INTRODUCTION TO GEODETIC ASTRONOMY

D. B. THOMSON

December 1981

LECTURE NOTES
49

PREFACE

In order to make our extensive series of lecture notes more readily available, we have scanned the old master copies and produced electronic versions in Portable Document Format. The quality of the images varies depending on the quality of the originals. In this version, images have been converted to searchable text.

INTRODUCTION TO GEODETIC ASTRONOMY

Donald B. Thomson

Department of Geodesy and Geomatics Engineering
University of New Brunswick
P.O. Box 4400
Fredericton, N.B.
Canada
E3B 5A3

September 1978
Reprinted with Corrections and Updates December 1981
Latest Reprinting January 1997

PREFACE

These notes have been written for undergraduate students in Surveying Engineering at the University of New Brunswick. The overall objective is the development of a set of practical models for the determination of astronomic azimuth, latitude and longitude that utilize observations made to celestial objects. It should be noted here that the emphasis in these notes is placed on the so-called second-order geodetic astronomy. This fact is reflected in the treatment of some of the subject matter. To facilitate the development of models, several major topics are covered, namely celestial coordinate systems and their relationships with terrestrial coordinate systems, variations in the celestial coordinates of a celestial body, time systems, timekeeping, and time dissemination.

Finally, the reader should be aware of the fact that much of the information contained herein has been extracted from three primary references, namely Mueller [1969], Robbins [1976], and Krakiwsky and Wells [1971]. These, and several other's, are referenced extensively throughout these notes.

D.B. Thomson.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

List of Figures Cont'd

List of Figures Cont'd

List of Figures cont'd

1. INTRODUCTION

Astronomy is defined as [Morris, 1975] "The scientific study of the universe beyond the earth, especially the observation, calculation, and theoretical interpretation of the positions, dimensions, distribution, motion, composition, and evolution of celestial bodies and phenomena". Astronomy is the oldest of the natural sciences dating back to ancient Chinese and Babylonian civilizations. Prior to 1609, when the telescope was invented, the naked eye was used for measurements.

Geodetic astronomy, on the other hand, is described as [Mueller, 1969] the art and science for determining, by astronomical observations, the positions of points on the earth and the azimuths of the geodetic lines connecting such points. When referring to its use in surveying, the terms practical or positional astronomy are often used. The fundamental concepts and basic principles of "spherical astronomy", which is the basis for geodetic astronomy, were developed principally by the Greeks, and were well established by the 2nd century A.D.

The treatment of geodetic astronomy in these notes is aimed at the needs of undergraduate surveying engineers. To emphasise the needs, listed below are ten reasons for studying this subject matter:

(i) a knowledge of celestial coordinate systems, transformations amongst them, and variations in each of them;

(ii) celestial coordinate systems define the "link" between satellite and terrestrial coordinate systems;

(iii) the concepts of time for geodetic purposes are developed;

(iv) tidal studies require a knowledge of geodetic astronomy;

(v) when dealing with new technologies (e.g. inertial survey systems) an understanding of the local astronomic coordinate system is essential;

(vi) astronomic coordinates of terrain points, which are expressed in a "natural" coordinate system, are important when studying 3-D terrestrial networks;

(vii) astronomically determined azimuths provide orientation for terrestrial networks;

(viii) the determination of astrogeodetic deflections of the vertical are useful for geoid determination, which in turn may be required for the rigorous treatment of terrestrial observations such as distances, directions, and angles;

(ix) geodetic astronomy is useful for the determination of the origin and orientation of independent surveys in remote regions;

(x) geodetic astronomy is essential for the demarcation of astronomically defined boundaries.

1.1 Basic definitions

In our daily work as surveyors, we commonly deal with three different surfaces when referring to the figure of the earth: (i) the terrain, (ii) an ellipsoid, and (iii) the geoid.

The physical surface of the earth is one that is extremely difficult to model analytically. It is common practice to do survey computations on a less complex and modelable surface. The terrain is, of course, that surface on or from which all terrestrially based observations are made.

The most common figure of the earth in use, since it best approximates the earth's size and shape is a biaxial ellipsoid. It is a purely mathematical figure, defined by the parameters a (semi-major axis) b (semi-minor axis) or a and f (flattening), where $f=(a-b)/a$ (Figure 1-1). This figure is commonly referred to as a "reference ellipsoid", but one should note that there are many of these for the whole earth or parts thereof. The use of a biaxial ellipsoid gives rise to the use of curvilinear geodetic coordinates - ϕ (latitude), λ(longitude), and h (ellipsoidal height) (Figure 1-2). Obviously, since this ellipsoid is a "mathematical" figure of the earth, its position and orientation within the earth body is chosen at will. Conventionally, it has been positioned non-geocentrically, but the trend is now to have a geocentric datum (reference ellipsoid). Conventional orientation is to have parallelism of the tertiary (Z) axis with the mean rotation axis of the earth, and parallelism of the primary (X) axis with the plane of the Greenwich Mean Astronomic Meridian.

