HEIGHTS ON A DEFORMING EARTH

GALO H. CARRERA

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PREFACE

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HEIGHTS ON A DEFORMING EARTH

Galo H. Carrera

Department of Surveying Engineering
University of New Brunswick
P.O. Box 4400
Fredericton, N.B.
Canada
E3B 5A3

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ABSTRACT

The purpose of this investigation is to

i) give a review of the present status and conceptual evolution of height networks in America and Europe,

ii) redefine path independent heights on a deforming earth,

iii) develop temporal corrections to be applied to present day networks,

iv) investigate the potential of terrestrial geodetic techniques as a tool to detect vertical crustal movements,

v) formulate a classification of mathematical models to extract and represent the vertical displacement field of the earth,

vi) present a survey of the changes that the geoid may experience in scale and shape, and

vii) perform an actual computational example of temporal corrections to a height network in eastern Canada with temporal inhomogeneities in its levelling observations and reference surface.

Recommendations are made for the redefinition of height networks regarding

i) a selection of a height system,

ii) the mathematical formulation of the adjustment,

iii) a selection of the different types of data to be used, and

iv) the number and type of corrections applied to the levelling data.

Several conclusions were reached:

i) path independence, or holonomy, can only be achieved when both heights or height differences and reference surfaces are homogeneous in time,

ii) the only rigorous approach to extract vertical crustal movements is that of a kinematic adjustment, when either scattered or connected segments are considered,

iii) the temporal cross-covariance matrix of any two sets of observations plays a key role in the design and adjustment of kinematic levelling nets,

iv) the geoid as a reference surface may experience changes in scale and shape, postglacial rebound being one of the most conspicuous.

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Last, a tribute to the person that shares my 'ups' and 'downs', and makes my life a fascinating adventure: Martha, my wife.

"What do you mean, Socrates? said Simmias. I have myself heard many descriptions of the earth, but I do not know, and I should very much like to hear the account in which you put faith.

"Well, Simmias, replied Socrates, it scarcely needs the art of Glaucus to give you a description; although I know that the art of Glaucus could prove the truth of my tale, which I myself should perhaps never be able to prove, and even if I could, I fear, Simmias, that my life would come to an end before the argument was completed. I may describe to you, however, the form and regions of the earth according to my conception of them.

"That, said Simmias, will be enough."

Plato, Phaedo 108 d-e

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Chapter I

GENERAL INTRODUCTION

Although determinations of heights date back early in history, it has been only since the work of Helmert (1880; 1884) that both height systems and their reference surfaces have been rigorously defined in space. It is Helmert's spatial concept that has prevailed during the last one hundred years in the establishment of height networks in the world.

A review of the evolution of different height concepts in America and Europe will serve as an adequate frame to present the main purpose of this work: A definition of path independent height systems in space and time.

A rigorous space-time definition of heights calls for a discussion of the entire vertical displacement field, i.e., changes in reference surfaces and heights themselves. It is worthwhile to recall in this respect Helmert's concept of the geoid:

> Specially important among the level surfaces is the surface of the oceans which one must consider here as subject only to the gravity of the earth and therefore calm, so that any movement due to the tides, winds, and other causes of ocean current is disregarded. This ideal ocean surface forms the visible part of a level surface. One calls it the mathematical surface of the earth or the geoid, in contrast to the real, physical surface of the earth. Through a system of channels leading from the ocean shore into the interior of the continents, one could also visualize the geoid there.

The geoid can be redefined as the equipotential surface of the gravity field that most closely approximates mean sea level in the least squares sense in space and time. However, since a continuous realization of the geoid in space-time is not feasible at present, epoch dependent solutions are useful alternatives (Mather, 1978; Castle and Vaníček, 1980).

Such an epoch dependent definition of height systems requires a complete description of the vertical displacement field of the earth. This task is not solely pursued by geodesy, but overlaps the field of other disciplines: relativity, paleomagnetism, structural geology, glacial geology and physics of the earth's interior. In this thesis, a survey of the results of the different approaches is presented.

Although a description of the vertical displacement field of the earth may be a useful piece of information about geologic processes (Lambert and Vaníček, 1979), no attempt is made here to provide a solution to the inverse geophysical problem posed by this kinematic description (Parker, 1977).

Finally, a case history in which space-time concepts are applied to achieve holonomity in the Maritime region of Canada is presented. Epoch dependent sea surface topography differences are obtained as a by-product of this work.

Chapter II

EVOLUTION OF HEIGHT NETWORKS IN AMERICA AND EUROPE

2.1 INTRODUCTION

Although levelling is probably the oldest geodetic technique, it has been during the last one hundred years that the most intensive research and widest collection of data has taken place. This can be best understood by tracing not only the incremental size, but also the underlying conceptual evolution of three different geodetic levelling networks: those of the United States, Canada and Western Europe.

2.2 GEODETIC LEVELLING NETWORKS IN THE UNITED STATES

A number of reports on the recent status of U.S. networks have been presented since the "First Symposium on Problems Related to the Redefinition of North American Networks" at the University of New Brunswick, Canada in 1974 (Whalen, 1978 1979, 1980; Lachapelle and Whalen, 1979; Lippold, 1980). Berry (1976) has given a chronology of levelling adjustments up to the General Adjustment of 1929. Table 2.1 summarises some of this information. This table shows a natural growth from a 1900 regional network in the eastern U.S. to the coast to coast network of 1929. Gravity corrections, as published by Bowie and Avers (1914), were applied to levelling data for the first time in the adjustment of 1912.

TABLE 2.1

First-Order Levelling in the United States

Year	Acummulated Levelling	Benchmarks	Benchmarks Fixed
1900	21 095 Km	4 200	5
1903	31 789	6 900	8
1907	38 359	9 100	9
1912	46 462	11 100	9
1929 +	106 724 ++	-	1 +++
1929 *	106 724 ++	-	26 **

+ Special Adjustment
++ 75 159 km of U.S. plus 31 565 km of Canadian levelling
+++ the tidal benchmark at Galveston, Texas
* General Adjustment
** 21 U.S. plus 5 Canadian benchmarks

By 1929 it was known that Mean Sea Level (MSL) was not a level (equipotential) surface. The Special Adjustment of 1929 only held fixed the height of the tide gauge in Galveston, Texas, for the sole purpose of studying the behaviour of MSL at different ports with respect to this 'levelling based' datum. On the other hand, in the adopted General Adjustment of 1929, MSL was held fixed at zero height at 21 U.S and 5 Canadian ports. The General Adjustment of 1929 was of a static nature, i.e., did not account for temporal variations heights. It was not homogeneous in time either, i.e., MSL determinations, as well as levelling observations, in general, were not updated to a common epoch.

It would be almost ten years before the first correction for levelling refraction appeared (Kukkamäki, 1938) and twenty years before the first tidal corrections were introduced (Jensen, 1949; Kukkamäki, 1949).

Different special adjustments have taken place since 1929. These have been of an experimental nature to investigate the behaviour of MSL with respect to 'levelling based' datums, e.g., the Special Adjustments of 1963 and 1980.

The Special Adjustment of 1980 was made for Mexico and Central America, holding one of five ties on the Mexico-U.S. boundary fixed. The positions resulting from this adjustment were then rigidly translated to match the U.S. positions of the 1963 Special Adjustment (Skaggs, 1980).

Lastly, it should be mentioned that regardless of these experimental adjustments, the levelling datum and heights in use in the U.S. today are those implied by the General Adjustment of 1929, to which all additional information has been referred. The Datums in use in Mexico and Central America are based on four different block adjustments made between 1959 and 1967.

2.3 GEODETIC LEVELLING NETWORKS IN CANADA

The status of Canadian networks has also been reviewed recently (Lachapelle et al. 1977; Lachapelle and Gareau, 1980). Cannon (1928, 1935) and Jones (1956) have given detailed accounts of the adjustments made between 1921 and 1952. And Dohler (1961, 1962) discussed the adoption of the International Great Lakes Datum in 1955. Table 2.2 summarises some of this information.

Nine adjustments were made between 1921 and 1928. The first of which was based on MSL held fixed in Halifax, Yarmouth, and Pointe au Père and a continental benchmark, Rouses Point (as obtained from MSL at Pointe au Pere), on the U.S.-Canada border. The eight additional adjustments added information successively, with previously adjusted positions not having been updated. This series of adjustments culminated in the 1928 adjustment of the whole network. Here, again, spatial variations of MSL were not accounted for, and MSL in five different ports, three on the Atlantic and two on the Pacific coasts, were held fixed at height zero. In this adjustment, the gravity correction to levelled height differences based on Bowie and Avers normal gravity was adopted (Nassar, 1977). The levelling datum and heights as implied by this adjustment are the ones in use today in Canada.

Several attempts to study the effect of levelling densification between 1930 and 1933 were made. They resulted in

TABLE 2.2

Annual First-Order Levelling in Canada (after Lachapelle, 1979)

Year	New Levelling Annual	New Levelling Acummulated	Relevelling
1907	374 km	374 km	
1908	998	1372	
1909	950	2322	
1910	692	3014	
1911	1216	4230	
1912	2024	6254	
1913	1706	7960	
1914	2665	10625	
1915	2903	13528	
1916	2211	15739	
1917	1255	16994	
1918	1337	18331	
1919	1677	20008	
1920	1783	21791	
1921	1585	23376	
1922	2102	25478	
1923	1125	26603	
1924	755	27358	
1925	916	28274	
1926	786	29060	
1927	1151	30211	
1928	721	30932	
1929	726	31658	
(Topo)	6619	38277	
(IBC)	790	39067	
1930	304	39371	
1931	155	39526	
1932	100	39626	
1933	111	39737	
1934	56	39793	
1935	1159	40952	
1936	462	41414	
1937	382	41796	
1938	362	42158	
1939	306	42464	
1940	0	42464	
1941	97	42561	
1942	0	42561	
1943	0	42561	
1944	1287	43848	
1945	1450	45298	

TABLE 2.2

Cont'd Annual First-Order Levelling and Rellevelling in Canada

	New Levelling		
Year	Annual	Acummulated	Relevelling
1946	1997	47295	193 km
1947	1477	48772	269
1948	1238	50010	42
1949	1375	51385	
1950	1095	52480	199
1951	1166	53646	19
1952	1500	55146	237
1953	1122	56268	517
1954	1535	57803	683
1955	1156	58959	322
1956	1414	60373	
1957	1483	61856	45
1958	1450	63306	
1959	1830	65136	23
1960	1949	67085	101
1961	1514	68599	1055
1962	1857	70456	572
1963	832	71288	2045
1964	1748	73036	828
1965	2602	75638	771
1966	1549	77187	1559
1967	2193	79380	1069
1968	3095	82475	660
1969	3796	86271	834
1970	2427	88698	776
1971	1551	90249	1578
1972	2789	93038	798
1973	2440	95478	1276
1974	2736	98214	781
1975	2791	101005	1429
1976	3014	104019	2048
1977	2753	106772	4235
1978	2975	109747	3080

various adjustments of the entire Canadian network. The first, 1930 'A', differed from the one of 1928 in that Rouses Point was not held fixed anymore. The three remaining, 1931 'B', 1932 'C', and 1933 'D', updated all levelled positions at their time. However, the heights of the five fundamental benchmarks adjacent to tidal stations, on which all these adjustments were based, were not updated. These heights were obtained at the time of the Low Water Datum determination (B. Tait and L. Ku personal communications, 1981): Halifax in 1895; Yarmouth in 1902; Pointe au Père in 1897; Vancouver in 1905; Prince Rupert in 1906.

Therefore, all levelling datums implied in all adjustments from 1921 to 1933, i.e., including the one of 1928 in current use, did not account for either spatial or temporal variations of MSL.

In order to have an insight into the distortions of the levelling net, because of neglecting MSL spatial variations, three more adjustments were made in 1936. American ports were also left out and the final positions did not change by more than 3 cm (Jones, 1956). A similar result was found in 1941.

Attention to the temporal homogeneity of MSL determinations was first paid in the experimental adjustments of 1952. In this year, new determinations of MSL were made for six Canadian ports. Two parallel adjustments with all additional data were performed. The first, held fixed only MSL at Yarmouth, in order to try to quantify the error induced by holding MSL at various other locations. The second, held MSL fixed at all six ports.

All adjustments, since 1928, were made using orthometric heights based on Bowie and Avers normal gravity. This made these levelling datums and heights useless for hydraulic purposes. An attempt to realize physically meaningful levelling datum and heights was madein 1955, which resulted in the International Great Lakes Datum (IGLD). This 1955 IGLD was established holding fixed MSL (averaged from 1941 to 1956) only at Pointe au Père (Dohler, 1970). Dynamic heights based on normal gravity, according to the U.S. Coast and Geodetic Survey, were used (Dohler, 1961; 1962).

2.4 GEODETIC LEVELLING NETWORKS IN WESTERN EUROPE

The establishment of levelling datums in Europe can be dated as far back as the XVII century, e.g., the Amsterdam Ordnance Datum in 1682. In addition, several re-levellings have taken place in individual countries in the past. Here, however, the Unified European Levelling Network of 1955 (UELN-55) is considered as a starting point.

The International Association of Geodesy (IAG) had already taken, at that time, an important stand with respect to key aspects of levelling nets. In Rome, 1954, the following resolution was adopted (IAG, 1955, p. 92):

> Considering the great scientific value of a simultaneous adjustment of the levelling nets, based on gravitational potential,
> Resolves, to appoint a Commission, which should meet at an early date, composed of qualified representatives of the levelling organizations of the different European countries, to undertake a simultaneous adjustment of the European levelling

> networks on the basis of gravitational potential; that European countries should be invited to designate their best qualified representatives, on the scale of one or two per country, to take part in that Commission.

The decision of performing the adjustment in geopotential numbers was apparently suggested by J.F. Baeschlin (Simonsen, 1955, p. 72). The basis of this decision was that no assumption with respect to the vertical gradient of gravity was required and that geopotential numbers would allow for transformations to any height system selected by the individual countries.

Two main goals should be reached (IAG, 1959, p. 3):

> i) Comparison of geopotential numbers for the MSL at the various mareographs attached to UELN or these geopotential numbers may be transformed into heights in metres in a European system by dividing each geopotential number by a local (observed) gravity at the tidal station in question.
>
> ii) Investigation of secular movement within the area covered by UELN by re-levelling of the network at certain fixed intervals, perhaps every 25 years.

Originally it was proposed in Florence that four institutions would carry out the adjustment (IAG, 1957, p. 60). In practice, the network was divided in two geographic blocks each of which was adjusted by one of two institutions. The northern block: Finland, Sweden and Norway, was placed in charge of the Finnish Geodetic Institute; and the rest of Western Europe: Denmark, Germany, The Netherlands, Belgium, Austria, Switzerland, Northern Italy, France, Spain, and Portugal was placed in charge of the Computing Centre of the Delft Technological University.

Two different philosophical approaches were followed by these institutions in their adjustments, and even though a detailed account will be given later, the following basic analogies and differences in performing them can be mentioned here:

i) As agreed, the units chosen by both centres were geopotential numbers. In the northern block, actual gravity values were obtained from maps of gravity anomalies (Sweden and Finland). In the southern block, geopotential numbers were based mostly on normal gravity.

ii) Neither of the two centres accounted quantitatively for spatial variations of MSL, i.e., Sea Surface Topography, at the tidal stations.

iii) Both centres computed differences of geopotential numbers between tidal stations, and statistical testing was made on them.

iv) For the first time, a correction for vertical crustal movements was applied to the northern block, i.e., the adjustment of that block was homogeneous in time. This was not the case with the southern block, very probably due to the lack of full data coverage.

The datum to which the whole network was referred was the NAP (Normal Amsterdam Peil) for the mean epoch of 1950, as an average from 1940 up to 1958 (Rossiter, 1960). Alberda (1963) acknowledged significant differences between NAP and MSL at 16 out of 44 European ports.

As a summary: The final adjustment was performed in geopotential numbers, all based on normal gravity as later agreed in Liverpool in 1959 (IAG, 1960, resolution No. 8), and based on MSL determined at one point only: NAP in Amsterdam. The system of heights finally chosen for practical purposes was that of Vignal (1954).

The need for a new adjustment including all additional data of the European network was decided upon in Brussels in 1973 (UELN-73) (IAG, 1973). Again two computing centres were chosen to perform independent adjustments but, this time, with the same data: the Computing Centre of the Delft Technological University and the Bayerische Akademie der Wissenschaften at Munich.

It was agreed that the adjustment be performed in two phases. First, a free adjustment carried out in geopotential numbers, and a second, information of MSL would be tested and somehow incorporated. Geopotential numbers are referred to the International Gravity Standarisation Net of 1971.

Corrections for vertical crustal movements have been applied so far to observations from Switzerland, Sweden and Finland, updated to the mean epoch of 1960 (Ehrnsperger, 1979). Up to now only preliminary reports of phase I have been presented (Kok et al, 1980).

2.5 A FEW CHARACTERISTICS OF PREVIOUS ADJUSTMENTS

The weighting scheme for different adjustments has changed from one to another and from network to network. For example, in the Canadian adjustment of 1928, levelling lines were weighted inversely to their length, a line of 20 miles being given a unit weight, and water transfers were regarded as errorless. In the Canadian experimental adjustment of 1952 lines were given the same weight, i.e., a line of 20 miles was given a unit weight, however, water transfers were now weighted according to their length, a unit weight being given for a length of 200 miles. In the adjustment of UELN-55, the adopted weight scheme was (IAG, 1957, p. 60; Kaariainen, 1960; Alberda, 1963):

$$P = \frac{200}{t^2 \, L_{km}} \,, \qquad (1\text{-}1)$$

where $t^2 = \sigma^2_{\Delta h}$ of 1 km of levelling in mm^2/km.

IAG (1959, p. 3) discussed the definition of t in 1957, and adopted it as the standard deviation of 1 km of levelling in mm^2/km instead of 10^{-3} g.p.u./$\sqrt{km}$ as used in some countries at that time.

UELN-73 has used this same a priori weighting scheme (Waalewijn, 1979; Kok et al, 1980). There is a very important assumption implied in all these schemes when weighting inversely to the length of the lines, namely that the height differences δh of the end points of individual segments within each line are statistically independent (Vaníček and Grafarend, 1980). The sum of the variances can then be written as

$$\sigma^2_{\Delta h} = \sum \sigma^2_{\delta h_i} \ . \tag{2-2}$$

Under the further assumption that all segments within a line are measured to the same accuracy, one can write

$$\sigma^2_{\delta h_i} = \sigma^2 \ell_i \ , \tag{2-3}$$

and then again from equation (2-2) one obtains

$$\sigma^2_{\Delta h} = \sigma^2 \sum \ell_i \ ,$$

$$= \sigma^2 L$$

or

$$\sigma_{\Delta h} = \sigma \sqrt{L} \ , \tag{2-4}$$

i.e., for the totally independent case the standard deviation propagates with the square root of the distance. However in the totally dependent case, the standard deviations instead propagate linearly with distance, i.e.,

$$\sigma_{\Delta h} = \sigma L \quad . \tag{2-5}$$

An analysis of the statistical dependence of the observations could explain perhaps why in most of the levelling nets the actual standard deviations coming from the adjustment are larger than the standard deviation of a levelled height difference obtained in the field.

The first approach, weighting inversely to the length of the line, was adopted by IAG in Oslo in 1948 (Braaten, et al, 1950) and has been applied since then. Much research about it is still taking place (Müller and Schneider, 1968; Lucht, 1972; Alberda, 1974; Remmer, 1975; Vaníček and Grafarend, 1980).

Different height systems have been used in different adjustments. Canadian and American national networks have used orthometric heights based on normal gravity as developed by Bowie and Avers, geopotential numbers have been used in Europe in UELN-55 and UELN-73, and later transformed into rigorous heights. The IGLD employed dynamic heights (based on normal gravity as developed again by Bowie and Avers (1914)) computed from orthometric heights. None of the formulae applied in these dynamic and orthometric height systems were rigorous (Nassar and Vaníček, 1975; Nassar, 1977).

MSL has been treated differently in Europe and America. While in UELN-55, only MSL at Amsterdam was taken as a reference, in most American and Canadian adjustments, MSL has been held fixed in several ports, probably due to the extensiveness of each network. MSL temporal variations have distorted particularly the American and Canadian nets, since MSL determinations have been heterogeneous in time. In UELN-55, this represents a systematic bias, because NAP does not actually coincide with present MSL at Amsterdam. MSL spatial variations remained as a common unsolved problem in American, Canadian and European adjustments. Corrections for refraction effects on level sightings were never applied to American or Canadian observations and only partially to European data, e.g., Finland (Kakkuri and Kääriäinen, 1977; Takalo, 1978). Corrections for vertical crustal movements in the northern block of UELN-55 have been reported by Kääriäinen (1960). A linear correction was applied to reduce all measurements to a mean epoch in order to perform an adjustment of epoch dependent height differences. Less frequent have been corrections to levelling observations for tidal effects such as those applied, for instance, in Denmark (Simonsen, 1950). The mathematical models employed in the Canadian adjustments, prior to the one of 1928, were of the parametric type (Cannon, 1928), i.e., they were formulated in the observation space as linear explicit models. Alernatively, the mathematical models used in the Canadian adjustment of 1928 and the U.S. of 1929 were formulated in the parameter space.

Regarding the European adjustment of UELN-55, two different approaches were followed in the formulation of its mathematical model. The northern block of the levelling net was adjusted in a parametric manner (Kääriäinen, 1960), and the southern part was adjusted following the phase method of J.M. Tienstra (1956), namely two models with common parameters but different observations. In phase one of this approach, the whole net was divided in four subnets, each of which was adjusted parametrically. In the second phase, the whole net was united in a conditional adjustment. Here the observations employed were linear combinations of all previously adjusted differences in geopotential numbers. Also, the four covariance matrices of the previously adjusted observations formed the covariance matrix of the 'new' observations.

Chapter III

TIME AND HEIGHTS

3.1 INTRODUCTION

The study of the earth's geometry is intrinsically a four dimensional anholonomic geodetic problem (Grafarend, 1976; 1978). This is a problem characterized by a path dependent triplet of spatial coordinates varying in time. Here, a definition of holonomic height systems is presented.

