IMPLEMENTATION OF TRIGONOMETRIC HEIGHT TRAVERSING IN GEODETIC LEVELLING OF HIGH PRECISION

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PREFACE

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PREFACE

This technical report is a reproduction of the final report for contract No. OST84-00226 (DSS File No. 22ST.23244-4-4005) prepared for the Geodetic Survey Division of Energy, Mines and Resources Canada in October 1985. The report covers a study on the implementation of trigonometric height traversing in geodetic levelling of high precision. Dr. Adam Chrzanowski served as the Principal Investigator of this project and Mr. Sandor Vamosi of Energy, Mines and Resources Canada was the Scientific Authority.

This report contains only Appendix I of the final contract report. All other appendices listed in the Table of Contents are available by application to Dr. Adam Chrzanowski at the Department of Surveying Engineering, University of New Brunswick.

The final contract report also has been published by Geodetic Survey of Canada as contract report No. 85-006 (October 1985).

Dr. A. Chrzanowski, P.Eng
Principal Investigator

ACKNOWLEDGEMENTS

Financial support for this project was obtained from Energy, Mines and Resources Canada, from the Natural Sciences and Engineering Research Council, and from the University of New Brunswick.

The Principal Investigator wishes to acknowledge with thanks the initiative and effort of Mr. S. Vamosi of the Geodetic Survey of Canada in instigating this investigation.

Several graduate and undergraduate students from the UNB Department of Surveying Engineering participated in the design and calibration of the equipment, development of the software, field surveys, and processing of the data. Among them, T. Greening, A. Kharaghani, J. Kornacki, and J. Secord are particularly thanked for the long days spent on the project, although the others, like C. Donkin, P. Romero, and Z. Shi, were also always ready to work overtime without pay.

Dr. A. Jarzymowski, a visiting research associate from Poland, helped in the study on the atmospheric refraction.

Kern Instruments of Canada, Kern & Co. Ltd. (Aarau), Wild Leitz Canada Ltd., Wild Heerbrugg Ltd., and AGA Geodimeter of Canada Ltd. are to be acknowledged for their generous provision of instrumentation and assistance.

Wendy Wells did an excellent job of editing and word processing this report.

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Available upon request:

1. INTRODUCTION

1.1 Background Information

As already discussed by many authors, the low speed and systematic errors of geometric levelling with its horizontal lines of sight are reason enough for trying to replace geometric levelling with a trigonometric method, with measured slope distances and vertical angles using either a reciprocal or leap-frog (balanced sights to targetted rods) mode of field operation. The targets in trigonometric height traversing can always be placed at the same height above the ground. Thus, the lengths of sight are not limited by the inclination of the terrain and the systematic errors from refraction are expected to become random because the back- and fore-sight lines pass through the same or similar layers of air when traversing along long slopes. By extending the lengths of sight to a few hundred metres, the number of set-ups per kilometre is minimized. This reduces the accumulation of errors due to the sinking of the instrumentation—another significant source of systematic error in geometric levelling.

Four years ago, the Principal Investigator and his associates at the University of New Brunswick (UNB) initiated a research programme to investigate what maximum accuracy and what speed can be achieved with the trigonometric height traversing in hilly and mountainous terrains.

During the first two years of the investigation, a leap-frog trigonometric method was developed at UNB using specially designed elevated rods (up to 5 metres high) with 3 to 4 targets at different heights [Chrzanowski, 1983]. During the summers of 1982 and 1983, encouraging results were obtained with the method using a Kern DKM2-A theodolite and a Kern DM502 EDM instrument with lines of sight up to 300 m long. The preliminary tests indicated that an accuracy of 2 mm $\sqrt{K}$ (K in kilometres) at the one sigma level could be achieved in hilly areas at a speed compatible with geometric levelling.

The results were presented by the Principal Investigator during a seminar at the Geodetic Survey of Canada (GSC), in Ottawa, in January 1984. This lead to a contract agreement (DSS contract No. 0ST83-00387) between the GSC and UNB to perform a feasibility study on the implementation of the trigonometric height traversing for precise levelling. The study was completed in August 1984 after a thorough evaluation of additional test surveys performed by the UNB group during the summer of 1984 and test surveys reported by the National Geographic Institute (IGN) in Paris [Kasser, 1983] who, concurrently with the studies at UNB, made field tests with reciprocal trigonometric height traversing.

The feasibility study included preliminary results of traversing with the electronic theodolites, Kern E2 and Wild T2000, in both the leap-frog and reciprocal modes. Special attention was paid to the influence of atmospheric refraction, including some field tests and theoretical simulations, which constituted a major part of an M.Sc. thesis [Greening, 1985] under the supervision of the Principal Investigator. The thesis was attached to the contract report as Supplement No. 1.

Results of the feasibility study were presented in an extensive report [Chrzanowski, 1984] to the GSC and summarized in a paper presented at the NAVD '85 Symposium [Chrzanowski et al., 1985].

The main conclusions and recommendations of the feasibility study have been a basis for further investigations which were contracted by the GSC to UNB on 22 August 1984, and which are a subject of this report.

1.2 Objectives and Tasks of the Project

As was mentioned above, the objectives of this investigation were based on the conclusions and recommendations arising from the preceding feasibility study. Extracts from the main conclusions are repeated below for the sake of completeness of this report.

- **Re: Achievable Accuracy:**

  Theoretically, when using the electronic theodolites Kern E2 or Wild T2000 or equivalent, the first-order accuracy ($\sigma = 1.44\sqrt{K}$ mm) of one-way traversing could be reached with four sets of angle measurements and with conventional short range EDM instruments keeping the maximum lengths of sight $s \leq 250$ m (for both trigonometric and leap-frog methods and in all weather conditions) and $\alpha \leq 5°$. By employing the special high accuracy EDM instruments, the inclination of the terrain could be increased to $\alpha \leq 10°$. In each case, the minimum clearance of the lines of sight above the ground should be 1.5 m. Due to a limited amount of experimental data with the electronic instruments, more investigations should be performed before implementing trigonometric traversing with electronic theodolites into first-order vertical control surveys.

- **Re: Influence of Atmospheric Refraction**

  On long runs, the influence is randomized and cancelled out when keeping the length of sights $s \leq 250$ m for the leap-frog method and $s \leq 300$ m for the reciprocal measurements. However, in short traverses, say, 10 km, the error may accumulate to about 1 mm/km with, perhaps, errors as large as up to 5 mm per individual set-up depending on the terrain configuration, particularly when the line of sight runs closer to the ground than 1.5 m within the first 100 m from the instrument.

  There is no practical method of correcting the measurements for the influence of refraction. The most reliable seems to be to measure gradients of temperature along the line of sight at all characteristic points of the terrain profile or, in the case of the known terrain profile, to calculate the gradients from an accepted model of the heat transfer and calculate the corrections. Both methods are impractical because they require additional and time-consuming measurements. There is an indication from the UNB tests that by employing at least two heights of measurements using, e.g., two targets at 2.0 m and 3.5 m, as well as avoiding measurements closer than 1.5 m above the ground and performing the measurements between 9:30 and 18:30, the influence of refraction will be cancelled out in most of the set-ups. More investigations are needed.

- **Re: Speed of the Survey**

  In the first- and second-order surveys, where 4 sets of angle measurements are required when using the precision electronic theodolites, a minimum of 7 to 9 minutes per set-up will be required in the reciprocal mode of operation and about 10 to 12 minutes in the leap-frog method. If an average length of sight distance is taken as 200 m for the leap-frog method (maximum 250 m for the first order) and 250 m for the reciprocal measurements, then one could expect a progress of about 2.2 km/h for each of the methods. Adding now the time needed for reconnaissance in the leap-frog method and for connecting surveys to the bench marks in the reciprocal method, a realistic progress for a 7-hour working day would be between 12 km and 14 km independent of the gradient of the route. This calculation is based on the assumption that a fully automatic recording and checking system is used in the field procedures.

- **Re: Reciprocal versus Leap-frog Method**

  Each method has its advantages and weak points.... Definitely more tests on the comparison of the performance of two methods is needed in order to optimize the trigonometric height traversing... .

Re: Overall recommendations for the implementation of the trigonometric height traversing. One can consciously say that the second-order accuracy with a speed of about 15 km/day without the accumulation of the systematic errors of refraction can be achieved. The first-order accuracy is perhaps achievable; however, more tests have to be done to verify this statement.

Since the above conclusions were drawn mainly from a theoretical pre-analysis or simulations and from a very small sample of the actual field surveys, the main objective of the further study, which is described in this report, has been to verify the conclusions and implement the above recommendations in additional field tests in the real-world environment using electronic theodolites of high precision in a motorized and computerized mode of operation. As a result, the following tasks had to be accomplished by the UNB group in order to meet the objective:

1. To design and construct a motorized survey system for both the leap-frog and reciprocal modes of the trigonometric height traversing.
2. To develop standards and tolerances for the field procedures, calibration of instruments, and computational checks.
3. To interface the electronic theodolites with microcomputers and to develop software for the on-line field checks and postmission data processing.
4. To make additional investigations on the influence of the atmospheric refraction and on other environmental sources of errors.
5. To perform extensive field tests of the developed system for its possible implementation in the geodetic vertical control surveys.

Since the theoretical aspects of trigonometric height traversing, such as the accuracy pre-analysis, theory of refraction, geodetic aspects, etc., have already been covered in the preceding feasibility study and in Greening [1985], this report concentrates mainly on the design and evaluation of the developed survey system and on the evaluation of the actual surveys which were performed by the UNB group between June and October, 1985.

2. DESIGN, CONSTRUCTION, AND CALIBRATION OF THE SURVEY SYSTEM

2.1 Equipment Requirements and Available Resources

An optimal design of the survey system required the following basic equipment.

(a) For the leap-frog method:

- one pick-up truck with an observing platform and two small vehicles for the target rods
- three electronic odometers (± 1 m accuracy of the travelled distance)
- one electronic theodolite (either Kern E2 or Wild T2000) with an adaptable short-range EDM instrument and accessories
- one battery-operated microcomputer (minimum 32K RAM) with external storage capability and with printer, interfaced with the theodolite and EDM instrument
- one specially designed high tripod with a mechanical or electro-hydraulic lifting device, and with a tiltable head
- two specially designed target rods
- four radio transmitters/receivers.

(b) For the reciprocal method:

- two pick-up trucks as in (a)
- two tripods as in (a)
- two electronic theodolites with adapted targets/reflectors and one EDM instrument as in (a)
- two microcomputers as in (a) with a radio-telemetry link
- four radio transmitters/receivers.

(c) For the investigations of the atmospheric refraction:

- two sets of high-precision (± 0.1°C or better) ΔT measuring systems with a minimum of five temperature sensors each and with automatic data recorders.

Except for one EDM instrument (Kern DM502) and one microcomputer (Epson HX20) which were available at UNB, all other equipment had to be either purchased, rented, or constructed for the project at an estimated cost of between $70 000 and $80 000. An additional $40 000 was needed to cover the labour (2 part-time research assistants and 3 part-time field assistants) and operational costs of the field surveys and laboratory tests. According to the contract agreement with the GSC, about $47 000 was available for the project. An additional $30 000 was assigned to the project by the Principal Investigator from other research grants, mainly from the Natural Sciences and Engineering Research Council. The remaining financial shortage had to be compensated for by partially cutting down the equipment requirements, the number of field tests, and the planned laboratory tests of the electronic theodolites.

As a compromise, it was decided to use only two vehicles in the leap-frog traversing; one vehicle with the observation platform and only one vehicle for the distribution of the forward and backward target rods. This, of course, slightly decreased the speed of the leap-frog method and required an additional person to drive the vehicle between the rod stations. On the other hand, the additional person could be used for the 'real time' reconnaissance to select the next station for the observing vehicle after setting up the forward rod and before returning to pick up the backward rod (see section 3.1). This partially compensated for the lost time and inconvenience due to the manual setting-up of the rods and delays when waiting for the vehicle at the backward target station. Another saving was made by giving up the radio-telemetry link between the microcomputers of the backward and forward stations in the reciprocal method. This turned out to considerably slow down the field checks of the observation data, as is discussed later on (section 3.2). Additional savings were made by decreasing the programme for the investigations on the atmospheric refraction by using only one set of ΔT instrumention with manual recording.

Finally, the following equipment was made available to the project:

- two 1/4-ton pick-up trucks (one new Toyota and one 7-year old Datsun) purchased by UNB and rented for one year to the project
- one theodolite Kern E2, rented for one month in 1984 and purchased (NSERC equipment grant) in April 1985
- one theodolite Wild T2000, rented for one month in the late summer of 1984, for two weeks (free of charge) in January 1985, and for one month in August 1985
- one Kern DM502 EDM instrument with two reflectors (owned by UNB)
- one Wild DI-5 EDM instrument (rented together with T2000) with two prisms
- one microcomputer EPSON HX20 (owned by UNB) and one microcomputer TRS80 model 100 with printer and cassette recorder, purchased with NSERC grant
- two target rods (designed and constructed at UNB)
- two high tripods (designed and constructed at UNB)
- one electronic odometer/tachometer (Halda Rally Computer) rented free of charge from the GSC
- four Motorola radio transmitters/receivers (rented for 4 months)
- six thermilinear temperature probes with a manually operated temperature indicator (purchased with NSERC grant in May 1985)
- various accessories, levels, levelling rods, meteorological instrumentation, etc. (available at UNB).

Details on the construction and/or adaptation of the above equipment are given below.

2.2 Adaptation of the Vehicles

The adaptations were made at the Engineering Faculty Workshop at UNB. Both the Toyota and Datsun pick-up trucks have a similar frame construction. Both required making holes in the floor of their platforms for the legs of the tripods. In order to minimize the damage to the rented vehicles, additional wooden platforms were made which extended, with two holes for the tripod legs, about 60 cm beyond the trucks' frames, as is shown in Figs. 1 and 2. A mechanical device for lifting and lowering the tripods (Fig. 3) was mounted on the wooden platform of each vehicle. The lifting devices were constructed according to the French (IGN) design with the permission from IGN [Kasser, 1984, personal communication]. Details are shown in Appendix I. Both trucks were equipped with extendible roof frames to support canvas for shading the instrumentation and, partially, the roof of the driver's cabin. Amber and red flashing lights were mounted on both trucks. The Datsun, which was supposed to serve as the reconnaissance and rod distributing vehicle in the leap-frog method, was equipped with the electronic tachometer (Halda Rally Computer), produced by Haldex, AB in Sweden (Fig. 4).

2.3 Design and Construction of the Tripods

In order to minimize the influence of the atmospheric refraction, the maximum possible height of the instrument above the ground was required. The Toyota and Datsun pick-up trucks have an advantage over some other models of small pick-ups in that their platforms are comparatively high, about 70 cm above the ground, allowing for a total height of the instrument of well over 2 metres.

There are no commercially available tripods of that height which would be rigid enough for precision surveys. Therefore, the tripods were designed and constructed at UNB using, with some modifications, the French (IGN) experience in designing the grooved flat footings (Figs. 5 and 6) to prevent sinking and sliding of the tripod legs on different surfaces. The supporting plate for interlocking the tripod with the lifting

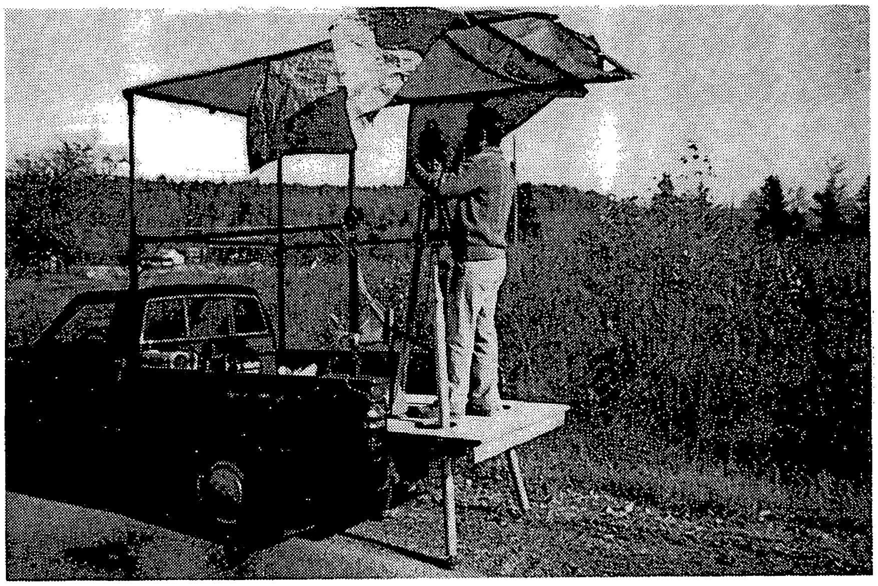


Fig. 1 Observing platform mounted on the Toyota truck.

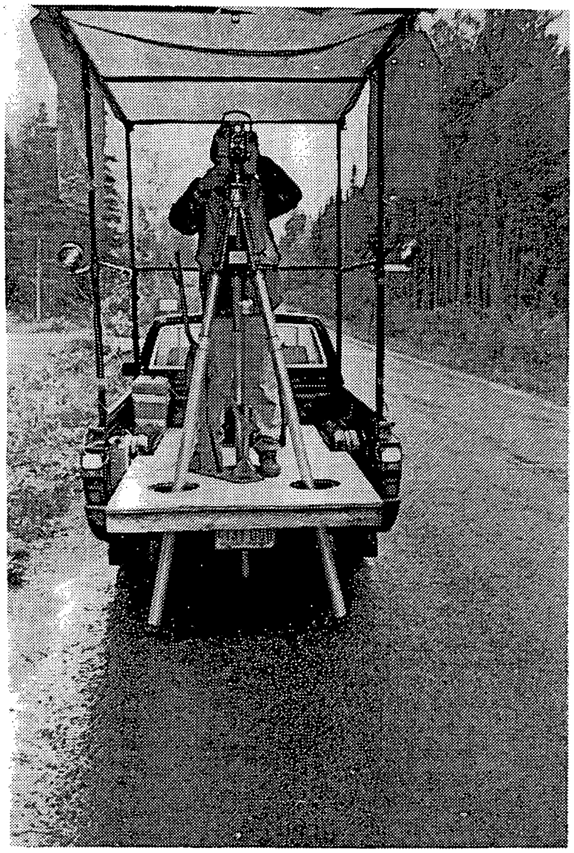


Fig. 2 Fully equipped observing vehicle.

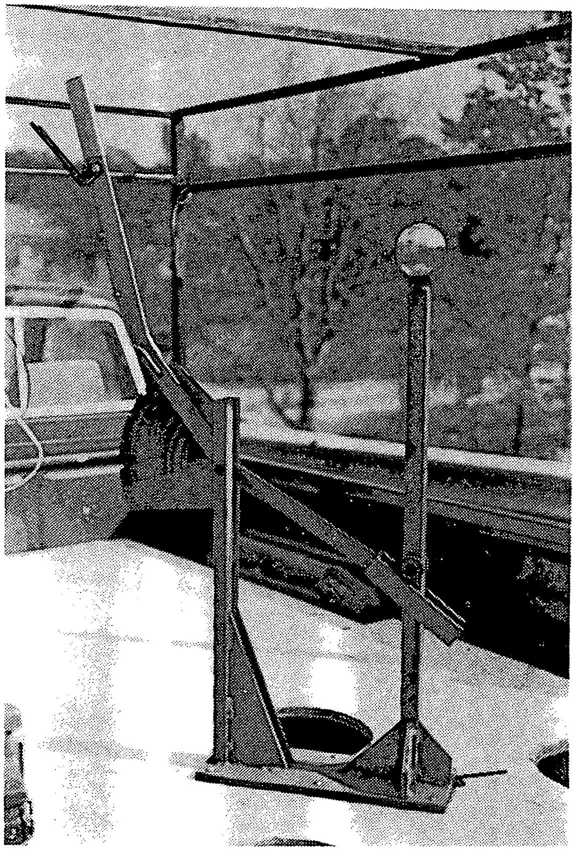


Fig. 3 Mechanical device for lifting and lowering the tripod.

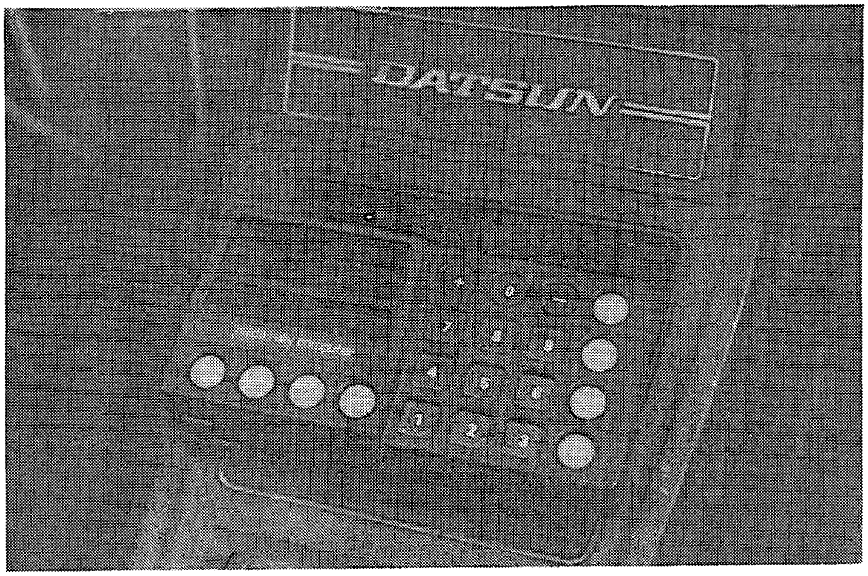


Fig. 4 Halda Rally Computer.

Fig. 5 Grooved footing of the tripod.

device (Figs. 7 and 8) was constructed in a way similar to that of a motorized survey system of the U.S. Geodetic Survey [Whalen, 1985]. The final version (after three iterations) of the UNB tripod is shown in Fig. 9.

The tripod is made mainly of aluminium pipe with the footings made of hardened steel. Centring plates from old Kern tripods were adapted as tilting heads to facilitate quick coarse levelling of the theodolites (Fig. 10). The total weight of the tripod is about 32 kg. Technical details and dimensions are given in Appendix I.

The stability of the tripod was tested on different types of ground surface by using the compensators of the Kern E2 theodolite as sensors of a possible differential sinking of the footings. The stability was found to be within a few hundreds of a millimetre. Nevertheless, the checks of the stability have always been made at each station during the field measurements (see section 3.1).

2.4 Design, Construction, and Calibration of the Target Rods

Several target plates with different patterns of various dimensions had been tested for accuracy of pointing in different atmospheric conditions over distances up to 350 m. A circular, black and yellow target (Figs. 11 and 12), with the overall diameter of 12 cm, was selected for both the leap-frog and reciprocal methods of traversing. It proved to give the accuracy of single pointing better than 30"/*M* (*M* being the magnification of the telescope) in average atmospheric conditions.

In the early experiments with the trigonometric height traversing at UNB, three or four targets were mounted on steel rods at different heights, up to 5 m, in order to minimize and randomize the effects of atmospheric refraction. The long rods were cumbersome to set up and required extreme care to maintain their verticality (see below for the effects of non-verticality). Therefore, a compromise had to be made between the

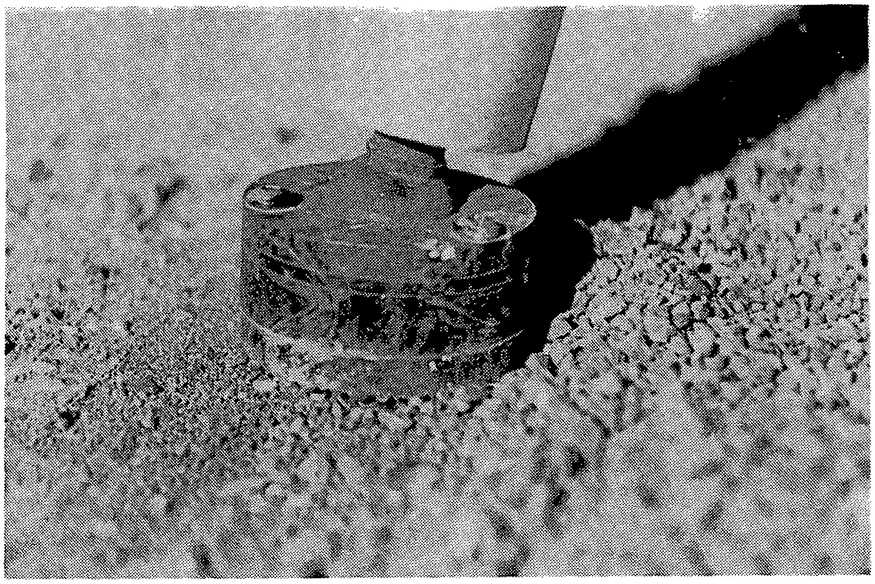


Fig. 6 Footing of the tripod "in action."

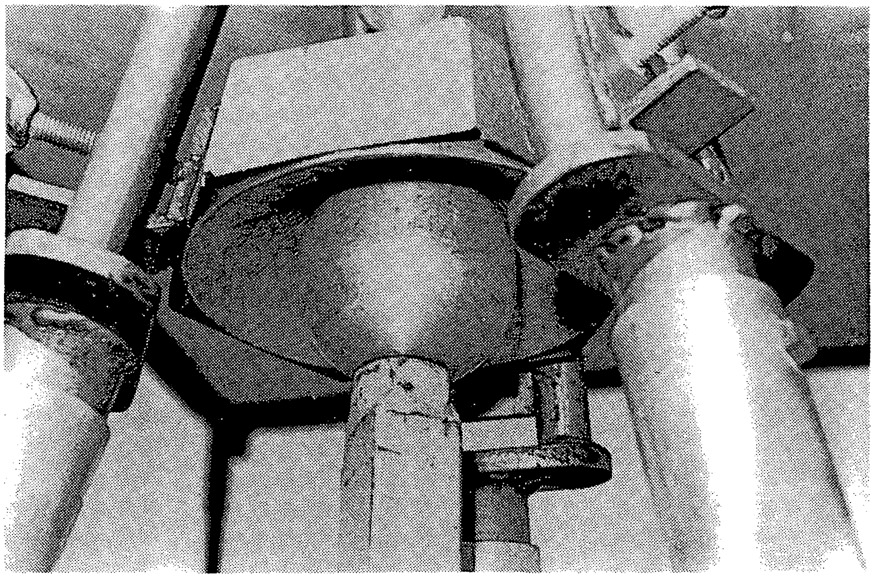


Fig. 7 Interlocking of the tripod with the lifting device.

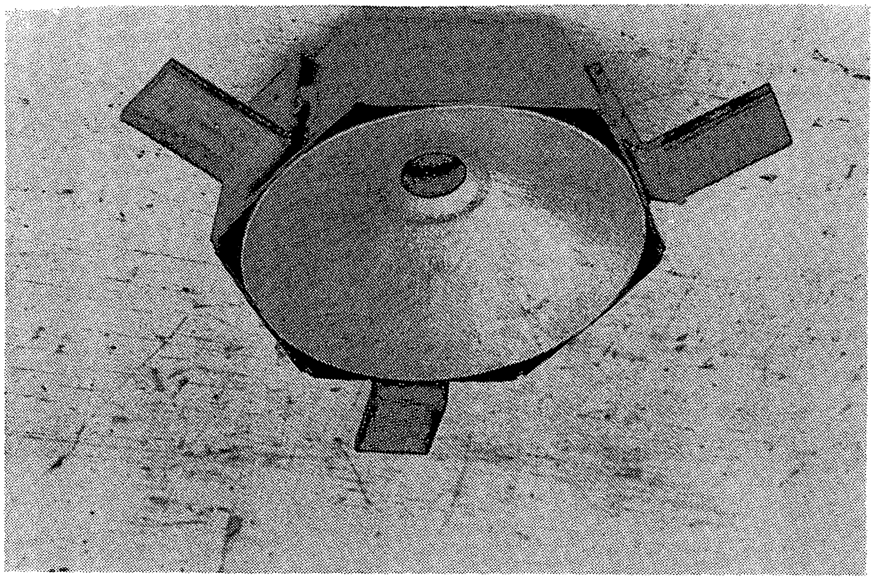


Fig. 8 Interlocking plate of the tripod.

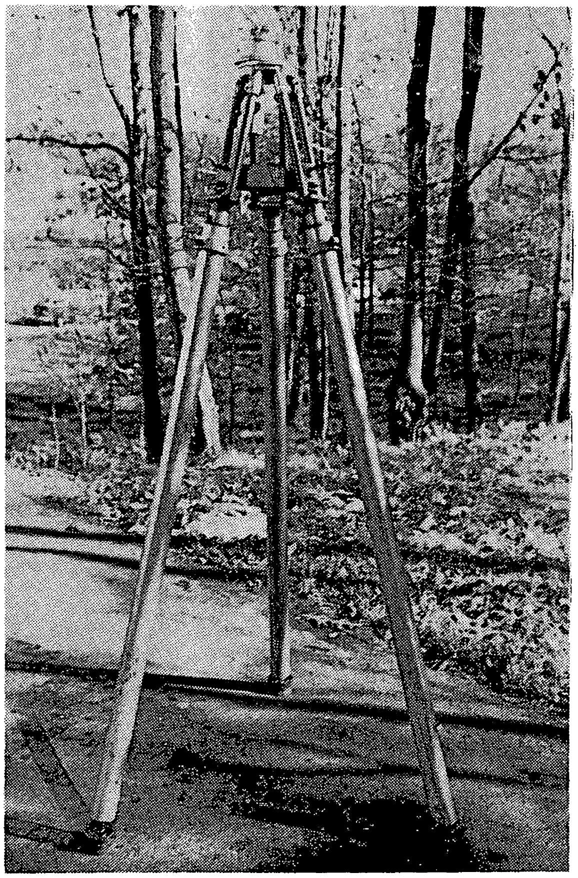


Fig. 9 Final version of the tripod.

