SECONDARY PHASELAG COMPUTATION

T. B. MAHMOOD

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PREFACE

SECONDARY PHASELAG COMPUTATION

Tuan Baharom Mahmood

Department of Surveying Engineering
University of New Brunswick
P.O. Box 4400
Fredericton, N.B.
Canada
E3B 5A3

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PREFACE

The technical report was originally issued in September, 1986, in partial fulfilment of the requirements for the degree of Master of Engineering in Surveying Engineering. This work was supported in part by a Strategic Grant entitled *Marine Geodesy Applications,* awarded by the Natural Sciences and Engineering Research Council of Canada to David Wells.

ABSTRACT

The success of using electronic positioning systems depends on a reasonably precise knowledge of the speed of propagation of the electromagnetic wave. In those systems utilizing low frequency radio waves, the ground wave becomes very dominant and therefore the speed of propagation is affected by the electrical characteristic of the ground path. For a homogeneous smooth earth model, this characteristic is mainly determined by the conductivity and dielectric constant (permittivity) of the ground, both of which give rise to the complicated problem of secondary phaselag. Using two existing programs developed by Brunavs and Gray, based on Johler's model to compute the secondary phaselag, a study on the method of computation and the behaviour of secondary phaselag with distance, conductivity and permittivity at 100KHz frequency is carried out here. The results of this study are then used to modify these programs for better efficiency.

The above method of computation has been found to be very complicated and time consuming. Brunavs had developed an approximate formula using coefficients to compute total phaselag at 100KHz frequency. However, the range of conductivity and permittivity in his tabulated coeficients is limited. This report expands Brunavs' work on the approximate formula so that it can be used for a wider range of conductivity and permittivity. An attempt is also made to fit the coefficients with another approximate formula so that interpolation can be made easily. Finally the results of this extension work are used to develop a commputer program to compute total phaselag at 100KHz frequency using the approximate formula over a conductivity range of 0.00001 to 5.5 siemens/metre and permittivity range of 3.0 to 80.0 esu respectively, which in most cases agree with the results using the Johler model to within 5.0m, for distances of 2 to 2000Km.

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This report has been supervised by Professor David E. Wells. I am very grateful to him for his patience and constant guidence throughout the course of this study. Without his support, this report would have not been successful.

In addition, I would like to thank Mr Paul Brunavs and Dave Gray of the Canadian Hydrographic Service for their generousity in providing their computer programs which form the major tool of this report, Mr See Hean Quek, Mr Iskandariah Roslan and Mr Ahmad Nawawi for their assistance in preparing this report, faculty, staff and fellow graduates at UNB for their continued assistance and encouragement.

I also would like to thank the University of Technology Malaysia for giving me the opportunity to further my education at UNB and providing the financial support throughout my stay here.

Finally, I would like to dedicate this report to my wife, Faridah and my children, Hanif, Hasif, Salina and Hafiz. Their patience and understanding have been the source of inspiration.

Chapter 1

INTRODUCTION

The development of electronic positioning system (EPS) was initiated during the second world war with the invention of radar (Radio Detection And Ranging) for direct ranging of passive targets using line of position (LOP) derived from signals transmitted from fixed ground station (transmitter). In this system, the range is obtained by measuring against an accurate built-in time base, the transit times of microwave signal pulse from the transmitting antenna to a passive reflector (target) and back. After the war, Electronic Positioning System was further developed with the development of atomic clocks, the semiconductor and later by micro-miniature circuitry. These developments have contributed to massive advances in accuracy, reliability and, of significant importance to surveyors and hydrographers, portability.

With the advancement of Electronic Positioning Systems, the use of visual methods (sextant and celestial observations) have been abandoned over the past 30 years and become secondary methods to be used only when electronic method proves impractical. However, the success of using Electronic Positioning Systems depends on a reasonably precise knowledge of the speed of propagation of electromagnetic wave. The speed is a value dependent upon the environmental factors and the effect of wave path on differing terrain, the effect of which varies with the carrier frequency of the signal in use.

1.1 FREQUENCY SPECTRUM OF RADIO WAVES.

The total spectrum of electromagnetic waves in electronic and electro-optical distance measurement systems ranges from wavelength of 5×10^{-7}m to 3×10^{4} m. Within the range of radio waves, the frequency spectrum is divided into bands in order to distinguish the different orders of frequencies used for different purposes. The classification commonly used in radio waves is given in Table1.1. The last two bands are combined together and usually called microwave band with SHF and EHF bands classified as S-band and X-band respectively. Radio waves are divided into two main groups called long wave and short wave. The former consists of LF and MF, and the latter VHF, UHF, SHF and EHF.

With the exception of satallite navigation and positioning systems, all offshore navigation or positioning systems use long waves. Within long waves group, there are certain frequency 'windows' reserved for navigation signals. They are 10KHz (Omega), 100KHz (Loran and Decca) and 2MHz (Hifix,Raydist and Argo). Shore or inshore positioning systems use either 400MHz (UHF systems) or 5GHz (microwave systems) frequency windows.

CLASSIFICATION	SYMBOL	FREQUENCY	WAVELENGTH(m)
Very Low Frequency	VLF	10-30 KHz	30000-10000
Low Frequency	LF	30-300 KHz	10000-1000
Medium Frequency	MF	300-3000 KHz	1000-100
High Frequency	HF	3-30 MHz	100-10
Very High Frequency	VHF	30-300 MHz	10-1.0
Ultra High Frequency	UHF	300-3000 MHz	1.0-0.1
Super High Frequency	SHF	3-30 GHz	0.1-0.01
Extremely High Frequency	EHF	30- GHz	0.01-

Table 1.1 Radio Frequency Spectrum and Classification.

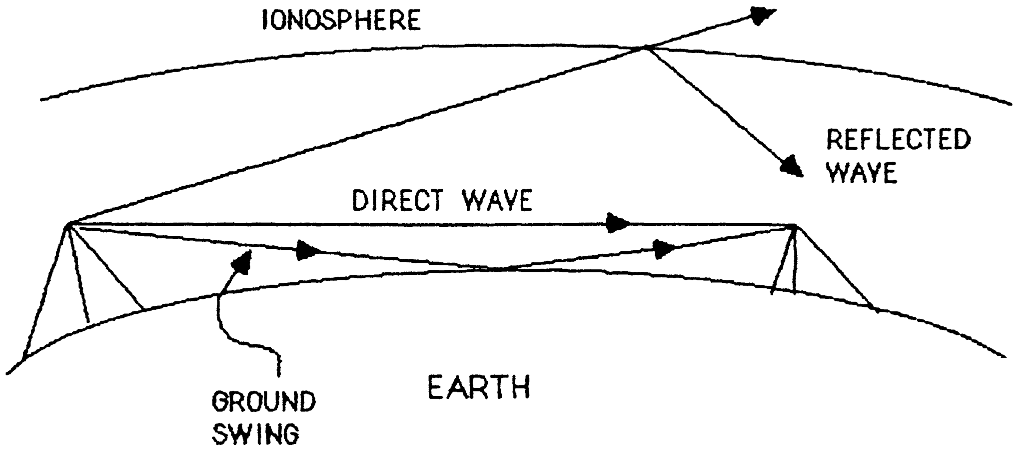


Figure 1.1:Short Waves Propagation

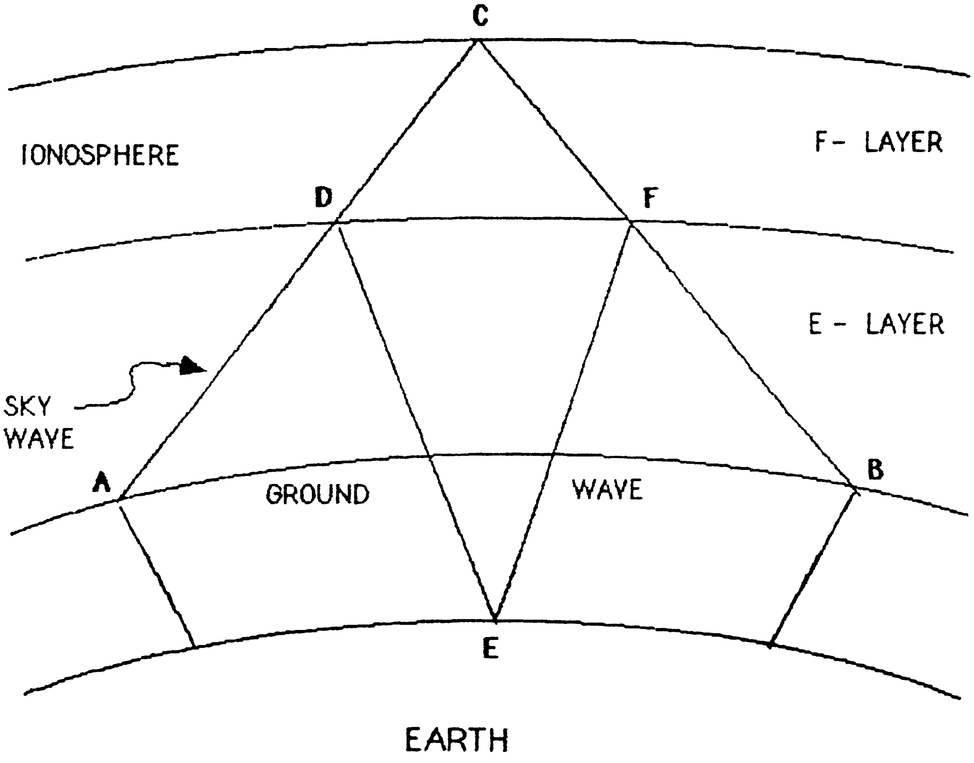


Figure 1.2 Long Waves Propagation

1.2 PROPAGATION OF RADIO WAVES

Radio waves may be propagated through or along the surface of the earth, through the atmosphere or by reflection or scattering from natural or artificial reflectors depending on the type of waves.

a) Short Waves - Short waves generally propagate along a line that is slightly bent and require line-of-sight condition. For this reason the measurement range is restricted to a relatively short one. The propagation is affected by the property of the atmosphere, that is, the vacuum velocityof light of 299 792 458 metre / sec (recommended by IUGG, 1975) is reduced in the atmosphere due to the density variation in the atmosphere. The velocity in the atmosphere is computed using the refractive index which indicates the dielectric properties of the atmosphere (Section2.3). Figure 1.1 illustrates the propagation of short waves. The presence of ground swing or sometimes called ground reflected wave, as shown in the figure should be removed in order to obtain satisfactory results.

b) Long Waves - Unlike short waves, antenas used for long waves systems are usually omnidirectional. For this reason the propagation is divided into two components; surface wave AB, normally called ground wave, and sky wave ACB as shown in Figure 1.2. Sky wave can be used for long range navigation. It is, however, not very stable due to the variation in the height of reflecting layers in the ionosphere. For shorter distances, the more stable ground wave is used in most systems utilizing low and medium frequencies. Ground wave not only travels along the surface of the earth but penetrates into the ground to some extent. Since the conductivity and dielectric constant (permittivity) of the ground vary considerably from those of the atmosphere, correction due to this effect must be considered (Sections1.4 and 2.4).

1.3 RADIO WAVES TRANSMISSIONS

Low and medium frequency waves may be transmitted by either of the following methods :

a) Pulse Type - In this case, the distance D is basically determined by observing the time t needed for pulses to travel from the transmitter to the target and back. Using the average velocity c of the electromagnetic propagation, the distance is computed simply by using the following formula :

$$D = ct / 2$$

b) Continuous Wave (CW) - In this case,the position of a point with respect to two known stations is basically determined by measuring the phase difference between signals transmitted by the two stations .

The phase comparison used in CW ground wave transmission may be contaminated by sky wave. Pulse type transmission is not affected by sky wave and therefore is very stable. Loran-C is an example of a system using pulse transmission. CW transmission is used in the Decca system.

1.4 PHASELAGS

Phaselags are the difference in phase between the real wave and a reference wave travelling at a specified constant velocity. This reference velocity can be the vacuum speed of light (299 792 458 metre / sec). In the case of the ground wave, the real wave travels through the atmosphere, and over the earth's surface of land and water, both of which have a retardation effect. The two effects vary considerably and they are termed as :

a) Primary phaselag - due to the atmosphere (Section 2.3)

b) Secondary phaselag - due to the ground (Section 2.4)

1.5 DEVELOPMENT OF SECONDARY PHASELAG COMPUTATIONS

Untill the end of 19th Century, electromagnetic waves were thought to propagate mainly in straight lines. The demonstration by Marconi in1901 that it was possible to transmit radio signals across the Atlantic (Smith-Rose,1957) made it necessary to study the propagation of electromagnetic waves along the surface of the earth. Since then, the problem of ground wave propagation over a homogeneous smooth earth has been researched under two different theories; plane earth and spherical earth.

Under the plane earth theory, the problem was first solved by Sommerfield in1909 and Newton in1937 presented this solution in a suitable form for computation. MacDonald in1903 was the first to present the solution under the spherical earth theory. His poorly convergent series was later converted into a highly convergent one by Watson in1918. This series was later modified by Van der Pol and Brammer in1939. Further researches were then carried out not only for a smooth homogeneous earth but also for the inhomogeneous case as well as the effect of terrain by authors like Millington in1949, Pressy in1953 and Hufford in1952 (see Samaddar, 1979 for further detail).

Most of the authors above only considered the amplitude of the ground waves in their computations. Johler and co-workers (1956) was the first to present the computation of the phase of low frequency ground wave based on the mathematical models presented by their predecessors. This work, which is of the intrest to hydrographers, has been used to compute the corrections to the measured distances in low frequency positioning and navigation systems.

Brunavs and Wells (1971) and again Brunavs (1973) confirmed the reliability of this computation using Decca Lambda. Brunavs (1977) presented two approximate formulas to replace the intricate Johler's model to compute the total (secondary + primary) phaselag using a number of coefficients.

1.6 OBJECTIVES AND SCOPE

The main objective of this report is to extend the work by Paul Brunavs (1977) on the approximate formulas where only a single value for land permittivity (15 esu) was considered. To support this objective, the followings are carried out :

i. The procedure of computing the secondary phaselag suitable for our purpose based on Johler's models is presented (Chapter 3).

ii. Two computer programs developed by Paul Brunavs and David Gray of the Canadian Hydrographic Service and modifications done to them are described and documented (Section 4.1.4)

iii. Secondary phaselag can be computed using plane and spherical earth theories. The validity of one over the other is examined (Section 4.1.1)

iv. τ_s represents one of the most important parameters in the computation of secondary phaselag using spherical earth theory (Section 3.1.1). It can be computed by two methods. Situations under which these methods are used are examined (Section 4.1.2).

v. The variation of secondary phaselag with input parameters: distance, conductivity and permittivity is studied (Section 4.2)

vi. The C coefficients for a wide range of conductivity and permittivity values, which are typically found in the field, are determined and tabulated (Chapter 5).

vii. An attempt is made to fit the C coefficients into a simple formula to facilitate interpolation (Section 6.1)

vii. Finally, a computer program is developed to compute the100KHz total phaselag using the results obtained for the approximate formula (Section 6.2).

To arrive at the above objectives, the computations are carried out for 100KHz frequency assuming that the earth is smooth and homogeneous. All formulas involved in Chapter 3 are accepted from Johler (1956) without further derivation and explaination. Distance, conductivity and permittivity are the main input parameters in the computation of secondary phaselag. However, methods of determination and accuracies of these input parameters are not within the scope of this report.

Chapter 2

GROUND WAVE PROPAGATION

2.1 COMPONENTS OF GROUND WAVE

As mentioned before, long waves may propagate as sky wave or ground wave. Ground wave is generally divided into three components (Fig. 2.1):

i) surface wave

ii) direct wave

iii) ground reflected wave.

Close to the surface, direct and ground reflected waves cancel each other leaving only the surface wave as the principle means of propagation. Surface wave is usually referred to as ground wave.

2.2 PENETRATION OF GROUND WAVE.

The earth acts like a capacitor and resistor in parallel in carrying current induced by the ground wave. This current is attenuated with depth into the earth. The effective depth of penetration is a function of conductivity and frequency; it decreases with frequency and conductivity (Fig. 2.2,Bigelow,1963). It is usually defined as the depth at which the intensity of the radiation has fallen to 1/e of its value at the surface (Gray,1975); i.e.

$$X_0 = (2 / \omega^2\mu_0\varepsilon[\{1 + \sigma^2/\omega^2\varepsilon^2\}^{1/2} - 1])^{1/2}$$

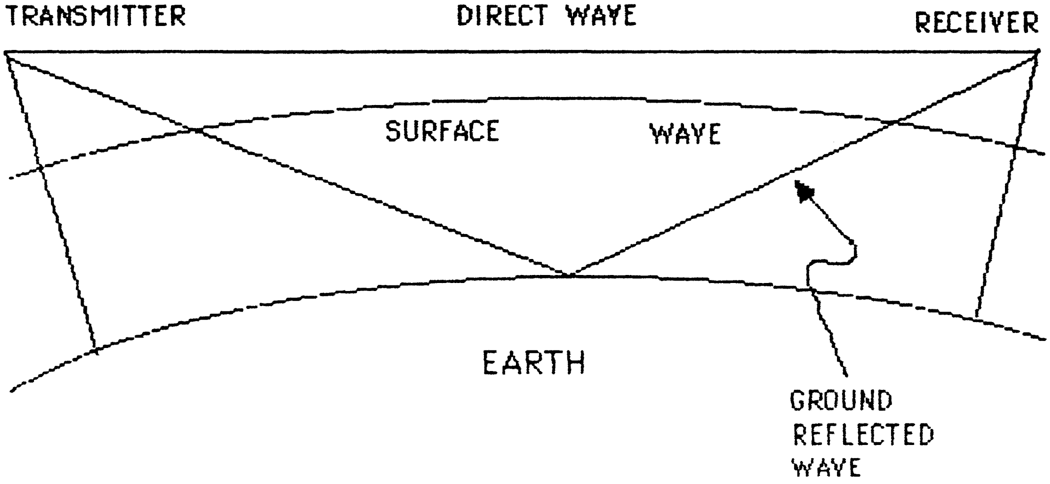


Figure 2.1: Ground Wave Propagation

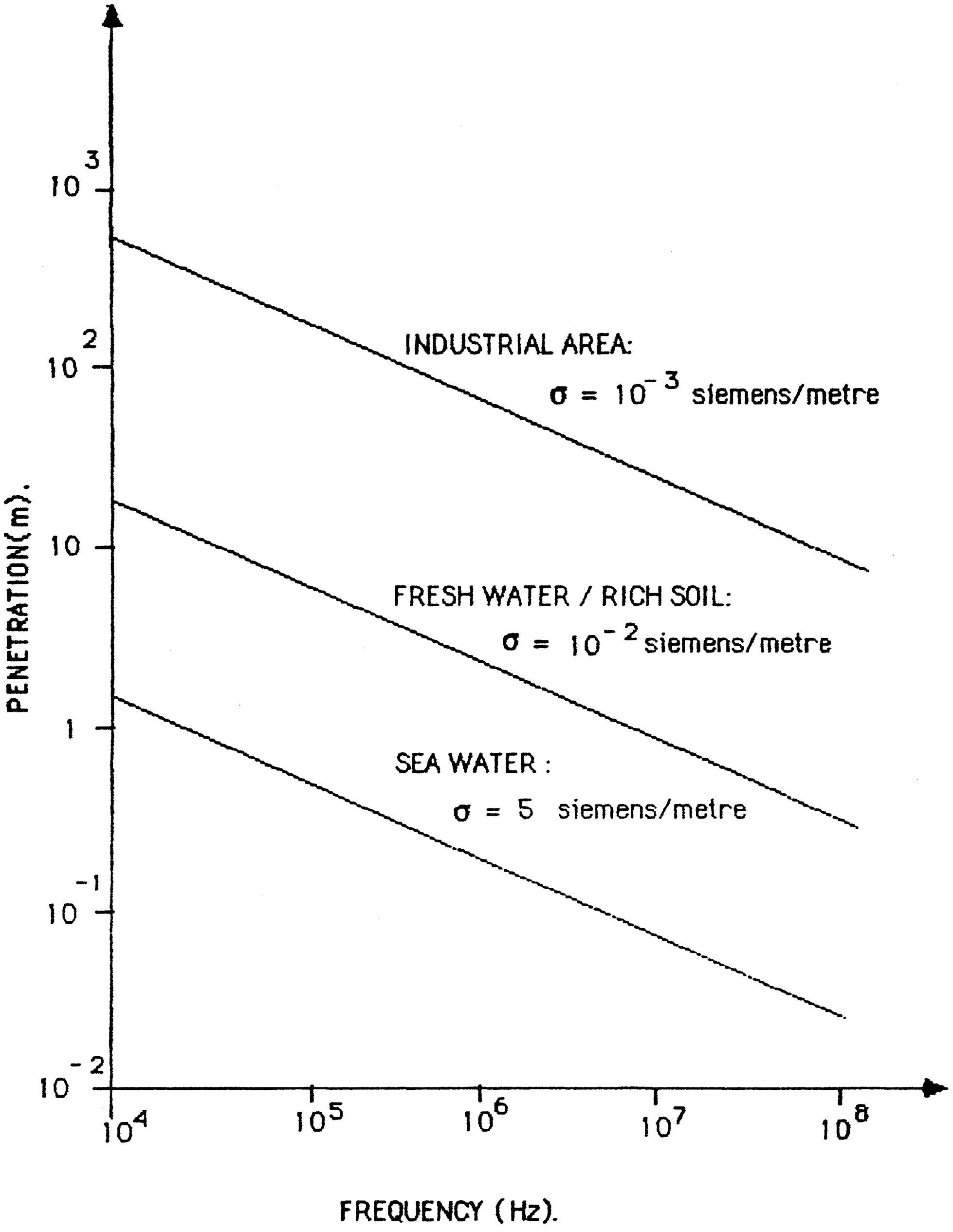


Figure 2.2: Penetration of Groundwave.

where X_0 = the depth (m)

ω = angular frequency (rad/sec)

μ_0 = permeability of free space

= $4\pi \times 10^{-7}$ henrys / m

ε = absolute permittivity

= $\varepsilon_{rel} \times 1/36\pi \times 10^{-9}$ farad / m

ε_{rel} = relative permittivity

σ = earth conductivity (siemens / metre)

2.3 PRIMARY PHASELAG

Ground wave is not confined to the earth surface or to the depth it penetrates into the earth. It also extend upwards a considerable height. The tropospheric refractivity slows down the signal. This is referred to as primary phaselag. This is accounted for by applying the refractive index correction in the same procedure as with microwave distance measurements. The correction is given by:

$$C_p = N \times 10^{-6} . d$$

where C_p = primary phase lag correction (metre)

N = refrectivity of the atmosphere (Essen and Froome, 1951)

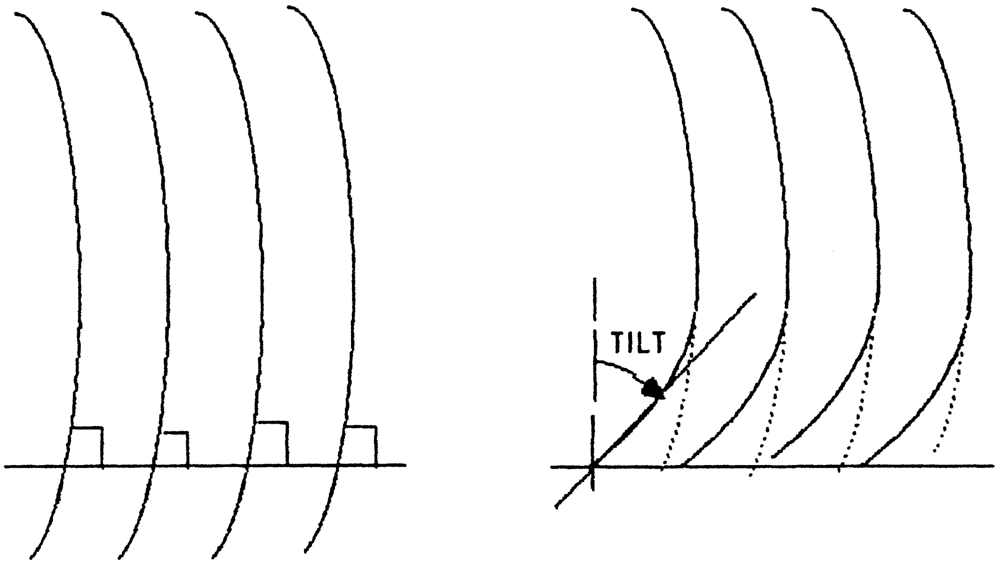
d = measured distance (metre)

2.4 SECONDARY PHASELAG

As mentioned earlier, groundwave induces charges in the earth that travel with the wave and so constitute a current. Therefore, the earth conductivity and dielectric constant describe its characteristics as a conductor. Because the earth surface is not a perfect conductor, the ground edge of the wavefront retards, causing it to tilt forward (Fig. 2.3). The tilt increases with frequency and decreases with conductivity (Table 2.1, Bigelow, 1963). For this reason, short waves are not capable of long range propagation; the propagation loss becomes excessive. The combined effect of the ground conductivity, curvature of the earth, dielectric constant and permittivity lapse rate give rise to the complicated problem of secondary phaselag which describes the disturbing influence of the earth surface on the propagation of ground wave.

	ANGLE OF TILT	
FREQUENCY	σ = 5 siemens/metre	$\sigma = 10^{-3}$ siemens/metre
20 KHz	0° 02.'5	4° 18'
200 KHz	0° 08.'0	13° 30'
2 MHz	0° 25.'0	32° 12'
20 MHz	1° 23.'0	35° 00'

Table 2.1 Angle of Tilt Variation With Frequency and Conductivity



a) Perfect Conductor

b) Poor Conductor.

Figure 2.3: Tilt of Groundwave.

2.5 GROUND CONDUCTIVITY

The electrical characteristic of the earth is described by its conductivity, dielectric constant (permittivity) and permeability. However, at long waves frequencies, the conductivity becomes the most important parameter in the determination of secondary phaselag. Since the ground wave may penetrate the earth, the effective conductivity is determined by the nature of the soil profile.

The conductivity and the dielectric constant of the soil vary with the nature of the soil, with moisture content as the main determining factor. An example of the effect of moisture on the conductivity is in the case of loam; 10^{-2} siemens/metre at normal condition and 10^{-4} siemens / metre when it is dry. Typical example of the variation in ground conductivity and dielectric constant is given in Table 2.2. (ITT, 1975). Figure 2.4 shows the conductivity map of Canada (Ireland, 1961).

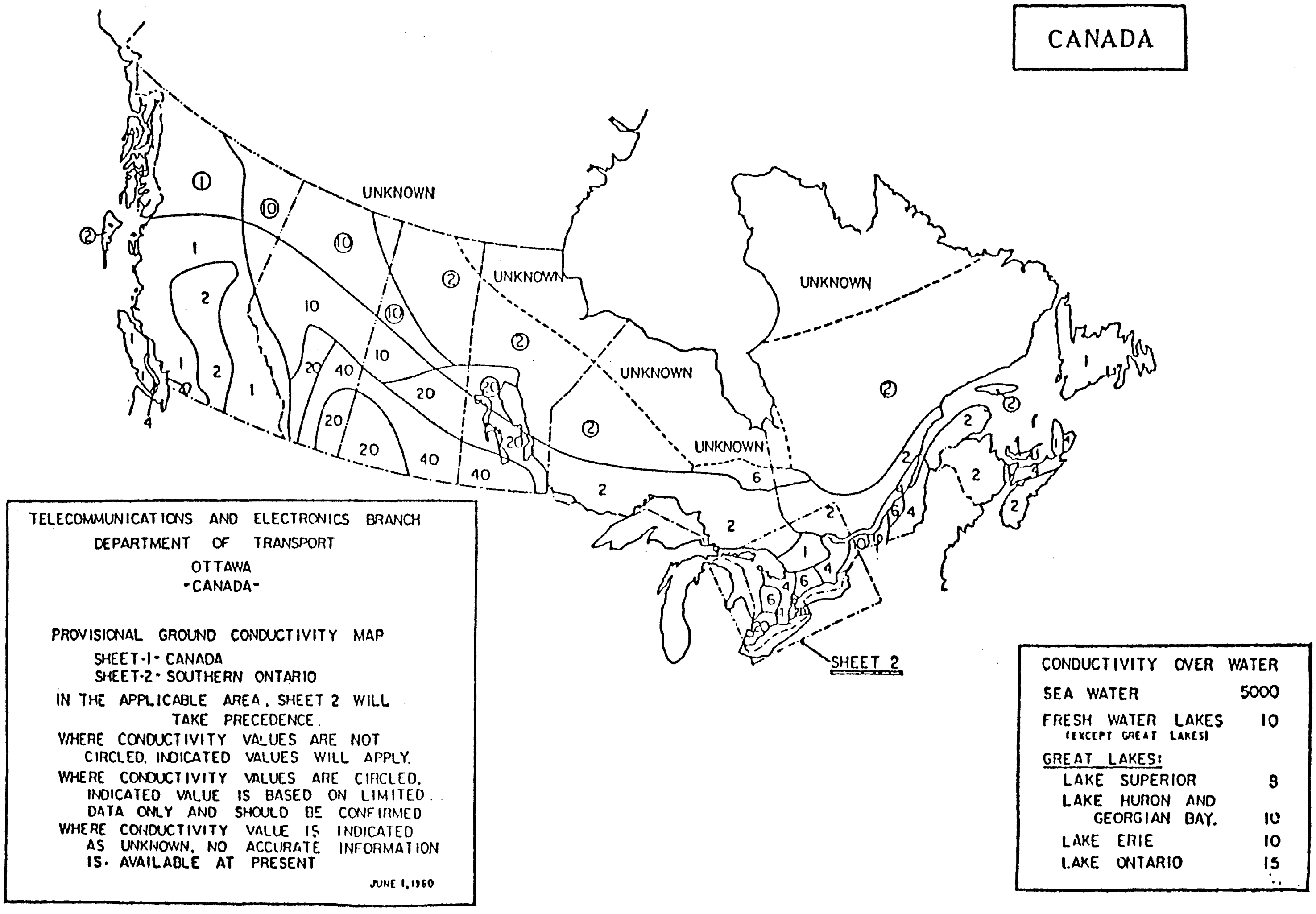
TYPE OF TERRAIN	CONDUCTIVITY (siemens/metre)	DIELECTRIC CONSTANTS (esu)
Sea water	5	80
Fresh water	8×10^{-3}	80
Dry, sandy, flat coastal land	2×10^{-3}	10
Marshy, forested flat land	8×10^{-3}	12
Rice agricultural land, low hill	1×10^{-2}	15
Pastoral land,medium hill and forestration	5×10^{-3}	13
Rocky land, steep hills	2×10^{-3}	10
Mountainous (hills up to 100m)	1×10^{-3}	5
Cities, industrial areas	1×10^{-4}	3

Table 2.2 Ground Conductivity and Dielectric Constants.

With offshore navigation, ground wave propagates mostly over the sea water. As shown in Table 2.2, sea water and fresh water differs greatly in conductivity. This clearly shows that conductivity of water varies with salinity and to a certain extent temperature as salinity varies with temperature. Typical variation of conductivity with temperature is given in Table 2.3 (Bigelow,1963).

TYPE OF WATER	SALINITY ($^0/_{00}$)	TEMPERATURE (^{0}C)	CONDUCTIVITY σ (siemens / metre)
SEA WATER	35	20	4.78
	20	10	2.29
FRESH WATER		20	4×10^{-2}
		10	3×10^{-2}
		0	2×10^{-2}

Table 2.3 Conductivity Variation with Salinity and Temperature.



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•CANADA•

PROVISIONAL GROUND CONDUCTIVITY MAP
SHEET-1- CANADA
SHEET-2- SOUTHERN ONTARIO

IN THE APPLICABLE AREA, SHEET 2 WILL TAKE PRECEDENCE.

WHERE CONDUCTIVITY VALUES ARE NOT CIRCLED, INDICATED VALUES WILL APPLY.

WHERE CONDUCTIVITY VALUES ARE CIRCLED, INDICATED VALUE IS BASED ON LIMITED DATA ONLY AND SHOULD BE CONFIRMED

WHERE CONDUCTIVITY VALUE IS INDICATED AS UNKNOWN, NO ACCURATE INFORMATION IS AVAILABLE AT PRESENT

JUNE 1, 1960

CONDUCTIVITY OVER WATER	
SEA WATER	5000
FRESH WATER LAKES (EXCEPT GREAT LAKES)	10
GREAT LAKES:	
LAKE SUPERIOR	8
LAKE HURON AND GEORGIAN BAY.	10
LAKE ERIE	10
LAKE ONTARIO	15

Figure 2.4 Conductivity Map of Canada

Chapter 3

COMPUTATION OF SECONDARY PHASELAG

Formulas and procedures in computing the secondary phaselag have been developed by Johler and co-workers (1956) and since then has been the accepted method. Using this method two programs have been developed by Paul Brunavs and Dave Gray. These programs were then modified and documented here (Section 4.1.4). In this chapter, these methods are laid out without further comment.

For better understanding, the main parameters used in the computation are first given here:-

Input d = distance (m)

f = frequency (Hz)

σ = conductivity of medium (siemens/m)

ε_2 = permittivity of earth (esu)

Assumed ε_1 = permittivity of atmosphere

= 1.000676 (esu)

a = radius of earth

= 6.367390×10^6 m

α = vertical lapse rate of permittivity of air

= 0.75

μ_0 = permeability of free space

= $4\pi \times 10^{-7}$ henry/m

c = velocity of electromagnetic wave

= 299 792 500 ms^{-1}

(note: in Chapters 5 and 6 we will be developing polynomials with coefficients labelled C_i. These are not to be confused with this velocity c)

Derived ω = angular frequency (= $2\pi f$)

k_1 = wave number of the atmosphere

k_2 = wave number of the earth

δ_e = the conductivity and permittivity parameters for a vertical dipole source

K_e = the modulus or amplitude of δ_e

τ_s = solution of Ricatti's equation

ϕ = secondary phaselag

3.1 SPHERICAL EARTH THEORY METHOD

The secondary phaselag ϕ_c is given as

$$\phi_c = \text{Arg}(F_r) = \tan^{-1}[\text{Imag}(F_r) / \text{Real}(F_r)] \quad \text{radians} \qquad (3.1)$$

or

$$\phi_c = \mathrm{Arg}\,(F_r) \times {}^{c}/_{\omega} \text{ metres} \tag{3.2}$$

where

$$F_r = [\,2\pi\alpha^{2/3}\,(k_1 a)^{1/3}\;{}^{d}/_{a}\,]^{1/2} \sum_{s=0}^{\infty} \{\,[\,[f_s(h_1)\,f_s(h_2)]\;/[2\tau_s - {}^{1}/\delta_e)^2]\,\}$$

$$\times\ \exp\{\,i[(k_1 a)^{1/3}\,\tau_s\alpha^{2/3}\;{}^{d}/_{a} + {}^{\alpha d}/_{2a} + {}^{\pi}/_{4}]\,\} \tag{3.3}$$

$f_s(h_1) = f_s(h_2) = 1$, both radiating source and observer are at the surface of the earth.

$$\delta_e = K_e \exp\{\,i\,[\,{}^{3\pi}/_{4} - \varphi_e\,]\,\} \tag{3.4}$$

$$K_e = [c\alpha\,/\varepsilon_1{}^2\omega\, a]^{1/3}\,[\varepsilon_2{}^2 + \sigma^2\mu_0{}^2\,c^4/\omega^2]^{1/2}\,/\,[\,(\varepsilon_2 - \varepsilon_1)^2$$

$$+\ \sigma^{\,2}\mu_0 c^4/\omega^2\,]^{1/4} \tag{3.5}$$

$$\varphi_e = \tan^{-1}[\omega\varepsilon_2/\mu_0 c^2\sigma] - {}^{1}/_{2}\,\tan^{-1}[\omega(\varepsilon_2 - \varepsilon_1)/\mu_0 c^2\sigma] \tag{3.6}$$

$$K_1 = {}^{\omega}/_{c}\sqrt{\varepsilon_1} \tag{3.7}$$

and

τ_s is the solution of Ricatti's equation (Section 3.1.1).

3.1.1 Computation of τ_s.

In order to solve equation (3.3), the values of τ_s , $s = 0, \infty$ have to be determined.

τ_s describes the special roots of Ricatti's differential equation:

$$d\delta_e / d\tau_s - 2\delta_e^2 \tau_s + 1 = 0 \qquad (3.8)$$

According to Johler and co-worker (1956, 1959), these roots are determined by expanding equation (3.8) above in power series:-

$$\tau_s = \sum_{n=0}^{\infty} {}^1/_{n!} [d^n\tau_s / d\delta_e^{\,n}]_{\delta e=0} \; \delta_e^{\,n} \quad ; \quad |\delta^2\tau_s| < {}^1/_2 \qquad (3.9)$$

and

$$d\tau_s / d\delta_e = {}^1/(2\tau_s\delta_e^{\,2} - 1)$$

or

$$\tau_s = \sum_{n=0}^{\infty} {}^1/_{n!} [d^n\tau_s / d(1/\delta_e)^n]_{\delta e=\infty} \; \delta_e^{\,-n} \; ; \; |d^2\tau_s| > {}^1/_2 \qquad (3.10)$$

and

$$d\tau_s / d(1/\delta_e) = -\delta_e^{\,2} \, d\tau_s / d\delta_e = {}^1/[1/\delta_e^{\,2} - 2\tau_s]$$

Equation (3.9) and (3.10) give (Johler et al, 1959):

$$\tau_s = \tau_s(0) - \delta_e - [{}^{2}/_{3}\tau_s(0)]\delta_e{}^{3} + {}^{1}/_{2}\delta_e{}^{4} - [{}^{4}/_{5}\tau^{2}{}_{s}(0)]\delta_e{}^{5} +$$

$$[{}^{14}/_{9}\,\tau_s(0)]\delta_e{}^{6} - {}^{1}/_{7}[5 + 8\tau^{3}{}_{s}(0)]\delta_e{}^{7} + \;.\;.\;.\;.\;.\;.\;.\;. \qquad (3.11)$$

where $\tau_s(0)$ is an initial approximation (actually the limiting root) to τ_s.

and

$$\tau_s = \tau_s(\infty) - [\,{}^{1}/2\,\tau_s(\infty)\,]\,{}^{1}/\delta_e - [\,{}^{1}/\tau^{3}{}_{s}(\infty)\,]\,{}^{1}/\delta_e - [\,{}^{1}/12\tau^{2}{}_{s}(\infty) +$$

$${}^{1}/16\tau^{5}{}_{s}(\infty)\,]\,{}^{1}/\delta_e{}^{3} - [{}^{7}/96\tau^{4}{}_{s}(\infty) + {}^{5}/128\tau^{7}{}_{s}(\infty)\,]\,{}^{1}/\delta_e{}^{4} - \qquad (3.12)$$

where $\tau_s(\infty)$ is an initial approximation (actually the limiting root) to τ_s.

$\tau_s(0)$ and $\tau_s(\infty)$ are computed as follows:-

$$\tau_s(0) = Y_1{}^{2/3}\,/2^{1/3}\,[1 + 5/48Y_1{}^{2} - 5/36Y_1{}^{4} - \;\ldots.] \qquad (3.13)$$

$$\tau_s(\infty) = Y_2{}^{2/3}\,/2^{1/3}\,[1 - 7/48Y_2{}^{2} + 35/288Y_2{}^{4} - \ldots..\,] \qquad (3.14)$$

where

$$Y_1 = {}^{3\pi}/_{8}\,(4s + 3) \qquad (3.15)$$

and

$$Y_2 = {}^{3\pi}/_{8}\,(4s + 1) \qquad (3.16)$$

These initial approximations, $\tau_s(0)$ and $\tau_s(\infty)$ for $\delta_e = 0$ and $\delta_e = \infty$ have been tabulated by Johler and co-workers (1959).

Howe (1960) modified equation (3.9) and (3.10) to cater for longer terms in (3.11) and (3.12) so that better accuracy could be obtained. His modified series were called Ascending and Decending Series and they were more practical when using computers.

a) Ascending Series

$$\tau_s = \sum_{j=0}^{\infty} B_j \tag{3.17}$$

where

$$B_0 = \tau_s(0),\ B_1 = -\delta_e \text{ and } B_2 = 0 \qquad \text{for } j < 3 \tag{3.18}$$

and

$$B_j = [(j-2)/j]\,\delta_e^2 \sum_{n=0}^{j-2} B_n B_{j-n-2} \qquad \text{for } j \geq 3 \tag{3.19}$$

b) Descending Series

$$\tau_s = \sum_{j=0}^{\infty} A_j \tag{3.20}$$

where

$$A_0 = \tau_s(\infty),\quad A_1 = (-1/2A_0)(1/\delta_e) \qquad \text{for } j < 2 \tag{3.21}$$

and

$$A_j = (-1/2A_0)\{\sum_{n=1}^{j-1} A_n A_{j-n} - [(j-2)/j]\, 1/\delta_e^2 A_{j-2}\} \quad \text{for } j \geq 2 \tag{3.22}$$

A_0 and B_0 are the constants of integration and may be chosen arbitrarily. However they have an amplitude of $\pi/3$ and therefore A_0 and B_0 are given the following values:

$$B_0 = \tau_s(0) \times \text{amplitude } \pi/3$$

and

$$A_0 = \tau_s(\infty) \times \text{amplitude } \pi/3 \tag{3.23}$$

where $\tau_s(0)$ and $\tau_s(\infty)$ are computed as in equations (3.13), (3.15), (3.14), (3.16) respectively.

In equation (3.3), the summation is shown to involve the computation of each of the τ_s, for $s = 0,\infty$. Since in practice we will sum only over a finite number of terms, we can choose either to start with using either equation (3.17) or (3.20) respectively. The computation of A_j or B_j are continued until A_j or B_j is small enough to be ignored. At this point the sum of all A_j or B_j gives the value of τ_s. However, if the series in use does not converge at a certain stage of computation, the computation should cease and switch to the other series. The convergence criterion to be used here is 10^{-6} or less (Gray and Brunavs used 10^{-7} to 10^{-9}).

These power series in δ_e^{n} and δ_e^{-n} seem to lose precision near the circle of convergence, $|\delta\tau_s^2| = 1/2$ (Walters and Johler, 1962), for a finite number of terms. This problem is highly prevalent at low conductivities (see 4.1.2). To overcome this problem Walters and Johler (1962) developed another method of computing τ_s using Hankel Functions. In this method, Ricatti's equation is expressed as a ratio of Hankel Functions:

$$1/(-2\tau_s)^{1/2}\{H^{(1)}{}_{1/3}[{}^1/_3(-2\tau_s)^{3/2}]/H^{(1)}{}_{2/3}[{}^1/_3(-2\tau_s)^{3/2}\}\exp[i^{\pi}/_3] + \delta_e = 0 \tag{3.24}$$

The Hankel Functions $H^{(1,2)}{}_{1/3,2/3}$ are then evaluated to obtain iterative solution of Ricatti's equation using the following:-

$$H_v^{(1)}(z) = {}^1/_\pi \int_0^\pi \exp[iz\sin\theta - vi\theta]d\theta + {}^1/_{\pi i}\int_0^1 \exp[{}^z/_2(u - {}^1/_u)][u^{-v-1} + u^{v-1}\exp(-v\pi i)]du \tag{3.25}$$

for $R_e(z) > 0$

and

$$H_v^{(2)}(z) = {}^1/\pi \int_0^\pi \exp[-iz\sin\theta + vi\theta]d\theta + {}^1/_\pi \int_0^1 \exp[{}^z/_2(u - {}^1/_u)][u^{-v-1} + u^{v-1}\exp(v\pi i)]du \tag{3.26}$$

for $R_e(z) < 0$

The integrals in (3.25) and (3.26) may be evaluated by Gaussian quadrature:

$$\int_a^b F(x)dx = \sum_{m=1}^{M} W_m F(Y_m) + \varepsilon(M) \qquad (3.27)$$

$$Y_m = {}^1/_2[(b-a)x_m + (b+a)] \qquad (3.28)$$

where

x_m = Gaussian abscissa

$\varepsilon(m)$ = error term

Detail procedure is explained in Waltrs and Johler (1962). It should be noted that this method is used only when the power series fail to converge. This is because the power series is morestraight forward and involves less of an approximation process as with the Hankel Function. That is for every τ_s, the power series are used first and only if both of them fail to converge, this method is then applied.

3.1.2 Method of Computation of τ_s used in Gray and Brunvs Program

a) Gray - using modified power series by Howe for τ_s, s = 0, 2500.

b) Brunav - using modified power series by Howe and Hankel Function for τ_s, s = 0, 620.

3.1.3 Restrictions in Spherical Earth Computation

The summation term in equation (3.3)

$$\sum_{s=0}^{\infty} 1 / (2\tau_s - 1/\delta_e^2) \exp\{ i[(k_1 a)^{1/3} \tau_s \alpha^{2/3}\, {}^{d}/_{a} + {}^{\alpha d}/_{2a} + {}^{\pi}/_{4}]\}$$

is dependent mainly on τ_s. It should be noted that the value of τ_s increases with s. Therefore as many s as possible should be used to obtain a good estimate. However, it is found that d(distance) plays a major role in the convergency of the summation equation. For great distances, a reasonably small number of terms are used but as the distance decreases the number of term begins to increase. For very short distances it requires too many terms. The Brunav and Gray programs demonstrate that even with 620 and 2500 number of terms, it does not converge. Another point to note is that if too many terms are used in the computation, the precision deteriorates and it takes too much computing time

3.2 PLANE EARTH THEORY METHOD.

As mentioned before (3.1.3), computation of secondary phaselag for short distances using the spherical earth theory is laborious and inaccurate. Therefore, it can be assumed that the earth is a plane. The secondary phaselag ϕ_z in this case is given by (Johler et al, 1956):

$$\phi_z = \mathrm{Arg}(Fz) = \tan^{-1}[\,\mathrm{Imag}(Fz)/\mathrm{Real}(Fz)\,] \quad \text{radians} \qquad (3.29)$$

or

$$\phi_z = \mathrm{Arg}(Fz) \times {}^{c}/_{W} \text{ metre}$$

where

$$F_z = Y(\rho_1) \int(\sigma,\varepsilon) - 1/ i k_1 d + 1/ (i k_1 d)^2 \tag{3.30}$$

$$Y(\rho_1) = i\sqrt{(\pi\rho_1)}\ e^{-\rho_1} + \sum_{n=0}^{\infty} [[(-1)^n 2^n (n!)] /2n!] (2\rho_1)^n \tag{3.31}$$

$$|\rho_1| << 1$$

or

$$Y(\rho_1) = -\sum_{n=1}^{\infty} [(2n!) /2^n(n!)] [1/(2\rho_1)^n] \tag{3.32}$$

$$|\rho_1| >> 1$$

$$\rho_1 = [(k_1 a)^{1/3}\ {}^{d}/_{a}\, \alpha^{2/3}] / 2i\,\delta_e^{\,2} \text{ radians} \tag{3.33}$$

$$\int(\sigma,\varepsilon) = 1 - (k_1/k_2)^2 + (k_1/k_2)^4 \tag{3.34}$$

$$k_2 = {}^{\omega}/c\, [\varepsilon_2 + i\sigma\mu_0 c^2/\omega]^{1/2} \tag{3.35}$$

and

k_1 and δ_e are as equations (3.7) and (3.4) respectively.

The validity of this method is given in Section 4.1.1.

Chapter 4

ANALYSIS ON SECONDARY PHASELAG COMPUTATION AND ITS VARIATION

With the objectives laid out in Section 1.4, it was therefore neccessary to compute the secondary phaselag value for different parameters, namely distance, conductivities and permittivity. For this purpose, a computer program was developed based on the two existing programs written by Paul Brunavs and Dave Gray. Paul Brunavs program incorporated Hankel function for the solution of τ_s but did not have the plain earth theory computation while Dave Gray's program was the reverse. Therefore the new program was developed to incorporate both of them.

In this program called SEPLAG, the values of secondary phaselag using both plane and spherical earth theories were computed for distances from 1 to 2000 km with conductivity and permittivity as the input parameters. Using the conductivity and permittivity values given in Table 2.2 as a guide, the computations were carried out for conductivity and permittivity values of 0.00001 to 5.5 siemens/metre and 3 to 80 esu respectively. With the results obtained from these computations, the following analysis can be made:-

4.1 ANALYSIS ON THE COMPUTATION OF SECONDARY PHASELAG.

In this section, the limitations of plane earth computation and computation of τ_s using Ascending and Decending series will be given together with a suggestion on how to reduce computing time.