3.2 THE DEFINITION OF HEIGHTS ON A DEFORMING EARTH

In general, a space-time change in position is given by

$$\begin{bmatrix} \Delta \Lambda \\ \Delta \Phi \\ \Delta W \end{bmatrix} = \begin{bmatrix} \Delta_s \Lambda + \Delta_t \Lambda \\ \Delta_s \Phi + \Delta_t \Phi \\ \Delta_s W + \Delta_t W \end{bmatrix} , \tag{3-1}$$

where the subscripts s and t indicate space and time changes respectively.

Differential expressions for the first two rows of equation (3-1) have been given by Grafarend (1978).

For the last row of equation (3-1)

$$\Delta W = \Delta_s W + \Delta_t W, \tag{3-2}$$

a commutative diagram can be formulated

$$\begin{tikzcd}
W(1,1) \arrow[r, "\Delta_s^1 W"] \arrow[d, "\Delta_t^1 W"'] & W(2,1) \arrow[d, "\Delta_t^2 W"] \\
W(1,2) \arrow[r, "\Delta_s^2 W"'] & W(2,2)
\end{tikzcd}$$

then

$$W(1,2) = W(1,1) + \Delta_t^1 W, \tag{3-3}$$

$$W(2,2) = W(2,1) + \Delta_t^2 W, \tag{3-4}$$

$$W(2,1) = W(1,1) + \Delta_s^1 W, \tag{3-5}$$

$$W(2,2) = W(1,2) + \Delta_s^2 W, \tag{3-6}$$

where $\Delta_t^1 W$ and $\Delta_t^2 W$ are the time changes at points P_1 and P_2, and $\Delta_s^1 W$ and $\Delta_s^2 W$ are the spatial differences between points P_1 and P_2 at epochs t_1 and t_2 respectively. For instance, the gravity potential at point P_2 and epoch t_2, $W(2,2)$, (cf. equation (3-4)) is equal to the gravity potential at point P_2 and epoch t_1, $W(2,1)$, plus the temporal change at point P_2 between epochs t_1 and t_2, $\Delta_t^2 W$.

A spatial difference in gravity potential can be written using equations (3-3) and (3-4) as

$$W(2,2) - W(1,2) = W(2,1) + \Delta_t^2 W - (W(1,1) + \Delta_t^1 W), \tag{3-7}$$

$$= W(2,1) - W(1,1) + \Delta_t^2 W - \Delta_t^1 W, \tag{3-8}$$

or

$$\Delta_s^2 W = \Delta_s^1 W + \Delta_t^2 W - \Delta_t^1 W. \tag{3-9}$$

Equation (3-9) can be viewed as a temporal transformation of a gravity potential difference between any two given points P_1 and P_2.

Alternatively equations (3-4) and (3-9) can be further generalized to transform the gravity potential at any point P_i, and any spatial gravity potential difference between points P_i and P_j, from any epoch t_k to any other epoch t_ℓ, i.e.,

$$W(i,\ell) = W(i,k) + \Delta_t^i W, \tag{3-10}$$

and

$$\Delta_s^\ell W = \Delta_s^k W + \Delta_t^j W - \Delta_t^i W. \tag{3-11}$$

On a uniformly rotating rigid earth model, i.e., the assumption implied if all temporal terms vanish, one obtains

$$\Delta_s^{\ell} W = \Delta_s^{k} W, \tag{3-12}$$

or equivalently

$$\int_{W(i,\ell)}^{W(j,\ell)} d_s W = \int_{W(i,k)}^{W(j,k)} d_s W \quad , \tag{3-13}$$

thus

$$\oint_c d_s W = \oint_c dW = 0, \tag{3-14}$$

a result first studied by Helmert (1880; 1884) and acknowledged by Marussi (1949), Hotine (1969), and Grafarend (1975) in their three dimensional approaches.

On a deforming earth, however, the condition

$$\Delta_t^{i} W = 0, \tag{3-15}$$

is not satisfied. A kinematic misclosure appears

$$\oint_c d_s W \neq 0, \tag{3-16}$$

$$\oint_{c(t)} d_s W = - \oint_{c(t)} d_t W, \tag{3-17}$$

since the integral, in practice, is never carried out instantaneously, i.e., invariant in time. Path independence, or holonomy, can only be achieved in space-time, therefore, writing

$$\oint_{c(t)} dW = \oint_{c(t)} d_s W + \oint_{c(t)} d_t W = 0, \tag{3-18}$$

where the second integral on the right hand side of equation (3-18) constitutes the sum of all temporal corrections along the closed curve c.

Let us define the space-time geopotential number as

$$C(i,k) = -\,(W(i,k) - W(0,k)), \tag{3-19}$$

where W(0,k) is the gravity potential at the geoid at epoch t_k.

A geopotential number temporal difference at point P_i is given by

$$\Delta_t^i C = -\,\{(W(i,\ell) - W(i,k)) - (W(0,\ell) - W(0,k))\}\ , \tag{3-20}$$

or

$$\Delta_t^i C = -\,(\Delta_t^i W - \Delta_t^0 W), \tag{3-21}$$

and a geopotential number spatial difference at epoch t_ℓ by

$$\Delta_s^\ell C = -\ (W(j,\ell) - W(i,\ell)), \tag{3-22}$$

or

$$\Delta_s^\ell C = -\ \Delta_s^\ell W.$$

Analogously to equation (3-10) and (3-11) one can now write

$$C(i,\ell) = C(i,k) + \Delta_t^i C, \tag{3-24}$$

and

$$\Delta_s^\ell C = \Delta_s^k C + (\ \Delta_t^j C - \Delta_t^i C). \tag{3-25}$$

Hence, any rigorous height system can be redefined on a deforming earth.

The dynamic height system will be defined as

$$H^d(i,\ell) = \frac{C(i,\ell)}{G}\ . \tag{3-26}$$

where G is a scale factor.

Its one-point temporal transformation can then be obtained from equations (3-24) and (3-26)

$$H^d(i,\ell) = \frac{1}{G}\,(C(i,k) + \Delta_t^i C), \tag{3-27}$$

$$= H^d(i,k) + \Delta_t^i H^d, \tag{3-28}$$

thus a height difference temporal transformation can be obtained

$$H^d(j,\ell) - H^d(i,\ell) = H^d(j,k) - H^d(i,k) + \Delta_t^j H^d - \Delta_t^i H^d, \tag{3-29}$$

$$\Delta_s^\ell H^d = \Delta_s^k H^d + \Delta_t^j H^d - \Delta_t^i H^d. \tag{3-30}$$

A space-time orthometric height system can be defined as

$$H^o(i,\ell) = \frac{C(i,\ell)}{\bar{g}(i,\ell)}\,, \tag{3-31}$$

where $\bar{g}(i,\ell)$ is the mean value of gravity along the actual plumbline at point P_i and epoch t_ℓ. The 'one-point' and 'two point' temporal transformations in this system are given by

$$= \frac{C(i,k)}{\bar{g}(i,k)} + \Delta_t^i H^o, \tag{3-32}$$

$$= H^o(i,k) + \Delta_t^i H^o, \tag{3-33}$$

and

$$H^o(j,\ell) - H^o(i,\ell) = H^o(j.k) - H^o(i,k) + \Delta_t^j H^o - \Delta_t^i H^o, \tag{3-34}$$

$$\Delta_s^\ell H^o = \Delta_s^k H^o + \Delta_t^j H^o - \Delta_t^i H^o. \tag{3-35}$$

Finally, the space-time normal height system can be summarized as

$$H^n(i,\ell) = \frac{C(i,\ell)}{\bar{\gamma}(i,\ell)}, \tag{3-36}$$

$$= H^n(i,k) + \Delta_t^i H^n, \tag{3-37}$$

$$\Delta_s^\ell H^n = \Delta_s^k H^n + \Delta_t^j H^n - \Delta_t^i H^n. \tag{3-38}$$

where is the mean value of normal gravity at point P .

All the above space-time height systems are constituted by path independent observables. They are referred to as holonomic of the first kind,

$$\oint_c dH^d = \oint_c d_s H^d + \oint_c d_t H^d = 0, \tag{3-39}$$

$$\oint_c dH^o = \oint_c d_s H^o + \oint_c d_t H^o = 0, \tag{3-40}$$

$$\oint_c dH^n = \oint_c d_s H^n + \oint_c d_t H^n = 0 \qquad (3\text{-}41)$$

The diagrams of the holonomic height systems: geopotential numbers-dynamic-orthometric and geopotential numbers-dynamic-normal, and of the anholonomic in space and time height systems: the time invariant geopotential numbers-dynamic-orthometric based either on rigorous or approximate normal gravity, show in figure 3.1 their space-time transformations. Nassar and Vaníček (1975), Nassar (1977) and Vanicek and Krakiwsky (1982) have studied exhaustively the spatial interprertation of these height systems. They have also provided spatial transformations between the height differences of the holonomic and anholonomic systems shown in the above diagrams. Their work forms here the framework from which the interpretation of temporal changes will be made.

The reference surface, or datum, for the geopotential numbers, dynamic and orthometric heights is the geoid W(0,i).

An interpretation of the change in time of a geopotential number can be made by means of equation (3-21)

$$\Delta_t^i C = - (\Delta_t^i W - \Delta_t^o W)$$

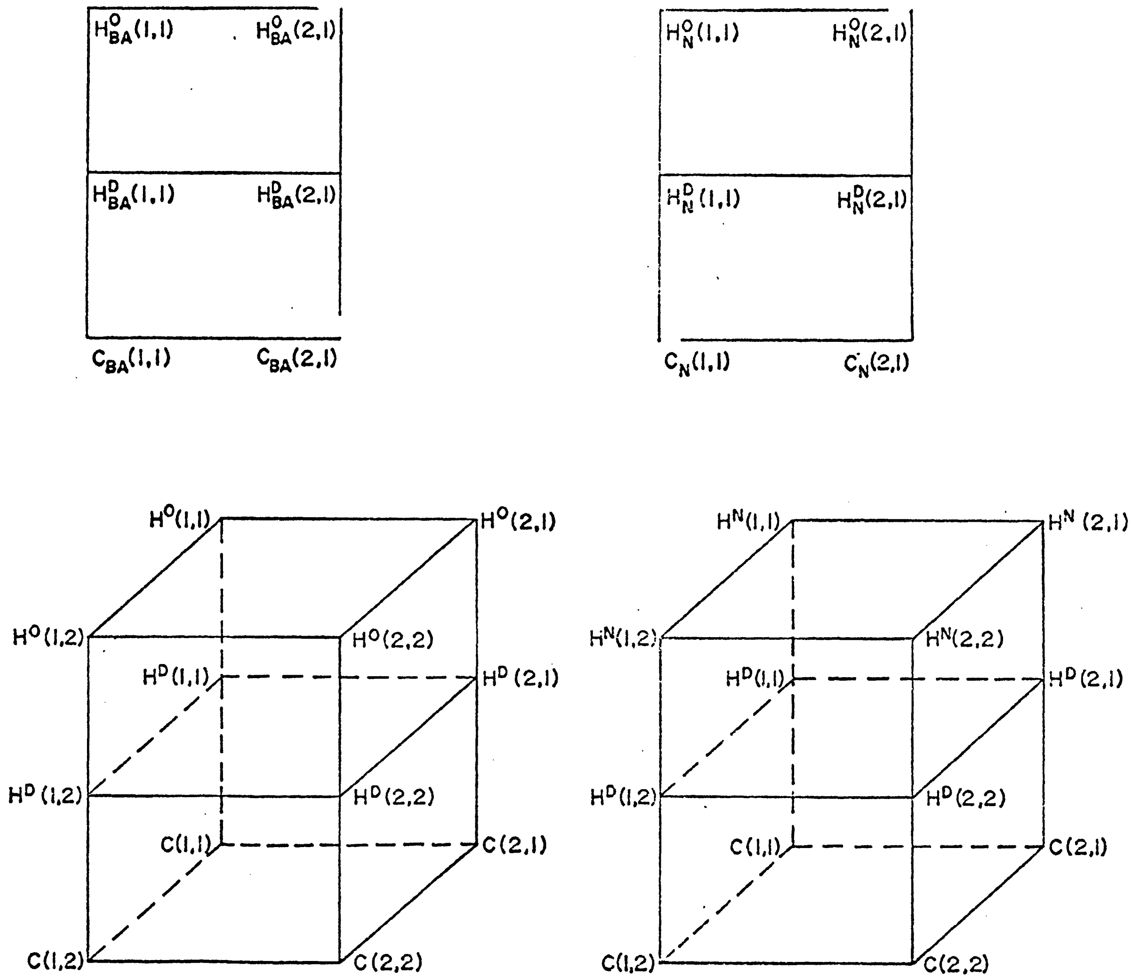


Figure 3.1 Space-Time Diagram of Height Systems

$\Delta_t^i C$ is a function of both the changes of gravity potential at point P_i and on the geoid.

Regarding dynamic heights the following statement can be made: A dynamic height difference between two equipotential surfaces of the earth's gravity field is always constant in space and time.

A geometric representation of a temporal change in height in the orthometric system is given in figure 3.2. Here a temporal height difference at point P_i can be understood as the difference of distances from the geoid up to point P_i along the actual 'new' and 'old' plumblines at epochs two and one respectively. It is of theoretical interest to note that heights in figure 3.2 have been drawn over plumblines with the same shape but changes in their curvature and torsion may occur.

A geometric representation of a temporal change in height in the normal system based on actual gravity is given in figure 3.3. The reference surface for this system is not the geoid but a (physically meaningless) time varying surface: the quasigeoid. A temporal normal height difference at point P_i can be understood as the difference in heights reckoned along the normal plumbline at epochs two and one respectively. Also, it is of theoretical interest to note that even when normal plumblines remain invariant in time, in general, different normal plumblines correspond to point P_i at the two epochs.

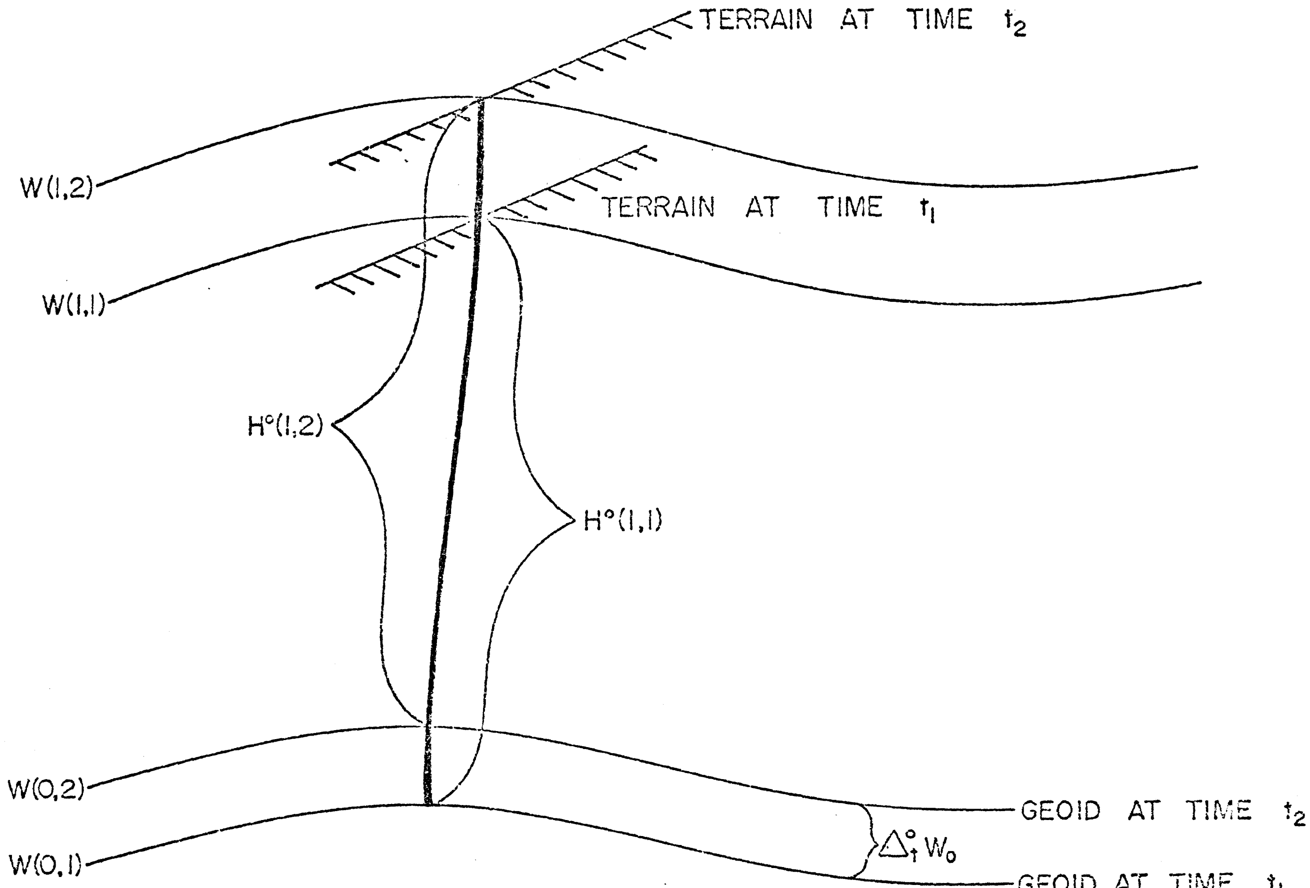


Figure 3.2 Temporal Height Change in the Orthometric System

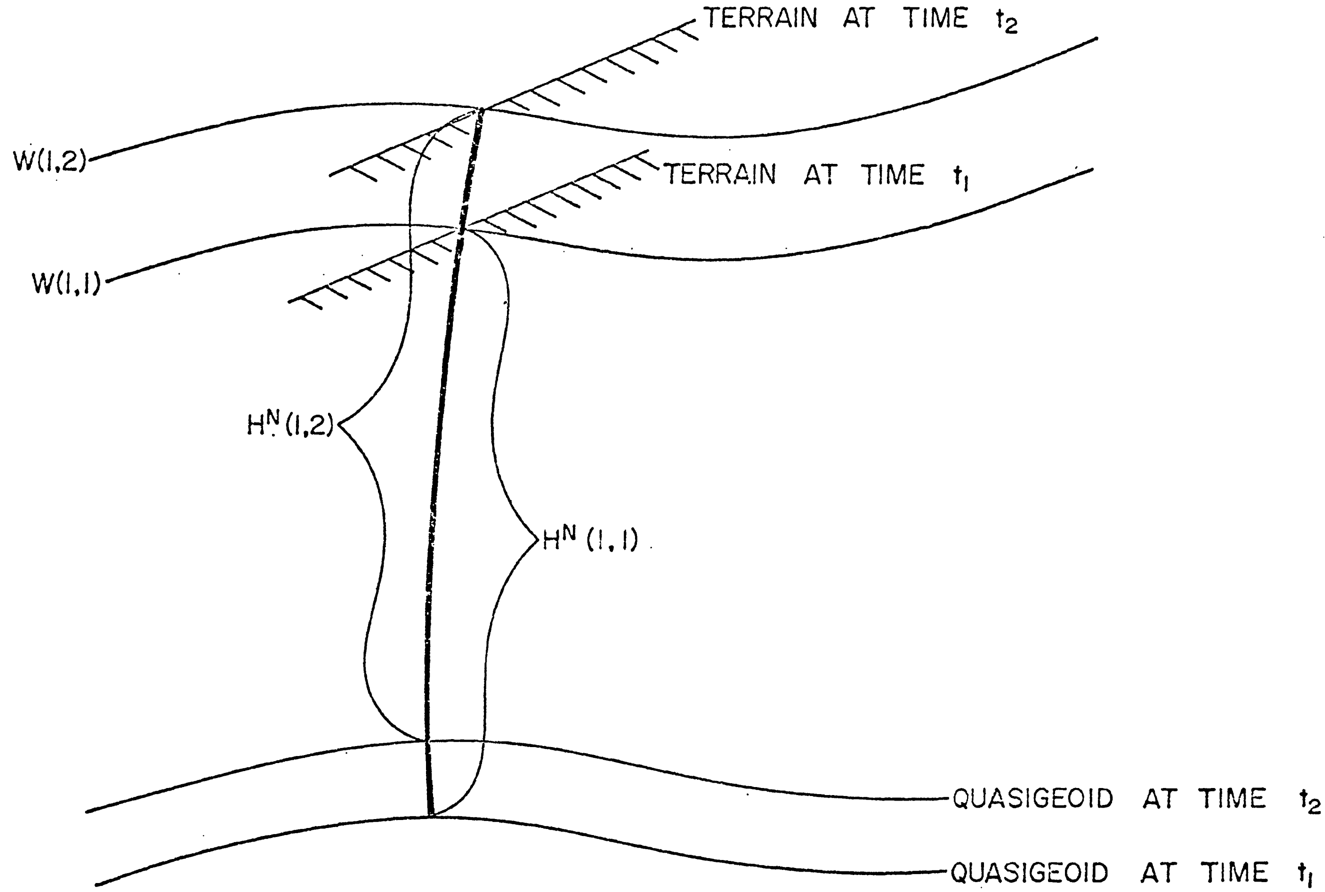


Figure 3.3 Temporal Height Change in the Normal System

Let us focus now on a particular 'two-point' temporal transformation: the height difference difference between the geoid and a tide gauge reference benchmark between epochs t_k and t_ℓ. Three types of temporal changes in quasiobservables are involved here:

i) in sea surface topography

$$\Delta_t^n SST = SST(n,\ell) - SST(n,k), \tag{3-42}$$

ii) in the height difference between local MSL and the conventional zero (CZ) of a tide gauge at point P_n

$$\Delta_t^n CZ = H(n,\ell) - MSL(n,\ell) - (H(n,k) - MSL(n,k)), \tag{3-43}$$

iii) and in the height difference between the conventional zero of a tide and the tide gauge reference benchmark at point P_m

$$\delta\Delta_s H = H(m,\ell) - H(n,\ell) - (H(m,k) - H(n,k)), \tag{3-44}$$

$$= \Delta_s^\ell H - \Delta_s^k H. \tag{3-45}$$

The total motion of a reference benchmark at point P_m with respect to the geoid can be expressed as

$$\Delta_t^m H = \Delta_t^n SST + \Delta_t^n CZ + \delta\Delta_s H. \tag{3-46}$$

Finally, each of equations (3-43), (3-44) and (3-46) can be expressed as temporal transformations between epochs t_k and t_ℓ

$$SST(n,\ell) = SST(n,k) + \Delta_t^n SST, \tag{3-47}$$

$$H(n,\ell) - MSL(n,\ell) = H(n,k) - MSL(n,k) + \Delta_t^n CZ, \tag{3-48}$$

$$\Delta_s^\ell H = \Delta_s^k H + \delta\Delta_s H. \tag{3-49}$$

3.3 TEMPORAL HOMOGENIZATION OF HEIGHT NETWORKS

It has been concluded from equation (3-18) that path independence, or holonomy, can only be achieved in space-time by adding a correction for time dependent effects. Therefore, disregarding all random and systematic errors, a levelling net can achieve holonomy not only when the geometry of the earth's gravity field has been accounted for, but also its variations in time.