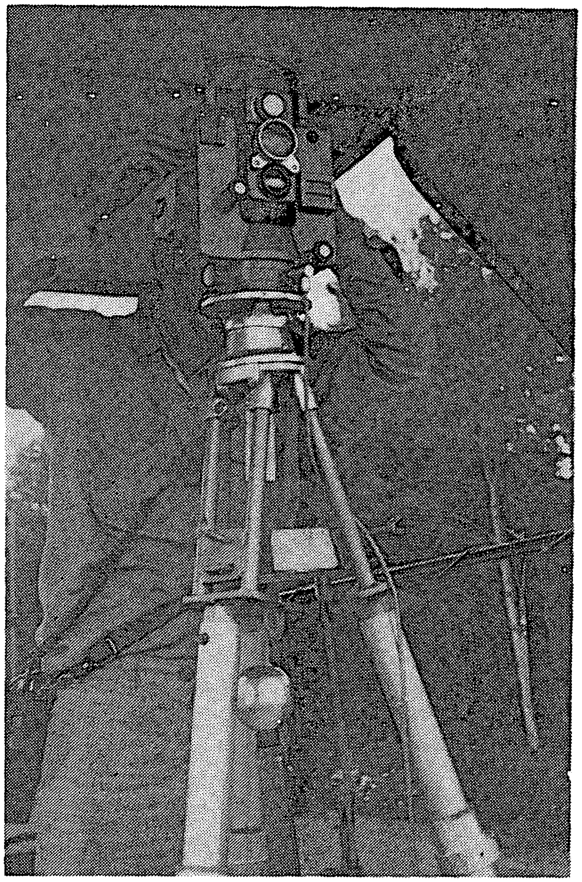


Fig. 10 Kern centring plate adapted to the UNB tripod.

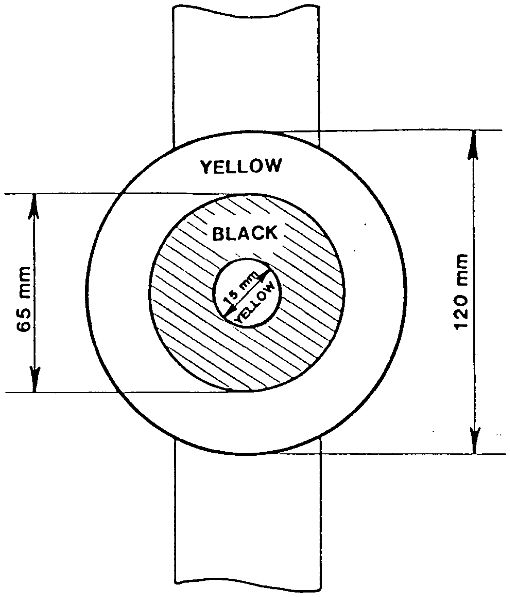


Fig. 11 Design of the target.

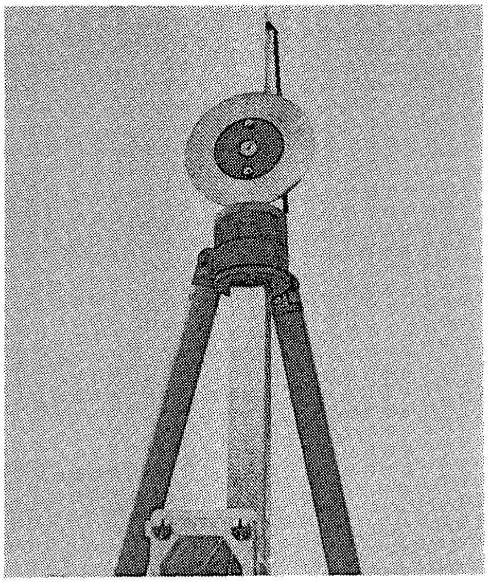


Fig. 12. The target mounted on the rod.

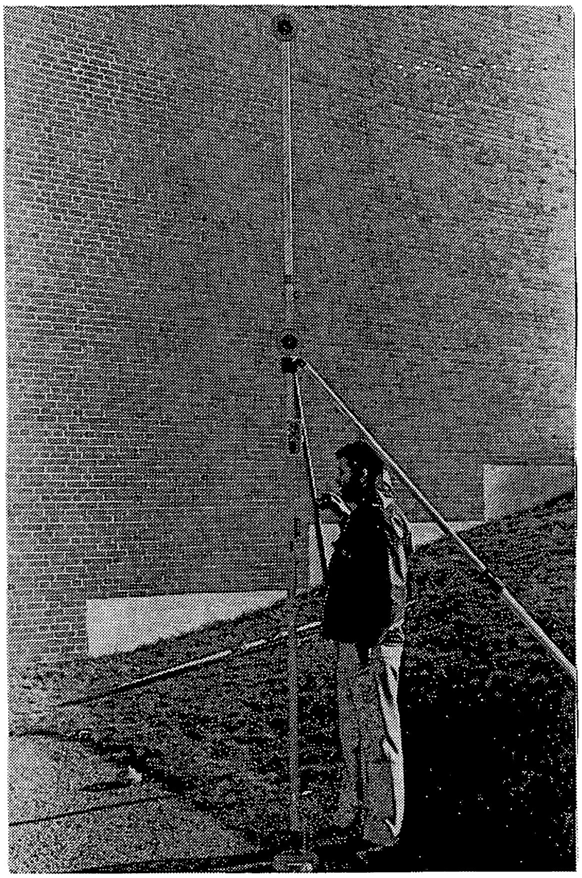


Fig. 13 The final version of the UNB target rod.

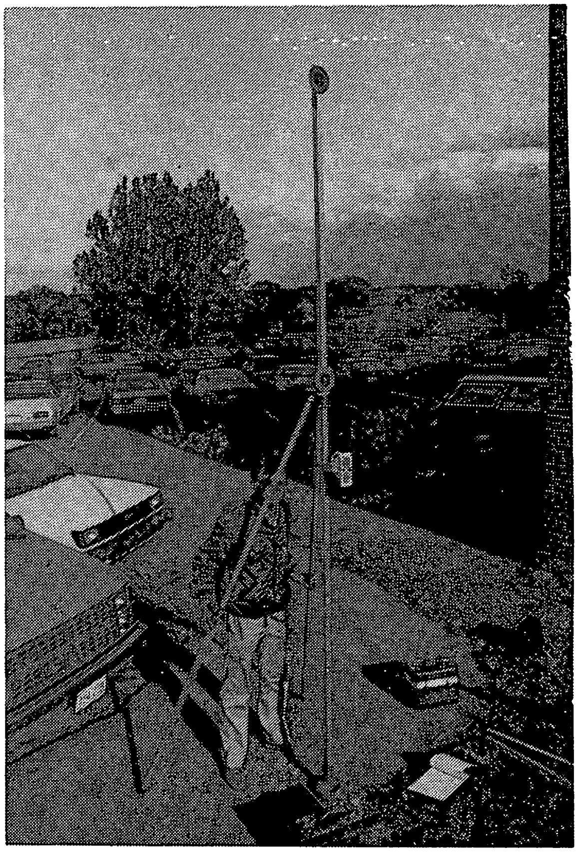


Fig. 14. Setting up the target rod on a turning point plate.

accuracy and economy. In the final design, the rods are made of a square (1.25" × 1.25") aluminum pipe 3.5 m in length, with only two targets mounted at 2.13 m and 3.50 m. The two heights of the targets were selected on the basis of some earlier UNB investigations on atmospheric refraction. The results of these investigations indicated that, in many cases, an inversion of the gradient of the air temperature occurs at heights of about 2.5 m to 3 m above the ground. Thus the measurements to the two targets at about 2 m and 3.5 m should help in the randomization of the refraction influence. Of course, possibly changing the instrument height between 2 m to 3.5 m would be more effective, but this was found difficult and uneconomical to implement in practice.

Figures 13 and 14 show the final version of the UNB rods which were used in the leap-frog trigonometric height traversing. Each rod is equipped with one reflector for EDM at a height of 1.70 m. Since in the UNB surveys, the Kern DM502 and DM503 instruments were used in the leap-frog method, the rods had to be equipped with the large Kern reflectors, as is shown in Fig. 15. When using, for example, Wild DI-4 or DI-5 EDM instruments, small diameter prisms could be used which would be a little more convenient. However, the type of reflector used is of no real importance as far as its mounting on the rod and offset calibration is concerned.

The UNB rods were constructed with supporting studs for manual setting up of the rods on bench marks and turning point plates. They could easily be adapted for the system used by the GSC in motorized geometrical levelling in which a mechanical system for setting up and for supporting the rod is mounted on the roof of the vehicle.

The supporting studs in the UNB system are attached to the rod by means of sliding rings (Fig. 12) which allow for a rotation of the rod between the fore- and back-sights without resetting the studs.

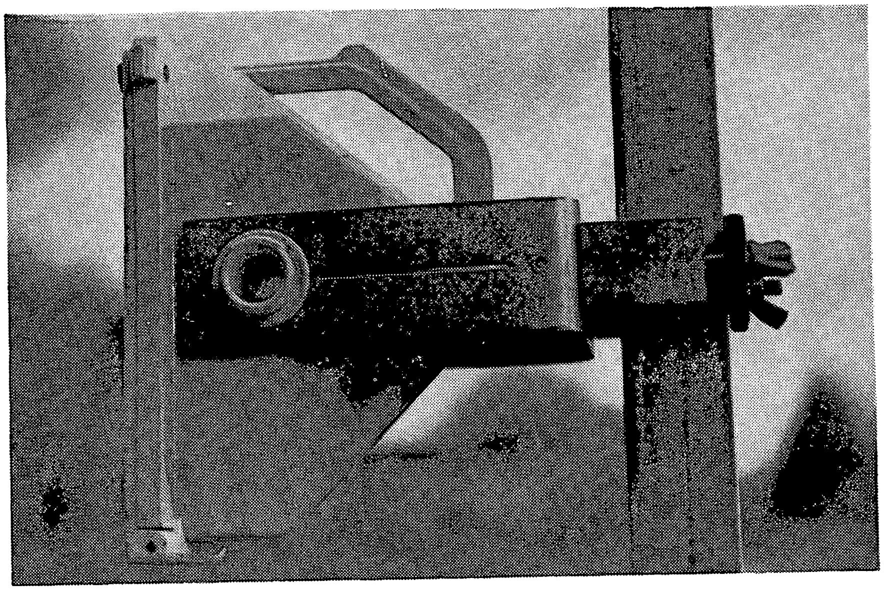


Fig. 15 Kern prism attached to the rod.

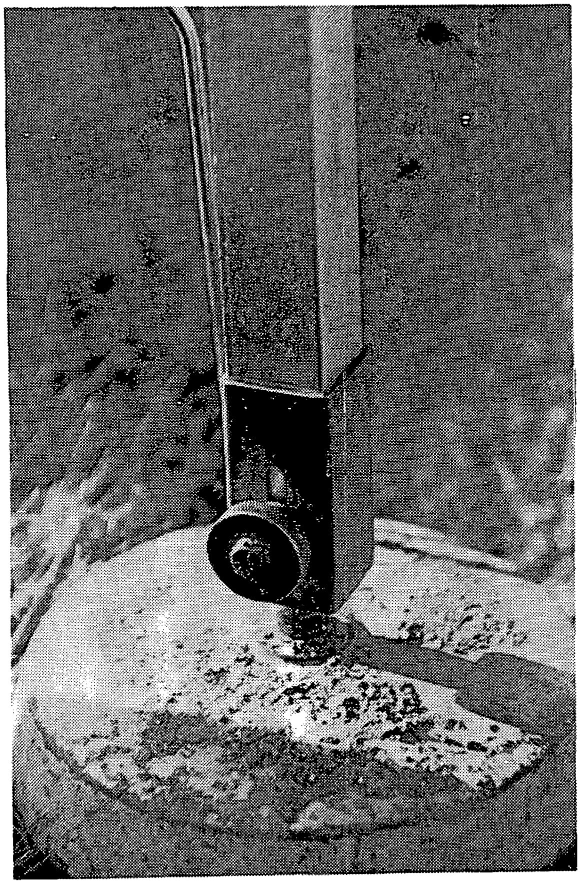


Fig. 16 Movable sleeve for preventing the rod from slipping off the turning point.

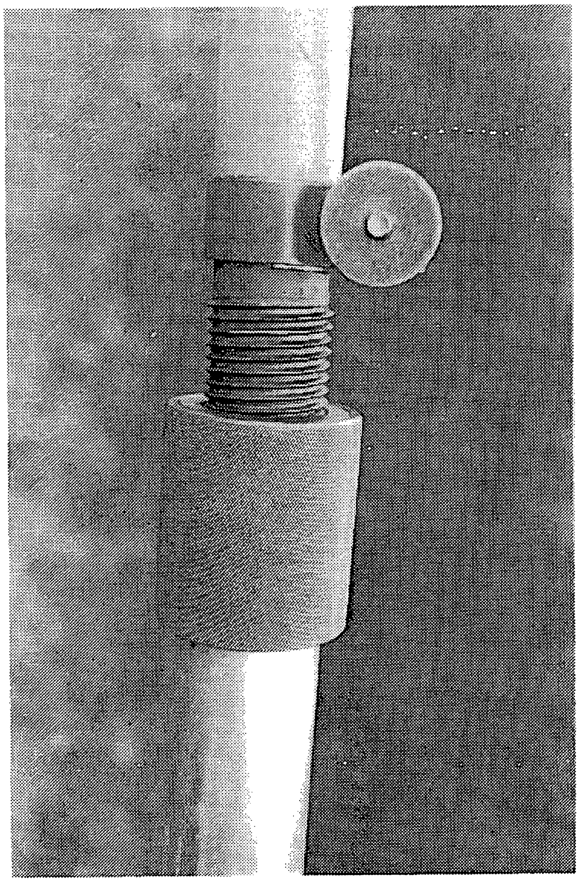


Fig. 17. Fine adjustment of the length of the supporting studs.

In order to prevent a possible slip of the rod from the turning point, a movable sleeve (Fig. 16) can be lowered a few millimetres below the flat surface of the bottom of the rod.

The supporting studs are made of two telescopically connected pipes to allow for an easy adjustment of their total length. A threaded connector is added (Fig. 17) for a fine adjustment of the length when setting the target rod in a vertical position.

The verticality of the rod must be maintained with high precision, particularly in the direction of the line-of-sight (Fig. 18). If the rod is tilted by an angle ω, the error ε_h in the height h_t of the target will be equal to:

$$\varepsilon_h = h_t \tan\omega \tan\alpha$$

where α is the inclination of the line-of-sight. For instance, in order to obtain $\varepsilon_h \leq 0.1$ mm when $\alpha = 6°$ and $h_t = 3.5$ m, the verticality of the rod must be kept within ± 60". The error will accumulate systematically when traversing on long inclined routes.

Each UNB rod is equipped with a tubular spirit level of 30" sensitivity mounted in the direction of the line-of-sight and with a fish-eye spirit level of 8' sensitivity (Fig. 19). In order to check and adjust the spirit levels, the rod is set up vertically by means of a theodolite by sighting from a distance of a few metres to the top and to the bottom of the rod in two positions of the telescope and in two positions (90° rotation) of the rod. The checking of the levels, which takes a few minutes of time, is done in the field at the beginning and at the end of each working day.

A similar procedure is used in a determination and checking of the horizontal offsets between the plumbing centre of the EDM prism and the centre of each target. The offset values are used in correcting the observed distances in the leap-frog traversing to obtain distances between the centre of the theodolite and centres of the targets. The offsets are

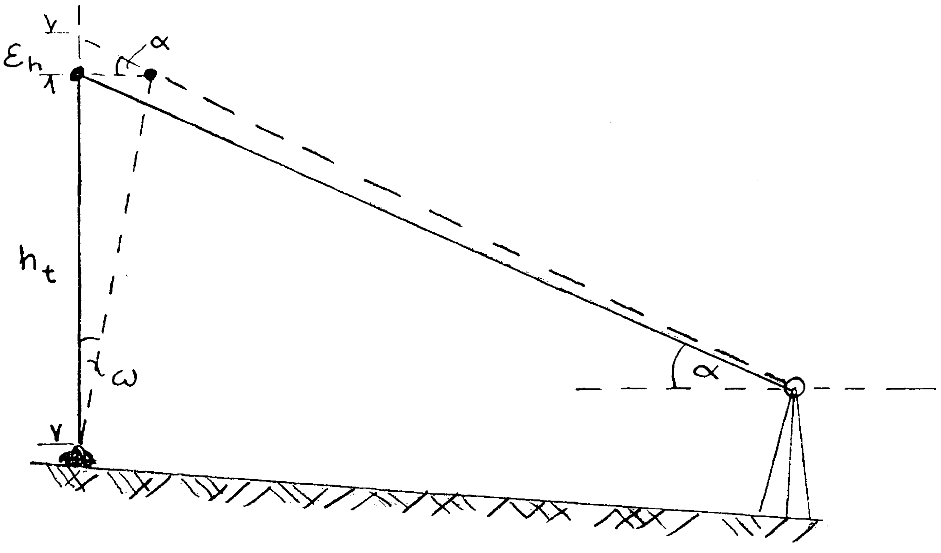


Fig. 18 Error in the height determination due to the tilt of the rod.

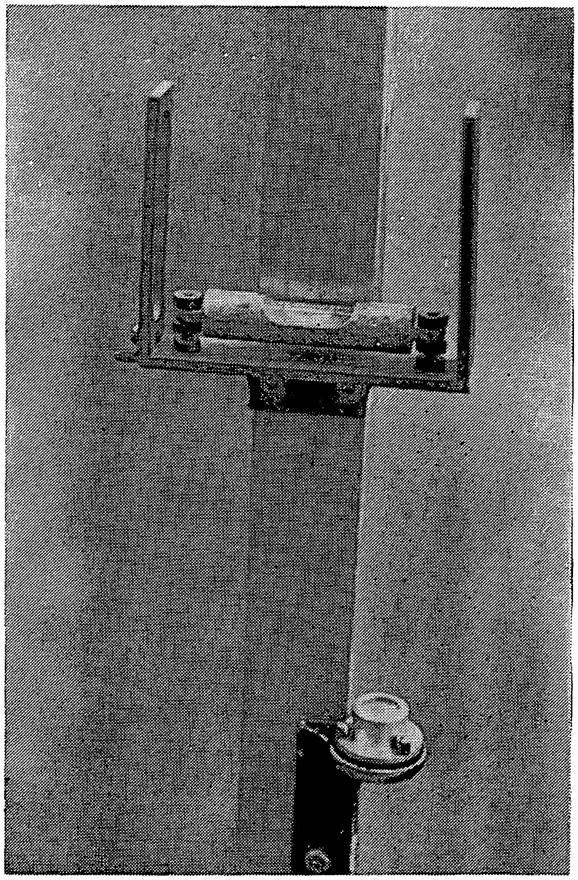


Fig. 19 Spirit levels mounted on the target rod.

determined with a theodolite which is set up a few metres from the rod by projecting the centres of the targets and of the EDM prism on a graduated scale (Fig. 20) in two positions of the telescope (several sets of pointings and readings are taken). The offset determination must be repeated from time to time, particularly when a suspicion arises that the rod could have been bent during the field surveys.

If two or more rods are used in the survey (commonly two rods are used in the leap-frog method), the heights of the targets on the rods must be compared in respect to a common turning point (bench mark). The absolute heights on each rod must also be determined, but they are less important than the height differences between the rods which must be determined with the highest possible precision.

The calibration of the target heights is made with a precision levelling instrument equipped with the parallel glass micrometer. The calibration is performed either in a stairwell or on sloping ground (Fig. 21). The corresponding targets on each rod are pre-set within a few millimetres at the same height when mounting them in the workshop. Later on, the locations of the targets can be vertically adjusted on the mounting screws during the calibration procedure.

The procedure is simple, though it takes about one hour for a comparison of two rods with two targets each. The rods are alternately placed on the same turning point a few metres from the levelling instrument which is set up approximately at the height of the targets. Since the targets are pre-set within a few millimetres at the same height on each rod, the height differences are directly determined by differences of the readings of the parallel glass plate micrometer. In order to determine the absolute height of each target, the target rod on the turning point is replaced by a levelling invar rod and a reading is taken with the levelling instrument with its line-of-sight set at the height of the previously calibrated target.

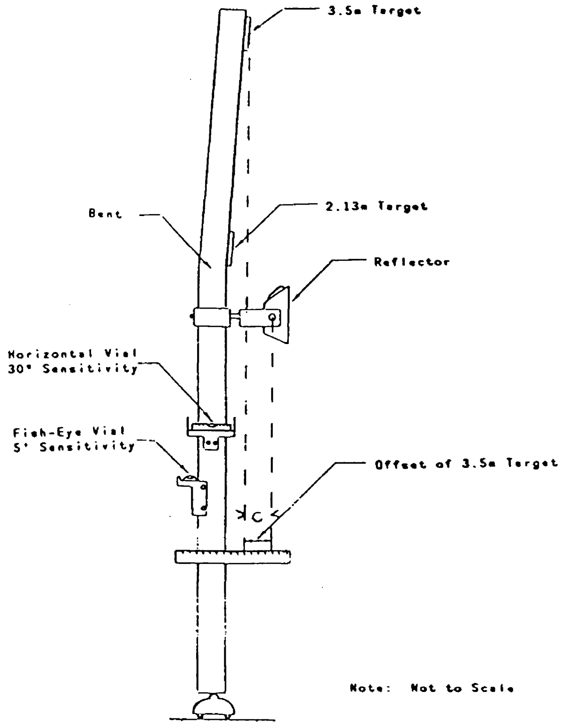


Fig. 20 Determination of the horizontal offsets of the EDM reflector.

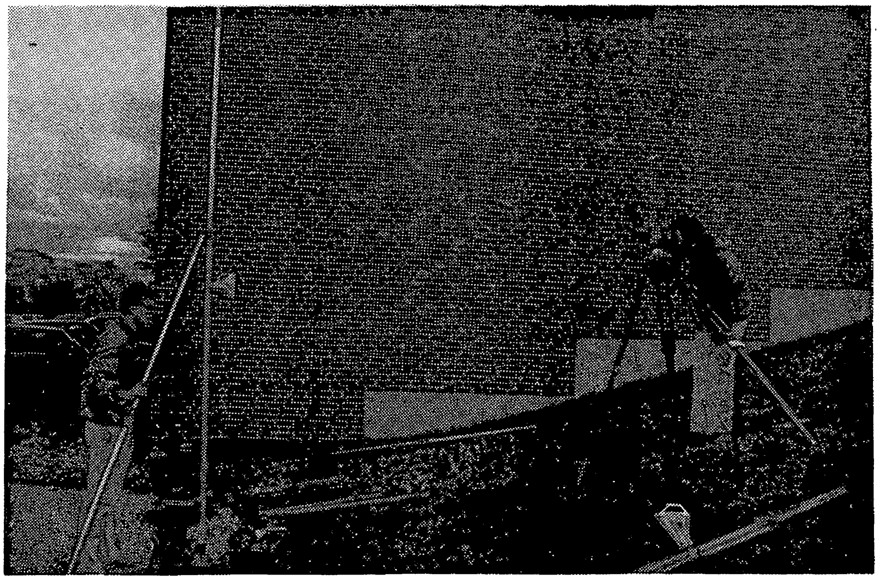


Fig. 21 Calibration of the targets.

When calibrating the targets at the 3.5 m height, an auxiliary turning point must be established at a higher elevation so that the 3 m invar rod will be within the pre-set horizontal line-of-sight of the levelling instruments. Later on an additional levelling is made between the main and the auxiliary turning points.

The targets are adjusted to be at the same height on each rod, within a few tens of a millimetre. The remaining small differences are entered into the processing of the field data as corrections to the calculated height differences (see section 3.1). The accuracy of these corrections is better than 0.1 mm.

Due to a thermal expansion of the rods, the heights of the targets may change by the amount:

$$\varepsilon_h = h_t \alpha (t - t_0)$$

where α is the thermal expansion coefficient, and t-t_0 is the difference between the actual and calibration temperatures. For aluminum, $\alpha = 2.4 \times 10^{-5}/^{\circ}C$. For example, for t-t_0 = 10°C, the absolute height of the 3.5 m target would change by $\varepsilon_h = +\ 0.8$ mm.

If the temperature of both the back- and fore-sight rods in the leap-frog method is the same, the expansion effect is cancelled out in the determination of the height difference. In practice, one may expect to have different temperatures when, for instance, one rod is exposed to the sun's radiation and another is shaded. However, UNB tests with temperature measurements using thermistors attached to the rods indicated that the temperature differences are not large, usually within 2°C, which corresponds to the differential error of only 0.16 mm for the 3.5 m targets.

Since the UNB rods are made of a shining (reflecting) aluminum and they have a comparatively small diameter (~ 30 mm), the heat absorption is minimized. In the process of traversing, the effect of a possible differential thermal expansion should be cancelled out because the fore-sight and back-sight targets are mounted at the same height on both rods. Therefore, the temperature of the rods was not measured in the UNB test surveys which were conducted mainly in open areas with a similar exposure of both rods to the sun's radiation at each set up.

The conventional Kern turning point plates, the same as used in geometric levelling, were employed in trigonometric height traversing.

2.5 Adaptation and Testing of the Electronic Theodolites

Three electronic instruments for the zenith angle measurements were tested for their use in trigonometric height traversing: Kern E2, Wild T2000, and AGA-142 total station.

An accuracy of 0.5" at the one sigma level was aimed at when testing the instruments for their adaptability to the precision height traversing. This error, if treated as a random error, would produce the following standard deviation of trigonometric height traversing in mm/km as a function of the sight distances s and angle of inclination α:

s[m] α°	50 m	100 m	200 m	300 m	400 m
0°	0.56	0.8	1.1	1.3	1.5
20°	0.63	0.9	1.2	1.5	1.7

According to the earlier results of the feasibility study, the Kern E2 and Wild T2000 theodolites were expected to achieve the 0.5" accuracy when measuring the angles in four sets.

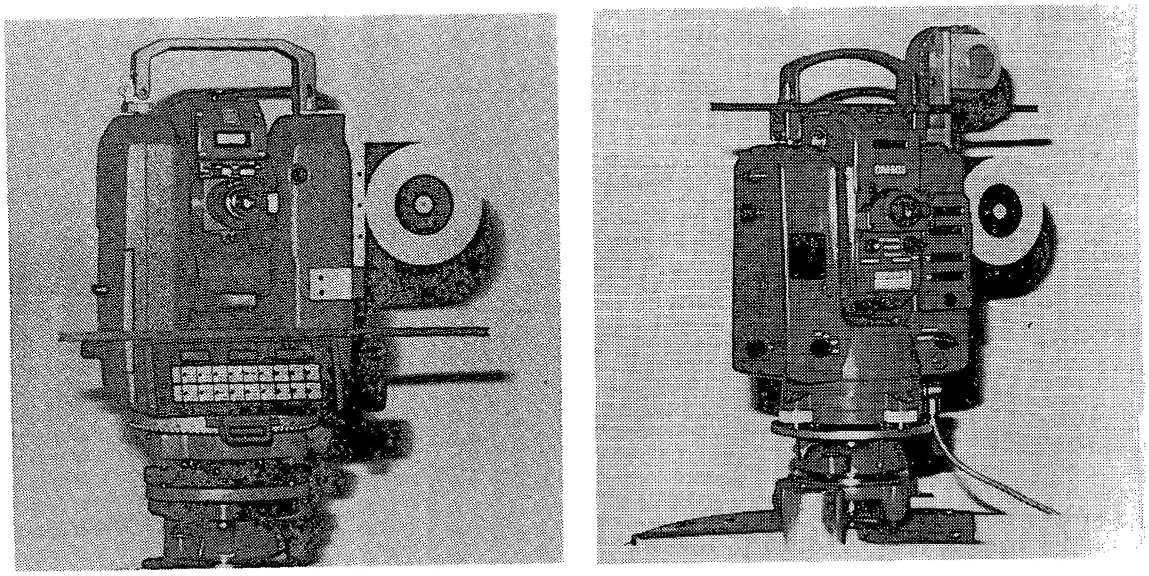
The AGA-142 instrument was briefly tested at UNB in the spring of 1985. Preliminary results immediately showed that the instrument, though demonstrating many interesting features and certainly ideal for many surveying applications, was less convenient and slower in operation than the E2 and T2000 theodolites in its application to the trigonometric height traversing. The basic resolution of angle measurements with the AGA-142, similarly as in the older AGA-140, is only 2". This resolution can be increased to about 1" by using the so-called mean value mode of measurements in which the reading of the angles is automatically repeated 20 times and the mean value is displayed. However, this mode requires double aiming at the target before the mode is activated and the averaging of the 20 readings takes over 15 seconds of time. The measurement in the second position of the telescope must be taken immediately after the first position to the same target. Besides, it appeared that in order to get the display of the vertical angle, the distance measurement would have to be made simultaneously. This would require placing EDM prisms at each target on the rods. Though the AGA-142 has not been discarded from possible future applications in trigonometric height traversing, it was decided to use only the E2 or T2000 theodolites in the project.

