SOME THEORETICAL AND PRACTICAL ASPECTS OF GEODETIC POSITIONING USING CARRIER PHASE DIFFERENCE OBSERVATIONS OF GPS SATELLITES

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PREFACE

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PREFACE

Carrier phase difference observations are potentially the most precise measurements that can be made using the Global Positioning System. In this report we discuss the development and use of a software package for processing such data. This software package was developed at the University of New Brunswick (UNB), Fredericton, Canada, between July 1983 and July 1984. It is the result of a common effort of the Department of Surveying Engineering of UNB and the Astronomical Institute of the University of Bern, Switzerland.

This software package has been used to process observations from Macrometer® V-1000 single-frequency receivers. Two campaigns, the Ottawa 1983 campaign (see Part A of this report) and the Quebec 1984 campaign (to be discussed in a subsequent work), have been processed to date.

The present software package will almost certainly be followed by a second generation version. This version will be capable of processing dual-frequency observations. The option of estimating orbital parameters will prove its full power only when dual-frequency observations on long baselines are available. However, the basic structure of the package will likely not change in these subsequent developments.

This report is arranged in three semi-autonomous parts. In Part A, we reprint, essentially unaltered, the paper "The Ottawa Macrometer® experiment: An independent analysis," which was presented at the 11th Annual Meeting of the Canadian Geophysical Union in Halifax, June 1984. Part B deals with the orbital aspects of positioning using GPS satellites. Part C concludes this report with some aspects of a technical nature, which did not fit neatly into either Part A or Part B.

Our work was supported by the Surveys and Mapping Branch and the Earth Physics Branch of Energy, Mines and Resources Canada, by the Natural Sciences and Engineering Research Council of Canada, the University of New Brunswick, and by the University of Bern, Switzerland.

Last but not least, we would like to thank Wendy Wells for the nerve-racking task of decoding and sometimes translating handwritten manuscripts and transferring them to the word processor.

PART A

THE OTTAWA MACROMETER® EXPERIMENT: AN INDEPENDENT ANALYSIS

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ABSTRACT

During the summer of 1983, the Earth Physics Branch of the federal Department of Energy, Mines and Resources conducted the first test in Canada of the Macrometer® V-1000 GPS receiver. Two receivers were used to determine the vector baselines between six points in the Ottawa area. The data obtained were processed by the Earth Physics Branch using software developed by Macrometrics Inc., the manufacturer of the Macrometer®, and, independently, at the University of New Brunswick (UNB).

The impetus for the UNB analysis was two-fold: (1) to corroborate the results obtained by the Earth Physics Branch, and (2) to develop an independent Canadian capability to process Macrometer® (and other types of) GPS observations. In this paper we describe the UNB analysis.

A computer software package was developed to process doubly-differenced phases from single or multiple observing sessions with the capability of estimating both receiver coordinates and satellite orbits. Our software yielded baseline components agreeing with the mean of those

obtained with Macrometrics' software to about 1 ppm. The corresponding agreement with terrestrial measurements is of the same order for longitude and latitude and, as may be expected, somewhat worse for the height (2 ppm to 20 ppm).

The software is also capable of directly yielding a network solution by processing all observation sessions on all baselines simultaneously. The network solution is significantly stronger than the solution based on single baseline processing.

RESUME

Au cours de l'été 1983, la Direction de la Physique du Globe du Ministère de l'Energie des Mines et des Ressources menait le premier test du récepteur GPS Macrometer® V-1000, au Canada. Deux récepteurs étaient utilisés pour déterminer les composantes des lignes de bases entre six points dans la région d'Ottawa. Les observations obtenues ont été traitées indépendamment par la Direction de la Physique du Globe en utilisant le logiciel développé par Macrometrics Inc., les fabricants du Macrometer®, et par l'Université du Nouveau-Brunswick (UNB).

Cet article décrit l'analyse effectuée à l'Université du Nouveau-Brunswick. Cette analyse avait deux buts principaux: (1) corroborer les résultats obtenus par la Direction de la Physique du Globe, et (2) de développer un logiciel Canadien indépendant pouvant traiter les observations GPS du Macrometer® (et d'autres types d'observations).

Le logiciel a été développé pour traiter les doubles différences de phases avec les options suivantes: combiner les différentes sessions d'observations ou les traiter indépendamment; estimer les coordonnées des

récepteurs ou les éléments orbitaux des satellites. Les résultats obtenus avec notre logiciel concordent avec la moyenne de ceux calculés avec le logiciel de Macrometrics à environ 1 ppm. La comparaison avec les mesures terrestres montre des résultats semblables pour les latitudes et les longitudes, cependant les différences en élévation sont de l'ordre de 2 à 20 ppm (résultats inférieurs prévisibles).

Le logiciel permet également de produire une solution pour un réseau complet en traitant toutes les observations de toutes les lignes de bases simultanéement. Cette dernière solution est significativement plus forte que la solution provenant de chacune des lignes de bases traitées individuellement.

1. INTRODUCTION

During the period from 19 July to 19 August 1983, the Earth Physics Branch (EPB) of the federal Department of Energy, Mines and Resources (EMR), with Herb Valliant as Chief Scientist, conducted the first test in Canada of Macrometrics' GPS surveying system (the Macrometer® Interferometric Surveyor). Two Macrometer® V-1000 single frequency receivers were used to determine the vector baselines between selected points of the Ottawa test network of the Surveys and Mapping Branch of EMR.

A detailed description and the results of the experiment (as obtained by the EPB with Macrometrics' software) have been recorded by Valliant [1984]. We therefore restrict ourselves to a very short description of the experiment.

A total of 30 observing sessions were conducted in as many days. The first two comprised 3 one-hour observation periods on short baselines (points 6A and 7, length 30 m; points 6A and 51, length 2230 m (see Table 1)). The remaining 28 sessions were of longer duration (24 of 5 hours, four of 3 hours) and on longer baselines (13 km to 66 km, see Table 1). Each observation period contains 60 equally spaced observation epochs where at each epoch up to six satellites are observed simultaneously. A sketch of the test area illustrating the long baselines is given in Figure 1. The short baselines were on the National Geodetic Base Line (NGBL).

Not all observing sessions yielded scientifically useful data. The observation schedule for those sessions producing useful results is given in Table 2.

2. THE MACROMETER OBSERVABLE

The measurements we have analysed are not the raw field data as recorded by single receivers. The most basic data available to us were those obtained from Macrometrics' INTERF or INTRFT computer programs (see Macrometrics [1983] or Counselman [1983]). These data usually are referred to as "interferometric phase differences between two receivers" or "single differences". In principle, one such measurement is the difference in the L_1 carrier phase of one GPS satellite measured at (nominally) the same time by the two receivers. This observable corresponds approximately to the range difference $\Delta\rho_i^j$ of satellite j of two receivers R_1 and R_2 at observation time t_i (see Figure 2). A more precise definition is given below in eqn. (1).

Tables 3(a) and 3(b) show two examples of the interferometric measurements derived with Macrometrics' program INTRFT. The measurements

TABLE 1

A Priori Coordinates for Station Positions
from Terrestrial Observations
(nominally on NAD 27)*

Station	Latitude	Longitude (west)	Orthometric Height (m)	Ellipsoidal Heights+
6A	45°23'55".79598	75°55'21".44516	77.085	78.754
7	45°23'55".13131	75°55'22".48157	76.629	78.298
51	45°23'07".16263	75°56'37".25020	70.190	71.859
Morris	45°26'34".29253	76°15'18".81735	89.806	91.164
Panmure	45°20'18".81549	76°11'04".58789	153.956	155.346
Metcalfe	45°14'34".01037	75°27'31".48309	102.590	104.823

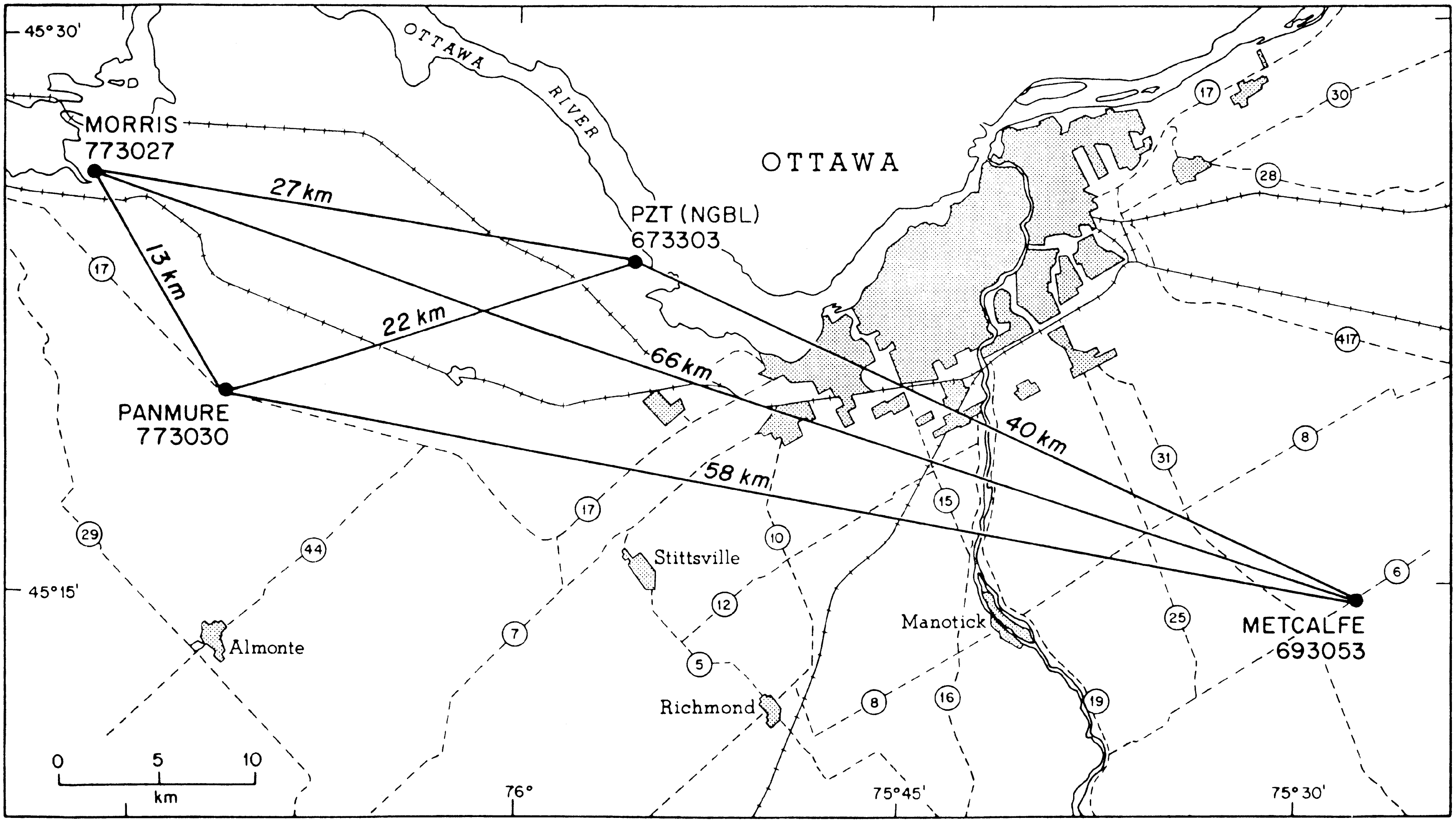
Approximate Baseline Lengths

Baseline	Length (m)
6A - 7	30
6A - 51	2230
6A - Metcalfe	40295
6A - Panmure	21590
6A - Morris	26489
Metcalfe - Panmure	57931
Metcalfe - Morris	66269
Panmure - Morris	12844

* from Valliant [1984, Table 6].

+ These values refer to the Clarke 1866 Ellipsoid of the North American Datum 1927 (NAD 27). They were calculated from the orthometric heights using the geoid undulations given with respect to the WGS-72 ellipsoid (see Valliant [1984, Table 7]) and assumed X, Y, and Z coordinate differences between the centres of the NAD-27 and WGS-72 ellipsoids of -22 m, 157 m, and 176 m respectively.

FIGURE 1



REPRINTED FROM VALLIANT [1984] WITH PERMISSION

TABLE 2

Summary of Observations.

No.	Baseline	N_s	d	$N_{s/s}$	$N_{o/s}$	N_{obs}
1	6A - 7	3	1/1/1	4/5/3	132/204/110	446
2	6A - 51	3	1/1/1	4/5/3	147/163/103	413
3	Pa - Mo	4	5/5/5/5	5/5/3/3	108/113/22/21	264
4	Pa - 6A	4	5/5/5/5	5/5/6/5	114/107/149/47	417
5	Mo - 6A	4	5/5/5/5	5/5/6/6	102/106/149/178	535
6	Me - 6A	4	5/5/5/5	5/5/6/6	124/91/163/86	464
7	Me - Pa	4	5/5/3/3	6/6/6/6	118/129/158/182	587
8	Me - Mo	3	5/5/5	5/6/6	117/151/145	413

N_s : Total number of observation sessions per baseline.

d : Duration of session (hours).

$N_{s/s}$: Number of satellites observed per session.

$N_{o/s}$: Number of (double difference) observations per baseline and per session.

N_{obs} : Number of (double difference) observations per baseline.

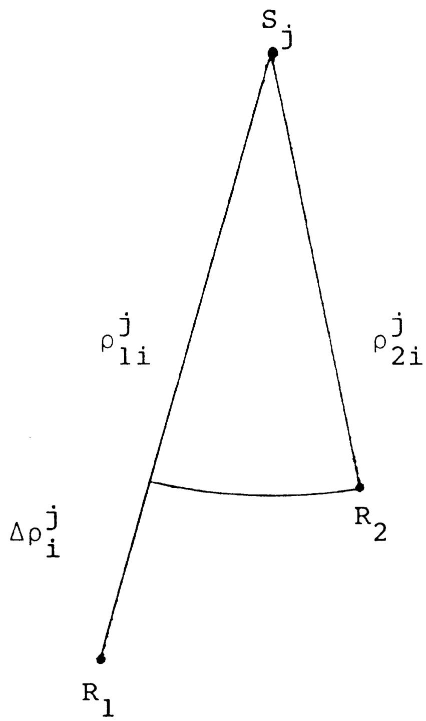
Mo : Morris

Pa : Panmure

Me : Metcalfe

FIGURE 2

THE MACROMETER OBSERVABLE (SIMPLIFIED)



S_j : Satellite at observation time t_i

R_1 , R_2 : Receivers at observation time t_i

ρ^j_{ki} : Distance of satellite j from receiver k at time t_i

$\Delta\rho^j_i = \rho^j_{1i} - \rho^j_{2i}$

N^j : Integer number (ambiguity parameter associated with satellite j)

λ : Wavelength of L_1 - carrier

$\Delta\rho^j_i + N^j \lambda/2$: Actual observable

TABLE 3A

*** INTERFEROMETRIC PHASES (M) ***
MACROMETER OBSERVATION FILE 8
STATIONS: 6A , 7 (30 M)
OBSERVATION DATE: 1983 7 19

	CHANNEL: 1	CHANNEL: 2	CHANNEL: 3	CHANNEL: 4	CHANNEL: 5	CHANNEL: 6
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0000	19454.6946	0.0000	0.0000	0.0000	0.0000
5	0.0000	19452.8690	-6866.7448	0.0000	0.0000	0.0000
6	0.0000	19451.1395	-6868.8989	0.0000	0.0000	0.0000
7	0.0000	19449.4256	-6871.0306	0.0000	0.0000	0.0000
8	0.0000	19447.7423	-6873.1327	0.0000	0.0000	0.0000
9	0.0000	19446.1177	-6875.1797	0.0000	0.0000	0.0000
10	0.0000	19444.6076	-6877.1130	0.0000	0.0000	0.0000
11	0.0000	19443.1234	-6879.0211	0.0000	0.0000	0.0000
12	0.0000	19441.6470	-6880.9233	0.0000	0.0000	0.0000
13	36032.5472	19440.1928	-6882.8050	0.0000	0.0000	0.0000
14	36030.7824	19438.7306	-6884.6988	0.0000	0.0000	0.0000
15	36028.9427	19437.1877	-6886.6684	0.0000	0.0000	0.0000
16	36027.0593	19435.6079	-6888.6768	0.0000	0.0000	0.0000
17	36025.0847	19433.9429	-6890.7701	0.0000	0.0000	0.0000
18	36023.0661	19432.2379	-6892.9121	0.0000	0.0000	0.0000
19	36021.0520	19430.5309	-6895.0494	0.0000	0.0000	0.0000
20	36019.0951	19428.8937	-6897.1172	0.0000	0.0000	0.0000
21	36017.3098	19427.4335	-6899.0123	0.0000	0.0000	0.0000
22	0.0000	19426.0782	-6900.8011	0.0000	0.0000	0.0000
23	36013.9856	19424.7567	-6902.5546	0.0000	0.0000	0.0000
24	36012.3719	19423.4784	-6904.2706	0.0000	0.0000	0.0000
25	36010.7424	19422.1863	-6905.9945	0.0000	0.0000	0.0000
26	36009.1388	19420.9221	-6907.6932	0.0000	0.0000	0.0000
27	36007.5760	19419.7063	-6909.3479	0.0000	0.0000	0.0000
28	36006.0228	19418.5004	-6910.9875	0.0000	0.0000	0.0000
29	36004.4457	19417.2711	-6912.6525	0.0000	263.5378	0.0000
30	36002.8951	19416.0795	-6914.2823	0.0000	262.2448	0.0000
31	36001.4252	19414.9584	-6915.8323	0.0000	261.0321	0.0000
32	35999.8508	19413.7493	-6917.4824	0.0000	0.0000	0.0000
33	35998.1667	19412.4249	-6919.2377	0.0000	0.0000	0.0000
34	35996.3508	19410.9778	-6921.1182	0.0000	256.7587	0.0000
35	35994.5052	19409.5090	-6923.0253	0.0000	255.1883	0.0000
36	35992.6336	19408.0024	-6924.9623	0.0000	253.5985	0.0000
37	35990.7021	19406.4495	-6926.9498	0.0000	251.9589	0.0000
38	35988.7792	19404.9000	-6928.9257	0.0000	250.3227	0.0000
39	35986.8597	19403.3653	-6930.8963	0.0000	248.7034	0.0000
40	35984.9693	19401.8500	-6932.8424	0.0000	247.1011	0.0000
41	35983.1436	19400.4131	-6934.7080	0.0000	245.5802	0.0000
42	35981.4083	19399.0694	-6936.4832	0.0000	244.1548	0.0000
43	35979.7006	19397.7525	-6938.2281	0.0000	242.7636	0.0000
44	35978.0365	19396.4799	-6939.9279	0.0000	241.4093	0.0000
45	35976.4159	19395.2574	-6941.5758	0.0000	240.1118	0.0000
46	35975.0240	19394.2629	-6942.9909	0.0000	239.0465	0.0000
47	35973.7151	19393.3568	-6944.3209	0.0000	238.0667	0.0000
48	35972.4233	19392.4696	-6945.6304	0.0000	237.1091	0.0000
49	35971.1391	19391.5903	-6946.9289	0.0000	236.1642	0.0000
50	35969.7055	19390.5627	-6948.3766	0.0000	235.0668	0.0000
51	35968.2192	19389.4859	-6949.8714	0.0000	233.9254	0.0000
52	35966.7274	19388.4036	-6951.3662	0.0000	232.7839	0.0000
53	35965.1775	19387.2677	-6952.9200	0.0000	231.5860	0.0000
54	35963.6773	19386.1843	-6954.4158	0.0000	230.4477	0.0000
55	35962.1425	19385.0626	-6955.9462	0.0000	229.2715	0.0000
56	35960.5677	19383.9113	-6957.5108	0.0000	228.0628	0.0000
57	35958.9282	19382.6926	-6959.1345	0.0000	226.7963	0.0000
58	35957.2950	19381.4797	-6960.7518	0.0000	225.5336	0.0000
59	35955.5843	19380.1876	-6962.4438	0.0000	224.2004	0.0000
60	35953.8828	19378.9122	-6964.1207	0.0000	222.8791	0.0000