4.1.1 Identical Results of Plane and Spherical Earth Theory Computations

In the secondary phaselag computation using both plane and spherical theories, it was found that at short distances the plane theory gives larger values than that of the spherical theory. At certain distances, these two values become identical and beyond those distances the spherical values become larger than that of the plane value. Table 4.2 shows where these two values coincide at various conductivities and permittivities. Beyond these distances, the difference between the two values increases with distance. It was also found that the spherical theory computation needs a great number of terms to compute the secondary phaselag at short distances but only a few in the plane theory computation. For this reason it was felt that errors may exist in the spherical theory computation especially at very low conductivity. Therefore for shorter distances than those in Table 4.1, values obtained from the plane theory computation should be used.

4.1.2 Limitation of Computing τ_s Using Ascending and Descending Series

Table 4.2 shows that Ascending and Descending Series (as explained in Section 3.1.1) can only be used for conductivity values higher than 0.001siemens/metre if permittivity is ignored. At 0.001siemens/metre and below τ_s converge only for values of s that are small or large. The gap of nonconvergence increases as conductivity and permittivity decreases. At these conductivity values, and for the nonconvergence gaps in s values τ_s should be computed using Hankel Function.

CONDUCTIVITY	IDENTICAL RESULTS OF PLANE AND SPHERICAL THEORY			
(σ)	D	TP	TS	
5.0	10	2	620	
3.0	10	2	620	
1.0	10	2	620	
0.8	10	2	620	INDEPENDENT
0.5	10	2	620	
0.3	10	2	620	OF
0.1	10	3	620	
0.08	10	3	620	PERMITTIVITY
0.05	10	3	620	
0.03	10	3	620	
0.01	10	4	620	

NOTES:

D - DISTANCE WHICH BOTH THEORIES GIVE IDENTICAL RESULT

TS- NUMBER OF TERMS USED IN SPHERICAL COMPUTATION

TP- NUMBER OF TERMS USED IN PLANE COMPUTATION

	$\varepsilon_2 = 3$			$\varepsilon_2 = 10$			$\varepsilon_2 = 20$		
	D	TP	TS	D	TP	TS	D	TP	TS
0.008	10	4	620	10	4	620	10	4	620
0.006	10	4	620	10	4	620	10	4	620
0.004	10.5	4	620	10.5	4	620	11	4	620
0.002	11	5	620	11	5	620	12	5	620
0.001	13	5	620	14	6	620	15	6	596
0.0008	15	6	606	16	6	558	18	6	479
0.0006	18	7	491	19	7	458	20	7	429
0.0004	23	9	412	25	9	358	26	9	338
0.0002	34	12	257	35	12	249	35	12	241
0.0001	43	19	194	41	17	210	40	14	199
0.00008	45	21	210	42	19	205	40	15	197
0.00006	45	25	210	42	20	200	40	16	194
0.00004	45	30	205	42	22	195	40	16	190
0.00002	44	39	225	42	24	200	40	16	185
0.00001	44	44	282	42	25	195	42	20	195

Table 4.1 Identical Results of Plane and Spherical Earth Theory Computation.

CONDUCTIVITY [σ] (in siemens/metre)	NON - CONVERGED τ_s USING ASCENDING/DECENDING SERIES FROM	TO	
5.0	NIL	NIL	
1.0	NIL	NIL	
0.8	NIL	NIL	
0.5	NIL	NIL	
0.3	NIL	NIL	INDEPENDENT
0.1	NIL	NIL	
0.08	NIL	NIL	OF
0.05	NIL	NIL	
0.03	NIL	NIL	PERMITTIVITY
0.01	NIL	NIL	
0.006	NIL	NIL	
0.002	NIL	NIL	

NOTE:

τ_s , s = 0,620 IS COMPUTED TO DETERMINE SECONDARY PHASELAG; FROM AND TO INDICATE WHERE τ_s DO NOT CONVERGE USING THESE SERIES.

	$\varepsilon_2 = 3$		$\varepsilon_2 = 10$		$\varepsilon_2 = 15$		$\varepsilon_2 = 20$	
	FROM	TO	FROM	TO	FROM	TO	FROM	TO
0.001	0	1	0	1	0	1	0	1
0.0006	0	2	0	2	0	2	0	2
0.0004	1	4	1	4	1	4	1	4
0.0002	5	13	5	12	4	12	4	11
0.0001	15	36	12	30	10	24	8	20
0.00008	21	49	15	37	11	29	9	22
0.00006	31	72	19	47	14	34	10	25
0.00004	54	116	25	58	16	39	11	27
0.00002	106	212	30	70	17	42	12	29
0.00001	137	265	32	73	18	43	12	29

Table 4.2: Non - Convergency of τ_s Using Ascending and Decending Series.

4.1.3 Number of Terms in Spherical Earth Theory Computation

Table 4.3 shows the number of terms required for the computation of secondary phaselag at different distances. This table was derived at permittivity of 15 esu and therefore slight changes in the values should be expected at different permittivity values. It should be noted here that 620 was the maximum number of terms used in the SEPLAG program.

From the table, it can be seen that short distances need greater number of terms which also means that greater computer time is needed. It was also found that there was no substantial improvement in the value if the number of terms were increased. For example, the maximum number of terms was increased to 2500 and the following results were obtained for 10km distance:-

i) At a conductivity of 5 siemens/metre and permittivity value of 80 esu, the secondary phaselag value was 26.7 metres needing 910 terms. The value computed using 620 terms was 25.9 metres and the plane computation was 25.9 metres.

ii) At conductivity of 0.001 siemens/metre and permittivity value of 15 esu, the value was 224.7 metres needing 1020 terms. The value computed using 620 terms was 224.6 metres and the plane computation was 226.4 metres.

For considerably longer distances, the table shows that much fewer number of terms were needed. It is therefore a waste of time to compute 620 values of τ_s when only, say 20, are needed.

4.1.4 Program SEPLAG

As mentioned earlier in this chapter, this program computes secondary phaselag using both the plane and spherical earth theories for distances from 1 to 2000 km with the following input and output data:

Input - 1. Conductivity (σ siemens/metre)

2. Permittivity (ε_2 esu)

Output - 1. Distance (km)

2. Plane secondary phaselag (m)

3. Spherical secondary phaselag (m)

With the results obtained in Table 4.1 and Table 4.3, the program was further modified to allow two options in computing the secondary phaselag.

OPTION 1:

To compute the secondary phaselag values for distances from 1 to 2000 km at given values of permittivity and conductivity. The output will be only one secondary phaselag value for each distance. In this option, the program computes the secondary phaselag using the spherical theory for distances of 50km and greater. For distances shorter than 50 km, both plane and spherical secondary phaselag are computed and the greater value is taken as the more accurate value (see Section 4.1.1).

OPTION 2:

To compute the secondary phaselag value for a given distance, permittivity and conductivity. With this option, the results obtained in Table 4.3 were used in the program to reduce computing time for distances greater than 50Km.

The listing of the program, its basic structure and user's guide are given in Appendix I

CONDUCTIVITY	NUMBER OF TERMS WITH DISTANCE (km) AT ε_2 = 15 esu							
(siemens/metre)	10	50	100	250	500	1000	1500	2000
5.0	620	167	72	25	14	9	8	8
3.0	620	168	71	25	14	9	8	8
1.0	620	167	71	26	14	9	8	8
0.5	620	168	73	26	14	9	8	8
0.3	620	168	73	26	14	9	8	8
0.1	620	167	73	26	14	10	8	8
0.08	620	168	73	25	14	10	8	8
0.05	620	168	72	26	14	10	8	8
0.03	620	167	72	26	14	10	8	8
0.01	620	166	74	27	15	10	9	8
0.008	620	106	73	27	15	10	9	8
0.005	620	109	74	27	15	10	9	8
0.003	620	114	73	28	15	10	9	8
0.001	620	127	72	29	16	11	9	8
0.0008	620	131	71	28	16	11	9	8
0.0005	620	139	61	30	17	11	9	8
0.0003	620	148	65	32	17	11	9	9
0.0001	620	155	92	33	17	11	9	9
0.00008	620	153	94	33	16	11	9	9
0.00005	620	147	95	32	16	11	9	9
0.00003	620	213	95	32	17	11	9	9
0.00001	620	215	94	32	17	11	9	9

Table 4.3 Number of Terms used in Spherical Earth Theory Computation.

4.2 VARIATION OF SECONDARY PHASELAG WITH INPUT PARAMETERS.

Using the results of the computation explained in this chapter, Table 4.4, Figures 4.1, 4.2, 4.3 and 4.4 were derived. From these tables and figures, the following analysis can be made:-

4.2.1 Secondary Phaselag Variation with Distance

Figures 4.1a and 4.1b show that at relatively long distances the secondary phaselag increases with distance. At very long, distances the secondary phaselag increases linearly with distance. At very short distances, however, the secondary phaselag decreases with distance (Figure 4.2). This phenomenon is absent with very low conductivities (Figure 4.3).

4.2.2 Secondary Phaselag Variation with Permittivity

Table 4.4 and Figure 4.4 show the secondary phaselag variation with permittivity. From Table 4.4, it can be seen that at high conductivities, the secondary phaselag does not vary with permittivity. However, at low conductivity, say 0.005 siemens/metre, the secondary phaselag increases as the permittivity decreases.

4.2.3 Secondary Phaselag Variation with Conductivity

Figures 4.1, 4.2, 4.3, and 4.4 and Table 4.4 show that the secondary phaselag increases as the conductivity decreases. However, this phenomenon is not true at very low conductivities (below 0.0005 siemens/metre). Figures 4.1a and 4.1b show that at relatively long distances (say over 200 km), the secondary phaselag decreases with conductivity. At very short distances this relationship is more complex.

This very low conductivity phenomenon was first thought to be due to problems in the summation series. However, by refering to Tables 4.2 and 4.3 it can be seen that, for low conductivities (e.g. at 0.0001 siemens/metre), distances above 1000 km, do not require more than 15 terms and τ_s can be computed using Ascending and Descending Series. This means that not much roundoff errors were expected. At very short distances where a large number of terms were needed, the summation series may have caused some roundoff errors together with roundoff errors in determining τ_s using Hankel Function to produce irregular variation in secondary phaselag as shown in Figures 4.1b and 4.3.

CONDUCTIVITY	SECONDARY PHASELAG(m) WITH PERMITTIVITY ε_2					
(siemens/metre)	$\varepsilon_2 = 15$	$\varepsilon_2 = 50$	$\varepsilon_2 = 75$	$\varepsilon_2 = 80$	$\varepsilon_2 = 85$	
5.0	531.1	531.1	531.1	531.1	531.1	
4.0	535.1	535.1	535.1	535.1	535.1	
3.0	541.0	541.0	541.0	541.0	541.0	
2.0	550.8	550.8	550.8	550.8	550.8	
	$\varepsilon_2 = 5$	$\varepsilon_2 = 10$	$\varepsilon_2 = 15$	$\varepsilon_2 = 20$	$\varepsilon_2 = 80$	
0.5	604.5	604.5	604.5	604.5	604.5	
0.1	736.8	736.8	736.8	736.8	736.8	
0.075	773.8	773.8	773.8	773.8	773.8	
	$\varepsilon_2 = 3$	$\varepsilon_2 = 5$	$\varepsilon_2 = 10$	$\varepsilon_2 = 15$	$\varepsilon_2 = 20$	$\varepsilon_2 = 40$
0.05	835.6	835.6	835.6	835.6	835.6	835.6
0.025	974.2	974.2	974.2	974.1	974.1	974.0
0.01	1243.4	1243.3	1243.1	1242.9	1242.6	1241.7
0.0075	1353.4	1353.3	1352.9	1352.5	1352.0	1350.2
0.005	1532.6	1532.2	1531.2	1530.1	1529.1	1524.7
	$\varepsilon_2 = 3$	$\varepsilon_2 = 5$	$\varepsilon_2 = 10$	$\varepsilon_2 = 15$	$\varepsilon_2 = 20$	$\varepsilon_2 = 30$
0.0025	1902.7	1901.0	1896.8	1892.5	1888.1	1879.1
0.001	2430.8	2421.9	2399.8	2377.7	2355.6	2311.8
0.0008	2529.1	2516.9	2486.7	2456.7	2427.1	2359.2
0.0005	2645.4	2625.5	2576.8	2529.4	2483.4	2395.9
0.0002	2625.3	2587.4	2496.9	2412.4	2335.9	2205.7
0.0001	2532.7	2470.0	2331.8	2221.4	2134.6	2010.2
0.00008	2493.6	2420.5	2267.8	2154.9	2071.3	1957.1
0.00005	2393.7	2298.1	2129.7	2026.6	1957.6	1868.9
0.00003	2255.5	2148.8	1998.9	1920.2	1870.0	1805.3
0.00001	1948.8	1904.5	1836.8	1800.5	1775.5	1739.3

Table 4.4 Secondary Phaselag Variation with Conductivity and Permittivity at 1000 km.

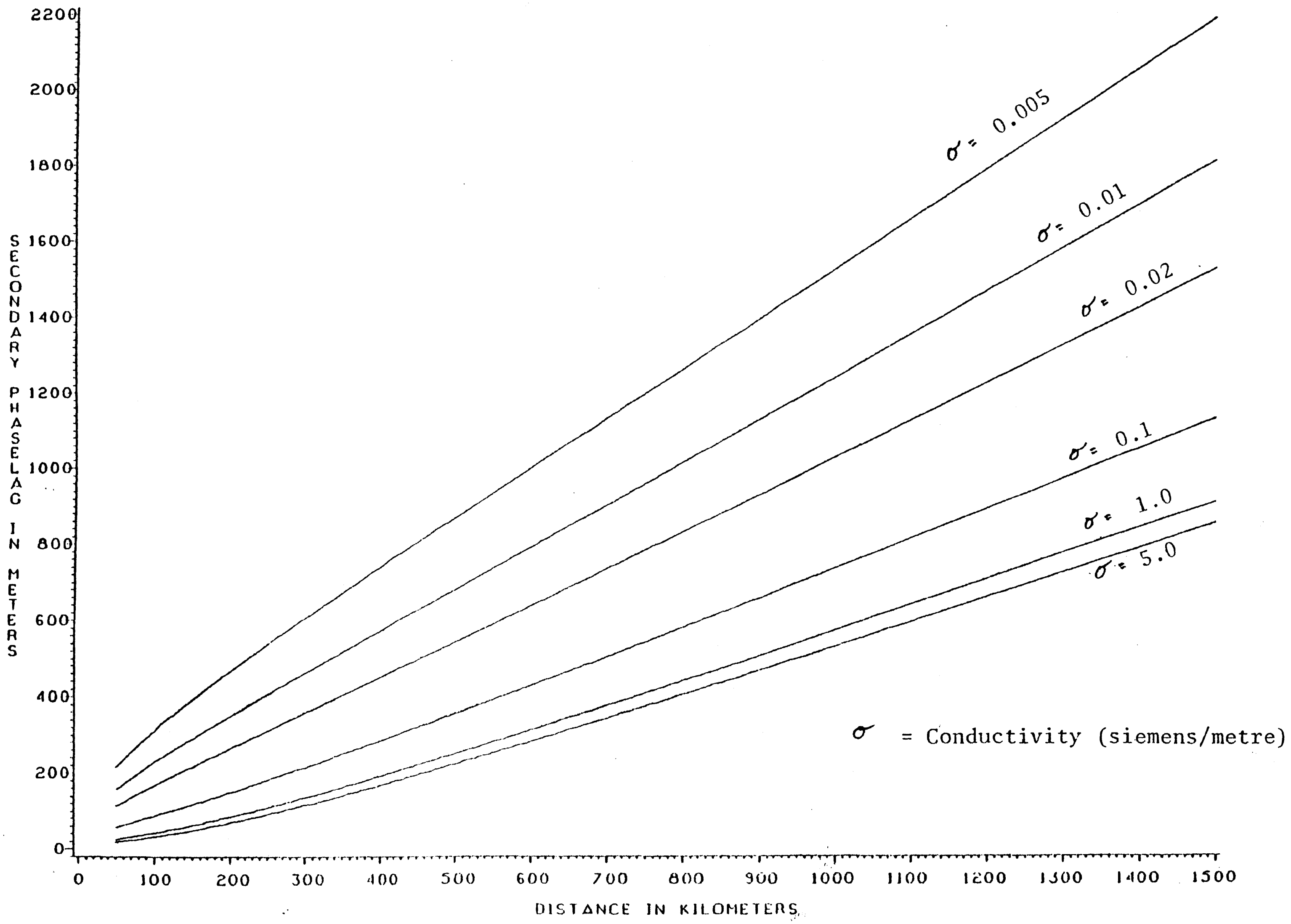

Figure 4.1a: Long Distance Variation of Secondary Phaselag at High Conductivity.

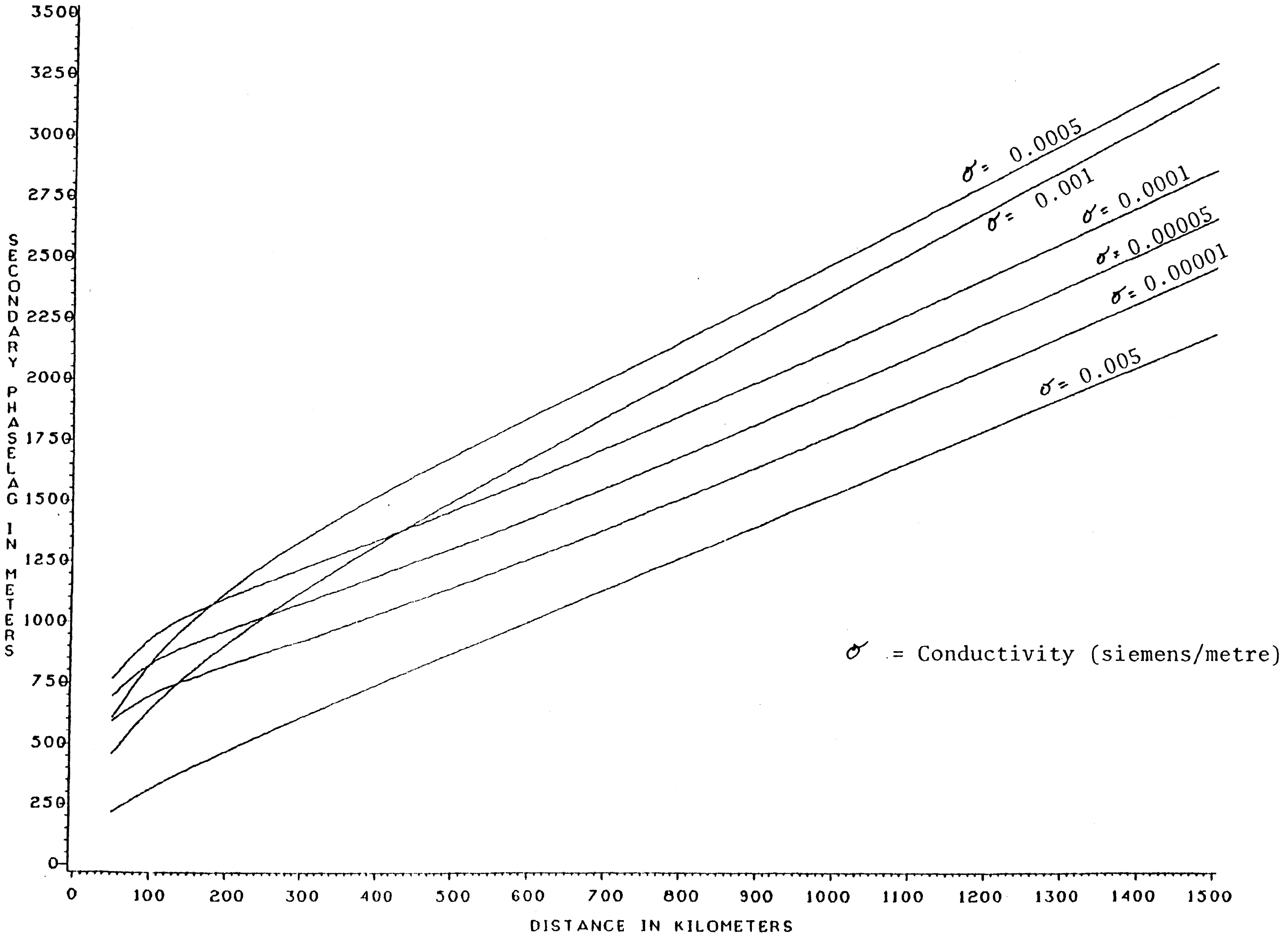

Figure 4.1b: Long Distance Variation of Secondary Phaselag at Low Conductivity (E_2=20 esu)

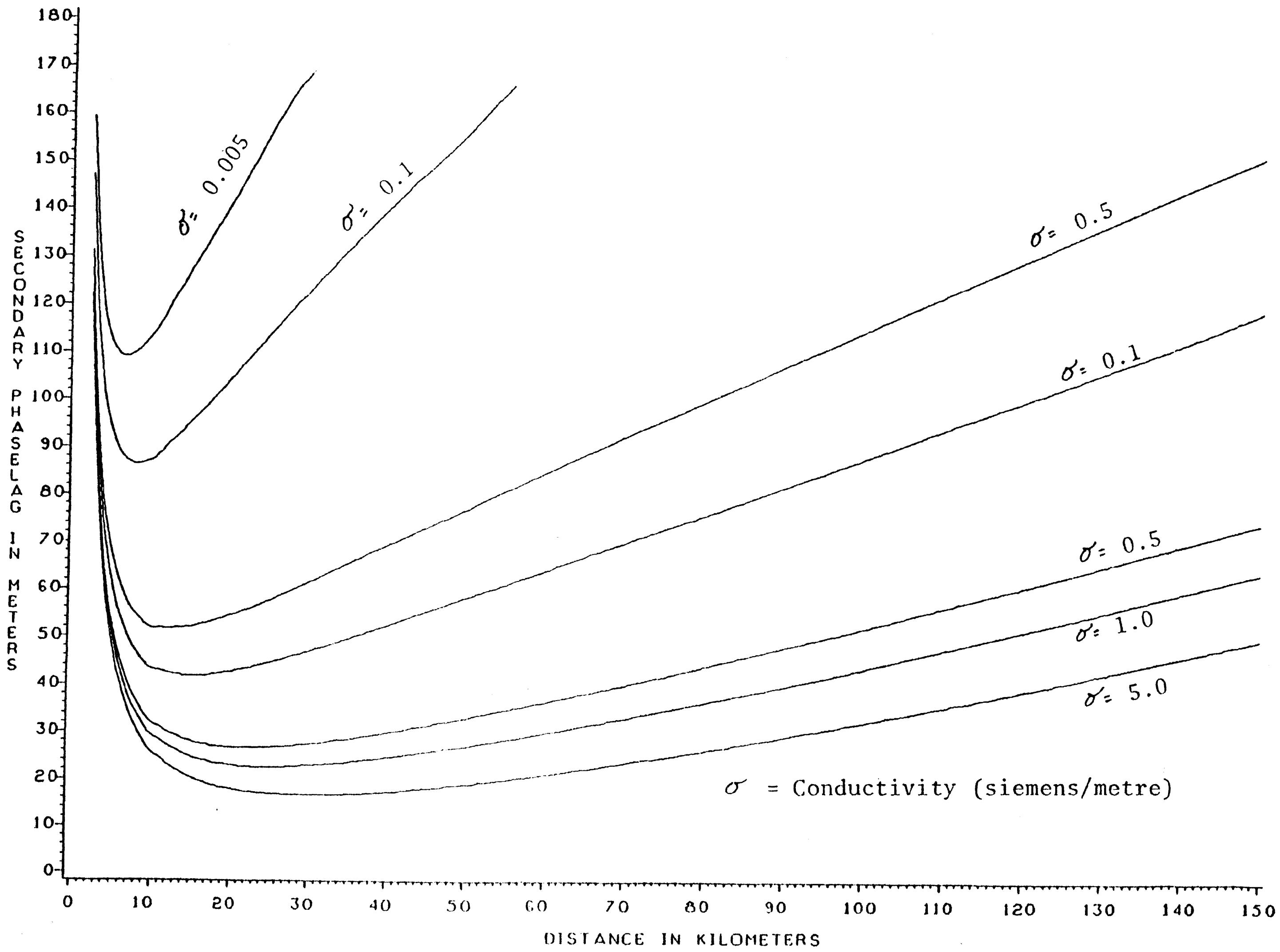

Figure 4.2: Short Distance Variation of Secondary Phaselag at High Conductivity.

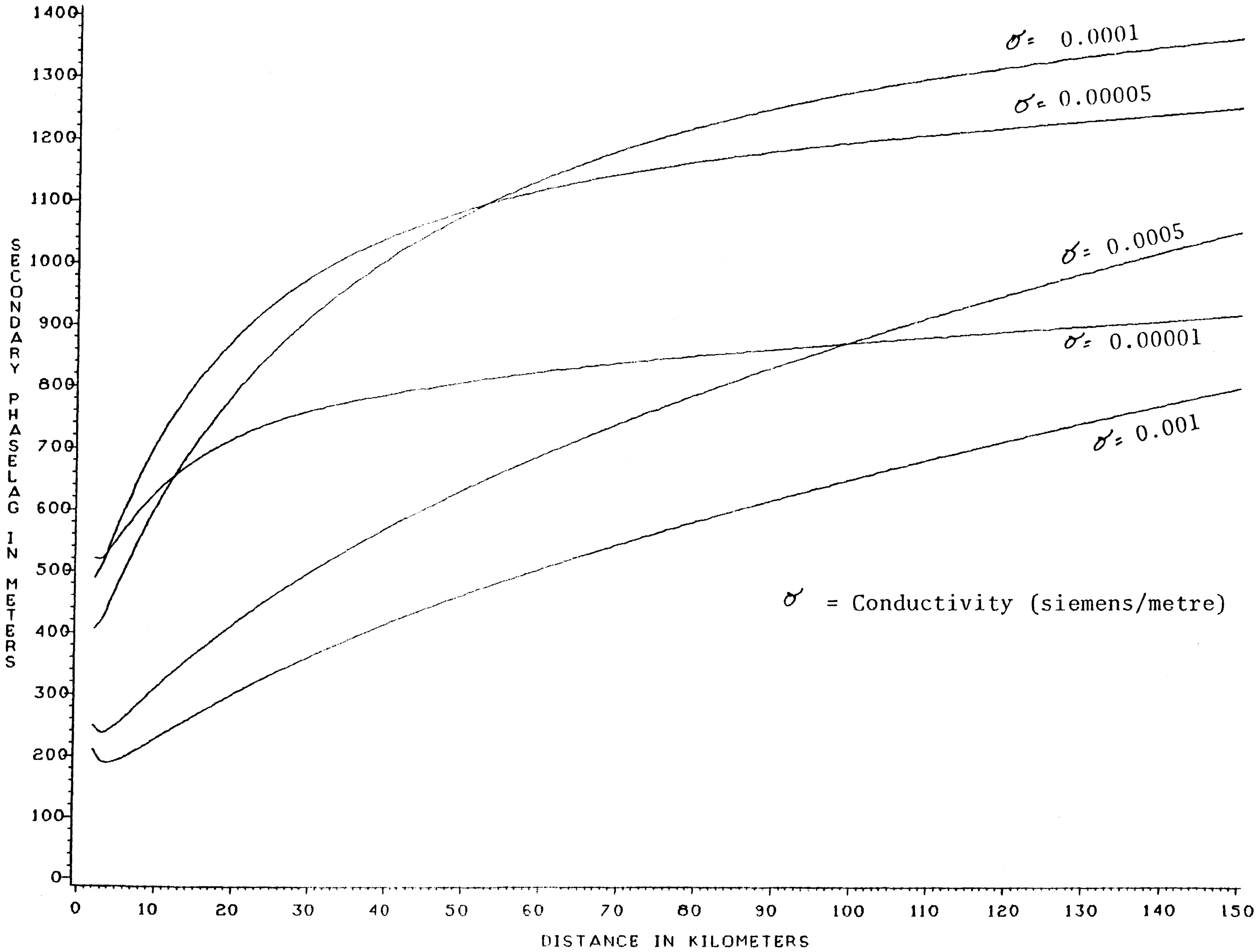

Figure 4.3: Short Distance Variation of Secondary Phaselag at Low Conductivity (E_2=5 esu).

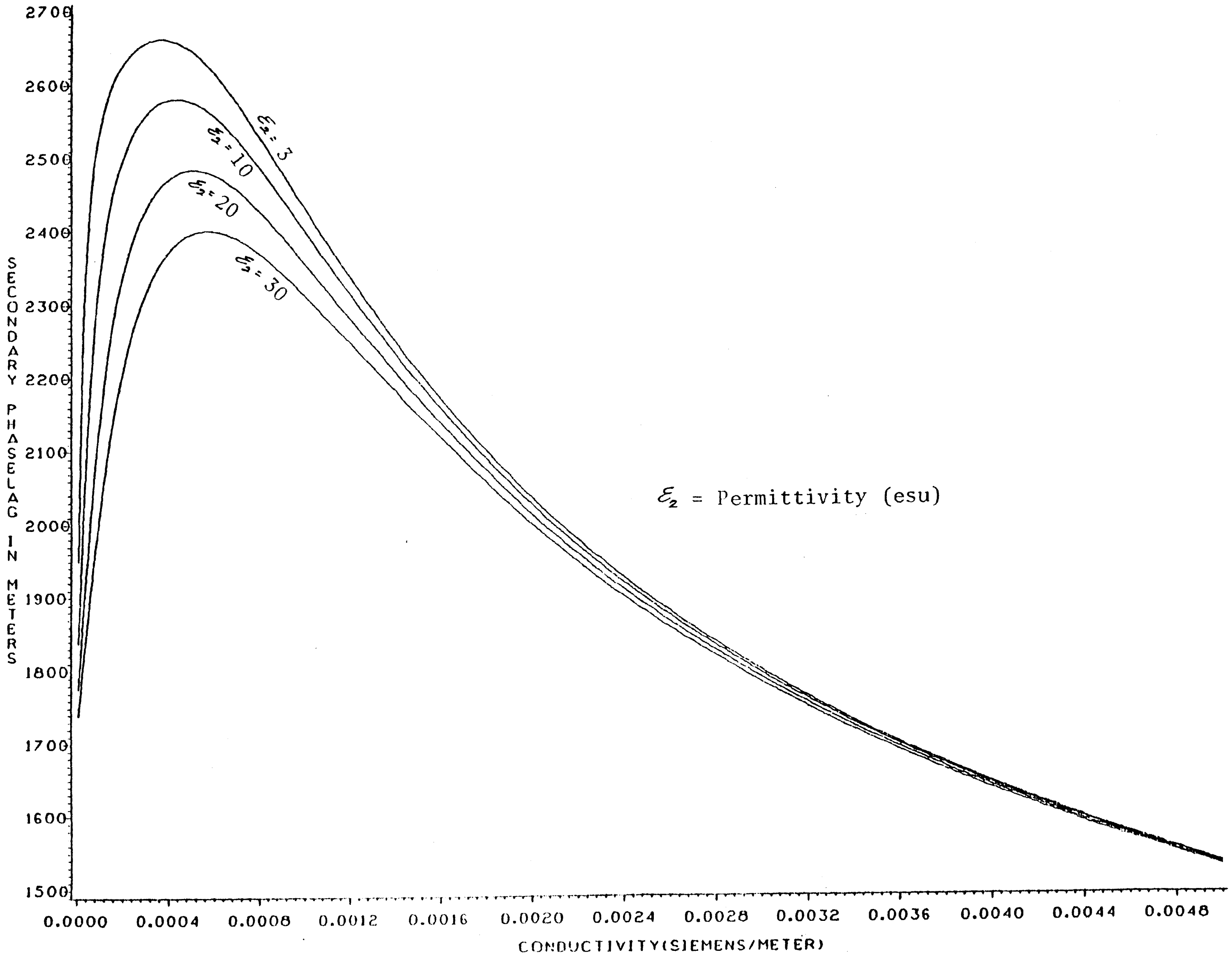

Figure 4.4: Secondary Phaselag Variation With Permittivity at 1000 Km Distance.

Chapter 5

APPROXIMATE FORMULA

From Chapters 3 and 4, it could be seen that the computation of secondary phaselag using plane earth theory is quite simple. However, it could only be used for very short distances (Table 4.1). For longer distances, the spherical earth method has to be used. This method, as shown in the last two chapters, requires a lot of computation time. Figures 4.1 to 4.3 showed that the secondary phaselag versus distance curves were smooth and therefore it would be possible to fit these curves with a simple model or an approximate formula.

5.1 EARLIER WORK BY BRUNAVS.

Paul Brunavs (1977) developed two approximate formulae to compute total phaselag (primary and secondary) and he named them Formula B and Formula C as shown below:-

Formula B: $TPL = B_1 + B_2S + (B_3S + B_4)e^{-S/2} + B_5/(1+6S) + 2.227/S$

Formula C: $TPL = C_1 + C_2S + (C_3S + C_4)e^{C_5S} + C_6/(1 + C_7S + C_8S^4) + 2.227/S$

where

TPL = Total phaselag

S = Distance(m)/100,000

B_i and C_i = Coefficients

He showed that Formulae B and C were accurate to ± 20m and ± 6m respectively when compared to the accurate computation of total phaselag for a range of 2 km to 5000 km distances. In this work he assumed the following:-

i) N = 338 for the primary phaselag value and can therefore be changed accordingly

ii) Sea water conductivity ranges from 2.0 to 5.5 siemens/metre with permittivity of 80 esu.

iii) Land conductivity ranges from 0.00001 to 0.03 siemens/metre with permittivity of 15 esu.

He has also tabulated the B and C coefficients for selected conductivities (see Table II.1 in Appendix II for C coefficients)

5.2 EXTENSION ON APPROXIMATE FORMULA.

There are a few reasons why these extension work needs to be done to fill in the gaps left by Brunavs' work so that his approximate formulae can be used in a wider scope. They are:-

i) As mentioned in Section 4.2.2, secondary phaselag varies with permittivity at conductivities lower than 0.005 siemens/metre. Therefore, the coefficients of the approximate formulae at permittivity values other than 15 esu tabulated by Brunavs have to be determined.

ii) Land conductivities can be as high as 1.0 siemens/metre (see Figure 5.1) and therefore the gap between 0.03 to 1 siemens/metre, left by Brunavs earlier work has to be computed.

iii) For completeness, the gap between 1.0 to 2.0 siemens/metre (sea water conductivities) left by Brunavs needs also to be filled.

iv) For fresh water whose conductivity and permittivity values of 0.005 siemens/metre and 80 esu (see Figure 5.1) respectively, a separate set of coefficients has to be determined. This is because the difference in the permittivity value with land permittivity values is too great.

Since Formula C is more accurate, it will be used in this extension work.

5.2.1 Selecting Corresponding Conductivity and Permittivity Values

Figure 5.1 (Wells et al, 1984) shows the correlation between conductivity and permittivity. From this figure it can be seen that all types of water (distilled, fresh or sea water) have a permittivity value of 80 esu while land permittivity varies with conductivity. For the purposes of this work, Table 5.1 can be derived from this figure for land conductivities.

CONDUCTIVITY (siemens/metre)	PERMITTIVITY (esu)
> 0.005	Independent of permittivity : taken at 15.
0.0001 to 0.001	3 to 30
0.00001 to 0.0001	3 to 5

Table 5.1 Correlation between Conductivity and Permittivity for Land.

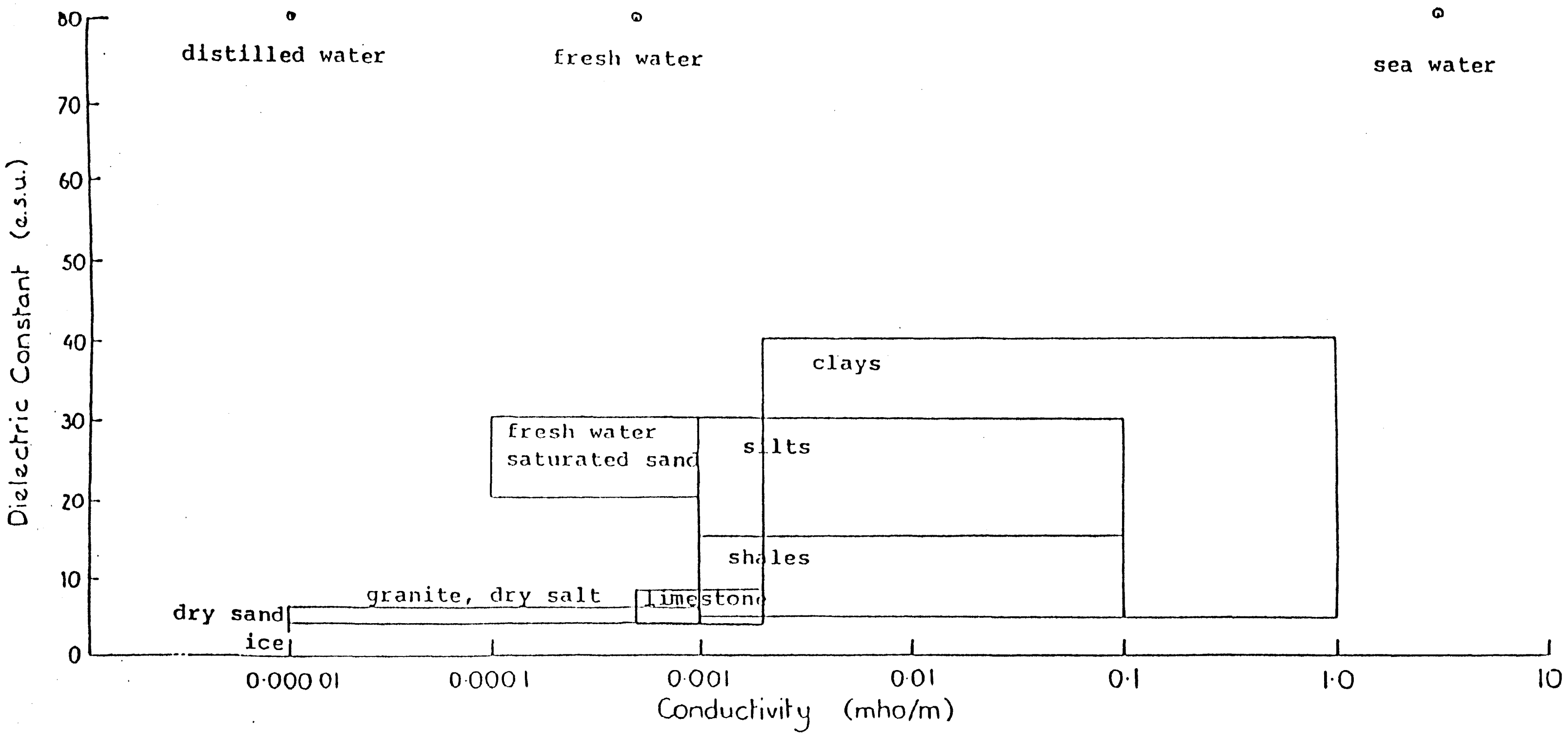

FIGURE 5.1

Collett's [1983] correlation of conductivity with permittivity.

With the help of Table 5.1 and Figure 5.1, it was decided to compute the C coefficients at the following permittivity values:-

i) For conductivities of 0.0001 to 0.005 siemens/metre, the permittivity values used are 3, 5, 10, 15, 20, 25 and 30 esu.

ii) For conductivities 0.00001 to 0.0001 siemens/metre, the permittivity values used are 3,4 and 5 esu.

5.2.2 Method of Computation

In his work, Brunavs did not mention the method used to determine the C coeffients. In this report, the normal method of parametric least square adjustment with weighted parameters and with the weight of observable set to unity is used. The mathematical model is his C formula:-

$$f(X,l) = C_1 + C_2S + (C_3S + C_4)e^{C_5S} + C_6/(1 + C_7S + C_8S^4) + 2.227/S - TPL = 0$$

and the least squares solution is:

$$\delta = (A^TA + P_x)^{-1}A^TW$$

$$X = X_0 + \delta$$

with residuals

$$r = -(A\delta + W)$$

where

A = first design matrix = $\partial f / \partial x$

W = $f(X, l)$

P_X = the weight of weighted parameters

X_o = initial values of X

X = $[\, C_1 \; C_2 \; C_3 \; C_4 \; C_5 \; C_6 \; C_7 \; C_8 \,]^T$

l = TPL

S = constant = Distance/100000 m

It should be noted here that all the C coefficients were weighted by 10% of the apriori values. The apriori coefficient values were taken from those determined by Brunavs . The main objective in this computation was to obtain the C coefficients that give residuals of less than 6 metres.

In Brunavs' work, he used distances from 2 to 5000 km. The program SEPLAG was developed for distances ranging from 1 to 2000 km only. Since low frequency positioning systems such as LORAN - C has a maximum range of 2000 Km, the use of distances from 2 to 2000 km is therefore more practical.

5.3 RESULTS.

During a test computation using a conductivity value already computed by Brunavs, it was found that there were some discrepancies in the coefficients obtained although the residuals were within the acceptable limit. It was first attributed to the difference in the range of input data. However, the result were almost identical when the same set of data input as used by Brunavs were used. Therefore it was decided to use the results obtained in this computation whenever necessary.

5.3.1 Conductivity between 0.0001 and 0.005 siemens/metre

The coefficients computed at this conductivity range for seven permittivity values are tabulated in Table II.2 to II.8 of Appendix II. Except for conductivities of 0.0001 and 0.0002 at permittivity values of 3,5 and 10 esu, all sets of coefficients were found to give residuals less than 3.0 metres. At those three permittivity and two conductivity values, the accuracy deteriorated but were still less than 5.0 metres. Examples of the difference between the accurate and approximate values are shown in Appendix III.

5.3.2 Conductivity between 0.00001 and 0.0001 siemens/metre

The coefficients computed at this conductivity range for three permittivity values are tabulated in Tables II.9 to II.11 of Appendix II. For these sets of coefficients, the worst discrepancy is 10.0 metres. Examples of the difference between the accurate and approximate values are shown in Appendix III.

5.3.3 Conductivity between 0.005 and 1.0 siemens/metre

For this conductivity range, the coefficients at conductivity values already computed by Brunavs were recomputed to be consistent with the new conductivity values ranging from 0.03 to 1.0 and are tabulated in Table II.12 of Appendix II. These new sets of coefficients were found to be accurate to within 3.0 metres. Example of the difference between accurate and approximate values are shown in Appendix III.

5.3.4 Sea Water Conductivity between 1 and 2 siemans/metre

Table II.12 shows that at high conductivity, C_6 approaches zero and therefore can be neglected. This puts the values of C_7 and C_8 also to zero. For this reason Brunavs tabulated C_6, C_7 and C_8 as zero. He went further ahead to make the computation of total phaselag for sea water easier by holding C_1,C_4 and C_5 constant for all conductivity values and only varied C_2 and C_3. This procedure was adopted to compute the C_2 and C_3 coefficients for conductivity values of 1.5 and 1.0 siemens/metre. At other conductivity values, Brunavs coefficients were adopted and the expanded coefficients are tabulated in Table II.13 of Appendix II. The differences between accurate and approximate values at these conductivities are also shown in Appendix III.

5.3.5 FRESH WATER CONDUCTIVITY.

The coefficients computed are tabulated in Table II.14 of Appendix II. The accuracy of this set of coefficients were found to be less than 2.0 metres (Appendix III).

Chapter 6

FITTING OF COEFFICIENTS AND PROGRAM TOPLAG.

In Chapter 5, the C coefficients were determined for a selected values of conductivity and permittivity only. In this chapter, an attempt is made to fit those coefficients with an approximate formula(e) so that interpolation can be made for other values.

6.1 FITTING OF COEFFICIENTS

From the results obtained in Chapter 5, it could be seen that, except for C_1, C_4, C_5, C_6, C_7 and C_8 at sea water conductivity, all coefficients vary with conductivity. For conductivities lower than 0.005 siemens/metre, they also vary with permittivity. To see the variation in each of the coefficients with conductivity, and permittivity at low conductivity, they were plotted in Figures IV.1 to IV.26 of Appendix IV

These figures show that C_2 and C_3 (Figures IV.1 to IV.2) at sea water conductivity and C_1 to C_8 (Figures IV.3 to IV.10) at land conductivity higher than 0.005 siemens/metre vary smoothly with conductivity . For low and very low land conductivity values, only some of the coefficients show uniform changes with conductivity and permittivity. For this reason, it was decided to fit only those coefficients at land conductivities higher than 0.005 siemens/metre, and C_2 and C_3 of the sea water conductivity. Due to their smooth variation with conductivity, a very simple function can be obtained.

6.1.1 Fitting of C_2 and C_3 at Sea Water Conductivity

Using the method of Least Squares, the two model curves were found to be best fitted by the following equation:

$$C = A_1 + A_2/\sigma_n + A_3/\sigma_n^{1/2} + A_4/\sigma_n^{1/4} \tag{6.1}$$

where $C = C_2$ or C_3 to be used in the approximate formula

A_i = coefficients as shown in Table 6.1 below

σ_n = conductivity x 10.0

C_i	A_1	A_2	A_3	A_4
C_2	97.882	- 11.437	40.006	- 13.576
C_3	- 18.496	63.283	5.844	7.714

Table 6.1 Coefficients for Fitting C_2 and C_3 for Sea Water.

The maximum residuals were found to be 0.04 and 0.12 respectively for C_2 aand C_3. These residuals are too small to affect the accuracy of the approximate formula. The examples of the difference between the accurate and approximate total phaselags where the C coefficients were calculated using this equation are shown in Appendix V.

6.1.2 Fitting of C_1 to C_8 for Land Conductivity above 0.005 siemens/metre

Figure IV.3 to IV.10 in Appendix IV show that all the curves except for C_7 and C_8 are quite smooth. However, C_7 and C_8 are not very sensitive and therefore will not affect the accuracy of the approximate formula computation. These figures also show that they have the same characteristic with those in Figure IV.1 and IV.2. For that reason they were fitted with the same equation (equation 6.1) given in Section 6.1.1 and found to give satisfactory result. Their coefficients and maximum residuals are shown in Table 6.2 .

C_i	A_1	A_2	A_3	A_4	MAXIMUM RESIDUAL
C_1	- 128.059	1.085	65.474	5.902	0.2
C_2	97.3809	- 0.615	20.238	- 4.107	0.07
C_3	19.048	0.049	- 4.812	22.139	0.3
C_4	126.620	- 0.901	- 38.415	- 11.674	0.3
C_5	- 0.5273	- 0.0025	0.0387	- 0.0927	0.002
C_6	- 4.055	- 0.024	- 30.266	23.080	0.7
C_7	8.421	- 0.494	5.677	- 5.644	0.11
C_8	315.57	22.37	- 346.46	448.31	40.0

Table 6.2 Coefficients for Fitting C Coefficients for Land Conductivity above 0.005 siemens/metre.

Chapter 7

CONCLUSIONS AND RECOMMENDATIONS

In this chapter, the results of this report are summarized. Conclusions and recommendations are made on the Johler's model for computing the secondary phaselag, secondary phaselag variation with input parameters and the approximate formula of computing the total phaselag.

7.1 COMPUTATION OF SECONDARY PHASELAG

Two methods of computing secondary phaselag have been studied (Chapters 3 and 4) and the following conclusions and recommendations are drawn.

7.1.1 Spherical Earth Theory

At distances over 250 km, this method has been shown to be quite easy, requiring not more than 50 terms. However, the method becomes more complicated as the distance decreases. For very short distances, say below 50 km, the number of terms needed to obtain an accurate result becomes very large and requires a lot of computing time. It has also been found that by increasing the maximum number of terms in the computation to greater than 620 used in this study, no substantial improvement was obtained. Therefore for distances of 10 km and below, plane earth theory computation should be used.

7.1.2 Methods of Computing τ_s

It has been found that the Ascending and Descending Series of computing τ_s were not suitable for conductivity values lower than 0.001 siemens/metre due to convergency problems (Table 4.2). This becomes more critical as conductivity and permittivity decrease. Therefore at certain stages where this method fails to converge, the method of using Hankel function should be used.