Preliminary tests with both instruments did not show any very significant advantages of one instrument in comparison with another. The T2000 when combined with the DI-4 or DI-5 EDM instruments has an advantage in the reciprocal mode of operation that only a small reflector has to be mounted on the opposite theodolite while when using the E2 theodolites with the DM502 or DM503 EDM instruments, the large Kern prism has to be mounted. The E2 theodolite with its two-directional compensator has an advantage over the T2000 in providing an easy check on the stability of the instrument. The signals from the compensator can be automatically displayed and recorded. Since one E2 was purchased by UNB in April, 1985, it was decided to use the E2/DM502 combination in the leap-frog measurements and in most of the experimental surveys (section 4). One T2000/DI-5 was rented later on to be used together with the E2 in reciprocal surveys.

Both instruments were adapted for the reciprocal surveys by mounting on the side of each instrument one circular target (Fig. 22) of the same dimensions as used in the leap-frog method. The E2 instrument was also equipped with one AGA prism. Since the T2000 theodolite did not belong to UNB, no permanent changes to its housing could be made which would be necessary if a Kern reflector had to be attached for the reciprocal surveys. Therefore, it was decided to measure the distances only one way, from the T2000 to the E2. The off-sets of the targets and of the reflector from the axes x, y, h (Fig. 23) of the theodolites were determined in the laboratory. The vertical offsets h_t of the targets were determined with a compensated level equipped with the parallel glass plate micrometer by measuring the height difference between the horizontal axis of the telescope of the theodolite and of the target.

An auxiliary millimetre scale was used for a direct measurement of the vertical offset h_r of the reflector mounted on the E2. The horizontal offsets x_t, y_t, x_r, and y_r were determined with an auxiliary theodolite by vertically projecting the centres of the target, of the reflector, and of the optical axis of the telescope on a horizontal graduated ruler, shown in Fig. 22. Later on, the calibration values were periodically checked throughout the duration of the project.

Initial tests with both theodolites indicated that their index errors in vertical angle measurements were not constant. Searching for an explanation, both instruments were tested for an influence of the changeable temperature. The graph in Fig. 24 shows results of one of the tests with the E2 theodolite in which the instrument was taken from a storage temperature of +22°C to an outdoor temperature of -5°C. Several determinations of the index errors were made, starting a few minutes after changing the location of the instrument. About 1.5 hours later, the instrument was taken back to the room with the temperature of +22°C and the index error was determined again several



a) T2000/DI-5

b) E2/DM502

Fig. 22 Electronic theodolites adapted for reciprocal height traversing.

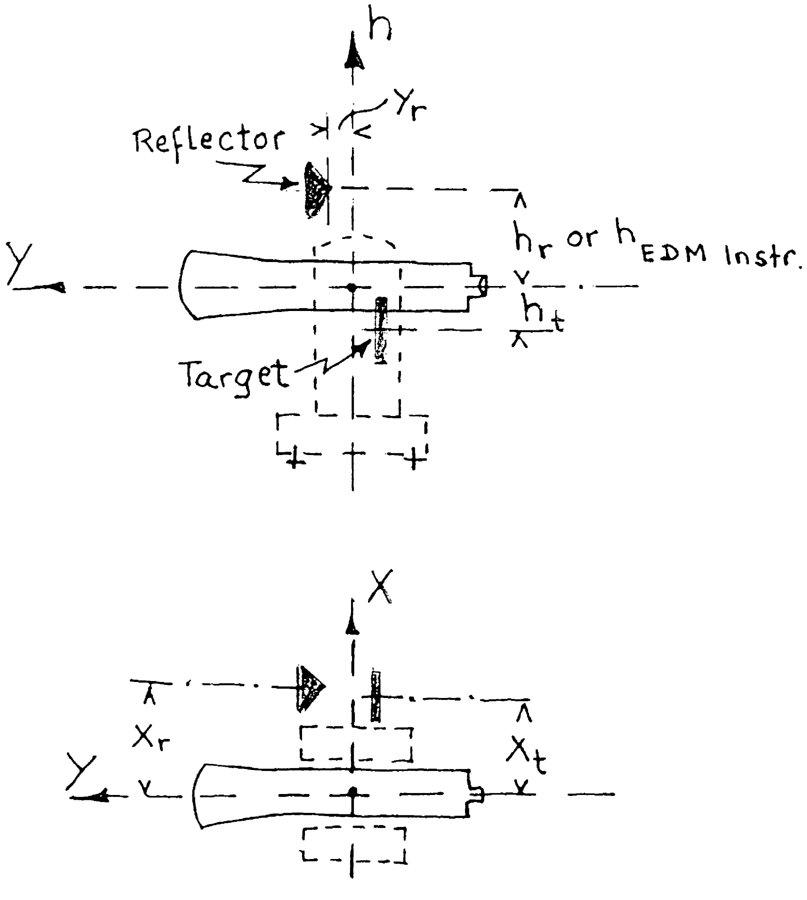


Fig. 23 Calibration constants.

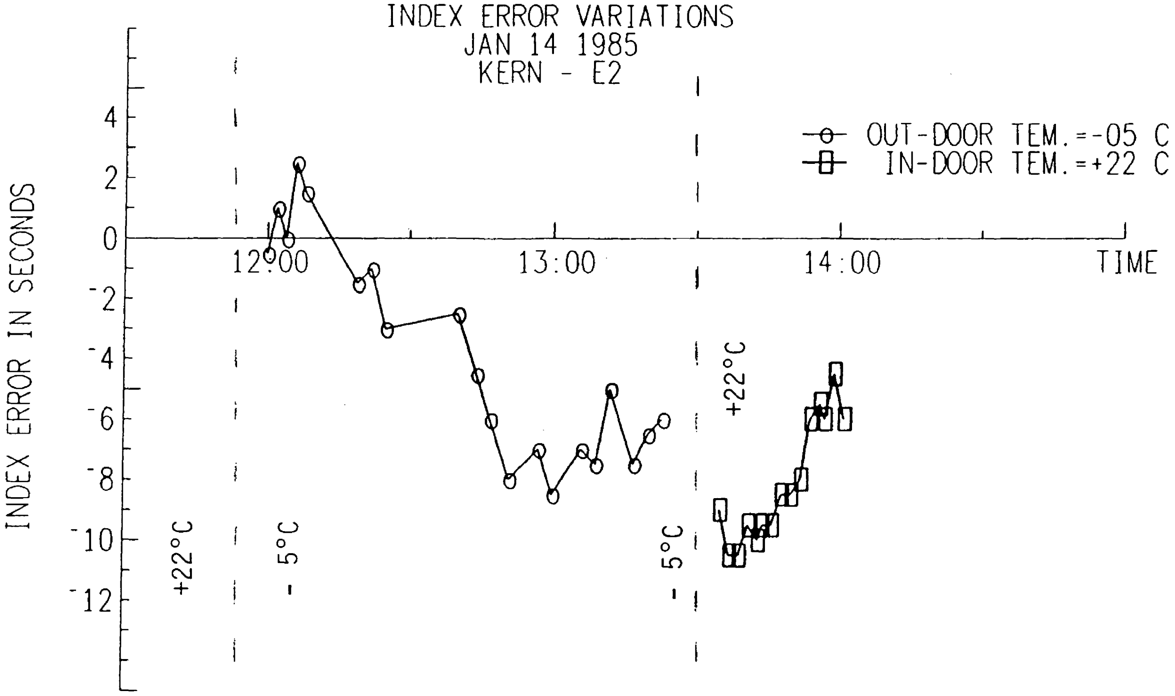


Fig. 24 Index error versus temperature (Kern E2).

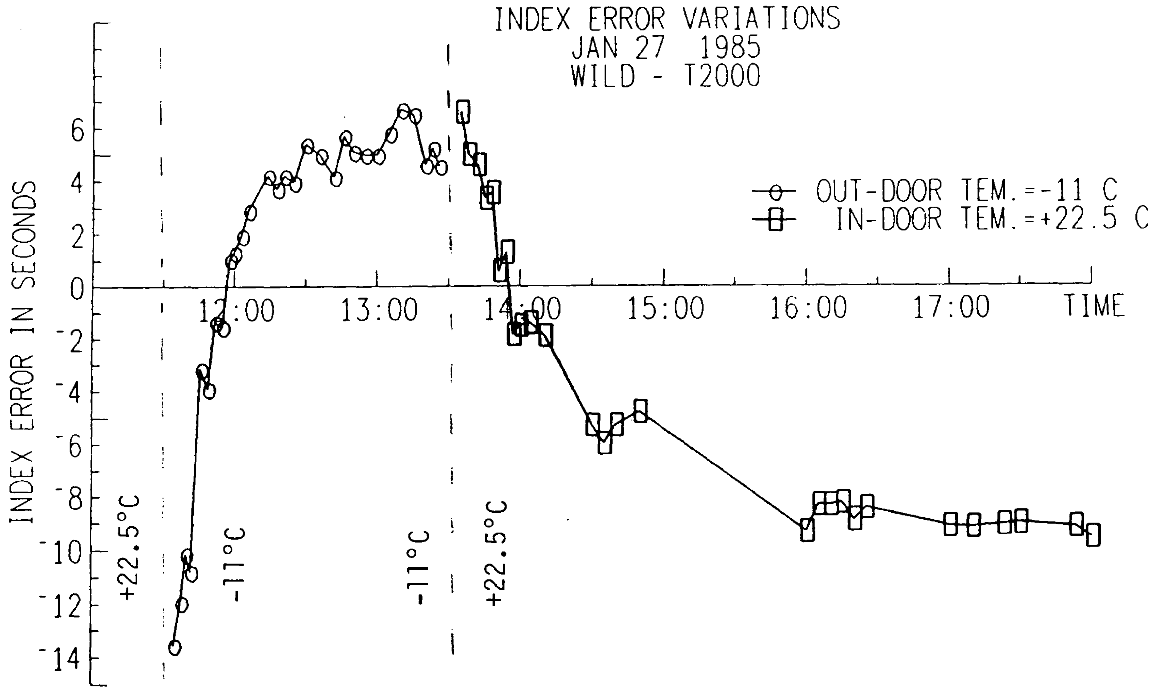


Fig. 25 Index error versus temperature (Wild T2000).

times. The change of the index error corresponded to about 0.3"/°C with a stabilization time of 1 h/27°C, or 2 minutes of time per one degree difference in the temperature. Similar tests were made with the T2000 theodolite moving the instrument from +22°C to -11°C and back to +22°C. In this case, the change of the index error (Fig. 25) was about 0.4"/°C with the stabilization time of about 2 minutes/1°C, i.e., the same as for the E2. In some other tests, the T2000 theodolite showed a change of the index error much larger, about 2"/°C. Generally, both theodolites demonstrated different rates of change of the index error in different temperature ranges. During the actual field surveys, changes of the index error between +6" to -6" were recorded in the T2000 theodolite and up to 30" in the E2. Lack of proper laboratory facilities at UNB and the financial limitations did not allow for more rigorous testing of the instruments. However, no obvious influence of the changeable index error on the accuracy of vertical angle measurements has been found as long as the angles were determined from readings in both positions of the telescope (both faces) taken within a short time interval. This was taken under consideration when designing the field procedures.

2.6 Calibration of EDM Instruments

Kern DM502, Kern DM503, and Wild DI-5 instruments were used in the project. Both manufacturers of the instruments claim the standard deviation of the distance measurements to be (in metres):

$$\sigma_s = [0.003^2 + (5 \times 10^{-6} s)^2]^{1/2} ,$$

if the instruments are properly calibrated. The above equation does not include errors arising from uncertainties in geometrical corrections of the measured distances. A full evaluation of all sources of errors in EDM is given in the Supplement No. 1 to the original contract report and is not repeated here. According to the results of the feasibility study, one may accept that all three instruments can give an accuracy of 4 mm or better at the one sigma level over the average distances of 200 m, as used in this project.

The error of 4 mm, if treated as a random error, will produce the following errors in mm/km of the trigonometric height determination as a function of inclination α and length s of the lines of sight in the traverse:

α \ s	50 m	100 m	200 m	300 m
0°	0.0	0.0	0.0	0.0
3°	0.9	0.7	0.5	0.4
6°	1.9	1.3	0.9	0.8
10°	3.2	2.2	1.6	1.3

The above clearly shows how critical is the accuracy of EDM in hilly terrain. Very short lines of sight should be avoided due to the dangerous propagation of the random errors and also due to the fact that most of the short range EDM instruments exhibit a deterioration of their accuracy on very short distances (<100 m) caused by pointing errors with the narrow beam and phase inhomogeneities across the radiating diode. It is contrary to the results reported by the U.S. Geodetic Survey [Whalen, 1985], who tested the trigonometric height traversing with lengths of sight of only about 60 m and obtained good results. However, their tests were made on comparatively flat terrain.

The use of EDM instruments with confidence, and to their ultimate accuracy, requires obtaining knowledge of their behaviour from calibration measurements. Such measurements are routinely done by the Department of Surveying Engineering using facilities in the department and a six pillar baseline near the fish hatchery at Mactaquac.

Normally, measurements are made to reveal the magnitude and influence of three types of error in the electromagnetic measurement of distance:

i) zero error;

ii) cyclic error; and

iii) scale.

Each of the above errors is determined separately. The zero error is the longitudinal discrepancy between the electro-optic and mechanical centres, to which the station marks, and hence the measured distance, are related. The value and sense of the correction for this error is unique to each mating of instrument and retro-reflector. The cyclic error is the sinusoidal variation of the instrument output which is periodic with the unit length of the EDM instrument and has an amplitude increasing with decreasing return signal strength. The scale is the factor by which the instrument output is brought to indicate the desired unit of measure after the meteorological reductions have been made.

When each of the EDM instruments used in this investigation of height traversing was received, measurements were made to determine the magnitude of each of these errors. Measurements were repeated for the significant errors to ensure that appropriate corrections were applied. The magnitude of the cyclic error was found to be well within the accuracy of each instrument, as claimed by its manufacturer. Hence, only the zero error and scale were pursued. The values of both of these errors were obtained from measurements on the six pillar calibration baseline at Mactaquac.

Because there is some doubt concerning the long-term stability of some of the concrete pillars at Mactaquac, the measurements by the subject EDM instruments were accompanied by additional observations using a Tellurometer model MA100. This afforded a reasonable comparison for determining the scale as well as the zero error corrections.

The results of the least-squares estimation of the zero and scale corrections are given below, with the estimated uncertainties at the one sigma level:

Instrument	Zero Error Correction	Scale Correction
DM502/E2 + Kern prisms	-4.1 mm ± 1.4 mm	+1.8 ppm ± 4 ppm
DM503/E2 + Kern prisms	+1.6 mm ± 0.7 mm	-12 ppm ± 4 ppm
DI-5/T2000 + AGA prism	+37 mm ± 1.3 mm	-5.5 ppm ± 4 ppm

Errors in the determination of the zero and scale corrections propagate as systematic errors in trigonometric height traversing. They cancel out along undulating routes with small height differences between the end bench marks. If the lengths of sights are approximately equal, then the error of the zero correction affects the accuracy of traversing in the same way as the scale error, i.e., directly proportional to the height difference. For instance, a scale error of 10 ppm in a traverse with the total $\Delta H = 100$ m, will produce an error of $10 \times 10^{-6} \Delta H = 1$ mm. Therefore, the systematic errors of distance measurements are particularly dangerous in short traverses with a large total height difference.

2.7 Interfacing of E2/DM502 and T2000/DI-5 with Microcomputers

The electronic systems T2000/DI-5 and ASB version of E2/DM502 are designed for interfacing them with microcomputers for data logging and for real-time data evaluation. Wild Co. recommends that their electronic data collector GRE3 be used with the T2000. However, the GRE3 provides very limited capability for the real-time data evaluation and is expensive (over $8000). Similarly, the Kern-suggested data collector, R48 and/or HP-41 with DIF41 interface, does not meet the requirements of the trigonometric height traversing for the real time pre-processing and evaluation of the observation data with an echoed hard copy. Therefore, it was decided to use other microcomputers in the project.

The Kern E2 (ASB version) has the following communication parameters for interfacing: baud rate 1200 (fixed) with the following configuration of ASCII characters: 7 data bits, even parity, and 2 stop bits. The T2000 theodolite has a changeable baud rate (2400 used in the project) and the configuration of ASCII characters of 7 data bits, even parity, and 2 stop bits.

Both instruments proved to be capable of being interfaced with other than recommended microcomputers. Two types, both inexpensive (~ $1500 each with printers and casette recording facilities) were selected for the field application: EPSON HX20 and Radio Shack TRS-80 Model 100. Both have 32K RAM. The Model 100 with 40 × 8 character LCD was used with an external TRS-80 TRP-100 thermal printer and with TRS-80 CCR-82 cassette recorder. The total set up is shown in Fig. 26. The EPSON HX20 has a 20 × 4 characters LCD and has a built-in dot matrix printer and a microcassette cartridge. It is shown in Fig. 27 interfaced to the E2 theodolite.

Both microcomputers can be interfaced with either E2 or with T2000 using the configurations as shown schematically in Figs. 28 and 29.

Due to a lack of experience and a lack of detailed information in the operating manuals of the theodolites, some problems were encountered in figuring out the pin connection in the cables and the configuration of the transmitted data. Major problems arose in transmitting the collected field data to the mainframe IBM computer at UNB. It was found indispensible to use a Data Line Monitor in the initial transmissions in order to solve the transmission problems. The telecommunciation software for the so-called

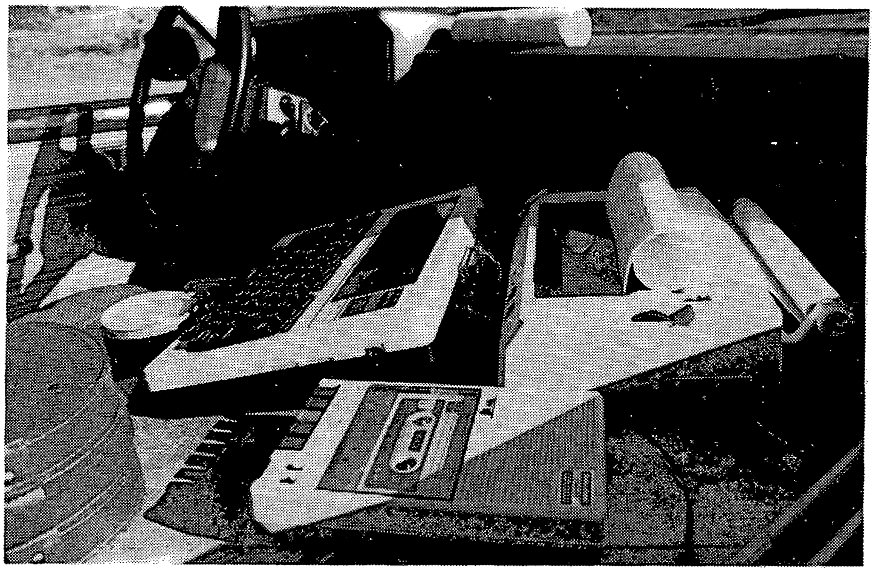


Fig. 26 TRS80 Model 100 with printer and cassette recorder mounted in the observing vehicle.

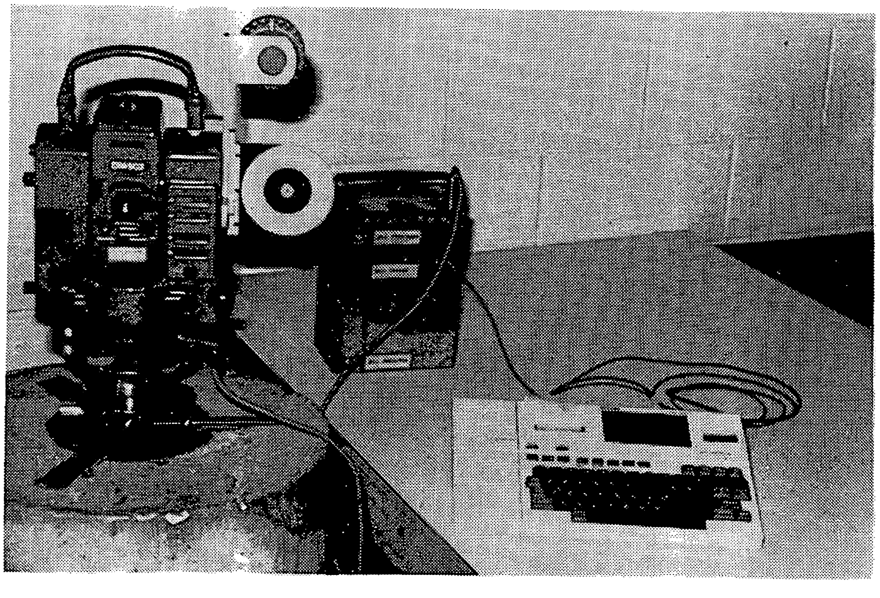


Fig. 27 EPSON HX20 interfaced with the E2/DM502 system.

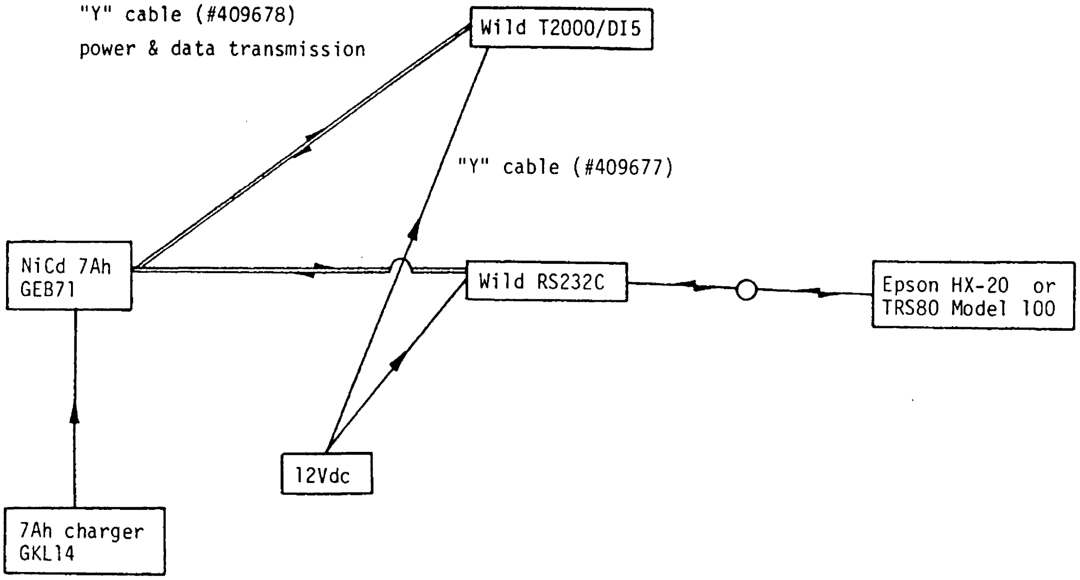


Fig. 28 Interfacing of T2000/DI-5 with microcomputers.

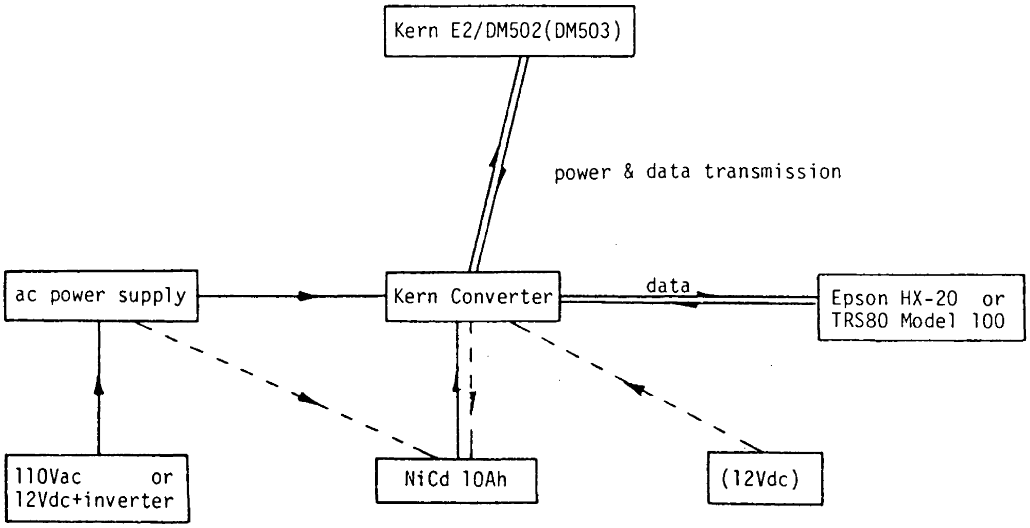


Fig. 29 Interfacing of E2/DM502 with microcomputers.

"hand shaking," supplied by the manufacturers of the TRS-80 Model 100, expected different characters from the IBM mainframe computer. Finally, a new programme had to be written in consultation with specialists from the Computer Centre at UNB. The EPSON HX-20 exhibited larger difficulties in transmitting the data to the IBM computer than the TRS-80 Model 100. It was found that the problem was in having too low a voltage at the output of the interface when operating on the internal batteries.

The solution was either to use a "line driver," which provides proper voltages or to keep the charger plugged in. The second case goes against the rules specified in the manual. Unrequired charging might shorten the life of the batteries. Therefore, later on, in order to avoid unnecessary problems, data obtained in the field was always first transferred to the TRS-80 Model 100 computer, and then to the IBM mainframe computer. This was a long procedure, but much more reliable.

Some problems also arose when interfacing the HX-20 with the T2000. Most probably, the difficulties were caused also by the aforementioned low voltages. Therefore, in the field surveys, the EPSON HX-20 was used only in reciprocal traversing interfaced with the E2. The TRS-80 was successfully used with both measuring systems.

The software for the data logging and for real-time data evaluation was written in Microsoft BASIC. It was developed at UNB as a part of the M.Sc. thesis [Kornacki, in prep.] which will be submitted to GSC as a Supplement No. 2 at a later date. It will include all the programmes developed at UNB for the trigonometric height traversing.

2.8 Instrumentation for Measurements of Temperature Gradients

A 4-metre rod with six temperature sensors was constructed for the investigations of the influence of atmospheric refraction. Standard YSI 705 probes, produced by Yellow Springs Instrument Co. Inc., have been used in the project. The probes consist of a YSI 44018 thermilinear thermistor composite housed in a stainless steel tube of 6 mm diameter and attached to a plasticized vinyl jacketed lead wire terminated with a phone plug. The probes have been connected to a manually operated 6 channel YSI model 4320 thermilinear temperature indicator with a digital readout (Fig. 30). The Yellow Springs Instrument Co. gives the following specifications for the sensors and for the indicator:

Sensors	**Indicator**
Range: -2°C to +38°C Accuracy and interchangeability: 0.13°C Time constant: 0.6 seconds to reach 63% of a changed temperature and 3 seconds to reach 99%.	Range: -50°C to +100°C Linearity: > 0.2% Temperature resolution: 0.1°C

The probes were laboratory compared in a constant temperature (wired together with a copper wire) with a precision (0.01°C resolution) HP temperature sensor. The differences and the absolute temperature determination were within the resolution of the indicator.

Based on previous experience with other thermistors and thermocouples and with various ΔT measuring systems (including the temperature sensors rented from GSC), the YSI system proved to be more reliable and gave more satisfactory results. Fig. 31 shows the complete UNB system with the sensors mounted at the heights of 0.3 m,

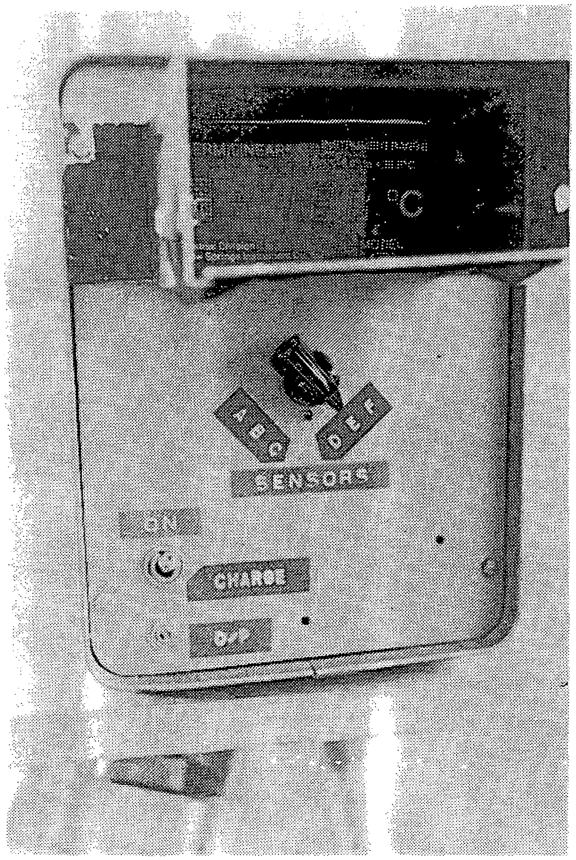


Fig. 30 Thermilinear YSI temperature indicator.

