STUDIES IN THE APPLICATION OF THE GLOBAL POSITIONING SYSTEM TO DIFFERENTIAL POSITIONING

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PREFACE

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PREFACE

This technical report is a revised version of the final contract report prepared for the Geodetic Survey of Canada (GSC) under the terms of contract OSU82-00370. The Scientific Authorities at GSC for this contract were David Boal and Robin Steeves. The Principal Investigator (Project Coordinator) at the University of New Brunswick was Richard Langley. Brad Nickerson, Petr Vaníček, and David Wells were co-Principal Investigators.

Part of the work contained herein was funded by three research grants from the Natural Sciences and Engineering Research Council (NSERC) of Canada. The first of these, held by Richard Langley, is an operating grant entitled "Applications of Radio Interferometry in Geodesy and Geodynamics". The second, also held by Richard Langley, is an NSERC International Scientific Exchange Award to support the visit of Gerhard Beutler who is on leave of absence from the Astronomical Institute of the University of Bern. The third, held by David Wells, is an operating grant entitled "Arctic Navigation/ Satellite Positioning." Support under the New Technology Employment program of Employment and Immigration Canada also contributed to this work.

We thank Herb Valliant of the Earth Physics Branch of Energy, Mines and Resources Canada for the Macrometer observations and associated data, which were independently analysed under the terms of the contract. We thank Thomas Inzinga who transferred the analysis software from the University of New Brunswick's IBM 3032 system to the Department of Surveying Engineering's HP-1000 and who checked most of the equations in the theoretical development leading to the operational software, and Derek Davidson for operational assistance with the HP-1000. We thank Nick Christou for drawing Figure 4.1, and Wendy Wells for word processing the report.

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CHAPTER 1

INTRODUCTION

In an earlier contract for the Geodetic Survey of Canada (OSU80-00311) it was made clear that the NAVSTAR Global Positioning System (GPS) must be used in a differential mode in order to fulfil the positioning requirements of the Canadian geodetic and geodynamic community [Wells et al., 1981]. There are four basic types of differential GPS measurements which have been suggested: interferometric time delay, differential pseudorange, differential carrier phase, and differential integrated Doppler measurements. In our last contract (OSU81-00314), we developed mathematical models and computer software to generate and process simulated observations of these data types, and used this software to evaluate the accuracy of the different data types for a particular network of ground stations. The simulations indicated [Davidson et al., 1983] that, given appropriate satellite constellations and observing time spans,

a) interferometric delay and differential carrier phase observations are capable of satisfying accuracy specifications for crustal movement monitoring (1 cm to 2 cm);
b) interferometric delay, differential carrier phase and P-code differential pseudorange are capable of satisfying accuracy specifications for mining subsidence (5 cm to 10 cm);
c) interferometric delay, differential carrier phase, and P-code and C/A-code differential pseudorange are capable of satisfying accuracy specifications for rural cadastral surveying (25 cm to 50 cm);
d) all of the differential techniques, probably including Doppler, are capable of satisfying accuracy specifications for small-scale (e.g.,

1:50,000) mapping control (5 m).

For the present contract, we extended our simulation work to investigate a number of aspects of GPS differential positioning not previously considered and performed some analyses of real data.

The simulation work was carried out with a new computer program called VECA (for VECtor Adjustment). This program was written to implement a novel approach for analysing differential GPS observations.

We had earlier looked at the geometry of GPS positioning from a vectorial point of view [Davidson et al., 1983]. We subsequently further developed these ideas [Vaníček et al., 1983] and have now extended the mathematical model to the case of many ground stations operating simultaneously. We have combined this geometrical model with an adjustment algorithm that is based on filter theory. The least-squares adjustment model is the filter, the observations are the input and the observation residuals and parameter estimates are the output of the filter. The construction of the basic filter for the case of one baseline is discussed in Chapter 2, and the extension to many baselines is discussed in Chapter 3.

The non-geometrical aspects of the model in VECA are more or less identical to those implemented in our old program DIGAP [Davidson et al., 1983]. However, we have added to VECA an ability to estimate parameters describing the orbits of the GPS satellites. The mathematical models for the estimation of orbital parameters is described in Chapter 4.

The mathematical models have been implemented on both the University of New Brunswick (UNB) IBM 3032 (now a 3081) mainframe computer and on the UNB Department of Surveying Engineering HP-1000/F minicomputer. The HP implementation is described in Chapter 5.

Using the VECA software package one can pose a variety of "what if" questions to determine the capabilities of differential GPS positioning under different conditions. For the present work we were particularly interested in using VECA to determine the answers to the following questions:

(1) How inaccurate can the a priori coordinates of the ground stations be, before an adjustment fails to converge?

(2) What is the best satellite-receiver geometry for differential GPS positioning?

(3) Is it practical or worthwhile to combine more than one kind of differential GPS measurement type?

(4) What is the effect of different "Denial of Accuracy" degradation scenarios on differential GPS positioning?

After some consideration we arrived at an answer to question (4) without actually performing any simulations. Our reasoning is outlined in Chapter 6. An attempt was made to answer the other three questions using VECA. The results of these attempts are documented in Chapter 7.

During the contract period, we were fortunate to participate in and to obtain data from the test of the Macrometer Interferometric Surveyor by the Earth Physics Branch of Energy, Mines and Resources Canada. Some of these data have been analysed at UNB with a special purpose suite of programs that were developed specifically to handle Macrometer data. The development of these programs and the results of their use are described in Chapters 8 and 9.

Conclusions and recommendations are presented in Chapter 10.

PART A: MODEL AND SOFTWARE DEVELOPMENT

CHAPTER 2

THE CONSTRUCTION OF A FILTER

2.1 Filter for Differential Ranges

Throughout this development we shall use the terminology and notation introduced in Vaníček et al. [1983] (see Appendix B). In this notation the observation equation for an observed differential range $\Delta\rho^i$ involving satellite position S^i and two points P_1, P_2 is

$$\frac{\vec{u}^i}{\vec{u}^i \cdot \vec{e}_1^i} \cdot \Delta\vec{R}_{12} = - \Delta\rho^i \quad , \tag{2.1}$$

where only the three coordinate differences $\Delta\vec{R} = \vec{R}_2 - \vec{R}_1$ are unknown, and $\vec{e}_1^i$, $\vec{u}^i$ are known approximately. Denoting now the design matrix composed of

$$\vec{A}^i = \vec{u}^i/(\vec{u}^i \cdot \vec{e}_1^i) \tag{2.2}$$

by $\underline{A}^T$, the vector of observed differential ranges by $\underline{\Delta\rho}$ and $\Delta\vec{R}_{12}$ by $\underline{\Delta R}$, the system of observation equations becomes

$$\underline{A}^T \, \underline{\Delta R} = - \underline{\Delta\rho} \quad . \tag{2.3}$$

(Note the different definition of design matrix, i.e., transpositon, compared with standard notion in adjustments. This notation is adopted because it remains valid even for $\underline{A}$ being composed of only one vector $\vec{A}^i = \vec{u}^i/(\vec{u}^i \cdot \vec{e}_1^i)$.)

Let us now consider two groups of observed differential ranges, namely $\underline{\Delta\rho}_1$ and $\underline{\Delta\rho}_2$. They give the following two systems of observation equations:

$$\underline{A}_1^T \, \underline{\Delta R} = - \, \underline{\Delta \rho}_1 \quad , \quad \underline{C}_1$$

$$\underline{A}_2^T \, \underline{\Delta R} = - \, \underline{\Delta \rho}_2 \quad , \quad \underline{C}_2 , \, \underline{C}_{12} \quad , \tag{2.4}$$

where $\underline{C}_1$, $\underline{C}_2$ are the covariance matrices of $\underline{\Delta\rho}_1$ and $\underline{\Delta\rho}_2$ respectively, and $\underline{C}_{12}$ is the corresponding crosscovariance matrix.

The first system of observation equations yields the following system of normal equations:

$$(\underline{A}_1 \underline{C}_1^{-1} \underline{A}_1^T) \underline{\Delta R}^{(1)} = - \, \underline{A}_1 \underline{C}_1^{-1} \underline{\Delta\rho}_1 \quad , \tag{2.5}$$

or

$$\underline{\Delta R}^{(1)} = - \, (\underline{A}_1 \underline{C}_1^{-1} \underline{A}_1^T)^{-1} \, \underline{A}_1 \underline{C}_1^{-1} \underline{\Delta\rho}_1 \quad . \tag{2.6}$$

Taken together, the two groups of observations give the complete system of normal equations:

$$(\underline{A}_1 \underline{P}_1 \underline{A}_1^T + \underline{A}_1 \underline{P}_{12} \underline{A}_2^T + \underline{A}_2 \underline{P}_{21} \underline{A}_1^T + \underline{A}_2 \underline{P}_2 \underline{A}_2^T) \underline{\Delta R}^{(2)}$$

$$= - \, \underline{A}_1 \underline{P}_1 \underline{\Delta\rho}_1 - \underline{A}_1 \underline{P}_{12} \underline{\Delta\rho}_2 - \underline{A}_2 \underline{P}_{21} \underline{\Delta\rho}_1 - \underline{A}_2 \underline{P}_2 \underline{\Delta\rho}_2 \, , \tag{2.7}$$

or, briefly,

$$(\underline{N}_1 + \underline{N}_{12} + \underline{N}_{21} + \underline{N}_2) \underline{\Delta R}^{(2)} = \underline{U}_1 \underline{\Delta\rho}_1 + \underline{U}_{12} \underline{\Delta\rho}_2 + \underline{U}_{21} \underline{\Delta\rho}_1 + \underline{U}_2 \underline{\Delta\rho}_2 , \tag{2.8}$$

where

$$\underline{P}_1 = \underline{C}_1^{-1} + \underline{C}_1^{-1} \underline{C}_{12} (\underline{C}_2 - \underline{C}_{21} \underline{C}_1^{-1} \underline{C}_{12})^{-1} \underline{C}_{21} \underline{C}_1^{-1} \quad , \tag{2.9}$$

$$\underline{P}_{12} = - \, \underline{C}_1^{-1} \underline{C}_{12} (\underline{C}_2 - \underline{C}_{21} \underline{C}_1^{-1} \underline{C}_{12})^{-1} = \underline{P}_{21}^T \quad , \tag{2.10}$$

$$\underline{P}_2 = (\underline{C}_2 - \underline{C}_{21} \underline{C}_1^{-1} \underline{C}_{12})^{-1} \quad . \tag{2.11}$$

Denoting now

$$\underline{\Delta R}^{(2)} = \underline{\Delta R}^{(1)} + \underline{\delta R} \tag{2.12}$$

and realizing that $\underline{N}_{12} = \underline{N}_{21}^T$ we can write eqn. (2.8) as

$$(\underline{N}_1 + \underline{N}_{12} + \underline{N}_{12}^T + \underline{N}_2) \underline{\Delta R}^{(1)} + (\underline{N}_1 + \underline{N}_{12} + \underline{N}_{12}^T + \underline{N}_2) \underline{\delta R}$$

$$= \underline{U}_1 \underline{\Delta\rho}_1 + \underline{U}_{21} \underline{\Delta\rho}_1 + \underline{U}_{12} \underline{\Delta\rho}_2 + \underline{U}_2 \underline{\Delta\rho}_2 \quad . \tag{2.13}$$

Taking eqn. (2.5), i.e.,

$$\underline{N}_1 \underline{\Delta R}^{(1)} = \underline{U}_1 \underline{\Delta\rho}_1 \quad , \tag{2.14}$$

into account, we have

$$(\underline{N}_1 + \underline{N}_{12} + \underline{N}_{12}^T + \underline{N}_2)\underline{\delta R} = -(\underline{N}_{12} + \underline{N}_{12}^T + \underline{N}_2)\underline{\Delta R}^{(1)}$$
$$+ \underline{U}_{21}\underline{\Delta\rho}_1 + (\underline{U}_{12} + \underline{U}_2)\underline{\Delta\rho}_2 \quad . \tag{2.15}$$

The vector of residuals $\underline{r}^{(1)}$ from the first adjustment (eqn. (2.6)) is

$$\underline{r}^{(1)} = \underline{A}_1^T\underline{\Delta R}^{(1)} + \underline{\Delta\rho}_1 \quad . \tag{2.16}$$

Substituting for $\underline{\Delta\rho}_1$ in eqn. (2.15) from eqn. (2.16) we get

$$(\underline{N}_1 + \underline{N}_{12} + \underline{N}_{12}^T + \underline{N}_2)\underline{\delta R} = -(\underline{N}_{12} + \underline{N}_{12}^T + \underline{N}_2)\underline{\Delta R}^{(1)}$$
$$+ \underline{U}_{21}(\underline{A}_1^T\underline{\Delta R}^{(1)} + \underline{r}^{(1)}) + (\underline{U}_{12} + \underline{U}_2)\underline{\Delta\rho}_2 \quad . \tag{2.17}$$

Realizing now that $\underline{U}_{21}\underline{A}_1^T = -\underline{A}_2\underline{P}_{21}\underline{A}_1^T = -\underline{N}_{21}$ we can simplify eqn. (2.17) to read

$$(\underline{N}_1+\underline{N}_{12}+\underline{N}_{12}^T+\underline{N}_2)\underline{\delta R} = (\underline{U}_{12}+\underline{U}_2)\underline{\Delta\rho}_2 - (\underline{N}_{12}+\underline{N}_2)\underline{\Delta R}^{(1)} - \underline{U}_{21}\underline{r}^{(1)} \quad . \tag{2.18}$$

Denoting $\underline{\Delta\rho}_2 + \underline{A}_2^T\underline{\Delta R}^{(1)}$ (observed minus computed differential range vector) by $\underline{\Delta}_2$ we obtain finally

$$\boxed{(\underline{N}_1 + \underline{N}_{12} + \underline{N}_{12}^T + \underline{N}_2)\underline{\delta R} = (\underline{U}_{12} + \underline{U}_2)\underline{\Delta}_2 - \underline{U}_{21}\underline{r}^{(1)}} \quad , \tag{2.19}$$

the equation for the increment $\underline{\delta R}$ to the solution $\underline{\Delta R}^{(1)}$ (of the first system of normal equations) as a linear function of the misclosures $\underline{\Delta}_2$, i.e., the <u>filter equation</u> we have been looking for. Here, because $E(\underline{r}^{(1)})=0$, we obtain $\underline{U}_{21}\underline{r}^{(1)} = 0$.

Equation (2.19) takes into account (rigorously) the correlation $\underline{C}_{12}$ between the first group of observations $\underline{\Delta\rho}_1$ and the second group $\underline{\Delta\rho}_2$. As a result the filter is unwieldy. For this reason, we will assume, from now on, the correlation between $\underline{\Delta\rho}_1$ and $\underline{\Delta\rho}_2$ to be nonexistent, and the crosscovariance matrix $\underline{C}_{12}$ to be zero. Under these circumstances, we get

$$(\underline{N}_1 + \underline{N}_2)\underline{\delta R} = \underline{U}_2\underline{\Delta}_2 \quad , \tag{2.20}$$

or, written in the usual form

$$\boxed{\underline{\delta R} = (\underline{N}_1 + \underline{N}_2)^{-1}\ \underline{U}_2\underline{\Delta}_2} \quad . \tag{2.21}$$

This equation can be written also for a second "group" of observations $\underline{\Delta\rho}_2$ consisting only of one observation, $\Delta\rho^i$, in which case it leads to a recursive formula for $\underline{\delta R}$. We denote

$\underline{C}_2^{-1}=\sigma_{\Delta\rho i}^{-2}=\sigma_i^{-2}$, $\underline{A}_2=\vec{A}^i$, $\underline{N}_1=\underline{N}^{(i-1)}$, $\underline{N}_2=\underline{\Delta N}_i$, $\underline{N}^{(i-1)}+\underline{\Delta N}_i=\underline{N}^i$, $\underline{\Delta R}^{(i)}=\underline{\Delta R}^{(i-1)}+\underline{\delta R}_i$,

and get

$$\underline{\delta R}_i = -\ (\underline{N}^{(i)})^{-1}\ \sigma_i^{-2}\vec{A}^i\overbrace{(\Delta\rho^i - \vec{A}^i \cdot \Delta\vec{R}^{(i-1)})}^{\Delta_i} \quad . \tag{2.22}$$

Other shapes of the filter are possible. It appears to us however that this particular form is the simplest from the mathematical point of view and thus particularly suitable for the later investigation of non-geometrical effects.

2.2 Convergence of Differential Range Filter

To study the rate of convergence of the sequence of solutions, let us first rewrite $\vec{A}^i$ as follows:

$$\vec{A}^i = \frac{\vec{u}^i}{\vec{u}^i \cdot \vec{e}_1^i} = \frac{\vec{u}^i}{\frac{1}{2}(\vec{e}_1^i + \vec{e}_2^i) \cdot \vec{e}_1^i} = \frac{\vec{u}^i}{\frac{1}{2}(1 + \cos\omega^i)} \quad , \tag{2.23}$$

where ω^i is the paralactical angle (the angle under which the baseline $\Delta\vec{R}$ is subtended, viewed from S^i). Realizing that

$$\frac{1}{2}(1 + \cos\omega^i) = \cos^2\frac{\omega^i}{2} \quad \text{and} \quad u^i = \cos\frac{\omega^i}{2} \quad ,$$

we get

$$\vec{A}^i = \frac{\vec{u}^i}{u^i \cos\frac{\omega^i}{2}} \quad . \tag{2.24}$$

Further, since $\delta\underline{N}_i$ can be written as

$$\delta\underline{N}_i = \sigma_i^{-2}\, \vec{A}^i \otimes \vec{A}^i = \sigma_i^{-2} \cos^{-2}\frac{\omega^i}{2}\; \frac{\vec{u}^i \otimes \vec{u}^i}{(u^i)^2} \quad , \tag{2.25}$$

where

$$\frac{\vec{u}^i \otimes \vec{u}^i}{\vec{u}^i \cdot \vec{u}^i} = \begin{bmatrix} \cos^2\alpha_1^i & \cos\alpha_1^i\cos\alpha_2^i & \cos\alpha_1^i\cos\alpha_3^i \\ \cos\alpha_1^i\cos\alpha_2^i & \cos^2\alpha_2^i & \cos\alpha_2^i\cos\alpha_3^i \\ \cos\alpha_1^i\cos\alpha_3^i & \cos\alpha_2^i\cos\alpha_3^i & \cos^2\alpha_3^i \end{bmatrix} \tag{2.26}$$

is the matrix of products of direction cosines of $\vec{u}^i$.

Now, the complete matrix $\underline{N}^{(i)}$ of normal equations is merely

$$\underline{N}^{(i)} = \sum_{j=0}^{i} \Delta\underline{N}_j = \sum_{j=0}^{i} \frac{1}{\sigma_j^2 \cos^2\frac{\omega^j}{2}} \cdot \frac{\vec{u}^j \otimes \vec{u}^j}{\vec{u}^j \cdot \vec{u}^j} \quad . \tag{2.27}$$

Let us assume, without any detriment of generality,

$$\forall j : \sigma_j = \sigma \quad . \tag{2.28}$$

We also realize that for a random distribution of S^j over the zenithal hemisphere

$$\lim_{i\to\infty} \sum_{j=0}^{i} \cos\alpha_k^j \cos\alpha_\ell^j = \begin{cases} 0 & k \neq \ell \\ 0.5 & k = \ell \end{cases} \quad . \tag{2.29}$$

Hence

$$\lim_{i\to\infty} \underline{N}^{(i)} \doteq \lim_{i\to\infty} \frac{1}{2\sigma^2} \sum_{j=0}^{i} \underline{I} = \lim_{i\to\infty} \frac{i}{2\sigma^2}\, \underline{I} \tag{2.30}$$

and, approximately

$$\delta\vec{R}_i \to -\frac{2}{i}\frac{\vec{u}^i}{u^i}\Delta_i \quad , \tag{2.31}$$

which clearly tends to $\vec{0}$ as i increases, since $\vec{u}/u$ is a unit vector and Δ is small for all i. This shows that the sequence of solutions converges. It must evidently converge to the best least-squares value since it is equivalent to a complete least-squares solution.

The convergence may be slow, when the initial $\underline{\Delta R}^{(o)}$ is very far away from the final solution. As a matter of fact the solution may not even converge to the right solution, because $\underline{N}^{(i-1)}$ in eqn. (2.22) does not have the benefit of using the best estimate $\underline{\Delta R}^{(i)}$ in its evaluation, and also the misclosure Δ_i should be computed from $\underline{\Delta R}^{(i)}$. Thus $\underline{N}^{(i-1)}$ (and perhaps even Δ_i) may have to be updated after each step particularly at the beginning of the process when the initial value $\underline{\Delta R}^{(o)}$ is far removed from the correct solution.

If the matrix $\underline{N}$ of normal equations has been updated at each step of the filter, then $\underline{N}^{(i-1)}$ reflects the knowledge $\underline{\Delta R}^{(i-1)}$ and should be updated for the effect of $\underline{\delta R}_i$. Let us first write the expression for $\underline{N}^{(i-1)}$ as follows:

$$\underline{N}^{(i-1)} = \sum_{j=0}^{i-1} \sigma_j^{-2}\vec{A}^j \otimes \vec{A}^j = \sum_{j=0}^{i-1} \underline{\Delta N}_j \quad . \tag{2.32}$$

In this expression, each $\vec{A}^j$ has to be corrected by $\delta\vec{A}^j$ caused by $\underline{\delta R}_i$ to obtain

$$\underline{\Delta N}_j^* = \underline{\Delta N}_j + \underline{\delta N}_j = \sigma_j^{-2}(\vec{A}^j + \delta\vec{A}^j) \otimes (\vec{A}^j + \delta\vec{A}^j) \quad . \tag{2.33}$$

Now, from eqn. (2.2) we can write (leaving out superscripts):

$$\begin{aligned}\vec{A} + \delta\vec{A} &= (\vec{u} + \delta\vec{u})/[(\vec{u} + \delta\vec{u}) \cdot \vec{e}_1] \\ &\doteq (\vec{u} + \delta\vec{u})/(\vec{u} \cdot \vec{e}_1)(1 - \frac{\delta\vec{u} \cdot \vec{e}_1}{\vec{u} \cdot \vec{e}_1}) \\ &\doteq \frac{\vec{u}}{\vec{u} \cdot \vec{e}_1} + \frac{\delta\vec{u}}{\vec{u} \cdot \vec{e}_1} - \frac{\vec{u}(\delta\vec{u} \cdot \vec{e}_1)}{(\vec{u} \cdot \vec{e}_1)^2} \\ &= \vec{A} + \frac{\delta\vec{u}}{\vec{u} \cdot \vec{e}_1} - \vec{A}\,\frac{(\delta\vec{u} \cdot \vec{e}_1)}{(\vec{u} \cdot \vec{e}_1)} \quad ,\end{aligned} \tag{2.34}$$

all under the assumption of $\delta u \ll u$. Realizing now that $\vec{u} \cdot \vec{e}_1 \sim 1$ we get

$$\delta\vec{A} \doteq \delta\vec{u} - \vec{A}(\delta\vec{u} \cdot \vec{e}_1) \quad . \tag{2.35}$$

Because $\delta\vec{u}$ is only due to the change $\delta\vec{e}_2$ in $\vec{e}_2$, we have (from $\vec{u}=1/2(\vec{e}_1+\vec{e}_2)$):

$$\delta\vec{u} = \frac{1}{2}\delta\vec{e}_2 \quad . \tag{2.36}$$

On the other hand, from the definition of $\vec{e}_2$ (i.e., $\vec{e}_2 = \vec{\rho}_2/\rho_2$) we have:

$$\vec{e}_2 + \delta\vec{e}_2 = \frac{\vec{\rho}_2 + \delta\vec{\rho}_2}{|\vec{\rho}_2 + \delta\vec{\rho}_2|} \quad . \tag{2.37}$$

From the definition of $\vec{\rho}_2$ (i.e., $\vec{\rho}_2 = \vec{r} - \vec{R}_2$) it follows that

$$\delta\vec{\rho}_2 = -\,\delta\vec{R} \quad . \tag{2.38}$$

We also have

$$|\vec{\rho}_2 + \delta\vec{\rho}_2| = \rho_2 - \vec{e}_2 \cdot \delta\vec{R} \quad , \tag{2.39}$$

and

$$\begin{aligned}
\vec{e}_2 + \delta\vec{e}_2 &\doteq \frac{\vec{\rho}_2 - \delta\vec{R}}{\rho_2}\left(1 + \frac{\vec{e}_2 \cdot \delta\vec{R}}{\rho_2}\right) \\
&= \left(\frac{\vec{\rho}_2}{\rho_2} - \frac{\delta\vec{R}}{\rho_2}\right)\left(1 + \frac{\vec{e}_2 \cdot \delta\vec{R}}{\rho_2}\right) \\
&\doteq \vec{e}_2 - \frac{\delta\vec{R}}{\rho_2} + \vec{e}_2\,\frac{\vec{e}_2 \cdot \delta\vec{R}}{\rho_2} \quad .
\end{aligned} \tag{2.40}$$

Hence

$$\delta\vec{e}_2 \doteq \vec{e}_2\frac{\vec{e}_2 \cdot \delta\vec{R}}{\rho_2} - \frac{\delta\vec{R}}{\rho_2} \quad , \tag{2.41}$$

$$\delta\vec{u} \doteq \frac{1}{2\rho_2}[(\vec{e}_2 \cdot \delta\vec{R})\vec{e}_2 - \delta\vec{R}] \quad , \tag{2.42}$$

and finally

$$\delta\vec{A} \doteq \frac{1}{2\rho_2}\,[(\vec{e}_2 \cdot \delta\vec{R})\vec{e}_2 - \delta\vec{R} - \vec{A}((\vec{e}_2 \cdot \delta\vec{R})(\vec{e}_2 \cdot \vec{e}_1) - \delta\vec{R} \cdot \vec{e}_1)] \quad . \tag{2.43}$$

Realizing that $\vec{e}_2 \cdot \vec{e}_1 \backsim 1$, $\vec{e}_1 \backsim \vec{e}_2 \backsim \vec{A}$ we can further reduce this equation to

$$\delta\vec{A} \doteq \frac{1}{2\rho_2}[2(\vec{A} \cdot \delta\vec{R})\vec{A} - \delta\vec{R}] \quad . \tag{2.44}$$

Substitution of eqn. (2.44) into eqn. (2.33) yields:

$$\begin{aligned}
\Delta\underline{N}_j + \delta\underline{N}_j &\doteq \sigma_j^{-2}\left(\vec{A}^j + \frac{(\vec{A}^j \cdot \delta\vec{R}_i)\vec{A}^j - \delta\vec{R}_i}{2\rho_2^j}\right) \otimes \left(\vec{A}^j + \frac{(\vec{A}^j \cdot \delta\vec{R}_i)\vec{A}^j - \delta\vec{R}_i}{2\rho_2^j}\right) \\
&\doteq \sigma_j^{-2}\left[\vec{A}^j \otimes \vec{A}^j + \frac{\vec{A}^j \cdot \delta\vec{R}_i}{2\rho_2^j}\,\vec{A}^j \otimes \vec{A}^j - \frac{1}{2\rho_2^j}(\vec{A}^j \otimes \delta\vec{R}_i + \delta\vec{R}_i \otimes \vec{A}^j)\right] \\
&= \Delta\underline{N}_j + \frac{\vec{A}^j \cdot \delta\vec{R}_i}{2\rho_2^j}\,\Delta\underline{N}_j - \frac{\sigma_j^{-2}}{\rho_2^j}\,\mathrm{sym}(\vec{A}^j \otimes \delta\vec{R}_i) \quad .
\end{aligned} \tag{2.45}$$

The total correction (update) $\underline{\delta N}_i$ to $\underline{N}^{(i-1)}$ is then given by

$$\underline{\delta N}_i = \sum_{j=0}^{i-1} \frac{\vec{A}^j \cdot \delta \vec{R}_i}{2\rho_2^j} \underline{\Delta N}_j - \mathrm{sym}\left(\sum_{j=0}^{i-1} \frac{\vec{A}^j}{\rho_2^j \sigma_j^2} \otimes \delta \vec{R}_i \right) \quad . \tag{2.46}$$

This equation is difficult to implement efficiently. Clearly, it may be more economical to always go all the way back and restart the process for j=0 with better and better initial approximation $\underline{\Delta R}^{(o)}$ until changes $\underline{\delta R}_i$ are "sufficiently small". This practice would also dispose of the problem with updating Δ_i. An estimate of what is sufficiently small may be obtained from eqn. (2.22), which can be written briefly as

$$\underline{\delta R} = - \underline{M}\ \underline{\Delta^*} \quad , \tag{2.47}$$

where $\underline{M}$ stands for $\underline{N}^{-1}\sigma^{-2}\vec{A}$ and $\underline{\Delta^*}$ is the observation misclosure. Clearly Δ^* is of the order of $\delta\underline{R}$ while the elements of $\underline{M}$ are at worst of the order of 1. Thus, if we want to determine $\underline{\delta R}$ to an accuracy of 1 mm, the product $\underline{dM\Delta}^*$, where $\underline{dM}$ is the admissible error in $\underline{M}$, should be smaller than 1 mm.

Now, disregarding the variances, the error $\underline{dM}$ would be of the same order of magnitude as that of $\underline{N}$, $\underline{dN}$. The error $\underline{dN}$, in turn, will be of the order of two times the error in direction cosines, i.e., of the order of $2\delta\underline{R}/\rho$. Taking $\rho \sim 2\times 10^7$ m, we get the final result that "sufficiently small" changes $\underline{\delta R}_i$ should be smaller than 100 m.

Another alternative to the rigorous filter update would be to make the first few observations (until $\delta R_i < 100$ m is reached) look less accurate than they actually are. This would give the new observations a better chance to change $\underline{\Delta R}$ to what it should be, i.e., to get the filter unstuck from a possibly biased value $\underline{\Delta R}$. A reasonable choice appears to be

$$\sigma_i^* = \sigma_i/\sqrt{i} \quad , \tag{2.48}$$

where σ_i is the actual standard deviation of ith observation and σ_i^* is the artificial value. The artificial standard deviation should be applied, instead of σ_i, in eqn. (2.22).

2.3 Filter for Other Observables

Let us consider here two more observables (in addition to differential ranges $\Delta\rho$): range differences ($\nabla\rho$) and ranges (ρ). The observation equations for range differences (Doppler) read:

$$\left.\begin{aligned} \nabla\vec{u}_1 \cdot \vec{R}_1 &= -\nabla\rho_1 + \vec{e}_1 \cdot \Delta\vec{r} + \nabla\vec{u}_1 \cdot \vec{r} \\ \nabla\vec{u}_2 \cdot \vec{R}_2 &= -\nabla\rho_2 + \vec{e}_2 \cdot \Delta\vec{r} + \nabla\vec{u}_2 \cdot \vec{r} \end{aligned}\right| \qquad (2.49)$$

(for notation and derivation see Vaníček et al. [1983], Appendix B). The observation equations for ranges are

$$\left.\begin{aligned} \vec{e}_1 \cdot \vec{R}_1 &= -\rho_1 + \vec{e}_1 \cdot \vec{r} \\ \vec{e}_2 \cdot \vec{R}_2 &= -\rho_2 + \vec{e}_2 \cdot \vec{r} \quad . \end{aligned}\right| \qquad (2.50)$$

Clearly, unless simultaneous observations are made at the two ground stations, i.e., unless $\Delta\vec{r}$'s and/or $\vec{r}$'s in the pairs of corresponding equations are the same, it would be obviously quite superficial to convert these equations to observation equations for $\Delta\vec{R}$. Thus the treatment of these observables is better left for Chapter 3, where we develop the observation equations and the filter equations for position vectors $\vec{R}_i$ of a network of ground stations. However, even if the observations are made simultaneously, we feel that the contribution of these two kinds of observables will be felt most strongly in the positioning of the baseline (or network) rather than in the length of the baseline (or relative positions of the network points). Hence we shall not even attempt to construct the filter equations for a baseline using these two additional observables. The rest of our investigations in this chapter will concentrate on differential ranging.

2.4 Clock Errors and Filtering

Let us assume now the most simple-minded (unknown) errors in the receivers' clocks: constant offsets ΔT_1, ΔT_2 with respect to the satellites' clocks. We shall show that the presence of these two unknown offsets can be rigorously accounted for simply by changing the weight matrix of the observed differential ranges.

Let us begin by rewriting observation equation (2.1) as follows:

$$- \vec{A}^i \cdot \Delta\vec{R} = \rho_2^i + c\Delta T_2 - \rho_1^i - c\Delta T_1 \quad , \tag{2.51}$$

where c is the speed of light. Denoting $\Delta T_2 - \Delta T_1$ by δT we get

$$- \vec{A}^i \cdot \Delta\vec{R} - c\delta T = \Delta\rho^i \quad . \tag{2.52}$$

The system of observation equations then becomes (cf. eqn. (2.3)):

$$\left[\underline{a} \;\middle|\; \underline{A}^T \right] \begin{bmatrix} \delta T \\ \hline \underline{\Delta R} \end{bmatrix} = - \underline{\Delta\rho} \quad , \tag{2.53}$$

where

$$\underline{a} = - c(1, 1, \ldots, 1)^T = - c\underline{\alpha} \quad . \tag{2.54}$$

The corresponding normal equations read:

$$\begin{bmatrix} \underline{a}^T \\ \hline \underline{A} \end{bmatrix} \underline{C}^{-1} \left[\underline{a} \;\middle|\; \underline{A}^T \right] \begin{bmatrix} \delta T \\ \hline \underline{\Delta R} \end{bmatrix} = - \begin{bmatrix} \underline{a}^T \\ \hline \underline{A} \end{bmatrix} \underline{C}^{-1} \underline{\Delta\rho} \quad , \tag{2.55}$$

or

$$\left[\begin{array}{c|c} \underline{a}^T\underline{C}^{-1}\underline{a} & \underline{a}^T\underline{C}^{-1}\underline{A}^T \\ \hline \underline{AC}^{-1}\underline{a} & \underline{AC}^{-1}A^T \end{array} \right] \begin{bmatrix} \delta T \\ \hline \underline{\Delta R} \end{bmatrix} = - \begin{bmatrix} \underline{a}^T \\ \hline \underline{A} \end{bmatrix} \underline{C}^{-1} \underline{\Delta\rho} \quad . \tag{2.56}$$

Rewriting these equations as

$$\begin{bmatrix} \underline{N}_{11} & | & \underline{N}_{12} \\ - & - & - \\ \underline{N}_{21} & | & \underline{N}_{22} \end{bmatrix} \begin{bmatrix} \delta T \\ - \\ \underline{\Delta R} \end{bmatrix} = - \begin{bmatrix} \underline{a}^T \\ - \\ \underline{A} \end{bmatrix} \underline{C}^{-1} \underline{\Delta\rho} \quad , \tag{2.57}$$

the solution is given in the following form:

$$\begin{bmatrix} \delta T \\ - \\ \underline{\Delta R} \end{bmatrix} = - \begin{bmatrix} \underline{M}_{11} & | & \underline{M}_{12} \\ - & - & - \\ \underline{M}_{21} & | & \underline{M}_{22} \end{bmatrix} \begin{bmatrix} \underline{a}^T \\ - \\ A \end{bmatrix} \underline{C}^{-1} \underline{\Delta\rho} \quad , \tag{2.58}$$

where

$$\begin{aligned} \underline{M}_{12} &= \underline{M}_{21}^T = - \underline{N}_{11}^{-1} \underline{N}_{12} \underline{M}_{22} \\ \underline{M}_{22} &= (\underline{N}_{22} - \underline{N}_{21} \underline{N}_{11}^{-1} \underline{N}_{12})^{-1} \quad . \end{aligned} \tag{2.59}$$

Substituting these into the equations for $\underline{\Delta R}$ (and forgetting δT) we obtain:

$$\begin{aligned} \underline{\Delta R} &= - \left[\underline{M}_{21} \;\; | \;\; \underline{M}_{22} \right] \begin{bmatrix} \underline{a}^T \\ - \\ \underline{A} \end{bmatrix} \underline{C}^{-1} \underline{\Delta\rho} \\ &= - \left[-\underline{M}_{22}\underline{N}_{21}\underline{N}_{11}^{-1} \;\; | \;\; \underline{M}_{22} \right] \begin{bmatrix} \underline{a}^T \\ - \\ \underline{A} \end{bmatrix} \underline{C}^{-1} \underline{\Delta\rho} \\ &= \underline{M}_{22} \begin{bmatrix} -\underline{N}_{21}\underline{N}_{11}^{-1}\underline{a}^T \\ - \\ \underline{A} \end{bmatrix} \underline{C}^{-1} \underline{\Delta\rho} \quad . \end{aligned} \tag{2.60}$$

Substitution for $\underline{M}_{22}$, $\underline{N}_{21}$ and $\underline{N}_{11}$ yields

$$\begin{aligned} \underline{\Delta R} &= - (\underline{AC}^{-1}\underline{A}^T - \underline{AC}^{-1}\underline{a}(\underline{a}^T\underline{C}^{-1}\underline{a})^{-1}\, \underline{a}^T\underline{C}^{-1}\underline{A}^T)^{-1} \times \\ &\quad \times (-\underline{AC}^{-1}\underline{a}(\underline{a}^T\underline{C}^{-1}\underline{a})^{-1}\underline{a}^T + \underline{A})\underline{C}^{-1}\underline{\Delta\rho} \\ &= - (\underline{AC}^{-1}\underbrace{(\underline{I} - \underline{a}(\underline{a}^T\underline{C}^{-1}\underline{a})^{-1}\underline{a}^T\underline{C}^{-1})}_{\underline{Q}}\underline{A}^T)^{-1} \times \\ &\quad \times \underline{AC}^{-1}\underbrace{(\underline{I} - \underline{a}(\underline{a}^T\underline{C}^{-1}\underline{a})^{-1}\underline{a}^T\underline{C}^{-1})}_{\underline{Q}}\underline{\Delta\rho} \\ &= - (\underline{AC}^{-1}(\underline{I} - \underline{Q})\underline{A}^T)^{-1}\, \underline{AC}^{-1}(\underline{I} - \underline{Q})\, \underline{\Delta\rho} \quad . \end{aligned} \tag{2.61}$$

We note that if we regard $\underline{C}^{-1}(\underline{I} - \underline{Q})$ as a modified weight matrix, the shape of these equations is exactly the same as the shape of the normal eqn. (2.5) for the case when the clock offsets are not considered. This then proves our original assertion that the presence of unknown clock offsets changes only the weight matrix of observations from $\underline{P}$ to

$$\underline{P}' = \underline{P}(\underline{I} - \underline{a}(\underline{a}^T\underline{P}\underline{a})^{-1}\underline{a}^T\underline{P}) \quad . \tag{2.62}$$

Taking into account eqn. (2.54) we can rewrite eqn. (2.62) as

$$\underline{P}' = \underline{P}(\underline{I} - \underline{\alpha}(\underline{\alpha}^T\underline{P}\underline{\alpha})^{-1}\underline{\alpha}^T\underline{P}) \quad . \tag{2.63}$$

Having a closer look at eqn. (2.63) we discover that $\underline{\alpha}^T\underline{P}\underline{\alpha} = \Sigma\sigma_i^{-2}$ (for uncorrelated observations) and

$$\underline{P}' = \frac{\underline{P}}{\Sigma\sigma_i^{-2}} \begin{bmatrix} \sum\limits_{i\neq 1}\sigma_i^{-2}, & -\sigma_2^{-2}, \ldots, & -\sigma_n^{-2} \\ -\sigma_1^{-2}, & \sum\limits_{i\neq 2}\sigma_i^{-2}, \ldots, & -\sigma_n^{-2} \\ -\sigma_1^{-2}, & -\sigma_2^{-2}, \ldots, & \sum\limits_{i\neq n}\sigma_i^{-2} \end{bmatrix} \quad . \tag{2.64}$$

For the special case of $\forall i$: $\sigma_i = \sigma$, we get

$$\underline{P}' = \frac{\underline{P}}{n} \begin{bmatrix} n-1, & -1, \ldots, & -1 \\ -1, & n-1, \ldots, & -1 \\ -1, & -1, \ldots, & n-1 \end{bmatrix} \quad , \tag{2.65}$$

and for large n: $\underline{P}' \to \underline{P} = \sigma_o^2\underline{I}$.

Clearly a similar treatment may be given to more complicated clock errors, e.g., linear or non-linear drift, with the same result, except that the $\underline{P}'$ matrix would be more complicated. In fact the shape of eqn. (2.62) for $\underline{P}'$ will be the same, only the $\underline{a}$ will no longer be a vector but a matrix with p columns, where p is the number of base functions used for the clock error modelling. It is interesting to note that

$$\underline{P}'\underline{a} = \underline{a}^T\underline{P}' = \underline{0} \quad , \tag{2.66}$$

i.e., the new weight matrix is orthogonal to the clock error design matrix.

Once the new $\underline{P}'$ matrix is assembled, the filter can be used in exactly the same way as in the case of no clock error. Let us just mention here, that clock error parameters may be treated as being part of the vector of unknown parameters, in which case the design matrix has to be changed accordingly. Once the design matrix is changed the filter equations are again applied the same way as before. This is the approach used in the next chapter.

2.5 Orbit Improvement and Filtering

Let us first have a look at the effect of incorrectly known satellite positions. We begin by assuming that the satellite position, at the instant of differential range measurement is $\vec{r} + \delta\vec{r}$ instead of $\vec{r}$. We wish to see the resulting effect $\underline{\delta A}$ and $\underline{\delta N}$, $\delta\vec{r}$ causes.

Evidently, $\delta\vec{r}$ changes the unit vectors $\vec{e}_1$, $\vec{e}_2$ to $\vec{e}_1 + \delta\vec{e}_1$, $\vec{e}_2 + \delta\vec{e}_2$ where, say

$$\begin{aligned}\vec{e}_1 + \delta\vec{e}_1 &= \frac{\vec{r} + \delta\vec{r} - \vec{R}_1}{|\vec{r} + \delta\vec{r} - \vec{R}_1|} = \frac{\vec{r} + \delta\vec{r} - \vec{R}_1}{\sqrt{[(\vec{r} + \delta\vec{r} - \vec{R}_1)\cdot(\vec{r} + \delta\vec{r} - \vec{R}_1)]}} \\ &\doteq \frac{\vec{r} + \delta\vec{r} - \vec{R}_1}{\sqrt{[(\vec{r} - \vec{R}_1)\cdot(\vec{r} - \vec{R}_1) + 2(\vec{r} - \vec{R}_1)\cdot\delta\vec{r}]}} \quad ,\end{aligned} \tag{2.67}$$

having assumed $\delta r << r$. Equation (2.67) can be rewritten as

$$\vec{e}_1 + \delta\vec{e}_1 \doteq \frac{\vec{r} + \delta\vec{r} - \vec{R}_1}{|\vec{r}-\vec{R}_1|\sqrt{[1 + \frac{2(\vec{r}-\vec{R}_1)\cdot\delta\vec{r}}{|\vec{r} - \vec{R}_1|^2}]}}$$

$$\doteq \frac{\vec{r} + \delta\vec{r} - \vec{R}_1}{|\vec{r} - \vec{R}_1|} (1 + \frac{(\vec{r} - \vec{R}_1)\cdot\delta\vec{r}}{|\vec{r} - \vec{R}_1|^2}) \tag{2.68}$$

$$= (\vec{e}_1 + \frac{\delta\vec{r}}{\rho_1})(1 - \vec{e}_1 \cdot \frac{\delta\vec{r}}{\rho_1})$$

$$\doteq \vec{e}_1 + \delta\vec{r}_1^{\,*} - \vec{e}_1(\vec{e}_1 \cdot \delta\vec{r}_1^{\,*}) \quad ,$$

where $\delta\vec{r}_1^{\,*} = \vec{\delta}r/\rho_1$. Analogously, we get

$$\delta\vec{e}_2 = \delta\vec{r}_2^{\,*} - \vec{e}_2(\vec{e}_2 \cdot \delta\vec{r}_2^{\,*}) \quad . \tag{2.69}$$

Since

$$\vec{A} = \frac{\vec{e}_1 + \vec{e}_2}{(\vec{e}_1+\vec{e}_2)\cdot\vec{e}_1} \quad , \tag{2.70}$$

we can write

$$\vec{A} + \delta\vec{A} \doteq \frac{\vec{e}_1+\delta\vec{r}^{\,*}-\vec{e}_1(\vec{e}_1\cdot\delta\vec{r}^{\,*})+\vec{e}_2+\delta\vec{r}^{\,*}-\vec{e}_2(\vec{e}_2\cdot\delta\vec{r}^{\,*})}{(\vec{e}_1+\delta\vec{r}^{\,*}-\vec{e}_1(\vec{e}_1\cdot\delta\vec{r}^{\,*})+\vec{e}_2+\delta\vec{r}^{\,*}-\vec{e}_2(\vec{e}_2\cdot\delta\vec{r}^{\,*}))\cdot(\vec{e}_2+\delta\vec{r}^{\,*}-\vec{e}_2(\vec{e}_2\cdot\delta\vec{r}^{\,*}))} \tag{2.71}$$

where by $\delta\vec{r}^{\,*}$ we denote $2\delta\vec{r}/(\rho_1+\rho_2)$. Retaining only the first-order terms in $\delta\vec{r}^{\,*}$ we can write further:

$$\vec{A}+\delta\vec{A} \doteq \frac{\vec{e}_1+\vec{e}_2+2\delta\vec{r}^*-\vec{e}_1(\vec{e}_1\cdot\delta\vec{r}^*)-\vec{e}_2(\vec{e}_2\cdot\delta\vec{r}^*)}{(\vec{e}_1+\vec{e}_2)\cdot\vec{e}_1+(2\delta\vec{r}^*-\vec{e}_1(\vec{e}_1\cdot\delta\vec{r}^*)-\vec{e}_2(\vec{e}_2\cdot\delta\vec{r}^*))\cdot\vec{e}_1+(\vec{e}_1+\vec{e}_2)\cdot(\delta\vec{r}^*-\vec{e}_1(\vec{e}_1\cdot\delta\vec{r}^*))}$$

$$\doteq (\vec{A} + \frac{2\delta\vec{r}^*-\vec{e}_1(\vec{e}_1\cdot\delta\vec{r}^*)-\vec{e}_2(\vec{e}_2\cdot\delta\vec{r}^*)}{(\vec{e}_1+\vec{e}_2)\cdot\vec{e}_1}) \times \tag{2.72}$$

$$\times(1 - \frac{2\delta\vec{r}^*\cdot\vec{e}_1-\vec{e}_1\cdot\delta\vec{r}^*-(\vec{e}_1\cdot\vec{e}_2)(\vec{e}_2\cdot\delta\vec{r}^*)+\vec{e}_1\cdot\delta\vec{r}^*+\vec{e}_2\cdot\delta\vec{r}^*-\vec{e}_1\cdot\delta\vec{r}^*-(\vec{e}_1\cdot\vec{e}_2)(\vec{e}_1\cdot\delta\vec{r}^*)}{(\vec{e}_1+\vec{e}_2)\cdot\vec{e}_1}).$$

In this equation, $(\vec{e}_1+\vec{e}_2)\cdot\vec{e}_1$ can be approximated by 2 (in the corrective terms) and we obtain

$$\vec{A}+\delta\vec{A}\doteq(\vec{A}+\delta\vec{r}^*-\vec{e}_1\frac{\vec{e}_1\cdot\delta\vec{r}^*}{2}-\vec{e}_2\frac{\vec{e}_2\cdot\delta\vec{r}^*}{2})(1-\frac{\vec{e}_1\cdot\delta\vec{r}^*}{2}-\frac{\vec{e}_2\cdot\delta\vec{r}^*}{2}+\frac{\vec{e}_1\cdot\vec{e}_2}{2}(\vec{e}_1\cdot\delta\vec{r}^*+\vec{e}_2\cdot\delta\vec{r}^*))$$

$$\doteq(\vec{A}+\delta\vec{r}^* - \frac{\vec{e}_1+\vec{e}_2}{2}(\vec{u}\cdot\delta\vec{r}^*))(1-\vec{u}\cdot\delta\vec{r}^*+\vec{u}\cdot\delta\vec{r}^*) \tag{2.73}$$

$$\doteq \vec{A}+\delta\vec{r}^* - \vec{u}(\vec{u}\cdot\delta\vec{r}^*) \quad .$$

Thus, we get finally

$$\begin{aligned}\delta\vec{A} &\doteq \delta\vec{r}^* - \vec{u}(\vec{u}\cdot\delta\vec{r}^*)\\ &= \frac{1}{\rho}(\underline{I} - \vec{u}\otimes\vec{u})\delta\vec{r} \quad ,\end{aligned} \tag{2.74}$$

where ρ is the mean range.

Realizing now that $\delta\vec{r}$ can be expressed as a linear function of the satellite position change $\underline{\delta k}$ expressed in Keplerian elements $\underline{k}$, i.e.,

$$\delta\vec{r} = \underline{S\delta k} \quad , \tag{2.75}$$

where $\underline{S}$ is the Jacobian of transformation from Keplerian elements into Cartesian coordinates, we get the resulting equation

$$\delta\vec{A} \doteq \frac{1}{\rho}(\underline{I} - \vec{u}\otimes\vec{u})\underline{S\delta k} = \underline{T}^*\underline{\delta k} \quad . \tag{2.76}$$

This is then easily transformed into $\underline{\delta N}$ as

$$\underline{\delta N} = \sigma_i^{-2} \delta\vec{A}_i \otimes \delta\vec{A}_i \quad , \tag{2.77}$$

if these quantities are of interest.

We shall now turn to the real problem of interest, namely, the evaluation of orbital biases $\underline{\delta k}$ from observed differential ranges. To solve the problem, let us consider again the system of observation equations (2.3). Clearly, since the design matrix $\underline{A}$ is wrong by

$$\underline{\delta A} = \begin{bmatrix} \delta\vec{A}_1 \\ \delta\vec{A}_2 \\ \cdot \\ \cdot \\ \cdot \\ \delta\vec{A}_n \end{bmatrix} = \begin{bmatrix} \underline{T}_1 \\ \hline \underline{T}_2 \\ \hline \cdot \\ \cdot \\ \cdot \\ \hline \underline{T}_n \end{bmatrix} \begin{bmatrix} \delta\underline{k}_1, & \delta\underline{k}_2, \ldots, & \delta\underline{k}_s \end{bmatrix}$$

$$= \underline{T}\ \underline{\Delta k} \quad , \tag{2.78}$$

where $\underline{T}_i$ are constructed from $\underline{T}$*s and zeros and $\delta\underline{k}_j$ belong to the s satellites used in the campaign. The computed $\underline{\Delta R}$ is wrong by $\underline{\delta R}$. We can thus rewrite eqn. (2.3) as

$$(\underline{A}^T + \delta\underline{A}^T)(\underline{\Delta R} + \underline{\delta R}) = -\ \underline{\Delta \rho} \quad . \tag{2.79}$$

Neglecting powers of higher order than one in the small quantities $\underline{\delta A}$, $\underline{\delta R}$ we obtain

$$\underline{A}^T\ \underline{\Delta R} + \underline{A}^T\ \underline{\delta R} + \underline{\delta A}^T\ \underline{\Delta R} \doteq -\ \underline{\Delta \rho} \quad . \tag{2.80}$$

Now a substitution for $\underline{\delta A}$ from eqn. (2.78) yields:

$$\underline{A}^T(\underline{\Delta R} + \underline{\delta R}) + \underline{\Delta k}^T\ \underline{T}^T\ \underline{\Delta R} \doteq -\ \underline{\Delta \rho} \quad , \tag{2.81}$$

which can be rewritten as

$$\underline{A}^T(\underline{\Delta R} + \underline{\delta R}) + \underline{\Delta R}^T\ \underline{T}\ \underline{\Delta k} \doteq -\ \underline{\Delta \rho} \quad , \tag{2.82}$$

or

$$\underline{A}^T(\underline{\Delta R} + \underline{\delta R}) + \underline{B}^T\ \underline{\Delta k} \doteq -\ \underline{\Delta \rho} \quad . \tag{2.83}$$

Assuming $\underline{P}$ to be the weight matrix of observed differential ranges, the system of normal equations for the unknowns $\underline{\Delta R} + \underline{\delta R}$ and $\underline{\Delta k}$ is

$$\underline{APA}^T(\underline{\Delta\hat{R}} + \underline{\delta\hat{R}}) + \underline{APB}^T\ \underline{\Delta\hat{k}} = -\ \underline{AP\Delta\rho} \tag{2.84}$$

$$\underline{BPA}^T(\underline{\Delta\hat{R}} + \underline{\delta\hat{R}}) + \underline{BPB}^T\ \underline{\Delta\hat{k}} = -\ \underline{BP\Delta\rho}\quad . \tag{2.84}$$

From the first set of equations we get

$$\underline{\Delta\hat{R}} + \underline{\delta\hat{R}} = \underbrace{(\underline{APA}^T)^{-1}}_{\underline{N}^{-1}}(-\ \underline{AP\Delta\rho} - \underline{APB}^T\ \underline{\Delta\hat{k}})\quad . \tag{2.85}$$

Substitution of this result into the second set of equations gives:

$$\underline{BPA}^T\underline{N}^{-1}(-\ \underline{AP\Delta\rho} - \underline{APB}^T\underline{\Delta\hat{k}}) + \underline{BPB}^T\underline{\Delta\hat{k}} = -\ \underline{BP\Delta\rho}\quad , \tag{2.86}$$

or

$$(-\underline{BPA}^T\underline{N}^{-1}\underline{APB}^T + \underline{BPB}^T)\underline{\Delta\hat{k}} = -\ (\underline{BP} - \underline{BPA}^T\underline{N}^{-1}\underline{AP})\underline{\Delta\rho}\quad . \tag{2.87}$$

This equation can be rewritten as

$$\underline{B}\underbrace{(\underline{P} - \underline{PA}^T\underline{N}^{-1}\underline{AP})}_{\underline{Y}}\underline{B}^T\underline{\Delta\hat{k}} = -\ \underline{B}\underbrace{(\underline{P} - \underline{PA}^T\underline{N}^{-1}\underline{AP})}_{\underline{Y}}\underline{\Delta\rho}\quad , \tag{2.88}$$

or, simply

$$\underline{BYB}^T\underline{\Delta\hat{k}} = -\ \underline{BY\Delta\rho}\quad . \tag{2.89}$$

Realizing now that in eqn. (2.82)

$$\underline{\Delta\hat{R}} = -\ \underline{N}^{-1}\underline{AP\Delta\rho}\quad , \tag{2.90}$$

we obtain

$$\underline{\delta\hat{R}} = -\ \underline{N}^{-1}\underline{APB}^T\underline{\Delta\hat{k}}\quad . \tag{2.91}$$

Equations (2.89) and (2.91) are the ones to use for the evaluation of the best estimates $\underline{\Delta\hat{k}}$ of orbital biases and the best estimate $\underline{\delta\hat{R}}$ of the baseline correction. If the orbital biases are regarded only as nuisance parameters to be eliminated then the correction $\underline{\delta\hat{R}}$ can be computed directly as a linear combination of the observed differential ranges as

$$\underline{\delta\hat{R}} = \underline{N}^{-1}\underline{APB}^T(\underline{BYB}^T)^{-1}\underline{BY\Delta\rho}\quad . \tag{2.92}$$

As such it can be obtained from modified filter equations.

It is interesting to note that the "weight matrix" $\underline{Y}$ in the normal eqns. (2.88) for $\underline{\Delta\hat{R}}$ is orthogonal to the design matrix $\underline{A}$. We get, clearly,

$$\underline{AY} = \underline{YA}^T = \underline{0} \quad . \tag{2.93}$$

We also note that for other observation modes, we get identical equations for $\underline{\Delta\hat{R}}$, where only the matrix $\underline{T}$, and thus $\underline{B}$, will have a different form.

2.6 Optimum Geometrical Configuration for Differential Ranging

To investigate the optimum configuration for differential ranging from the geometrical point of view it is expedient to take the final system of normal equations rather than the equations for the filter. Clearly, if the filter is applied properly, the end results ($\underline{\Delta\hat{R}}$) from both techniques should be identical.

Now, the most accurate (best) result will be obtained for the case when Tr($\underline{N}$) is the maximum and the off-diagonal elements of $\underline{N}$ are as close to zero as possible. In the first approximation, for shorter baselines, the contribution to the matrix of normal equations from an observed (ith) differential range is (uncorrelated case):

$$\underline{\Delta N}_i = \sigma_i^{-2} \vec{A}^i \otimes \vec{A}^i \doteq \sigma_i^{-2} \vec{u}^i \otimes \vec{u}^i/(u^i)^2$$

$$\doteq \sigma_i^{-2} \begin{bmatrix} \cos^2\alpha_1^i & \cos\alpha_1^i\cos\alpha_2^i & \cos\alpha_1^i\cos\alpha_3^i \\ \cos\alpha_2^i\cos\alpha_1^i & \cos^2\alpha_2^i & \cos\alpha_2^i\cos\alpha_3^i \\ \cos\alpha_3^i\cos\alpha_1^i & \cos\alpha_3^i\cos\alpha_2^i & \cos^2\alpha_3^i \end{bmatrix} , \tag{2.94}$$

where the elements are obviously expressed as products of directional cosines. Thus the upper triangular part of $\underline{N}$ can be written as

$$\underline{N} = \begin{bmatrix} \sum_i \sigma_i^{-2} \cos^2 \alpha_1^i & \sum_i \sigma_i^{-2} \cos\alpha_1^i \cos\alpha_2^i & \sum_i \sigma_i^{-2} \cos\alpha_1^i \cos\alpha_3^i \\ & \sum_i \sigma_i^{-2} \cos^2 \alpha_2^i & \sum_i \sigma_i^{-2} \cos\alpha_2^i \cos\alpha_3^i \\ & & \sum_i \sigma_i^{-2} \cos^2 \alpha_3^i \end{bmatrix} . \qquad (2.95)$$

From eqn. (2.94) it is not difficult to see that for three differential ranges the optimal configuration is achieved for $\vec{u}^1 \perp \vec{u}^2$, $\vec{u}^1 \perp \vec{u}^3$, $\vec{u}^2 \perp \vec{u}^3$, i.e., for an orthogonal triad of mean vectors $\vec{u}^1$, $\vec{u}^2$, $\vec{u}^3$. This is the same result as obtained for the geometrical configuration optimal for ranging [Spilker, 1978].

It appears to us that an algorithm for selecting satellites could be designed such that $\underline{N}$ would tend to the most ideal case:

$$\underline{N} = \frac{1}{3} \sum_i \sigma_i^{-2} \underline{I} \quad , \qquad (2.96)$$

since for satellites theoretically available at any desired position we would get

$$\sum_i \cos\alpha_s^i \cos\alpha_t^i = \begin{cases} 1/3 & s=t \\ 0 & s \neq t \end{cases} \quad . \qquad (2.97)$$

Of course, the question remains whether such a selection would be really desirable; clearly, the accuracy of the solution also can be improved simply by augmenting the number of observations.

2.7 Optimum Geometrical Configuration for Differential Range Differences

Whereas in the case of the differential Doppler determination of $\nabla\rho$ the satellite locations S^j, S^k are separated by about 10^5 m (for one 30-second Doppler integration interval) along one pass, for the differential range differencing techniques the optimal satellite configuration would require S^j and S^k to subtend a large angle (e.g., 90^o) at the baseline. Thus, while the paralactical angles for one Doppler measurement are of the order of 5×10^{-3} radians they would optimally be close to 90^o for the differential range differencing. Considering the observation equation for differenced Doppler observations [Vaníček et al., 1983]

$$-\nabla\vec{u}\cdot\Delta\vec{R} = \nabla^2\rho - \Delta\vec{u}\cdot\Delta\vec{r} + \Delta^2\vec{u}\cdot(\vec{R}_m - \vec{r}^m) \qquad (2.98)$$

we can see that clearly, differenced Doppler observations $\nabla^2\rho$ would have to be measured with an accuracy at least two orders of magnitude greater than the differential range observations $\Delta\rho$. The geometric disadvantage would tend to disappear, of course, when the Doppler integration interval is extended; more than one hour of integration would be needed, however, to get a good configuration [Fell, 1980]. The effect of imperfect knowledge of $\Delta\vec{r}$ can be minimized by selecting passes that are approximately normal to $\Delta\vec{R}$. In such cases $\nabla\vec{u}$ tends to be normal to $\Delta\vec{r}$ and the second term on the right-hand side of eqn. (2.98) will go to zero. It is interesting to see that under these circumstances even $\Delta^2\vec{u}$ tends to $\vec{0}$ and the third term does not contribute appreciably either.

Obviously, not much is achieved from the geometrical point of view when differential range differences (or differenced range differences) are used instead of just differential ranges. On the other hand, the best satellite configuration for the differential range differencing can only bring $\nabla\vec{u}$ close to a unit vector and make the effect of errors in $\nabla^2\rho$ on $\Delta\vec{R}$ as small as that of differential ranges. On the other hand, there are the additional terms that generally will reduce the accuracy of $\Delta\vec{R}$. It is important to bear in mind that the argument in favour of differential range differences is based on the elimination of clock errors.

CHAPTER 3

MATHEMATICAL MODELS OF MULTI-STATION DIFFERENTIAL OBSERVATIONS

In this section we seek to formulate the mathematical models relating the multistation solution (instead of the interstation vector $\Delta\vec{R}$) to differential ranges, range differences, or differential range differences observations. To do so we shall deviate somewhat from the previous formulation choosing to formulate the mathematical models in terms of coordinate components (position vectors) for each station rather than coordinate differences (interstation vectors).

3.1 Multistation Differential Range Mathematical Model

Let us start from the single point P_α mathematical model for ranging

$$\vec{e}^{\,i}_\alpha \cdot \vec{R}_\alpha = -\rho^i_\alpha + \vec{e}^{\,i}_\alpha \cdot \vec{r}^{\,i} \quad , \tag{3.1}$$

where the subscript indicates the participating station and the superscript the participating satellite position S^i. For a pair of ground stations P_α, P_β we get

$$\vec{e}^{\,i}_\alpha \cdot \vec{R}_\alpha - \vec{e}^{\,i}_\beta \cdot \vec{R}_\beta = \Delta\rho^i_{\alpha\beta} - \Delta\vec{u}^{\,i}_{\alpha\beta} \cdot \vec{r}^{\,i} \quad , \tag{3.2}$$

which may also be written as

$$\begin{bmatrix} (\vec{e}^{\,i}_\alpha)^T & \vert & -(\vec{e}^{\,i}_\beta)^T \end{bmatrix} \begin{bmatrix} \vec{R}_\alpha \\ \\ \vec{R}_\beta \end{bmatrix} = \Delta\rho^i_{\alpha\beta} - \Delta\vec{u}^{\,i}_{\alpha\beta} \cdot \vec{r}^{\,i} = \Delta^i_{\alpha\beta} \quad , \tag{3.3}$$

where $\Delta^i_{\alpha\beta}$ is the misclosure. For several ground stations P_α, P_β, ..., P_ω observing simultaneously and for many satellite positions S^i, S^j, ..., S^n we have

$$\underline{A}_\Delta \, \underline{R} = \underline{\Delta} \quad , \tag{3.4}$$

where the rows of design matrix $\underline{A}_{\Delta}$ are composed of pairs of unit vectors (see Figure 3.1), $\underline{\Delta}$ is a vector of misclosures $\Delta^{i}_{\alpha\beta}$ and $\underline{R}^T = [\vec{R}_{\alpha}, \vec{R}_{\beta}, \ldots, \vec{R}_{\omega}]$.

3.2 Multistation Range Difference Mathematical Model

The equation for range difference (Doppler) observation $\nabla\rho$ can be obtained from the following two range equations:

$$\begin{aligned} \vec{e}^{\,i}_{\alpha} \cdot \vec{R}_{\alpha} &= -\vec{\rho}^{\,i}_{\alpha} + \vec{e}^{\,i}_{\alpha} \cdot \vec{r}^{\,i} \\ \vec{e}^{\,j}_{\alpha} \cdot \vec{R}_{\alpha} &= -\vec{\rho}^{\,j}_{\alpha} + \vec{e}^{\,j}_{\alpha} \cdot \vec{r}^{\,j} \end{aligned} \tag{3.5}$$

as

$$\begin{aligned} \nabla\vec{u}^{\,ij}_{\alpha} \cdot \vec{R}_{\alpha} &= -\nabla\rho^{ij}_{\alpha} + \vec{e}^{\,j}_{\alpha} \cdot \vec{r}^{\,j} - \vec{e}^{\,i}_{\alpha} \cdot \vec{r}^{\,i} \\ &= -\nabla\rho^{ij}_{\alpha} + \vec{e}^{\,j}_{\alpha} \cdot \Delta\vec{r}^{\,ij} + \nabla\vec{u}^{\,ij}_{\alpha} \cdot \vec{r}^{\,i} \\ &= -\nabla^{ij}_{\alpha} \quad , \end{aligned} \tag{3.6}$$

where ∇^{ij}_{α} is the misclosure of the observed range difference $\nabla\rho^{ij}_{\alpha}$. Considering several ground stations, the system of observation equations becomes

$$\underline{A}_{\nabla}\ \underline{R} = \underline{\nabla} \quad , \tag{3.7}$$

where the rows of the design matrix contain just only $-\nabla\vec{u}$'s corresponding to the appropriate $\vec{R}$'s and to the appropriate pairs of satellite positions for which the corresponding range difference is observed.

3.3 Multistation Differential Range Difference Mathematical Model

Since differential range differences and differenced differential ranges (double differences) are the same [Vaníček et al., 1984] we can derive their observation equations from either section 3.1 or section 3.2.

$$
\begin{bmatrix}
(\vec{e}_1^{\,i})^T & -(\vec{e}_2^{\,i})^T & & & & & \\
(\vec{e}_1^{\,i})^T & & -(\vec{e}_3^{\,i})^T & & & & \\
(\vec{e}_1^{\,i})^T & & & -(\vec{e}_4^{\,i})^T & & & \\
\vdots & & & & \ddots & & \\
(\vec{e}_1^{\,i})^T & & & & & & -(\vec{e}_\ell^{\,i})^T \\
& (\vec{e}_2^{\,i})^T & -(\vec{e}_3^{\,i})^T & & & & \\
& (\vec{e}_2^{\,i})^T & & -(\vec{e}_4^{\,i})^T & & & \\
& \vdots & & & \ddots & & \\
& (\vec{e}_2^{\,i})^T & & & & & -(\vec{e}_\ell^{\,i})^T \\
& & (\vec{e}_3^{\,i})^T & -(\vec{e}_4^{\,i})^T & & & \\
& & \vdots & & \ddots & \ddots & \\
& & & & & (\vec{e}_{\ell-1}^{\,i})^T & -(\vec{e}_\ell^{\,i})^T
\end{bmatrix}
\begin{bmatrix}
\vec{R}_1 \\ \vec{R}_2 \\ \vec{R}_3 \\ \vec{R}_4 \\ \vdots \\ \vec{R}_\ell
\end{bmatrix}
=
\begin{bmatrix}
\Delta_{12} \\ \Delta_{13} \\ \Delta_{14} \\ \vdots \\ \Delta_{1\ell} \\ \Delta_{23} \\ \Delta_{24} \\ \vdots \\ \Delta_{2\ell} \\ \Delta_{34} \\ \vdots \\ \Delta_{\ell-1,\ell}
\end{bmatrix}
$$

FIGURE 3.1

Example of a set of observation equations
for differential ranges
observed simultaneously from a network of ℓ stations.

We shall use eqn. (3.6) to start with.

Writing two observation equations (3.6) for range differences $\nabla\rho_{\alpha}^{ij}$ and $\nabla\rho_{\beta}^{ij}$ (observed simultaneously from two ground stations) and subtracting the second from the first we get

$$\nabla\vec{u}_{\alpha}^{ij}\cdot\vec{R}_{\alpha}-\nabla\vec{u}_{\beta}^{ij}\cdot\vec{R}_{\beta} = \nabla^{2}\rho_{\alpha\beta}^{ij}+(\vec{e}_{\alpha}^{j}-\vec{e}_{\beta}^{j})\cdot\Delta\vec{r}^{ij}+(\nabla\vec{u}_{\alpha}^{ij}-\nabla\vec{u}_{\beta}^{ij})\cdot\vec{r}^{i} \quad . \qquad (3.8)$$

This can be rewritten as

$$\left[(\nabla\vec{u}_{\alpha}^{ij})^{T} \;\; | \;\; -(\nabla\vec{u}_{\beta}^{ij})^{T}\right] \begin{bmatrix} \vec{R}_{\alpha} \\ \vec{R}_{\beta} \end{bmatrix} = \nabla^{2}\rho_{\alpha\beta}^{ij} - \Delta\vec{u}_{\alpha\beta}^{j}\cdot\Delta\vec{r}^{ij} - \Delta^{2}\vec{u}_{\alpha\beta}^{ij}\cdot\vec{r}^{i} = \nabla_{\alpha\beta}^{2ij} \quad . \qquad (3.9)$$

It can easily be shown that a difference of two eqns. (3.2) formulated for P_{α} P_{β} S^{i} and P_{α} P_{β} S^{j}, gives an equation identical to eqn. (3.9), as it should.

It is clear that the system of observation equations for several ground stations is

$$\underline{A}_{\nabla^{2}}\ \underline{R} = \underline{\nabla}^{2} \quad , \qquad (3.10)$$

where the design matrix $\underline{A}_{\nabla^{2}}$ has rows containing vectors $\nabla\vec{u}_{\alpha}^{ij}$, $-\nabla\vec{u}_{\beta}^{ij}$ and $\underline{\nabla}^{2}$ is the vector of misclosures given by eqn. (3.9).

At the moment, only double differences pertaining to one satellite (i.e., differential range differences or differential Doppler can be processed with the VECA package (see Chapter 5). Double differences involving two satellites have to be processed using PRMAC-3 (see Chapter 9).

3.4 Mathematical Model Expansion to Include Clock Errors

The previous models, eqns. (3.2), (3.6) and (3.9), can be easily modified to include clock error parameters in the solution vector. For this purpose we have assumed that both satellite and receiver clock errors can be represented by an algebraic polynomial in time as (cf. Davidson et al. [1983])

$$\Delta t = a_o + a_1(t - t_o) + \alpha_2(t - t_o)^2 \quad , \tag{3.11}$$

and

$$\Delta T = A_o + A_1(T - T_o) + A_2(T - T_o)^2 \quad , \tag{3.12}$$

where t_o and T_o are some reference time epochs for the particular set of coefficients. In view of such errors, a range to a satellite can be expressed as

$$\rho = \tilde{\rho} + c\Delta t - c\Delta T \quad , \tag{3.13}$$

where c is the speed of light and $\tilde{\rho}$ represents the measured pseudorange.

Usually satellite clock errors can be accounted for by using the clock coefficients supplied in the navigation message, so that eqn. (3.13) can be simplied to

$$\rho = \tilde{\rho}* - c\Delta T \tag{3.14}$$

with the understanding that the satellite clock error has been included as part of the "reduced" observation $\tilde{\rho}*$.