Recalling equations (3-31), (3-36) and (3-39) one may find a way to transform rigorous height differences from any epoch t_ℓ to a reference epoch t_r as

$$\Delta_s^r H^d = \Delta_s^\ell H^d - (\Delta_t^j H^d - \Delta_t^i H^d), \tag{3-50}$$

$$\Delta^{r}_{s}H^{o} = \Delta^{\ell}_{s}H^{o} - (\Delta^{j}_{t}H^{o} - \Delta^{i}_{t}H^{o}), \tag{3-51}$$

$$\Delta^{r}_{s}H^{n} = \Delta^{\ell}_{s}H^{n} - (\Delta^{j}_{t}H^{n} - \Delta^{i}_{t}H^{n}). \tag{3-52}$$

All these temporal corrections may take one of four forms:

i) A continuous function in space and time,

ii) a continuous function in space and discontinuous in time,

iii) a discontinuous function in space and continuous in time, and

iv) a discontinuous function in space and time.

Here the vertical displacement field $\Delta_t H$ can be expressed as

$$\Delta_t H(x,y,t) = \sum_{k=1}^{n_t} C_{ok}\, T_k(t) + \sum_{i=0}^{n_x} \sum_{j=0}^{n_y} \sum_{k=1}^{n_t} C_{ijk}\, x^i y^j T_k(t), \tag{3-53}$$

where $T_K(t)$ may be a continuous or discontinuous function of time and the value of the coefficients C and C may may be dependent upon the region covered to model spatial discontinuities.

It is of theoretical interest to note that, rigorously, the vertical displacement field $\Delta_t H(x,y,t)$ should be expressed in the same height system of the quasiobservables.

The practical realization in space-time of the geoid is carried out at the tide gauges, and it is there connected to the height network. The temporal transformations of the height differences between the geoid, local MSL, the conventional zero of the tide gauge, and the reference benchmark are given by equations (3-47), (3-48) and (3-49), respectively. An analogous set of equations to (3-50), (3-51) and (3-52) can now be derived to transform these three height differences at each tide gauge from an arbitrary epoch t_ℓ to a reference epoch t_r

$$SST(n,r) = SST(n,\ell) - \Delta_t^n SST, \tag{3-54}$$

$$H(n,r) - MSL(n,r) = H(n,\ell) - MSL(n,\ell) - \Delta_t^n CZ, \tag{3-55}$$

$$\Delta_s^r H = \Delta_s^\ell H - \delta\Delta_s H. \tag{3-56}$$

Equation (3-54), although of theoretical value, is never used due to our ignorance, at present, of both SST and, mainly, its variations in time. Equation (3-55) on the other hand can be readily evaluated from the analysis of sea level records. Finally, equation (3-56) usually reflects the local stability between the tide gauge and the reference benchmark, and it is periodically reevaluated. The sum of all three corrections in the above equations may be referred to as a geoid-tidal benchmark correction.

Chapter IV

TEMPORAL VARIATIONS OF THE GEOID

4.1 INTRODUCTION

A definition of heights on a deforming earth calls for a temporal definition of the reference surfaces from which they are reckoned: the geoid or the quasigeoid.

The geoid may experience changes in both scale and shape, i.e., in the value of the gravity potential, and in the value of all coefficients of its spherical harmonic expansion.

4.2 VARIATIONS IN THE SCALE OF THE GEOID

Variations of the gravitational constant , G, as well as of the semi-major axis, a, lead to temporal scale changes of the geoid.

There are various metric theories of gravitation that, in contradiction to general relativity, postulate the existence of a preferred reference frame and an anisotropic gravitational constant G(t) (Ni, 1972; Norvedt and Will, 1972). If this is the case, the geoid would experience a change in scale with a period prescribed by the earth's motion with respect to the assumed preferred frame.

The theoretical viability of temporal changes in G has been examined, for example, by Will (1971), and Norvedt and Will (1972).

Recent attempts to quantitatively assess such temporal changes have been made, for example, by Shapiro et al (1971), and Warburton and Goodkind (1976). Shapiro's results have set an upper limit of 4×10^{-10} for the annual rate change. Warburton and Goodkind's results were also unable to confirm any change in G. This result was due to the high imposed noise of earth, oceanic, and atmospheric tides, and, probably, resonances in the liquid core on the gravity measurements made with the superconducting gravimeter.

The semi-major axis, a, is another parameter whose change could also induce a change in the scale of the geoid. Different paleomagnetic techniques were designed during the 1960's to quantify these changes over millions of years (Hospers and Van Andel, 1970). However, their results are now thought to be spurious (Wesson, 1975, p. 354).

4.3 VARIATIONS IN THE SHAPE OF THE GEOID

Different phenomena lead to changes in the shape of the geoid, for example,

i) Rotational effects,

ii) tides,

iii) loads, and

iv) other sources of crustal movements.

4.3.1 Variations of the Geoid Due to Rotational Effects

The centrifugal potential $W_c(t)$ can be expressed for a rigid earth model as (Vaníček and Krakiwsky, 1982, p. 83):

$$W_c(t) = \frac{1}{2}\,\omega^2 p_a^2, \tag{4-1}$$

where

$$\omega^2 = \underline{\omega} \cdot \underline{\omega}, \tag{4-2}$$

$$= |\,\underline{\omega}\,|^2, \tag{4-3}$$

and

$$p_a^2 = (\underline{x}_a \quad \underline{x}_a) - \left(\frac{\underline{\omega} \cdot \underline{x}_a}{\omega}\right)^2, \tag{4-4}$$

or

$$p_a^2 = r^2 - \left(\frac{\underline{\omega} \cdot \underline{x}_a}{\omega}\right)^2, \tag{4-5}$$

where $\underline{\omega}$ and $\underline{x}_a$ are vectors that describe the earth's variable angular velocity and the time invariant position of point a, respectively.

Substituting equation (4-5) in (4-1), one obtains (Rochester and Smylie, 1974):

$$W_c(t) = \frac{1}{2}\,\omega^2 r^2 - \frac{1}{2}\,(\underline{\omega} \cdot \underline{x}_a)^2. \tag{4-6}$$

Explicitly, equation (4-6) reads (Munk and Macdonald, 1960, p. 25)

$$W_c(t) = \frac{1}{3}\omega^2 r^2 + \frac{1}{6}[\omega_1(x_2^2 + x_3^2 - 2x_1^2) + \omega_2(x_3^2 + x_1^2 - 2x_2^2)$$

$$+ \omega_3(x_1^2 + x_2^2 - 2x_3^2) - 6\omega_1\omega_2 x_1 x_2 - 6\omega_2\omega_3 x_2 x_3 - 6\omega_3\omega_1 x_3 x_1] ,$$

or

$$W_c(t) = \frac{1}{3}\omega^2 r^2 + \frac{1}{6}[x_1^2(\omega_2^2 + \omega_3^2 - 2\omega_1^2) + x_2^2(\omega_3^2 + \omega_1^2 - 2\omega_2^2)$$

$$+ x_3^2(\omega_1^2 + \omega_2^2 - 2\omega_3^2) - 6\omega_1\omega_2 x_1 x_2 - 6x_3(\omega_2\omega_3 x_2 + \omega_3\omega_1 x_1)] , \tag{4-7}$$

which can be expressed in terms of zonal, tesseral, and sectorial spherical harmonics as

$$W_c(t) = \frac{1}{3}\omega^2 r^2 + \frac{r^2}{6}((\omega_1^2 + \omega_2^2 - 2\omega_3^2)\ P_{20}(\cos\theta)$$

$$- 2(\omega_2\omega_3 \sin\lambda + \omega_3\omega_1 \cos\lambda)\ P_{21}(\cos\theta)$$

$$+ \frac{1}{2}((\omega_2^2 - \omega_1^2) \cos 2\lambda - 2\omega_1\omega_2 \sin 2\lambda)\ P_{22}(\cos\theta)) , \tag{4-8}$$

an equivalent result to that given by Lambeck (1980, p. 41).

The direction cosines of $\underline{\omega}$ with respect to the CT system are given by (Munk and Macdonald, 1960, p. 14)

$$m_1 = \frac{\omega_1}{\Omega}$$

$$m_2 = \frac{\omega_2}{\Omega} , \tag{4-9}$$

$$1 + m_3 = \frac{\omega_3}{\Omega}$$

where ω_1 and ω_2 are the components of polar motion, ω_3 is the variable spin velocity, and Ω is the mean diurnal angular velocity of the earth. m_1 and m_2 are of the order of 10^{-6}, and m_3 is of the order of 10^{-8}.

Equation (4-8) can be rewritten as

$$\begin{aligned} W_c(t) = \tfrac{1}{3}\Omega^2 r^2 \{ & [m_1^2 + m_2^2 + (1 + m_3)^2] P_{00}(\cos\theta) \\ & + \tfrac{1}{2}[m_1^2 + m_2^2 - 2(1 - m_3)^2] P_{20}(\cos\theta) \\ & - [m_2(1 + m_3)\sin\lambda + (1 + m_3) m_1 \cos\lambda] P_{21}(\cos\theta) \\ & + \tfrac{1}{4}[(m_2^2 - m_1^2)\cos 2\lambda - 2m_1 m_2 \sin 2\lambda] P_{22}(\cos\theta)\} . \end{aligned} \quad (4\text{-}10)$$

This is a linear combination of stationary and non-stationary terms of the form

$$W_c(t) = W_s + W_s(t) + W_p(t) + W_{sp}(t), \quad (4\text{-}11)$$

where W_s and $W_s(t)$ are the stationary and non-stationary components of the spin velocity potential, $W_p(t)$ is the polar motion potential, and $W_{sp}(t)$ is the effect of the earth's variable spin velocity imposed on polar motion. Thus

$$W_s = \frac{1}{3}\Omega^2 r^2 [P_{00}(\cos\theta) - P_{20}(\cos\theta)], \quad (4\text{-}12)$$

$$W_s(t) = \frac{1}{3}\Omega^2 r^2 [(2m_3 + m_3^2) P_{00}(\cos\theta) - (2m_3 + m_3^2) P_{20}(\cos\theta)] , \quad (4\text{-}13)$$

$$W_p(t) = \frac{1}{3}\,\Omega^2 r^2 [(m_1^2 + m_2^2) P_{00}(\cos\theta)$$

$$+ \frac{1}{2}(m_1^2 + m_2^2) P_{20}(\cos\theta)$$

$$-((m_2 \sin\lambda) - (m_1 \cos\lambda))\, P_{21}(\cos\theta)$$

$$+ \frac{1}{4}\,((m_2^2 - m_1^2)\cos 2\lambda - 2m_1 m_2 \sin 2\lambda)\, P_{22}(\cos\theta)], \qquad (4\text{-}14)$$

$$W_{sp}(t) = -\frac{1}{3}\,\Omega^2 r^2 [(m_3 m_2)\sin\lambda + (m_3 m_1)\cos\lambda] P_{21}(\cos\theta). \qquad (4\text{-}15)$$

In common gravity reductions only the stationary spin veloccity component W_s is eliminated.

The centrifugal potential $W_c(t)$ can be further generalized for a deforming earth. Its components are given as

$$W_s = \frac{2}{3}\,\Omega^2 r^2 \; P_{00}(\cos\theta) - P_{20}(\cos\theta) \quad , \qquad (4\text{-}16)$$

for the stationary spin velocity,

$$W_s(t) = \frac{1}{3}\,\Omega^2 r^2 [\,(2m_3 + m_3^2)(1 + k_0) P_{00}(\cos\theta)$$

$$- (2m_3 + m_3^2)(1 + k_2) P_{20}(\cos\theta)] \, , \qquad (4\text{-}17)$$

for the non-stationary spin velocity,

$$W_p(t) = \frac{1}{3}\,\Omega^2 r^2 [\,(m_1^2 + m_2^2)(1 + k_0) P_{00}(\cos\theta) + \frac{1}{2}(m_1^2 + m_2^2)$$

$$(1 + k_2) P_{20}(\cos\theta) - (m_2 \sin\lambda - m_1 \cos\lambda)(1 + k_2) P_{21}(\cos\theta)$$

$$+ \frac{1}{4}\,((m_2^2 - m_1^2)\cos 2\lambda - 2m_1 m_2 \sin 2\lambda)(1 + k_2) P_{22}(\cos\theta)] \, ,$$

$$(4\text{-}18)$$

for polar motion, and

$$W_{sp}(t) = -\frac{1}{3}\,\Omega^2 r^2 \Big[(m_2 m_3 \sin\lambda + m_3 m_1 \cos\lambda)$$

$$(1 + k_2) P_{21}(\cos\theta)\Big] , \tag{4-19}$$

for spin velocity on polar motion. Where in all equations k_0 and k_2 are frequency dependent Love numbers depicting the rheologic behaviour of an earth model (rigid, elastic, maxwellian, etc.).

4.3.2 Variations of the Geoid Due to Tidal Effects

The lunar tidal potential for a rigid earth model is given by (Vaníček and Krakiwsky, 1982, p. 126):

$$W_t(t) = \frac{GM}{\rho^{ℂ}(t)} \sum_{n=2}^{\infty} \left(\frac{r_a}{\rho^{ℂ}(t)}\right)^n P_n(\cos Z_a^{ℂ}(t)) . \tag{4-20}$$

Analogous expressions can be written for any other celestial body (sun, planets, etc.).

Equation (4-20) can be decomposed in terms of latitude ϕ, declination δ, and hour angle h by means of

$$\cos Z_a(t) = \sin\phi_a \sin\delta(t) + \cos\phi_a \cos\delta(t) \cos h(t), \tag{4-21}$$

or in terms of colatitude θ_a

$$\theta_a = 90 - \phi_a,$$

we have

$$\cos Z_a(t) = \cos \theta_a \sin \delta(t) + \sin \theta_a \cos \delta(t) \cos h(t), \qquad (4\text{-}22)$$

Substituting equation (4-22) in (4-20), and reordering terms, the result is (Groten, 1980, p. 534):

$$W_t(t) = \frac{GM}{r_a} \sum_{n=2}^{\infty} \left(\frac{r_a}{\rho^{\prime}(t)}\right)^{n+1} \{P_n(\cos \theta_a)\, P_n\, (\cos \delta(t))$$

$$+ 2 \sum_{m=1}^{\infty} \frac{(n-m)!}{(n+m)!} P_{nm}(\cos \theta_a)\, P_{nm}(\cos \delta(t)) \cos mh(t)\}, \qquad (4\text{-}23)$$

the declination δ and hour angle h, functions of time, contain a large spectrum of periodicities. Among these, a zero frequency term, usually named after Honkasalo, and the diurnal and semidiurnal bands have the largest amplitudes.

Equation (4-23) can be generalized to a deforming earth model by introducing Love numbers in a manner similar to that of the centrifugal potential.

4.3.3 Variations of the Geoid Due to Loading

Loading is the term used to describe the combination of the three following phenomena:

i) The deformation of the earth's crust due to the stress applied by the load,

ii) the gravitational attraction of the mass of the load, and

iii) the change in gravitational attraction due to the change in mass density distribution caused by the loading deformation, usually referred to as the indirect effect.

The easiest effect to compute is the gravitational attraction of the load W_a. This can be done in two fashions, either, by means of a series of spherical harmonics or of a convolution integral.

The spherical harmonic expansion reads (Vaníček and Krakiwsky, 1982, p. 597):

$$W_a(\phi_a,\lambda_a) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (W_a)_{nm} , \tag{4-24}$$

where the potential coefficients are given as

$$\begin{pmatrix} A_{nm} \\ \\ B_{nm} \end{pmatrix} = -2G\sigma_L \frac{n-m}{n+m} \oiint_B Z(\phi,\lambda) \begin{pmatrix} \cos m\lambda \\ \\ \sin m\lambda \end{pmatrix} P_{nm}(\sin \phi)\ d\nu, \tag{4-25}$$

and the convolution integral is written as (Vaníček and Krakiwsky, 1982, p. 598):

$$W_a(\phi_a,\lambda_a) = \quad K_a(\phi_a,\lambda_a,\phi,\lambda)\ Z(\phi,\lambda,t)\ d\nu, \tag{4-26}$$

where the kernel K_a is given by

$$K_a = -\ G\sigma_L R \sum_{n=0}^{\infty} P_n(\cos \psi), \tag{4-27}$$

here σ_L is the mass density of the load, R the mean radius of the earth, Z the amplitude of the load, ψ is a spherical angle between (ϕ_a, λ_a) and the dummy point (ϕ, λ) and $d\nu$ is a solid angle element.

The gravitational attraction of the load can serve as a basis to compute the indirect effect W_I as

$$W_I = \sum_{n=0}^{\infty} k'_n (W_a)_n, \tag{4-28}$$

where k'_n is a load number defined as

$$k'_n = \frac{(u_I)_n}{(u_a)_n}, \tag{4-29}$$

where $(u_I)_n$ and $(u_a)_n$ are vertical displacements associated with the indirect effect and the attraction of the load respectively.

The total change due to loading in the gravity potential is given by

$$W_L(t) = W_a(t) + W_I(t). \tag{4-30}$$

Different loading sources induce temporal changes of the geoid:

i) Atmospheric variations,

ii) tidal waters,

iii) ice and ice melts, and

iv) sedimentation.

Atmospheric variations induce a very small deformation over the earth's crust, however, their gravitational attraction has already been measured with the superconducting gravimeter (Goodkind, 1979). A local gravity gradient of 0.3 μgal/mbar has been associated to these changes (Warburton and Goodkind, 1977).

Tidal water loading can be described by the mathematical apparatus of equations (4-24) and (4-26). It should be mentioned, however, that the integration of infinite series is made through Green's functions (Farrell, 1972) and given that load numbers are not known, a certain rheology has to be postulated by means of an earth model (Bullen, 1975).

The implication of ice and rock loading with sea level are the object of the following section.

4.3.3.1 Ice Loading and Sea Level

Glaciations and deglaciations represent large time scale loading and unloading phenomena. In North America, for example, the postglacial rebound due to the retreat of the Wisconsin ice sheet still takes place today.

The basic source of information about the kinematic character of this rebound is provided by radiocarbon dating of organic materials (Faure, 1977, ch. 17). The method involves three steps:

i) The identification of former mean sea levels,

ii) the measurement of their actual height, and

iii) the dating of organic material associated with them.

The identification of former mean sea levels may prove to be one of the more difficult tasks. That is, many organisms are able to live under the higher high water line but above mean sea level. Therefore, a correction for the local tidal amplitude must be applied.

Dating of rocks is based on the fundamental equation of geochronology (York and Farquahr, 1972):

$$t = \frac{1}{\lambda} \ln \left(1 + \frac{D}{P}\right), \tag{4-31}$$

where t is time, λ the decay constant of a radiactive isotope, D is equal to the number of initial parent atoms minus P the number of parent atoms after time t.

From this method relaxation curves can be obtained. They usually have the forms (Andrews, 1970):

$$u(t) = C_1(1 - \exp(-st)), \tag{4-32}$$

or

$$u(t) = A\,\frac{(1 - i^t)}{(1 - i)}, \tag{4-33}$$

where C is the amount of total uplift, s is a constant, A is the amount of uplift recovered in the first 1000 years and i is the uplift that remains after the first 1000 years.

It is interesting to compare these relaxation curves with present day VCM as obtained by means of geodetic techniques. In North America and in particular in Atlantic Canada, for example, the works of Andrews (1970), Grant (1970, 1977, 1980), Kranck (1972), Thomas _et al_ (1973) and Walcott (1972) show the same tendency as the map of VCM of Vaníček and Nagy (1980). Even the 'zero contour line' (dividing the province of New Brunswick) shows a clear resemblance in both types of studies (Walcott, 1972; Vaníček, 1976).

The mathematical apparatus described in the previous section may be used also to obtain gravity potential variations, induced by ice and ice melt, i.e.,

$$W_I = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (1 + k_n') (W_a)_{nm}. \tag{4-34}$$

However, given the long duration of the stresses, a generalized load number has to be used. For instance, Peltier (1974) has suggested the use of a Maxwell rheology where

$$k_n' = (k_n')_e + (k_n'(t))_v$$

in which $(k_n')_e$, $(k_n'(t))_v$ are the elastic and viscous components of the generalized Love number k_n'.

Based on this principle, Peltier et al (1978) calculated the sea level rise due to the ice sheet melting since the last glacial maximum 18 000 years ago. Their explicit expression reads

$$\begin{aligned} SL(t,r) = & \iint_O G^e(r-r')\ \sigma_w\ SL(t,r')\ d\nu + \iint_I G^e(r-r')\ \sigma_I\ I(t,r')\ d\nu \\ & + \int_o^t d\tau \iint_{o+I} G^v(t-\tau,r-r')(\sigma_w SL(\tau,r') + \sigma_I\ I(\tau,r'))\ d\nu \\ & - k_E(t) - k_c(t), \end{aligned} \tag{4-36}$$

where G^e is an elastic response Green function, G^v is the time-dependent Green function, σ_w is the density of water, σ_I is the density of ice, SL and I are the amplitudes of the sea and ice loads, d is a solid angle element, k_E corrects for eustatic rise and k_c insures conservation of mass.

The first term in equation (4-36) is the elastic depression of the lithosphere caused by changes in water load, the second term accounts for the elastic rebound due to change in ice load, and the third term represents the viscoelastic deformation of the earth due to both ice and water loads.

Five different regions are characterized by five distinct sea level curves: in region I the land rises continuously, relative to sea level, through the 15 000 thousand year interval; in region II there is a continuous submergence due to the presence of the collapsing forebulge; in region III the land first sinks relative to the ocean surface and then emerges slightly; in region IV there is continuous submergence; finally, in region V the relative sea level curve is of smaller magnitudes and not monotonic.