7.1.3 Plane Earth Theory

It has been found that, at higher conductivity values (above 0.005 siemens/metre), the plane earth theory computation gives identical result with the spherical earth method for distance of 10 km (Table 4.1). However, at lower conductivity values, the distance where the two methods of computation give identical results, increases as conductivity decreases.

7.1.4 General Remarks

It has been found that the conductivity value where computation of τ_s using Ascending and Descending series does not converge and where the identical results of plane and spherical theory computation becomes greater than 10 km, is almost the same (Tables 4.1 and 4.2). As conductivity decreases, the non-convergency of the series increases and the distance where the two methods of computing secondary phaselag give identical results also increases The number of spherical terms however decreases. It is therefore felt that the spherical earth computation, due to the approximation in the Hankel Function which forces the computation of τ_s to converge a little too early, is not quite accurate.

7.2 SECONDARY PHASELAG VARIATIONS WITH INPUT PARAMETERS

The study on these variations have been made in Chapter 4 and the following conclusions are drawn.

7.2.1 Variation With Distances

At distances greater than 30 km, secondary phaselag increases with distance. At long distances (over 500 km), the increment is linear. In between this two distances, the variation pattern differs depending on the conductivity values. At very short distances, the variation differs greatly depending on the conductivity values. For higher conductivity values, there is a pattern where secondary phaselag decreases with distance. This pattern which sometimes called near-field pattern and varies with conductivity. In another words, the near-field pattern decreases in distance as conductivity decreases.

7.2.2 Variation With Conductivity

Generally, secondary phaselag increases as conductivity decreases. However, this pattern is reversed at very low conductivities (below 0.0005 siemens/metre).

7.2.3 Variation With Permittivity

At high conductivities, permittivity has no effect on the secondary phaselag. For low conductivities (lower than 0.005 siemens/metre), secondary phaselag increases as permittivity decreases.

7.3 APPROXIMATE FORMULA

The scope on the use of approximate C Formula developed by Brunavs has been expanded in Chapters 5 and 6. Brunavs original 40 sets of eight C coefficients (Table II.1) has now been expanded to 206 sets. All these are collected together in Appendix II. With the results obtained in these chapters, the following conclusion and recommendation can be made.

7.3.1 Scope of Use

This C Formula can now be used to cover a much wider scope of conductivity and permittivity. Additional C coefficients have been computed in this report and tabulated for conductivity values from 0.00001 to 5.5 siemens/metre and permittivity values from 3 to 80 esu. These conductivity and permittivity ranges are typical of those usually found in the field.

7.3.2 Accuracy

For conductivity higher than 0.0001 siemens/metre, the coefficients computed for the approximate formula are capable of giving accuracy better than 5.0 metres. If this formula is used for distances between 3 to 2000 km, an accuracy of better than 2.0m can be expected. For conductivity between 0.00001 and 0.0001, the accuracy has been found to deteriorate. For this range of conductivity, the worst discrepancy is 10.0 metres.

7.3.3 Interpolation

The coefficients for the approximate formula have determined for selected conductivity and permittivity values which therefore gives rise to interpolation problems. However, for conductivities higher than 0.005 siemens/metre, this problem has been taken care off with the development of 'A' Formula (equation 6.1) in Sections 6.1.1 and 6.1.2.

For conductivities lower than 0.005 siemens/metre, the variation of some coefficients with conductivity and permittivity were found to be rather irregular. Therefore simple function as equation 6.1 could not be used. However, the selected conductivity and permittivity values as shown in Section 5.2.1 are very close together which make interpolation quite easy. If accurate results are required, compute at least four values straddling the required conductivity and plot these results to form a smooth curve. A guide for the interpolation in between permittivity is that for conductivities higher than 0.0005 siemens/metre, secondary phaselag varies linearly with permittivity.

7.4 SUGGESTIONS FOR FUTURE WORKS

The following areas are recommended for further investigations.

1. As mentioned in Section 7.1.4, the method of computing τ_s using Hankel Function is suspected quite inaccurate. Therefore it should be investigated to improve the accuracy or another method should be looked into as an alternative.
2. There are some interpolation problems in using approximate formula for conductivities below 0.005 siemens/metre as mentioned in Section 7.3.2. Therefore a study should be carried out to obtain a simple function quite simillar to that of eqution 6.1so that interpolation can be done mathematically.
3. Refering to Figures 4.1a, 4.1b, 4.2 and 4.3, a new approximate formula should be developed for relatively long distances. For short distances, plane computation can be used. Separate algorithms should be considered for short and long lines.

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Appendix I

PROGRAM SEPLAG

I.1 INTRODUCTION

This program computes the secondary phaselag at a given distance,conductivity and permittivity. It has two options as follows:

1. To compute the secondary phaselag for distances from 1 to 2000Km for a given set of conductivity and permittivity values.
2. To compute the secondary phaselag for a given distance,conductivity and permittivity values only.

Since frequency forms one of the input data, Option 1 can also be used for frequencies other than 100KHz. Option 2 incorporates the finding of this report at 100KHz frequency and therefore some errors can be expected at other frequencies. It should be noted that Option 2 is designed to reduce computing time especially at large distances and therefore necessary care should be given so that it serves the purpose.

The user's guide to this program is given in Section I.2 and its listing and basic structure are given in Section I.3 and Figure I.1 respectively.

I.2 USER'S GUIDE TO PROGRAM SEPLAG

1. PURPOSE To compute the secondary phaselag correction to measured distances in electronic positioning systems employing low and medium frequency radio waves at a given conductivity and permittivity values.

2. OPTIONS This programs allows the user to select one of the two options available for different purposes.

   1. OPTION 1 : To compute the secondary phaselag for distances from 1 to 2000 Km for a given set of conductivity and permittivity. This option should only be used if the computations are needed at many distances within the range ot 1 to 2000 km where all these distances have the same values of conductivity and permittivity.

   2. OPTION 2: To compute the secondary phaselag at a very specific distance with a specific set of conductivity and permittivity values. To save some computing time,this option also allows the computation at few other distances if and only if the conductivity and permittivity remain the same. However, the user must be very careful with the input data.

3. INPUT DATA All input data are free formatted and therefore a space(blank) is required in between data.

.........First data The first data input is 1 OR 2 to select the option

.... <u>Option 1</u> If the <u>OPTION is 1</u> the the followings should be followed:

Enter NCODE,frequency(Hz),conductivity(siemens/metre), permittivity(esu).

e.g. 0 100000.0 0.005 15.0

NCODE indicates the end of data if entered as 1 or greater (integer).

If there is only one set of data,the input data should be as below:

.... sigle set 1

0 100000.0 0.005 15.0

1 0.0 0.0 0.0

if there are two or more sets of data, the following example can be used:

....multiple set 1

0 100000.0 0.005 15.0

0 100000.0 1.5 80.0

0 100000.0 0.0005 10.0

1 0.0 0.0 0.0

.... <u>Option 2</u> If the <u>OPTION is 2,</u> the followings should be followed:

a) frequency(Hz),conductivity(siemens/metre),permittivity(esu)

b) distance(metres)

e.g.

......one distance 2

100000.0 0.005 15.0

100000.0

0.0

......few distances 0.0 indicates the end of distance input.

if there are afew distances to be computed at the same conductivity and permittivity,just add the distance input and end it with 0.0. However the user must remember that the distance input must be in increasing order.

e.g.

```
2
100000.0 0.005 15.0
50000.0
100000.0
150000.0
500000.0
1000000.0
0.0
```

If the order is not right, an error maasage will be given.

4. HOW TO USE AND JCL CARDS

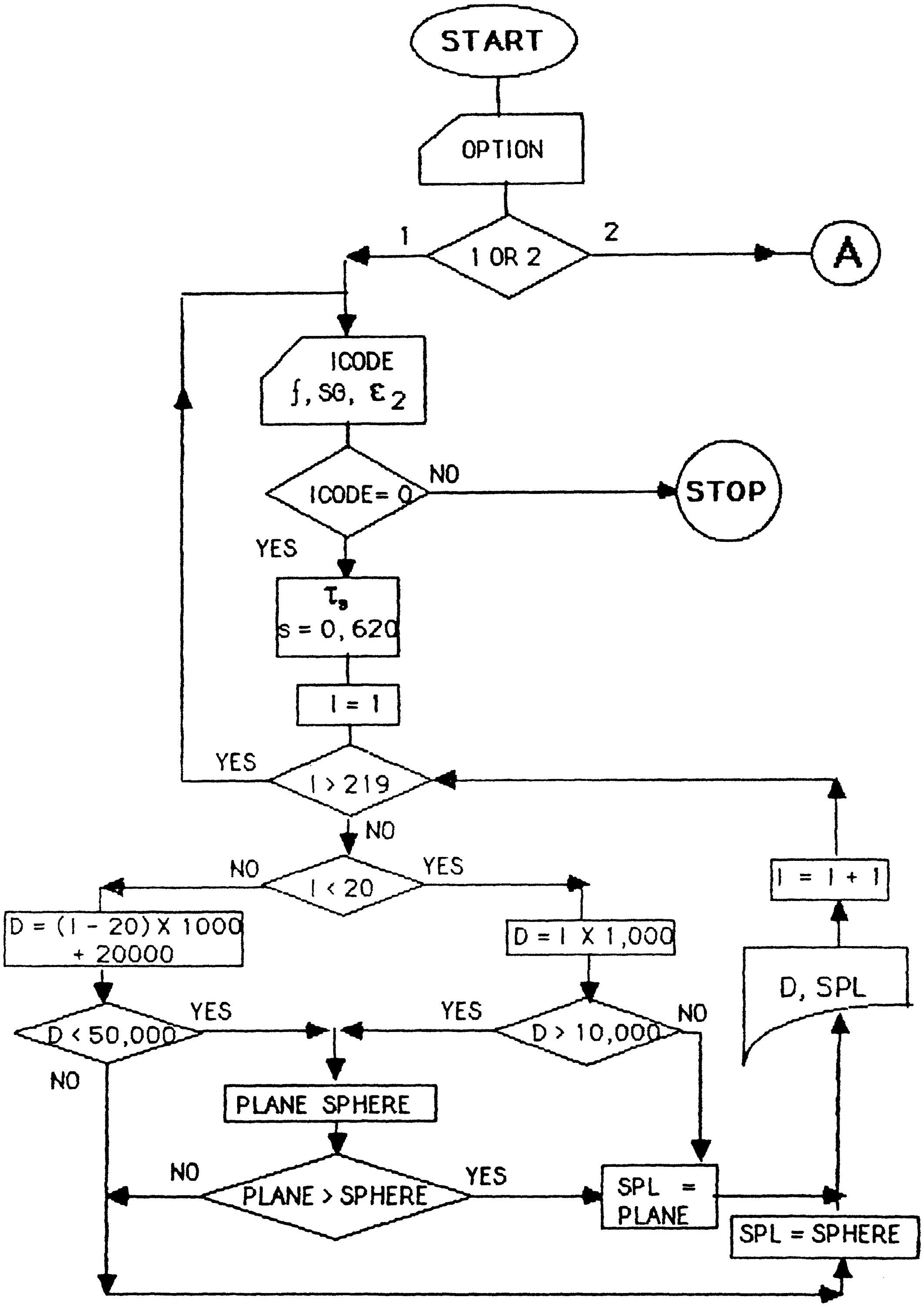
This program is in the department's library and therefore the following JCL cards can be used :

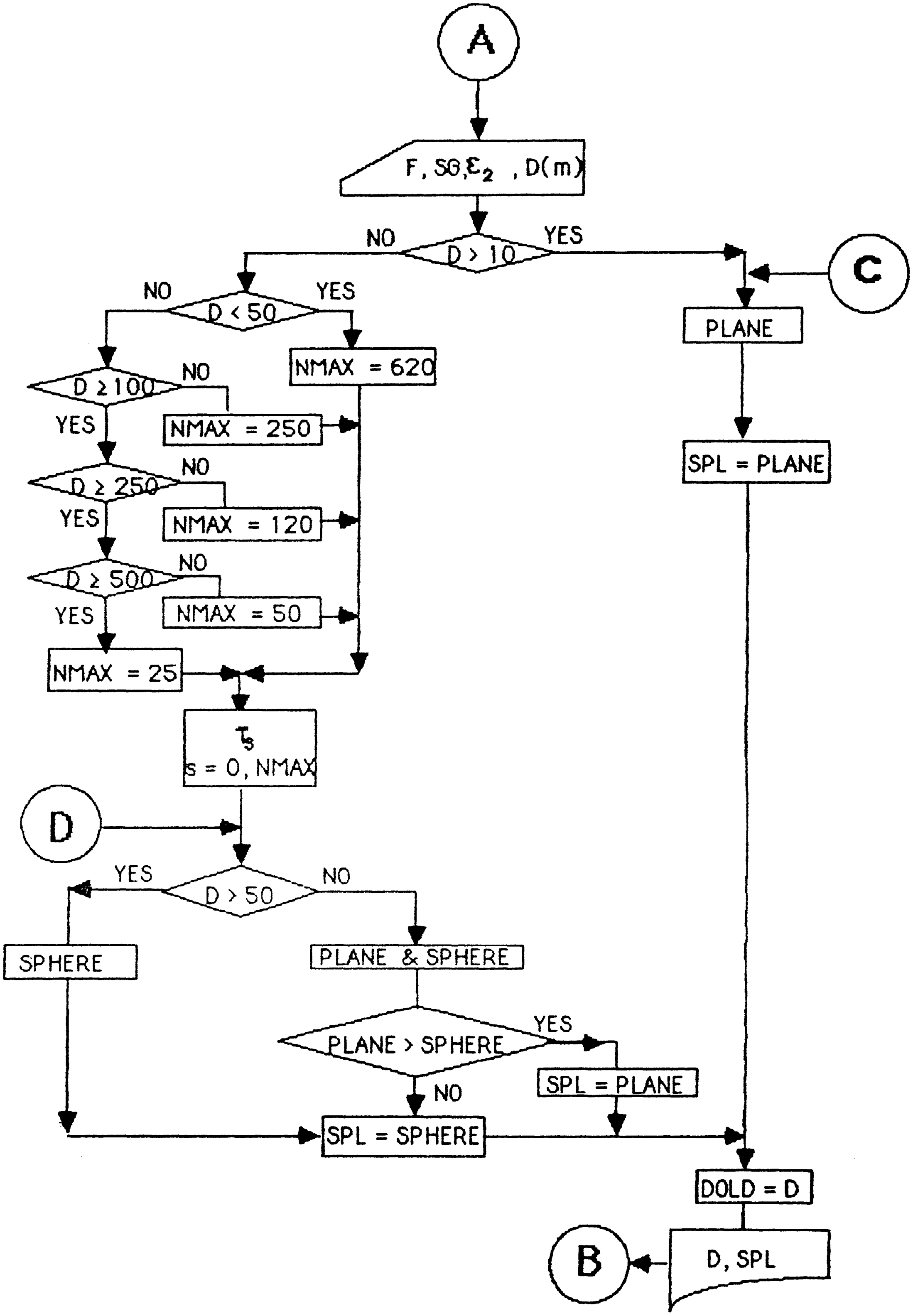
```
//                JOB ,SE1234
/*JOBPARM         S=5,L=99,R=512K
/*SERVICE         -4
//*
//                EXEC FORTVG,RG=512K,GOPGM=SEPLAG
//STEPLIB         DD  DSN=A.M12129.SELIBOJ,DISP=SHIR
//SYSIN           DD  *

DATA FOR THE PROGRAM

//
```

5. REMARKS

i. Option 2 should be used if there are only a few distances need to be computed at a set of conductivity and permittivity. It was designed to eliminate unneccessary computation. Since the design was based on findings at 100KHz frequency, some errors are to be expected if used at other frequencies.

ii. Option 1 can be used for frequencies other than 100KHz without expecting much problems. It was desinged for research works and should therefore be used sparely to avoid unneccessary wasting of computing time.


START
OPTION
1 OR 2
1
2
A
ICODE
∫, SG, ε2
ICODE= 0
NO
STOP
YES
τs
s = 0, 620
I = 1
YES
I > 219
NO
NO
I < 20
YES
D = (I - 20) X 1000 + 20000
D = I X 1,000
I = I + 1
D, SPL
D < 50,000
YES
NO
YES
D > 10,000
NO
PLANE SPHERE
NO
PLANE > SPHERE
YES
SPL = PLANE
SPL = SPHERE


A
F, SG, ε_2 , D(m)
D > 10
NO
YES
C
D < 50
NO
YES
PLANE
NMAX = 620
D ≥ 100
NO
YES
NMAX = 250
SPL = PLANE
D ≥ 250
NO
YES
NMAX = 120
D ≥ 500
NO
YES
NMAX = 50
NMAX = 25
τ_s
s = 0, NMAX
D
D > 50
YES
NO
SPHERE
PLANE & SPHERE
PLANE > SPHERE
YES
NO
SPL = PLANE
SPL = SPHERE
DOLD = D
B
D, SPL

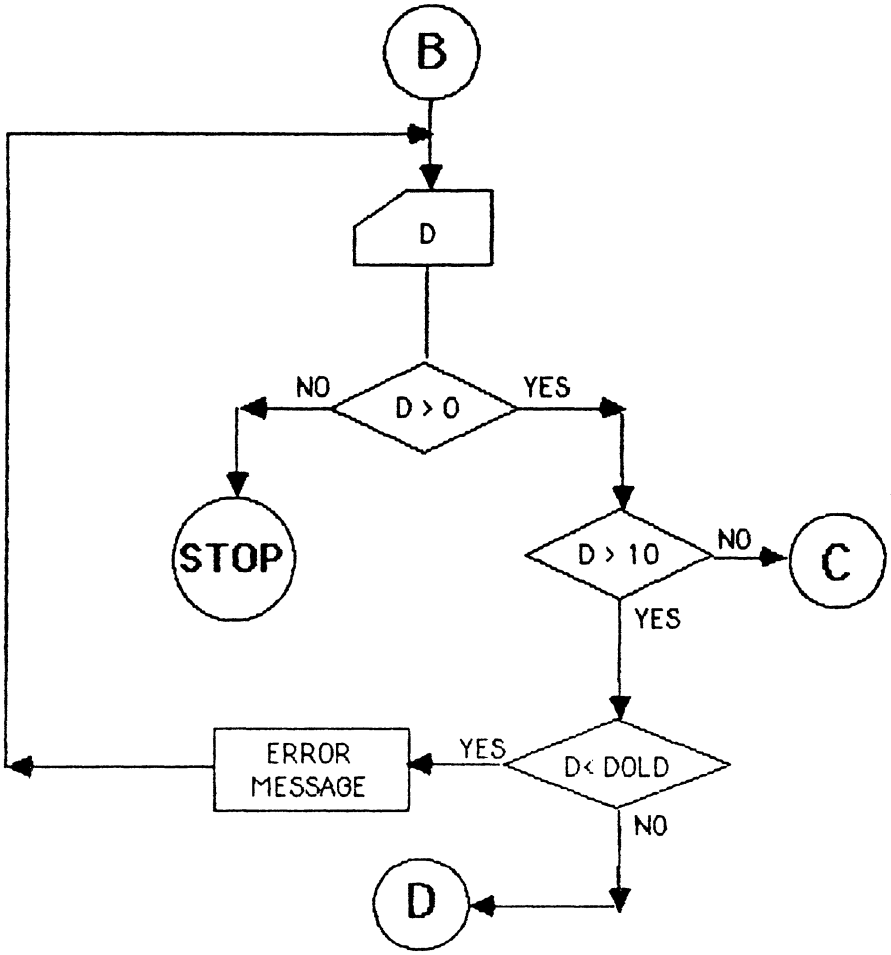


Figure I.1 Basic Structure of Program SEPLAG.

I.3 - Program Listing

```
      IMPLICIT REAL *8(A-B,D-H,O-Z),COMPLEX *16(C)
      REAL *8 K1,KE
      DIMENSION CTS(625),SPL(250)
C*********************************************************************
C* NAME            SEPLAG                                            *
C*                                                                   *
C* TYPE            MAIN                                              *
C*                                                                   *
C* PURPOSE         TO COMPUTE THE SECONDARY PHASELAG CORRECTION(M)   *
C*                 USING SPHERICAL AND PLANE EARTH THEORIES          *
C*                                                                   *
C* AUTHOR          P.BRUNAVS,1977                                    *
C* MODIFIED BY     T.MAHMOOD,AUGUST 1986                             *
C*                                                                   *
C* EXTERNALS       TSZERO,TSINTY,QUAD,ASYMP,PLANE,SPHERE,TRAPS       *
C*                 NOTE:TRAPS IS AN IBM ROUTINE TO TRAP              *
C*                      EXPONENT UNDERFLOW                           *
C*                                                                   *
C* PARAMETERS                                                        *
C*                 NOPTN = OPTION : 1 OR 2                           *
C*                 ICODE = END OF DATA(GT.0) FOR OPTION 1            *
C*                 FRQ   = FREQUANCY (HZ)                            *
C*                 SG    = CONDUCTIVITY (MHOS/METER)                 *
C*                 V     = VELOCITY (METERS/SEC)                     *
C*                 A     = RADIUS OF THE EARTH(METER)                *
C*                 ALF   = VERTICAL LAPSE RATE OF THE OF             *
C*                         THE PERMITTIVITY OF THE ATMOSPHERE        *
C*                 D     = DISTANCE (METER)                          *
C*                 W     = ANGULAR FREQUENCY (RADIANS/SEC)           *
C*                 E1    = PERMITTIVITY OF THE AIR  (E.S.U)          *
C*                       = 1.000676 (ASSUMED)                        *
C*                 E2    = DIELECTRIC CONSTANT OR PERMITIVITY        *
C*                         OF THE EARTH (E.S.U)                      *
C*                 K1    = WAVE NUMBER OF THE ATMOSPHERE (RAD/METER) *
C*                 CDE   = CONDUCTIVITY AND PERMITTIVITY PARAMETER   *
C*                         FOR A VERTICAL DIPOLE SOURCE              *
C*                 KE    = THE MODULUS OR AMPLITUDE OF CDE           *
C*                 CTS   = TAU S ;SOLUTION OF RICATTI EQUATION       *
C*                 IS    = SUBSCRIPT ASSOCIATED WITH CTS             *
C*                 LFA   = CODES USED TO DETERMINE THE CONVERGENCY   *
C*                         OF THE METHODS OF COMPUTING TAU S         *
C*                       = 1  DOES NOT CONVERGE,>0.1                 *
C*                       = 2  DOES NOT CONVERVE                      *
C*                       = 3  DOES NOT CONVERGE AFTER FULL ITERATION *
C*                       = 4  CONVERGE AFTER FULL ITERATION          *
C*                       = 5  CONVERGE, <CTS*10**-9                  *
C*                         (USING ASCENDING/DESCENDING SERIES)       *
C*                       > 32 ...USING HANKEL FUNCTION               *
C*                 SPL   = SECONDARY PHASELAG (METERS)               *
C*                 SPLINC= SECONDARY PHASELAG INCREMENT (METER)      *
C*                 NC    = DENOTES THE METHOD USED IN THE COMPUTATION*
C*                       = 1 ..PLANE EARTH THEORY                    *
C*                       = 2 ..SPHERICAL EARTH THEORY                *
C*                                                                   *
C* LANGUAGE        FORTRAN                                           *
C*                                                                   *
C* REFERENCES      JOHLER ET AL (1956) : NBS CIRCULAR 573            *
C*                 HOWE (1960) : NBS JOURNAL,VOL.64B,#2              *
C*                 WALTERS ET AL (1962) : NBS JOURNAL,VOL.66D,#1     *
C*********************************************************************
      DATA IR,IP,IW/5,6,6/
      READ(IR,*) NOPTN
      IF(NOPTN.EQ.1) GO TO 100
      READ(IR,*) FRQ,SG,E2
      READ(IR,*) D
```

```
      IF(D.LT.50000.0D0)                         NMX = 620
      IF(D.LT.100000.D0.AND.D.GE.50000.D0)       NMX = 250
      IF(D.LT.250000.D0.AND.D.GE.100000.D0)      NMX = 120
      IF(D.LT.500000.D0.AND.D.GE.250000.D0)      NMX = 50
      IF(D.GE.500000.D0)                         NMX = 25
      GO TO 101
  100 READ (IR,*) ICODE,FRQ,SG,E2
      IF(ICODE.GE.1)GO TO 200
      NMX = 620
  101 CC1  = DCMPLX(1.D0,0.D0)
      CC2  = DCMPLX(0.D0,1.D0)
      C1   = DCMPLX(0.5D0,-0.5D0*DSQRT(3.D0))
      C2   = DCMPLX(-0.5D0,-0.5D0*DSQRT(3.D0))
      PI   = 4.D0*DATAN(1.D0)
      V    = 2.997925D+8
      E1   = 1.000676D0
      A    = 6.36739D+6
      ALF  = 0.75D0
      W    = 2.D0*PI*FRQ
      K1   = (W/V)*DSQRT(E1)
      SMC  = (SG*PI*V*V*4.0D-7)/W
      KE   = (((V*ALF)/(A*W*E1*E1))**4*(E2*E2+SMC*SMC)**6
     &        /((E2-E1)**2+SMC*SMC)**3)**(1.D0/12.D0)
      PSI  = DATAN(E2/SMC)-0.5D0*DATAN((E2-E1)/SMC)
      PP   = PI*0.75D0-PSI
      CPPI = DCMPLX(0.D0,PP)
      CDE  = DCMPLX(KE,0.D0)*CDEXP(CPPI)
      FRC  = (K1*A*ALF*ALF)**(1.D0/3.D0)
C
      IF(NOPTN.EQ.2.AND.D.LE.10000.0D0) GO TO 121
C
C*********  COMPUTATION OF TAU S(CTS)  **********
C           """""""""""""""""""""""""
      ER3  = 0.1D0
      KP   = 1
      KOUT = 0
      N    = 0
  102 N   =N+1
      IS = N-1
      AMT= ER3
      CTS(N)= DCMPLX(0.D0,0.D0)
      CTSST = DCMPLX(0.D0,0.D0)
      CDIFT = DCMPLX(0.D0,0.D0)
      IF(KP.EQ.2) GO TO 105
C
C...........  USING ASCENDING SERIES  ............
C             """"""""""""""""""""""
      CALL TSZERO(IS,CDE,CTSV,AMA,LFA)
      CTSST=CTSV
      AMT  =AMA
      GO TO (103,200,104,114,114),LFA
  103 KOUT=1
  104 KP  =2
C
C.............    USING DESCENDING SERIES  ...........
C                 """""""""""""""""""""""
  105 CALL TSINTY(IS,CDE,CTSV,AMA,LFA)
      GO TO (106,200,107,114,114),LFA
  106 IF (KOUT.EQ.1) GO TO 200
  107 KP=1
      IF(AMA.GT.AMT) GO TO 108
      KP=2
      CTSST=CTSV
      AMT  =AMA
  108 CTSS =CTSST
```

```
      TIM  =0.0001D0
      TRE  =0.0001D0
      J    =25
      ITS  =0
C
C.............  USING HANKEL FUNCTIONS  .............
C               **********************
  109 ITS  =ITS+1
      CTSI =CTSS
      DO 112 IK=1,3
           IF(IK.EQ.2) CTSI=CTSS+DCMPLX(TRE,0.D0)
           IF(IK.EQ.3) CTSI=CTSS+DCMPLX(0.D0,TIM)
           CZH  =(CDSQRT(CTSI*2)**3*DCMPLX(0.D0,-1.D0))/3.D0
           CRUTS=CDSQRT(-CTSI*2)
           KIND =1
           CKNDM=DCMPLX(0.D0,1.D0)
C
C.....     IF(KIND.EQ.2) CKNDM=-CKNDM
C
C.....     EVALUATION OF HANKEL FUNCTIONS
C
           DO 110 KV=1,2
                VK3=DFLOAT(KV)/3.D0
                IF(N.LE.J) CALL QUAD(CKNDM,VK3,CZH,CHNK)
                IF(N.GE.(J+1)) CALL ASYMP(CKNDM,VK3,CZH,CHNK,IRT)
                IF(KV.EQ.1) CHNK1=-CHNK
  110      CONTINUE
           CHNK2=-CHNK
           CZH=DCONJG(CZH)
           DO 111 KV=1,2
                VK3=DFLOAT(KV)/3.D0
                IF(N.LE.J) CALL QUAD(CKNDM,VK3,CZH,CHNK)
                IF(N.GE.(J+1)) CALL ASYMP(CKNDM,VK3,CZH,CHNK,IRT)
                IF(KV.EQ.1)CHNK1=CHNK1-DCONJG(CHNK)*C1
  111      CONTINUE
           CZH  =DCONJG(CZH)
           CHNK2=CHNK2+DCONJG(CHNK)*C2
           CHTS =(CHNK1/CHNK2)*DCMPLX(0.5D0,0.5D0*DSQRT(3.D0))/CRUTS
           IF(IK.EQ.1) CDEH=CHTS
           IF(IK.EQ.2) CAA =CHTS-CDEH
  112 CONTINUE
      CBB  =CHTS-CDEH
      CC   =CDE-CDEH
      DTRE =(DIMAG(CC/CBB)/DIMAG(CAA/CBB))*TRE
      DTIM =(DIMAG(CC/CAA)/DIMAG(CBB/CAA))*TIM
      CDTSI=DCMPLX(DTRE,DTIM)
      CTSS =CTSS+CDTSI
      IF(CDABS(CDTSI).LT.1.0D-7) GO TO 113
      IF(ITS.LT.10) GO TO 109
      LFA   =1
  113 LFA   =LFA+32
      CTSV  =CTSS
  114 CTS(N)=CTSV
      IF(N.LT.NMX) GO TO 102
C
C*********  COMPUTATION OF SECONDARY PHASELAG  **********
C           *********************************
      IF (NOPTN.EQ.2) GO TO 121
C
C.............   OPTION 1  .............
C                ********
      WRITE(IW,1001)
      WRITE(IW,1002) FRQ,SG,E2
      WRITE(IW,1003)
      FYP = 0.0D0
```

```
      DO 120 K=1,219
           IF(K.LT.20) D=K*1000.D0
           IF(K.GE.20) D=(K-20)*10000.D0+20000.D0
           IF(D.LE.10000.0D0) GO TO 118
           IF(D.GT.50000.0D0) GO TO 117
           CALL PLANE(D,FRQ,V,SG,E1,E2,ALF,A,PLP)
           CALL SPHERE(D,V,A,CDE,CTS,K1,FRC,W,NMX,FYP,FYN,PLS)
           IF(PLS.GE.PLP) GO TO 116
           SPL(K) = PLP
           NC     = 1
           GO TO 119
  116      SPL(K) = PLS
           NC     = 2
           GO TO 119
  117      CALL SPHERE(D,V,A,CDE,CTS,K1,FRC,W,NMX,FYP,FYN,SPL(K))
           NC     = 2
           GO TO 119
  118      CALL PLANE(D,FRQ,V,SG,E1,E2,ALF,A,SPL(K))
           NC     = 1
  119      IF(K.EQ.1) SPLINC = 0.0D0
           IF(K.GT.1) SPLINC = SPL(K)-SPL(K-1)
           FYP = FYN
           WRITE(IW,1004) D,SPL(K),SPLINC,NC
  120 CONTINUE
      GO TO 100
C
C............  OPTION 2  ..............
C              ********
  121 WRITE(IW,1001)
      WRITE(IW,1002) FRQ,SG,E2
      WRITE(IW,1005)
      FYP  = 0.0D0
      IF(D.LE.10000.0D0) GO TO 125
      CALL SPHERE(1000000.D0,V,A,CDE,CTS,K1,FRC,W,NMX,FYP,FYN,SPL0)
  122 IF(D.GT.50000.0D0) GO TO 124
      CALL PLANE(D,FRQ,V,SG,E1,E2,ALF,A,PLP)
      CALL SPHERE(D,V,A,CDE,CTS,K1,FRC,W,NMX,FYP,FYN,PLS)
      IF(PLS.GE.PLP) GO TO 123
      SPL1 = PLP
      NC   = 1
      GO TO 126
  123 SPL1 = PLS
      NC   = 2
      GO TO 126
  124 IF (D.LT.1000000.D0) GO TO 1241
      CALL SPHERE(D,V,A,CDE,CTS,K1,FRC,W,NMX,FYP,FYN,SPL1)
      IF(SPL1.LT.SPL0) SPL1 =SPL1+V/W
      GO TO 1242
 1241 CALL SPHERE(D,V,A,CDE,CTS,K1,FRC,W,NMX,FYP,FYN,SPL1)
 1242 NC   = 2
      GO TO 126
  125 CALL PLANE(D,FRQ,V,SG,E1,E2,ALF,A,SPL1)
      NC   = 1
  126 WRITE(IW,1006)D,SPL1,NC
      DOLD = D
  127 READ(IR,*)D
      IF(D.LE.0.0D0) GOTO 200
      IF(D.LE.10000.0D0) GO TO 125
      IF(D.GT.DOLD) GO TO 128
      WRITE(IW,1007)D
      GO TO 127
  128 GO TO 122
 1001 FORMAT('1',' ')
 1002 FORMAT(///10X,'FREQUENCY     =',F9.1,3X,'HZ'/10X,'CONDUCTIVITY =',
     &  F10.6,2X,'SEIMENS/METER'/10X,'PERMITIVITY  =',F5.1,7X,'E.S.U'//)
```

```
 1003 FORMAT(10X,'DISTANCE',10X,'SEC.PHASELAG',10X,'INCREMENT',
     &  10X,'1 = PLANE'/13X,'(M)',17X,'(M)',17X,'(M)',13X,'2 = SPHERE'/)
 1004 FORMAT(9X,F9.1,13X,F6.1,14X,F6.1,16X,I1)
 1005 FORMAT(10X,'DISTANCE',10X,'SEC.PHASELAG',10X,'1 = PLANE'/13X,
     &  '(M)',17X,'(M)',14X,'2 = SPHERE'/)
 1006 FORMAT(8X,F10.2,13X,F6.1,17X,I1)
 1007 FORMAT(/10X,'*****  ERROR .... ERROR *****'/
     &        10X,'YOUR DISTANCE:',F10.2,' NOT IN ORDER......'/)
  200 WRITE(IW,1001)
      STOP
      END
      SUBROUTINE TSINTY(IS,CDE,CTSV,AMA,LFA)
C*********************************************************************
C* NAME            TSINTY                                            *
C*                                                                   *
C* TYPE            SUBROUTINE                                        *
C*                                                                   *
C* PURPOSE         TO COMPUTE TAU S(CTS) USING DESCENDING SERIES     *
C*                                                                   *
C* AUTHOR          P.BRUNAVS, 1977                                   *
C* MODIFIED BY     T.MAHMOOD, OCT.1981                               *
C*                                                                   *
C* EXTERNALS       NONE                                              *
C*                                                                   *
C* CALLING         CALL TSINTY(IS,CDE,CTSV,AMA,LFA)                  *
C*                                                                   *
C* PARAMETERS      ALL EXPLAINED IN THE MAIN                         *
C*                 INPUT : IS,CDE                                    *
C*                 OUTPUT: CTSV,AMA,LFA                              *
C*                                                                   *
C* LANGUAGE        FORTRAN                                           *
C*                                                                   *
C* REFERENCES      JOHLER ET AL(1956) : NBS CIRCULAR 573             *
C*                 HOWE (1960) : NBS JOURNAL,VOL.64B,#2              *
C*********************************************************************
      IMPLICIT REAL *8(A-B,D-H,O-Z),COMPLEX *16(C)
      DIMENSION CA(70),TU(6)
      TF(Y)=((Y*Y/2.D0)**(1.D0/3.D0))*(1.D0-(7.D0/(48.D0*Y*Y))
     &      +(35.D0/(288.D0 *Y*Y*Y*Y)))
      TU(1)=0.808616516D0
      TU(2)=2.57809613D0
      TU(3)=3.82571528D0
      TU(4)=4.89182029D0
      TU(5)=5.85130097D0
      TU(6)=6.73731638D0
      PI   =4.D0*DATAN(1.D0)
      N    =IS+1
      TAU  =TF((DFLOAT(IS)*4.D0+1.D0)*PI*0.375D0)
      IF(N.LE.6) TAU=TU(N)
      CTAUC=DCMPLX(0.5D0,0.5D0*DSQRT(3.0D0))*TAU
      ER   =TAU*1.0D-9
      ER2  =TAU*1.0D-7
      EMX  =1.0D0
      ER3  =0.1D0
      CAD  =-0.5D0/CTAUC
      CDE2 =1.D0/CDE**2
      CA(1)=CTAUC
      CA(2)=CAD/CDE
      CTSV =CA(1)+CA(2)
      LFA  =1
      DO 22 I=3,50
           DCI =(DFLOAT(I)-3.D0)/(DFLOAT(I)-1.D0)
           ME  =I-1
           CSUM=DCMPLX(0.D0,0.D0)
           DO 21 M=2,ME
```

```
   21       CSUM =CSUM+CA(M)*CA(ME-M+2)
          CA(I)=(CSUM-CA(I-2)*CDE2*DCI)*CAD
          CTSV =CTSV+CA(I)
          AMA  =CDABS(CA(I))
          IF(AMA.GT.EMX)GO TO 24
          IF(AMA.LT.ER )GO TO 23
   22 CONTINUE
      IF(AMA.GE.ER3) GO TO 24
      LFA=3
      IF(AMA.LT.ER2) LFA=4
      GO TO 24
   23 LFA=5
   24 RETURN
      END
      SUBROUTINE TSZERO(IS,CDE,CTSV,AMA,LFA)
C*********************************************************************
C* NAME            TSZERO                                            *
C*                                                                   *
C* TYPE            SUBROUTINE                                        *
C*                                                                   *
C* PURPOSE         TO COMPUTE TAU S(CTS) USING ASCENDING SERIES      *
C*                                                                   *
C* AUTHOR          P.BRUNAVS, 1977                                   *
C* MODIFIED BY     T.MAHMOOD, OCT.1981                               *
C*                                                                   *
C* EXTERNALS       NONE                                              *
C*                                                                   *
C* CALLING         CALL TSZERO(IS,CDE,CTSV,AMA,LFA)                  *
C*                                                                   *
C* PARAMETERS      ALL EXPLAINED IN THE MAIN                         *
C*                 INPUT : IS,CDE                                    *
C*                 OUTPUT:CTSV,AMA,LFA                               *
C*                                                                   *
C* LANGUAGE        FORTRAN                                           *
C*                                                                   *
C* REFRENCES       JOHLER ET AL (1956) : NBS CIRCULAR 573            *
C*                 HOWE (1960) : NBS JOURNAL,VOL.64B,#2              *
C*********************************************************************
      IMPLICIT REAL *8(A-B,D-H,O-Z),COMPLEX *16(C)
      DIMENSION CA(70),TU(6)
      TF(X)=((X*X/2.D0)**(1.D0/3.D0))*(1.D0+(5.D0/(48.D0*X*X))
     &     -(5.DC/(36.D0*X*X*X*X)))
      TU(1)=1.85575708D0
      TU(2)=3.24460762D0
      TU(3)=4.38167124D0
      TU(4)=5.38661378D0
      TU(5)=6.30526301D0
      TU(6)=7.16128272D0
      N    =IS+1
      PI   =4.D0*DATAN(1.D0)
      TAU  =TF((DFLOAT(IS)*4.D0+3.D0)*PI*0.375D0)
      IF(N.LE.6) TAU=TU(N)
      CTAUC=DCMPLX(0.5D0,0.5D0*DSQRT(3.D0))*TAU
      ER   =TAU*1.0D-9
      ER2  =TAU*1.0D-7
      EMX  =1.D0
      ER3  =0.1D0
      CDES =CDE**2
      CA(1)=CTAUC
      CA(2)=-CDE
      CA(3)=DCMPLX(0.D0,0.D0)
      CTSV =CA(1)+CA(2)+CA(3)
      LFA  = 1
      DO 32 I=4,50
           DCI=(DFLOAT(I)-3.D0)/(DFLOAT(I)-1.D0)
```

```
          ME =I-2
          CSUM=DCMPLX(0.D0,0.D0)
          DO 31 M=1,ME
  31          CSUM=CSUM+CA(M)*CA(ME-M+1)
          CA(I)=CSUM*CDES*DCI
          CTSV =CTSV+CA(I)
          AMA  =CDABS(CA(I))
          IF(AMA.GT.EMX)GO TO 34
          IF(AMA.LT.ER )GO TO 33
  32 CONTINUE
     IF(AMA.GE.ER3) GO TO 34
     LFA=3
     IF(AMA.LT.ER2) LFA=4
     GO TO 34
  33 LFA= 5
  34 RETURN
     END
     COMPLEX FUNCTION CSX(CKNDM,VK3,CZH,T)
C*********************************************************************
C* NAME              CSX                                             *
C*                                                                   *
C* TYPE              COMPLEX FUNCTION (*16)                          *
C*                                                                   *
C* PURPOSE           TO COMPUTE THE VALUE OF THE FIRST TERM OF       *
C*                   THE HANKEL FUNCTION                             *
C*                                                                   *
C* AUTHOR            P.BRUNAVS, 1977                                 *
C* MODIFIED BY       T.MAHMOOD, OCT.1981                             *
C*                                                                   *
C* EXTERNALS         NONE                                            *
C*                                                                   *
C* CALLING           X=CSX(CKNDM,VK3,CZH,T)                          *
C*                                                                   *
C* PARAMETERS        ALL EXPLAINED IN THE MAIN AND SUBROUTINE QUAD   *
C*                                                                   *
C* LANGUAGE          FORTRAN                                         *
C*                                                                   *
C* REFERENCES        WALTERS ET AL (1962) :NBS JOURNAL,VOL.66D,#1    *
C*********************************************************************
      IMPLICIT REAL *8(A-B,D-H,O-Z),COMPLEX *16(C)
      CALL TRAPS(0,0,500,0,0)
      CSX=CDEXP((CZH*DCMPLX(DSIN(T),0.D0)-DCMPLX(T,0.D0)*
     &    DCMPLX(VK3,0.D0))*CKNDM)
      RETURN
      END
      COMPLEX FUNCTION CSU(CKNDM,VK3,CZH,VV)
C*********************************************************************
C* NAME              CSU                                             *
C*                                                                   *
C* TYPE              COMPLEX FUNCTION (*16)                          *
C*                                                                   *
C* PURPOSE           TO COMPUTE THE VALUE OF THE SECOND TERM         *
C*                   OF THE HANKEL FUNCTION                          *
C*                                                                   *
C* AUTHOR            P.BRUNAVS, 1977                                 *
C* MODIFIED BY       T.MAHMOOD, OCT.1981                             *
C*                                                                   *
C* EXTERNALS         NONE                                            *
C*                                                                   *
C* CALLING           CALL CSU(CKNDM,VK3,CZH,VV)                      *
C*                                                                   *
C* PARAMETERS        ALL EXPLAINED IN THE MAIN AND SUBROUTINE QUAD   *
C*                                                                   *
C* LANGUAGE          FORTRAN                                         *
C*                                                                   *
```

```
C* REFRENCES        WALTERS ET AL (1962) : NBS JOURNAL,VOL.66D,#1      *
C*********************************************************************
      IMPLICIT REAL *8(A-B,D-H,O-Z),COMPLEX *16(C)
      PI =4.DO*DATAN(1.DO)
      FV =(VV-1.DO/VV)*0.5DO
      PV =-PI*VK3
      U  =VV**VK3
      CU =DCMPLX(U,0.DO)
      CPV=CKNDM*PV
      CEZ=CZH*FV
      CSU=DCMPLX(0.DO,0.DO)
      IF((DREAL(CEZ)+670.DO).LT.0.DO) GO TO 50
      CALL TRAPS(0,0,500,0,0)
      CM =(CKNDM*CDEXP(CEZ))/VV
      CALL TRAPS(0,0,500,0,0)
      CSU=CM/CU+(CM*CDEXP(CPV))*CU
   50 RETURN
      END
      SUBROUTINE QUAD(CKNDM,VK3,CZH,CHNK)
C*********************************************************************
C* NAME             QUAD                                               *
C*                                                                     *
C* TYPE             SUBROUTINE                                         *
C*                                                                     *
C* PURPOSE          EVALUATION OF DEFINITE INTERGRAL OF F(X)*DX FROM   *
C*                  X=A TO X=B USING GAUSS-LEGENDRE QUADRATURE         *
C*                  OF ORDER 48 TO EVALUATE THE HANKEL FUNCTIONS       *
C*                  OF ORDER VK3                                       *
C*                                                                     *
C* AUTHOR           P.BRUNAV, 1977                                     *
C* MODIFIED BY      T.MAHMOOD, OCT.1981                                *
C*                                                                     *
C* EXTERNALS        CSU,CSX                                            *
C*                                                                     *
C* CALLING          CALL QUAD(CKNDM,VK3,CZH,CHNK)                      *
C*                                                                     *
C* PARAMETERS       ALL EXPLAINED IN THE MAIN +                        *
C*                  R = GAUSSIAN ABSCISSAS TABULATED BY KOPAL          *
C*                  W = GAUSSIAN WEIGHT TABULATED BY KOPAL             *
C*                  INPUT : CKNDM,VK3,CZH                              *
C*                  OUTPUT: CHNK                                       *
C*                                                                     *
C* LANGUAGE         FORTRAN                                            *
C*                                                                     *
C* REFRENCES        WALTERS ET AL (1962) : NBS JOURNAL,VOL.66D,#1      *
C*                  F.KOPAL (1955) : NUMERICAL ANALYSIS                *
C*                                   (J.WILEY & SON,N.Y)               *
C*********************************************************************
      IMPLICIT REAL *8(A-B,D-H,O-Z),COMPLEX *16(C)
      DIMENSION R(24),W(24)
      EXTERNAL CSU,CSX
      PI=4.DO*DATAN(1.DO)
      DATA N/24/
      DATA R/.03238 01709 62869, .09700 46992 09463, .16122 23560 68892,
     &       .22476 37903 94689, .28736 24873 5546 , .34875 58862 9216 ,
     &       .40868 64819 9072 , .46690 29047 5096 , .52316 09747 2223 ,
     &       .57722 47260 8397 , .62886 73967 7651 , .67787 23796 3266 ,
     &       .72403 41309 2382 , .76715 90325 1574 , .80706 62040 2944 ,
     &       .84358 82616 2439 , .87657 20202 7425 , .90587 91367 1557 ,
     &       .93138 66907 0655 , .95298 77031 6043 , .97059 15925 4625 ,
     &       .98412 45837 2283 , .99353 01722 6635 , .99877 10072 5243 /
      DATA W/.06473 76968 12684, .06446 61644 35950, .06392 42385 84648,
     &       .06311 41922 86254, .06203 94231 59893, .06070 44391 65894,
     &       .05911 48396 98396, .05727 72921 00403, .05519 95036 99984,
     &       .05289 01894 85194, .05035 90355 53854, .04761 66584 92490,
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     &       .04467 45608 56694, .04154 50829 43465, .03824 13510 65831,
     &       .03477 72225 64770, .03116 72278 32798, .02742 65097 08357,
     &       .02357 07608 39324, .01961 61604 57356, .01557 93157 22944,
     &       .01147 72345 79235, .00732 75539 01276, .00315 33460 52306/
      CZ =DCMPLX(0.D0,0.D0)
C
C.....EVALUATION OF THE FIRST TERM OF THE HANKEL FUNCTION
C
      XMD=0.25D0*PI
      SCL=0.25D0*PI
      XMI=0.75D0*PI
      CQX=CZ
      DO 61 K=1,N
           S=R(K)*SCL
   61  CQX=CQX+(CSX(CKNDM,VK3,CZH,XMD+S)+CSX(CKNDM,VK3,CZH,XMD-S)+
     &      CSX(CKNDM,VK3,CZH,XMI+S)+CSX(CKNDM,VK3,CZH,XMI-S))*
     &      DCMPLX(W(K),0.D0)
      CQX=CQX*SCL
C
C.....EVALUATION OF THE SECOND TERM OF THE HANKEL FUNCTION
C
      XMD=0.5D0
      SCL=0.5D0
      CQU=CZ
      DO 62 K=1,N
           S=R(K)*SCL
   62  CQU=CQU+(CSU(CKNDM,VK3,CZH,XMD+S)+CSU(CKNDM,VK3,CZH,XMD-S))*
     &      DCMPLX(W(K),0.D0)
      CQU =CQU*SCL
      CHNK=(CQX-CQU )/DCMPLX(PI,0.D0)
      RETURN
      END
      SUBROUTINE ASYMP(CKNDM,VK3,CZH,CHNK,IRT)
C*********************************************************************
C* NAME               ASYMP                                          *
C*                                                                   *
C* TYPE               SUBROUTINE                                     *
C*                                                                   *
C* PURPOSE            EVALUATION OF HANKEL FUNCTION                  *
C*                    USING ASSYMTOTIC EXPANSION                     *
C*                                                                   *
C* AUTHOR             P.BRUNAVS, 1977                                *
C* MODIFIED BY        T.MAHMOOD, OCT.1981                            *
C*                                                                   *
C* EXTERNAL           NONE                                           *
C*                                                                   *
C* CALLING            CALL ASYMP(CKNDM,VK3,CZH,CHNK,IRT)             *
C*                                                                   *
C* PARAMETERS         ALL EXPLAINED IN THE MAIN                      *
C*                    INPUT :CKNDM,VK3,CZH                           *
C*                    OUTPUT:CHNK,IRT                                *
C*                                                                   *
C* LANGUAGE           FORTRAN                                        *
C*                                                                   *
C* REFRENCES          UNKNOWN                                        *
C*********************************************************************
      IMPLICIT REAL *8(A-B,D-H,O-Z),COMPLEX *16(C)
      PI   =4.D0*DATAN(1.D0)
      CEPX =(CZH-DCMPLX((0.5D0*VK3+0.25D0)*PI,0.D0))*CKNDM
      CM   =CDSQRT(DCMPLX(2.D0/PI,0.D0)/CZH)
      CZ8  =DCMPLX(0.125D0,0.D0)/CZH
      V42  =VK3**2*4.D0
      CSUMA=DCMPLX(1.D0,0.D0)
      CT1A =DCMPLX(1.D0,0.D0)
      DO 71 L=1,50
```

```
           FL  =DFLOAT(L)
           VNA =(V42-(FL*2.DO-1.DO)**2)/FL
           CT2A=(CZ8*CKNDM**L)*VNA
           IF(CDABS(CT2A).GT.0.99D0) GO TO 72
           CT2A =CT1A*CT2A
           CSUMA=CSUMA+CT2A
           DCRIT=CDABS(CT2A)/CDABS(CSUMA)
           IF(DCRIT.LT.1.0D-11) GO TO 73
           CT1A=CT2A
           KEA =L
   71 CONTINUE
   72 IRT =3
      GO TO 74
   73 CALL TRAPS(0,0,500,0,0)
      CHNK=CSUMA*CM*CDEXP(CEPX)
      IRT =5
   74 RETURN
      END
      SUBROUTINE PLANE  (D,FRQ,V,SG,E1,E2,ALF,A,SPL)
C*********************************************************************
C* NAME            PLANE                                             *
C*                                                                   *
C* TYPE            SUBROUTINE                                        *
C*                                                                   *
C* PURPOSE         TO COMPUTE SECONDARY PHASELAG                     *
C*                 USING PLANE EARTH THEORY                          *
C*                                                                   *
C* AUTHOR          D.GRAY,1977/78                                    *
C* MODIFIED BY     T.MAHMOOD, SEPT.1986                              *
C*                                                                   *
C* EXTERNALS       DCBRT                                             *
C*                                                                   *
C* CALLING         CALL PHASE2(D,FRQ,V,SG,E1,E2,ALF,A,SPL)           *
C*                                                                   *
C* PARAMETERS      ALL EXPLAINED IN THE MAIN                         *
C*                 INPUT :D,FRQ,V,SG,E1,E2,ALF,A                     *
C*                 OUTPUT:SPL                                        *
C*                                                                   *
C* LANGUAGE        FORTRAN                                           *
C*                                                                   *
C* REFRENCES       JOHLER ET AL (1956) : NBS CIRCULAR 573            *
C*********************************************************************
      IMPLICIT REAL *8(A-B,D-H,O-Z),COMPLEX *16(C)
      REAL *8 K1, KE, MUO
      PI     = 4.DO*DATAN(1.DO)
      MUO    = 4.OD-07*PI
      W      = 2.DO*PI*FRQ
      K1     = W*DCBRT(E1)/V
      AK     = SG*MUO*V*V/W
      CK2    = (W/V)*CDSQRT(DCMPLX(E2,AK))
      IF(AK.NE.0.DO) GO TO 201
      PSIE   = 0.5DO*PI
      GO TO 202
 201  PSIE   = DATAN(E2/AK)-0.5DO*DATAN((E2-E1)/AK)
 202  KE     = DCBRT(V*ALF/(E1*E1*W*A))*DSQRT(E2*E2+AK*AK)/((E2-E1)*
     &         (E2-E1)+AK*AK)**0.25DO
      PP     = 0.75DO*PI-PSIE
      CPPI   = DCMPLX(0.DO,PP)
      CALL TRAPS(0,0,500,0,0)
      CDELE = DCMPLX(KE,0.DO)*CDEXP(CPPI)
      CRHOC = -D*DCBRT(K1*A*ALF*ALF)/(A*2.DO*CDELE*CDELE)
      CRHO1 = CRHOC*DCMPLX(0.DO,1.DO)
      CFUNC = 1.DO-K1*K1/(CK2*CK2)+(K1/CK2)**4
      CALL TRAPS(0,0,500,0,0)
      CY     = (CDSQRT(PI*CRHO1)*CDEXP(-CRHO1))*DCMPLX(0.DO,1.DO)+
```

```
     _       DCMPLX(1.D0,0.D0)
      CADD  = DCMPLX(1.D0,0.D0)
      DO 203 I = 1,1000
          KEY2 = I
          CADD = CADD*(-4.D0*CRHO1)/DFLOAT(4*I-2)
          CY   = CY+CADD
          IF(CDABS(CADD) .LT. 1.0D-7*CDABS(CY)) GO TO 204
 203  CONTINUE
 204  DCORR = 1.D0/(K1*D)
      FZR   = DREAL(CY*CFUNC)-DCORR*DCORR
      FZI   = DIMAG(CY*CFUNC)+DCORR
      FYCT  = (DATAN(FZI/FZR))/W
      SPL   = FYCT*V
 205  IF(SPL.GT.0.D0) GO TO 206
      SPL = SPL+V/(2.D0*FRQ)
      GO TO 205
 206  RETURN
      END
      SUBROUTINE SPHERE(D,V,A,CDE,CTS,K1,FRC,W,NMX,FYP,FYN,SPL)
C*******************************************************************
C* NAME            SPHERE                                          *
C*                                                                 *
C* TYPE            SUBROUTINE                                      *
C*                                                                 *
C* PURPOSE         TO COMPUTE SECONDARY PHASELAG                   *
C*                 USING SPHERICAL EARTH THEORY                    *
C*                                                                 *
C* AUTHOR          P.BRUNAVS,1977                                  *
C* MODIFIED BY     T.MAHMOOD, SEPT.1986                            *
C*                                                                 *
C* EXTERNALS       NONE                                            *
C*                                                                 *
C* CALLING         CALL SPHERE(D,V,CDE,CTS,K1,FRC,W,NMX,FYP,FYN,SPL) *
C*                                                                 *
C* PARAMETERS      ALL EXPLAINED IN THE MAIN                       *
C*                 INPUT  : D,V,A,CDE,CTS,K1,FRC,W,NMX             *
C*                 OUTPUT : FYP,SPL                                *
C*                                                                 *
C* REFERENCE       JOHLER ET AL (1956) : NBS CIRCULAR 573          *
C*******************************************************************
      IMPLICIT REAL *8(A-B,D-H,O-Z),COMPLEX*16(C)
      REAL *8 K1
      DIMENSION CTS(620)
      PI   = 4.0D0*DATAN(1.0D0)
      CC1  = DCMPLX(1.0D0,0.0D0)
      DBIG = 50000.0D0
      DCR  = 1.D0/(K1*D)
      FWC  = DSQRT(2.D0*PI*FRC*D/A)
      CSUM = DCMPLX(0.D0,0.D0)
      ALF  = 0.75D0
      FYI  = 0.0D0
      MLT  = 0
      DO 101 M = 1,NMX
             CT   = CTS(M)
             CFX  = CT*DCMPLX(0.D0,FRC*D/A)+DCMPLX(0.D0,0.5D0*ALF*D/
                    A+0.25D0*PI)
             CALL TRAPS(0,0,900,0,0)
             CSUM = CSUM+CDEXP(CFX)/(CT*DCMPLX(2.D0,0.D0)-CC1/CDE**2)
             CFR  = CSUM*FWC*DCMPLX(1.D0-DCR**2,DCR)
             FY   = DATAN2(DIMAG(CFR),DREAL(CFR))
             MR   = M
             IF(DABS(FY-FYI).LT.0.000001D0) MLT=MLT+1
             IF(MLT.GT.5) GO TO 102
             FYI = FY
  101 CONTINUE
```

```
  102 IF(FY.LT.0.D0) FY = FY+PI*2.D0
      IF(D.LT.DBIG) GO TO 104
      DO 103 I = 1,3
             IF(FY.GE.FYP) GO TO 104
             FY = FY+PI*2.0D0
  103 CONTINUE
  104 SPL = FY*V/W
      FYN = FY
      RETURN
      END
      FUNCTION DCBRT(X)
C*********************************************************************
C* NAME               DCBRT(X)                                        *
C*                                                                    *
C* TYPE               DOUBLE PRECISION FUCTION                        *
C*                                                                    *
C* PURPOSE            TO COMPUTE THE CUBE ROOT OF A REAL NUMBER       *
C*                                                                    *
C* AUTHOR             D.GRAY 1977/78                                  *
C*                                                                    *
C* PARAMETERS         X=THE NUMBER TO BE COMPUTED                     *
C*                                                                    *
C* LANGUAGE           FORTRAN                                         *
C*********************************************************************
      IMPLICIT REAL *8(A-H,O-Z)
      IF(X.GE.0.D0) DCBRT=X**(1.D0/3.D0)
      IF(X.LT.0.D0) DCBRT=-(DABS(X))**(1.D0/3.D0)
      RETURN
      END
```