---------------------- MACROMETER OBSERVATION FILE 8 ----------------------

Table 3b

*** INTERFEROMETRIC PHASES (M) ***
MACROMETER OBSERVATION FILE 14
STATIONS: METCALFE , MORRIS (66 KM)
OBSERVATION DATE: 1983 7 22

	CHANNEL: 1	CHANNEL: 2	CHANNEL: 3	CHANNEL: 4	CHANNEL: 5	CHANNEL: 6
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.0000	-321584.3996	-151480.2507	0.0000	0.0000	0.0000
10	0.0000	-324264.0496	-155789.9306	0.0000	0.0000	0.0000
11	0.0000	-327376.3508	-161489.2377	0.0000	0.0000	0.0000
12	0.0000	-330917.5478	-168580.7074	0.0000	0.0000	0.0000
13	0.0000	-334884.5494	-177067.7843	0.0000	0.0000	0.0000
14	0.0000	-339273.9333	-186950.5627	0.0000	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
16	0.0000	-349295.3403	-210890.3735	0.0000	0.0000	0.0000
17	0.0000	-354926.2093	-224954.9952	0.0000	0.0000	0.0000
18	0.0000	-360966.7338	-240414.4832	0.0000	0.0000	0.0000
19	-31175.3692	-367413.9693	-257266.0687	0.0000	0.0000	0.0000
20	-35984.3920	-374265.8165	-275506.7880	0.0000	0.0000	0.0000
21	-41294.2189	-381519.0190	-295131.7072	0.0000	0.0000	0.0000
22	-47111.9496	-389171.7306	-316137.8739	0.0000	0.0000	0.0000
23	-53441.4384	-397221.6756	-338519.6331	0.0000	152283.0142	0.0000
24	-60295.2225	-405667.1112	-362272.4609	0.0000	144440.0905	0.0000
25	-67676.6516	-414508.3290	-387390.9650	0.0000	136175.0187	0.0000
26	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
27	-84044.6454	-433370.5343	-441695.2708	0.0000	118381.5543	0.0000
28	-93042.9095	-443393.1979	-470871.7521	0.0000	108854.9352	0.0000
29	-102590.2305	-453810.1650	-501389.7626	0.0000	98908.6583	0.0000
30	-112691.3417	-464623.1581	-533242.2915	0.0000	88542.8291	0.0000
31	-123349.9526	-475833.2253	-566421.9440	0.0000	77757.5715	0.0000
32	-134568.7811	-487441.9784	-600922.2362	0.0000	66552.2796	0.0000
33	-146349.3499	-499450.5455	-636734.7360	0.0000	54926.7390	0.0000
34	-158694.6074	-511861.4338	-673852.5732	0.0000	42880.3162	0.0000
35	-171604.3054	-524676.5710	-712268.4365	0.0000	30412.1642	0.0000
36	-185079.8890	-537898.8487	-751974.6052	0.0000	17521.2096	0.0000
37	-199119.6435	-551530.8864	-792964.0406	0.0000	4207.8464	0.0000
38	-213724.0774	-565574.8817	-835228.9354	0.0000	-9531.9277	0.0000
39	-228890.4011	-580033.9818	-878763.0584	693706.0607	-23699.0068	0.0000
40	-244616.7978	-594912.7678	-923557.5828	736307.4557	-38294.2218	0.0000
41	-260901.1065	-610212.3311	-969607.7295	780063.8701	-53321.1126	0.0000
42	-277739.9618	-625937.4695	-1016907.7417	824974.1449	-68781.5052	0.0000
43	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
44	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
45	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
46	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
47	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
48	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
49	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
50	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
51	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
52	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
53	3506495.2641	0.0000	0.0000	2463899.9526	1786481.3832	0.0000
54	3503735.4991	0.0000	0.0000	2462037.4988	1785929.8288	0.0000
55	3500939.5681	0.0000	0.0000	2460059.9398	1785346.3658	0.0000
56	3498116.5612	0.0000	0.0000	2457971.8607	1784729.3834	0.0000
57	3495276.3135	0.0000	0.0000	2455779.0905	1784078.0701	0.0000
58	3492428.6451	0.0000	0.0000	2453487.7012	0.0000	0.0000
59	3489583.1362	0.0000	0.0000	2451104.0730	0.0000	0.0000
60	3486748.0176	0.0000	0.0000	2448633.6927	0.0000	0.0000

----------------------- MACROMETER OBSERVATION FILE 14 -----------------------

in Table 3(a) were recorded on the first day of the experiment on the shortest baseline (points 6A and 7, 19 July 1983). Only four satellites were observed during this session. The measurements of Table 3(b) were obtained three days later on the longest baseline, Metcalfe-Morris. Five satellites were observed during this session.

The measurements in each column of Tables 3(a) and 3(b) contain a large bias. This bias is an arbitrary initial phase term which is different for each satellite. Because of the way the Macrometer® works, this constant is actually an integer multiple of $\lambda/2$, where $\lambda \doteq 19$ cm is the wavelength of the L_1 carrier transmitted by the GPS satellites. This constant is usually referred to as the "ambiguity term." If neither receiver loses lock during the observation period, there is only one such ambiguity per satellite. Such was the case for the session of Table 3(a). If there are breaks in the data, two or more different ambiguities per satellite may result. Such breaks are clearly evident in the data of Table 3(b).

Several observation equations for these kinds of measurements have been published (e.g., Davidson et al. [1983]; Goad and Remondi [1983]). One explicit formulation is that of Bauersima [1983]. Equation (1) below is basically his equation (38) somewhat simplified and specifically modified to account for the behaviour of the Macrometer® V-1000 receivers.

$$\Delta\rho_i^j + (c-\dot{\rho}_{1i}^j)\ \Delta t_i + d(d\rho^j)_{ion} + d(d\rho^j)_{trop} + \frac{\lambda}{2} N^j - \Delta\rho_i^{j'} = v_i^j \quad (1)$$

$$i = 1,2,\ldots,n_b$$

$$j = 1,2,\ldots,n_s$$

where

c is the speed of light;

λ is the nominal wavelength of the L_1 carrier;

n_s is the number of satellites;

n_b is the number of observation times;

t_i, $i=1,2,\ldots,n_b$ are the observation times (UTC);

$\rho^j_{ki} = \rho^j_k(t_i)$, $k=1,2$ is the distance of satellite j at time $t_i-\rho^j_{ki}/c$ to receiver k at time t_i;

$\dot{\rho}^j_{ki}$ is the range rate at time t_i;

$\Delta\rho^j_i = \rho^j_{1i} - \rho^j_{2i}$;

N^j are integer numbers;

$(d\rho^j_k)_{ion}$, $k=1,2$ is the ionospheric refraction correction to phase observation of satellite j as observed from receiver k;

$(d\rho^j_k)_{trop}$, $k=1,2$ is the tropospheric refraction correction;

$d(d\rho^j)_{ion} = (d\rho^j_1)_{ion} - (d\rho^j_2)_{ion}$;

$d(d\rho^j)_{trop} = (d\rho^j_1)_{trop} - (d\rho^j_2)_{trop}$;

Δt_i is the clock synchronization error of receiver clock 2 with respect to receiver clock 1;

$\Delta\rho^{j'}_i$ is the recorded phase difference measurement;

v^j_i is the residual in range difference $\Delta\rho^j_i$.

In eqn. (1) we usually express distances and phases in metres, times in seconds, and velocities in metres per second.

3. GOALS OF THE ANALYSIS

The most important motive for this analysis was the development of a software package for analysing Macrometer® data independent of Macrometrics' programs. This was possible only to some extent as, so far, we have had no access to the raw measurements recorded by the individual receivers. We therefore were not in a position to develop any software for computing the interferometric distance difference from the single receiver data.* Rather, we accepted the output of Macrometrics' INTRFT program as our observations.

Apart from this point, the programs developed at UNB are completely independent of Macrometrics' software.

The other goals of this analysis may be summarized as follows:

- Check of the results obtained by the Earth Physics Branch using Macrometrics' software.
- Comparison of the quality of our solutions with Macrometrics' solutions.
- Independent estimation of the influence of modelling errors (ionospheric, orbital errors) on baseline estimations.
- Development of an improved processing mode for high-precision surveys by the following techniques.

Instead of producing one set of coordinates for each observation session held on the same baseline and then computing the final results by averaging these partial results (the technique used with Macrometrics' software), our programs may process all observations on the same baseline in one program run. Apart from obvious advantages from the theoretical point of view, this kind of processing greatly facilitates the resolution of the so-called ambiguity problem.

The data may be processed in a "network mode" by combining all observations on all baselines in one program run. This program option makes sense for the long baselines of the Ottawa campaign, although only two receivers were operating simultaneously, because all the sides and diagonals of the quadrilateral, 6A-Morris-Panmure-Metcalfe had been

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* This step should be a simple differencing of the phases from the individual receivers.

measured. Naturally this option will be of even greater importance for campaigns with more than two simultaneously operating receivers.

4. MACROMETER® DATA PROCESSING

The processing of Macrometer® data at UNB may be logically divided into (a) a preprocessing step, and (b) a parameter estimation step. The goal of preprocessing is the detection and, if possible, the removal of data breaks within the observation series pertaining to one satellite (columns in Tables 3(a), 3(b)) using very simple mathematical tools. The main goal of the parameter estimation step, of course, consists of the estimation of the relative position of one receiver with respect to the other using purely physical models.

4.1 Data Preprocessing

In the first step, single difference preprocessing, we analyse the values

$$\varepsilon_i^j = \Delta\rho_i^{j'} - \Delta\rho_i^{jo}, \; i=1,2,\ldots,n_b \quad , \tag{2}$$

separately for each satellite j, where

$\Delta\rho_i^{j'}$ are the actual (interferometric) measurements (single differences);

$\Delta\rho_i^{jo}$ are the theoretical values for the distance differences $\Delta\rho_i^j$ using approximated orbits and station coordinates.

We now distinguish two cases (where actually the first is a special case of the second):

(a) There are no breaks in the data: we fit the ε_i^j (separately for each satellite) by low degree algebraic polynomials (the degree q typically chosen was between 4 and 6).

$$P(t) = \sum_{k=0}^{q} p_k \, t^k \quad . \tag{3}$$

A typical example of the residuals after a polynomial fit is given in Figure 3(a) (see discussion below).

(b) There are n_{br} breaks in the data of satellite j: the total observation period is divided into n_{br} break-free subintervals I_k, $k=1,2,\ldots,n_{br}$ (see below). The ε_i^j are now fitted by the following piecewise continuous function:

$$\tilde{p}(t) = \tilde{p}_{o\ell} + \sum_{k=1}^{q} \tilde{p}_{jk} \, t^k , \; t \in I_\ell \; ; \; \ell=1,2,\ldots,n_{br} \quad . \tag{4}$$

This first step is followed by the <u>double difference preprocessing</u>. Here we analyse the so-called double differences, $\Delta\rho_i^{j'} - \Delta\rho_i^{\ell'}$, forming the values

$$E_i^{j\ell} = (\Delta\rho_i^{j'} - \Delta\rho_i^{jo}) - (\Delta\rho_i^{\ell} - \Delta\rho_i^{\ell o}), \; \ell \neq j, \; i=1,2,\ldots,n_b \quad . \tag{5}$$

The method applied is then identical with the method for single differences. A typical example for the residuals after a polynomial fit to double differences is given in Figure 3(b).

If we compare the residuals of a single difference analysis (Figure 3(a)) with those of a double difference analysis (Figure 3(b)) for the same observation period, we note some remarkable dissimilarities. First, we have a difference in the scale of the residuals (roughly a factor of 100). Second, the distribution of the residuals is clearly systematic for single differences but more or less random for the double differences.

The reason for this diversity becomes obvious if we compare the observation equations (1) for single differences with those for double differences (difference of two equations of the form of eqn. (1) for the

FIGURE 3A

SINGLE DIFFERENCES

DATE : 1983 JULY 19 STATIONS : 6A - 7

FILE : MAC08 CHANNEL : 2 (NAVSTAR 3)

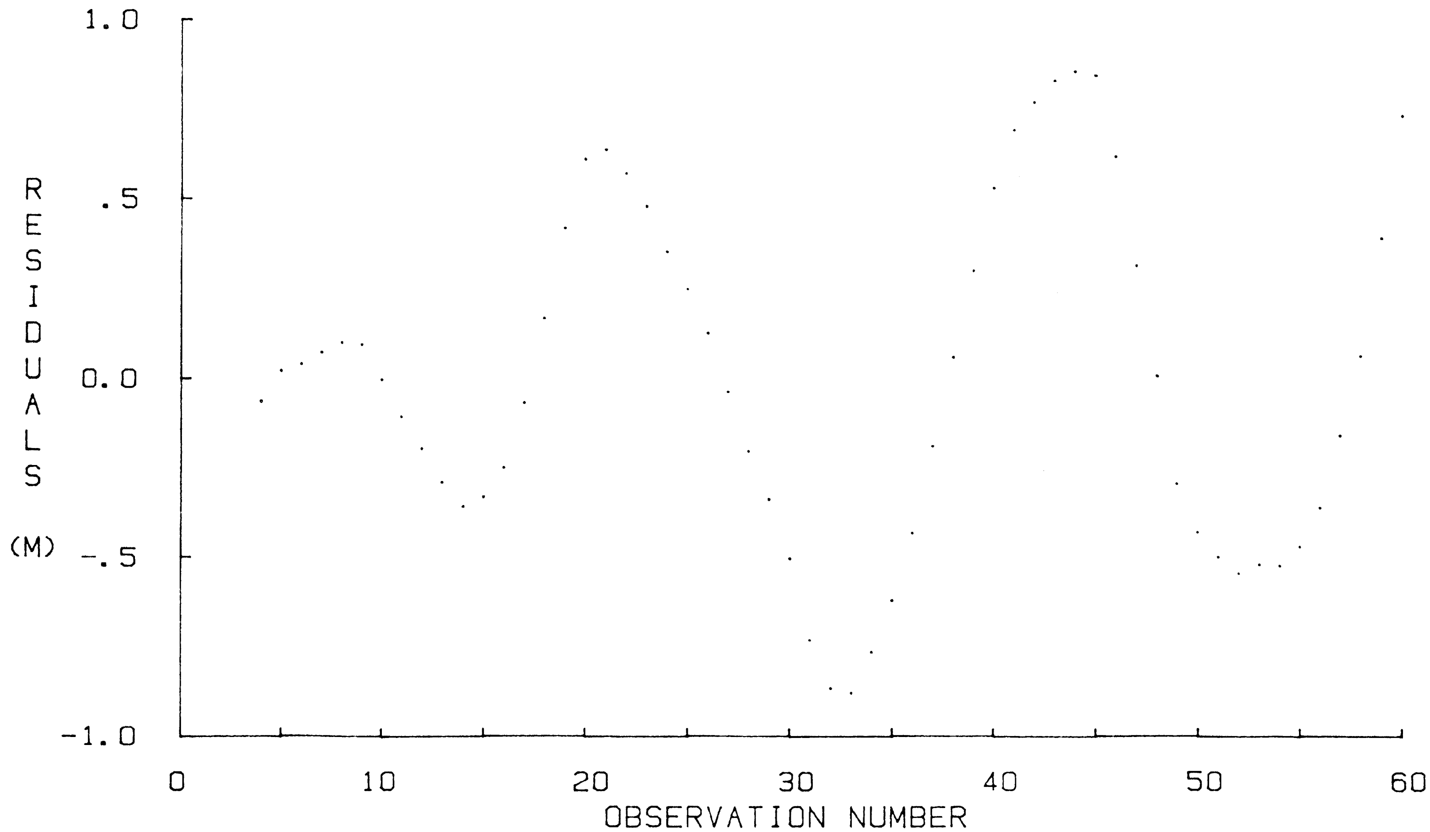
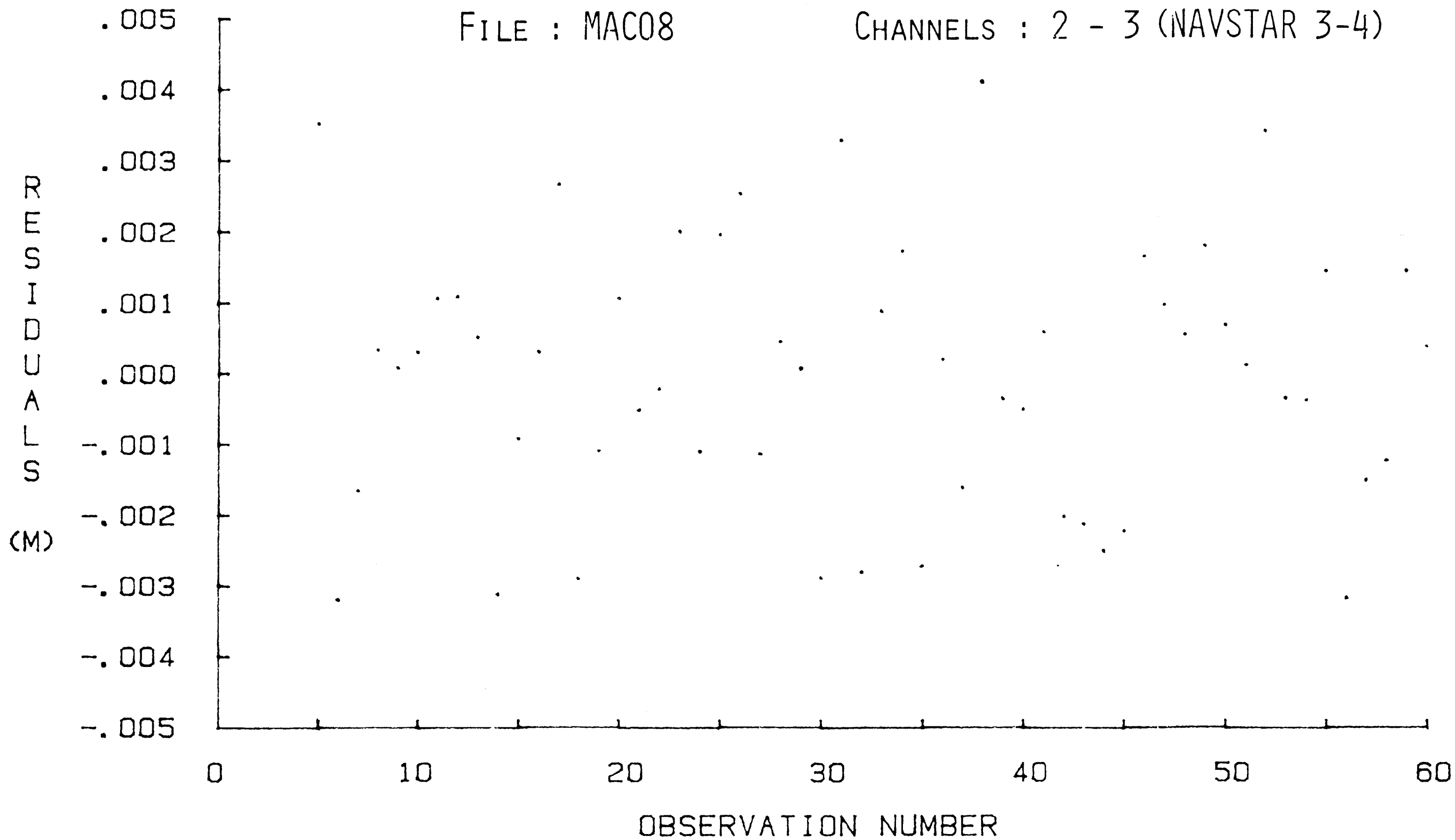


FIGURE 3B

DOUBLE DIFFERENCES

DATE : 1983 JULY 19   STATIONS : 6A - 7

FILE : MAC08   CHANNELS : 2 - 3 (NAVSTAR 3-4)



same subscript i but two different superscripts j, k):

$$(\Delta\rho_i^j-\Delta\rho_i^k)-(\dot{\rho}_{1i}^j - \dot{\rho}_{1i}^k)\Delta t_i+d(d\rho^j)_{ion}-d(d\rho^k)_{ion}$$

$$+d(d\rho^j)_{trop}-d(d\rho^k)_{trop}+\frac{\lambda}{2}(N^j-N^k)-(\Delta\rho_i^{j'} - \Delta\rho_i^{k'}) = w_i^{jk} \qquad (6)$$

$$i = 1,2,\ldots,n_b$$

$$j,k = 1,2,\ldots,n_s,\ k\neq j.$$

In eqn. (6) the contribution of the clock synchronization error Δt_i has been reduced roughly by a factor of $5\cdot 10^6$ as compared to eqn. (1) due to the elimination of the term c Δt_i. The systematic variation in Figure 3(a), therefore, reflects the (nonpolynomial) errors characteristic of the crystal-oscillator controlled clocks in the Macrometer® V-1000.

