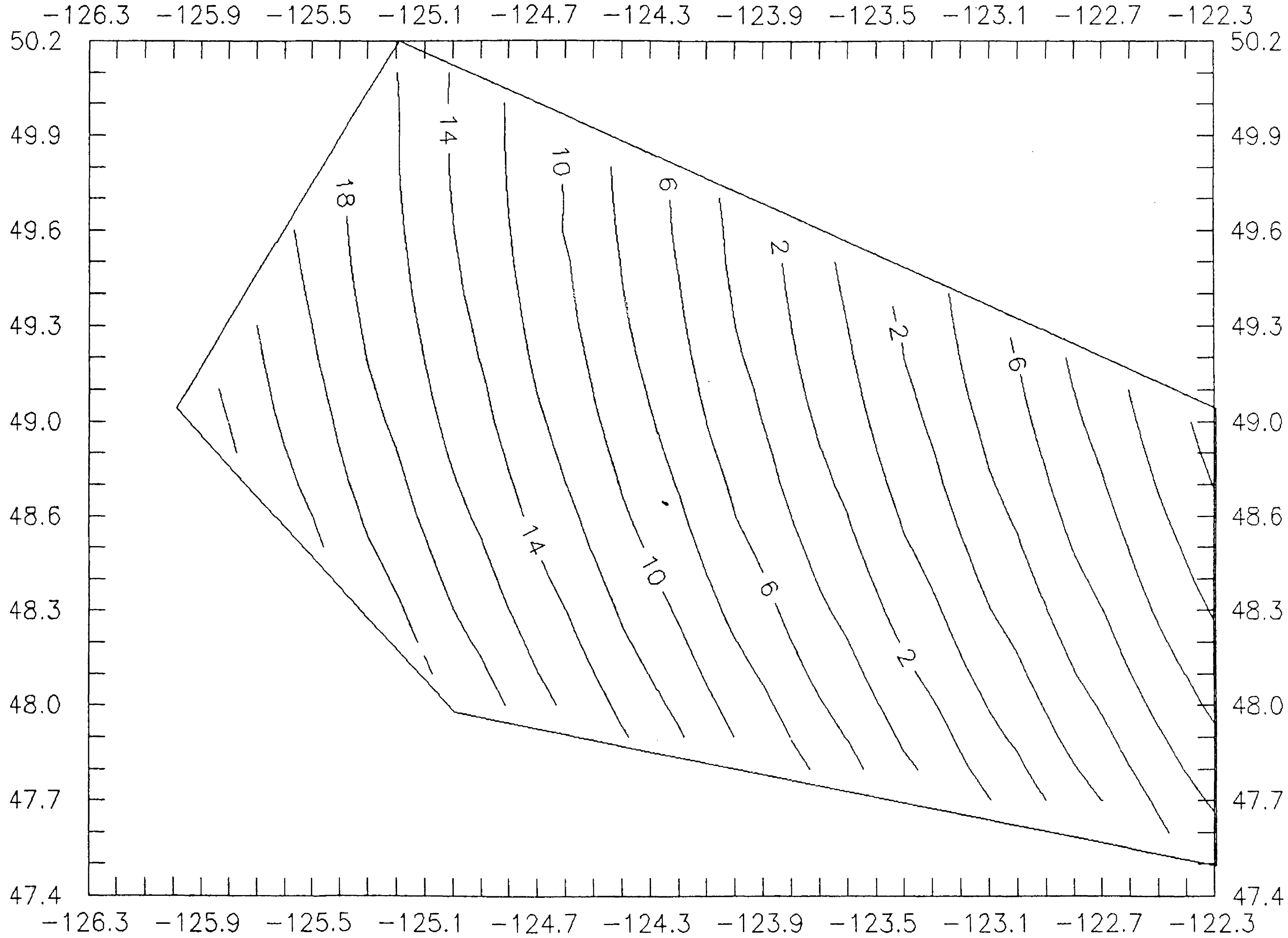


Uplift in the Southern British Columbia Region (5x3)

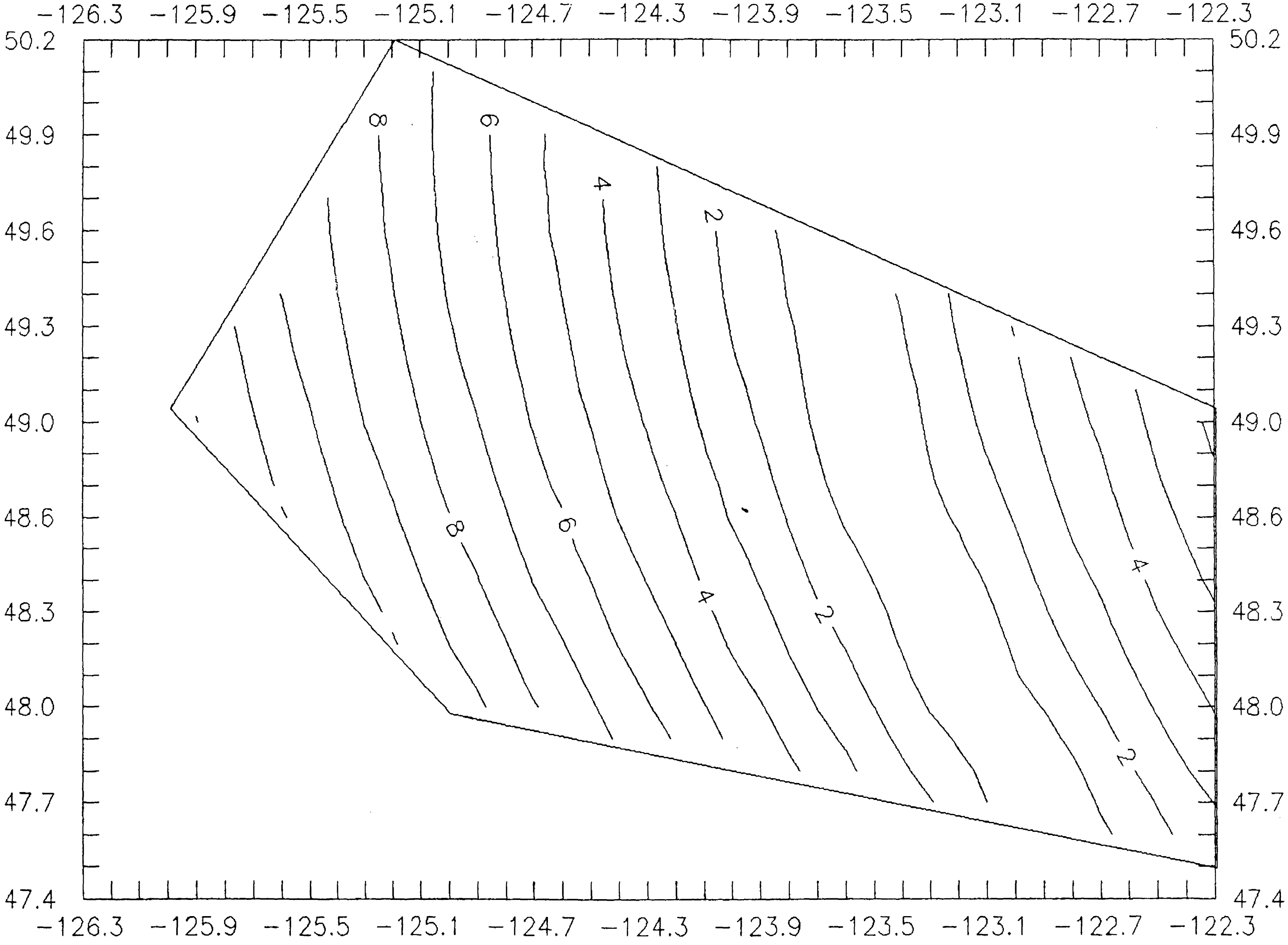