With eqn. (3.14) in mind, a double difference can be expressed as

$$\begin{aligned} \nabla^2\rho^{ij}_{\alpha\beta} &= \nabla\rho^{ij}_{\beta} - \nabla\rho^{ij}_{\alpha} \\ &= (\rho^{j}_{\beta} - \rho^{i}_{\beta}) - (\rho^{j}_{\alpha} - \rho^{i}_{\alpha}) \\ &= \nabla^2\tilde{\rho}* - c[\Delta T_{\beta}(\tau^{j}) - \Delta T_{\beta}(\tau^{i})] + c[\Delta T_{\alpha}(\tau^{j}) - \Delta T_{\alpha}(\tau^{i})] \end{aligned} \tag{3.15}$$

where $\nabla^2\tilde{\rho}*$ represents the "reduced" observation and τ is used to denote the GPS time scale (cf. Davidson et al. [1983]). Substituting eqn. (3.15) into eqn. (3.5), the model for differenced range difference observations including receiver clock errors can be written as

$$\nabla \vec{u}^{ij}_{\alpha} \cdot \vec{R}_{\alpha} - \nabla \vec{u}^{ij}_{\beta} \cdot \vec{R}_{\beta} = \nabla^2 \tilde{\rho}* - c[\Delta T_{\beta}(\tau^j) - \Delta T_{\beta}(\tau^i)] + c[\Delta T_{\alpha}(\tau^j) - \Delta T_{\alpha}(\tau^i)] - \Delta \vec{u}^j_{\alpha\beta} \cdot \Delta \vec{r}^{ij} + \Delta^2 \vec{u}^{ij}_{\alpha\beta} \cdot \vec{r}^i \tag{3.16}$$

or using eqn. (3.12) and transferring the terms in brackets to the left-hand side

$$\begin{aligned} &\nabla \vec{u}^{ij}_{\alpha} \cdot \vec{R}_{\alpha} - \nabla \vec{u}^{ij}_{\beta} \cdot \vec{R}_{\beta} + c\{[A_{\beta o} + A_{\beta 1}(T_{\beta}(\tau^j) - T_{\beta o}) + A_{\beta 2}(T_{\beta}(\tau^j) - T^2_{\beta o})] \\ &\quad - [A_{\beta o} + A_{\beta 1}(T_{\beta}(\tau^i) - T_{\beta o}) + A_{\beta 2}(T_{\beta}(\tau^i) - T_{\beta o})^2]\} \\ &\quad - c\{[A_{\alpha o} + A_{\alpha 1}(T_{\alpha}(\tau^j) - T_{\alpha o}) + A_{\alpha 2}(T_{\alpha}(\tau^j) - T_{\alpha o})^2] \\ &\quad - [A_{\alpha o} + A_{\alpha 1}(T_{\alpha}(\tau^i) - T_{\alpha o}) + A_{\alpha 2}(T_{\alpha}(\tau^i) - T_{\alpha o})^2]\} \\ &\quad = \nabla^2 * \quad , \end{aligned} \tag{3.17}$$

where the first subscript in the coefficients $A_{\ell,o}$, $A_{\ell,1}$, $A_{\ell,2}$ and $T_{\ell,o}$ is used to denote the station to which they refer and $\nabla^2 *$ is the reduced misclosure. Defining the vectors

$$\vec{B}_{\ell} = [b_{\ell 1} \quad b_{\ell 2} \quad b_{\ell 3}] \; , \; \ell = \alpha\beta \quad , \tag{3.18}$$

where

$$b_{\ell 1} = 0 \tag{3.19}$$

(i.e., constant time offset cannot be determined from double differences),

$$\begin{aligned} b_{\ell 2} &= c[(T_{\ell}(\tau^j) - T_{\ell o}) - (T_{\ell}(\tau^i) - T_{\ell o})] \\ &= c[T_{\ell}(\tau^j) - T_{\ell}(\tau^i)] \end{aligned} \tag{3.20}$$

$$b_{\ell 3} = c[(T_{\ell}(\tau^j) - T_{\ell o})^2 - (T_{\ell}(\tau^i) - T_{\ell o})^2] \tag{3.21}$$

eqn. (3.17) can be simplified to

$$\nabla \vec{u}_{\alpha} \cdot \vec{R}_{\alpha} - \nabla \vec{u}_{\beta} \cdot \vec{R}_{\beta} + \vec{B}_{\alpha} \cdot \vec{Q}_{\alpha} + \vec{B}_{\beta} \cdot \vec{Q}_{\beta} = \nabla^2 * \quad , \tag{3.22}$$

where

$$\vec{Q}_\alpha = [A_{\alpha o} \quad A_{\alpha 1} \quad A_{\alpha 2}]^T \tag{3.23}$$

$$\vec{Q}_\beta = [A_{\beta o} \quad A_{\beta 1} \quad A_{\beta 2}]^T \quad . \tag{3.24}$$

For several ground stations eqn. (3.22) may be written in a more convenient matrix form:

$$\underline{A}_{\nabla^2}\ \underline{R} + \underline{B}_{\nabla^2}\ \underline{Q} = \nabla^{2*} \quad , \tag{3.25}$$

where

$$B_{\nabla^2} = [\vec{B}_\alpha \;\vdots\; \vec{B}_\beta] \tag{3.26}$$

is the second design matrix, and

$$Q = [\vec{Q}_\alpha \;\vdots\; \vec{Q}_\beta]^T \tag{3.27}$$

is the solution vector for the 2ℓ clock parameters. Similarly, using eqn. (3.13) a differential range can be expressed as

$$\begin{aligned}
\Delta\rho &= \rho_\beta - \rho_\alpha \\
&= \tilde{\rho}_\beta - \tilde{\rho}_\alpha - c[\Delta T_\beta - \Delta T_\alpha] \\
&= \Delta\tilde{\rho} - c[A_{\beta o} + A_{\beta 1}(T_\beta - T_{\beta o}) + A_{\beta 2}(T_\beta - T_{\beta o})^2] \\
&\quad + c[A_{\alpha o} + A_{\alpha 1}(T_\alpha - T_{\alpha o}) + A_{\alpha 2}(T_\alpha - T_{\alpha o})^2] \quad .
\end{aligned} \tag{3.28}$$

Substituting eqn. (3.28) into eqn. (3.2) yields

$$\vec{e}_\alpha \cdot \vec{R}_\alpha - \vec{e}_\beta \cdot \vec{R}_\beta = \Delta\tilde{\rho} - \vec{B}_\beta\ \vec{Q}_\beta + \vec{B}_\alpha \cdot \vec{Q}_\alpha \quad , \tag{3.29}$$

where

$$\vec{B}_\ell = [b_{\ell 1} \quad b_{\ell 2} \quad b_{\ell 3}] \ , \ \ell = \alpha, \beta \tag{3.30}$$

with

$$b_{\ell 1} = c \tag{3.31}$$

$$b_{\ell 2} = c[T_\ell - T_{\ell o}] \tag{3.32}$$

$$b_{\ell 3} = c[T_\ell - T_{\ell o}]^2 \tag{3.33}$$

and

$$\vec{Q}_\ell = [A_{\ell o} \quad A_{\ell 1} \quad A_{\ell 2}]^T \quad . \tag{3.34}$$

Recalling eqn. (3.4) and after some simple manipulations, eqn. (3.29) can be simplified to

$$\underline{A}_\Delta \ \underline{R} + \underline{B}_\Delta \ \underline{Q} = \underline{\Delta}^* \quad , \tag{3.35}$$

where the second design matrix is defined as

$$\underline{B}_\Delta = [-\underline{B}_1 \vdots \underline{B}_2] \tag{3.36}$$

and $\underline{\Delta}^*$ is a vector of reduced misclosures.

For range differences we end up with an expanded observation equation for point P_α:

$$- \nabla \vec{u}^{ij}_\alpha \cdot \vec{R}_\alpha = \nabla^*_\alpha + A_{\alpha 1} \ c(T_\alpha(\tau^j) - T_\alpha(\tau^i)) \tag{3.37}$$

$$+ A_{\alpha 2} \ c(T^2_\alpha(\tau^j) - 2T_{\alpha o}(T_\alpha(\tau^j) - T_\alpha(\tau^i)) - T^2_\alpha(\tau^i)) \quad ,$$

where ∇^* is the misclosure reduced for satellite time correction. For several ground stations, we have again

$$\underline{A}_\nabla \ \underline{R} + \underline{B}_\nabla \ \underline{Q} = \underline{\nabla}^* \quad , \tag{3.38}$$

where $\underline{B}_\nabla$ is a two-row matrix of coefficients

$$c[T_\alpha(\tau^j) - T_\alpha(\tau^i)] \ c[T^2_\alpha(\tau^j) - 2T_{\alpha o}(T_\alpha(\tau^j) - T_\alpha(\tau^i)) - T^2_\alpha(\tau^i)] \tag{3.39}$$

and $\underline{Q}$ has two columns of $A_{\alpha 1}$ and $A_{\alpha 2}$ for $\alpha = 1,2,\ldots$. We again note that constant time shift $A_{\alpha o}$ cannot be determined for range differences alone.

CHAPTER 4

ESTIMATION OF ORBITAL PARAMETERS

4.1 General Considerations

It is clear that every observation of a satellite is a function of the satellite's position at that time. This functional relationship is given through orbital parameters which describe the motion of the satellite around the earth in a unique way. However, the number and choice of such parameters are by no means unique.

The orbit of every satellite is a particular solution of a system of second-order differential equations:

$$\ddot{\vec{r}} = \vec{f}(t;\ \vec{r},\ \dot{\vec{r}},\ p_1,\ p_2,\ \ldots,\ p_n) \qquad (4.1)$$

where

$\vec{r} = \vec{r}(t)$ is the position of the satellite in an inertial reference frame,

$\dot{\vec{r}}$ is the satellite's velocity, and $\ddot{\vec{r}}$ is the acceleration,

p_i, $i=1,2,\ldots,n$ are parameters defining the forces acting on the satellite.

The parameters p_i describe, for example, the gravity field, drag and radiation pressure experienced by the satellite. Their choice and their number mainly depend on the length of the orbital arcs considered, and as such are not unique.

Of course, eqn. (4.1) does not define a satellite's orbit uniquely either. We have to furnish additional conditions, and again we have several possibilities. We only describe here the option used in VECA: it calls for specified initial values of position and velocity (initial conditions), specified themselves as functions of a set of six parameters, the so-called osculating orbital elements k_i, $i=1,2,\ldots,6$ at reference epoch t_o. This osculation epoch may be chosen arbitrarily in principle; in VECA it is always associated with the middle of the observation interval. The choice of the kind of elements used in VECA is discussed in section 4.2.

$$\vec{r}(t_o) = \vec{r}_o(k_1, k_2, \ldots, k_6)$$
$$\dot{\vec{r}}(t_o) = \dot{\vec{r}}_o(k_1, k_2, \ldots, k_6) \quad (4.2)$$

If we specify the values of orbital elements k_i, and those of the "dynamical" parameters $p_1, p_2, \ldots, p_n$, the orbit of the satellite under consideration is then uniquely defined.

A completely general parameter estimation program should be able to solve for the best estimates of values of any combination of the orbital parameters

$$p_1,\ p_2,\ \ldots,\ p_n,\ k_{i1},\ k_{i2},\ \ldots,\ k_{i6},\ i=1,2,\ldots,n_s \quad (4.3)$$

where n_s is the number of satellites or, more specifically, the number of satellite orbital arcs observed, and $k_{i\ell}$, $\ell=1,2,\ldots,6$ are the six osculating elements at epoch t_o (see eqn. (4.2)) for satellite ℓ. Such complete generality is not provided for in VECA. Only relatively short contiguous orbital arcs (typically shorter than 10 hours) will be processed with this program. This means that we are allowed to model $\vec{f}$ in eqn. (4.1) using only very few parameters. Moreover we are allowed to assume that these parameters (for example, low order potential coefficients, lunar and solar gravity) are known a priorily. Therefore, in VECA we are left only with the necessity to determine or update the set of orbital elements $k_{i\ell}$. Often, this way of processing data is referred to as the semi-dynamical approach.

Actually even this set may contain more free parameters than required. If the observations originate from a relatively small area on the earth's surface (within a diameter of, say, less than 100 km) and if the number of simultaneously operating receivers is small (say, 2 or 3) it probably will not make sense to solve for all these elements. Often we will be able to estimate only one element (responsible for a possible along-track error) per satellite with any degree of certainty.

For these reasons it is possible in VECA to define the subset of elements to be estimated for each satellite. Moreover an option exists to introduce a priori information concerning these parameters by specifying an input variance-covariance matrix.

These options make VECA an ideal instrument for answering, by simulations, questions of the following kind: What orbital accuracy is needed when a certain positional accuracy is needed? How do these requirements change with the number of receivers and their separations? How do these results change if we assume the positions of a subset of receivers to be known?

4.2 Coordinate Systems and Satellite Position at Osculation Epoch

The apparent place coordinate system defined by the true equator and equinox corresponding to the middle of the observation epoch, t_o, is chosen as the reference frame for the orbital elements.

Osculating Keplerian elements at time t_o are used, where (see Figure 4.1)

k_1 = a, semimajor axis of the orbit

k_2 = e, eccentricity

k_3 = i, inclination of orbital plane with respect to equatorial plane

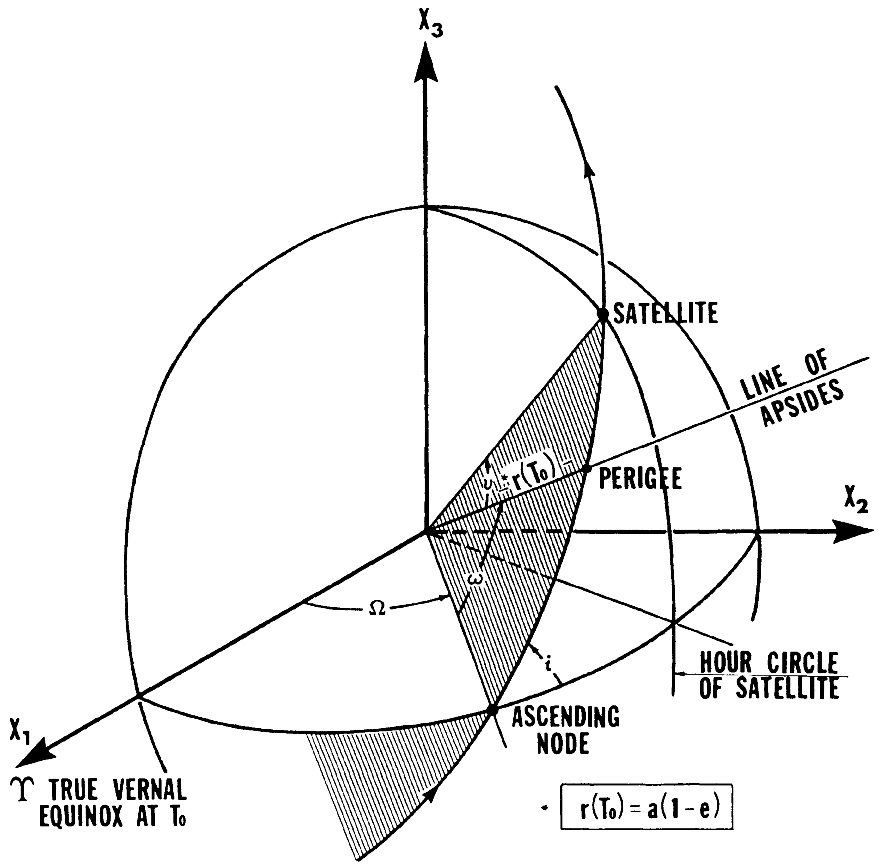

FIGURE 4.1 Keplerian Elements

$k_4 = \Omega$, right ascension of ascending node

$k_5 = \omega$, argument of perigee

$k_6 = T_o$, time of perigee passage.

If $\vec{r}^*(t_o)$ and $\vec{r}(t_o)$ are the position vectors of the satellite at osculation epoch expressed in the conventional terrestrial and apparent place coordinate systems, respectively, we have

$$\vec{r}^*(t_o) = \underline{X}^T(t_o)\ \underline{\theta}(t_o)\ \vec{r}(t_o) \tag{4.4}$$

where

$$\underline{X}^T(t_o) = \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & -y \\ -x & y & 1 \end{bmatrix} \tag{4.5}$$

and x,y are the displacements of the instantaneous rotation pole with respect to the CIO at t_o, where

$$\underline{\theta}(t_o) = \begin{bmatrix} \cos\theta^*(t_o) & \sin\theta^*(t_o) & 0 \\ -\sin\theta^*(t_o) & \cos\theta^*(t_o) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.6}$$

and Θ^* = Greenwich Apparent Sidereal Time at t_o, where

$$\vec{r}(t_o) = \underline{R}_3(-\Omega)\underline{R}_1(-i)\underline{R}_3(-\omega) \begin{bmatrix} r\cos(f) \\ r\sin(f) \\ 0 \end{bmatrix} \tag{4.7}$$

and

$$r = a(1-e^2)/(1+e\cos f) \quad , \tag{4.8}$$

$$f = 2\arctan\left[\left(\frac{1+e}{1-e}\right)^{1/2} \tan\left(\frac{E}{2}\right)\right] \tag{4.9}$$

and the Kepler equation

$$E = \left(\frac{GM}{a^3}\right)^{1/2} (t_o - T_o) + e \sin E \quad , \tag{4.10}$$

gives E, the eccentric anomaly. f is the true anomaly at time t_o.

4.3 Transformation Equations

It was shown in Chapter 2 that the Jacobian matrix of transformation from the system of Keplerian elements into the conventional terrestrial system is needed to solve our problem. To derive the Jacobian let us first define the following functions:

$$\vec{z}_i(t) = \frac{\partial \vec{r}}{\partial k_i} , \quad i=1,2,\ldots,6 \quad , \tag{4.11}$$

really the elements of the Jacobian matrix of transformation from Keplerian elements into the apparent place system. It is easy to define implicitly these functions by taking the total derivatives of eqns. (4.1) and (4.2) with respect to these parameters. The result (together with the corresponding initial conditions) is usually called the system of variational equations for the orbital elements

$$\ddot{\vec{z}}_i = \underline{A}_o \vec{z}_i + \underline{A}_1 \dot{\vec{z}}_i \quad , \tag{4.12}$$

$$\vec{z}_i(t_o) = \frac{\partial \vec{r}_o}{\partial k_i}$$

$$\dot{\vec{z}}_i(t_o) = \frac{\partial \dot{\vec{r}}_o}{\partial k_i} \quad . \tag{4.13}$$

where the matrices $\underline{A}_o$ and $\underline{A}_1$ are defined by their elements in the following way:

$$\underline{A}_{o,ik} = \frac{\partial f_i}{\partial r_k} \; ; \; \underline{A}_{1,ik} = \frac{\partial f_i}{\partial \dot{r}_k} , \quad i=1,2,3; \; k=1,2,3 \quad . \tag{4.14}$$

Every orbit determination is actually an orbit improvement process, which means that we always have approximate orbits at our disposal. In practice we evaluate the partial derivatives for these known orbits, which means that we may assume the matrices $\underline{A}_o$ and $\underline{A}_1$ in eqn. (4.12) to be known.

Equation (4.12) is a system of second-order differential equations for each of the elements k_i. As opposed to the original system of equations (4.1), the variational system of eqn. (4.12) is linear and homogeneous. These properties may be used to produce very accurate and very powerful numerical solution algorithms (see Beutler [1982]).

However, the partials of eqn. (4.11) are calculated approximately in VECA since only short arcs are being considered here. It should be pointed out that the benefit stemming from the approximations given below is not a saving of computing time, but a simpler program structure.

It is well known that eqn. (4.1) has an analytical solution if we approximate $\vec{f}$ by

$$\vec{f} = - GM \frac{\vec{r}}{r^3} \quad . \tag{4.15}$$

This analytical solution is given by eqns. (4.4) to (4.10) where these expressions may actually be used for any time t and not only for $t = t_o$ (the time for which eqns. (4.4) to (4.10) are explicitly written). In VECA, the partials defined in eqn. (4.11) are approximated by taking the derivatives of eqn. (4.7) with respect to the elements (and not by solving the initial value problems defined by eqns. (4.12) and (4.13)).

The derivatives of the satellite position vector $\vec{r}*(t)$ in the conventional terrestrial system (see eqn. (4.4)) with respect to the elements k_i are given as follows:

Let $\Omega^* = \Theta^* - \Omega$ (4.16)

$$\tilde{r}^T = r \cdot (\cos f, \sin f) \tag{4.17}$$

$$\underline{M}_o = \begin{bmatrix} \cos\Omega^*\cos\omega - \sin\Omega^*\cos i\sin\omega, & -\cos\Omega^*\sin\omega - \sin\Omega^*\cos i\cos\omega \\ \sin\Omega^*\cos\omega + \cos\Omega^*\cos i\sin\omega, & -\sin\Omega^*\sin\omega + \cos\Omega^*\cos i\cos\omega \\ \sin i\sin\omega, & \sin i\cos\omega \end{bmatrix} \tag{4.18}$$

$$\underline{M}_i = \begin{bmatrix} \sin\Omega^*\sin i\sin\omega, & \sin\Omega^*\sin i\cos\omega \\ -\cos\Omega^*\sin i\sin\omega, & -\cos\Omega^*\sin i\cos\omega \\ \cos i\sin\omega, & \cos i\cos\omega \end{bmatrix} \tag{4.19}$$

$$\underline{M}_\Omega = \begin{bmatrix} -\sin\Omega^*\cos\omega - \cos\Omega^*\cos i\sin\omega, & \sin\Omega^*\sin\omega - \cos\Omega^*\cos i\cos\omega \\ \cos\Omega^*\cos\omega - \sin\Omega^*\cos i\sin\omega, & -\cos\Omega^*\sin\omega - \sin\Omega^*\cos i\cos\omega \\ 0, & 0 \end{bmatrix} \tag{4.20}$$

$$\underline{M}_\omega = \begin{bmatrix} -\cos\Omega^*\sin\omega - \sin\Omega^*\cos i\cos\omega, & -\cos\Omega^*\cos\omega + \sin\Omega^*\cos i\sin\omega \\ -\sin\Omega^*\sin\omega + \cos\Omega^*\cos i\cos\omega, & -\sin\Omega^*\cos\omega - \cos\Omega^*\cos i\sin\omega \\ \sin i\cos\omega, & -\sin i\sin\omega \end{bmatrix} \tag{4.21}$$

$$E_a = \frac{dE}{da} = -\frac{3}{2}\left(\frac{GM}{a^3}\right)^{1/2}\frac{1}{r}(t - T_o) \tag{4.22}$$

$$E_e = \frac{dE}{de} = \frac{a}{r}\sin E \tag{4.23}$$

$$E_{T_o} = \frac{dE}{dT} = \left(\frac{GM}{a^3}\right)^{1/2}\frac{a}{r} \quad . \tag{4.24}$$

We then have

$$\frac{\partial \vec{r}^*}{\partial a} = \underline{X}^T \underline{M}_o \left\{ \frac{1}{a} \underline{\tilde{r}} + \begin{bmatrix} -a\sin E \, E_a \\ +a(1-e^2)^{1/2} \cos E \, E_a \end{bmatrix} \right\}$$

$$\frac{\partial \vec{r}^*}{\partial e} = \underline{X}^T \underline{M}_o \begin{bmatrix} -a(1+\sin E \, E_e) \\ a(e/(1-e^2)^{1/2} \sin E + (1-e^2)^{1/2} \cos E \, E_e) \end{bmatrix}$$

$$\frac{\partial \vec{r}^*}{\partial i} = \underline{X}^T \underline{M}_i \underline{\tilde{r}} \qquad (4.25)$$

$$\frac{\partial \vec{r}^*}{\partial \Omega} = \underline{X}^T \underline{M}_\Omega \underline{\tilde{r}}$$

$$\frac{\partial \vec{r}^*}{d\omega} = \underline{X}^T \underline{M}_\omega \underline{\tilde{r}}$$

$$\frac{\partial \vec{r}^*}{\partial T_o} = \underline{X}^T \underline{M}_o \begin{bmatrix} -a\sin E \, E_{T_o} \\ a(1-e^2)^{1/2} \cos E \, E_{T_o} \end{bmatrix} .$$

The proper working of the procedure outlined here has been tested in VECA using elliptical orbits. These tests have been successful, demonstrating that the implementation is correct. It may be desirable to replace the orbital modelling presently used in VECA (relying basically on the GPS messages) by a more accurate procedure, based on numerical integration.

CHAPTER 5

IMPLEMENTATION OF MATHEMATICAL MODELS

In this chapter we outline the software that has been written to implement the models described in earlier chapters. Since we have attempted to make the software self-documenting, here we take a block-diagram approach.

The software has been implemented on both the UNB IBM 3081 mainframe computer, and on the UNB Surveying Engineering HP-1000/F minicomputer. The latter implementation was used for the simulations reported in Chapter 7. It is the version that will be under continued active development, and which is compatible with hardware elsewhere than at UNB. Hence we describe only the HP implementation here. The two implementations do not differ significantly, however, particularly in terms of the mathematical models. The main differences are the interactive capabilities of the HP implementation, which do not exist for the IBM implementation.

The functions of the four main programs involved are:

GPS Interactively set up and schedule n runs of FOROB, VECA, and VEPLT.

FOROB Select the desired subset of the data on the input observation magnetic tape, for processing by VECA.

VECA Vector GPS adjustment. The mathematical models described earlier are all implemented in VECA. This is the only one of these four programs also implemented on the IBM version.

VEPLT Plot the results of one VECA run, using the Autoplot feature of the HP2648A terminal.

Normally GPS will be the only program run by the operator. However, it is possible to "manually" run the other program, as long as the appropriate input files have been set up.

These programs use many disk data files. The # character is used as the first character of a data file. There are four input data files which must be available before any of these progams can be run. They can be set up using the HP Editor, or for some external data source. They are:

#DEFLT Default values for all interactive options for program GPS.

#STATN A priori station coordinates and covariance matrix.

#EPHEM Satellite ephemerides for all satellites to be used.

Observation tape.

If GPS sets up and schedules n runs of FOROB, VECA, and VEPLT, then 4n+1 temporary data files will be set up and used. These are:

#FORii Run instructions for the ith run of FOROB

#VECii Run instructions for the ith run of VECA

#PLTii Run instructions for the ith run of VEPLT

#RESii Results of the ith run of VECA

#VEOBS Input data selected by FOROB for each run of VECA. Due to the size of this file, it is overwritten for each run.

Figure 5.1 shows the overall interaction between the four programs and all of these files. Figures 5.2, 5.3, 5.4, and 5.5 are block diagrams of GPS, FOROB, VEPLT, and VECA, respectively. Tables 5.1, 5.2, 5.3, and 5.4 are LOADR maps of these four progams, with short descriptions of each subroutine used.

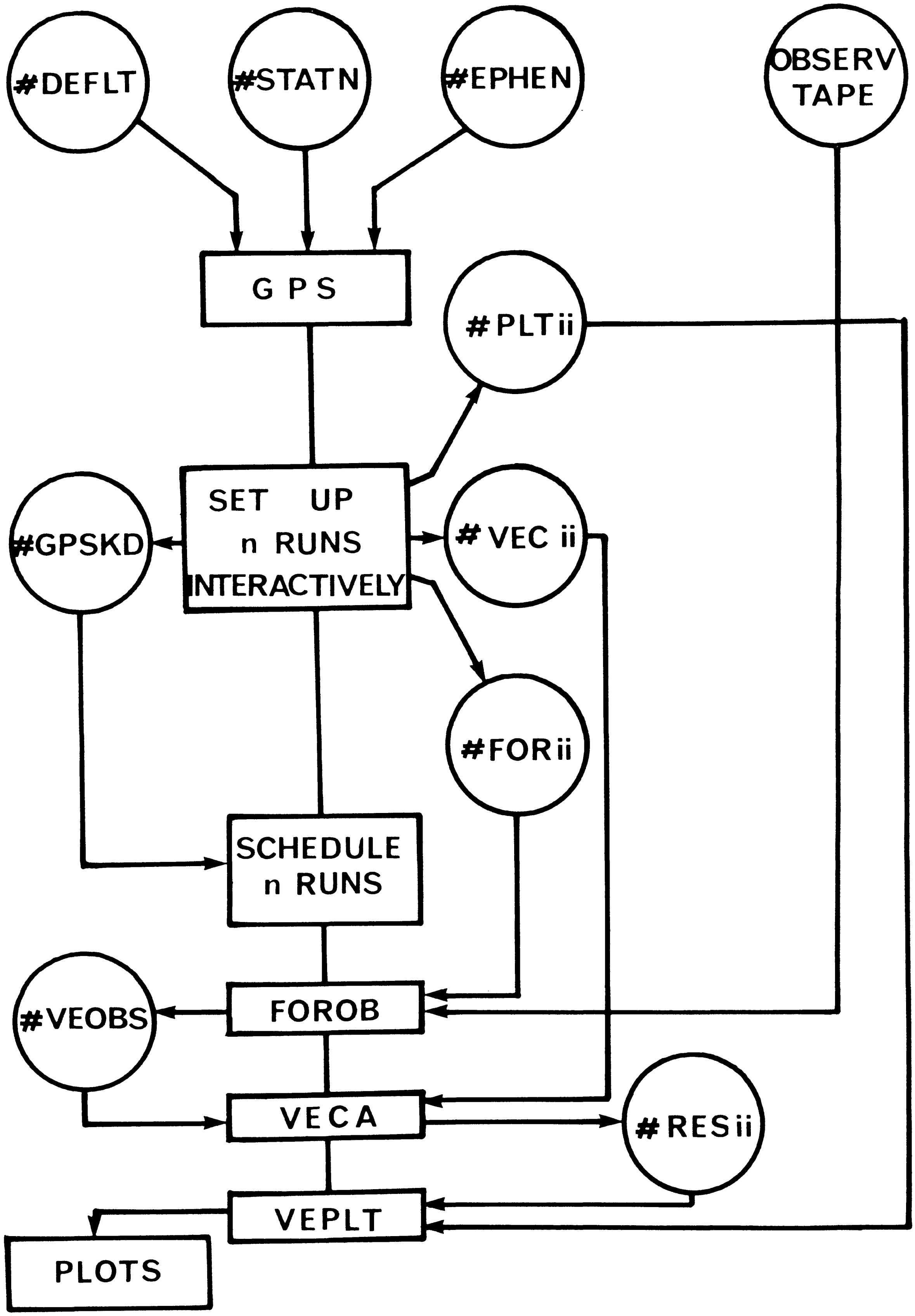

Fig. 5.1 Overall interaction between the program and the files

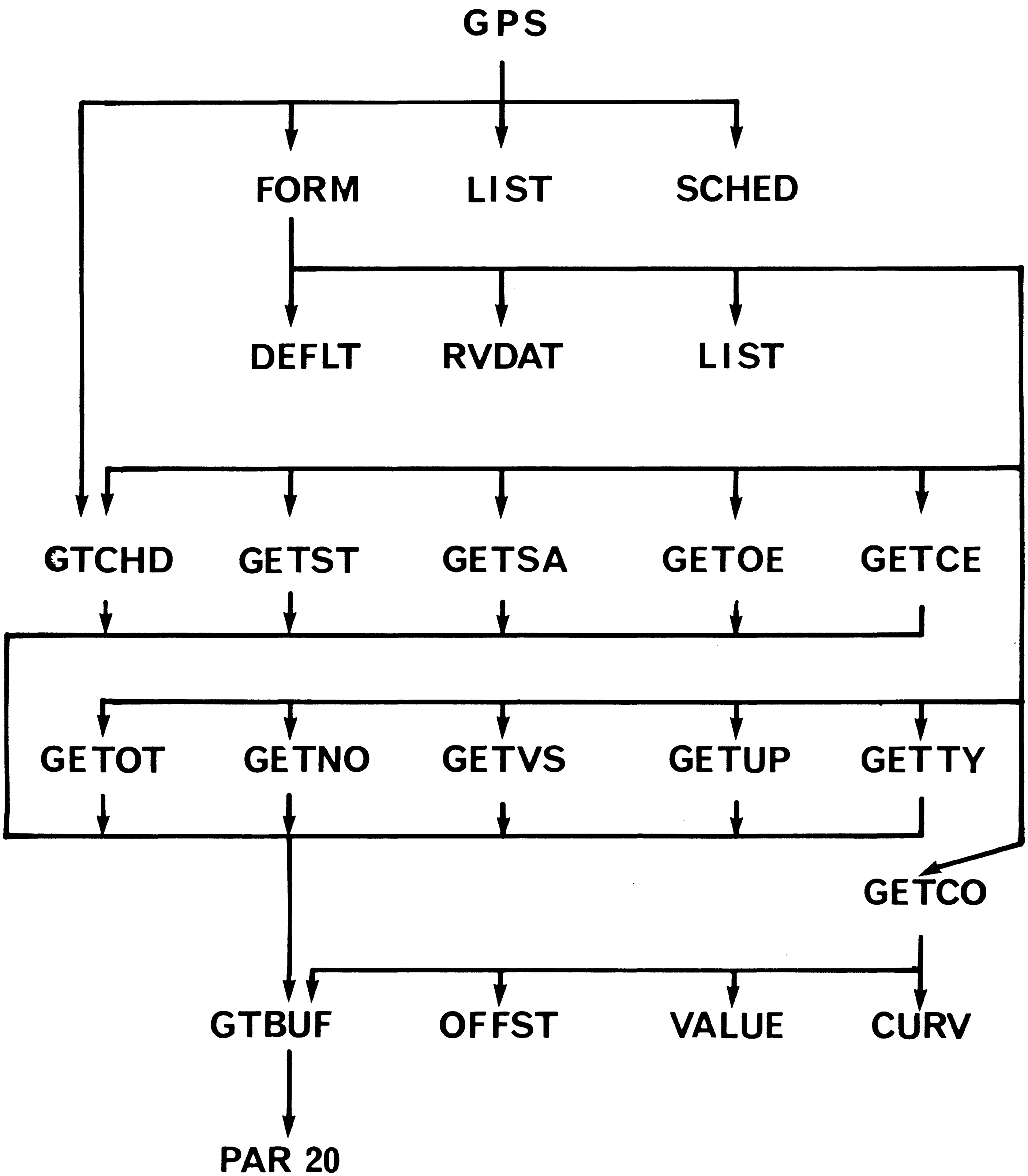

Fig. 5.2 Block diagram of GPS

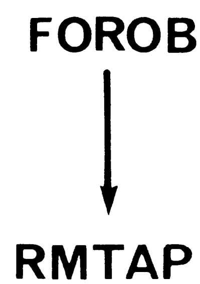

Fig. 5.3 Block diagram of FOROB

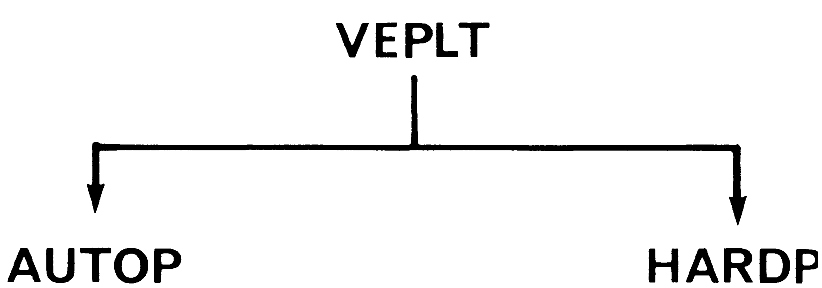

Fig. 5.4 Block diagram of VEPLT

```
VECA
|
|           DASET
|           ZEROE
|
|-- 2       DATUM
|-- 3       REDXR   DECDG, PLXYZ, ZERO, ERR3D, DASTE, MOUTE, SPINE, COMRM
|-- 4       GTEPH   READE, DASET
|-- 5       ZERO
|-- 5       IZERO
|-- 6       SRTUV   ZCONT, DASET, CKCOR, OSCIC, UNITV, VMEAN
|-- 7       DSGAR   TRPCR, DOTVC
|-- 8       EXTAR
|-- 9       PRLSA   ROWSE, SATDR, DOTVC
|-- 10      BATCH   LSA, MATE2, MATE3
|-- 11, 14  EXTPX
|-- 12, 15  COMPR
|-- 13      SEQSL   SPINE, MATE3
|-- 16      REDOB   LCSTA, RION, CION
|-- 17      RPART
|-- 18, 19  UPDAT   RANGE, SCMUL, UNITV, DOTVC, ROWSE, SPINE, XYZPL
L-- 20, 21  COMSY   XYZPL, ROTRF, PROP, BASEL


BASEL       CARTL
CKCOR       ANML2
COMRM       VMEAN, DASET, MOUTD
DERIV       ROTRF
LSA         SPINE
OSCIC       ANMLY, ROTRF, DASET, SCMUL, RANGE, SCDOT, VECSM
READE       NCLOK, DASET, NEPHM, CLKAN
SATDR       VECSM, ROWSE
TRPCR       DERIV, TROP
VMEAN       SCMUL, VECSM

CLKAN       ANMLZ
NCLOK       DASET
NEPHM       DASET
```

FIGURE 5.5

Block Diagram of VECA.

GPS2	12042	26755	VECA INTERACTIVE INPUT PROGRAM WITH VEPLT	<840128.1616>
FORM	26756	30137	INTERACTIVE INPUT FOR ONE FOROB / VECA RUN	<840128.1616>
RVDAT	30140	30601	READ/EDIT &VDAT1 FILE (STN COORDS/EPHEM)	<840128.1616>
DEFLT	30602	31202	SET INPUT PARAMETERS TO DEFAULT VALUES	<840128.1616>
LIST	31203	32202	LIST INPUT PARAMETER VALUES	<840128.1616>
GTCMD	32203	33550	GET INTERACTIVE DATA ENTRY COMMAND	<840128.1616>
GETST	33551	34101	ENTER STATION NUMBERS	<840128.1616>
GETSA	34102	34435	ENTER SATELLITE NUMBERS	<840128.1616>
GETOE	34436	34652	ENTER NUMBER OF ORBIT PARAMETERS TO BE EST.	<840128.1616>
GETCE	34653	35053	ENTER STATION NUMBERS	<840128.1616>
GETOT	35054	35607	ENTER OBSERVATION TIME SPAN AND INTERVAL	<840128.1616>
GETNO	35610	36005	ENTER NUMBER OF OBSERVATIONS PER BATCH	<840128.1616>
GETVS	36006	36206	ENTER NUMBER OF SATELLITES ACTUALLY VISIBLE	<840128.1616>
GETUP	36207	36657	ENTER UPDATE SWITCH AND LIMITS	<840128.1616>
GETTY	36660	37346	ENTER OBSERVATION TYPES	<840128.1616>
GETCO	37347	40751	STATION COORDINATE OFFSET AND SIGMAS	<840128.1616>
GETAU	40752	41155	ENTER AUTOPLOT OUTPUT FILE OPTION	<840128.1616>
CURV	41156	41474	COMPUTE LAT/LON RADIAN TO METRE CONVERSION	<840128.1616>
OFFST	41475	41667	ADD RADIANS TO ANGLE IN D/M/S	<840128.1616>
VALUE	41670	42016	FUNCTION TO RETURN D.P. VALUE FROM IPBUF	<840128.1616>
GTBUF	42017	42221	GET A PARSED INPUT PARAMETER STRING	<840128.1616>
PAR20	42222	43640	PARSE INPUT STRING OF UP TO 20 PARAMS	<840128.1616>
SCHED	43641	44154	SCHEDULE PROGRAM (WITH/WITHOUT WAIT/QUEUE)	<840128.1616>

Table 5.1 LOADR of GPS

FOROB	30042	74144	SELECT SUBSET OF OBS DATA FOR VECA INPUT	<840130.1615>
RMTAP	74145	74215	READ AND UNBLOCK ASCII DATA FROM TAPE	<840130.1615>

Table 5.2 LOADR of FOROB

VEPLT	30042	31533	AUTOPLOT VECA OUTPUT	<840130.1609>
AUTOP	31534	32033	SET UP AUTOPLOT FUNCTIONS AUTOMATICALLY	<840130.1609>
HARDP	32034	32135	COPY HP2648A GRAPHICS MEMORY TO PLOTTER	<840130.1609>

Table 5.3 LOADR of VEPLT

```
COM     12042 22054
VECA1   22055 30720
DASET   30721 30755  VSUB - COPY DBLE PREC VECTOR (VIS, NO EMA) <840218.0
ZEROE   30756 31021  VSUB - ZERO REAL*8 MATRIX IN EMA (VIS)     <840217.1

RMPAR   31022 31066  92068-1X025 REV.2101 800919
ERR0    31067 31074  24998-1X250 REV.2140 810506
.EXIT   31075 31150  24998-1X320 REV.2101 800731
.FFRW   31151 31274  24998-1X297 REV.2226 820413 N
.FMER   31275 31346  24998-1X352 REV.2226 820412
.EIO.   31347 31421  24998-1X329 REV.2226 820503
.FMCN   31422 31503  24998-1X345 REV.2226 820107
.IOER   31504 31651  24998-1X321 REV.2140 810506
.FMFP   31652 33115  24998-1X346 REV.2226 820426
.FMO?   33116 33177  24998-1X351 REV.2140 810415
.IIO.   33200 33353  24998-1X343 REV.2140 810422
.UFMP   33354 33371  24998-1X296 REV.2226 820426
.FMCV   33372 34631  24998-1X333 REV.2303 830103
.FMUI   34632 35764  24998-1X349 REV.2140 810416
.FMGB   35765 36241  24998-1X353 REV.2226 820420
.FMIO   36242 36464  24998-1X348 REV.2226 820420
.FPAU   36465 36570  24998-1X324 REV.2101 800731
PAU.E   36571 36571  24998-1X254 REV.2001 750701
.IOOP   36572 36600  24998-1X300 REV.2101 800805
.IOCM   36601 36644  24998-1X327 REV.2101 801007
.FFOP   36645 40064  24998-1X301 REV.2226 820414 N
.IOCL   40065 40166  24998-1X305 REV.2101 800731
.FOP?   40167 40244  24998-1X326 REV.2101 800729
.FFCL   40245 40605  24998-1X306 REV.2226 820414 N
.FIOI   40606 40676  24998-1X322 REV.2226 820629
.SQRT   40677 40770  24998-1X128 REV.2226 820414
.YINT   40771 41016  24998-1X133 REV.2001 780424
.TENT   41017 41132  24998-1X160 REV.2001 780424
DVWMV   41133 41160  12824-1X043 REV.2026 800506
LOGLU   41161 41236  92067-1X297 REV.2013 790228
REIO    41237 41363  92067-1X275 REV.2140 810805
OPEN    41364 41744  92067-16125 REV.2101 810615
CLOSE   41745 42161  92067-16125 REV.2140 810616
NAMR    42162 42461  92068-1X021 REV.2226 820225
$SMVE   42462 42554  92067-1X483 REV.2013 800129
LURQ    42555 43167  92067-1X270 REV.2013 791024
POST    43170 43216  92067-16125 REV.1903 740801
OVRD.   43217 43217  92067-16125 REV.1903 780526
RWNDF   43220 43304  92067-16125 REV.1903 780724
CREAT   43305 43674  92067-16125 REV.2226 820420
```

Table 5.4 LOADR of VECA

```
LOCF    43675 44175  92067-16125 REV.1903 781110
LUTRU   44176 44304  92067-1X308 REV.2013 790223
SESSN   44305 44322  92067-16125 REV.1903 780413
R/W$    44323 44461  92067-16125 REV.2101 801013
P.PAS   44462 44510  92067-16125 REV.1903 740801
IFTTY   44511 44576  92067-1X295 REV.2013 790118
PNAME   44577 44647  92068-1X035 REV.2101 800919
$ALRN   44650 44765  92067-1X271 REV.2013 770715
LIMEM   44766 45027  92067-1X477 REV.2226 820326
$OPEN   45030 45204  92067-16125 REV.1903 790103
READF   45205 46517  92067-16125 REV.2226 820114
RW$UB   46520 47071  92067-16125 REV.2101 800303
RWND$   47072 47224  92067-16125 REV.2226 820114
.OPN?   47225 47250  24998-1X325 REV.2101 800803
ER0.E   47251 47251  24998-1X249 REV.2001 750701
.FMIN   47252 47532  24998-1X344 REV.2226 820420
.WCOM   47533 50104  12824-1X045 REV.2026 800506
NAM..   50105 50201  92067-16125 REV.1903 740801
COR.A   50202 50222  92067-1X277 REV.2013 770621
.LWAS   50223 50223  92067-1X592 REV.2226 820326


VECA2   50224 50242
DATUM   50243 50525  VSUB - INITIALIZE DATUM PARAMETERS          <840218.0045>


VECA3   50224 50254
REDXR   50255 53404  VSUB - READ STATION FILE                    <840310.1546>
SPINE   53405 54274  VSUB - MATRIX INVERSION IN EMA              <840217.1126>
ZERO    54275 54333  VSUB - ZERO REAL*8 MATRIX (VIS)             <840217.1126>
COMRM   54334 54656  VSUB - MEAN ALL STN VECTOR PAIRS            <840218.0043>
ERR3D   54657 60351  VSUB - CARTESIAN COV TO GEODETIC AND VV     <840311.1019>
DASTE   60352 60436  VSUB - COPY DBLE PREC VECTOR (VIS, EMA OUT)<840218.0044>
DECDG   60437 60557  VSUB - DEG/MIN/SEC TO DEGREES               <840218.0045>
MOUTE   60560 60730  VSUB - PRINT OUT MATRIX FROM EMA            <840218.0051>
PLXYZ   60731 61175  VSUB - ELLIPSOIDAL TO CARTESIAN COORDS      <840310.1545>
VMEAN   61176 61256  VSUB - COMPUTE MEAN OF TWO VECTORS          <840217.1126>
SCMUL   61257 61311  VSUB - MULTIPLY VECTOR BY SCALAR            <840217.1126>
VECSM   61312 61372  VSUB - VECTOR SUM OR DIFFERENCE (VIS)       <840217.1126>

.DMAP   61373 61563  92068-1X046 REV.2101 800919
.FIO.   61564 61624  24998-1X330 REV.2140 810414
```

Table 5.4 Continued

```
.FMLD   61625 62720  24998-1X347 REV.2226 820423
.SST    62721 63036  24998-1X336 REV.2140 810812
.TTOT   63037 63142  24998-1X132 REV.2013 791019
.LOG    63143 63274  24998-1X158 REV.2001 790417
.EXP    63275 63371  24998-1X156 REV.2001 780921
.TSCS   63372 63562  24998-1X131 REV.2001 790417
.ABS    63563 63604  24998-1X030 REV.2001 781016
/EXTH   63605 63715  24998-1X175 REV.2001 790417
.LOG0   63716 63741  24998-1X125 REV.2001 780424
.VDRP   63742 64013  12824-1X047 REV.2026 800506
DWDOT   64014 64020  12824-1X042 REV.2026 800506
.4ZRO   64021 64024  24998-1X183 REV.2001 780424


VECA4   50224 50241
GTEPH   50242 50765  VSUB - READ EPHEMERIS AND CORRECT CLOCK    <840218.0048>
READE   50766 51403  VSUB - READ EPHEMERIS DISC FILE            <840217.2219>
CLKAN   51404 51724  VSUB - CORRECT CLOCK COEFFICIENTS          <840218.0042>
NCLOK   51725 52116  VSUB - EXTRACT CLOCK INFO FROM EPHEMERIDES <840218.0051>
NEPHM   52117 52421  VSUB - EXTRACT EPHEM INFO FROM INPUT RECORD<840218.0051>
ANML2   52422 53115  VSUB - ECCTY/MEAN ANOM  TO  ECC/TRUE ANOM  <840218.0040>

.TTOT   53116 53221  24998-1X132 REV.2013 791019
.LOG    53222 53353  24998-1X158 REV.2001 790417
.EXP    53354 53450  24998-1X156 REV.2001 780921
.TSCS   53451 53641  24998-1X131 REV.2001 790417
.ATA2   53642 53756  24998-1X118 REV.2101 800421
.ABS    53757 54000  24998-1X030 REV.2001 781016
.ATAN   54001 54201  24998-1X154 REV.2001 790417
/EXTH   54202 54312  24998-1X175 REV.2001 790417
.MOD    54313 54353  24998-1X058 REV.2001 781016
.4ZRO   54354 54357  24998-1X183 REV.2001 780424


VECA5   50224 50761
ZERO    50762 51020  VSUB - ZERO REAL*8 MATRIX (VIS)            <840217.1126>
IZERO   51021 51100  VSUB - ZERO INTEGER MATRIX                 <840217.1126>


VECA6   50224 50261
SRTUV   50262 52520  VSUB - SORT OUT UNIT VECTORS               <840217.1126>
```

Table 5.4 Continued

```
UNITV   52521 52641   VSUB - SAT/USER COORDS TO UNIT VECTOR (VIS)<840217.1126
VMEAN   52642 52722   VSUB - COMPUTE MEAN OF TWO VECTORS         <840217.1126
ZCONT   52723 53144   VSUB - GIVEN TIME, COMPUTE Z-COUNT         <840217.1126
CKCOR   53145 54263   VSUB - CORRECT CLOCK FROM EPHEMERIS VALUES <840218.0042
OSCIC   54264 55453   VSUB - SAT EARTH FIXED POSN/VEL FROM EPHEM <840217.2219
RANGE   55454 55577   VSUB - STATION TO SATELLITE RANGE          <840217.2219
ROTRF   55600 56626   VSUB - ROTATION/REFLECTION PRODUCT MATRIX  <840217.1126
SCDOT   56627 60713   VSUB - SAT VEL (AVG TERR) FROM EPHEM       <840217.1126
SCMUL   60714 60746   VSUB - MULTIPLY VECTOR BY SCALAR           <840217.1126
VECSM   60747 61027   VSUB - VECTOR SUM OR DIFFERENCE (VIS)      <840217.1126
ANMLY   61030 61607   VSUB - EPHEMERIDES TO MEAN/ECC/TRUE ANOM   <840218.0039
ANML2   61610 62303   VSUB - ECCTY/MEAN ANOM  TO  ECC/TRUE ANOM  <840218.0040

.DMAP   62304 62474   92068-1X046 REV.2101 800919
.TSCS   62475 62665   24998-1X131 REV.2001 790417
.ATA2   62666 63002   24998-1X118 REV.2101 800421
.ABS    63003 63024   24998-1X030 REV.2001 781016
.ATAN   63025 63225   24998-1X154 REV.2001 790417
.MOD    63226 63266   24998-1X058 REV.2001 781016
PDMOD   63267 63314   92832-16700 REV.2101 801010
.VDRP   63315 63366   12824-1X047 REV.2026 800506
.4ZRO   63367 63372   24998-1X183 REV.2001 780424


VECA7   50224 50250
DSGAR   50251 50574   VSUB - DIFFERENTIAL RANGE DESIGN MATRIX    <840218.0047
TRPCR   50575 51674   VSUB - TROPO CORRECTION FOR CARRIER PHASE  <840310.1550
TRPRG   51675 52232   VSUB - TROPO CORRECTION FOR RANGES         <840310.1551
DERIV   52233 53541   VSUB - STN/SAT RNG + DERIV WRT LAT/LON/ELEV<840218.0046
DOTVC   53542 53574   VSUB - SCALAR PROD OF TWO POSN VEC (VIS)   <840218.0046
ROTRF   53575 54623   VSUB - ROTATION/REFLECTION PRODUCT MATRIX  <840217.1126

.DMAP   54624 55014   92068-1X046 REV.2101 800919
.TSCS   55015 55205   24998-1X131 REV.2001 790417
.ATA2   55206 55322   24998-1X118 REV.2101 800421
.ABS    55323 55344   24998-1X030 REV.2001 781016
.ATAN   55345 55545   24998-1X154 REV.2001 790417
.4ZRO   55546 55551   24998-1X183 REV.2001 780424


VECA8   50224 50235
EXTAR   50236 50455   VSUB - EXTEND A MATRIX FOR STN CLOCK PARAMS<840218.0048
```

Table 5.4 Continued

```
.TTOJ   50456 50511  24998-1X258 REV.2101 800303
.TTOI   50512 50630  24998-1X070 REV.2013 791019
.4ZRO   50631 50634  24998-1X183 REV.2001 780424


VECA9   50224 50270
PRLSA   50271 51652  VSUB - PREPARE DESIGN MAT/MISCL VEC FOR LSA<840217.2219>
ROWSE   51653 51754  VSUB - PLACE VECTOR INTO MATRIX ROW (EMA)  <840217.1126>
SATDR   51755 52510  VSUB - PREPARE DESIGN MATRIX FOR DOPPLER   <840217.1126>
VECSM   52511 52571  VSUB - VECTOR SUM OR DIFFERENCE (VIS)      <840217.1126>
DOTVC   52572 52624  VSUB - SCALAR PROD OF TWO POSN VEC (VIS)   <840218.0046>

.DMAP   52625 53015  92068-1X046 REV.2101 800919
.VDRP   53016 53067  12824-1X047 REV.2026 800506


VEC10   50224 50273
BATCH   50274 50723  VSUB - SOLN/COVAR FROM FIRST OBSERV BATCH  <840218.0411>
LSA     50724 53564  VSUB - LEAST SQUARES APPROXIMATION SOLUTION<840218.0050>
MATE2   53565 54234  VSUB - MATMY FROM EMA TO NORMAL            <840218.0050>
MATE3   54235 54736  VSUB - MATMY FROM EMA TO EMA               <840218.0050>
SPINE   54737 55626  VSUB - MATRIX INVERSION IN EMA             <840217.1126>

.DMAP   55627 56017  92068-1X046 REV.2101 800919
.TTOT   56020 56123  24998-1X132 REV.2013 791019
.LOG    56124 56255  24998-1X158 REV.2001 790417
.EXP    56256 56352  24998-1X156 REV.2001 780921
.ABS    56353 56374  24998-1X030 REV.2001 781016
/EXTH   56375 56505  24998-1X175 REV.2001 790417
.LOG0   56506 56531  24998-1X125 REV.2001 780424
DWDOT   56532 56536  12824-1X042 REV.2026 800506
.4ZRO   56537 56542  24998-1X183 REV.2001 780424


VEC11   50224 50247
EXTPX   50250 51053  VSUB - EXTEND PX FOR ADDED ORBIT PARAMS    <840218.0048>

.DMAP   51054 51244  92068-1X046 REV.2101 800919


VEC12   50224 50257
```

Table 5.4 Continued

```
COMPR   50260 54005  VSUB - COMPARE ESTIMATED AND APRIORI COORDS<840218.0043>

.DMAP   54006 54176  92068-1X046 REV.2101 800919


VEC13   50224 50314
SEQSL   50315 52545  VSUB - SOLN/COVAR SEQUENTIAL UPDATE         <840217.1126>
SPINE   52546 53435  VSUB - MATRIX INVERSION IN EMA              <840217.1126>
MATE3   53436 54137  VSUB - MATMY FROM EMA TO EMA                <840218.0050>

.DMAP   54140 54330  92068-1X046 REV.2101 800919
.TTOT   54331 54434  24998-1X132 REV.2013 791019
.LOG    54435 54566  24998-1X158 REV.2001 790417
.EXP    54567 54663  24998-1X156 REV.2001 780921
.ABS    54664 54705  24998-1X030 REV.2001 781016
/EXTH   54706 55016  24998-1X175 REV.2001 790417
.LOG0   55017 55042  24998-1X125 REV.2001 780424
DWADD   55043 55047  12824-1X024 REV.2026 800506
.VDRP   55050 55121  12824-1X047 REV.2026 800506
DWDOT   55122 55126  12824-1X042 REV.2026 800506
.4ZRO   55127 55132  24998-1X183 REV.2001 780424


VEC14   50224 50250
EXTPX   50251 51054  VSUB - EXTEND PX FOR ADDED ORBIT PARAMS     <840218.0048>

.DMAP   51055 51245  92068-1X046 REV.2101 800919


VEC15   50224 50261
COMPR   50262 54007  VSUB - COMPARE ESTIMATED AND APRIORI COORDS<840218.0043>

.DMAP   54010 54200  92068-1X046 REV.2101 800919


VEC16   50224 50240
REDOB   50241 52454  VSUB - READ OBSERVATION FILE                <840217.2219>
RION    52455 52545  VSUB - PSEUDORANGE IONOSPHERIC CORRECTION   <840217.1126>
CION    52546 52646  VSUB - CARRIER PHASE IONOSPHERIC CORRECTION<840218.0042>
LCSTA   52647 53040  VSUB - ARRAY VALUES FROM STN OR SAT INDEX   <840218.0049>
```

Table 5.4 Continued

```
VEC17   50224 50256
RPART   50257 52520  VSUB - DERIV OF POSN VECTOR WRT ORB ELEM   <840217.1126>

.TSCS   52521 52711  24998-1X131 REV.2001 790417


VEC18   50224 50304
UPDAT   50305 53742  VSUB - UPDATE NORMAL EQUATIONS              <840217.1126>
XYZPL   53743 54416  VSUB - CARTESIAN TO ELLIPSOIDAL COORDS      <840217.1126>
DOTVC   54417 54451  VSUB - SCALAR PROD OF TWO POSN VEC (VIS)    <840218.0046>
RANGE   54452 54575  VSUB - STATION TO SATELLITE RANGE           <840217.2219>
ROWSE   54576 54677  VSUB - PLACE VECTOR INTO MATRIX ROW (EMA)   <840217.1126>
SCMUL   54700 54732  VSUB - MULTIPLY VECTOR BY SCALAR            <840217.1126>
SPINE   54733 55622  VSUB - MATRIX INVERSION IN EMA              <840217.1126>
UNITV   55623 55743  VSUB - SAT/USER COORDS TO UNIT VECTOR (VIS)<840217.1126>

.DMAP   55744 56134  92068-1X046 REV.2101 800919
.TTOT   56135 56240  24998-1X132 REV.2013 791019
.LOG    56241 56372  24998-1X158 REV.2001 790417
.EXP    56373 56467  24998-1X156 REV.2001 780921
.TSCS   56470 56660  24998-1X131 REV.2001 790417
.ATA2   56661 56775  24998-1X118 REV.2101 800421
.ABS    56776 57017  24998-1X030 REV.2001 781016
.ATAN   57020 57220  24998-1X154 REV.2001 790417
/EXTH   57221 57331  24998-1X175 REV.2001 790417
.LOG0   57332 57355  24998-1X125 REV.2001 780424
.VDRP   57356 57427  12824-1X047 REV.2026 800506
DWDOT   57430 57434  12824-1X042 REV.2026 800506
.4ZRO   57435 57440  24998-1X183 REV.2001 780424


VEC19   50224 50304
UPDAT   50305 53742  VSUB - UPDATE NORMAL EQUATIONS              <840217.1126>
XYZPL   53743 54416  VSUB - CARTESIAN TO ELLIPSOIDAL COORDS      <840217.1126>
DOTVC   54417 54451  VSUB - SCALAR PROD OF TWO POSN VEC (VIS)    <840218.0046>
RANGE   54452 54575  VSUB - STATION TO SATELLITE RANGE           <840217.2219>
ROWSE   54576 54677  VSUB - PLACE VECTOR INTO MATRIX ROW (EMA)   <840217.1126>
SCMUL   54700 54732  VSUB - MULTIPLY VECTOR BY SCALAR            <840217.1126>
SPINE   54733 55622  VSUB - MATRIX INVERSION IN EMA              <840217.1126>
UNITV   55623 55743  VSUB - SAT/USER COORDS TO UNIT VECTOR (VIS)<840217.1126>
```

Table 5.4 Continued

.DMAP	55744	56134	92068-1X046 REV.2101 800919
.TTOT	56135	56240	24998-1X132 REV.2013 791019
.LOG	56241	56372	24998-1X158 REV.2001 790417
.EXP	56373	56467	24998-1X156 REV.2001 780921
.TSCS	56470	56660	24998-1X131 REV.2001 790417
.ATA2	56661	56775	24998-1X118 REV.2101 800421
.ABS	56776	57017	24998-1X030 REV.2001 781016
.ATAN	57020	57220	24998-1X154 REV.2001 790417
/EXTH	57221	57331	24998-1X175 REV.2001 790417
.LOG0	57332	57355	24998-1X125 REV.2001 780424
.VDRP	57356	57427	12824-1X047 REV.2026 800506
DWDOT	57430	57434	12824-1X042 REV.2026 800506
.4ZRO	57435	57440	24998-1X183 REV.2001 780424

VEC20	50224	50253	
COMSY	50254	55633	VSUB - COMPARE ADJUSTED - APRIORI COORDS <840218.0044>
PROP	55634	56124	VSUB - COVARIANCE PROPAGATION <840217.2219>
ROTRF	56125	57153	VSUB - ROTATION/REFLECTION PRODUCT MATRIX <840217.1126>
XYZPL	57154	57627	VSUB - CARTESIAN TO ELLIPSOIDAL COORDS <840217.1126>
BASEL	57630	60016	VSUB - COMPARE TRUE/COMP BASELINE LEN/AZ/EL<840218.0040>
CARTL	60017	60441	VSUB - CONVERT CARTESIAN TO LOCAL BASELINE <840218.0411>
.DMAP	60442	60632	92068-1X046 REV.2101 800919
.TSCS	60633	61023	24998-1X131 REV.2001 790417
.ATA2	61024	61140	24998-1X118 REV.2101 800421
.ABS	61141	61162	24998-1X030 REV.2001 781016
.DASN	61163	61324	24998-1X383 REV.2101 800222
.ATAN	61325	61525	24998-1X154 REV.2001 790417
.4ZRO	61526	61531	24998-1X183 REV.2001 780424

VEC21	50224	50253	
COMSY	50254	55633	VSUB - COMPARE ADJUSTED - APRIORI COORDS <840218.0044>
PROP	55634	56124	VSUB - COVARIANCE PROPAGATION <840217.2219>
ROTRF	56125	57153	VSUB - ROTATION/REFLECTION PRODUCT MATRIX <840217.1126>
XYZPL	57154	57627	VSUB - CARTESIAN TO ELLIPSOIDAL COORDS <840217.1126>
BASEL	57630	60016	VSUB - COMPARE TRUE/COMP BASELINE LEN/AZ/EL<840218.0040>
CARTL	60017	60441	VSUB - CONVERT CARTESIAN TO LOCAL BASELINE <840218.0411>
.DMAP	60442	60632	92068-1X046 REV.2101 800919
.TSCS	60633	61023	24998-1X131 REV.2001 790417