So far, we have assumed that the divisions of an observation period into break-free subintervals for each satellite j are known a priori. This, however, is not the case. Such breaks are often not detected at the time of observation, and a close examination of the data is necessary to detect them. Although completely automatic "break-detection and -removal software" could be developed, we opted for a semi-automatic, interactive preprocessing program (similar to that available with the Macrometrics software) using a computer graphics package and the mathematical tools given in the preceding paragraphs. A brief description of the use of this program follows.

First, it is assumed by default that a new subinterval begins if one or more zeroes are encountered in the observation series of one satellite. An adjustment using eqn. (4) is performed, and the residuals are displayed on a graphics terminal. The operator has the ability to redefine the interval boundaries and to reject outliers. This process may be repeated until a satisfactory solution is found.

Essentially the same procedure is repeated with the double differences. At this stage it is usually easy to remove any data breaks present. Normally, the estimated values (see eqn.(4))

$$M_{\ell k} = (\tilde{p}_{o\ell} - \tilde{p}_{o1})/(\lambda/2), \quad \ell = 2,3,\ldots,n_{br} \tag{7}$$

are close to integer numbers. If, moreover, the estimated r.m.s. errors of the $M_{\ell k}$ are much smaller than 1, it is safe to remove the data breaks by correcting the double difference observations in the subintervals $2,3,\ldots,n_{br}$ by adding to them the values $M_{\ell k}\cdot(\lambda/2)$. Processing the corrected data without interval subdivision shows the success or the failure of the process. Actually, the half-cycle slips found in the double differences are applied to one of the two single differences forming the particular double difference series. Which of the single differences has to be corrected follows by analysing, in principle, all possible double differences.

This preprocessing is done on the HP-1000 computer of the Department of Surveying Engineering at UNB. Program DPLOT [Davidson, 1984] was already available on the HP-1000 enabling interactive plotting of ASCII data files on an HP-7470 plotter or a Cybernex 1012 graphics terminal. Program DPLOT was modified, renamed GPSPL, and used to allow interactive preprocessing of the GPS data. Adaptations were made to directly produce the plots without as much operator intervention as would be required with DPLOT. The ability to create plots on the screen for interactive processing, or on paper as a permanent record, was particularly attractive. Figures 3(a) and 3(b) were generated on the HP-7470 plotter.

One would expect that this non-parametric preprocessing technique might fail if the $\Delta\rho_i^{jo}$ are only very poor approximations of the "true" distance differences $\Delta\rho_i^j$. Tests have shown that, even for the longest baselines analysed, station offsets of up to 500 m and orbital errors of the order of some kilometres did not compromise this approach.

4.2 Parameter Estimation

Two parameter estimation programs, PRMAC3 and PRMNET, have been developed. The former is used for single baseline estimations, and the latter for network analyses wherein observations on many baselines may be processed simultaneously.

The programs in their present form may be used to estimate (almost any combination of) the following types of parameters:

(a) Receiver coordinates in the conventional terrestrial system.

(b) Ambiguity parameters as defined by eqns. (10), below.

(c) Clock synchronization parameters c_o, c_1 (offset and drift) from the following model:

$$\Delta t_i = c_o + c_1(t_i - t_1),\ i=1,2,\ldots,n_b \quad . \tag{8}$$

(d) A maximum of six orbital parameters per satellite arc.

By choosing different combinations of parameters for estimation, quite different problems may be treated.

(a) If no orbital parameters are estimated, the programs actually reduce to pure (relative) positioning procedures.

(b) Assuming all receiver positions are known and estimating all the orbital parameters, the programs may be used as pure orbit determination programs.

It should be mentioned, however, that option (b) does not make sense for the data analysed here (too few receivers too close to each other and only one frequency observed).

The modelling of receiver coordinates is straightforward. The estimation of the other parameter types, however, is more complex, and some discussion of our philosophy in handling these parameters is required.

Figure 3(a) clearly shows that the use of eqns. (1) as observation equations implies sophisticated models for the clock performances. In the authors' opinion, the best way of modelling is the following: define a statistical model of the clock performances using available information on clock offset, drift, and jitter. This leads to a simple stochastic differential equation for the phase differences of the two receiver clocks or, even more directly, to an equation for the clock synchronization error as a function of time. The Δt_i in eqns. (1) may then be interpreted as the solution of this stochastic equation at the observation times t_i. Of course, this approach complicates matters considerably. Instead of more or less standard least-squares solutions, one would have to apply methods of "optimal filtering" or "optimal smoothing." Although this approach is advantageous from a theoretical point of view, its application would have required a considerable investment of time which was not available. Nevertheless, this technique should be kept in mind for future studies.

The next best approach to follow is to deny all functional models for the errors Δt_i, and to introduce them as unknowns into a least-squares adjustment. Although there are no objections from the theoretical point of view, there is a strong objection from the practical point of view: the number of unknowns tends to increase dramatically. One gets into the problem of manipulations with large matrices, which cause a significant increase in computation time and the use of large storage areas.

An alternative approach to those already mentioned is to implicitly eliminate the clock synchronization term by using the double difference equations (6) as observation equations. This approach--which was followed by Macrometrics [1983], Goad and Remondi [1983], and by ourselves--has some important advantages from the practical point of view. Two major implications should be mentioned:

(a) We are no longer in a position to solve for all the ambiguity parameters N^j, $j=1,2,\ldots,n_s$. This clearly follows from the fact that only the differences

$$N^{jk} = N^j - N^k \qquad j \neq k,\ j,k=1,2,\ldots,n_s \tag{9}$$

figure in eqns. (6). Of all the values in eqn. (9), only n_s-1 are actually independent, and so only one value of $k=k_o$ need be selected:

$$N^{jk_o} = N^j - N^{k_o},\ j \neq k_o,\ j=1,2,\ldots,n_s, \qquad k_o \in \{1,2,\ldots,n_s\} \quad . \tag{10}$$

It is an intrinsic feature of the double difference approach that we are free to choose k_o.

(b) As a double difference is formed with two single difference observations for the same time t_i, the double differences for different satellites for the same time t_i, are actually correlated.

In our software, this mathematical correlation may be included or neglected. The precise formulation of this problem will be presented in Part C of this report.

If we want to model each satellite arc with, at most, six physically meaningful parameters, we have to represent all satellite arcs as solutions of known ordinary differential equations. The unknown orbital parameters are essentially the initial conditions (actually we chose osculating Keplerian elements as unknowns). Further details of our approach to orbit modelling and estimation will also be dealt with in Part B of this report.

The orbits we used for the present analysis were derived from the so-called Macrometrics T-files using methods of numerical integration. Our approximation of the T-files was typically better than ± 5 m, which means that, for the present analysis, the T-files and our orbits may be considered to be identical. No serious attempt was made to improve the orbits.

The tropospheric refraction is represented by the Saastamoinen model [1971] using the following standard surface values for the entire observation period for all sites: temperature = 291° Kelvin, atmospheric pressure = 1013.2 mbar, water vapour pressure = 10 mbar.

For the ionosphere we used two approaches. In the first approach we completely neglected the ionosphere. In the second, we used the model developed by Geckle and Feen [1982] to correct single frequency Transit observations for the daytime GPS observations. The daily solar fluxes required for the model were obtained from the Herzberg Institute of Astrophysics [1983]. For nighttime observations, an exponential decrease of the vertical electron content was assumed with the initial value at "sunset" fixed at the value predicted by the daytime model. A reduction of electron content by a factor of 10 over 12 hours was assumed.

In the first part of the parameter estimation programs, all parameters selected as unknowns, including the ambiguity parameters, are estimated by the standard least-squares technique. The values of these parameters and their corresponding estimated r.m.s. errors are printed.

In the second part of the program, an attempt is made to resolve the so-called ambiguity problem by assigning known <u>integer</u> values to the ambiguity parameters. Some of the different strategies that may be used to solve this problem are given in Langley et al. [1984].

We have found that for short baselines (shorter than 20 km), it is quite easy to determine the ambiguity if two or more observation periods are combined into the same program run. As a matter of fact, the simplest strategy, that of rounding the non-integer ambiguities estimated in the first part of the program to the nearest integers, is good enough in all such cases. For longer baselines, the ambiguity problem is more complicated due to the corrupting effect of the ionosphere on single frequency observations. Obviously there is a region where it is questionable or even subjective whether an attempt should be made to solve this problem. We accepted the solutions of the second part of the program for baseline lengths up to 22 kilometres. For the longer baselines, no attempt was made to fix the ambiguity parameters at integer values.

5. RESULTS

The decision as to what to include in this section was not a simple one. Many different program options produce a variety of slightly different results, and it is often difficult to say which result is the best. The final selection was based on the following considerations:

(a) We wanted to include a useful comparison with the results obtained using Macrometrics' software.

(b) We wanted to discuss the influence of the ionosphere.

(c) We decided to minimize the number of program runs by <u>not</u> varying the orbits (see Part B of this report) and by <u>not</u> estimating the clock synchronization errors.

As the baselines analysed are relatively short and as the clock synchronization was supervised very carefully during the Ottawa compaign, the two simplifications (c) are fully justified. Some experiences with the estimation of orbit parameters will be presented in Part B of this report.

5.1 Comparison with the Results Using Macrometrics' Software

In order to obtain a meaningful comparison we had to neglect the ionospheric refraction completely (as was done in Macrometrics' software), and we had to use the orthometric heights in Table 1 as ellipsoidal heights referred to Clarke's 1866 ellipsoid (as was assumed in the processing with Macrometrics' software [Valliant, 1984]). As the difference between ellipsoidal and orthometric heights are fortuitously only of the order of 1 to 2 metres in the test area, this neglect of geoidal height is harmless for the analysis of the present compaign. It should be stated, however, that the errors introduced into the baseline estimates by incorrect geocentric coordinates of the fixed station(s) are of the same order of magnitude as those introduced by orbital errors: a bias of up to 0.4 ppm in the baseline may result from an error of 10 m in the orbit or in the position of the fixed station.

The differences between our solution and that obtained using the Macrometrics software, for latitude ϕ, longitude λ, height h, for station pairs, and for length ℓ of the baselines, are given in Table 4. The absolute values of the vectorial differences and the differences in the lengths of the baselines between the two solutions as a function of the length of the baseline are given in Figures 4(a) and 4(b) respectively.

If we take into account that there were differences in the data editing and that we used the first halves of three sessions (two on baseline 3, one on baseline 4) completely rejected by Valliant [1984, Table 2, days 219, 224, 225], the agreement between the two solutions is quite good. This is particularly true for the lengths of the baselines for which

TABLE 4

Comparison of UNB and EPB Solutions.

PRMAC - 3 minus Macrometrics Solution					
No.	Baseline	$\Delta\phi$ (cm)	$\Delta\lambda$ (cm)	Δh (cm)	$\Delta\ell$ (cm)
1	6A - 7 (30 m)	-0.2	0.0	0.3	0.1
2	6A - 51 (2 km)	0.4	- 0.2	-0.3	- 0.1
3	Pa - Mo (13 km)	0.5	- 0.6	2.3	0.8
4	Pa - 6A (22 km)	-0.2	1.0	1.1	1.0
5	Mo - 6A (27 km)	-0.9	- 1.5	-0.1	- 1.5
6	Me - 6A (40 km)	2.8	0.1	7.2	1.2
7	Me - Pa (58 km)	-9.0	13.7	0.3	-16.3
8	Me - Mo (66 km)	3.2	- 4.2	5.2	5.0

Pa : Panmure; Mo : Morris; Me : Metcalfe.

ϕ, λ, h : Ellipsoidal coordinates

ℓ : Length of baseline

For baselines 1 to 4, the solution of part 2 (ambiguities resolved) of program PRMAC-3 was accepted as the final solution. For baselines 5 to 8, the solution of part 1 (without ambiguity resolution) was selected.

FIGURE 4A

LENGTH OF THE DIFFERENCE VECTOR BETWEEN OUR SOLUTION AND THAT USING MACROMETRICS' SOFTWARE

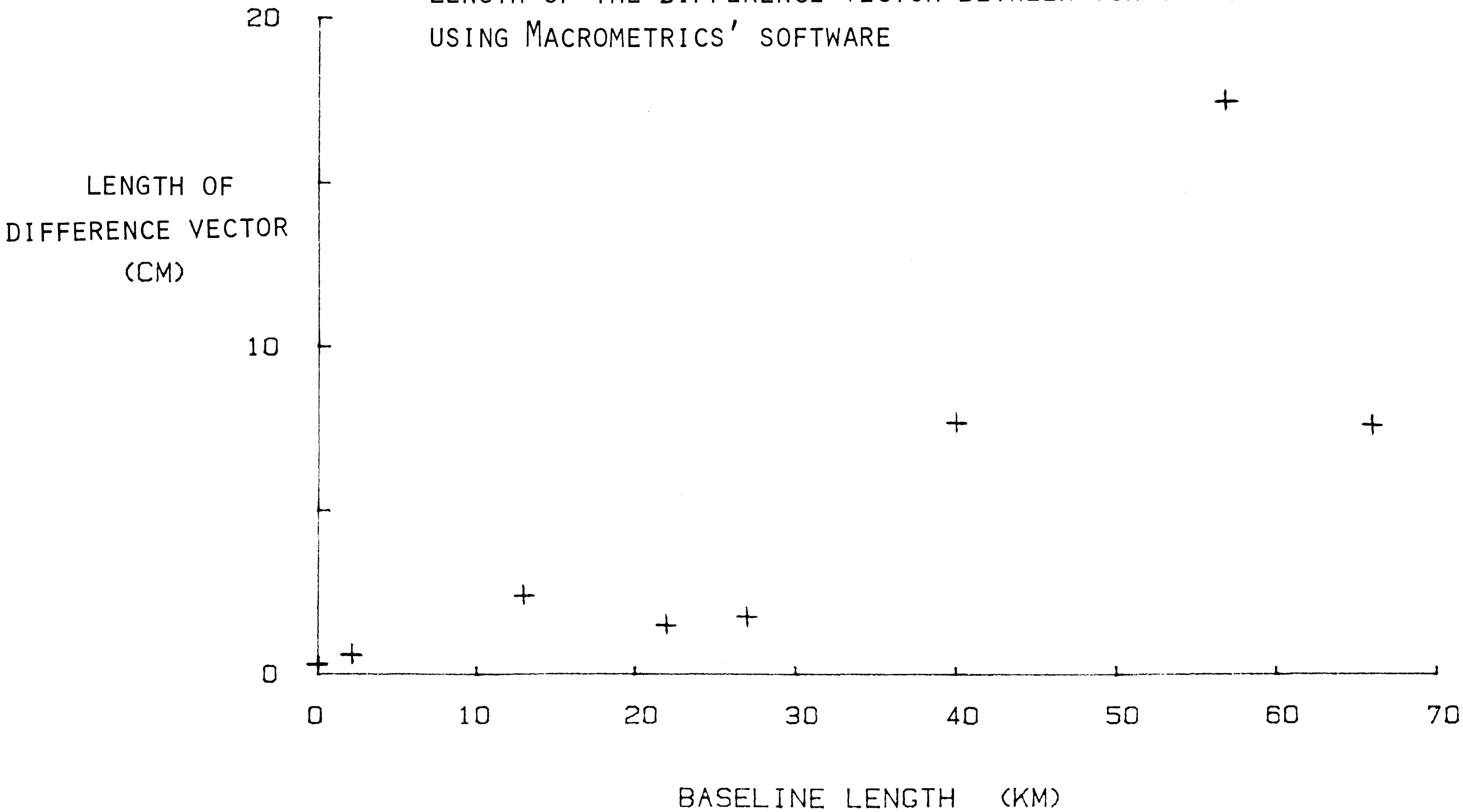
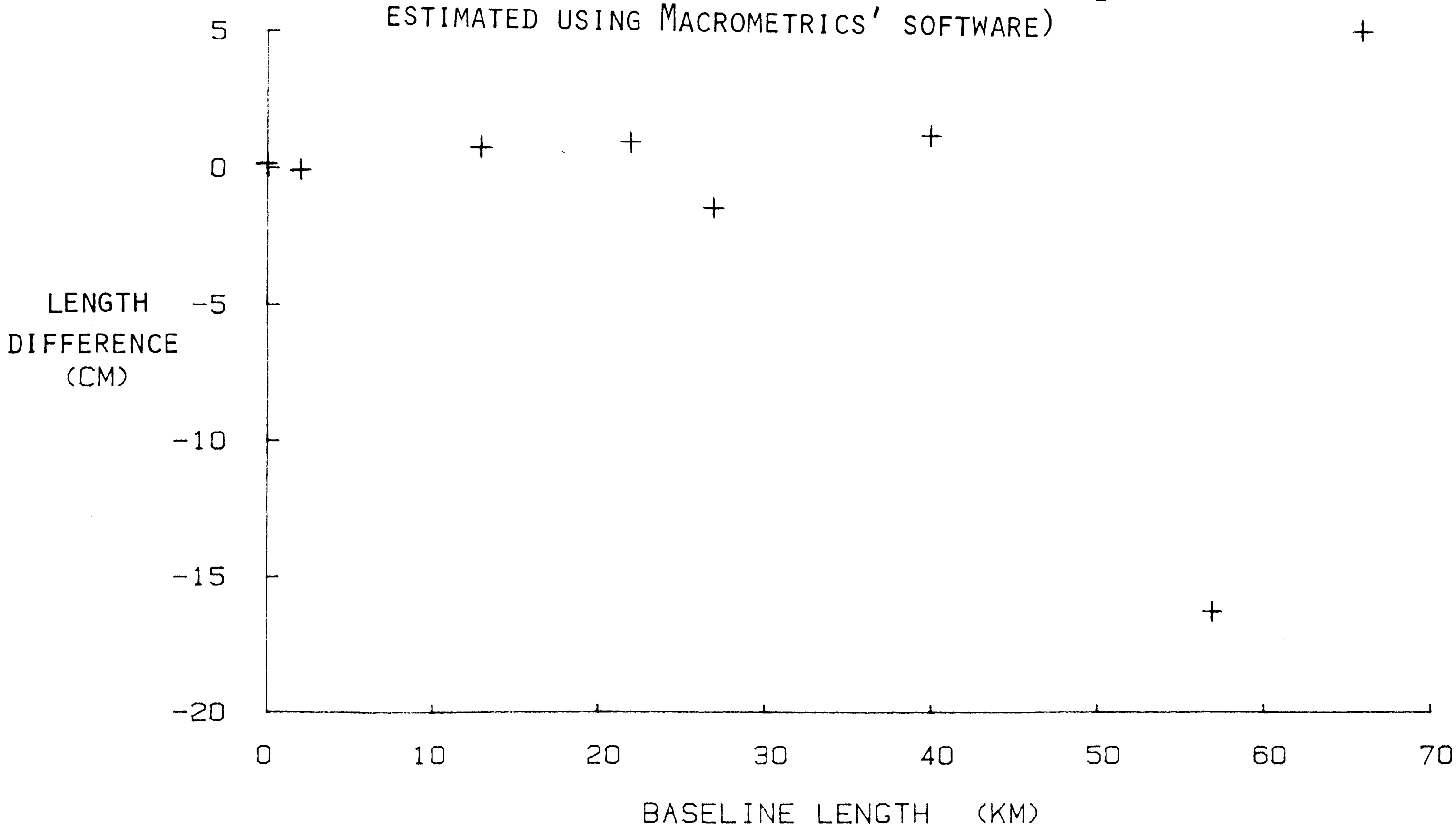