Std Dev in the Southern British Columbia Region (5x3)



Uplift in the Southern Vancouver Island Region (2x2)



Std Dev in the Southern Vancouver Island Region (2x2)



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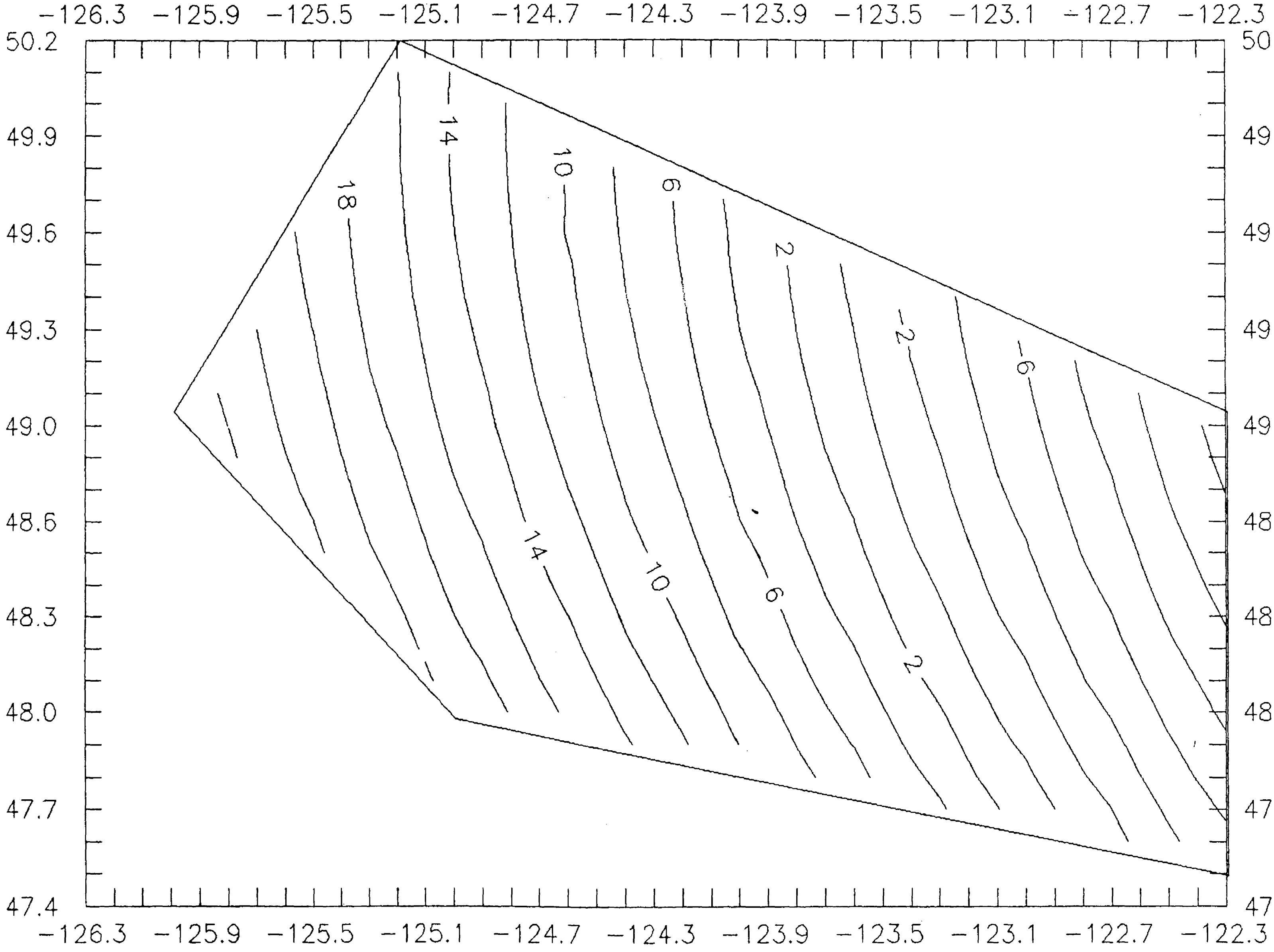

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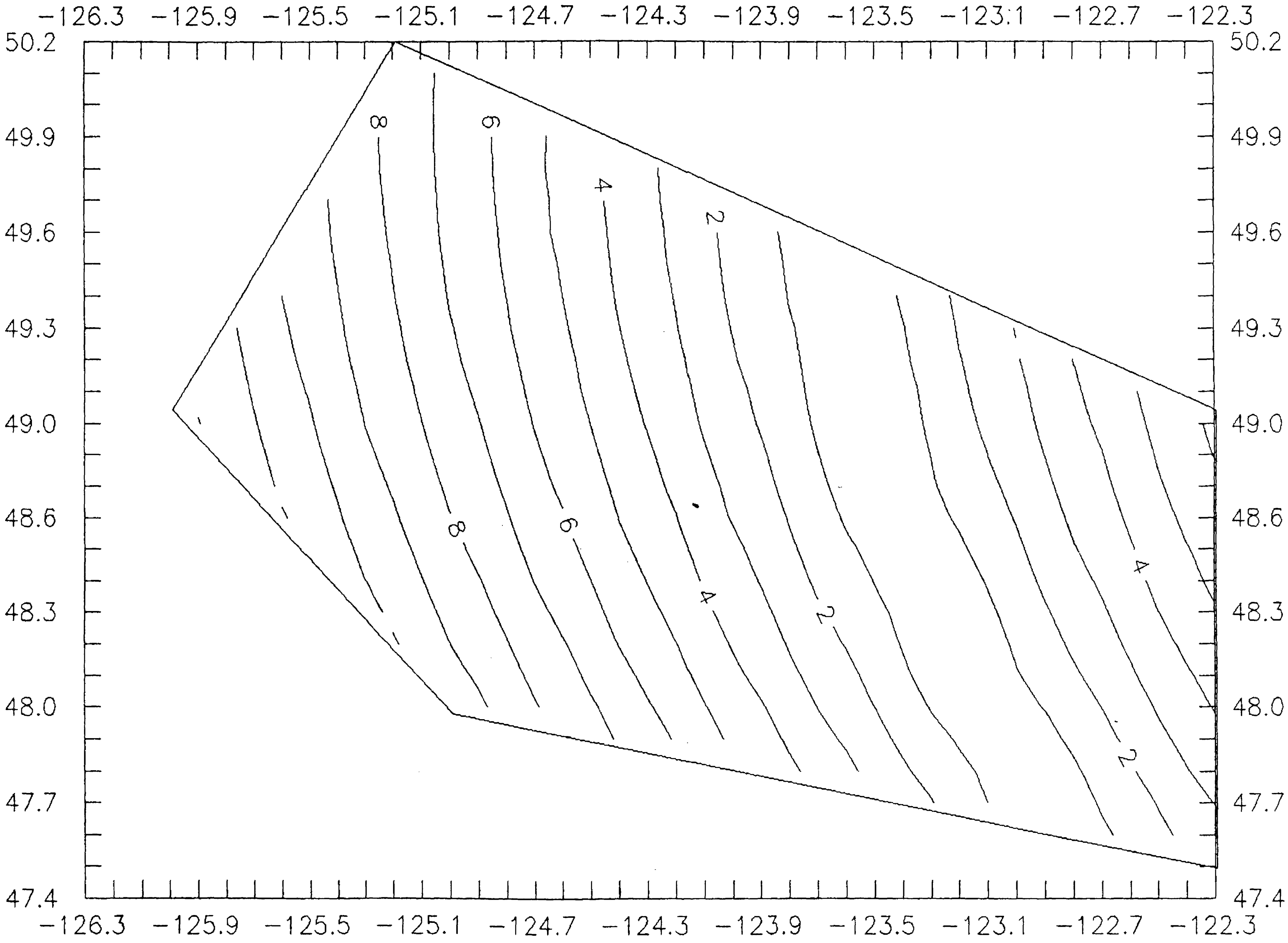