Table 5.4 Continued

```
.ATA2   61024 61140  24998-1X118 REV.2101 800421
.ABS    61141 61162  24998-1X030 REV.2001 781016
.DASN   61163 61324  24998-1X383 REV.2101 800222
.ATAN   61325 61525  24998-1X154 REV.2001 790417
.4ZRO   61526 61531  24998-1X183 REV.2001 780424

23 PAGES RELOCATED    91 PAGES REQ'D    68 PAGES EMA        3 PAGES MSEG
LINKS:BP     PROGRAM:LB   LOAD:TE   COMMON:NC
 /LOADR:VECA1  READY AT 10:40 AM  MON., 12  MAR., 1984

 /LOADR:$END
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Table 5.4 Continued

PART B: PERFORMANCE ANALYSIS

CHAPTER 6

DENIAL OF ACCURACY

It is the announced policy of the U.S. Department of Defense [USDOD/DOT, 1983] that upon GPS being declared operational in the late 1980s, access to standard non-differential GPS will be of two types. Precise Positioning Service (PPS) users will have access to the P-code and to the full accuracy capability of GPS. Standard Positioning Service (SPS) users will not have access to the P-code, but only to the C/A-code. Further, the C/A-code will be artificially degraded to limit the accuracy available to SPS users. The policy of imposing this degradation has been called Denial of Accuracy (DOA) and Selective Availability (SA). Following a policy change during 1983, the present policy is that this degradation will be such that an SPS user would obtain the following instantaneous navigational accuracies at the 95% confidence level (over time and space):

	Horizontal	Vertical
Predictability	100 m	160 m
Repeatability	100 m	160 m
Relative Accuracy	10 m	16 m

where predictability measures the uncertainty in the relationship between a position and some well-defined coordinate system; repeatability measures the uncertainty in the capability of returning to the same point; and relative accuracy measures the uncertainty in position with respect to a differential monitor.

These are the prospects for instantaneous non-differential GPS navigation. What are the prospects for time-averaged differential geodetic

GPS positioning? Rather than speculating about such institutional issues as who will have access to PPS, let us consider the mechanism by which the DOA degradation may be implemented, and its implications.

Little official information concerning DOA is available. However we know the following:

(1) The production GPS system will broadcast the same navigation message as is being broadcast at present. This message is changed only once per hour [Payne, 1982]. PPS and SPS use the same message.

(2) Tests with simulated DOA data [Kalafus, 1983] indicate that the characteristic period of the DOA degradation will likely be of the order of tens of seconds.

DOA degradation must involve some mismatch between the actual satellite status (position, clock, signal status, etc.), our knowledge of the satellite status as represented by the satellite messate, and by our measurements. The message is updated only hourly, and is common to both PPS and SPS, so that DOA at a ten-second period cannot be obviously implemented via message degradation.

The only way to implement a mismatch in <u>position</u> with a period of tens of seconds would be to actually physically move the satellite around in orbit. This is not a realistic possibility.

It would be possible to electronically dither the satellite reference clock <u>frequency</u> or <u>epoch</u> so as to depart from the clock model contained within the navigation message. However, this would affect both PPS and SPS users. While it is possible that this kind of dither could be unscrambled by all PPS receivers, but not by SPS receivers, this would involve unnecessary complications. A frequency dither in particular would introduce other complications (in refraction modelling, for example).

The most likely mechanism for DOA is to introduce a dither only in the C/A-code epochs, leaving the carrier, P-Code, and message unaffected. This would impact only on SPS users, and would affect only their pseudorange measurements.

Differential users, whether static or dynamic, will be unaffected by DOA, only as long as both stations in a differential pair make simultaneous measurements (at the same "phase" of the DOA dither).

If this line of reasoning is valid, then the effect of DOA on differential C/A-code positioning will be nil. The effect of DOA on differential P-code positioning will also be nil (provided access to the P-code is available). However, it is more difficult to assess the possible effects on code-independent differential positioning methods, such as the Macrometer and SERIES techniques, since the methods for recovering reconstructed carrier phase without knowledge of the codes are so far proprietary secrets. However, if these methods involve assumptions about the coherence between the carrier and codes, then DOA may cause some problems, since the C/A-code will no longer be coherent (derived from the same basic oscillator) with the carrier and P-code. However, the P-code/carrier coherence would be preserved.

CHAPTER 7

SIMULATION RESULTS

Program VECA was used to estimate station positions from a simulated data set in an effort to obtain answers to the following questions:

(1) How inaccurate can the a priori coordinates of the ground stations be, before an adjustment fails to converge?

(2) What is the best satellite-receiver geometry for differential GPS positioning?

(3) Is it practical or worthwhile to combine more than one kind of differential GPS measurement type?

7.1 Simulation Procedure

The simulation procedure used is as described in Chapter 12 of Davidson et al. [1983]:

(1) "True" values were assigned to the ground and satellite coordinates involved.

(2) The "true" coordinates were used to generate "errorless" observations.

(3) The "errorless" observations were corrupted to account for clock and atmospheric effects, and for measurement noise.

(4) These simulated noisy observations were used as input to the adjustment.

(5) Either the "true" ground station coordinates, or values offset by exactly one kilometre from them, were used as a priori coordinates in the adjustment.

(6) The output from each simulation run consisted of the vector displacements between the adjusted ground station coordinates, and the

a priori coordinates, together with the covariance matrix for these vector displacements.

The ground stations used were stations 1, 3, 4, and 8 of the Point Sapin network (Figure 7.1). Only data on baselines 1-3, 1-4, and 1-8 were used. A priori standard deviations assigned to the "true" coordinate values were always 100 metres. When the one-kilometre offset was applied to the true values, an a priori standard deviation of one kilometre was used. For station 1 (the "fixed" station), an a priori standard deviation of one millimetre on all components was used.

The satellite constellation used was a hypothetical 18-satellite GPS constellation. The simulated observation period was 1800 to 1900 UT on 12 November 1981, during which time six of these 18 GPS satellites were visible from the Point Sapin network. Figure 7.2 shows a polar plot of the azimuth and elevation of each satellite, for this observation period, as seen from station 6 (the centre) of the Point Sapin network.

Observations were generated at six-second intervals, for each of five data types: interferometric delays, differential carrier phase, differential P-code and C/A-code pseudoranges, and differential integrated Doppler. The simulated data was created by programs DIFGPS and FOROBS [Davidson et al., 1983] and stored on file OBSERV44. Because of present limitations of the hardware and operating system of the HP-1000/F computer, the Doppler observations were not used in this analysis.

The station vector displacements (or "discrepancy vectors") resulting from the adjustments are actually the adjusted minus a priori baseline vectors. They can be interpreted as position displacements, however, since we designed the simulations to hold fixed one end of all baselines involved (station 1). Exceptions are runs 3 and 4, discussed below. These

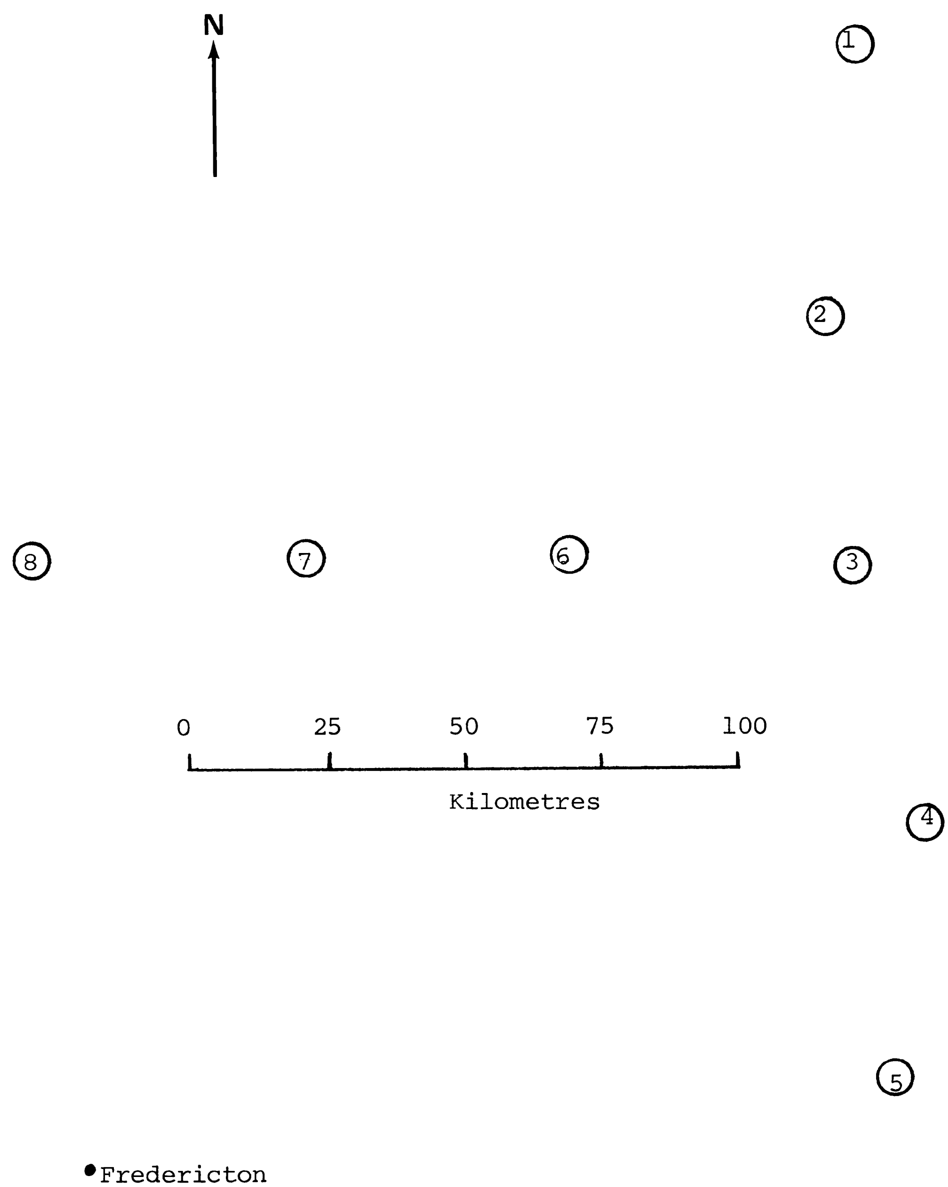


Figure 7.1
Point Sapin Network used for Simulations

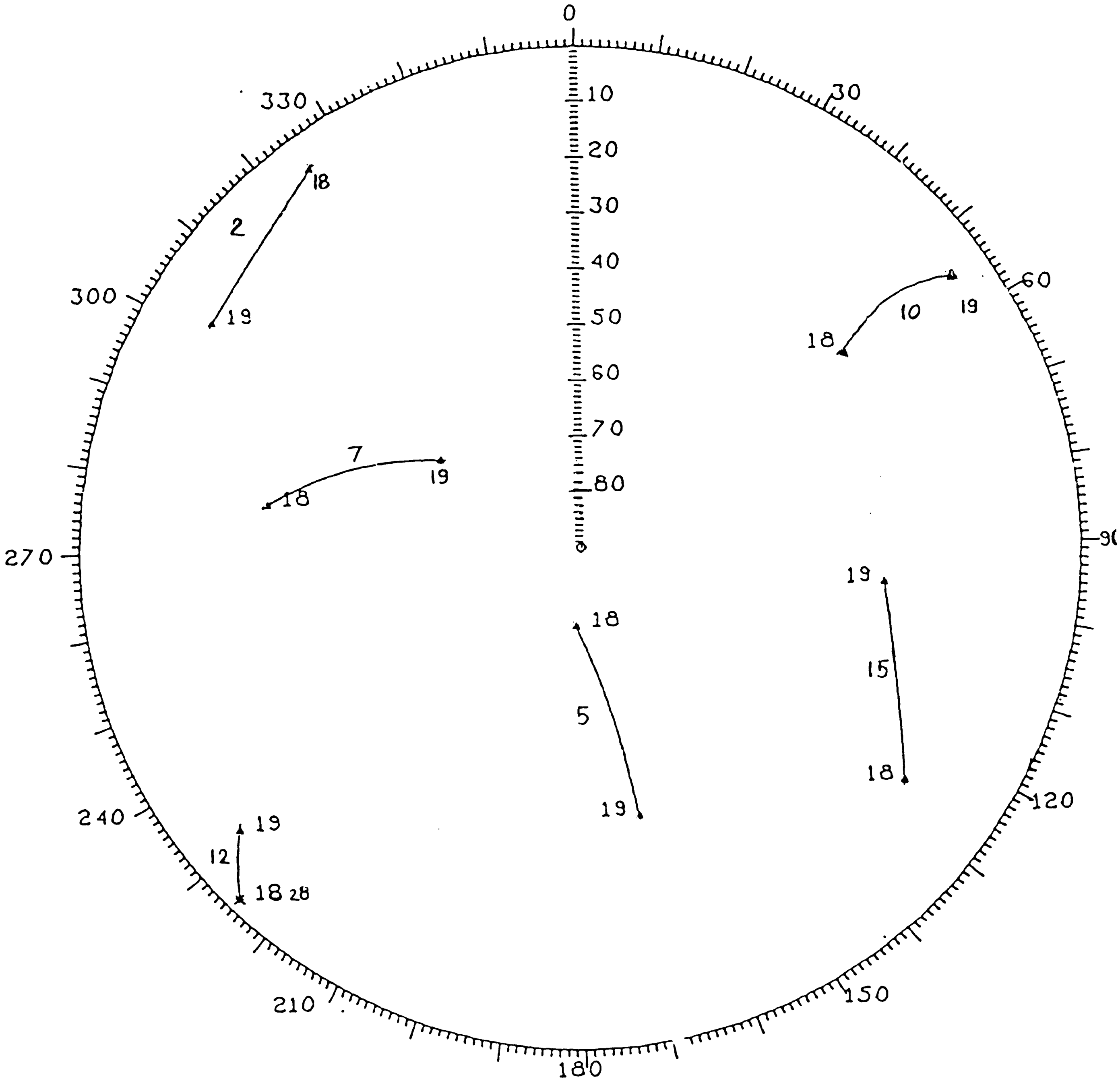


FIGURE 7.2

Polar Plots of Satellite Azimuth and Elevation
as seen from Point Sapin Network Station 6,
for the Period 1800 to 1900 UT, 12 November 1981,
for the Proposed 18-Satellite Constellation.

discrepancy vectors, and their covariance matrices, were presented in three coordinate systems: the geocentric Cartesian system (delta X, delta Y, delta Z), the local topocentric system (delta latitude, delta longitude, delta height), and a system aligned to the a priori baselines (delta length, delta azimuth, delta elevation). The length of the discrepancy vector was also presented.

A total of 19 simulation runs were made, as listed in Table 7.0. Tables 7.1 to 7.19 present the final results for each of the 19 runs, in all three coordinate systems. Shown for each coordinate system are the displacement vector components and length, followed by their standard deviations in parentheses. All values in each table are in millimetres (except for the baseline lengths shown in the bottom right-hand corner, which are in metres).

Figures A.1 to A.19 in Appendix A also present the results for each of the 19 runs. Each figure represents a time history of selected discrepancy vector components (in millimetres) as a function of the accumulated observation time (in seconds). Covariance information is not shown. The figure captions are coded as follows:

-- the discrepancy vector component plotted (e.g., $D\phi$ = delta latitude)
-- observation type (e.g., P-code = differential P-code)
-- satellites used (e.g., 2 5 7 10 12 15, includes all 6 in Figure 7.2)

Station numbers are noted on the plots. A set of four plots (3 components and length of the discrepancy vectors) comprise each of the 19 figures.

7.2 General Results and Conclusions

Runs 1 to 4 (see Tables 7.1 to 7.4 and Figures A.1 to A.4) were designed to consider the first of the above three questions. This test was limited to offsetting either one or two of the a priori components by one kilometre and finding out how well the "true" values were recovered by VECA. In the case of one "bad" component, the true value was recovered to within about 50 mm. In the case of two "bad" components (one for each of the two stations), the baseline components were recovered to within about 50 mm using carrier phase observations, and to within about one metre using P-code pseudorange observations.

Runs 5 to 16 (see Tables 7.5 to 7.16 and Figures A.5 to A.16) were designed to consider the second of the above questions. Using three subsets of the six available satellites shown in Figure 7.2, it was found that the accuracy with which VECA could recover the "true" coordinate values varied from between 10 mm and 100 mm.

Runs 17 to 19 (see Tables 7.17 to 7.19 and Figures A.17 to A.19) were designed to consider the third and last of the above questions, but were limited to the combination of P-code and carrier phase. Results based on the combination of P-code and carrier phase (run 19) differ little from those based on carrier phase alone, since the carrier phase observations are an order of magnitude better than the P-code observations.

These results represent most of the information content of the 19 runs:

(1) One km offsets affect convergence only to the 50 mm to 1 metre level.

(2) Constellation changes affect results at the 10 mm to 100 mm level.

(3) If carrier phase is available, other less accurate observations improve the results very little.

(4) While these are neither surprising nor exhaustive results, perhaps the main conclusion to be derived from them is that VECA performed as expected.

In the following three sections, we discuss in more detail the results of the runs related to each of the above three questions.

7.3 Convergence Tests

The convergence capability of VECA was tested using data from one baseline, that involving stations 1 and 8.

First the a priori latitude of station 8 was offset from the true value by 1 kilometre. Two runs were made; one using P-code pseudorange and one using carrier phase data. The convergence is illustrated in Figures A.1 and A.2 and the final results are tabulated in Tables 7.1 and 7.2.

Referring to Figure A.1, convergence using the pseudo-range data is initially quite rapid. After only a few observations, the bulk of the offset is recovered. After one hour, the final offsets in latitude, longitude, and height are 39, 28, and 45 mm, respectively. Figure A.1 shows that, using pseudo-range data, convergence to about 100 mm of the true position is achieved after about 300 seconds of observations. After one hour, the final offsets in latitude, longitude, and height are less than 50 millimetres.

Figure A.2 shows that, using phase data, convergence to within 50 mm is achieved after 300 seconds of observations.

Next the a priori longitude of station 1 and latitude of station 8 were offset by 1 kilometre. The corresponding figures are Figures A.3 and A.4; the corresponding tables are Tables 7.3 and 7.4.

From Table 7.3 (P-code pseudoranges), we obtain the discrepancy in the 1-8 baseline components by differencing the DLAT, DLON, and DHGHT values, and subtracting the 1 km offsets in latitude and longitude. The results indicate that the "true" baseline is recovered with an accuracy of

-977 mm in latitude, -985 mm in longitude, and +536 mm in height. The corresponding values from Table 7.4 (carrier phase), indicate a "true" baseline recovery accuracy of -59 mm in latitude, +36 mm in longitude, and +33 mm in height.

Figures A.3 and A.4 illustrate the time histories of the discrepancy vectors for stations 1 and 8 separately. Whereas the components of the individual discrepancy vectors show large variations over the observation period, the plots for the two stations track in unison. This indicates that although the absolute positions of the two stations are not well determined, even after one hour of data, the baseline vector between the two stations is well determined.

7.4 Satellite-Receiver Geometry Tests

In Chapters 2 and 3 we discussed, theoretically, the optimum selection of satellites for a particular configuration of ground stations. We used VECA to gain some "practical" insight into the effects of selecting different subsets of the available satellites.

During the 1 hour observation period, a total of 6 satellites were visible. We selected 3 subsets of 4 satellites and processed separately the interferometric delay, differential carrier phases, differential P-code and differential C/A-code pseudoranges. Because the carrier phase observable is likely to be the most accurate GPS observable available, at least in the near term, we have concentrated our attention on those results.

The constellation of satellites 2, 5, 7, and 10 is displaced slightly to the northern half of the sky but is well positioned in the east-west direction. Using the phase observable (Figure A.6 and Table 7.6), baselines 1-3 and 1-4 are more poorly determined than baseline 1-8 which has a large east-west component. For baseline 1-3, convergence is to within 100 mm after 40 minutes and to within 70 mm after one hour. No one component of the baselines is determined better than the others.

The interferometric delay results (Figure A.5 and Table 7.5) closely parallel the carrier phase results, both in the rate of convergence and accuracy of the final results. The 1-8 baseline, however, appears in this case to be no better than the 1-3 and 1-4 baselines.

The P-code results (Figure A.7 and Table 7.7) and the C/A-code results (Figure A.8 and Table 7.8) are essentially identical, and about five times worse than the carrier phase (final convergence to within a few hundred mm, rather than 70 mm).

The constellation of satellites 5, 10, 12, and 15 is more offset to the east and south and suffers from satellite 12 being available only during the last half hour of the observations. As expected, convergence is only achieved after half an hour (Figure A.10 and Table 7.10). However, after one hour of observations, baselines 1-3 and 1-4 are determined, using the phase observable, to about 10 mm and baseline 1-8 to about 30 mm (mostly in the horizontal components).

The interferometric delay results (Figure A.9 and Table 7.9) also show convergence after half an hour. Baseline 1-8 is recovered as accurately as for carrier phase, however baselines 1-3 and 1-4 are recovered only to the 20 cm level.

The P-code results (Figure A.11 and Table 7.11) and C/A-code results (Figure A.12 and Table 7.12) are again roughly five times worse than the carrier phase results. Again, convergence is achieved only in the second half hour. The 1-4 baseline using P-code is recovered to 150 mm, and the other two baselines to 60 mm. The C/A-code results are between 40 mm and 100 mm, with 1-8 being the worst.

The constellation of satellites 5, 7, 10, and 15 provides few low elevation observations. Although convergence using the phase observable is to within 70 mm after 10 minutes on all three baselines, convergence improves to only 50 mm after one hour. It appears that the heights of the stations are poorly determined using this constellation (Figure A.14 and Table 7.14).

The interferometric delay results (Figure A.13 and Table 7.13) are very similar to the carrier phase results, not improving significantly after the first 15 minutes. Baseline 1-3, however, was the worst determined here, as compared to 1-8 for carrier phase.

The P-code results (Figure A.15 and Table 7.15) and C/A-code results (Figure A.16 and Table 7.16) are again about five times worse than carrier phase. They also require the first 15 minutes to achieve best convergence, however start diverging again after about 35 minutes.

7.5 Tests of the Effect of Combining Two Observation Types

We tested the effect of combining P-code pseudorange observations with carrier phase observations to determine whether there may be some advantage in using these two observation types simultaneously.

We first obtained solutions using the pseudoranges and carrier phases separately. All six visible satellites were used. The results are presented in Figures A.17 and A.18 and Tables 7.17 and 7.18. The final results using the phase observable are slightly worse (by 10 mm or so) than the results when the constellation of satellites 5, 10, 12, and 15 was used (Figure A.10 and Table 7.10). This may not be statistically significant given the estimated standard deviations of the results (up to 9 mm). As might be expected, convergence is initially much faster with the six satellite constellation.

The results of analysing the combined data types are presented in Figures A.19 and Table 7.19. Since the phase results are one order of magnitude better than the P-code results, it is not surprising that the combined results differ little from the phase results. However, P-code data may be useful for other purposes, such as helping to resolve cycle ambiguities in phase data. This was not tested here.

TABLE 7.0

SIMULATION RUNS

Run	
1	Discrepancies, station 8 ϕ offset + 1 km, P-code.
2	Discrepancies, station 8 ϕ offset + 1 km, carrier phase.
3	Discrepancies, station 1 λ offset + 1 km, station 8 ϕ offset + 1 km, P-code.
4	Discrepancies, station 1 λ offset + 1 km, station 8 ϕ offset + 1 km, carrier phase.
5	Discrepancies, sat 2, 5, 7, 10 interferometric delay.
6	Discrepancies, sat 2, 5, 7, 10 differential carrier phase.
7	Discrepancies, sat 2, 5, 7, 10 differential P-code.
8	Discrepancies, sat 2, 5, 7, 10 differential C/A-code.
9	Discrepancies, sat 5, 10, 12, 15 interferometric delay.
10	Discrepancies, sat 5, 10, 12, 15 differential carrier phase.
11	Discrepancies, sat 5, 10, 12, 15 differential P-code.
12	Discrepancies, sat 5, 10, 12, 15 differential C/A-code.
13	Discrepancies, sat 5, 7, 10, 15 interferometric delay.
14	Discrepancies, sat 5, 7, 10, 15 differential carrier phase.
15	Discrepancies, sat 5, 7, 10, 15 differential P-code.
16	Discrepancies, sat 5, 7, 10, 15 differential C/A-code.
17	Discrepancies, sat 2, 5, 7, 10, 12, 15 differential P-code.
18	Discrepancies, sat 2, 5, 7, 10, 12, 15 differential carrier phase.
19	Discrepancies, sat 2, 5, 7, 10, 12, 15 P-code + carrier phase.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(1)	0(1)
2	PTSAPIN8	291205(44)	-670293(69)	-682521(101)	999966(56)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(1)	0(1)
2	PTSAPIN8	-999961(56)	28(40)	45(110)	999962(56)

Table 7.1 Final results station 8 offset ϕ + 1 km P-code.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(2)	0(2)
2	PTSAPIN8	291173(4)	-670322(5)	-682623(8)	1000045(5)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(2)	0(2)
2	PTSAPIN8	-1000041(5)	-12(3)	-18(9)	1000042(5)

Table 7.2 Final results station 8 offset ϕ + 1 km carrier phase.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	-923384(27893)	-398499(28394)	-49237(41585)	1006909(26728)
2	PTSAPIN8	273286(27884)	-644716(28308)	-731629(41552)	1012733(23284)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	-10148(22407)	-1005231(27723)	-57263(45198)	1006913(26728)
2	PTSAPIN8	-1011125(22296)	-6216(27707)	-56727(45173)	1012735(23286)

Table 7.3 Final results station 1 λ offset + 1 km, station 8 ϕ offset + 1 km P-code.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	-901358(2135)	-422203(2174)	-10063(3185)	995393(2106)
2	PTSAPIN8	295331(2135)	-668255(2168)	-692657(3182)	1006758(1724)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	-6657(1715)	-995340(2123)	-7547(3462)	995392(2106)
2	PTSAPIN8	-1006716(1707)	4624(2121)	-7514(3460)	1006755(1724)

Table 7.4 Final results station 1 λ offset + 1 km, station 8 ϕ offset + 1 km carrier phase.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)		DY (SD-DY)		DZ (SD-DZ)		DR (SD-DR)	
1	PTSAPIN1	F I X E D		S T A T I O N					
2	PTSAPIN3	11(	5)	-21(	6)	38(	16)	45(	16)
3	PTSAPIN4	17(	5)	-10(	6)	0(	16)	20(	6)
4	PTSAPIN8	4(	5)	-23(	6)	-44(	16)	51(	13)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)		DLON (SD-DLON)		DHGT (SD-DHGT)		DR (SD-DR)	
1	PTSAPIN1	F I X E D		S T A T I O N					
2	PTSAPIN3	7(	9)	1(	4)	44(	15)	45(	16)
3	PTSAPIN4	-11(	9)	11(	4)	12(	15)	20(	6)
4	PTSAPIN8	-47(	9)	-5(	4)	-15(	15)	51(	13)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN (SD-DLEN)		DAZ (SD-DAZ)		DELEV (SD-DELEV)		BASELINE (IN M)
1	PTSAPIN1	F I X E D		S T A T I O N				
2	PTSAPIN3	-6(	9)	0(	4)	45(	15)	92429
3	PTSAPIN4	13(	9)	-8(	4)	12(	15)	142000
4	PTSAPIN8	34(	7)	-33(	7)	-16(	15)	154584

Table 7.5 Final results sat 2, 5, 7, 10 interferometric delay.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)		DY (SD-DY)		DZ (SD-DZ)		DR (SD-DR)	
1	PTSAPIN1	F I X E D		S T A T I O N					
2	PTSAPIN3	19(	5)	-24(	6)	60(	16)	68(	17)
3	PTSAPIN4	23(	5)	-19(	6)	29(	16)	42(	15)
4	PTSAPIN8	5(	5)	-28(	6)	-14(	16)	33(	6)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)		DLON (SD-DLON)		DHGT (SD-DHGT)		DR (SD-DR)	
1	PTSAPIN1	F I X E D		S T A T I O N					
2	PTSAPIN3	18(	9)	7(	4)	66(	15)	68(	17)
3	PTSAPIN4	0(	9)	12(	4)	40(	15)	42(	15)
4	PTSAPIN8	-30(	9)	-6(	4)	9(	15)	33(	6)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN (SD-DLEN)		DAZ (SD-DAZ)		DELEV (SD-DELEV)		BASELINE (IN M)
1	PTSAPIN1	F I X E D		S T A T I O N				
2	PTSAPIN3	-16(	9)	-7(	4)	66(	15)	92429
3	PTSAPIN4	2(	9)	-11(	4)	40(	15)	142000
4	PTSAPIN8	24(	7)	-19(	7)	9(	15)	154584

Table 7.6 Final results sat 2, 5, 7, 10 differential carrier phase.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)		DY (SD-DY)		DZ (SD-DZ)		DR (SD-DR)	
1	PTSAPIN1	F I X E D		S T A T I O N					
2	PTSAPIN3	-67(	71)	35(	94)	-30(	300)	82(	182)
3	PTSAPIN4	59(	71)	-139(	94)	549(	301)	570(	310)
4	PTSAPIN8	71(	71)	-39(	93)	530(	300)	536(	306)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)		DLON (SD-DLON)		DHGT (SD-DHGT)		DR (SD-DR)	
1	PTSAPIN1	F I X E D		S T A T I O N					
2	PTSAPIN3	23(	160)	-46(	60)	-63(	274)	82(	182)
3	PTSAPIN4	267(	162)	-5(	60)	503(	273)	570(	310)
4	PTSAPIN8	315(	160)	49(	60)	431(	273)	536(	306)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN (SD-DLEN)		DAZ (SD-DAZ)		DELEV (SD-DELEV)		BASELINE (IN M)
1	PTSAPIN1	F I X E D		S T A T I O N				
2	PTSAPIN3	-25(	157)	45(	62)	-63(	275)	92429
3	PTSAPIN4	-260(	158)	-21(	64)	506(	274)	142000
4	PTSAPIN8	-227(	115)	217(	123)	434(	274)	154584

Table 7.7 Final results sat 2, 5, 7, 10 differential P-code.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	F I X E D	S T A T I O N		
2	PTSAPIN3	-92(62)	24(75)	-200(210)	223(209)
3	PTSAPIN4	32(62)	-90(75)	115(210)	150(199)
4	PTSAPIN8	4(62)	-102(74)	270(209)	289(213)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	F I X E D	S T A T I O N		
2	PTSAPIN3	-91(113)	-73(53)	-188(195)	223(209)
3	PTSAPIN4	10(114)	-9(53)	150(194)	150(199)
4	PTSAPIN8	114(113)	-36(53)	263(194)	289(213)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN (SD-DLEN)	DAZ (SD-DAZ)	DELEV (SD-DELEV)	BASELINE (IN M)
1	PTSAPIN1	F I X E D	S T A T I O N		
2	PTSAPIN3	87(111)	79(54)	-188(195)	92429
3	PTSAPIN4	-8(111)	9(56)	150(195)	142000
4	PTSAPIN8	-37(88)	113(87)	263(195)	154584

Table 7.8 Final results sat 2, 5, 7, 10 differential C/A-code.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX	(SD-DX)	DY	(SD-DY)	DZ	(SD-DZ)	DR	(SD-DR)
1	PTSAPIN1	0	1	0	1	0	2	0	2
2	PTSAPIN3	-10	7	-10	7	14	14	21	13
3	PTSAPIN4	-15	7	-7	7	12	13	21	12
4	PTSAPIN8	-10	7	-30	7	-7	14	33	5

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT	(SD-DLAT)	DLON	(SD-DLON)	DHGT	(SD-DHGT)	DR	(SD-DR)
1	PTSAPIN1	0	1	0	1	0	2	0	2
2	PTSAPIN3	6	8	-14	7	14	13	21	13
3	PTSAPIN4	8	8	-17	7	9	13	21	12
4	PTSAPIN8	-22	8	-21	7	10	13	33	5

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN	(SD-DLEN)	DAZ	(SD-DAZ)	DELEV	(SD-DELEV)	BASELINE (IN M)
1	PTSAPIN1	0	1	0	1	0	2	0
2	PTSAPIN3	-5	9	15	7	14	13	92429
3	PTSAPIN4	-9	9	17	6	9	13	142000
4	PTSAPIN8	32	4	-3	10	10	13	154584

Table 7.9 Final results sat 5, 10, 12, 15 interferometric delay.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(2)	0(2)
2	PTSAPIN3	-4(7)	0(7)	-5(14)	8(9)
3	PTSAPIN4	-4(7)	-4(7)	-1(14)	7(5)
4	PTSAPIN8	-12(7)	-27(7)	-11(14)	34(4)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	0(2)	0(1)	0(2)	0(2)
2	PTSAPIN3	-2(9)	-4(7)	-4(13)	8(9)
3	PTSAPIN4	-2(9)	-6(7)	1(13)	7(5)
4	PTSAPIN8	-23(8)	-22(7)	5(13)	34(4)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN (SD-DLEN)	DAZ (SD-DAZ)	DELEV (SD-DELEV)	BASELINE (IN M)
1	PTSAPIN1	0(2)	0(1)	0(2)	0
2	PTSAPIN3	3(9)	5(7)	-4(13)	92429
3	PTSAPIN4	2(9)	7(6)	1(13)	142000
4	PTSAPIN8	33(4)	-3(10)	5(13)	154584

Table 7.10 Final results sat 5, 10, 12, 15 differential carrier phase.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	F I X E D	S T A T I O N		
2	PTSAPIN3	16(92)	-7(93)	57(175)	60(169)
3	PTSAPIN4	-24(92)	-14(92)	146(175)	149(184)
4	PTSAPIN8	1(93)	60(93)	1(176)	60(91)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	F I X E D	S T A T I O N		
2	PTSAPIN3	28(107)	11(90)	51(168)	60(169)
3	PTSAPIN4	99(108)	-28(90)	108(167)	149(184)
4	PTSAPIN8	41(106)	25(90)	-36(169)	60(91)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN (SD-DLEN)	DAZ (SD-DAZ)	DELEV (SD-DELEV)	BASELINE (IN M)
1	PTSAPIN1	F I X E D	S T A T I O N		
2	PTSAPIN3	-26(110)	-12(85)	52(169)	92429
3	PTSAPIN4	-99(114)	18(81)	109(168)	142000
4	PTSAPIN8	-44(49)	17(130)	-35(169)	154584

Table 7.11 Final results sat 5, 10, 12, 15 differential P-code.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(1)	0(1)
2	PTSAPIN3	30(92)	-32(93)	0(175)	45(95)
3	PTSAPIN4	-7(92)	-51(92)	59(175)	79(181)
4	PTSAPIN8	-56(93)	-10(92)	-63(176)	86(113)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(1)	0(1)
2	PTSAPIN3	-31(107)	14(90)	28(168)	45(95)
3	PTSAPIN4	9(108)	-28(89)	73(167)	79(181)
4	PTSAPIN8	-34(106)	-56(90)	-54(169)	86(113)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN (SD-DLEN)	DAZ (SD-DAZ)	DELEV (SD-DELEV)	BASELINE (IN M)
1	PTSAPIN1	0(1)	0(1)	0(1)	0
2	PTSAPIN3	33(110)	-11(85)	28(169)	92429
3	PTSAPIN4	-10(114)	28(81)	73(168)	142000
4	PTSAPIN8	65(49)	8(130)	-55(169)	154584

Table 7.12 Final results sat 5, 10, 12, 15 differential C/A-code.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(2)	0(1)
2	PTSAPIN3	7(11)	-18(9)	35(21)	40(23)
3	PTSAPIN4	3(11)	-4(9)	4(21)	7(21)
4	PTSAPIN8	1(11)	-25(9)	-23(21)	36(9)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(2)	0(1)
2	PTSAPIN3	9(8)	0(7)	39(23)	40(23)
3	PTSAPIN4	-1(8)	1(7)	7(23)	7(21)
4	PTSAPIN8	-34(8)	-8(8)	0(23)	36(9)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN (SD-DLEN)	DAZ (SD-DAZ)	DELEV (SD-DELEV)	BASELINE (IN M)
1	PTSAPIN1	0(1)	0(1)	0(2)	0
2	PTSAPIN3	-8(7)	1(8)	39(23)	92429
3	PTSAPIN4	2(7)	0(8)	7(23)	142000
4	PTSAPIN8	29(10)	-20(5)	0(23)	154584

Table 7.13 Final results sat 5, 7, 10, 15 interferometric delay.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(2)	0(2)
2	PTSAPIN3	6(11)	-12(9)	28(21)	31(24)
3	PTSAPIN4	5(11)	-7(9)	5(21)	10(21)
4	PTSAPIN8	17(12)	-37(9)	13(21)	44(18)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(2)	0(2)
2	PTSAPIN3	9(8)	0(8)	30(23)	31(24)
3	PTSAPIN4	-3(8)	1(8)	10(23)	10(21)
4	PTSAPIN8	-21(8)	1(8)	38(23)	44(18)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN (SD-DLEN)	DAZ (SD-DAZ)	DELEV (SD-DELEV)	BASELINE (IN M)
1	PTSAPIN1	0(1)	0(1)	0(2)	0
2	PTSAPIN3	-8(7)	0(8)	30(23)	92429
3	PTSAPIN4	4(7)	0(8)	10(23)	142000
4	PTSAPIN8	13(10)	-17(5)	38(23)	154584

Table 7.14 Final results sat 5, 7, 10, 15 differential carrier phase.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(1)	0(1)
2	PTSAPIN3	40(148)	-22(117)	107(272)	117(315)
3	PTSAPIN4	-25(148)	0(117)	124(273)	127(240)
4	PTSAPIN8	149(149)	-71(117)	375(273)	410(316)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(1)	0(1)
2	PTSAPIN3	46(100)	-26(97)	104(301)	117(315)
3	PTSAPIN4	93(101)	-23(97)	82(300)	127(240)
4	PTSAPIN8	165(101)	108(102)	360(299)	410(316)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN (SD-DLEN)	DAZ (SD-DAZ)	DELEV (SD-DELEV)	BASELINE (IN M)
1	PTSAPIN1	0(1)	0(1)	0(1)	0
2	PTSAPIN3	-42(95)	-28(100)	104(301)	92429
3	PTSAPIN4	-93(93)	14(103)	83(301)	142000
4	PTSAPIN8	-181(123)	62(68)	362(301)	154584

Table 7.15 Final results sat 5, 7, 10, 15 differential P-code.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(1)	0(1)
2	PTSAPIN3	68(148)	-63(117)	100(272)	137(313)
3	PTSAPIN4	17(148)	-63(117)	115(272)	133(300)
4	PTSAPIN8	140(149)	-169(116)	393(272)	451(316)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	0(1)	0(1)	0(1)	0(1)
2	PTSAPIN3	5(99)	35(97)	133(300)	137(313)
3	PTSAPIN4	32(101)	-11(97)	128(300)	133(300)
4	PTSAPIN8	114(101)	61(102)	432(299)	451(316)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN (SD-DLEN)	DAZ (SD-DAZ)	DELEV (SD-DELEV)	BASELINE (IN M)
1	PTSAPIN1	0(1)	0(1)	0(1)	0
2	PTSAPIN3	-1(95)	-34(100)	133(300)	92429
3	PTSAPIN4	-31(93)	9(102)	129(300)	142000
4	PTSAPIN8	-112(123)	52(68)	433(300)	154584

Table 7.16 Final results sat 5, 7, 10, 15 differential C/A-code.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	F I X E D	S T A T I O N		
2	PTSAPIN3	-46(45)	17(71)	44(104)	66(59)
3	PTSAPIN4	-6(45)	-52(71)	146(104)	155(113)
4	PTSAPIN8	25(45)	7(71)	78(104)	82(101)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	F I X E D	S T A T I O N		
2	PTSAPIN3	56(58)	-34(41)	8(113)	66(59)
3	PTSAPIN4	68(58)	-28(41)	137(113)	155(113)
4	PTSAPIN8	50(58)	26(41)	59(113)	82(101)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN (SD-DLEN)	DAZ (SD-DAZ)	DELEV (SD-DELEV)	BASELINE (IN M)
1	PTSAPIN1	F I X E D	S T A T I O N		
2	PTSAPIN3	-57(57)	32(41)	8(114)	92429
3	PTSAPIN4	-68(57)	21(42)	137(113)	142000
4	PTSAPIN8	-49(49)	24(51)	60(113)	154584

Table 7.17 Final results sat 2, 5, 7, 10, 12, 15 differential P-code.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX (SD-DX)	DY (SD-DY)	DZ (SD-DZ)	DR (SD-DR)
1	PTSAPIN1	F I X E D	S T A T	I O N	
2	PTSAPIN3	9(4)	-17(6)	7(8)	22(7)
3	PTSAPIN4	14(4)	-15(6)	-1(8)	22(5)
4	PTSAPIN8	0(4)	-25(6)	-27(8)	39(5)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT (SD-DLAT)	DLON (SD-DLON)	DHGT (SD-DHGT)	DR (SD-DR)
1	PTSAPIN1	F I X E D	S T A T	I O N	
2	PTSAPIN3	-9(5)	1(3)	19(9)	22(7)
3	PTSAPIN4	-15(5)	6(3)	13(9)	22(5)
4	PTSAPIN8	-36(5)	-9(3)	-3(9)	39(5)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN (SD-DLEN)	DAZ (SD-DAZ)	DELEV (SD-DELEV)	BASELINE (IN M)
1	PTSAPIN1	F I X E D	S T A T	I O N	
2	PTSAPIN3	10(5)	0(3)	19(9)	92429
3	PTSAPIN4	17(5)	-3(3)	13(9)	142000
4	PTSAPIN8	31(4)	-22(4)	-4(9)	154584

Table 7.18 Final results sat 2, 5, 7, 10, 12, 15 differential carrier phase.

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED CARTESIAN COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DX	(SD-DX)	DY	(SD-DY)	DZ	(SD-DZ)	DR	(SD-DR)
1	PTSAPIN1	0(	1)	0(	1)	0(	2)	0(	1)
2	PTSAPIN3	10(	4)	-13(	6)	7(	9)	18(	8)
3	PTSAPIN4	14(	4)	-15(	6)	0(	9)	20(	6)
4	PTSAPIN8	2(	4)	-22(	6)	-29(	9)	36(	6)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED GEODETIC COORDINATES IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLAT	(SD-DLAT)	DLON	(SD-DLON)	DHGT	(SD-DHGT)	DR	(SD-DR)
1	PTSAPIN1	0(	2)	0(	1)	0(	2)	0(	1)
2	PTSAPIN3	-7(	5)	3(	4)	16(	10)	18(	8)
3	PTSAPIN4	-14(	5)	7(	4)	13(	10)	20(	6)
4	PTSAPIN8	-35(	5)	-7(	4)	-7(	10)	36(	6)

DISCREPANCY BETWEEN A PRIORI AND ADJUSTED BASELINE COMPONENTS IN MM (ADJUSTED MINUS A PRIORI)

STN	NAME	DLEN	(SD-DLEN)	DAZ	(SD-DAZ)	DELEV	(SD-DELEV)	BASELINE (IN M)
1	PTSAPIN1	0(	2)	0(	1)	0(	2)	0
2	PTSAPIN3	7(	5)	-3(	4)	16(	10)	92429
3	PTSAPIN4	15(	5)	-5(	4)	13(	10)	142000
4	PTSAPIN8	27(	4)	-23(	4)	-8(	10)	154584

Table 7.19 Final results sat 2, 5, 7, 10, 12, 15 P-code + carrier phase.

PART C: PRELIMINARY ANALYSES OF OTTAWA MACROMETER TEST DATA

CHAPTER 8

NON-PARAMETRIC ANALYSIS

8.1 Introduction

In the period from 19 July to 19 August 1983, the Earth Physics Branch of the Federal Department of Energy, Mines and Resources, with Herb Valliant as Chief Scientist, conducted the first test of Macrometrics' GPS surveying system (the Macrometer Interferometric Surveyor) in Canada. Two Macrometer V-1000 single frequency receivers were used to determine the vector baselines between selected points of the Geodetic Survey's Ottawa test network.

A general description and the results of the experiment (as obtained with Macrometrics' software) have been recorded by Valliant [1983a; 1983b]. We therefore restrict ourselves to a very short description of the experiment.

A total of thirty observing sessions were conducted in as many days. The first two comprised three one-hour observation periods on short baselines (points 6A, 7, length 30 m; points 6A, 51, length 2230 m (see Table 8.1)). The remaining 28 sessions were longer in duration (24 of 5 hours, 4 of 3 hours) and on longer baselines (13 km to 66 km, see Table 8.1). Four of these sessions provided no data due to operational difficulties.

We obtained the observations for an independent analysis. Preliminary results of the analysis look very promising. Here we present some of these results computed to date and an outline of the methods used to generate

TABLE 8.1

A priori coordinates for station positions.
(nominally NAD 27)*

Station	Latitude	Longitude	Geodetic Height (m)
6A	45°23'55".79598	75°55'21".44516	77.085
7	45°23'55".13131	75°55'22".48157	76.629
51	45°23'07".16263	75°56'37".25020	70.190
Morris	45°26'34".29253	76°15'18".81735	89.806
Panmure	45°20'18".81549	76°11'04".58789	153.956
Metcalfe	45°14'34".01037	75°27'31".48309	102.590

Approximate baseline lengths

Baseline	Length (m)
6A - 7	30
6A - 51	2230
6A - Metcalfe	40295
6A - Morris	26489
Metcalfe-Panmure	57930
Metcalfe-Morris	66268
Panmure-Morris	12843

*from Valliant [1983b, Table 4].

them. A final report on the UNB analysis of the Macrometer test data will be presented in a future publication.

8.2 Methods of Analysis

Initially the analysis was severely handicapped by the lack of good ephemerides of the GPS satellites; only the predicted ephemerides from the NASA bulletins [NASA, 1983] were available. This of course was a serious limitation: one cannot expect high precision in the estimation of even comparatively short baselines without proper knowledge of the satellite orbits. On the other hand, the following goals could be achieved even with poor ephemeridal information:

(1) Proper understanding of the Macrometer observable.

(2) Quality and consistency checks of the recorded data.

(3) Development and testing of a parameter estimation program for processing the Macrometer observations.

In order to achieve these goals, three computer programs were developed: PRMAC-1, PRMAC-2, and PRMAC-3 (names stand for PRocessing of MACrometer observations). The purpose of these programs is briefly described in Table 8.2. These programs were developed independently of the work on the VECA program solely for the efficient analysis of the Macrometer data and are not intended to be general purpose programs. It is our intention to process the data with the VECA program and to compare the results at a later date.

The PRMAC programs were tested with the NASA predicted ephemerides and this proved to be sufficient for the kind of analysis performed with PRMAC-2. It could be verified that no phase jumps were present in the measurements pertaining to the two short baselines and that the rms error

TABLE 8.2

Functions of Computer Programs.

Program Name	Description
PRMAC-1	Lists the observations (not documented).
PRMAC-2	Non-parametric quality check of data based on polynomial fit of "observed quantities minus approximate theoretical values of observations". See section 8.4.
PRMAC-3	Parameter estimation program. The observations are modelled as functions of the physical parameters. For a description, see Chapter 9.

for a single observation was of the order of some millimetres (see Chapter 9).

The high quality of the recorded data for the short baselines facilitated the first tests of the parameter estimation program, PRMAC-3. The development of "phase jump removal software" could be postponed; essentially only one so-called ambiguity parameter per satellite had to be estimated (see Chapter 9).

Whereas the proper performance of the parameter estimation program could be tested without problems using the NASA predicted ephemerides, the quality of the baseline estimates, as expected, was rather poor; the uncertainty was of the order of centimetres for the 30 m baseline, of the order of decimetres for the 2 km baseline. The reason is clear: in addition to receiver coordinates and ambiguity parameters some of the orbital parameters for each satellite also had to be estimated; it is quite obvious however that it is not possible to determine simultaneously receiver coordinates and satellite orbits with a high accuracy from observations of two receivers separated only by 30 m or 2 kilometres.

This situation drastically changed when better ephemerides, originating from the Naval Surface Weapons Center [O'Toole, 1976], became available to us. With the orbits now assumed known, PRMAC-3 estimated the (relative) receiver coordinates with a precision in the sub-centimetre region for the two short baselines (see Chapter 9).

In our subsequent analyses, even better orbital information will be at our disposal; the so-called Macrometer T-files (see Counselman [1983]) have been made available by Macrometrics. These T-files contain geocentric rectangular coordinates of the satellites in tabular form. Of course, the short baselines will be reprocessed with these best available orbits.

However, in view of the shortness of the baselines, it is not expected that the results will be essentially better than those reported in Chapter 9.

8.3 The Observation Equation

The measurements we deal with here are not the raw field data as recorded by single receivers. The most basic data available to us were those obtained from Macrometrics' INTERF or INTRFT computer programs (see Macrometrics [1983] or Counselman [1983]). These data usually are referred to as "interferometric phase differences between two receivers"; in principle one such measurement is the difference in the L_1 carrier phase of one GPS satellite measured at (nominally) the same time by the two receivers.

Several observation equations for these kinds of measurements have been published (e.g., Davidson et al. [1983]; Goad and Remondi [1983]). One explicit formulation is that of Bauersima [1983b]. The observation equation (8.1) below is basically his equation (38) somewhat simplified.

Expressing all quantities in metres, they read

$$(c-\dot{\rho}^{j}_{1i})\Delta t_i + \Delta\rho^{j}_{i} + d(d\rho^{j})_{ion} + d(d\rho^{j})_{trop} + \lambda N^{j}_{i} - \Delta\rho^{j'}_{i} = v^{j}_{i} \qquad (8.1)$$

$$i = 1,2,\ldots,n_b$$

$$j = 1,2,\ldots,n_s$$

where

c is the speed of light;

λ is the nominal wavelength of the L_1-carrier;

n_s is the number of satellites;

n_b is the number of observation times;

t_i, $i=1,2,\ldots,n_b$ are the observation times (UTC);

$\rho^j_{ki} = \rho^j_k(t_i)$, k=1,2 is the distance of satellite j at time $t_i - \rho^j_{ki}/c$ to receiver k at time t_i. (Note: in Part C of this report, the index j in ρ^j_{ki} specifies a satellite; k specifies a receiver; and i specifies an observation time);

$\dot{\rho}^j_{ki}$ is the range rate at time t_i;

$\Delta\rho^j_i = \rho^j_{1i} - \rho^j_{2i}$;

N^j_i are integer numbers;

$(d\rho^j_k)_{ion}$, k=1,2 is the ionospheric refraction correction to phase observation of satellite j as observed from receiver k;

$(d\rho^j_k)_{trop}$, k=1,2 is the tropospheric refraction correction;

$d(d\rho^j)_{ion} = (d\rho^j_1)_{ion} - (d\rho^j_2)_{ion}$;

$d(d\rho^j)_{trop} = (d\rho^j_1)_{trop} - (d\rho^j_2)_{trop}$;

Δt_i is the clock synchronization error of receiver clock 2 with respect to receiver clock 1;

$\Delta\rho^{j'}_i$ is the recorded phase difference measurement;

v^j_i is the residual in range difference $\Delta\rho^j_i$.

These observation equations have been deduced in a purely theoretical way. They are applicable to any receivers making phase difference measurements. The observation equations pertaining to the Macrometer V-1000 receivers differ in two points from the (more general) eqns. (8.1):

(1) The Macrometer keeps track of the number of integer wavelengths of the L_1 signal between observation times (with a "finite number" of exceptions, the so-called cycle-slips or phase jumps, which may be removed more or less easily). Therefore after some preprocessing ("phase jump removal software") we may assume

$$N^j_i = N^j \ , \ i=1,2,\ldots,n_b \qquad (8.2)$$

This means that there is only one unknown "ambiguity parameter" N^j per satellite per observing session. This of course simplifies matters considerably.

(2) Due to the manner in which the Macrometer works (in reconstructing the carrier phase the frequency is doubled), the ambiguity parameter is actually the number of half cycles.

This leads us finally to the following set of observation equations:

$$(c-\dot{\rho}^j_{1i})\Delta t_i+\Delta\rho^j_i+d(d\rho^j)_{ion}+d(d\rho^j)_{trop}+\frac{\lambda}{2}\ N^j - \Delta\rho^{j'}_i = v^j_i \qquad (8.3)$$

$$i = 1,2,\ldots,n_b$$

$$j = 1,2,\ldots,n_s$$

These observation equations were used for the analyses reported here. Moreover, for the two short baselines, the tropospheric and ionospheric correction terms in eqn. (8.3) are so small that they were completely neglected.

8.4 Non-parametric Analysis

As already stated, the program PRMAC-2 was developed to give a first impression of the quality and the consistency of the observational data from the Ottawa campaign. The method used is very simple: If we look at the unknown terms of eqns. (8.3) for one satellite j, clearly the term $\Delta\rho^j_i$ shows the "strongest" time dependence ($\Delta\rho^{j'}_i$ are the known observations). We therefore approximated this term as follows: Let $\Delta\rho^{jo}_i$ be the approximation of the term $\Delta\rho^j_i$ in eqn. (8.3), calculated with the provisional values for the receiver coordinates and with the NASA predicted orbits (of course, better orbital information may be used if available). Next we made the following assumption: The values

$$\xi^j_i = (c-\dot{\rho}^j_{1i})\Delta t_i+(\Delta\rho^j_i-\Delta\rho^{jo}_i)+d(d\rho^j)_{ion}+d(d\rho^j)_{trop}+\frac{\lambda}{2}\ N^j \qquad (8.4)$$

for $i=1,2,\ldots,n_b$ are values of a low degree algebraic polynomial (in time):

$$\sum_{k=0}^{q} p_k^j \, (t_i)^k = \xi_i^j \ , \ i=1,2,\ldots,n_b \qquad (8.5)$$

where q is the degree of the polynomial. Equations (8.3) may then be rewritten as follows:

$$\sum_{k=0}^{q} p_k^j \, (t_i)^k - (\Delta\rho_i^{j'} - \Delta\rho_i^{jo}) = v_i^j \qquad (8.6)$$

$$i = 1,2,\ldots,n_b; \ j=1,2,\ldots,n_s.$$

From eqn. (8.6) the polynomial coefficients p_k^j for each satellite j are estimated by a conventional least-squares technique.

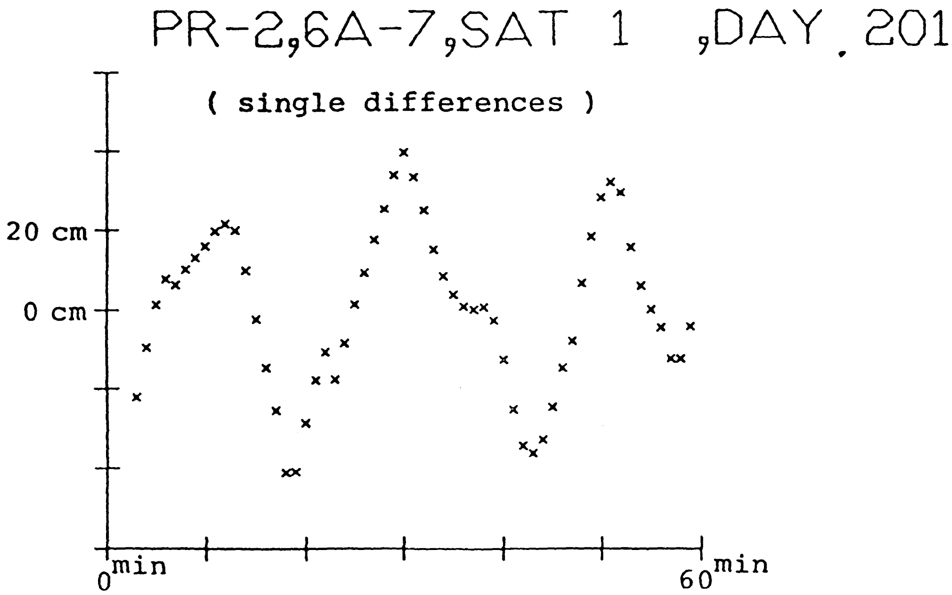
The assumption made in eqn. (8.5) clearly holds if all the terms on the right-hand side of eqn. (8.4) (for one value of j) can be modelled by low degree polynomials. This certainly is true for the last term, $\frac{\lambda}{2} N^j$, which is a constant for every satellite. In view of the short observation sessions (a maximum of 5 hours corresponding to less than one-half of the satellites' orbital periods), experience indicates that the same assumption holds sufficiently well for other than the first term of eqn. (8.4) provided we choose the polynomial degree $q \geq 4$. Whether or not the assumption is true for the first term depends on the performances of the clocks in the two receivers.

8.5 Single Difference Results

The non-parametric analysis was applied to the observations of all the satellites in the observing sessions on the two short baselines.

Instead of giving a complete list of results, we only give one example in Figure 8.1. This figure shows the "residuals" as produced by PRMAC-2 for the interferometric phase observations (single differences) as

Figure 8.1



recorded on 20 July (day 201 of year 1983) on the shortest baseline. These residuals show a clear systematic variation. We therefore have to draw the conclusion that assumption of eqn. (8.5) is not valid. Figure 8.1 reflects the (non-polynomial) errors to be expected from the crystal clocks in the Macrometer V-1000. The quality of the results obtained with this analysis is consistent with the results published by Goad and Remondi [1983].

In principle there are two techniques to overcome this "clock synchronization problem":

(1) use of better frequency standards in the receivers;

(2) use of more sophisticated models to describe the clock performances.

Whereas the first technique will be applied probably in the next generation of receivers, for the present analysis better modelling had to be looked for.

In the authors' opinion the best way of modelling is the following: define a statistical model of the clock performances using the known facts on clock offset, drift and jitter. This leads to a simple stochastic differential equation for the phase differences of the two receiver clocks or, even more directly, to an equation for the clock synchronization error as a function of time. The Δt_i, $i=1,2,...,n_b$ in eqns. (8.3) may then be interpreted as the solution of the stochastic equation at the observation times t_i, $i=1,2,...,n_b$. Of course, this approach complicates matters. Instead of more or less standard least-squares solutions, one would have to apply methods of "optimal filtering" or "optimal smoothing". Although this approach is advantageous from a theoretical point of view, its application would have required a considerable investment of time which was not available. Nevertheless, this technique should be kept in mind for future studies.

The next best approach to follow is to deny all functional models for the errors Δt_i, $i=1,2,\ldots,n_b$ and to introduce them as unknowns into a least-squares adjustment. Although there are no objections from the theoretical point of view, there is a strong objection from the practical point of view: the number of unknowns tends to increase dramatically. One gets into the problem of manipulations with large matrices, which cause a significant increase of computation time and the use of large storage areas. These requirements more or less restrict the processing to large main frame computers.

8.6 Use of Double Differences

An alternative approach to those already mentioned is to implicitly eliminate the clock synchronization term by using the differences of two eqns. (8.3) with the <u>same</u> subscript i but different superscript j. One easily sees that the main contribution of the clock synchronization error $(c\Delta t_i)$ is eliminated and one gets:

$$-(\dot{\rho}^j_{1i}-\dot{\rho}^k_{1i})\Delta t_i+(\Delta\rho^j_i-\Delta\rho^k_i)+d(d\rho^j)_{ion}-d(d\rho^k)_{ion}$$

$$+d(d\rho^j)_{trop}-d(d\rho^k)_{trop}+\frac{\lambda}{2}(N^j-N^k)-(\Delta\rho^{j'}_i-\Delta\rho^{k'}_i) = w^{jk}_i \qquad (8.7)$$

$$i = 1,2,\ldots, n_b$$

$$j,k = 1,2,\ldots, n_s,\ k\neq j.$$

Again for simplicity we have assumed that the same number n_s of satellites is observed at each observation time. In practice this assumption is usually not satisfied, which leads to a slightly more complex program logic.

It should also be pointed out that the time synchronization errors have <u>not</u> been removed completely by the use of eqns. (8.7) rather than eqn.

(8.3). However the order of magnitude of these terms in eqn. (8.7) is quite different from that in eqn. (8.3).

Since $|\dot{\rho}^j_{1i}| < 0.8$ km/s [Bauersima, 1983a], the ratio of the coefficients of Δt_i in the two equations is smaller than 5×10^{-6}. This means that the effect of imperfect clocks, so predominant in the "single differences" (Figure 8.1), will be much smaller in the so-called "double differences" of eqn. (8.7). It also implies that the time synchronization error remaining in eqn. (8.7) may be modelled very simply by a first-degree algebraic polynomial (representing clock offset and drift).

If we use eqn. (8.7) as the observation equation, we are no longer in a position to solve for all the ambiguity parameters N^k, $k=1,2,\ldots,n_s$. This clearly follows from the fact that only the differences

$$N^{jk} = N^j - N^k \qquad (8.8)$$

figure in eqn. (8.7) and only these can be solved for.

One option of the program PRMAC-2 is to analyse these "double differences". The procedure is very similar to the one used for the "single differences" as described above. A brief summary of the principles is therefore sufficient.

The terms $\Delta\rho^j_i$, $\Delta\rho^k_i$ in eqn. (8.7) are approximated in the same way as in the last section. Then the assumption embodied in eqn. (8.5) is replaced by the following assumption. The

$$\zeta^{jk}_i = \xi^j_i - \xi^k_i, \quad i=1,2,\ldots,n_b \qquad (8.9)$$

(see eqn. (8.4)) are values of a low degree algebraic polynolial (in time):

$$\sum_{\ell=0}^{q} p^{*jk}_{\ell} \, (t_i)^{\ell} = \zeta^{jk}_i, \quad i=1,2,\ldots,n_b \qquad (8.10)$$

where q is the degree of the polynomial. Using eqns. (8.9), (8.10), and (8.4), we may rewrite eqns. (8.7) in the following way:

$$\sum_{\ell=0}^{q} p_{\ell}^{*jk} (t_i)^{\ell} - [(\Delta\rho_i^{j'} - \Delta\rho_i^{jo}) - (\Delta\rho_i^{k'} - \Delta\rho_i^{ko})] = w_i^{jk} \qquad (8.11)$$

$i=1,2,\ldots,n_b$

$j,k=1,2,\ldots,n_s,\ k\neq j.$

For every pair of indices j,k, eqns. (8.11) may be used to determine the polynomial coefficients p_{ℓ}^{*jk}, $\ell=0,1,\ldots,q$ by a conventional least-squares technique.

In practice this of course is not done for every possible index combination j,k. The program PRMAC-2 identifies the index j with the satellite that has the most observations, then k is varied to cover the other satellites.

8.7 Double Difference Results

The double difference analysis was applied to all satellite pairs mentioned above in the observing sessions on the two short baselines.

The residuals for two typical examples, one for a satellite pair observed on the 30 m baseline and one for a satellite pair observed on the 2 km baseline, are given in Figures 8.2a and 8.2b.

First we see that there are positively no phase jumps which would amount to a multiple of 9.5 cm in the residuals, and secondly that a significant reduction in the scale of the residuals is obtained (from approximately 20 cm in Figure 8.1 to 2 mm and 6 mm in Figures 8.2a, 8.2b, respectively). Moreover the residuals seem to be reasonably random (at least in Figure 8.2a) thus supporting the assumption behind eqn. (8.11).

We conclude this (non-parametric) analysis with the following remarks:

Figure 8.2a

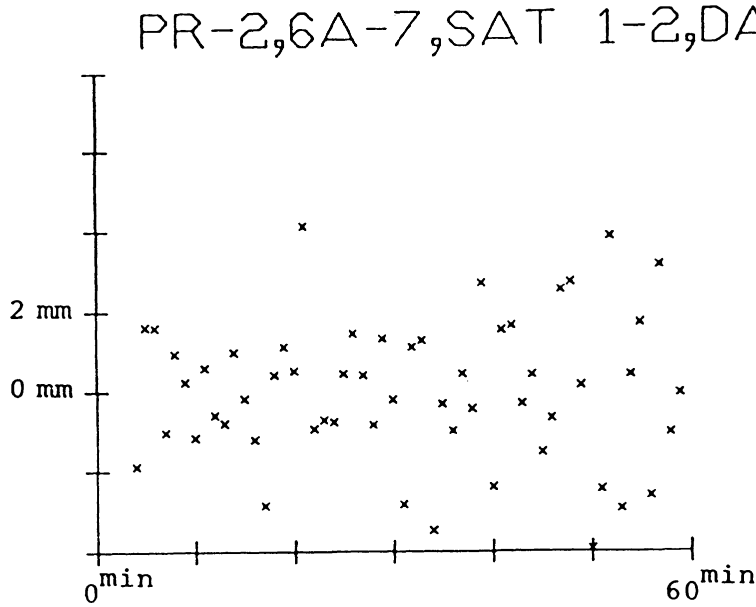
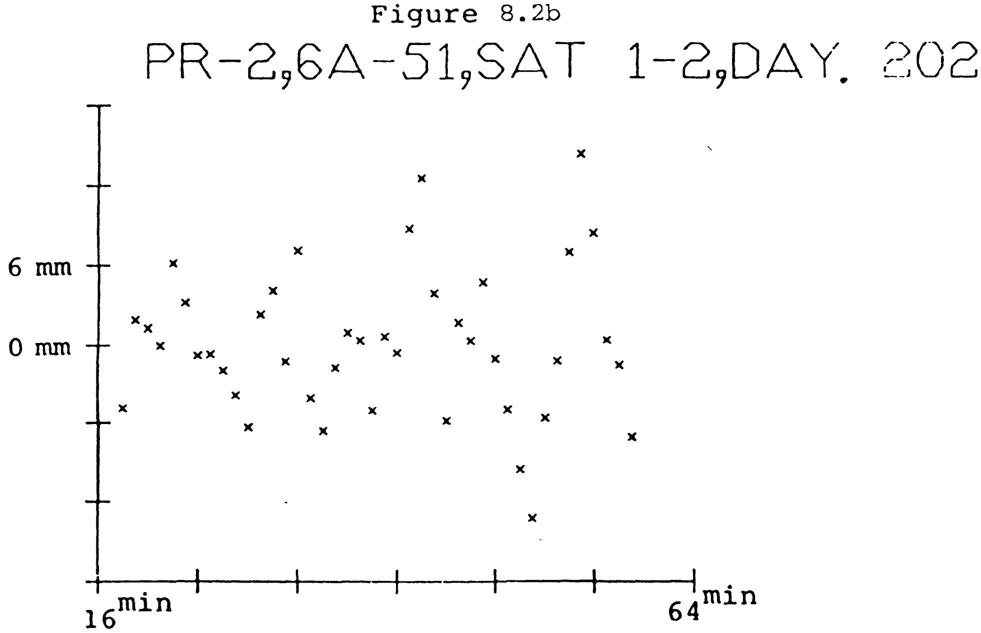


Figure 8.2b



(1) Macrometer V-1000 single frequency receivers are capable of producing high quality measurements. For very short baselines, the rms errors for a single observation is of the order of 3 mm.

(2) There is, however, an important contribution from the clock synchronization error when dealing with the original interferometric phase observations ("single differences"). This difficulty can be overcome either by physical modelling leading to stochastic differential equations, or by introducing one unknown clock parameter for each observation time, or by working with so-called "double differences" as shown above. These options are given in decreasing order of theoretical desirability but in increasing order of practical feasibility.

CHAPTER 9

PARAMETRIC ANALYSIS

9.1 Parameter Estimation

Having seen the excellent quality of Macrometer data using the preprocessing methods (Chapter 8), the next step was to write a parameter estimation program able to produce receiver coordinates. Rather than employing VECA which, at the moment, does not process double-difference data, we developed the program PRMAC-3 (cf. Table 8.2). Some of the features of this program will subsequently be implemented in VECA.

The present version of PRMAC-3 is limited by the following assumptions:

(a) It is assumed that only two receivers are operating simultaneously. (Only one baseline is estimated in one program run.)

(b) The "double difference approach" is used: the linearized versions of eqns. (8.7) are used as observation equations. Furthermore these observations are assumed to be uncorrelated.

(c) The satellite orbits are assumed to be purely "Keplerian" during each observation period.

As actually only two receivers took part in the Ottawa compaign, the restriction of assumption (a) is irrelevant for this study.

Assumption (b) helps to reduce computation times, storage areas, and program logic. That this approach is by no means the best one was stated in the previous chapter. However using a more sophisticated approach will likely have only a minor effect on the results.

Concerning assumption (c), the quality of orbits needed to obtain baseline estimates of a certain precision depends highly on the length of the baseline under consideration [Bauersima, 1983a]:

$$\frac{d\Delta R}{\Delta R} \backsim \frac{dr}{\rho} \tag{9.1}$$

where $d\Delta R$ is the baseline error, dr the orbit error, ΔR is the baseline length, ρ the range (receiver-satellite).

We can calculate the order of magnitude of an orbital error giving rise to a baseline error of 2 mm for the two short baselines. Using ρ = 25,000 km, we have

dr = 1700 m for the 30 m baseline

dr = 25 m for the 2 km baseline.

These errors may be compared to the orbital errors to be expected through adoption of assumption (c).

The NSWC elements are osculating elements where the osculation epochs correspond to the middle of the observation periods. As these periods were one hour for the two short baselines, we must estimate the effect of assumption (c) after 1/2 hour. To do so we use Table 2 from van Dierendonck et al. [1978] giving the maximum acceleration due to the J_2 gravity field coefficient as

$$a = 5.3 \times 10^{-5}\ ms^{-1} \quad .$$

The maximum error neglecting this influence after 30 minutes therefore will be

$$dr^* = \frac{1}{2}\, a\, \Delta t^2 \simeq 90\ m \tag{9.2}$$

Comparing this with the permissible errors given above, we conclude that assumption (c) is fully justified for the 30 m baseline, and that we have a questionable case for the 2 km baseline. Bearing in mind however that we have made the worst case estimation (in three respects: (a) the large errors occur only at the beginning and at the end of the observation period; (b) the acceleration, a, is a maximum value; (c) the estimations from eqn. (9.1) are rather pessimistic), the use of assumption (c) probably will not bias our results significantly. It will be interesting to see the difference in the results, when the 2 km baseline is reprocessed with better ephemerides with the next version of PRMAC-3.

Apart from the limitations due to the above of assumptions, the present version of PRMAC-3 is a general parameter estimation program which can solve for (almost) any combination of the physical parameters in eqns. (8.7). These parameters are

(a) Receiver coordinates in the conventional terrestrial system.

(b) Ambiguity parameters as defined by eqn. (8.8).

(c) Clock synchronization parameters c_o, c_1 (offset and drift) from the following model:

$$\Delta t_i = c_o + c_1(t_i - t_1),\ i=1,2,...,n_b \tag{9.3}$$

(t_i = observation times).

(d) A maximum of six orbital parameters are allowed per satellite. They figure implicitly in the second term of eqns. (8.7).

(e) Parameters describing tropospheric and ionospheric refraction (terms 3, and 4 in eqns. (8.7)) are neglected in the present analysis.

For obvious reasons (see assumption above) the Keplerian elements defined in Chapter 4 are used to represent the orbits. In addition, it is possible to specify an a priori variance covariance matrix for these parameters. The reference plane is the true equator of date (at the midpoint of the observation period). This description of the orbits will be kept in the next version of the program where better orbital models will be used. We then will have to specify that the elements are osculating elements.

The observation equations (8.7) are linear in the clock parameters (see eqn. (9.3)) and in the ambiguity parameters (see eqn. (8.8)); they are nonlinear in both the receiver coordinates and the orbital parameters. In PRMAC-3 a linearized version of eqns. (8.7) is used, where only the second term has to be linearized. This is done in the conventional way, using a Taylor's series expansion.

As almost any combination of the parameters mentioned above may form the vector of unknowns, PRMAC-3 must accommodate many options. The following are the main options:

(a) It may be used for pure positioning, assuming the orbits to be perfectly known.

(b) It may be used for pure orbit estimation, assuming all receiver positions to be known. (It is questionable however whether this option would produce reasonable results in the case of single frequency receivers.)

(c) It is possible to process data originating from different observation sessions of the same baseline in the same run.

9.2 Principles of Operation of PRMAC-3

PRMAC-3 has two parts: In part 1 the parameters chosen as unknowns are estimated with a conventional least-squares technique. In part 2 the integrality of the ambiguity parameters is enforced, and the best integer set of ambiguity parameters is determined. We shall first describe the functions of part 1 in detail.

In matrix notation the linearized version of eqns. (8.7) may be written

$$\underline{A}\,\underline{x} - \underline{w} = \underline{v} \quad , \qquad (9.4)$$

where

n_p is the number of parameters;

n_b is the number of observations;

$\underline{A}$ is the design matrix (n_p columns, n_b rows);

$\underline{x}$ is the vector of unknown parameters (n_p elements);

$\underline{w}$ is the vector of n_b "misclosures" (observed minus computed values);

$\underline{v}$ is the vector of n_b residuals.

The least-squares solution is

$$\hat{\underline{x}} = (\underline{A}^T \underline{A})^{-1} \underline{A}^T \underline{w} \quad . \tag{9.5}$$

PRMAC-3 calculates $\hat{\underline{x}}$ and it also gives the standard deviations σ_i for these elements as square roots of the diagonal elements of $(\underline{A}^T\underline{A})^{-1}$. The a posteriori variance factor is given by

$$\hat{\sigma}_o^2 = (\underline{v}^T \underline{v})/(n_b - n_p) \quad . \tag{9.6}$$

We will find it most convenient later to partition the vector of unknowns into:

$$\hat{\underline{x}}_1^T = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_{n_o}) \tag{9.7}$$

and

$$\hat{\underline{x}}_2^T = (\hat{x}_{n_o+1}, \ldots, \hat{x}_{n_p}), \text{ ambiguity parameters} \tag{9.8}$$

where

n_p is the total number of parameters;

n_s is the the number of ambiguity parameters;

$n_o = n_p - n_s$.

The vector $\hat{\underline{x}}_2$ contains the n_s ambiguity parameters, and $\hat{\underline{x}}_1$ the remaining n_o parameters. With eqns. (9.7) and (9.8), we rewrite eqn. (9.4) as

$$\underline{A}_1 \underline{x}_1 + \underline{A}_2 \underline{x}_2 - \underline{w} = \underline{v} \quad , \tag{9.9}$$

where $\underline{A}_1$ is the matrix formed with the first n_o columns of matrix $\underline{A}$, $\underline{A}_2$ is the matrix formed with the last n_s columns of $\underline{A}$. Equation (9.5) may be written as

$$\left[\begin{array}{c|c} \underline{A}_1^T \underline{A}_1 & \underline{A}_1^T \underline{A}_2 \\ \hline \underline{A}_2^T \underline{A}_1 & \underline{A}_2^T \underline{A}_2 \end{array}\right] \left[\begin{array}{c} \hat{\underline{x}}_1 \\ \hline \hat{\underline{x}}_2 \end{array}\right] = \left[\begin{array}{c} \underline{A}_1^T \\ \hline \underline{A}_2^T \end{array}\right] \underline{w} \qquad (9.10)$$

Introducing

$$\underline{N}_{11} = \underline{A}_1^T \underline{A}_1 \quad , \quad \underline{N}_{12} = \underline{A}_1^T \underline{A}_2 \quad , \quad \underline{N}_{22} = \underline{A}_2^T \underline{A}_2$$
$$\underline{u}_1 = \underline{A}_1^T \underline{w} \quad , \quad \underline{u}_2 = \underline{A}_2^T \underline{w} \qquad (9.11)$$

then eqn. (9.10) becomes

$$\left[\begin{array}{c|c} \underline{N}_{11} & \underline{N}_{12} \\ \hline \underline{N}_{12}^T & \underline{N}_{22} \end{array}\right] \left[\begin{array}{c} \hat{\underline{x}}_1 \\ \hline \hat{\underline{x}}_2 \end{array}\right] = \left[\begin{array}{c} \underline{u}_1 \\ \hline \underline{u}_2 \end{array}\right] . \qquad (9.12)$$

Part 2 of PRMAC-3 is devoted to the solution of the ambiguity problem. Whereas we know from the very beginning that the ambiguity parameters are integer numbers, there appears to be no simple way of utilizing this knowledge in the solution algorithm of the first part of the program. Ideally, the standard deviations associated with these parameters will be small (hopefully << 1). If this is so, then it will not be too difficult to find the correct set of integer ambiguity parameters in the second part of the program.

All strategies for resolving the ambiguity problem have one characteristic in common. The ambiguity vector $\hat{\underline{x}}_2$, or at least some of the elements of this vector, is no longer considered to be unknown. To some or all of these elements, known integer values are assigned _a priori_. These values are chosen to be "in the vicinity" of the non-integer values $\hat{\underline{x}}_2$

estimated in part 1 of the program, the "vicinity" being somehow limited by the corresponding standard deviations. Having made a choice for the integral values of the ambiguity parameters, we can use a standard least-squares solution to obtain values for the remaining unknown parameters $\hat{\underline{x}}_1$ and the corresponding residual square sum. The latter will be used to judge the a priori choice for $\hat{x}_2$.

As the case where some of the elements of $\hat{\underline{x}}_2$ are assumed to be known may be made formally equivalent to the case where the entire $\hat{\underline{x}}_2$ is known simply by transferring some elements of $\hat{\underline{x}}_2$ into the vector $\hat{\underline{x}}_1$, we only deal with the latter case.

Let $\underline{x}_2^*$ be an integer valued approximation of $\underline{x}_2$. We are now looking for the best values for $\hat{\underline{x}}_1$ in the least-squares sense, provided $\hat{\underline{x}}_2$ is approximated by $\underline{x}_2^*$.

The observation equations for this new problem may be simply transcribed from (9.9), using primes (') to distinguish the matrices of the second part of the program from those in the first. We get:

$$\underline{A}_1 \, \underline{x}_1' - \underline{w}' = \underline{v}' \quad , \tag{9.13}$$

where

$$\underline{w}' = \underline{w} - \underline{A}_2 \, \underline{x}_2^* \quad . \tag{9.14}$$

The least-squares solution is

$$\hat{\underline{x}}_1' = (\underline{A}_1^T \; \underline{A}_1) \, \underline{A}_1^T \, \underline{w}' \tag{9.15}$$

Using eqns. (9.11), $\hat{\underline{x}}_1'$ reads as:

$$\hat{\underline{x}}_1 = \underline{N}_{11}^{-1} \, \underline{u}_1' \quad , \tag{9.16}$$

where

$$\underline{u}_1' = \underline{A}_1^T \, \underline{w}' \quad . \tag{9.17}$$

As the quality of the choice of values for $\underline{x}_2^*$ will be measured by the sum of the squared residuals, $\underline{v}'^T \, \underline{v}'$, and as many different trials may have to be checked, it is absolutely mandatory to have a rapid algorithm to calculate this number. Starting from eqn. (9.13) we have

$$\underline{v}'^T \underline{v}' = (\hat{\underline{x}}_1'\, \underline{A}_1^T - \underline{w}'^T)\, (\underline{A}_1\, \hat{\underline{x}}_1' - \underline{w}') \quad , \tag{9.18}$$

which may be brought easily into the well-known standard form

$$\underline{v}'^T \underline{v}' = \underline{w}'^T \underline{w}' - \underline{u}_1'^T \hat{\underline{x}}_1' \quad . \tag{9.19}$$

Replacing the primed quantities on the right-hand side using eqns. (9.14), (9.15) leads to the following simple results:

$$\underline{v}'^T \underline{v}' = d_1 + \underline{d}_2^T \underline{x}_2^* + \underline{x}_2^{*T} \underline{D}_3 \underline{x}_2^* \quad , \tag{9.20}$$

where

$$\begin{aligned} d_1 &= \underline{w}^T \underline{w} - \underline{u}_1^T \underline{N}_{11}^{-1} \underline{u}_1 \\ \underline{d}_2^T &= -2(\underline{u}_2^T - \underline{u}_1^T \underline{N}_{11}^{-1} \underline{N}_{12}) \\ \underline{D}_3 &= \underline{N}_{22} - \underline{N}_{12}^T \underline{N}_{11}^{-1} \underline{N}_{12} \quad . \end{aligned} \tag{9.21}$$

As d_1, $\underline{d}_2$, $\underline{D}_3$ are functions of quantities already appearing in part 1 of the program and may therefore be calculated once and for all, eqn. (9.20) is an efficient tool for the calculation of $\underline{v}'^T \underline{v}'$. Moreover it is quite simple to derive powerful recursion relations starting from eqn. (9.20), if $\underline{x}_2^*$ is varied systematically. Clearly, $\hat{\underline{x}}_1'$ is computed only for the final choice of $\underline{x}_2^*$.

This brings us to the last open question of this chapter. What strategy should we use to find the best set $\underline{x}_2^*$ of integers, i.e., the one yielding the smallest value for $\underline{v}'^T\underline{v}'$? Three strategies have been used in PRMAC-3 in a hierarchical algorithm. If the first one fails, the second is followed; if the second fails the third is followed. The main difference between the strategies is in the number of checks performed with different trial choices of $\underline{x}_2^*$.

Strategy 1: Select

$$\underline{x}_2^{*T} = (x^*_{n_o+1},\ x^*_{n_o+2},\ \ldots,\ x^*_{n_p})_i \qquad x^*_i = \mathrm{rnd}(\hat{x}_i) \tag{9.22}$$

Here, the non-integer results $\hat{x}_i$ from the first part of the program are simply rounded to the nearest integer value. This choice may be considered to be the final one, if the sum of the squared residuals associated with this choice is only insignificantly larger than $\underline{v}^T\,\underline{v}$, the value obtained in the first part of the program.

The criterion for accepting this choice for $\underline{x}_2^*$ is

$$\frac{\underline{v}'\ \underline{v}'^T - \underline{v}\ \underline{v}^T}{\underline{v}\ \underline{v}^T} < \varepsilon \tag{9.23}$$

where ε must be specified explicitly in the computer program. For the moment we are working with

$$\varepsilon = 0.1 \quad . \tag{9.24}$$

We intend to replace the criterion given by eqn. (9.23), (9.24) by a χ^2-test in the future.

It is worth noting that there are two possible causes for condition (9.23), (9.24) to hold:

(a) The non-integer guesses in the first part of the program have all been very close to integer values.

(b) The standard deviations associated with $\hat{\underline{x}}_2$ from the first part of the program are comparatively large. This implies that the squared residual sum is not very sensitive to a change in these parameters.

In case (a) we claim to have solved the ambiguity problem, and we terminate by calculating the rest of the unknowns using eqns. (9.16).

In case (b) we have to conclude that with the data available it is not possible to resolve the problem. The results of part one of the program may then be taken to be the final results.

Strategy 2: Let

$$n_k = \text{rnd}\,(3\sigma_k),\ k = n_o+1,\ n_o+2,\ \dots,\ n_p \quad . \tag{9.25}$$

The values n_k correspond to the 3σ limits of the ambiguity parameters rounded to the nearest integer.

A maximum of

$$m_1 = \sum_{k=1}^{n_s} (2n_{n_o+k} + 1) \tag{9.26}$$

trials are performed, where during each trial only one of the elements of $\hat{\underline{x}}_2$ is held fixed as x^*_{ik}:

$$x^*_{ik} = x^*_i + k;\ k = -n_i,\ \dots,\ 0,\ \dots,\ n_i \tag{9.27}$$

$$i = n_o+1,\ n_o+2,\ \dots,\ n_p$$

The other elements are determined to minimize $\underline{v}'^T\,\underline{v}'$. The solutions are then rounded to the nearest integer, the resulting $\underline{v}'^T\,\underline{v}'$ is calculated. We call this strategy a suboptimal strategy. We consider it to be successful if the inequality of eqns. (9.23), (9.24) holds for at least one choice of $\underline{x}^*_{ik}$.

Strategy 3

This is the most straight forward, the most secure, but also the most time consuming of the three strategies mentioned here. We simply check every possible combination of integer ambiguities (in the vicinity of $\hat{\underline{x}}_2$)

$$\underline{x}_2^{*T} = (x^*_{n_o+1} + i_{n_o+1},\ x^*_{n_o+2} + i_{n_o+2},\ \dots,\ x^*_{n_p} + i_{n_p}) \tag{9.28}$$

where $i_k \in \{-n_k,\ \dots,\ 0,\ \dots,\ n_k\}$, $k = n_o+1,\ \dots,\ n_p$. That this strategy tends to be time consuming for large numbers n_s is indicated by the fact that m_2, the number of different choices for $\underline{x}_2^*$, is calculated by

$$m_2 = \prod_{k=1}^{n_s} (2n_{n_o+k} + 1) \tag{9.29}$$

This strategy is feasible if we do not combine observations from different observation sessions. As the total number of GPS satellites available today is 6, $n_s \leq 5$ results. This is the approach we believe is implemented in Macrometrics' software (programs LSQ, LSQT, see Counselman [1983]). This strategy is only used as a last resort in PRMAC-3.

We note that other strategies are also possible.

9.3 Results

The results obtained by PRMAC-3 on the short baselines are presented in Tables 9.1 to 9.4. Comparisons with the results obtained using Macrometrics' software [Valliant, 1983b] are given in Tables 9.5 and 9.6. Although PRMAC-3 produces graphical output (figures of residuals for satellite pairs of a given observation period) we have decided not to present it here. The reason for this decision is that the figures produced by PRMAC-3 are visually indistinguishable from the corresponding figures produced by PRMAC-2 and reproduced in Figures 8.2a, 8.2b.

The structure of the output is the same for all the tables. The observations pertaining to different observation periods are stored in different disk files. File numbers and the corresponding observation times (midpoint of observation interval) are given in Table 9.6.

In all computer runs only the coordinates of one receiver and the ambiguity parameters were designated as unknowns. No orbital biases were estimated. It is worth mentioning that the receiver coordinates estimated in the first part of the program are quite precise (observe the rms

TABLE 9.1