FIGURE 4B

DIFFERENCES $\ell_1 - \ell_2$ IN THE LENGTHS OF THE ESTIMATED BASELINES (ℓ_1: ESTIMATE FROM OUR PROGRAM PRMAC-3, ℓ_2: BASELINE LENGTHS ESTIMATED USING MACROMETRICS' SOFTWARE)



we have agreement within 1.5 cm for baselines up to 40 km in length. The differences for the 58 km baseline, Panmure-Metcalfe, are significantly larger than those for the other baselines. We believe the source of this anomaly lies in the editing of the data by EPB. One of the EPB solutions for this baseline differs in height with respect to the other solutions by about 25 centimetres. This solution has a significant effect on the mean positions with which we have compared our solutions.

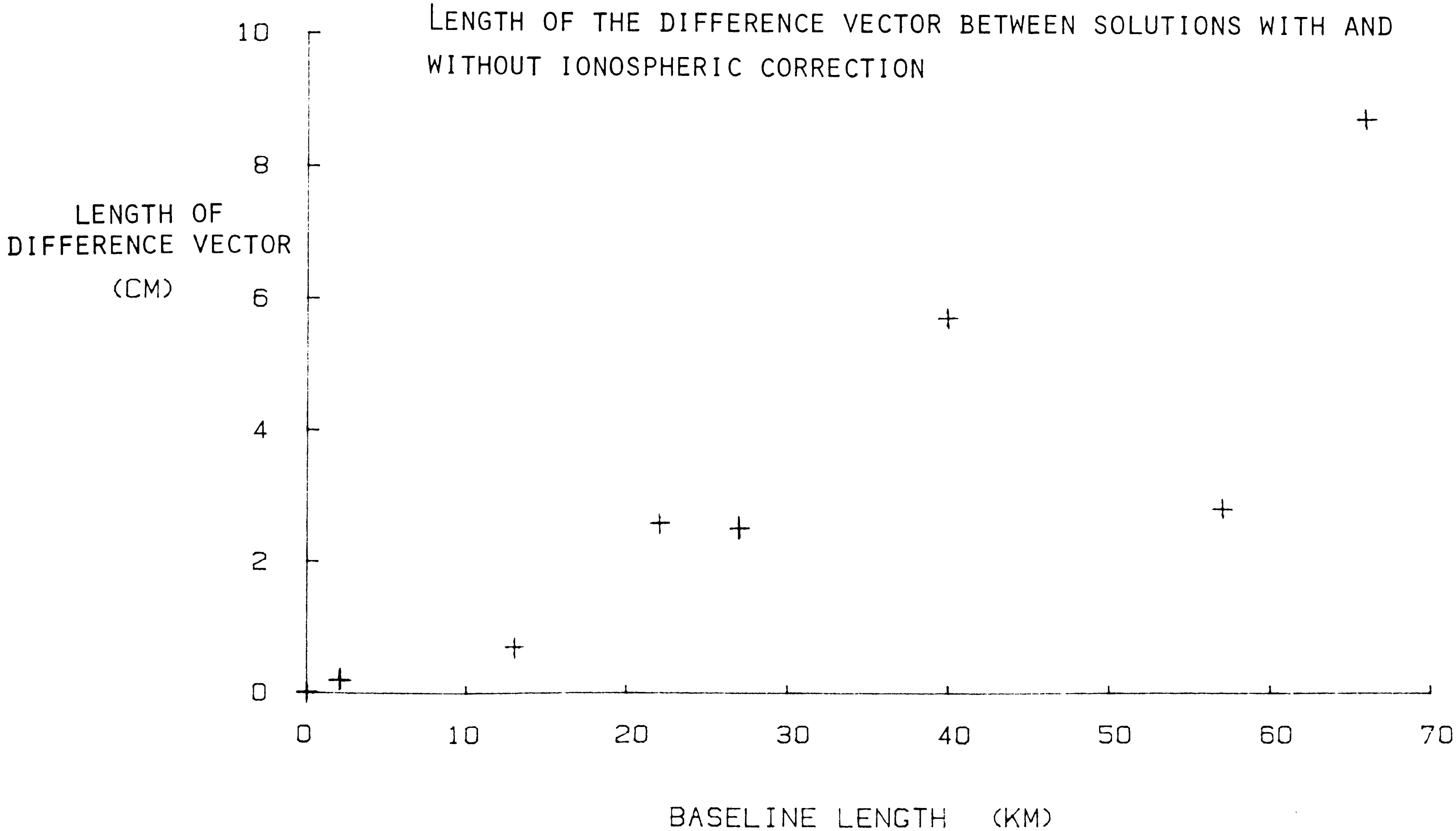
We may conclude at this stage that there appear to be no gross errors in either the Macrometrics or UNB software packages. A further test, where we use identical observations and orbits as those used with the Macrometrics software, may elucidate the residual discrepancies. That there are relatively large differences between the two solutions for the longer baselines is not surprising. Here we found it very difficult to do objective data editing. We believe that an improvement in preprocessing is possible, provided we have access to the raw field data.

5.2 Ionospheric Refraction

Ionospheric refraction certainly is the most severe limiting factor for any single frequency receiver. We include Figure 5, therefore, giving the absolute values of the vectorial position differences between the solutions using the ionospheric model given in section 4.2 and the solution neglecting the ionosphere completely. As the ionospheric model used does not account for any local variations, we cannot hope to model the real ionosphere accurately. We may, however, estimate from Figure 5 the order of magnitude of the mean effect. This figure indicates that improper modelling of the ionosphere may induce baseline errors ranging from 0.7 cm for the 13 km baseline up to 8.7 cm for the 66 km baseline.

FIGURE 5

LENGTH OF THE DIFFERENCE VECTOR BETWEEN SOLUTIONS WITH AND WITHOUT IONOSPHERIC CORRECTION



During the Ottawa campaign, we operated a dual-frequency Transit satellite receiver in the vicinity of point 6A. The Doppler measurements obtained may be used to estimate directly the vertical electron content below the satellite during the Transit passes. Unfortunately, due to hardware difficulties, only a few Transit passes were recorded during the GPS observation sessions. A separate report will deal with the baseline results obtained using this potentially more accurate ionospheric modelling.

5.3 Quality of Baseline Solutions

The formal accuracy of the solutions is given by the a posteriori estimates of the r.m.s. errors of the parameters. We give these values in Figures 6(a), 6(b), 6(c) for latitude ϕ, for longitude λ, and for the ellipsoidal height h as a function of the length of the baseline.

The estimates of the formal precision are incredibly small. We should bear in mind, however, that they are based on presumptions that are not satisfied here. We know, for example, that our modelling of the ionosphere is far from being perfect, and we have already seen in Figure 5 that solution differences of the order of 10 times the r.m.s. estimates in Figures 6(a), 6(b), and 6(c) may occur, if we modify the ionospheric modelling for the longer baselines. Figures 6 show, however, what precision may be expected from interferometric phase measurements, if the refraction effects are under control (for dual frequency receivers). From Figures 6 we also may conclude that, even without resolving the ambiguities, we may expect excellent results, at least if we process all observation sessions in the same program run.

A further, and more realistic, check of different baseline solutions

FIGURE 6A

FORMAL RMS-ERROR σ_ϕ IN LATITUDE ϕ

"+" IS USED FOR VALUES BEFORE , "×" FOR VALUES AFTER RESOLVING THE AMBIGUITY PROBLEM

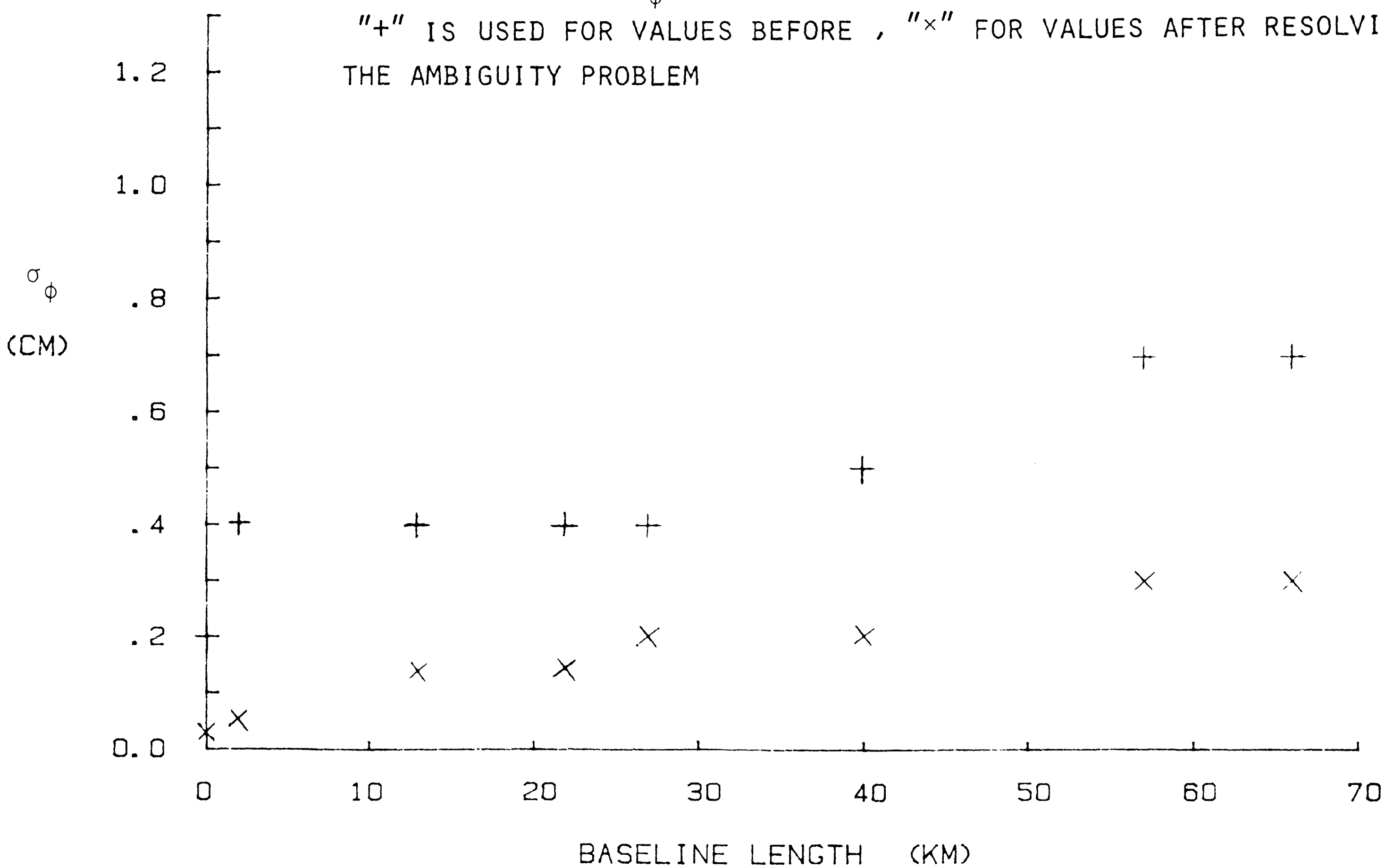


FIGURE 6B

FORMAL RMS-ERRORS σ_λ IN LONGITUDE λ
"+" IS USED FOR VALUES BEFORE, "×" FOR VALUES AFTER RESOLVING THE AMBIGUITY PROBLEM

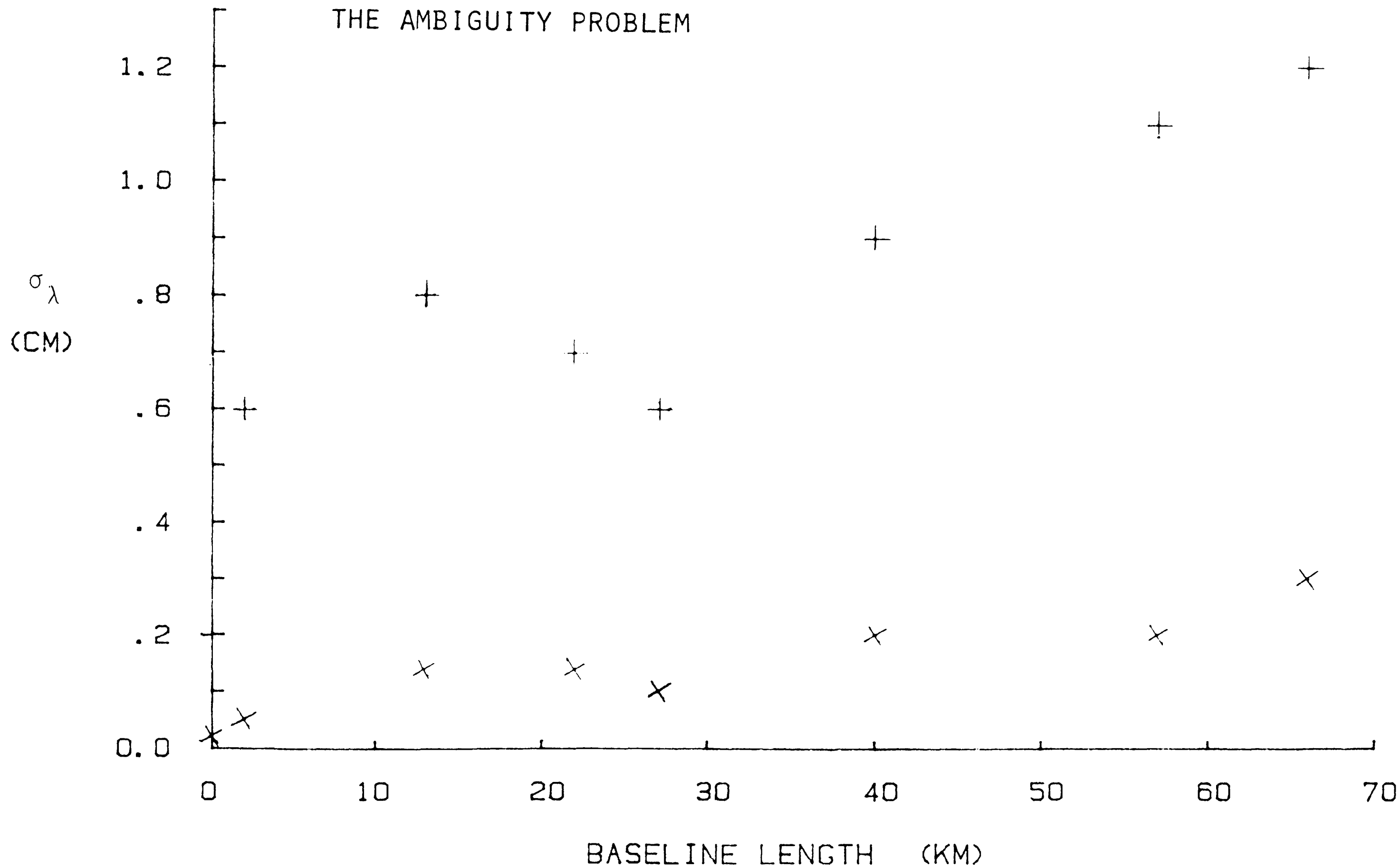
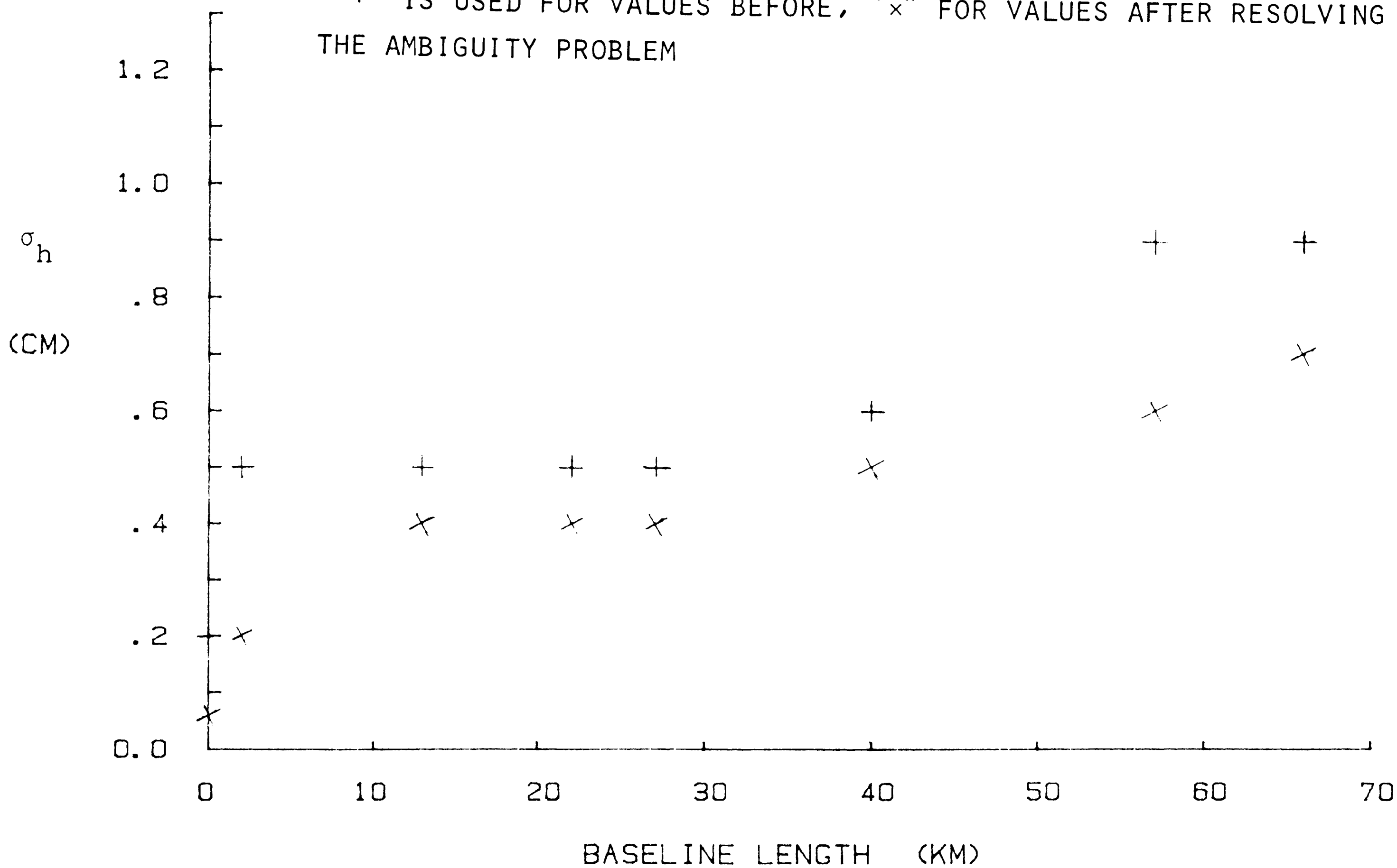