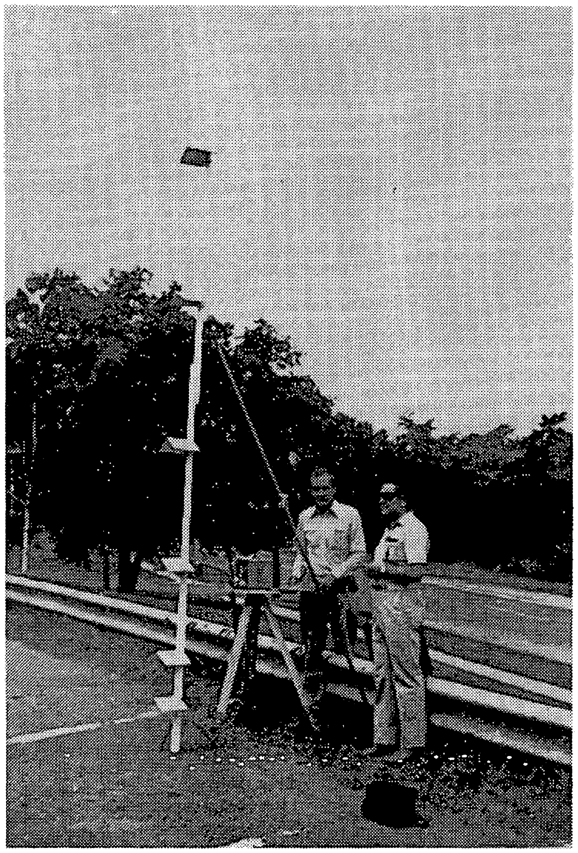


Fig. 31 The total temperature sensing system.

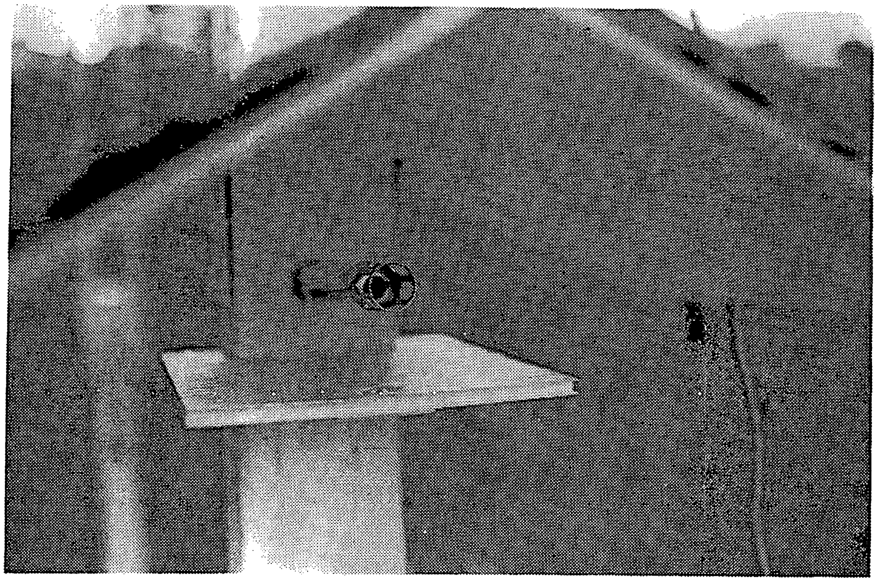


Fig. 32 One of the temperature sensors mounted on the rod.

0.6 m, 1.2 m, 2.0 m, 3.0 m, and 4.0 m. The sensors were shaded (Fig. 32) from a direct and reflected sun's radiation. Due to the short response time of the sensors, it was decided not to apply any forced air circulation around the sensors because the previous experience with any type of electric fans and additional tubing around the sensors usually gave unsatisfactory (distorted turbulence) results.

A portable weather station, with a barometer, wet and dry thermometer, and wind speed and direction measurements, was used as auxiliary equipment in the refraction investigations.

3. FIELD PROCEDURES AND TRAVERSE COMPUTATIONS

3.1 Leap-Frog Method

A typical set up of the instrumentation in the leap-frog method is shown in Fig. 33. The zenith angles z_F and z_B and the distances s_F and s_B were measured with the Kern E2/DM502 instruments interfaced with the TRS80 Model 100 microcomputer. Two vehicles were used: the observing vehicle (OV), and the rod distributing and reconnaissance vehicle (RDV). The survey crew consisted of five persons:

- one observer
- one recorder/driver
- two rodmen
- one driver of the RDV.

The E2 theodolite, when switched from 'off' to 'on,' requires an initialization of the readout system which takes over one minute of time. In order to avoid this procedure at each set up, the theodolite was usually left in the 'on' mode all the time when changing the set ups.

The distances between the OV and the rods were kept approximately 200 m or, if necessary, shorter. The set-up points for the rods and for the OV were flagged ahead by the RDV which was equipped with the Halda Rally Computer, electronic odometer. In most cases, the flagged back- and fore-sight distances were balanced to within 2 metres. Sometimes, on a winding road, the set up of the forward rod had to be adjusted with the DM502 from the observing vehicle to balance the distances to ± 5 m, which was used as a tolerance.

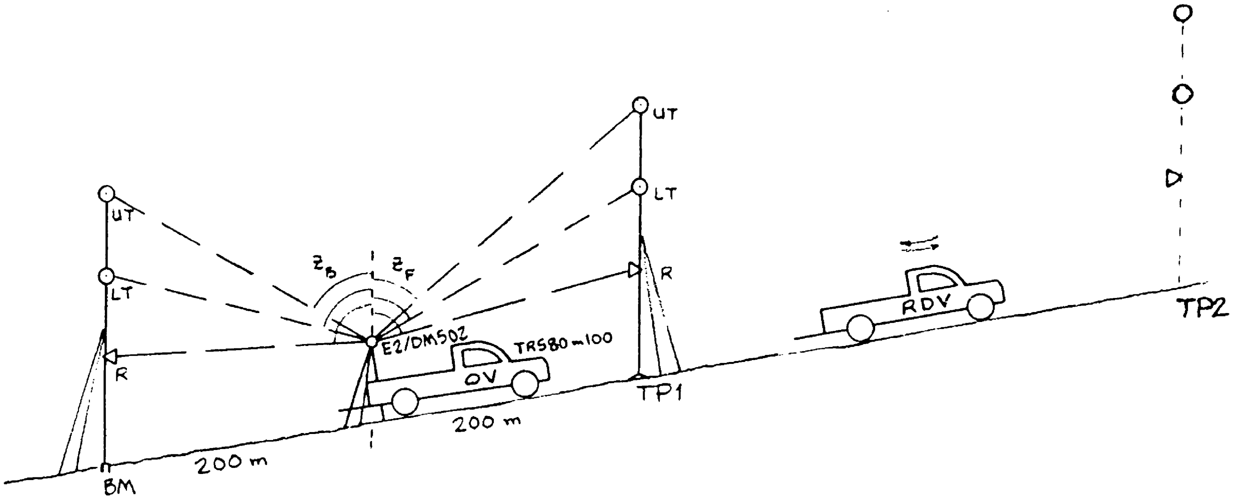


Fig. 33 Leap-frog traversing with two vehicles.

The average measuring time at each set up was about ten minutes. During that time, the driver of the RDV, after helping to set up the forward rod, would do the reconnaissance, mark the next set up, and return to pick up the rodmen and the rod at the back-sight set up. Generally, this procedure did not produce any delays because the observations at the next set up would always start to the back-sight rod which would be only rotated on the turning point plate and relevelled giving enough time for the set up of the second rod at the new forward turning point.

At each set up, four sets of zenith angles in both positions of the telescope were measured to both the back-sight and fore-sight targets with two sets to the lower and two sets to the upper target. In addition, one set of zenith angles was measured to the forward and backward retroreflectors for the reduction of the distances. The distances were measured in two sets with two readings each.

All the readings were automatically recorded and echoprinted at a command from the computer after the verbal signal from the observer: "take".

The observations followed the sequence:

1. Check levelling of the instrument.
2. Sighting to the **back-sight rod** in the **direct position** of the telescope, pointing to and automatic recording of:
   - zenith angle to the retroreflector;
   - distance (two readings) and automatic recording of the compensator readouts (if two readings of distances differ more than 5 mm, measure again);
   - zenith angle to the lower target;
   - zenith angle to the upper target.
3. **Reverse** the telescope, point to the **back-sight** rod, and measure:
   - zenith angle to the retroreflector;
   - zenith angle to the lower target;
   - zenith angle to the upper target.
4. Sight to the **fore-sight rod** in the **reverse position** of the telescope and measure as in (3) above.
5. Turn the telescope to the **direct position**, sight to the **forward rod**, and measure as in (2) above. This is the end of set number one.

6. The second set is measured using the same procedures as above, however, the measurements commence to the fore-sight rod.
7. If the calculated height differences from two sets differ more than 1.5 mm, take one more set of observations following (1) to (5).
8. Read temperature and barometric pressure and enter comments.

An example of the computer output at one survey station is given in Appendix II. The calibration constants of the targets and reflectors were stored in advance in the computer. While driving to the next set up, the computer would print a summary of all the observations, including calculated index errors. Due to the limited RAM, the field evaluation of the data was limited to one set up at a time. The data which could not be stored in the computer's RAM was 'dumped' on the casette tape. Since it is rather a slow process, a disk drive would be recommended in future applications. Recently, Radio Shack released on the market a portable disk drive which utilizes 3.5 inch disks. The cost is under $300.00.

Each day, the field data recorded on the audio tape was transmitted via a telephone line to a file in the mainframe IBM computer. In order to perform a final computation of height differences in traverses, each of the files required minor editing. Observed distances were corrected for the refractive index of air, zero error correction, and scale to obtain the final values s_0 of distances between the EDM instrument and the retroreflectors. Then distances s_F and s_B (Fig. 34) between the centre of the theodolite and individual targets were calculated by introducing geometrical and offset calibration (section 2.4) corrections:

$$s = (s_0 \sin z_r + c) \sin z_t$$

where z_r and z_t are zenith angles to the retroreflector and to the target, respectively, and c is the horizontal offset of the reflector in respect to the target (Fig. 20).

The final differences in elevation between the fore-sight and back-sight turning points were calculated separately from observations to the lower and upper targets from:

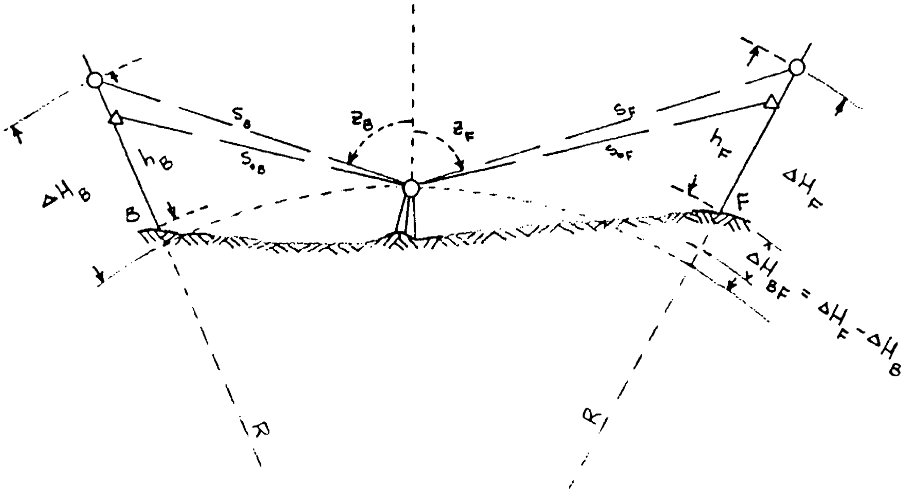


Fig. 34 Leap-frog determination of height determinations.

$$\Delta H = \Delta h_F - \Delta h_B = s_F \cos z_F - s_B \cos z_B + (s_F^2 \sin^2 z_F - s_B^2 \sin^2 z_B)/(2R) + (h_B - h_F),$$

where $(h_B - h_F)$ is the calibration correction for the difference in heights of the corresponding targets, and R is the mean radius of the earth for the area of the survey.

The software for the calculations was developed at UNB. A full listing of the programme is given in Kornacki [in prep.]. An example of the computer output of a traverse calculation is given in Appendix III. The computer programme accepts observation data for up to four targets at the back-sight and fore-sight rods and computes height differences for each corresponding pair of targets (of the same height on both rods) as well as mean height differences computed from different combinations of two and three targets. The final traverse value is calculated as a mean from all targets. In the surveys of this project, only two targets were used on each rod at the heights of 2.13 m and 3.5 m. They are coded in the computer output in Appendix III as targets 2.5 m and 3.5 m. The observation, calibration, and ΔH calculation values for the two more optional targets (not used), which are coded as 1.5 m and 4.5 m, are given as 0.00. At the end of the output of the traverse computations, a summary of all the set ups is given, and an evaluation of the survey such as comparison with other survey (e.g., geodetic levelling), loop misclosures, etc., as is self-explanatory in the given example.

In this project, the geodetic aspects, such as influence of the deflections of the vertical and calculations of ellipsoidal heights, were ignored because, as can be seen in Greening [1985], the geodetic corrections would be within the accuracy of the surveys and they would not influence the comparison of the trigonometric leap-frog traversing with the reciprocal traversing and with the geodetic geometric levelling.

3.2 Reciprocal Method

A typical set up of the instrumentation in the reciprocal method is shown in Fig. 35. The height traversing consists of two basic procedures: connecting surveys to the initial and end bench marks of the traverse, and the measurements of the traverse itself. After trying and analysing a few possible methods for connecting surveys, the method proposed by Rueger and Brunner [1982] was selected for the UNB field procedures. In this method, the height difference h (Fig. 36) between the trunnion axis of the theodolite and the bench mark is determined from zenith angle measurements to at least two marked lines of a graduated rod. Two more readings should always be taken as a check. From one pair of readings, the value of h is calculated from:

$$h = \frac{(l_2 \cot z_1 - l_1 \cot z_2)}{(\cot z_1 - \cot z_2)} .$$

Assuming that errors of l_i and z_i are random and uncorrelated, and applying the law of the propagation of variances to the above equation, leads to the expression [Rueger and Brunner, 1982] for the variance of h:

$$\sigma_h^2 = ((l_1 - h)^2 + (l_2 - h)^2)\, \sigma_l^2 \,/\, ((l_1 - l_2)^2) + 1 \,/\, ((l_1 - l_2)^2)$$

$$\{4(l_1 - h)^2(l_2 - h)^2 + [(l_1 - h)^2 + (l_2 - h)^2][D^4 + (l_1 - h)^2(l_2 - h)^2] \,/\, D^2\}\, \sigma_z^2$$

where σ_l^2 is the variance of the length l_1 of a rod graduation, and σ_z^2 is the variance of the zenith angle z_i. It can be shown that the selection of rod readings which are symmetrically spaced about the horizontal line of sight leads to the smallest σ_h^2.

For instance, if: $l_1 = 2.80$ m, $l_2 = 2.00$ m, $h = 4.00$ m, $D = 30$ m, $\sigma_l = 0.1$ mm, and $\sigma_z = 2''$, then $\sigma_h = 0.9$ mm. If the readings in this example would be symmetrical, i.e., $h = 2.40$ m, then $\sigma_h = 0.2$ mm. From two independent pairs of readings in the above

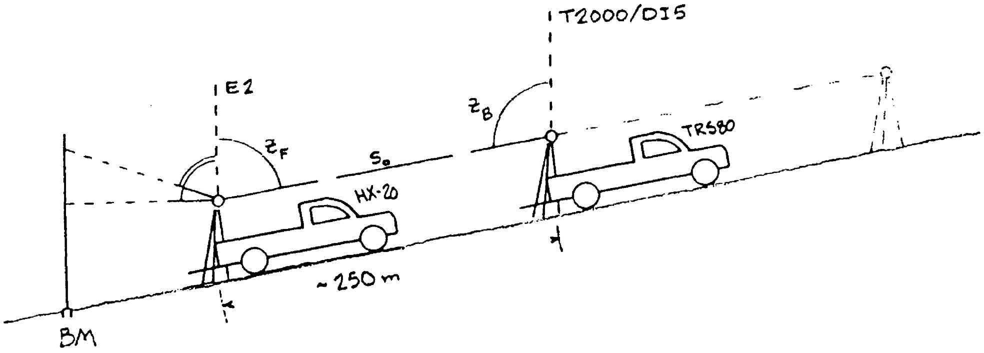


Fig. 35 Reciprocal height traversing.

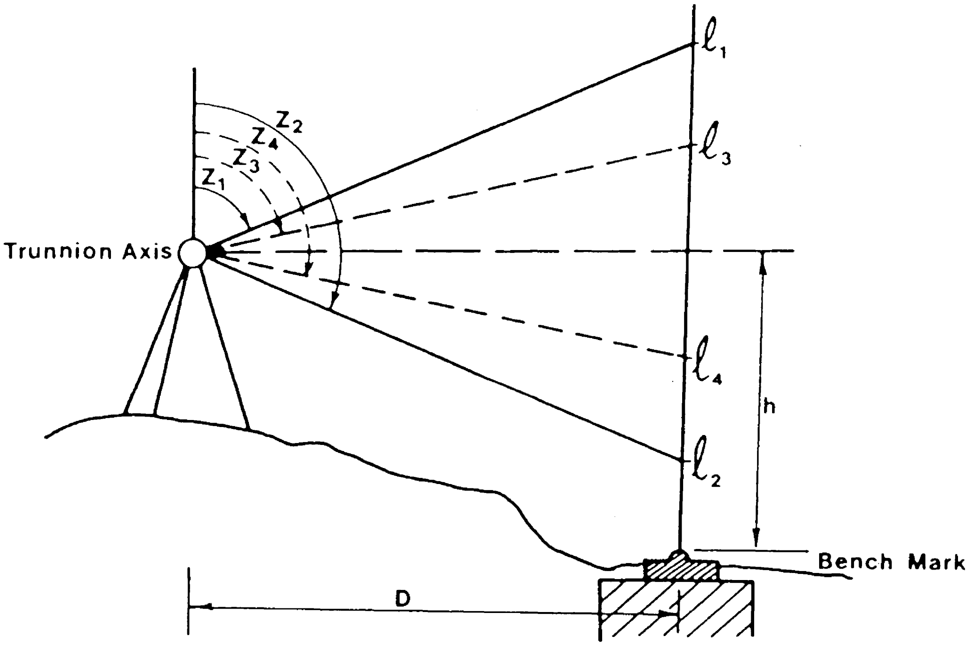


Fig. 36 Connecting survey to the bench mark in the reciprocal height traversing.

examples, the accuracy of the mean value of h would be $\sigma_h = 0.6$ mm and 0.14 mm, respectively.

In the UNB surveys, a 3 m invar rod with double graduation was used for the connecting surveys. Zenith angles to two pairs (on both scales) of graduation lines were measured in two positions of the telescope (one set). The separation between the upper and lower lines was usually taken 60 cm for the symmetrical case and at least one metre when the bench mark was significantly below or above the observing station, not allowing for the symmetrical sighting in respect to the horizontal line of sight. The theodolite was usually set up not more than 20 m away from the bench mark. Fig. 37 shows a typical connecting survey during field tests of the survey system. The readings were processed and evaluated in real time using one of the microcomputers employed in the reciprocal height traversing. If two determinations of h from two pairs of zenith angles differed by more than 0.3 mm, an additional set of connecting measurements was made. An example of the field computer output is given in Appendix IV.

Once the connecting surveys to the initial bench mark were completed, the reciprocal traverse measurements would commence by a survey crew of four persons using the T2000/DI-5 interfaced with the TRS80 Model 100 and the E2 interfaced with the EPSON HX20. Two sets of simultaneous reciprocal zenith angles in forward and backward directions were measured (a total of 4 sets per each line of the traverse) and two sets of distances measured only in one direction by the DI-5.

The station with the T2000/DI-5, from which the distances were measured, acted as a master station. This station would radio the value of the reduced horizontal distance (not corrected for the meteorological conditions) to the E2 station which would calculate the difference in elevation after each set of angle measurements and dictate it back to the

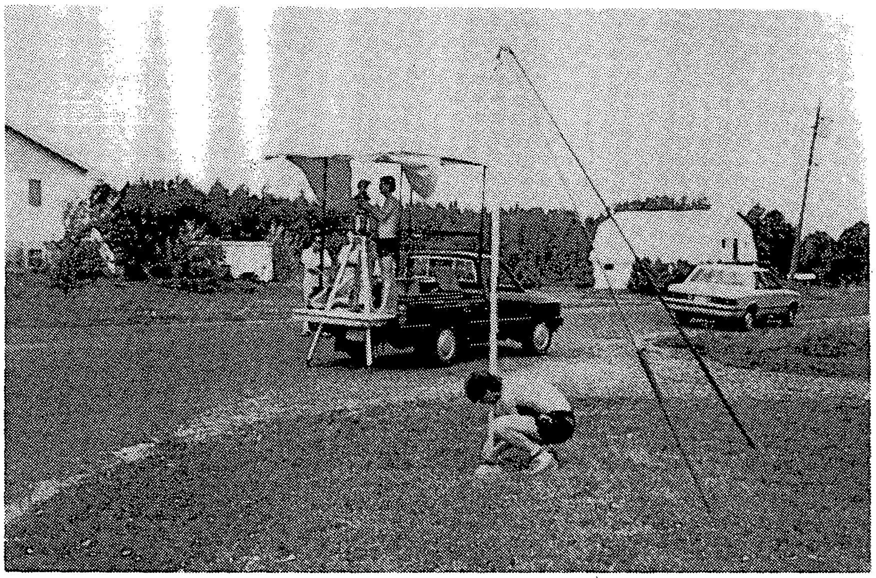


Fig. 37 Set up of instruments for a connecting survey to a bench mark in reciprocal traversing.

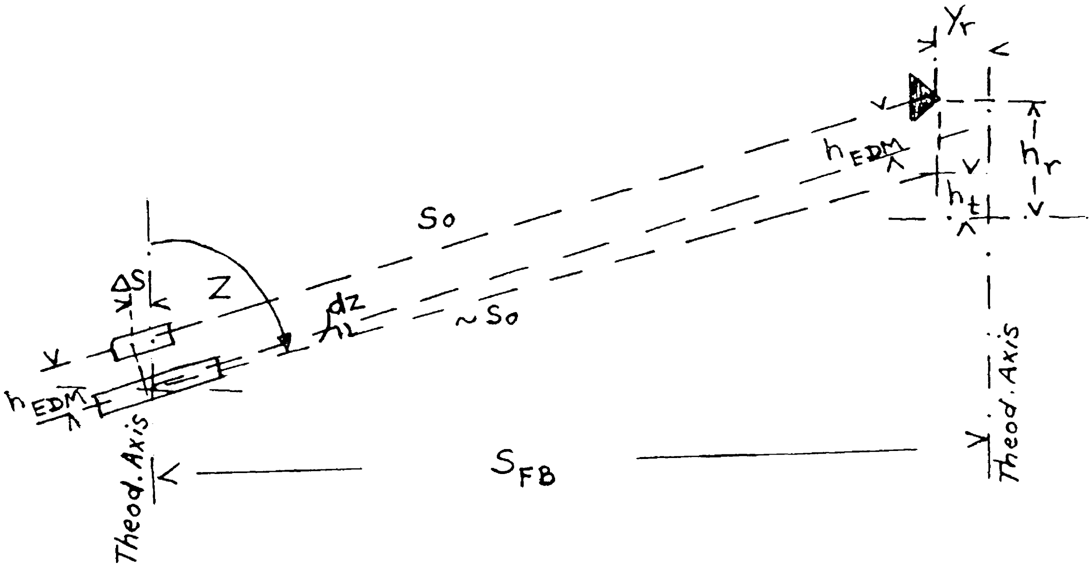


Fig. 38 Distance reductions.

master station. If the mean height differences from each set would differ by more than 1.5 mm (for an average distance of 250 m), another set of measurements was taken.

Since the distances in the reciprocal traversing were generally longer (up to 300 m) than in the leap-frog method measurements, more than one pointing to the targets had to be made in each single sighting on hot days in turbulent air conditions. The software for the real-time data processing could accept up to four pointings. The number of required pointings was agreed upon between the observers before starting the measurements, and it was entered in both computers.

The sequence of measurements was as follows:

1. The connecting surveys to the initial bench mark of the traverse by the backward station. In the meantime, the second observing vehicle goes forward about 250 m to 300 m and chooses a location for the first forward station from which another forward station, preferably at a similar distance, is visible. The observer decides on the number of required pointings and signals it to the back-sight station.
2. Two stations point to each other and the distance is measured and dictated to the E2/EPSON station.
3. Two sets of angles are measured in direct-reverse-reverse-direct sequence of the telescope positions. The moments of exact pointing at each others targets are established by the observers using hand signals (it was found more convenient than using the radio communciation). Between the two sets of angle measurements, the E2 station dictates to the T2000 the height difference calculated from the first set of observations.
4. The second set of distance measurements is taken and the value of the reduced horizontal distance is dictated to the E2 station, which calculates the second height difference and sends it back to the TRS80. In the meantime, meteorological data and comments are entered in the computers.
5. If two mean height differences, calculated for each set separately, agree to within 1.5 mm, the backward vehicle moves to the next forward station.

An example of field records is given in Appendix V (available upon request from UNB).

Similarly as in the leap-frog method, the field data was transferred each day to the mainframe computer for the final traverse calculations. The height differences between the centres of both theodolites were calculated from:

$$\Delta H = [s_{FB}(\cot z_F - \cot z_B) + h_B - h_F] / 2 ,$$

where h_F and h_B are vertical offsets (marked h_t in Fig. 23) of the targets in respect to the trunnion axes of the corresponding forward and backward theodolites, and s_{FB} is the horizontal distance between the vertical axes of the theodolites. Following the notations in Figs. 23 and 38, the horizontal distance is obtained from:

$$s_{FB} = s_0 \sin(z - dz) + y_r ,$$

where s_0 is the observed distance corrected for zero error, cyclic error, and scale. In the case of the DI-5 mounted on the top of the T2000, an additional correction had to be entered arising from the tilt of the EDM instrument aiming at the vertically fixed AGA prism (Fig. 38). The tilt correction Δs is calculated from:

$$\Delta s = - h_{EDM} \cot z .$$

The correction dz to the zenith angle for the reduction of s_0 to s_{FB} is calculated with a sufficient accuracy from the approximation:

$$dz = (h_r - h_t - h_{EDM}) \frac{\sin z}{s_0} ,$$

where the value of z is taken as measured with the T2000 theodolite to the target at E2.

In the case of the DI-5 distance measurements, the vertical offset h_{EDM} of the EDM optical axis in respect to the optical axis of the telescope was +40 mm which, for an inclination of the line-of-sight of only 6°, would produce the appreciable $\Delta s = -4$ mm.

The total height difference between the end bench marks of the traverse of n lines is calculated from:

$$\Delta H_{TOTAL} = \sum_{1}^{n} \Delta H_{F_i B_i} + h_{start} - h_{end} .$$

A full example of a traverse computation using the programme developed at UNB [Kornacki, in prep.] is given in Appendix VI.

4. TEST SURVEYS AT UNB

4.1 Test Area and Scope of the Tests

The main purpose of the test surveys was to confirm in practice the designed resolution (precision) of the developed survey system in controlled field conditions and to add to the knowledge of the influence of the atmospheric refraction in trigonometric height traversing.

The studies were performed in a selected area at the UNB campus which, more or less, could be considered as representing average conditions of the real-life implementation of the trigonometric height determination when using 200 m lines of sight, height above the ground between 1.2 m and 3.5 m, and having three typical ground surfaces: gravel, grass, and asphalt.

Three bench marks were established in the selected area (Fig. 39) which were visible from one set up of the instrument (marked IP) with the three lines of sight of the required 200 m length and inclinations up to 2°30'. Fig. 40 shows ground profiles along the lines of sight to the targetted rods, as designed for the leap-frog method, with the targets at 2.13 m and 3.50 m, and the height of the instrument of about 2.30 m.