```
OTTAWA-TEST JULY 1983, 30 M BASELINE , FILE 14-16 , NO PERTURBATIONS
--------------------------------------------------------------------------------

RESULTS OF PROGRAM PRMAC-3 (PART 1)
-----------------------------------

BASELINE ANALYZED : 6A       , 70

MEAN ERROR OF UNIT WEIGHT=     0.0026 M

         OLD             NEW        DIFF.       +-

    1091191.207    1091191.209     0.002     0.002
   -4351475.228   -4351475.231    -0.002     0.002
    4518591.093    4518591.092    -0.000     0.001

RESULTS FOR FILE-NR. 14
-----------------------

AMB. PARAMETER 1 =          -147.01 +-     0.01
AMB. PARAMETER 2 =             1.01 +-     0.01
AMB. PARAMETER 3 =          -417.03 +-     0.02

RESULTS FOR FILE-NR. 15
-----------------------

AMB. PARAMETER 1 =             1.00 +-     0.01
AMB. PARAMETER 2 =           -15.98 +-     0.02
AMB. PARAMETER 3 =          -115.00 +-     0.01
AMB. PARAMETER 4 =           -34.01 +-     0.01

RESULTS FOR FILE-NR. 16
-----------------------

AMB. PARAMETER 1 =           -14.02 +-     0.01
AMB. PARAMETER 2 =            -0.02 +-     0.01


RESULTS OF PROGRAM PRMAC-3 (PART 2)
-----------------------------------

FINAL ESTIMATION OF AMBIGUITIES
-------------------------------

AMB.NR.  1 =   -147
AMB.NR.  2 =      1
AMB.NR.  3 =   -417
AMB.NR.  4 =      1
AMB.NR.  5 =    -16
AMB.NR.  6 =   -115
AMB.NR.  7 =    -34
AMB.NR.  8 =    -14
AMB.NR.  9 =      0

MEAN ERROR OF UNIT WEIGHT=     0.0026 M

FINAL ESTIMATION OF RECEIVER COORDINATES
----------------------------------------

         OLD             NEW        DIFF.       +-

    1091191.207    1091191.207    -0.000     0.000
   -4351475.228   -4351475.230    -0.001     0.000
    4518591.093    4518591.092    -0.001     0.000