FIGURE 6C

FORMAL RMS-ERROR σ_h IN HEIGHT h
"+" IS USED FOR VALUES BEFORE, "×" FOR VALUES AFTER RESOLVING THE AMBIGUITY PROBLEM



(EPB's and our solutions without and with ionospheric models) may be found in Table 5, where we give the absolute values of the misclosure vectors resulting from the different solutions for all triangles and for the contour quadrilateral in Figure 1. First we see that the misclosures are all of the same order of magnitude (ranging from 0.33 ppm to 1.38 ppm). We see, moreover, that our misclosures are more favourable for all figures except those including the longest baseline, Morris-Metcalfe.

It is somewhat disappointing to see that our solutions including a simple ionospheric model produce systematically larger misclosures than the solutions neglecting the ionosphere completely. One possible explanation is that spatial and temporal variations of the ionosphere lead to significantly different effects than those predicted by our smoothly varying model. This, of course, is an argument favouring Macrometrics' approach to this problem, which is to ignore the ionosphere completely on such short baselines.

5.4 Network Solutions

We feel that processing all observations on all baselines of Figure 1 from all observing sessions in one program run and computing one set of coordinates for Morris, Panmure, and Mecalfe with respect to point 6A, is the best possible way of analysing the data. As in this processing mode the closure of the triangles of Figure 1 is enforced, we checked the stability of the solutions by varying the ionospheric model.

Table 6 gives the result of this network analysis. For solution A we used no ionospheric modelling; for solution B, the augmented Geckle and Feen [1982] model was used. It is worth noting that the influence of different ionospheric modelling on the receiver coordinates is greatly

TABLE 5

Figure Misclosures.

Figure Contours	Length	Macrometrics		PRAMC - 3 Without Ionosphere		PRAMC - 3 With Ionosphere	
		cm	ppm	cm	ppm	cm	ppm
6A - Mo - Pa	62 km	5.6	0.93	2.5	0.42	4.2	0.68
6A - Pa - Me	119 km	13.7	1.15	3.9	0.33	9.7	0.82
6A - Mo - Me	133 km	10.7	0.81	11.4	0.86	11.6	0.87
Mo - Pa - Me	136 km	11.1	0.81	13.5	0.99	18.9	1.38
6A - Mo - Pa - Me	137 km	15.8	1.15	6.2	0.45	13.8	1.00

TABLE 6

Network Solution for Quadrilateral
6A - Morris - Panmure - Metcalfe
Solution A: No ionosphere.
Solution B: Ionospheric model given in section 4.2.

Solution	Station	ϕ	λ	h (m)
	6A (fixed)	45°23'55".79598	-75°55'21".44516	78.754
A	Morris	45°26'34".29501±".00013	-76°15'18".81750±".00034	90.965±.005
	Panmure	45°20'18".81718±".00013	-76°11'4".58665±".00038	155.371±.005
	Metcalfe	45°14'34".01436±".00013	-75°27'31".48094±".00034	104.948±.005
B	Morris	45°26'34".29524±".00013	-76°15'18".81871±".00034	90.959±.005
	Panmure	45°20'18".81648±".00013	-76°11'4".58667±".00038	155.378±.005
	Metcalfe	45°14'34".01346±".00013	-75°27'31".47919±".00034	104.965±.005

TABLE 7

Network Solution for Triangle
6A - Morris - Panmure
Solution C: No ionosphere.
Solution D: Ionospheric model given in section 4.2

Solution	Station	ϕ	λ	h (m)
	6A (fixed)	45°23'55".79598	-75°55'21".44516	78.754
C	Morris	45°26'34".29480±".00010	-75°15'18".81653±".00026	90.948±.004
	Panmure	45°20'18".81755±".00012	-76°11'4".58825±".00032	155.362±.004
D	Morris	45°26'34".29500±".00010	-76°15'18".81772±".00026	90.942±.004
	Panmure	45°20'18".81702±".00012	-76°11'4".58859±".00032	155.361±.004

reduced in comparison to the baseline-by-baseline processing technique.

As the influence of the ionosphere on the baselines of triangle 6A-Morris-Panmure is comparatively small, we also processed the observations of this triangle alone in the network mode. The results of this analysis, giving probably the most reliable results for Morris and Panmure, are given as solutions C and D in Table 7.

In Table 8, we list the differences in station coordinates between our network solutions A, B, C, and D and the positions of Table 1.

6. CONCLUSIONS

The Ottawa test was one of a number of tests of the Macrometer® designed to assess its capabilities for precise positioning. Unlike other tests, our test also included a separate analysis of the Macrometer® data using a software processing package independent of that developed by Macrometrics.

The formal one sigma uncertainties of the estimated coordinates for Morris, Panmure, and Metcalfe with respect to station 6A (as given in Table 6) are about 4 mm, 7mm, and 5mm for latitude, longitude, and height respectively.

Given the effect of the ionosphere as the likely source of the largest systematic errors, the actual uncertainties are much larger, perhaps up to about 9 cm for the longest baseline, when we are working in the single baseline mode. These uncertainties are reduced if we process the data in the network mode. Typically, we estimate uncertainties of 2 cm to 4 cm for the network 6A, Morris, Panmure, Metcalfe, and typically 1 to 1.5 cm for the smaller network, 6A, Morris, Panmure.

TABLE 8

Differences in cm between network solutions of Tables 6, 7 and ground values of Table 1.

Solution	Station	$\Delta\phi$	$\Delta\lambda$	Δh
A	Morris	7.7	-0.3	-19.9
	Panmure	5.2	2.7	2.5
	Metcalfe	12.4	-6.7	12.5
B	Morris	8.4	-3.0	-20.5
	Panmure	3.1	2.7	3.2
	Metcalfe	9.6	-8.6	14.2
C	Morris	7.0	-1.8	-21.6
	Panmure	6.4	-0.8	1.6
D	Morris	7.7	-0.8	-22.2
	Panmure	4.7	-1.5	1.5

The agreement between the coordinates of stations obtained using Macrometrics' and UNB's software is about 1 ppm. The agreement in horizontal coordinates between our values and those obtained from terrestrial measurements is of the order of 1 ppm to 3 ppm. The differences in vertical coordinates are somewhat larger due, in part, to the uncertainties of the geoid undulations in the vicinity of Ottawa.

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PART B

MODELLING AND ESTIMATING THE ORBITS OF GPS SATELLITES

1. INTRODUCTION

Although only partially deployed, the Global Positioning System (GPS) is already being exploited for high-precision geodetic surveys (see Part A). The Macrometer® V-1000, a first generation interferometric receiver using only the L1 carrier phase of the GPS signal, has clearly demonstrated its ability in differential positioning. It is clear that the next generation, dual-frequency receivers, will have the potential of measuring baselines on a continental scale with an accuracy of a few centimetres--provided the orbits of the satellites are known to a precision of a few decimetres (see section 2).

These orbital precision requirements suggest that an extended worldwide tracking network equipped with high-precision receivers will be required. Such a network is not now available to civilian users nor will one be available in the immediate future. Plans for several regional civilian tracking networks are presently being developed and/or implemented, and these may provide acceptable orbits over certain regions. However, the geodesist using GPS for large-scale, high-precision surveys may not be able to assume that the GPS orbits are known to sufficiently high accuracy. He may have to estimate so-called orbital biases along with the parameters in which he is actually interested, i.e., the relative coordinates of the receivers.

Methods to estimate such orbital biases have been developed and widely used in the processing of Transit Doppler data. With the exception of some of the so-called short arc procedures, these algorithms do not describe the orbits by physical parameters. For example, one technique is to parallely shift and rotate the orbit. What usually results is nonphysical in the sense that the resulting orbit is not a particular solution of the equations of motion of the satellite.

Here we attempt to demonstrate that there is no need for such nonphysical methods in the determination of GPS satellite orbits. As a matter of fact, it is quite simple to model the orbital biases for these satellites to any precision required in a purely physical way. Because of the orbital characteristics of GPS satellites (almost circular orbits with semimajor axes of about 26,500 km), we may assume the earth's gravity field to be known. As a rigorous modelling of the gravitational attraction due to the sun and the moon is not a serious problem, the only significant external force on the satellites which is not adequately known a priori is radiation pressure. That radiation pressure has an appreciable influence on GPS orbits is stated by van Dierendonck et al. [1980].

We have subdivided our discussion into the five subsequent sections. We discuss the orbital precision requirements for geodetic applications in section 2. We then develop the theory for our general algorithms in sections 3 and 4. These algorithms are generalizations of short arc methods in as much as (a) there are no constraints on the arc length, and (b) it is possible to solve not only for initial or boundary conditions but also for dynamical parameters (e.g., parameters governing the effect of radiation pressure). In section 5 we discuss short arc applications with specific reference to the Ottawa Macrometer® test campaign (see also Part A). Finally in section 6 we discuss some aspects of the general orbit determination problem.

2. PRECISION REQUIREMENTS

The accuracy of orbits needed to obtain baseline estimates of a certain accuracy depends mainly on the length of the baseline (see Bauersima [1983, eqn. (84)]):

$$\frac{db}{b} \doteq \frac{dr}{\rho} \tag{2.1}$$

where b is the length of the baseline

ρ is the range (receiver to satellite)

dr is the orbit error

db is the induced baseline error.

For GPS satellites we have approximately

$$\rho = 25{,}000 \text{ km} \quad . \tag{2.2}$$

Accepting, for example,

$$db = 1 \text{ cm} \tag{2.3}$$

as a maximum for the baseline error introduced by the orbit, we obtain the values in Table 2.1 for the maximum orbit error, dr, allowed.

Table 2.1

Maximum permissible orbit error, dr, for an accuracy of 1 cm in a baseline of length b.

b (km)	dr (m)
0.1	2500.0
1.0	250.0
10.0	25.0
100.0	2.5
1000.0	.25

Rather than assuming a specific value for a baseline error independent of baseline length, we may wish to talk about a relative error expressed as parts per million (ppm) of the baseline length. Table 2.2 gives the values of dr for a number of relative baseline errors.

Table 2.2

Maximum permissible orbit error, dr, for a certain relative accuracy in the baseline.

$\frac{db}{b}$ (ppm)	dr (m)
5	125.0
1	25.0
0.5	12.5
0.1	2.5

Summarizing Tables 2.1 and 2.2:

(a) The orbital accuracy required for surveying depends highly on the length of the baselines to be measured.

(b) For "local surveys" (diameter of surveyed region smaller than 100 km), an orbital accuracy of 5 m to 10 m will be sufficient in most cases.

3. PRINCIPLES OF ORBIT DETERMINATION

3.1 Statement of the Problem

The orbit of every satellite is a particular solution of a system of second-order differential equations:

$$\vec{r}^{(2)} = \vec{f}(t;\vec{r},\vec{r}^{(1)},p_1,p_2,\ldots,p_n) \tag{3.1}$$

where $\vec{r} = \vec{r}(t)$ is the position of the satellite in a nonrotating (with respect to inertial space), geocentric coordinate system.

$\vec{r}^{(i)}$, i=1,2, are the first and second time derivatives of $\vec{r}(t)$

p_i, i=1,2,...,n are parameters defining the forces acting on the satellite ("dynamical" parameters, e.g., coefficients of the expansion of the earth's gravitational field into spherical harmonics; parameters describing drag, radiation pressure).

To define an orbit uniquely (a particular solution of eqn. (3.1)), additional information has to be supplied. The two simplest formalisms for introducing this information are:

(a) Formulation as an initial value problem:

$$\begin{aligned} \vec{r}(t_o) &= \vec{r}_o(k_1,k_2,\ldots,k_6) \\ \vec{r}^{(1)}(t_o) &= \vec{r}_o^{(1)}(k_1,k_2,\ldots,k_6) \end{aligned} \tag{3.2a}$$

(b) Formulation as a simple boundary value problem:

$$\begin{aligned} \vec{r}(t_1) &= \vec{r}_1(k_1,k_2,\ldots,k_6) \\ \vec{r}(t_2) &= \vec{r}_2(k_1,k_2,\ldots,k_6) \end{aligned} \tag{3.2b}$$

where k_i, i=1,2,...,6 are six parameters uniquely specifying the vectors on the right-hand sides of eqns. (3.2a) or (3.2b). Possible choices for these parameters are:

Components of vectors $\vec{r}_o$, $\vec{r}_o^{(1)}$ or $\vec{r}_1$, $\vec{r}_2$.

Osculating orbital elements at time t_o for eqns. (3.2a).

If we know the right-hand sides of eqns. (3.2a) or (3.2b) (parameters k_i, i=1,2,...,6) and the dynamical parameters p_i, i=1,2,...,n, the orbit of a satellite is uniquely defined. We therefore may state:

(a) Orbit determination in its usual, more restricted, sense is defined as the problem of determining the six parameters k_i, i=1,2,...,6 defining the initial values or boundary values on the right-hand sides of eqns. (3.2a) or (3.2b).

(b) Orbit determination in its most general sense is the problem of determining the six parameters k_i, $i=1,2,\ldots,6$ defining the initial values or boundary values <u>and</u> the dynamical parameters p_i, $i=1,2,\ldots,n$.

For the application we have in mind here (modelling the orbits of GPS satellites) most of the dynamical parameters in eqn. (3.1) may be assumed to be known (coefficients of the earth's gravity field, gravitational force of the sun and the moon). For utmost accuracy, however, we will have to estimate some of the dynamical parameters such as those defining radiation pressure.

3.2 Observations

To solve an orbit determination problem, at some point one needs observations. An observation may be defined as a value of a function of satellite positions, ground positions, and nuisance parameters, like clock offsets or clock drifts of satellite and receiver clocks. More specifically, the following GPS observables have been instrumented in presently available receivers:

- pseudoranges using the C/A-code
- pseudoranges using the P-code
- Doppler measurements
- phase measurements of carriers.

The carrier phase measurements are potentially the most powerful measurements as they can be made with the greatest precision. We therefore assume that we will be dealing with measurements of this kind for orbit determination problems.

3.3 Principles of Solution of Orbit Determination Problems

Every orbit determination is actually an orbit improvement process using observations such as those given in section 3.2. These observations are nonlinear functions of the satellite position $\vec{r}(t_i)$ (and possibly of the velocity $\vec{r}^{(1)}(t_i)$) at observation time t_i. $\vec{r}(t_i)$ in turn is a nonlinear function of the parameters of the orbit. The linearization of the orbit determination problem is therefore done in two steps: (a) the observation has to be approximated as a linear function of $\vec{r}(t_i)$, which is straight forward; and (b) $\vec{r}(t_i)$ has to be represented by a linear function of the unknown parameters k_i (and possibly some of the p_i). This is done in the following way. Since eqn. (3.1) is nonlinear, we must linearize it and determine our orbit iteratively, where in each iteration step we assume we have a known approximate orbit $\vec{r}_a(t)$ at our disposal (for the sake of simplicity we are dealing only with an initial value problem here):

$$\begin{aligned} \vec{r}_a^{(2)} &= \vec{f}(t;\vec{r}_a,\vec{r}_a^{(1)},p_1,p_2,\ldots,p_n) \\ \vec{r}_a(t_o) &= \vec{r}_{ao}(k_{a1},k_{a2},\ldots,k_{a6}) \\ \vec{r}_a^{(1)}(t_o) &= \vec{r}_{ao}^{(1)}(k_{a1},k_{a2},\ldots,k_{a6}) \quad , \end{aligned} \tag{3.3}$$

where the k_{ai}, $i=1,2,\ldots,6$ are approximate values of the unknown parameters.

The true, initially unknown, orbit is now assumed to be a linear(ized) function of the parameters k_i, $i=1,2,\ldots,6$:

$$\vec{r}(t) = \vec{r}_a(t) + \sum_{i=1}^{6} \vec{z}_i(t)(k_i - k_{ai}) \tag{3.4}$$

where $\vec{z}_i(t) = \left.\frac{\partial \vec{r}_a}{\partial k_i}\right|_{k=k_a}$.

If we deal with the more general problem, additional terms involving the dynamical parameters appear on the right-hand side of eqn. (3.4). Let us assume, for the sake of simplicity, that only one of the dynamical parameters, p_1, has to be estimated. Equation (3.4) is now replaced by:

$$\vec{r}(t) = \vec{r}_a(t) + \sum_{i=1}^{6} \vec{z}_i(t)(k_i - k_{ai}) + \vec{z}_7(t)(p_1 - p_{a1}) \tag{3.5}$$

where $\vec{z}_7(t) = \left.\frac{\partial \vec{r}_a}{\partial p_1}\right|_{k=k_a, p_1=p_{a1}}$.