The surveys involved long-term (up to 38 hours), continuous determination of height differences between the bench marks. Geodetic levelling of high precision between the bench marks was repeated 3 times during the summer of 1985: on 18 June, 22 July, and 30 July. Bench marks BM2 and BM3 were additionally checked by 3 reference marks established about 30 m away. The surveys indicated that during the test surveys, which were conducted mainly between 19 July and 29 July, all the bench marks were stable to within ± 0.2 mm at one sigma level, with differences in elevations:

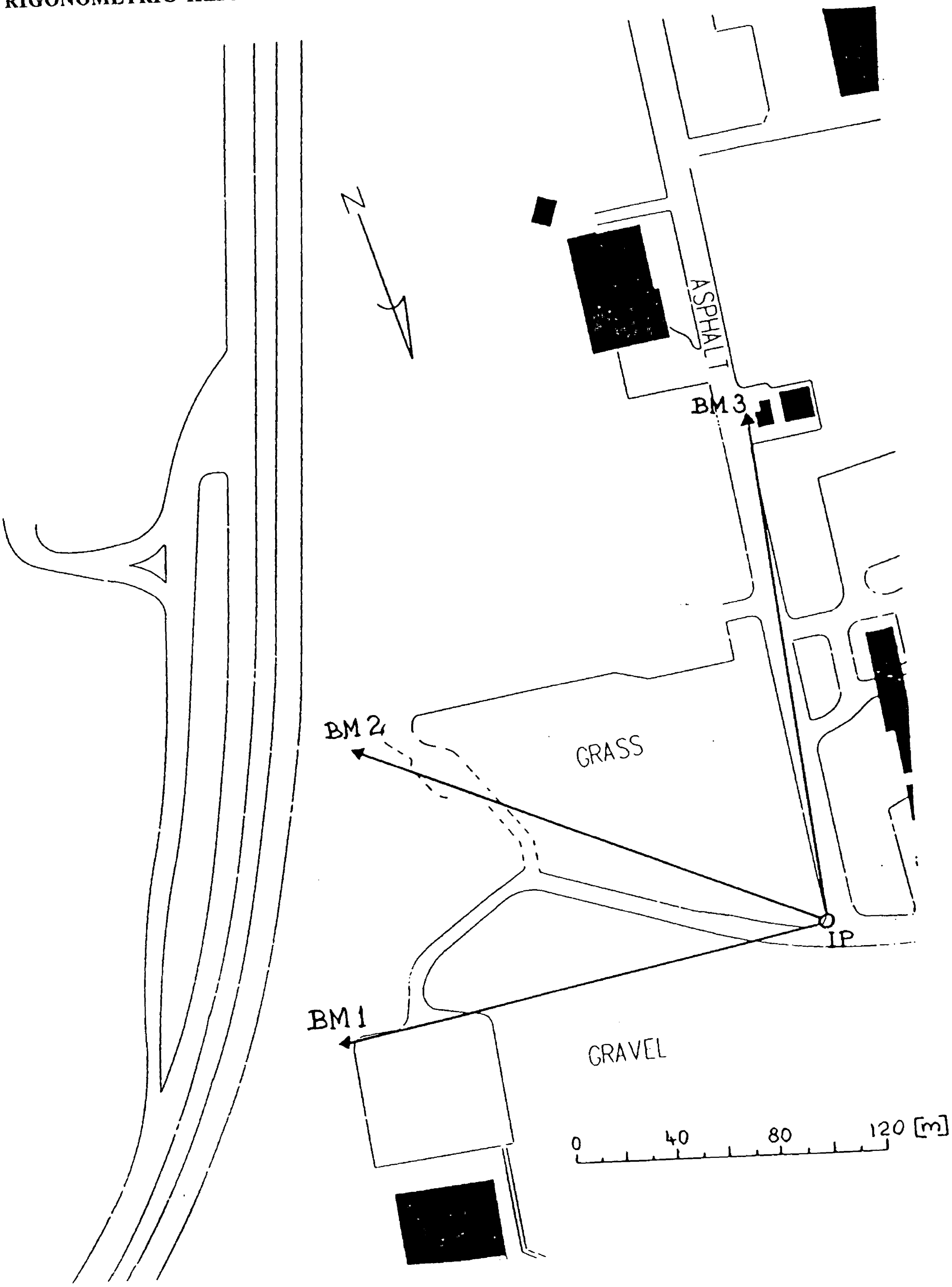


FIG. 39 UNB Test Area

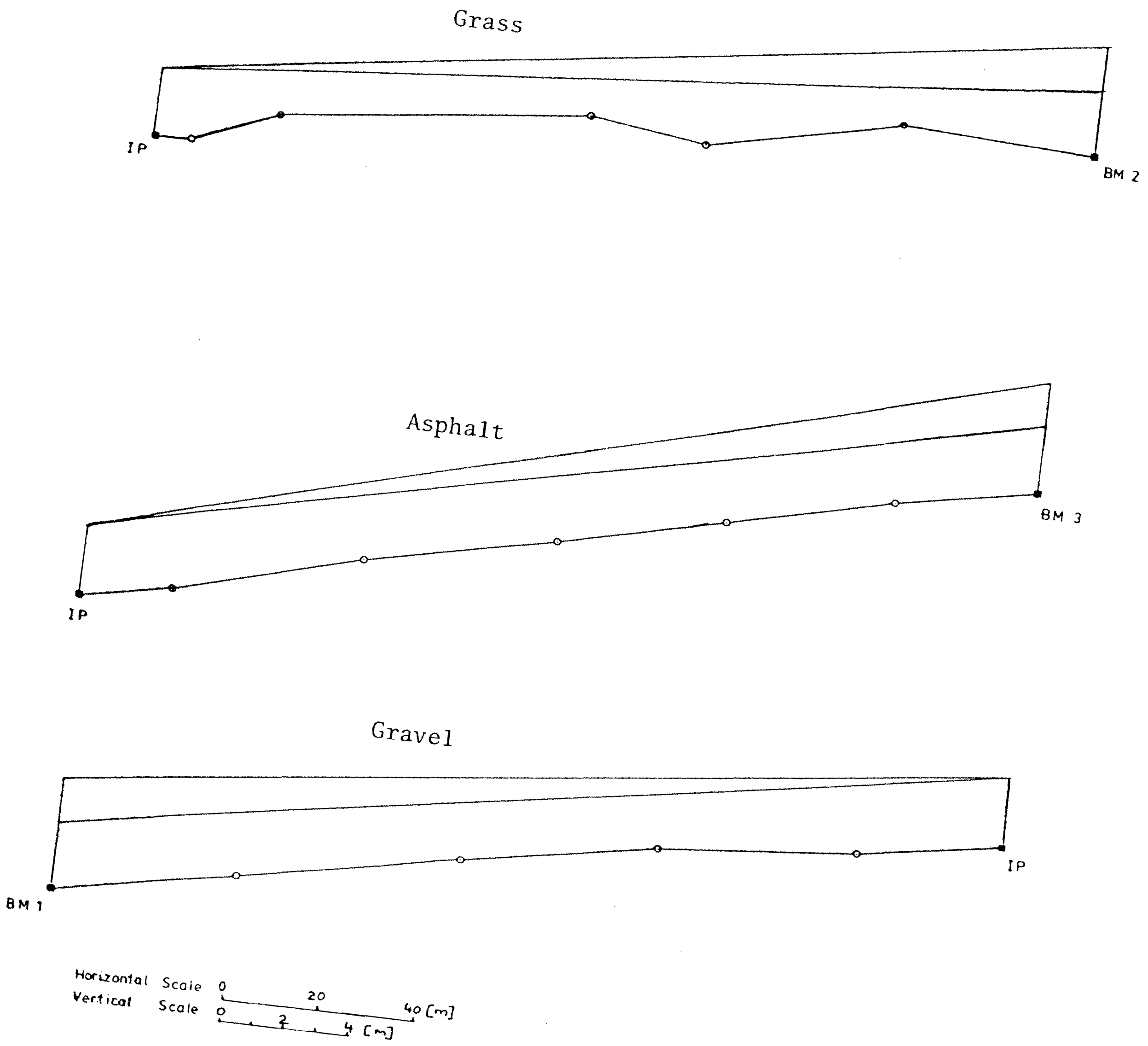


FIG.40 Profiles of the UNB Test Lines

BM1 to BM2 : + 6.7817 m
BM2 to BM3 : + 4.3234 m
BM1 to BM3 : + 11.1051 m .

The test surveys were conducted on:

20 June, 4 hours between 10:20 and 14:30
19/20 July, 13 hours between 11:10 and 0.30
23/24 July, 38 hours between 9:30 and 23:40
29 July, 6 hours between 11:40 and 17:30,

using the field procedures of the leap-frog method with a consecutive determination of height differences between BM1 to BM2, BM2 to BM3, and BM1 to BM3. The Kern E2/DM502 was used in all the tests. The Wild T2000 was used only on 29 July, also in the leap-frog mode with both E2 and T2000 located within a few metres of each other (on the observing vehicles) so that the two systems could be compared in strongly correlated conditions. The first test survey on 20 June was conducted only between BM1 and BM2.

In all the test surveys, gradients of the temperatures were measured on all three lines, about 50 m from the observing vehicle using the thermilinear system described in section 2.8. The system was transferred every 0.5 hours from one line to another. The measurements of the temperatures would commence about 10 minutes after setting up the thermistor rod and ten sets of readings of all 6 thermistors were taken within 15 minutes of time. Then the system was transferred to the next line, and so on, coming back to the initial line after about 60 to 70 minutes of time. In addition to the air temperature profile measurements, the temperature of the ground surface (gravel, asphalt, grass) was measured as well as speed and direction of wind and barometric pressure.

4.2 Summary of Results

The determination of the height differences between the three bench marks is summarized in Tables 1 to 5. The results are shown in a form of differences between the values ΔH_G, obtained from the repeated geodetic levelling, and ΔH_T, obtained from the trigonometric heighting. The 38-hour test survey on 23/24 July gave valuable material for the evaluation of systematic effects of refraction. The results of the survey are plotted in Fig. 41, together with a graph of measured gradients of temperature at the average heights of the lines of sight within the distance of 100 m from the observing station. The average heights were: 2.30 m for asphalt, 2.10 m for gravel, and 1.80 m for grass. Similarly, fluctuations of the lines of sight (changes in the zenith angles) to the targets at heights of 2.1 m and 3.5 m are plotted in Fig. 42.

All field records and computational results are available upon request from the Principal Investigator.

4.3 Evaluation of the Results

At the time of writing this report, not all the observed data has yet been evaluated, particularly the vast amount of temperature profile measurements for the refraction studies. The data will be used in two theses being written in partial fulfilment of M.Sc. degrees in Surveying Engineering; one, already mentioned, by W.J. Kornacki and the other by A. Kharaghani. Both theses will be made available to the Geodetic Survey of Canada in February, 1986, as Suppplements Nos. 2 and 3 to this report.

The preliminary evaluation, which is given in this report, is sufficient to draw meaningful conclusions within the scope of the project as outlined in the Introduction.

Table 1

Preliminary test measurements (leap-frog) at UNB test line from BM1 (gravel) to BM2 (grass) with Kern E2.

Date: 20 June 1985 Lines of sight: 200 m

Time Start	Weather t°C, overcast %, remarks	$\Delta H_{\mathrm{GEOD}} - \Delta H_{\mathrm{TRIG}}$ [mm]
10:25	21°C, 50%, sunny	-1.0
10.45	22°C, 50%, sunny, windy	-0.4
11:02	22°C, 50%, sunny, windy	0.8
11:16	21°C, 70%	-0.9
11:32	21°C, 90%	-1.4
11:54	21°C, 100%	-2.9
12:12	20°C, 100%, windy	-1.5
12:30	20°C, 75%, sunny periods, windy	-0.6
13:01	21°C, 50%, sunny periods, windy	-1.8
13:15	21°C, 50%, sunny periods, windy	0.5
13:47	22°C, 75%, windy	-4.0
14:00	22°C, 75%, windy	-2.3
14:13	22°C, 75%, windy	-0.9
Mean difference		-1.3 mm

Table 2

Continuous (12 hours) leap-frog measurements at UNB test lines between BM1 (gravel), BM2 (grass), and BM3 (asphalt) with Kern E2.

Date: 19 July 1985 Lines of sight: 200 m

Differences $d = \Delta H_{\mathrm{GEOD}} - \Delta H_{\mathrm{TRIG}}$ in [mm]

Time Start	Weather t°C, overcast %, remarks	BM1 to BM2	BM1 to BM3	BM2 to BM3
11:13	23°C, 75%	-0.5	-	-1.6
12:09	25°C, 75%, shimmer	-0.7	-1.7	-2.1
13:24	27.5°C, 25%, sunny, windy	-2.0	-1.2	0.7
14:54	26°C, 75%, shower, wind	-2.3	-2.4	-0.1
15:49	26°C, 100%	-2.4	-0.8	-0.1
16:32	26°C, 100%	-	-1.3	-
18:06	26°C, 100%	-1.6	-	0.1
18:56	25°C, 100%	-1.6	-0.2	0.6
19:31	24°C, 100%	-0.9	-0.1	1.3
20:09	24°C, 100%	-1.5	0.0	2.0
21:56	21°C, 100%, after rain, calm	-1.9	-	0.3
22:47	20°C, 100%	-1.3	-0.5	0.7
23:25	20°C, 100%	-0.7	-0.4	0.5
00:05	20°C, 100%	-	-0.5	-
Mean: $\Delta H_{\mathrm{GEOD}} - \Delta H_{\mathrm{TRIG}}$ =		-1.5 mm	-0.8 mm	0.2 mm

Table 3

Continuous (38 hours) leap-frog measurements at UNB test lines between BM1 (gravel), BM2 (grass), and BM3 (asphalt) using Kern E2.

Date: 23/24 July 1985 Lines of sight: 200 m

Differences: $d = \Delta H_{GEOD} - \Delta H_{TRIG}$ in [mm]

Start Time	Weather *t*°C, overcast %, other	BM1 to BM2	BM1 to BM3	BM2 to BM3
9:45	18°C, 0%, windy	-0.3	-	-1.4
10:29	20°C, 25%, wind gusts	0.4	-1.7	0.2
11:30	21°C, 50%, windy	-1.9	-1.5	0.1
12:03		-	-1.0	-
12:29	20°C, 74%, windy	-1.6	-2.7	-1.1
12:54		-	-1.9	-
13:22	20°C, 60%, strong wind	-1.3	-2.3	-0.1
13:50	21°C, 60%, strong wind	-	-1.3	-
14:25	23°C, 60%, strong wind	-1.7	-2.7	1.3
15:30	21°C, 40%	-2.1	-2.8	-2.6
16:01	21°C, 50%	-	-2.9	-
18:04	20°C, 60%, windy	-2.4	-	0.2
19:00	20°C, 50%	-1.8	-1.9	-
19:22	20°C, 50%	-2.9	-1.5	1.8
20:16	20°C, 0%	-3.7	-1.2	1.6
21:09	18°C, 0%	-2.6	0.1	1.9
22:57	16°C, 0%	-	1.1	1.9
23:34	16°C, 0%	-1.6	-0.2	0.5
0:39	16°C, 0%	-	-0.8	1.9
0:55	15°C, 0%	-	0.5	1.8
1.28	15°C, 0%	-2.0	0.8	2.8
2.14	15°C, 0%	-1.6	0.0	-
2:44	14°C, 0%	-	0.0	1.4
3:11	13°C, 0%	-1.9	-	1.9
4:04	13°C, 0%, breeze	-3.3	-0.9	0.8
4:49	12°C, 0%	-2.3	-0.5	1.6
5:36	12°C, 0%	-2.6	-0.8	0.3
6:14	12°C, 0% sunrise	-1.3	-1.0	0.5
7:02	13°C, 0%	-1.2	-0.6	0.9
7:54	14°C, 0%, calm	-0.6	-1.3	-1.7
9:04	17°C, 0%, breeze	-0.3	-2.6	-2.2
9:55	19°C, 0%	0.1	-0.7	-1.4
10:37	20°C, 0%	-0.6	-2.6	-1.7
11:37	22°C, 0%	0.2	-	-
12:02	23°C, 0%	-	-	-0.5
12:27	23°C, 0%	-1.2	-2.0	-2.7
13:10	23°C, 0%	-0.7	-0.8	-0.8
14:11	25°C, 0%	0.0	-2.8	-0.1
14:48	25°C, 0%	-2.8	-3.0	-1.7
15:41	26°C, 0%	-1.4	-1.7	0.0
16:19	26°C, 0%	-1.2	-1.0	1.2
17:37	25°C, 0%, windy	-2.5	-2.2	-0.8
18:22	25°C, 0%	-2.5	-1.8	1.6
19:10	24°C, 10%, strong shimmer over grass	-2.5	-0.9	1.4
20:07	22°C, 5%, strong shimmer over grass	-2.5	0.0	3.2
21:10	20°C, 0%	-2.4	1.4	-
21:40	18°C,	-	-0.5	1.9
22:04	19°C, 0%, windy	-1.7	-	1.4
22:53	19°C, 0%	-1.0	-0.1	1.0
23:33		-	0.8	-
Mean difference		-1.6	-1.1	0.4

Table 4

Simultaneous leap-frog measurements with Kern E2 and Wild T2000 at UNB test lines between BM1 (gravel), BM2 (grass), and BM3 (asphalt).

Date: 29 July 1985 Lines of sight: 200 m

Differences: $d = \Delta H_{\mathrm{GEOD}} - \Delta H_{\mathrm{TRIG}}$ in [mm]

Start		Line: BM1 to BM2		Line: BM1 to BM3		Line: BM2 to BM3	
Time	Weather	E2	T2000	E2	T2000	E2	T2000
11:42	sunny, windy, 25°C	-1.5	0.0	-1.5	-0.5	0.1	-0.5
12:05	sunny, windy, 26°C	-	-0.4	-	-1.0	-	-0.6
12:20	sunny, windy, 26°C	-2.0	0.1	-2.5	-1.1	-0.6	-1.2
12:37	sunny, windy, 26°C	-	-1.4	-	-2.6	-	-1.2
13:22	sunny, windy, 27°C	0.3	-1.3	1.0	-1.4	0.7	-0.1
13:45	sunny, windy, 27°C	0.7	-1.1	-1.1	-2.2	-1.8	-1.2
14:05	sunny, shimmer, 27°C	0.3	-0.5	-0.6	-1.6	-0.8	-0.9
14:30	sunny, shimmer, 27°C	-1.5	-1.3	-2.1	-2.8	-0.6	-0.7
14:50	sunny, shimmer, 27°C	-0.5	0.0	-1.9	-2.0	-2.1	-0.8
15:13	sunny, shimmer, 27°C	1.3	-	-2.1	-	-	-
15:25	sunny, shimmer, 28°C	-0.9	-1.2	-1.8	-1.3	-0.4	-0.1
15:47	sunny, shimmer, 28°C	-2.0	-2.2	-1.9	-0.3	0.0	1.9
16:00	cloud 50%, wind, 28°C	-2.0	-1.3	-1.3	-1.4	0.6	-0.1
16:12	clouds, wind, 28°C	-1.1	-	-1.7	-	0.5	-
16:25	cloud 80%, 28°C	-	-1.5	-	-0.6	-	0.9
16:45	cloud 80%, 29°C	-0.3	-1.1	-2.3	-1.7	-2.0	-0.6
17:05	cloud 80%, 29°C	-0.5	-2.2	-1.4	-1.3	-0.9	0.9
Mean *d* [mm]		-0.7	-1.0	-1.4	-1.5	-0.6	-0.3

Table 5

Mean $\Delta H_{\mathrm{GEOD}} - \Delta H_{\mathrm{TRIG}}$ in [mm] at UNB test lines on different days between 10:30 a.m. and 2:30 p.m.

Date	BM1 to BM2	BM1 to BM3	BM2 to BM3
20 June 1985	-1.3	-	-
19 July 1985	-1.4	-1.8	-0.8
23 July 1985	-1.2	-2.2	0.1
24 July 1985	-0.5	-2.1	-1.2
29 July 1985 (E2)	-0.6	-1.1	-0.5
29 July 1985 (T2000)	-0.6	-1.7	-0.8
Average	-0.9 mm	-1.8 mm	-0.6 mm
Maximum spread	0.9 mm	1.1 mm	1.3 mm

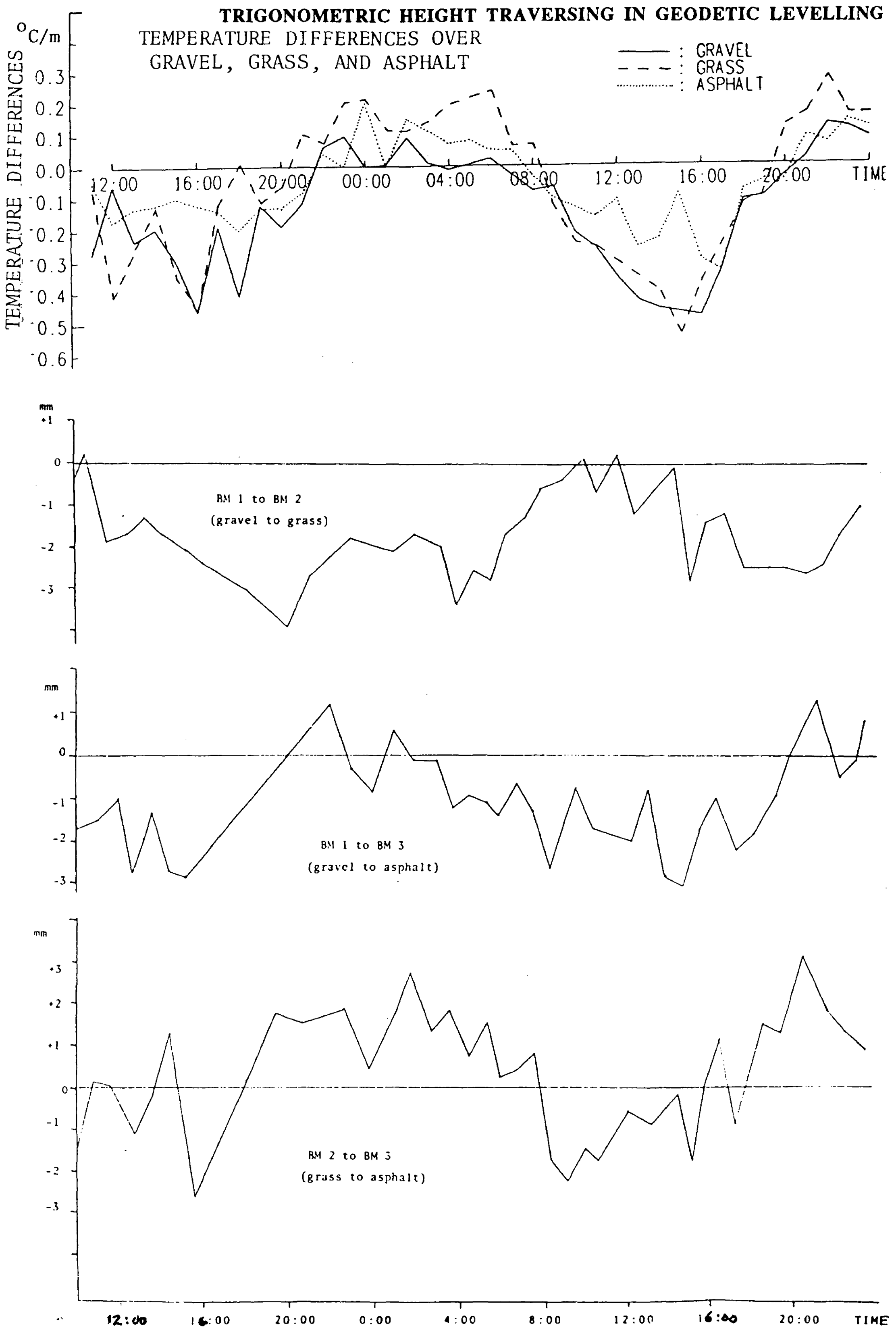


Fig. 41 Trigonometric height differences vs. geodetic levelling in millimetres and vs. gradients of temperature.

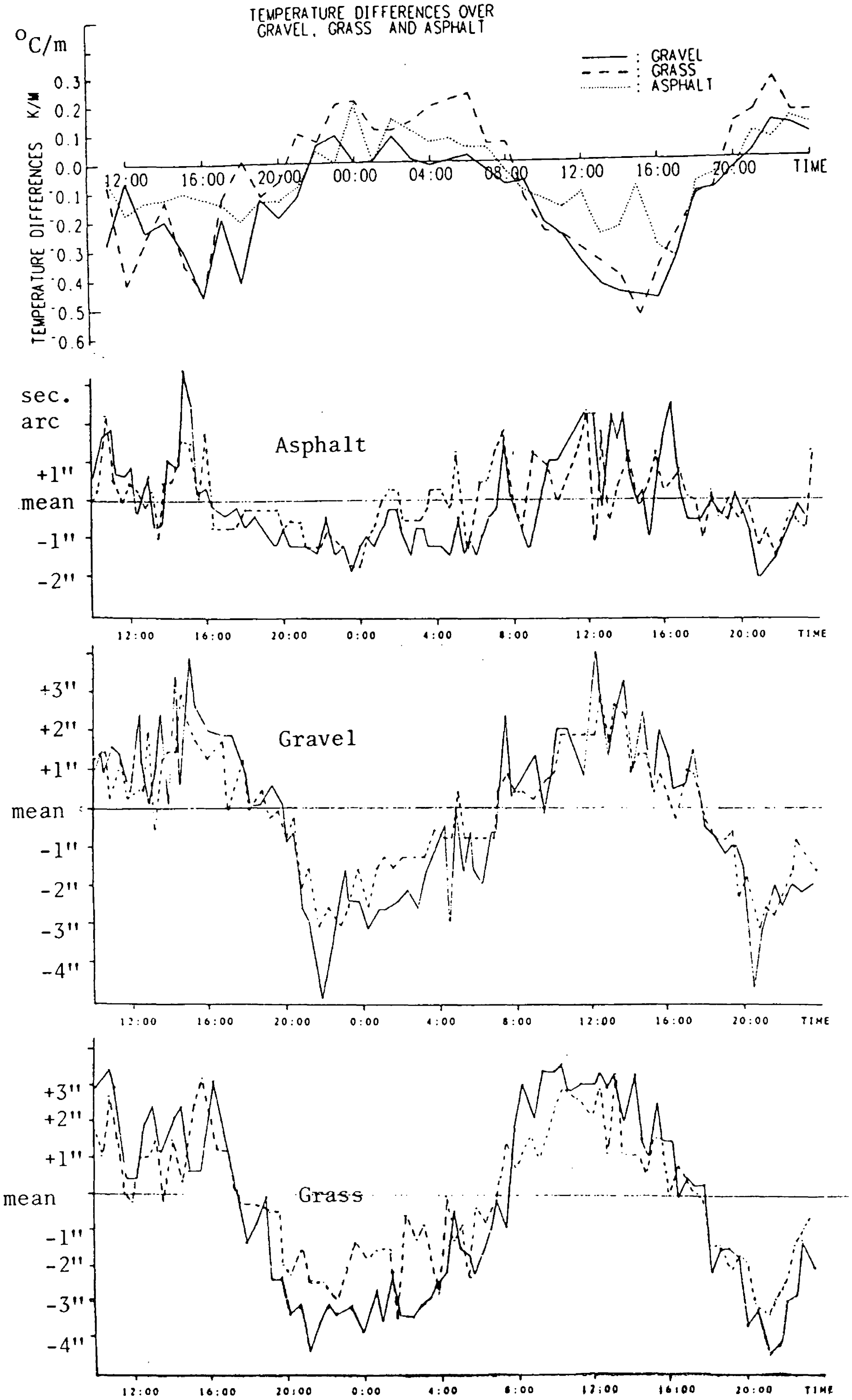


Fig. 42 Fluctuations of the lines of sight in seconds of arc vs. changes of dt/dh. ----------- = target at 3.5 m ———————— = target at 2 m.

4.3.1 Evaluation of the accuracy of angle measurements

According to the preceding feasibility study [Chrzanowski, 1984], the theodolites E2 and T2000 should give 0.5" r.m.s. error, or smaller, if the angles are measured in four sets.

The test survey supplied sufficient data for the estimation of the actual accuracy in the real conditions of trigonometric height traversing. The most objective evaluation, not affected by systematic errors of refraction, was obtained from the data of 29 July, listed in Table 4.

The height differences were calculated from simultaneous, or almost simultaneous (about 2 minutes of time variations), independent observations with the E2 and T2000 theodolites. Since the theodolites were located close to each other, systematic effects of refraction were eliminated in the differences $d = \Delta H_{E2} - \Delta H_{T2000}$ obtained from the listed results.

Assuming the same accuracy of both theodolites and taking the differences from individual pairs of observations as true errors, the standard deviation of each ΔH_i could be estimated from:

$$\hat{\sigma}_{\Delta H} = \left(\sum_{1}^{n} \frac{\Delta H_i^2}{2n} \right)^{1/2} .$$

The following values were obtained:

for BM1 to BM2: $\hat{\sigma}_{\Delta H} = 0.85$ mm,
for BM2 to BM3: $\hat{\sigma}_{\Delta H} = 0.73$ mm, and
for BM1 to BM3: $\hat{\sigma}_{\Delta H} = 0.78$ mm,

with an overall average:

$\bar{\sigma}_{\Delta H} = 0.79$ mm.

Since the lengths of the lines of sights were 200 m, the estimated standard deviation of an angle measured in four sets (the standard field procedure used in the test surveys) is equal to:

$$\hat{\sigma}_Z = \frac{0.79''}{\sqrt{2}} = 0.56'' .$$

The result is very close to the a priori estimated value. Since the angle measurements with the E2 and T2000 were not exactly synchronized in time, the differences in ΔH determinations were perhaps affected by short-term refraction changes which were not included in the pre-analysis of angle measurements. Therefore, the accuracy of 0.5" can be accepted as having been confirmed.