ELLIPSOIDAL COORDINATES OF SECOND RECEIVER
------------------------------------------

LATITUDE  =  45 23 55.13142
LONGITUDE =- 75 55 22.48161
HEIGHT    =  76.756 M

LENGTH OF BASELINE(OLD)=               30.483 M
LENGTH OF BASELINE(NEW)=               30.485 M
```

TABLE 9.2a

OTTAWA-TEST JULY 1983, 30 M BASELINE , FILE 14 , NO PERTURBATIONS

RESULTS OF PROGRAM PRMAC-3 (PART 1)

BASELINE ANALYZED : 6A , 70

MEAN ERROR OF UNIT WEIGHT= 0.0026 M

OLD	NEW	DIFF.	+-
1091191.207	1091191.177	-0.030	0.016
-4351475.228	-4351475.223	0.005	0.006
4518591.093	4518591.089	-0.003	0.004

RESULTS FOR FILE-NR. 14

AMB. PARAMETER 1 = -146.87 +- 0.07
AMB. PARAMETER 2 = 0.86 +- 0.07
AMB. PARAMETER 3 = -416.87 +- 0.08

RESULTS OF PROGRAM PRMAC-3 (PART 2)

FINAL ESTIMATION OF AMBIGUITIES

AMB.NR. 1 = -147
AMB.NR. 2 = 1
AMB.NR. 3 = -417

MEAN ERROR OF UNIT WEIGHT= 0.0026 M

FINAL ESTIMATION OF RECEIVER COORDINATES

OLD	NEW	DIFF.	+-
1091191.207	1091191.206	-0.001	0.001
-4351475.228	-4351475.229	-0.001	0.001
4518591.093	4518591.093	0.000	0.001

ELLIPSOIDAL COORDINATES OF SECOND RECEIVER

LATITUDE = 45 23 55.13145
LONGITUDE =- 75 55 22.48163
HEIGHT = 76.756 M

LENGTH OF BASELINE(OLD)= 30.483 M
LENGTH OF BASELINE(NEW)= 30.484 M

TABLE 9.2b

OTTAWA-TEST JULY 1983, 30 M BASELINE , FILE 15 , NO PERTURBATIONS

RESULTS OF PROGRAM PRMAC-3 (PART 1)

BASELINE ANALYZED : 6A , 70

MEAN ERROR OF UNIT WEIGHT= 0.0028 M

OLD	NEW	DIFF.	+-
1091191.207	1091191.202	-0.005	0.007
-4351475.228	-4351475.226	0.003	0.005
4518591.093	4518591.093	0.001	0.002

RESULTS FOR FILE-NR. 15

AMB. PARAMETER 1 =	0.96	+-	0.04
AMB. PARAMETER 2 =	-16.02	+-	0.05
AMB. PARAMETER 3 =	-114.97	+-	0.03
AMB. PARAMETER 4 =	-34.00	+-	0.01

RESULTS OF PROGRAM PRMAC-3 (PART 2)

FINAL ESTIMATION OF AMBIGUITIES

AMB.NR. 1 =	1
AMB.NR. 2 =	-16
AMB.NR. 3 =	-115
AMB.NR. 4 =	-34

MEAN ERROR OF UNIT WEIGHT= 0.0028 M

FINAL ESTIMATION OF RECEIVER COORDINATES

OLD	NEW	DIFF.	+-
1091191.207	1091191.208	0.001	0.001
-4351475.228	-4351475.232	-0.004	0.001
4518591.093	4518591.093	0.000	0.001

ELLIPSOIDAL COORDINATES OF SECOND RECEIVER

LATITUDE = 45 23 55.13138
LONGITUDE =- 75 55 22.48158
HEIGHT = 76.758 M

LENGTH OF BASELINE(OLD)= 30.483 M
LENGTH OF BASELINE(NEW)= 30.485 M

TABLE 9.2c

OTTAWA-TEST JULY 1983, 30 M BASELINE , FILE 16 , NO PERTURBATIONS

RESULTS OF PROGRAM PRMAC-3 (PART 1)

BASELINE ANALYZED : 6A , 70

MEAN ERROR OF UNIT WEIGHT= 0.0018 M

OLD	NEW	DIFF.	+-
1091191.207	1091191.202	-0.005	0.007
-4351475.228	-4351475.180	0.048	0.021
4518591.093	4518591.082	-0.011	0.002

RESULTS FOR FILE-NR. 16

AMB. PARAMETER 1 = -14.00 +- 0.03
AMB. PARAMETER 2 = -0.14 +- 0.06

RESULTS OF PROGRAM PRMAC-3 (PART 2)

FINAL ESTIMATION OF AMBIGUITIES

AMB.NR. 1 = -14
AMB.NR. 2 = 0

MEAN ERROR OF UNIT WEIGHT= 0.0019 M

FINAL ESTIMATION OF RECEIVER COORDINATES

OLD	NEW	DIFF.	+-
1091191.207	1091191.206	-0.001	0.001
-4351475.228	-4351475.217	0.011	0.004
4518591.093	4518591.085	-0.008	0.002

ELLIPSOIDAL COORDINATES OF SECOND RECEIVER

LATITUDE = 45 23 55.13153
LONGITUDE =- 75 55 22.48151
HEIGHT = 76.742 M

LENGTH OF BASELINE(OLD)= 30.483 M
LENGTH OF BASELINE(NEW)= 30.481 M

TABLE 9.3

OTTAWA-TEST JULY 1983, 2 KM BASELINE , FILE 11-13 , NO PERTURBATIONS

RESULTS OF PROGRAM PRMAC-3 (PART 1)

BASELINE ANALYZED : 6A , 51

MEAN ERROR OF UNIT WEIGHT= 0.0075 M

OLD	NEW	DIFF.	+-
1089868.674	1089868.661	-0.013	0.009
-4352888.811	-4352888.799	0.012	0.006
4517546.554	4517546.554	0.000	0.004

RESULTS FOR FILE-NR. 11

AMB. PARAMETER 1 =	-12.97	+-	0.04
AMB. PARAMETER 2 =	0.03	+-	0.08
AMB. PARAMETER 3 =	-5.91	+-	0.07
AMB. PARAMETER 4 =	-12.92	+-	0.05

RESULTS FOR FILE-NR. 12

AMB. PARAMETER 1 =	-13.06	+-	0.06
AMB. PARAMETER 2 =	0.99	+-	0.03

RESULTS FOR FILE-NR. 13

AMB. PARAMETER 1 =	-0.00	+-	0.03
AMB. PARAMETER 2 =	0.02	+-	0.05
AMB. PARAMETER 3 =	-102.87	+-	0.06

RESULTS OF PROGRAM PRMAC-3 (PART 2)

FINAL ESTIMATION OF AMBIGUITIES

AMB.NR. 1 =	-13
AMB.NR. 2 =	0
AMB.NR. 3 =	-6
AMB.NR. 4 =	-13
AMB.NR. 5 =	-13
AMB.NR. 6 =	1
AMB.NR. 7 =	0
AMB.NR. 8 =	0
AMB.NR. 9 =	-103

MEAN ERROR OF UNIT WEIGHT= 0.0077 M

FINAL ESTIMATION OF RECEIVER COORDINATES

OLD	NEW	DIFF.	+-
1089868.674	1089868.670	-0.004	0.001
-4352888.811	-4352888.810	0.001	0.001
4517546.554	4517546.555	0.001	0.001

ELLIPSOIDAL COORDINATES OF SECOND RECEIVER

LATITUDE = 45 23 7.16356
LONGITUDE =- 75 56 37.25089
HEIGHT = 70.309 M

LENGTH OF BASELINE(OLD)= 2230.111 M
LENGTH OF BASELINE(NEW)= 2230.113 M

TABLE 9.4a

OTTAWA-TEST JULY 1983, 2 KM BASELINE , FILE 11 , NO PERTURBATIONS

RESULTS OF PROGRAM PRMAC-3 (PART 1)

BASELINE ANALYZED : 6A , 51

MEAN ERROR OF UNIT WEIGHT= 0.0086 M

OLD	NEW	DIFF.	+-
1089868.674	1089868.639	-0.035	0.023
-4352888.811	-4352888.805	0.007	0.017
4517546.554	4517546.557	0.003	0.008

RESULTS FOR FILE-NR. 11

AMB. PARAMETER 1 =	-12.94	+-	0.14
AMB. PARAMETER 2 =	-0.22	+-	0.17
AMB. PARAMETER 3 =	-5.79	+-	0.21
AMB. PARAMETER 4 =	-12.85	+-	0.16

RESULTS OF PROGRAM PRMAC-3 (PART 2)

FINAL ESTIMATION OF AMBIGUITIES

AMB.NR. 1 =	-13
AMB.NR. 2 =	0
AMB.NR. 3 =	-6
AMB.NR. 4 =	-13

MEAN ERROR OF UNIT WEIGHT= 0.0087 M

FINAL ESTIMATION OF RECEIVER COORDINATES

OLD	NEW	DIFF.	+-
1089868.674	1089868.669	-0.005	0.002
-4352888.811	-4352888.809	0.003	0.003
4517546.554	4517546.554	0.000	0.003

ELLIPSOIDAL COORDINATES OF SECOND RECEIVER

LATITUDE = 45 23 7.16357
LONGITUDE =- 75 56 37.25089
HEIGHT = 70.307 M

LENGTH OF BASELINE(OLD)= 2230.111 M
LENGTH OF BASELINE(NEW)= 2230.113 M

TABLE 9.4b

OTTAWA-TEST JULY 1983, 2 KM BASELINE , FILE 12 , NO PERTURBATIONS

RESULTS OF PROGRAM PRMAC-3 (PART 1)

BASELINE ANALYZED : 6A , 51

MEAN ERROR OF UNIT WEIGHT= 0.0056 M

OLD	NEW	DIFF.	+-
1089868.674	1089868.578	-0.096	0.029
-4352888.811	-4352888.576	0.235	0.084
4517546.554	4517546.562	0.009	0.007

RESULTS FOR FILE-NR. 12

AMB. PARAMETER 1 = -13.41 +- 0.13
AMB. PARAMETER 2 = 0.05 +- 0.32

RESULTS OF PROGRAM PRMAC-3 (PART 2)

# (FIXED)	PAR.1	PAR.2	RMS
1	-14	-1	0.0061
1	-13	1	0.0058
1	-12	3	0.0082
2	-14	-1	0.0061
2	-13	0	0.0065
2	-13	1	0.0058

FINAL ESTIMATION OF AMBIGUITIES

AMB.NR. 1 = -13
AMB.NR. 2 = 1

MEAN ERROR OF UNIT WEIGHT= 0.0058 M

FINAL ESTIMATION OF RECEIVER COORDINATES

OLD	NEW	DIFF.	+-
1089868.674	1089868.671	-0.002	0.001
-4352888.811	-4352888.824	-0.012	0.009
4517546.554	4517546.566	0.012	0.005

ELLIPSOIDAL COORDINATES OF SECOND RECEIVER

LATITUDE = 45 23 7.16349
LONGITUDE =- 75 56 37.25095
HEIGHT = 70.326 M

LENGTH OF BASELINE(OLD)= 2230.111 M
LENGTH OF BASELINE(NEW)= 2230.115 M

TABLE 9.4c

OTTAWA-TEST JULY 1983, 2 KM BASELINE , FILE 13 , NO PERTURBATIONS

RESULTS OF PROGRAM PRMAC-3 (PART 1)

BASELINE ANALYZED : 6A , 51

MEAN ERROR OF UNIT WEIGHT= 0.0069 M

OLD	NEW	DIFF.	+-
1089868.674	1089868.655	-0.019	0.037
-4352888.811	-4352888.785	0.027	0.014
4517546.554	4517546.540	-0.014	0.009

RESULTS FOR FILE-NR. 13

AMB. PARAMETER 1 =	0.08	+-	0.17
AMB. PARAMETER 2 =	0.01	+-	0.16
AMB. PARAMETER 3 =	-102.77	+-	0.20

RESULTS OF PROGRAM PRMAC-3 (PART 2)

FINAL ESTIMATION OF AMBIGUITIES

AMB.NR. 1 = 0
AMB.NR. 2 = 0
AMB.NR. 3 = -103

MEAN ERROR OF UNIT WEIGHT= 0.0072 M

FINAL ESTIMATION OF RECEIVER COORDINATES

OLD	NEW	DIFF.	+-
1089868.674	1089868.669	-0.005	0.002
-4352888.811	-4352888.810	0.001	0.002
4517546.554	4517546.550	-0.004	0.003

ELLIPSOIDAL COORDINATES OF SECOND RECEIVER

LATITUDE = 45 23 7.16344
LONGITUDE =- 75 56 37.25092
HEIGHT = 70.305 M

LENGTH OF BASELINE(OLD)= 2230.111 M
LENGTH OF BASELINE(NEW)= 2230.116 M

TABLE 9.5

Differences in latitude ($\Delta\phi$), longitude ($\Delta\lambda$), height (Δh) and length ($\Delta\ell$) for Batch Processing.

(a) 30 m baseline		
	PRMAC-3 minus Macrometrics[1]	PRMAC-3 minus "Ground Truth"[2]
$\Delta\phi$	-0".00005 (-1.5)	0".00009 (2.7)
$\Delta\lambda$	0".0000	– 0".00004
Δh	0.003 m	0.001 m
$\Delta\ell$ *	0.002 m	-0.002 m

(b) 2 km baseline		
	PRMAC-3 minus Macrometrics[1]	PRMAC-3 minus "Ground Truth"[2]
$\Delta\phi$	0".00008 (2.4)	0".00091 (27.3)
$\Delta\lambda$	-0".00018 (3.8)	-0".00065 (-13.8)
Δh	0.001 m	-0.008 m
$\Delta\ell$ *	0.002 m	-0.008 m

1) The "mean" value as published by Valliant [1983b, Table 2], was used as the Macrometrics solution.
2) Ground truth as published by Valliant [1983b, Table 2] was used.
*) Difference in baseline length.
Numbers in parentheses are in millimetres.

TABLE 9.6

Coordinate differences in the sense "PRMAC-3 minus Macrometrics" when processing each observation period separately with PRMAC-3.

(a) 30 m baseline

UTC of mid-point of observation period Day	Hr	Min	File No.	Number of Satellites	$\Delta\phi$	$\Delta\lambda$	Δh
200	23	59	14	4	-0".00004	0".00006	.001 m
201	01	15	15	5	-0".00002	0".00001	.001 m
201	02	30	16	3	0".00000	0".00003	.002 m

(b) 2 km baseline

UTC of mid-point of observation period Day	Hr	Min	File No.	Number of Satellites	$\Delta\phi$	$\Delta\lambda$	Δh
202	01	11	11	5	0".00004	-0".00023	.001 m
202	02	20	12	3	0".00000	-0".00017	.006 m
201	23	56	13	4	0".00003	0".00022	.006 m

errors). The same coordinates would result if range difference observations (integrated Doppler) of one satellite in the time intervals $t_i - t_1$, $i=2,3,\ldots,n_b$) were to be processed in the double difference mode. Therefore the rms errors of the receiver coordinates in the first part of the output indicate what baseline quality likely could be expected from integrated Doppler observations of a precision of 2 mm, corresponding to 0.01 cycles of the L_1 carrier phase.

In all cases but one (Table 9.4b) the non-integer ambiguity estimates in part one are immediately followed by the results of part two. In all these cases the finally accepted integer values for the ambiguities were the ones defined by eqn. (9.22). In the case of Table 9.4b the "suboptimal search algorithm" described in the previous section was invoked. In none of the examples given here was it necessary to invoke the general search algorithm (eqn. (9.28)).

As one may judge by comparing the rms errors for the coordinates from the two parts of the program, the resolution of the ambiguity problem in part 2 significantly increased the quality of the results. As in this second part of the program, the observations are actually the range differences from the satellite to the two receivers (processed in the double difference mode). The results also indicate the superiority of these measurements compared with the integrated Doppler observation approach used in the first part of the program (same measurement errors assumed in both cases).

A further common characteristic: It is worth noting that the a posteriori variance factor in the second part of the program differs only by very small amounts from that of part one.

It should be mentioned that in the computer runs presented in Tables

9.1 to 9.4, the mean coordinates obtained using Macrometrics' software [Valliant, 1983b] were used as a priori ("old") coordinates (where the antenna heights were added to the geodetic heights of the stations). To establish the comparison with the "ground truth", the coordinates of Table 8.1 were used as a priori coordinates. The output corresponding to this comparison is not presented here. We would like to point out that the a priori coordinates need not be known precisely; offsets in the initial coordinates of up to 100 m in each coordinate (of the receiver whose position is to be estimated) produce results identical to those presented here. If larger initial coordinate offsets are used, a further iteration step is necessary.

In Tables 9.1, 9.2 the 30 m baseline, and in tables 9.3, 9.4 the 2 km baseline was processed.

In Tables 9.1 and 9.3 <u>all</u> observations on the same baseline were processed in one run, whereas for Tables 9.2a,b,c, 9.4a,b,c the observations for each one-hour observation period were processed separately.

The advantage of the "batch processing" is most obvious for the estimation of the ambiguity parameters in the first part of the program; these numbers are very close to integers with very small rms errors (Tables 9.1, 9.3). Neither a suboptimal nor a general search was necessary. Acceptance of the rounded values as the final solution is fully justified. Comparisons with "ground truth" and with solutions using Macrometrics' software are shown in Tables 9.5 and 9.6. In Table 9.5 we see that the difference between our combined solutions and the mean of the Macrometrics' solutions are very small. There seems to be, however, a significant difference of $\sim$ 4 mm in the longitude λ in the case of the 2 km baseline.

The reason for this might be our somewhat too simple orbital models.

The comparison with the so-called ground truth (see Valliant [1983b]) is, as might be expected, somewhat less favourable.

We should point out that Valliant [1983a,b] used the so-called horizontal distance as a measure of the distance between the end points of baselines, whereas we have used the geometric distance in three-dimensional space. Therefore horizontal distance in Valliant's Table 2 [Valliant, 1983b] is not directly comparable with the baseline lengths in our Tables 9.1 to 9.4.

For the sake of completeness, we give the differences between the PRMAC-3 solutions (processing each observation period separately) and the Macrometrics' solutions. It should be noted that, with the exception of $\Delta\lambda$ for the 2 km baseline, these differences are very small.

PART D: SUMMARY

CHAPTER 10

CONCLUSIONS AND RECOMMENDATIONS

During the lifetime of our present contract, we developed a fairly sophisticated software package that contains many options and has a large degree of flexibility. However, the package, because of its size and flexibility, takes a significant amount of computer time to run, and its computer memory requirements are substantial. The package has been designed basically as a research tool.

We would like to convert the VECA package into a production tool on the HP 1000 computer for differential/point GPS positioning. We propose tackling this task along three parallel lines:

(1) Reduction of CPU time and core memory requirements. This is to be achieved by taking better advantage of the possible direct formulation of the normal equations inverse. The elements of the inverse matrix of normal equations are functions of the defining vectors of the tetrahedrons involved in the geometrical configurations. There is a strong possibility that the geometrical formulation used in the VECA package would admit this approach which should significantly improve the speed of execution.