Appendix II

TABULATION OF C COEFFICIENTS

In this appendix all results obtained in Chapter 5 are tabulated. They are arranged as follows

Table II.1	Brunavs' original C coefficients
Table II.2 - II.8	C coefficients for conductivity between 0.0001 and 0.005 siemens/metre at seven different permittivity values.
Table II.9 - II.11	C coefficients for conductivity between 0.00001 and 0.0001 siemens/metre at three different permittivity values.
Table II.12	C coefficients for land conductivity higher than 0.005 siemens/metre.
Table II.13	C coefficients for sea water.
Table II.14	C coefficients for fresh water.

100 KHz
GROUND WAVE PHASE LAGS

DATUM VELOCITY C=299792.5 KM/SEC
ATMOSPHERE E1=1.000676, ALFA=0.75

COEFFICIENT FORMULA
P=PHASE LAG,METRES, D=DIST.,METRES, S=D/100,000.
FOR D > 2000 M

P=C1+C2*S+(C3*S+C4)*EXP(C5*S)+C6/(1.+C7*S+C8*S**4)+2.277/S

CONDCT. MHO/M	C1	C2	C3	C4	C5	C6	C7	C8
SEAWATER (E2=81)								
5.5	-111.0	98.08	-13.75	112.8	-.254	0.0	0.00	0
5.0	-111.0	98.20	-13.51	112.8	-.254	0.0	0.00	0
4.5	-111.0	98.35	-13.23	112.8	-.254	0.0	0.00	0
4.0	-111.0	98.53	-12.90	112.8	-.254	0.0	0.00	0
3.5	-111.0	98.75	-12.50	112.8	-.254	0.0	0.00	0
3.0	-111.0	99.01	-12.00	112.8	-.254	0.0	0.00	0
2.5	-111.0	99.35	-11.36	112.8	-.254	0.0	0.00	0
2.0	-111.0	99.80	-10.50	112.8	-.254	0.0	0.00	0
LAND (E2=15)								
.03000	1.9	125.77	43.7	36.9	-.600	-30.3	13.64	310
.02500	15.4	129.63	43.9	29.2	-.600	-35.2	14.08	310
.02000	31.0	133.49	45.1	18.4	-.600	-40.0	14.30	310
.01750	42.1	135.98	45.7	11.0	-.600	-43.0	14.47	310
.01500	55.7	139.05	46.6	2.1	-.600	-48.1	14.42	290
.01250	73.6	142.94	47.0	-8.9	-.600	-54.3	14.13	270
.01000	98.0	148.11	47.0	-24.0	-.600	-60.8	14.00	245
.00750	133.7	155.47	49.0	-47.0	-.600	-72.5	13.20	226
.00600	166.0	161.67	49.2	-66.7	-.600	-83.3	12.67	167
.00500	195.5	167.04	48.9	-83.9	-.598	-94.3	12.26	151
.00400	236.2	173.89	47.5	-105.9	-.587	-108.7	10.76	96
.00300	297.1	182.95	48.3	-143.1	-.556	-127.7	10.42	74
.00250	341.3	188.62	47.5	-163.8	-.534	-146.4	9.16	56
.00200	402.7	195.13	48.8	-195.3	-.508	-169.3	8.29	31
.00175	442.6	198.63	51.4	-218.0	-.496	-183.5	8.02	28
.00150	492.2	202.13	54.3	-236.5	-.457	-208.2	7.16	21
.00140	515.5	203.43	55.3	-241.1	-.433	-223.2	6.62	17
.00130	541.1	204.70	57.6	-249.7	-.415	-237.4	6.36	14
.00120	569.1	205.82	61.6	-263.1	-.406	-249.3	6.25	13
.00110	599.8	206.75	67.3	-280.2	-.402	-259.7	6.22	13
.00100	633.3	207.42	75.0	-299.4	-.400	-271.4	6.30	13
.00090	669.3	207.73	85.3	-319.7	-.399	-285.7	6.48	13
.00080	707.5	207.58	98.9	-339.9	-.401	-302.3	6.74	14
.00070	746.5	206.87	118.2	-361.1	-.409	-316.9	6.97	16
.00060	785.1	205.45	145.8	-382.5	-.426	-328.0	7.31	20
.00050	819.4	203.26	182.6	-392.5	-.450	-347.2	7.78	25
.00040	845.8	200.16	216.5	-357.8	-.473	-401.6	7.58	16
.00030	855.7	196.04	219.5	-217.3	-.488	-523.4	5.64	7
.00020	832.2	190.58	190.3	-8.7	-.498	-685.7	5.23	6
.00010	717.7	183.06	164.2	132.0	-.510	-690.6	7.25	19
.00005	573.3	178.07	158.7	165.4	-.516	-568.4	9.61	55
.00001	393.6	173.46	155.7	186.4	-.522	-402.6	11.29	111

Table II.1 Brunavs C Coefficients

σ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.005	199.7	167.01	43.2	- 85.4	-0.596	-90.0	9.97	142.0
0.004	240.6	173.99	42.0	-107.8	-0.583	-105.8	9.46	94.0
0.003	302.1	183.31	41.7	-142.4	-0.556	-128.5	8.97	69.0
0.0025	347.1	189.22	42.0	-166.8	-0.534	-146.2	8.60	53.0
0.0020	410.2	196.10	42.6	-199.1	-0.507	-172.7	8.02	33.0
0.00175	452.6	199.87	44.3	-223.7	-0.495	-187.8	7.85	29.0
0.0015	503.7	203.85	49.7	-246.6	-0.457	-211.9	7.36	21.0
0.0014	527.2	205.45	52.5	-253.0	-0.434	-226.8	6.98	16.0
0.0013	553.9	206.95	55.2	-261.6	-0.417	-242.5	6.66	12.0
0.0012	584.4	208.32	59.7	-279.4	-0.410	-253.2	6.65	12.0
0.0011	618.9	209.49	65.3	-300.6	-0.406	-264.3	6.68	12.0
0.0010	656.6	210.44	73.7	-325.1	-0.402	-275.2	6.79	13.0
0.0009	698.4	211.03	84.3	-350.5	-0.399	-288.9	6.87	13.0
0.0008	743.1	211.19	100.8	-381.3	-0.398	-300.0	7.16	17.0
0.0007	790.4	210.77	124.5	-415.5	-0.402	-310.1	7.61	23.0
0.0006	838.8	209.64	158.1	-449.1	-0.414	-321.5	8.21	34.0
0.0005	886.0	207.70	205.2	-475.0	-0.436	-338.3	8.85	44.0
0.0004	933.1	204.65	271.2	-492.0	-0.478	-362.2	9.61	54.0
0.0003	976.7	200.32	335.8	-426.2	-0.534	-453.1	8.12	15.0
0.0002	1016.8	194.08	401.8	-285.3	-0.632	-605.5	7.11	14.0
0.0001	985.5	187.47	160.7	229.7	-0.514	-1040.9	6.62	30.0

Table II.2 C Coefficients at $\varepsilon_2 = 3$ esu for Conductivity between 0.0001 and 0.005 siemens/metre.

σ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.005	199.4	167.00	43.9	-85.4	-0.596	-89.6	10.03	145.0
0.004	240.5	173.94	42.1	-107.8	-0.584	-105.6	9.49	95.0
0.003	301.7	183.23	42.0	-142.4	-0.556	-128.2	9.01	71.0
0.0025	346.7	189.09	42.2	-166.6	-0.534	-145.9	8.59	54.0
0.0020	409.5	195.91	42.9	-198.5	-0.507	-172.4	7.99	33.0
0.00175	451.5	199.64	44.9	-222.7	-0.495	-187.6	7.85	29.0
0.0015	502.3	203.53	50.1	-245.4	-0.457	-211.7	7.36	21.0
0.0014	525.5	205.10	53.1	-251.6	-0.434	-226.5	6.99	16.0
0.0013	552.1	206.56	55.5	-259.7	-0.417	-242.4	6.65	12.0
0.0012	581.9	207.90	60.3	-276.7	-0.410	-253.2	6.64	12.0
0.0011	615.5	279.04	66.0	-297.0	-0.406	-264.4	6.66	12.0
0.0010	652.5	209.94	74.4	-320.8	-0.402	-275.2	6.77	13.0
0.0009	693.4	210.47	84.9	-345.3	-0.399	-288.8	6.85	13.0
0.0008	737.2	210.56	100.6	-374.1	-0.399	-301.0	7.09	16.0
0.0007	782.6	210.12	124.2	-405.9	-0.403	-311.4	7.52	22.0
0.0006	829.1	208.95	156.9	-437.2	-0.416	-323.1	8.10	31.0
0.0005	873.9	206.98	202.1	-459.8	-0.437	-340.6	8.70	40.0
0.0004	914.2	204.08	254.5	-443.9	-0.467	-387.3	8.42	25.0
0.0003	956.6	199. 52	320.3	-398.4	-0.532	-459.1	7.87	14.0
0.0002	984.0	193.45	359.4	-233.5	-0.609	-621.9	6.76	14.0
0.0001	930.9	186.62	164.8	201.7	-0.512	-959.2	6.92	30.0

Table II.3 C Coefficients at ε_2 = 5 esu for Conductivity between 0.0001 and 0.005 siemens/metre.

σ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.005	199.1	166.92	44.1	-85.5	-0.597	-89.2	10.07	149.0
0.004	240.2	173.81	42.3	-107.8	-0.585	-105.4	9.54	96.0
0.003	301.2	182.99	42.3	-142.2	-0.556	-127.8	9.05	73.0
0.0025	345.7	188.76	42.7	-165.9	-0.534	-145.6	8.63	55.0
0.0020	408.0	195.41	43.2	-196.8	-0.507	-172.5	7.98	33.0
0.00175	449.3	199.01	45.5	-220.6	-0.495	-187.2	7.83	29.0
0.0015	498.9	202.74	51.1	-242.5	-0.457	-211.0	7.37	21.0
0.0014	521.2	204.24	54.3	-248.3	-0.434	-225.3	7.02	16.0
0.0013	546.9	205.61	56.7	-255.0	-0.417	-241.7	6.66	12.0
0.0012	575.6	206.85	61.4	-270.6	-0.409	-252.7	6.62	12.0
0.0011	607.6	207.88	67.5	-289.7	-0.405	-263.5	6.66	12.0
0.0010	642.5	208.68	75.9	-310.8	-0.401	-274.8	6.74	13.0
0.0009	680.9	209.10	86.3	-333.0	-0.399	-288.2	6.82	13.0
0.0008	721.4	209.08	101.6	-358.0	-0.400	-300.7	7.03	15.0
0.0007	763.2	208.52	123.4	-384.1	-0.405	-313.0	7.38	19.0
0.0006	805.3	207.24	153.6	-409.8	-0.419	-325.6	7.89	26.0
0.0005	844.9	205.14	194.0	-425.1	-0.442	-344.6	8.38	32.0
0.0004	877.9	202.15	237.0	-397.9	-0.469	-394.4	7.97	20.0
0.0003	906.2	197.66	284.1	-333.7	-0.523	-470.2	7.38	13.0
0.0002	908.4	191.76	293.9	-161.0	-0.574	-618.6	6.64	13.0
0.0001	811.6	184.68	166.1	154.7	-0.507	-798.3	7.31	28.0

Table II.4 C Coefficient at $\varepsilon_2 = 10$ esu for Conductivity between 0.0001 and 0.005 siemens/metre.

σ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.005	198.9	166.84	44.2	-85.3	-0.598	-89.2	10.10	149.0
0.004	239.9	173.68	42.4	-107.4	-0.585	-105.3	9.51	96.0
0.003	300.5	182.76	42.6	-141.6	-0.556	-127.7	9.04	73.0
0.0025	344.7	188.43	43.1	-165.1	-0.534	-145.3	8.63	55.0
0.0020	406.2	194.91	43.7	-195.3	-0.507	-172.2	7.97	33.0
0.00175	446.8	198.39	46.3	-218.6	-0.495	-186.7	7.84	30.0
0.0015	495.2	201.96	52.0	-239.2	0.457	-210.5	7.36	21.0
0.0014	516.8	203.38	55.3	-244.4	-0.434	-224.7	7.02	17.0
0.0013	541.0	204.69	58.7	-251.4	-0.416	-239.4	6.73	13.0
0.0012	568.2	205.85	63.5	-264.8	-0.407	-251.0	6.66	13.0
0.0011	598.8	206.78	69.5	-282.2	-0.403	-262.0	6.68	13.0
0.0010	632.3	207.44	77.2	-301.1	-0.401	-274.0	6.73	13.0
0.0009	668.2	207.75	87.4	-320.7	-0.400	-287.4	6.79	13.0
0.0008	705.9	207.62	101.7	-341.6	-0.401	-300.9	6.95	14.0
0.0007	744.4	206.94	122.0	-364.1	-0.409	-313.5	7.27	18.0
0.0006	782.2	205.56	150.5	-385.9	-0.424	-325.6	7.77	24.0
0.0005	816.5	203.38	187.1	-396.0	-0.447	-344.2	8.21	30.0
0.0004	842.2	200.32	222.3	-360.3	-0.472	-394.9	7.74	18.0
0.0003	857.6	195.91	257.9	-289.9	-0.517	-464.9	7.28	14.0
0.0002	839.6	190.14	260.6	-135.9	-0.555	-577.8	6.91	16.0
0.0001	717.0	183.06	162.6	124.5	-0.501	-681.9	7.38	24.0

Table II.5 C Coefficients at ε_2 = 15 esu for Conductivity between 0.0001 and 0.005 siemens/metre.

σ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.005	198.7	166.76	44.3	-85.2	-0.598	-89.1	10.09	149.0
0.004	239.5	173.55	42.6	-107.1	-0.585	-105.2	9.51	97.0
0.003	299.7	182.53	42.9	-140.9	-0.556	-127.5	9.03	73.0
0.0025	343.6	188.10	43.4	-164.1	-0.534	-145.1	8.62	55.0
0.0020	404.4	194.42	44.4	-193.7	-0.507	-171.7	7.97	33.0
0.00175	444.4	197.77	46.9	-216.5	-0.495	-186.2	7.84	30.0
0.0015	491.4	201.18	52.9	-235.9	-0.457	-209.8	7.36	21.0
0.0014	512.4	202.52	56.1	-240.3	-0.434	-224.0	7.01	17.0
0.0013	535.5	203.76	59.5	-246.3	-0.416	-238.8	6.72	13.0
0.0012	561.5	204.83	64.5	-258.7	-0.407	-250.2	6.66	13.0
0.0011	590.6	205.66	70.5	-274.6	-0.404	-261.0	6.67	13.0
0.0010	622.1	206.22	78.2	-291.5	-0.401	-273.0	6.70	13.0
0.0009	655.5	206.45	88.3	-308.7	-0.400	-286.2	6.77	13.0
0.0008	689.9	206.24	102.3	-326.4	0.402	-299.7	6.91	14.0
0.0007	724.9	205.45	121.1	-344.2	-0.410	-313.3	7.17	17.0
0.0006	758.9	203.97	146.8	-361.0	-0.427	-326.2	7.59	21.0
0.0005	788.5	201.69	179.8	-367.4	-0.451	-343.9	8.04	27.0
0.0004	806.9	198.61	209.5	-327.6	0.473	-391.6	7.61	18.0
0.0003	810.8	194.28	234.3	-249.7	-0.509	-458.0	7.14	14.0
0.0002	777.7	188.62	234.7	-116.5	-0.540	-538.5	7.09	18.0
0.0001	643.3	181.72	157.5	103.0	-0.494	-596.0	7.30	21.0

Table II.6 C Coefficients at ε_2 = 20 esu for Conductivity between 0.0001 and 0.005 siemens/metre.

σ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.005	198.4	166.68	44.4	-84.7	-0.597	- 89.2	10.06	145.0
0.004	239.0	173.42	42.8	-106.6	-0.584	-105.2	9.51	95.0
0.003	299.0	182.30	43.1	-140.1	-0.556	-127.5	9.02	71.0
0.0025	342.5	187.76	43.8	-163.0	-0.534	-144.9	8.61	54.0
0.0020	402.5	193.91	45.0	-192.3	-0.506	-171.2	7.98	33.0
0.00175	441.7	197.15	57.8	-214.6	-0.495	-185.2	7.86	31.0
0.0015	487.6	200.40	53.6	-232.3	-0.457	-209.3	7.35	21.0
0.0014	508.1	201.63	56.6	-236.6	-0.435	-223.3	7.01	17.0
0.0013	530.2	202.81	59.9	-241.1	-0.417	-238.4	6.69	13.0
0.0012	555.8	203.75	64.3	-253.2	-0.412	-249.6	6.62	12.0
0.0011	583.5	204.49	70.4	-268.2	-0.408	-260.0	6.65	12.0
0.0010	613.1	204.96	78.1	-283.8	-0.406	-271.5	6.69	12.0
0.0009	644.1	205.09	88.3	-299.4	-0.405	-284.0	6.78	13.0
0.0008	676.2	204.77	102.4	-317.4	-0.410	-294.9	7.00	15.0
0.0007	709.0	203.84	121.9	-339.5	-0.423	-302.6	7.48	22.0
0.0006	738.8	202.28	146.4	-352.8	-0.439	-314.9	7.90	27.8
0.0005	762.7	199.99	176.3	-352.7	-0.461	-333.2	8.25	33.0
0.0004	774.4	196.90	203.6	-314.0	-0.482	-373.9	7.96	25.0
0.0003	767.9	192.70	218.6	-226.9	-0.506	-439.5	7.27	16.0
0.0002	722.4	187.23	215.2	-102.8	-0.528	-500.4	7.23	19.0
0.0001	585.9	180.58	153.6	84.7	-0.490	-529.5	7.28	20.0

Table II.7 C Coefficients at $\varepsilon_2 = 25$ esu for Conductivity between 0.0001 and 0.005 siemens/metre.

σ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.005	198.0	166.60	44.5	-84.3	-0.596	-89.2	10.00	143.0
0.004	238.5	173.29	43.0	-106.1	-0.583	-105.1	9.48	95.0
0.003	298.3	182.05	43.1	-139.2	-0.556	-127.6	8.98	70.0
0.0025	341.4	187.42	44.0	-161.9	-0.534	-144.9	8.59	53.0
0.0020	400.4	193.43	45.8	-190.6	-0.506	-170.6	7.99	34.0
0.00175	439.0	196.53	48.6	-212.5	-0.495	-184.6	7.88	31.0
0.0015	483.4	199.64	54.5	-228.8	-0.457	-208.5	7.36	21.0
0.0014	502.9	200.83	57.9	-232.7	-0.436	-221.9	7.03	17.0
0.0013	524.5	201.89	60.4	-235.7	-0.418	-237.9	6.66	12.0
0.0012	548.7	202.76	65.1	-246.8	-0.412	-248.8	6.61	12.0
0.0011	575.1	203.40	71.0	-260.3	-0.409	-259.3	6.63	12.0
0.0010	602.7	203.79	78.7	-274.3	-0.407	-270.5	6.68	12.0
0.0009	632.0	203.81	88.7	-289.3	-0.408	-281.8	6.79	13.0
0.0008	661.6	203.39	102.8	-307.4	-0.415	-290.5	7.09	17.0
0.0007	691.6	202.36	121.9	-328.7	-0.431	-296.1	7.62	24.0
0.0006	717.2	200.76	144.3	-336.8	-0.446	-309.3	7.96	30.0
0.0005	736.3	198.43	171.2	-332.1	-0.466	-327.3	8.26	34.0
0.0004	741.8	195.35	193.9	-290.83	-0.483	-365.3	7.95	26.0
0.0003	727.4	191.23	205.2	-207.0	-0.503	-420.5	7.37	18.0
0.0002	673.8	185.93	199.7	-92.5	-0.519	-465.9	7.33	20.0
0.0001	540.3	179.57	148.7	71.4	-0.486	-479.1	7.20	18.0

Table II.8 C Coefficients at ε_2 = 30 esu for Conductivity between 0.0001 and 0.005 siemens/metre.

σ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.00001	533.9	174.44	135.0	274.8	-0.517	-513.8	27.00	3386.0
0.00002	697.8	176.50	151.2	260.7	-0.538	-707.1	24.43	2621.0
0.00003	803.9	178.34	165.4	246.8	-0.554	-827.4	19.64	1485.0
0.00004	867.2	180.10	164.4	245.3	-0.547	-906.1	15.68	685.0
0.00005	901.0	182.02	126.5	276.7	-0.492	-979.6	12.36	258.0
0.00006	925.9	183.53	117.8	286.4	-0.477	-1019.6	10.28	136.0
0.00007	946.0	184.73	120.0	283.2	-0.477	-1041.3	8.88	84.0
0.00008	961.9	185.75	129.1	271.9	-0.484	-1050.6	7.88	57.0
0.00009	975.3	186.64	144.1	252.7	-0.500	-1049.3	7.17	41.0
0.00010	985.9	187.45	162.0	228.2	-0.516	-1039.7	6.63	30.0

Table II.9 C Coefficients at $\varepsilon_2 = 3$ esu for Conductivity between 0.00001 and 0.0001 siemens/metre.

σ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.00001	513.8	174.20	146.7	255.9	-0.531	-494.2	23.17	2197.0
0.00002	649.6	176.09	160.7	242.6	-0.547	-644.3	21.62	1728.0
0.00003	749.9	177.88	171.4	231.2	-0.557	-757.3	18.44	1072.0
0.00004	814.4	179.74	157.0	241.4	-0.533	-847.8	15.04	479.0
0.00005	853.7	181.64	126.2	269.5	-0.487	-922.9	12.10	203.0
0.00006	885.6	183.03	121.4	273.1	-0.478	-964.7	10.26	119.0
0.00007	910.5	184.20	126.3	266.6	-0.481	-988.7	8.98	79.0
0.00008	930.4	185.22	137.5	252.1	-0.492	-999.3	8.05	56.0
0.00009	946.2	186.13	151.5	232.9	-0.505	-1000.8	7.35	41.0
0.00010	959.1	186.97	168.4	208.5	-0.520	-993.9	6.82	31.0

Table II.10 C Coefficients at $\varepsilon_2 = 4$ esu for Conductivity between 0.00001 and 0.0001 siemens/metre.

σ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.00001	495.0	174.03	153.3	241.7	-0.537	-477.9	20.68	1509.0
0.00002	612.0	175.80	164.1	230.2	-0.549	-602.2	19.55	1165.0
0.00003	703.5	177.61	163.6	228.1	-0.543	-710.2	17.06	682.0
0.00004	757.3	180.02	100.3	285.4	-0.449	-830.5	13.29	191.0
0.00005	811.2	181.30	122.4	264.9	-0.479	-874.5	11.71	157.0
0.00006	848.0	182.58	124.0	261.6	-0.478	-915.0	10.17	104.0
0.00007	877.0	183.71	131.6	251.2	-0.486	-940.0	9.03	73.0
0.00008	899.8	184.72	142.8	235.8	-0.496	-952.8	8.17	54.0
0.00009	918.2	185.65	157.4	215.2	-0.509	-955.8	7.51	41.0
0.00010	932.9	186.51	172.8	191.9	-0.522	-951.9	6.99	32.0

Table II.11 C Coefficients at $\varepsilon_2 = 5$ esu for Conductivity between 0.00001 and 0.0001 siemens/metre.

σ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
1.0	-104.0	101.37	30.3	108.0	-0.569	-0.7	6.90	460
0.75	-100.4	102.19	30.6	105.4	-0.569	-1.1	7.01	460
0.50	-94.6	103.58	31.5	101.4	-0.571	-2.2	7.20	460
0.25	-81.5	106.72	33.4	92.5	-0.577	-4.8	7.39	460
0.10	-55.6	112.93	36.4	75.6	-0.584	-11.4	7.88	460
0.075	-44.6	115.53	37.4	68.4	-0.586	-14.3	8.29	440
0.050	-26.3	119.88	38.9	56.4	-0.588	-19.0	8.70	420
0.040	-14.5	122.66	39.4	49.1	-0.589	-22.9	9.03	380
0.030	2.9	126.70	40.4	38.0	-0.590	-28.3	9.51	320
0.025	15.5	129.56	40.9	30.0	-0.591	-32.3	9.79	320
0.020	32.5	133.39	41.6	19.1	-0.592	-38.0	10.21	320
0.0175	43.7	135.88	42.1	11.5	-0.592	-40.4	10.45	320
0.0150	57.8	138.92	42.4	2.6	-0.593	-45.4	10.78	300
0.0125	75.8	142.81	43.0	-8.9	-0.594	-51.1	11.09	270
0.010	100.3	147.97	43.6	-24.6	-0.595	-59.0	11.38	250
0.0075	136.8	155.28	44.4	-48.3	-0.596	-70.6	11.79	225
0.006	169.1	161.49	45.1	-68.5	-0.598	-81.5	11.86	170
0.005	199.0	166.83	45.3	-87.9	-0.601	-91.1	12.05	155

Table II.12 Coefficients for Land at high Conductivities

σ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
5.5	-111.0	98.08	-13.75	112.8	-0.254	0.0	0.0	0.0
5.0	-111.0	98.20	-13.51	112.8	-0.254	0.0	0.0	0.0
4.5	-111.0	98.35	-13.23	112.8	-0.254	0.0	0.0	0.0
4.0	-111.0	98.53	-12.90	112.8	-0.254	0.0	0.0	0.0
3.5	-111.0	98.75	-12.50	112.8	-0.254	0.0	0.0	0.0
3.0	-111.0	99.01	-12.00	112.8	-0.254	0.0	0.0	0.0
2.5	-111.0	99.35	-11.36	112.8	-0.254	0.0	0.0	0.0
2.0	-111.0	99.80	-10.50	112.8	-0.254	0.0	0.0	0.0
1.5	-111.0	100.58	-8.75	112.8	-0.254	0.0	0.0	0.0
1.0	-111.0	101.75	-6.00	112.8	-0.254	0.0	0.0	0.0

Table II.13 Coefficients for Sea Water Conductivities.

σ	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
0.005	195.0	165.72	46.0	-82.4	-0.600	-87.8	10.13	150.0

Table II.14 Coefficients for Fresh Water (ε_2 = 80 esu).

Appendix III

DIFFERENCE BETWEEN ACCURATE AND APPROXIMATE COMPUTATIONS

In this appendix, a few examples of the difference in the values of total phaselag between accurate and approximate computations are given. These examples are taken at different conductivity and permittivity values and some of them are at the worst attainable accuracy. From this examples, it is hoped that some measure of confidence could be given so that this approximate formula can be used without much hesitation. As mentioned in Section 7.3.2, the worst accuracy occured at very low conductivity, i.e, below 0.0001 siemens/metre where the expected accuracy is within 10.0 metres. At higher conductivity,the worst can be expected is in the region of 5.0 metres but in most cases it is within 2.0 metres if distances greater than 2Km are used in the computation.

Appendix III - Cont'd

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                  ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG


Frequency    = 100000.0    Hz
Conductivity = 1.500000    siemens/meter
Permitivity  = 80.0        esu

Coefficients C(I), I=1 to 8 are :

  -111.000   100.580    -8.750   112.800
    -0.254     0.000     0.000     0.000

DISTANCE              ACCURATE              APPROXIMATE             DIFFERENCE
  (M)                   (M)                     (M)                    (M)

  2000.0               121.2                  116.9                   4.3
  3000.0                81.1                   79.6                   1.5
  4000.0                62.1                   61.3                   0.9
  5000.0                51.2                   50.5                   0.7
  6000.0                44.2                   43.6                   0.6
  7000.0                39.4                   38.8                   0.6
  8000.0                35.9                   35.4                   0.6
  9000.0                33.4                   32.8                   0.6
 10000.0                31.5                   30.9                   0.5
 11000.0                30.8                   29.5                   1.2
 12000.0                29.9                   28.4                   1.5
 13000.0                29.3                   27.6                   1.6
 14000.0                28.7                   27.0                   1.7
 15000.0                28.3                   26.6                   1.7
 16000.0                28.0                   26.3                   1.8
 17000.0                27.9                   26.1                   1.8
 18000.0                27.8                   26.0                   1.8
 19000.0                27.8                   26.0                   1.8
 20000.0                27.9                   26.1                   1.9
 30000.0                30.8                   28.9                   1.9
 40000.0                35.5                   33.7                   1.8
 50000.0                41.0                   39.3                   1.7
 60000.0                47.1                   45.5                   1.6
 70000.0                53.4                   52.0                   1.4
 80000.0                60.0                   58.7                   1.3
 90000.0                66.7                   65.5                   1.1
100000.0                73.6                   72.6                   1.0
200000.0               148.3                  148.6                  -0.3
300000.0               230.7                  231.9                  -1.2
400000.0               318.4                  320.1                  -1.7
500000.0               410.0                  411.7                  -1.8
600000.0               504.4                  506.0                  -1.6
700000.0               600.9                  602.1                  -1.2
800000.0               698.8                  699.5                  -0.7
900000.0               797.6                  797.9                  -0.3
1000000.0              897.1                  897.0                   0.1
1100000.0              997.0                  996.6                   0.4
1200000.0             1097.2                 1096.5                   0.6
1300000.0             1197.4                 1196.7                   0.7
1400000.0             1297.7                 1297.0                   0.7
1500000.0             1398.1                 1397.4                   0.6
1600000.0             1498.4                 1498.0                   0.5
1700000.0             1598.8                 1598.5                   0.3
1800000.0             1699.1                 1699.1                   0.0
1900000.0             1799.5                 1799.7                  -0.2
2000000.0             1899.8                 1900.3                  -0.5
```

Appendix III - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 1.000000 siemens/meter
Permitivity = 80.0 esu

Coefficients C(I), I=1 to 8 are :

-111.000 101.750 -6.000 112.800
-0.254 0.000 0.000 0.000

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	121.7	117.0	4.8
3000.0	81.7	79.7	2.0
4000.0	62.9	61.4	1.5
5000.0	52.1	50.7	1.3
6000.0	45.1	43.8	1.3
7000.0	40.4	39.1	1.3
8000.0	37.0	35.7	1.3
9000.0	34.5	33.2	1.3
10000.0	32.6	31.3	1.3
11000.0	32.0	29.9	2.1
12000.0	31.2	28.9	2.3
13000.0	30.6	28.1	2.5
14000.0	30.1	27.6	2.5
15000.0	29.8	27.2	2.6
16000.0	29.5	26.9	2.6
17000.0	29.4	26.7	2.7
18000.0	29.4	26.7	2.7
19000.0	29.5	26.7	2.7
20000.0	29.6	26.8	2.8
30000.0	32.9	30.0	2.9
40000.0	37.9	35.1	2.8
50000.0	43.7	41.1	2.6
60000.0	50.0	47.6	2.4
70000.0	56.5	54.4	2.2
80000.0	63.3	61.4	1.9
90000.0	70.3	68.6	1.7
100000.0	77.3	75.9	1.5
200000.0	153.7	154.3	-0.6
300000.0	237.4	239.3	-1.9
400000.0	326.2	328.7	-2.5
500000.0	418.9	421.5	-2.6
600000.0	514.4	516.6	-2.3
700000.0	611.8	613.5	-1.7
800000.0	710.7	711.8	-1.0
900000.0	810.6	811.0	-0.4
1000000.0	911.1	910.9	0.2
1100000.0	1011.9	1011.3	0.6
1200000.0	1113.0	1112.1	0.9
1300000.0	1214.3	1213.2	1.1
1400000.0	1315.6	1314.5	1.1
1500000.0	1416.9	1415.9	1.0
1600000.0	1518.3	1517.4	0.8
1700000.0	1619.6	1619.0	0.6
1800000.0	1721.0	1720.7	0.3
1900000.0	1822.3	1822.4	-0.1
2000000.0	1923.6	1924.1	-0.4

Appendix III - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

```
Frequency     = 100000.0    Hz
Conductivity  = 0.750000    siemens/meter
Permitivity   = 15.0        esu

Coefficients C(I), I=1 to 8 are :

  -100.400    102.190     30.600    105.400
    -0.569     -1.100      7.010    460.000
```

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	122.2	119.3	2.9
3000.0	82.3	82.2	0.1
4000.0	63.5	64.0	-0.5
5000.0	52.8	53.4	-0.6
6000.0	45.9	46.5	-0.6
7000.0	41.2	41.9	-0.7
8000.0	37.9	38.6	-0.7
9000.0	35.5	36.2	-0.7
10000.0	33.6	34.4	-0.8
11000.0	33.1	33.1	-0.1
12000.0	32.3	32.1	0.2
13000.0	31.7	31.4	0.3
14000.0	31.3	30.9	0.3
15000.0	31.0	30.6	0.4
16000.0	30.8	30.4	0.4
17000.0	30.7	30.3	0.4
18000.0	30.7	30.4	0.4
19000.0	30.8	30.4	0.4
20000.0	31.0	30.6	0.4
30000.0	34.6	34.3	0.3
40000.0	39.8	39.8	0.0
50000.0	45.9	46.0	-0.1
60000.0	52.4	52.7	-0.2
70000.0	59.2	59.5	-0.3
80000.0	66.2	66.6	-0.4
90000.0	73.3	73.8	-0.5
100000.0	80.5	81.1	-0.5
200000.0	158.2	158.5	-0.3
300000.0	243.0	242.7	0.3
400000.0	332.8	332.3	0.5
500000.0	426.4	426.0	0.4
600000.0	522.7	522.6	0.1
700000.0	621.1	621.2	-0.1
800000.0	720.8	721.1	-0.3
900000.0	821.5	821.8	-0.3
1000000.0	922.8	923.1	-0.3
1100000.0	1024.5	1024.7	-0.2
1200000.0	1126.4	1126.6	-0.1
1300000.0	1228.5	1228.6	-0.1
1400000.0	1330.6	1330.6	0.0
1500000.0	1432.8	1432.7	0.1
1600000.0	1535.0	1534.8	0.1
1700000.0	1637.2	1637.0	0.2
1800000.0	1739.3	1739.2	0.2
1900000.0	1841.5	1841.3	0.2
2000000.0	1943.7	1943.5	0.2

Appendix III - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.025000 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

15.500 129.560 40.900 30.000
-0.591 -32.300 9.790 320.000

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	137.1	135.4	1.7
3000.0	100.6	101.0	-0.4
4000.0	84.7	85.3	-0.6
5000.0	76.4	77.0	-0.5
6000.0	71.8	72.3	-0.4
7000.0	69.2	69.6	-0.3
8000.0	67.8	68.1	-0.3
9000.0	67.2	67.4	-0.2
10000.0	67.1	67.3	-0.2
11000.0	68.0	67.6	0.4
12000.0	68.9	68.1	0.8
13000.0	69.8	68.9	0.9
14000.0	70.8	69.8	0.9
15000.0	71.9	70.9	1.0
16000.0	73.0	72.1	1.0
17000.0	74.3	73.3	0.9
18000.0	75.5	74.6	0.9
19000.0	76.9	76.0	0.9
20000.0	78.2	77.4	0.8
30000.0	92.4	92.4	0.0
40000.0	106.8	107.2	-0.4
50000.0	120.7	121.1	-0.4
60000.0	134.4	134.6	-0.2
70000.0	147.8	147.8	-0.1
80000.0	160.9	160.9	0.0
90000.0	173.8	173.7	0.1
100000.0	186.6	186.5	0.1
200000.0	309.7	310.0	-0.4
300000.0	430.7	430.9	-0.1
400000.0	552.8	552.5	0.3
500000.0	676.4	676.0	0.5
600000.0	801.6	801.2	0.4
700000.0	928.0	927.8	0.2
800000.0	1055.3	1055.4	-0.1
900000.0	1183.5	1183.7	-0.3
1000000.0	1312.1	1312.5	-0.4
1100000.0	1441.2	1441.6	-0.4
1200000.0	1570.4	1570.8	-0.4
1300000.0	1699.9	1700.2	-0.3
1400000.0	1829.4	1829.7	-0.3
1500000.0	1959.0	1959.1	-0.2
1600000.0	2088.6	2088.7	-0.1
1700000.0	2218.2	2218.2	0.0
1800000.0	2347.8	2347.7	0.1
1900000.0	2477.5	2477.3	0.2
2000000.0	2607.1	2606.8	0.3

Appendix III - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.005000 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

199.000	166.830	45.300	-87.900
-0.601	-91.100	12.050	155.000

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	159.9	156.8	3.0
3000.0	128.4	128.0	0.4
4000.0	116.8	117.1	-0.3
5000.0	112.3	113.0	-0.7
6000.0	111.1	112.0	-0.9
7000.0	111.6	112.7	-1.0
8000.0	113.1	114.3	-1.1
9000.0	115.2	116.4	-1.2
10000.0	117.7	118.9	-1.2
11000.0	120.8	121.6	-0.9
12000.0	124.0	124.5	-0.5
13000.0	127.2	127.5	-0.2
14000.0	130.4	130.5	-0.1
15000.0	133.6	133.5	0.0
16000.0	136.7	136.6	0.2
17000.0	139.9	139.7	0.3
18000.0	143.1	142.8	0.3
19000.0	146.3	145.9	0.4
20000.0	149.4	148.9	0.5
30000.0	179.6	179.1	0.5
40000.0	207.4	207.2	0.1
50000.0	233.2	233.2	0.0
60000.0	257.5	257.3	0.2
70000.0	280.7	280.2	0.5
80000.0	303.0	302.1	0.8
90000.0	324.5	323.4	1.1
100000.0	345.4	344.2	1.2
200000.0	534.5	534.6	0.0
300000.0	707.6	708.2	-0.5
400000.0	875.2	875.3	-0.1
500000.0	1040.8	1040.5	0.3
600000.0	1205.9	1205.4	0.5
700000.0	1371.0	1370.5	0.4
800000.0	1536.4	1536.2	0.2
900000.0	1702.1	1702.2	-0.1
1000000.0	1868.1	1868.4	-0.3
1100000.0	2034.5	2034.9	-0.4
1200000.0	2201.0	2201.5	-0.5
1300000.0	2367.7	2368.2	-0.5
1400000.0	2534.5	2534.9	-0.4
1500000.0	2701.4	2701.7	-0.3
1600000.0	2868.4	2868.5	-0.1
1700000.0	3035.3	3035.3	0.1
1800000.0	3202.3	3202.1	0.3
1900000.0	3369.4	3368.9	0.5
2000000.0	3536.4	3535.7	0.7

Appendix III - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.001000 siemens/meter
Permitivity = 10.0 esu

Coefficients C(I), I=1 to 8 are :

642.500	208.680	75.900	-310.800
-0.401	-274.800	6.740	13.000

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	211.3	211.6	-0.3
3000.0	191.1	191.2	-0.1
4000.0	188.9	188.5	0.5
5000.0	192.7	192.0	0.6
6000.0	198.9	198.4	0.6
7000.0	206.3	205.9	0.3
8000.0	214.1	214.1	0.1
9000.0	222.2	222.4	-0.2
10000.0	230.3	230.8	-0.5
11000.0	238.3	239.1	-0.8
12000.0	246.3	247.3	-1.0
13000.0	254.1	255.3	-1.2
14000.0	261.9	263.2	-1.3
15000.0	269.6	270.8	-1.2
16000.0	277.1	278.3	-1.2
17000.0	284.6	285.7	-1.1
18000.0	291.8	292.9	-1.0
19000.0	299.0	299.9	-0.9
20000.0	306.0	306.8	-0.8
30000.0	370.2	369.4	0.8
40000.0	426.2	424.6	1.6
50000.0	476.6	475.1	1.5
60000.0	522.7	522.1	0.6
70000.0	565.7	566.1	-0.4
80000.0	606.2	607.4	-1.2
90000.0	644.5	646.2	-1.7
100000.0	681.1	682.9	-1.8
200000.0	989.2	988.5	0.7
300000.0	1244.3	1244.1	0.2
400000.0	1475.8	1476.3	-0.5
500000.0	1695.1	1695.6	-0.5
600000.0	1907.7	1908.0	-0.3
700000.0	2116.9	2116.9	0.0
800000.0	2324.4	2324.2	0.2
900000.0	2531.2	2531.0	0.2
1000000.0	2737.8	2737.7	0.2
1100000.0	2944.6	2944.5	0.1
1200000.0	3151.7	3151.7	0.0
1300000.0	3359.1	3359.2	-0.1
1400000.0	3566.7	3566.9	-0.2
1500000.0	3774.7	3774.9	-0.2
1600000.0	3982.8	3983.0	-0.2
1700000.0	4191.1	4191.3	-0.2
1800000.0	4399.5	4399.6	-0.1
1900000.0	4608.0	4608.1	0.0
2000000.0	4816.6	4816.6	0.0

Appendix III - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.000500 siemens/meter
Permitivity = 5.0 esu

Coefficients C(I), I=1 to 8 are :

873.900	206.980	202.100	-459.800
-0.437	-340.600	8.700	40.000

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	250.0	250.0	0.0
3000.0	238.3	238.1	0.2
4000.0	243.0	242.6	0.4
5000.0	252.9	252.5	0.4
6000.0	264.6	264.5	0.1
7000.0	277.0	277.1	-0.1
8000.0	289.5	289.9	-0.4
9000.0	301.9	302.5	-0.6
10000.0	314.1	314.8	-0.7
11000.0	326.0	326.8	-0.8
12000.0	337.6	338.5	-0.8
13000.0	349.0	349.8	-0.8
14000.0	360.0	360.7	-0.7
15000.0	370.8	371.4	-0.6
16000.0	381.3	381.8	-0.5
17000.0	391.6	391.9	-0.4
18000.0	401.6	401.8	-0.2
19000.0	411.4	411.5	-0.1
20000.0	420.9	420.9	0.0
30000.0	508.7	506.9	1.8
40000.0	583.8	582.3	1.4
50000.0	650.1	650.2	-0.1
60000.0	710.0	711.6	-1.5
70000.0	765.1	767.2	-2.1
80000.0	816.1	818.2	-2.0
90000.0	863.9	865.5	-1.5
100000.0	909.0	909.8	-0.8
200000.0	1266.2	1265.3	0.9
300000.0	1534.5	1535.0	-0.5
400000.0	1762.5	1763.1	-0.6
500000.0	1971.0	1971.2	-0.2
600000.0	2171.0	2170.8	0.1
700000.0	2368.1	2367.9	0.2
800000.0	2565.2	2565.1	0.1
900000.0	2763.6	2763.6	0.0
1000000.0	2963.5	2963.7	-0.1
1100000.0	3165.0	3165.3	-0.3
1200000.0	3367.9	3368.2	-0.3
1300000.0	3571.9	3572.2	-0.3
1400000.0	3776.7	3777.0	-0.3
1500000.0	3982.2	3982.4	-0.2
1600000.0	4188.1	4188.3	-0.1
1700000.0	4394.4	4394.5	-0.1
1800000.0	4600.9	4600.9	0.0
1900000.0	4807.5	4807.5	0.0
2000000.0	5014.2	5014.2	0.0

Appendix III - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