An integration with a detailed grid of $1^{\circ} \times 1^{\circ}$, and $5^{\circ} \times 5^{\circ}$ for outer zones, and time dependent loads (changing at intervals of 1000 years) has been made for Atlantic Canada by Quinlan and Beaumont (1981, 1982). In their first study they used two different deglaciation histories, those of Flint (1971) and Grant (1977). Again, their results for relative sea level (RSL) agree, for present times with those obtained through geodetic techniques (Vaníček, 1976; Vaníček and Nagy, 1981). The publication (Quinlan and Beaumont, 1982) describes an attempt to find an optimum deglaciation history given their former RSL results.

4.4 THE DETERMINATION OF THE GEOID IN TIME

There is a large spectrum of techniques to determine spatial variations of the geoid (Heiskanen and Moritz, 1967; Vaníček and Krakiwsky, 1982). However, among these, only one, involving terrestrial techniques, can provide results accurate enough to detect temporal variations of the geoid: The solution of the geodetic boundary value problem (g.b.v.p.).

4.4.1 The Geodetic Boundary Value Problem in Time

According to Moritz (1980, p. 239), the g.b.v.p. may be defined as the determination of the earth's physical surface from the values of the gravity vector and the gravity potential both given on it.

Different linear and non-linear solutions have been attempted (Moritz, 1980, part D). All, however, use the assumption that the gravity field remains invariant in time. In four dimensional studies, two alternatives arise to overcome this limitation:

i) The reformulation of a continuous in time g.b.v.p., or

ii) the solution of the g.b.v.p. for kinematically adjusted parameters and observations, i.e., homogeneous in time geodetic quantities.

In the first alternative, the displacement field could be represented in a similar fashion to that made in the treatment of earth's normal modes (Gilbert, 1970; Aki, 1980, p.p. 337-347). However, this treatment requires continuous gravity information in time homogeneously distributed all over the earth.

A far less stringent requirement applies to the determinations of solutions from kinematically estimated 'point' height and gravity variations. Here, an analysis of the quasiobservables employed in the solution in time of the g.b.v.p., geoidal gravity anomalies changes, is made

The free air anomaly change at point P_i takes the form

$$\delta\Delta g = \Delta_t^i g - \Gamma \cdot \Delta_t^i H^o, \tag{4-37}$$

where

$$\Delta_t^i g = g(i,2) - g(i,1), \tag{4-38}$$

$$\Delta_t^i H^o = H^o(i,2) - H^o(i,1), \tag{4-39}$$

are the gravity and orthometric height changes at point P_i in between epochs t_2 and t_1, and Γ is the time invariant free air gradient.

A large range of values in the vertical gravity gradient $\Delta_t^i g/\Delta_t^i H$ can be found associated with different phenomena (Jachens, 1978). If $\Delta_t^i g/\Delta_t^i H$ is equal to the free air gradient, i.e.,

$$\frac{\Delta_t^i g}{\Delta_t^i H^o} = \Gamma, \tag{4-40}$$

substitution of equation (4-40) in (4-37), gives

$$\delta\Delta g = 0. \tag{4-41}$$

This is, if a gravity gradient associated to changes in gravity and height is found to be equal to the free air gradient, the free air anomaly remains invariant in time.

It is, only, when vertical gradients associated with local changes in gravity and heights are different from the free air gradient, that the resulting change in free air gravity anomaly does not vanish. Specifically, if

$$\frac{\Delta_t^i g}{\Delta_t^i H^o} \neq \Gamma, \tag{4-42}$$

and

$$\Delta_t^i g \neq 0, \tag{4-43}$$

then

$$\delta\Delta g \neq 0. \tag{4-44}$$

Equations (4-42) and (4-43), represent necessary but not sufficient conditions to lead to temporal variations of geoidal heights. Using Stokes approximation, for example, it is also necessary that the spatial average of $\delta\Delta(\psi,\alpha)$ does not cancel out along the azimuth α

$$\overline{\delta\Delta g}(\psi) = \frac{1}{2} \int_0^{2\pi} \delta\Delta g(\psi,\alpha)\, d\alpha \neq 0, \tag{4-45}$$

and that the 'weighted' average of $\delta\Delta g(\psi)$ does not cancel out either along the spherical distance ψ

$$\Delta_t^i N = \frac{R}{4\pi g} \int_0^{2\pi} \overline{\delta\Delta g}(\psi)\ S(\psi)\ \sin\psi\ d\psi \neq 0, \tag{4-46}$$

where $\Delta_t^i N$ represents the change of geoidal height at point P_i.

Finally, a full account of height changes can be made by writing

$$\Delta_t^i h = \Delta_t^i H^o + \Delta_t^i N\ , \tag{4-47}$$

where $\Delta_t^i h$ represents the change in geometric height at point P_i.

Chapter V

VERTICAL CRUSTAL MOVEMENTS

5.1 INTRODUCTION

The vertical displacement field of the earth has been regarded so far as perfectly known. Here, an investigation on the sources that induce it and an analysis of the different terrestrial techniques employed in its determination are made.

VCM are induced by a large space-time spectrum of physical phenomena. In space they range from zero degree changes in a global spherical harmonic expansion to phenomena of the order of tens of metres. In time, they range from permanent deformations to phenomena with period of a fraction of a second. Their amplitude may reach, over geologic time, several kilometers, or be as small as to be detected only by by seismometers.

The thermodynamics of VCM is not fully reconciled in both a macro and a microscopic scale (Kaula, 1980). On a microscopic scale, it is not clear if the creep mechanism of the mantle is Newtonian, i.e., the stress and shear strain rate are directly proportional, or non-Newtonian (Weertam, 1978; Peltier, 1981).

On a macroscopic scale, among the different Newtonian rheologies, the linear steady state Maxwell body has proved very succesful to solve problems related to postglacial rebound (Peltier, 1974; Farrell and Clark, 1976; Peltier, 1976). This same body, however, can only succesfully fit seismic data if a radically different viscosity value is employed. What this suggests is that a more sophisticated body has to be developed. Peltier (1981) has suggested the use of a four element body: a Burgers model, i.e., a Maxwell and a Kelvin body in series (Fiendley et al, 1976).

On the earth's surface, vertical movements are continuous or discontinuous in space and time:

VCM continuous in space and time are induced, for example, by free oscillations, centrifugal forces, postglacial glacial rebound, tides and tidal loading.

The three remaining cases: VCM continuous in space and discontinuous in time, discontinuous in space and continuous in time, and discontinuous in space and time may be associated with pre, post, and co-seismic activity. Pre and co-seismic movements have been detected, for example, by Castle et al (1974, 1975). Post-seismic movements appear as aseismic or seismic slip (Kanamori, 1977).

Geodetic measurements have a 'window' in space-time that allow to sense most of these movements. While extraterrestrial techniques like Very Long Base Interferometry (VLBI), Laser Ranging (LR) either to satellites or to the moon, and the Global Positioning System (GPS) may determine the nature of global phenomena, terrestrial techniques, already in use for more than one hundred years, can give an account of regional and local phenomena, i.e., within the extent of present geodetic networks.

Four different types of vertical kinematic information can be extracted from terrestrial techniques: point height changes, height difference differences, tilt and gravity variations.

5.2 POINT HEIGHT VARIATIONS

The analysis of tide gauge records is the only terrestrial geodetic technique capable of providing point height variations (Lennon, 1978). This is usually done through the determinations of MSL and its variations in time.

Mean Sea Level has been a concept used in a very generalized manner to specify a certain mean over a given period of the instantaneous sea level sampled at prescribed times. Its ultimate goal is to represent the surface that the sea in hydrostatic equilibrium would attain, i.e., free from all dynamic effects. It is by means of this determination that the practical realization of the geoid is achieved.

Sea level instrumentation has been thoroughly investigated in Europe (e.g., Lennon, 1970; Saeger et al. 1970; Cartwright, 1977), the U.S. (Barbee, 1965; Baker, 1981), and Canada (Dohler and Ku, 1970; Ku, 1970).

Non-harmonic methods, e.g., arithmetic mean values and regressions, have been mainly used to detect sea level trends.

MSL computed by arithmetic mean values are straight averages of sea level observations over a day, month, or year (or up to 19 years) sampled at every 1, 3, 4, 6, or 8 hours per day. Two drawbacks of this procedure are that it invariably leads to aliasing and that useful kinematic information is also averaged. A search for point height variations may be attempted with this technique by direct intercomparison of monthly, yearly or up to 19 years mean sea level values (Kääriäinen, 1975; Wyss, 1975).

An alternative approach to find sea level trends is that of comparing MSL values obtained by means of time domain numerical filters.

Such numerical filters have been designed since the 1920's under the assumption, not always acknowledged, that the spectrum of observed tidal records is a line spectrum (Doodson, 1921; Lecolazet, 1956; Suthons, 1959). Among these the X_o filter has found the widest application (Rossiter, 1958, 1960). It can be expressed as (Doodson, 1928, p. 265):

$$\begin{aligned} S_o = {} & S(0) + S(2) + S(5) + S(7) + S(8) + 2\,S(10) + S(12) \\ & + S(13) + 2\,S(15) + S(16) + S(17) + S(18) + 2\,S(20) \\ & + S(21) + S(22) + S(23) + S(25) + S(26) + 2\,S(28) \\ & + S(31) + S(33) + S(36) + S(38)\ , \end{aligned} \tag{5-1}$$

where S is the daily MSL value obtained from the linear combination of thirty nine hourly sea level observations S(n).

Here, point height variations may be found by differencing yearly MSL values.

Arithmetic mean values are clearly incapable to extract trends from many physical parameters which influence sea level: currents, water density, temperature, salinity, air pressure, tangential surface wind stress, tides, river discharge, bathymetric configuration, shape of the shore line, crustal movements, precipitation and evaporation.

Different regressions to evaluate these physical influences have been designed by different authors:

Gordon and Suthons (1965) proposed a model which takes into account mean annual anomalies of local air pressure P, mean annual anomalies of local air temperature T for a year Y, and a linear secular change aY.

$$Z_y = a\,Y + b\,P + c\,T \tag{5-2}$$

Rossiter (1972) proposed a model which takes into account the secular variation of the sea level $a_p\ Y^p$, meteorological contributions (air pressure and wind stress) $b_r B_r$, the effect of the nodal tide $C_1 \cos N + C_2 \sin N$ and a residual of all other sources ϕ_y:

$$Z_y = \sum_p a_p\ Y^p + \sum_r b_r B_r + C_1 \cos N + C_2 \sin N + \phi_y$$

Vaníček (1978) proposed a model which takes into account the datum bias C_a, a linear trend $C_1 t_i$, pressure variations $C_p \delta P(t_i)$, temperature variations $C_t \delta T(t_i)$, river discharge $C_d \delta D(t_i)$, tidal variations $A_j \cos(\omega_j t_i - \phi_j)$, and a result of all other sources $R(t_i)$:

$$S(t_i) = C_a + C_1 t_i + C_p \delta P(t_i) + C_t \delta T(t_i) + C_d \delta D(t_i) + \sum_j A_j \cos(\omega_j t_i - \phi_j) + R(t_i)\ . \qquad (5\text{-}4)$$

Here point height variations are clearly included as parameters into the model.

Finally, a remark on the common trend usually found in the kinematic analysis of sea level records, i.e., the eustatic water rise, appears to be in order. Eustatic water rise was thought, for many years, to be an independent linear factor affecting worldwide sea level observations (Lisitzin, 1972; 1974). Gutenberg (1941) and Munk and Revelle (1952) estimated 'eustatic rise' on sea level to be 1.1 and 1.0 mm/yr respectively.

Recently, Peltier et al (1978) have shown that such concept is wrongly based on the assumption that stable points exist on the earth's surface. They have shown, also, that the ocean volume has remained unchanged for the last 5 000 years. On the other hand, Emery (1980) has carried out the last worldwide estimate of linear secular change in sea level at 247 presently available tide gauges with long enough records to fit linear trends with, at least, an 80% confidence level. His results, independent from those of Peltier, show expected displacements predicted by the linear viscoelastic response at formerly glaciated areas on the northern hemisphere. However, on the southern hemisphere only 19 records were available and no significant conclusions could be drawn.

5.3 HEIGHT DIFFERENCE DIFFERENCES

Repeated measurements of levelling techniques provide height difference defferences. These levelling techniques are (Montgomery, 1969):

i) Geodetic,

ii) hydrostatic,

iii) steric, and

iv) geostrophic.

VCM can be detected by means of geodetic and hydrostatic levelling. On the other hand, steric and geostrophic levellings are unable to provide any kinematic information about the earth's crust but they have proved useful to suggest the presence of systematic effects in geodetic levelling nets (Sturges, 1974; Castle and Elliot, 1982), and to compare geodetic reference surfaces along sea channels (Cartwright and Crease, 1963).

Geodetic levelling constitutes, by large, the bulk of data from which VCM can be extracted. Different aspects of it are discussed in the next sections.

5.3.1 Geodetic Levelling

Geodetic levelling has been reviewed recently by Vaníček et al (1980) and exhaustive analysis of time invariant height systems can be found in Nassar and Vaníček (1975) and Nassar (1977). Here, two topics will be further discussed:

i) The optimum design of levelling nets to extract VCM, and

ii) systematic effects in levelling.

5.3.1.1 Optimum Design of Kinematic Levelling Nets

The problem of optimum design of kinematic levelling nets is a problem of preanalysis from which several particular cases arise (Grafarend, 1974):

i) The zero order design problem; the singularity problem,

ii) the first order design problem; the design matrix problem,

iii) the second order design problem; the weight matrix problem, and

iv) the third order design problem; the densification problem.

A simultaneous solution to the first and second order design problems can be regarded as a combined design problem (Vaníček and Krakiwsky, 1982, p.243).

The zero order design problem in kinematic levelling nets arises when only relative measurements are available, e.g., height difference differences or tilt information. A singular system of normal equations results and one of two practices are followed:

i) The use of generalized inverses (Rao and Mitra, 1971; Bjerhammar, 1973), or

ii) the removal of the singularity, i.e., the vertical indeterminacy of the whole net, by the introduction of absolute or weighted constraints.

In the first approach there is a large number of alternatives (Rao and Mitra, 1971, p.p. 14-15). Among them the Moore-Penrose has proved to be the most popular choice in geodesy (Grafarend and Schaffrin, 1974).

The introduction of constraints is often used as an alternative approach. Here, only a minimum constraint frees the solution of as many unnecessary physical hypothesis as it is possible.

The first order design problem poses the question of which is the best configuration a net can acquire, and also, which is the optimum observation scheme in terms of prescribed accuracies. All that information is contained in the design matrix of the net. Simulations to solve this problem have been attempted, for example, by Niemeier and Rhode (1981).

The second order design problem arises when an accuracy is prescribed in the parameters, i.e., uplifts, a design matrix of the experiment is given, and the accuracy in the observations to satisfy the above two requirements is sought. Two characteristics are usually prescribed in the covariance matrix of the parameters: that all absolute error intervals within the net show the same magnitude, and no preferred directions of weakness. The work of Borre and Meissl (1974) should be noted in this respect. A direct solution to this problem can be attempted in two different fashions:

i) Solving for two or more static epochs, or

ii) solving for the kinematic case, i.e., simultaneously for the covariance matrix of the height difference differences $\underline{C}_{\delta\Delta_s H}$.

In the first case one obtains

$$\underline{C}_{\Delta_s^m H} = \underline{A}\,\underline{C}_{H_m}\underline{A}\ , \qquad (5\text{-}5)$$

and

$$\underline{C}_{\Delta_s^{\ell} H} = \underline{A}\ \underline{C}_{H_{\ell}} \underline{A}^T , \tag{5-6}$$

adding (5-5) and (5-6) we get

$$\underline{C}_{\Delta_s^m H} + \underline{C}_{\Delta_s^{\ell} H} = \underline{A}\ (\ \underline{C}_{H_m} + \underline{C}_{H_{\ell}}\ )\ \underline{A}^T . \tag{5-7}$$

In the second approach

$$\underline{C}_{\delta\Delta_s H} = \underline{C}_{\Delta_s^m H} - 2\ \underline{C}_{\Delta_s^m H \Delta_s^{\ell} H} + \underline{C}_{\Delta_s^{\ell} H} , \tag{5-8}$$

where $\underline{C}_{\Delta_s^m H}$ and $\underline{C}_{\Delta_s^{\ell} H}$ are the covariance matrices of the height differences at epochs t_m and t_{ℓ} and $\underline{C}_{\Delta_s^m H \Delta_s^{\ell} H}$ is the temporal cross-covariance matrix. One obtains

$$\underline{C}_{\delta\Delta_s H} = \underline{A}\ \underline{C}_{\Delta_t H}\ \underline{A}^T , \tag{5-9}$$

substituting (5-8) in (5-9)

$$\underline{C}_{\Delta_s^m H} - 2\ \underline{C}_{\Delta_s^m H \Delta_s^{\ell} H} + \underline{C}_{\Delta_s^{\ell} H} = \underline{A}\ \underline{C}_{\Delta_t H}\ \underline{A}^T , \tag{5-10}$$

since by prescription, we know that from equations (5-7) and (5-10)

$$\underline{A}\,(\,\underline{C}_{H_m} + \underline{C}_{H_\ell}\,)\,\underline{A}^T = \underline{A}\,\underline{C}_{\Delta_t H}\,\underline{A}^T\ , \tag{5-11}$$

a clear spurious component is introduced in equation (5-7) by neglecting the temporal cross-covariance matrix. Three cases may arise in theory:

i) If all the elements of $\underline{C}_{\Delta_s^m H \Delta_s^\ell H}$ are positive, i.e., if all the observations are positively correlated, employing equation (5-7) an unnecesarily stringent requirement of accuracy in the observations is imposed,

ii) If all the elements of $\underline{C}_{\Delta_s^m H \Delta_s^\ell H}$ are negative, i.e., if all the observations are negatively correlated, the accuracy prescribed by $C_{\Delta_t H}$ will not be satisfied employing the first approach, and

iii) if $\underline{C}_{\Delta_s^m H \Delta_s^\ell H} = \underline{0}$ a proper assessment of the accuracy in the observations to fit the requirements is also made by the first technique.

The comparison of the above results allows us to conclude that an optimum network configuration and observation scheme designed to provide optimum heights is not necessarily an optimum network from which the best height difference differences are to be extracted. The cross-correlation between any two sets of observations remains here as a key factor.

On the other hand, provided that an assessment of the temporal cross-covariance is made, the formal structure of other optimization techniques for the static case, like linear programming (Cross and Thapa, 1979), can be also used to solve the kinematic case. This is done by introducing a prescribed covariance matrix of uplifts instead of the one of heights.

The densification problem in a kinematic adjustment is referred to as a third order design problem: given a network at an epoch t_ℓ what extra observations or stations can be added to it at an epoch t_m to increase the accuracy of the computed displacements. In other words: what is the optimum modification that can be performed in time to the design matrix of the experiment.

Two cases may arise:

i) When the space of parameters $\mathcal{X}$ remains invariant in time and an optimum collection of extra observations l is sought, and

ii) when both the spaces of the parameters and the observations do not remain invariant in time, and optimum additional subspaces of parameters $\mathcal{X}'$ and observations $\mathcal{L}'$ are sought.

Very little attention has been paid to this third order design problem in the literature, and only rigorous formulations of it have been attempted (Grafarend, 1977).

5.3.1.2 Systematic Effects in Levelling

Two main groups of errors can be distinguished in geodetic levelling: those of instrumental origin, and those inherent to the measuring system. Both can be categorized in their nature as blunders, systematic, and random. Comprehensive compilations of them have been made, for example, by Rappleye (1948), Karren (1964), Whalen and Balazs (1977), Takalo (1978), and Vaníček et al (1980).

Blunders may be introduced by erroneous procedures of data acquisition, e.g., wrong readings or recordings of the observations.

Systematic errors may be defined as those errors that prevent the observations being regarded as a sample of an unbiased normal probability distribution. Systematic errors of instrumental origin are those inherent in the mechanical design of the instrument. In the level, the static collimation error may be the result of several design originated internal problems (Jones, 1964 a,b,c). Among these problems, inaccurate compensation is the most important (Karren, 1964; Berry, 1976; Kivioja, 1980). In the rod two errors may be present: rod scale (Vamosi, 1980), and index errors (Takalo, 1978).

Systematic errors inherent to the measuring system are those induced by the environment and the observation procedure. These may be: centering of spherical levels (Jones, 1964), thermal expansion of invar rod instrument (Takalo, 1978), time dependent collimation error induced by temperature changes (Takalo, 1978), collimation change with refocussing and reticle housing (Karren, 1964); settling of rods (Kukkamäki, 1980); thermal expansion (Vaníček et al. 1980); discrete representation of the gravity field along the levelling line (Krakiwsky, 1966; Tscherning, 1980); rod verticality (Kukkamäki, 1980); differential refraction (Kukkamäki, 1938); biasing due to the geomagnetic field (Kukkamäki, 1980); and instability of pins and plates (Vaníček et al. 1980).

A different group of systematic effects is that which characterizes the geometry of the space in which the observations are made: anholonomity (Vaníček, 1980); non-conservative gravity field (Grafarend, 1980); VCM (Kääriäinen, 1960; Vaníček, 1980); tidal phenomena (Kukkamäki, 1949; Vaníček, 1980); and tidal loading (Farrell, 1972).

Random errors can be defined as those which do not lead to biased determinations of the mean value of the observations. These are: Scintilation of short and long period (Kukkamäki, 1950); pointing errors (Kivioja, 1980); and rod scale error in individual graduations (Witte, 1980).

Among the three types of effects the systematic are the most difficult to eliminate. Different calibration and observational procedures are specially designed to avoid them (Phillips, 1980); when this is not feasible several corrections are applied to the data.

Refraction constitutes the most conspicuous source of systematic errors in levelling. It affects every single levelling observation. However, since geodetic levelling is a differential technique of height determination, it is not refraction but the amount of it which is not canceled in the difference of two observations that must be corrected. This difference is defined as differential refraction.

Refraction behaves differently in three distinct atmospheric conditions, these are:

i) Unstable, if $H > 0$,

ii) neutral, if $H = 0$, and

iii) stable, if $H < 0$,

where H is the heat flux in between the ground and air masses. Geodetic levelling is carried out under the second, and, mostly, the first conditions.

Two radically different approaches to account for differential refraction have been proposed: the actual physical modelling of differential refraction through measurements in the field, and the statistical approach which extracts the bulk of differential refraction from the observations within the adjustment of the net.