At present VECA admits batching of obvservations (simultaneous processing) that can be changed from one at a time to 28. We wish to investigate the optimal batching from the time consumption point of view. A systematic implementation of the HP Vector Instruction Set should further increase the computational speed.

(2) Model and option improvement. There is, naturally, room for improvement of the software performance. These improvements should be aimed at

(a) increasing the accuracy in computed position, and

(b) cutting down the necessary observing time and/or the number of observations necessary to determine positions to a specified accuracy.

Specifically, we initiated the study of the problem of correlations among observations under the terms of the present contract. This very complex problem, which involves temporal correlations, spatial correlations, and correlations among different types of observables, has, to our knowledge, not been solved by any research group working with GPS. We thus propose to look into the various possibilities and implement, in the VECA package, whatever can reasonably be implemented, along these lines, with the aim of making the best use of the collected data.

We have investigated in this report different options for the orbital bias modelling. These options should be implemented in the production version of VECA to provide the flexibility needed to cope with the various forms and kinds of ephemerides that will be available under different circumstances.

(3) Testing of software performance under 'real' conditions. To test the performance of VECA under production conditions, we propose to carry out comprehensive tests with data sets collected with the Macrometer, Texas Instruments, and possibly SERIES receivers. The performance of VECA,

using different option combinations, should be thoroughly analysed with the aim of establishing the optimal processing modes for specific receiver systems. In this context, we would also like to test approaches to resolving the ambiguity inherent in the reconstructed carrier phase difference other than the one used by the Macrometer software.

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APPENDIX A
DETAILED SIMULATION RESULTS

List of Figures

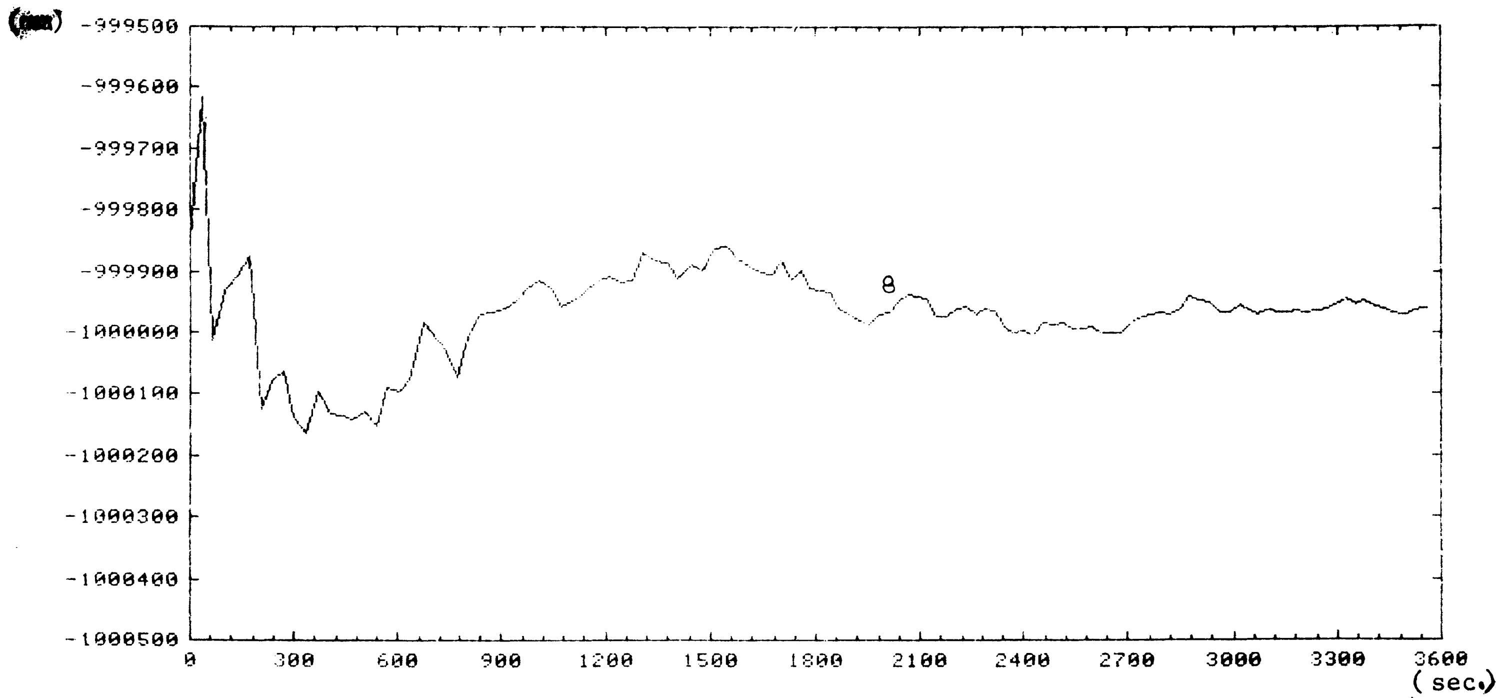
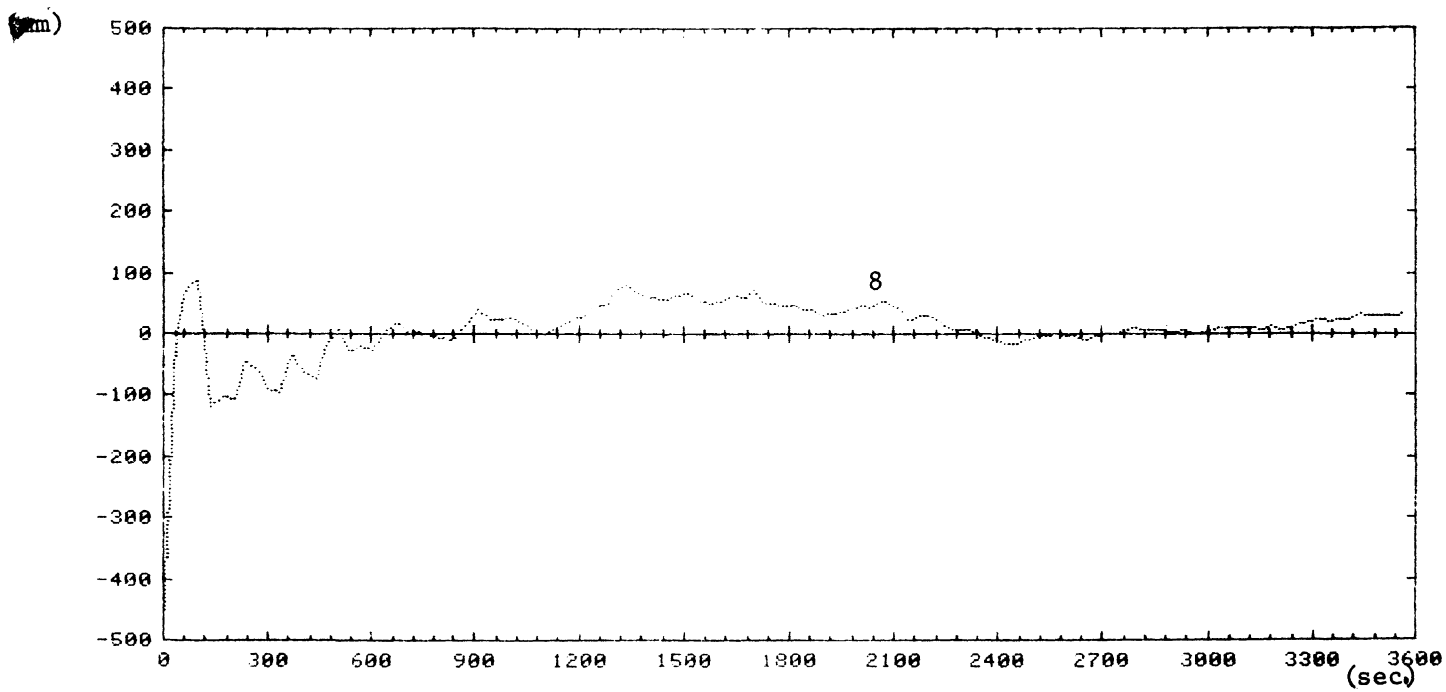


Fig. A1. Discrepancy Dϕ P code 2 5 7 10 12 15
Figures A.1 to A.19 are coded as follows: Discrepancy, station coordinate plotted, observation type, satellites used. The station number(s) is(are) noted on the plot(s).



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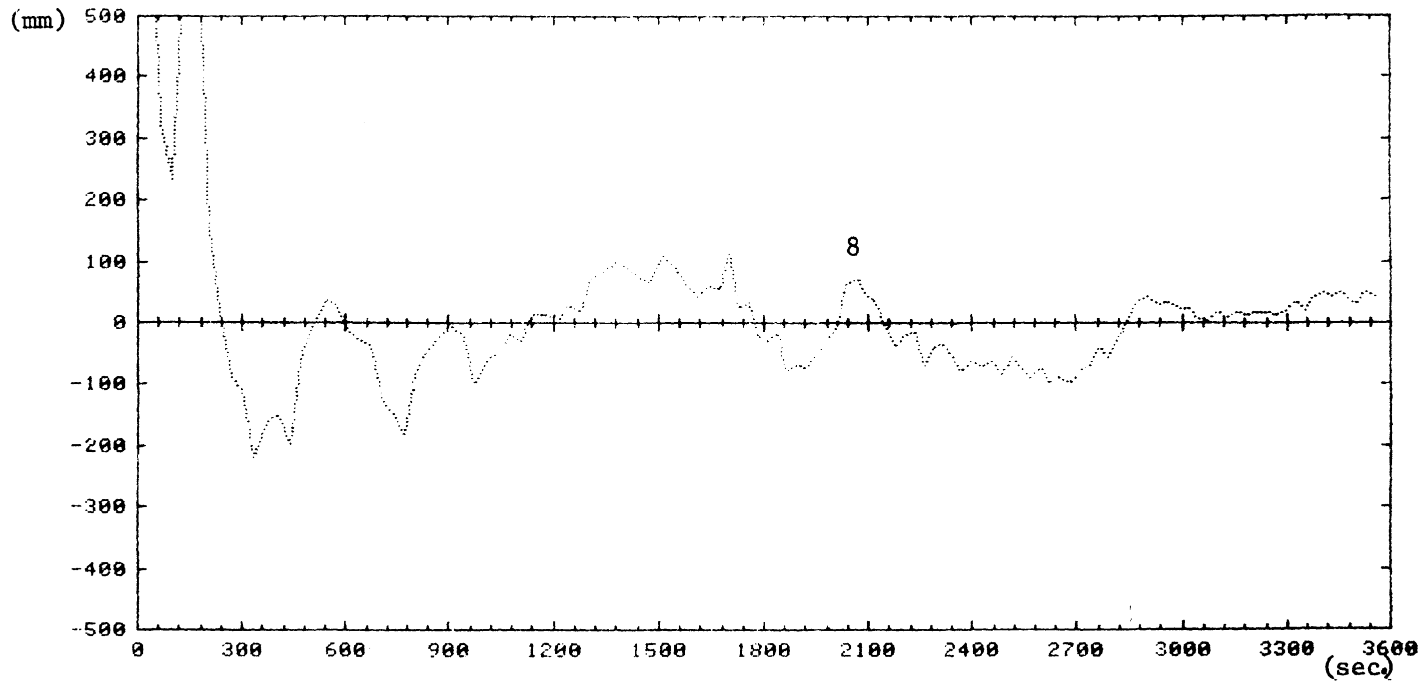

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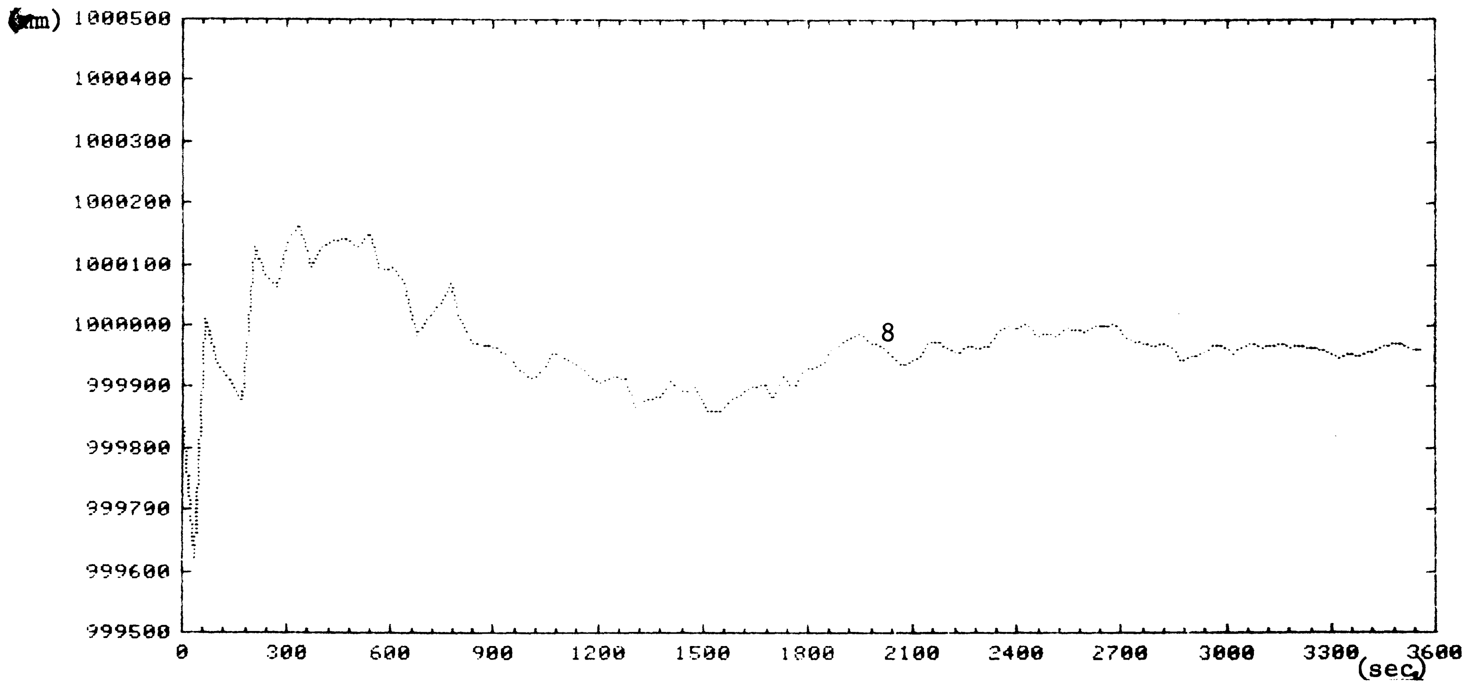

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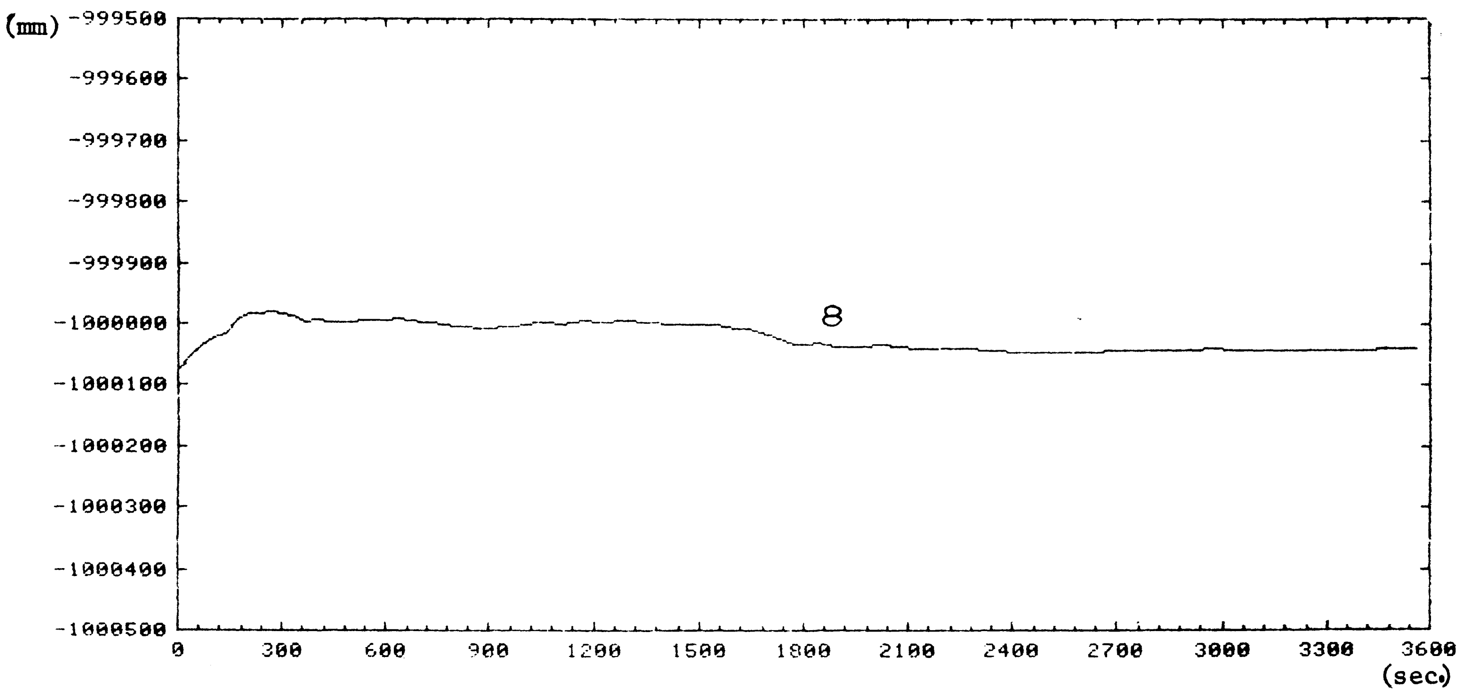
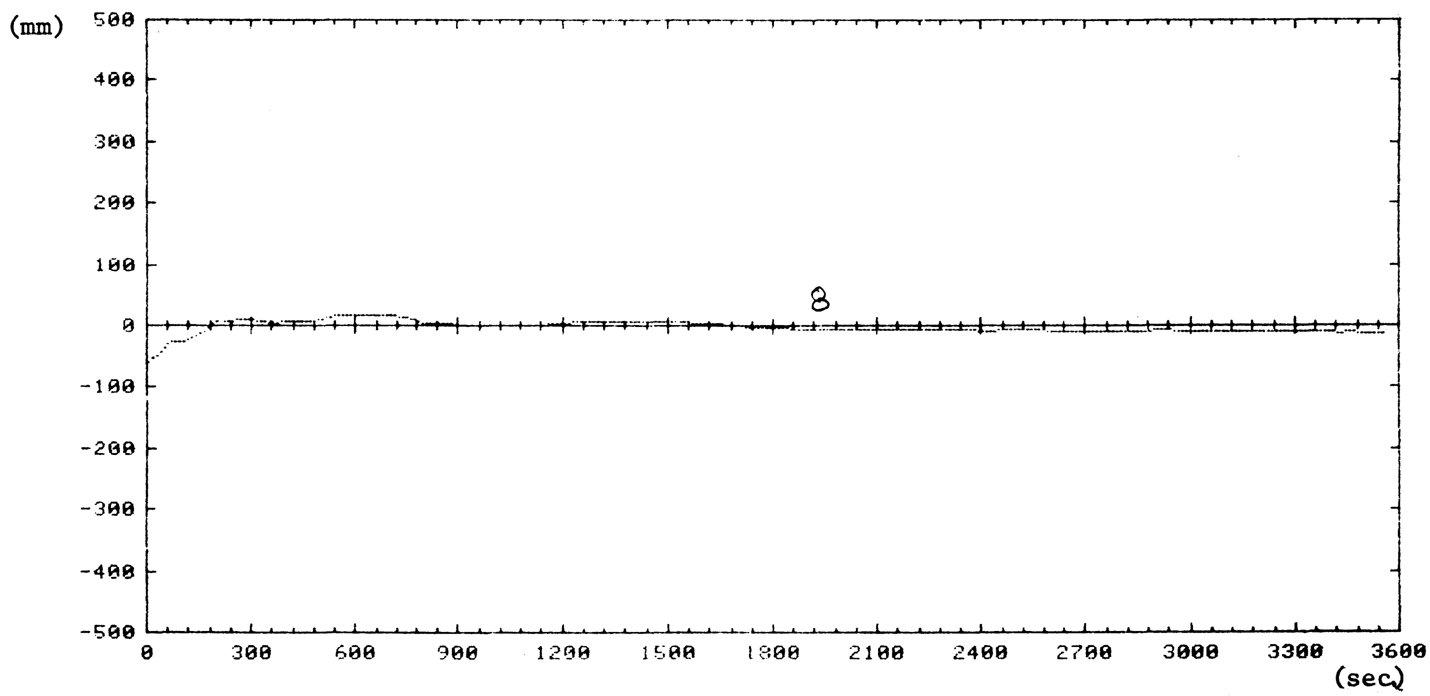


Fig. A2. Discrepancy Dϕ phase 2 5 7 10 12 15



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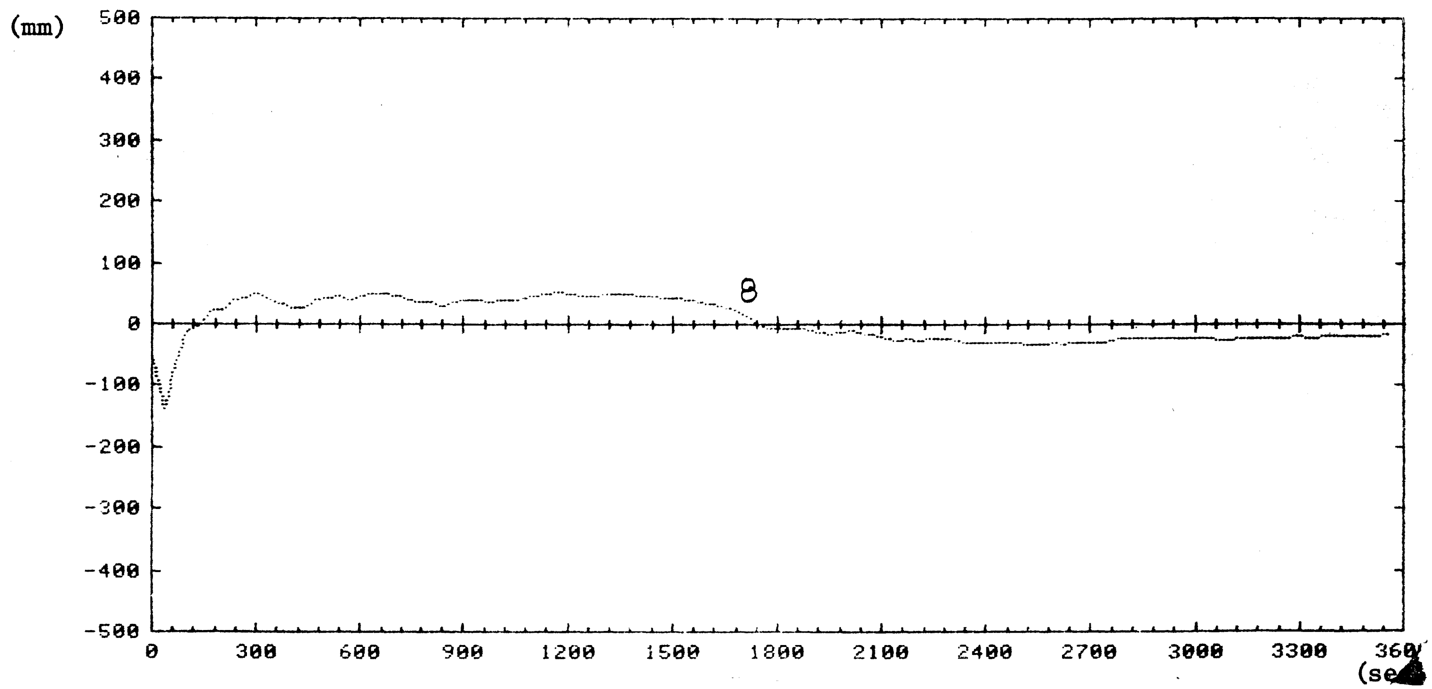

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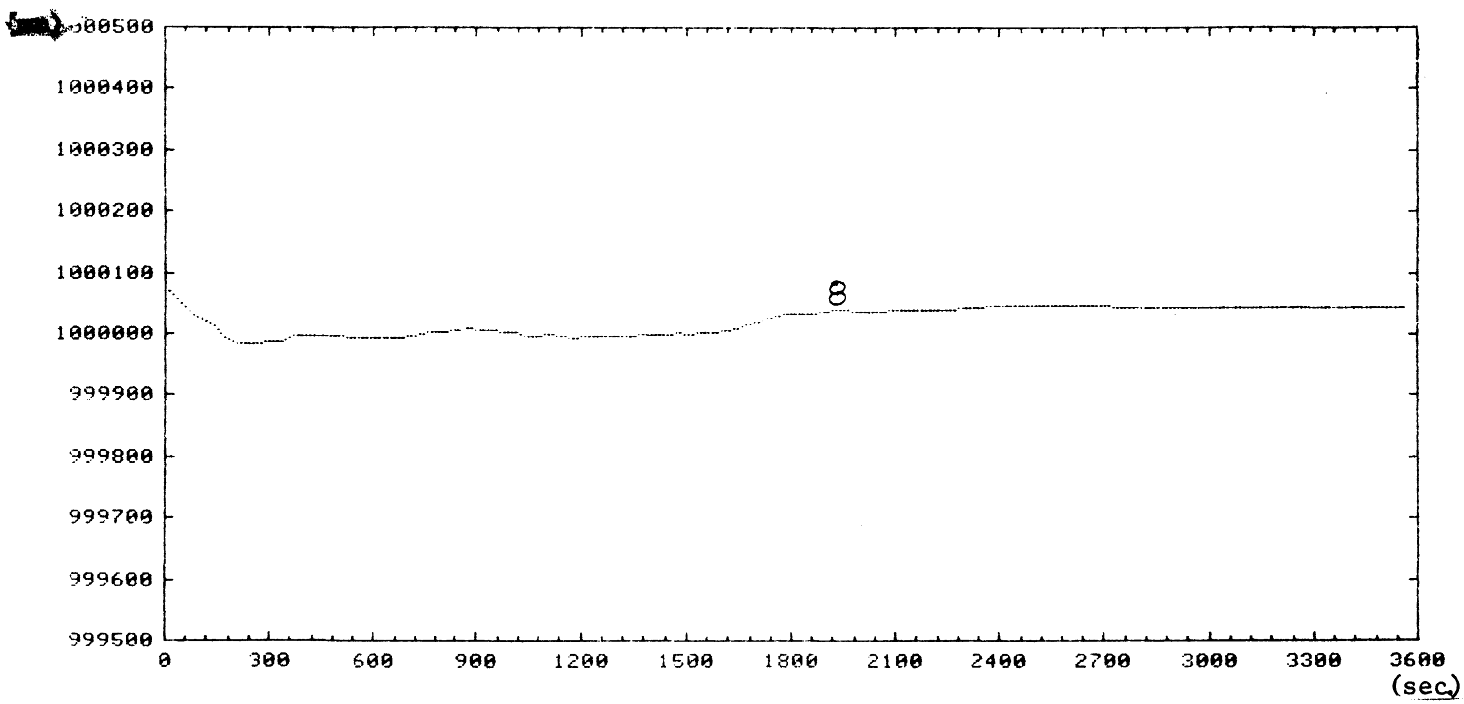
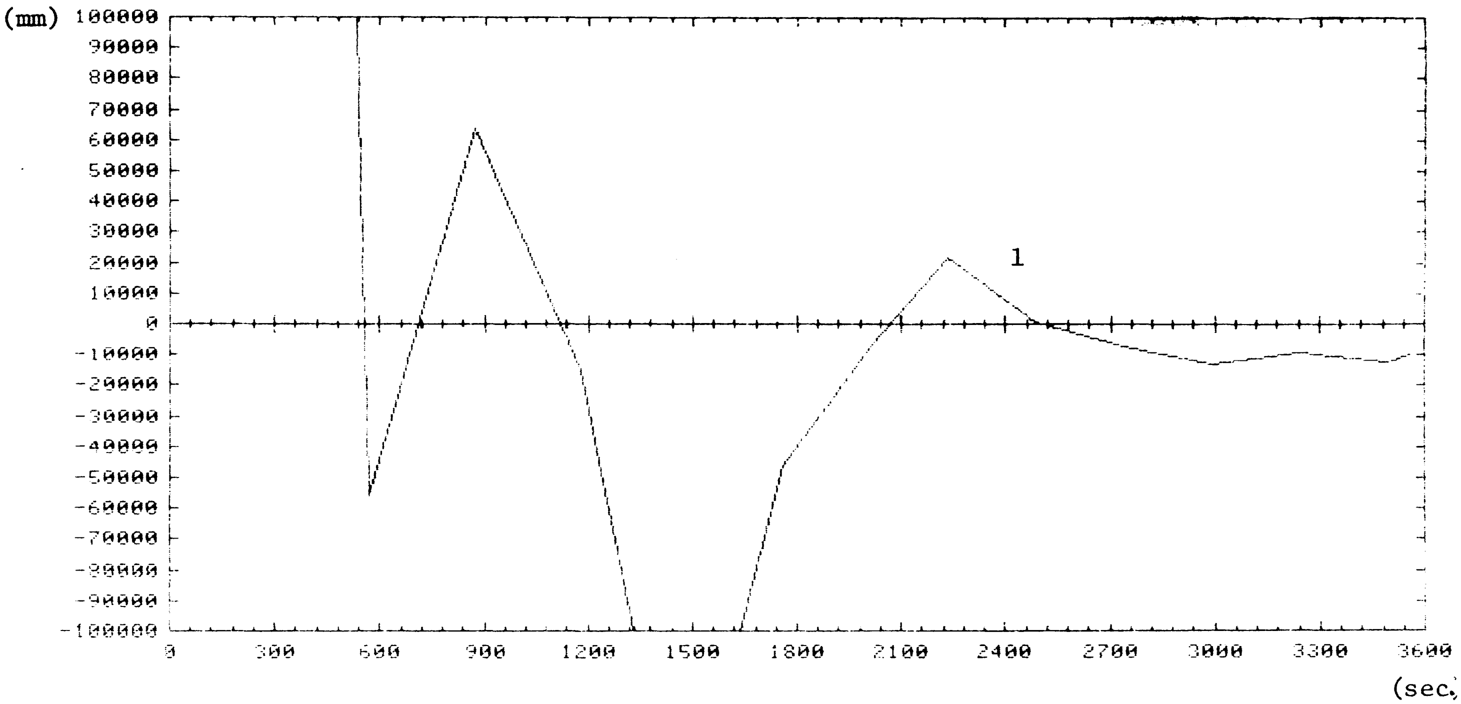


Fig. A2. Discrepancy DR phase 2 5 7 10 12 15



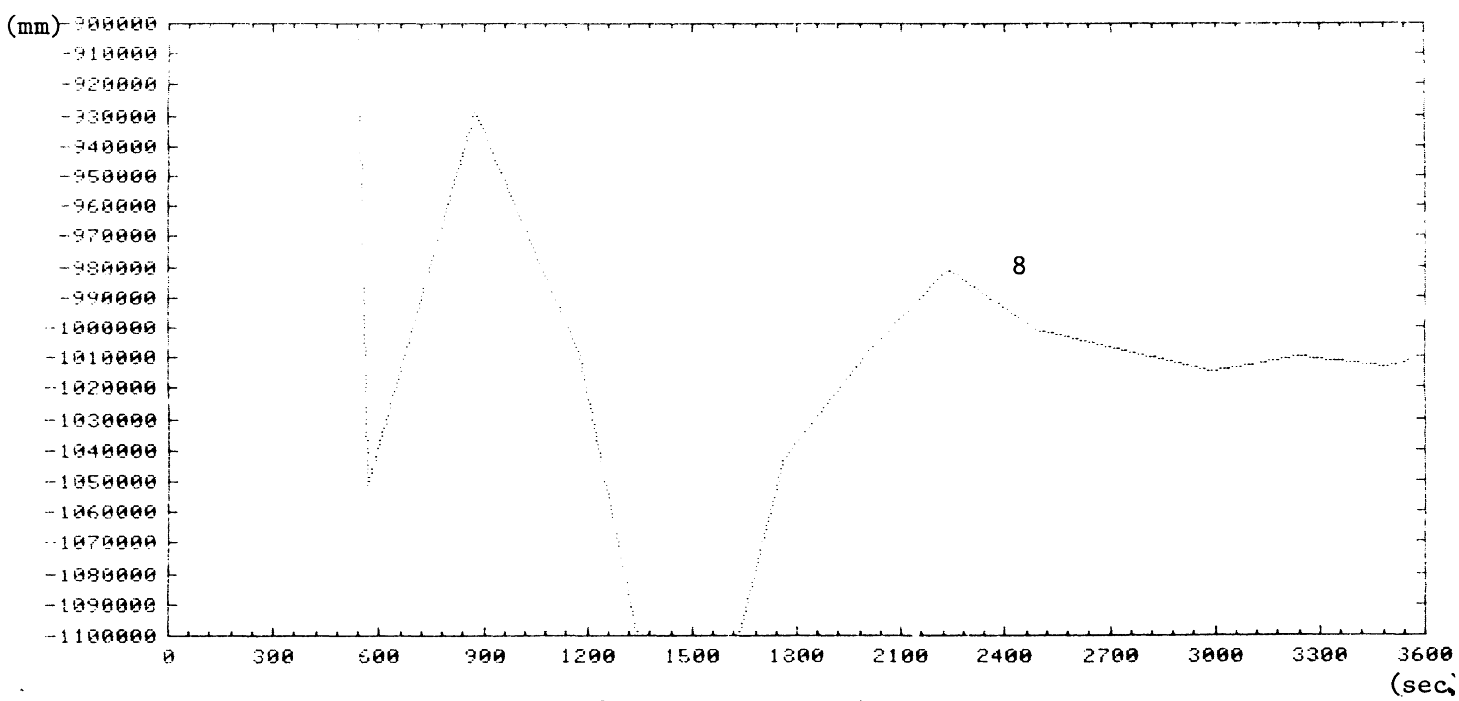
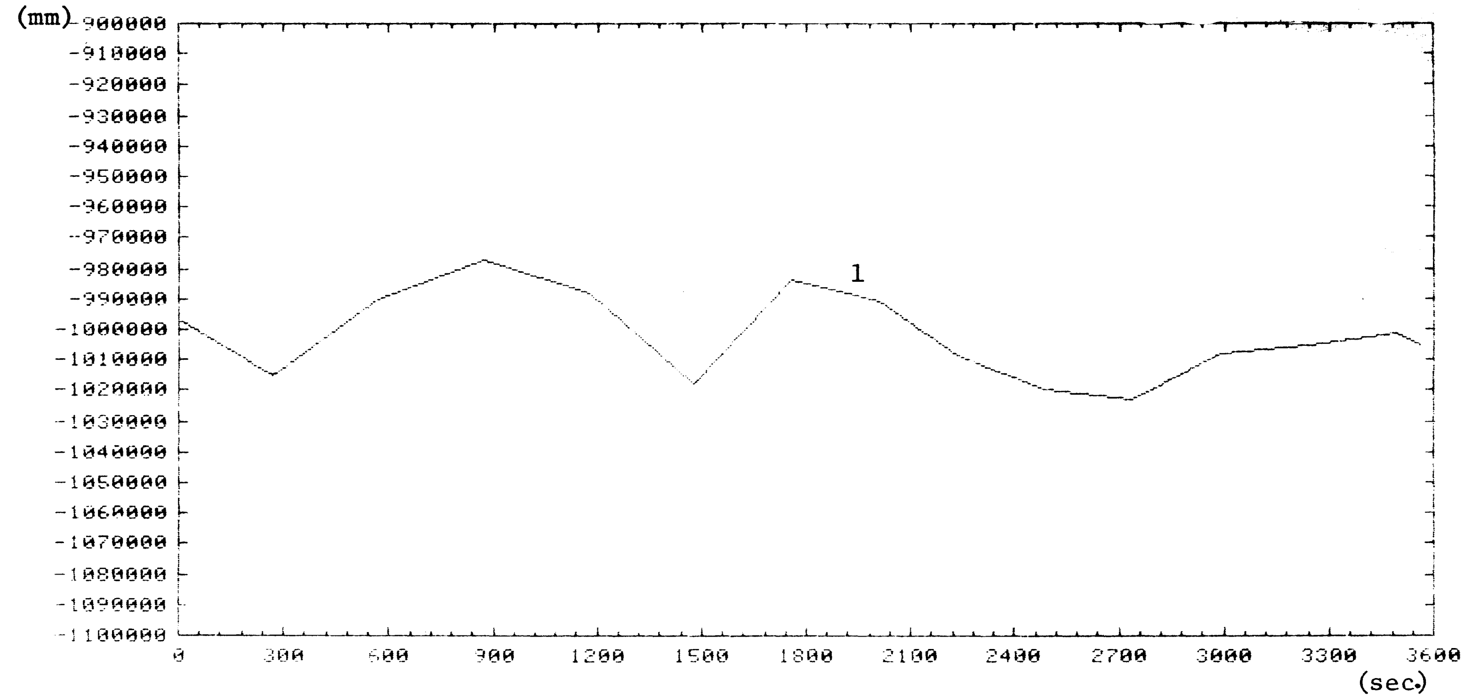


Fig. A3. Discrepancy Dϕ P code



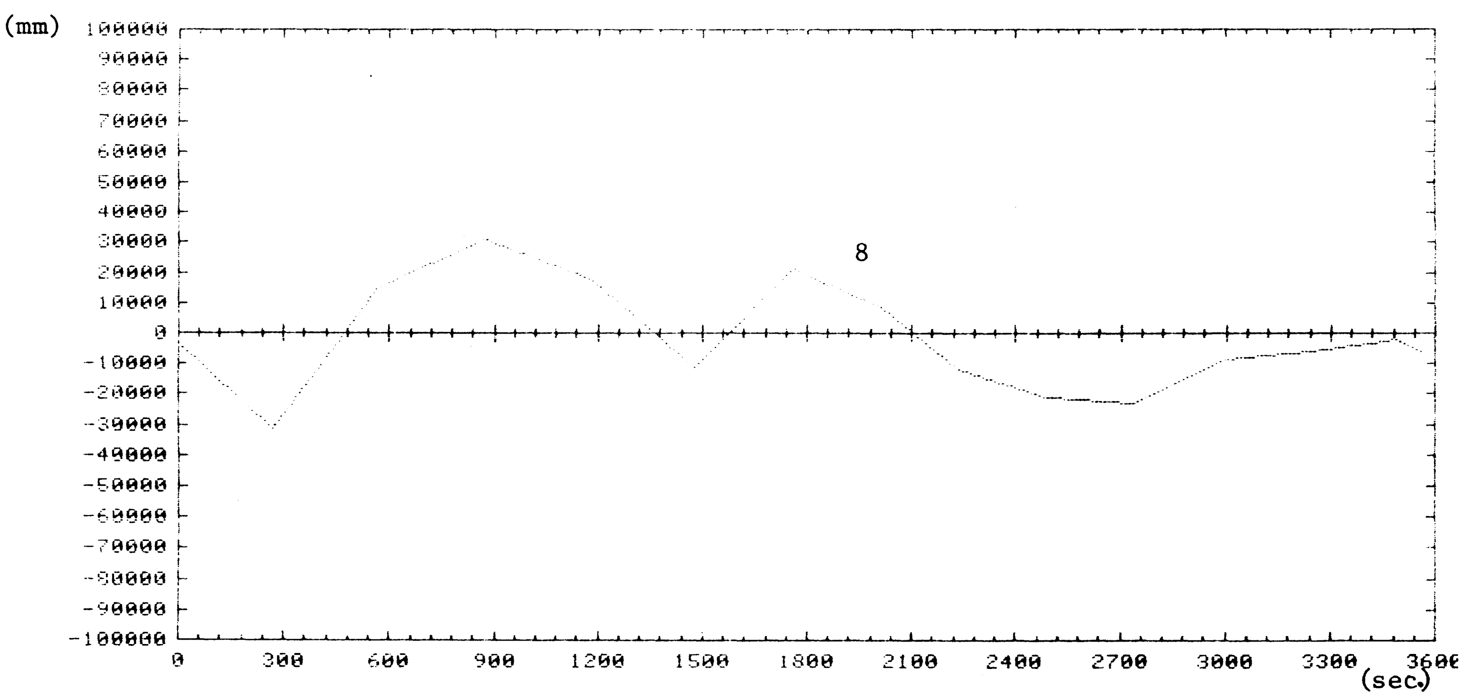
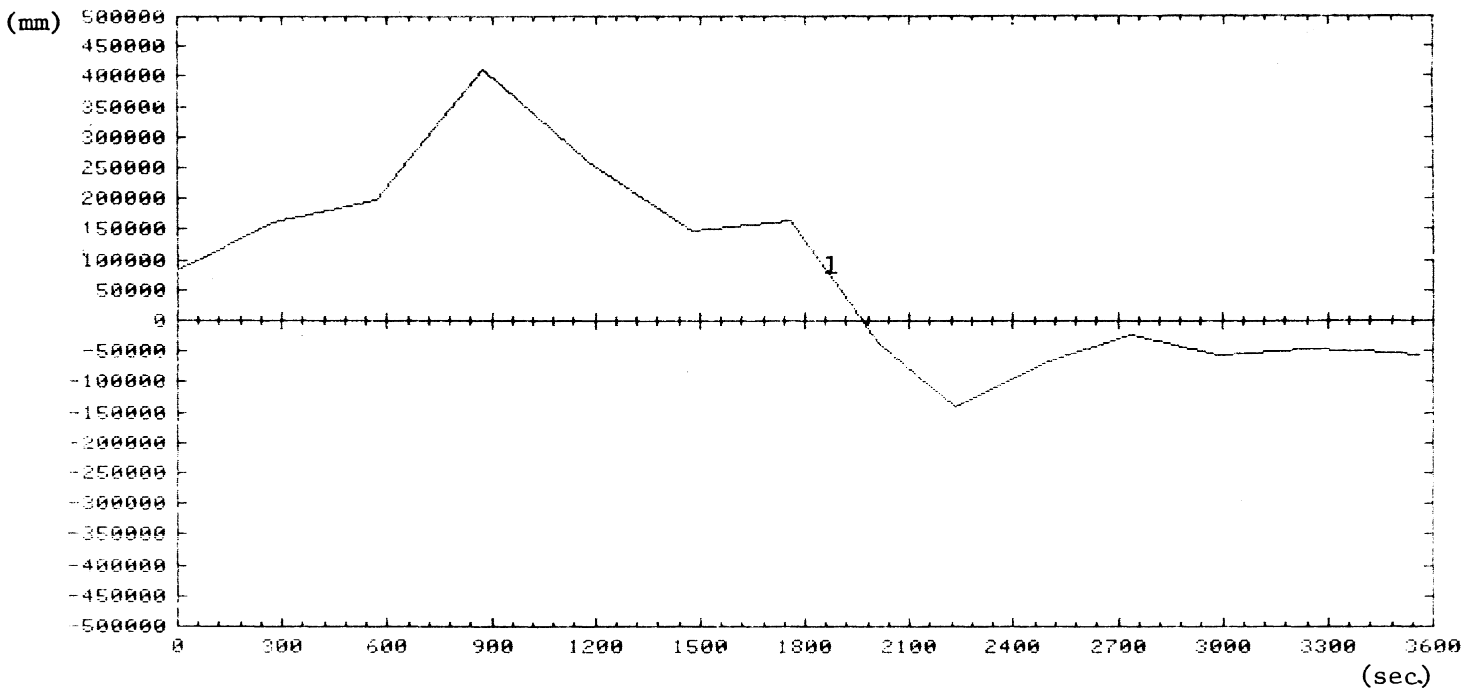
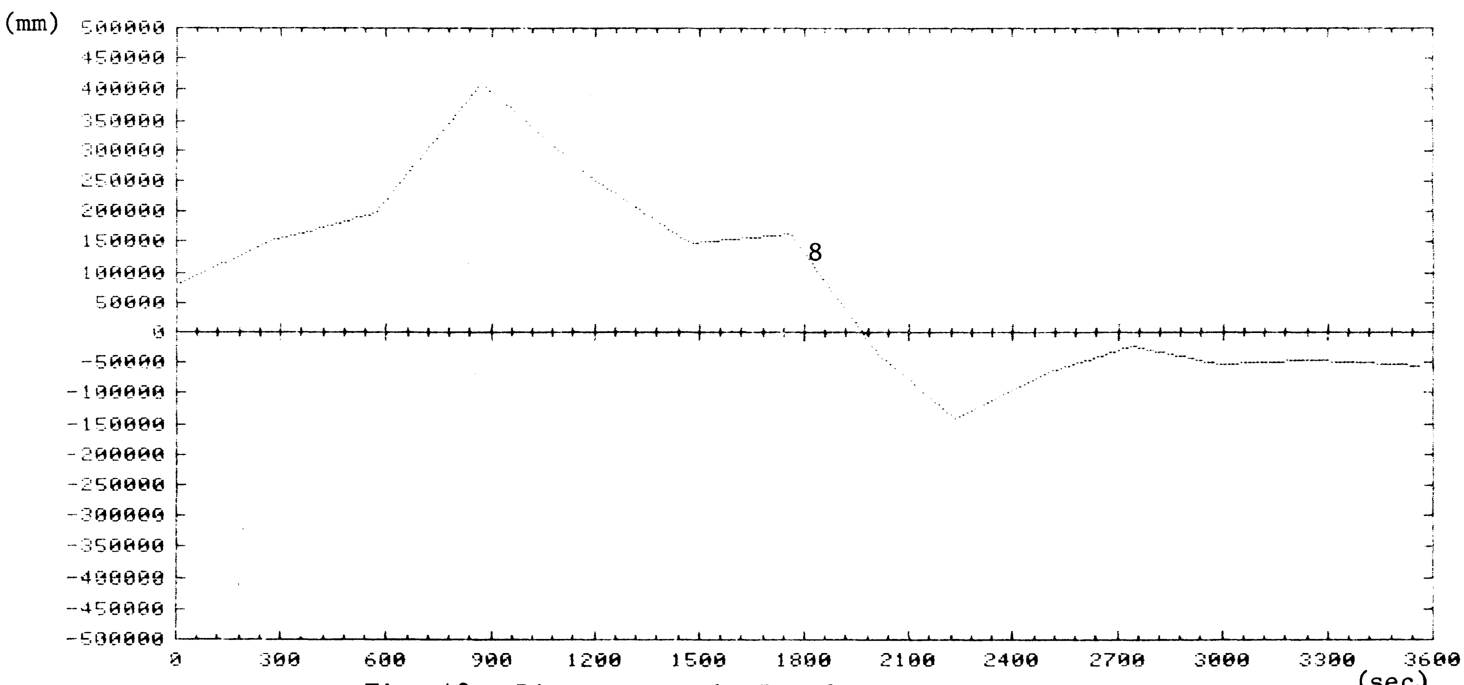


Fig. A3. Discrepancy Dλ P code





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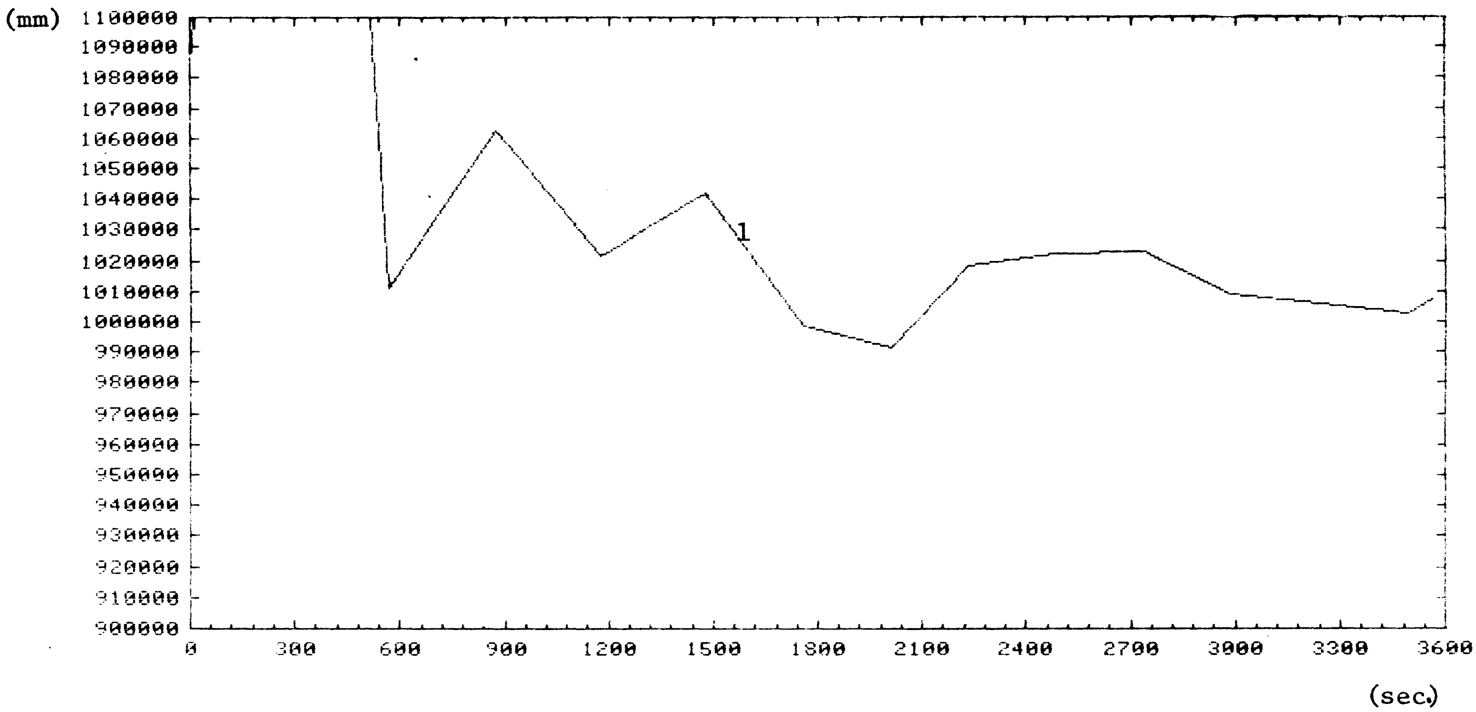

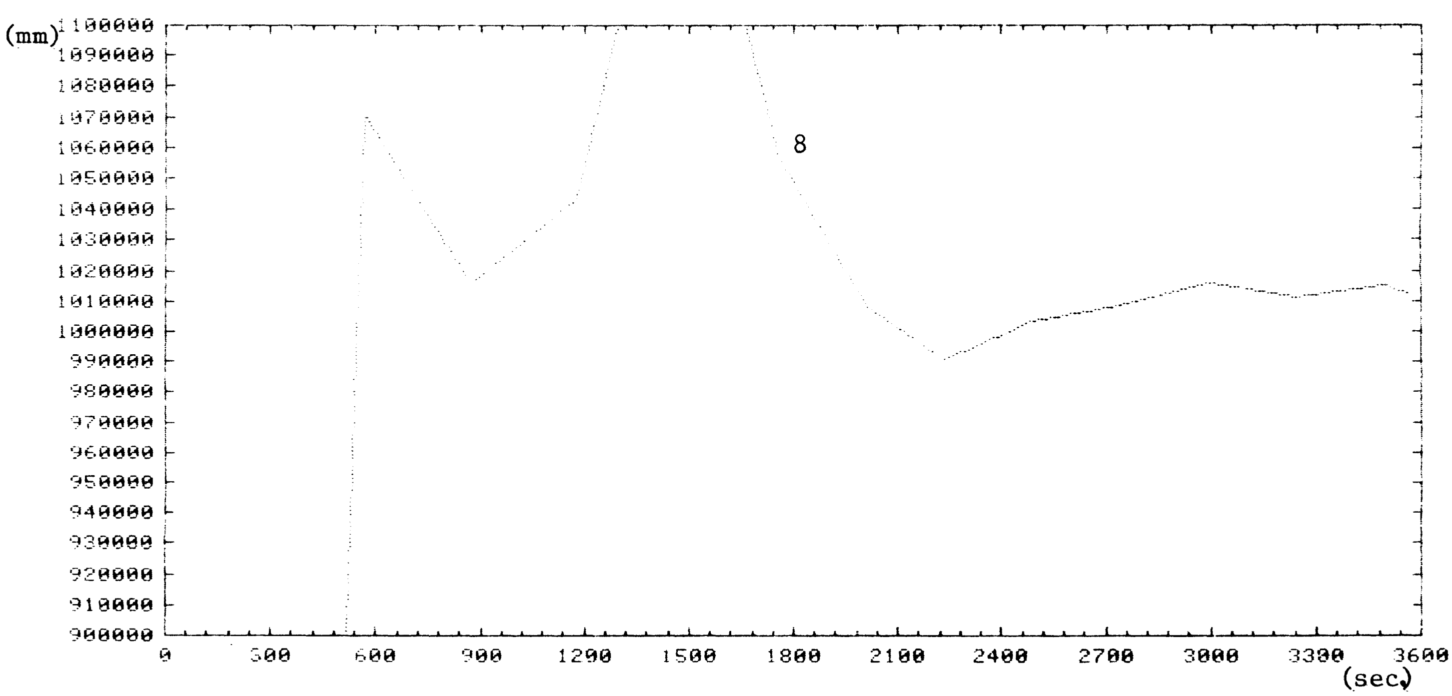

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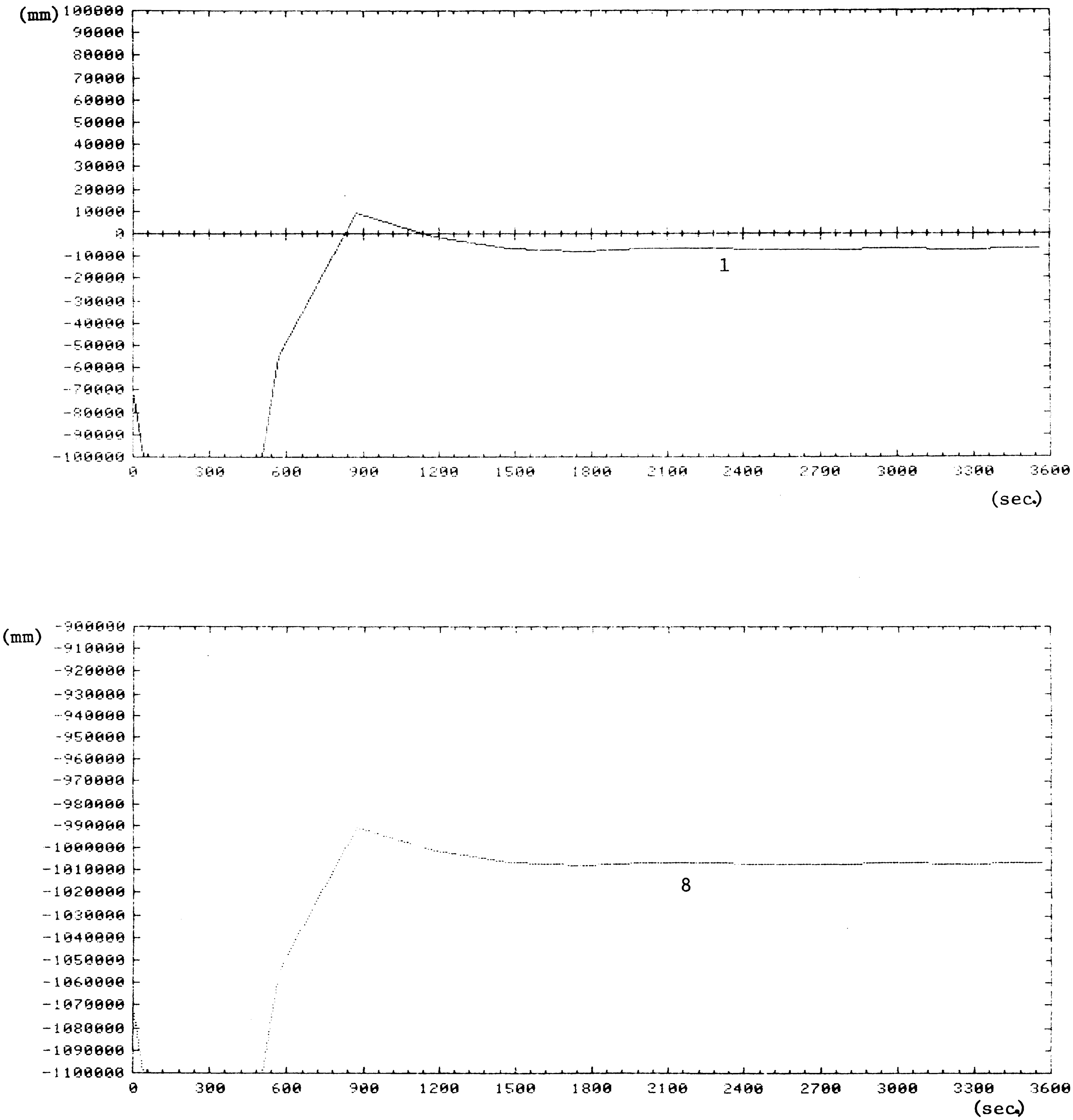
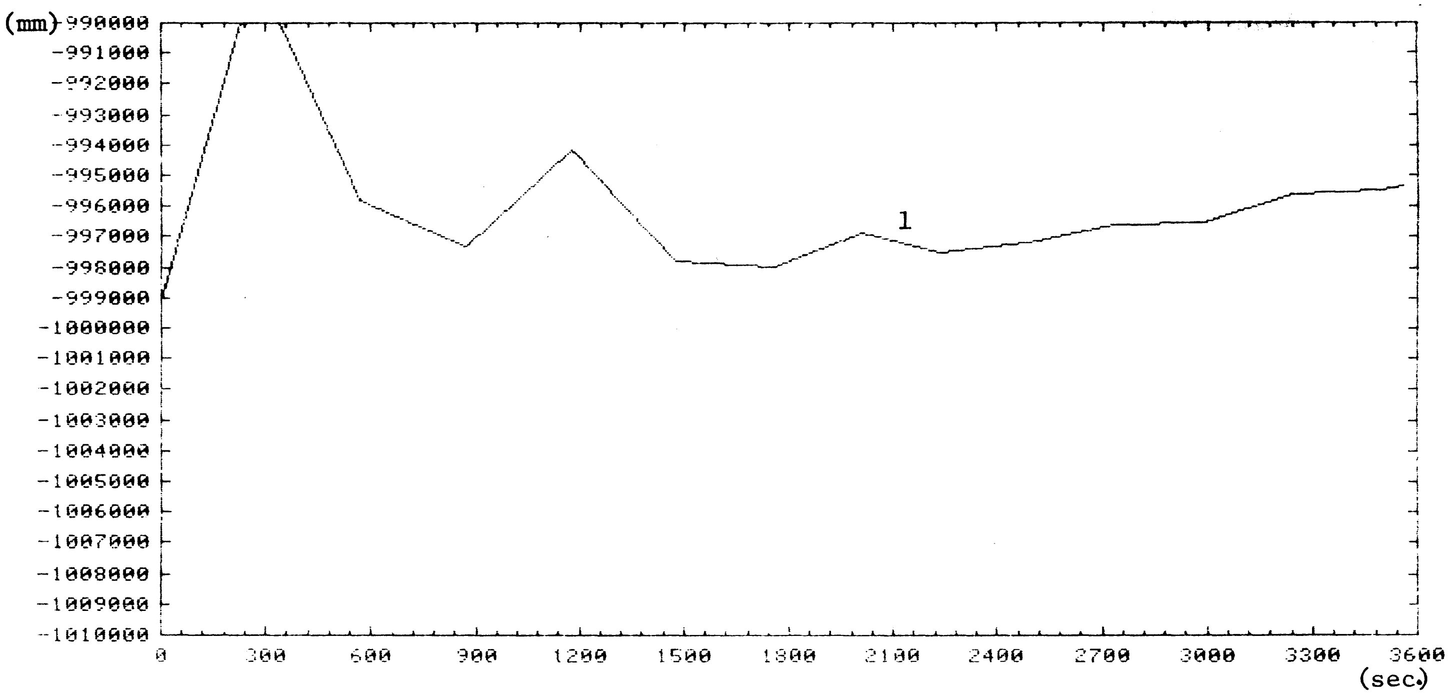


Fig. A4. Discrepancy Dϕ phase 2 5 7 10 12 15



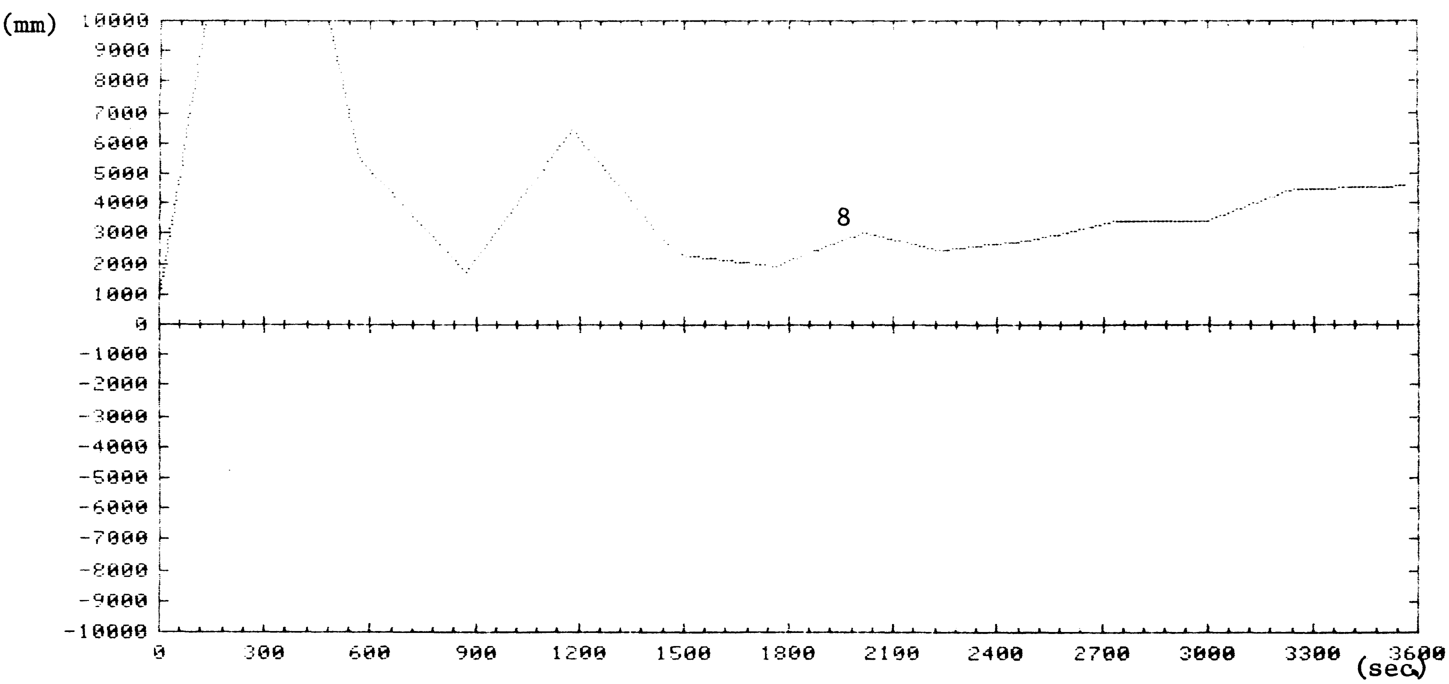
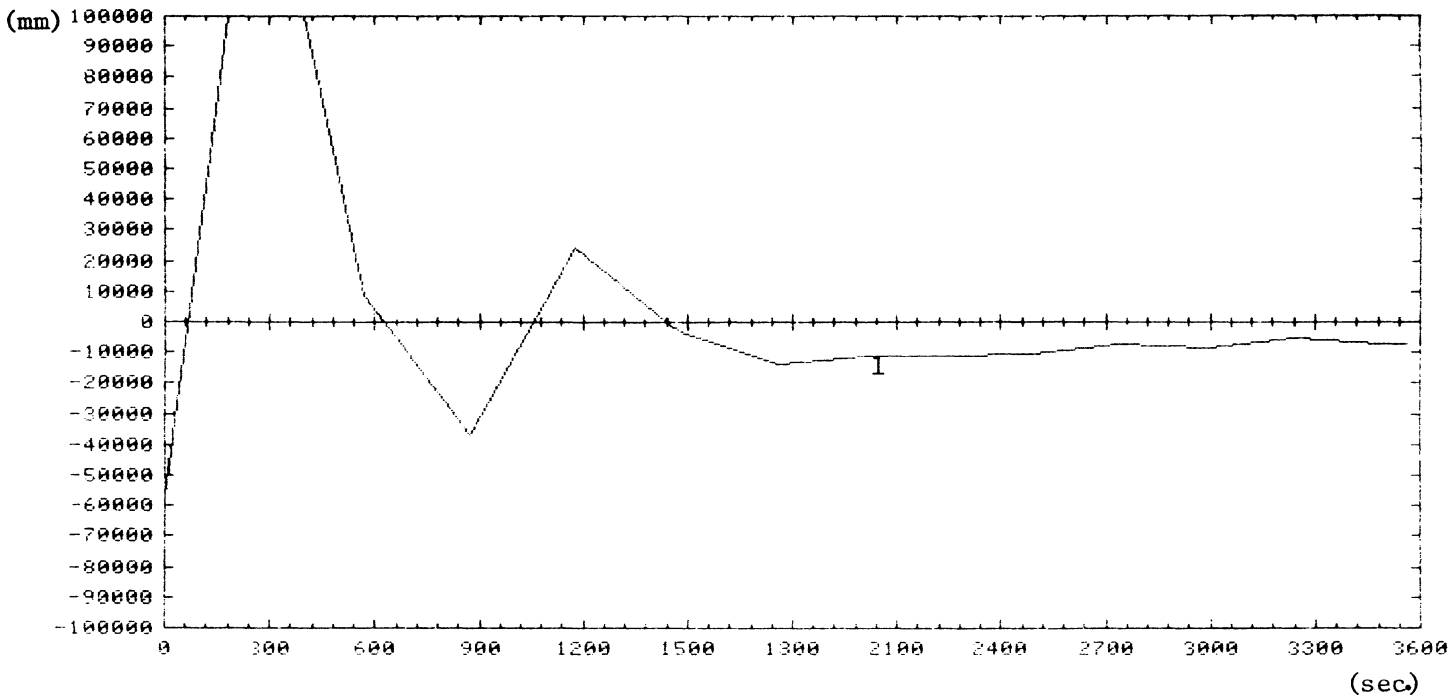
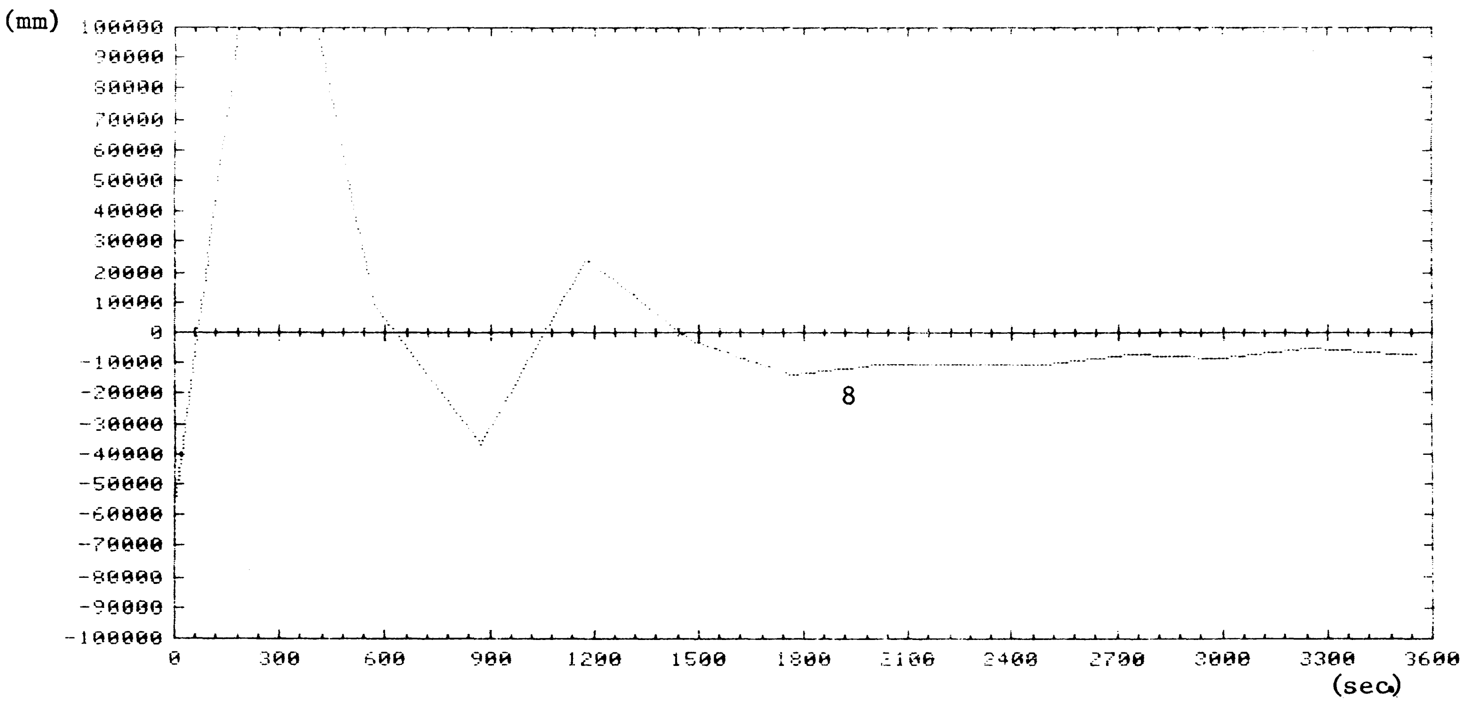


Fig. A4. Discrepancy Dλ phase





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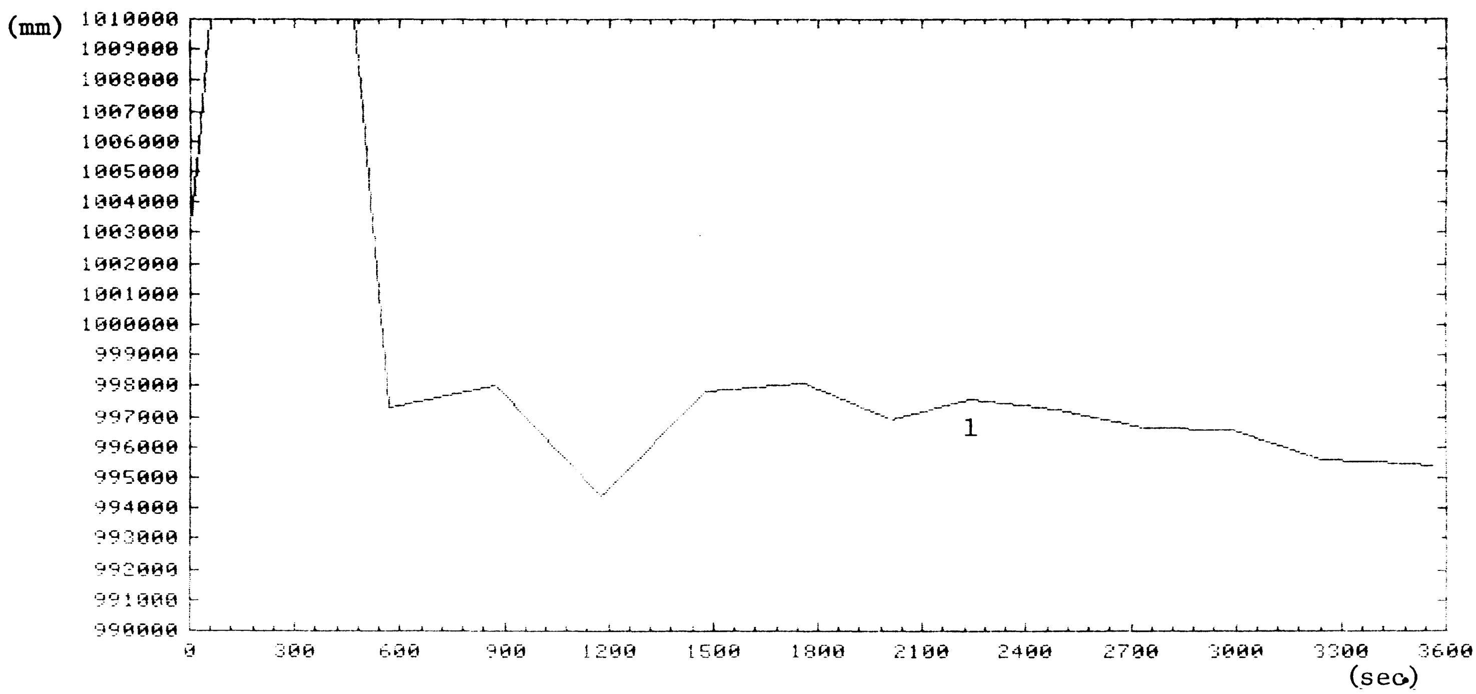


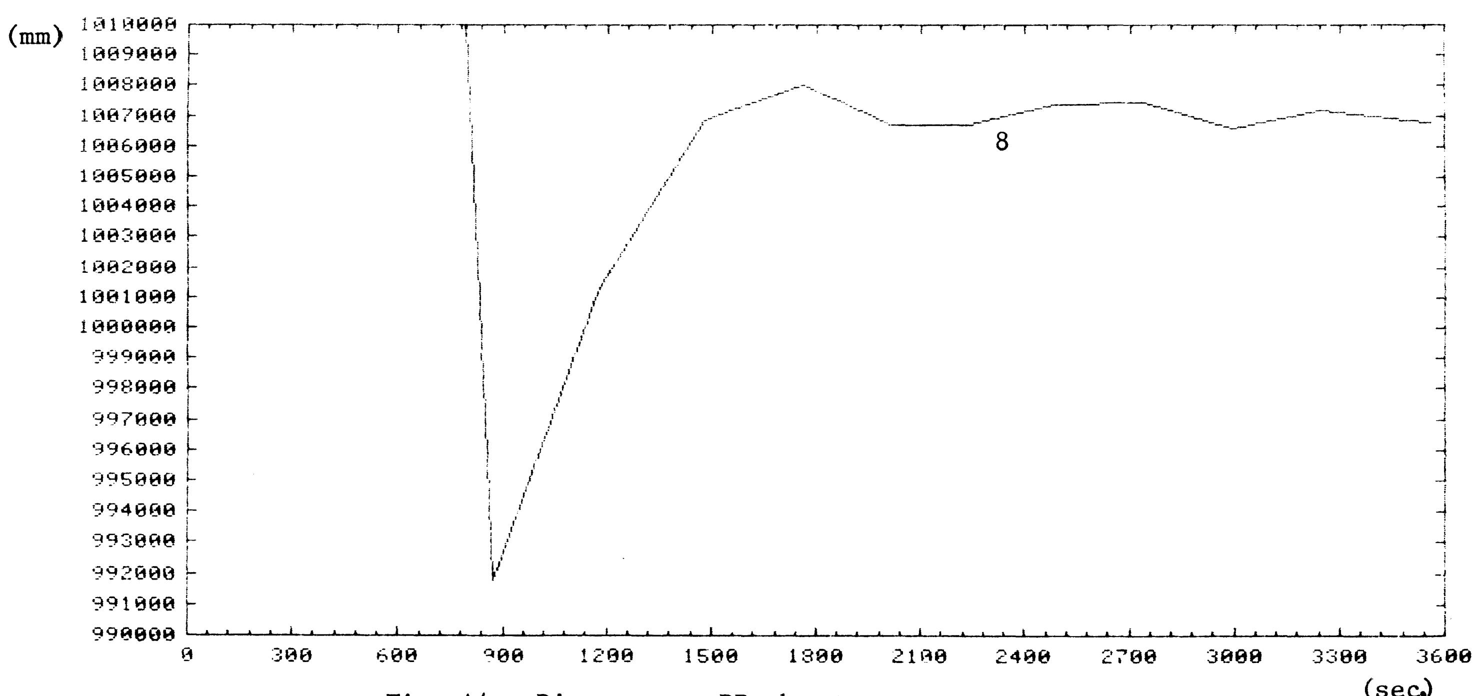
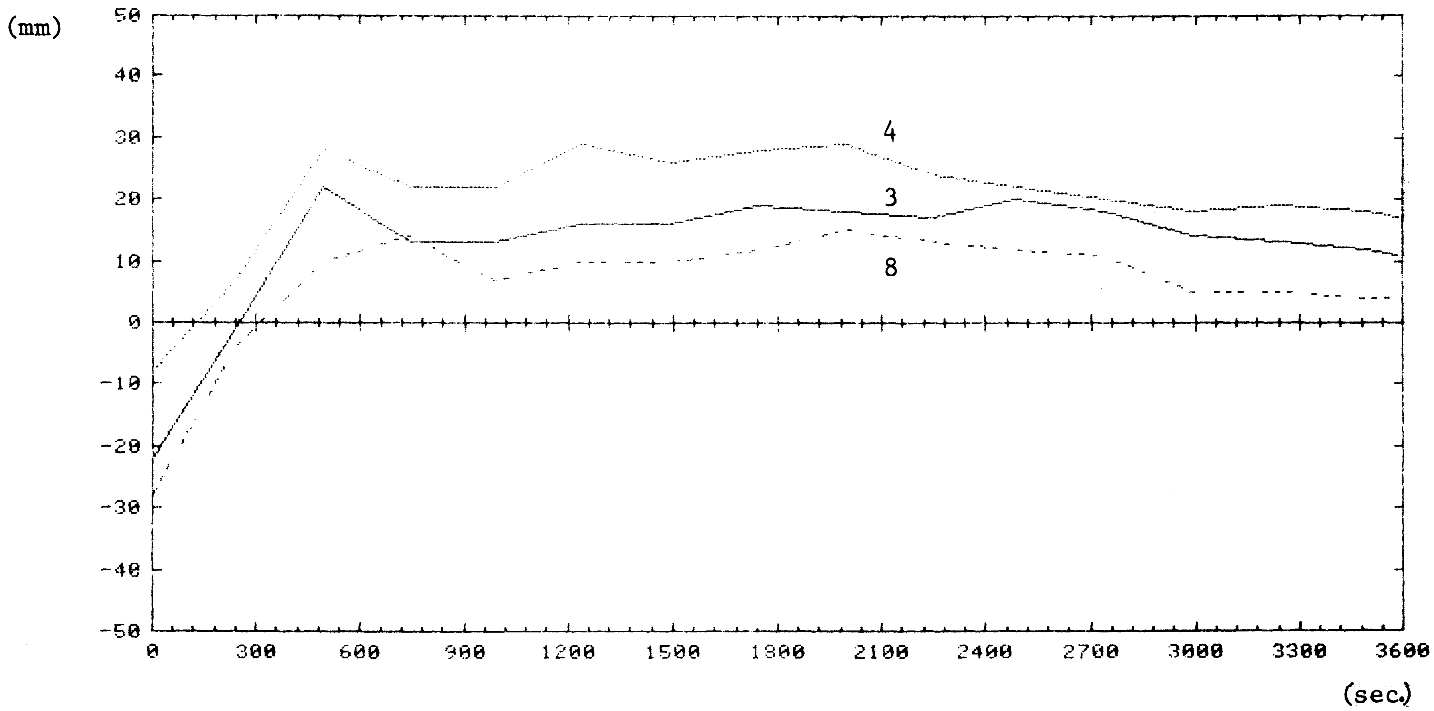


Fig. A4. Discrepancy DR phase



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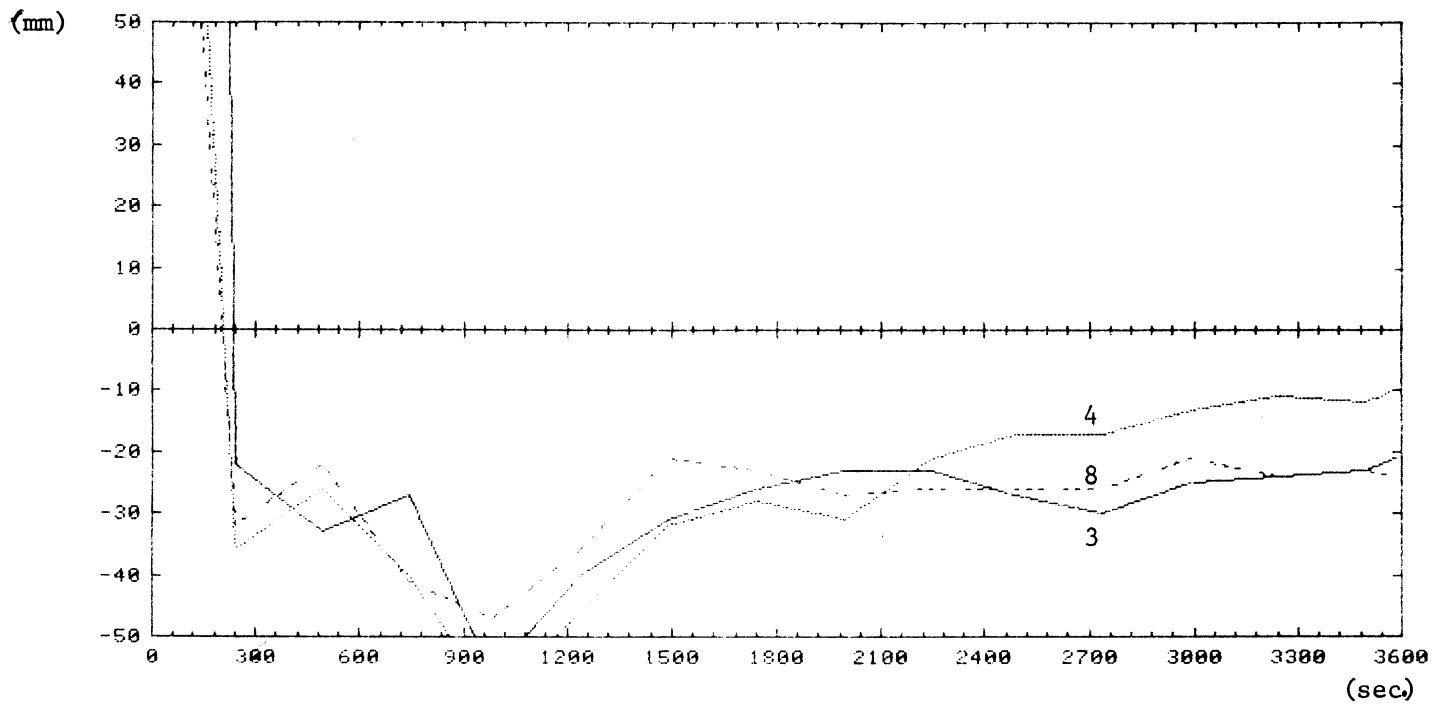
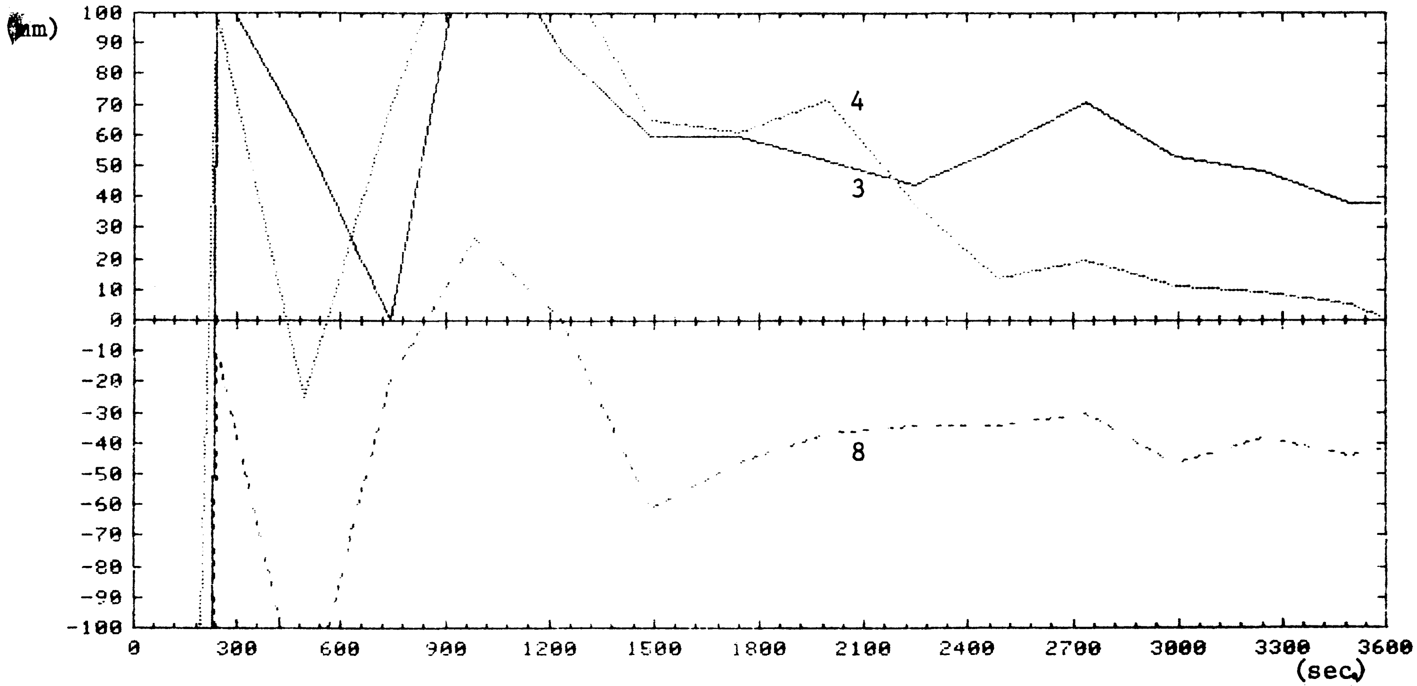


Fig. A5. Discrepancy DY interf. 2 5 7 10



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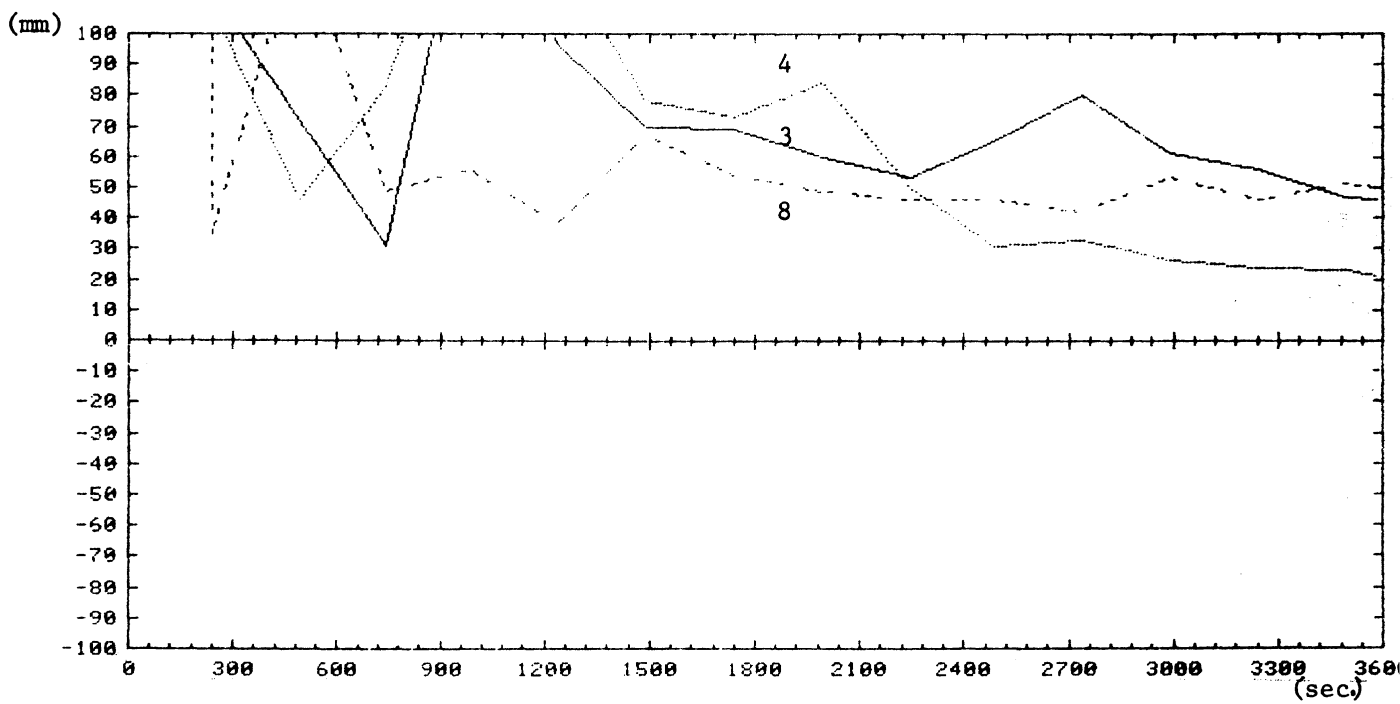

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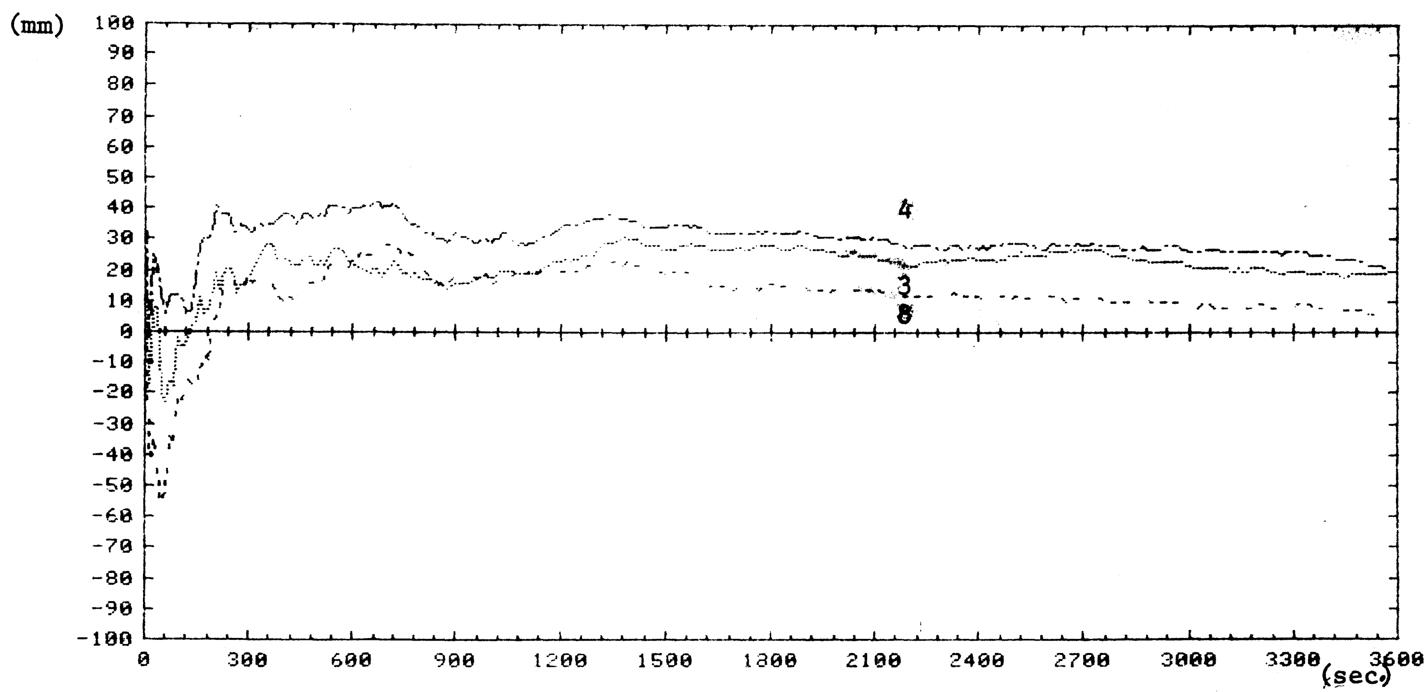

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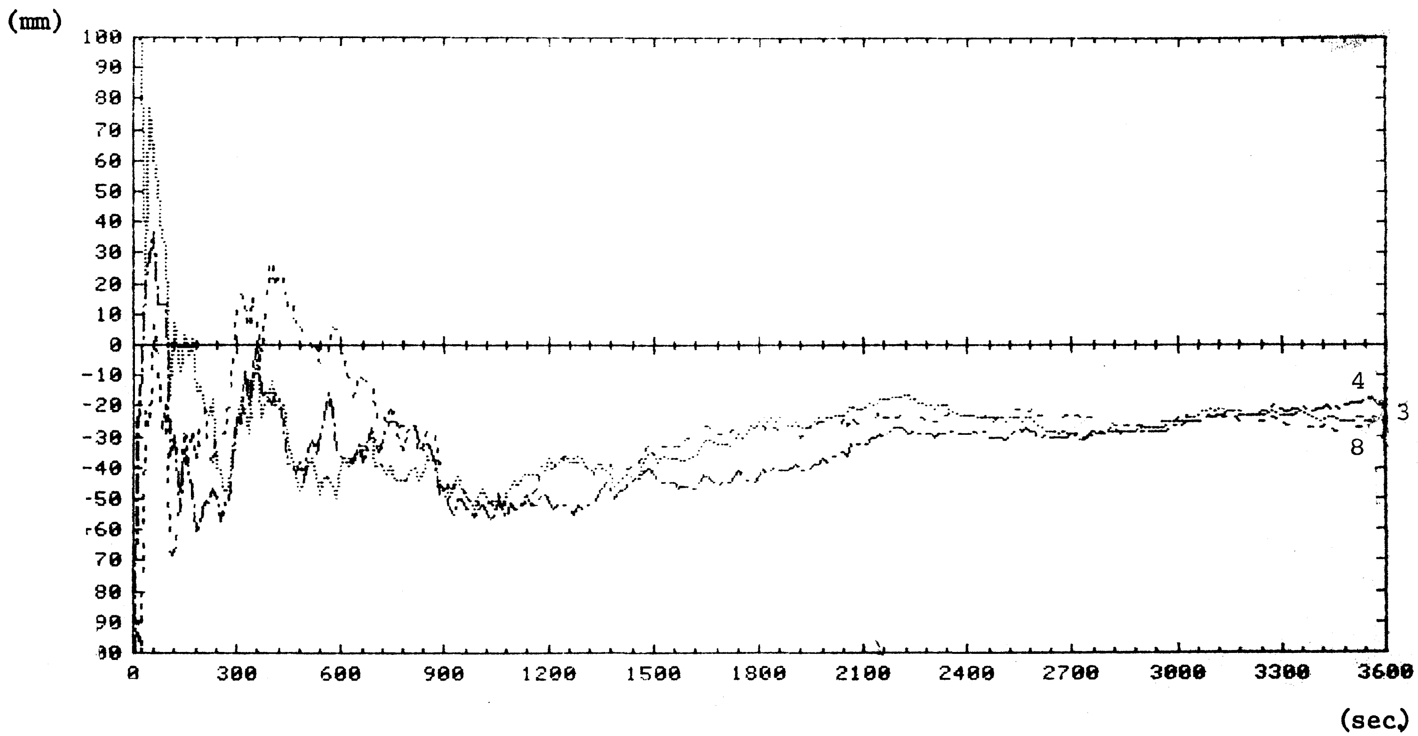

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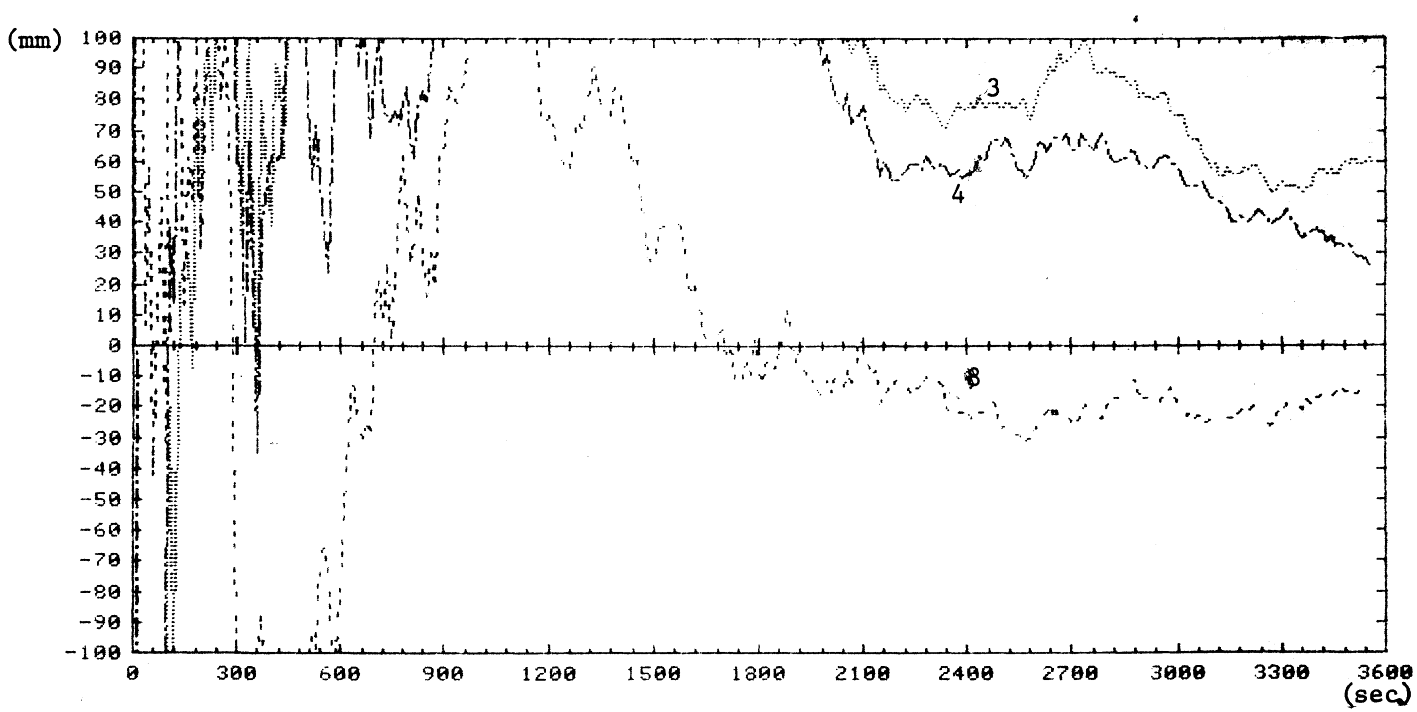

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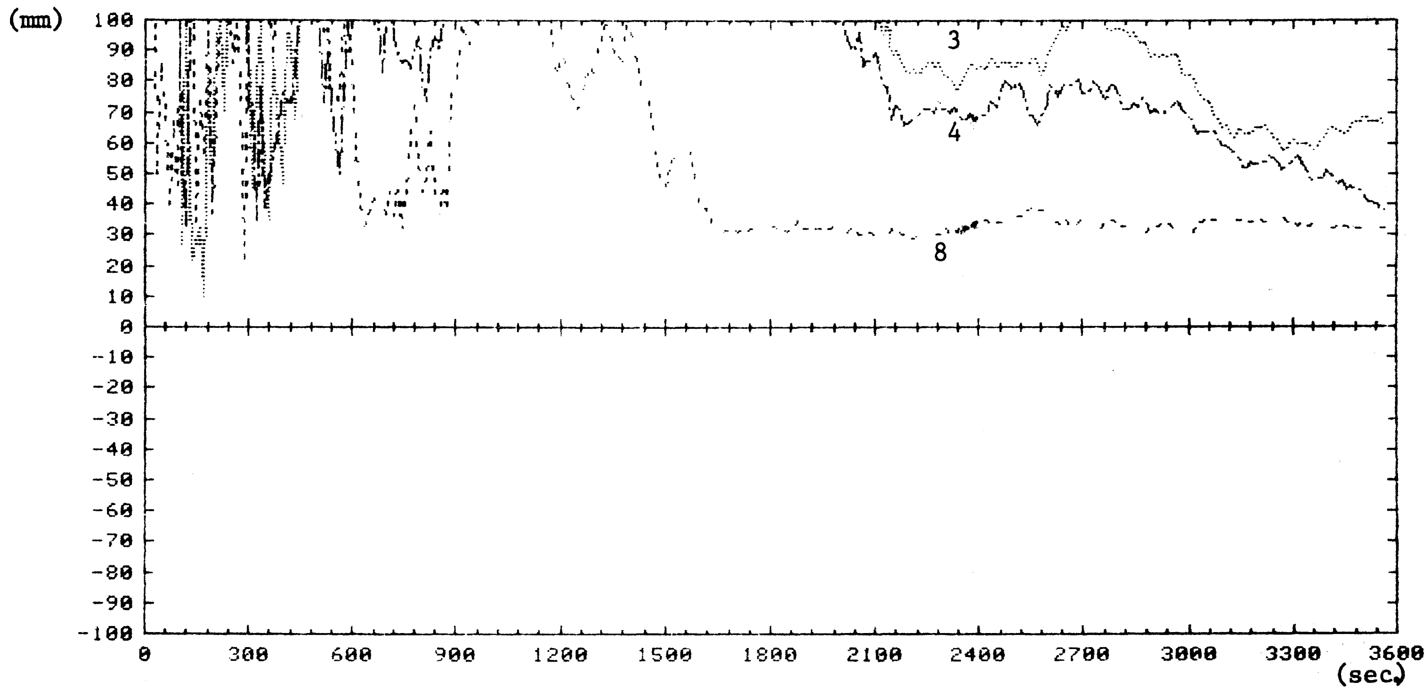

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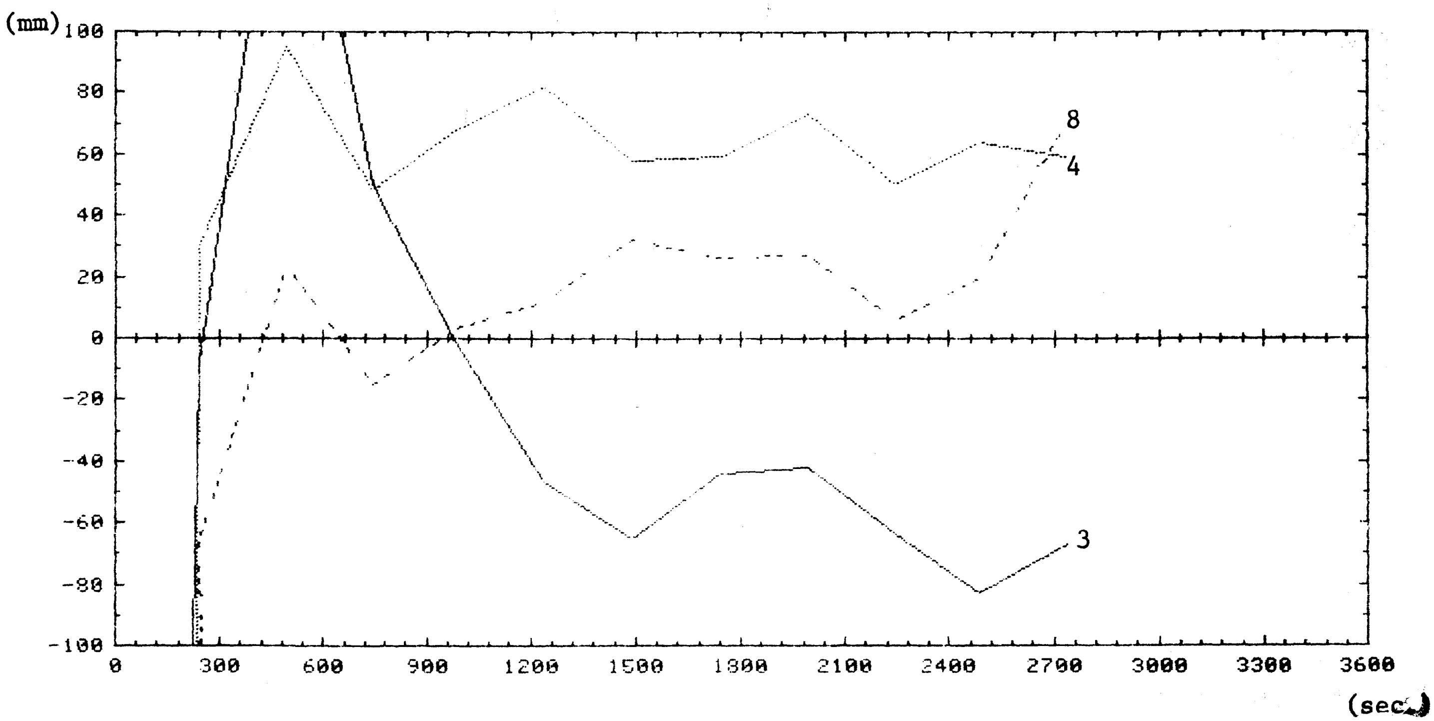

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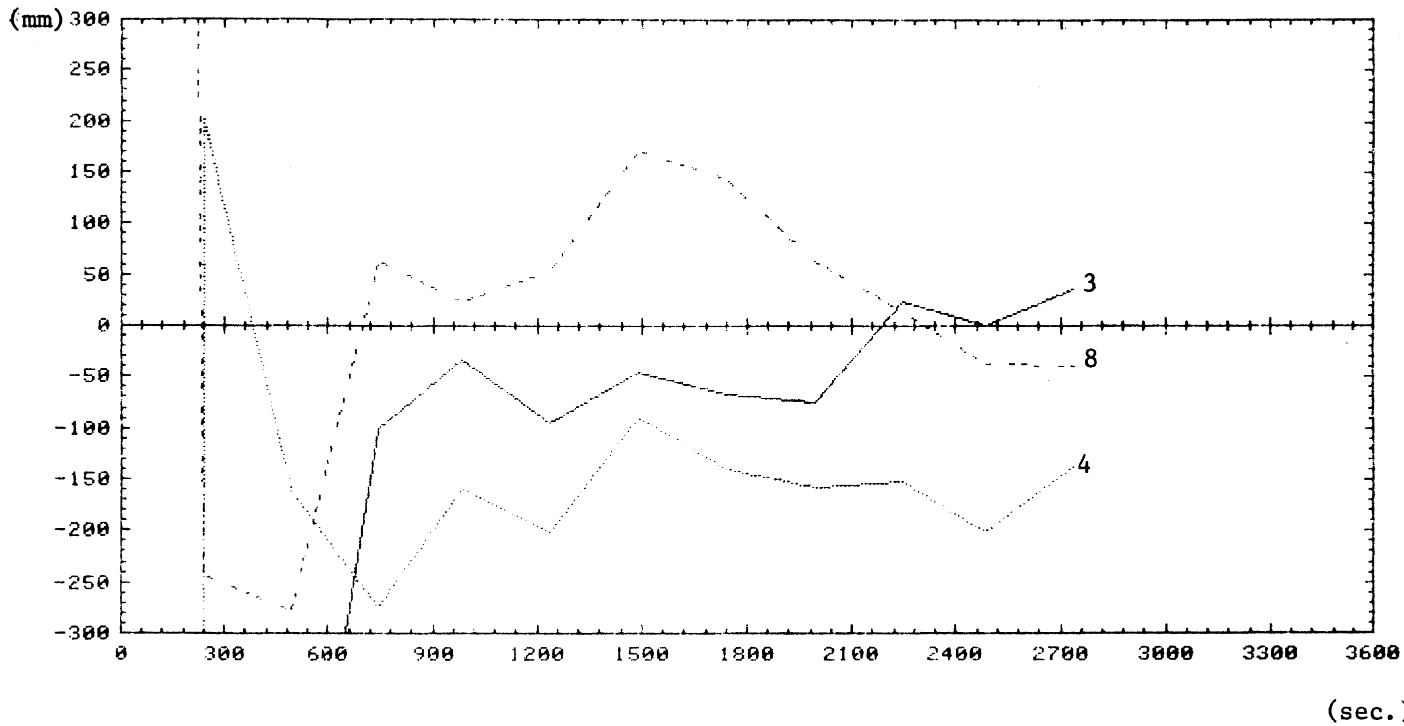

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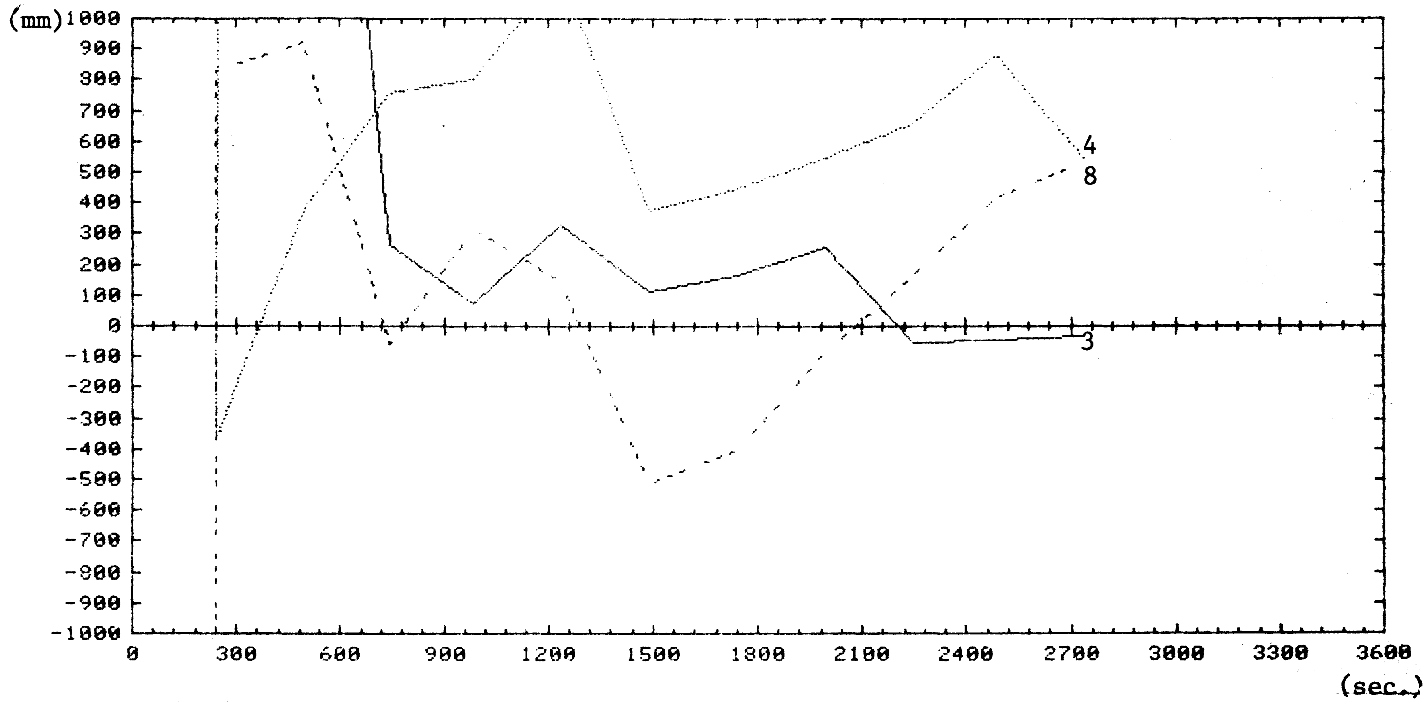

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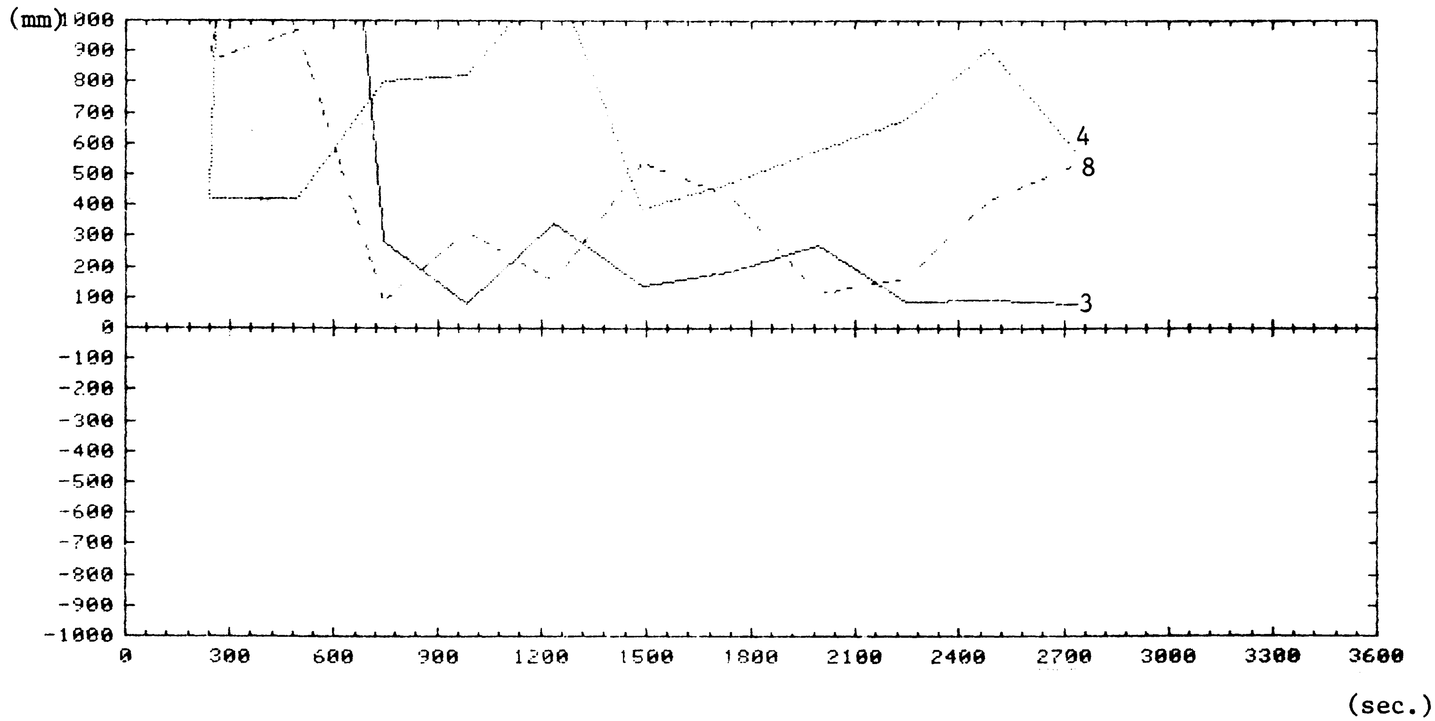

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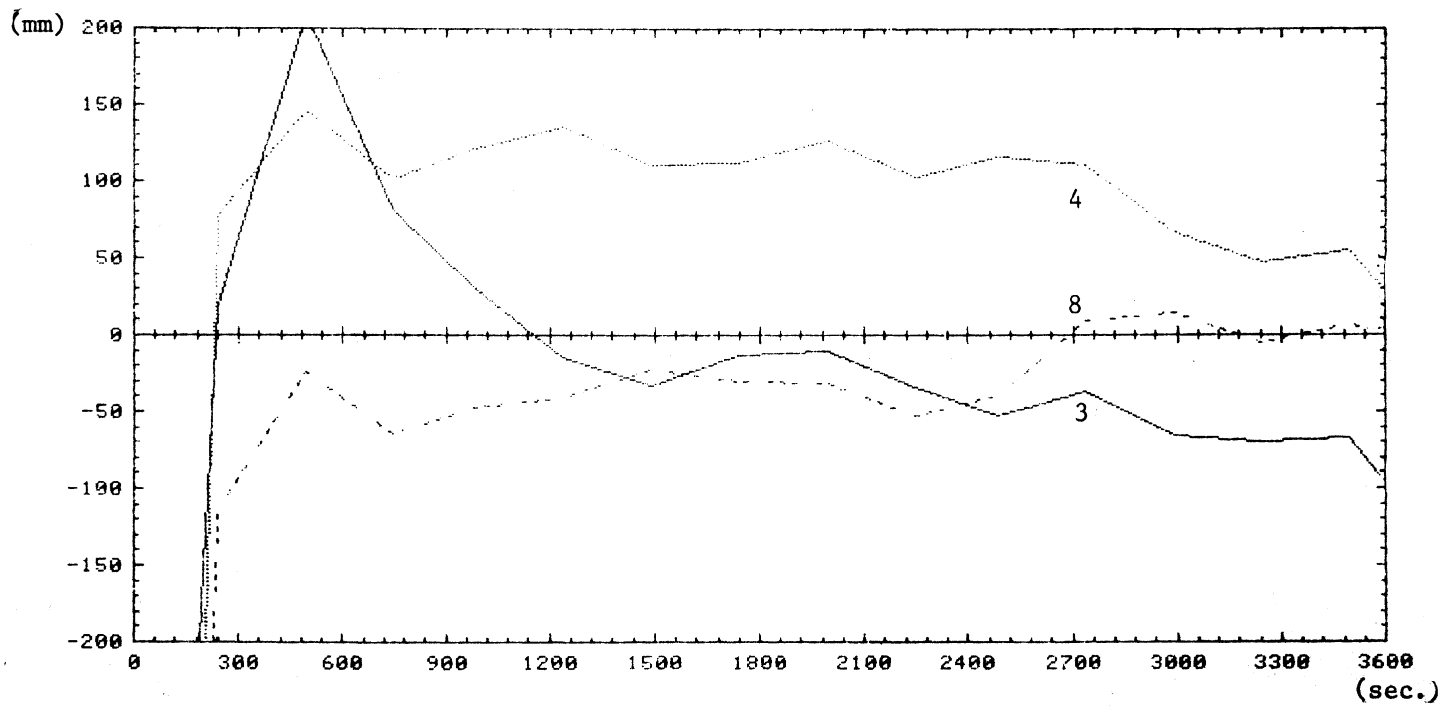

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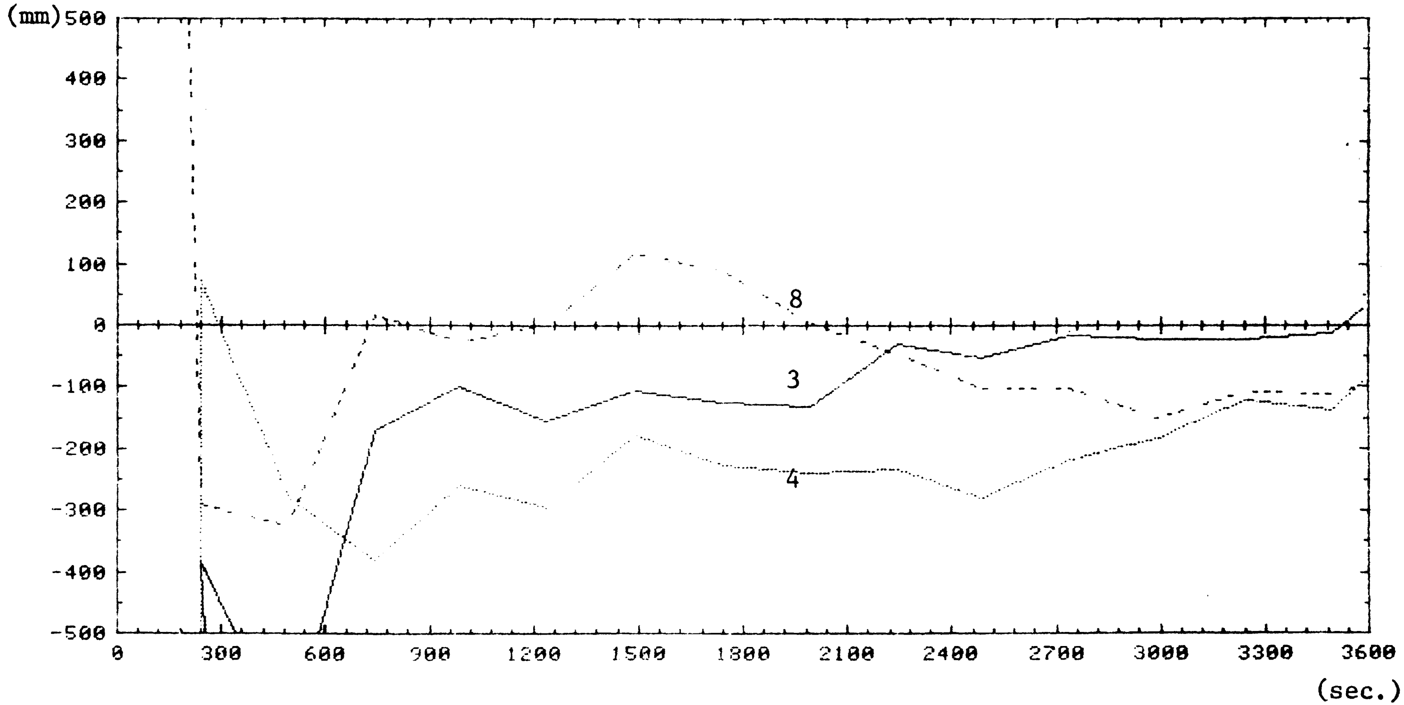

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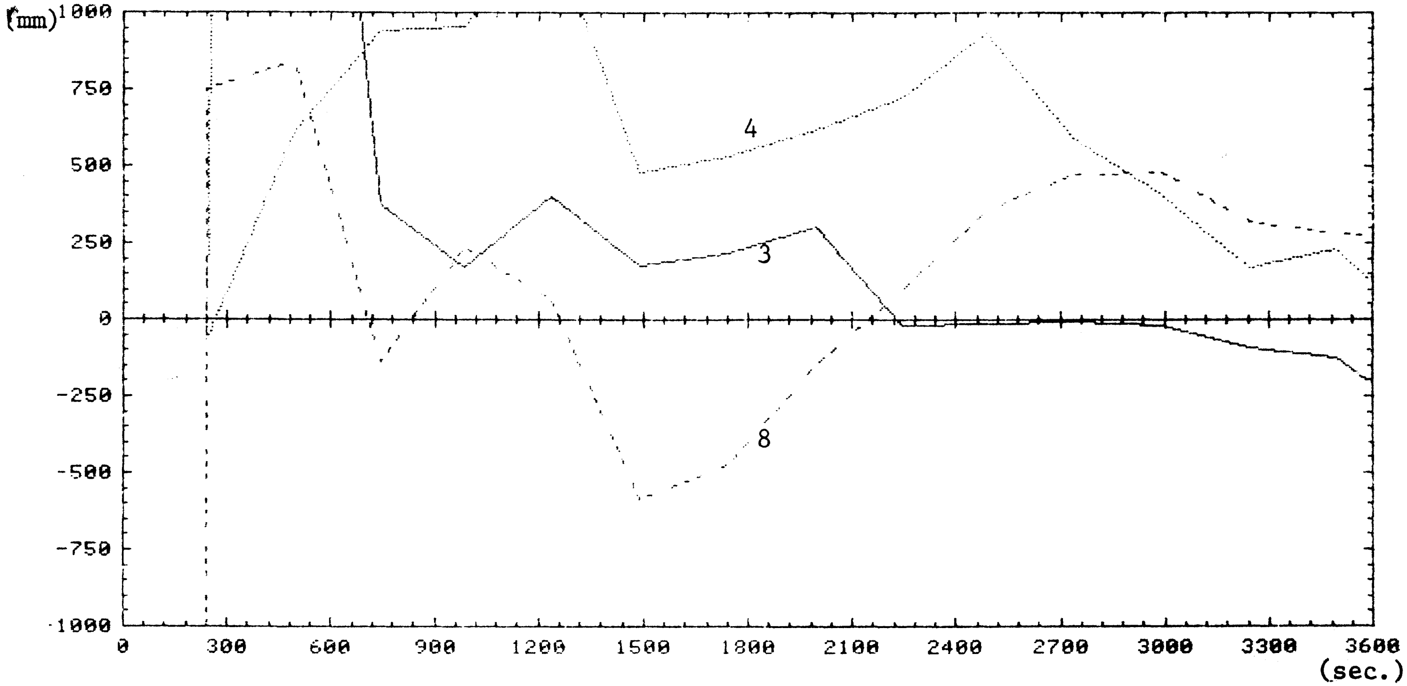

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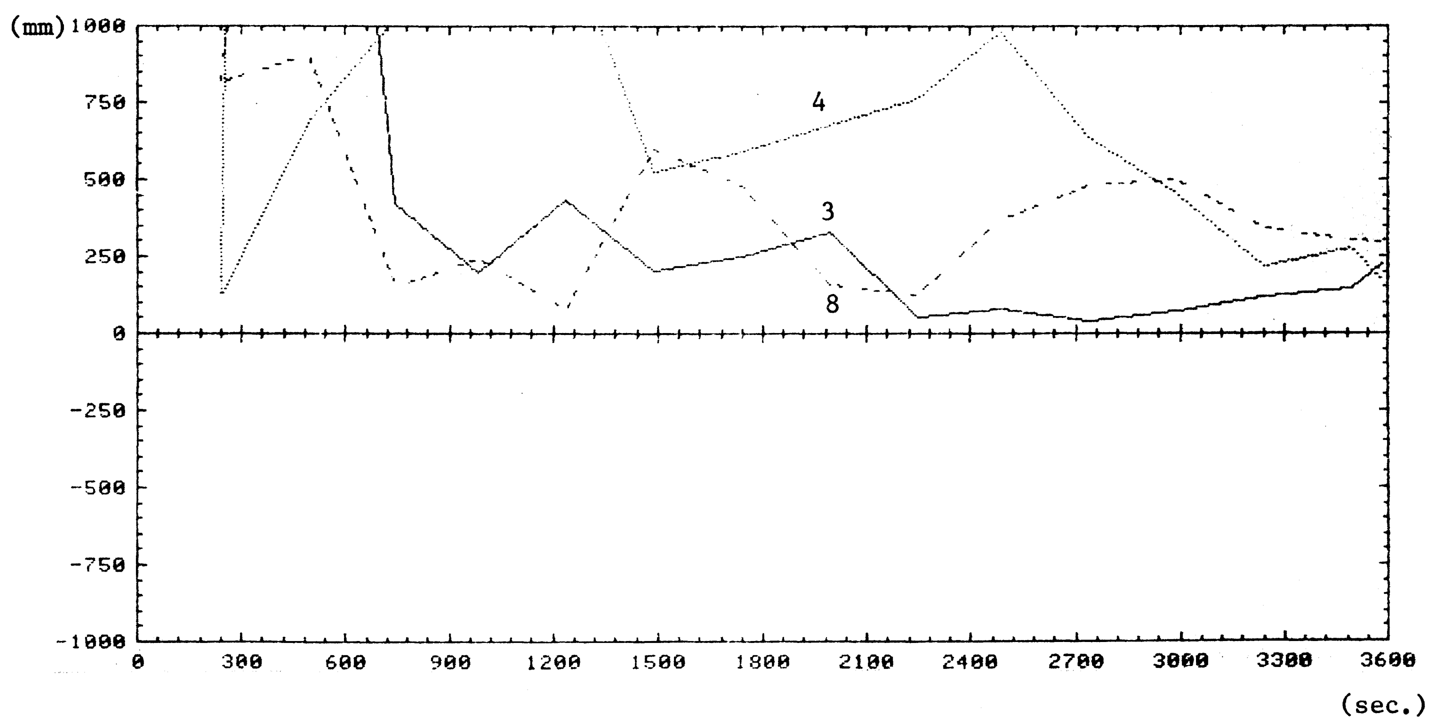

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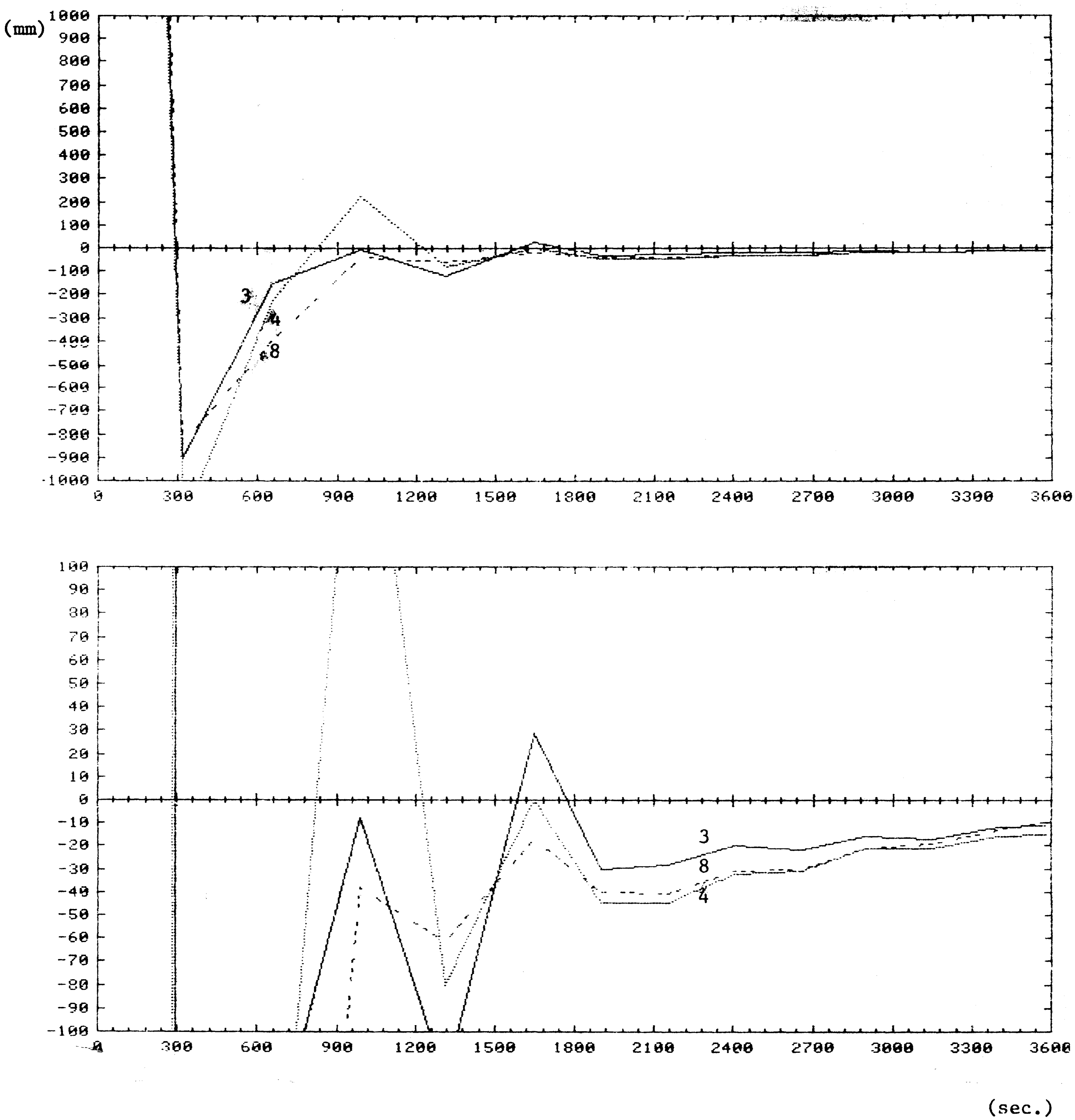

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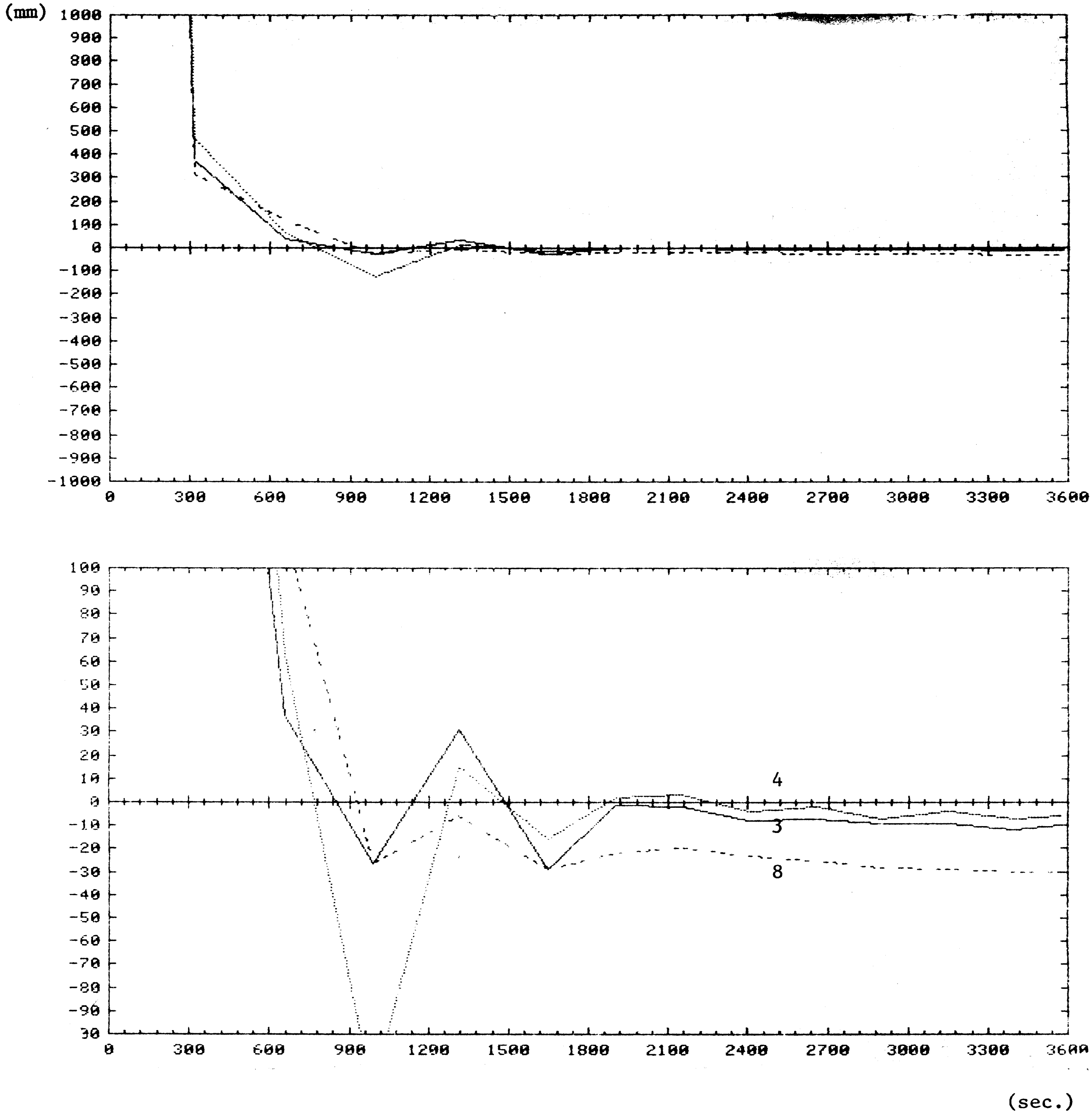

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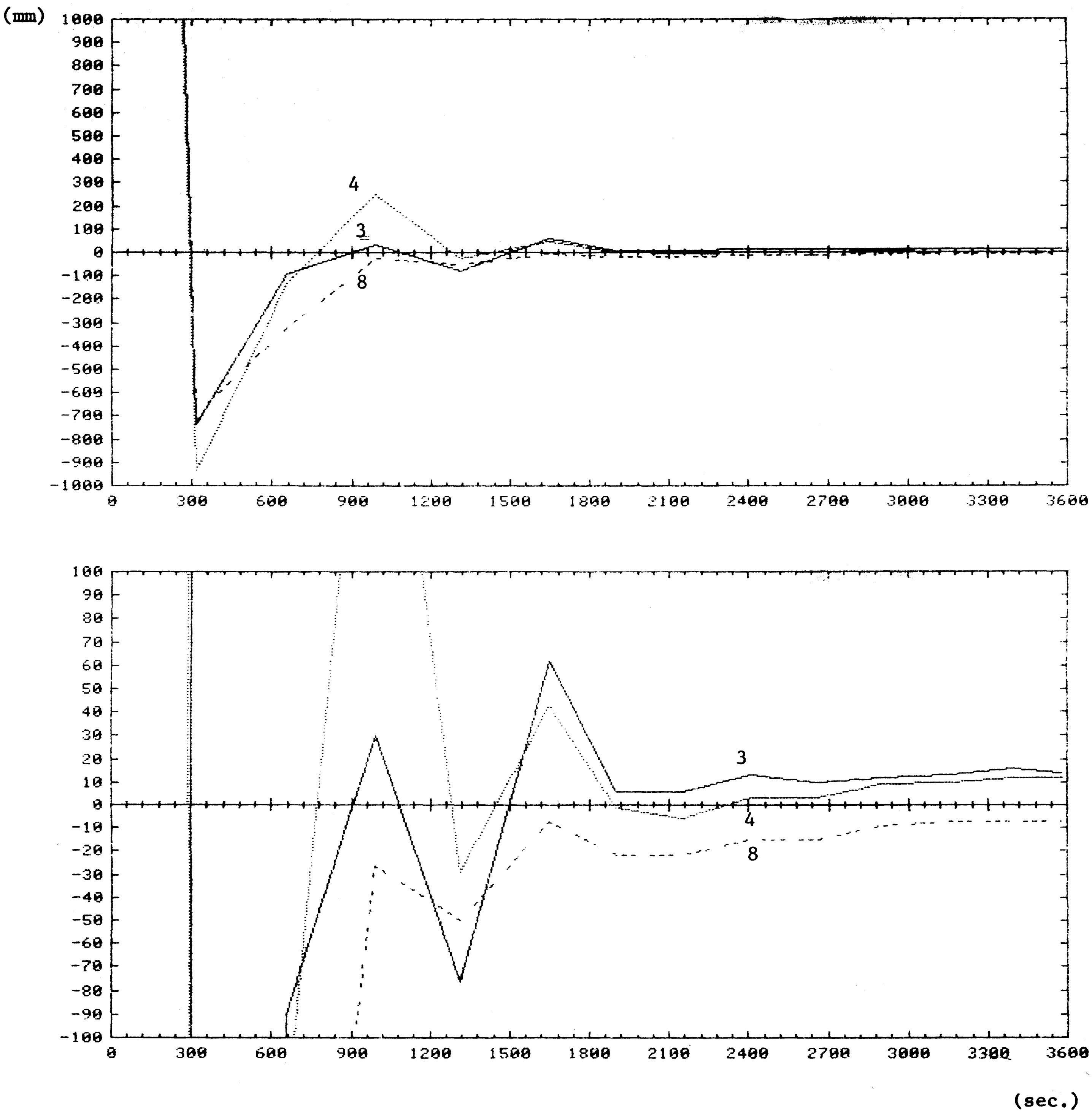

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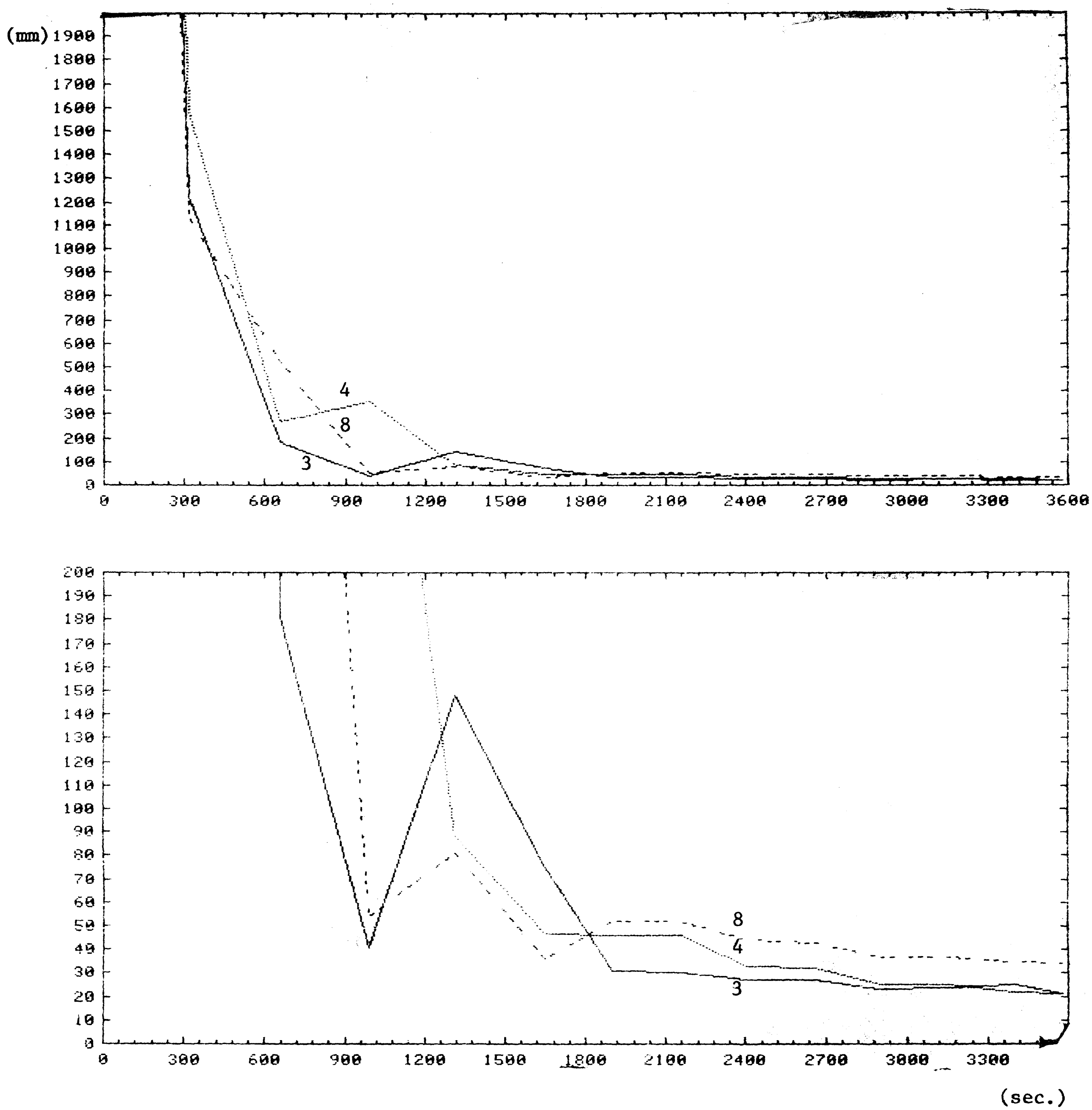

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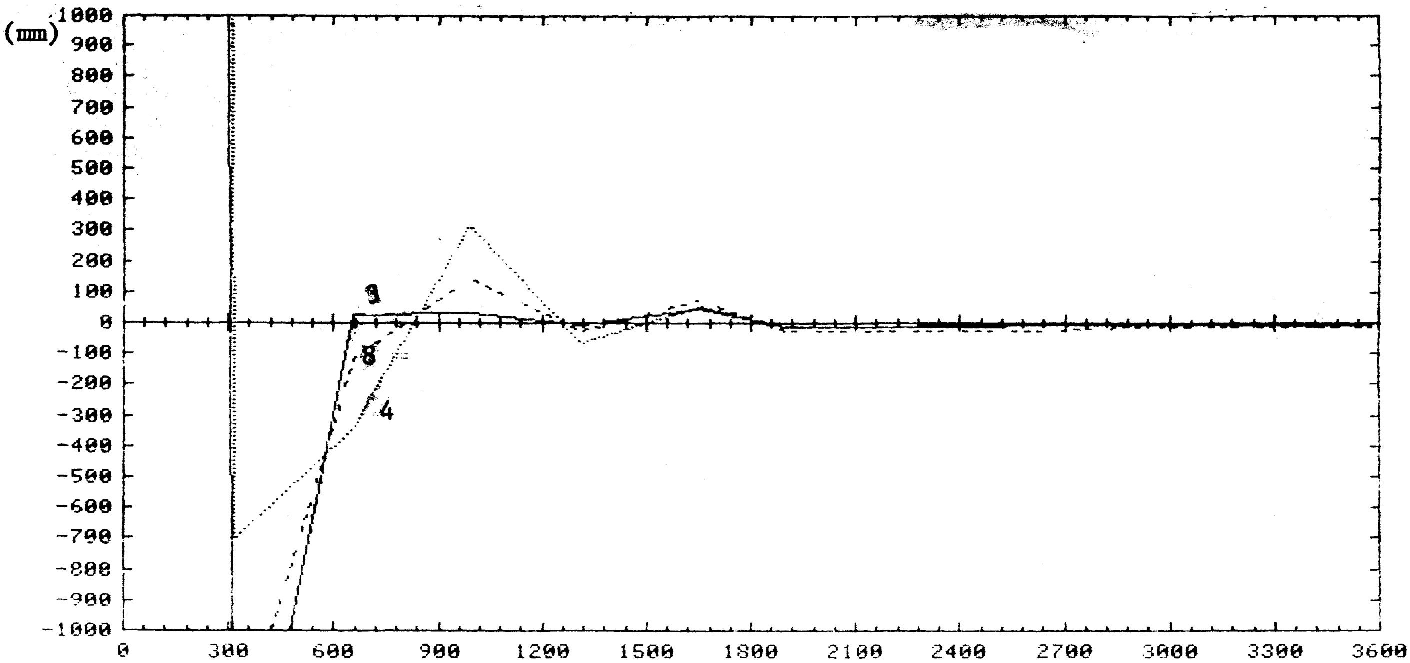

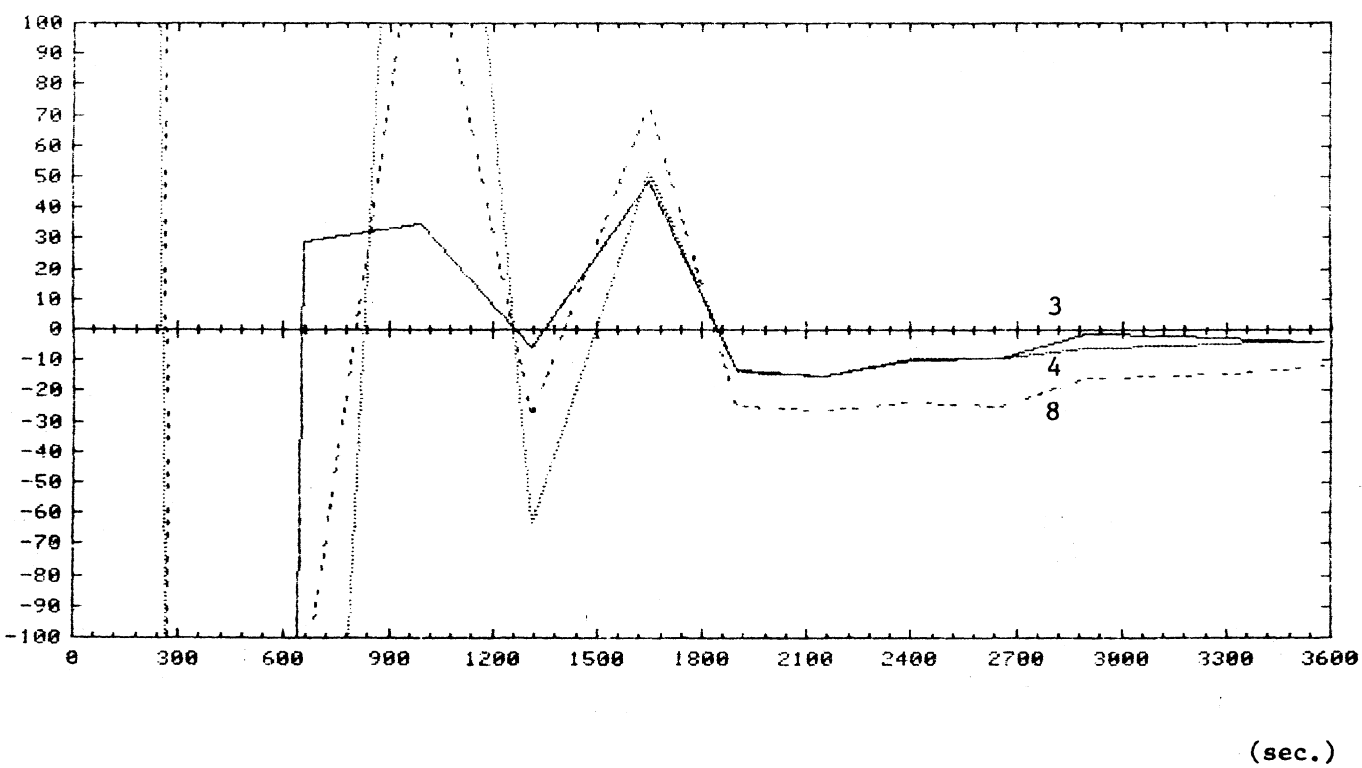

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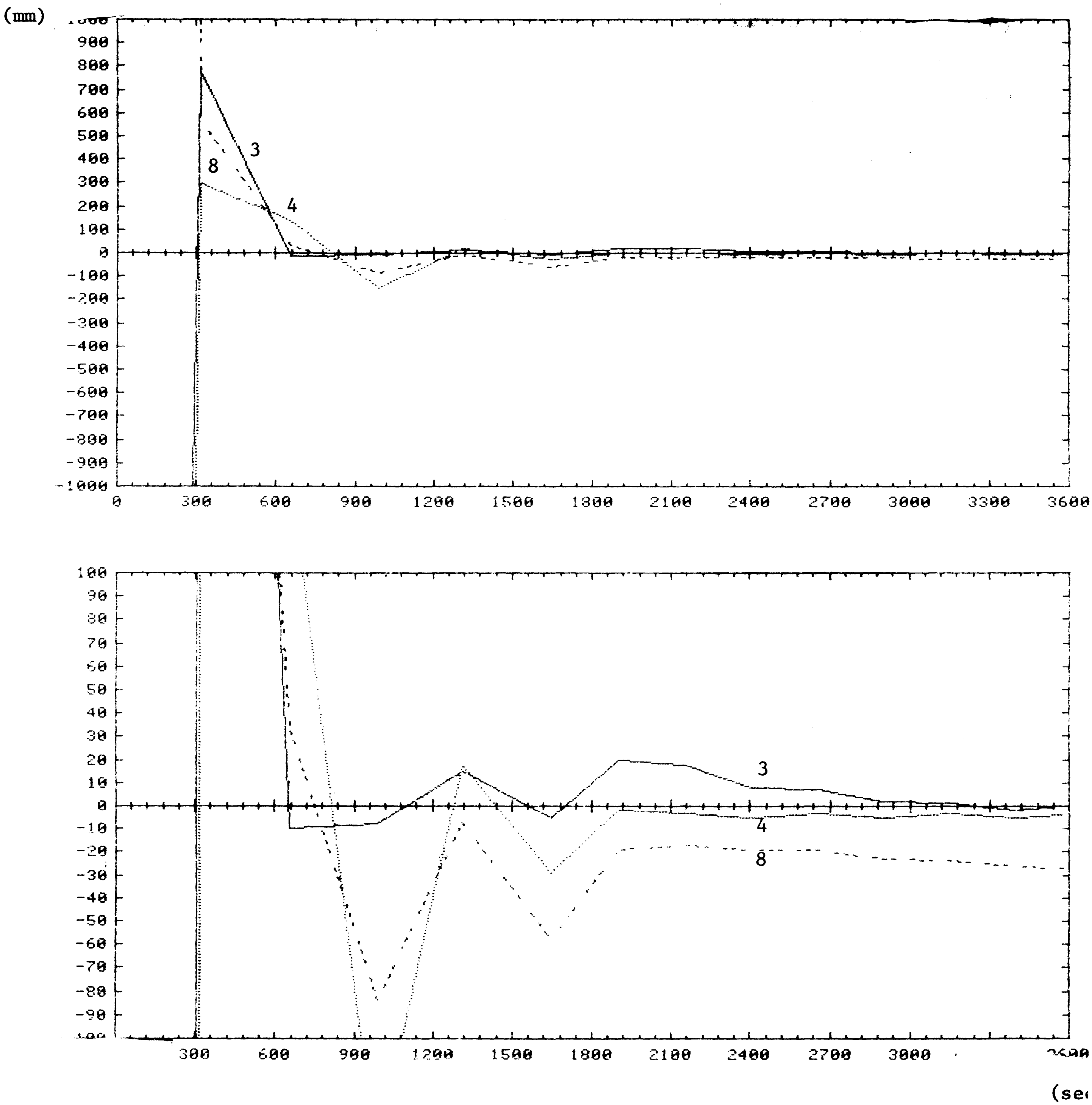

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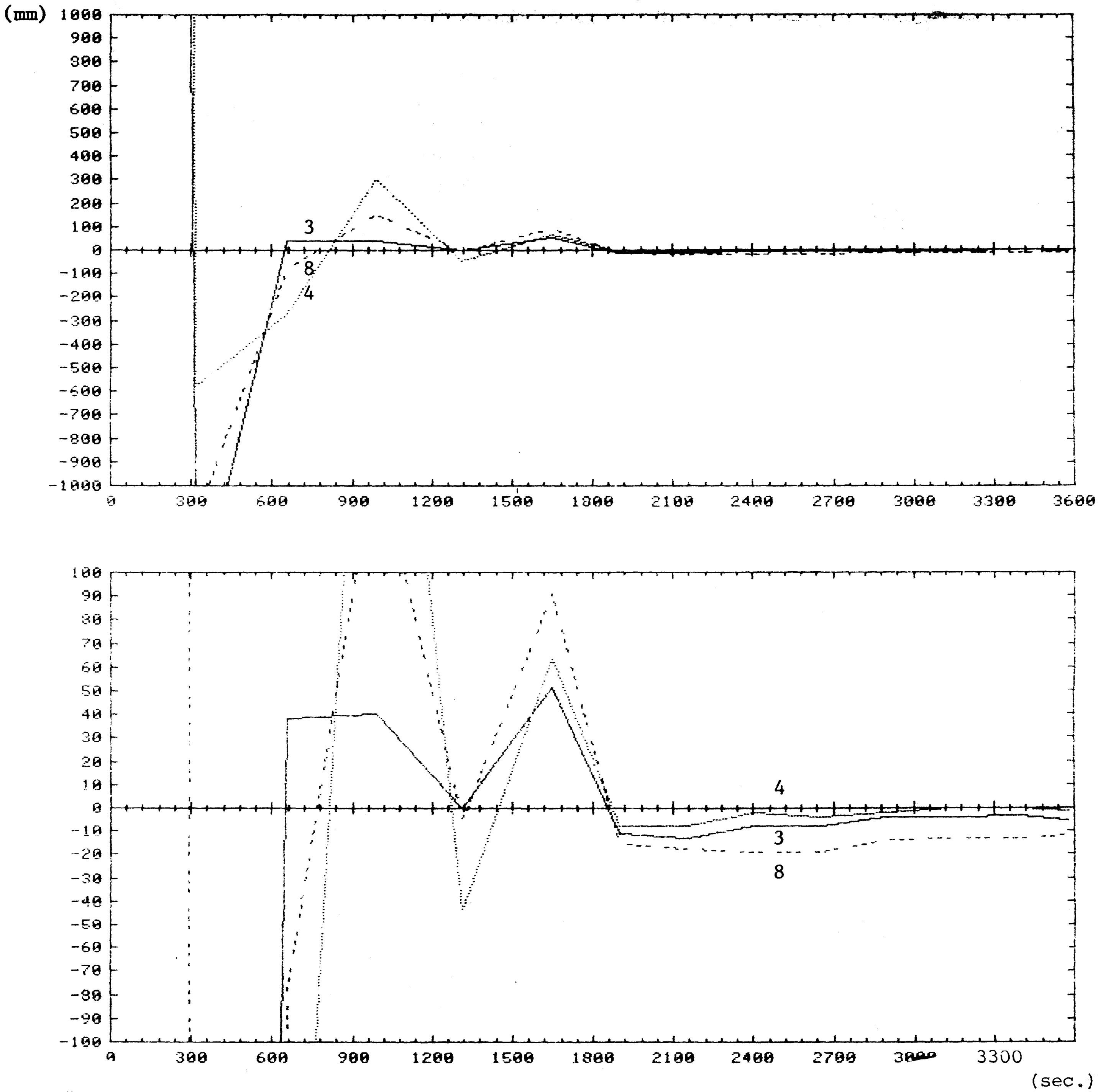

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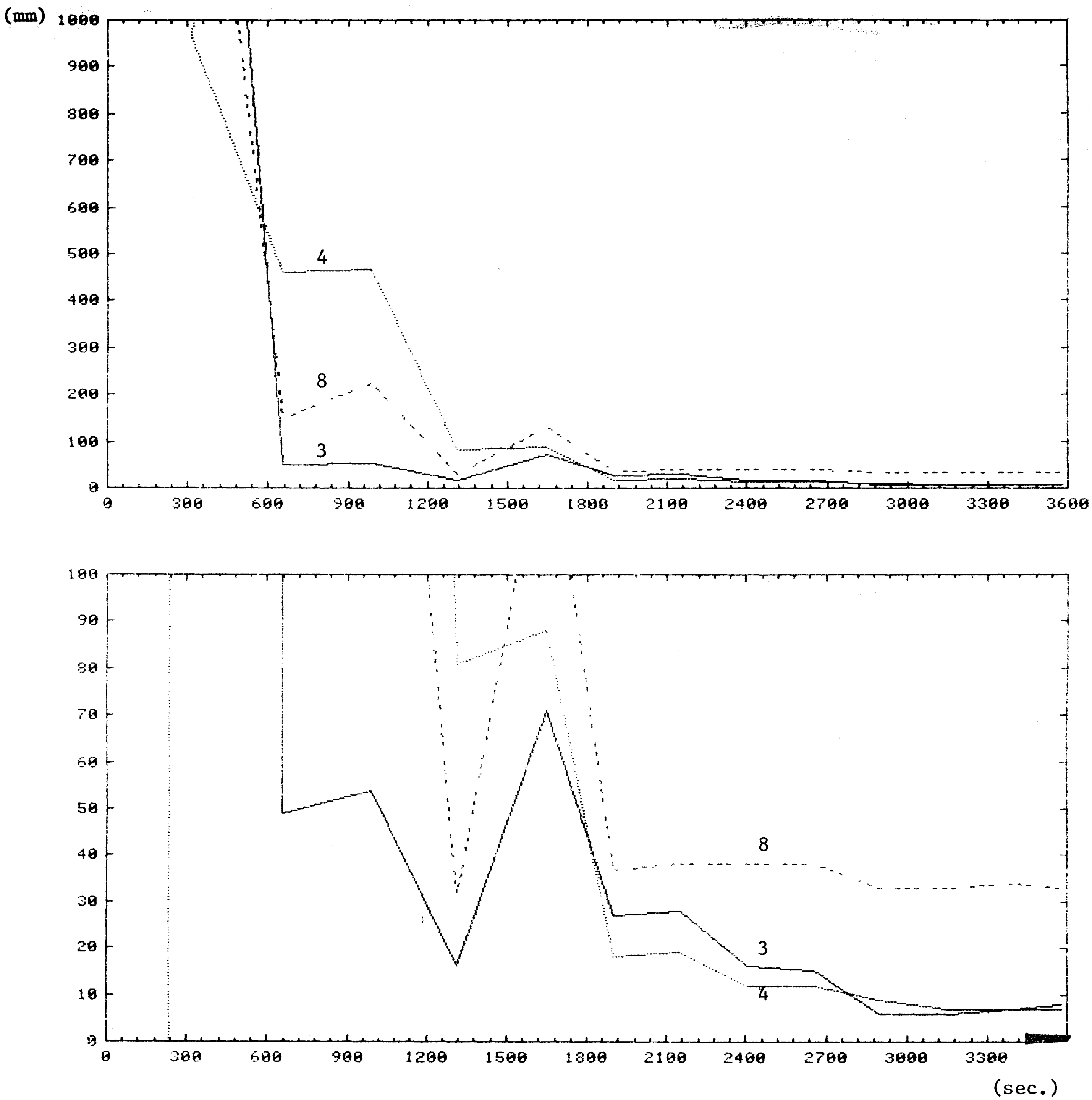

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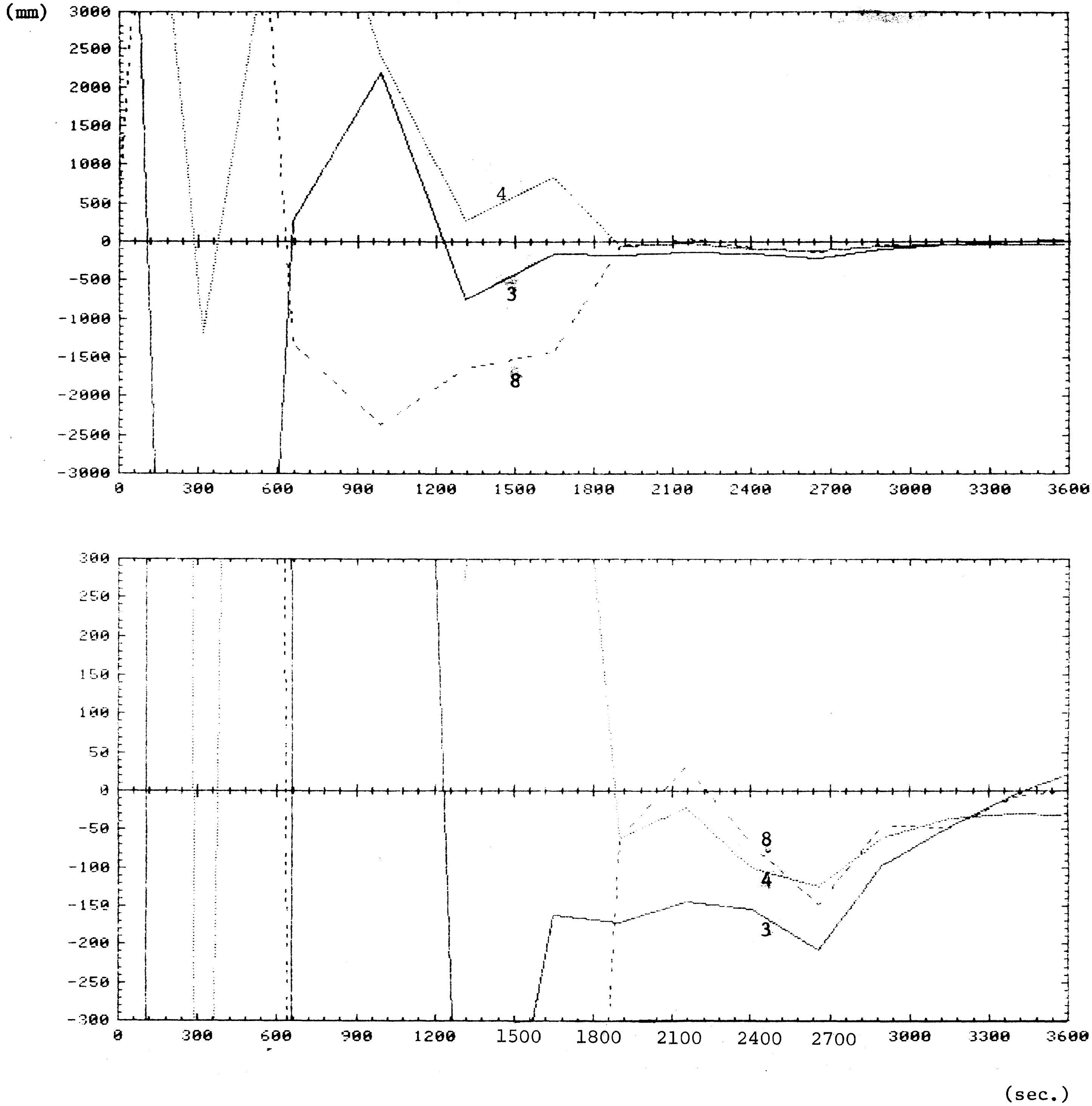

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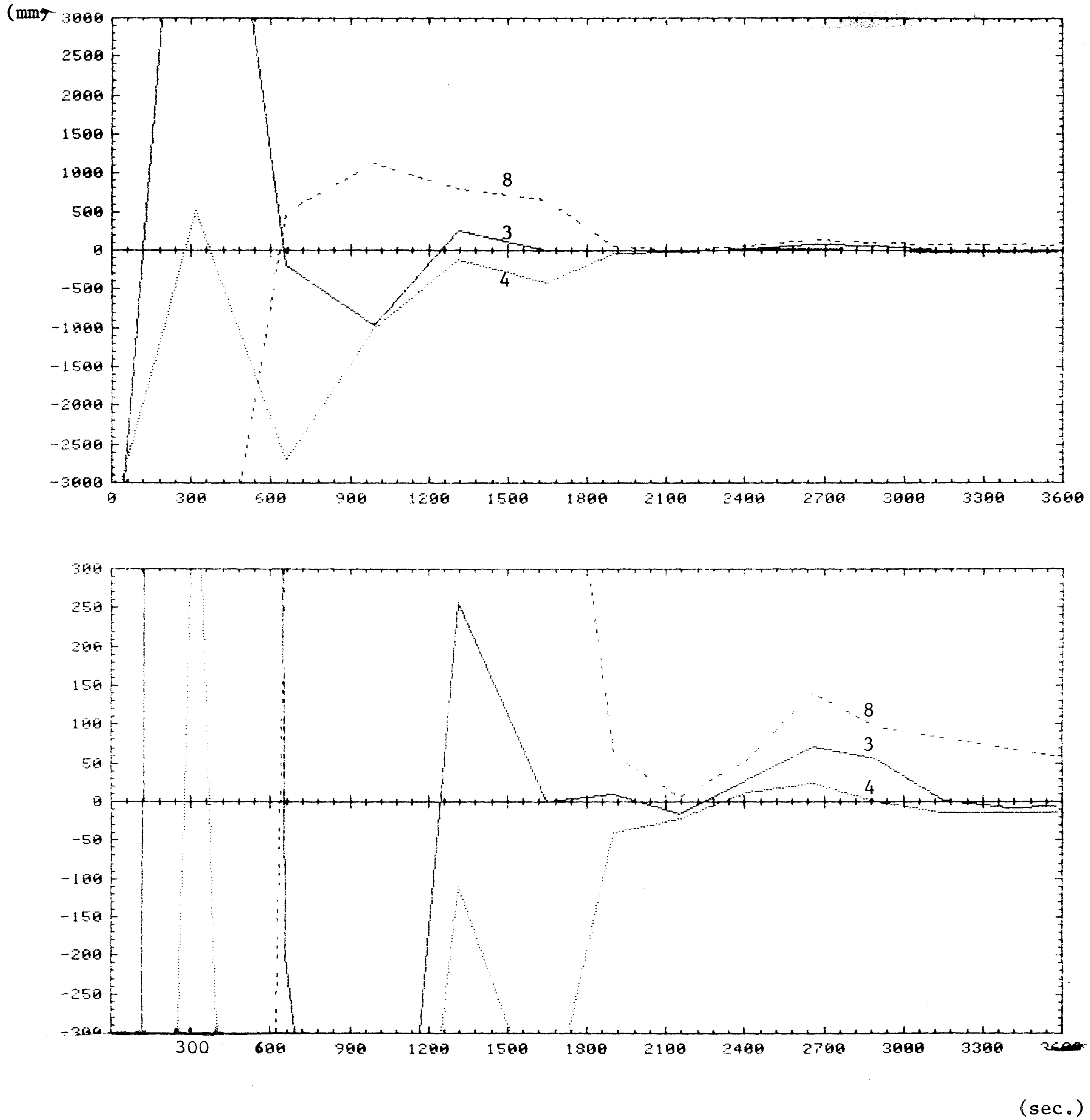

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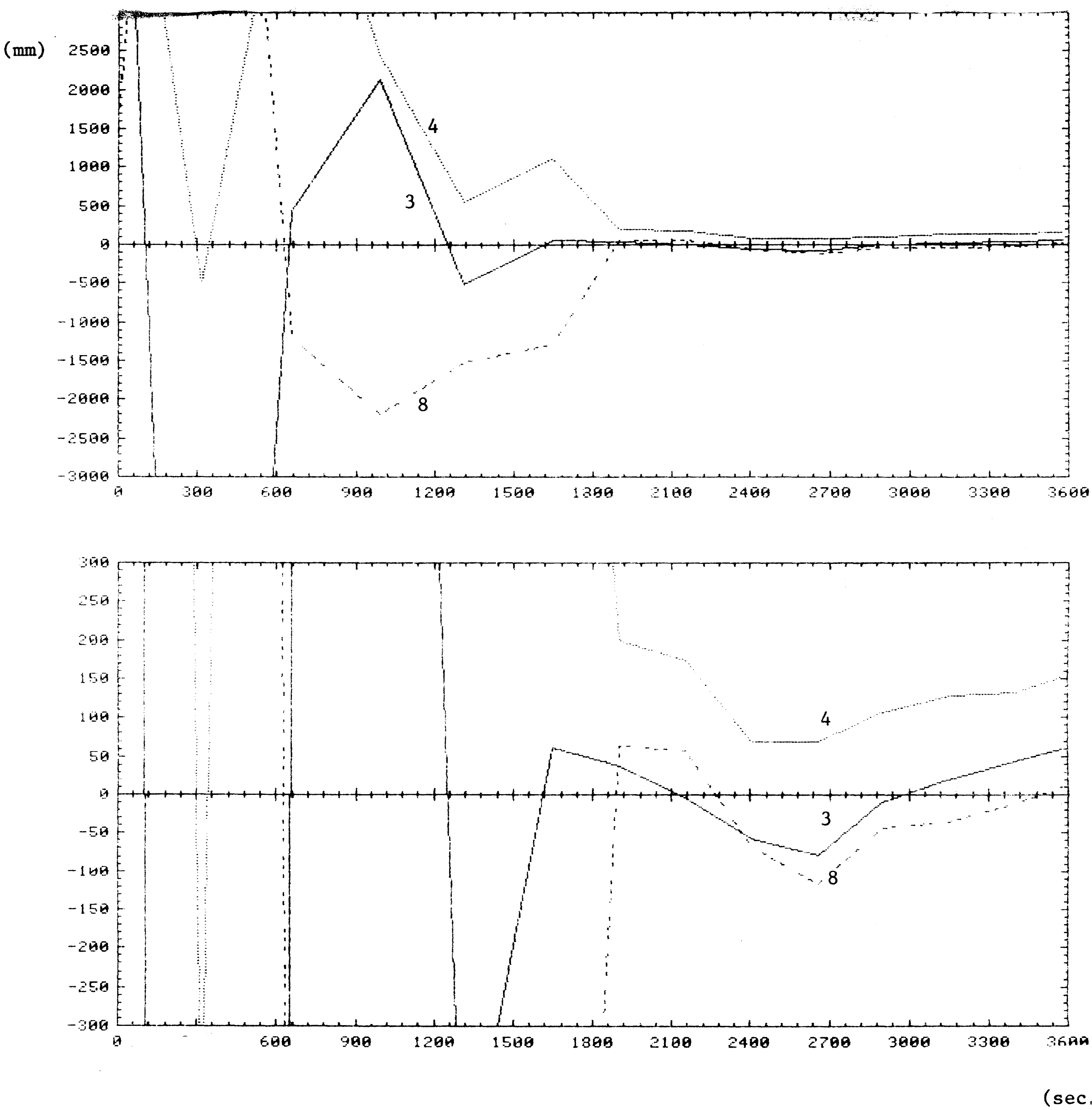

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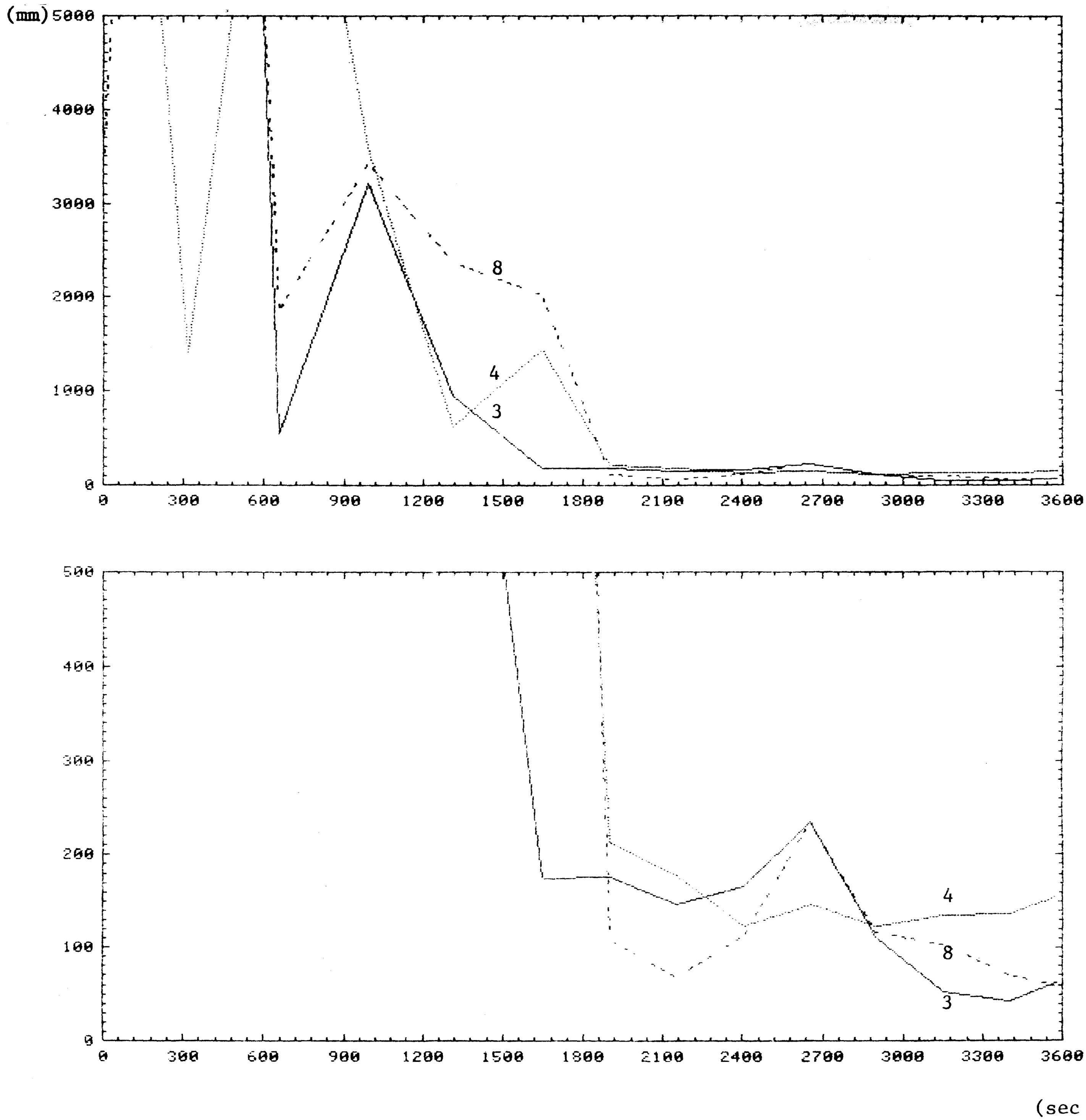

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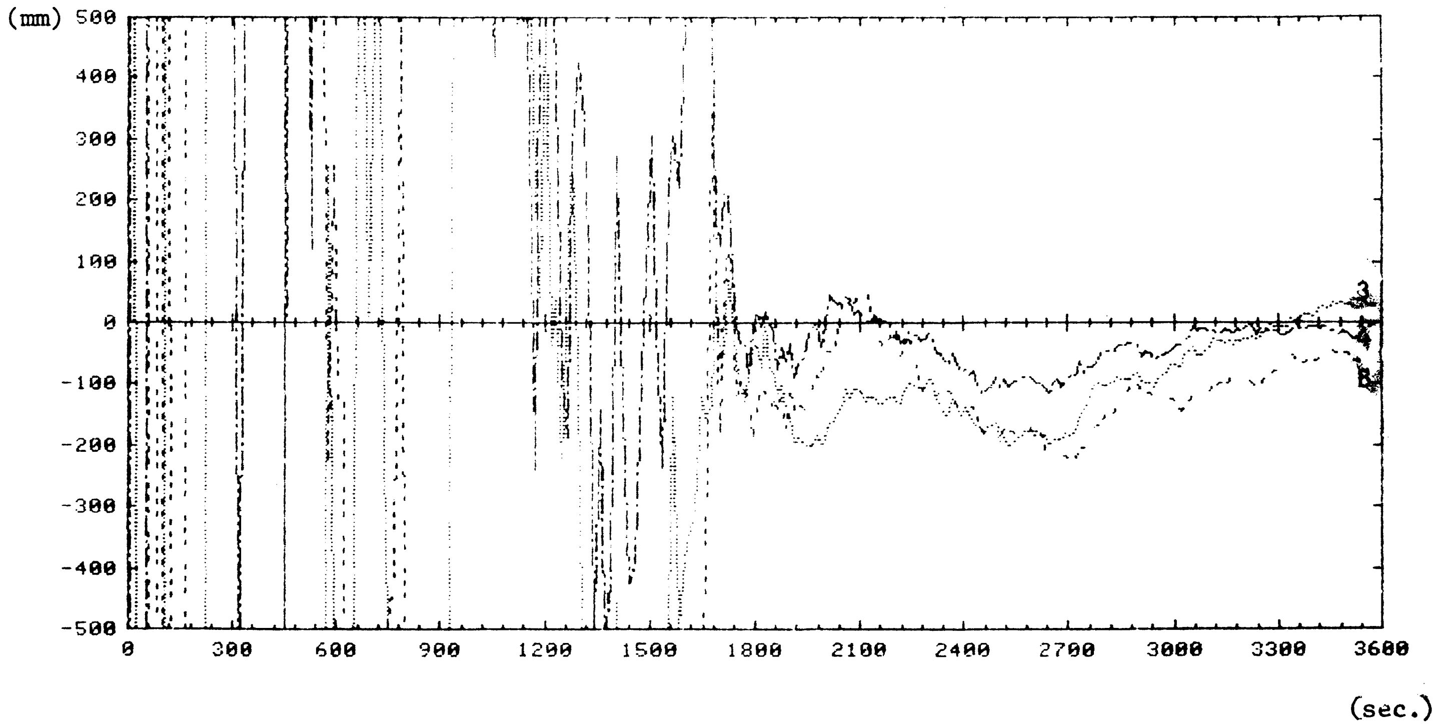

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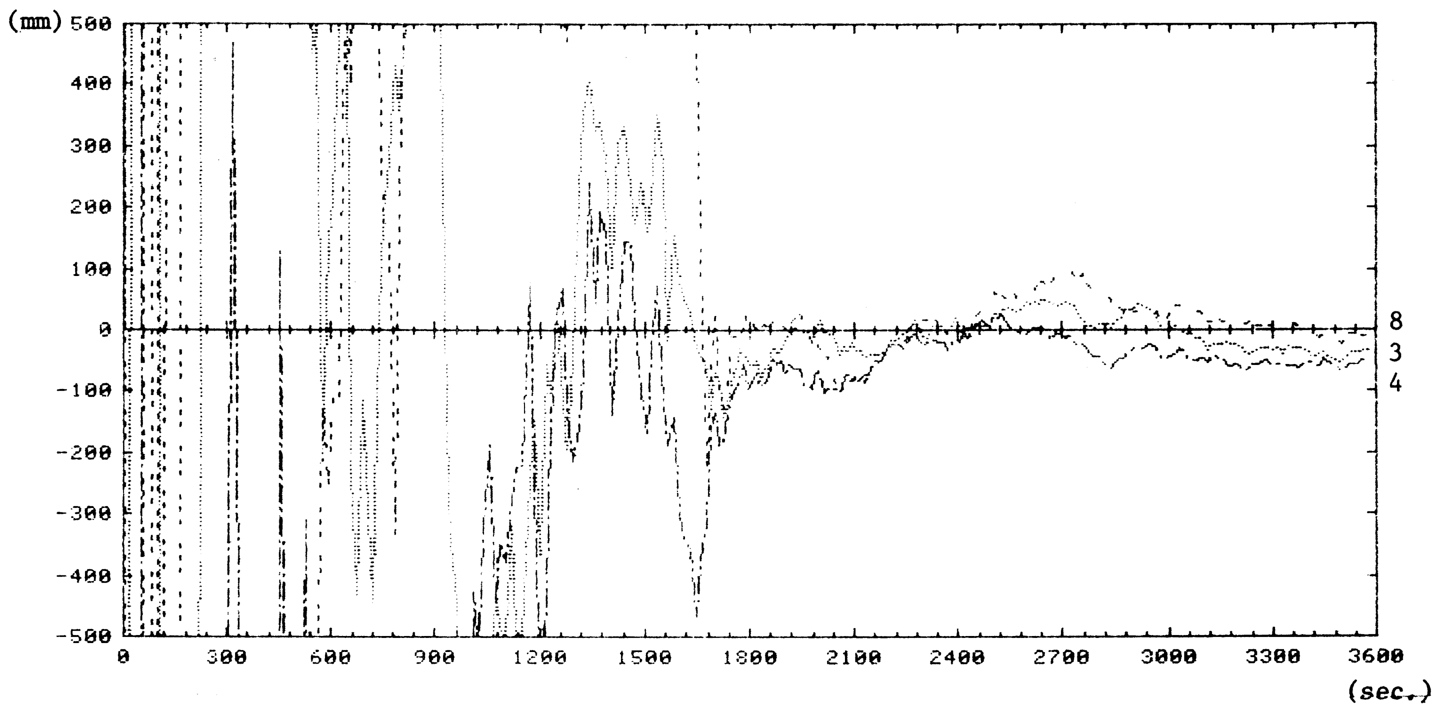

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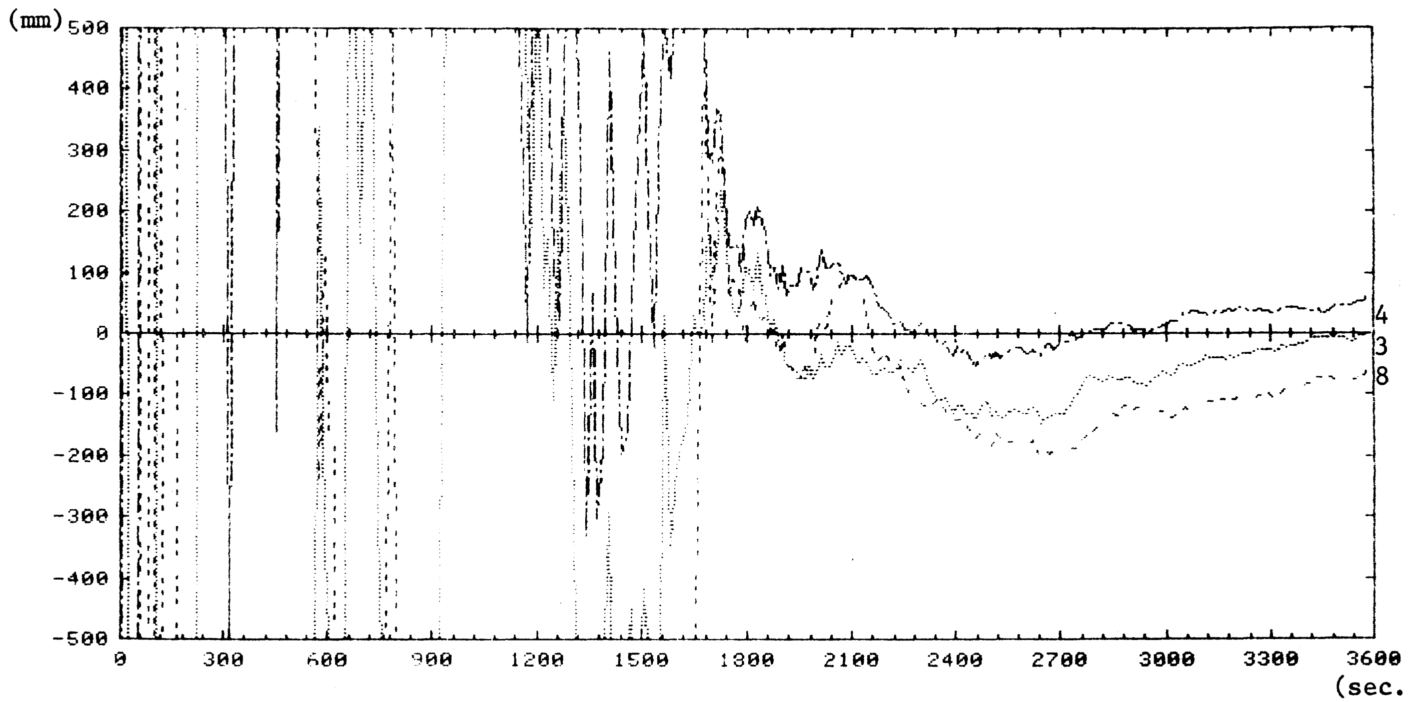

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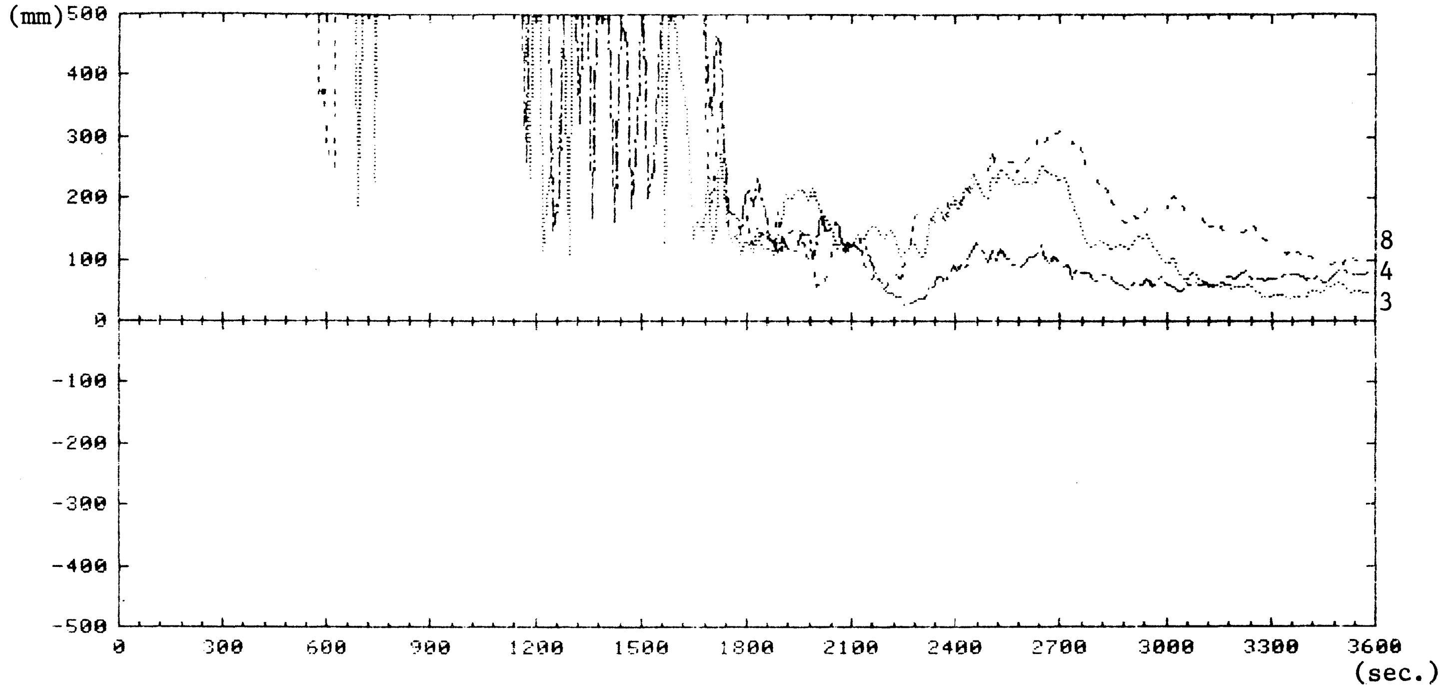

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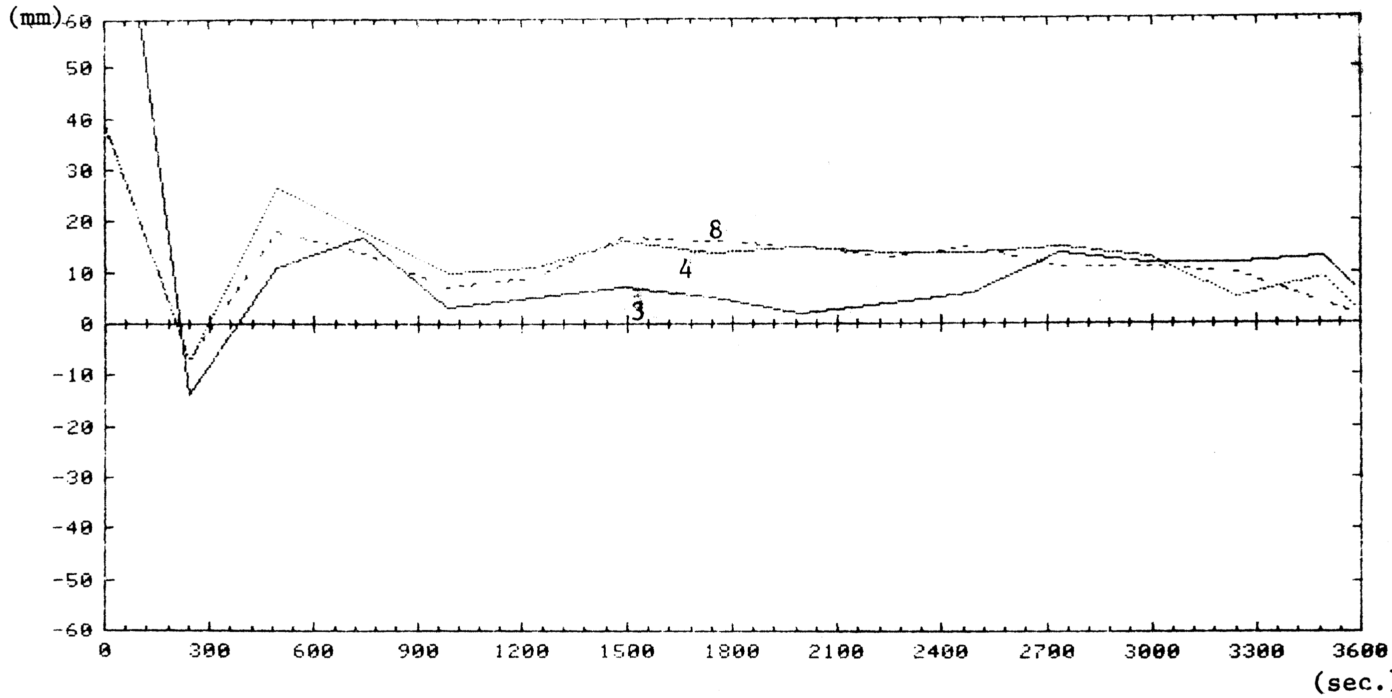

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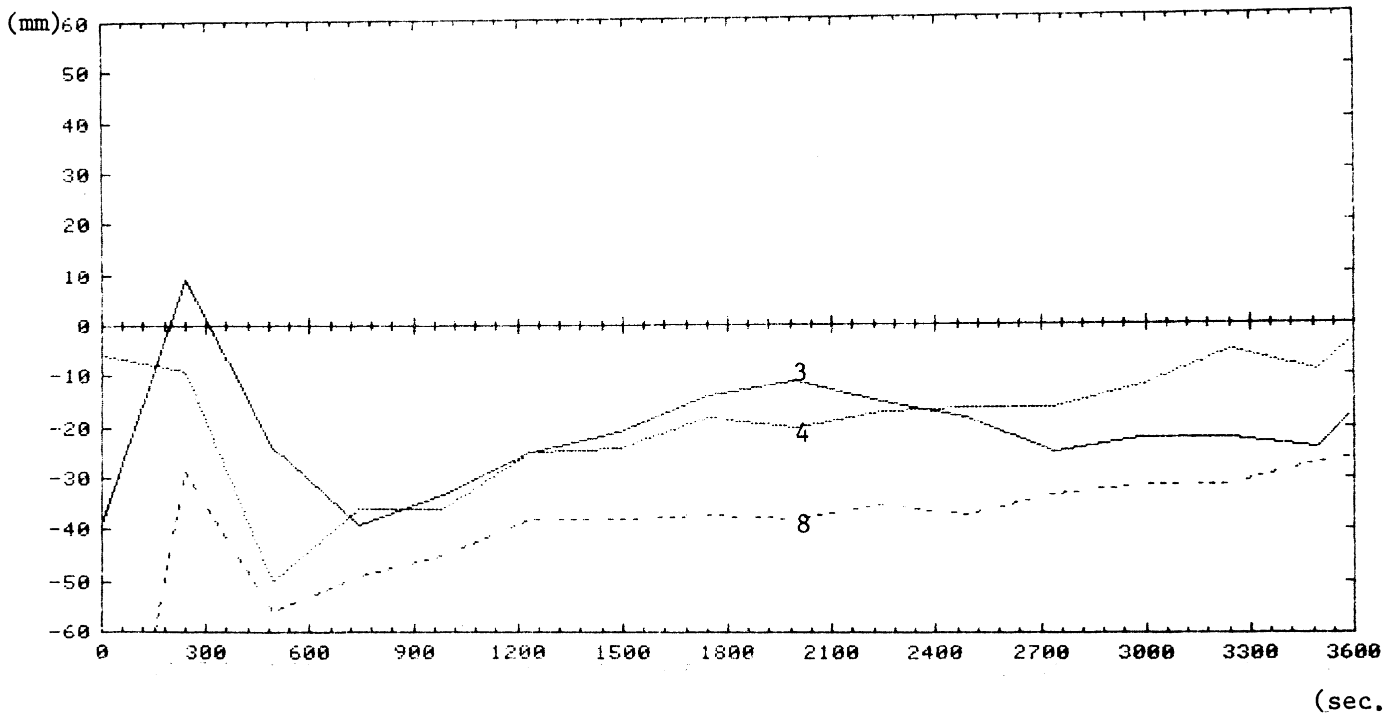

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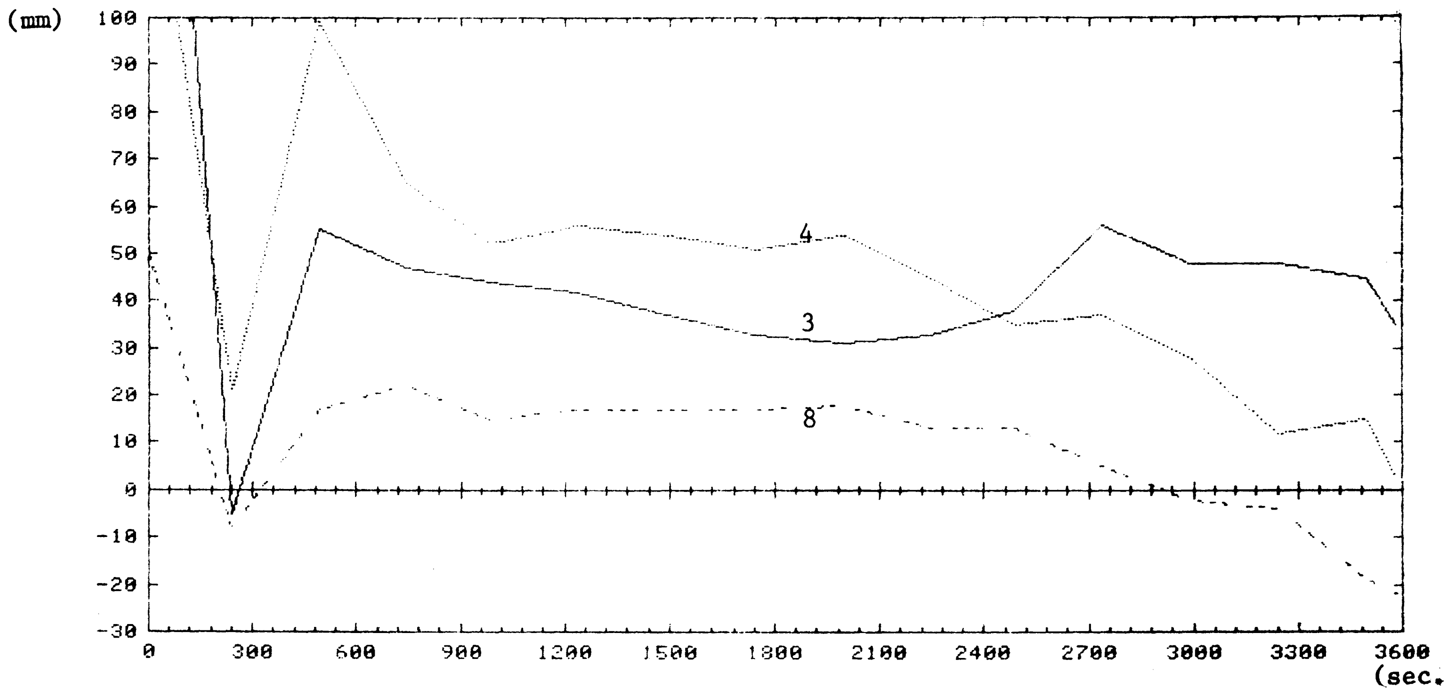

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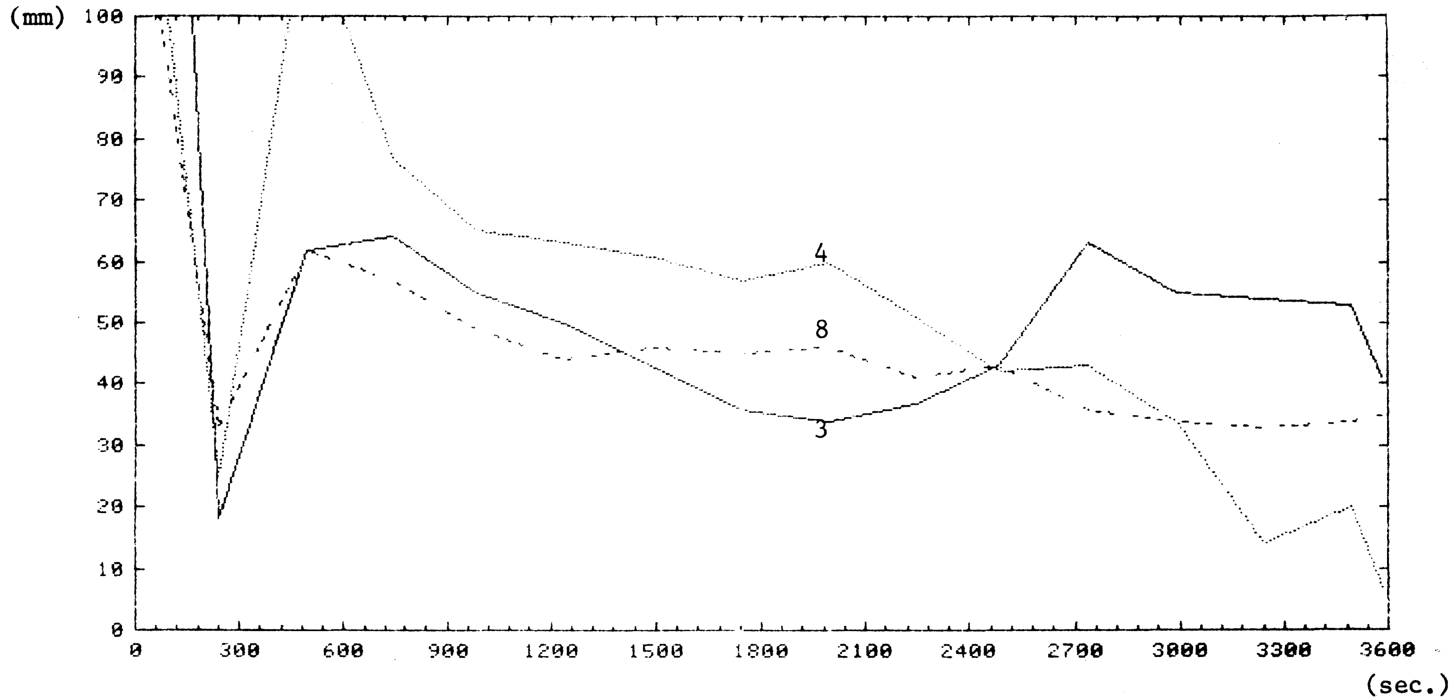

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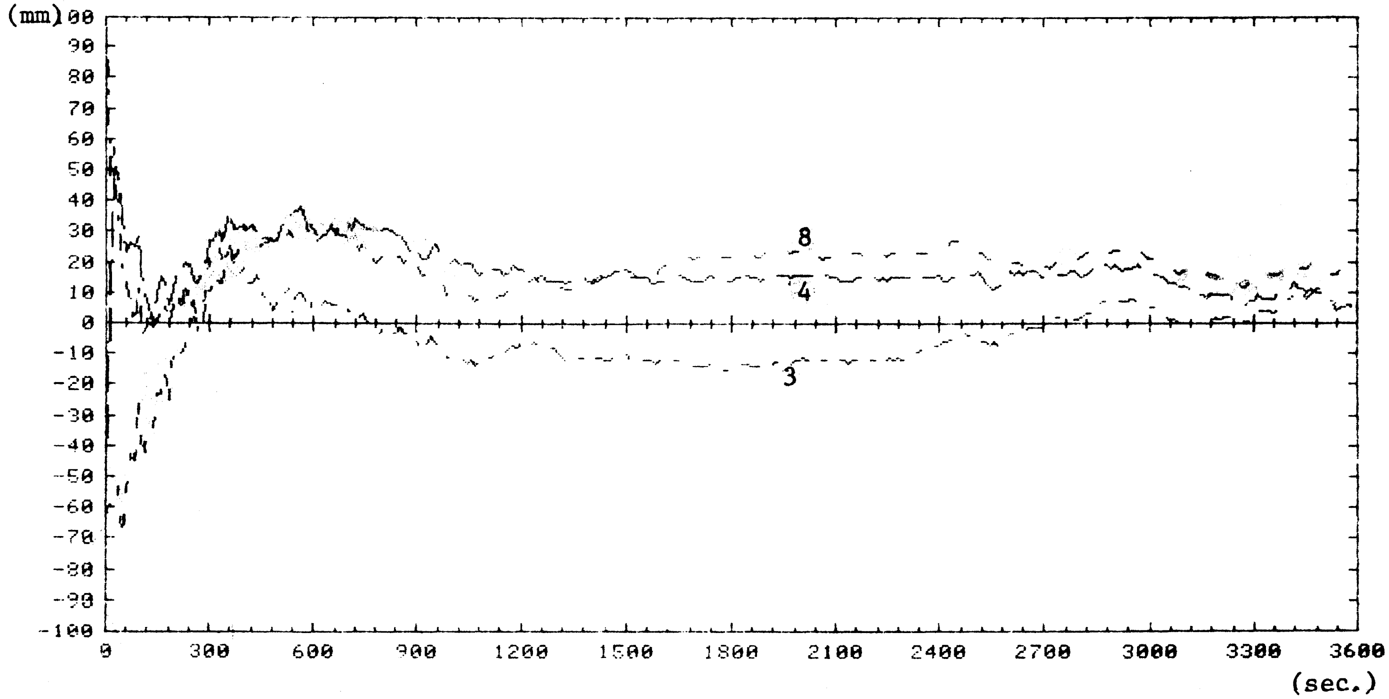

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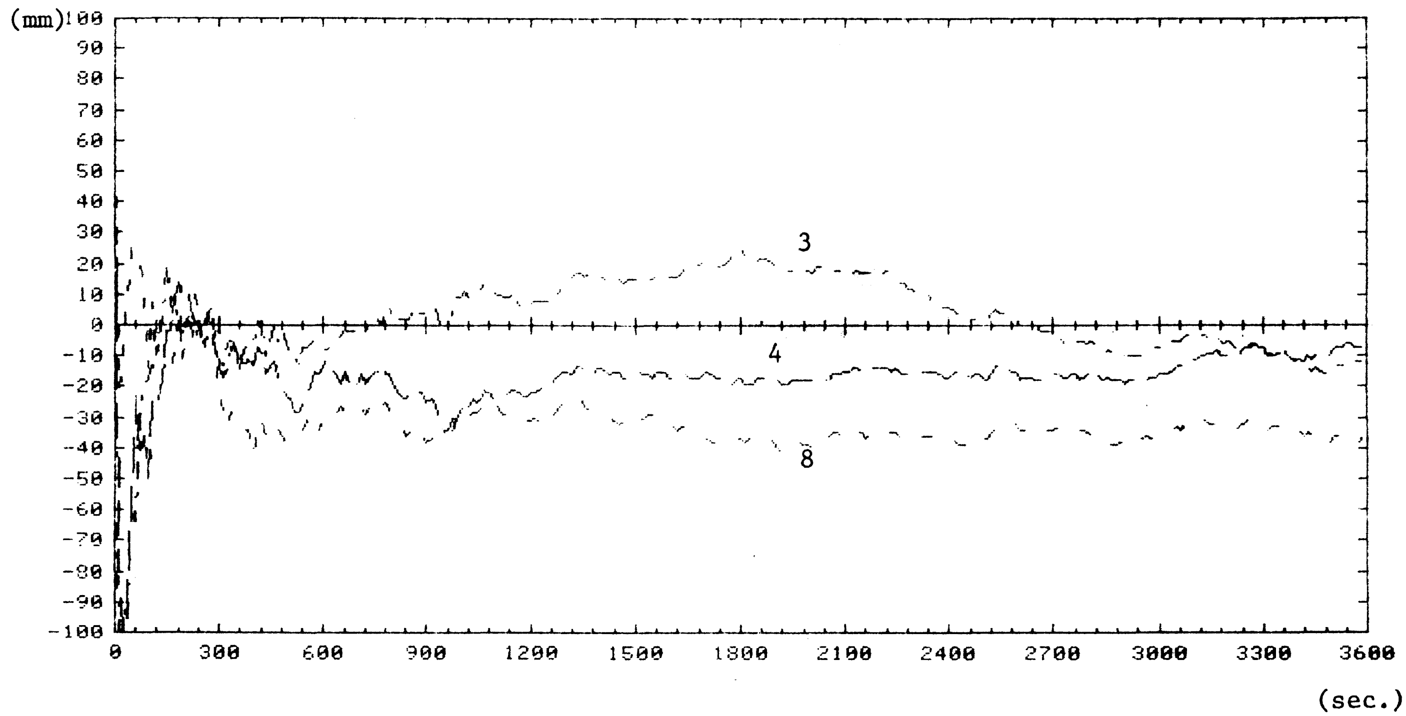

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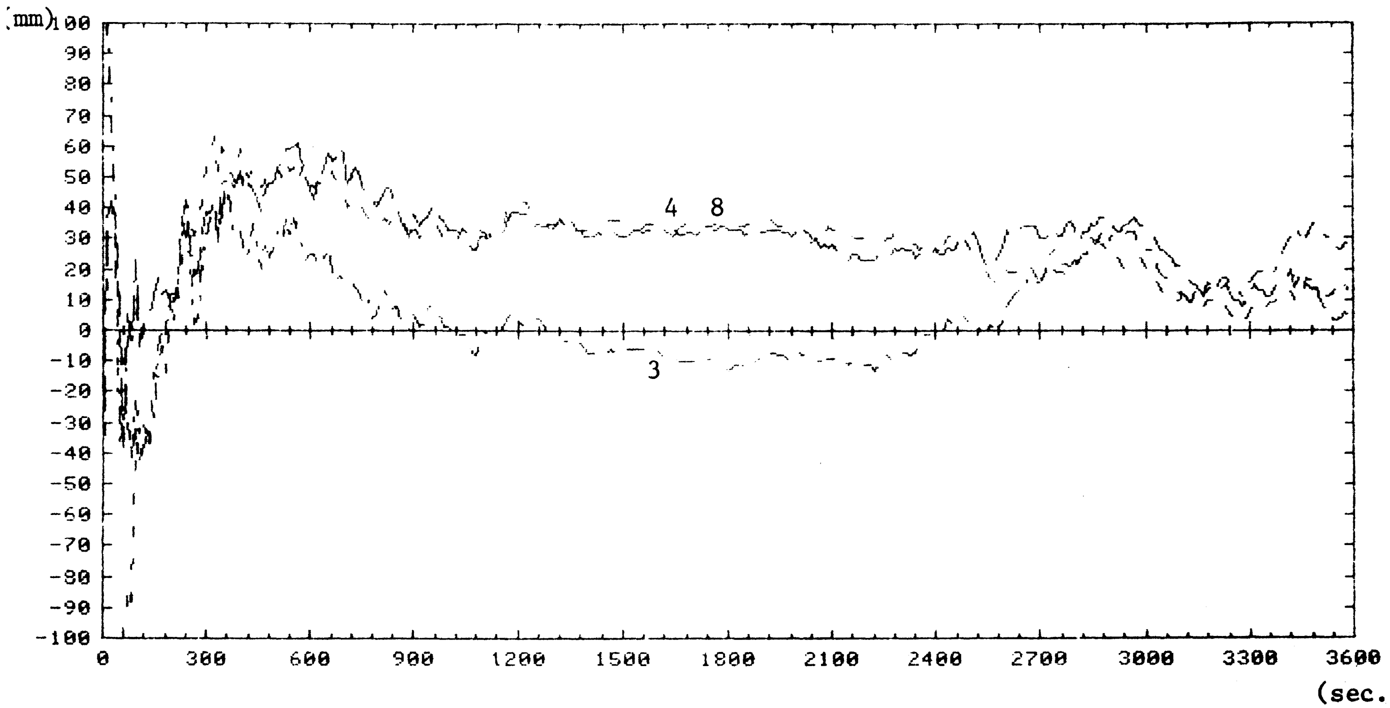

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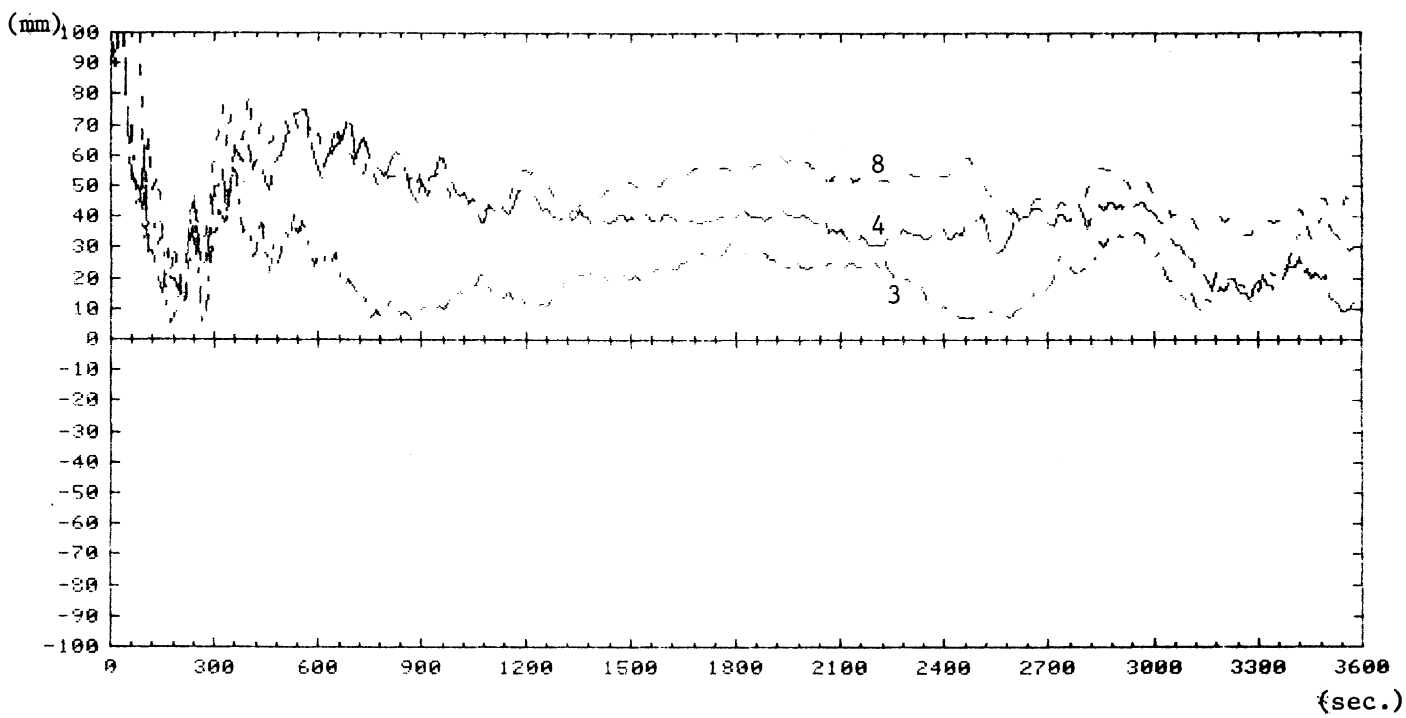

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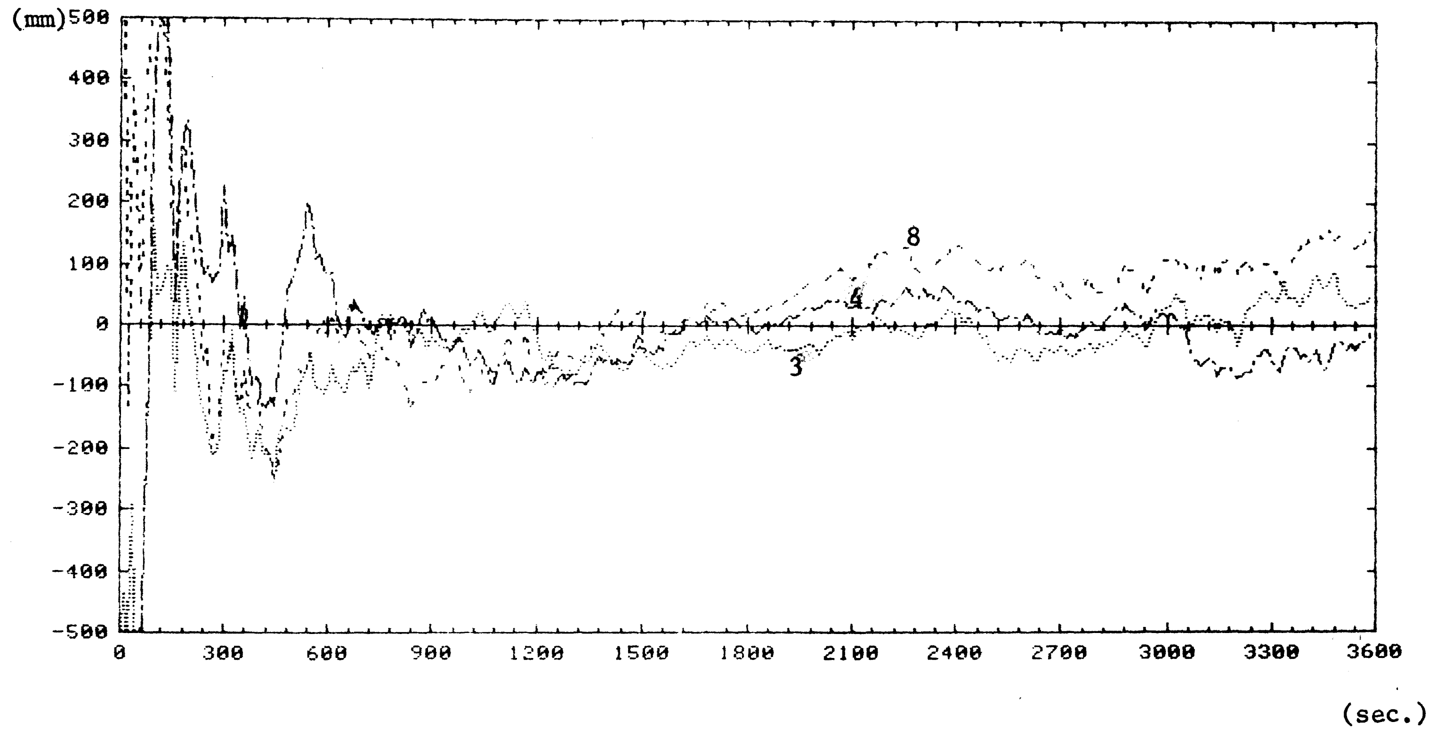

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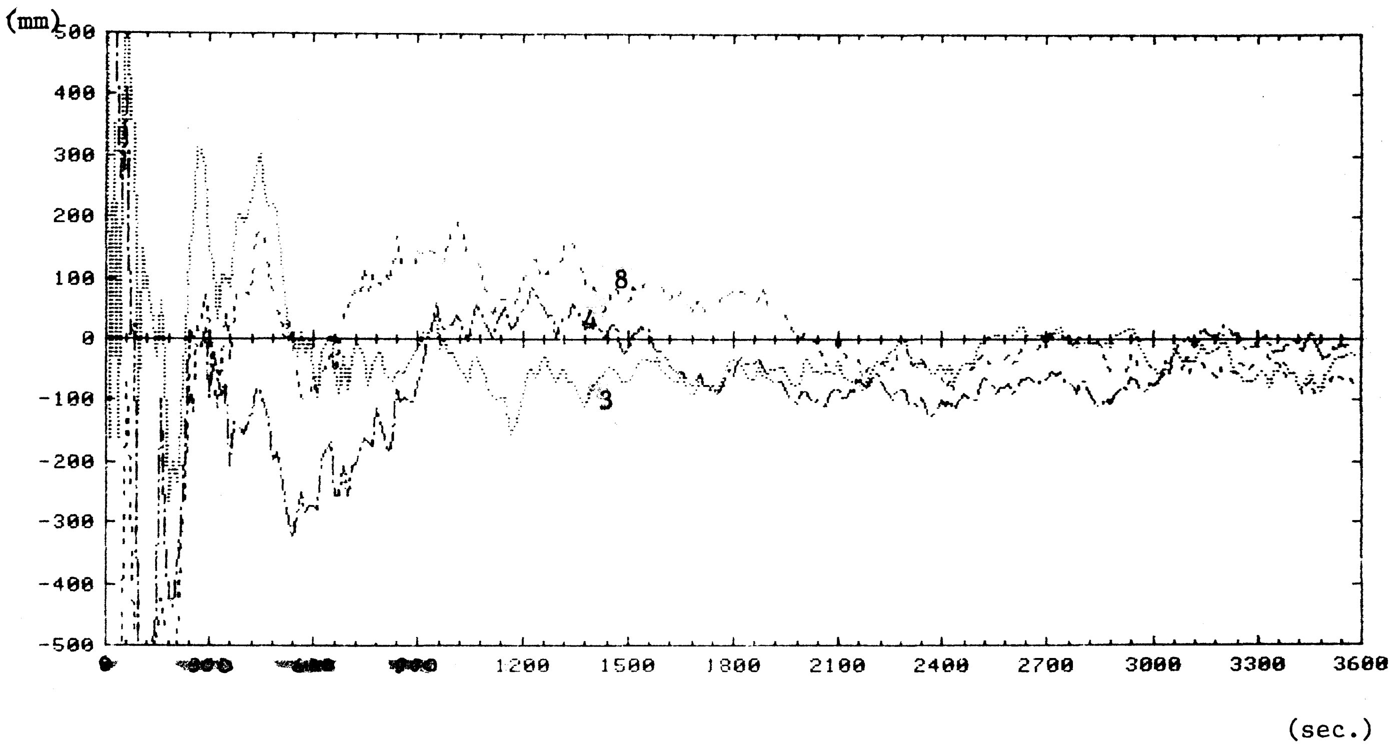

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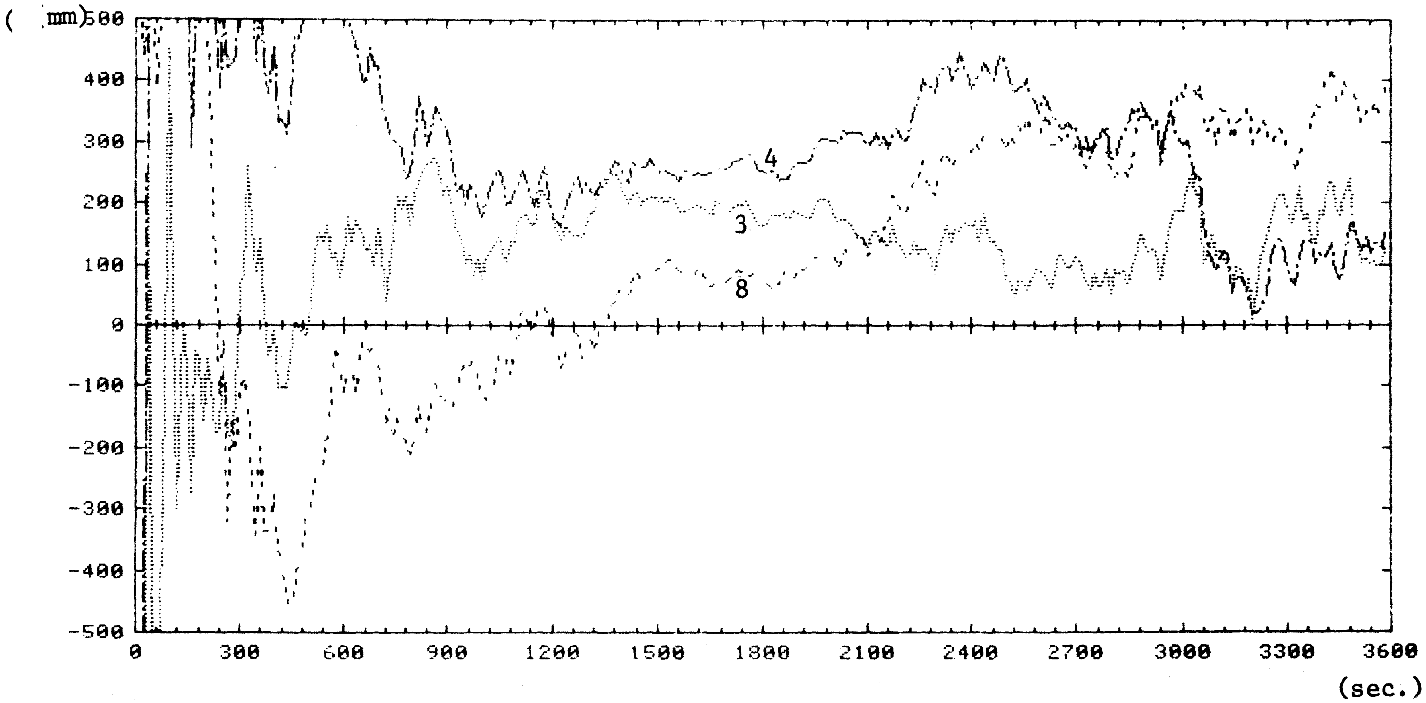

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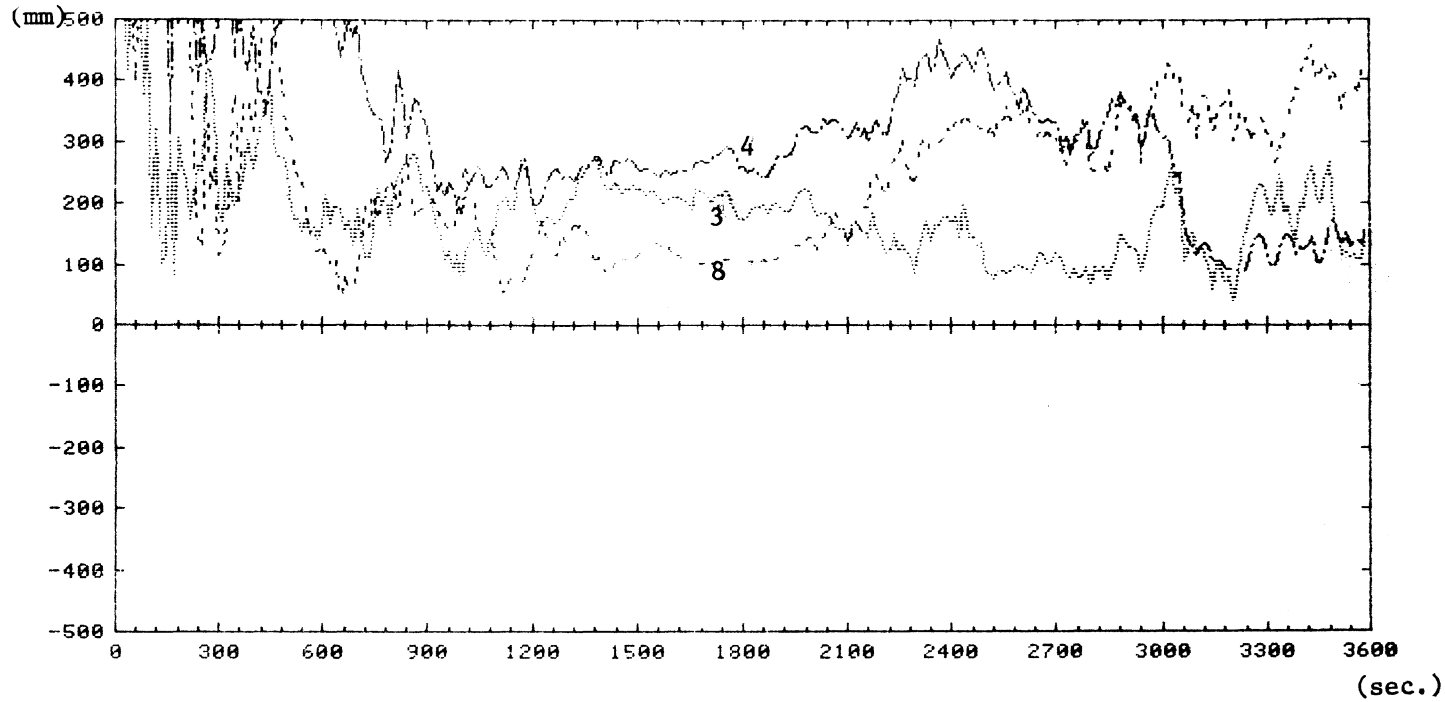

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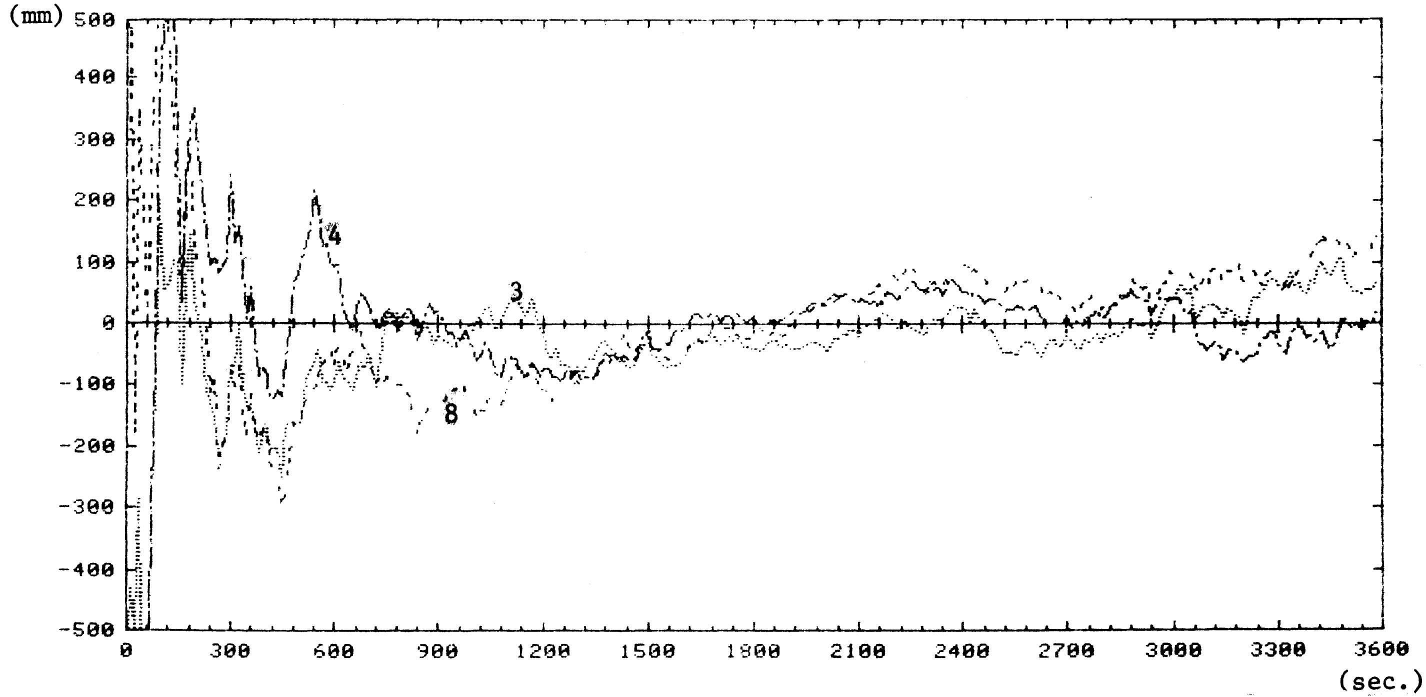

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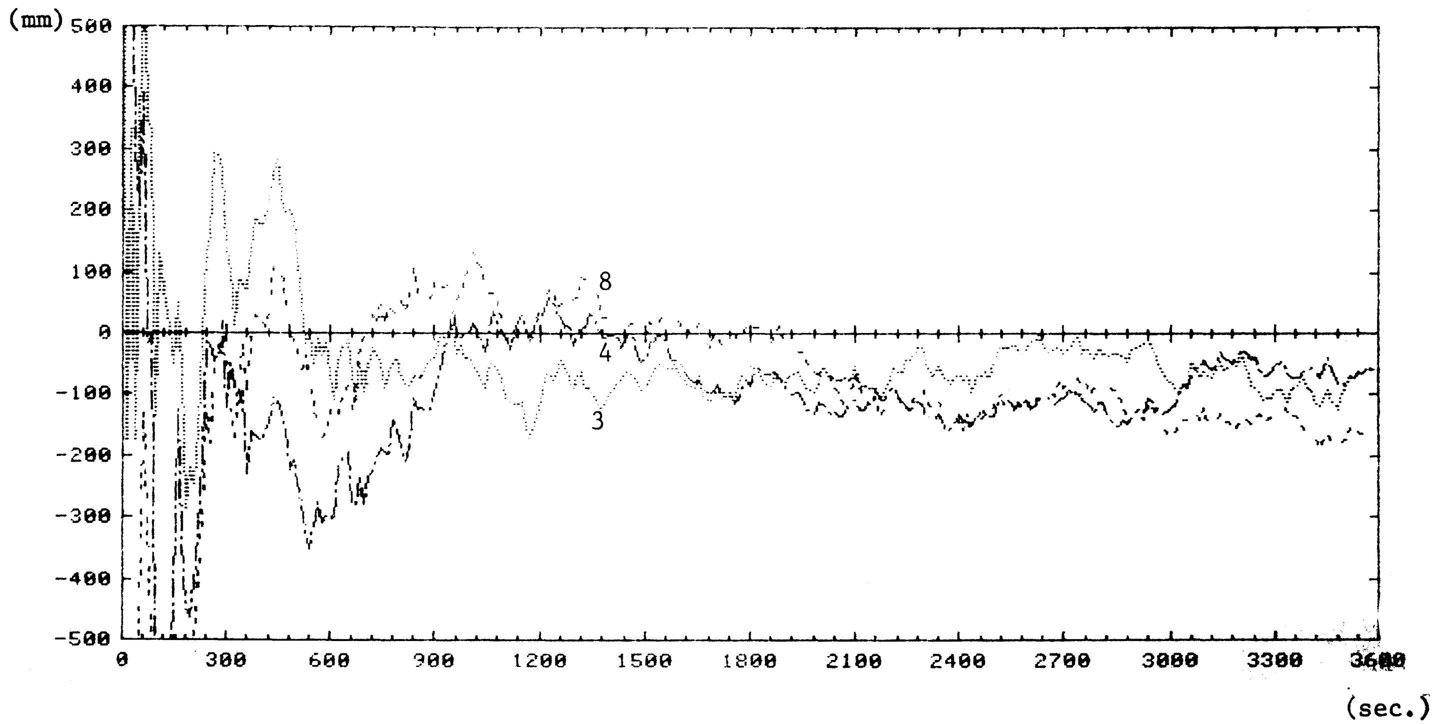

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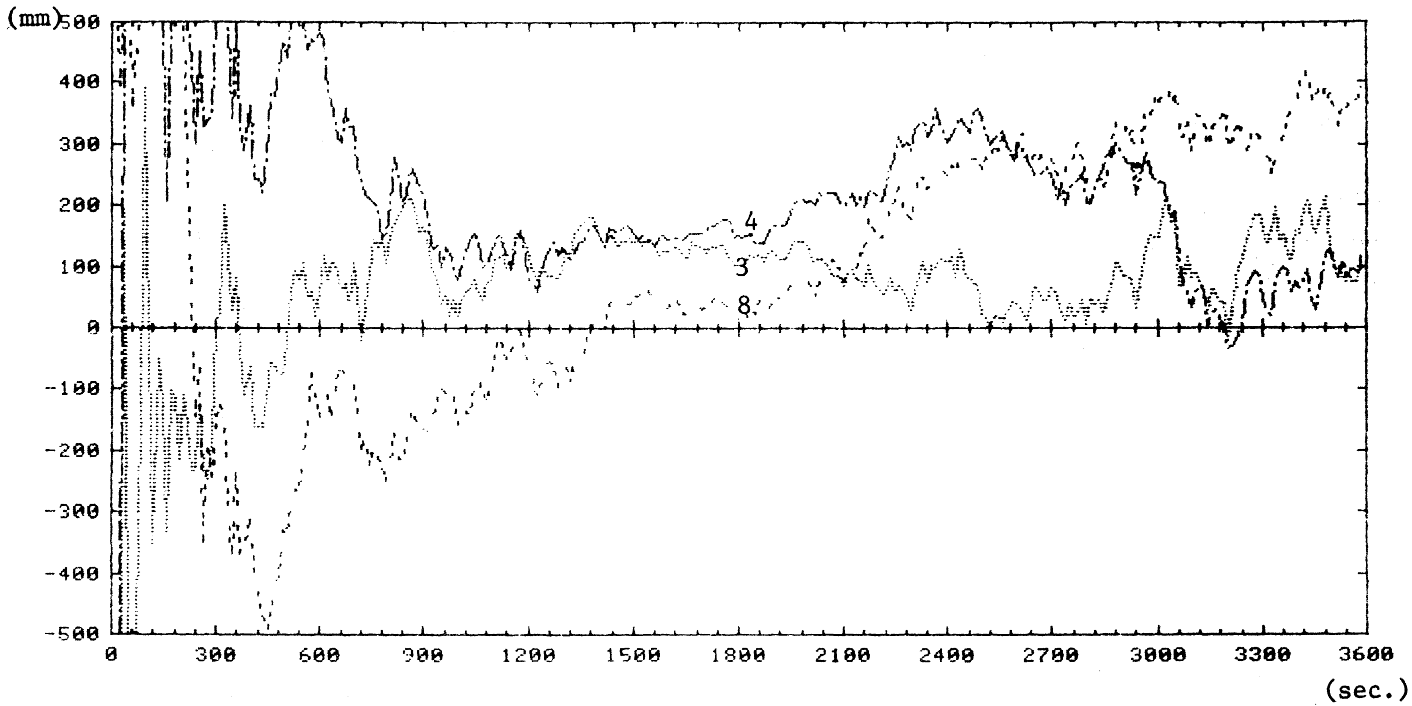

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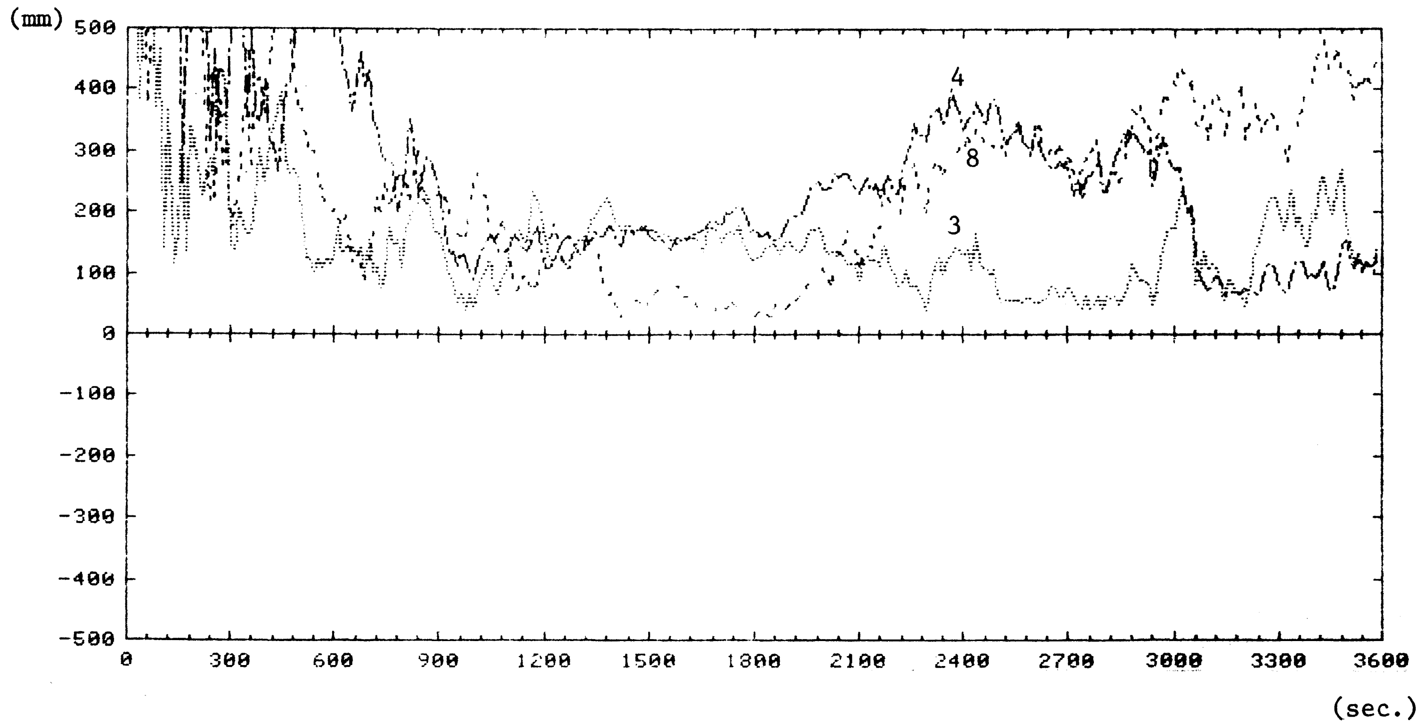

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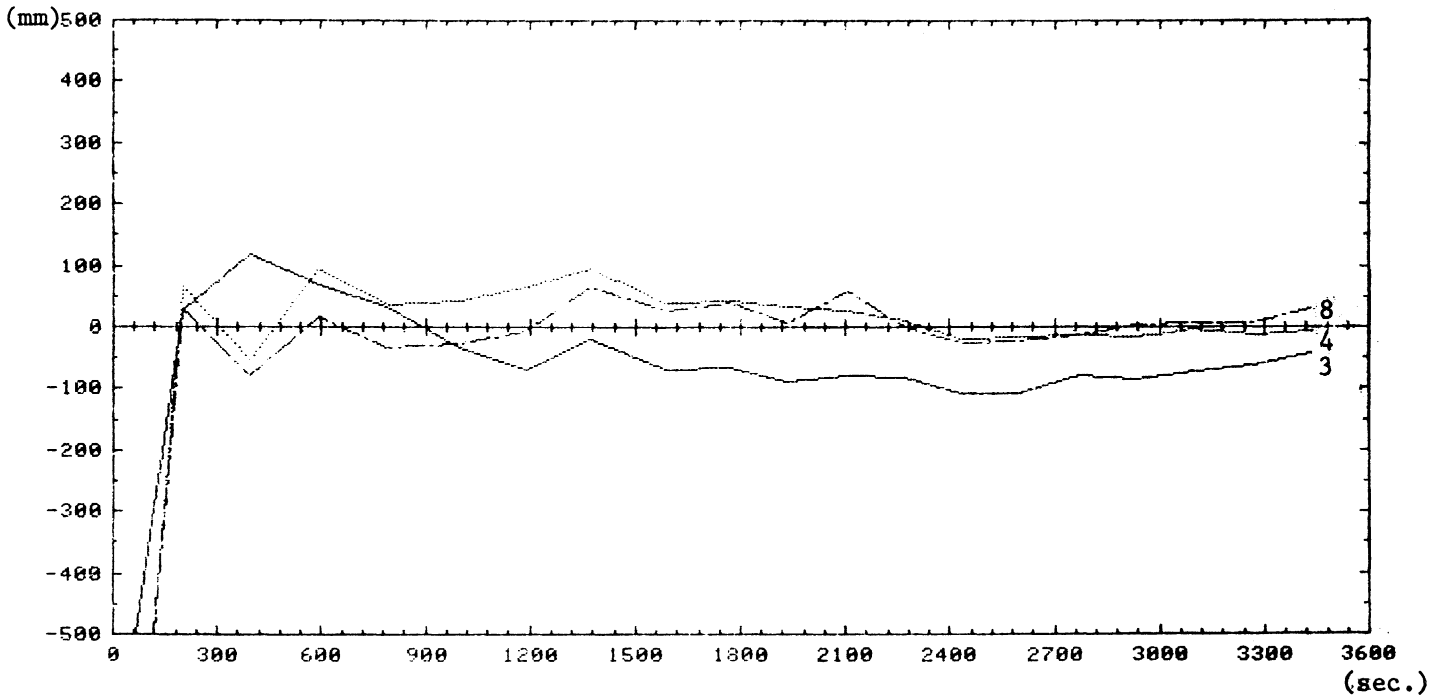

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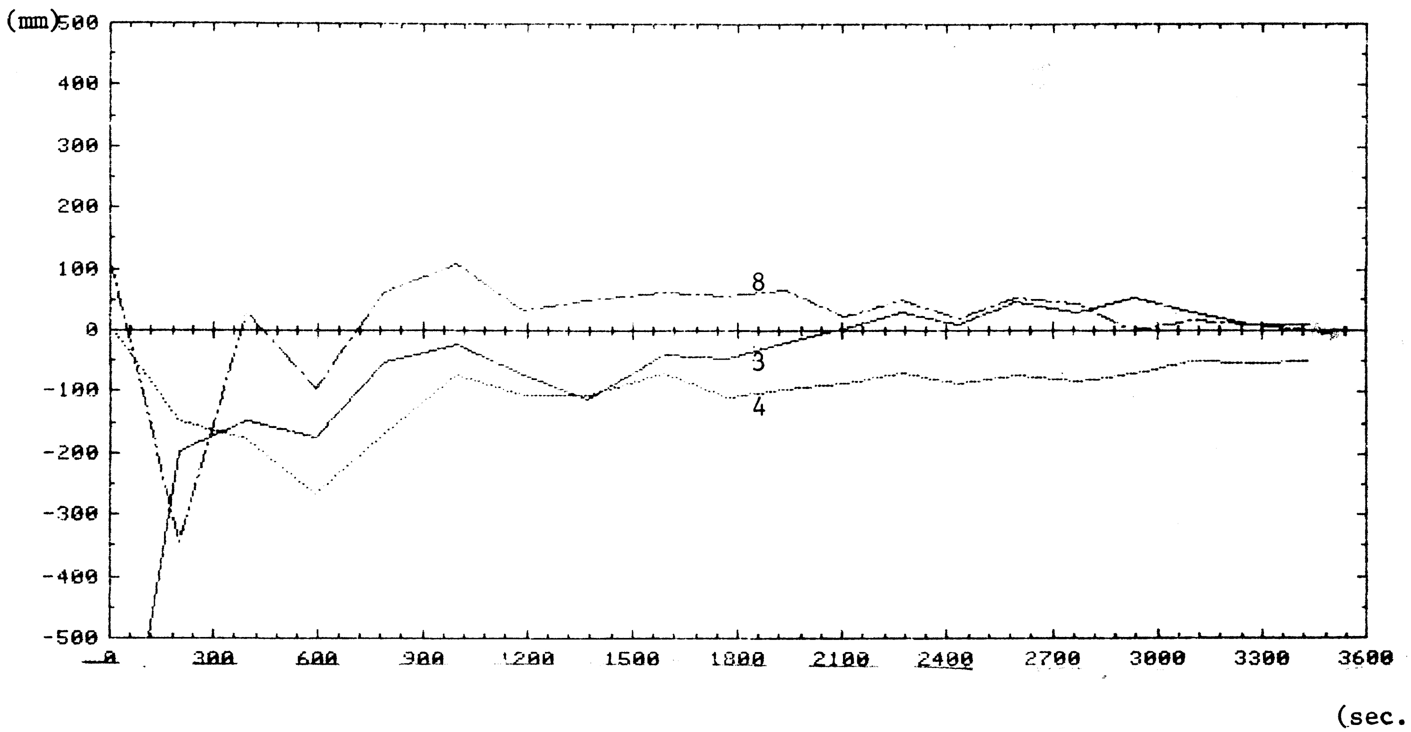

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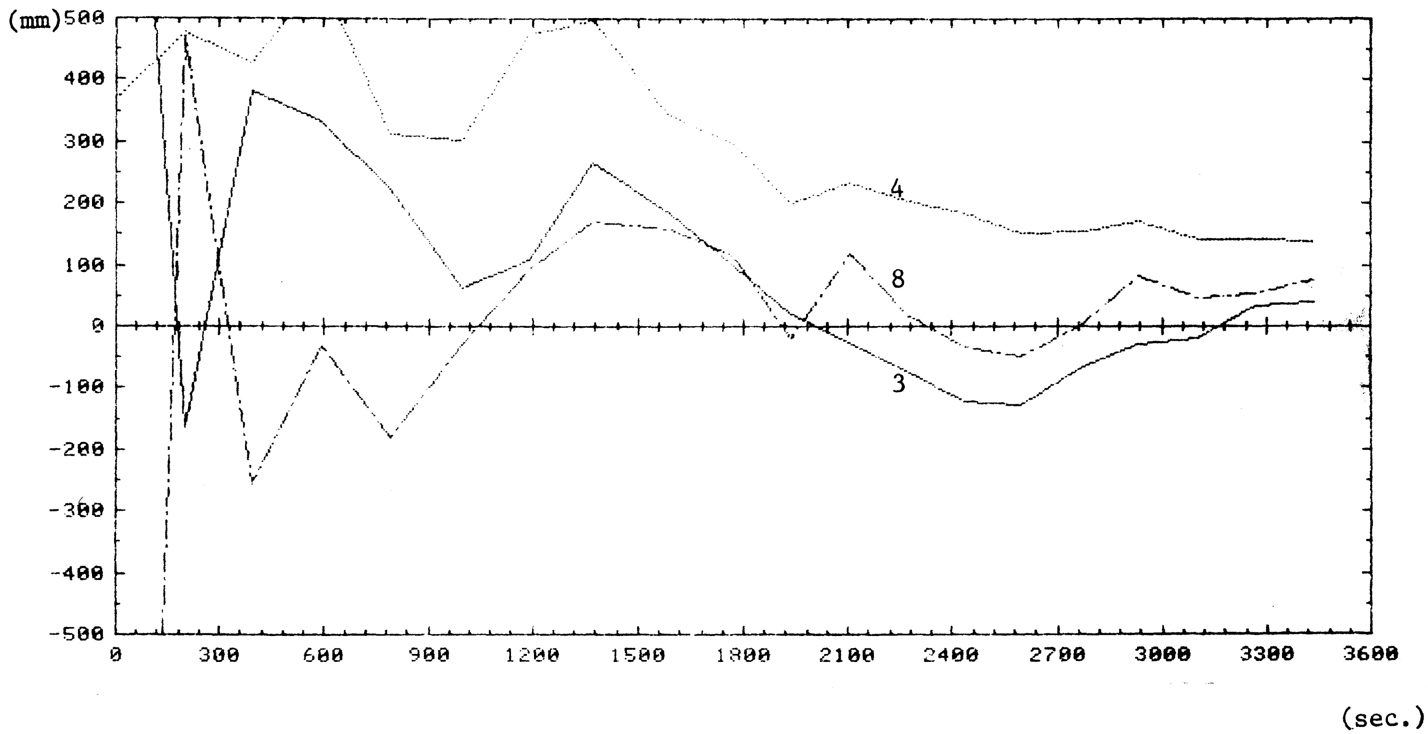

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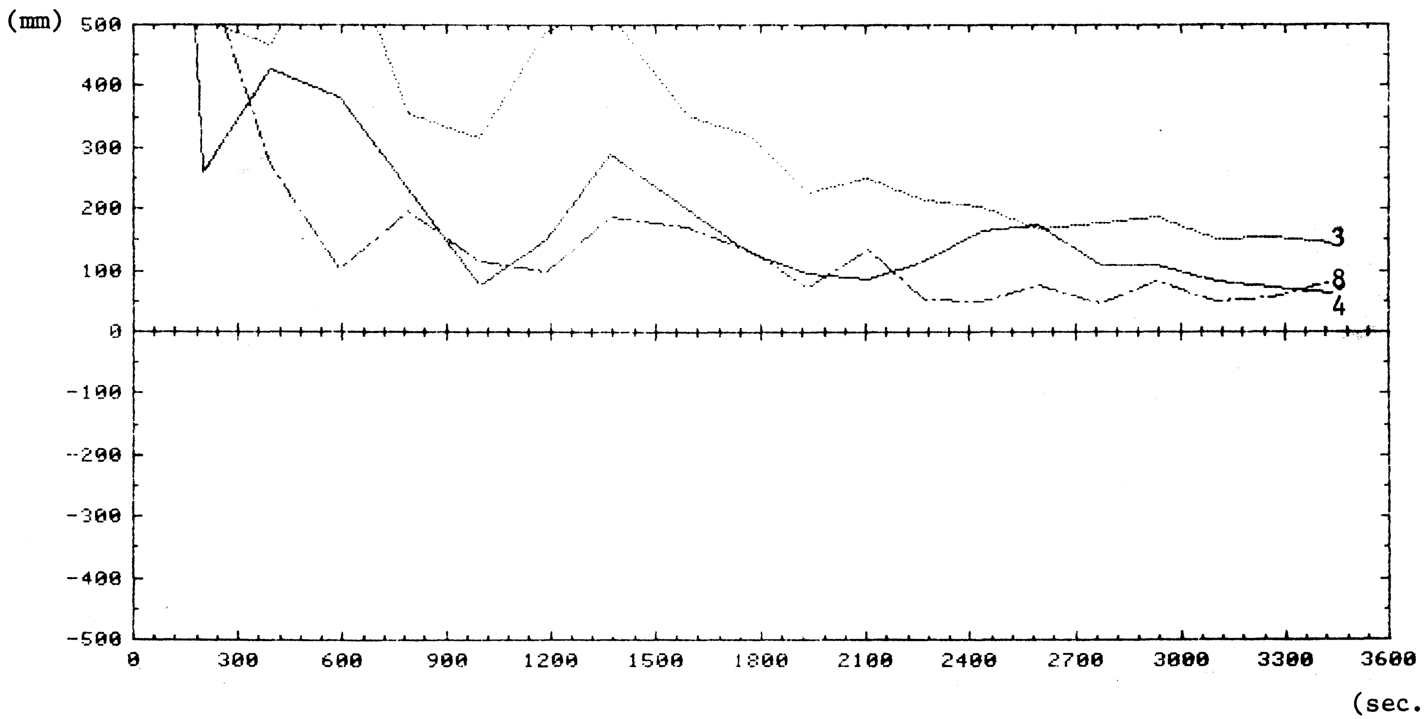

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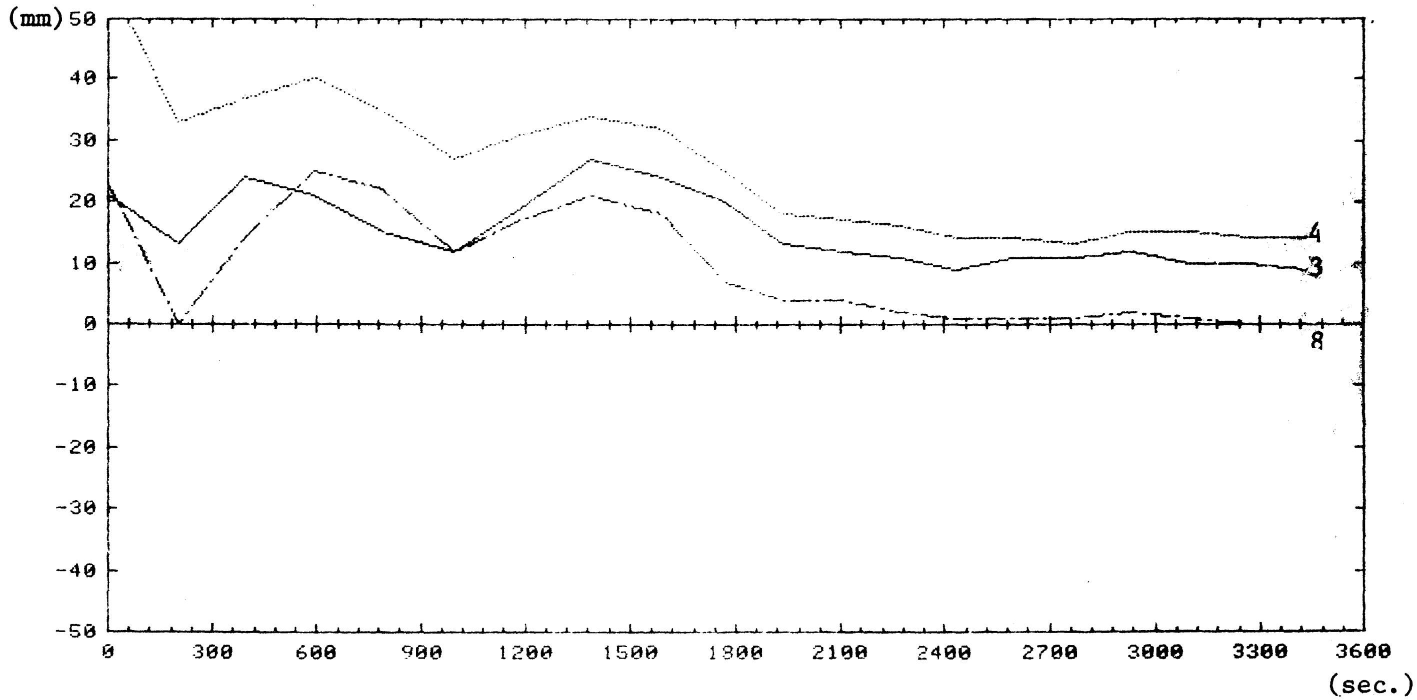

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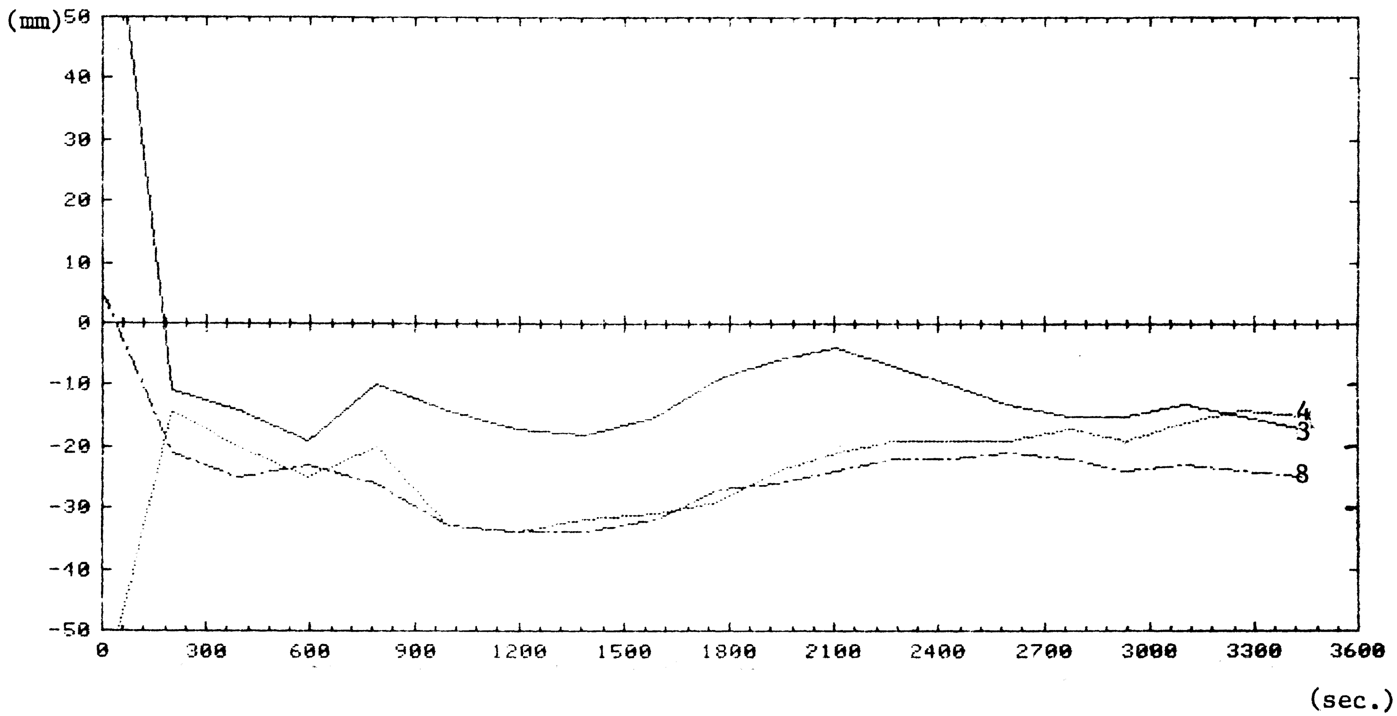

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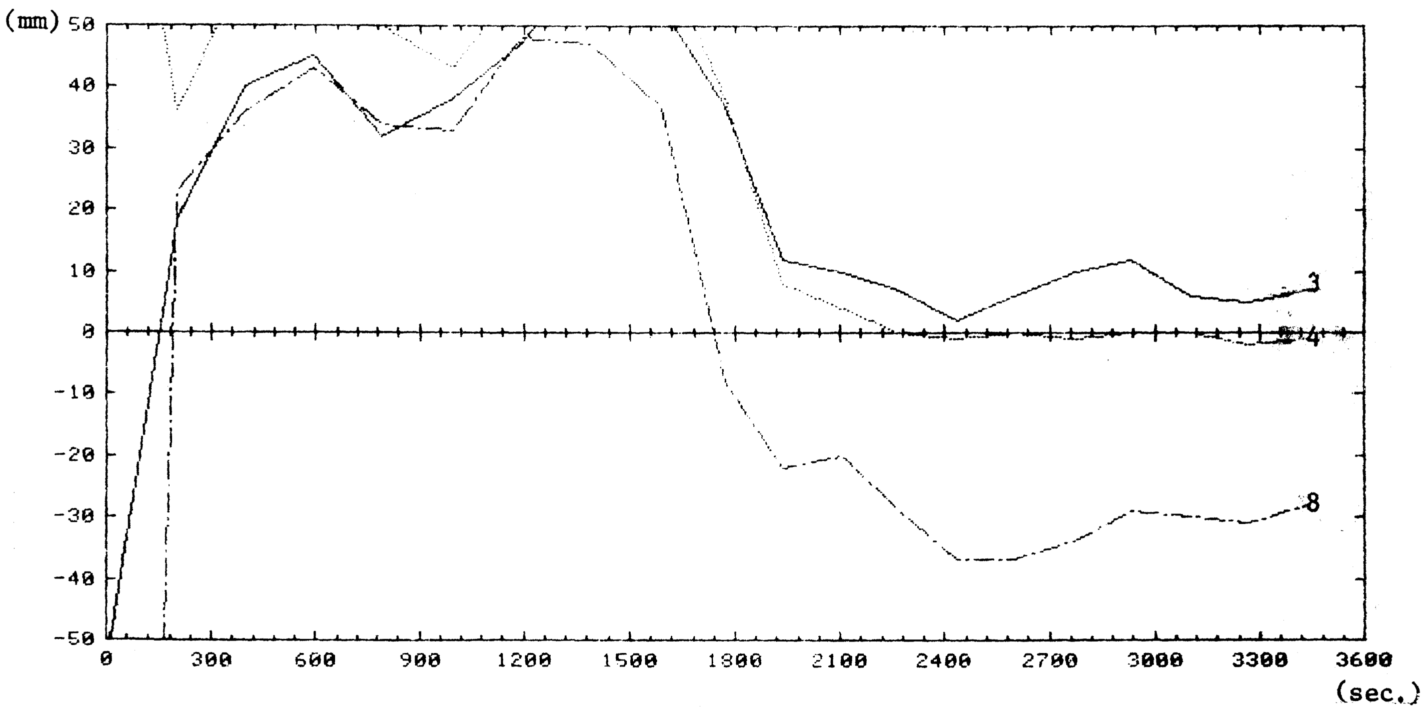

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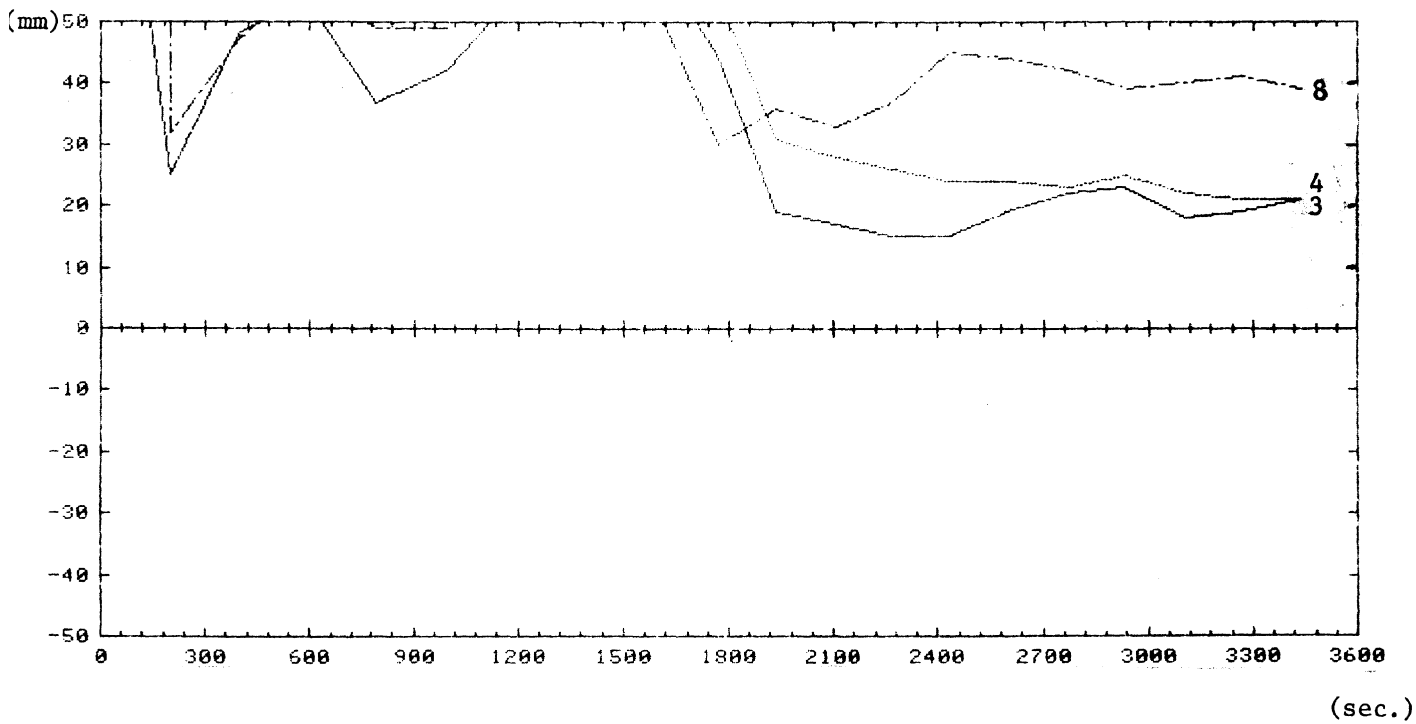

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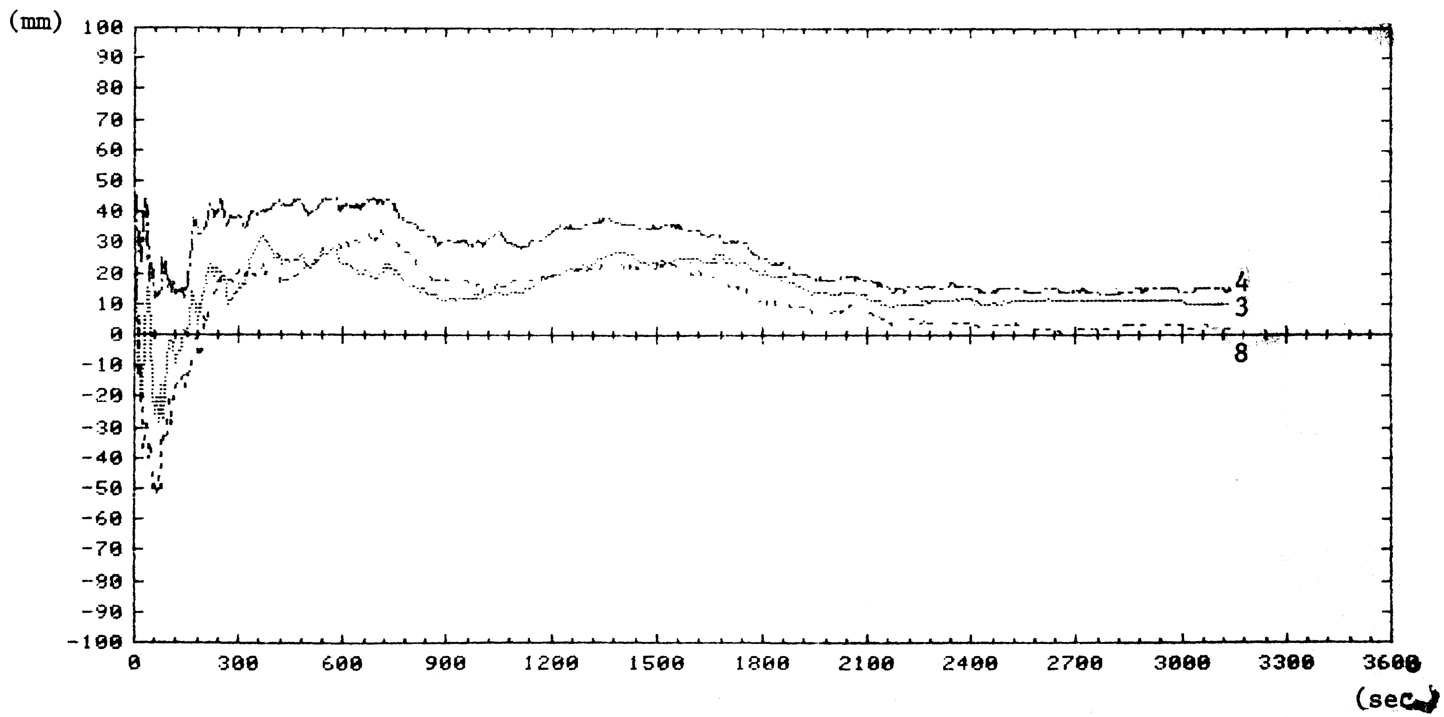

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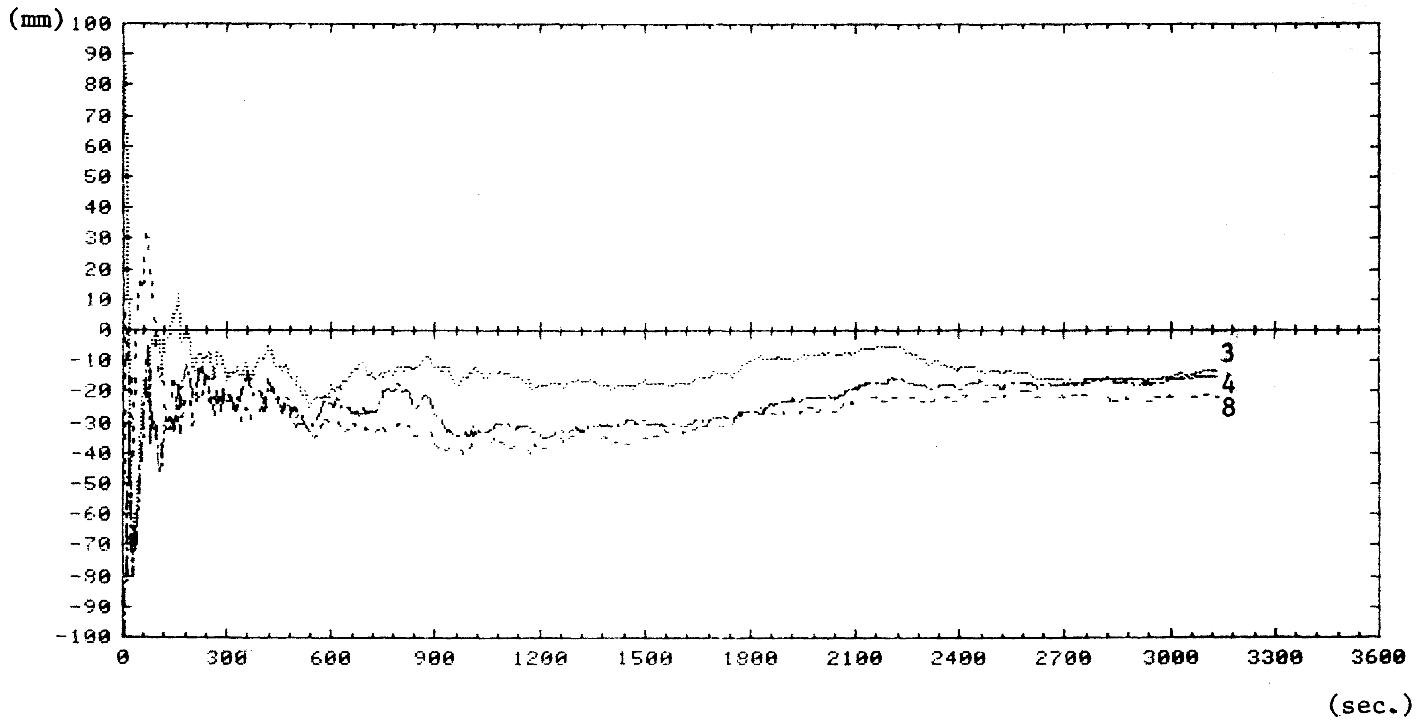

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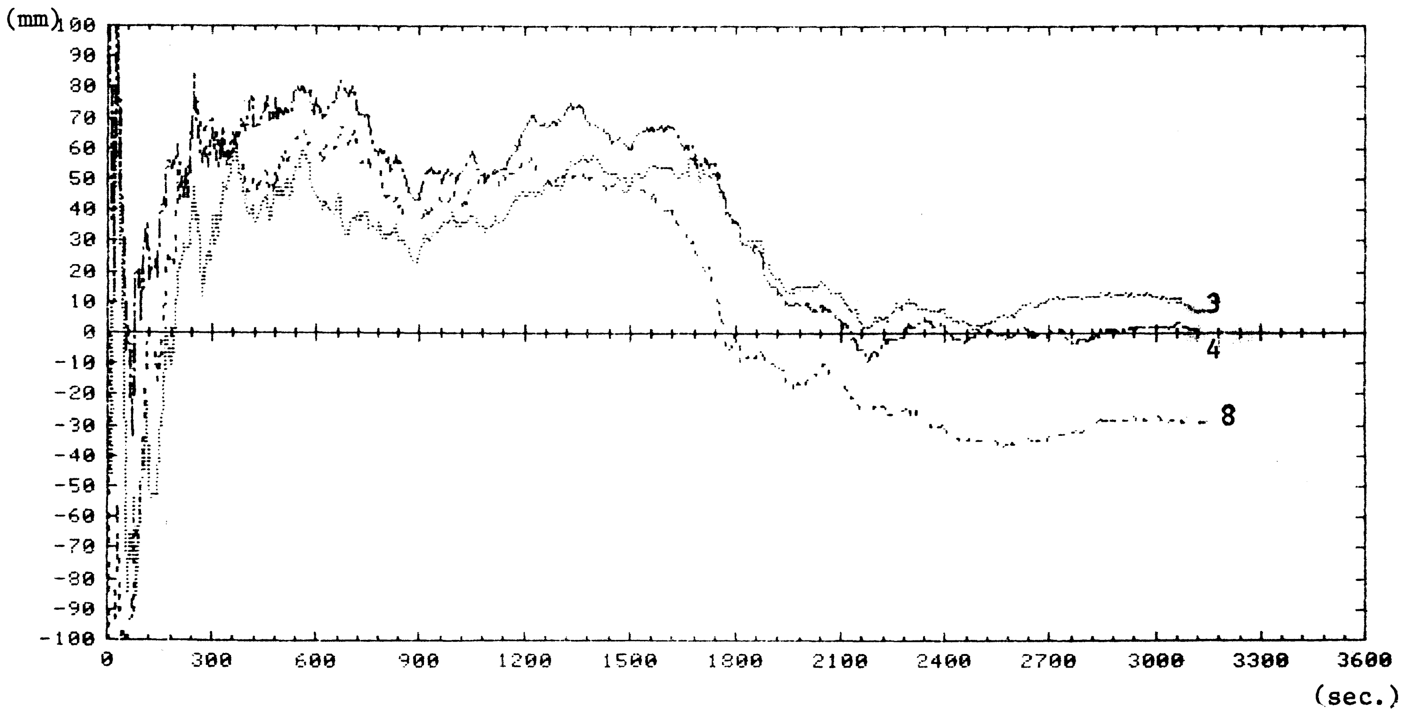

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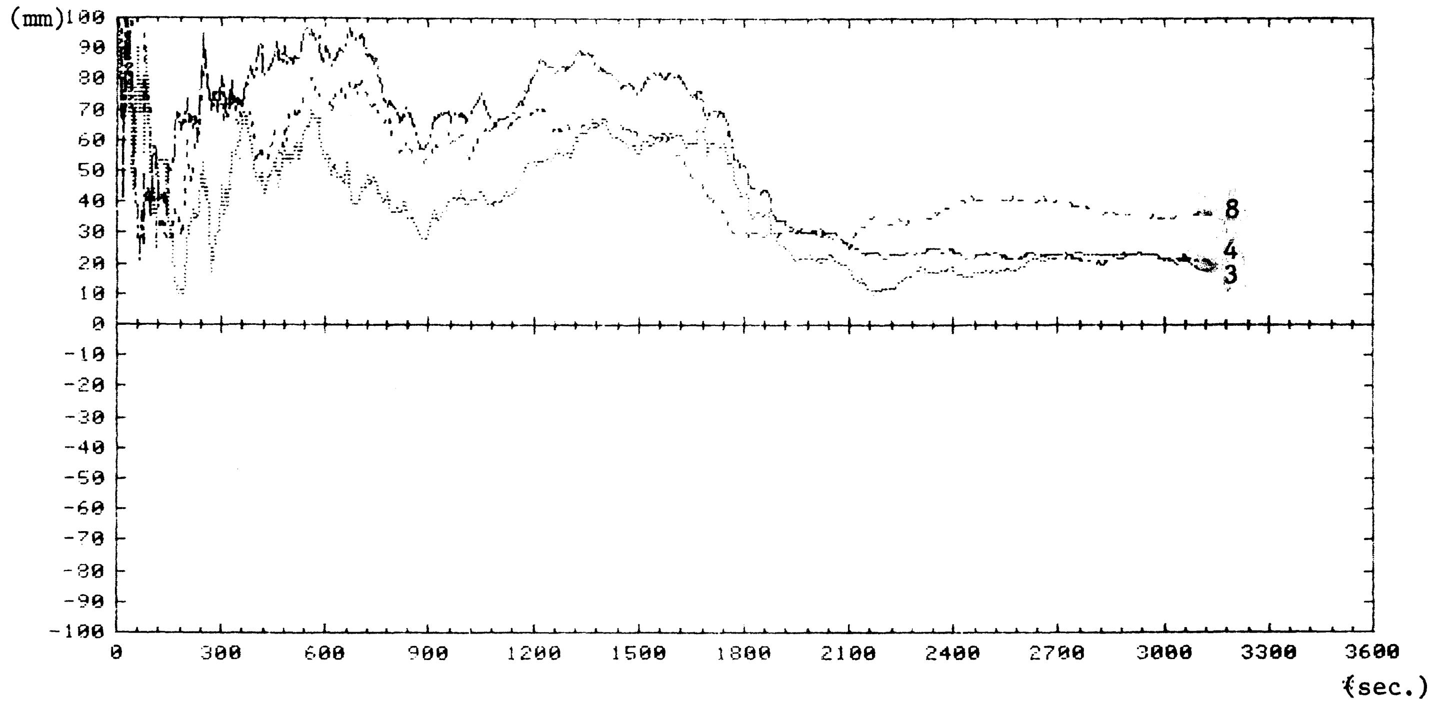

Fig. A19. Discrepancy DR P code + phase 2 5 7 10 12 15

APPENDIX B

Geometrical Aspects of Differential GPS Positioning

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ABSTRACT

Differential GPS positioning is considered from the purely geometric point of view. The tetrahedron formed by two ground stations and two satellite locations is the basic geometrical building block for differential satellite positioning. Relationships between the various vectors involved in this tetrahedron are described. These relationships are used to develop linear mathematical models which relate the vector baseline between the two ground stations to various kinds of differential GPS observations. Geometrically, all proposed observation types can be considered as either differential range observations or differential range difference observations. In the absence of instrumental and refraction effects, it is found that differential range observations are geometrically superior to differential range difference observations. Some implications of these geometrical considerations to practical differential GPS positioning are discussed.

INTRODUCTION

The NAVigation Satellite Timing And Ranging (NAVSTAR)/Global Positioning System (GPS) will become fully operational by about 1990. One of the many applications of this system will be geodetic positioning. In fact, several precise positioning experiments using the partially deployed system have already been conducted [Anderle and Evans, 1982; Counselman et al., 1982; Greenspan et al., 1982; Lachapelle and Wade, 1982; MacDoran et al., 1982; Hothem and Fronczek, 1983]. Both point positioning and relative positioning are possible with GPS. However because of uncertainties in satellite positions, clock behaviour, and transmission media delays, "absolute" point positioning will likely be obtained with accuracies not much better than about 50 cm [Anderle, 1979; Senus and Hill, 1981]. For the most demanding geodetic work, such as monitoring crustal motions, it will be necessary to use GPS in a differential mode [Davidson et al., 1983].

In a differential mode, two or more GPS receivers simultaneously receive signals from the same set of satellites. Subsequently the resulting observations are processed to obtain the components of the interstation baseline vectors. There are four types of measurements of GPS signals which have been suggested for differential use: pseudorange, integrated Doppler frequency, carrier phase, and interferometric time delay [Wells et al., 1981]. Although these observables are instrumented differently, they are all functions of the instantaneous ranges between satellites and ground stations and their time derivatives. These quantities in turn reflect the relative geometry of the ground stations and the satellites. In the absence of instrumental and transmission media effects, it would be this geometry which would control the precision with which relative positions could be obtained. Unfortunately the non-geometrical effects play an important role in determining the precision of relative positions and these effects must also be considered in any complete analysis of potential GPS measurements. However, as a first step, in this paper we restrict our attention to the purely geometrical considerations of differential GPS positioning in order to first understand that part of the problem. We will keep in mind, however, that purely geometrical strength and the cancellation of non-geometrical effects will often involve conflicting design criteria.

GEOMETRY OF A TETRAHEDRON

A tetrahedron is formed by two ground stations P_1, P_2 and two satellite positions S^j, S^k (see Figure 1). Such a tetrahedron is the basic geometrical "building block" in any investigation of differential positioning by satellites. To facilitate setting up geometrical models, we will first introduce the vectorial quantities one will need to describe geometrical relations within a tetrahedron.

We have selected notational conventions which allow us to develop the geometrical concepts as clearly as possible. These conventions are shown in Table 1. In our conventions, we have tried to reflect the geometrical properties in an obvious way. For example, we distinguish ground points (low points) by varying a subscript index, and distinguish satellite points (high points) by varying a superscript index. Similarly, differences between quantities involving two satellite points and one ground point (two high and one low point) are generally denoted by ∇, and differences involving two ground points and one satellite point by Δ. To make the notation even clearer, we generally use numerals for ground station indices and letters for satellite indices.