Of course, for general problems of this kind, we have to replace p_1 by p_{a1} in the system of differential equations (3.3):

$$\vec{r}_a^{(2)} = \vec{f}(t; \vec{r}_a, \vec{r}_a^{(1)}, p_{a1}, p_2, \ldots, p_n) \tag{3.6}$$

The functions $\vec{z}_i(t)$ are solutions of an initial (or boundary) value problem, which follows from the primary problem (3.3) by taking the (total) derivatives of all the equations in (3.3) with respect to k_i, i=1,2,...,6 and p_1. The resulting sets of differential equations are usually called the systems of variational equations:

For i=1,2,...,6, we get:

$$\begin{aligned} \vec{z}_i^{(2)} &= A_o \vec{z}_i + A_1 \vec{z}_i^{(1)} \\ \vec{z}_i(t_o) &= \frac{\partial \vec{r}_{ao}}{\partial k_i}, \quad \vec{z}_i^{(1)}(t_o) = \frac{\partial \vec{r}_{ao}^{(1)}}{\partial k_i} \end{aligned} \tag{3.7}$$

For i = 7 we get:

$$\vec{z}_7^{(2)} = A_o\vec{z}_7 + A_1\vec{z}_7^{(1)} + \frac{\partial\vec{f}}{\partial p_1}$$

$$\vec{z}_7(t_o) = \vec{0} \quad , \quad \vec{z}_7^{(1)}(t_o) = \vec{0} \quad . \tag{3.8}$$

The symbols, A_o and A_1, used above are 3 × 3 matrices whose elements are defined by:

$$A_{o,ik} = \partial f_i/\partial r_{a,k} \ , \ A_{1,ik} = \partial f_i/\partial r_{a,k}^{(1)} \ , \ i=1,2,3; \ k=1,2,3 \tag{3.9}$$

where $r_{a,k}$ and $r_{a,k}^{(1)}$ are the kth components of the $\vec{r}_a$ and $\vec{r}_a^{(1)}$ vectors. The systems of differential equations (3.7) and (3.8) are linear; the systems (3.7) are, in addition, homogeneous.

Let us summarize: In every iteration step of the orbit improvement process, we have to solve one system of nonlinear differential equations of type (3.3), six linear systems of type (3.7), and, if dynamical parameters are considered, some of type (3.8).

4. THE USE OF NUMERICAL INTEGRATION IN ORBIT DETERMINATION

4.1 Statement of the Problem

With numerical integration techniques, the initial or boundary value problems which we have to solve in each iteration step of the orbit improvement process may be treated directly without any transformations.

Numerical integration, properly understood, is a special branch of approximation theory, the underlying mathematical theorems being the Weierstrass Approximation Theorem and Taylor's Theorem.

The true solution $\vec{r}(t)$ of eqns. (3.1), (3.2a), or (3.2b) is approximated in a certain time interval Δt by a finite series, $\vec{r}^*(t)$, of known base functions $g_i(t)$:

$$\vec{r}^*(t) = \sum_{i=0}^{q} \vec{a}_i g_i(t) \quad , \tag{4.1}$$

where q is the order of the approximation, and $\vec{a}_i$, i=0,1,...,q are the q+1 unknown coefficient vectors of the series.

In conventional integration techniques, the base functions, $g_i(t)$, are algebraic polynomials in t:

$$g_i(t) = (t - t_o)^i \ , \ i=0,1,\ldots,q. \tag{4.2}$$

In principle, the time t_o may be chosen arbitrarily. However, when solving initial value problems, t_o usually is chosen to coincide with the initial epoch.

The problem of numerical integration consists of determining the coefficients $\vec{a}_i$. Usually this is done in the following way:

(a) The approximating function $\vec{r}^*(t)$ is asked to satisfy the initial conditions, eqn. (3.2a), or the boundary conditions, eqn. (3.2b). This gives us two linear (vectorial) condition equations for the unknown vectors $\vec{a}_i$.

(b) The approximating function $\vec{r}^*(t)$ is asked to satisfy eqn. (3.1) at q-1 different times t_i, i=1,2,...,q-1, in the integration interval Δt. From this step we get (q-1) independent algebraic equations for the unknowns.

We now have reduced the problem of the solution of a system of <u>differential</u> equations to the solution of a system of nonlinear <u>algebraic</u> equations. It can be demonstrated that most of the classical integration methods (deserving that name) are actually special cases of the general concept developed here (e.g., the methods of Adams, Moulton, Størmer, etc.).

Of course there are different techniques for practically solving this nonlinear problem. The results, however, should be identical. We wish to stress that the direct result of a modern numerical integration technique is <u>not</u> a set of equi-distantly (or otherwise) spaced coordinates of the satellite. The result consists of one or more sets of coefficients $\vec{a}_i$, i=0,1,...,q, which enable us to compute $\vec{r}^*(t)$ or any of its derivatives for any time t in the integration interval Δt.

For long integration intervals (e.g., in the case of low eccentricity orbits longer than a quarter of a revolution), this interval Δt has to be subdivided into smaller intervals and one approximate function of type (4.1) has to be determined per subdivision. The link between subsequent intervals is obtained by requiring the satellite position and velocity to be continuous functions at the interval boundaries.

Seen from this point of view, numerical integration techniques are very transparent and very easy to use. That they are universally applicable, is generally acknowledged.

4.2 Transformations

The system of differential equations (3.1) may be transformed into a system of six first-order equations for the six osculating Keplerian elements, a, e, i, Ω, ω, and T_o (see Figure 4.1):

$$\begin{aligned} a^{(1)} &= g_1(t;a,e,\ldots,T_o,p_1,p_2,\ldots,p_n) \\ e^{(1)} &= g_2(t;a,e,\ldots,T_o,p_1,p_2,\ldots,p_n) \\ i^{(1)} &= g_3(t;a,e,\ldots,T_o,p_1,p_2,\ldots,p_n) \\ \Omega^{(1)} &= g_4(t;a,e,\ldots,T_o,p_1,p_2,\ldots,p_n) \\ \omega^{(1)} &= g_5(t;a,e,\ldots,T_o,p_1,p_2,\ldots,p_n) \\ T_o^{(1)} &= g_6(t;a,e,\ldots,T_o,p_1,p_2,\ldots,p_n) \quad , \end{aligned} \tag{4.3}$$

where $a^{(1)}$ is the first derivative of <u>a</u> with respect to time, and so on.

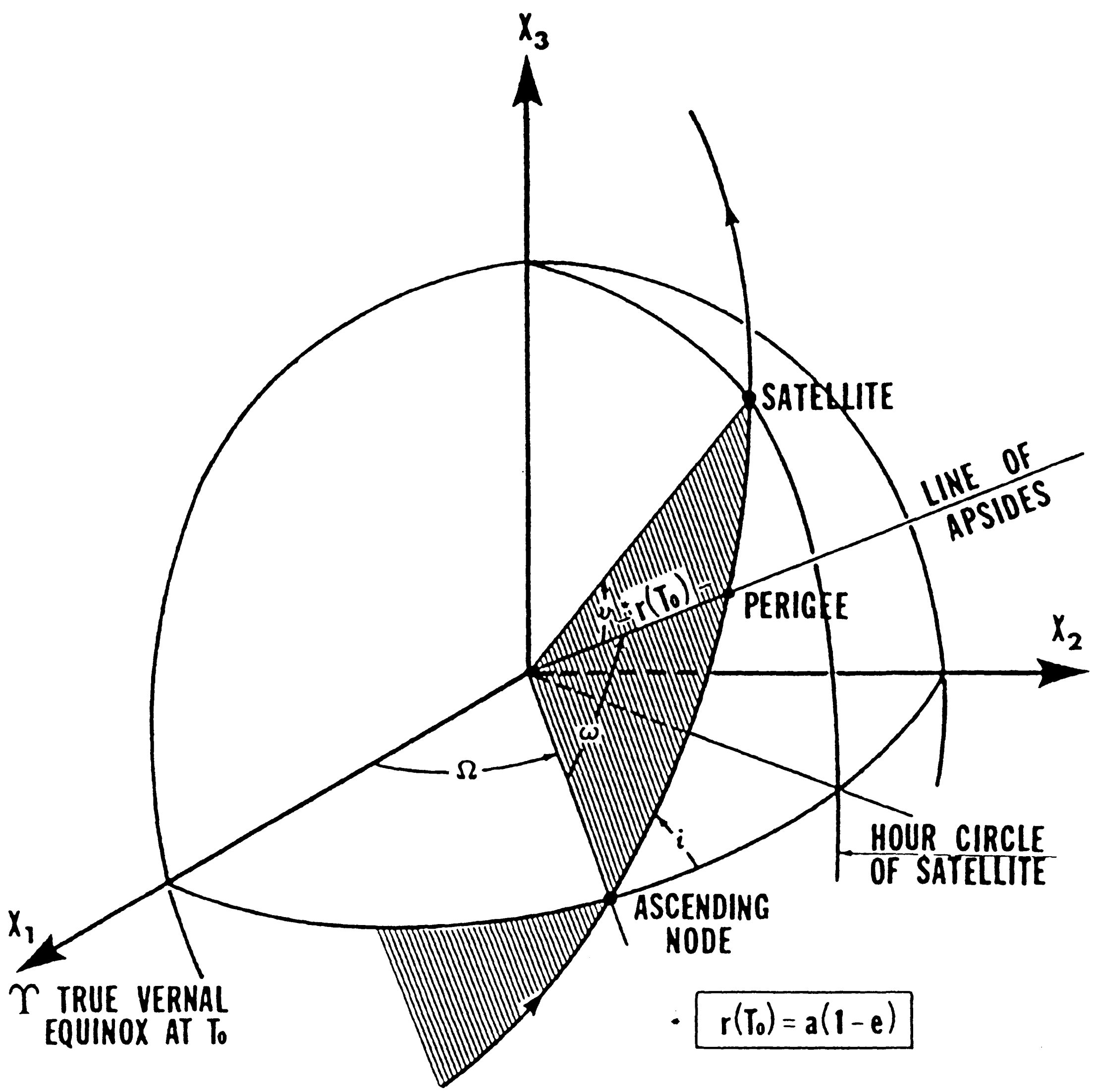


FIGURE 4.1 Keplerian Elements

As the values of the functions g_i are "small" (perturbations of the central force field), eqns. (4.3) have two simple approximate solutions:

(a) Keplerian approximation

$$a(t) = a_o,\ e(t) = e_o,\ \ldots,\ T_o(t) = T_{o,o} \quad . \tag{4.4}$$

(b) First-order perturbations.

In first-order perturbation theory, the approximations (4.4) (actually the osculating elements at a time t_o) are used on the right-hand sides of (4.3). This reduces the problem of solving a system of six coupled differential equations to the problem of evaluating six separate integrals.

4.3 Numerical Methods versus Analytical Methods

We may solve the integrals analytically by series expansions. Heavy algebraic manipulations, however, are involved. Alternatively we may use numerical methods. These have the following advantages:

(a) There is no heavy algebra involved (we only need code a subroutine giving the right-hand sides of eqns. (3.1), (3.7), and (3.8)).

(b) They are easy to generalize: modelling an additional force merely introduces a new term in eqn. (3.1).

(c) After the intrinsic integration procedure, the approximate functions and their time derivatives are readily available (the same is true for the partials).

(d) Apart from round-off errors, the approximation may be generated as precisely as desired.

The disadvantages of numerical integration are:

(a) The integration process itself is a heavy consumer of computer time; this is especially true if many partials $\vec{z}_i(t)$ have to be computed.

(b) Numerical methods are said to be less transparent.

We may contrast these advantages and disadvantages with those of analytical methods. Analytical methods have the following advantages:

(a) The solutions are explicitly given as functions of the unknown parameters. Therefore, the partials with respect to these unknowns follow simply by differentiating the solutions. Moreover (and more importantly) it is easy to give simple approximations for these partials.

(b) Values of the solution for "very different times" are readily available.

Disadvantages of analytical methods are:

(a) Heavy algebra is involved; generalizations are not easy to implement.

(b) If the solution of eqn. (4.3) is needed for many instants of time, many trigonometric functions have to be evaluated.

(c) The solutions actually are always approximations (Keplerian, first-, second-, ..., nth-order perturbations).

4.4 Combination of Numerical and Analytical Methods

It is, of course, tempting to create a "new" method combining the positive aspects of both numerical and analytical methods. For relatively short arcs, this is possible by the following procedure:

(a) Choose the osculating elements at time t_o to define the initial state vectors.

(b) Solve the primary problem of eqns. (3.1), (3.2a), or (3.2b) by rigorous numerical integration using as sophisticated a force field as necessary.

(c) Use a very simple analytical approximation for the orbit to generate approximate partials by differentiating this approximate orbit with respect to the osculating elements of time t_o. For orbits not longer than half a revolution, even the Keplerian approximation is good enough. Whereas this procedure may reduce the convergence speed of the orbit improvement process, it does not affect the final result.

5. DETERMINING ORBITS OF GPS SATELLITES

5.1 Program TRNSNEW

A FORTRAN computer program, TRNSNEW, was developed to implement the techniques of section 4.4 as a first step in developing a capability to determine the orbits of GPS satellites. We wished to investigate, in a quantitative way, how imprecise orbits affected baseline determinations. We had access to the set of GPS observations from the Ottawa Macrometer® campaign and the so-called T-files (see Part A) consisting of the approximate state vectors of the GPS satellites during the observation periods. We therefore developed TRNSNEW to generate (degraded) orbits of a certain chosen precision, starting with the Macrometer® T-files which are believed to have an accuracy of 30 m or better.

Designed specifically for GPS orbits, program TRNSNEW uses a simplified force model. It is well known that the force field acting on an artificial satellite is rather complex. Concentrating only on the most important effects, we have

- gravitational attraction of the earth (usually an expansion into spherical harmonics is used)
- solar and lunar gravitation (known point mass attraction)

- atmospheric drag
- radiation pressure.

GPS satellites are in high, low eccentricity orbits (semimajor axis of about 26,500 km; eccentricity less than 0.01). We may therefore neglect atmospheric drag even for long arcs (several days). Moreover, it can be assumed that the coefficients of the earth's gravity field are known from the analysis of low orbiting satellites (terms up to degree and order 12 are reasonably well established). We may even expect that for short arcs (half a revolution or less) an approximation using only very few terms of the earth's gravity field should be sufficient for many applications.

Program TRNSNEW presently contains only the J_2, J_3, J_4 terms of the earth's gravitational field and the solar and lunar point mass gravitational fields.

In TRNSNEW, the user defines the time interval over which he wants to approximate the GPS orbits. The program then extracts the positions of all GPS satellites in this time interval from the T-files. These positions (Cartesian coordinates in the equatorial system 1950.0) are interpreted as observations of the satellites. An orbit improvement process is invoked, giving as the result the best orbit in the sense of the method of least squares. Then the orbits are generated by numerical integration using the methods of section 3. The program user defines the forces acting on the satellites by coding the subroutine DERIV. In TRNSNEW the osculating elements of Figure 4.1 (osculation epoch t_o is automatically defined by the program) are chosen as parameters $k_1, k_2, \ldots, k_6$ defining the initial or boundary conditions. Starting values for these parameters are obtained by first solving a boundary value problem using two T-file positions as boundary positions, then computing $\vec{r}(t_o)$, $\vec{r}^{(1)}(t_o)$ using the resulting approximations (4.1), and finally transforming these vectors into osculating elements. As in the near future only relatively short arcs will likely be analysed with TRNSNEW, the partials of the orbit with respect to the osculating elements are approximated by the method outlined in section 4.4.

5.2 Tests Using Program TRNSNEW

Using T-files from the Ottawa test, we made two sets of runs using TRNSNEW. In the first set, we examined the successive improvement for an arc of fixed length of the orbit approximation using the following force fields:

(1) pure central force ($GM = 3.9860047 \cdot 10^{14}\ m^3 s^{-2}$)

(2) (1) plus J_2 term ($J_2 = 1082.627 \cdot 10^{-6}$, $a_e = 6378.140$ km)

(3) (2) plus solar and lunar gravitation ($GMS = 1.32712438 \cdot 10^{20} m^3 s^{-2}$, $GMM = 4.90278888 \cdot 10^{12} m^3 s^{-2}$)

(4) (3) plus J_3, J_4 terms ($J_3 = -2.536 \cdot 10^{-6}$, $J_4 = -1.623 \cdot 10^{-6}$).

Numerical values for GM, J_2, J_3, J_4 were taken from Lerch et al. [1979], the values for lunar and solar gravitation constants are taken from Beutler [1982, eqn. (200a)]. The results are presented in Tables 5.1a to 5.1d. In each case, the force field used is specified in the header of the tables; the time relative to the first T-file position used is given in minutes; DX, DY, DZ are the differences of the numerically integrated position using the known approximate orbit of the present iteration step minus the T-file positions in metres for the first two iterations. "INT.-ERROR" is an estimation for the truncation introduced by numerical integration.