Kukkamäki (1938) was the first to develop a physical correction for differential refraction

$$r_f - r_b = \frac{d \quad s}{(Z_f - Z_b)} \; \frac{\upsilon}{Z_b^c - Z_f^c} \left[\frac{1}{c+1} \left(Z_b^{c+1} - Z_f^{c+1} \right) - Z_o^c \left(Z_b - Z_f \right) \right] , \qquad (5\text{-}12)$$

$$d = - 10^{-6} \left[0.933 - 0.0064 \, (T - 20) \right] \frac{P}{760} , \qquad (5\text{-}13)$$

where

$$\nu = T_2 - T_1 , \qquad (5-14)$$

$$T = a + b\, Z^c \qquad (5-15)$$

T is temperarure in °C, P is pressure in mm Hg, s is the sight length in m, Z is the height of the instrument in cm, c is the exponent in the vertical distribution function and Z_b and Z_f are the two heights of the sight line on the back and fore rods above the ground.

The basic equation of a vertical refraction correction to a levelling sight line is expressed as (Angus-Leppan, 1980):

$$r = - 10^{-6} \int_0^S \left(\frac{\partial N}{\partial h} \right)(s - x)dx , \qquad (5-16)$$

where N is the refractivity defined as

$$N = 10^6 (n - 1) , \qquad (5-17)$$

and n is the refractivity index, x is the integration variable along the sight line, and S is the sight line's length.

The difference in between several refraction correction resides in the formulation of the vertical refractivity gradient.

Angus-Leppan (1979) has proposed for unstable conditions

$$\frac{\partial N}{\partial h} = \frac{P}{T^2} \left(2.8 + 82 \frac{\partial T}{\partial h} \right) , \qquad (5-18)$$

where P is pressure in mb, T is temperature in °K and $\partial T/\partial h$ is the a vertical temperature gradient function in K/m.

Brunner (1979, 1980) based also on the surface similarity theory gives

$$\frac{\partial N}{\partial h} = - \frac{\partial N}{\partial T} \left(0.0244 + \frac{T_*}{k\, h_p} \phi_h \right) , \qquad (5\text{-}19)$$

where T_* is the scaling temperature, k is the Von-Karman constant, h_ρ is the average height of the path above the ground, and $\phi_h > 0$ is the flux-profile function for unstable atmospheric conditions.

Two other approaches using the same physical apparatus of equations (5-18), and (5-19) are those of Garfinkel (1979), and Shaw et al (1982) respectively.

Corrections also to historic records for which specific information was not collected have been proposed by Holdahl (1980). Here the temperature distribution function T is reconstructed from one temperature value in the field T and an estimate of the heat flux as follows

$$T = T_o + 3 \left(\frac{H^2 \quad T_o}{(\rho\, C_p)\; gZ} \right)^{1/3} - 0.0098\, Z , \qquad (5\text{-}20)$$

where T is given in °K, C_p is thespecific heat of air at constant pressure, ρ is air density, g is gravity, Z is height above the ground and H is heat flux computed via

$$H = S_n - G - \lambda E \tag{5-21}$$

where S is the net solar radiation, G the heat flux into the ground, and λE the evaporation flux are parameters computed from observed solar radiation and precipitation values.

On the other hand, Remmer (1979, 1980) has proposed a statistical scheme to account for the overall effect of differential refraction in levelling networks. This, as a consequence of two main objections to the physical approach, namely:

i) The difficulty in postulating a truly representative temperature distribution function, and that

ii) temperature can not be measured in the field with enough accuracy.

Instead, Remmer (1980) has proposed, basically, the addition of a nuisance parameter to remove the remanent systematic effect from the residuals (Vaníček and Krakiwsky, 1982, p.p. 181-188).

A few words about this seem to be in order. First, by adding a nuisance parameter to the adjustment only the systematic component left in the residuals is removed. This is, a space-time average of differential refraction from the entire network is removed. Furthermore, any systematic effect, or a combination of them, positively or negatively correlated with differential refraction will affect the value of both the estimated parameters and the nuisance parameter. In general, any departure from the average of differential refraction in space and time will not be corrected.

The success of this approach lies in three factors:

i) The removal of other systematic effects,

ii) the symmetry of the levelling profile along nodal points, and

iii) a small dispersion of field temperature values.

5.3.2 Hydrostatic Levelling

Two techniques to detect VCM by means of hydrostatic levelling can be distinguished:

i) Water level transfers, and

ii) pipeline levelling.

According to Forrester (1980) a water transfer may be defined as the differential positioning technique by which elevation is transferred from benchmark to benchmark across a lake using the surface of the lake as a common reference surface.

Most errors in this technique arise due to departures of the lake surface from one in hydrostatic equilibrium.

The following physical parameters may influence the lake surface:

i) Water density,

ii) air pressure,

iii) tangential surface wind stress,

iv) river discharge,

v) currents, and

vi) precipitation and evaporation.

The last two, evaporation and precipitation, are easily eliminated in the differential process. However the first four could only be accounted for if the lake surface response is evaluated in a similar fashion as that made by Merry and Vaníček (1983) for sea level records.

Water transfers without any correction have been employed extensively in the past in the establishment of the Canadian first order levelling net.

An attempt to compute height difference differences has been made by subtracting the values recorded by pairs of gaues over a certain span of time (Coordinating Commitee on Great Lakes Basic Hydraulic and Hydrologic Data, 1977).

The difficulty to eliminate non-hydrostatic effects on the the sea and lake surfaces lead to the development of the pipeline levelling technique. Pipeline levelling, theoretically, can be carried out between any two points on the surface of the earth, however, it has found its widest use along lakes and sea channels where geodetic levelling can not be carried out. Only recently, Kivioja (1980) has proposed its use as a standard levelling technique on land.

There are several instrumental as well as procedural variations in pipeline levelling. The pipeline can be of one of two types:

i) A floating hose (Kakkuri and Kääriäinen, 1977), or

ii) a sunken hose (Waalewijn, 1964).

Also, the measurements can be carried out either optically (Kakkuri and Kääriäinen, 1977) or by means of pressure gauges (Sneddon, 1979c).

The following effects induce non-hydrostatic conditions along the pipeline:

i) Temperature changes in space and time (Waalewijn, 1964; Sneddon, 1974, 1975, 1979a),

ii) non-equilibrium end conditions (Sneddon, 1979b),

iii) motion of the pipeline (Waalewijn, 1964; Kakkuri and Kääriäinen, 1977), and

iv) air bubbles (Waalewijn, 1964; Kakkuri and Kääriäinen, 1977).

Mercury and ethyl-alcohol binary systems in a hose have been proposed as optimum combinations to avoid temperature induced non-static effects (Sneddon, 1979a).

Determinations of height difference differences by means of this technique have been actually carried out, for example, in Finland by Kukkamäki (1950).

5.4 TILT

Tilt measurements can be performed by means of a number of terrestrial techniques:

i) Geodetic levelling,

ii) hydrostatic levelling, and

iii) tiltmeters.

Scattered relevelled segments constitute in many countries the bulk of data from which tilt information can be extracted. Paradoxically enough, this information is available as a by-product of the maintenance of national levelling networks and not as a goal in itself. Scattered relevelled segments are distributed rather randomly in space and time, and constitute the main body of information from which a characterization of the displacement field can be made. Small levelling arrays to compute tilt in several directions have been proposed, for example, by Gagnon et al (1980).

In countries like Canada differencing lake level records proves to be another source of abundant tilt information that arises as a by-product of the hydraulic control of numerous lakes. Its determination has been discussed earlier within the context of hydrostatic levelling.

Long base, short base, and bore-hole tiltmeters are instruments explicitly designed to determine tilt as an observable. Here, two types of errors can also be distinguished: instrumental, and those inherent in the measuring system.

Among several instrumental errors one finds (Allen et al. 1973; Mortensen, 1978):

i) Telemetry problems,

ii) calibration errors, and

iii) relaxation.

On the other hand, errors induced through the measuring system, for example, are:

i) Rainfall,

ii) temperature changes,

iii) local surface instability, and

iv) material inhomogeneity.

Rainfall produces differential soil expansion, differential loading due to non-uniform drainage, and increases thermal conductivity.

Temperature changes induce thermoelastic deformations in the instrument and on the ground surrounding the instrument.

It is not difficult to find irregular responses due to coupling of these effects. Harrison (1976) and Harrison and Herbst (1977) have found cavities and irregular topography to be responsible also for the introduction of noise in the data.

Finally, Savage et al (1979) have shown that tilt can be transformed into meaningful strain information when a physical interpretation of the process is attempted.

5.5 GRAVITY VARIATIONS

The geometry of the earth's gravity field is completely determined by:

i) The gravity potential W,

ii) the gravity vector W_i, and

iii) the gravity gradient tensor W_{ij} .

Among these W_z and only W_{xy} , W_{xz} , W_{yz} , and $W_{yy} - W_{xx}$, are observable quantities on the surface of the earth in a topocentric cartesian reference system. Still, only W_z can be measured accurately enough at present to detect temporal changes in the earth's shape.

Gravity can be measured in two modes: absolute and relative . The first, and less abundant type of information, can be obtained by means of three instruments:

i) Pendulums (Hytonen, 1972),

ii) free fall devices (Faller et al. 1980), and

iii) rise and fall devices (Cannizzo et al. 1978).

Pendulum observations were still used in the establishment of the International Gravity Standarization Net of 1971 (IGSN71) (IAG, 1974), but have not been used as a tool to detect VCM.

Free fall and rise-and-fall devices appear to have reached a relative accuracy of 5 and 1 parts in 10^8 respectively (Faller et al. 1980), enough to be employed systematically in geodynamic studies. However their use so far has been very limited.

Relative gravimetry, instead, has proved to be the only supplier of temporal changes of gravity differences. The most commonly used gravimeters have been Lacoste and Romberg models G and D (Torge,1982). Both have been the object of numerous technical evaluations (Brein et al. 1977; Boedecker, 1978; Lambert et al. 1979; Krieg, 1981; Nakagawa et al.

1982; Kanngieser, 1982). The following sources of instrumental errors can be cited:

i) Drift,

ii) lack of calibration,

iii) effect of varying voltage, and

iv) non linearity due to screw effect.

Errors inherent to the measuring system are related to

i) atmospheric temperature and pressure changes,

ii) biasing due to the geomagnetic field,

iii) transportation vibration and shocks, and

iv) levelling of the instrument.

A breakthrough in the accuracy of relative gravimetry was made by the introduction of the cryogenic gravimeter (Prothero and Goodkind, 1968). Its use, however, has been restricted to just a few locations.

A number of phenomena lead to variations in the value of gravity. Here, a non-exhaustive account of studies to illustrate them is presented.

Earth tides are the most conspicuous source of changes in gravity. However, in the determination of VCM they are regarded as noise to be corrected for and several formulae and tables (e.g., Longman, 1959; Cartwright and Tayler, 1971; Cartwright and Edden, 1973; Durcame _et al_. 1978), special field procedures (Kiviniemi, 1974), and actual determinations (Torge and Wenzel, 1976) have been used to avoid them. The effects of ocean loading have been both predicted (Farrell, 1972, 1973; Goad, 1980), and observed (Warburton _et al_, 1975). On the other hand, gravity variations of rotational origin have been only predicted (Lambeck, 1973; Mansinha _et al_. 1976).

Gravity variations due to redistribution of masses within the earth have been observed, e.g., ground water movement (Lambert and Beaumont, 1977), and volcanic activity (Hagiwara, 1977; Torge, 1981). For the first time, the effect of short period atmospheric temporal density variations has been measured (Warburton _et al_. 1977). Long period phenomena like postglacial rebound has shown, in the time window of geodetic measurements, a linear trend (Kiviniemi, 1974).

Different models of changes in gravity due to deformation (Walsh _et al_. 1979), topographic changes (Walsh, 1982) and seismic events (e.g., Barnes, 1966) have been proposed.

Finally, a review of vertical gravity gradients associated with different phenomena has been made by Jachens (1978).

Chapter VI

MATHEMATICAL MODELS FOR THE DETECTION OF VERTICAL CRUSTAL MOVEMENTS

6.1 INTRODUCTION

Four available sources of information to detect vertical crustal movements have been discussed so far: sea level records, levelled height differences, tilt, and gravity. None of these techniques is capable of providing continuous information in space, thus any areal characterization of vertical crustal movements is spatially limited. Also, a complete kinematic description can not be extracted from the data, thus temporal limitations are also imposed.

Spatial limitations arise due to the number of sources available and due to the amount of data coverage. For instance, if sea level information is available crustal movements are referred to as absolute. If only levelling, tilt and gravity information is available vertical crustal movements are referred to as relative. When levelling information is considered, it may be by means of connected or scattered segments.

Temporal limitations arise due to the distribution in time of the data. For instance, if only two levellings are available constant velocities can only be extracted. Higher order movements in time should be obtained from more than two levellings.

Different models have been developed according to the temporal and spatial limitations imposed on the data. These are:

i) Absolute linear or higher order in time vertical crustal movements from connected segments,

ii) relative linear or higher order in time vertical crustal movements from connected segments,

iii) absolute linear or higher order in time vertical crustal movements from scattered segments, and

iv) relative linear or higher order in time vertical crustal movements from scattered segments.

6.2 DETECTION OF VERTICAL CRUSTAL MOVEMENTS FROM CONNECTED SEGMENTS

If a height network is completely relevelled two or more times, each occasion in a such short span of time that can be considered instantaneously levelled, the idea of comparing two or more sets of separately adjusted vertical positions may prove tempting. However, several assumptions implied when following this approach may not be obvious. First, that the reference system, or the heights datum, to which both levelling nets refer is the same in space. This is practically realized when, at least, one common point p within the network is considered fixed at different epochs t_m and t_ℓ :

$$H(p,m) = H(p,\ell), \tag{6-1}$$

and relative temporal height differences at all points p_i of the network are obtained

$$\Delta_t^i H = H(i,m) - H(i,\ell). \tag{6-2}$$

Refering all vertical displacements to point p_p calls for calls for a bias in vertical translation $\Delta_t^p H$. This can be alleviated if the vertical displacement of point p_p

$$\Delta_t^p H = H(p,m) - H(p,\ell), \tag{6-3}$$

is referred to the geoid. All vertical displacements within the network $\Delta_t^i H$ for which this bias is solved will be referred to as absolute.

An assessment of the accuracy of all point temporal changes when positions are compared can be made by writing

$$\underline{C}_{\Delta_t^i H} = \underline{C}_{H(i,m)} + \underline{C}_{H(i,\ell)}, \tag{6-4}$$

which reflects the degree of confidence of the determined displacements with respect to the preselected datum: the geoid or any other surface implied by a fixed reference benchmark.

This technique requires that the vector of parameters in both adjustments must belong to the same parameter space, but not necessarily that the vectors of observations belong to the same observation space.

An alternative more rigorous method to extract vertical crustal movements from repeatedly measured height networks is their direct evaluation by means of a common, or kinematic, adjustment. Here, instead of observations, observation differences, i.e., height difference differences $\delta\Delta_s H$ are adjusted:

$$\delta\Delta_s H = \Delta_s^2 H - \Delta_s^1 H. \tag{6-5}$$

They can be regarded as quasiobservations each of which provides an observation equation of the form

$$\begin{aligned} \delta r &= H(2,2) - H(1,2) - H(2,1) + H(1,1) - \delta\Delta_s H, \\ &= H(2,2) - H(2,1) - H(1,2) + H(1,1) - \delta\Delta_s H, \\ &= \Delta_t^2 H - \Delta_t^1 H - \delta\Delta_s H. \end{aligned} \tag{6-6}$$

The system of equations for the whole network can be written as

$$\underline{\delta r} = \underline{A}\ \underline{\Delta_t H} - \underline{\delta\Delta_s H}, \tag{6-7}$$

where A is the first design matrix. The system of normal equations is given by

$$\underline{A}^T \underline{C}^{-1}_{\delta\Delta_s H} \underline{A} \underline{\hat{\Delta_t}H} = \underline{A}^T \underline{C}^{-1}_{\delta\Delta_s H} \underline{\delta\Delta_s H}, \tag{6-8}$$

where $C_{\delta\Delta_s H}$ is the variance-covariance matrix of $\delta\Delta_s H$.

If absolute vertical crustal movements are to be computed the above equation can be rewritten as

$$[A^T \vdots B^T] \; \underline{C}^{-1}_{A,B} \; [\underline{A} \vdots \underline{B}]^T \; \underline{\hat{\Delta_t}H} = [A^T \vdots B^T] \; \underline{C}^{-1}_{A,B} \left[\underline{\delta\Delta_s H} \vdots \underline{\Delta_t H}\right], \tag{6-9}$$

where $\underline{B}$ is the second design matrix. Then the residuals and the a posteriori variance factor can be computed

$$\underline{\hat{\delta r}} = \underline{A} \; \underline{\hat{\Delta_t}H} - \underline{\delta\Delta_s H},$$

$$\sigma^2 = \frac{\underline{\hat{\delta r}}^T \underline{C}_{AB} \underline{\hat{\delta r}}}{m - u}, \tag{6-10}$$

where m and u are the number of observations and parameters respectively.

It should be noted that the covariance matrix of the 'observed' height difference differences reads

$$\underline{C}_{\delta\Delta_s H} = \underline{C}_{\Delta^1_s H} - 2\,\underline{C}_{\Delta^1_s H \Delta^2_s H} + \underline{C}_{\Delta^2_s H}\,. \tag{6-11}$$

However, the second term of the right hand side is usually neglected. A further account of this effect will be given in section 6.4.

A kinematic adjustment of repeatedly measured networks can also be formulated in the parameter space. All loops provide a system of equations of the form

$$\underline{B}\ \underline{\delta\Delta_s H} + \underline{\delta\omega} = \underline{0}, \tag{6-12}$$

where $\underline{\delta\omega}$ is the vector of loop misclosures. This system can be rewritten for the adjustment as

$$\underline{B}\ (\underline{\delta\Delta_s H} + \underline{\delta r}) + \underline{\delta\omega} = \underline{0}, \tag{6-13}$$

then

$$\underline{\hat{\delta r}} = -\ \underline{C}_{\delta r}\ \underline{B}^T\ \underline{\hat{k}}, \tag{6-14}$$

$$= -\ \underline{C}_{\delta r}\ \underline{B}^T\ \underline{M}\ \underline{\delta\omega}, \tag{6-15}$$

where $C_{\delta r}$ is the variance-covariance matrix of the residuals δr, and

$$\underline{M} = (\underline{B}\ \underline{C}_{\delta r}\ \underline{B}^T)^{-1}. \tag{6-16}$$

The adjusted height difference differences are given by

$$\underline{\delta\hat{\Delta}_s H} = \underline{\delta\Delta_s H} + \underline{\hat{\delta r}}. \tag{6-17}$$

So far, it has been assumed in the three different techniques that all height networks have been observed instantaneously. As this is never the case, different techniques have been developed to overcome the temporal inhomogeneity of the observations.

Frost and Lilly (1966) compared two separately adjusted sets of positions of the same net. They adopted an iterative solution which can be summarized as follows: two sets of observations of the same net, one covering a span of time from 1919 to 1938, and the second from 1962 to 1964, were separately adjusted. Each time the heights obtained from the Canadian adjustment of 1928 of four benchmarks were held fixed. All relative vertical movements were computed with respect to the datum implied by those benchmarks. Uplift rates were computed to reduce the first set of observations to a mean epoch of 1925. Then a whole iteration of the process was made: a new adjustment of the so reduced first set of observations was performed, and a new comparison of heights was made leading to the final uplift rates of the entire network.

Gale (1970) attempted a kinematic parametric adjustment with Frost and Lilly's data. A solution to the problem posed by heterogeneous observations in time was attempted by establishing, in our notation, observation equations of the form

$$\delta r = \Delta_t^2 H - \Delta_t^1 H - \delta\Delta_s H, \tag{6-18}$$

where the observed spatial height differences at any epochs t_n and t_m are linearly reduced to epochs t_2 and t_1 by

$$\Delta_s^2 H - \Delta_s^1 H = (\Delta_s^n H - \Delta_s^m H)\,\frac{(t_2 - t_1)}{(t_n - t_m)}\,. \tag{6-19}$$

A kinematic adjustment with heterogeneous observations in time could also be formulated in the parameter space as

$$\underline{B}\ (\underline{\delta\Delta_s H} - \underline{\delta r}) + \underline{\delta\omega} = \underline{0} \tag{6-20}$$

where equation (6-19) can be used to ensure homogeneity in time.

Kukkamäki (1939) and Kääriäinen (1949, 1953) developed a method which is formally a combination of two previously discussed: A conditional adjustment of observation differences and a direct comparison of vertical separately adjusted positions.

Here, computed uplift rates $\delta\dot{\Delta}_s H$ are regarded as observations. They are formed from observed height differences and the ellapsed time in between their measurement

$$\frac{\Delta_s^2 H - \Delta_s^1 H}{t_2 - t_1} = \frac{\Delta_s^n H - \Delta_s^m H}{t_n - t_m}, \tag{6-21}$$

$$= \delta \dot{\Delta}_s H. \tag{6-22}$$

The velocity field is assumed to be linear, not only in between epochs t_n and t_m, but also t_2 and t_1. This takes care of all time inhomogeneities in the original observations.

A kinematic adjustment of the uplift rates can then be formulated as follows

$$\underline{B}\ (\underline{\delta \dot{\Delta}_s H} + \underline{\delta \dot{r}}) + \underline{\delta \dot{\omega}} = \underline{0}. \tag{6-23}$$

The adjusted uplift rates are then used to reduce each set of originally observed height differences $\Delta_s^i H$ to their mean epoch of measurement t_m.

Since

$$\frac{\Delta_s^m H - \Delta_s^i H}{(t_m - t_i)} = \delta \dot{\Delta}_s H, \tag{6-24}$$

we have

$$\Delta_s^m H = \Delta_s^i H - \delta\dot{\Delta}_s H(t_i - t_m). \tag{6-25}$$

Given that the only adjusted quantity in equation (6-25) is the uplift rate, a new adjustment using two homogeneous in time sets of observations has to be performed. This results in two sets of epoch dependent vertical positions which themselves are compared to produce new uplift rates. This last step can be iterated. Kääriäinen (1955) found after three iterations land uplifts that did not change by more than one hundredth of 1 mm.