The accuracy of angle measurements was also determined from the 38-hour test from two sets of ΔH determinations from two sets of angle measurements, separately for the upper and lower targets and separately from the daylight (06:00 to 21:00) and night (21:00 to 06:00) observations. The results are given below as an r.m.s. error of **one set** of observations (in two positions of the telescope).

Daylight observations to the lower target:	$\hat{\sigma} = 0.96''$
Daylight observations to the upper target:	$\hat{\sigma} = 0.88''$
Night observations to the lower target:	$\hat{\sigma} = 0.75''$
Night observations to the upper target:	$\hat{\sigma} = 0.62''$

which, as one could expect, indicates higher accuracy in the night observations (no shimmer and better accuracy of pointing to illuminated targets), as well as an overall better accuracy in the measurements to the upper target. The overall average r.m.s. error for one set of measurements becomes: 0.8" which gives 0.4" for four sets of measurements. The result is better than from previous calculations, but in the latter case, the observations had larger correlation (the same theodolite, the same observer).

4.3.2 Evaluation of the choice of target height

The above evaluation of the accuracy of the angle measurements has already indicated that the results to the upper targets are slightly better with an overall r.m.s. error of one set of measurements equal to 0.75" versus 0.85" for the lower targets.

The graphs in Fig. 42 also lead to a conclusion that the angles to the upper targets are less affected by systematic deviations from the mean direction which are, certainly, produced by the cyclic changes of the refraction effects. This is particularly visible on the line above gravel surface to BM1. The amplitude of the cyclic variations of the line of sight to the lower target is about 7.5" while for the upper target it is about 5.5". The random short-term oscillations to the upper target also have smaller magnitude than to the lower targets, at least above gravel and grass. The line over asphalt does not show a significant difference. This is, perhaps, due to mixing of air over the asphalt because of the car traffic on that line.

If one looks at actual results of the height differences, the difference in accuracy from observations to the upper and lower targets becomes less visible.

The question arises, therefore, whether the 3.5 m targets are really needed in the leap-frog method. If not, then perhaps a rod with a compromised height of about 2.5 m, with only one target on the top, would give practically the same results. It would have the advantages of being easier to handle and set up, and there would be a decreased demand for the maintenance of its verticality. However, the two well-separated targets give a good field check on any gross errors or on irregularities in the atmospheric refraction (aforementioned inversions of dT / dz at 2.5 m to 3 m elevations). In the latter case, a change in the height of the instrument, rather than the second target on the rod, would be much more effective. A change in the height of the instrument by only 30 cm to 40 cm would, perhaps, be much more effective than having two targets 1.5 m apart. No work in that direction has been done within this project.

4.3.3 Evaluation of temperature profile measurements

The study of the atmospheric refraction has been a side product of the project, and it has mainly been sponsored by NSERC. Therefore, a full evaluation of the results is beyond the scope of the contract agreement between the GSC and UNB. Nevertheless, some preliminary results are presented here and a more detailed evaluation of the results obtained will be given in Supplement No. 3.

The study concentrated mainly on the determination of an optimal model of the temperature profiles up to 4 m above the ground.

As was already discussed in the feasibility study report, the Kukkamaki empirical formula for the vertical profile of air temperature is used by most researchers involved in the studies of the influence of refraction in levelling. The empirical formula which reads:

$$T(z) = a + bz^c ,$$

where z is the height above the ground, and a, b, c are determined empirically, has been confirmed in certain conditions by the free convection theory of heat transfer in which, for turbulent air conditions, the coefficient c is taken as a constant and equal to -0.33. As already indicated in Chrzanowski [1984] and Greening [1985], there are some limitations in the actual field conditions in which the Kukkamaki model approximates reasonably well the actual temperature profile. In the investigations at UNB, in some cases an inversion of the gradient of temperature occurred in daytime conditions at elevations over 2 m above the ground. In such cases, the Kukkamaki model as well as the free convection models fail. The c coefficient has been found to differ significantly from the generally accepted value of -0.33.

Tables 6 and 7 give a summary of partial results obtained from temperature measurements over the gravel surface on the test line to BM1.

In the first case (Table 6), the coefficients *b* and *c* of the Kukkamaki model were determined through the least-squares fitting to 5 temperature differences obtained from 6 points of temperature measurements at heights of 0.3 m, 0.6 m, 1.2 m, 2.0 m, 3.0 m, and 4.0 m. The results show that only observations at 07:00 to 08:00 hours and at 16:00 gave a good agreement with the theoretical value of $c = -0.33$. In the second case (Table 7), the value of *c* was held equal to -0.33, with *b* as the only parameter estimated through least-squares fitting. In this case, the residuals were slightly larger than in the first calculation and, of course, the values for *b* differed.

As one can see from Tables 6 and 7 and from the graphs in Figs. 43 and 44, the calculated coefficients of refraction from the two solutions show the same general trend but they significantly differ up to a factor of 2 or even more. Some other models, for instance third-order polynomial, have also been tried in approximating the temperature profile through least-squares fitting to the observed data. They seem to give better approximation than Kukkamaki's model in some cases. The study has not yet been completed. Presently, only one thing has been confirmed: that the coefficient of refraction may reach values of -10 and even larger at about 0.5 m above the ground and even at the elevation of 2.5 m, may reach -1 and more which could cause an error of about -3 mm in sighting to a target at a distance of 200 m. This is confirmed in the evaluation given below.

```
    TEMPERATURE MEASURMENT OVER "GRAVEL"     DATE:  JULY 24 1985
-----------------------------------------------------------------------------

  #  TIME ----------DIFFERENCES--------- -----------RESIDUALS----------   ITE  ST DV    B      C    POINT' COEFFICIENT OF REFR.
                                                                                                  K0.45   K.9  K1.6  K2.5  K3.5

* 1  7.00 -0.18 -0.19 -0.11 -0.03          -0.02  0.03  0.01 -0.04          9   0.03  0.76 -0.29    -3.6  -1.4  -0.5  -0.2  -0.1
* 2  8.05 -0.22 -0.29 -0.10 -0.02          -0.04  0.09 -0.02 -0.06         11   0.07  0.79 -0.35    -4.7  -1.7  -0.7  -0.3  -0.1
* 3  9.20 -0.65 -0.13 -0.20 -0.18           0.03 -0.13  0.11  0.14         17   0.13  0.23 -1.28   -10.5  -2.0  -0.4  -0.0   0.1
* 4 10.40 -0.36  0.05 -0.26 -0.21           0.14 -0.27  0.10  0.09         40   0.19  7.71 -0.04    -4.0  -1.8  -0.9  -0.5  -0.3
@ 5 11.40    -0.53    -0.57 -0.07 -0.01       -0.05     0.33 -0.13 -0.14   31   0.22 -3.64  0.12    -5.0  -2.7  -1.5  -1.0  -0.7
@ 6 12.50    -0.96    -0.51 -0.27 -0.15       -0.02     0.13 -0.04 -0.07   18   0.09 *****  0.06    -8.8  -4.5  -2.5  -1.6  -1.1
@ 7 13.50    -0.93    -0.49 -0.34 -0.15       -0.02     0.10  0.01 -0.09   19   0.08 -5.88  0.12    -8.2  -4.4  -2.6  -1.7  -1.2
@ 8 15.00    -1.02    -0.49 -0.38 -0.24       -0.01     0.05  0.00 -0.04   16   0.04 -4.98  0.16    -8.8  -4.8  -2.9  -1.9  -1.4
@ 9 16.00    -1.21    -0.41 -0.19 -0.07       -0.01     0.09 -0.03 -0.07    6   0.07  2.20 -0.33   -11.9  -4.6  -2.0  -1.0  -0.6
*10 17.10 -0.63 -0.20 -0.20  0.00           0.01 -0.06  0.11 -0.04         10   0.08  0.23 -1.27   -10.0  -1.9  -0.4  -0.0   0.1
*11 18.20 -0.32 -0.06 -0.11 -0.07           0.01 -0.06  0.07  0.05         14   0.06  0.10 -1.32    -4.8  -0.8  -0.1   0.1   0.2
*12 19.15 -0.29 -0.08 -0.10  0.04           0.00 -0.02  0.07 -0.05          8   0.05  0.08 -1.47    -4.4  -0.6  -0.0   0.1   0.2


     ----------------------------------------------------------------------------------------------
         * FIVE TEMPERATURE SENSORS, SENSORS ARE AT HEIGHTS 0.3, 0.6, 1.2, 2.0 AND 3.0 METRES.
         @ FIVE TEMPERATURE SENSORS, SENSORS ARE AT HEIGHTS 0.3, 1.2, 2.0, 3.0 AND 4.0 METRES.
```

TABLE 6

Determination of coefficients of refraction from the gradients of temperature (Kukkamaki's model with b and c as unknown parameters).

TEMPERATURE MEASURMENT OVER "GRAVEL"   DATE: JULY 24 1985

#	TIME	DIFFERENCES				RESIDUALS				ITE	ST DV	B	C	POINT' COEFFICIENT OF REFR.				
														K0.45	K.9	K1.6	K2.5	K3.5
* 1	7.00	-0.18	-0.19	-0.11	-0.03	-0.02	0.03	0.01	-0.04	2	0.03	0.65	-0.33	-3.7	-1.3	-0.5	-0.2	-0.0
* 2	8.05	-0.22	-0.29	-0.10	-0.02	-0.04	0.09	-0.02	-0.06	2	0.07	0.83	-0.33	-4.7	-1.7	-0.7	-0.3	-0.1
* 3	9.20	-0.65	-0.13	-0.20	-0.18	0.19	-0.24	-0.02	0.03	2	0.18	1.50	-0.33	-8.5	-3.2	-1.4	-0.7	-0.4
* 4	10.40	-0.36	0.05	-0.26	-0.21	0.10	-0.26	0.14	0.12	2	0.19	0.85	-0.33	-4.6	-1.7	-0.7	-0.3	-0.1
@ 5	11.40	-0.63	-0.57	-0.07	-0.01	-0.09	0.40	-0.04	-0.06	2	0.24	1.13	-0.33	-6.1	-2.3	-1.0	-0.4	-0.2
@ 6	12.50	-0.96	-0.51	-0.27	-0.15	-0.08	0.23	0.08	0.03	2	0.15	1.88	-0.33	-10.3	-4.0	-1.7	-0.9	-0.5
@ 7	13.50	-0.93	-0.49	-0.34	-0.15	-0.09	0.22	0.15	0.03	2	0.16	1.85	-0.33	-10.1	-3.9	-1.7	-0.8	-0.5
@ 8	15.00	-1.02	-0.49	-0.38	-0.24	-0.10	0.19	0.18	0.11	2	0.17	2.02	-0.33	-11.0	-4.2	-1.9	-0.9	-0.5
@ 9	16.00	-1.21	-0.41	-0.19	-0.07	-0.01	0.09	-0.03	-0.07	2	0.07	2.21	-0.33	-11.9	-4.6	-2.0	-1.0	-0.6
*10	17.10	-0.63	-0.20	-0.20	0.00	0.18	-0.16	-0.02	-0.15	2	0.16	1.46	-0.33	-7.9	-3.0	-1.3	-0.6	-0.3
*11	18.20	-0.32	-0.06	-0.11	-0.07	0.09	-0.12	0.00	-0.00	2	0.09	0.73	-0.33	-3.9	-1.4	-0.6	-0.2	-0.1
*12	19.15	-0.29	-0.08	-0.10	0.04	0.09	-0.08	0.01	-0.10	2	0.09	0.64	-0.33	-3.4	-1.2	-0.5	-0.2	-0.0

* FIVE TEMPERATURE SENSORS, SENSORS ARE AT HEIGHTS 0.3, 0.6, 1.2, 2.0 AND 3.0 METRES.

@ FIVE TEMPERATURE SENSORS, SENSORS ARE AT HEIGHTS 0.3, 1.2, 2.0, 3.0 AND 4.0 METRES.

TABLE 7

Determination of coefficients of refraction from the gradients of temperature (Kukkamaki's model with c = -0.33).

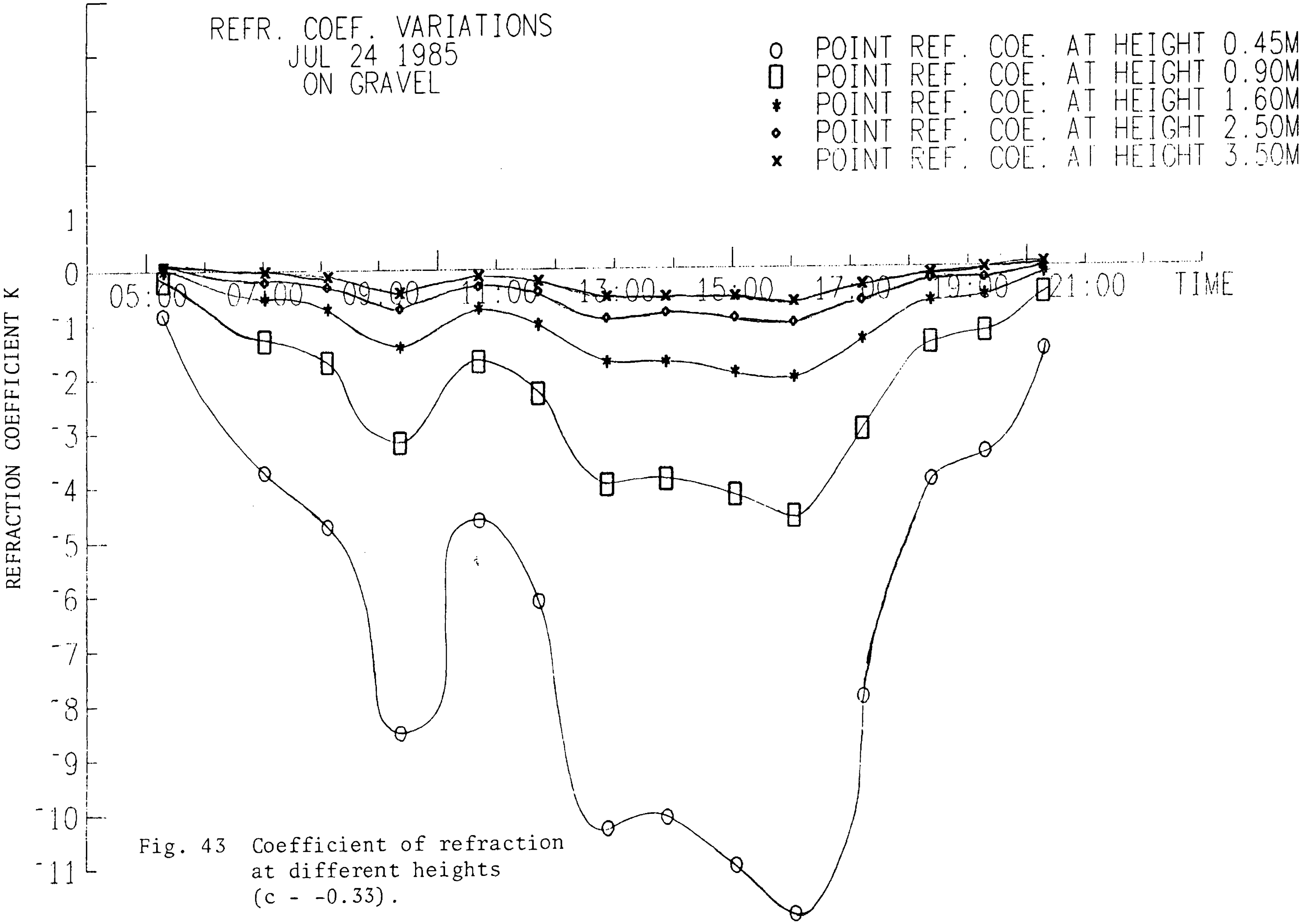


Fig. 43 Coefficient of refraction at different heights (c - -0.33).

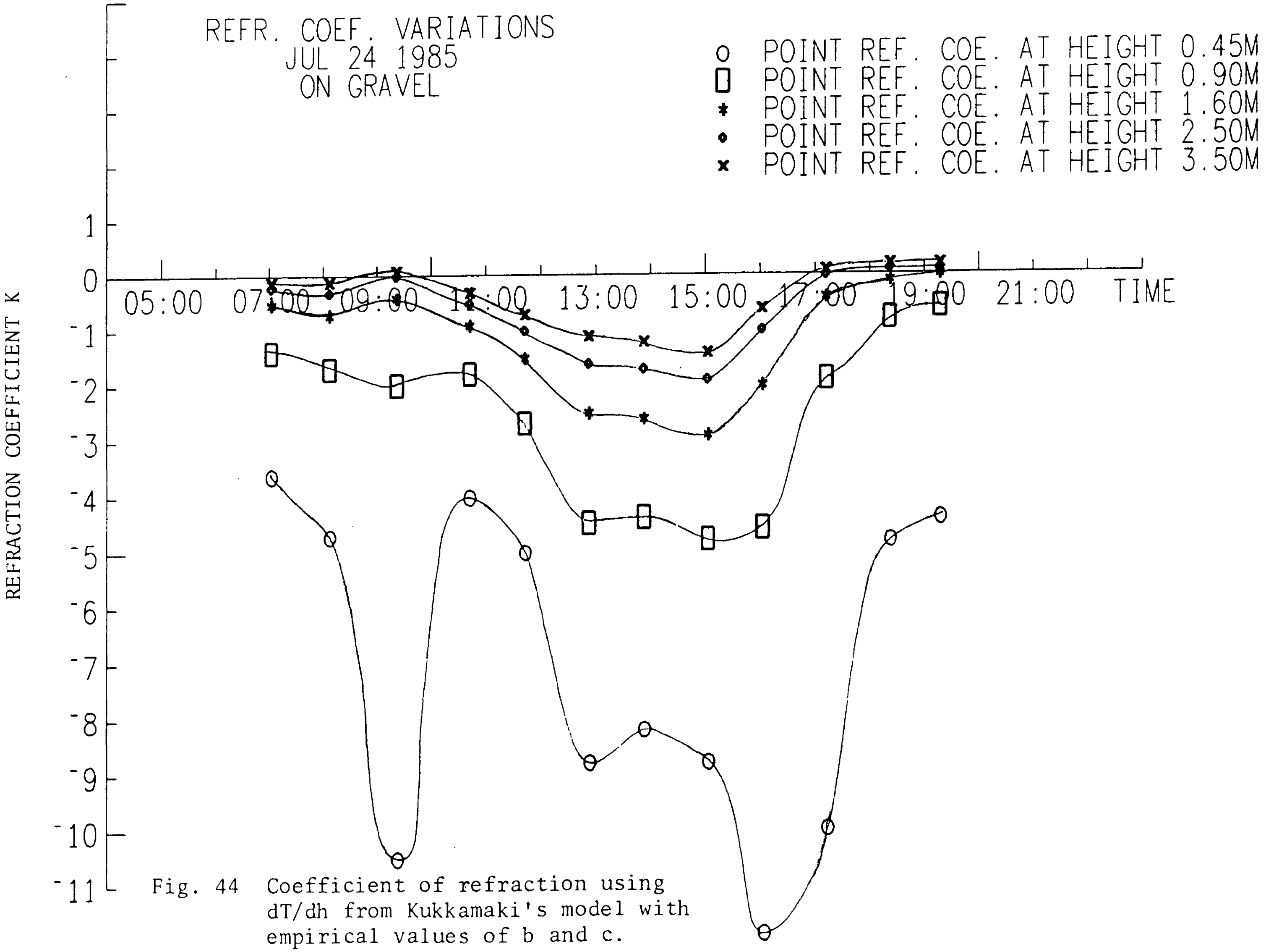


Fig. 44 Coefficient of refraction using dT/dh from Kukkamaki's model with empirical values of b and c.

4.3.4 Evaluation of the effects of atmospheric refraction

The tabulated results and the graphs in Fig. 41 clearly show systematic deviations of the trigonometric height differences from those calculated from several geodetic levellings. The cyclic changes of these deviations clearly correspond to the cyclic changes of the gradients of temperature determined at the average heights of the lines of sight.

The results demonstrate that in the unfavourable conditions of trigonometric traversing with 200 m lines, when the fore-sight line passes over different types of ground surface than the back-sight line, the maximum error (combined systematic and random effects) in height differences may reach 4 mm. Besides the pronounced cyclic nature of the errors, there is an evident systematic shift of all the results. This is shown in Table 5 which summarizes survey results of 5 different days averaged over a period of about 4 hours each. This is clearly visible in the 38-hour test (Table 3) where the total average between BM1 to BM2 (gravel to grass) is shifted by -1.6 mm. An extract of a 24-hour cycle of observations from the same data (from 20:00 on 23 July to 20:00 on 24 July) gives the same average shift of -1.6 mm. Generally, no specific time of the day could be identified as the best for the observations. The largest error, 4 mm, occurred at 14:00 on a windy day (20 June) with 75% overcast and t = 22°C. The second largest error, 3.7 mm, was at 20:00 (23 July) with clear skies and t = 20°C. The third largest, 3.3 mm, was at 04:00 (24 July) with clear skies, breeze, and t = 13°C. On the hot sunny day of 29 July, with t = 27°C and shimmering sighting conditions, the maximum error was 2.8 mm. On the cloudy day (100% overcast) of 19 July, the maximum error was still 2.4 mm, except for the period after the rain in calm conditions later in the evening when the systematic deviations became much smaller. Though the above discussion relates directly to the leap-frog method, similar results could be expected with the reciprocal method, if the ground surfaces near each of the instruments should be different [Chrzanowski, 1984].

In order to confirm that the above systematic effects were produced by atmospheric refraction, the average gradients of temperature (mean of the 38 hour measurements) were derived directly from the measured mean temperatures at the average heights of the lines of sight. They were:

$(dT / dz)_1 = -0.139°\text{C/m}$ for gravel (BM1),

$(dT / dz)_2 = -0.067°\text{C/m}$ for grass (BM2), and

$(dT / dz)_3 = -0.059°\text{C/m}$ for asphalt (BM3),

which corresponded to the following coefficients of refraction (see method of calculation in Greening [1985] or in the feasibility study report) at the given meteorological conditions: $k_1 = -0.61$; $k_2 = -0.19$; and $k_3 = -0.15$. Assuming for these approximate calculations a homogeneous refraction on each line, the errors of Δh between the instrument and each bench mark would be: $\varepsilon_{\Delta h} = ks^2 / 2R$, which gives:

for BM1, $\varepsilon_{\Delta h} = -1.9$ mm,

for BM2, $\varepsilon_{\Delta h} = -0.6$ mm, and

for BM3, $\varepsilon_{\Delta h} = -0.5$ mm.

Thus one could expect to have the average systematic error of the difference in elevation from BM1 to BM2 equal to -1.3 mm, for BM1 to BM3 equal -1.4 mm, and for BM2 to BM3 equal -0.1 mm. These values agree very well with the actually obtained average shifts of -1.6 mm, -1.1 mm, and 0.4 mm, respectively.

More detailed evaluation of the observation data from the point of view of the systematic influence of refraction is in progress and will be presented in Supplement No. 3.

5. IMPLEMENTATION OF THE DEVELOPED SYSTEM TO A GEODETIC NETWORK SURVEY

5.1 Description of the Survey and Summary of Results

A major test of the developed survey system was its application to the determination of heights in a network (Fig. 45) consisting of 21 bench marks with a total length of the interconnecting lines of over 70 km. There were several reasons to choose this particular area for the main test:

- The area is within only half an hour's drive from UNB.
- It is hilly with road grades of up to 7°.
- It already had 15 established bench marks.
- NB Power has been interested in studying the stability of the area due to its close proximity to the Mactaquac head pond.
- Part of the network (line from N-81-10 through 68-166 to 68-182) was supposed to be measured by the levelling group of GSC during the summer of 1985. This would have given a direct comparison between the trigonometric and geometric levellings.
- The dense road network allowed having several survey loops in the network for the self-determination of the internal accuracy of the trigonometric height traversing.

In order to have better checks and more loops in the network, six more bench marks (marked UNB) were established in the area during May and June 1985, giving a total of 21 points.

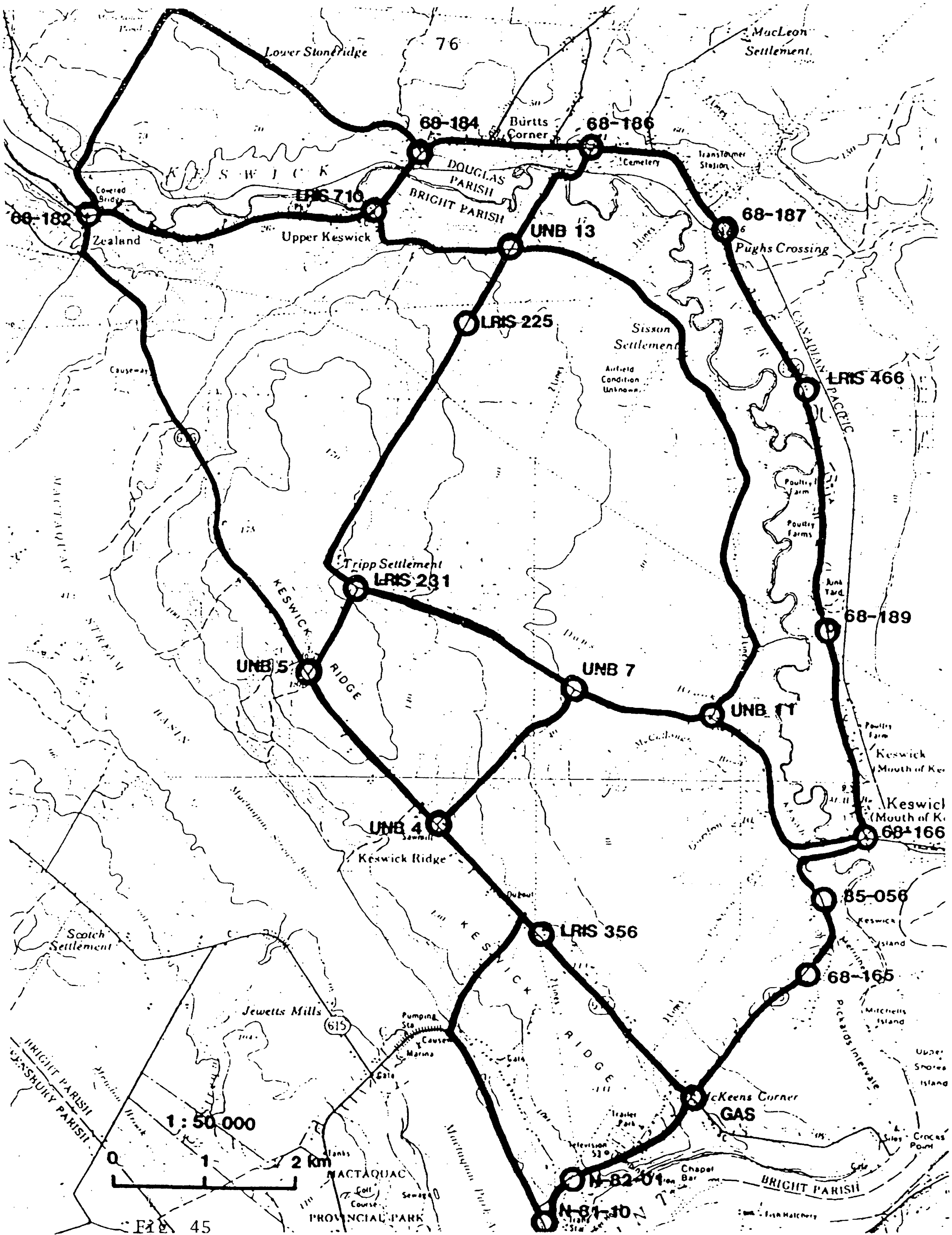

76
Lower Stoneridge
MacLeon Settlement
68-184
Burtts Corner
68-186
Cemetery
Transformer Station
KESWICK
DOUGLAS PARISH
BRIGHT PARISH
LRIS 710
68-182
Covered Bridge
Zealand
Upper Keswick
UNB 13
68-187
Pughs Crossing
LRIS 225
Sisson Settlement
Airfield Condition Unknown
CANADIAN PACIFIC
LRIS 466
Poultry Farm
Poultry Farms
Junk Yard
Causeway
616
Tripp Settlement
LRIS 231
68-189
KESWICK RIDGE
UNB 5
UNB 7
UNB 11
Poultry Farm
Keswick Mouth of Keswick
Keswick (Mouth of K
68-166
UNB 4
Sawmill
Keswick Ridge
85-056
Keswick Island
LRIS 356
Scotch Settlement
68-165
Mitchells Island
Jewetts Mills
615
Pumping Sta
Causeway
Marina
KESWICK RIDGE
Upper Shores Island
McKeens Corner
GAS
Pickards Intervale
BRIGHT PARISH
QUEENSBURY PARISH
1 : 50 000
0
1
2 km
MACTAQUAC
Golf Course
Sewage
PROVINCIAL PARK
Television
Trailer Park
N-82-01
Chapel Bar
BRIGHT PARISH
Crocks Point
N-81-10
Fish Hatchery
Mactaquac Park
MACTAQUAC STREAM BASIN

Fig 45

Initial plans called for measurements of all the lines using two-way leap-frog and two-way reciprocal surveys. However, due to adverse weather conditions in June and part of July and due to financial restrictions, the network was measured only in one direction with each method. This still gave a good sample of a total of over 140 km of survey with 8 control loops.