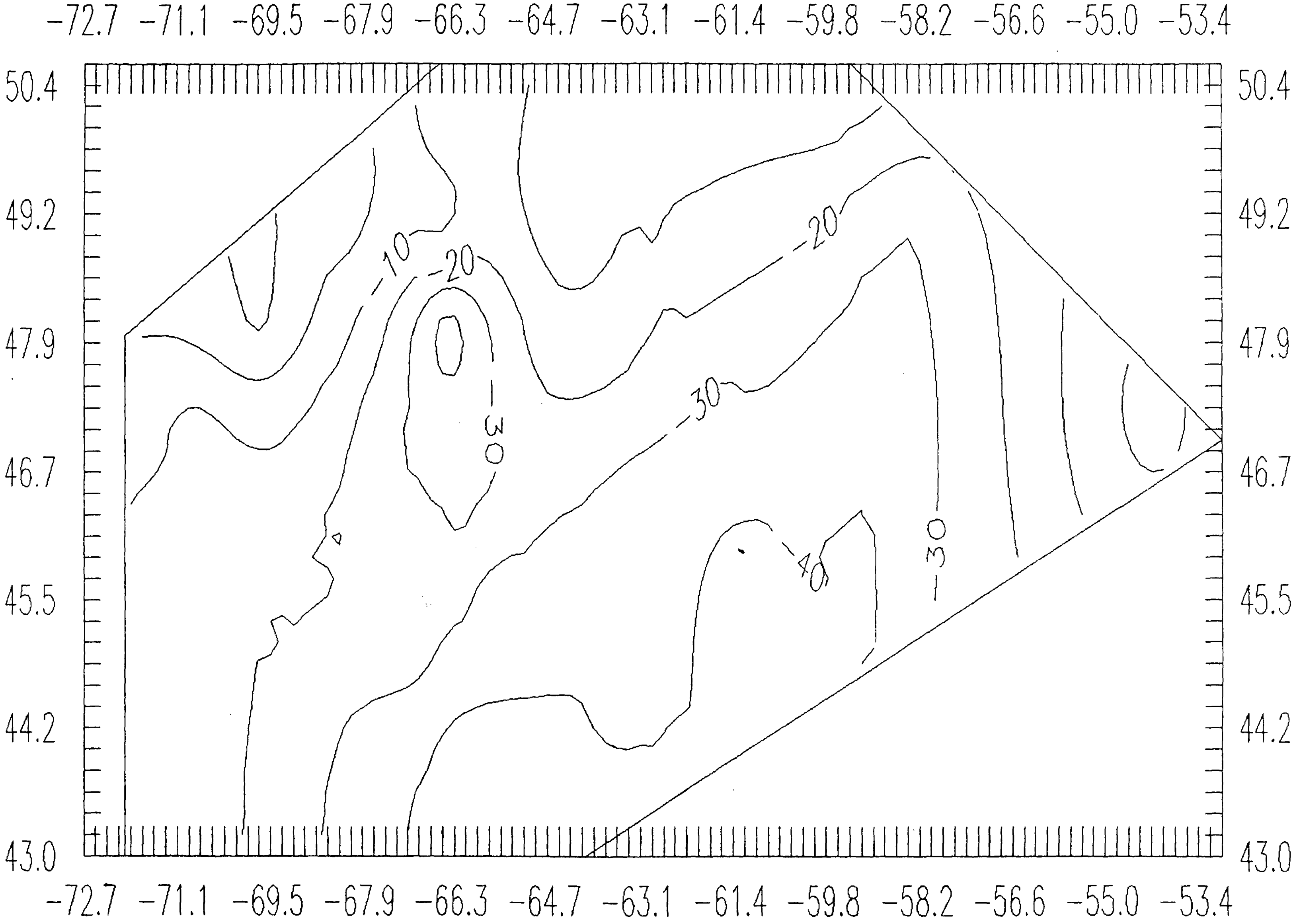
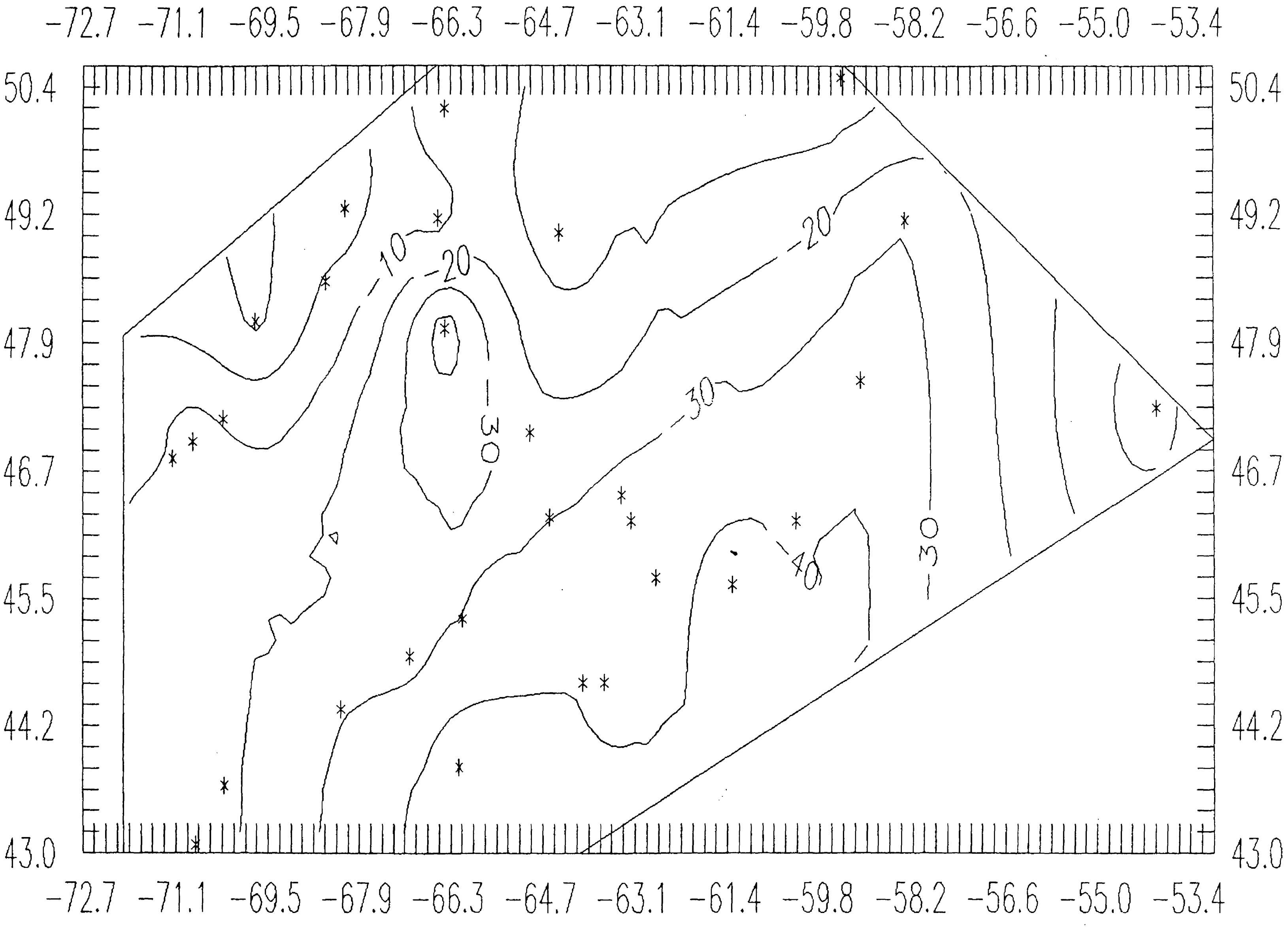


Kriging over Gauge Sites in Atlantic Canada and New England



Kriging over Gauge Sites in Atlantic Canada and New England



Kriging over Gauge Sites in Southern Vancouver I.