```
Frequency     = 100000.0    Hz
Conductivity  = 0.000200    siemens/meter
Permitivity   =   3.0       esu

Coefficients C(I), I=1 to 8 are :

  1016.800    194.080    401.800  -285.300
    -0.632   -605.500      7.110    14.000

DISTANCE              ACCURATE              APPROXIMATE             DIFFERENCE
  (M)                   (M)                    (M)                     (M)

  2000.0               327.3                  330.6                   -3.4
  3000.0               331.4                  331.4                    0.0
  4000.0               349.2                  347.6                    1.7
  5000.0               370.5                  368.4                    2.1
  6000.0               392.4                  390.5                    1.8
  7000.0               414.0                  412.7                    1.3
  8000.0               435.0                  434.3                    0.7
  9000.0               455.2                  455.2                    0.1
 10000.0               474.8                  475.3                   -0.5
 11000.0               493.6                  494.6                   -1.0
 12000.0               511.7                  513.1                   -1.4
 13000.0               529.2                  530.9                   -1.7
 14000.0               546.1                  548.0                   -1.9
 15000.0               562.4                  564.4                   -2.0
 16000.0               578.2                  580.3                   -2.0
 17000.0               593.6                  595.6                   -2.0
 18000.0               608.5                  610.4                   -2.0
 19000.0               622.9                  624.8                   -1.9
 20000.0               637.0                  638.7                   -1.7
 30000.0               759.8                  759.8                    0.0
 40000.0               860.9                  859.3                    1.6
 50000.0               947.4                  945.4                    2.0
 60000.0              1022.3                 1021.3                    1.0
 70000.0              1088.2                 1088.5                   -0.3
 80000.0              1146.8                 1148.0                   -1.2
 90000.0              1199.3                 1200.7                   -1.5
100000.0              1246.6                 1247.7                   -1.1
200000.0              1552.5                 1550.0                    2.5
300000.0              1735.1                 1737.4                   -2.4
400000.0              1897.1                 1899.0                   -2.0
500000.0              2061.7                 2060.7                    1.0
600000.0              2232.5                 2229.6                    2.9
700000.0              2409.0                 2406.0                    3.0
800000.0              2590.2                 2588.4                    1.9
900000.0              2775.3                 2775.0                    0.2
1000000.0             2963.3                 2964.5                   -1.3
1100000.0             3153.4                 3155.8                   -2.4
1200000.0             3345.3                 3348.3                   -3.0
1300000.0             3538.2                 3541.3                   -3.1
1400000.0             3732.0                 3734.8                   -2.8
1500000.0             3926.4                 3928.6                   -2.2
1600000.0             4121.1                 4122.5                   -1.4
1700000.0             4316.0                 4316.4                   -0.4
1800000.0             4511.1                 4510.4                    0.6
1900000.0             4706.2                 4704.5                    1.7
2000000.0             4901.3                 4898.5                    2.8
```

Appendix III - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.000050 siemens/meter
Permitivity = 3.0 esu

Coefficients C(I), I=1 to 8 are :

901.000	182.020	126.500	276.700
-0.492	-979.600	12.360	258.000

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	514.3	509.6	4.7
3000.0	545.8	544.2	1.5
4000.0	585.0	586.3	-1.2
5000.0	623.8	627.0	-3.2
6000.0	660.3	664.5	-4.2
7000.0	694.4	698.7	-4.3
8000.0	726.0	729.8	-3.8
9000.0	755.4	758.2	-2.8
10000.0	782.8	784.3	-1.5
11000.0	808.4	808.4	0.0
12000.0	832.3	830.8	1.6
13000.0	854.8	851.7	3.1
14000.0	875.9	871.4	4.5
15000.0	895.7	889.9	5.8
16000.0	914.4	907.5	6.9
17000.0	932.1	924.3	7.8
18000.0	948.7	940.3	8.5
19000.0	964.5	955.6	8.9
20000.0	979.4	970.3	9.1
30000.0	1093.2	1090.6	2.7
40000.0	1163.7	1170.3	-6.6
50000.0	1211.9	1220.3	-8.5
60000.0	1248.3	1253.1	-4.7
70000.0	1275.7	1276.8	-1.1
80000.0	1297.5	1296.0	1.5
90000.0	1315.9	1312.8	3.1
100000.0	1332.1	1328.2	3.9
200000.0	1463.7	1464.0	-0.3
300000.0	1595.8	1597.7	-1.9
400000.0	1738.1	1739.0	-0.9
500000.0	1889.5	1889.2	0.3
600000.0	2048.4	2047.6	0.8
700000.0	2213.2	2212.6	0.6
800000.0	2382.7	2382.6	0.1
900000.0	2555.8	2556.3	-0.5
1000000.0	2731.7	2732.7	-1.0
1100000.0	2909.6	2910.9	-1.3
1200000.0	3088.9	3090.3	-1.4
1300000.0	3269.3	3270.6	-1.3
1400000.0	3450.5	3451.5	-1.1
1500000.0	3632.0	3632.8	-0.8
1600000.0	3813.9	3814.3	-0.4
1700000.0	3995.9	3996.0	-0.1
1800000.0	4178.1	4177.9	0.2
1900000.0	4360.3	4359.7	0.5
2000000.0	4542.5	4541.7	0.8

Appendix III - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.000050 siemens/meter
Permitivity = 4.0 esu

Coefficients C(I), I=1 to 8 are :

853.700	181.640	126.200	269.500
-0.487	-922.900	12.100	203.000

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	502.1	497.5	4.6
3000.0	528.9	527.3	1.6
4000.0	564.5	565.5	-1.0
5000.0	600.1	602.9	-2.8
6000.0	633.9	637.8	-3.8
7000.0	665.5	669.6	-4.1
8000.0	694.9	698.6	-3.7
9000.0	722.3	725.2	-2.9
10000.0	747.8	749.5	-1.8
11000.0	771.6	772.0	-0.5
12000.0	793.9	792.9	1.0
13000.0	814.8	812.4	2.4
14000.0	834.5	830.7	3.8
15000.0	853.0	847.9	5.1
16000.0	870.5	864.3	6.2
17000.0	887.0	879.8	7.2
18000.0	902.6	894.6	8.0
19000.0	917.3	908.8	8.6
20000.0	931.3	922.4	9.0
30000.0	1038.8	1034.3	4.5
40000.0	1106.7	1111.8	-5.1
50000.0	1154.7	1163.0	-8.4
60000.0	1191.6	1197.5	-5.9
70000.0	1219.9	1222.7	-2.8
80000.0	1242.8	1242.9	-0.1
90000.0	1262.3	1260.5	1.8
100000.0	1279.4	1276.5	2.9
200000.0	1415.1	1414.9	0.2
300000.0	1548.1	1549.7	-1.6
400000.0	1690.5	1691.2	-0.7
500000.0	1841.7	1841.2	0.4
600000.0	2000.2	1999.2	1.0
700000.0	2164.6	2163.6	0.9
800000.0	2333.6	2333.1	0.5
900000.0	2506.2	2506.3	0.0
1000000.0	2681.6	2682.1	-0.5
1100000.0	2859.0	2859.8	-0.8
1200000.0	3037.8	3038.7	-0.9
1300000.0	3217.7	3218.6	-0.9
1400000.0	3398.3	3399.1	-0.7
1500000.0	3579.4	3579.9	-0.5
1600000.0	3760.8	3761.0	-0.3
1700000.0	3942.3	3942.3	0.0
1800000.0	4123.9	4123.7	0.2
1900000.0	4305.6	4305.2	0.3
2000000.0	4487.3	4486.8	0.5

Appendix III - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.000040 siemens/meter
Permitivity = 5.0 esu

Coefficients C(I), I=1 to 8 are :

757.300 180.020 100.300 285.400
-0.449 -830.500 13.290 191.000

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	508.9	503.5	5.4
3000.0	530.6	529.5	1.2
4000.0	562.0	563.6	-1.6
5000.0	593.8	597.2	-3.4
6000.0	624.1	628.3	-4.2
7000.0	652.3	656.6	-4.3
8000.0	678.5	682.2	-3.8
9000.0	702.7	705.5	-2.8
10000.0	725.2	726.8	-1.6
11000.0	746.2	746.4	-0.2
12000.0	765.7	764.5	1.2
13000.0	783.9	781.2	2.7
14000.0	801.0	796.9	4.1
15000.0	816.9	811.6	5.3
16000.0	832.0	825.5	6.5
17000.0	846.1	838.6	7.5
18000.0	859.4	851.1	8.3
19000.0	872.0	863.0	9.0
20000.0	883.8	874.4	9.4
30000.0	973.6	967.5	6.1
40000.0	1029.6	1032.9	-3.3
50000.0	1070.1	1077.5	-7.4
60000.0	1102.1	1108.5	-6.3
70000.0	1127.4	1131.5	-4.1
80000.0	1148.4	1150.2	-1.8
90000.0	1166.7	1166.6	0.1
100000.0	1183.1	1181.7	1.4
200000.0	1317.5	1316.2	1.3
300000.0	1449.6	1450.5	-0.9
400000.0	1590.6	1591.9	-1.2
500000.0	1740.3	1741.2	-0.9
600000.0	1897.2	1897.8	-0.6
700000.0	2059.9	2060.4	-0.5
800000.0	2227.1	2227.7	-0.6
900000.0	2397.9	2398.6	-0.7
1000000.0	2571.5	2572.2	-0.7
1100000.0	2747.0	2747.7	-0.7
1200000.0	2924.0	2924.5	-0.6
1300000.0	3102.0	3102.4	-0.4
1400000.0	3280.7	3280.9	-0.2
1500000.0	3459.9	3459.9	0.0
1600000.0	3639.3	3639.2	0.1
1700000.0	3818.9	3818.7	0.2
1800000.0	3998.7	3998.4	0.2
1900000.0	4178.4	4178.2	0.2
2000000.0	4358.2	4358.1	0.1

Appendix III - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

```
Frequency    = 100000.0    Hz
Conductivity = 0.005000    siemens/meter
Permitivity  = 80.0        esu

Coefficients C(I), I=1 to 8 are :