The leap-frog traversing was made between 4 July and 16 July and the reciprocal survey between 2 August and 28 August. In October, an additional leap-frog survey was made of the traverse: UNB11-UNB13-LRIS225-LRIS231-UNB5-68/182 and geometric levelling of two lines: UNB13 to 68/186 and LRIS710 to 68/184.

Unfortunately, the levelling plans of NB Power and of the GSC group changed and their geometric levelling (August/September) did not overlap with the UNB lines as had been anticipated. The only common lines for a possible comparison were from N-82-01 to 68-165 and from 68-165 to 68-166.

A summary of the results of the July and August measurements at individual set-ups is given in Appendices VII and VIII (available upon request from UNB) for the leap-frog and reciprocal surveys, respectively. Table 8 and Figs. 46 and 47 give a summary of results of the network lines. They are entitled "initial results" in Table 8 because they do not included the additional October surveys, which are discussed later on.

Inclinations along the traversing routes in the networks were generally smaller than 5°. Therefore, according to the theoretical pre-analysis of the feasibility study [Chrzanowski et al., 1985], the overall accuracy of the network results was expected to be 1.5 mm/km or better at the one sigma level of a one-way levelling. This gave a tolerance for differences between the leap-frog and reciprocal surveys to be within $1.5\sqrt{2} \times 1.96\sqrt{L} = 4.2\sqrt{L}$ [mm] and for loop closures $3\sqrt{L}$ [mm] at 95% confidence level except for a few lines on which the inclinations were up to 7° (lines 68-182 to 68-184, UNB5 to

TABLE 8

Network measurements. Summary of initial results

	LEAP-FROG			RECIPROCAL			
Line From/To (km)	Date (No. of setups)	Weather t°C, overcast % (remarks)	ΔH_{LF} [m] (–> <–)	Date (No. of setups)	Weather t°C, overcast % (remarks)	ΔH_R [m] (–> <–)	Diff. $\lvert\Delta H\rvert_{LF} - \lvert\Delta H\rvert_R$ [mm]
68-165/GAS (1.7)	5 July (5)	~31°C, 0%-25%	- 50.7242 (<–)	28 Aug. (6)	~22°C, 90%-50% (breeze, sunny at end)	50.7227 (–>)	+ 1.5
GAS/LRIS356 (1.9)	5 July (6)	~31°C, 10%	- 37.6566 (<–)	15 Aug. (9)	~23°C, 100% (good obs. conditions)	37.6580 (–>)	- 1.4
LRIS/UNB4 (2.1)	5 July (6)	~30°C, 20%	- 10.8675 (<–)	15 Aug. (8)	26°C-28°C, 20%-80% (strong shimmer)	10.8688 (–>)	- 1.3
UNB4/UNB5 (2.1)	5 July (6)	~27°C, 0%	- 54.1619 (<–)	2 Aug. (9)	~24°C, 0% (strong shimmer)	54.1593 (–>)	+ 2.6
UNB5/68-182 (5.7)	10 July (17)	16°C-19°C, 100% (some rain)	- 152.2761 (–>)	9 Aug. (27)	19°C-24°C, 0%-90% (windy, new observes)	- 152.2617 (–>)	+ 14.4
68-182/68-184 (6.1)	12 July (20)	~23°C, 0%-90% (some wind,shimmer)	- 3.7078 (–>)	12 Aug. (27)	~23°C, 0%-60% (strong shimmer)	- 3.7160 (–>)	- 8.2
68-184/68-186 (1.7)	16 July (5)	~24°C, 100%-75% (windy)	3.5966 (<–)	23 Aug. (7)	~22°C, 50%	- 3.6003 (–>)	- 3.7
68-186/68-187 (1.9)	16 July (6)	~22°C, 50%-100%	4.3523 (<–)	26 Aug. (9)	~19°C, 100% (good conditions)	- 4.3512 (–>)	+ 1.1
68-187/LRIS466 (2.0)	15 July (14)	26°C-29°C, 95%-25%	+ 7.5293 (<–)	26 Aug. (7)	~20°C, 100%	0.5831 (–>)	- 6.4
LRIS466/68-189 (2.7)				27 Aug. (9)	~18°C, 100% (rain at end)	- 8.1188 (–>)	
68-189/68-166 (2.2)	15 July (6)	~26°C, 100%	- 1.8622 (<–)	27 Aug. (9)	~18°C, 100% (shower)	1.8607 (–>)	+ 1.5
68-166/85-056 (1.1)	15 July (6)	22°C-25°C, 90%	- 16.1865 (<–)	27 Aug. (4)	~18°C, 100% (showers)	- 2.1340 (–>)	+ 4.1
85-056/68-165 (1.0)				28 Aug. (5)	~20°C, 100% (windy)	18.3164 (–>)	
GAS/N-82-01 (1.9)	15 July (5)	~21°C, 100% (good conditions)	10.7525 (<–)	28 Aug. (8)	~23°C, 25%-50% (windy)	- 10.7530 (–>)	- 0.5
N-82-01/N-81-10 (0.5)	13 July (2)	~22°C, 75%	16.3627 (<–)	28 Aug. (2)	~23°C, 25%-50% (windy, shimmer)	- 16.3611 (–>)	+ 1.6
N-81-10/LRIS356 (4.7)	13 July (14)	~22°C, 0%-50%	- 64.7741 (<–)	13 Aug. (18)	16°C-20°C, 10% (strong shimmer)	64.7724 (–>)	+ 1.7
68-166/UNB11 (2.6)	8 July (7)	~21°C, 100% (showers at start)	14.4184 (–>)	21 Aug.	18°C-25°C, 0%-50% (strong shimmer)	- 14.4177 (<–)	+ 0.7
UNB11/UNB7 (1.5)	8 July (5)	~22°C, 100%-80% (rain start, sun end)	13.2862 (–>)	8 Aug. (9)	~18°C, 100% (rain)	13.2859 (–>)	+ 0.3
UNB7/UNB4 (2.1)	4 July (6)	~24°C, 65%-80%	- 87.7281 (<–)	2 Aug. (10)	~22°C, 0% (strong shimmer)	87.7292 (–>)	- 1.1
UNB7/LRIS231 (2.6)	4 July (8)	~28°C, 25%	96.3559 (–>)	22 Aug. (11)	18°C-24°C, 0%-75% (strong shimmer)	- 96.3594 (<–)	- 3.5
LRIS231/UNB5 (1.1)	12 July (3)	~29°C, 25% (windy)	45.5334 (–>)	22 Aug. (4)	~16°C, 0% (light wind)	- 45.5348 (<–)	- 1.4
LRIS231/LRIS225 (3.2)	8 July (8)	~21°C, 100% (rain at end)	- 55.7043 (–>)	5 Aug. (12)	~29°C, 0% (shimmer)	- 55.6969 (–>)	+ 7.4
LRIS225/UNB13 (1.0)	9 July (3)	~19°C, 100%	- 46.1570 (–>)	5 Aug. (4)	29°C-32°C, 0% (strong shimmer)	- 46.1536 (–>)	+ 3.4
UNB13/LRIS710 (2.1)	10 July (7)	~21°C, 80%-100% (showers)	10.4767 (<–)	14 Aug. (10)	~26°C, 0%-70% (shimmer)	- 10.4765 (–>)	+ 0.2
UNB13/68-186 (1.5)	12 July (6)	14°C-19°C, 0%	- 12.1878 (–>)	14 Aug. (7)	~21°C, 0% (good conditions)	12.1793 (<–)	+ 8.5
LRIS710/68-184 (0.8)	16 July (3)	~26°C, 75%-50% (windy)	- 1.8912 (<–)	23 Aug. (5)	~20°C, 100% (showers)	1.8961 (–>)	- 4.9
LRIS710/68-184 (0.8)	16 July (3)	~26°C, 75%-50% (windy)	- 1.8912 (<–)	23 Aug. (5)	~20°C, 100% (showers)	1.8892 (–>)	+ 2.0
LRIS710/68-182 (3.3)	16 July (9)	~28°C, 25% (windy)	5.5968 (–>)	23 Aug. (13)	17°C-23°C, 0%-10% (strong shimmer)	- 5.5993 (<–)	- 2.5
UNB11/UNB13 (6.7)	9 July (19)	19°C-26°C,100%-10% (sunny, hot at end)	- 7.7807 (<–)	7 Aug. (29)	17°C-29°C, 0% (fog start, shimmer)	- 7.7945 (<–)	- 13.8

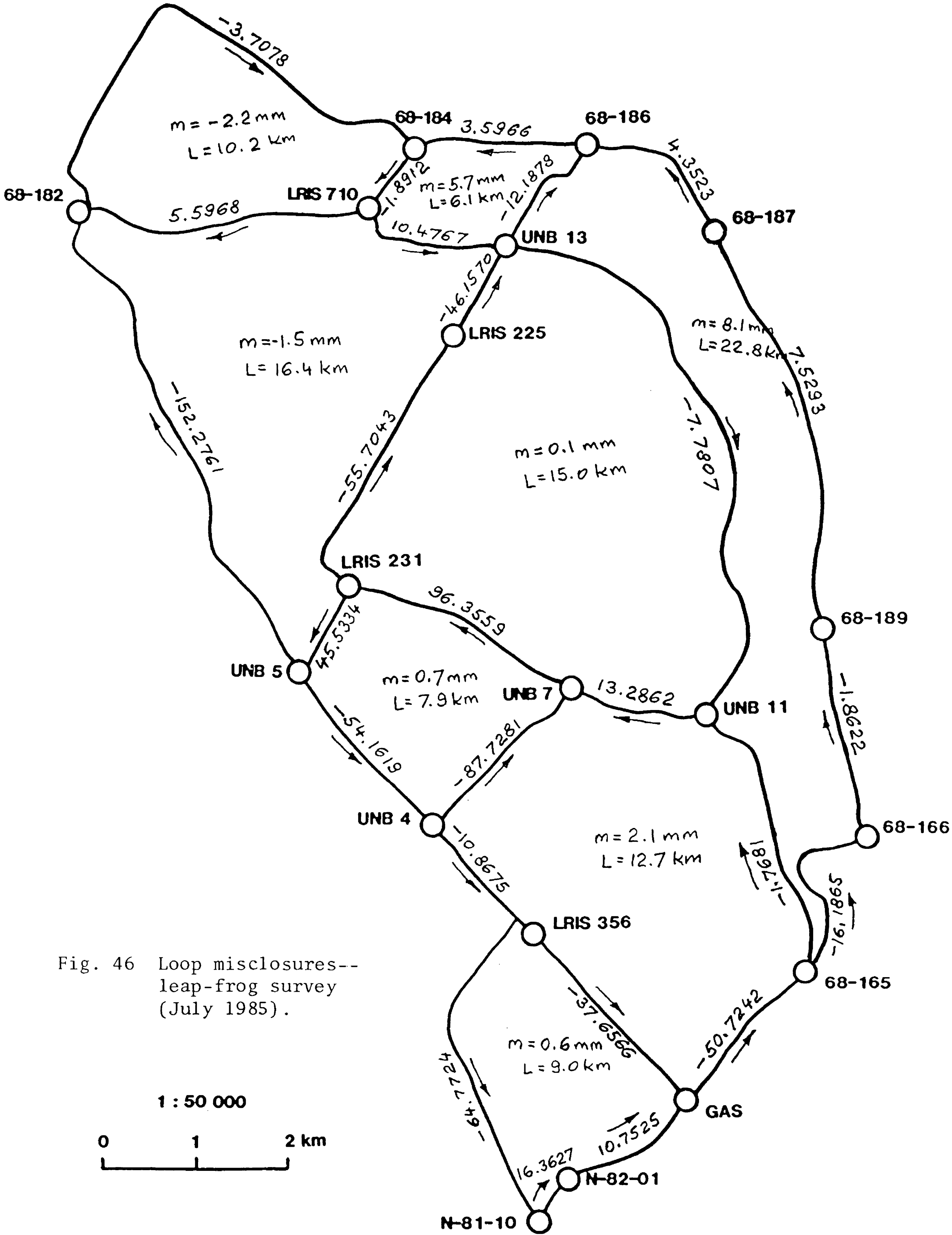


Fig. 46 Loop misclosures--leap-frog survey (July 1985).

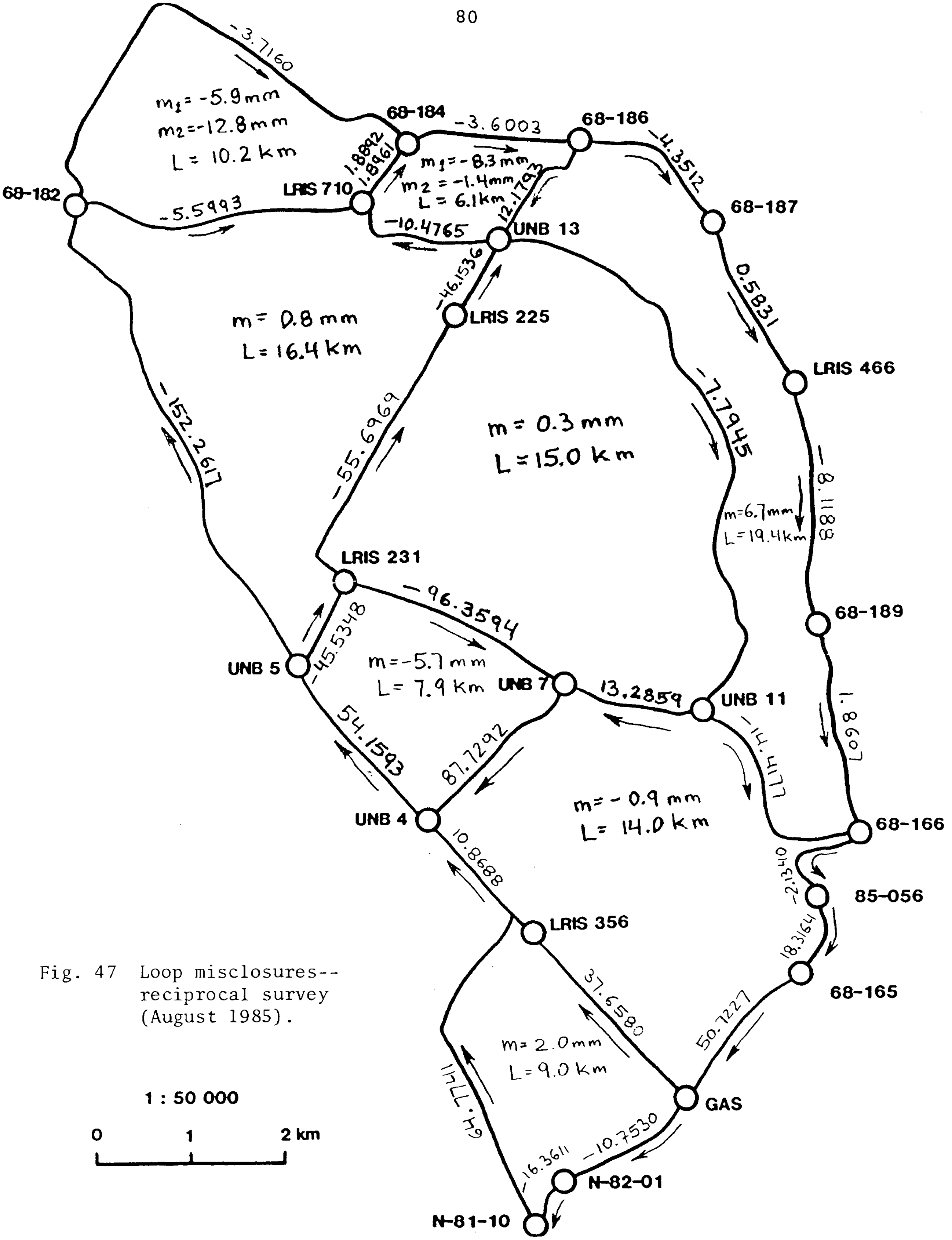


Fig. 47 Loop misclosures--reciprocal survey (August 1985).

68-182, N-81-10 to LRIS356, and UNB11 to UNB13) on which one could expect slightly worse results, about $4.7\sqrt{L}$ [mm] for the differences of the runs and $3.4\sqrt{L}$ for loop closures where L is the length of the surveyed line in kilometres.

5.2 Internal Evaluation of the Leap-Frog Traversing

The survey with only two vehicles did not cause any major delays. The work went smoothly in spite of the fact that some members of the survey crew (not always the same throughout the course of the survey) had no previous experience in trigonometric height traversing and none had any experience in using the computerized and motorized survey system.

Progress of up to 14 km/day was achieved. The initial results, which are shown in Fig. 46, exceeded the expectations.

The loop misclosures gave the estimated standard deviation of the one-way traversing equal to:

$$1.07\sqrt{L} \text{ [mm]} \ .$$

The total closure of the perimeter traverses was 3.2 mm over the distance of 32 km.

The least-squares adjustment of the network with weighting of the lines between bench marks as $p_i = 1/L_i$ gave the estimated standard deviation of one kilometre line: $\hat{\sigma}_0 = 1.0$ mm. One line, UNB13 to 86-186, was rejected at 95% probability (τ_{max} test). This line was later re-measured using geodetic levelling of special order accuracy (result: 12.1794 m) which revealed that, indeed, a comparatively large error of 8.6 mm was made over this short, 1.5 km, line.

After rejecting the trigonometric observation of that line, a new least-squares adjustment of the network was performed resulting in $\hat{\sigma}_0 = 0.5$ mm/km (versus the expected 1.5 mm/km) of one-way traversing. A summary of the results of the final least-squares adjustment is given in Appendix IX (available upon request from UNB).

5.3 Internal Evaluation of Reciprocal Traversing and Comparison with the Leap-Frog Method

The reciprocal surveys did not proceed as smoothly as in the leap-frog method. The following reasons can be given:

- The survey crew had no previous experience in the reciprocal surveys.
- Different people participated throughout the field surveys; some new crew members did not have any experience with the electronic theodolites and microcomputers.
- One of the two tripods was the first prototype, differing in some details from the discussed final design; special care had to be taken to properly fasten its upper parts together; its stability was not as good as that of the second tripod.

The maximum daily progress achieved was about 10 km. The connecting surveys to the bench marks were more time-consuming than had been anticipated. The computer field checks required an inter-station radio transmittance and manual inputting of the values of the one-way observed distances and calculated height differences. This produced considerable delays. A radio telemetry system and distance measurements in both directions would considerably improve the speed of the field procedures. Also both vehicles should have been equipped with electronic odometers.

Since the observing instrument with the heavy tripod occupied the forward station a few minutes longer than in the leap-frog method, a possibility for the sinking effects was greater in the reciprocal method. The stability of the E2 was controlled by the double compensator, but the stability check on the T2000 was less reliable.

The results of the network measurements are given in Fig. 47, and a summary of results at individual set-ups is given in Appendix VIII (available upon request from UNB). Most of the loop misclosures were as small as in the leap-frog surveys. However, the overall standard deviation estimated from the misclosures was worse: 1.56 mm/km for one-way traversing.

The least-squares adjustment gave $\hat{\sigma}_0 = 2.4$ mm/km with one observation (the value of 1.8961 m for the line LRIS710 to 68/184) rejected. After the rejection, the estimated standard deviation improved to $\hat{\sigma}_0 = 1.8$ mm/km. The extract from the least-squares adjustment output is given in Appendix X (available upon request from UNB).

A combined adjustment of all the observed data from both the reicprocal and leap-frog surveys was performed without the previously rejected one observation in each of them. First, equal weights were given to the data of both methods. This led to the estimated $\hat{\sigma}_0 = 1.7$ mm/km (see Appendix XI). A second adjustment was performed differentiating the weights in both methods according to the ratio of their variance factors obtained from individual adjustments, i.e., by a factor of $0.5^2/1.7^2$. This adjustment gave $\hat{\sigma}_0 = 0.6$ mm/km.

So, according to the least-squares estimation, if both methods would be accepted as giving the same accuracy, one could say that in the worst case the accuracy of 1.7 mm/km at the one sigma level of one-way traversing has been achieved. The leap-frog method alone definitely gave better accuracy than that pessimistic value obtained from the combined adjustment.

A direct comparison (Table 8) between the leap-frog and reciprocal surveys gave three lines on which the differences exceeded the tolerance value of 4.2 mm $\sqrt{L}$ at 95% confidence level. These were:

line from UNB11 to UNB13
UNB5 to 68/182
LRIS710 to 68/184, and
UNB13 to 68/186.

The last discrepancy was clarified by the aforementioned geodetic levelling which indicated that there was a significant error in the leap-frog survey on that line. The line from LRIS710 to 68/184 was also resurveyed with geodetic levelling of high precision giving: 1.8927 m versus two results of the reciprocal surveys of 1.8961 m and 1.8892 m; and one result of the leap-frog survey: 1.8912 m. This did not quite resolve which of the two reciprocal surveys was worse. However, the good agreement between geodetic and leap-frog trigonometric surveys indicated that the reciprocal result of 1.8961 m should be rejected. This was in agreement with the above least-squares analysis.

In order to clarify the other two unaccepted results, the aforementioned additional leap-frog traverse was run from UNB11 through UNB13, UNB7 to 68/182 in October. The results are listed in Table 9 together with the results of the July and August surveys for the sake of an easier comparison.

TABLE 9

Repeated Surveys in the Network

Line	L[km]	New Leap-Frog ΔH_N [m] (October)	Old Leap-Frog ΔH_0 [m] (July)	Reciprocal ΔH_R [m] (August)	ΔH_N-ΔH_0 [mm]	ΔH_N-ΔH_R [mm]
UNB11-UNB13	6.7	7.7842	7.7807	7.7945	3.5	10.3
UNB13-LRIS225	1.0	46.1569	46.1570	46.1536	0.1	3.3
LRIS225-LRIS231	3.2	55.6929	55.7043	55.6969	11.4	4.0
LRIS231-UNB5	1.1	45.5359	45.5334	45.5348	2.5	1.1
UNB5-68/182	5.7	152.2682	152.2761	152.2617	7.9	6.5

The large discrepancy of 11.4 mm between the old and new leap-frog results on the LRIS225-LRIS231 line came as a surprise. The July survey of that line was not rejected in either of the least-squares adjustments, and the loop misclosures of the July survey containing that line were very small. Hopefully, the difference is due to a gross error in the new survey. Otherwise, all the optimistic expectations based on the results of the internal accuracy analysis would be jeopardized. Although the meteorological conditions during the October survey were quite different from the conditions in July, it would be extremely pessimistic and rather impossible to blame the 11.4 mm difference on changes of the refraction effects along a comparatively smooth route with a homogeneous surface. Other October surveys differed from the July surveys well within the tolerance of 4.2 mm $\sqrt{L}$.

If the new survey of the line LRIS225 to LRIS231 were rejected as an outlier, this would bring a total of rejections in the leap-frog survey to 2 out of a total of 32 (old and new surveys combined) or, if counting in kilometres of the rejected surveys, 4.7 km over a total of about 85 km which means about 5%. This is a very good score considering that it was a pioneering use of the method.

In reciprocal surveys, in addition to one survey of the LRIS710 to 68/184 line, the surveys of UNB11 to UNB13 and, perhaps, also UNB5 to 68/182 should be rejected which would bring a total of rejections to 3 over a total of 28 measured lines or, in kilometres, 13.2 km over about 70 km, i.e., about 20%. This is rather disappointing.

The rejection of the reciprocal survey of UNB5 to 68/182 is rather subjective because a comparison with the October resurvey does not indicate which of the old surveys was better, the leap-frog or the reciprocal. One should point out that the area of the network may be subjected to ground tilts due to the changeable water level in the nearby reservoir. Therefore, a possibility that relative heights of some of the bench marks changed between the July, August, and October surveys cannot be rejected. Definitely, more meaningful conclusions on the comparison could have been drawn if both reciprocal and leap-frog surveys had been made simultaneously and in both directions each.

5.4 Comparison with Geodetic Levelling

Only two lines of the network could be compared with the GSC survey of 1985. The geometric levelling was performed in both directions according to the first-order field specifications. The results are given in Table 10.

TABLE 10

Comparison with Geodetic Levelling

Line	L [km]	GSC 1985 ΔH_G [m]	Leap-frog Trig. ΔH_T [m]	Reciprocal Trig. ΔH_R [m]	ΔH_G-ΔH_T [mm]	ΔH_G-ΔH_R [mm]
N-82-01 to 68/165	3.6	-39.9636	-39.9717	-39.9697	8.1	6.1
68/166-68/165	2.2	16.1819	16.1865	16.1824	4.6	0.5

In both cases, the reciprocal results gave a better agreement with the geodetic levelling than the leap-frog trigonometric, though the differences within trigonometric heightings were within the given tolerances. The time interval between geodetic and leap-frog trigonometric was longer than between the geodetic and reciprocal surveys. As has already been mentioned above, the question is: How stable are the bench marks near Mactaquac? NB Power suspects some movements of the points. In addition, the influence of the atmospheric refraction in geodetic levelling should be considered, as will be discussed in more detail in Suplement No. 3. The difference in elevation between N-82-01 and 68/165 is about 40 m and the route is along asphalt road. The line was measured twice in 1984 by the UNB group using geodetic levelling of high precision and the results (June and September) were: -39.9624 and -39.9704. At that time, the second survey was rejected as an outlier, but its result agrees very well with the trigonometric values of 1985. In 1984 the same line was also measured with the UNB leap-frog manual system with the DKM2A/DM502 obtaining four results on four different days:

39.9676 m
39.9715 m
39.9734 m
39.9686 m

All the above results have the maximum spread of 5.8 mm which is within the expected value of $4.2\sqrt{L} = 8.0$ mm. However, later in the fall of 1984 the same line was measured with the first prototype of the reciprocal system with the T2000 and E2 obtaining:

39.9668 m, and
39.9643 m

which brought the maximum spread of all the trigonometric surveys to 9.1 mm over the distance of 3.6 km.

Another comparison of the network surveys with geodetic surveys was made by using 1968 GSC results between some of the bench marks (marked 68') which were included in the test network.

The GSC results were taken from published (to 1 mm) listings. Table 11 gives the comparison:

TABLE 11

Line	L[km]	GSC 1968 ΔH_G	UNB Leap-frog 1985 ΔH_{LF}	UNB Reciprocal 1985 ΔH_R	$\lvert\Delta H_G\rvert - \lvert\Delta H_{LF}\rvert$ [mm]	$\lvert\Delta H_G\rvert - \lvert\Delta H_R\rvert$ [mm]
68/166 to 68/189	2.2	-1.860	-1.8622	-1.8607	-2.2	-0.7
68/189 to 68/187	5.0	7.534	7.5293	7.5358	4.7	-1.8
68/187 to 68/186	1.9	4.346	4.3523	4.3512	6.3	5.2
68/186 to 68/184	1.7	3.603	3.5966	3.6003	6.4	2.7
68/184 to 68/182	5.0	3.700	3.7078	3.7160	7.8	16.0
Total:						
68/166 to 68/182	16.3	17.323	17.3238	17.3426	-0.8	19.6

Taking into consideration that the geodetic results are 17 years old and that according to the 1985 GSC re-survey of other 1968 bench marks in that area some of them indicate vertical movements of several millimetres, the results are very good, except the last two comparisons of the reciprocal trigonometric traversing. The last two values include the line 68/189 to 68/182 on which the leap-frog and reciprocal surveys differ by 8.2 mm (Table 8) over a distance of 6.1 km. Though the difference is within the accepted tolerance (4.2 $\sqrt{6.1}$ = 10.3 mm), the better agreement of the leap-frog trigonometric result with the 1968 geodetic survey places the reciprocal result again in an inferior position.

6. SUMMARY OF RESULTS AND CONCLUSIONS

Generally, all the tasks of the project as set out in section 1.2 have been accomplished and even exceeded. The amount of field survey in the actual implementation of the trigonometric height traversing in the network, with 70 km of traverse lines, has been greater than initially proposed in the contract agreement.

Due to financial limitations, more tests were made with the leap-frog method than with the reciprocal traversing because only one electronic theodolite was continuously available, while the second one could be rented only for short periods of time. This was not considered as a problem in drawing conclusions regarding the achievable accuracy of height traversing because, theoretically, the reciprocal method should give better accuracy if the lengths of sight would be compatible with those of the leap-frog method. Therefore, an assumption was made that if a good understanding of propagation of errors in the leap-frog method were achieved, then the evaluation of the reciprocal surveys could also be made even with fewer field results.

Even though the number of performed field tests exceeded the initial proposal, it was mainly a question of what would be the maximum achievable accuracy of trigonometric height traversing in its implementation to levelling of high precision which cannot be fully answered.