In the second set, we examined the decreasing quality of the

TABLE 5.1A

(T-FILE POSITION)-(TRNSNEW POSITION (ONLY GM - TERM))

PROGRAM TRNSNEW: APPROXIMATION OF MACROMETRICS' T-FILES

TERRESTRIAL-,SOLAR,LUNAR GRAVITY CONSTANTS :
GM= 0.39860047D+15 GMS= 0.0000000D+00 GMM= 0.0000000D+00
J2= 0.0000000D+00 J3= 0.0000000D+00 J4= 0.0000000D+00

SATELLITE NR. 1 ITERATION NR. 1

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	-0.000	0.000	-0.000	0.0001
22.50	-108.327	48.834	-84.174	0.0001
45.00	-202.201	86.970	-86.338	0.0001
67.50	-245.248	99.023	-34.104	0.0001
90.00	-217.952	82.724	27.792	0.0001
112.50	-126.086	45.942	51.281	0.0001
135.00	-0.000	-0.000	0.000	0.0001
157.50	115.094	-47.936	-139.808	0.0000
180.00	174.376	-100.255	-356.351	0.0000
202.50	148.698	-169.869	-618.240	0.0000
225.00	34.336	-276.142	-887.010	0.0000
247.50	-146.270	-437.173	-1131.555	0.0000

SATELLITE NR. 1 ITERATION NR. 2

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	87.804	-200.870	-257.162	0.0001
22.50	-35.420	-107.436	-259.743	0.0001
45.00	-141.010	-22.051	-170.759	0.0001
67.50	-189.901	37.735	-22.168	0.0001
90.00	-162.802	68.777	135.947	0.0001
112.50	-68.947	78.700	250.907	0.0001
135.00	55.175	78.322	284.083	0.0001
157.50	156.884	73.150	222.924	0.0000
180.00	184.476	57.271	84.294	0.0000
202.50	104.600	12.071	-91.958	0.0000
225.00	-86.092	-89.541	-256.833	0.0000
247.50	-358.919	-274.713	-368.578	0.0000

TABLE 5.1B

(T-FILE POSITION)-(TRNSNEW POSITION (INCL. J2))

PROGRAM TRNSNEW: APPROXIMATION OF MACROMETRICS' T-FILES

TERRESTRIAL-,SOLAR,LUNAR GRAVITY CONSTANTS :
GM= 0.39860047D+15 GMS= 0.0000000D+00 GMM= 0.0000000D+00
J2= 0.1082627D-02 J3= 0.0000000D+00 J4= 0.0000000D+00

SATELLITE NR. 1 ITERATION NR. 1

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	-0.000	0.000	-0.000	0.0001
22.50	0.670	9.153	-3.046	0.0001
45.00	0.998	14.475	-3.237	0.0001
67.50	0.941	15.919	-1.718	0.0001
90.00	0.653	13.646	0.233	0.0001
112.50	0.307	8.102	1.247	0.0001
135.00	-0.000	-0.000	0.000	0.0001
157.50	-0.223	-9.758	-4.678	0.0000
180.00	-0.286	-20.142	-13.790	0.0000
202.50	-0.017	-30.008	-28.274	0.0000
225.00	0.794	-38.045	-49.156	0.0000
247.50	2.186	-42.680	-77.677	0.0000

SATELLITE NR. 1 ITERATION NR. 2

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	-4.825	-24.717	-16.314	0.0001
22.50	-3.729	-11.181	-14.501	0.0001
45.00	-2.654	-0.770	-9.316	0.0001
67.50	-1.651	6.282	-2.142	0.0001
90.00	-0.706	9.966	5.488	0.0001
112.50	0.094	10.554	12.004	0.0001
135.00	0.503	8.550	15.998	0.0001
157.50	0.203	4.584	16.366	0.0000
180.00	-1.022	-0.666	12.331	0.0000
202.50	-3.138	-6.487	3.307	0.0000
225.00	-5.843	-12.058	-11.316	0.0000
247.50	-8.739	-16.308	-32.389	0.0000

TABLE 5.1C

(T-FILE POSITION)-(TRNSNEW POSITION) (INCL. J2 , SUN+MOON)

PROGRAM TRNSNEW: APPROXIMATION OF MACROMETRICS' T-FILES

TERRESTRIAL-,SOLAR,LUNAR GRAVITY CONSTANTS :
GM= 0.39860047D+15 GMS= 0.1327125D+21 GMM= 0.4902789D+13
J2= 0.1082627D-02 J3= 0.0000000D+00 J4= 0.0000000D+00

SATELLITE NR. 1 ITERATION NR. 1

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	-0.000	-0.000	-0.000	0.0001
22.50	-1.049	3.210	-0.238	0.0001
45.00	-1.827	5.184	-0.147	0.0001
67.50	-2.236	5.818	0.110	0.0001
90.00	-2.151	5.089	0.351	0.0001
112.50	-1.443	3.083	0.380	0.0001
135.00	-0.000	-0.000	0.000	0.0001
157.50	2.257	-3.859	-0.992	0.0000
180.00	5.331	-8.115	-2.809	0.0000
202.50	9.104	-12.343	-5.652	0.0000
225.00	13.309	-16.112	-9.655	0.0000
247.50	17.542	-19.050	-14.834	0.0000

SATELLITE NR. 1 ITERATION NR. 2

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	3.173	-9.123	-1.768	0.0001
22.50	0.576	-4.033	-1.662	0.0001
45.00	-1.426	-0.031	-0.945	0.0001
67.50	-2.834	2.798	0.063	0.0001
90.00	-3.676	4.418	1.089	0.0001
112.50	-3.991	4.862	1.919	0.0001
135.00	-3.812	4.227	2.408	0.0001
157.50	-3.146	2.674	2.455	0.0000
180.00	-1.994	0.420	1.971	0.0000
202.50	-0.386	-2.274	0.864	0.0000
225.00	1.576	-5.133	-0.942	0.0000
247.50	3.713	-7.904	-3.475	0.0000

TABLE 5.1D

(T-FILE POSITION)-(TRNSNEW POSITION) (INCL. J2,J3,J4 , SUN+MOON)

PROGRAM TRNSNEW: APPROXIMATION OF MACROMETRICS' T-FILES

TERRESTRIAL-,SOLAR,LUNAR GRAVITY CONSTANTS :
GM= 0.39860047D+15 GMS= 0.1327125D+21 GMM= 0.4902789D+13
J2= 0.1082627D-02 J3= -0.2536000D-05 J4= -0.1623000D-05

SATELLITE NR. 1 ITERATION NR. 1

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	-0.000	-0.000	-0.000	0.0001
22.50	-1.098	3.247	-0.161	0.0001
45.00	-1.868	5.224	0.005	0.0001
67.50	-2.231	5.842	0.290	0.0001
90.00	-2.101	5.096	0.499	0.0001
112.50	-1.390	3.082	0.454	0.0001
135.00	-0.000	-0.000	0.000	0.0001
157.50	2.161	-3.856	-1.027	0.0000
180.00	5.132	-8.109	-2.820	0.0000
202.50	8.836	-12.326	-5.584	0.0000
225.00	13.027	-16.061	-9.479	0.0000
247.50	17.302	-18.922	-14.544	0.0000

SATELLITE NR. 1 ITERATION NR. 2

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	3.160	-9.117	-1.836	0.0001
22.50	0.529	-3.998	-1.662	0.0001
45.00	-1.453	0.003	-0.877	0.0001
67.50	-2.803	2.811	0.157	0.0001
90.00	-3.587	4.408	1.151	0.0001
112.50	-3.881	4.837	1.910	0.0001
135.00	-3.731	4.196	2.328	0.0001
157.50	-3.128	2.640	2.339	0.0000
180.00	-2.038	0.386	1.872	0.0000
202.50	-0.453	-2.297	0.826	0.0000
225.00	1.545	-5.113	-0.901	0.0000
247.50	3.771	-7.789	-3.365	0.0000

approximation for increasing arc length, keeping the same force terms. We chose arc lengths of about 1.9, 4.1, 7.1, 10.1, and 14.5 hours. The results are presented in Tables 5.2a to 5.2e.

We may conclude from the results that for short arcs we indeed may represent the orbits using very simple models for the force field. In view of Tables 2.1 and 2.2, it is even evident that for very short baselines (1 km or less) and very short observation time spans (1 hour or less), even the simplest model for the force field (central force field of the earth) is good enough.

Observation sessions of the Ottawa Macrometer® campaign (see Part A) lasted up to 5 hours. The Macrometer® V-1000 receiver is capable of providing baselines of 1 ppm accuracy. Table 2.2 indicates that for such a relative baseline accuracy an orbital precision of 25 m is required. This requirement is completely fulfilled using the most elaborate of the force field approximations discussed here. It is also clear, however, that the next generation of receivers will ask for better orbit models. A future program modification, therefore, will allow one to specify the earth's gravity field in terms of spherical harmonics with a user-specified upper limit. Furthermore, a radiation pressure model with some a priori known and some adjustable parameters (as discussed in section 3.3) will be implemented.

5.3 The Influence of Orbit Errors on Baseline Estimations

The tests concerning the orbits presented above were not performed in order to produce better baseline results or in order to estimate better orbits than the ones defined by Macrometrics' T-files. The tests were proposed to give answers to the following questions:

TABLE 5.2A

(T-FILE POSITION) -(TRNSNEW POSITION) (ARC-LENGTH = 1.9 HOURS)

PROGRAM TRNSNEW: APPROXIMATION OF MACROMETRICS' T-FILES

TERRESTRIAL-,SOLAR,LUNAR GRAVITY CONSTANTS :
GM= 0.39860047D+15 GMS= 0.1327125D+21 GMM= 0.4902789D+13
J2= 0.1082627D-02 J3= -0.2536000D-05 J4= -0.1623000D-05

SATELLITE NR. 1 ITERATION NR. 1

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	-0.000	-0.000	-0.000	0.0000
22.50	-0.955	2.586	-0.290	0.0000
45.00	-1.553	3.909	-0.233	0.0000
67.50	-1.675	3.893	-0.028	0.0000
90.00	-1.197	2.549	0.118	0.0000
112.50	-0.000	-0.000	0.000	0.0000

SATELLITE NR. 1 ITERATION NR. 2

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	0.905	-2.121	0.167	0.0000
22.50	-0.268	0.432	-0.229	0.0000
45.00	-0.962	1.720	-0.211	0.0000
67.50	-1.056	1.686	-0.010	0.0000
90.00	-0.428	0.359	0.136	0.0000
112.50	1.032	-2.123	-0.005	0.0000

TABLE 5.2B

(T-FILE POSITION)- (TRNSNEW POSITION) (ARC-LENGTH = 4.1 HOURS)

PROGRAM TRNSNEW: APPROXIMATION OF MACROMETRICS' T-FILES

TERRESTRIAL-,SOLAR,LUNAR GRAVITY CONSTANTS :
GM= 0.39860047D+15 GMS= 0.1327125D+21 GMM= 0.4902789D+13
J2= 0.1082627D-02 J3= -0.2536000D-05 J4= -0.1623000D-05

SATELLITE NR. 1 ITERATION NR. 1

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	-0.000	-0.000	-0.000	0.0001
22.50	-1.098	3.247	-0.161	0.0001
45.00	-1.868	5.224	0.005	0.0001
67.50	-2.231	5.842	0.290	0.0001
90.00	-2.101	5.096	0.499	0.0001
112.50	-1.390	3.082	0.454	0.0001
135.00	-0.000	-0.000	0.000	0.0001
157.50	2.161	-3.856	-1.027	0.0000
180.00	5.132	-8.109	-2.820	0.0000
202.50	8.836	-12.326	-5.584	0.0000
225.00	13.027	-16.061	-9.479	0.0000
247.50	17.302	-18.922	-14.544	0.0000

SATELLITE NR. 1 ITERATION NR. 2

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	3.160	-9.117	-1.836	0.0001
22.50	0.529	-3.998	-1.662	0.0001
45.00	-1.453	0.003	-0.877	0.0001
67.50	-2.803	2.811	0.157	0.0001
90.00	-3.587	4.408	1.151	0.0001
112.50	-3.881	4.837	1.910	0.0001
135.00	-3.731	4.196	2.328	0.0001
157.50	-3.128	2.640	2.339	0.0000
180.00	-2.038	0.386	1.872	0.0000
202.50	-0.453	-2.297	0.826	0.0000
225.00	1.545	-5.113	-0.901	0.0000
247.50	3.771	-7.789	-3.365	0.0000

TABLE 5.2C

(T-FILE POSITION) - (TRNSNEW POSITION)(ARC-LENGTH = 7.1 HOURS)

PROGRAM TRNSNEW: APPROXIMATION OF MACROMETRICS' T-FILES

TERRESTRIAL-,SOLAR,LUNAR GRAVITY CONSTANTS :
GM= 0.39860047D+15 GMS= 0.1327125D+21 GMM= 0.4902789D+13
J2= 0.1082627D-02 J3= -0.2536000D-05 J4= -0.1623000D-05

SATELLITE NR. 1 ITERATION NR. 1

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	-0.000	0.000	-0.000	0.0010
22.50	-1.175	3.934	0.021	0.0010
45.00	-2.050	6.587	0.346	0.0010
67.50	-2.578	7.854	0.763	0.0010
90.00	-2.708	7.711	1.080	0.0010
112.50	-2.379	6.227	1.141	0.0010
135.00	-1.509	3.569	0.822	0.0010
157.50	0.000	-0.000	0.000	0.0010
180.00	2.213	-4.131	-1.476	0.0001
202.50	5.101	-8.423	-3.785	0.0001
225.00	8.480	-12.461	-7.070	0.0001
247.50	12.024	-15.871	-11.381	0.0001
270.00	15.315	-18.381	-16.612	0.0001
292.50	17.929	-19.847	-22.502	0.0001
315.00	19.517	-20.265	-28.665	0.0000
337.50	19.861	-19.768	-34.635	0.0000
360.00	18.908	-18.582	-39.928	0.0000
382.50	16.792	-16.994	-44.089	0.0000
405.00	13.817	-15.304	-46.750	0.0000
427.50	10.444	-13.791	-47.681	0.0000

SATELLITE NR. 1 ITERATION NR. 2

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	4.672	-17.175	-4.140	0.0010
22.50	0.773	-12.406	-4.292	0.0010
45.00	-2.187	-8.319	-3.470	0.0010
67.50	-4.253	-4.947	-2.134	0.0010
90.00	-5.566	-2.298	-0.644	0.0010
112.50	-6.307	-0.350	0.769	0.0010
135.00	-6.633	0.931	2.004	0.0010
157.50	-6.620	1.590	3.034	0.0010
180.00	-6.276	1.639	3.821	0.0001
202.50	-5.579	1.132	4.296	0.0001
225.00	-4.541	0.215	4.387	0.0001
247.50	-3.229	-0.975	4.020	0.0001
270.00	-1.763	-2.324	3.158	0.0001
292.50	-0.288	-3.756	1.807	0.0001
315.00	1.057	-5.234	0.011	0.0000
337.50	2.145	-6.756	-2.153	0.0000
360.00	2.872	-8.339	-4.578	0.0000
382.50	3.173	-10.008	-7.126	0.0000
405.00	3.027	-11.786	-9.623	0.0000
427.50	2.482	-13.695	-11.886	0.0000

TABLE 5.2D

(T-FILE POSITION)-(TRNSNEW POSITION) (ARC-LENGTH = 10.1 HOURS)

PROGRAM TRNSNEW: APPROXIMATION OF MACROMETRICS' T-FILES

TERRESTRIAL-,SOLAR,LUNAR GRAVITY CONSTANTS :
GM= 0.39860047D+15 GMS= 0.1327125D+21 GMM= 0.4902789D+13
J2= 0.1082627D-02 J3= -0.2536000D-05 J4= -0.1623000D-05

SATELLITE NR. 1 ITERATION NR. 1

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	-0.000	0.000	-0.000	0.0010
22.50	-1.175	3.934	0.021	0.0010
45.00	-2.050	6.587	0.346	0.0010
67.50	-2.578	7.854	0.763	0.0010
90.00	-2.708	7.711	1.080	0.0010
112.50	-2.379	6.227	1.141	0.0010
135.00	-1.509	3.569	0.822	0.0010
157.50	0.000	-0.000	0.000	0.0010
180.00	2.213	-4.131	-1.477	0.0005
202.50	5.101	-8.425	-3.786	0.0005
225.00	8.480	-12.468	-7.073	0.0005
247.50	12.024	-15.889	-11.387	0.0005
270.00	15.315	-18.417	-16.625	0.0005
292.50	17.930	-19.907	-22.523	0.0005
315.00	19.518	-20.356	-28.694	0.0005
337.50	19.865	-19.882	-34.668	0.0001
360.00	18.917	-18.714	-39.958	0.0001
382.50	16.810	-17.144	-44.114	0.0001
405.00	13.851	-15.478	-46.768	0.0001
427.50	10.502	-13.999	-47.694	0.0001
450.00	7.338	-12.919	-46.853	0.0001
472.50	4.959	-12.339	-44.420	0.0001
495.00	3.899	-12.223	-40.787	0.0000
517.50	4.510	-12.407	-36.519	0.0000
540.00	6.873	-12.608	-32.262	0.0000
562.50	10.749	-12.482	-28.608	0.0000
585.00	15.602	-11.716	-25.971	0.0000
607.50	20.697	-10.119	-24.477	0.0000

SATELLITE NR. 1 ITERATION NR. 2

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	6.630	-8.265	0.357	0.0010
22.50	1.883	-7.880	-1.707	0.0010
45.00	-1.980	-7.823	-2.690	0.0010
67.50	-4.933	-8.020	-2.980	0.0010
90.00	-7.059	-8.372	-2.893	0.0010
112.50	-8.508	-8.762	-2.625	0.0010
135.00	-9.424	-9.086	-2.247	0.0010
157.50	-9.892	-9.254	-1.755	0.0010
180.00	-9.928	-9.428	-1.225	0.0005
202.50	-9.511	-9.737	-0.762	0.0005
225.00	-8.643	-10.060	-0.416	0.0005
247.50	-7.374	-10.293	-0.263	0.0005
270.00	-5.805	-10.369	-0.362	0.0005
292.50	-4.068	-10.259	-0.754	0.0005
315.00	-2.307	-9.974	-1.456	0.0005
337.50	-0.669	-9.552	-2.459	0.0001
360.00	0.695	-9.062	-3.732	0.0001
382.50	1.657	-8.590	-5.200	0.0001
405.00	2.118	-8.226	-6.743	0.0001
427.50	2.042	-8.065	-8.202	0.0001
450.00	1.478	-8.192	-9.407	0.0001
472.50	0.551	-8.664	-10.214	0.0001
495.00	-0.558	-9.494	-10.538	0.0000
517.50	-1.667	-10.650	-10.368	0.0000
540.00	-2.647	-12.047	-9.742	0.0000
562.50	-3.461	-13.569	-8.693	0.0000
585.00	-4.161	-15.123	-7.200	0.0000
607.50	-4.838	-16.682	-5.134	0.0000

TABLE 5.2E

(T-FILE POSITION) - (TRNSNEW POSITION) (ARC-LENGTH = 14.5 HOURS)

PROGRAM TRNSNEW: APPROXIMATION OF MACROMETRICS' T-FILES

TERRESTRIAL-,SOLAR,LUNAR GRAVITY CONSTANTS :
GM= 0.39860047D+15 GMS= 0.1327125D+21 GMM= 0.4902789D+13
J2= 0.1082627D-02 J3= -0.2536000D-05 J4= -0.1623000D-05

SATELLITE NR. 1 ITERATION NR. 1

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	-0.000	0.000	-0.000	0.0010
22.50	-1.175	3.934	0.021	0.0010
45.00	-2.050	6.587	0.346	0.0010
67.50	-2.578	7.854	0.763	0.0010
90.00	-2.708	7.711	1.080	0.0010
112.50	-2.379	6.227	1.141	0.0010
135.00	-1.509	3.569	0.822	0.0010
157.50	0.000	-0.000	0.000	0.0010
180.00	2.213	-4.131	-1.476	0.0001
202.50	5.101	-8.423	-3.785	0.0001
225.00	8.480	-12.461	-7.070	0.0001
247.50	12.024	-15.871	-11.381	0.0001
270.00	15.315	-18.381	-16.612	0.0001
292.50	17.929	-19.847	-22.502	0.0001
315.00	19.517	-20.265	-28.665	0.0001
337.50	19.861	-19.770	-34.636	0.0001
360.00	18.908	-18.590	-39.931	0.0001
382.50	16.792	-17.014	-44.096	0.0001
405.00	13.819	-15.343	-46.763	0.0001
427.50	10.449	-13.857	-47.704	0.0001
450.00	7.255	-12.763	-46.873	0.0001
472.50	4.836	-12.165	-44.445	0.0000
495.00	3.726	-12.042	-40.812	0.0000
517.50	4.280	-12.230	-36.539	0.0000
540.00	6.582	-12.445	-32.265	0.0000
562.50	10.395	-12.341	-28.580	0.0000
585.00	15.187	-11.602	-25.894	0.0000
607.50	20.231	-10.037	-24.335	0.0007
630.00	24.754	-7.677	-23.706	0.0007
652.50	28.146	-4.793	-23.522	0.0007
675.00	30.105	-1.865	-23.125	0.0007
697.50	30.743	0.502	-21.849	0.0007
720.00	30.572	1.693	-19.178	0.0007
742.50	30.435	1.190	-14.878	0.0007
765.00	31.373	-1.334	-9.069	0.0002
787.50	34.471	-5.965	-2.272	0.0002
810.00	40.678	-12.495	4.614	0.0002
832.50	50.631	-20.414	10.384	0.0002
855.00	64.480	-28.934	13.665	0.0002
877.50	81.794	-37.072	13.112	0.0002

TABLE 5.2E (CONTINUED)

SATELLITE NR. 1 ITERATION NR. 2

TIME(MIN)	DX	DY	DZ	INT.-ERROR
0.00	-31.633	26.478	24.988	0.0010
22.50	-28.273	18.632	21.321	0.0010
45.00	-25.437	12.118	17.112	0.0010
67.50	-23.343	6.888	12.685	0.0010
90.00	-22.096	2.864	8.383	0.0010
112.50	-21.688	-0.046	4.558	0.0010
135.00	-21.975	-1.935	1.546	0.0010
157.50	-22.676	-2.899	-0.395	0.0010
180.00	-23.413	-3.708	-1.420	0.0001
202.50	-23.785	-5.023	-1.840	0.0001
225.00	-23.467	-6.687	-1.843	0.0001
247.50	-22.287	-8.491	-1.709	0.0001
270.00	-20.243	-10.216	-1.732	0.0001
292.50	-17.484	-11.661	-2.157	0.0001
315.00	-14.270	-12.673	-3.150	0.0001
337.50	-10.927	-13.176	-4.769	0.0001
360.00	-7.798	-13.162	-6.955	0.0001
382.50	-5.182	-12.698	-9.534	0.0001
405.00	-3.284	-11.911	-12.233	0.0001
427.50	-2.162	-10.983	-14.736	0.0001
450.00	-1.704	-10.104	-16.744	0.0001
472.50	-1.642	-9.449	-18.052	0.0000
495.00	-1.615	-9.133	-18.611	0.0000
517.50	-1.259	-9.171	-18.537	0.0000
540.00	-0.304	-9.462	-18.085	0.0000
562.50	1.331	-9.819	-17.568	0.0000
585.00	3.522	-10.021	-17.265	0.0000
607.50	5.962	-9.891	-17.321	0.0007
630.00	8.255	-9.382	-17.697	0.0007
652.50	10.054	-8.596	-18.162	0.0007
675.00	11.172	-7.775	-18.359	0.0007
697.50	11.667	-7.252	-17.905	0.0007
720.00	11.841	-7.369	-16.490	0.0007
742.50	12.204	-8.415	-13.961	0.0007
765.00	13.404	-10.571	-10.370	0.0002
787.50	16.144	-13.884	-6.013	0.0002
810.00	21.073	-18.228	-1.440	0.0002
832.50	28.673	-23.285	2.569	0.0002
855.00	39.120	-28.554	5.076	0.0002
877.50	52.200	-33.388	5.109	0.0002

(a) How does insufficient orbital modelling influence baseline results?