A two component adjustment with a prediction of the signal has been attempted by Hein (1979). The mathematical model can be expressed as

$$\underline{A}\ \underline{\Delta_t H} + \underline{B}\ (\underline{\delta s} + \underline{\delta v}) + \underline{\delta \omega} = \underline{0}, \tag{6-26}$$

then the estimated signal for such an explicit model is given by

$$\underline{\delta\Delta_s H_p} = -\underline{C}_{s_p s}\ \underline{C}_{\delta r}^{-1}\ \underline{\delta\Delta_s H}, \tag{6-27}$$

where $\underline{C}_{s_p s}$ is the cross-covariance matrix of the predicted signal, and $\underline{C}_{\delta r}^{-1}$ is the inverse of the total covariance matrix formed by the inverse of the sum of the signal and noise covariance matrices

$$\underline{C}_{\delta r}^{-1} = (\underline{C}_{\delta s} + \underline{C}_{\delta v})^{-1}. \tag{6-28}$$

Here $\underline{C}_{\delta v}$ is derived from the sum of the covariance matrices of the observed height differences

$$\underline{C}_{\delta v} = \underline{C}_{\delta\Delta_s H} = \underline{C}_{\Delta_s^m H} + \underline{C}_{\Delta_s^\ell H}, \tag{6-29}$$

the term $\underline{C}_{\Delta_s^m H \Delta_s^\ell H}$ in covariance matrix $\underline{C}_{\delta\Delta_s H}$ again being neglected.

The covariance function employed to prescribe the stochastic characteristics of the covariance matrix of the signal is of the Hirvonen type

$$C(s) = \frac{C_o}{1 + (\frac{s}{d})^2}. \tag{6-30}$$

With the exception of the two component adjustment, all previously discussed techniques to detect VCM provide point vertical displacements or uplift rates only at levelled points in the network. If a continuous characterisation of the vertical velocity field is desired an areal prediction may be attempted. Practically any surface fitting technique developed in approximation theory may be used.

Holdahl and Hardy (1979), for example, have employed multiquadric kernel functions. The velocity surface is represented by a linear combination of the form

$$V(x,y) = \sum_{i=1}^{j} a_i \, Q(x,y,x_i,y_i), \tag{6-31}$$

where the hyperboloid

$$Q(x,y,x_i,y_i) = \{(x - x_i)^2 + (y - y_i)^2 + D^2\}^{1/2}, \tag{6-32}$$

and the inverse hyperboloid

$$Q(x,y,x_i,y_i) = \{(x - x_i)^2 + (y - y_i)^2 + D^2\}^{-1/2}, \tag{6-33}$$

are used as quadric forms.

The advantage of using multiquadric kernels are:

i) The data may be approximated to any desired degree of accuracy, and

ii) they are computationally convenient models, for instance, the reciprocal hyperboloid tends to zero outside of the region containing nodal points, i.e., avoid problems of extrapolation at the edges of the map.

Their disadvantages are:

i) Nodal points should be located at maxima or minima whose location will not be a priori known, and

ii) the value of D in equations (6-32), and (6-33) which controls the shape of the hyperboloid and the inverse hyperboloid is usually chosen a priori.

An attempt has been made to overcome the subjective selection of the number of nodal points by means of an iterative scheme (Holdahl and Hardy, 1979).

So far, models from which constant velocities could only be extracted have been discussed. However, if more than two levellings are available the same technique can be applied to any two sets of measurements. The final product being epoch dependent velocity values. An alternative approach is to construct constant acceleration models in which the observations are expressed in terms of finite differences (Vaníček and Krakiwsky, 1982, p. 643)

$$\frac{(\underline{\delta\Delta_s^{mn}}H - \underline{\delta\Delta_s^{\ell m}}H)}{(t_n - t_m)\,(t_m - t_\ell)} = \frac{(\underline{\Delta_s^{n}}H - 2\underline{\Delta_s^{m}}H + \underline{\Delta_s^{\ell}}H)}{(t_n - t_m)\,(t_m - t_\ell)}, \tag{6-34}$$

where

$$\underline{\delta\Delta_s^{mn}}H = \underline{\delta\Delta_s^{n}}H - \underline{\delta\Delta_s^{m}}H \,, \tag{6-35}$$

and

$$\underline{\delta\Delta_s^{\ell m}H} = \underline{\delta\Delta_s^{m}H} - \underline{\delta\Delta_s^{\ell}H}. \tag{6-36}$$

6.3 DETECTION OF VERTICAL CRUSTAL MOVEMENTS FROM SCATTERED SEGMENTS

When there is geophysical evidence that the vertical displacement field over an area is continuous in space, not only a continuous surface can be used to represent it but less restrictions are imposed on the data. One such restriction: the connected relevelled segments, is not imposed any more. This proves to be very useful when only scattered relevelled segments exist over an area.

A set of relevelled segments provides us with a system of observation equations of the form

$$\underline{r} = \underline{\tilde{\Delta\Phi}}^T \underline{c}_u - \underline{\delta\Delta_s H}, \tag{6-37}$$

if a displacement surface representation is sought.

Similarly, a system of observation equations can be formulated for a velocity, or uplift rate, surface (Vaníček and Christodulidis, 1974):

$$\underline{\dot{r}} = \underline{\tilde{\Delta\Phi}}^T \underline{c}_v - \underline{\delta\dot{\Delta}_s H}, \tag{6-38}$$

where

$$\underline{\widetilde{\Delta\Phi}}^T = \underline{\widetilde{\Phi}}^T(x_j,y_j) - \underline{\widetilde{\Phi}}^T(x_i,y_i), \tag{6-39}$$

$\underline{\widetilde{\Phi}}^T$ represents a system of selected base functions, and c and $\underline{c}_v$ are two vectors of coefficients.

Observation equations of a similar type can be formed to include other sources of tilt information, e.g., lake level tilts and long base tiltmeters.

The system of normal equations for the displacement model reads

$$\underline{\widetilde{\Delta\Phi}}\ C^{-1}_{\delta\Delta_s H}\ \underline{\widetilde{\Delta\Phi}}^T\ \underline{c}_u = \underline{\widetilde{\Delta\Phi}}\ C^{-1}_{\delta\Delta_s H}\ \underline{\delta\Delta_s H}, \tag{6-40}$$

and for the velocity model

$$\underline{\widetilde{\Delta\Phi}}\ C^{-1}_{\delta\Delta s H}\ \underline{\widetilde{\Delta\Phi}}^T\ \underline{c}_v = \underline{\widetilde{\Delta\Phi}}\ C^{-1}_{\delta\Delta_s H}\ \underline{\delta\Delta_s H}, \tag{6-41}$$

their solution can be written respectively as

$$\hat{\underline{c}}_u = \underline{N}^{-1}\underline{u}, \tag{6-42}$$

or

$$\underline{\hat{c}}_v = \underline{M}^{-1} \underline{v}, \tag{6-43}$$

where

$$\underline{N} = \underline{\widetilde{\Delta\Phi}} \; \underline{C}^{-1}_{\delta\Delta_s H} \; \underline{\widetilde{\Delta\Phi}}^T, \tag{6-44}$$

$$\underline{u} = \underline{\widetilde{\Delta\Phi}} \; \underline{C}^{-1}_{\delta\Delta_s H} \; \underline{\delta\Delta_s H}, \tag{6-45}$$

$$\underline{M} = \underline{\widetilde{\Delta\Phi}} \; \underline{C}^{-1}_{\delta\dot{\Delta}_s H} \; \underline{\Delta\Phi}^T, \tag{6-46}$$

and

$$\underline{v} = \underline{\widetilde{\Delta\Phi}} \; \underline{C}^{-1}_{\delta\dot{\Delta}_s H} \; \underline{\delta\dot{\Delta}_s H}, \tag{6-47}$$

$\underline{M}$ and $\underline{N}$ are Gram matrices composed of products of linearly independent functions. Vaníček and Christodulidis (1974), Vaníček (1976), and Vaníček and Nagy (1981) have used generalized algebaric polynomials as base functions.

A reduction is usually implemented: A statistical filtering of the coefficients. This is done by means of an orthogonalization of the base functions. A new set of optimal coefficients $\underline{c}^o$ is computed and then tested against its own variance. The significant coefficients are then deorthogonalized into the original space. This finally results in a smoother yet statistically significant displacement

$$\Delta_t H(x,y) = \underline{\tilde{\Phi}}^{oT}\ \underline{\hat{c}}^o_u, \tag{6-48}$$

or velocity solution

$$\dot{\Delta}_t H(x,y) = \underline{\tilde{\Phi}}^{oT}\ \underline{\hat{c}}^o_v, \tag{6-49}$$

the covariance matrix of the surface is given by

$$\underline{C}_{\Delta_t H}(x,y) = \underline{\tilde{\Phi}}^{oT}\ \underline{C}_{\hat{c}^o_u}\ \underline{\tilde{\Phi}}^o, \tag{6-50}$$

or

$$\underline{C}_{\dot{\Delta}_t H}(x,y) = \underline{\tilde{\Phi}}^{oT}\ \underline{C}_{\hat{c}^o_v}\ \underline{\tilde{\Phi}}^o, \tag{6-51}$$

where $\underline{C}_{c_u}$ and $\underline{C}_{c_v}$ are the estimated covariance matrices of the estimated significant coefficients given by

$$\underline{\hat{C}}_{\hat{c}^o_u} = \frac{\underline{\hat{r}}_u^T\ \underline{C}^{-1}_{\delta\Delta_s H}\ \underline{\hat{r}}_u}{m - u^o}\ \underline{N}^{-1}, \tag{6-52}$$

or

$$\hat{\underline{C}}_{\underline{c}^{\circ}_v} = \frac{\underline{r}_v^T \, C_{\delta\Delta_s H} \, \underline{r}_v}{m - u^{\circ}} \, \underline{M}^{-1} , \tag{6-53}$$

where

$$\hat{\underline{r}}_u = \underline{\tilde{\Delta\Phi}}^T \, \underline{c}_u - \underline{\delta\Delta_s H} , \tag{6-54}$$

and

$$\hat{\underline{r}}_v = \underline{\tilde{\Delta\Phi}}^T \, \underline{c}^{\circ}_v - \underline{\delta\dot{\Delta}_s H} , \tag{6-55}$$

and m and u are the number of relevelled segments and the number of filtered coefficients.

From the two former models only relative VCM can be extracted. If absolute linear VCM are to be computed at least one, but preferably several point temporal changes $\Delta_t H$ must be introduced to solve for the indeterminacy in vertical translation. This is done by introducing sea level information to the model. The system of normal equations is augmented such that, for instance, for the displacement model reads

$$\left[\underline{\widetilde{\Delta\Phi}} \mid \underline{\widetilde{\Phi}}\right] \; C^{-1}_{\delta\Delta_s H \Delta_t H} \; \left[\underline{\widetilde{\Delta\Phi}}^T \mid \underline{\widetilde{\Phi}}^T\right]^T \; \hat{\underline{C}}_u = \left[\underline{\widetilde{\Delta\Phi}} \mid \underline{\widetilde{\Phi}}\right] \underline{C}_{\delta\Delta_s H \Delta_t H} \left[\underline{\delta\Delta_s H} \mid \underline{\Delta_t H}\right]^T , \tag{6-56}$$

where

$$\underline{C}_{\delta\Delta_s H \Delta_t H} = \begin{bmatrix} \underline{C}_{\delta\Delta_s H} & 0 \\ 0 & \underline{C}_{\Delta_t H} \end{bmatrix} \tag{6-57}$$

If the distribution in time of the data allows for the determination of more kinematic information a four dimensional modelling of VCM may be attempted. This has been done by Vaníček, Elliott and Castle (1979). The uplift surface is given in the form

$$\Delta_t H(x,y,t) = \sum_{k=1}^{n_t} c_{ok} \, T_k(t) + \sum_{i=0}^{n_x} \sum_{j=0}^{n_y} \sum_{k=1}^{n_t} c_{ijk} \, x^i y^j \, T_k(t) , \tag{6-58}$$

where

$$T_k(t) = t^k \tag{6-59}$$

$$T_k(t) = \begin{cases} 0 & t < b_k \\ \dfrac{t - b_k}{e_k - b_k} & b_k \leq t \leq e_k \qquad k = n_p + 1, n_p + n_e \\ 1 & t > e_k \end{cases} \tag{6-60}$$

Each relevelled segment provides an observation equation of the form

$$r = \sum_{i=0}^{n_x} \sum_{j=0}^{n_y} \sum_{k=1}^{n_t} c_{ijk} \, (x_2^i y_2^j - x_1^i y_1^j)\{T_k(t_2) - T_k(t_1)\} - \delta\Lambda_s H, \tag{6-61}$$

the system of which can be written as

$$\underline{r} = \underline{u}^T \, \underline{\tilde{\Delta\Phi}} \, \underline{\tilde{\Delta T}}^T \, \underline{\tilde{c}} \, \underline{u} - \underline{\delta\Lambda_s H}, \tag{6-62}$$

where

$\underline{\tilde{\Delta\Phi}}$ is a m_ϕ vector that represents the system of differences of space base functions,

$\underline{\tilde{\Delta T}}$ is a m_t vector that represents the system of differences of time base functions,

$\underline{\tilde{c}}$ is a $m_t \times m_\phi$ matrix of space-time coefficients, and

$\underline{u}$ is a m_ϕ vector of ones.

Equation (6-62) can be rewritten as

$$\underline{r} = \underline{B}^T \, \underline{c} - \underline{\delta\Delta_s H}, \tag{6-63}$$

where

$$\underline{B}^T = \underline{u}^T \, \underline{\tilde{\Delta\Phi}} \, \underline{\tilde{\Delta T}}^T, \tag{6-64}$$

and

$$\underline{c} = \underline{\tilde{c}}\ \underline{u}. \tag{6-65}$$

If sea level information is also available an absolute four dimensional modelling can be carried out. The augmented system of observation equations reads

$$r = \underline{u}^T \left[\underline{\tilde{\Delta\Phi}} \vdots \underline{\tilde{\Phi}}\right] \left[\underline{\tilde{\Delta T}}^T \vdots \underline{\tilde{T}}^T\right] \underline{\tilde{c}}\ \underline{u} - \left[\underline{\delta\Delta_s H} \vdots \underline{\Delta_t H}\right]^T, \tag{6-66}$$

or

$$\underline{r} = \underline{A}^T \underline{c} - \left[\underline{\delta\Delta_s H} \vdots \underline{\Delta_t H}\right]^T, \tag{6-67}$$

where

$$\underline{A}^T = \underline{u}^T \left[\underline{\tilde{\Delta\Phi}} \vdots \underline{\tilde{\Phi}}\right] \left[\underline{\tilde{\Delta T}}^T \vdots \underline{\tilde{T}}^T\right], \tag{6-68}$$

and

$$\underline{c} = \underline{\tilde{c}}\ \underline{u}. \tag{6-69}$$

The system of normal equations is given by

$$\underline{A}\ \underline{C}^{-1}_{\delta\Delta_s H, \Delta_t H}\ \underline{A}^T\ \underline{\hat{c}} = \underline{A}\ \underline{C}^{-1}_{\delta\Delta_s H, \Delta_t H} \left[\underline{\delta\Delta_s H} \vdots \underline{\Delta_t H}\right]^T, \tag{6-70}$$

where

$$\underline{C}_{\delta\Delta_s H,\Delta_t H} = \left|\begin{array}{c|c} \underline{C}_{\delta\Delta_s H} & 0 \\ \hline 0 & \underline{C}_{\Delta_t H} \end{array}\right| . \tag{6-71}$$

Finally, it should be remarked that the power of this four-dimensional technique does not only lie in the fact of making use of scattered relevelled segments in space but also in time.

6.4 A COMPARATIVE ANALYSIS OF DIFFERENT ADJUSTMENTS

A systematic approach to analyse the results from different mathematical models to detect VCM from connected segments can be performed by discussing their different stochastical assumptions. In all cases these assumptions are specified by the covariance matrix of the observations.

Several alternatives arise (Vaníček and Krakiwsky, 1982, p. 640):

i) If $C_{\Delta_s^{\ell} H} = k\, C_{\Delta_s^{m} H}$ $\quad k > 0$, and $C_{\Delta_s^{\ell} H \Delta_s^{m} H} = 0$, (6-72)

ii) If $C_{\Delta_s^{\ell} H} = k\, C_{\Delta_s^{m} H}$ $\quad k > 0$, and $C_{\Delta_s^{\ell} H \Delta_s^{m} H} \neq 0$, (6-73)

iii) If $C_{\Delta_s^{\ell} H} \neq k\, C_{\Delta_s^{m} H}$ $\quad k > 0$, and $C_{\Delta_s^{\ell} H \Delta_s^{m} H} = 0$, (6-74)

iv) If $C_{\Delta_s^{\ell} H} \neq k\, C_{\Delta_s^{m} H}$ $\quad k > 0$, and $C_{\Delta_s^{\ell} H \Delta_s^{m} H} \neq 0$, (6-75)

One finds that

i) Under the specific conditions of equations (6-72) a comparison of separately adjusted positions and a kinematic adjustment, both formulated either in the observation or the parameter space, lead to equivalent results. Also, the covariance matrices of the adjusted results in the two cases are the same. On the other hand, different estimated covariance matrices are found for both approaches.

ii) Under the specific conditions of equations (6-73) a comparison of separately adjusted positions and a kinematic adjustment, both formulated either in the observation or the parameter space, do not lead to equivalent results. Neither the covariance matrices, nor the estimated covariance matrices of the adjusted results are equivalent in both approaches.

iii) Under the specific conditions of equations (6-74) a comparison of separaterly adjusted positions and a kinematic adjustment, both either formulated in the observation or the parameter space, do not lead to equivalent results. On the other hand, the covariance matrices of the adjusted results in both cases are the same. This, however, does not occur between the estimated covariance matrices which show different values.

iv) Finally, under the specific conditions of equations (6-75) a comparison of separately adjusted positions and a kinematic adjustment, both either formulated in the observation space or the parameter space, do not lead to equivalent results. Neither the covariance, nor the estimated covariance matrices of the adjusted results are equal.

It is only in the first case discussed above that a comparison of a separately adjusted height network can provide rigorous results. The conditions described by equation (6-72) are, in practice, very difficult to find therefore its use should be avoided.

A kinematic adjustment, on the other hand, is a rigorous approach that permits the introduction of a complete temporal covariance matrix.

6.5 STATISTICAL TESTING AND VARIANCE COVARIANCE ESTIMATION

Proper assessments of kinematic adjustments are suppossed to answer the question if, in fact, VCM have occured or if the intrinsic lack of exact repeatibility of the results has its origin in the design and instrumentation of the experiment.

Two aspects play a crucial role here:

i) Statistical testing, and

ii) variance-covariance estimation.

The following information can be extracted from statistical testing (Vaníček and Krakiwsky, 1982, p. 220):

i) If the postulated probability density function (p.d.f.) for the experiment is likely to have been correctly postulated,

ii) if the estimated value of a population parameter is to be trusted, and

iii) if the estimated value of a population parameter is consistent with the known (a priori) value of the parameter, if it is available.

Testing of VCM can be performed on mathematical models of

i) Continuously connected relevelled segments whose design matrix has remained either invariant during the experiment or, alternatively, that has suffered modifications in time, or

ii) scattered relevelled segments heterogeneously distributed not only in space but also in time.

Tests on the first type of these two possible models have been made, for example, by Vaníček and Hamilton (1972) on the p.d.f., showing bimodality, and the correlation coefficient of the uplift and the topography, showing uncorrelated results.

Other testing approaches are clearly applicable, particularly the tests leading to the determination of the reliability of the net (Baarda, 1968). The studies of a sensitivity analysis designed by Hein (1981) following the work of Moritz (1972) for kinematic adjustments with a time invariant design matrix, and of Schaffrin (1981) for time dependent design and covariance matrices should be mentioned in this respect.

A problem that has to be also solved is the proper estimation of the observations' covariance matrix of each epoch

(Wassef, 1979) and the temporal cross-covariance matrix in between any two sets of observations. The two limiting cases for the propagation of errors, totally statistical dependence and independence, for positively correlated observations have been shown by Vaníček and Grafarend (1980). Autocorrelation techniques have shown promising results to solve this problem (Vaníček and Craymer, 1983).

On the other hand, high correlation coefficients in between forward and backward levellings have been found (Remmer, 1975), probably an analysis of such type could prove useful to evaluate, at least, the main diagonal of the temporal cross-covariance matrix of any two epochs.

Chapter VII

EXAMPLE OF A TEMPORAL HOMOGENIZATION OF LEVELLING OBSERVATIONS

7.1 INTRODUCTION

It was shown in Chapter III that complete path independence can only be achieved if a height network is homogeneous in time. Here, an example of such an homogeneization for a height network in the Maritime Provinces is presented.