The field tests performed at UNB allow for drawing only the following conclusions regarding the accuracy:

i) The standard deviation of vertical angle measurements with the electronic theodolites, Wild T2000 and Kern E3, is 0.5" if the angles are measured in four sets in two positions of the telescope each to properly designed targets at distances of about 200 m.

ii) Coefficients, k, of atmospheric refraction reach values of -10, and even larger, at 0.5 m height above typical ground surfaces, and reach the value of -1, or even larger, at 2.5 m above the ground. The model of the vertical distribution of temperature and, related to it, the values of k are not adequately known and, therefore, any application of refraction corrections to trigonometric heights is not reliable.

iii) In typical survey conditions along roads and highways, the maximum error of trigonometric height traversing at individual set ups with $s = 200$ m in the leap-frog method may reach 4 mm with an average value of about 1.5 mm if the foreward and backward lines of sight pass over different types of ground surface but not lower than about 1.5 m above the ground. This error may be considerably reduced in actual surveys if a proper reconnaissance would be made to avoid the extreme conditions of the different types of ground surfaces. This type of error would be expected to at least partially cancel out over long traverses. However, no answer can be made at this stage as to how to actually model the propagation of refraction errors in long traverses.

The above conclusion (i) on the accuracy of angle measurements sets a practical limit on the achievable accuracy of trigonometric height traversing to 0.77 mm/km, 1.1 mm/km, and 1.3 mm/km when using lengths of sight of 100 m, 200 m, and 300 m, respectively, and when disregarding all other sources of errors, such as errors of distance measurements which become very critical in hilly and mountainous terrain, errors due to the atmospheric refraction, and others, which are discussed in Supplement No. 1.

Any higher accuracy of angle measurements would require many more sets of observations which would make the trigonometric method uneconomical.

The above limit of the accuracy has been contradicted by the results of the network measurements with the leap-frog method which gave the estimated standard deviation of 0.5 mm/km of one-way traversing with most of the lengths of sight between 150 m and 200 m. The total a priori estimated accuracy for the given field conditions was 1.5 mm/km. The higher than expected accuracy obtained should be treated with caution and should not lead to over optimistic conclusions. It points out that in trigonometric height traversing the propagation of errors does not necessarily follow the well established law of propagation of variances. Perhaps a large correlation between individual set ups exists which has not yet been identified. This could be an explanation for the sudden large discrepancies between some repeated surveys. Unfortunately, the small number of repeated surveys and small number of lines on which the trigonometric levelling could be compared with the conventional geodetic levelling do not enable clarification of how reliable is the higher accuracy obtained from the least-squares estimation of the leap-frog height traversing.

The obtained accuracy of 1.8 mm/km, as estimated for the reciprocal network traversing, is statistically equal to the a priori estimation of 1.5 mm/km for the given degrees of freedom. Therefore, even though the value looks much worse than the result obtained from the leap-frog method, it may be more reliable. Some large discrepancies were obviously produced by some gross errors which, perhaps, arose from the inexperience of the survey crew. Again, the one-way traversings have not supplied as much material for the full evaluation of the reciprocal surveys as one would have wish.

Summarizing, one can perhaps say that despite the extremely high accuracy estimated from the leap-frog network measurements, which could lead some other investigators to over optimistic conclusions, the real accuracy of trigonometric height traversing at the one sigma level is somewhere between 1 mm/km and 2 mm/km when the described field procedures are employed.

Many more surveys will definitely have to be done to derive a realistic and more definite error model for trigonometric heighting. The knowledge about the actual error propagation in this type of survey is practically nil. This should not be taken by the readers of this report as a criticism of the work done within this small project. One should remember that in conventional geodetic levelling, which has been a subject of immense investigation by hundreds of researchers for over one hundred years, not all the questions on modelling of errors and their propagation have already been answered. The research on trigonometric height traversing has just begun. The claims made by some authors of extreme accuracies obtained at fantastic speeds of several kilometres per hour should be treated with a great deal of caution.

The experience gained from this project indicates that, in the surveys of desired high accuracy, the speed of trigonometric height traversing is in the realm of 12 km to 18 km per working day with properly designed equipment. Higher speeds could be attained only at a cost of lower accuracy.

Detailed conclusions and recommendations regarding the design of the equipment, field procedures, and desired improvements in them were discussed, or at least mentioned, in the appropriate sections of the main text, and they are not repeated here.

Definitely, trigonometric height traversing is a logical replacement for conventional geometric levelling of high precision. However, the full answer to the actual accuracy and error modelling will not come until the method is implemented into production surveys and hundreds of kilometres of actual field observations become available for analysis.

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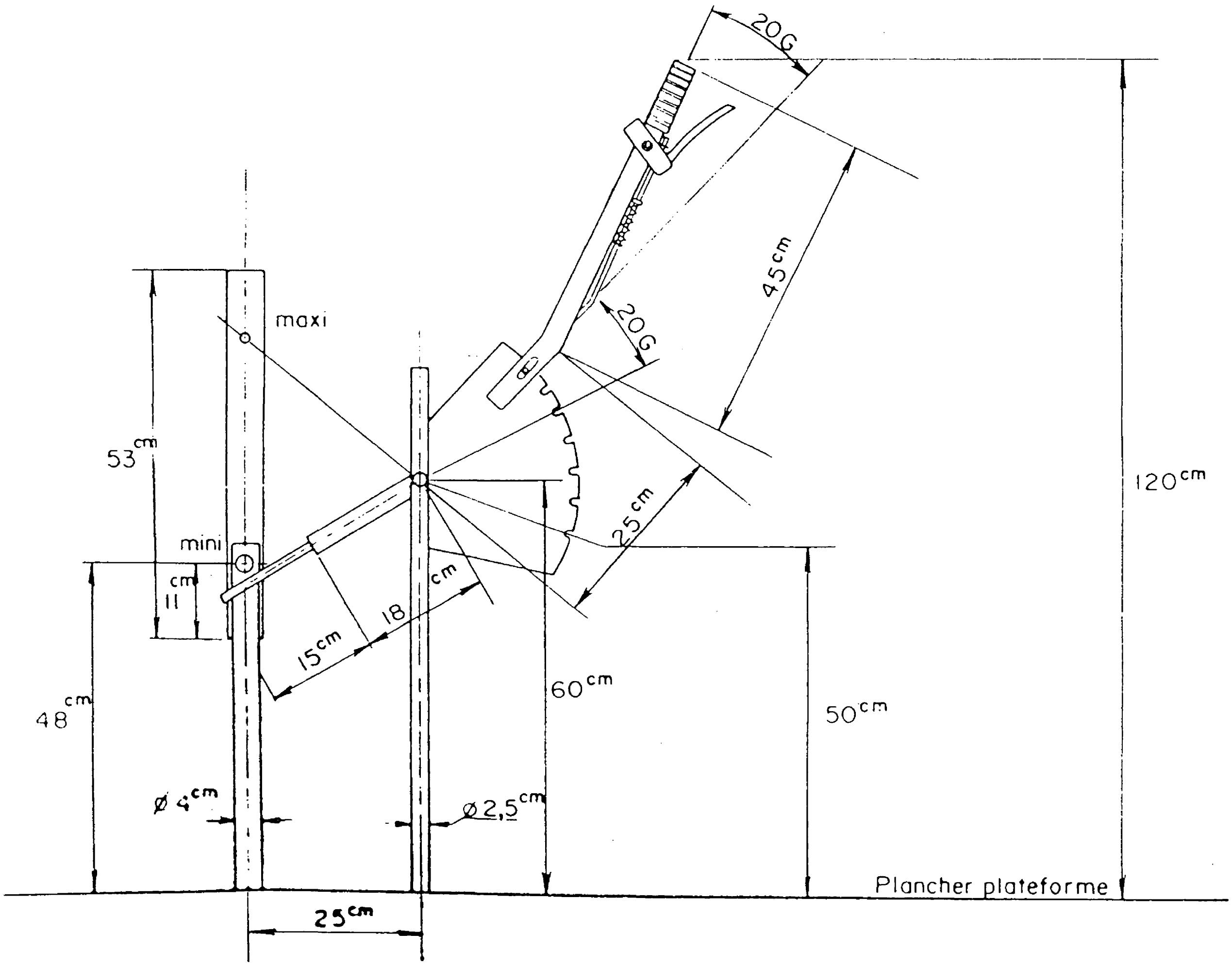
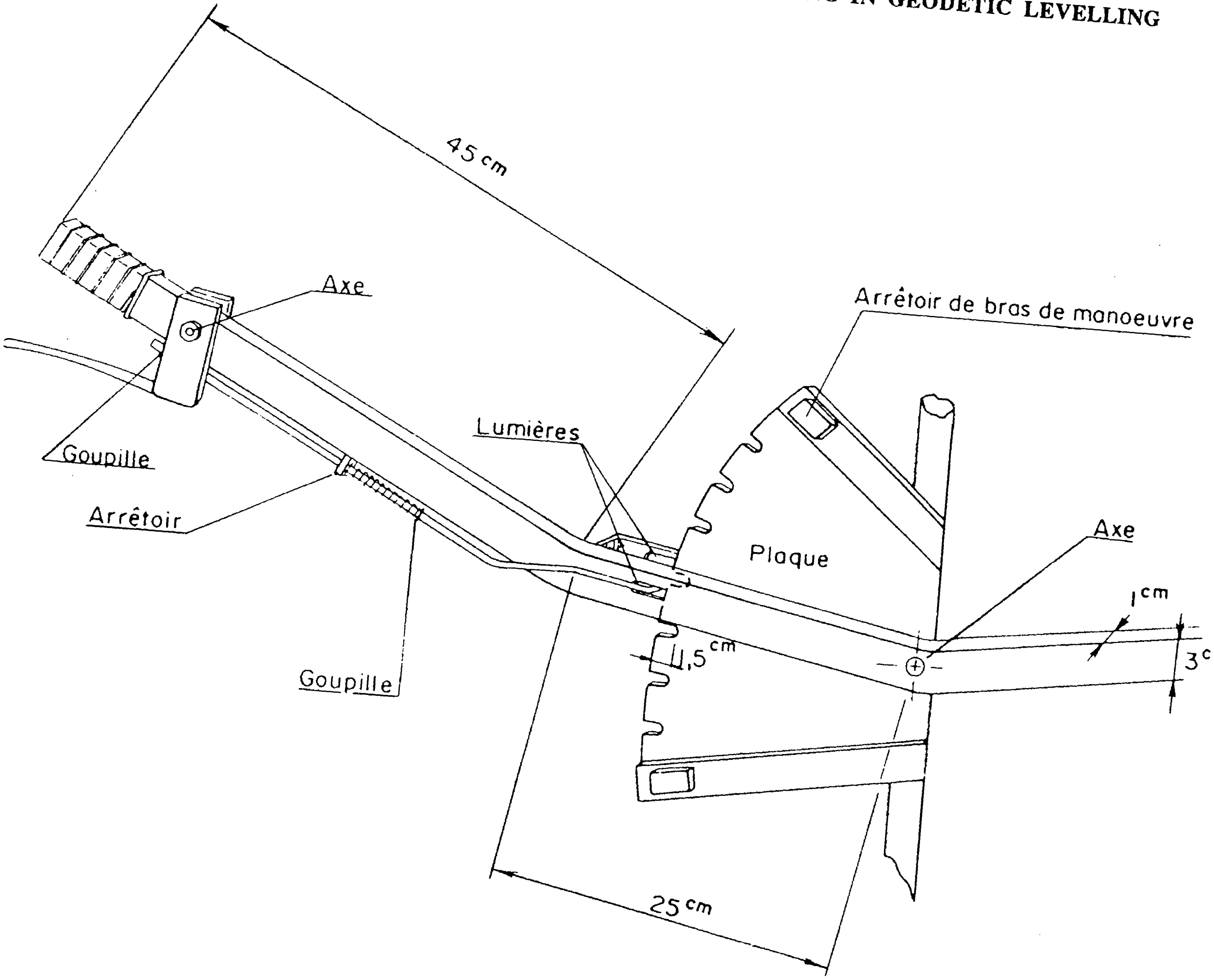
APPENDIX I

TECHNICAL DETAILS OF THE TRIPOD AND LIFTING MECHANISM

Mechanism for lifting the tripod
as designed by IGN, Paris.

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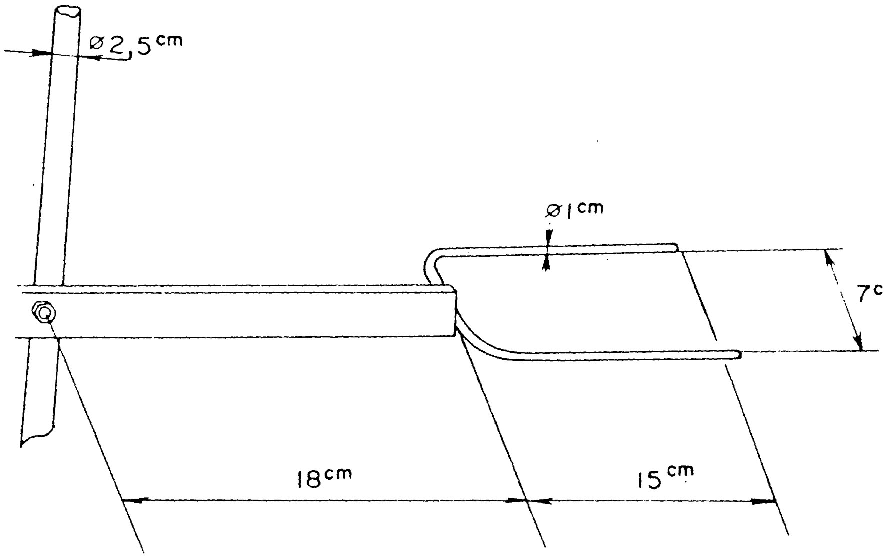
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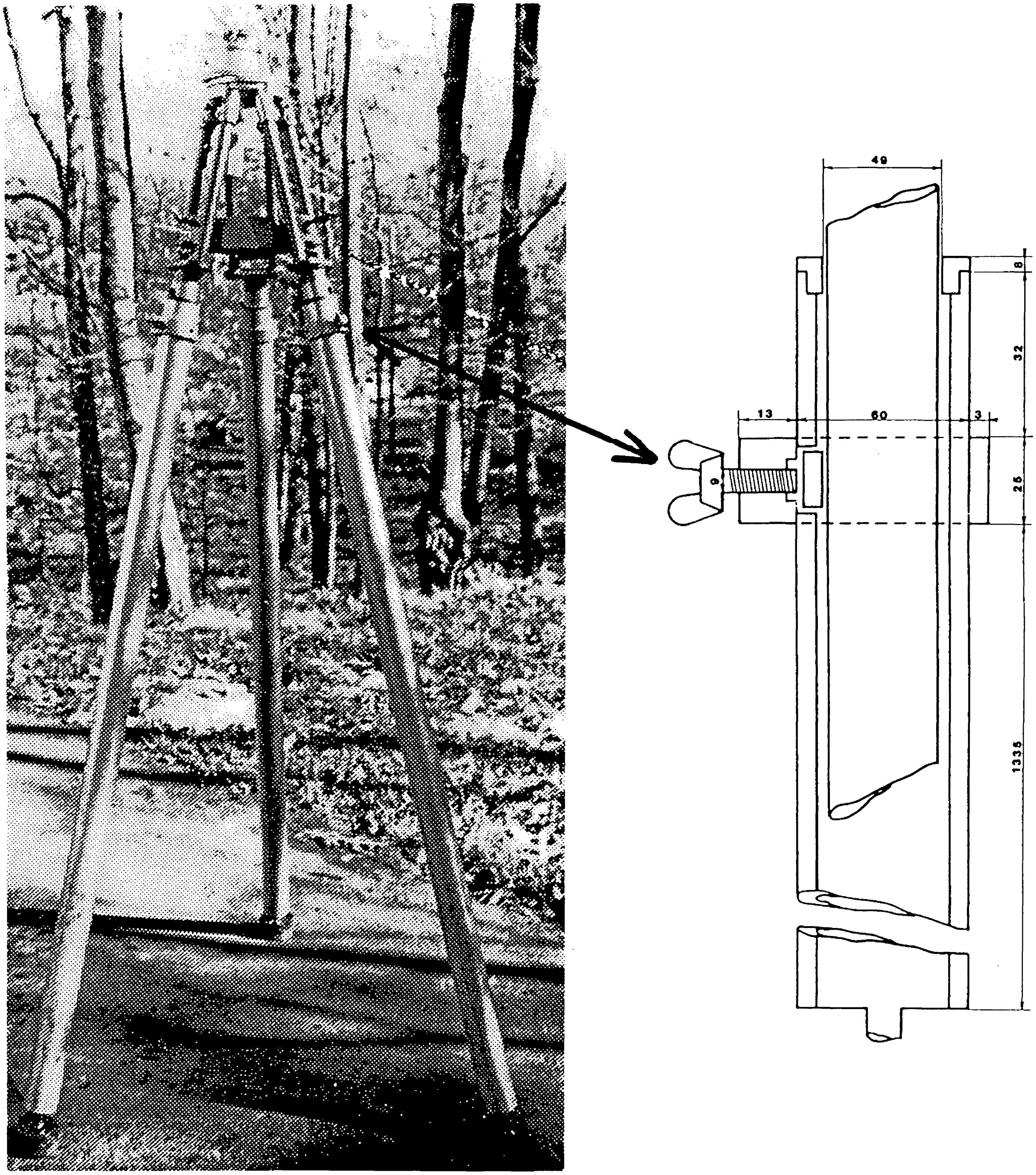
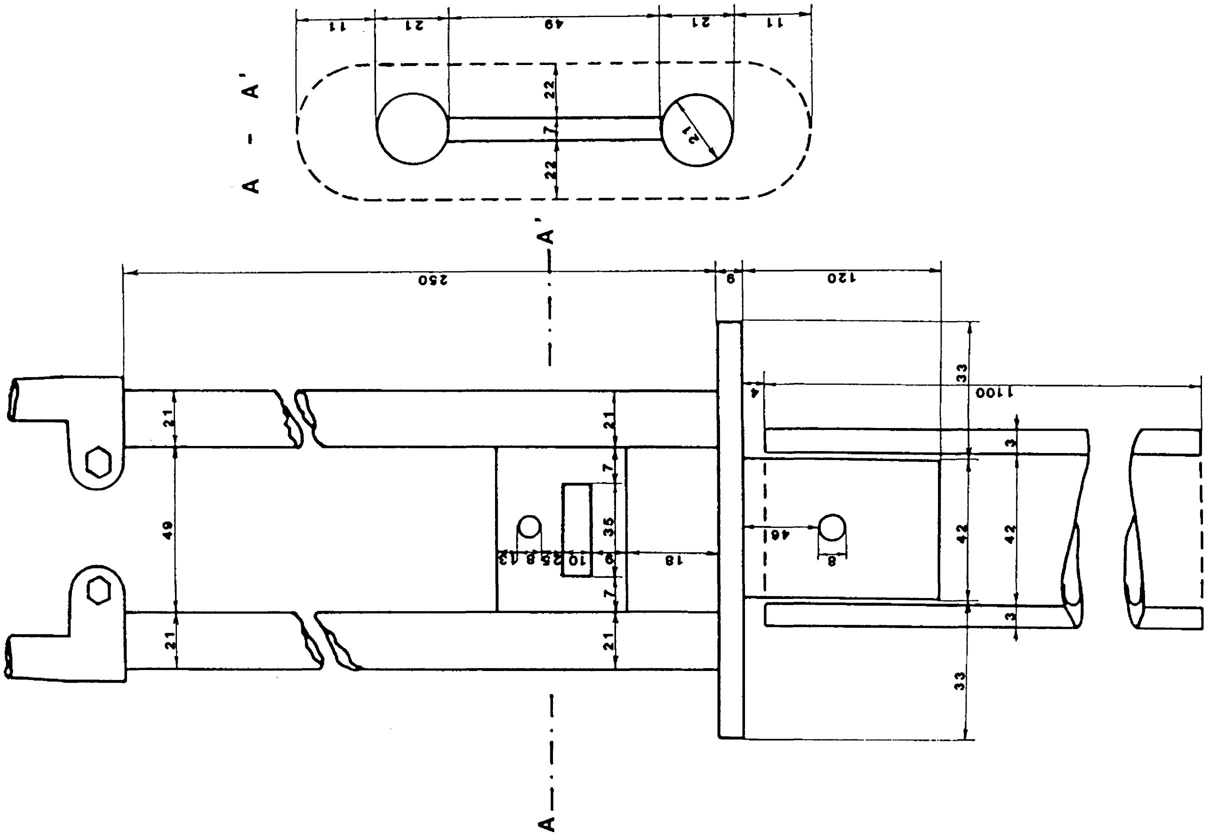
Tripod for trigonometric height traversing (UNB design).

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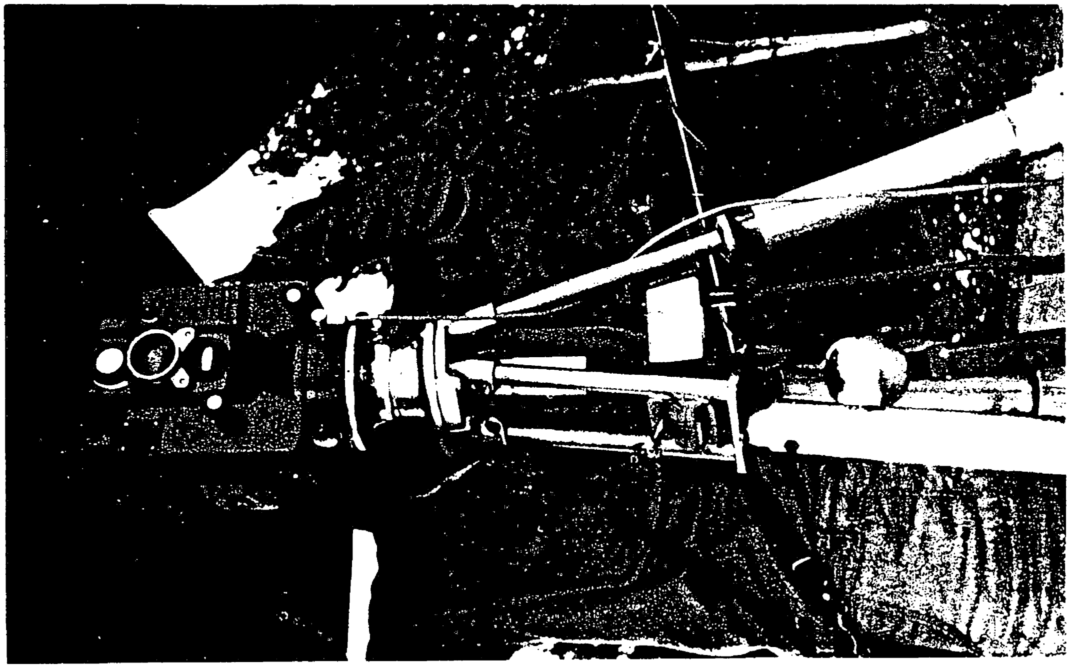

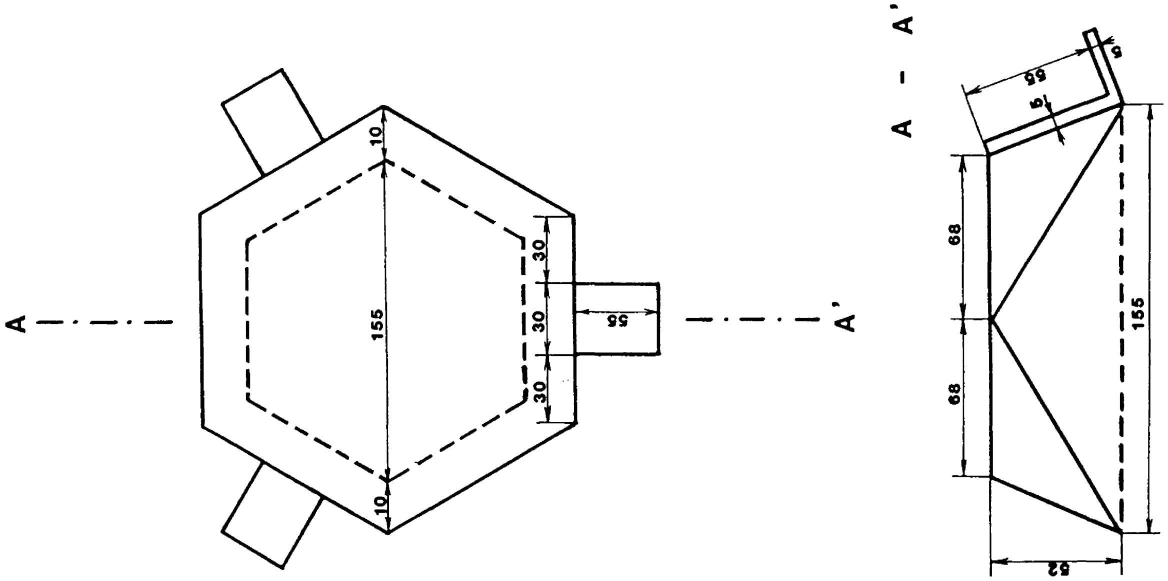
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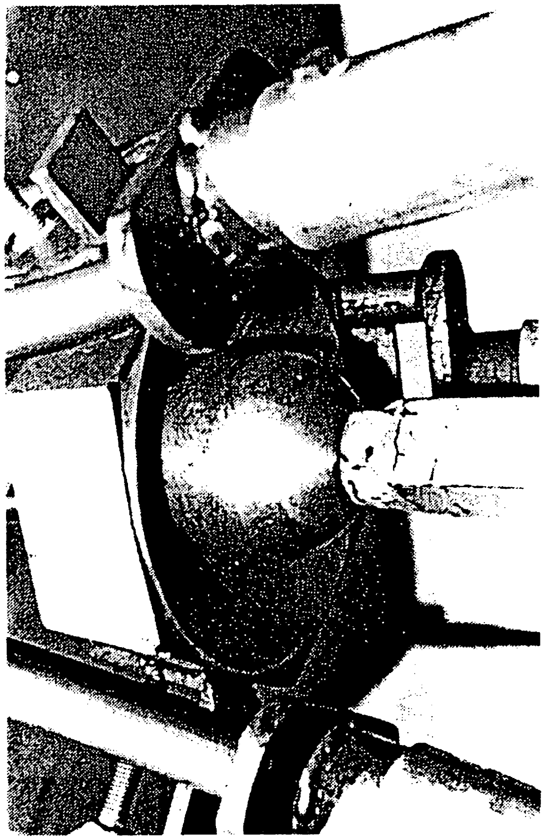

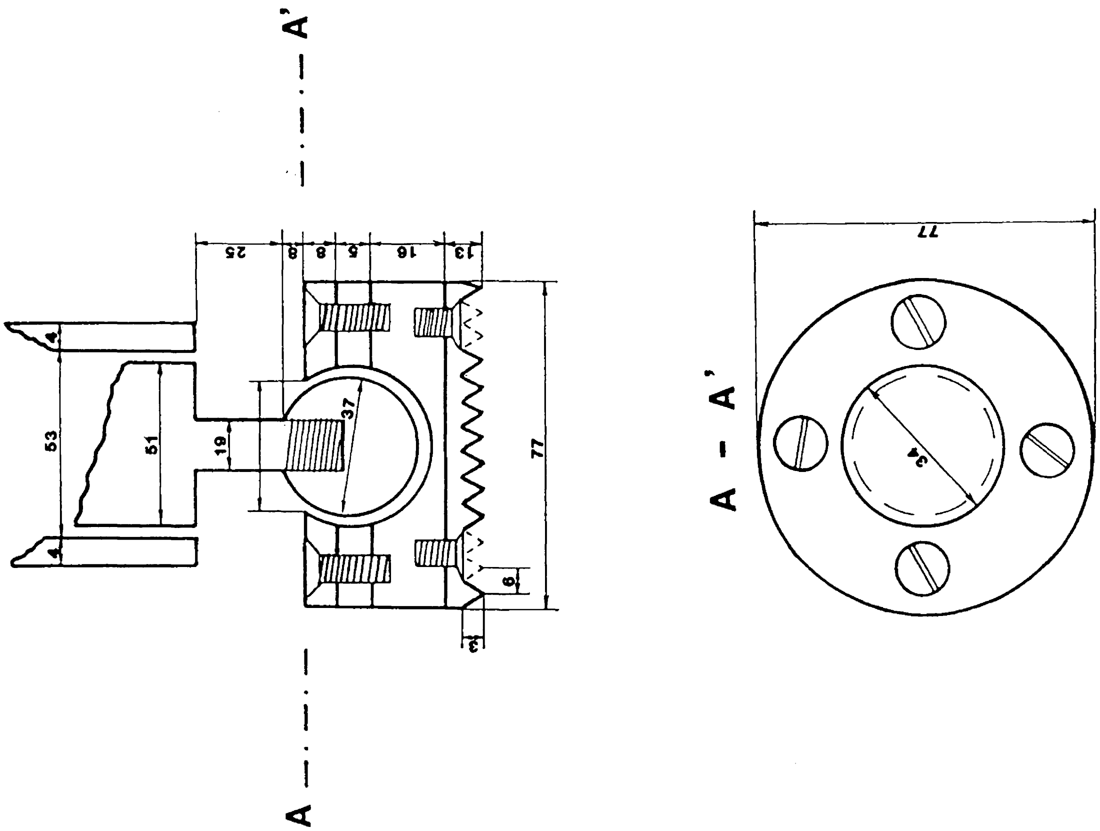
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34