   195.000   165.720    46.000   -82.400
    -0.600   -87.800    10.130   150.000
```

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFEREN (M)
2000.0	160.1	158.6	1.4
3000.0	128.5	129.0	-0.4
4000.0	116.8	117.4	-0.6
5000.0	112.3	112.8	-0.6
6000.0	111.0	111.5	-0.5
7000.0	111.5	111.9	-0.5
8000.0	112.9	113.4	-0.4
9000.0	115.0	115.4	-0.4
10000.0	117.4	117.8	-0.4
11000.0	120.3	120.4	-0.1
12000.0	123.5	123.2	0.3
13000.0	126.7	126.2	0.5
14000.0	129.8	129.2	0.7
15000.0	133.0	132.2	0.8
16000.0	136.1	135.3	0.9
17000.0	139.3	138.4	0.9
18000.0	142.5	141.5	1.0
19000.0	145.6	144.6	1.0
20000.0	148.7	147.7	1.0
30000.0	178.6	178.3	0.3
40000.0	206.2	206.8	-0.6
50000.0	231.8	232.7	-0.9
60000.0	255.9	256.7	-0.8
70000.0	278.8	279.3	-0.4
80000.0	300.9	301.0	0.0
90000.0	322.2	322.0	0.2
100000.0	342.9	342.5	0.4
200000.0	530.2	530.4	-0.2
300000.0	701.6	702.1	-0.5
400000.0	867.7	867.7	0.0
500000.0	1031.9	1031.4	0.5
600000.0	1195.6	1195.0	0.6
700000.0	1359.4	1359.0	0.4
800000.0	1523.5	1523.4	0.1
900000.0	1688.1	1688.2	-0.2
1000000.0	1853.0	1853.4	-0.4
1100000.0	2018.1	2018.7	-0.6
1200000.0	2183.6	2184.2	-0.6
1300000.0	2349.1	2349.7	-0.6
1400000.0	2514.8	2515.4	-0.5
1500000.0	2680.6	2681.0	-0.4
1600000.0	2846.5	2846.7	-0.3
1700000.0	3012.3	3012.4	-0.1
1800000.0	3178.2	3178.1	0.1
1900000.0	3344.1	3343.8	0.3
2000000.0	3510.1	3509.5	0.5

Appendix IV

COEFFICIENT VARIATION WITH CONDUCTIVITY AND PERMITTIVITY

In this appendix, those coefficients that vary with conductivity are shown. They were plotted using UNB GRAPH PACK available in the main frame which joins the data points with spline curves. For sea water conductivity and land conductivity above 0.005 siemens/metre, each C_i coefficients was plotted individually. For land conductivity between 0.0001 and 0.005 siemens/metre,the same C_i coefficients at seven different permittivity values were plotted together (7 graphs in each plot) to show their variation with conductivity and permittivity. This method was also used for land conductivity between 0.00001 and 0.0001 siemens/metre for three different permittivity values. These plots are showns by the following figures:

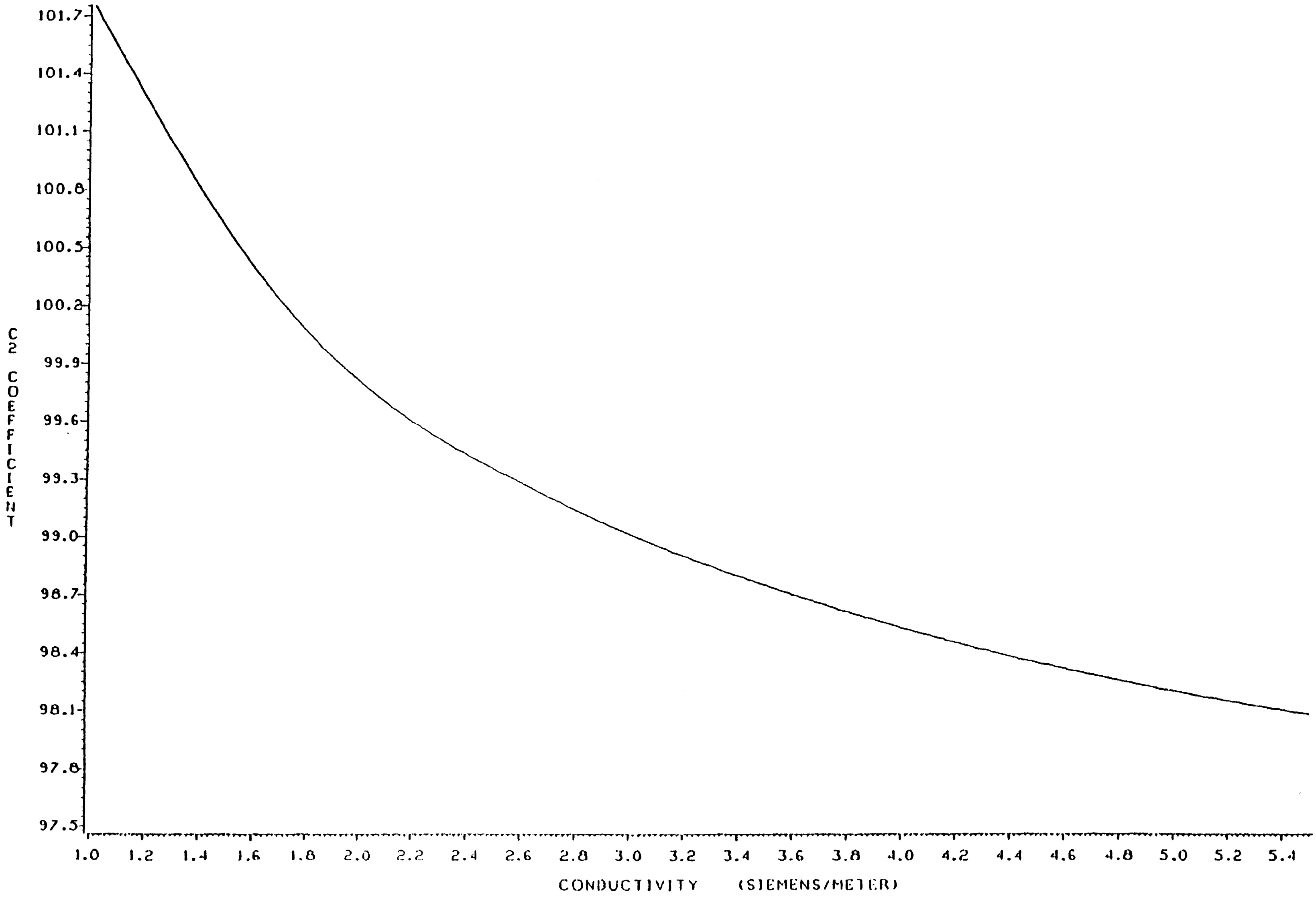

Figure IV.1 : C_2 Variation with Conductivity for Sea Water

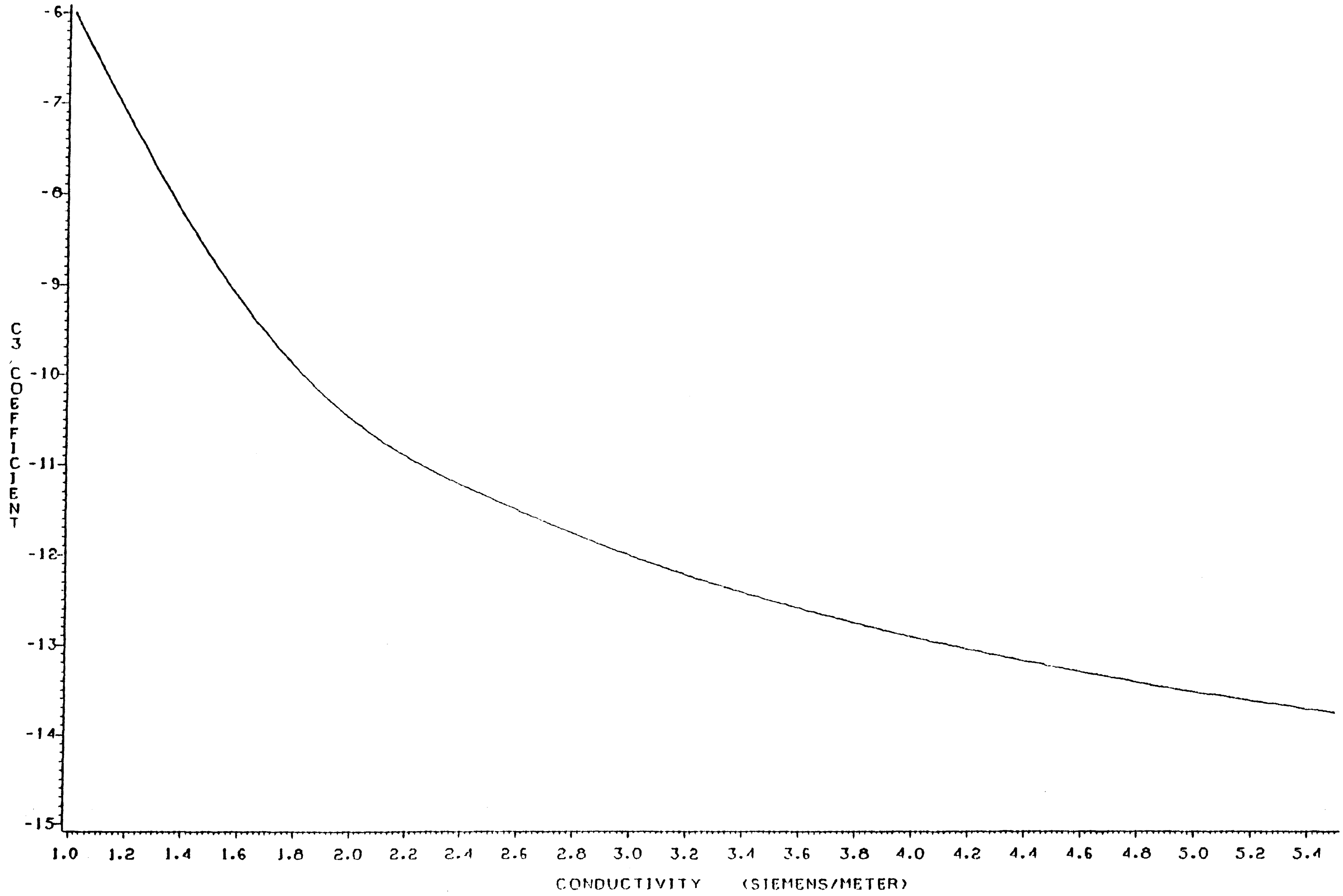

Figure IV.2 : C_3 Variation with Conductivity for Sea Water

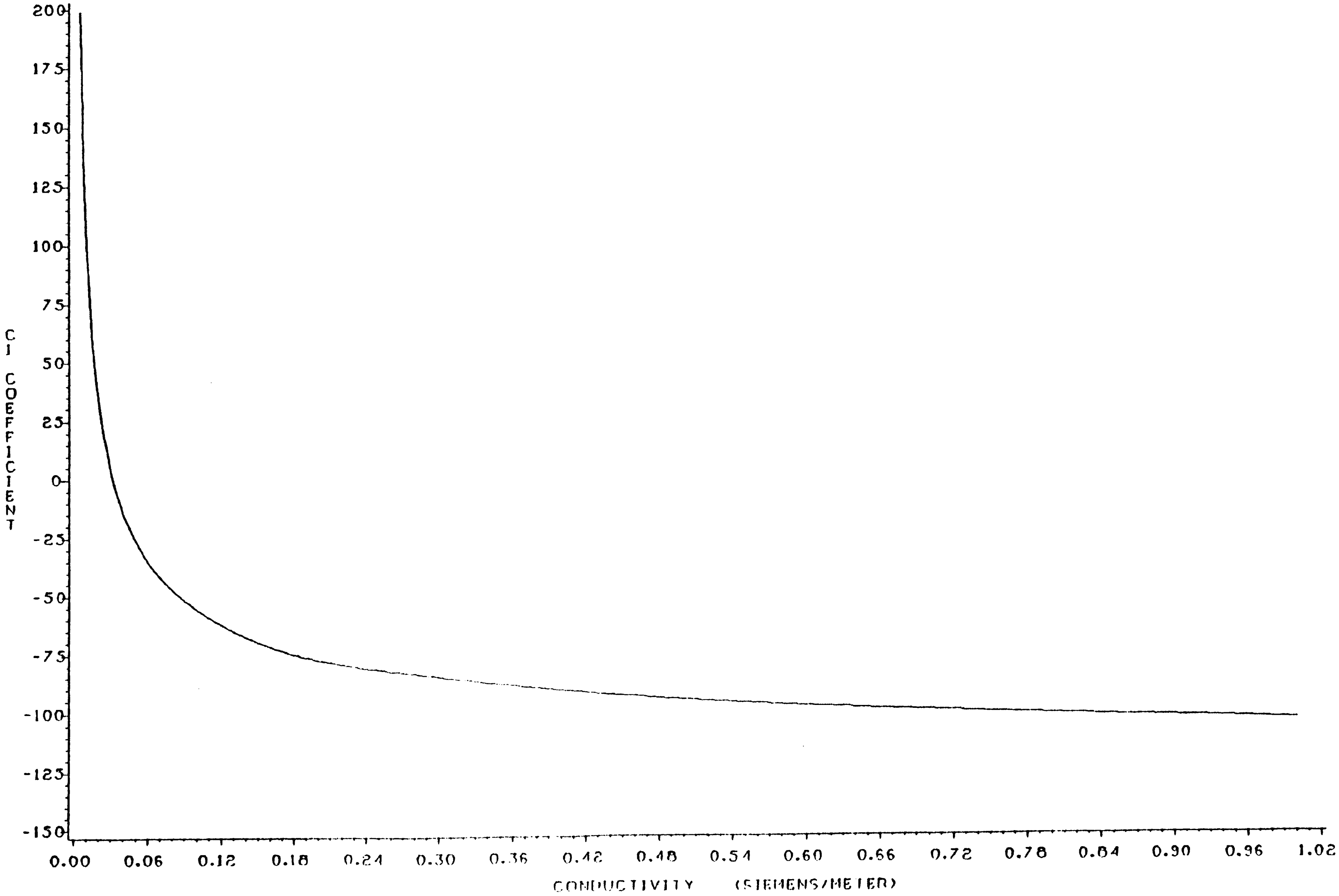

Figure IV.3 : C_1 Variation with Land Conductivity above 0.005 siemens/metre

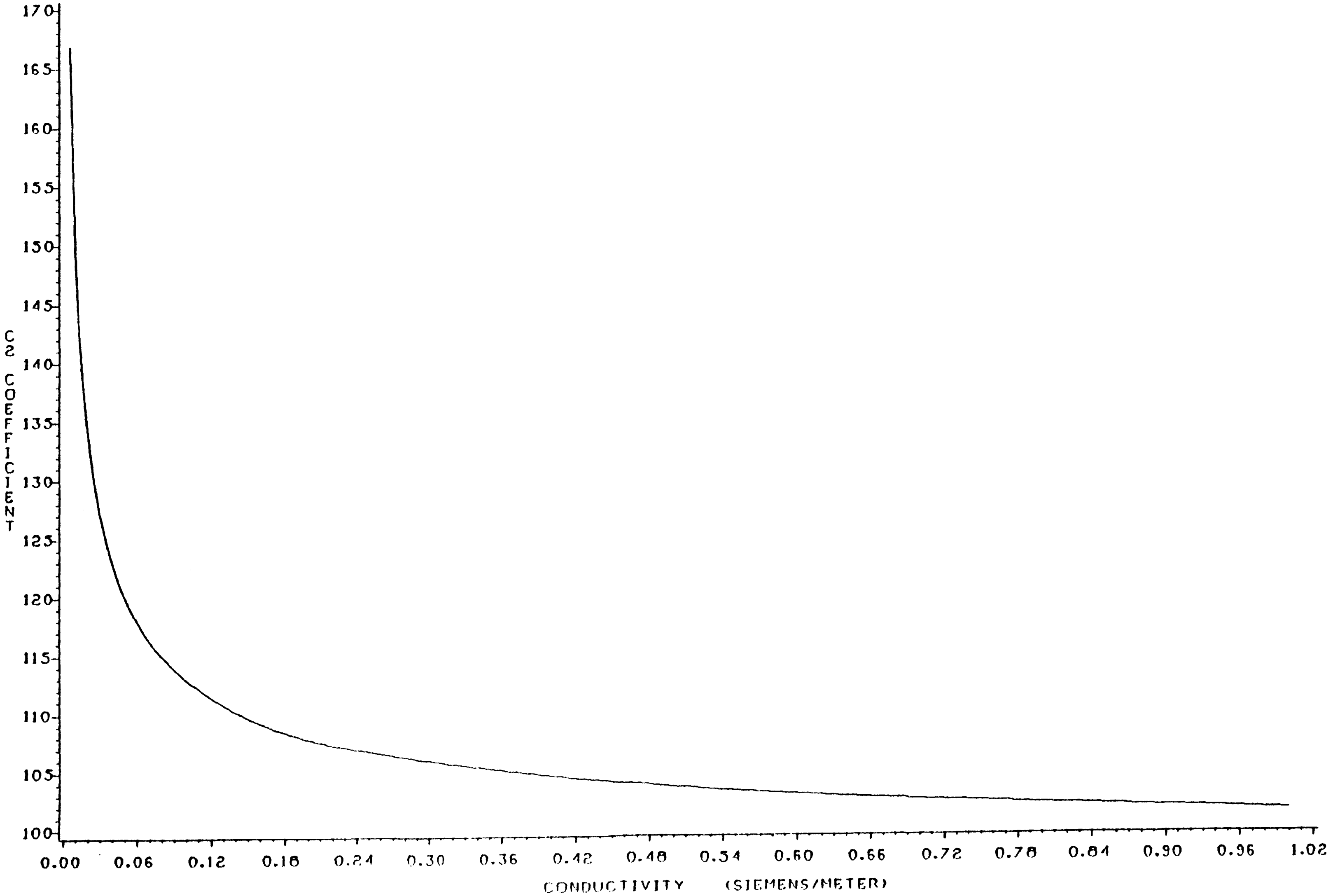

Figure IV.4 : C_2 Variation with Land Conductivity above 0.005 siemens/metre

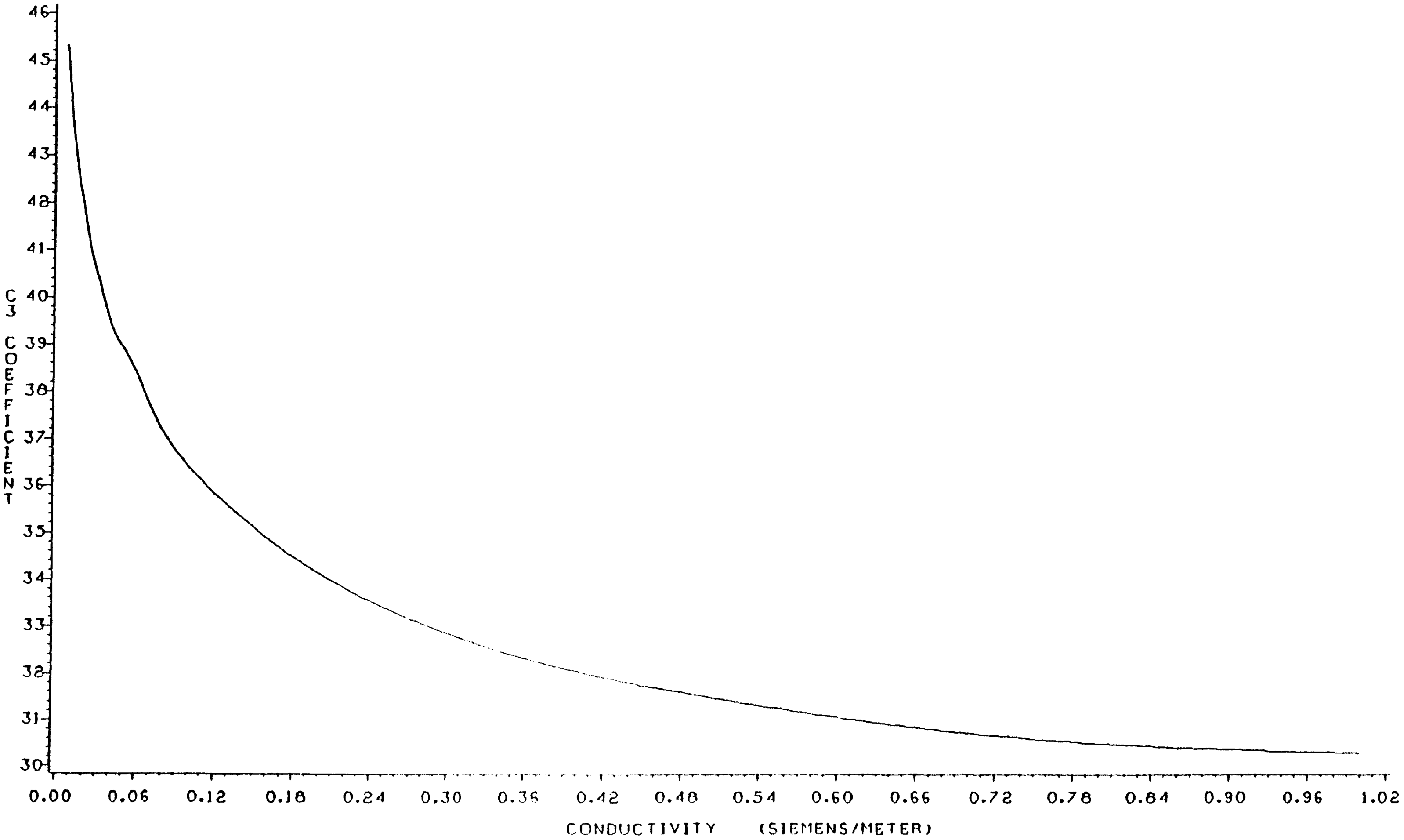

Figure IV.5 : C_3 Variation with Land Conductivity above 0.005 siemens/metre

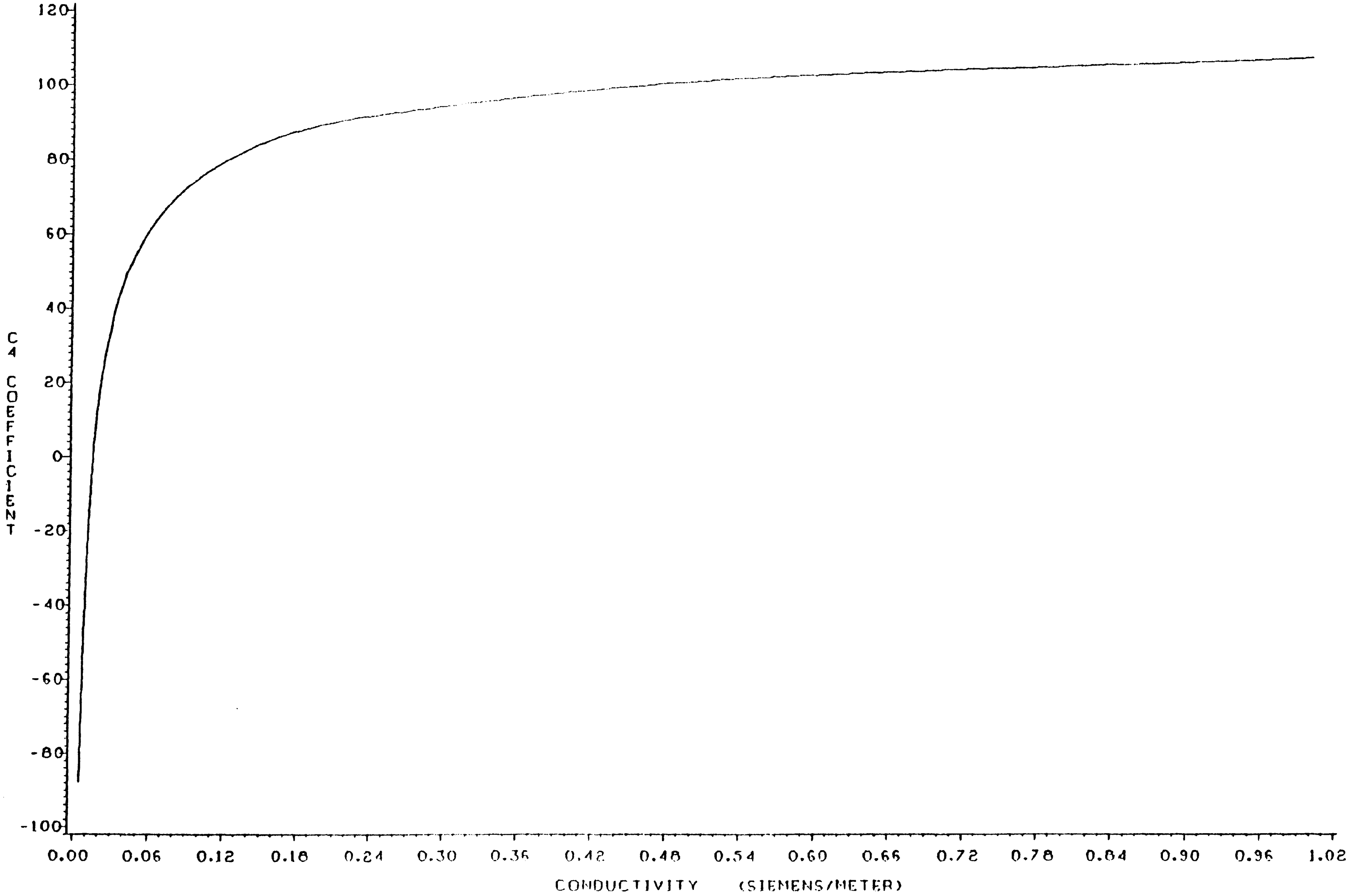

Figure IV.6 : C_4 Variation with Land Conductivity above 0.005 siemens/metre

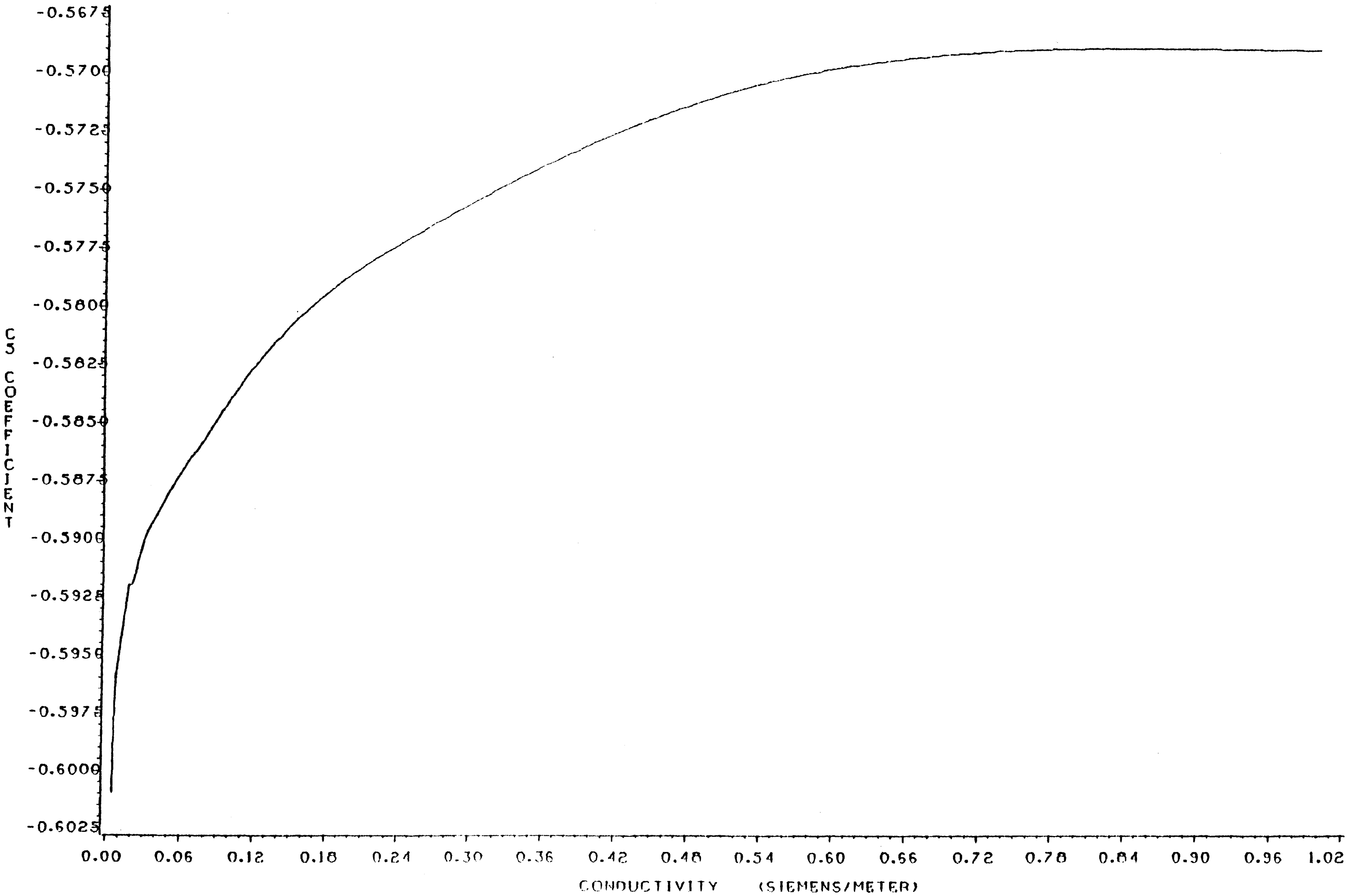

Figure IV.7 : C_5 Variation with Land Conductivity above 0.005 siemens/metre

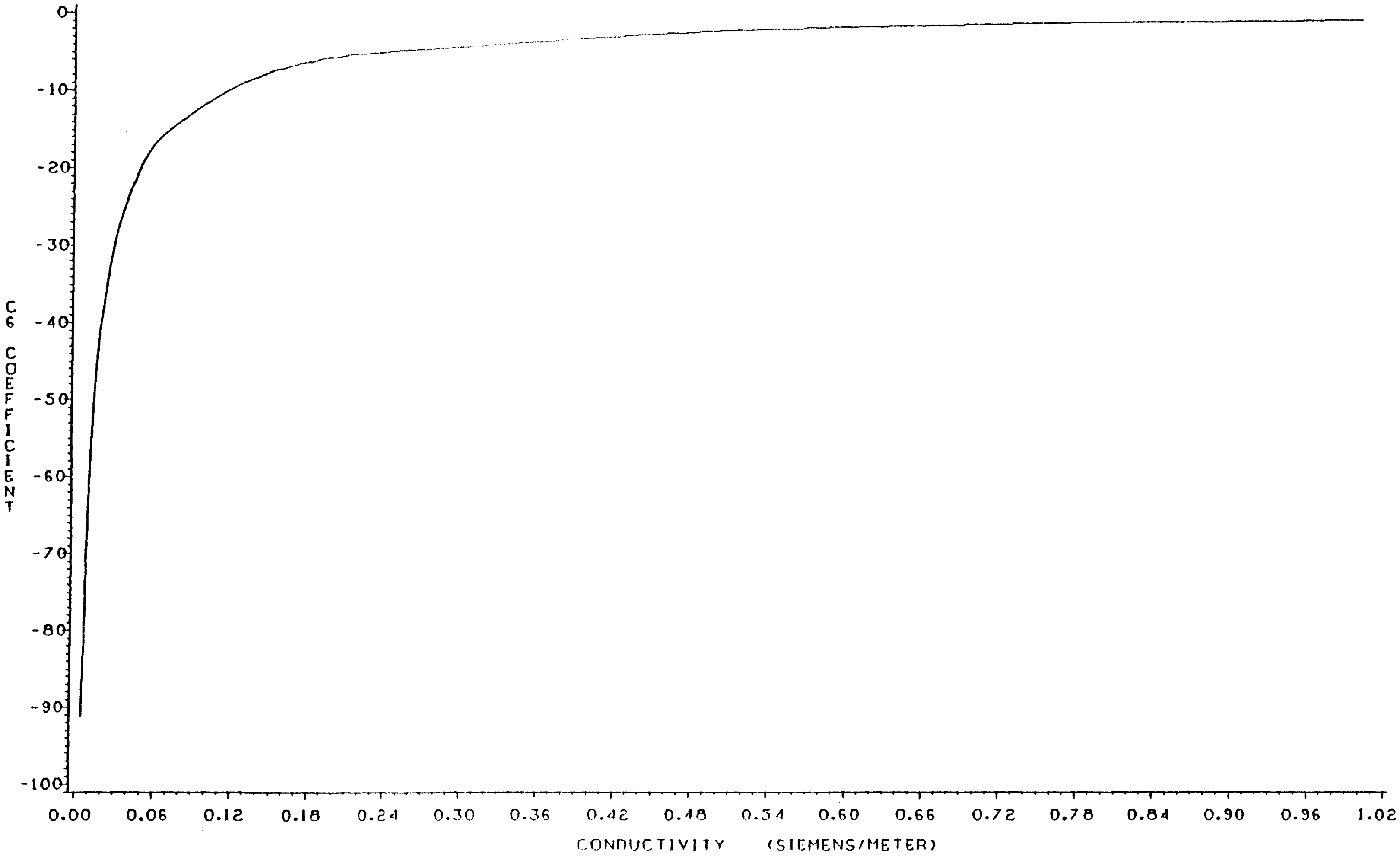

Figure IV.8 : C_6 Variation with Land Conductivity above 0.005 siemens/metre

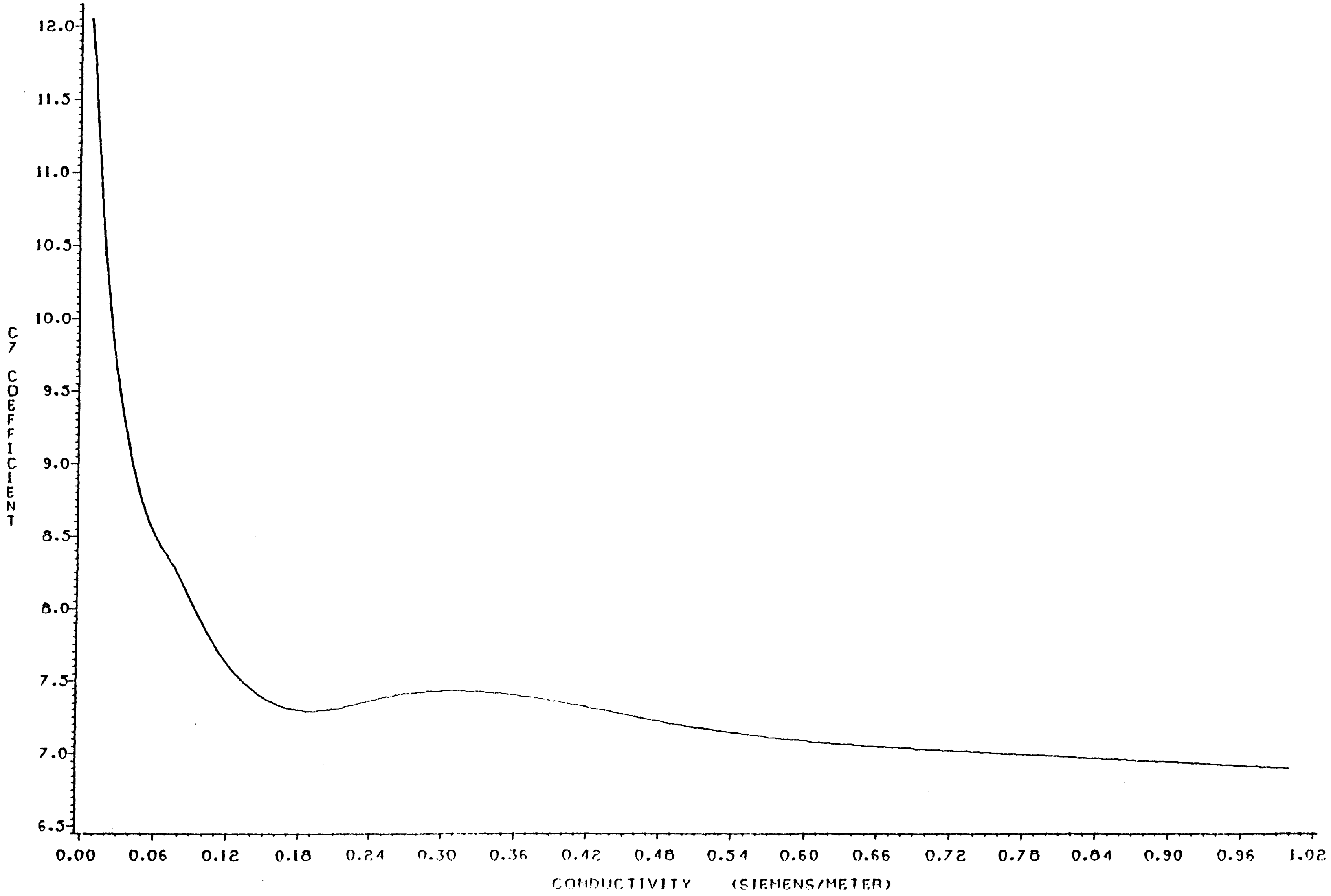

Figure IV.9 : C_7 Variation with Land Conductivity above 0.005 siemens/metre

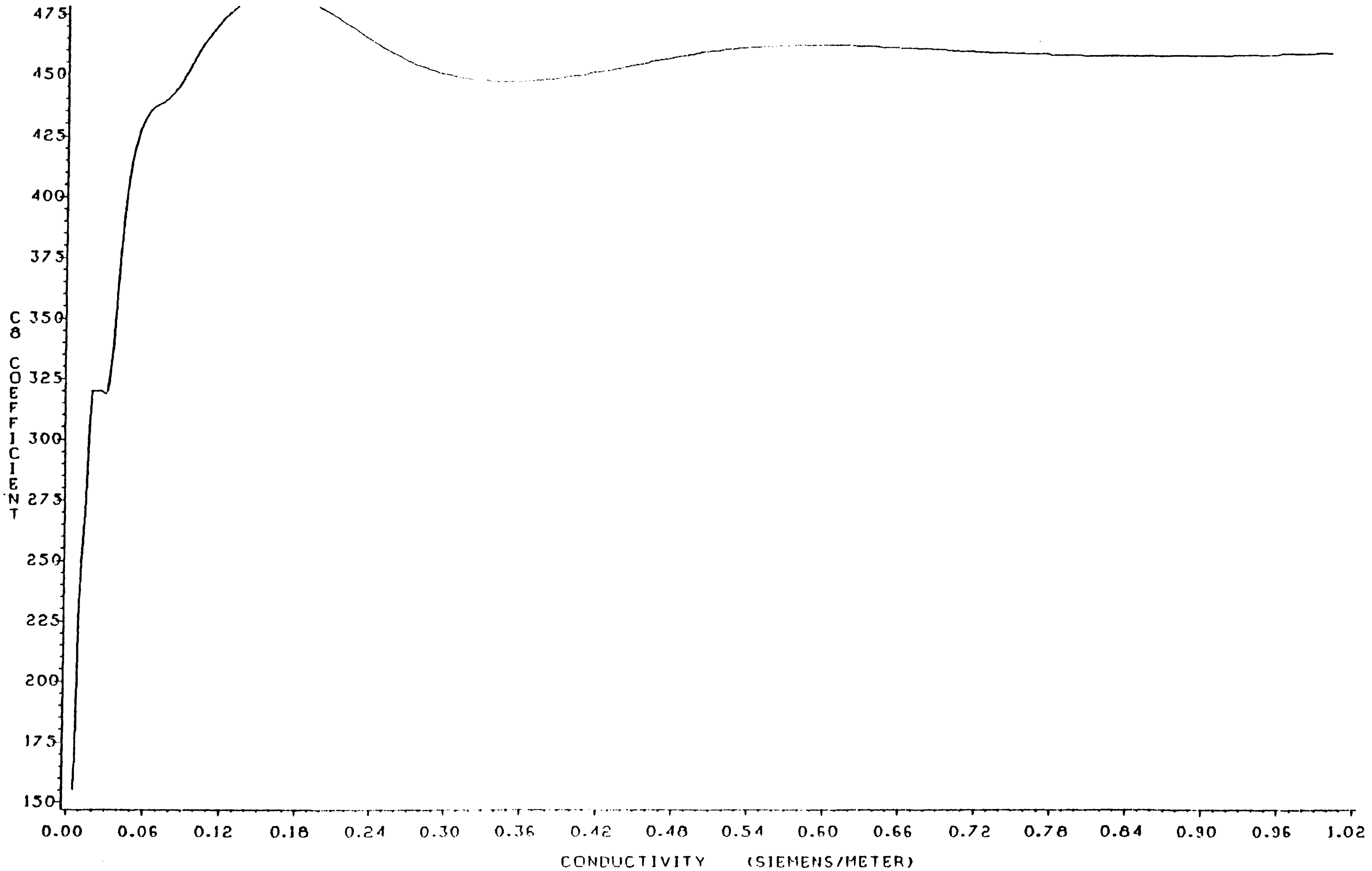

Figure IV.10 : C_8 Variation with Land Conductivity above 0.005 siemens/metre

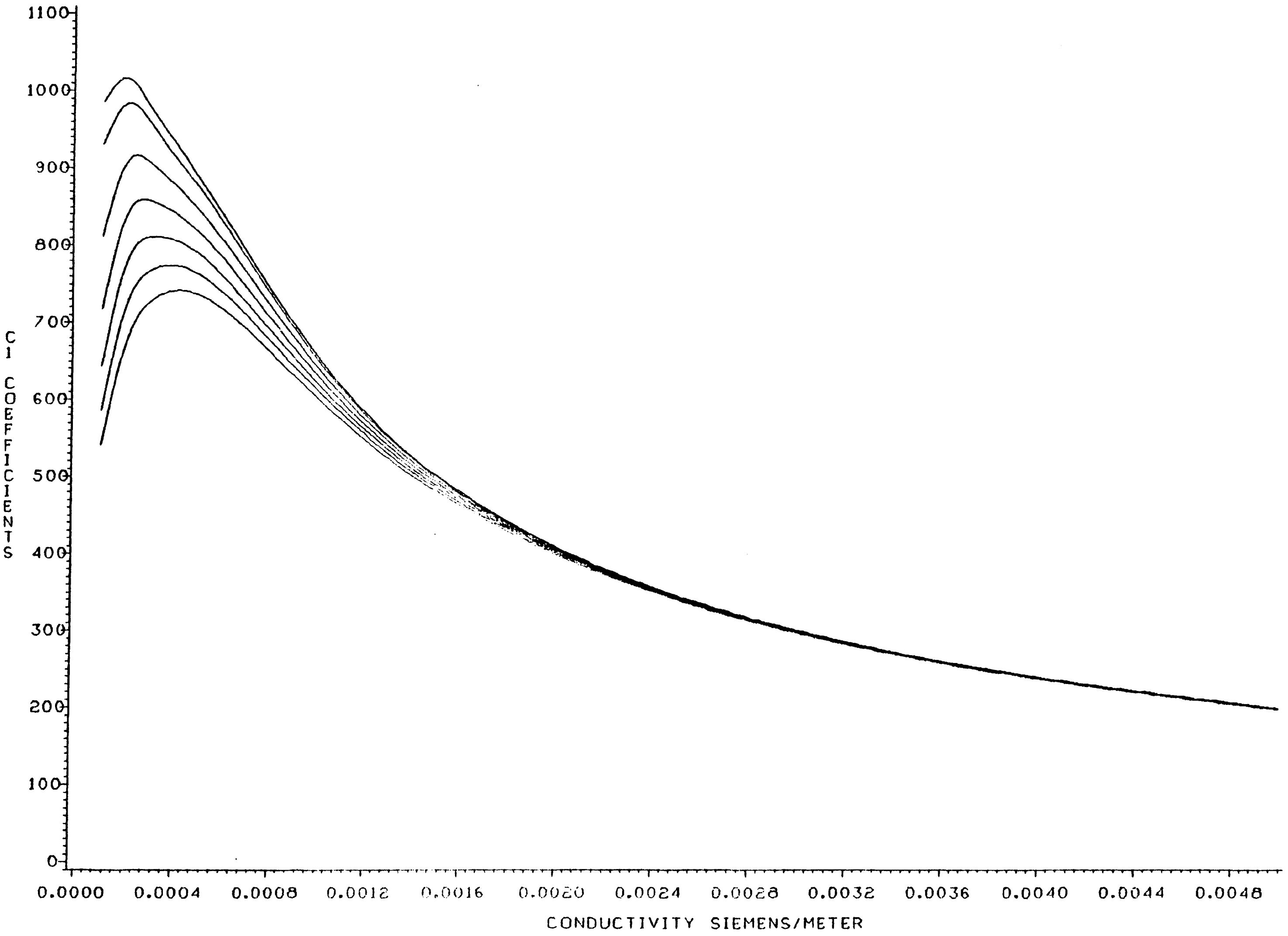

Figure IV.11 : C_1 Variation with Conductivity and Permittivity (0.0001 to 0.005 siemens/metre)

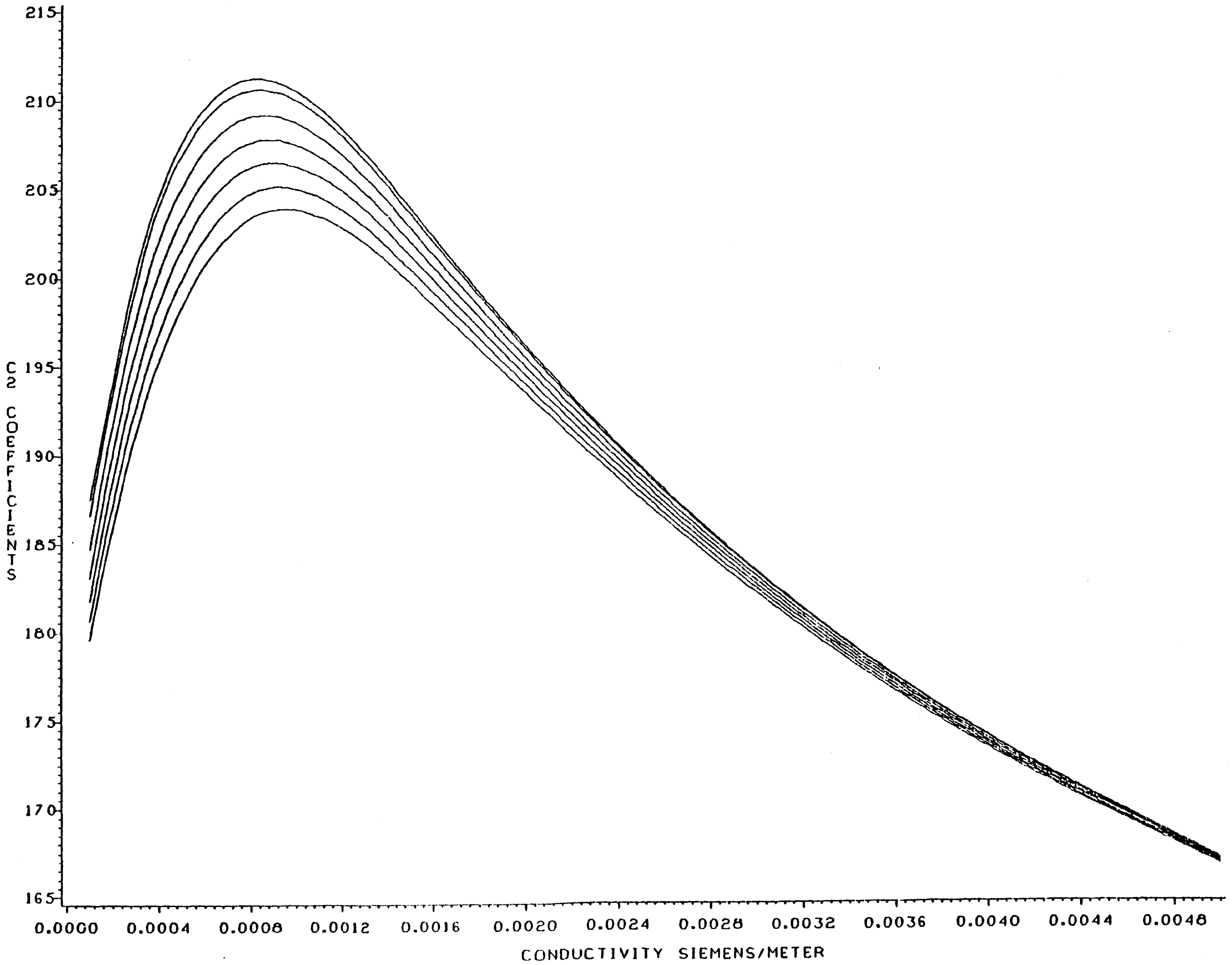

Figure IV.12 : C_2 Variation with Conductivity and Permittivity (0.0001 to 0.005 siemens/metre)

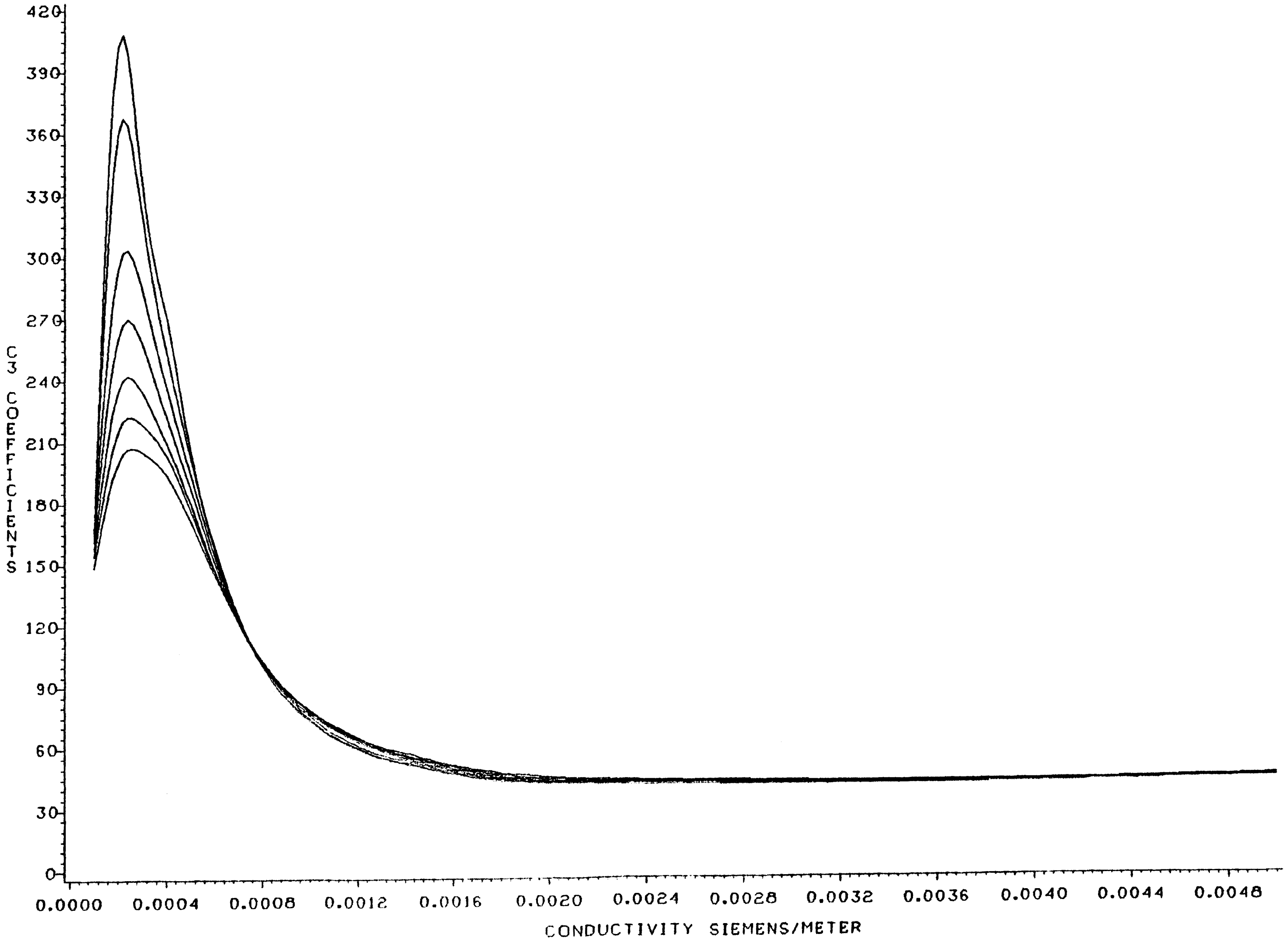

Figure IV.13 : C_3 Variation with Conductivity and Permittivity (0.0001 to 0.005 siemens/metre)

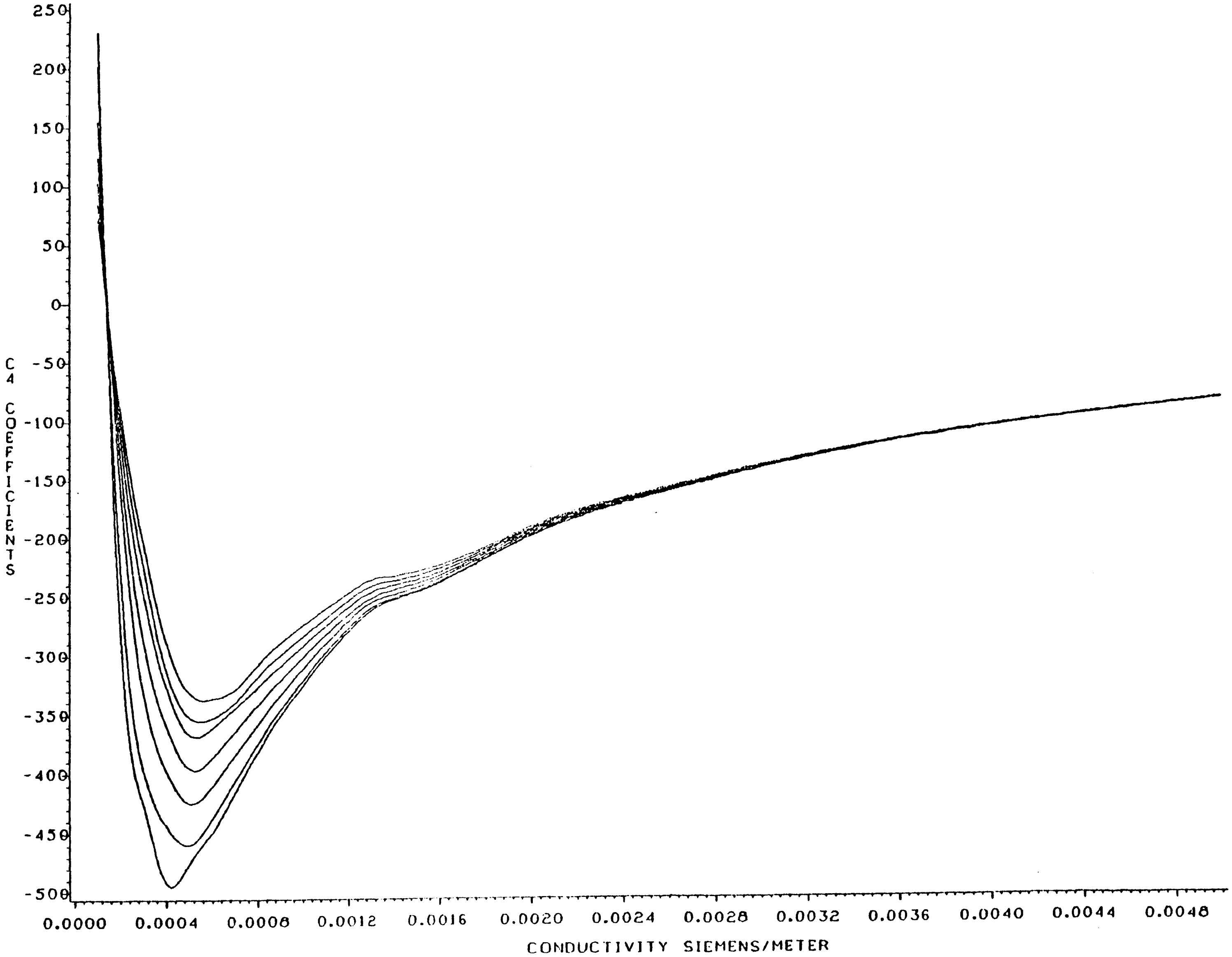

Figure IV.14 : C_4 Variation with Conductivity and Permittivity (0.0001 to 0.005 siemens/metre)

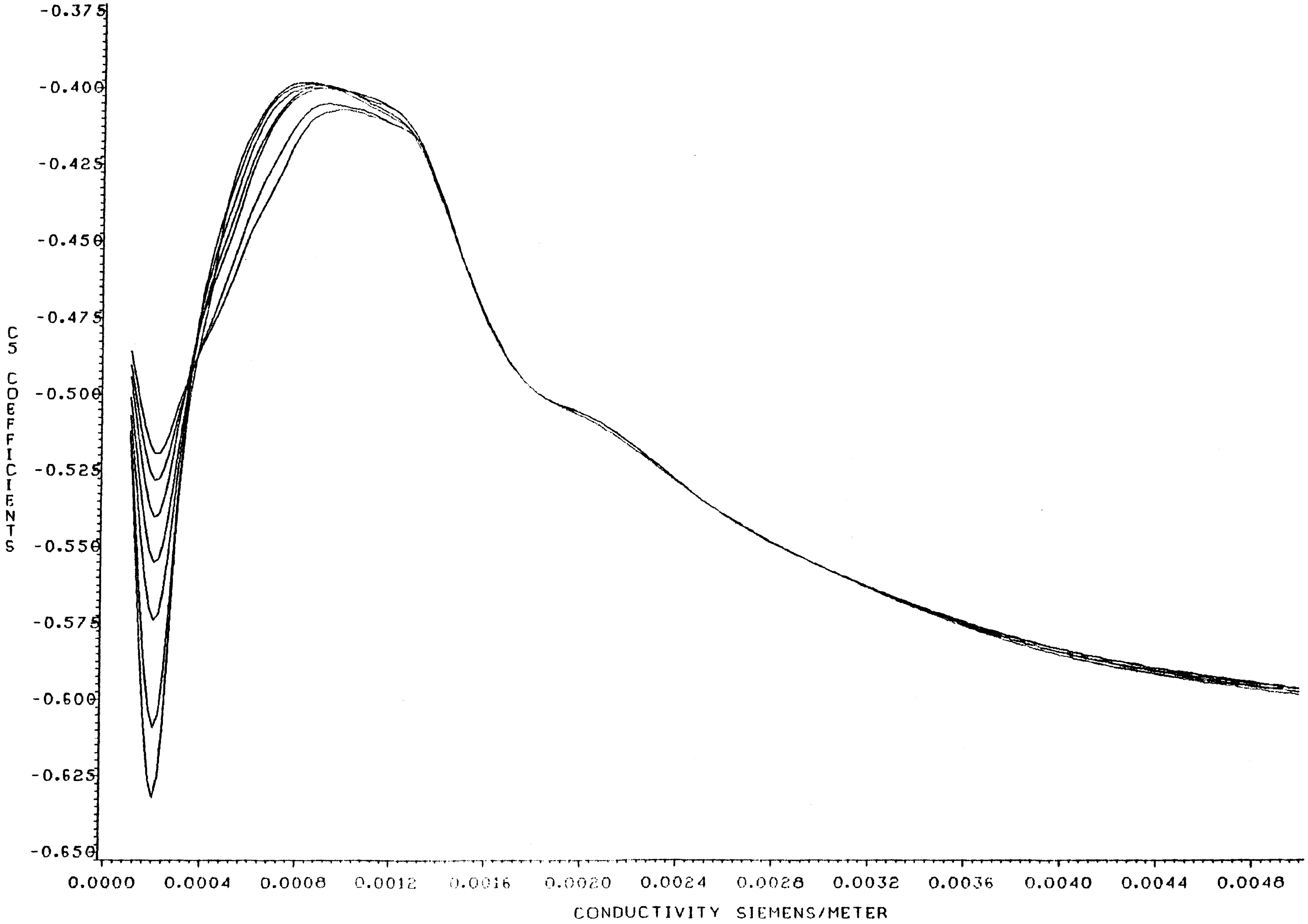

Figure IV.15 : C_5 Variation with Conductivity and Permittivity (0.0001 to 0.005 siemens/metre)

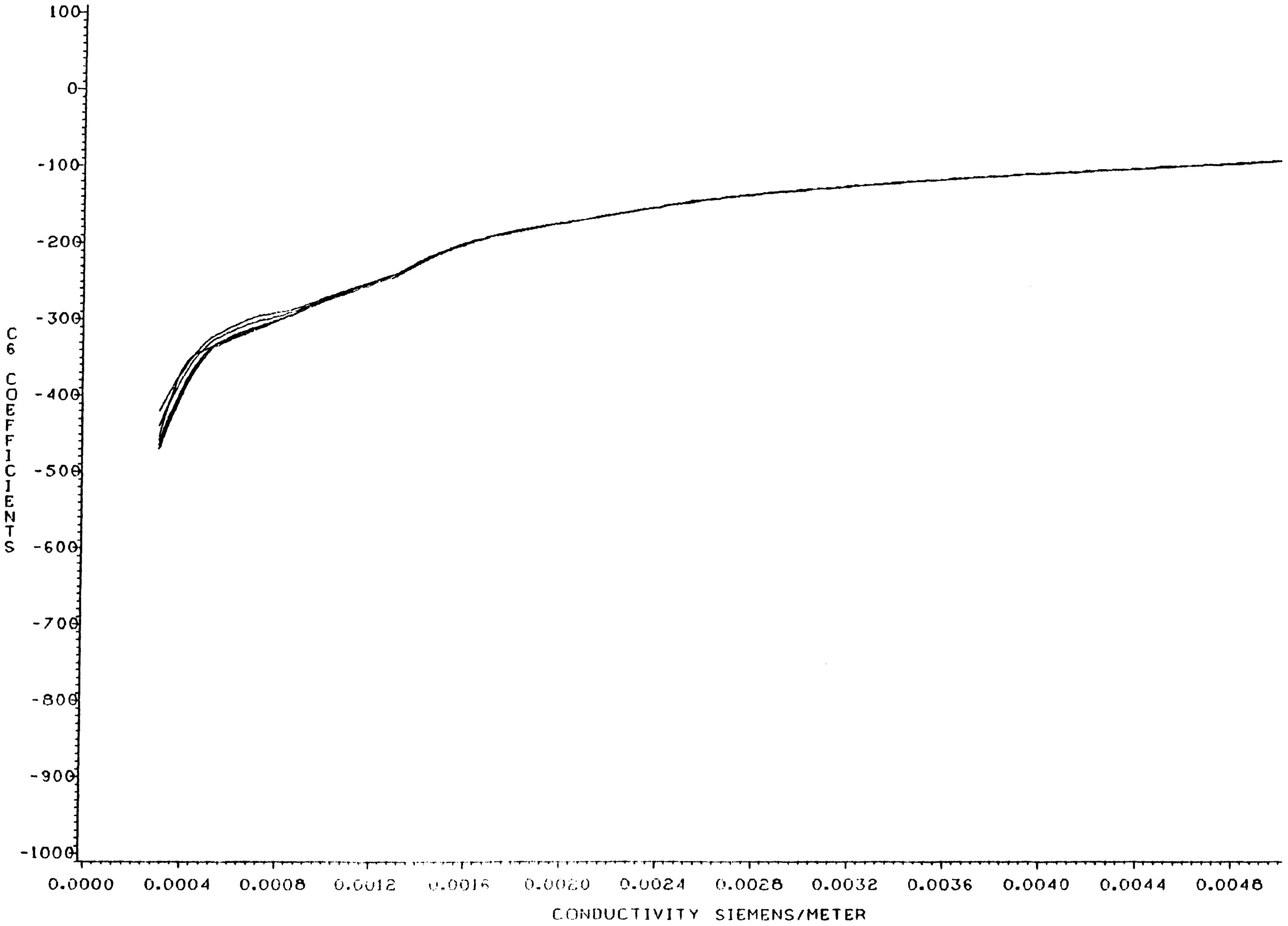

Figure IV.16 : C_6 Variation with Conductivity and Permittivity (0.0001 to 0.005 siemens/metre)

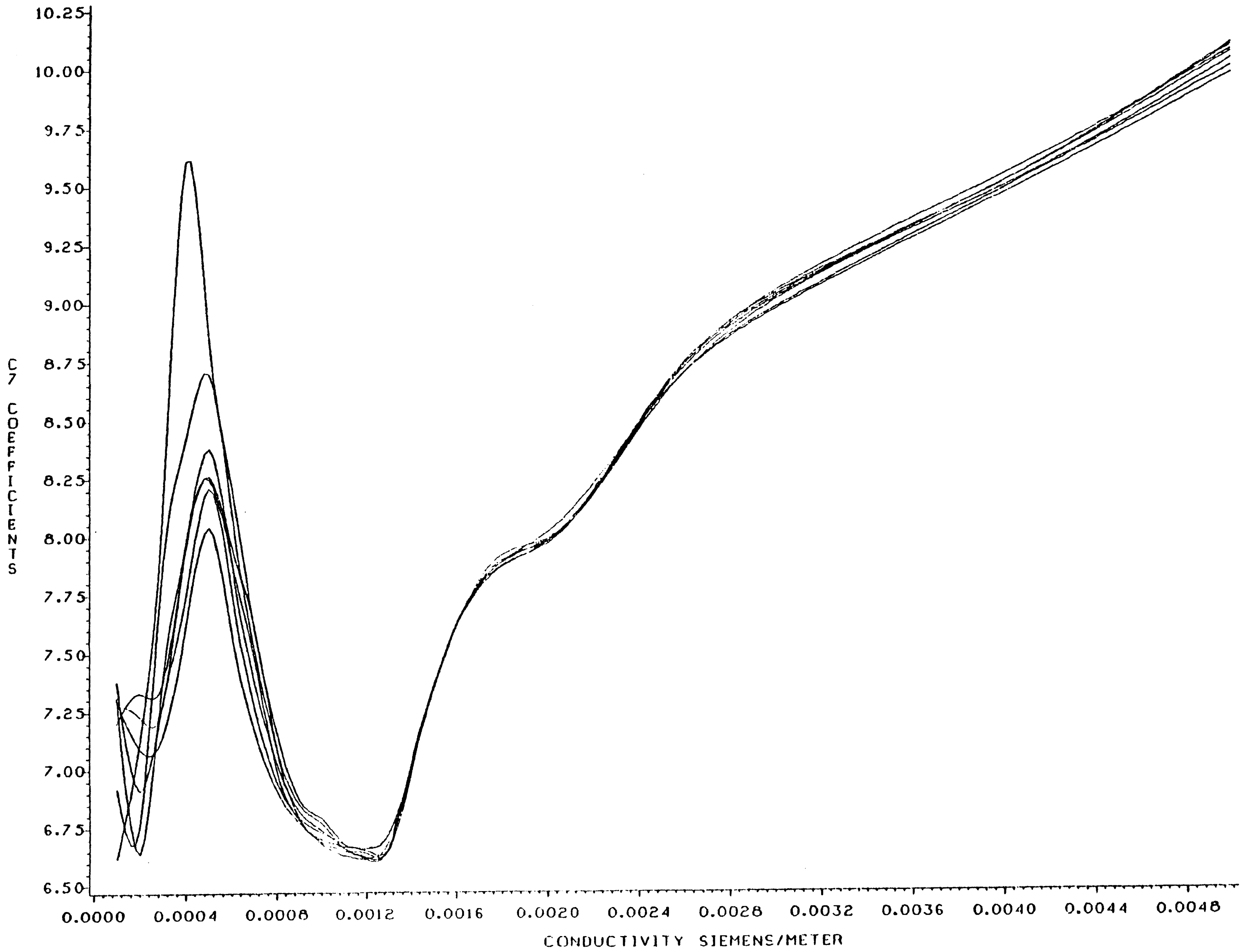

Figure IV.17 : C_7 Variation with Conductivity and Permittivity (0.0001 to 0.005 siemens/metre)

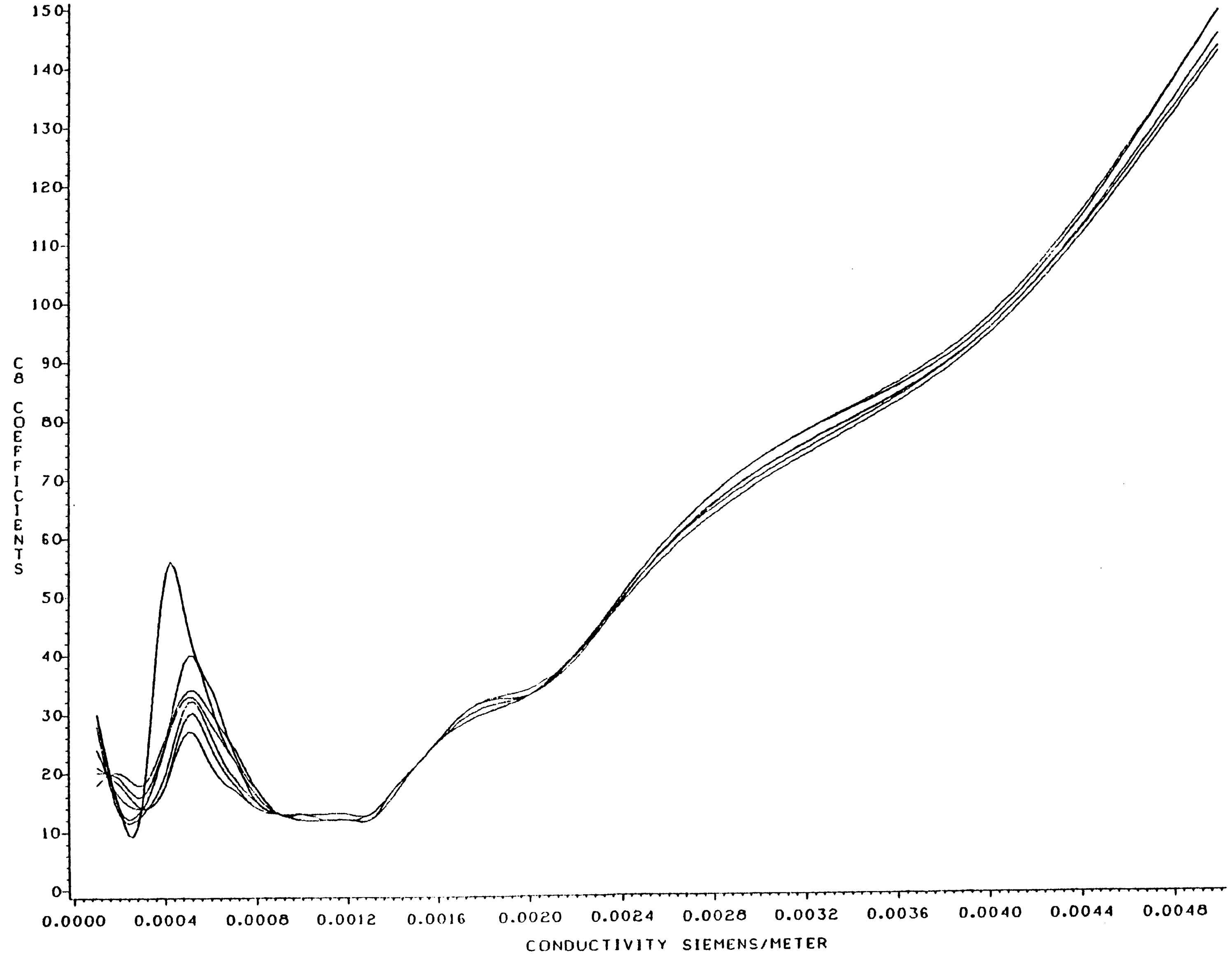

Figure IV.18 : C_8 Variation with Conductivity and Permittivity (0.0001 to 0.005 siemens/metre)

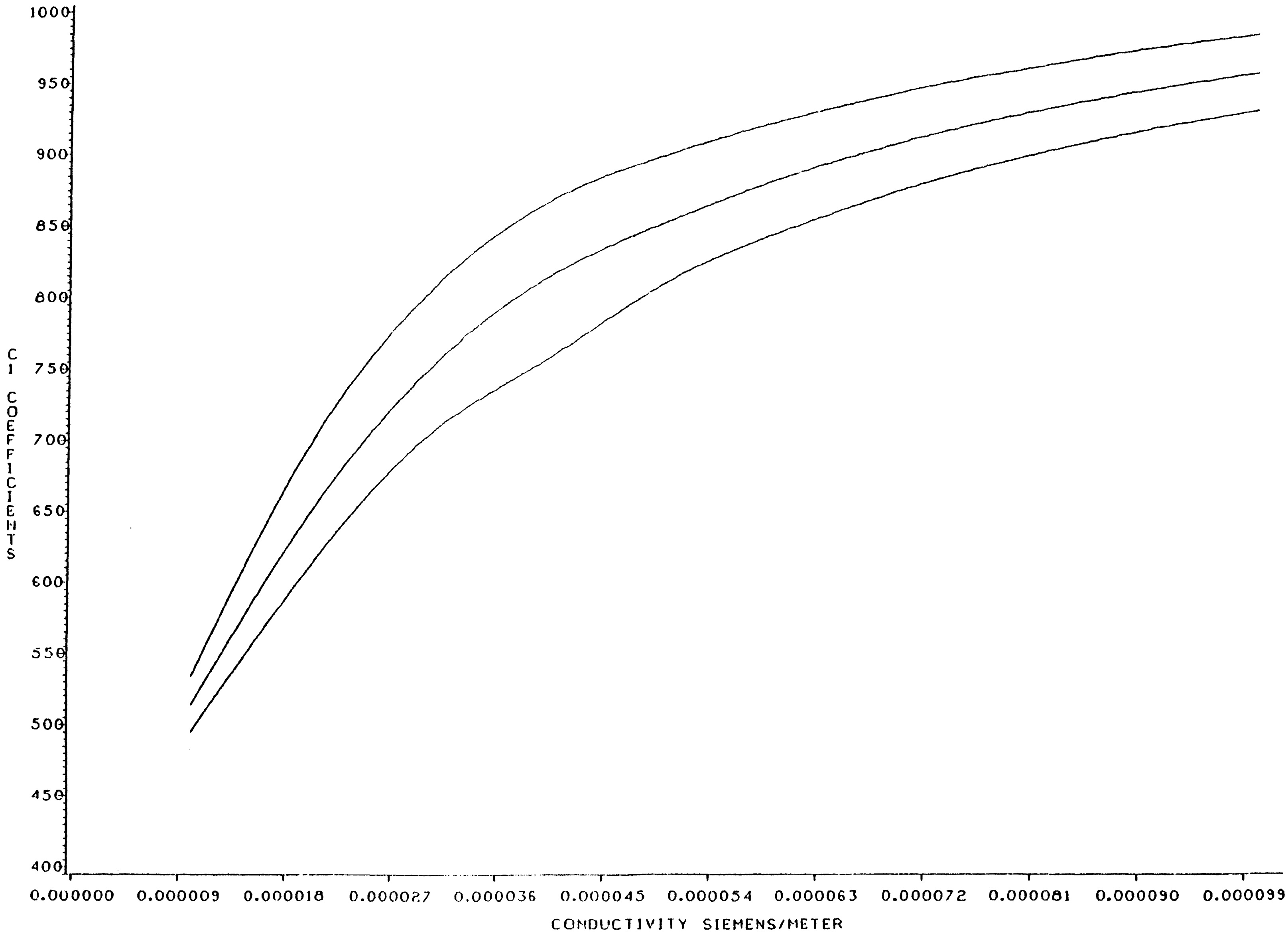

Figure IV.19 : C_1 Variation with Conductivity and Permittivity (0.00001 to 0.0001 siemens/metre)

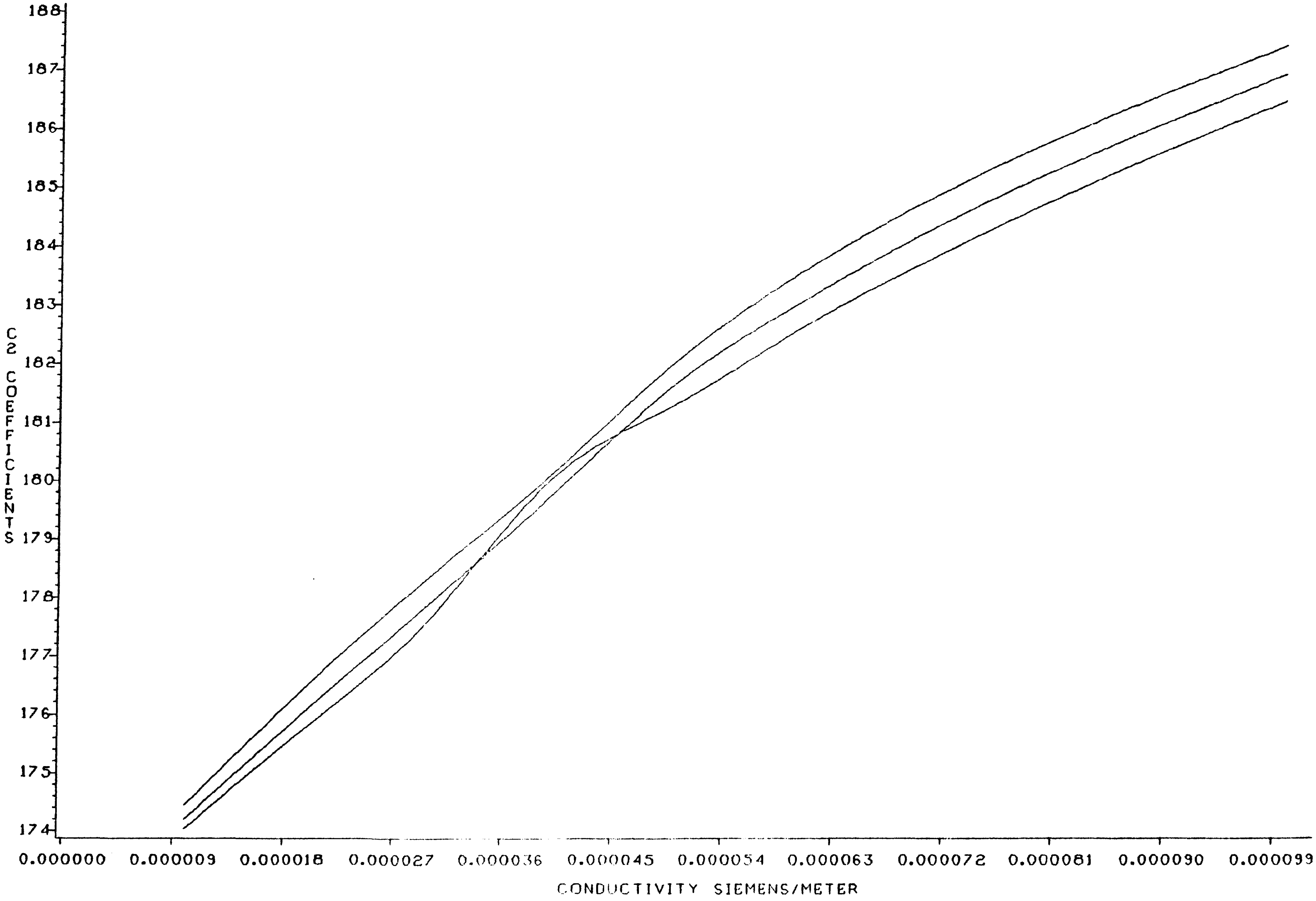

Figure IV.20 : C_2 Variation with Conductivity and Permittivity (0.00001 to 0.0001 siemens/metre)

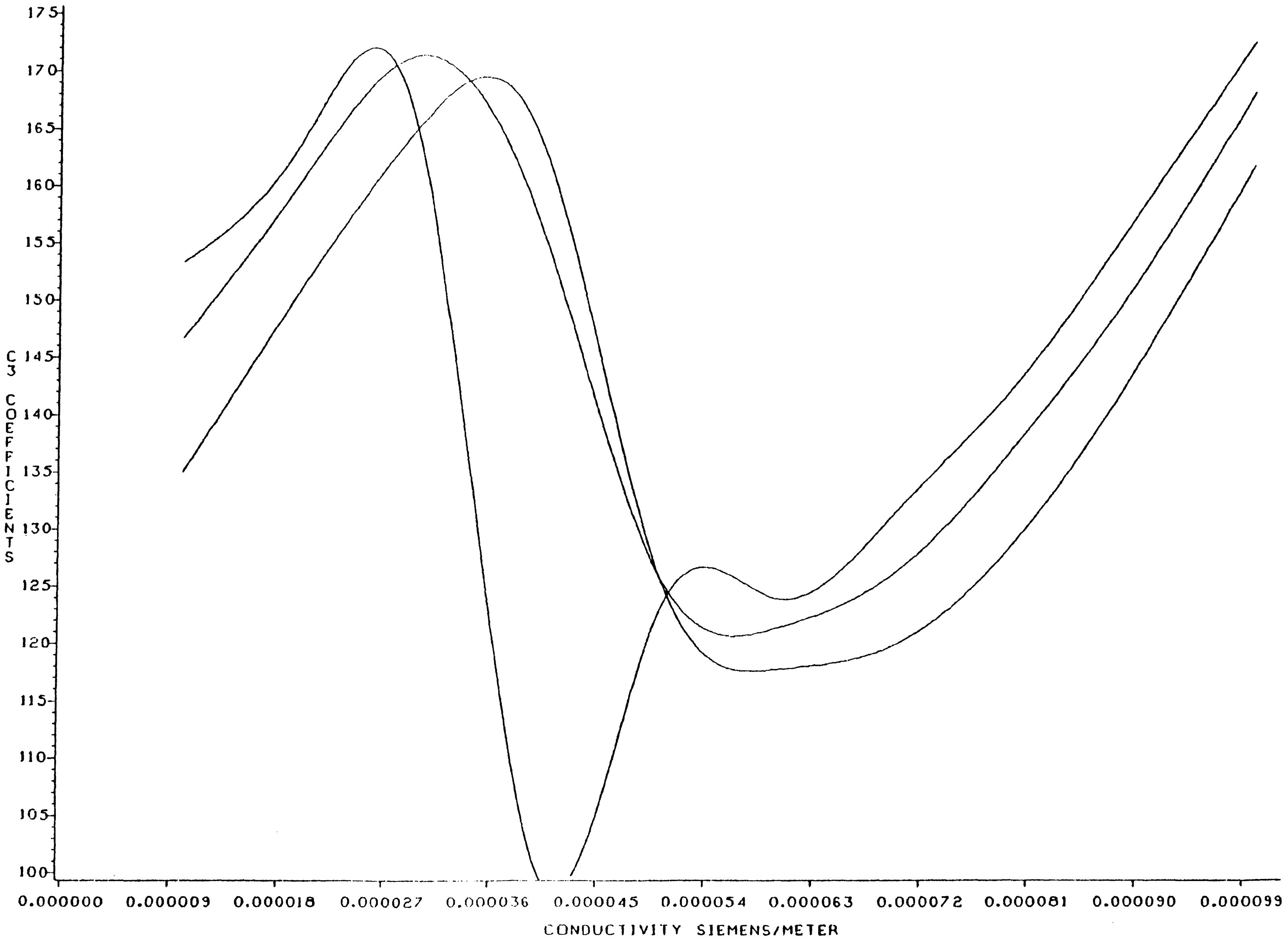

Figure IV.21 : C_3 Variation with Conductivity and Permittivity (0.00001 to 0.0001 siemens/metre)

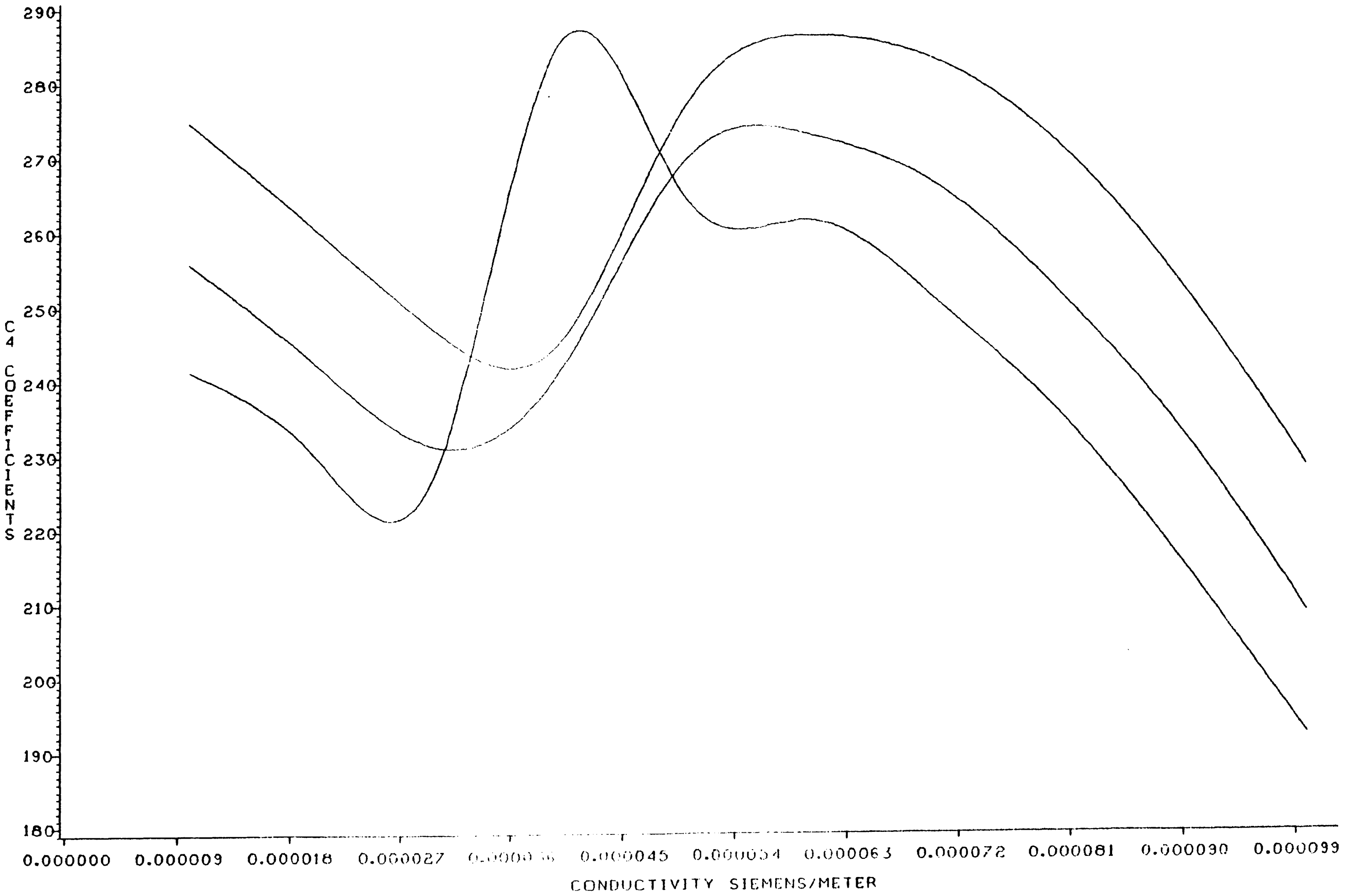

Figure IV.32 : C_4 Variation with Conductivity and Permittivity (0.00001 to 0.0001 siemens/metre)

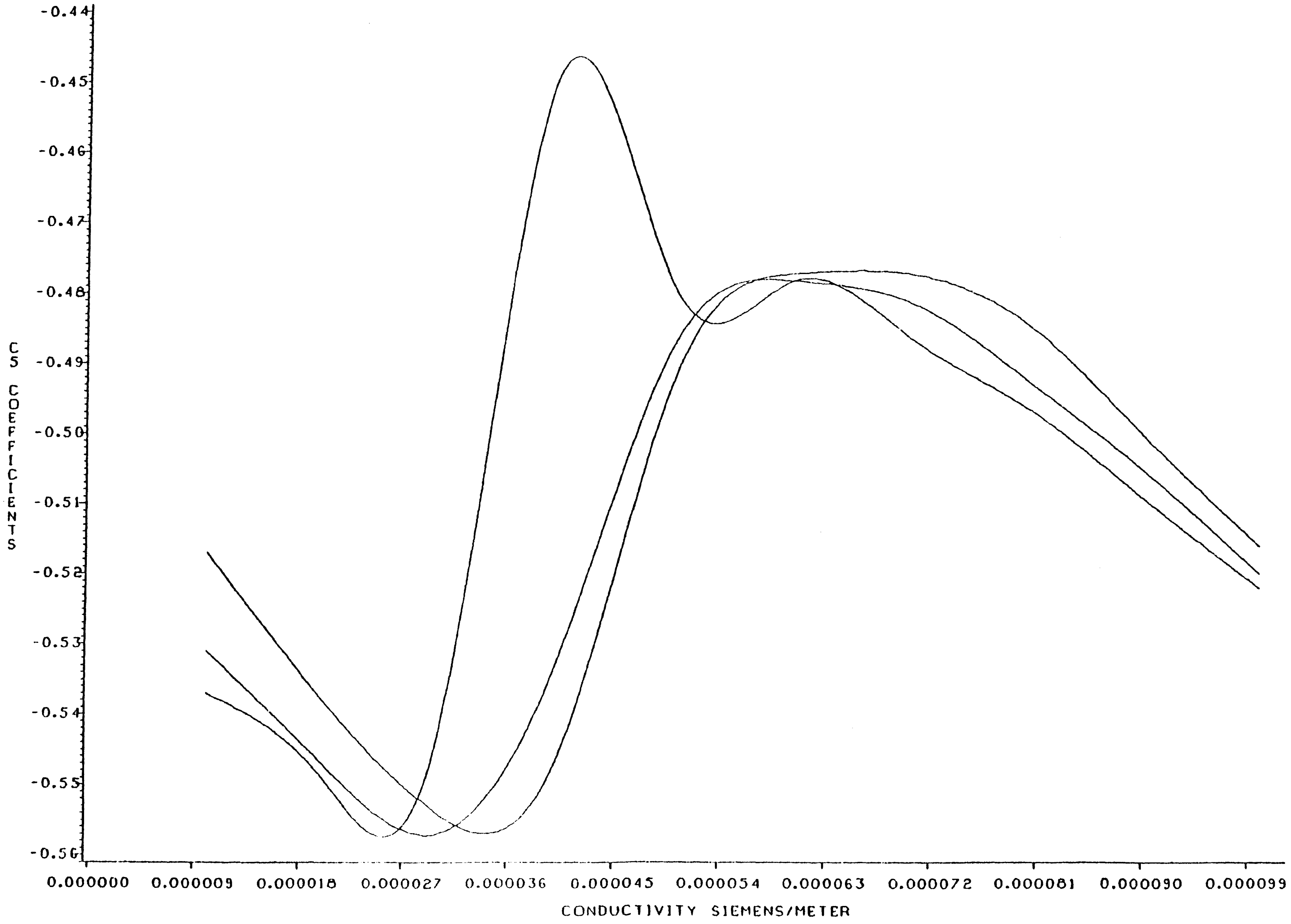

Figure IV.23 : C_5 Variation with Conductivity and Permittivity (0.00001 to 0.0001 siemens/metre)

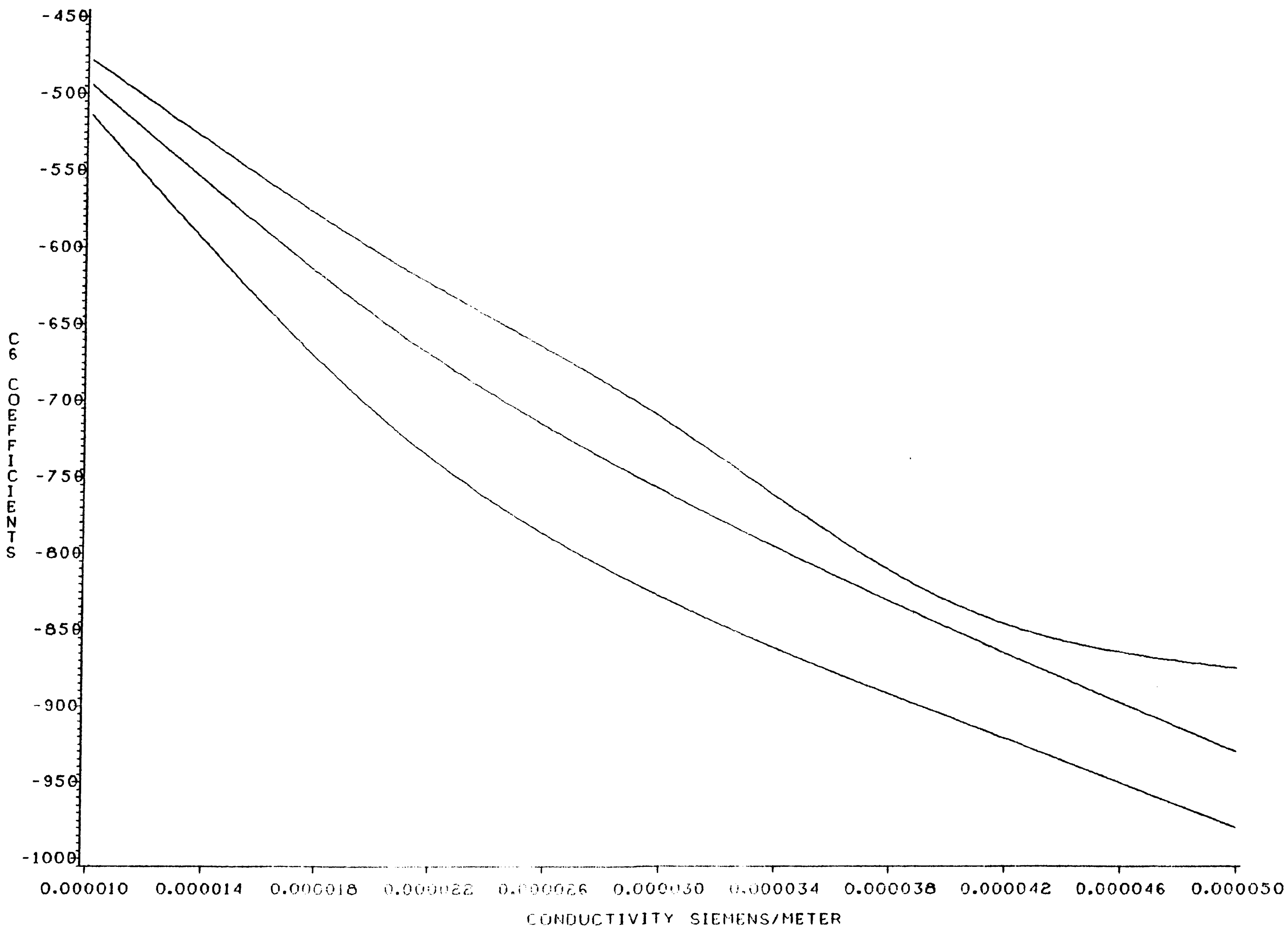

Figure IV.24 : C_6 Variation with Conductivity and Permittivity (0.00001 to 0.0001 siemens/metre)

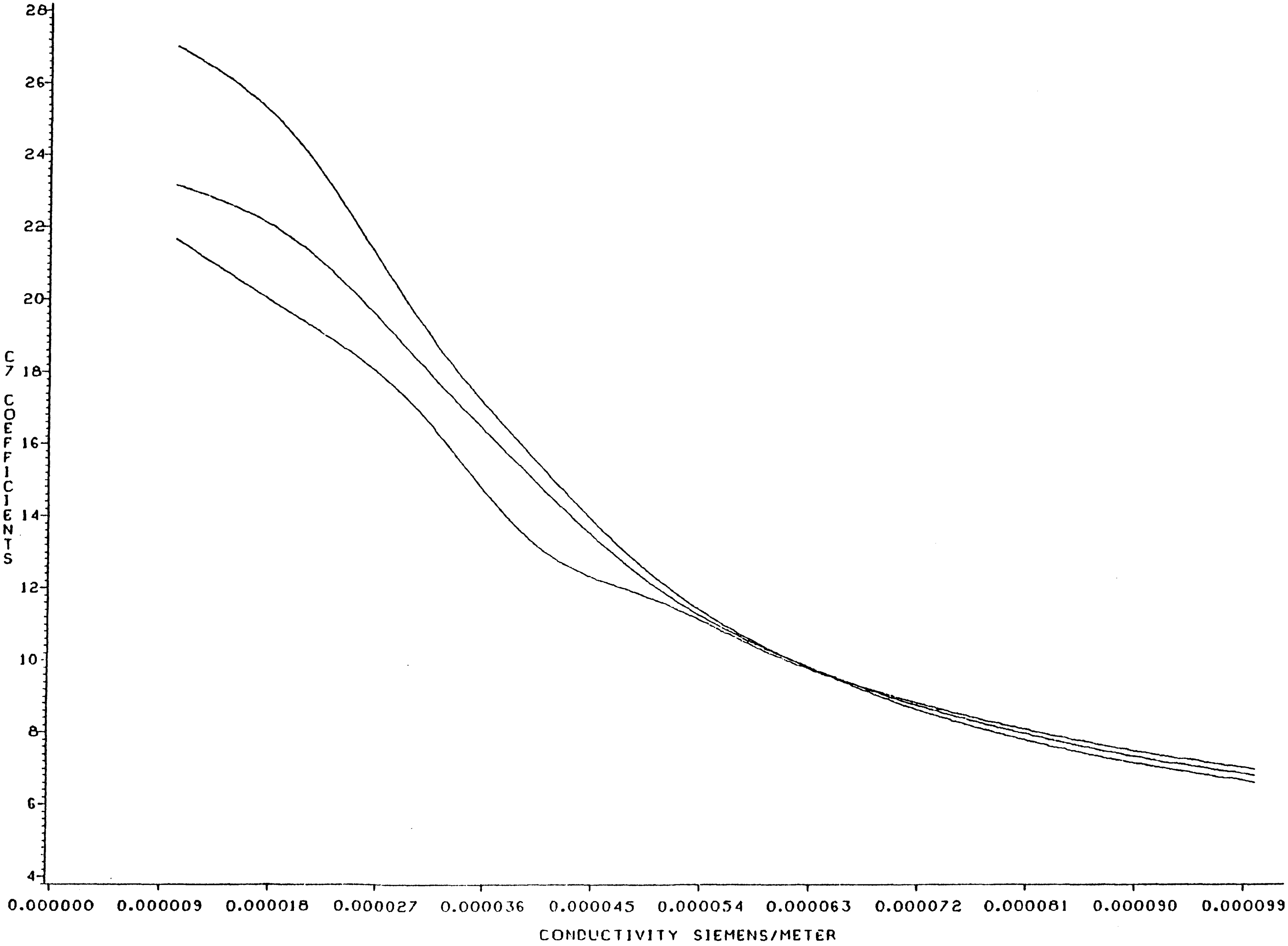

Figure IV.25 : C_7 Variation with Conductivity and Permittivity (0.00001 to 0.0001 siemens/metre)

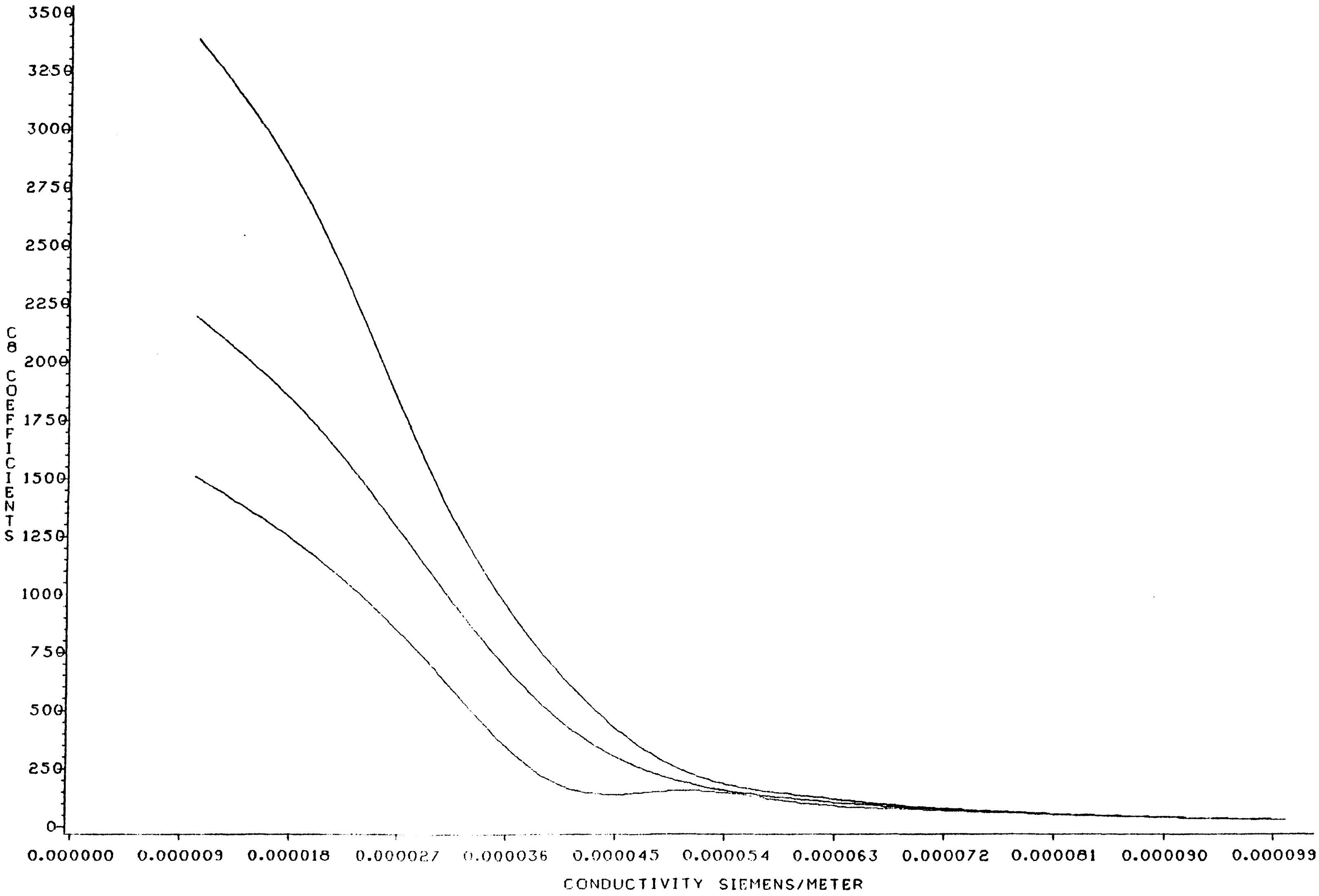

Figure IV.26 : C_8 Variation with Conductivity and Permittivity (0.00001 to 0.0001 siemens/metre)

Appendix V

DIFFERENCE BETWEEN ACCURATE AND APPROXIMATE COMPUTATIONS WHERE THE C COEFFICIENTS WERE COMPUTED USING 'A' FORMULA

In this appendix, a few examples of the difference between accurate and approximate computations are given. In this case, the C coefficients were derived from the 'A' Formula developed in Chapter 6.

It is hoped that the results shown here will give a measure of accuracy of the 'A' Formula so that it can be used with confidence.

Appendix V - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 5.000000 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

-111.000	98.206	-13.503	112.800
-0.254	0.000	0.000	0.000

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	120.2	116.8	3.4
3000.0	79.8	79.4	0.4
4000.0	60.6	61.0	-0.4
5000.0	49.5	50.2	-0.6
6000.0	42.4	43.1	-0.8
7000.0	37.4	38.3	-0.9
8000.0	33.8	34.8	-1.0
9000.0	31.1	32.2	-1.1
10000.0	29.1	30.2	-1.2
11000.0	28.3	28.8	-0.5
12000.0	27.3	27.6	-0.3
13000.0	26.5	26.7	-0.2
14000.0	25.9	26.0	-0.2
15000.0	25.4	25.5	-0.2
16000.0	25.0	25.2	-0.2
17000.0	24.8	24.9	-0.2
18000.0	24.6	24.8	-0.1
19000.0	24.5	24.7	-0.1
20000.0	24.5	24.7	-0.1
30000.0	26.7	26.8	-0.2
40000.0	30.7	31.0	-0.3
50000.0	35.7	36.1	-0.4
60000.0	41.2	41.6	-0.4
70000.0	47.1	47.5	-0.4
80000.0	53.2	53.7	-0.5
90000.0	59.5	60.0	-0.5
100000.0	66.0	66.5	-0.5
200000.0	137.5	138.2	-0.7
300000.0	217.2	218.1	-0.9
400000.0	302.6	303.7	-1.1
500000.0	392.0	393.2	-1.2
600000.0	484.4	485.5	-1.1
700000.0	578.8	579.9	-1.0
800000.0	674.7	675.6	-0.8
900000.0	771.6	772.2	-0.6
1000000.0	869.1	869.5	-0.4
1100000.0	967.0	967.3	-0.2
1200000.0	1065.2	1065.3	-0.1
1300000.0	1163.4	1163.5	-0.1
1400000.0	1261.8	1261.9	-0.1
1500000.0	1360.1	1360.2	-0.1
1600000.0	1458.5	1458.7	-0.2
1700000.0	1556.9	1557.1	-0.2
1800000.0	1655.2	1655.5	-0.2
1900000.0	1753.6	1753.9	-0.3
2000000.0	1851.9	1852.2	-0.3

Appendix V - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 3.750000 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

-111.000	98.624	-12.737	112.800
-0.254	0.000	0.000	0.000

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	120.4	116.8	3.6
3000.0	80.0	79.4	0.6
4000.0	60.9	61.0	-0.1
5000.0	49.8	50.2	-0.4
6000.0	42.7	43.2	-0.5
7000.0	37.8	38.4	-0.6
8000.0	34.2	34.9	-0.7
9000.0	31.5	32.3	-0.8
10000.0	29.5	30.4	-0.9
11000.0	28.7	28.9	-0.1
12000.0	27.8	27.7	0.1
13000.0	27.0	26.9	0.2
14000.0	26.4	26.2	0.2
15000.0	25.9	25.7	0.2
16000.0	25.6	25.4	0.2
17000.0	25.4	25.1	0.2
18000.0	25.2	25.0	0.2
19000.0	25.2	24.9	0.3
20000.0	25.2	24.9	0.3
30000.0	27.4	27.2	0.3
40000.0	31.6	31.4	0.1
50000.0	36.7	36.6	0.1
60000.0	42.3	42.3	0.0
70000.0	48.3	48.3	0.0
80000.0	54.4	54.5	0.0
90000.0	60.8	60.9	-0.1
100000.0	67.4	67.5	-0.1
200000.0	139.5	139.9	-0.4
300000.0	219.7	220.4	-0.7
400000.0	305.6	306.5	-0.9
500000.0	395.4	396.4	-1.0
600000.0	488.1	489.0	-0.9
700000.0	583.0	583.7	-0.7
800000.0	679.2	679.7	-0.5
900000.0	776.5	776.7	-0.2
1000000.0	874.4	874.3	0.1
1100000.0	972.7	972.4	0.3
1200000.0	1071.2	1070.8	0.4
1300000.0	1169.8	1169.3	0.5
1400000.0	1268.5	1268.0	0.5
1500000.0	1367.2	1366.8	0.5
1600000.0	1466.0	1465.6	0.4
1700000.0	1564.7	1564.4	0.4
1800000.0	1663.5	1663.2	0.3
1900000.0	1762.2	1761.9	0.3
2000000.0	1860.9	1860.7	0.2

Appendix V - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 3.000000 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

-111.000	99.004	-12.024	112.800
-0.254	0.000	0.000	0.000

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	120.5	116.8	3.7
3000.0	80.2	79.5	0.8
4000.0	61.2	61.1	0.1
5000.0	50.1	50.3	-0.1
6000.0	43.0	43.3	-0.3
7000.0	38.1	38.4	-0.3
8000.0	34.6	35.0	-0.4
9000.0	31.9	32.4	-0.5
10000.0	29.9	30.5	-0.6
11000.0	29.1	29.0	0.2
12000.0	28.3	27.9	0.4
13000.0	27.5	27.0	0.5
14000.0	26.9	26.4	0.5
15000.0	26.4	25.9	0.5
16000.0	26.1	25.5	0.6
17000.0	25.9	25.3	0.6
18000.0	25.7	25.2	0.6
19000.0	25.7	25.1	0.6
20000.0	25.7	25.1	0.6
30000.0	28.1	27.5	0.6
40000.0	32.4	31.9	0.5
50000.0	37.6	37.1	0.5
60000.0	43.3	42.9	0.4
70000.0	49.3	48.9	0.4
80000.0	55.6	55.3	0.3
90000.0	62.0	61.8	0.3
100000.0	68.7	68.5	0.2
200000.0	141.3	141.5	-0.3
300000.0	222.0	222.6	-0.6
400000.0	308.2	309.0	-0.8
500000.0	398.4	399.3	-0.9
600000.0	491.4	492.3	-0.8
700000.0	586.6	587.2	-0.6
800000.0	683.2	683.5	-0.3
900000.0	780.8	780.8	0.0
1000000.0	879.0	878.7	0.3
1100000.0	977.6	977.1	0.5
1200000.0	1076.4	1075.7	0.7
1300000.0	1175.4	1174.6	0.8
1400000.0	1274.4	1273.6	0.8
1500000.0	1373.5	1372.7	0.8
1600000.0	1472.6	1471.8	0.7
1700000.0	1571.6	1571.0	0.7
1800000.0	1670.7	1670.1	0.6
1900000.0	1769.8	1769.3	0.5
2000000.0	1868.8	1868.4	0.4

Appendix V - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 2.750000 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

-111.000	99.167	-11.712	112.800
-0.254	0.000	0.000	0.000

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	120.6	116.8	3.8
3000.0	80.3	79.5	0.8
4000.0	61.3	61.1	0.2
5000.0	50.2	50.3	-0.1
6000.0	43.1	43.3	-0.2
7000.0	38.2	38.5	-0.2
8000.0	34.7	35.0	-0.3
9000.0	32.1	32.4	-0.4
10000.0	30.1	30.5	-0.4
11000.0	29.3	29.0	0.3
12000.0	28.4	27.9	0.5
13000.0	27.7	27.1	0.6
14000.0	27.1	26.4	0.6
15000.0	26.6	25.9	0.7
16000.0	26.3	25.6	0.7
17000.0	26.1	25.4	0.7
18000.0	26.0	25.2	0.7
19000.0	25.9	25.2	0.7
20000.0	26.0	25.2	0.7
30000.0	28.4	27.6	0.8
40000.0	32.7	32.0	0.7
50000.0	38.0	37.3	0.6
60000.0	43.7	43.1	0.6
70000.0	49.7	49.2	0.5
80000.0	56.0	55.6	0.4
90000.0	62.5	62.1	0.4
100000.0	69.2	68.9	0.3
200000.0	142.1	142.2	-0.2
300000.0	222.9	223.5	-0.6
400000.0	309.3	310.1	-0.9
500000.0	399.6	400.5	-0.9
600000.0	492.8	493.6	-0.8
700000.0	588.1	588.7	-0.6
800000.0	684.9	685.1	-0.2
900000.0	782.6	782.5	0.1
1000000.0	880.9	880.6	0.4
1100000.0	979.7	979.1	0.6
1200000.0	1078.7	1077.9	0.8
1300000.0	1177.8	1176.9	0.9
1400000.0	1276.9	1276.0	0.9
1500000.0	1376.1	1375.3	0.9
1600000.0	1475.3	1474.5	0.8
1700000.0	1574.6	1573.8	0.7
1800000.0	1673.8	1673.1	0.6
1900000.0	1773.0	1772.4	0.6
2000000.0	1872.2	1871.7	0.5

Appendix V - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.750000 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

-100.440 102.206 30.675 105.418
-0.570 -1.163 7.018 462.945

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	122.2	119.3	2.9
3000.0	82.3	82.1	0.2
4000.0	63.5	63.9	-0.4
5000.0	52.8	53.3	-0.5
6000.0	45.9	46.5	-0.6
7000.0	41.2	41.8	-0.6
8000.0	37.9	38.5	-0.6
9000.0	35.5	36.1	-0.7
10000.0	33.6	34.4	-0.7
11000.0	33.1	33.1	0.0
12000.0	32.3	32.1	0.2
13000.0	31.7	31.4	0.4
14000.0	31.3	30.9	0.4
15000.0	31.0	30.6	0.4
16000.0	30.8	30.4	0.4
17000.0	30.7	30.3	0.4
18000.0	30.7	30.3	0.4
19000.0	30.8	30.4	0.4
20000.0	31.0	30.6	0.4
30000.0	34.6	34.3	0.3
40000.0	39.8	39.8	0.1
50000.0	45.9	46.0	-0.1
60000.0	52.4	52.6	-0.2
70000.0	59.2	59.5	-0.3
80000.0	66.2	66.6	-0.4
90000.0	73.3	73.7	-0.5
100000.0	80.5	81.0	-0.5
200000.0	158.2	158.5	-0.3
300000.0	243.0	242.7	0.3
400000.0	332.8	332.3	0.5
500000.0	426.4	426.1	0.4
600000.0	522.7	522.7	0.1
700000.0	621.1	621.3	-0.2
800000.0	720.8	721.2	-0.4
900000.0	821.5	821.9	-0.4
1000000.0	922.8	923.2	-0.4
1100000.0	1024.5	1024.9	-0.4
1200000.0	1126.4	1126.7	-0.3
1300000.0	1228.5	1228.7	-0.2
1400000.0	1330.6	1330.8	-0.2
1500000.0	1432.8	1432.9	-0.1
1600000.0	1535.0	1535.1	-0.1
1700000.0	1637.2	1637.2	-0.1
1800000.0	1739.3	1739.4	-0.1
1900000.0	1841.5	1841.6	-0.1
2000000.0	1943.7	1943.8	-0.1

Appendix V - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.600000 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

-97.377 102.915 31.237 103.328
-0.571 -1.668 7.050 464.300

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	122.6	119.8	2.7
3000.0	82.7	82.7	0.0
4000.0	64.1	64.6	-0.5
5000.0	53.4	54.0	-0.6
6000.0	46.6	47.2	-0.7
7000.0	42.0	42.6	-0.7
8000.0	38.7	39.4	-0.7
9000.0	36.3	37.0	-0.7
10000.0	34.5	35.3	-0.8
11000.0	34.0	34.0	0.0
12000.0	33.3	33.1	0.2
13000.0	32.7	32.4	0.3
14000.0	32.3	31.9	0.4
15000.0	32.1	31.7	0.4
16000.0	31.9	31.5	0.4
17000.0	31.9	31.5	0.4
18000.0	31.9	31.5	0.4
19000.0	32.1	31.6	0.4
20000.0	32.2	31.8	0.4
30000.0	36.1	35.8	0.3
40000.0	41.6	41.5	0.1
50000.0	47.9	48.0	-0.1
60000.0	54.6	54.8	-0.2
70000.0	61.5	61.8	-0.3
80000.0	68.7	69.0	-0.4
90000.0	75.9	76.4	-0.5
100000.0	83.3	83.8	-0.5
200000.0	162.2	162.5	-0.2
300000.0	248.0	247.6	0.3
400000.0	338.7	338.1	0.6
500000.0	433.0	432.6	0.4
600000.0	530.1	529.9	0.2
700000.0	629.2	629.3	-0.1
800000.0	729.7	729.9	-0.2
900000.0	831.1	831.4	-0.3
1000000.0	933.1	933.4	-0.2
1100000.0	1035.6	1035.7	-0.2
1200000.0	1138.2	1138.3	-0.1
1300000.0	1241.0	1241.0	0.0
1400000.0	1343.9	1343.8	0.1
1500000.0	1446.8	1446.6	0.2
1600000.0	1549.7	1549.5	0.2
1700000.0	1652.6	1652.4	0.3
1800000.0	1755.5	1755.3	0.3
1900000.0	1858.4	1858.2	0.3
2000000.0	1961.4	1961.1	0.3

Appendix V - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.150000 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

-68.543	109.783	35.157	84.105
-0.581	-7.928	7.627	452.689

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	126.3	124.5	1.9
3000.0	87.3	87.9	-0.6
4000.0	69.4	70.3	-0.9
5000.0	59.3	60.2	-0.9
6000.0	53.1	53.8	-0.8
7000.0	48.9	49.7	-0.7
8000.0	46.1	46.8	-0.7
9000.0	44.2	44.8	-0.6
10000.0	42.9	43.5	-0.6
11000.0	42.7	42.6	0.1
12000.0	42.4	42.0	0.4
13000.0	42.3	41.7	0.5
14000.0	42.2	41.6	0.6
15000.0	42.3	41.7	0.6
16000.0	42.5	41.9	0.6
17000.0	42.8	42.2	0.6
18000.0	43.1	42.5	0.6
19000.0	43.5	43.0	0.6
20000.0	44.0	43.5	0.5
30000.0	50.6	50.4	0.2
40000.0	58.4	58.4	0.0
50000.0	66.6	66.7	-0.1
60000.0	75.1	75.2	-0.1
70000.0	83.7	83.9	-0.2
80000.0	92.3	92.6	-0.3
90000.0	101.1	101.4	-0.3
100000.0	109.8	110.2	-0.4
200000.0	200.1	200.5	-0.3
300000.0	295.0	294.7	0.3
400000.0	393.8	393.1	0.6
500000.0	495.7	495.0	0.7
600000.0	600.1	599.6	0.5
700000.0	706.2	705.9	0.3
800000.0	813.7	813.5	0.2
900000.0	922.0	921.9	0.1
1000000.0	1031.0	1030.8	0.1
1100000.0	1140.3	1140.1	0.2
1200000.0	1249.9	1249.5	0.3
1300000.0	1359.6	1359.1	0.5
1400000.0	1469.4	1468.8	0.6
1500000.0	1579.2	1578.5	0.7
1600000.0	1689.0	1688.2	0.9
1700000.0	1798.9	1797.9	1.0
1800000.0	1908.8	1907.7	1.1
1900000.0	2018.6	2017.5	1.1
2000000.0	2128.5	2127.2	1.2

Appendix V - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.050000 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

-26.276	119.887	38.669	56.608
-0.588	-19.459	8.750	403.462

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	131.8	130.1	1.6
3000.0	94.0	94.6	-0.6
4000.0	77.1	77.8	-0.8
5000.0	67.9	68.6	-0.7
6000.0	62.5	63.0	-0.5
7000.0	59.2	59.6	-0.4
8000.0	57.1	57.4	-0.3
9000.0	55.8	56.1	-0.3
10000.0	55.1	55.3	-0.3
11000.0	55.5	55.0	0.5
12000.0	55.8	55.0	0.7
13000.0	56.2	55.3	0.9
14000.0	56.6	55.7	0.9
15000.0	57.2	56.3	0.9
16000.0	57.9	57.0	0.9
17000.0	58.7	57.8	0.9
18000.0	59.5	58.7	0.8
19000.0	60.4	59.6	0.8
20000.0	61.3	60.6	0.7
30000.0	71.7	71.6	0.1
40000.0	82.8	83.0	-0.2
50000.0	93.9	94.2	-0.2
60000.0	105.1	105.2	-0.2
70000.0	116.1	116.2	-0.1
80000.0	127.0	127.1	-0.1
90000.0	137.8	137.9	-0.1
100000.0	148.6	148.8	-0.1
200000.0	255.5	256.0	-0.4
300000.0	363.7	363.7	-0.1
400000.0	474.3	474.0	0.3
500000.0	587.2	586.8	0.4
600000.0	702.2	701.9	0.2
700000.0	818.6	818.6	0.0
800000.0	936.2	936.4	-0.2
900000.0	1054.6	1055.0	-0.4
1000000.0	1173.6	1174.1	-0.5
1100000.0	1293.0	1293.4	-0.5
1200000.0	1412.6	1413.0	-0.4
1300000.0	1532.3	1532.7	-0.4
1400000.0	1652.1	1652.5	-0.3
1500000.0	1772.0	1772.3	-0.2
1600000.0	1891.9	1892.1	-0.2
1700000.0	2011.9	2012.0	-0.1
1800000.0	2131.8	2131.8	0.0
1900000.0	2251.7	2251.7	0.0
2000000.0	2371.7	2371.6	0.1

Appendix V - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