7.2 DATA

The original heterogeneities in time raw data to be corrected are:

i) Observed height differences in between tidal benchmarks, and

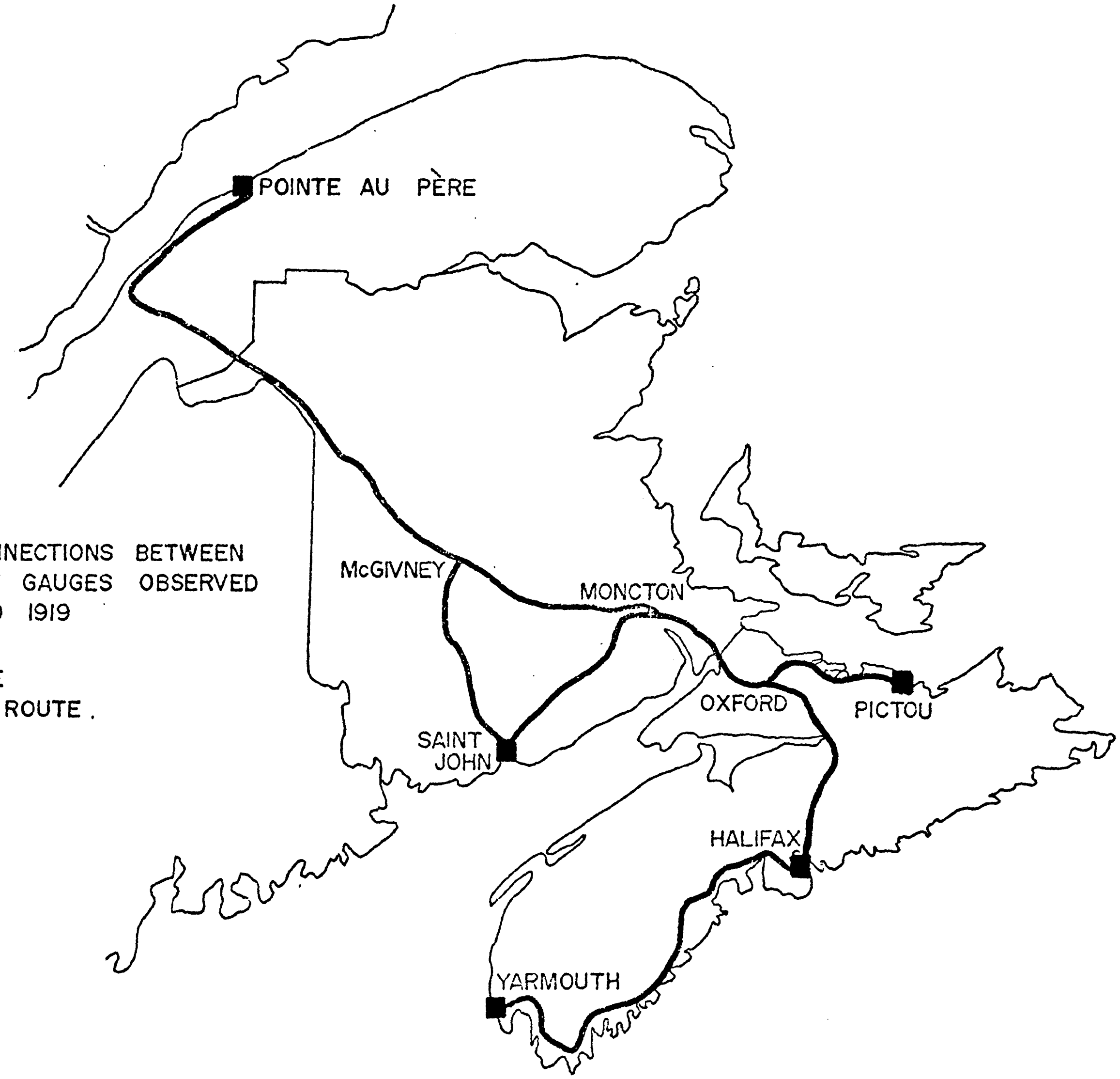
ii) height connections between local chart Datums and the Geodetic Survey of Canada Heights Datum.

Observed height differences of lines connecting different eastern ports are available from two different levellings: the 'old' levelling carried out between 1909 and 1923, and the 'new' levelling carried out between 1952 and 1978. Levelling lines from both surveys connecting the locations of Pointe au Père, Que.; Saint John, N.B.; Pictou, N.S.; Halifax, N.S.; and Yarmouth, N.S. were selected. Their configurations are shown in figures 7.1 and 7.2.

Figure 7.1

LEVELLING CONNECTIONS BETWEEN
MARITIME TIDE GAUGES OBSERVED
FROM 1909 TO 1919

■ TIDE GAUGE
━ LEVELLING ROUTE.



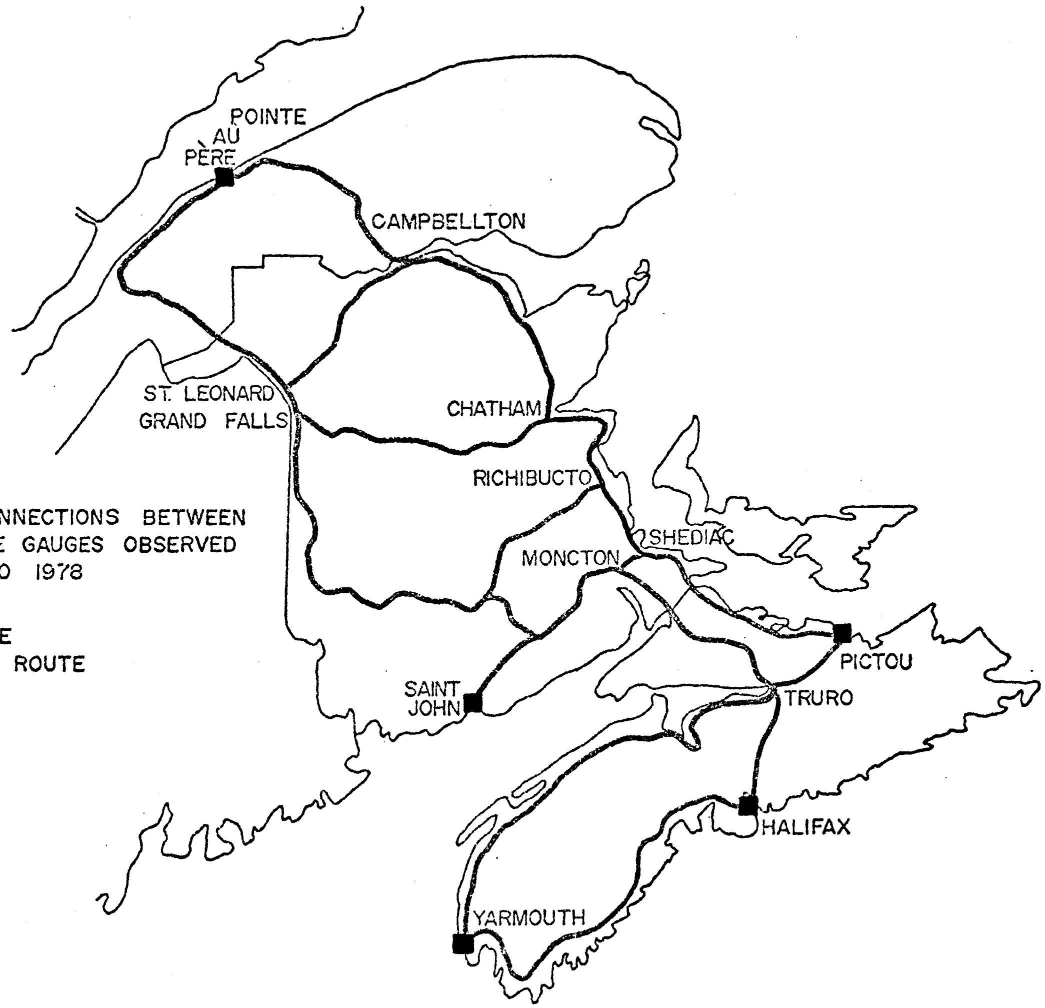


Figure 7.2

LEVELLING CONNECTIONS BETWEEN MARITIME TIDE GAUGES OBSERVED FROM 1952 TO 1978

■ TIDE GAUGE
▬ LEVELLING ROUTE

The height connections between local Chart Datums and the heights' datum can be split into:

i) The height difference between local MSL and the conventional zero of the tide gauge, and

ii) the height difference between the conventional zero of the tide gauge and the reference benchmark.

The first were determined in eastern Canada at the time of the Low Water Datum determination: Pointe au Père in 1897, Saint John in 1895, Pictou in 1898, Halifax in 1895, and Yarmouth in 1902.

The height difference between the conventional zero of the tide gauges and the reference benchmark of the ports mentioned above has been routinely observed on a yearly basis since the determination of the Low Water Datum. None has shown variations in time.

The source of information to evaluate all temporal corrections is the Map of VCM of Canada computed by Vaníček and Nagy (1981).

7.3 TEMPORAL CORRECTIONS

Two corrections to homogenize levelling networks in time were introduced in Chapter III:

i) Levelling corrections, and

ii) geoid-tidal benchmark corrections.

Levelling corrections were introduced in equations (3-50), (3-51), and (3-52) for the dynamic, orthometric and normal height systems based on actual gravity.

In practice, two temporal corrections may be applied to the levelling data of a height network:

i) An intersurvey correction to reduce all levelling segments within a line to their mean epoch of measurement, and

ii) a reduction of all network lines from their individual mean epochs of measurement to a common reference epoch for the entire net.

The intersurvey correction of a net may be interpreted to be the reduction for all movements occurring in between its different segments with respect to its mean epoch. Clearly, if a line is formed by a single segment, i.e., the line is levelled in a single campaign, its intersurvey correction vanishes.

The reduction of all lines in a network from their individual mean epochs of measurement to a common reference epoch may be interpreted as the correction for the amount of movement that each individual, homogeneous in time, levelling line would have experienced between their individual mean and the network reference epoch.

Geoid-benchmark corrections were included in equations (3-54), (3-55), and (3-56). However, a correction for SST temporal variations is not available at the present time, and, in this case, the corrections for relative movements between the conventional zeros of the tide gauges and the tidal benchmark vanishes in all ports. Then the only correction left is given by equation (3-55).

$$H(n,r) - MSL(n,r) = H(n,\ell) - MSL(n,\ell) - \Delta_t^n CZ, \qquad (3\text{-}56)$$

which corrects for MSL temporal variations.

There are two ways to check the quality of observations homogenized in time:

i) Connecting local MSL between different ports, and

ii) studying the misclosures of levelling loops homogeneous in time.

7.4 DETERMINATION OF SEA SURFACE TOPOGRAPHY DIFFERENCES

Levelling connections between MSL at different ports leads to values of SST differences. These values can only be checked if there is a value of SST differences a priori known from an independent source. Here, the zero frequency response analysis of Merry and Vaníček (1981) provides such an independent approach.

Table 7.1 shows the results of SST differences using the 'old' and 'new' levellings reduced both to a common epoch of 1962.

The standard deviation associated to the observed height differences were obtained using 1.4 $\sqrt{\text{km}}$, as obtained by Nassar (1977).

A comparison of the SST computed by means of levelling and those which resulted from the response method is given

Table 7.1 Results of Geodetic Levelling

PORT	MEAN EPOCH OF FIELD WORK	LEVELLED HEIGHT DIFFERENCE mm	CORRECTION FOR NORMAL GRAVITY mm	INTERSURVEY CORRECTION FOR CRUSTAL MOVEMENTS mm	REDUCTION TO EPOCH OF 1962 mm	TIDAL BENCHMARK MOVEMENT CORRECTION mm	EPOCH OF MSL DETERMINATION mm	MSL HEIGHT DIFFERENCE IN 1962 mm
Halifax	1914	- 33 ± 29	8	1	57	- 92	1895	- 59
Yarmouth	1975	15 ± 27	8	1	- 15		1902	- 83
Halifax	1913	76 ± 31	- 2	0	17	- 24	1895	67
St. John	1971	101 ± 30	+ 1	- 1	- 3		1895	74
Halifax	1970	- 42 ± 19	- 2	0	- 8	- 70	1895	- 122
Pictou							1898	
Pictou	1913	231 ± 40	- 22	6	157		1898	165
	1967	(220*)±40	- 2	27	- 13	- 207		(25*)
Father's Point	1973	(100*)±40	- 17	- 13	- 35		1897	(-172*)
St. John	1966	- 126 ± 38	11	14	- 3	- 69	1895	- 173
Yarmouth							1902	
St. John	1912	- 96 ± 26	- 2	18	30		1895	- 96
	1973	- 141 ± 29	- 2	12	- 7	- 46		- 184
Pictou	1974	- 108 ± 30	- 1	- 21	- 7		1898	- 183
St. John	1916	139 ± 33	- 32	12	175		1895	41
	1972	(138*)±34	- 24	- 7	- 38	- 253		(-184*)
Father's Point	1975	(221*)±34	- 22	1	- 49		1897	(-102*)

(*) Height differences questionable (inacceptable loop closures) in table 7.2. With exception of the 'new' levelling connections to Pointe au Père, all other results are encouraging. A substantial agreement between both approaches is found.

TABLE 7.2

Comparison of Partial SST Differences With Levelling

PORT	MEAN EPOCH OF FIELD WORK	MSL HEIGHT DIFFERENCES* mm	PARTIAL SST DIFFERENCE+ mm	DIS-CREPANCY mm
Halifax	1914	- 59	- 112	53
Yarmouth	1975	- 83		29
Halifax	1913	67	26	41
St. John	1971	74		48
Halifax	1970	- 122	- 134	12
Pictou				
Pictou	1913	165		63
Father's Point	1967	(25**)	228	-
	1973	(-172**)		-
St. John	1966	- 173	- 138	- 35
Yarmouth				
St. John	1912	- 96		64
Pictou	1973	- 184	- 160	- 24
	1974	- 183		- 23
St. John	1916	41		- 27
Father's Point	1972	(-184**)	68	-
	1975	(-102**)		-
Average (absolute values)		113	124	38

* From Table 7.1

** Height differences disregarded (inacceptable loop closure

\+ From Merry and Vaníček (1981).

Let us now turn to discuss the large discrepancies found when Pointe au Père was included using the 'new' levelling.

Figure 7.3 shows all loop misclosures employing raw 'new' levelling observations. Clearly, these loop misclosures are not supposed to vanish. However, the large values of some of them in central New Brunswick confirm the fact that those lines should hardly be trusted.

Figure 7.4 shows all loop misclosures when all observations are homogeneous in time. In figure 7.5 also gravity corrections based on normal gravity have been added. Still, unacceptable loop misclosures are found.

In order to locate areas with high geodynamic activity in time, maps of earthquake locations in eastern Canada from 1535 to 1959 are shown in figures 7.6 and 7.7. An attempt to investigate a relationship between loop misclosures with areas of high seismic activity is made in figure 7.8. Two locations should be pointed:

i) The valley of the St. Lawrence River, and

ii) Central New Brunswick.

Seismicity in these areas has been already recognized as one of the most challenging problems of intra-plate tectonics (Wetmiller, 1975; Yang and Aggarwal, 1981).

The high historic activity located in Central New Brunswick, along the levelling line from Grand Falls to Chatham, correlates also with recent seismic activity registered between 1981 and 1982, as shown in figure 7.9.