We begin with the four basic unit vectors $\vec{e}_1^{\,j}$, $\vec{e}_2^{\,j}$, $\vec{e}_1^{\,k}$, $\vec{e}_2^{\,k}$ which indicate the directions of satellite positions j and k with respect to ground station positions 1 and 2 (see Figure 1). The shape of the tetrahedron is uniquely determined by these vectors. (The size of the tetrahedron can be derived if the length of any of the involved vectors is known, simply by scaling the corresponding side of the configuration.) All other vectors of interest can be expressed in terms of the unit vectors. The mean "station" and "satellite" vectors are defined as

$$\left.\begin{aligned} \vec{u}_1 &= \tfrac{1}{2}(\vec{e}_1^{\,j} + \vec{e}_1^{\,k}) \\ \vec{u}_2 &= \tfrac{1}{2}(\vec{e}_2^{\,j} + \vec{e}_2^{\,k}) \\ \vec{u}^{\,j} &= \tfrac{1}{2}(\vec{e}_1^{\,j} + \vec{e}_2^{\,j}) \\ \vec{u}^{\,k} &= \tfrac{1}{2}(\vec{e}_1^{\,k} + \vec{e}_2^{\,k}) \end{aligned}\right| \quad , \qquad (1)$$

and the total mean vector $\vec{u}$ is defined as

$$\vec{u} = \tfrac{1}{4}(\vec{e}_1^{\,j} + \vec{e}_2^{\,j} + \vec{e}_1^{\,k} + \vec{e}_2^{\,k}) \quad . \qquad (2)$$

Through elementary operations, it can be shown that

$$\vec{u} = \tfrac{1}{2}(\vec{u}^{\,j} + \vec{u}^{\,k}) = \tfrac{1}{2}(\vec{u}_1 + \vec{u}_2) \quad . \qquad (3)$$

This completes the definition of vectors. We note that only the four basic vectors are unit vectors. For a station and satellite configuration of, say, baseline $\Delta R \approx 100$ km long; range, $\rho \approx 2.3 \times 10^4$ km; and satellite spacing, $\Delta r \approx 100$ km (which corresponds to about half a minute difference between the satellite positions S^j and S^k on the same pass), all of the vectors are very close to being unit vectors. On the other hand, Δr could be as large as 4×10^4 km (for two satellite positions on different passes), in which case u_1, u_2 and u will be significantly smaller than 1.

Next, we define the following vector differences:

$$
\begin{aligned}
\Delta\vec{u}^j &= \vec{e}_2^{\,j} - \vec{e}_1^{\,j} \\
\Delta\vec{u}^k &= \vec{e}_2^{\,k} - \vec{e}_1^{\,k} \\
\nabla\vec{u}_1 &= \vec{e}_1^{\,k} - \vec{e}_1^{\,j} \\
\nabla\vec{u}_2 &= \vec{e}_2^{\,k} - \vec{e}_2^{\,j} \quad .
\end{aligned} \qquad (4)
$$

From these we can construct the mean differences

$$
\begin{aligned}
\Delta\vec{u} &= \frac{1}{2}(\Delta\vec{u}^j + \Delta\vec{u}^k) \\
\nabla\vec{u} &= \frac{1}{2}(\nabla\vec{u}_1 + \nabla\vec{u}_2) \quad .
\end{aligned} \qquad (5)
$$

Obviously, while Δu is more sensitive to the baseline length, ∇u is more sensitive to the satellite spacing: Δu goes to zero when ΔR does, whereas ∇u goes to zero with Δr. Through elementary means, we can show that

$$
\begin{aligned}
\Delta\vec{u} &= \vec{u}_2 - \vec{u}_1 = \frac{1}{2}(-\vec{e}_1^{\,j} + \vec{e}_2^{\,j} - \vec{e}_1^{\,k} + \vec{e}_2^{\,k}) \\
\nabla\vec{u} &= \vec{u}^k - \vec{u}^j = \frac{1}{2}(-\vec{e}_1^{\,j} - \vec{e}_2^{\,j} + \vec{e}_1^{\,k} + \vec{e}_2^{\,k})
\end{aligned} \quad . \qquad (6)
$$

The total mean difference can be defined as

$$
D\vec{u} = \frac{1}{2}(\nabla\vec{u} + \Delta\vec{u}) \quad . \qquad (7)
$$

Analogously, we define a symmetric quantity:

$$
d\vec{u} = \frac{1}{2}(\nabla\vec{u} - \Delta\vec{u}) \quad . \qquad (8)
$$

The last two differences can also be written,

$$
\begin{aligned}
D\vec{u} &= \frac{1}{2}(\vec{e}_2^{\,k} - \vec{e}_1^{\,j}) \\
d\vec{u} &= \frac{1}{2}(\vec{e}_1^{\,k} - \vec{e}_2^{\,j}) \quad .
\end{aligned} \qquad (9)
$$

We note that for Δr and $\Delta R \approx 100$ km, the magnitudes of all of the above differences are of the order of 4×10^{-3} or less; i.e., none of the components of these vector differences would be larger than 4×10^{-3}. As Δr approaches 4×10^{4} km, Δu still remains near 4×10^{-3} while ∇u may approach 2.

The following scalar products involving first differences are useful in the derivation of the geometrical models:

$$
\left.\begin{aligned}
\vec{u}\Delta\vec{u} &= \frac{1}{4}(\vec{e}_2^{\,j}\,\vec{e}_2^{\,k} - \vec{e}_1^{\,j}\,\vec{e}_1^{\,k}) \\
\vec{u}\nabla\vec{u} &= \frac{1}{4}(\vec{e}_1^{\,k}\,\vec{e}_2^{\,k} - \vec{e}_1^{\,j}\,\vec{e}_2^{\,j}) \\
\vec{u}D\vec{u} &= \frac{1}{8}(\vec{e}_1^{\,k} + \vec{e}_2^{\,j})(\vec{e}_2^{\,k} - \vec{e}_1^{\,j}) \\
\vec{u}d\vec{u} &= \frac{1}{8}(\vec{e}_1^{\,j} + \vec{e}_2^{\,k})(\vec{e}_1^{\,k} - \vec{e}_2^{\,j}) \\
\nabla\vec{u}\ \Delta\vec{u} &= \frac{1}{2}(\vec{e}_1^{\,k}\,\vec{e}_2^{\,j} - \vec{e}_2^{\,k}\,\vec{e}_1^{\,j}) \\
D\vec{u}\ d\vec{u} &= \frac{1}{4}(\vec{e}_2^{\,k} - \vec{e}_1^{\,j})(\vec{e}_1^{\,k} - \vec{e}_2^{\,j})
\end{aligned}\right| \quad . \qquad (10)
$$

The magnitude of these scalar products will be discussed later.

A natural extension of these developments gives second vector differences:

$$
\left.\begin{aligned}
\Delta^2\vec{u} &= \Delta\vec{u}^{\,k} - \Delta\vec{u}^{\,j} \\
\nabla^2\vec{u} &= \nabla\vec{u}_2 - \nabla\vec{u}_1 \quad .
\end{aligned}\right| \qquad (11)
$$

It is easily shown that

$$\nabla^2\vec{u} = \Delta^2\vec{u} = \vec{e}_1^{\,j} - \vec{e}_2^{\,j} - \vec{e}_1^{\,k} + \vec{e}_2^{\,k} . \tag{12}$$

As for the first differences, no valid estimates for the components of the second vector differences can be obtained without specifying the shape of the tetrahedron. However, the second vector differences may be as large as some of the first vector differences.

We have defined fourteen different linear combinations of the four unit vectors with which we started. We illustrate in Figure 2 the relationships among these vectors. It can be seen from this diagram that the selected scheme is a natural one; the eight "second level" vectors are obtained from the four unit vectors (1st level) through natural (i.e., with either one subscript or one superscript common) averaging or differencing. Natural averaging and differencing of the eight "second level" vectors results in only four independent "third level" vectors: $\vec{u}$, $\nabla\vec{u}$, $\Delta\vec{u}$, $\Delta^2\vec{u}$ which we shall call the <u>defining vectors</u>. The two other differences, $D\vec{u}$, $d\vec{u}$ (eqns. (7) and (8)), are introduced simply because they are found useful as alternatives to $\nabla\vec{u}$ and $\Delta\vec{u}$.

The role of the four unit vectors can be taken over by the four "defining" vectors. This becomes clear from the fact that the unit vectors can be expressed uniquely as linear combinations of the defining vectors. From eqns. (2), (6), (9) and (12):

$$\left.\begin{aligned}
\vec{e}_1^{\,j} &= \vec{u} - \frac{1}{2}\nabla\vec{u} - \frac{1}{2}\Delta\vec{u} + \frac{1}{4}\Delta^2\vec{u} = \vec{u} - D\vec{u} + \frac{1}{4}\Delta^2\vec{u} \\
\vec{e}_2^{\,j} &= \vec{u} - \frac{1}{2}\nabla\vec{u} + \frac{1}{2}\Delta\vec{u} - \frac{1}{4}\Delta^2\vec{u} = \vec{u} - d\vec{u} - \frac{1}{4}\Delta^2\vec{u} \\
\vec{e}_1^{\,k} &= \vec{u} + \frac{1}{2}\nabla\vec{u} - \frac{1}{2}\Delta\vec{u} - \frac{1}{4}\Delta^2\vec{u} = \vec{u} + d\vec{u} - \frac{1}{4}\Delta^2\vec{u} \\
\vec{e}_2^{\,k} &= \vec{u} + \frac{1}{2}\nabla\vec{u} + \frac{1}{2}\Delta\vec{u} + \frac{1}{4}\Delta^2\vec{u} = \vec{u} + D\vec{u} + \frac{1}{4}\Delta^2\vec{u}
\end{aligned}\right| \tag{13}$$

It should be noted that eight independent (i.e., arbitrarily selectable) quantities are needed to define the shape of the tetrahedron uniquely—four unit vectors contain eight independent components. On the other hand, the four defining vectors have twelve components altogether; therefore, there must exist four independent relations among the defining vectors that must be satisfied under any circumstances, i.e., for a tetrahedron of any shape. It can be shown that the following relations always hold:

$$\left.\begin{aligned}
&\vec{u}\,\vec{u} + \frac{1}{4}(\nabla\vec{u}\;\nabla\vec{u} + \Delta\vec{u}\;\Delta\vec{u}) + \frac{1}{16}\,\Delta^2\vec{u}\;\Delta^2\vec{u} = 1\\
&\vec{u}\;\Delta^2\vec{u} = -\,\nabla\vec{u}\;\Delta\vec{u}\\
&\nabla\vec{u}\;\Delta^2\vec{u} = -\,4\vec{u}\;\Delta\vec{u}\\
&\Delta\vec{u}\;\Delta^2\vec{u} = -\,4\vec{u}\;\nabla\vec{u}\quad.
\end{aligned}\right| \qquad (14)$$

If the vector differences $D\vec{u}$ and $d\vec{u}$ are used instead of $\nabla\vec{u}$ and $\Delta\vec{u}$, eqns. (14) are replaced by

$$\left.\begin{aligned}
&\vec{u}\,\vec{u} + \frac{1}{2}(D\vec{u}\;D\vec{u} + d\vec{u}\;d\vec{u}) + \frac{1}{16}\,\Delta^2\vec{u}\;\Delta^2\vec{u} = 1\\
&\vec{u}\;\Delta^2\vec{u} = d\vec{u}\;d\vec{u} - D\vec{u}\;D\vec{u}\\
&D\vec{u}\;\Delta^2\vec{u} = -\,4\vec{u}\;D\vec{u}\\
&d\vec{u}\;\Delta^2\vec{u} = 4\vec{u}\;d\vec{u}\quad.
\end{aligned}\right| \qquad (15)$$

We also note that the last two equations (15) may be rewritten as follows:

$$\left.\begin{aligned}
&D\vec{u}(\vec{u} + \frac{1}{4}\,\Delta^2\vec{u}) = 0\\
&d\vec{u}(\vec{u} - \frac{1}{4}\,\Delta^2\vec{u}) = 0\quad.
\end{aligned}\right| \qquad (16)$$

To gain additional insight into the meaning of the defining vectors, let us see what can be learnt about them from some special configurations of the tetrahedron.

(i) As the intersatellite distance, Δr, shortens, we get the following tendencies:

$$\nabla\vec{u} \to \vec{0},\ \Delta^2\vec{u} \to \vec{0},\ D\vec{u} \to \frac{1}{2}\Delta\vec{u},\ d\vec{u} \to -\frac{1}{2}\Delta\vec{u},\ \vec{u}\,\Delta\vec{u} \to 0, \text{ and } u^2 \to 1 - \frac{1}{4}\Delta u^2.$$

(ii) As the interstation distance, ΔR, shortens, the following trends become apparent:

$$\Delta\vec{u} \to \vec{0},\ \Delta^2\vec{u} \to \vec{0},\ D\vec{u} \to \frac{1}{2}\nabla\vec{u},\ d\vec{u} \to \frac{1}{2}\nabla\vec{u},\ \vec{u}\,\nabla\vec{u} \to 0, \text{ and } u^2 \to 1 - \frac{1}{4}\nabla u^2 .$$

(iii) If the tetrahedron is normal and symmetric (i.e., $\Delta\vec{r}$ is perpendicular to the plane defined by $\Delta\vec{R}$ and the mid-satellite point S, and $\Delta\vec{R}$ is perpendicular to the plane defined by $\Delta\vec{r}$ and the mid-ground point P), then $\Delta^2\vec{u} \to \vec{0}$ and $\vec{u}$, $\nabla\vec{u}$, $\Delta\vec{u}$ make an orthogonal triad.

The overall tendency is for $\vec{u}$ to be in the PS direction and to be the closer to a unit vector the more elongated is the tetrahedron in the PS direction. $\Delta\vec{u}$ tends to be in the S^jS^kP plane and its length shrinks with ΔR, while $\nabla\vec{u}$ tends to be in the P_1P_2S plane and its length shrinks with Δr. $\Delta^2\vec{u}$ tends to disappear when the tetrahedron becomes symmetric.

DIFFERENTIAL GPS OBSERVATIONS

The basic measurable quantity of GPS point positioning is the magnitude of the instantaneous range vector between a ground station and a satellite. In differential positioning, suitable differences of range vector magnitudes form the observables. Although a receiving system may not be instrumented to perform direct differencing of ranges, its operation may be mathematically described as such. Actually, because of timing errors and delays in the satellite and ground station equipment and the effects of signal propagation through the ionosphere and troposphere, the measured signal is a "pseudorange". We will neglect all timing and refraction errors in the following analyses and hence refer to the observations simply as ranges.

RANGE DIFFERENCE MATHEMATICAL MODEL

We are interested in forming equations which relate a baseline vector, $\Delta\vec{R}$, to suitable differences of ranges. Let us look first at the geometry of differenced range differences. By range difference we mean the difference in range to two positions of a satellite from a single ground station (see Figure 3(a)). Such differences are typically obtained by integrating the Doppler shift of the received signal over the time period required for the satellite to move from one position to the other. By subtracting the range differences observed at two stations, we create differenced range differences which can be used to determine the baseline between the ground stations (see Figure 3(b)).

The range vector between a ground station P_α and satellite position S^β is

$$\vec{\rho}^{\,\beta}_{\alpha} = \vec{r}^{\,\beta} - \vec{R}_{\alpha} \quad , \tag{17}$$

where $\vec{r}^{\,\beta}$ is the position vector of the satellite and $\vec{R}_\alpha$ is the position vector of the ground station. The unit vector $\vec{e}^{\beta}_{\alpha}$ transforms between $\vec{\rho}^{\beta}_{\alpha}$ and its length ρ^{β}_{α} according to

$$\left.\begin{aligned} \vec{e}^{\beta}_{\alpha}\,\vec{\rho}^{\beta}_{\alpha} &= \vec{e}^{\beta}_{\alpha}(\vec{r}^{\beta} - \vec{R}_{\alpha}) = \rho^{\beta}_{\alpha} \\ \vec{e}^{\beta}_{\alpha}\,\rho^{\beta}_{\alpha} &= \vec{e}^{\beta}_{\alpha}(\vec{e}^{\beta}_{\alpha}\,\vec{\rho}^{\beta}_{\alpha}) = (\vec{e}^{\beta}_{\alpha}\,\vec{e}^{\beta}_{\alpha})\,\vec{\rho}^{\beta}_{\alpha} = \vec{\rho}^{\beta}_{\alpha} \end{aligned}\right| \quad . \tag{18}$$

The difference in the length of the two range vectors from a specific ground station P_1 to specific satellite positions S^j and S^k, called here the range difference $\nabla\rho_1$, is

$$\vec{e}^{k}_{1}(\vec{r}^{k} - \vec{R}_{1}) - \vec{e}^{j}_{1}(\vec{r}^{j} - \vec{R}_{1}) = \rho^{k}_{1} - \rho^{j}_{1} \quad . \tag{19}$$

Denoting

$$\nabla\rho_1 = \rho^{k}_{1} - \rho^{j}_{1} \quad , \tag{20}$$

rearranging the terms, and using eqns. (4), we get the equation for "Doppler" point positioning

$$\boxed{-\nabla\vec{u}_1\,\vec{R}_1 = \nabla\rho_1 - \vec{e}^{k}_{1}\,\vec{r}^{k} + \vec{e}^{j}_{1}\,\vec{r}^{j}} \quad . \tag{21}$$

This equation may be used as a mathematical model for the position $\vec{R}_1$.

DIFFERENCED RANGE DIFFERENCE MATHEMATICAL MODEL

To determine the baseline vector

$$\Delta\vec{R} = \Delta\vec{R}_{12} = \vec{R}_2 - \vec{R}_1 \quad , \tag{22}$$

we shall assume that two range differences, $\nabla\rho_1$, $\nabla\rho_2$ were observed simultaneously from P_1 and P_2. It is not necessary to define simultaneity here other than by saying that the ranges refer to unique positions S^j and

S^k of the satellite. We can then write two equations like eqn. (21) and subtract one from the other, obtaining

$$\nabla\vec{u}_1\,\vec{R}_1 - \nabla\vec{u}_2\,\vec{R}_2 = \nabla\rho_2 - \nabla\rho_1 - (\vec{e}_2^{\,k} - \vec{e}_1^{\,k})\vec{r}^{\,k} + (\vec{e}_2^{\,j} - \vec{e}_1^{\,j})\vec{r}^{\,j} \quad . \tag{23}$$

Using the expressions defined earlier for the tetrahedron defined by P_1, P_2, S^j, S^k we can rewrite this equation as:

$$\begin{aligned}\nabla\vec{u}\;\;\vec{R}_1 - \tfrac{1}{2}\Delta^2\vec{u}\;\vec{R}_1 - \nabla\vec{u}\;\;\vec{R}_2 - \tfrac{1}{2}\Delta^2\vec{u}\;\vec{R}_2 \\ = \nabla\rho_2 - \nabla\rho_1 - \Delta\vec{u}\;\;\vec{r}^{\,k} - \tfrac{1}{2}\Delta^2\vec{u}\;\vec{r}^{\,k} + \Delta\vec{u}\;\;\vec{r}^{\,j} - \tfrac{1}{2}\Delta^2\vec{u}\;\vec{r}^{\,j} \quad .\end{aligned} \tag{24}$$

Rearranging this equation we get

$$-\nabla\vec{u}\;\Delta\vec{R} = \nabla\rho_2 - \nabla\rho_1 - \Delta\vec{u}(\vec{r}^{\,k} - \vec{r}^{\,j}) + \tfrac{1}{2}\Delta^2\vec{u}(\vec{R}_1 + \vec{R}_2 - \vec{r}^{\,k} - \vec{r}^{\,j}) \quad . \tag{25}$$

Denoting

$$\left.\begin{aligned}\nabla^2\rho &= \nabla\rho_2 - \nabla\rho_1 \\ \Delta\vec{r} &= \vec{r}^{\,k} - \vec{r}^{\,j} \\ \vec{R}_m &= \tfrac{1}{2}(\vec{R}_1 + \vec{R}_2) \\ \vec{r}^{\,m} &= \tfrac{1}{2}(\vec{r}^{\,k} + \vec{r}^{\,j})\end{aligned}\right| \quad , \tag{26}$$

we obtain the equation we are seeking:

$$\boxed{-\nabla\vec{u}\;\Delta\vec{R} = \nabla^2\rho - \Delta\vec{u}\;\Delta\vec{r} + \Delta^2\vec{u}\;(\vec{R}_m - \vec{r}^{\,m})} \quad . \tag{27}$$

We note that this equation is linear in the unknown $(\Delta\vec{R})$ as well as in the observations $(\nabla^2\rho)$ and is (geometrically) exact. Clearly, for the solution of $\Delta\vec{R}$ we have to have at least three range differences and an appropriate geometry. Knowledge of the intersatellite vector $\Delta\vec{r}$, mean station position $\vec{R}_m$, mean satellite position $\vec{r}^m$ and all the involved direction cosines (components of the aforementioned unit vectors) is required which makes this equation somewhat inconvenient to use directly.

DIFFERENTIAL RANGE MATHEMATICAL MODEL

Here we shall seek to formulate the linear mathematical model that relates the interstation vector $\Delta\vec{R}$ to differential ranges (obtained either by differencing the ranges obtained via the L_1/L_2 timing--using either the code timing [Spilker, 1978] or the reconstructed carrier timing [Bossler et al., 1980]--or directly by the interferometric technique [Counselman et al., 1982]) (see Figure 3(c)).

From eqn. (4) and Figure 4 we use the relation $\Delta\vec{u}^j = \vec{e}_2^j - \vec{e}_1^j$ to substitute as follows in the basic differential range equation:

$$
\begin{aligned}
\Delta\vec{R} &= \vec{e}_1^j \rho_1^j - \vec{e}_2^j \rho_2^j \\
&= \vec{e}_1^j \rho_1^j - (\Delta\vec{u}^j + \vec{e}_1^j) \rho_2^j \\
&= - \Delta\vec{u}^j \rho_2^j - \vec{e}_1^j (\rho_2^j - \rho_1^j) \\
&= - \Delta\vec{u}^j \rho_2^j - \vec{e}_1^j \Delta\rho^j \ ,
\end{aligned}
\qquad (28)
$$

where we denote the differential range to be observed by $\Delta\rho^j = \rho_2^j - \rho_1^j$. Multiplication of eqn. (28) by $\vec{u}^j$ results in

$$
\vec{u}^j \Delta\vec{R} = - \vec{u}^j \Delta\vec{u}^j \rho_2^j - \vec{u}^j \vec{e}_1^j \Delta\rho^j \ . \qquad (29)
$$

Rewriting the coefficient of ρ_2^j as

$$\vec{u}^j \, \Delta\vec{u}^j = \frac{1}{2}(\vec{e}_1^j + \vec{e}_2^j)(\vec{e}_2^j - \vec{e}_1^j) = \frac{1}{2}(\vec{e}_2^j \, \vec{e}_2^j - \vec{e}_1^j \, \vec{e}_1^j) \quad , \tag{30}$$

it is easy to see that it is identically equal to zero. Thus the final equation reads

$$\boxed{\vec{u}^j \, \Delta\vec{R} = - \vec{u}^j \vec{e}_1^j \Delta\rho^j} \quad . \tag{31}$$

It represents an exact linear relation between observed differential range $\Delta\rho^j$ and the unknown baseline vector $\Delta\vec{R}$. We note that to solve for the baseline vector we have to know, apart from the (observed) differential ranges, only the direction cosines of the unit vectors. No other information is required.

DIFFERENTIAL RANGE DIFFERENCE MATHEMATICAL MODEL

Let us now see if we can take advantage of the combination of two observed differential ranges, $\Delta\rho^j$, $\Delta\rho^k$. Since, compared to the range difference model above, the tetrahedron here would involve two satellite positions S^j, S^k that do not have to be on the same pass (i.e., typically two different satellites), we may obtain a more favourable geometry (see Figure 3(d)). Writing two equations like eqn. (31) and subtracting one from the other we get

$$\vec{u}^j \Delta\vec{R} - \vec{u}^k \Delta\vec{R} = - \vec{u}^j \vec{e}_1^j \Delta\rho^j + \vec{u}^k \vec{e}_1^k \, \Delta\rho^k \ . \tag{32}$$

This can be rewritten as

$$- \nabla\vec{u} \, \Delta\vec{R} = \vec{u}^k \vec{e}_1^k \Delta\rho^k - \vec{u}^j \vec{e}_1^j \, \Delta\rho^j \ . \tag{33}$$

Substituting $\vec{u} + 1/2 \, \nabla\vec{u}$ for $\vec{u}^k$ and $\vec{u} - 1/2 \, \nabla\vec{u}$ for $\vec{u}^j$ and using eqns. (13) we get

$$- \nabla\vec{u}\ \Delta\vec{R} = (\vec{u} + \frac{1}{2}\nabla\vec{u})\ (\vec{u} + d\vec{u} - \frac{1}{4}\Delta^2\vec{u})\ \Delta\rho^k$$

$$- (\vec{u} - \frac{1}{2}\nabla\vec{u})(\vec{u} - D\vec{u} + \frac{1}{4}\Delta^2\vec{u})\Delta\rho^j \quad . \tag{34}$$

Using eqns. (7) and (8), we can also write eqn. (34) as

$$- \nabla\vec{u}\ \Delta\vec{R} = [\vec{u}(\vec{u} - \frac{1}{2}\Delta\vec{u}) + \frac{1}{2}\nabla\vec{u}(\frac{1}{2}\nabla\vec{u} - \frac{1}{4}\Delta^2\vec{u})](\Delta\rho^k - \Delta\rho^j)$$

$$+ [\vec{u}(\frac{1}{2}\nabla\vec{u} - \frac{1}{4}\Delta^2\vec{u}) + \frac{1}{2}\nabla\vec{u}(\vec{u} - \frac{1}{2}\Delta\vec{u})]\ (\Delta\rho^k + \Delta\rho^j) \quad . \tag{35}$$

Denoting now

$$\left.\begin{aligned} \Delta\rho^m &= \frac{1}{2}(\Delta\rho^k + \Delta\rho^j), \quad \text{and} \\ \Delta^2\rho &= \Delta\rho^k - \Delta\rho^j \end{aligned}\right| \tag{36}$$

we obtain

$$- \nabla\vec{u}\ \Delta\vec{R} = ((\vec{u})^2 - \frac{1}{2}\vec{u}\Delta\vec{u} + \frac{1}{4}\nabla\vec{u}\ \nabla\vec{u} - \frac{1}{8}\nabla\vec{u}\ \Delta^2\vec{u})\Delta^2\rho$$

$$+ (\vec{u}\nabla\vec{u} - \frac{1}{2}\vec{u}\Delta^2\vec{u} + \vec{u}\nabla\vec{u} - \frac{1}{2}\Delta\vec{u}\ \nabla\vec{u})\ \Delta\rho^m \quad . \tag{37}$$

Using eqns. (14), it can be shown that the expression in the first set of parentheses in eqn. (37) is equal to $[(\vec{u})^2 + \frac{1}{4}(\nabla\vec{u})^2]$ whereas the expression in the second set reduces to $2\vec{u}\nabla\vec{u}$. We thus get finally:

$$\boxed{- \nabla\vec{u}\ \Delta\vec{R} = [(\vec{u})^2 + \frac{1}{4}(\nabla\vec{u})^2]\ \Delta^2\rho + 2\vec{u}\nabla\vec{u}\ \Delta\rho^m} \quad . \tag{38}$$

We note that to obtain the solution $\Delta\vec{R}$ we not only have to know the differential range differences $\Delta^2\rho$ but also the mean differential ranges $\Delta\rho^m$. Using the definition of $\Delta^2\rho$ and $\nabla^2\rho$ we can show the obvious:

$$\Delta^2\rho = \nabla^2\rho \quad . \tag{39}$$

Thus eqns. (27) and (38) should be considered equivalent and are reducible to each other. Either equation can be used for either differential range difference observations $\Delta^2\rho$, or for differenced range difference observations $\nabla^2\rho$. The choice is between parameterization using $\Delta\vec{u}$, $\Delta^2\vec{u}$, $\Delta\vec{r}$, $\vec{R}_m$ and $\vec{r}^m$ in the case of (27) or $\vec{u}$ and $\Delta\rho^m$ for (38). Undoubtedly other parameterizations are possible.

We shall now have a look at the three models from the point of view of suitability for differential positioning.

COMPARISON OF THE DEVELOPED MODELS

All three equations for the baseline vector, using the differenced range differences (27), differential ranges (31) or differential range differences (38) are exact and linear in both the unknowns and observables. There is however a considerable difference between the three equations.

Whereas in the case of the differential Doppler determination of $\nabla\rho$ the satellite locations S^j, S^k are separated by about 10^5 m (for one 30-second Doppler integration interval) along one pass, for the differential range differencing techniques the optimal satellite configuration would require S^j and S^k to subtend a large angle (e.g., 90^o) at the baseline. Thus, while ω_1, ω_2 (see Figure 1) for one Doppler measurement are of the order of 5×10^{-3} radians they would optimally be close to 90^o for the differential range differencing. Clearly, differenced Doppler observations $\nabla^2\rho$ would have to be measured with an accuracy at least two orders of magnitude greater than either the differential range observations $\Delta\rho^j$ in eqn. (31) or the differential range difference observations $\Delta^2\rho$. The geometric disadvantage would tend to disappear, of course, when the Doppler integration interval is extended; more than one hour of integration would be needed however to get a good configuration [Fell, 1980]. The effect of imperfect knowledge of $\Delta\vec{r}$ can be minimized by selecting passes that are approximately normal to $\Delta\vec{R}$. In such cases $\nabla\vec{u}$ tends to be normal to $\Delta\vec{r}$ and the second term on the right hand side of eqn. (27) will go to zero. It is interesting to see that under these circumstances even $\Delta^2\vec{u}$ tends to $\vec{0}$ and the third term does not contribute appreciably either.

Obviously, not much is achieved from the geometrical point of view when differential range differences (or differenced range differences) are used instead of just differential ranges. On the one hand, the best satellite configuration for the differential range differencing can only bring $\nabla\vec{u}$ close to a unit vector and make the effect of errors in $\Delta^2\rho$ $(\nabla^2\rho)$ on $\Delta\vec{R}$ as small as that of differential ranges. On the other hand, there are the additional terms that generally will reduce the accuracy of $\Delta\vec{R}$. It is important to bear in mind that the argument in favour of differential range differences is based on elimination of a non-geometrical effect we have not considered here; that of imperfect clocks.

Let us now have a closer look at eqn. (31). If an accuracy of 1 cm in $\Delta\vec{R}$ is to be achieved then $\vec{u}^j$ must be known to a relative accuracy of at least 10^{-7}. This, in turn, implies a required accuracy of at least 1 m in $\vec{r}^j$, $\vec{R}_1$ and $\vec{R}_2$ which is achievable only in an iterative fashion.

We can rewrite the coefficient of $\Delta\rho^j$ as follows:

$$\vec{u}^j\vec{e}_1^j = \frac{1}{2}(\vec{e}_1^j + \vec{e}_2^j)\vec{e}_1^j = \frac{1}{2}(1 + \vec{e}_2^j\vec{e}_1^j)$$

$$= \frac{1}{2}(1 + \cos\omega^j) = 1 - \sin^2\frac{\omega^j}{2} \quad . \tag{40}$$

Since the parallactic angle ω^j will be of the order of 4×10^{-3} radians for $\Delta R \simeq 10^5$ m, we can expand the $\sin^2$ function in a power series obtaining

$$\vec{u}^j\vec{e}_1^j = 1 - \frac{(\omega^j)^2}{4} + \ldots. \tag{41}$$

and take only the first two terms, if an accuracy of no better than 1 cm in $\Delta\vec{R}$ is required. We note, however, that the second term must not be neglected since it is of the order of 5×10^{-6}.

To an accuracy of about 10^{-8} then, the range difference mathematical model is

$$\vec{u}^j\Delta\vec{R} = -(1 - \frac{(\omega^j)^2}{4})\,\Delta\rho^j \quad . \tag{42}$$

This equation has been derived, using a different approach, by Bossler et al. [1980].

CONCLUSIONS

We have considered differential GPS positioning from the purely geometrical point of view. The basic geometrical building block of differential satellite positioning is the tetrahedron. We have shown that the shape of any tetrahedron can be uniquely described by four defining vectors which have some definite geometrical meaning. These vectors provide a very versatile tool for the design of mathematical models, which turn out to be linear and exact.

From the geometrical point of view, all differential GPS measurements can be classified either as differential ranges or as differential range differences. We have shown mathematically what is known intuitively: that differential ranges have more geometrical strength than differential range differences, and that differential range differences (double differences) are equivalent to differential Doppler (differenced range difference) measurements.

To translate these geometrical insights into practical differential GPS positioning tools, two steps remain to be undertaken. The first step is to introduce the effect of non-geometric considerations, such as imperfect clocks and non-simultaneous observations, refraction, and errors in assumed satellite positions. The second step is to extend our investigation to the geometry of many tetrahedrons (i.e., as in the adjustment of GPS observations from many ground stations involving many satellite points).

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final draft.

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TABLE 1

Notational Conventions.

$\vec{r}^j$	position vector of satellite position S^j.
$\vec{R}_i$	position vector of ground station P_i.
$\vec{\rho}_i^j$	range vector from $\vec{R}_i$ to $\vec{r}^j$. ρ_i^j is the length of $\vec{\rho}_i^j$.
$\vec{e}_i^j$	unit vector from $\vec{R}_i$ to $\vec{r}^j$. A tetrahedron involves four such unit vectors.
$\vec{u}$	(with or without super and subscripts and prefixed Δ, ∇, Δ^2 or ∇^2) some linear combination of the four unit vectors in a tetrahedron.
prefix ∇	difference between two quantities involving two satellite positions.
prefix Δ	difference between two quantities involving two ground stations.
prefix ∇^2	difference between two differential quantities involving ∇ prefixes.
prefix Δ^2	difference between two differential quantities involving Δ prefixes.
prefix D, d	difference between two quantities.

LEGENDS TO FIGURES

Figure 1: Differential GPS Tetrahedron.

Figure 2: Scheme for Deriving the Defining Vectors.

Figure 3: Four Basic Differential Ranging Modes.

Figure 4: Differential Range Geometry.

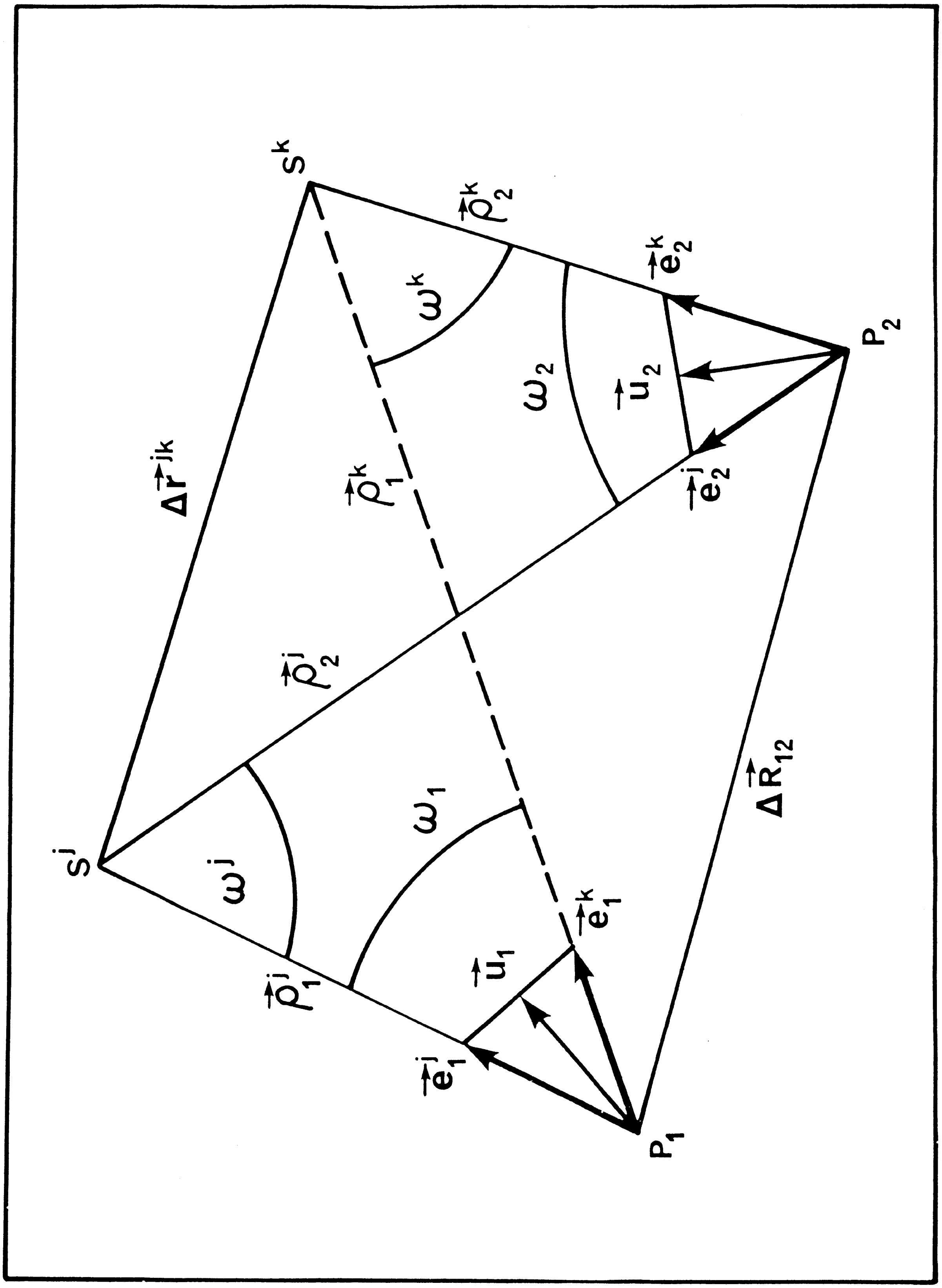

S^k
S^j
P_1
P_2
$\Delta \vec{r}^{jk}$
$\Delta \vec{R}_{12}$
$\vec{\rho}^k_2$
$\vec{\rho}^k_1$
$\vec{\rho}^j_2$
$\vec{\rho}^j_1$
$\vec{e}^k_2$
$\vec{e}^j_2$
$\vec{e}^k_1$
$\vec{e}^j_1$
$\vec{u}_2$
$\vec{u}_1$
ω^k
ω^j
ω_2
ω_1

Figure 1

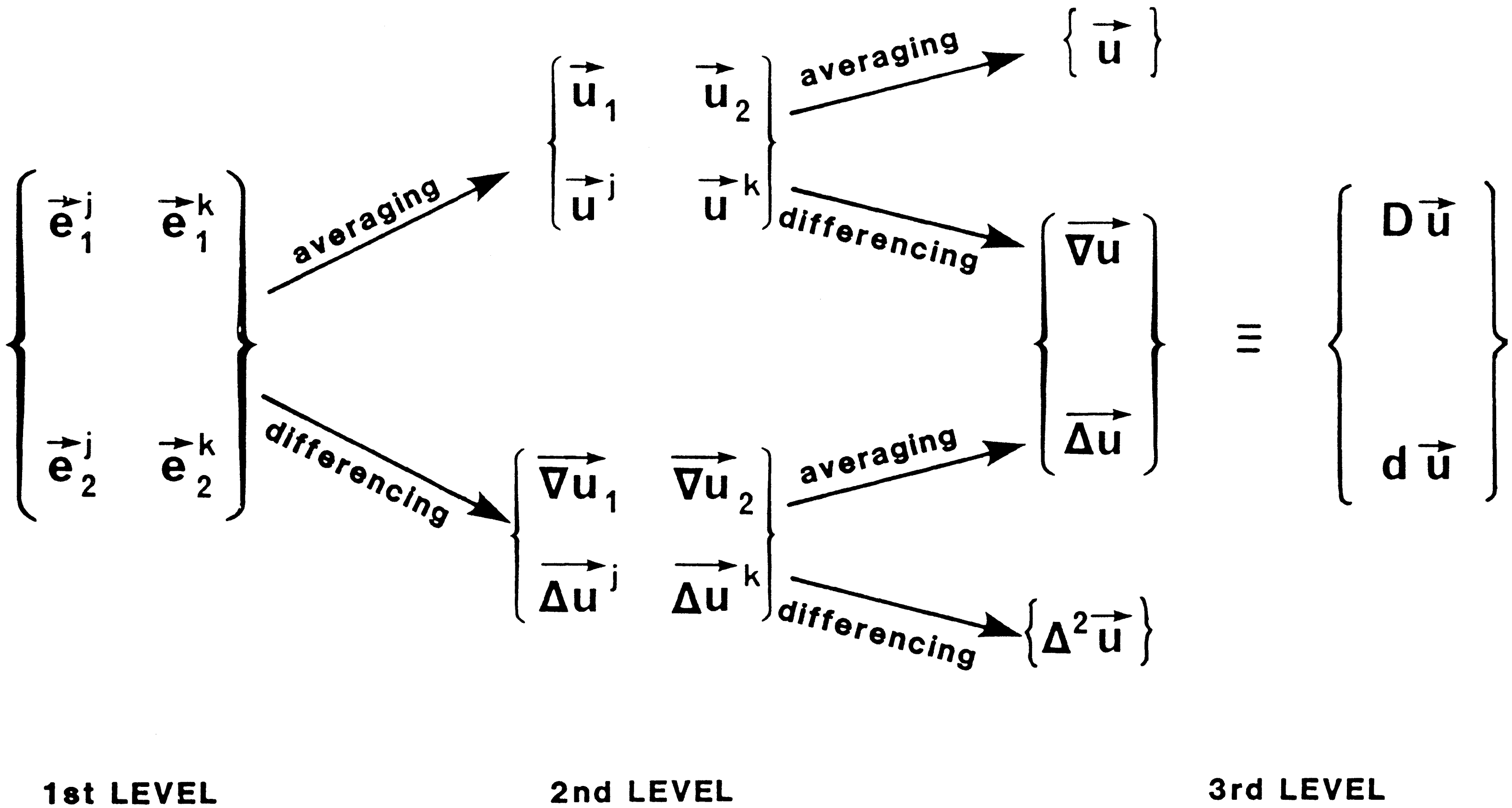

averaging
differencing
averaging
differencing
averaging
differencing
1st LEVEL
2nd LEVEL
3rd LEVEL

Figure 2

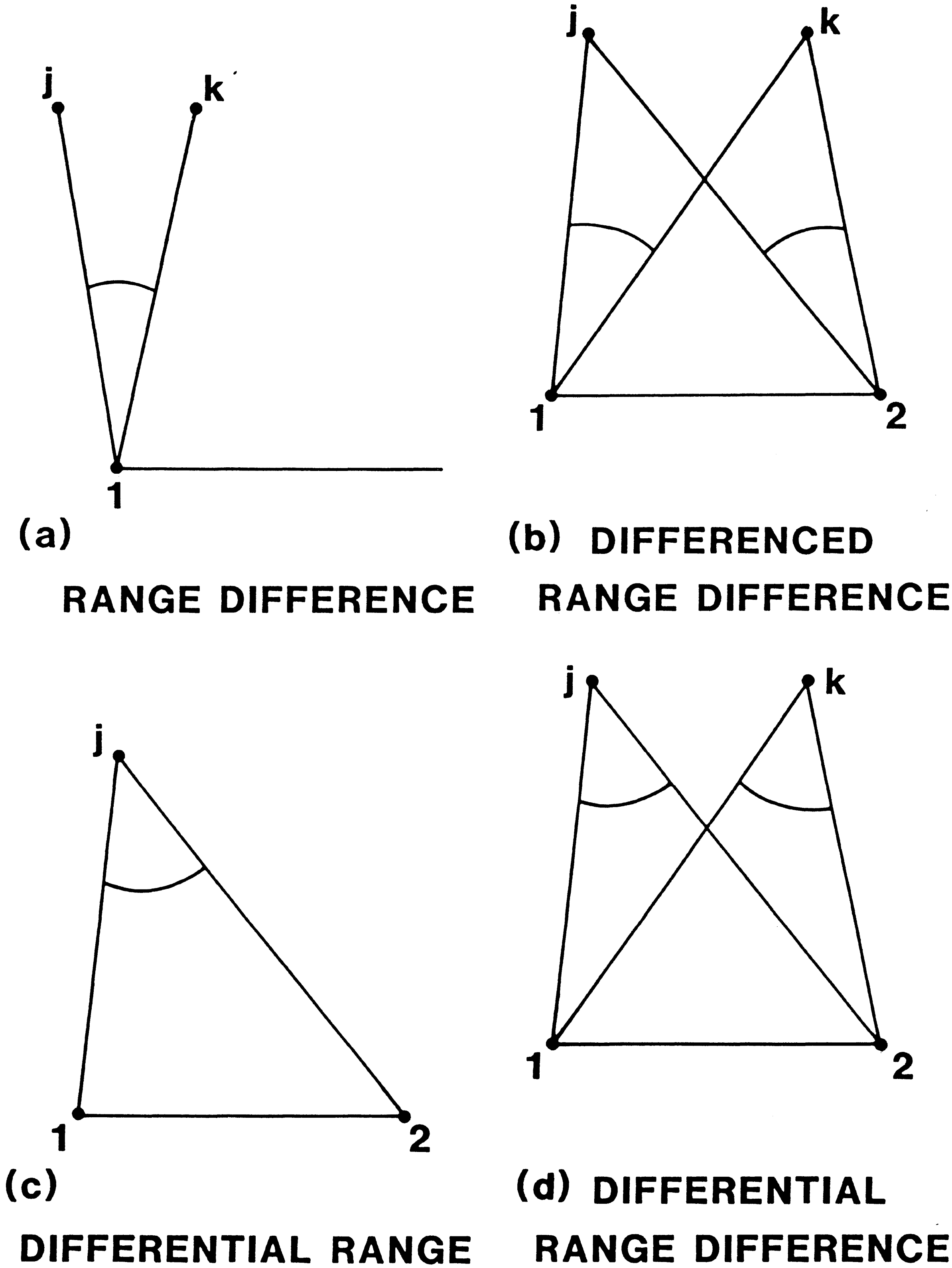
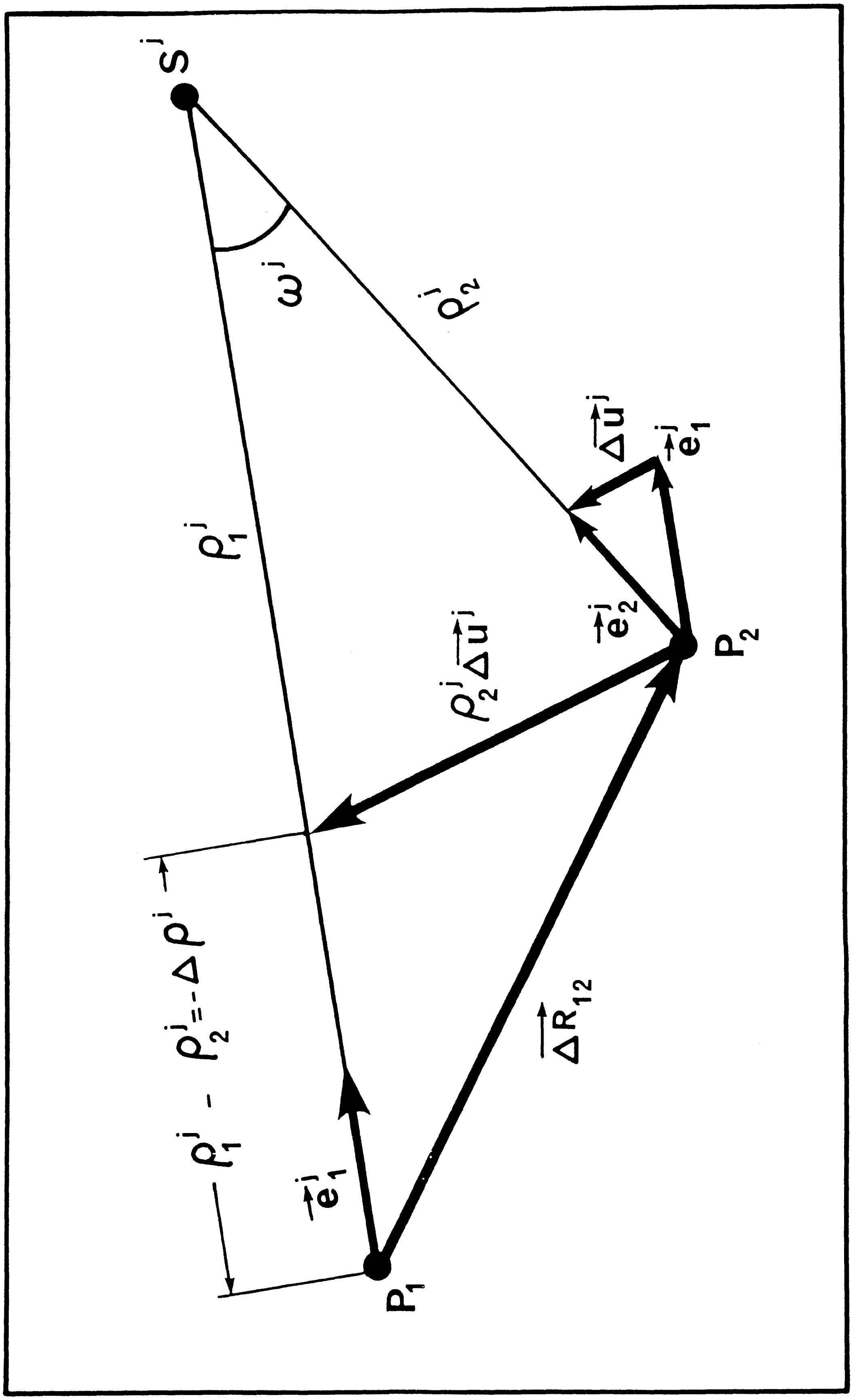

j
k
1
(a)
RANGE DIFFERENCE
j
k
1
2
(b) DIFFERENCED
RANGE DIFFERENCE
j
1
2
(c)
DIFFERENTIAL RANGE
j
k
1
2
(d) DIFFERENTIAL
RANGE DIFFERENCE

Figure 3



S^j
ω^j
ρ_2^j
ρ_1^j
$\overrightarrow{\Delta u}^j$
$\vec{e}_1^{\,j}$
$\vec{e}_2^{\,j}$
P_2
$\rho_2^j \overrightarrow{\Delta u}^j$
$\overrightarrow{\Delta R}_{12}$
$\rho_1^j - \rho_2^j = -\Delta\rho^j$
$\vec{e}_1^{\,j}$
P_1

Figure 4

Appendix C

GPS MASTER REFERENCE LIST

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