```
Frequency     = 100000.0    Hz
Conductivity  = 0.045000    siemens/meter
Permitivity   = 15.0        esu

Coefficients C(I), I=1 to 8 are :

   -20.839   121.168    39.014    53.099
    -0.588   -21.047     8.895   396.158

DISTANCE            ACCURATE           APPROXIMATE         DIFFERENCE
  (M)                 (M)                  (M)                 (M)

   2000.0             132.5               130.8                1.6
   3000.0              94.9                95.4               -0.5
   4000.0              78.1                78.8               -0.7
   5000.0              69.0                69.7               -0.6
   6000.0              63.7                64.2               -0.5
   7000.0              60.5                60.9               -0.4
   8000.0              58.5                58.8               -0.3
   9000.0              57.3                57.5               -0.2
  10000.0              56.7                56.9               -0.2
  11000.0              57.1                56.6                0.5
  12000.0              57.5                56.7                0.8
  13000.0              57.9                57.0                0.9
  14000.0              58.5                57.5                0.9
  15000.0              59.1                58.2                0.9
  16000.0              59.9                58.9                0.9
  17000.0              60.7                59.8                0.9
  18000.0              61.6                60.7                0.9
  19000.0              62.5                61.7                0.8
  20000.0              63.5                62.8                0.8
  30000.0              74.4                74.4                0.1
  40000.0              86.0                86.2               -0.2
  50000.0              97.4                97.7               -0.3
  60000.0             108.9               109.0               -0.2
  70000.0             120.2               120.3               -0.1
  80000.0             131.4               131.5               -0.1
  90000.0             142.5               142.6               -0.1
 100000.0             153.6               153.7               -0.1
 200000.0             262.6               263.1               -0.5
 300000.0             372.5               372.5               -0.1
 400000.0             484.6               484.3                0.3
 500000.0             598.9               598.6                0.4
 600000.0             715.2               715.0                0.2
 700000.0             832.9               833.0                0.0
 800000.0             951.8               952.1               -0.3
 900000.0            1071.5              1072.0               -0.4
1000000.0            1191.8              1192.3               -0.5
1100000.0            1312.4              1313.0               -0.6
1200000.0            1433.3              1433.8               -0.5
1300000.0            1554.3              1554.8               -0.5
1400000.0            1675.4              1675.8               -0.4
1500000.0            1796.6              1796.9               -0.3
1600000.0            1917.8              1918.0               -0.3
1700000.0            2039.0              2039.2               -0.2
1800000.0            2160.2              2160.3               -0.1
1900000.0            2281.4              2281.5               -0.1
2000000.0            2402.6              2402.6                0.0
```

Appendix V - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.030000 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

3.071	126.730	40.340	37.707
-0.590	-28.207	9.513	363.326

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	135.5	133.8	1.7
3000.0	98.6	99.1	-0.4
4000.0	82.4	83.0	-0.6
5000.0	73.9	74.4	-0.5
6000.0	69.1	69.5	-0.4
7000.0	66.2	66.5	-0.3
8000.0	64.6	64.8	-0.2
9000.0	63.8	64.0	-0.2
10000.0	63.5	63.7	-0.2
11000.0	64.3	63.8	0.5
12000.0	65.0	64.2	0.8
13000.0	65.7	64.8	1.0
14000.0	66.6	65.6	1.0
15000.0	67.5	66.5	1.0
16000.0	68.5	67.5	1.0
17000.0	69.6	68.7	1.0
18000.0	70.8	69.9	0.9
19000.0	72.0	71.1	0.9
20000.0	73.2	72.4	0.8
30000.0	86.3	86.3	0.0
40000.0	99.7	100.0	-0.3
50000.0	112.8	113.1	-0.3
60000.0	125.7	125.8	-0.2
70000.0	138.3	138.4	0.0
80000.0	150.8	150.8	0.1
90000.0	163.1	163.1	0.1
100000.0	175.3	175.3	0.0
200000.0	293.6	294.0	-0.5
300000.0	410.8	411.0	-0.2
400000.0	529.5	529.3	0.2
500000.0	650.0	649.7	0.3
600000.0	772.1	771.9	0.1
700000.0	895.5	895.6	-0.1
800000.0	1020.0	1020.4	-0.4
900000.0	1145.3	1145.9	-0.6
1000000.0	1271.1	1271.8	-0.7
1100000.0	1397.3	1398.0	-0.8
1200000.0	1523.7	1524.5	-0.8
1300000.0	1650.3	1651.0	-0.7
1400000.0	1776.9	1777.6	-0.7
1500000.0	1903.6	1904.3	-0.6
1600000.0	2030.4	2030.9	-0.6
1700000.0	2157.1	2157.6	-0.5
1800000.0	2283.9	2284.4	-0.5
1900000.0	2410.7	2411.1	-0.4
2000000.0	2537.4	2537.8	-0.3

Appendix V - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.017500 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

43.779	135.894	42.054	11.593
-0.592	-40.857	10.443	308.304

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	140.7	138.8	1.9
3000.0	105.0	105.3	-0.3
4000.0	89.7	90.3	-0.6
5000.0	82.1	82.6	-0.5
6000.0	78.0	78.4	-0.4
7000.0	75.9	76.3	-0.4
8000.0	75.0	75.3	-0.3
9000.0	74.8	75.0	-0.3
10000.0	75.1	75.3	-0.3
11000.0	76.4	76.0	0.4
12000.0	77.6	76.9	0.7
13000.0	78.9	78.0	0.8
14000.0	80.2	79.3	0.9
15000.0	81.6	80.7	0.9
16000.0	83.1	82.2	0.9
17000.0	84.7	83.7	0.9
18000.0	86.2	85.4	0.9
19000.0	87.8	87.0	0.8
20000.0	89.5	88.7	0.8
30000.0	106.2	106.2	0.0
40000.0	122.7	123.1	-0.4
50000.0	138.5	138.9	-0.4
60000.0	153.9	154.1	-0.1
70000.0	168.9	168.8	0.1
80000.0	183.5	183.2	0.3
90000.0	197.8	197.4	0.3
100000.0	211.8	211.5	0.4
200000.0	345.6	346.0	-0.3
300000.0	475.2	475.5	-0.3
400000.0	604.8	604.7	0.1
500000.0	735.5	735.2	0.3
600000.0	867.3	867.1	0.2
700000.0	1000.2	1000.2	0.0
800000.0	1134.0	1134.3	-0.3
900000.0	1268.5	1269.0	-0.5
1000000.0	1403.5	1404.1	-0.6
1100000.0	1538.8	1539.5	-0.7
1200000.0	1674.4	1675.1	-0.7
1300000.0	1810.2	1810.8	-0.6
1400000.0	1946.0	1946.6	-0.6
1500000.0	2081.9	2082.4	-0.5
1600000.0	2217.9	2218.3	-0.4
1700000.0	2353.8	2354.1	-0.3
1800000.0	2489.8	2490.0	-0.2
1900000.0	2625.8	2625.9	-0.1
2000000.0	2761.8	2761.8	0.0

Appendix V - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.010000 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

100.333 147.925 43.690 -24.629
-0.595 -58.962 11.397 240.831

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	147.8	145.7	2.1
3000.0	113.7	113.8	-0.2
4000.0	99.8	100.4	-0.6
5000.0	93.3	94.0	-0.7
6000.0	90.3	91.0	-0.7
7000.0	89.1	89.8	-0.7
8000.0	89.1	89.8	-0.7
9000.0	89.8	90.4	-0.7
10000.0	90.9	91.6	-0.7
11000.0	92.9	93.0	-0.1
12000.0	94.8	94.6	0.2
13000.0	96.8	96.4	0.4
14000.0	98.8	98.3	0.5
15000.0	100.9	100.3	0.6
16000.0	103.0	102.4	0.6
17000.0	105.2	104.5	0.6
18000.0	107.4	106.7	0.6
19000.0	109.5	108.9	0.7
20000.0	111.7	111.1	0.6
30000.0	133.5	133.4	0.1
40000.0	154.2	154.5	-0.4
50000.0	173.7	174.1	-0.3
60000.0	192.5	192.5	0.0
70000.0	210.5	210.2	0.3
80000.0	228.0	227.4	0.6
90000.0	245.0	244.3	0.7
100000.0	261.6	260.8	0.8
200000.0	416.3	416.4	-0.1
300000.0	562.5	562.7	-0.2
400000.0	706.7	706.5	0.2
500000.0	850.8	850.3	0.5
600000.0	995.5	995.0	0.6
700000.0	1141.0	1140.5	0.4
800000.0	1287.1	1286.8	0.3
900000.0	1433.7	1433.7	0.1
1000000.0	1580.9	1580.9	0.0
1100000.0	1728.3	1728.4	-0.1
1200000.0	1876.0	1876.0	0.0
1300000.0	2023.8	2023.8	0.1
1400000.0	2171.8	2171.6	0.2
1500000.0	2319.8	2319.4	0.4
1600000.0	2467.9	2467.3	0.5
1700000.0	2616.0	2615.2	0.7
1800000.0	2764.1	2763.1	0.9
1900000.0	2912.2	2911.0	1.1
2000000.0	3060.3	3058.9	1.3

Appendix V - Cont'd

ACCURATE VERSUS APPROXIMATE TOTAL PHASELAG

Frequency = 100000.0 Hz
Conductivity = 0.005000 siemens/meter
Permitivity = 15.0 esu

Coefficients C(I), I=1 to 8 are :

198.931	166.902	45.326	-87.885
-0.600	-91.080	11.994	161.505

DISTANCE (M)	ACCURATE (M)	APPROXIMATE (M)	DIFFERENCE (M)
2000.0	159.9	156.7	3.1
3000.0	128.4	127.9	0.6
4000.0	116.8	117.0	-0.2
5000.0	112.3	112.8	-0.5
6000.0	111.1	111.8	-0.7
7000.0	111.6	112.5	-0.9
8000.0	113.1	114.1	-1.0
9000.0	115.2	116.3	-1.0
10000.0	117.7	118.8	-1.1
11000.0	120.8	121.5	-0.7
12000.0	124.0	124.4	-0.4
13000.0	127.2	127.3	-0.1
14000.0	130.4	130.4	0.0
15000.0	133.6	133.4	0.1
16000.0	136.7	136.5	0.2
17000.0	139.9	139.6	0.3
18000.0	143.1	142.7	0.4
19000.0	146.3	145.8	0.5
20000.0	149.4	148.9	0.5
30000.0	179.6	179.1	0.5
40000.0	207.4	207.3	0.0
50000.0	233.2	233.3	-0.1
60000.0	257.5	257.4	0.1
70000.0	280.7	280.2	0.5
80000.0	303.0	302.2	0.8
90000.0	324.5	323.5	1.0
100000.0	345.4	344.2	1.1
200000.0	534.5	534.7	-0.1
300000.0	707.6	708.3	-0.7
400000.0	875.2	875.6	-0.4
500000.0	1040.8	1040.8	0.0
600000.0	1205.9	1205.7	0.1
700000.0	1371.0	1371.0	0.0
800000.0	1536.4	1536.7	-0.3
900000.0	1702.1	1702.7	-0.7
1000000.0	1868.1	1869.1	-0.9
1100000.0	2034.5	2035.6	-1.1
1200000.0	2201.0	2202.3	-1.3
1300000.0	2367.7	2369.0	-1.3
1400000.0	2534.5	2535.8	-1.3
1500000.0	2701.4	2702.7	-1.3
1600000.0	2868.4	2869.5	-1.2
1700000.0	3035.3	3036.4	-1.1
1800000.0	3202.3	3203.3	-1.0
1900000.0	3369.4	3370.2	-0.8
2000000.0	3536.4	3537.1	-0.7

Appendix VI

PROGRAM TOPLAG

VI.1 INTRODUCTION

This program computes the total (secondary + primary) phaselag using approximate formula for the following conductivity and permittivity values at 100KHz frequency:

i. at any sea water conductivity between 1.0 and 5.5 siemens/metre (ε_2= 80 esu)

ii. at fresh water conductivity of 0.005 siemens/metre(ε_2= 80 esu)

iii. at any land conductivity between 0.005 and 1.0 siemens/metre and permittivity between 5 and 40 esu.

iv. at 21 selected conductivity and 7 permittivity values for land conductivity between 0.0001 and 0.005 siemens/metre.

v. at 10 selected conductivity and 3 permittivity values for land conductivity between 0.00001 and 0.0001 siemens/metre.

For i and iii above, the C coefficients are derived from the 'A' Formula developed in Chapter 6. Therefore any value in the range of 0.005 to 5.5 seimens/metre can be used . For this conductivity range a subroutine was developed to compute the C coefficients at any input conductivity.

For ii,iv and v above, a subroutine was developed at each permittivity value to store the C coefficients for the selected conductivity values. The user's guide to this program and its listing are given in Sections VI.2 and VI.3 respectively.

VI.2 USER'S GUIDE TO PROGRAM TOPLAG

1. PURPOSE To compute the 100KHz total (primary + secondary) phaselag at a given distance between 2 and 2000Km and at any refractivity index (N) value for the the following conductivity and permittivity values:

   i. at any conductivity between 0.005 and 5.5 siemens/metre.

   ii. at the following 21 conductivity and 7 permittivity values for land conductivity between 0.0001 and 0.005 siemens/metre:

   Conductivity=0.0001,0.0002,0.0003,0.0004,0.0005,0.0006,
   0.0007,0.0008,0.0009,0.0010,0.0011,0.0012,
   0.0013,0.0014,0.0015,0.00175,0.002,0.0025,
   0.003,0.004,0.005.

   Permittivity = 3.0,5.0,10.0,15.0,20.0,25.0,30.0.

   iii. at the following 10 conductivity and 3 permittivity values for land conductivity between 0.00001 and 0.0001 siemens/metre:

   Conductivity= 0.00001,0.00002,0.00003,0.00004,0.00005,
   0.00006,0.00007,0.00008,0.00009,0.00010.

   Permittivity= 3.0,4.0,5.0.

   iv. at fresh water conductivity and permittivity values of 0.005 siemens/metre and 80.0 esu respectively.

   Note:

   For ii and iii above, the user must select the pair of conductivity and permittivity values required as data input. For an example, in ii above, the pair can be 0.0004 and 3.0 or 0.0004 and 5.0......

2. INPUT DATA All input data are free formatted and therefore a space in between each of them is required. The required data are conductivity, permittivity, refractivity index (N) and distance in that order. As many sets of data input can be run together with this program but it must end with 0.0 0.0 0.0 0.0 as the last data input. Below are two examples on how to input the data :

………. 1 set

0.005 15.0 338.0 100000.0
0.0 0.0 0.0 0.0

………. many sets

0.001 15.0 340.0 50000.0
1.5 80.0 330.0 12000.0
0.0002 20.0 340.0 1000000.0
0.0 0.0 0.0 0.0

<u>Note:</u>

For land conductivity between 0.00001 and 0.005, the user must be very careful to input only the listed conductivity with corresponding listed permittivity, otherwise error massage will be given.

3. HOW TO USE AND JCL CARDS This program is in the Surveying Engineering computer library and the following JCL cards can be used:

```
//                JOB,SE1234

/*JOBPARM         S=5,L=99,R=512

/*SERVICE         -4

//*

//                EXEC FORTVG,RG=512K,GEOPGM=TOPLAG

//STEPLIB         DD  DSN=A.M12129.SELIBOJ,DISP=SHIR

//SYSIN           DD  *


DATA FOR THE PROGRAM


//
```

4. INTERPOLATION

For conductivity higher than 0.005 siemens/metre, total phaselag is not affected by permittivity. Therefore this program can be used for any value from 0.005 to 5.5 siemens/metre. If there is a need to do the computation at conductivity or permittivity or both which are not listed, the user is advised to do the computation at few close-by values and plot the results. These points are then joined with a smooth curve (as it should be) and obtain the required result from the plot. As a guide in the interpolation between permittivity, total phaselag variation with permittivity for conductivity above 0.0005 siemens/metre is quite uniform.

VI.3 - Program Listing

```
      IMPLICIT REAL *4(A-H,O-Z)
      DIMENSION CL1(21),CL2(10),PL1(7),PL2(3)
C***********************************************************************
C*                                                                     *
C* NAME        TOPLAG                                                  *
C*                                                                     *
C* TYPE        MAIN                                                    *
C*                                                                     *
C* PURPOSE     TO COMPUTE TOTAL PHASELAG USING APPROXIMATE FORMULA:     *
C*                                                                     *
C*             TPL =C1+C2S+(C3S+C4)EXP(C5S)+C6/(1+C7S+C8S**4)+2.277/S *
C*                                                                     *
C*             AT 100 KHZ FREQUENCY FOR THE FOLLOWING GROUND PATH       *
C*             AT RESPECTIVE CONDUCTIVITY AND PERMITTIVITY VALUES:      *
C*                                                                     *
C*             GROUND WAVE      CONDUCTIVITY        PERMITTIVITY        *
C*                PATH         (SIEMENS/METRE)          ( ESU )         *
C*             """""""""""      """"""""""""         """""""""""        *
C*             1.SEA WATER       1.0 TO 5.5              80             *
C*             2.FRESH WATER       0.005                 80             *
C*             3.LAND SOIL      0.005 TO 1.0      ANY VALUE(3 TO 40)    *
C*             4.LAND SOIL     0.0001 TO 0.005    3,5,10,15,20,25,30    *
C*             5.LAND SOIL    0.00001 TO 0.0001         3,4,5           *
C*                                                                     *
C*           ====> FOR PATH 4,THE SELECTED CONDUCTIVITY VALUES ARE :   *
C*                  0.0001,0.0002,0.0003,0.0004,0.0005,0.0006,0.0007,  *
C*                  0.0008,0.0009,0.001,0.0011,0.0012,0.0013,0.0014,   *
C*                  0.0015,0.00175,0.002,0.0025,0.003,0.004,0.005.     *
C*           ====> FOR PATH 5,THE SELECTED CONDUCTIVITY VALUES ARE :   *
C*                  0.00001,0.00002,0.00003,0.00004,0.00005,0.00006,   *
C*                  0.00007,0.00008,0.00009,0.00010.                   *
C*                                                                     *
C* AUTHOR      TUAN BAHAROM MAHMOOD, SEPTEMBER 1986                     *
C*                                                                     *
C* EXTERNALS   COEF00,COEF03,COEF05,COEF10,COEF15,COEF20,COEF25,       *
C*             COEF30,COEF3L,COEF4L,COEF5L,COEFFW                      *
C*             NOTE: SUBROUTINE COEF00 COMPUTES THE COEFFICIENTS       *
C*             """"  TO BE USED IN THE APPROXIMATE FORMULA FOR         *
C*                   PATH 1 AND 3 ABOVE.                               *
C*                   OTHER SUBROUTINES STORE THESE COEFFICIENTS        *
C*                   AT THIER RESPECTIVE PERMITTIVITY VALUES.          *
C*                                                                     *
C* PARAMETERS                                                          *
C*             COND     = GROUND/WATER CONDUCTIVITY (SIEMENS/METRE)    *
C*             PER      = GROUND/WATER PERMITTIVITY (ESU)              *
C*             REF      = REFRACTIVITY INDEX (N),E.G: N = 338.0        *
C*             DIST     = DISTANCE (METRES)                            *
C*             C1 TO C8 = COEFFICIENTS TO BE USED IN THE FORMULA       *
C*             PPL      = PRIMARY PHASELAG                             *
C*             SPL      = SECONDARY PHASELAG                           *
C*             TPL      = TOTAL PHASELAG                               *
C*             S        = DISTANCE/100000.0                            *
C*             N        = TO DETERMINE THE COEFFICIENTS TO BE USED     *
C*                        WITH RESPECTIVE CONDUCTIVITY AND             *
C*                        PERMITTIVITY FOR PATH 4 AND 5                *
C*                                                                     *
C*             DATA INPUT : COND,PER,REF,DIST                          *
C*                 OUTPUT : PPL,SPL,TPL                                *
C*                                                                     *
C* LANGUAGE    FORTRAN                                                 *
C*                                                                     *
C* REFERENCE   MAHMOOD,T.B(1986) "COMPUTATION OF SECONDARY PHASELAG",  *
C*             MASTER OF ENGINEERING REPORT, DEPT. OF SURVEYING        *
C*             ENGINEERING, UNIVERSITY OF NEW BRUNSWICK.               *
C*                                                                     *
```

```
C*********************************************************************
      DATA CL1/0.0001,0.0002,0.0003,0.0004,0.0005,0.0006,0.0007,0.0008,
     &         0.0009,0.001,0.0011,0.0012,0.0013,0.0014,0.0015,0.00175,
     &         0.002,0.0025,0.003,0.004,0.005/
      DATA CL2/0.00001,0.00002,0.00003,0.00004,0.00005,0.00006,0.00007,
     &         0.00008,0.00009,0.0001/
      DATA PL1/3.0,5.0,10.0,15.0,20.0,25.0,30.0/
      DATA PL2/3.0,4.0,5.0/
      WRITE(6,1000)
  100 READ(5,*)COND,PER,REF,DIST
      IF(COND.LE.0.0) GO TO 400
      IF(COND.LE.5.5) GO TO 1
      WRITE(6,1002) COND
      GO TO 300
    1 IF(COND.LT.0.005) GO TO 101
      CALL COEF00(COND,C1,C2,C3,C4,C5,C6,C7,C8)
      GO TO 200
  101 ID1 = 0
      ID2 = 0
      IF(COND.EQ.0.005.AND.PER.GT.30.0) GO TO 109
      IF(COND.LE.0.0001.AND.PER.LE.5.0) GO TO 105
      DO 10 I=1,21
           IF(COND.NE.CL1(I)) GO TO 10
           ID1 = 1
           N   = I
           GO TO 102
   10 CONTINUE
  102 DO 20 I=1,7
           IF(PER.NE.PL1(I)) GO TO 20
           ID2 = 1
           GO TO 103
   20 CONTINUE
  103 IF(ID1.EQ.1.AND.ID2.EQ.1) GO TO 104
      IF(ID1.EQ.0.AND.ID2.EQ.0) WRITE (6,1001) COND,PER
      IF(ID1.EQ.0.AND.ID2.EQ.1) WRITE (6,1002) COND
      IF(ID1.EQ.1.AND.ID2.EQ.0) WRITE (6,1003) PER
      GO TO 300
  104 IF(PER.EQ.3.0) CALL COEF03(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IF(PER.EQ.5.0) CALL COEF05(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IF(PER.EQ.10.0) CALL COEF10(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IF(PER.EQ.15.0) CALL COEF15(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IF(PER.EQ.20.0) CALL COEF20(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IF(PER.EQ.25.0) CALL COEF25(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IF(PER.EQ.30.0) CALL COEF30(N,C1,C2,C3,C4,C5,C6,C7,C8)
      GO TO 200
  105 DO 30 I=1,10
           IF(COND.NE.CL2(I)) GO TO 30
           ID1 = 1
           N   = I
           GO TO 106
   30 CONTINUE
  106 DO 40 I=1,3
           IF(PER.NE.PL2(I)) GO TO 40
           ID2 = 1
           GO TO 107
   40 CONTINUE
  107 IF(ID1.EQ.1.AND.ID2.EQ.1) GO TO 108
      IF(ID1.EQ.0.AND.ID2.EQ.0) WRITE(6,1001) COND,PER
      IF(ID1.EQ.0.AND.ID2.EQ.1) WRITE(6,1002) COND
      IF(ID1.EQ.1.AND.ID2.EQ.0) WRITE(6,1003) PER
      GO TO 300
  108 IF(PER.EQ.3.0) CALL COEF3L(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IF(PER.EQ.4.0) CALL COEF4L(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IF(PER.EQ.5.0) CALL COEF5L(N,C1,C2,C3,C4,C5,C6,C7,C8)
      GO TO 200
```

```
 109 CALL COEFFW(C1,C2,C3,C4,C5,C6,C7,C8)
 200 S   = DIST/100000.0
     TPL = C1+C2*S+(C3*S+C4)*EXP(C5*S)+C6/(1.0+C7*S+C8*S**4)+2.277/S
     PPL = (DIST/1000.0)*0.338
     SPL = TPL-PPL
     IF(REF.EQ.338.0) GO TO 110
     PPL = (DIST/1000.0)*(REF/1000.0)
     TPL = SPL+PPL
 110 WRITE(6,1004) COND,PER,REF
     WRITE(6,1005)C1,C2,C3,C4,C5,C6,C7,C8
     WRITE(6,1006) DIST,PPL,SPL,TPL
 300 GO TO 100
 400 WRITE(6,1000)
1000 FORMAT('1',' ')
1001 FORMAT(////10X,'*****   E R R O R  *****'//10X,
    &        'YOUR CONDUCTIVITY',F10.6,' AND PERMITTIVITY',F5.1,
    &        ' NOT IN THE LIST'/10X,'**********************************',
    &        '*********************************')
1002 FORMAT(////10X,'*****  E R R O R  *****'//10X,
    &        'YOUR CONDUCTIVITY',F10.6,' NOT IN THE LIST'/10X,
    &        '********************************************')
1003 FORMAT(////10X,'*****  E R R O R  *****'//10X,
    &        'YOUR PERMITTIVITY',F5.1,' NOT IN THE LIST'/10X,
    &        '***************************************')
1004 FORMAT(////10X,'CONDUCTIVITY =',F9.6,2X,'SIEMENS/METRE'/
    &            10X,'PERMITTIVITY =',F5.1,7X,'ESU'/
    &            10X,'REFRACTIVITY =',F6.1//)
1005 FORMAT(10X,'THE COEFFICIENTS C(I),I = 1 TO 8 ARE:'//
    &        10X,4F10.3/10X,4F10.3//)
1006 FORMAT(10X,'DISTANCE(M)',5X,'PRI.PHASELAG(M)',
    &         5X,'SEC.PHASELAG(M)',5X,'TOT.PHASELAG(M)'//11X,F9.1,
    &         10X,F6.1,15X,F6.1,13X,F6.1)
     STOP
     END
     SUBROUTINE COEFOO(COND,C1,C2,C3,C4,C5,C6,C7,C8)
     IMPLICIT REAL *4(A-H,O-Z)
C****************************************************************************
C*                                                                          *
C* NAME       COEFOO                                                        *
C*                                                                          *
C* TYPE       SUBROUTINE                                                    *
C*                                                                          *
C* PURPOSE    TO COMPUTE THE C COEFFIENTS TO BE USED IN THE                 *
C*            COMPUTATION OF TOTAL PHASELAG USING APPROXIMATE               *
C*            FORMULA FOR SEA WATER AND LAND CONDUCTIVITY ABOVE             *
C*            0.005 SIEMENS/METRE.                                          *
C*                                                                          *
C* AUTHOR     TUAN BAHAROM MAHMOOD                                          *
C*                                                                          *
C* EXTERNALS  NONE                                                          *
C*                                                                          *
C* PARAMETERS ALL EXPLAINED IN THE MAIN                                     *
C*            INPUT : COND                                                  *
C*            OUTPUT: C1,C2,C3,C4,C5,C6,C7,C8                               *
C*                                                                          *
C* CALLING    CALL COEFOO(COND,C1,C2,C3,C4,C5,C6,C7,C8)                     *
C*                                                                          *
C* LANGUAGE   FORTRAN                                                       *
C*                                                                          *
C* REFERENCE  MAHMOOD,T.B(1986) "COMPUTATION OF SECONDARY PHSELAG"          *
C*            M.ENG REPORT,DEPT. OF SURVEYING ENG.,UNB                      *
C*                                                                          *
C****************************************************************************
     CN   = COND*10.0
     CN12 = CN**0.5
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```
      CN14 = CN**0.25
      IF(COND.LT.1.0) GO TO 10
      C1    = -111.0
      C2    = 97.882-11.437/CN+40.006/CN12-13.576/CN14
      C3    = -18.496+63.283/CN+5.844/CN12+7.714/CN14
      C4    = 112.8
      C5    = -0.254
      C6    = 0.0
      C7    = 0.0
      C8    = 0.0
      GO TO 20
   10 C1    = -128.059+1.085/CN+65.474/CN12+5.902/CN14
      C2    = 97.380-0.615/CN+20.238/CN12-4.107/CN14
      C3    = 19.048+0.049/CN-4.812/CN12+22.139/CN14
      C4    = 126.620-0.901/CN-38.415/CN12-11.674/CN14
      C5    = -0.5273-0.0025/CN+0.0387/CN12-0.0927/CN14
      C6    = -4.055-0.024/CN-30.266/CN12+23.080/CN14
      C7    = 8.421-0.494/CN+5.677/CN12-5.644/CN14
      C8    = 315.570+22.365/CN-346.462/CN12+448.310/CN14
   20 RETURN
      END
      SUBROUTINE COEF03(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IMPLICIT REAL *4(A-H,O-Z)
C*********************************************************************
C*                                                                   *
C*    THIS SUBROUTINE STORES THE C COEFFICIENTS FOR THE APPROXIMATE  *
C*          FORMULA AT SELECTED CONDUCTIVITIES (SEE MAIN)            *
C*          ......... AND PERMITTIVITY OF 3 ESU .........            *
C*      N IS DETERMINED IN THE MAIN TO INDICATE THE POSITION         *
C*    OF THE INPUT CONDUCTIVITY WITHIN THE LIST AND C1 TO C8 ARE     *
C*                      RETURNED TO THE MAIN.                        *
C*                                                                   *
C*********************************************************************
      DIMENSION CC1(21),CC2(21),CC3(21),CC4(21),CC5(21)
      DIMENSION CC6(21),CC7(21),CC8(21)
      DATA CC1/985.5,1016.8,976.7,933.1,886.0,838.8,790.4,
     &         743.1,698.4,656.6,618.9,584.4,553.9,527.2,
     &         503.7,452.6,410.2,347.1,302.1,240.6,199.7/
      DATA CC2/187.47,194.08,200.32,204.65,207.70,209.64,210.77,
     &         211.19,211.03,210.44,209.49,208.32,206.95,205.45,
     &         203.85,199.87,196.10,189.22,183.31,173.99,167.01/
      DATA CC3/160.7,401.8,335.8,271.2,205.2,158.1,124.5,
     &         100.8,84.3,73.7,65.3,59.7,55.2,52.5,
     &         49.7,44.3,42.6,42.0,41.7,42.0,43.2/
      DATA CC4/229.7,-285.3,-426.2,-492.0,-475.0,-449.1,-415.5,
     &         -381.3,-350.5,-325.1,-300.6,-279.4,-261.6,-253.0,
     &         -246.6,-223.7,-199.1,-166.8,-142.4,-107.8,-85.4/
      DATA CC5/-0.514,-0.632,-0.534,-0.478,-0.436,-0.414,-0.402,
     &         -0.398,-0.399,-0.402,-0.406,-0.410,-0.417,-0.434,
     &         -0.457,-0.495,-0.507,-0.534,-0.556,-0.583,-0.596/
      DATA CC6/-1040.9,-605.5,-453.1,-362.3,-338.3,-321.5,-310.1,
     &         -300.0,-288.9,-275.2,-264.3,-253.2,-242.5,-226.8,
     &         -211.9,-187.8,-172.7,-146.2,-128.5,-105.8,-90.0/
      DATA CC7/6.62,7.110,8.12,9.61,8.85,8.21,7.61,
     &         7.16,6.87,6.79,6.68,6.65,6.66,6.98,
     &         7.36,7.85,8.02,8.60,8.97,9.46,9.97/
      DATA CC8/30.0,14.0,15.0,54.0,44.0,34.0,23.0,
     &         17.0,13.0,13.0,12.0,12.0,12.0,16.0,
     &         21.0,29.0,33.0,53.0,69.0,94.0,142.0/
      C1 = CC1(N)
      C2 = CC2(N)
      C3 = CC3(N)
      C4 = CC4(N)
      C5 = CC5(N)
      C6 = CC6(N)
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      C7 = CC7(N)
      C8 = CC8(N)
      RETURN
      END
      SUBROUTINE COEF05(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IMPLICIT REAL *4(A-H,O-Z)
C*********************************************************************
C*                                                                   *
C*  THIS SUBROUTINE STORES THE C COEFFICIENTS FOR THE APPROXIMATE    *
C*  FORMULA AT THE SELECTED CONDUCTIVITY AND PERMITTIVITY OF 5 ESU   *
C*                                       """"""""""""    """"""      *
C*********************************************************************
      DIMENSION CC1(21),CC2(21),CC3(21),CC4(21),CC5(21)
      DIMENSION CC6(21),CC7(21),CC8(21)
      DATA CC1/930.9,984.0,956.6,914.2,873.9,829.1,782.6,
     &         737.2,693.4,652.5,615.5,581.9,552.1,525.5,
     &         502.3,451.5,409.5,346.7,301.7,240.5,199.4/
      DATA CC2/186.62,193.45,199.52,204.08,206.98,208.95,210.12,
     &         210.56,210.47,209.94,209.04,207.90,206.56,205.10,
     &         203.53,199.64,195.91,189.09,183.23,173.94,167.00/
      DATA CC3/164.8,359.4,320.3,254.5,202.1,156.9,124.2,
     &         100.6,84.9,74.4,66.0,60.3,55.5,53.1,
     &         50.1,44.9,42.9,42.2,42.0,42.1,43.9/
      DATA CC4/201.7,-233.5,-398.4,-443.9,-459.8,-437.2,-405.9,
     &         -374.1,-345.3,-320.8,-297.0,-276.7,-259.7,-251.6,
     &         -245.4,-222.7,-198.5,-166.6,-142.4,-107.8,-85.4/
      DATA CC5/-0.512,-0.609,-0.532,-0.467,-0.437,-0.416,-0.403,
     &         -0.399,-0.399,-0.402,-0.406,-0.410,-0.417,-0.434,
     &         -0.457,-0.495,-0.507,-0.534,-0.556,-0.584,-0.596/
      DATA CC6/-959.2,-621.9,-459.1,-387.3,-340.6,-323.1,-311.4,
     &         -301.0,-288.8,-275.2,-264.4,-253.2,-242.4,-226.5,
     &         -211.7,-187.6,-172.4,-145.9,-128.2,-105.6,-89.6/
      DATA CC7/6.92,6.76,7.87,8.42,8.70,8.10,7.52,
     &         7.09,6.85,6.77,6.66,6.64,6.65,6.99,
     &         7.36,7.85,7.99,8.59,9.01,9.49,10.03/
      DATA CC8/30.0,14.0,14.0,25.0,40.0,31.0,22.0,
     &         16.0,13.0,13.0,12.0,12.0,12.0,16.0,
     &         21.0,29.0,33.0,54.0,71.0,95.0,145.0/
      C1 = CC1(N)
      C2 = CC2(N)
      C3 = CC3(N)
      C4 = CC4(N)
      C5 = CC5(N)
      C6 = CC6(N)
      C7 = CC7(N)
      C8 = CC8(N)
      RETURN
      END
      SUBROUTINE COEF10(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IMPLICIT REAL *4(A-H,O-Z)
C*********************************************************************
C*                                                                   *
C*   THIS SUBROUTINE STORES THE C COEFFICIENTS FOR THE APPROXIMATE   *
C*   FORMULA AT THE SELECTED CONDUCTIVITY AND PERMITTIVITY OF 10 ESU *
C*                                        """"""""""""    """"""     *
C*********************************************************************
      DIMENSION CC1(21),CC2(21),CC3(21),CC4(21),CC5(21)
      DIMENSION CC6(21),CC7(21),CC8(21)
      DATA CC1/811.6,908.4,906.2,877.9,844.9,805.3,763.2,
     &         721.4,680.9,642.5,607.6,575.6,546.9,521.2,
     &         498.9,449.3,408.0,345.7,301.2,240.2,199.1/
      DATA CC2/184.68,191.76,197.66,202.15,205.14,207.24,208.52,
     &         209.08,209.10,208.68,207.88,206.85,205.61,204.24,
     &         202.74,199.01,195.41,188.76,182.99,173.81,166.92/
      DATA CC3/166.1,293.9,284.1,237.0,194.0,153.6,123.4,
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     &         101.6,86.3,75.9,67.5,61.4,56.7,54.3,
     &         51.1,45.5,43.2,42.7,42.3,42.3,44.1/
      DATA CC4/154.7,-161.0,-333.7,-397.9,-425.1,-409.8,-384.1,
     &         -358.0,-333.0,-310.8,-289.7,-270.6,-255.0,-248.3,
     &         -242.5,-220.6,-196.8,-165.9,-142.2,-107.8,-85.5/
      DATA CC5/-0.507,-0.574,-0.523,-0.469,-0.442,-0.419,-0.405,
     &         -0.400,-0.399,-0.401,-0.405,-0.409,-0.417,-0.434,
     &         -0.457,-0.495,-0.507,-0.534,-0.556,-0.585,-0.597/
      DATA CC6/-798.3,-618.6,-470.2,-394.4,-344.6,-325.6,-313.0,
     &         -300.7,-288.2,-274.8,-263.5,-252.7,-241.7,-225.3,
     &         -211.0,-187.2,-172.5,-145.6,-127.8,-105.4,-89.2/
      DATA CC7/7.31,6.64,7.38,7.97,8.38,7.89,7.38,
     &         7.03,6.82,6.74,6.66,6.62,6.66,7.02,
     &         7.37,7.83,7.98,8.63,9.05,9.54,10.07/
      DATA CC8/28.0,13.0,13.0,20.0,32.0,26.0,19.0,
     &         15.0,13.0,13.0,12.0,12.0,12.0,16.0,
     &         21.0,29.0,33.0,55.0,73.0,96.0,149.0/
      C1 = CC1(N)
      C2 = CC2(N)
      C3 = CC3(N)
      C4 = CC4(N)
      C5 = CC5(N)
      C6 = CC6(N)
      C7 = CC7(N)
      C8 = CC8(N)
      RETURN
      END
      SUBROUTINE COEF15(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IMPLICIT REAL *4(A-H,O-Z)
C*********************************************************************
C*                                                                   *
C*   THIS SUBROUTINE STORES THE C COEFFICIENTS FOR THE APPROXIMATE   *
C*   FORMULA AT THE SELECTED CONDUCTIVITY AND PERMITTIVITY OF 15 ESU *
C*                                          """"""""""""    """""    *
C*********************************************************************
      DIMENSION CC1(21),CC2(21),CC3(21),CC4(21),CC5(21)
      DIMENSION CC6(21),CC7(21),CC8(21)
      DATA CC1/717.0,839.6,857.6,842.2,816.5,782.2,744.4,
     &         705.9,668.2,632.3,598.8,568.2,541.0,516.8,
     &         495.2,446.8,406.2,344.7,300.5,239.9,198.9/
      DATA CC2/183.06,190.14,195.91,200.32,203.38,205.56,206.94,
     &         207.62,207.75,207.44,206.78,205.85,204.69,203.38,
     &         201.96,198.39,194.91,188.43,182.76,173.68,166.84/
      DATA CC3/162.6,260.6,257.9,222.3,187.1,150.5,122.0,
     &         101.7,87.4,77.2,69.5,63.5,58.7,55.3,
     &         52.0,46.3,43.7,43.1,42.6,42.4,44.2/
      DATA CC4/124.5,-135.9,-289.9,-360.3,-396.0,-385.9,-364.1,
     &         -341.6,-320.7,-301.1,-282.2,-264.8,-251.4,-244.4,
     &         -239.2,-218.6,-195.3,-165.1,-141.6,-107.4,-85.3/
      DATA CC5/-0.501,-0.555,-0.517,-0.472,-0.447,-0.424,-0.409,
     &         -0.401,-0.400,-0.401,-0.403,-0.407,-0.416,-0.434,
     &         -0.457,-0.495,-0.507,-0.534,-0.556,-0.585,-0.598/
      DATA CC6/-681.9,-577.8,-464.9,-394.9,-344.2,-325.6,-313.5,
     &         -300.9,-287.4,-274.0,-262.0,-251.0,-239.4,-224.7,
     &         -210.5,-186.7,-172.2,-145.3,-127.7,-105.3,-89.2/
      DATA CC7/7.38,6.91,7.28,7.74,8.21,7.77,7.27,
     &         6.95,6.79,6.73,6.68,6.66,6.73,7.02,
     &         7.36,7.84,7.97,8.63,9.04,9.51,10.10/
      DATA CC8/24.0,16.0,14.0,18.0,30.0,24.0,18.0,
     &         14.0,13.0,13.0,13.0,13.0,13.0,17.0,
     &         21.0,30.0,33.0,55.0,73.0,96.0,149.0/
      C1 = CC1(N)
      C2 = CC2(N)
      C3 = CC3(N)
      C4 = CC4(N)
```

```
      C5 = CC5(N)
      C6 = CC6(N)
      C7 = CC7(N)
      C8 = CC8(N)
      RETURN
      END
      SUBROUTINE COEF20(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IMPLICIT REAL *4(A-H,O-Z)
C**********************************************************************
C*                                                                    *
C*   THIS SUBROUTINE STORES THE C COEFFICIENTS FOR THE APPROXIMATE    *
C*   FORMULA AT THE SELECTED CONDUCTIVITY AND PERMITTIVITY OF 20 ESU *
C*                                           """"""""""""    """""" *
C**********************************************************************
      DIMENSION CC1(21),CC2(21),CC3(21),CC4(21),CC5(21)
      DIMENSION CC6(21),CC7(21),CC8(21)
      DATA CC1/643.3,777.7,810.8,806.9,788.5,758.9,724.9,
     &         689.9,655.5,622.1,590.6,561.5,535.5,512.4,
     &         491.4,444.4,404.4,343.6,299.7,239.5,198.7/
      DATA CC2/181.72,188.62,194.28,198.61,201.69,203.97,205.45,
     &         206.24,206.45,206.22,205.66,204.83,203.76,202.52,
     &         201.18,197.77,194.42,188.10,182.53,173.55,166.76/
      DATA CC3/157.5,234.7,234.3,209.5,179.8,146.8,121.1,
     &         102.3,88.3,78.2,70.5,64.5,59.5,56.1,
     &         52.9,46.9,44.4,43.4,42.9,42.6,44.3/
      DATA CC4/103.0,-116.5,-249.7,-327.6,-367.4,-361.0,-344.2,
     &         -326.4,-308.7,-291.5,-274.6,-258.7,-246.3,-240.3,
     &         -235.9,-216.5,-193.7,-164.1,-140.9,-107.1,-85.2/
      DATA CC5/-0.494,-0.540,-0.509,-0.473,-0.451,-0.427,-0.410,
     &         -0.402,-0.400,-0.401,-0.404,-0.407,-0.416,-0.434,
     &         -0.457,-0.495,-0.507,-0.534,-0.556,-0.585,-0.598/
      DATA CC6/-596.0,-538.5,-458.0,-391.6,-343.9,-326.2,-313.3,
     &         -299.7,-286.2,-273.0,-261.0,-250.2,-238.8,-224.0,
     &         -209.8,-186.2,-171.7,-145.1,-127.5,-105.2,-89.1/
      DATA CC7/7.30,7.09,7.14,7.61,8.04,7.59,7.17,
     &         6.91,6.77,6.70,6.67,6.66,6.72,7.01,
     &         7.36,7.84,7.97,8.62,9.03,9.51,10.09/
      DATA CC8/21.0,18.0,14.0,18.0,27.0,21.0,17.0,
     &         14.0,13.0,13.0,13.0,13.0,13.0,17.0,
     &         21.0,30.0,33.0,55.0,73.0,97.0,149.0/
      C1 = CC1(N)
      C2 = CC2(N)
      C3 = CC3(N)
      C4 = CC4(N)
      C5 = CC5(N)
      C6 = CC6(N)
      C7 = CC7(N)
      C8 = CC8(N)
      RETURN
      END
      SUBROUTINE COEF25(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IMPLICIT REAL *4(A-H,O-Z)
C**********************************************************************
C*                                                                    *
C*   THIS SUBROUTINE STORES THE C COEFFICIENTS FOR THE APPROXIMATE    *
C*   FORMULA AT THE SELECTED CONDUCTIVITY AND PERMITTIVITY OF 25 ESU *
C*                                           """"""""""""    """""" *
C**********************************************************************
      DIMENSION CC1(21),CC2(21),CC3(21),CC4(21),CC5(21)
      DIMENSION CC6(21),CC7(21),CC8(21)
      DATA CC1/585.9,722.4,767.9,774.4,762.7,738.8,709.0,
     &         676.2,644.1,613.1,583.5,555.8,530.2,508.1,
     &         487.6,441.7,402.5,342.5,299.0,239.0,198.4/
      DATA CC2/180.58,187.23,192.70,196.90,199.99,202.28,203.84,
     &         204.77,205.09,204.96,204.49,203.75,202.81,201.63,
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     &         200.40,197.15,193.91,187.76,182.30,173.42,166.68/
      DATA CC3/153.6,215.2,218.6,203.6,176.3,146.4,121.9,
     &         102.4,88.3,78.1,70.4,64.3,59.9,56.6,
     &         53.6,47.8,45.0,43.8,43.1,42.8,44.4/
      DATA CC4/84.7,-102.8,-226.9,-314.0,-352.7,-352.8,-339.5,
     &         -317.4,-299.4,-283.8,-268.2,-253.2,-241.1,-236.6,
     &         -232.3,-214.6,-192.3,-163.0,-140.1,-106.6,-84.7/
      DATA CC5/-0.490,-0.528,-0.506,-0.482,-0.461,-0.439,-0.423,
     &         -0.410,-0.405,-0.406,-0.408,-0.412,-0.417,-0.435,
     &         -0.457,-0.495,-0.506,-0.534,-0.556,-0.584,-0.597/
      DATA CC6/-529.5,-500.4,-439.5,-373.9,-333.2,-314.9,-302.6,
     &         -294.9,-284.0,-271.5,-260.0,-249.6,-238.4,-223.3,
     &         -209.3,-185.2,-171.2,-144.9,-127.5,-105.2,-89.2/
      DATA CC7/7.28,7.23,7.27,7.96,8.25,7.90,7.48,
     &         7.00,6.78,6.69,6.65,6.62,6.69,7.01,
     &         7.35,7.86,7.98,8.61,9.02,9.51,10.06/
      DATA CC8/20.0,19.0,16.0,25.0,32.0,28.0,22.0,
     &         15.0,13.0,12.0,12.0,12.0,13.0,17.0,
     &         21.0,31.0,33.0,54.0,71.0,95.0,145.0/
      C1 = CC1(N)
      C2 = CC2(N)
      C3 = CC3(N)
      C4 = CC4(N)
      C5 = CC5(N)
      C6 = CC6(N)
      C7 = CC7(N)
      C8 = CC8(N)
      RETURN
      END
      SUBROUTINE COEF30(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IMPLICIT REAL *4(A-H,O-Z)
C**********************************************************************
C*                                                                    *
C*   THIS SUBROUTINE STORES THE C COEFFICIENTS FOR THE APPROXIMATE    *
C*   FORMULA AT THE SELECTED CONDUCTIVITY AND PERMITTIVITY OF 30 ESU  *
C*                                           """"""""""""    """"""   *
C**********************************************************************
      DIMENSION CC1(21),CC2(21),CC3(21),CC4(21),CC5(21)
      DIMENSION CC6(21),CC7(21),CC8(21)
      DATA CC1/540.3,673.8,727.4,741.8,736.3,717.2,691.6,
     &         661.6,632.0,602.7,575.1,548.7,524.5,502.9,
     &         483.4,439.0,400.4,341.4,298.3,238.5,198.2/
      DATA CC2/179.57,185.93,191.23,195.35,198.43,200.76,202.36,
     &         203.39,203.81,203.79,203.40,202.76,201.89,200.83,
     &         199.64,196.53,193.43,187.42,182.05,173.29,166.60/
      DATA CC3/148.7,199.7,205.2,193.9,171.2,144.3,121.9,
     &         102.8,88.7,78.7,71.0,65.1,60.4,57.9,
     &         54.5,48.6,45.8,44.0,43.1,43.0,44.5/
      DATA CC4/71.4,-92.5,-207.0,-290.3,-332.1,-336.8,-328.7,
     &         -307.4,-289.3,-274.3,-260.3,-246.8,-235.7,-232.7,
     &         -228.8,-212.5,-190.6,-161.9,-139.2,-106.1,-84.3/
      DATA CC5/-0.486,-0.519,-0.503,-0.483,-0.466,-0.446,-0.431,
     &         -0.415,-0.408,-0.407,-0.409,-0.412,-0.418,-0.436,
     &         -0.457,-0.495,-0.506,-0.534,-0.556,-0.583,-0.596/
      DATA CC6/-479.1,-465.9,-420.5,-365.3,-327.3,-309.3,-296.1,
     &         -290.5,-281.8,-270.5,-259.3,-248.8,-237.9,-221.9,
     &         -208.5,-184.6,-170.6,-144.9,-127.6,-105.1,-89.2/
      DATA CC7/7.20,7.33,7.37,7.95,8.26,7.96,7.62,
     &         7.09,6.79,6.68,6.63,6.61,6.66,7.03,
     &         7.36,7.88,7.99,8.59,8.98,9.48,10.00/
      DATA CC8/18.0,20.0,18.0,26.0,34.0,30.0,24.0,
     &         17.0,13.0,12.0,12.0,12.0,12.0,17.0,
     &         21.0,31.0,34.0,53.0,70.0,95.0,143.0/
      C1 = CC1(N)
      C2 = CC2(N)
```

```
      C3 = CC3(N)
      C4 = CC4(N)
      C5 = CC5(N)
      C6 = CC6(N)
      C7 = CC7(N)
      C8 = CC8(N)
      RETURN
      END
      SUBROUTINE COEF3L(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IMPLICIT REAL *4(A-H,O-Z)
C*****************************************************************
C*                                                               *
C*   THIS SUBROUTINE STORES THE C COEFFICIENTS FOR THE APPROXIMATE   *
C*   FORMULA AT THE SELECTED CONDUCTIVITY AND PERMITTIVITY OF 3 ESU  *
C*                                            """"""""""""    """""   *
C*****************************************************************
      DIMENSION CC1(10),CC2(10),CC3(10),CC4(10),CC5(10)
      DIMENSION CC6(10),CC7(10),CC8(10)
      DATA CC1/533.9,697.8,803.9,867.2,901.0,925.9,946.0,961.9,
     &         975.3,985.9/
      DATA CC2/174.44,176.50,178.34,180.10,182.02,183.53,184.73,
     &         185.75,186.64,187.45/
      DATA CC3/135.0,151.2,165.4,164.4,126.5,117.8,120.0,129.1,
     &         144.1,162.0/
      DATA CC4/274.8,260.7,246.8,245.3,276.7,286.4,283.2,271.9,
     &         252.7,228.2/
      DATA CC5/-0.517,-0.538,-0.554,-0.547,-0.492,-0.477,-0.477,
     &         -0.484,-0.500,-0.516/
      DATA CC6/-513.8,-707.1,-827.4,-906.1,-979.6,-1019.6,-1041.3,
     &         -1050.6,-1049.3,-1039.7/
      DATA CC7/27.00,24.43,19.64,15.68,12.36,10.28,8.88,7.88,7.17,6.63/
      DATA CC8/3386.0,2621.0,1485.0,685.0,258.0,136.0,84.0,57.0,
     &         41.0,30.0/
      C1 = CC1(N)
      C2 = CC2(N)
      C3 = CC3(N)
      C4 = CC4(N)
      C5 = CC5(N)
      C6 = CC6(N)
      C7 = CC7(N)
      C8 = CC8(N)
      RETURN
      END
      SUBROUTINE COEF4L(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IMPLICIT REAL *4(A-H,O-Z)
C*****************************************************************
C*                                                               *
C*   THIS SUBROUTINE STORES THE C COEFFICIENTS FOR THE APPROXIMATE   *
C*   FORMULA AT THE SELECTED CONDUCTIVITY AND PERMITTIVITY OF 4 ESU  *
C*                                            """""""""""""""   *
C*****************************************************************
      DIMENSION CC1(10),CC2(10),CC3(10),CC4(10),CC5(10)
      DIMENSION CC6(10),CC7(10),CC8(10)
      DATA CC1/513.8,649.6,749.9,814.4,853.7,885.6,910.5,930.4,
     &         946.2,959.1/
      DATA CC2/174.20,176.09,177.88,179.74,181.64,183.03,184.20,
     &         185.22,186.13,186.97/
      DATA CC3/146.7,160.7,171.4,157.0,126.2,121.4,126.3,137.5,
     &         151.5,168.4/
      DATA CC4/255.9,242.6,231.2,241.4,269.5,273.1,266.6,252.1,
     &         232.9,208.5/
      DATA CC5/-0.531,-0.547,-0.557,-0.533,-0.487,-0.478,-0.481,
     &         -0.492,-0.505,-0.520/
      DATA CC6/-494.2,-644.3,-757.3,-847.8,-922.9,-964.7,-988.7,
     &         -999.3,-1000.8,-993.9/
```

```
      DATA CC7/23.17,21.62,18.44,15.04,12.10,10.26,8.98,8.05,7.35,6.82/
      DATA CC8/2197.0,1728.0,1072.0,479.0,203.0,119.0,79.0,56.0,
     &         41.0,31.0/
      C1 = CC1(N)
      C2 = CC2(N)
      C3 = CC3(N)
      C4 = CC4(N)
      C5 = CC5(N)
      C6 = CC6(N)
      C7 = CC7(N)
      C8 = CC8(N)
      RETURN
      END
      SUBROUTINE COEF5L(N,C1,C2,C3,C4,C5,C6,C7,C8)
      IMPLICIT REAL *4(A-H,O-Z)
C*********************************************************************
C*                                                                   *
C*   THIS SUBROUTINE STORES THE C COEFFICIENTS FOR THE APPROXIMATE   *
C*   FORMULA AT THE SELECTED CONDUCTIVITY AND PERMITTIVITY OF 5 ESU  *
C*                                            *****************     *
C*********************************************************************
      DIMENSION CC1(10),CC2(10),CC3(10),CC4(10),CC5(10)
      DIMENSION CC6(10),CC7(10),CC8(10)
      DATA CC1/495.0,612.0,703.5,757.3,811.2,848.0,877.0,899.8,
     &         918.2,932.9/
      DATA CC2/174.03,175.80,177.61,180.02,181.30,182.58,183.71,
     &         184.72,185.65,186.51/
      DATA CC3/153.3,164.1,163.6,100.3,122.4,124.0,131.6,142.8,
     &         157.4,172.8/
      DATA CC4/241.7,230.2,228.1,285.4,264.9,261.6,251.2,235.8,
     &         215.2,191.9/
      DATA CC5/-0.537,-0.549,-0.543,-0.449,-0.479,-0.478,-0.486,
     &         -0.496,-0.509,-0.522/
      DATA CC6/-477.9,-602.2,-710.2,-830.5,-874.5,-915.0,-940.0,
     &         -952.8,-955.8,-951.9/
      DATA CC7/20.68,19.55,17.06,13.29,11.71,10.17,9.03,8.17,7.51,6.99/
      DATA CC8/1509.0,1165.0,682.0,191.0,157.0,104.0,73.0,54.0,
     &         41.0,32.0/
      C1 = CC1(N)
      C2 = CC2(N)
      C3 = CC3(N)
      C4 = CC4(N)
      C5 = CC5(N)
      C6 = CC6(N)
      C7 = CC7(N)
      C8 = CC8(N)
      RETURN
      END
      SUBROUTINE COEFFW(C1,C2,C3,C4,C5,C6,C7,C8)
      IMPLICIT REAL *4(A-H,O-Z)
C*********************************************************************
C*                                                                   *
C*   THIS SUBROUTINE STORES THE C COEFFICIENTS FOR THE APPROXIMATE   *
C*     FORMULA FOR FRESH WATER,CONDUCTIVITY AND PERMITTIVITY OF      *
C*                 0.005 SIEMENS/METRE AND 80 ESU                    *
C*                 ******************************                    *
C*********************************************************************
      C1 = 195.0
      C2 = 165.72
      C3 = 46.0
      C4 = -82.4
      C5 = 0.600
      C6 = -87.8
      C7 = 10.13
      C8 = 150.0
```

```
      RETURN
      END
$ENTRY
5.0 80.0 350.0 1000000.0
0.005 15.0 338.0 1000000.0
0.0005 10.0 338.0 1000000.0
0.00005 5.0 338.0 1000000.0
0.003 15.0 338.0 1000000.0
0.0 0.0 0.0 0.0
```

The residual in C_8 seems fairly large. However due to rather small values in C_6, it will not deteriorate the accuracy of the approximate formula. Other residuals are also too small to affect the accuracy of the approximate formula.The examples of the difference between the accurate and approximate total phaselags where the C coefficients were calculated using equation 6.1 are shown in Appendix V.

6.4 PROGRAM TOPLAG.

Using the results obtained in Sections 6.1.1 and 6.1.2, Tables II.2 to II.11 and Table II.14 in Appendix II, a program called TOPLAG was developed. There were 12 subroutines whose main function is to supply the C coefficients to the main program. The first subroutine is called COEF00. It incorporates the results obtained in Sections 6.1 .1 and 6.1.2. Each of the other subroutines stores the coefficients computed at different permittivity values. This program also allows for different refractive indexes to be used other than N = 338 previously used in the approximate formula.

Using this program, a user can obtain the total phaselag (primary and secondary phaselags are also given) at any distance (2 to 2000 km) and refractive index (N) for the following:

i. Any conductivity value between 0.005 and 5.5siemens/metre.

ii. Fresh water conductivity and permittivity values of 0.005 siemens/metre and 80 esu respectively.

iii. At selected conductivity values between 0.0001 and 0.005 siemens/metre and permittivity values between 3 to 30 esu as shown in Tables II.2 to II.8.

iv. At selected conductivity values between 0.00001 and 0.0001 siemens/metre and permittivity values between 3 and 5 as shown in Tables II.9 to II.11.

The data input for this program are conductivity, permittivity, refractive index and distance. If any input is not in the specified range or list, an error message will be given. Therefore the user has to be very careful with the data input, especially for iii. and iv. above. The listing and user's guide for this program are given in Appendix VI.