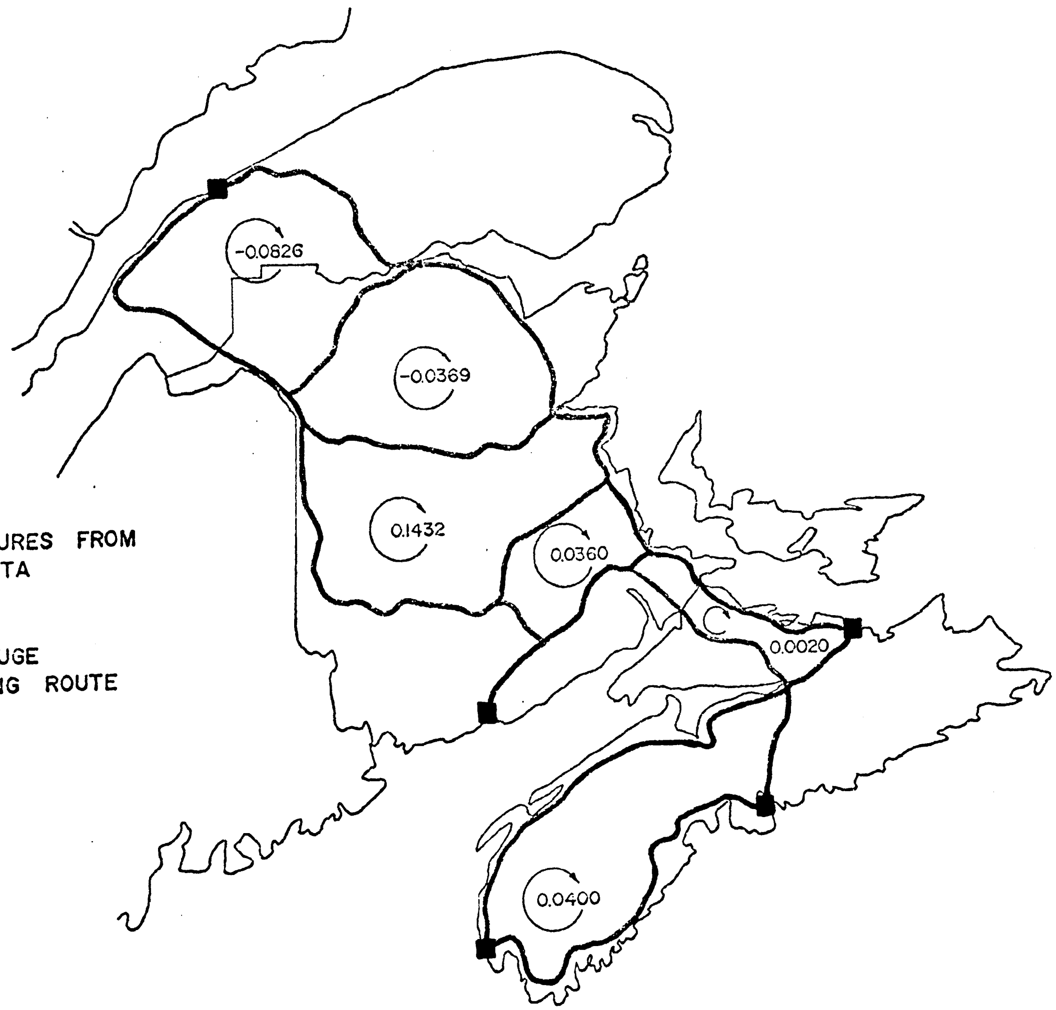


Figure 7.3
LOOP MISCLOSURES FROM OBSERVED DATA

■ TIDE GAUGE
— LEVELLING ROUTE

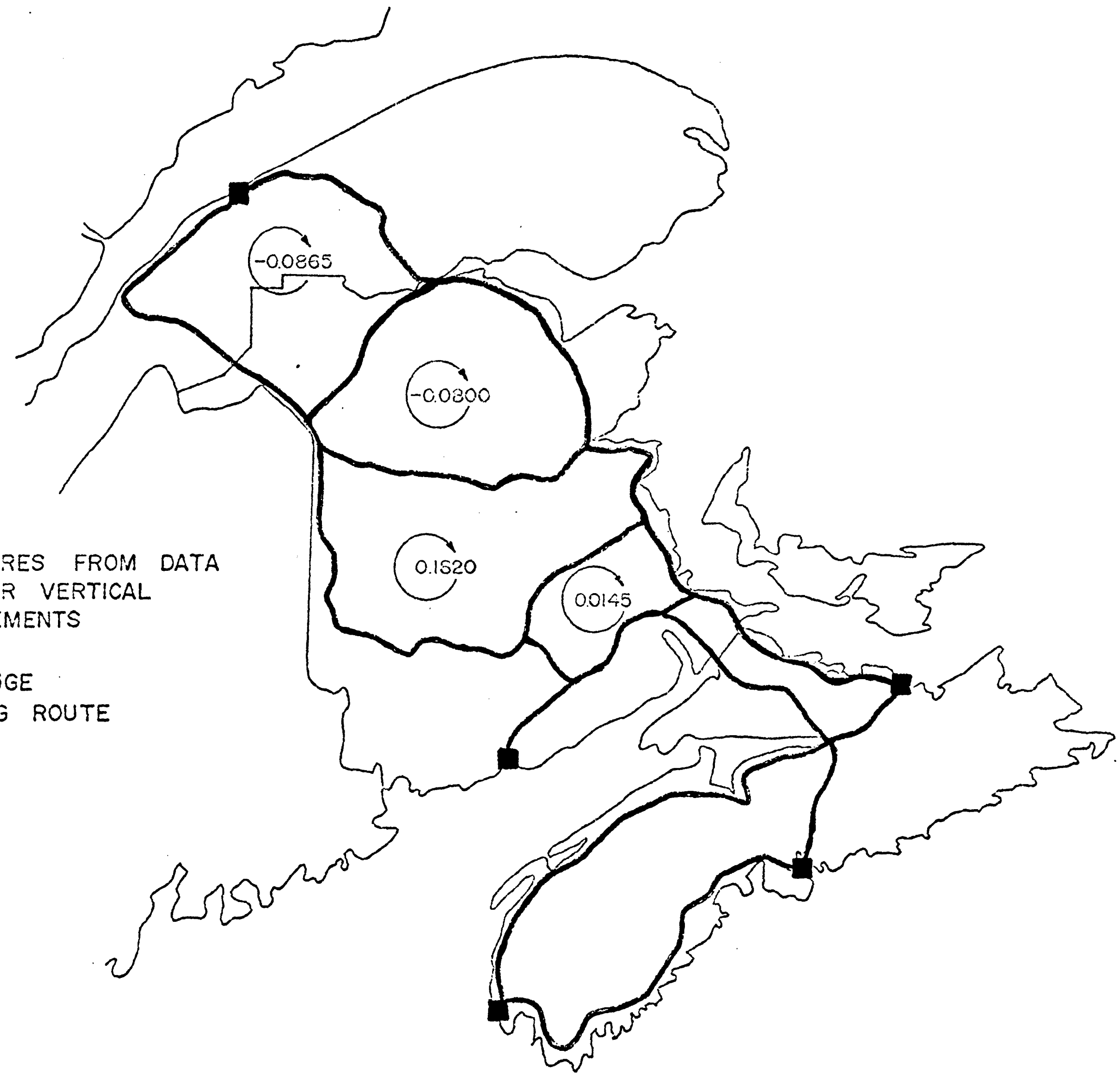


Figure 7.4

LOOP MISCLOSURES FROM DATA CORRECTED FOR VERTICAL CRUSTAL MOVEMENTS

■ TIDE GAUGE
━ LEVELLING ROUTE

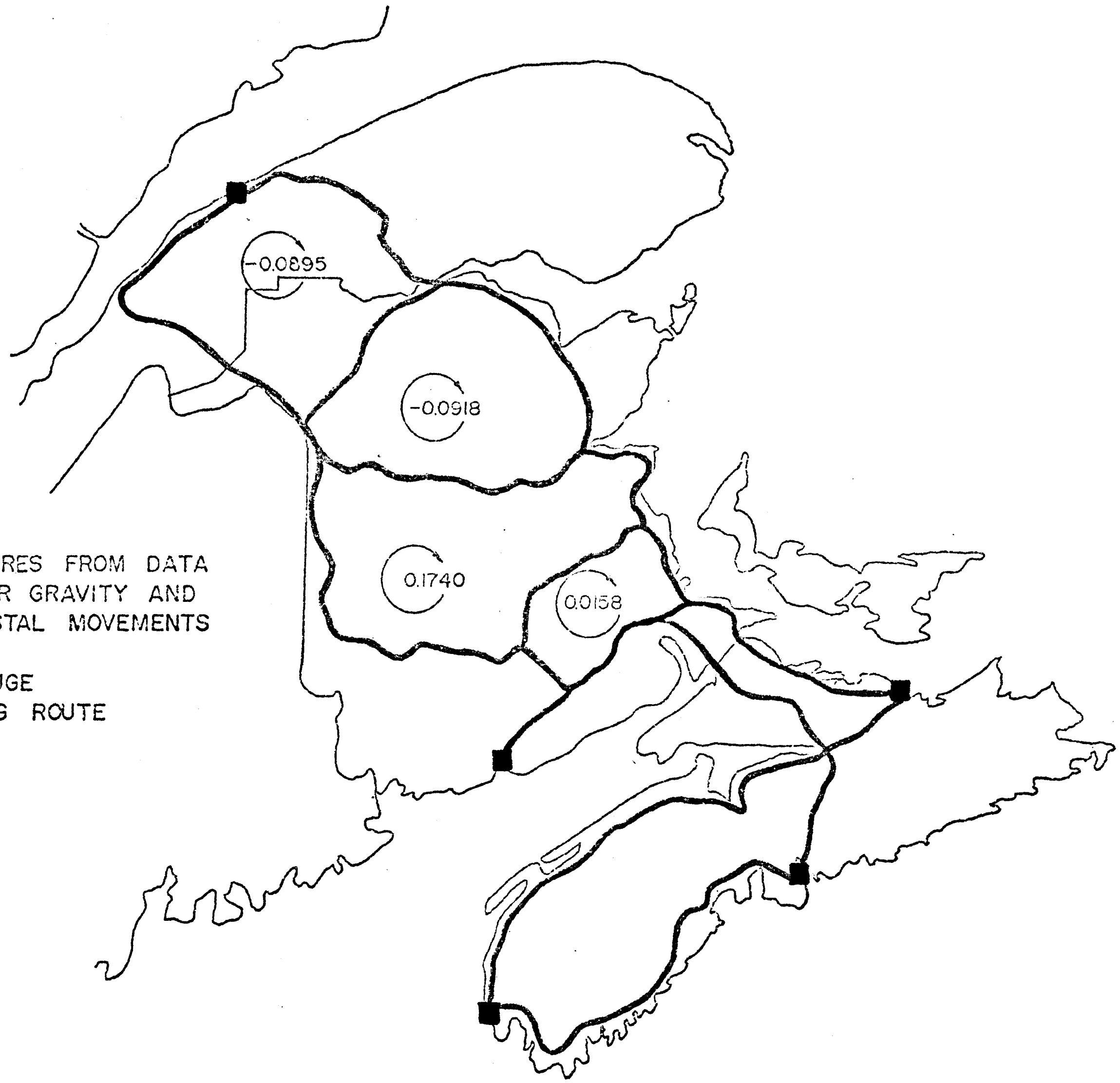


Figure 7.5

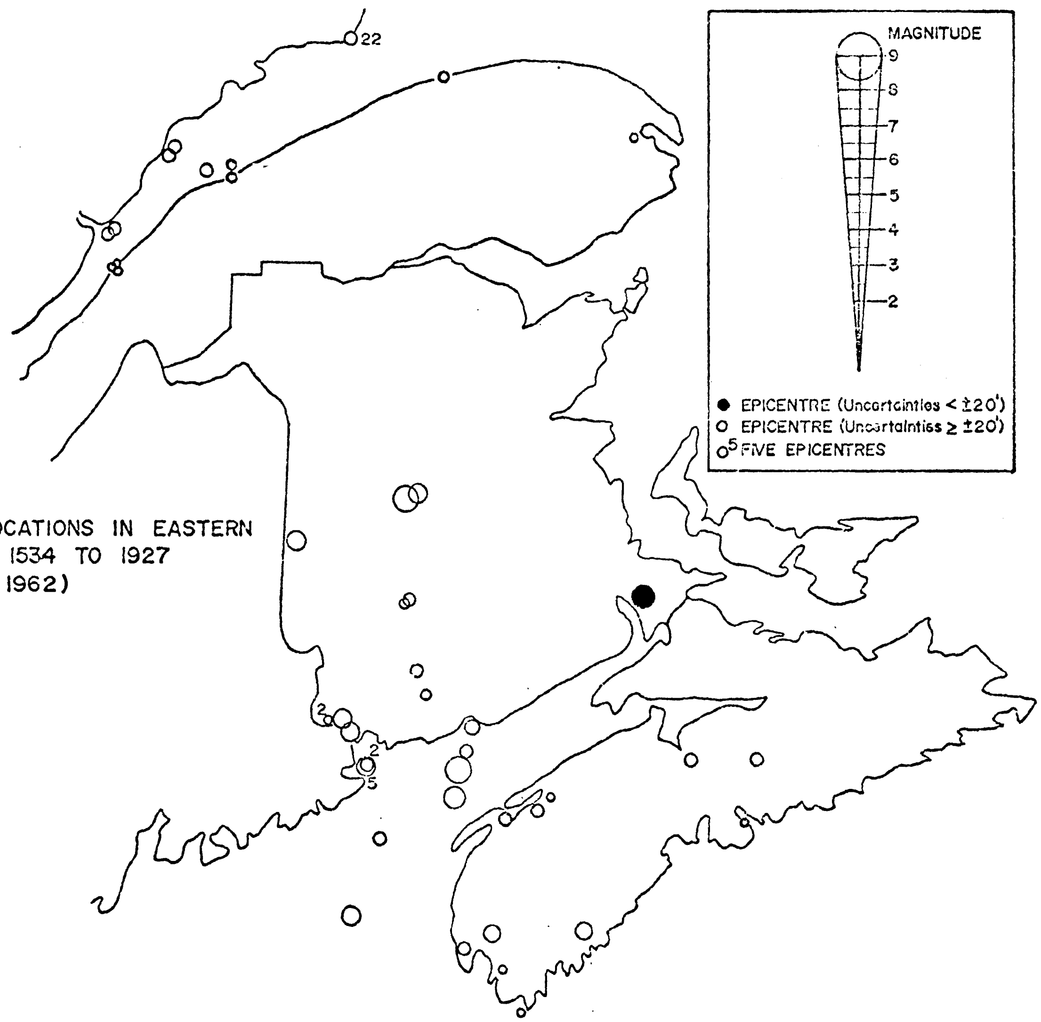
LOOP MISCLOSURES FROM DATA CORRECTED FOR GRAVITY AND VERTICAL CRUSTAL MOVEMENTS

■ TIDE GAUGE

—— LEVELLING ROUTE

Figure 7.6

EARTHQUAKE LOCATIONS IN EASTERN CANADA FROM 1534 TO 1927 (AFTER SMITH 1962)



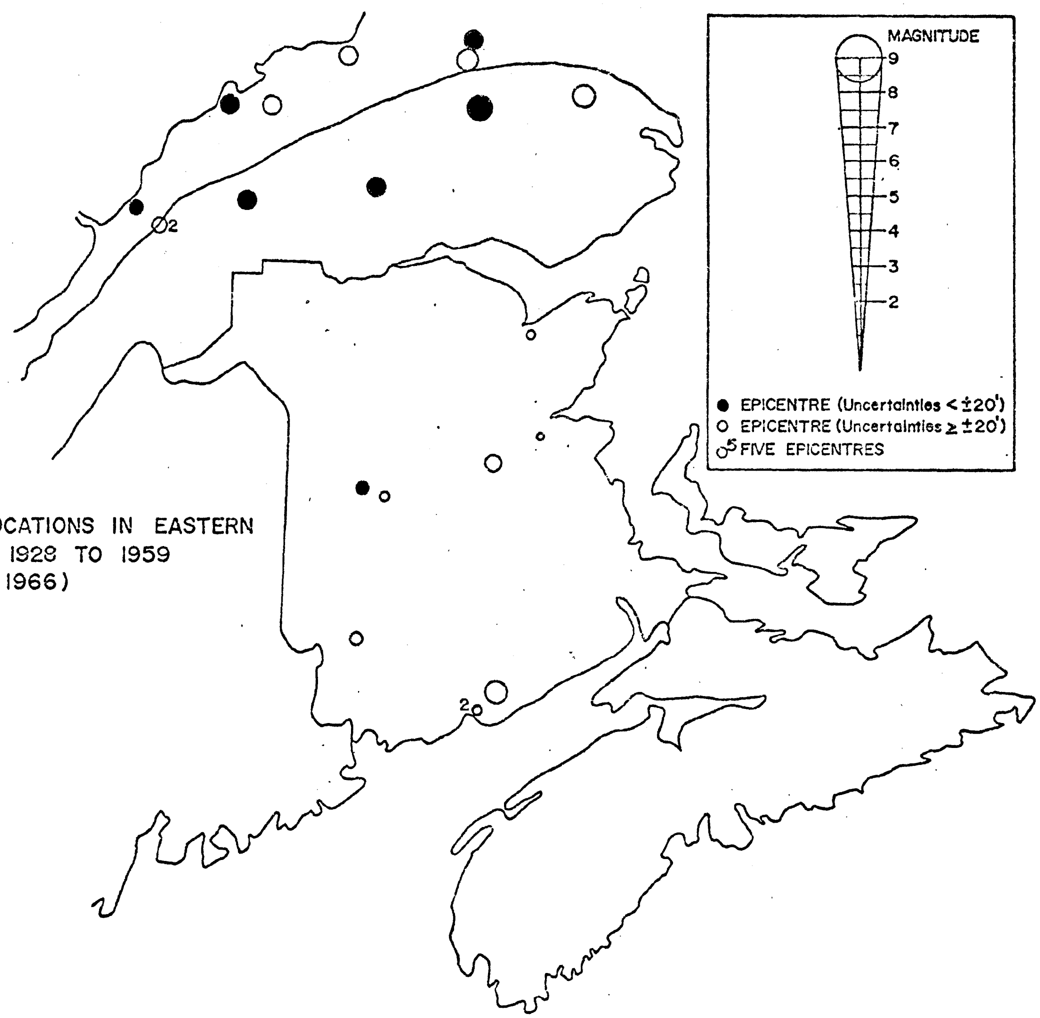


Figure 7.7

EARTHQUAKE LOCATIONS IN EASTERN CANADA FROM 1928 TO 1959 (AFTER SMITH 1966)

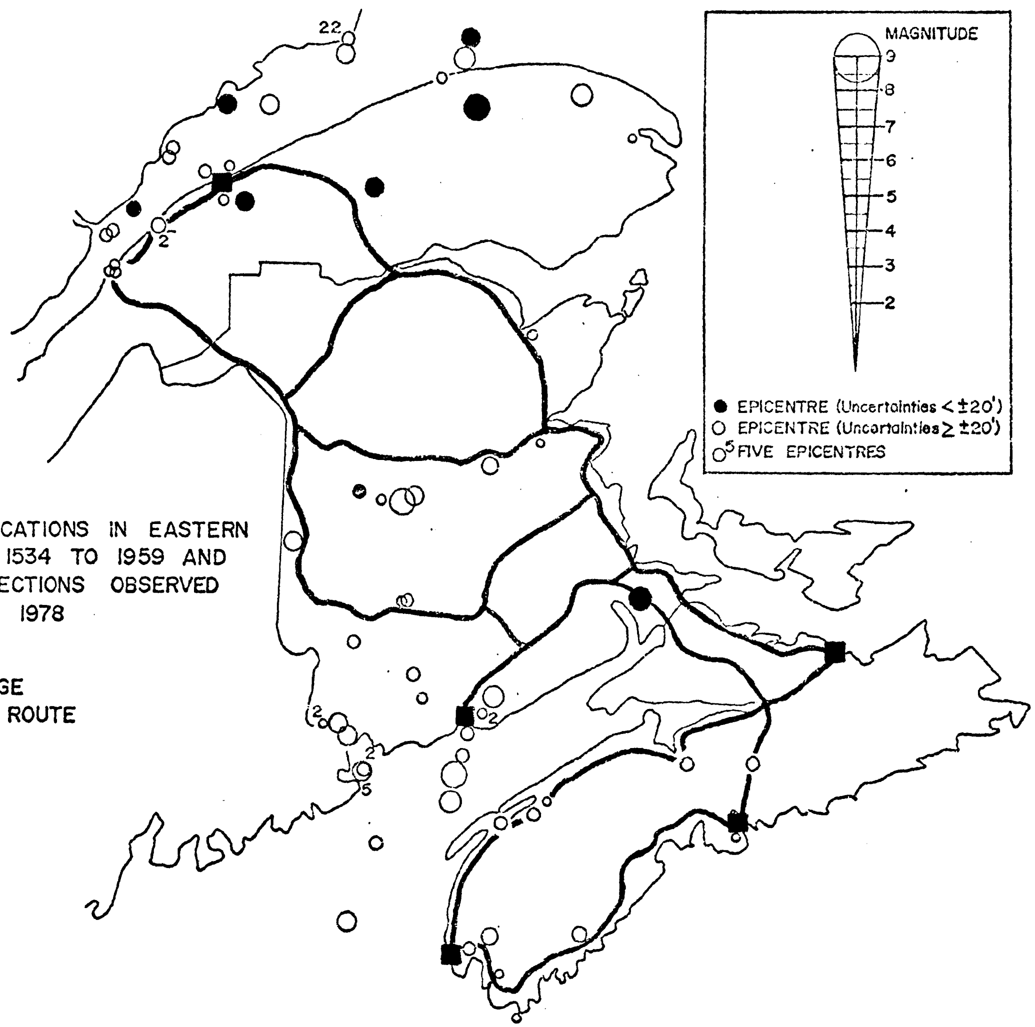


Figure 7.8

EARTHQUAKE LOCATIONS IN EASTERN CANADA FROM 1534 TO 1959 AND LEVELLING CONNECTIONS OBSERVED FROM 1952 TO 1978

TIDE GAUGE
LEVELLING ROUTE

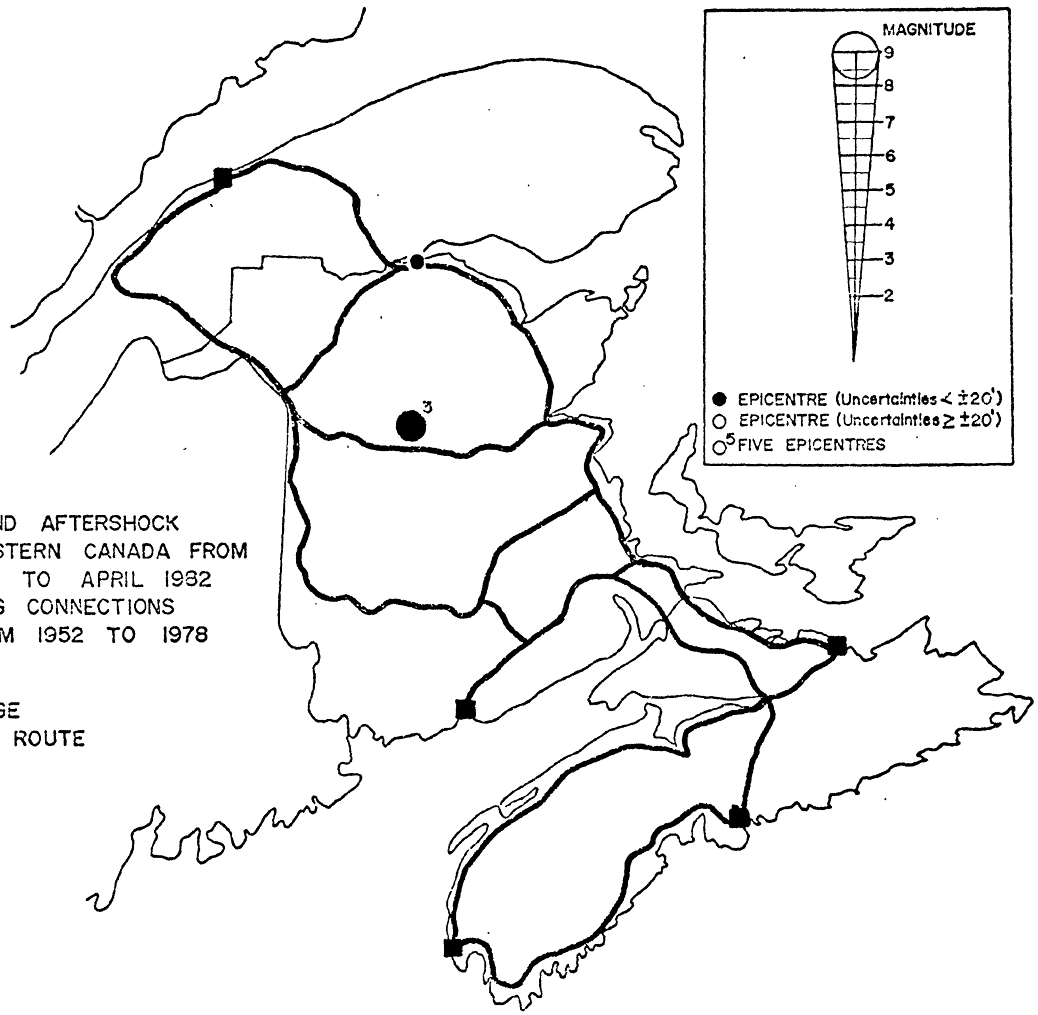


Figure 7.9

EARTHQUAKE AND AFTERSHOCK ACTIVITY IN EASTERN CANADA FROM NOVEMBER 1981 TO APRIL 1982 AND LEVELLING CONNECTIONS OBSERVED FROM 1952 TO 1978

■ TIDE GAUGE
—— LEVELLING ROUTE

It is not adventurous, then, to suggest that some of the large misclosures found in the new levelling may very well be a consequence of VCM discontinuous in space and time.

This is one example of the type of interdisciplinary considerations that must be made in each geodetic study (Castle and Vaníček, 1980).

Chapter VIII

CONCLUSIONS AND RECOMMENDATIONS

From the review of levelling nets in America and Europe, presented in Chapter I, several points regarding their future redefinition arise:

i) A selection of a height system, i.e., reference surface and heights,

ii) the mathematical formulation of the adjustment,

iii) a selection of the different types and amount of data to be used, and

iv) the number and type of corrections applied to the levelling data.

The question of what height system should be used may be answered having in mind the different needs of the various users. Theoretically, of course, any rigorous height system based on actual gravity can be selected, given that all are path-independent (Vaníček, 1982). However, if extraterrestrial information is somehow incorporated the orthometric height system proves to be a better choice.

The question of what mathematical model should be used in the adjustment has also several equivalent answers: Parametric, conditional, and combined. Any one could be used.

A more involved question is what type of constraints should be imposed in the adjustment model. This may be answered by selecting the types of data to be adjusted:

i) Levelling and water transfers,

ii) sea level, and

iii) Doppler data.

Two alternatives arise if water transfers and levelling observations are only included in the model: A free adjustment, or and adjustment with an absolute minimum constraint. The first was advocated in the past by Vaníček <u>et al</u> (1972).

A better alternative may be if, in addition to levelling, sea level information is also included in the adjustment as a weighted constraint. This is supported by the recent success of the evaluation of partial sea surface topography at different ports by Merry and Vaníček (1981, 1983). Another type of weighted constraints which could be imposed in the adjustment are orthometric heights resulting from Doppler data, provided that an accurate description of geoidal heights (Vaníček and John, 1983) is available.

The final question which may be raised with respect to the adjustment is the assembling of the covariance matrix of the observations:

Water transfers can be weighted in a realistic manner if an account of the many dynamic factors affecting lake surfaces is made.

The weight given to the sea level constraints may be obtained from the standard deviations of the computed partial SST at different ports (Merry and Vaníček, 1981).

The weight of Doppler derived orthometric heights as constraints may readily be obtained from the standard deviations of geoidal and Doppler determined geometric heights.

On the other hand, no definite answer can be given at the present time to what the optimum scheme to weight levelling observations may be. Their statistical dependence remains here as a key factor (Vaníček and Craymer, 1983).

Finally, the question of what corrections should be applied to the levelling data has multiple answers, many of which have been discussed in section 5.3.1. One of these corrections: the one for kinematic effects has been the main motivation of this investigation.

It has been shown in Chapter III that complete path independence can only be achieved when both the reference surfaces and heights, are homogeneous in time. Explicit expressions of temporal corrections for a height network have been explicitly developed in section 3.3. An actual example of temporal corrections has been worked in Chapter VII.

The homogeneization in time of a height network requires a knowledge of the entire vertical displacement field of the earth. Chapter V is totally devoted to discuss the validity of geodetic terrestrial techniques as a tool to detect such displacements. The conclusion that can be obtained from this analysis is that even when extraterrestrial techniques may provide an abundant quantity of data in the future, terrestrial techniques still will prove to be an indispensable complement in any kinematic study.

The investigation on the optimum design of levelling networks to detect VCM shows the unnecesary requirement of accuracy made in the observations when the temporal cross-covariance matrix of the observations is neglected. In other words, an optimum network configuration and observation scheme designed to provide optimum heights are not necessarily optimum to detect temporal variations in heights.

Regarding what scheme to use to correct for differential refraction in levelling, either for positioning or kinematic studies, no definite answer can be given at this time. However, nothing prevents the use of both the physical and the statistical approach together. If differential refraction is indeed entirely corrected by means of physical modelling, the value of the nuisance parameter in the statistical approach should be zero. This is true, provided that all systematic effects that correlate with refraction are also removed.

From the analysis made of the mathematical models designed to extract vertical crustal movements in Chapter VI, the kinematic adjustment of a net proves to be a superior technique to that of a comparison of separately adjusted positions. Furthermore, a strong remark must be made: The only rigorous method to detect VCM and their estimated accuracies is that of a kinematic adjustment, either when connected or scattered relevelled segments are available.

A chapter on temporal variations of the geoid was thought to be justified to show possible temporal changes in scale and shape. One such change in shape: the viscoelastic response due to ice unloading, or postglacial rebound, leads to the conclusion that the old belief of an 'eustatic rise' of sea level is a misconception. Instead, eustatic changes with different histories as a function of position and time are experienced.

Finally, the temporal homogeneization of the Canadian levelling net in the Maritime Provinces shows consistent results in both the 'old' and 'new' levelling observations, except for the 'new' levelling lines that run from southern New Brunswick to Pointe au Père in Quebec. Seismic activity is likely to have induced VCM discontinuous in space and time. Further relevelling of those lines would be required before further conclusions could be drawn. Also, a useful piece of kinematic information could have been obtained from the tide gauge at Point Sapin, N.B., this gauge, however, was discontinued in 1975. Its reactivation becomes indispensable for any future study.

The Chart datums of all tide gauges in this study were found to be without a statistical assesement of their confidence. A new determination of all and their standard deviations would help to assess confidence intervals for any type of study on heights.

Mean Sea Level determinations in all ports were found to be old and heterogeneous in time. The value of their corrections constitutes the bulk of the temporal homogenization.

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