(b) What precision in the baseline estimates may be achieved, if only relatively poor orbits are known, but if we estimate orbital parameters along with the baselines?

In order to answer the first question, the 6 long baselines of the Ottawa test (see Part A) were processed using the orbits produced by program TRNSNEW, when only the central force term was retained to model the force field. From Table 5.1a, we know that orbital errors of the order of several hundred metres may occur.

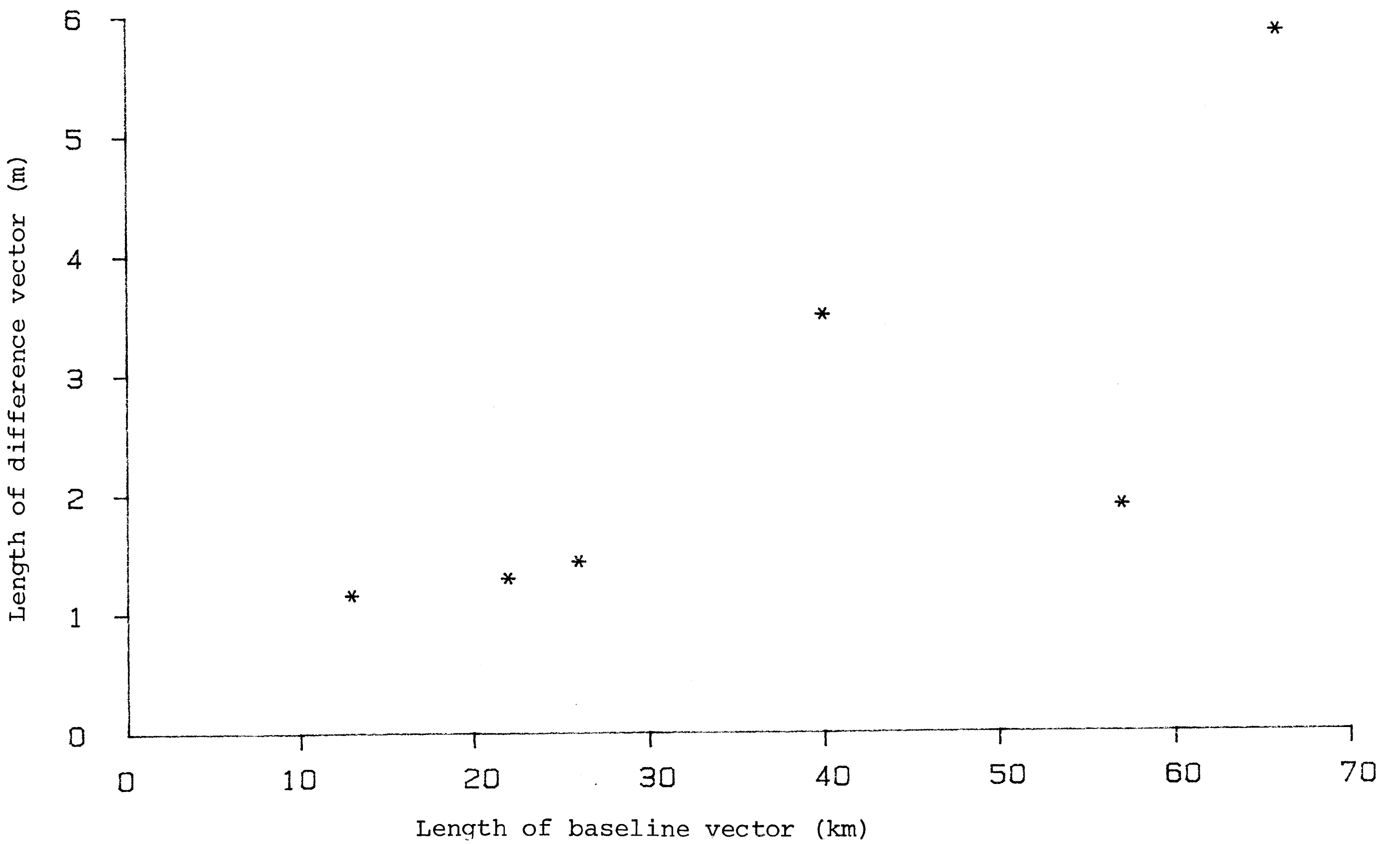
The effect of this improper modelling may be seen in Figure 5.1: errors of the order of 50 ppm to 100 ppm are introduced into the baseline estimates. This is somewhat larger than the effect we would expect from Table 2.2 (orbital errors of 625 m are supposed to give rise to baseline errors of only 25 ppm). If we bear in mind that in establishing eqn. (2.1) the orbital errors had to be considered as infinitesimal quantities (see Bauersima [1983]) which certainly is not true for the orbital errors involved here, the agreement between Figure 5.1 and Table 2.2 is not too bad.

In order to study the accuracy of the estimated baselines as a function of the accuracy of the orbits, we used the option of estimating orbital parameters and baselines simultaneusly with program PRMAC-3. The general characteristics of this program are given in Part A of this report. The orbital modelling and the orbit determination follows exactly the pattern given in the preceding sections. We therefore may restrict ourselves to the description of the practical aspects.

The program user first selects the sequence in which the osculating orbital elements (see Figure 4.1) are estimated. Next he chooses for each

FIGURE 5.1

Length of difference vector between baseline vector solutions using the best available orbits (central force plus J_2, J_3, J_4 plus luni-solar gravitation) and the simplest orbits (only central force).



observation session the number of orbital elements (between 0 and 6) to be estimated. The program user assigns a priori variances to each of the element types. (Actually it would be preferable and easy to implement in the program, fully populated variance-covariance matrices for the orbital elements. However, as this information at present is not readily available, the present version of PRMAC-3 does not have this option.)

The observations on the baselines of the Ottawa test were processed three times assuming three different sets of uncertainties for the orbital elements (see Table 5.3).

For the first trial, we assumed perfectly known orbits (as was done in Part A of this report). Actually no orbital elements were estimated there. In the second and third trials, five orbital elements per satellite were determined assuming the a priori uncertainties of Table 5.3. The argument of perigee was not estimated since, for short arcs of very nearly circular orbits, the time of perigee passage and the argument of perigee are almost perfectly correlated.

As the velocity of GPS satellites is very roughly 4 km s^{-1}, we are dealing with along-track errors of about 400 m and 4000 m for trials two and three, respectively. The cross-track and out-of-plane errors are roughly a factor of 4 smaller.

The a posteriori estimated variances for the coordinates of the free ends of the baselines (expressed in latitude ϕ, longitude λ, and height h) are compiled in Table 5.4.

It is amazing and encouraging to see that even with very pessimistic guesses for the orbit accuracy, it is still possible to estimate baselines of up to 70 km in length at the decimetre level when interferometric observations are used.

TABLE 5.3

A priori variances for the osculating orbital elements.

Trial	σ_a (m)	σ_e	σ_i (")	σ_Ω (")	σ_{T_o} (sec)
1	0	0	0	0	0
2	10	$5 \cdot 10^{-6}$	1	1	0.1
3	100	$5 \cdot 10^{-5}$	10	10	1.0

TABLE 5.4

Estimated r.m.s. errors in centimetres for the free ends of the baselines for the sets of uncertainties of Table 5.3
ϕ = latitude, λ = longitude, h = height.

Baseline Length (km)	Trial #1			Trial #2			Trial #3		
	σ_ϕ	σ_λ	σ_h	σ_ϕ	σ_λ	σ_h	σ_ϕ	σ_λ	σ_h
13	0.4	0.8	0.5	1.1	3.0	1.9	4.0	10.7	6.1
22	0.4	0.7	0.5	1.8	2.5	1.8	3.5	5.4	6.2
26	0.4	0.6	0.5	1.5	2.4	1.4	3.2	5.0	3.5
40	0.5	0.9	0.6	2.4	5.6	3.2	5.3	13.1	6.8
57	0.8	1.1	0.9	3.7	5.0	3.0	5.0	9.0	4.7
66	0.7	1.2	0.9	3.3	6.0	3.5	4.8	9.1	5.8

6. DESIGN OF A FUTURE GENERAL ORBIT DETERMINATION SOFTWARE PACKAGE FOR GPS SATELLITES

We have stated in the introductory section that many nonphysical approaches for estimating orbital parameters have been developed for processing Transit Doppler data. These techniques are inadequate if the orbits are of primary interest. The only degrees of freedom an investigator has in orbit determination are choice of arc lengths, the formulation of the problem (initial or boundary value problem), and the number of parameters to be estimated per satellite (six parameters defining initial or boundary conditions and perhaps additional dynamical parameters).

One of the most powerful features of the GPS system is the possibility of observing more than one satellite virtually simultaneously. This means that the contribution of the receiver-clock error to the observable cancels out if the observations of two different satellites made by the same receiver at the same time are subtracted. Any orbit determination program, which wants to make full use of the potential of these measurements, has to take the clock errors into account. This in turn implies that for n simultaneously observed orbits we are not dealing with separate orbit determination problems but with one problem where 6n orbital parameters have to be estimated (together with receiver-clock errors, station coordinates, etc.).

It is evident that a general simulation program is absolutely mandatory for complex problems of this kind. Among many others the following questions can only be answered by simulation techniques:

- Choice of a proper force field as a function of the arc length (see section 5.2 for a discussion of short arc applications).
- Influence of imperfectly modelled effects (e.g., residual ionosphere, troposphere, receiver coordinates, clock errors).
- Optimum choice of receiver sites for a permanent tracking network.

An orbit determination program for the orbits of GPS satellites should fulfill the following minimum requirements:

- The program should be designed to process all observations of all simultaneously observed satellites in the same program run.
- It must be possible to solve for any combination of the following parameters:
    - orbit parameters (6 for initial or boundary conditions, one or more dynamical parameters)
    - receiver coordinates
    - observation type-specific parameters, such as clock and ambiguity parameters for phase observations.
- It must be possible to introduce a priori information for the above listed parameters via variance-covariance matrices.
- The program should be general enough to accept additional observations of nonpermanent high-precision receivers in order to produce networks of highest possible precision.
- For users of the orbits determined by such a program, variance-covariance matrices for the orbital elements should be made available.
- In a first stage, the program system may be designed for relatively short arcs (one half of a revolution or less), allowing an approximate computation of the partials as indicated in section 4.4. The program

structure should make it easy to switch to the correct computation of these quantities, as given in section 3.3, in the future, when longer arcs may be processed.

7. CONCLUSIONS

In this part of the report we have been dealing with some of the orbital aspects of positioning using GPS. Following the introductory remarks and the discussion of precision requirements, the principles of modern orbit determination and the tools of numerical integration were developed in sections 3 and 4. It was demonstrated in section 5 that the problem of modelling orbital biases may be solved correctly and efficiently without resorting to nonphysical modelling techniques. The question of adequate physical modelling of short satellite arcs (for short baselines) was discussed in section 5.2, and an illustration of the influence of degraded orbit models on baseline results was given in section 5.3. In the same section we dealt with the problem of estimating orbit parameters together with other parameters (receiver coordinates, ambiguity parameters, etc.). Table 5.4 illustrated the precision that may be expected for the baselines of the Ottawa Macrometer® campaign, if we assume that the orbits are known a priori only to a certain precision. Finally, in section 6 we dealt with some aspects of a future general orbit determination program.

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PART C

MISCELLANEOUS REMARKS

1. CORRELATIONS BETWEEN DOUBLE DIFFERENCE OBSERVATIONS

Our parameter estimation programs, PRMAC-3 and PRMNET, process so-called double difference observations: the difference between two interferometric phase observations (single differences) of two different satellites made at the same time. The observation equations of these observation types are given by eqns. (1) and (6) of Part A of this report.

Different double difference observations pertaining to the same observation time are mathematically correlated. In our programs, this correlation may be accounted for as follows: Let us assume that single differences for n satellites have been observed at the same time t_i. Let us further assume that the observation errors ε_i, i=1,2,...,n of these n single differences are independent, normally distributed random errors with zero means and with a common variance σ_o^2. Starting from these single differences, n(n-1)/2 different double differences may be formed, where only n-1 of these are linearly independent. In our software, we always form the differences 1-2, 2-3,..., (n-1)-n. If we designate the errors of these double differences with δ_i, i=1,2,...,n-1, the relation between the ε_i and the δ_i may be stated by the following matrix relation:

$$\underline{\delta}_n = \underline{G}_n \, \underline{\varepsilon}_n \quad , \tag{1}$$

where

$$\underline{\delta}_n = (\delta_1, \delta_2, \ldots, \delta_{n-1}) \quad , \tag{2a}$$

$$\underline{\varepsilon}_n = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n) \quad , \tag{2b}$$

$$\underline{G}_n = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdots & 1 & -1 \end{bmatrix} \tag{2c}$$

The covariance matrix $\text{cov}(\underline{\delta}_n)$ is routinely formed by first computing the outer product $\underline{\delta}_n \, \underline{\delta}_n^T$, then taking the expectation value of this matrix equation:

$$\underline{\delta}_n \, \underline{\delta}_n^T = \underline{G}_n \, \underline{\varepsilon}_n \, \underline{\varepsilon}_n^T \, \underline{G}_n^T \quad , \tag{3}$$

$$\text{cov}(\underline{\delta}_n) = \underline{G}_n \, \text{cov}(\underline{\varepsilon}_n) \, \underline{G}_n^T \quad , \tag{4}$$

where, according to our assumptions concerning the errors ε_i, $i=1,2,\ldots,n$, we have:

$$\text{cov}(\underline{\delta}_n) = \sigma_o^2 \, \underline{G}_n \, \underline{G}_n^T \quad . \tag{5}$$

Finally, we define the following weight matrix:

$$\underline{P}_n = \sigma_o^{-2} \, (\underline{G}_n \, \underline{G}_n^T)^{-1} \quad . \tag{6}$$

These weight matrices are implemented in our programs in the following way. The linearized version of the double difference observation equations (see Part A, eqn. (6)) may be written in matrix notation as:

$$\underline{A} \, \underline{x} - \underline{w} = \underline{v} \quad , \tag{7}$$

where $\underline{A}$ is the design matrix,

$\underline{x}$ is the vector of unknown parameters,

$\underline{w}$ is the vector of misclosures,

$\underline{v}$ is the residual vector.

The least-squares solution is:

$$\hat{\underline{x}} = (\underline{A}^T \, \underline{P} \, \underline{A})^{-1} \, \underline{A}^T \, \underline{P} \, \underline{w} \quad , \tag{8}$$

where the total weight matrix $\underline{P}$ has a block diagonal structure, the nonzero blocks being defined by eqn. (6) above:

$$P = \begin{bmatrix} \underline{P}_{n_1} & & & 0 \\ & \underline{P}_{n_2} & & \\ & & \ddots & \\ 0 & & & P_{n_m} \end{bmatrix} \tag{9}$$

where n_i, $i=1,2,\ldots,m$ is the number of single difference observations at observation time t_i.

m is the number of observation epochs.

2. AMBIGUITY RESOLUTION

The first version of our ambiguity resolution software was outlined in Langley et al. [1984]. Since that time, the following three modifications have been made:

(a) A program option was implemented to divide the observation periods into subintervals and to estimate one set of ambiguity parameters (see Part A, eqn. (10)) in each subinterval. This option is used to remove cycle slips, which were not distinguished during the preprocessing phase.

(b) Originally we developed three strategies to remove the ambiguities. Strategy 3 has since been removed, as in all examples considered so far, strategies 1 and 2 solved the problem satisfactorily.

(c) The formulae used for ambiguity resolution had to be generalized in order to account for the weight matrix $\underline{P}$.

As the generalizations are straight forward, we only present here, without proof, those equations allowing an efficient computation of the sum of squared residuals weighted with matrix $\underline{P}$. Equation (9.20) in Langley et al. [1984] has to be modified as follows:

$$\underline{v}'^T \, \underline{P} \, \underline{v}' = e_1 + \underline{e}_2^T \, \underline{x}_2^* + \underline{x}_2^{*T} \, \underline{E}_3 \, \underline{x}_2^* \quad , \tag{10}$$

where $e_1 = \underline{w}^T \, \underline{P} \, \underline{w} - \underline{u}_1^T \, \underline{N}_{11}^{-1} \, \underline{u}_1$,

$\underline{e}_2^T = - \, 2(\underline{u}_2^T - \underline{u}_1^T \, \underline{N}_{11}^{-1} \, \underline{N}_{12})$,

$\underline{E}_3 = \underline{N}_{22} - \underline{N}_{12}^T \, \underline{N}_{11}^{-1} \, \underline{N}_{12}$,

and where

$\underline{w}$ is the misclosure vector (see Langley et al. [1984, eqn. (9.4)]

$\underline{P}$ is the weight matrix defined by eqn. (9) above

$\underline{u}_i = \underline{A}_i^T \, \underline{P} \, \underline{w}$, i=1,2

$\underline{N}_{ik} = \underline{A}_i^T \, \underline{P} \, \underline{A}_k$, i,k = 1,2

$\underline{A}_1$ and $\underline{A}_2$ are defined by eqn. (9.9) in Langley et al. [1984].

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