Equipotential surfaces (Figure 1-3), of which there are an infinite number for the earth, can be represented mathematically. They account for the physical properties of the earth such as mass, mass distribution, and rotation, and are "real" or physical surfaces of the earth. The common equipotential surface used is <u>the geoid</u>, defined as [Mueller, 1969] "that equipotential surface that most nearly coincides with the undisturbed mean surface of the oceans". Associated with these equipotential surfaces is the <u>plumbline</u> (Figure 1-3). It is a line of force that is everywhere normal to the equipotential surfaces; thus, it is a spatial curve.

We now turn to definitions of some fundamental quantities in geodetic astronomy namely, the astronomic latitude (Φ), astronomic longitude (Λ), and orthometric height (H). These quantities are sometimes referred to as "natural" coordinates, since, by definition, they are given in terms of the

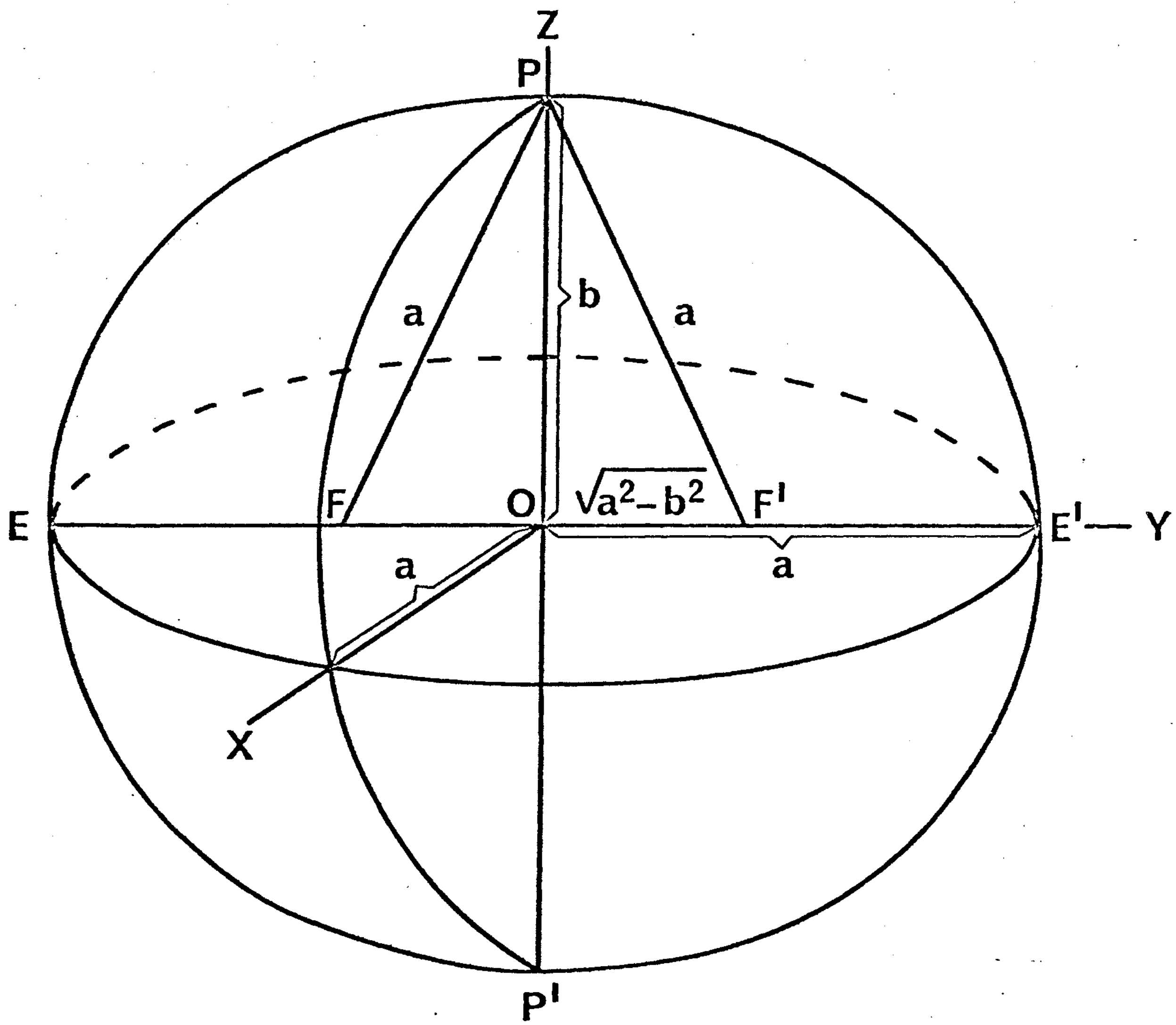


Figure 1-1

Biaxial Ellipsoid

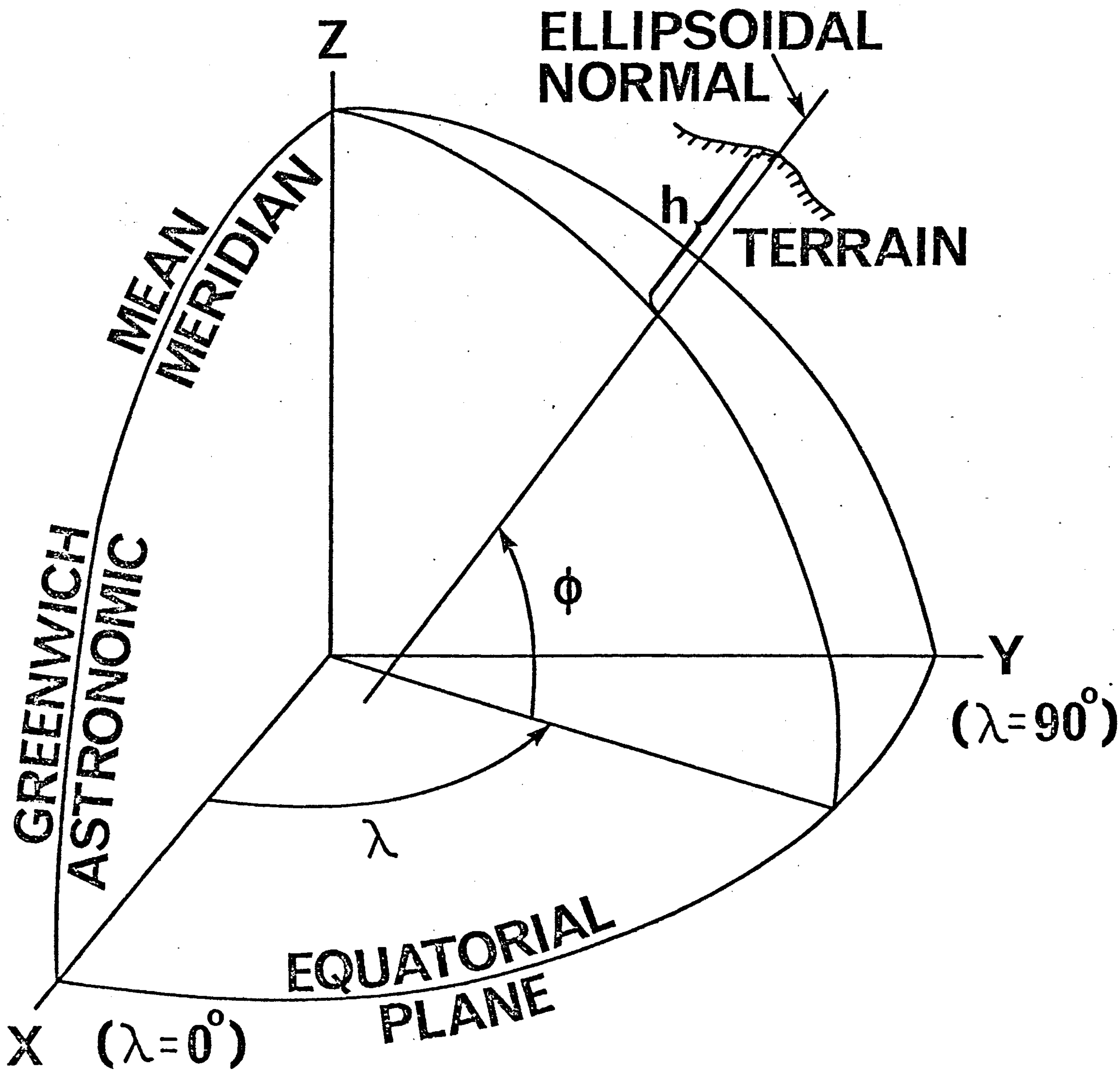


Figure 1-2

Geodetic Latitude, Longitude, and Ellipsoidal Height

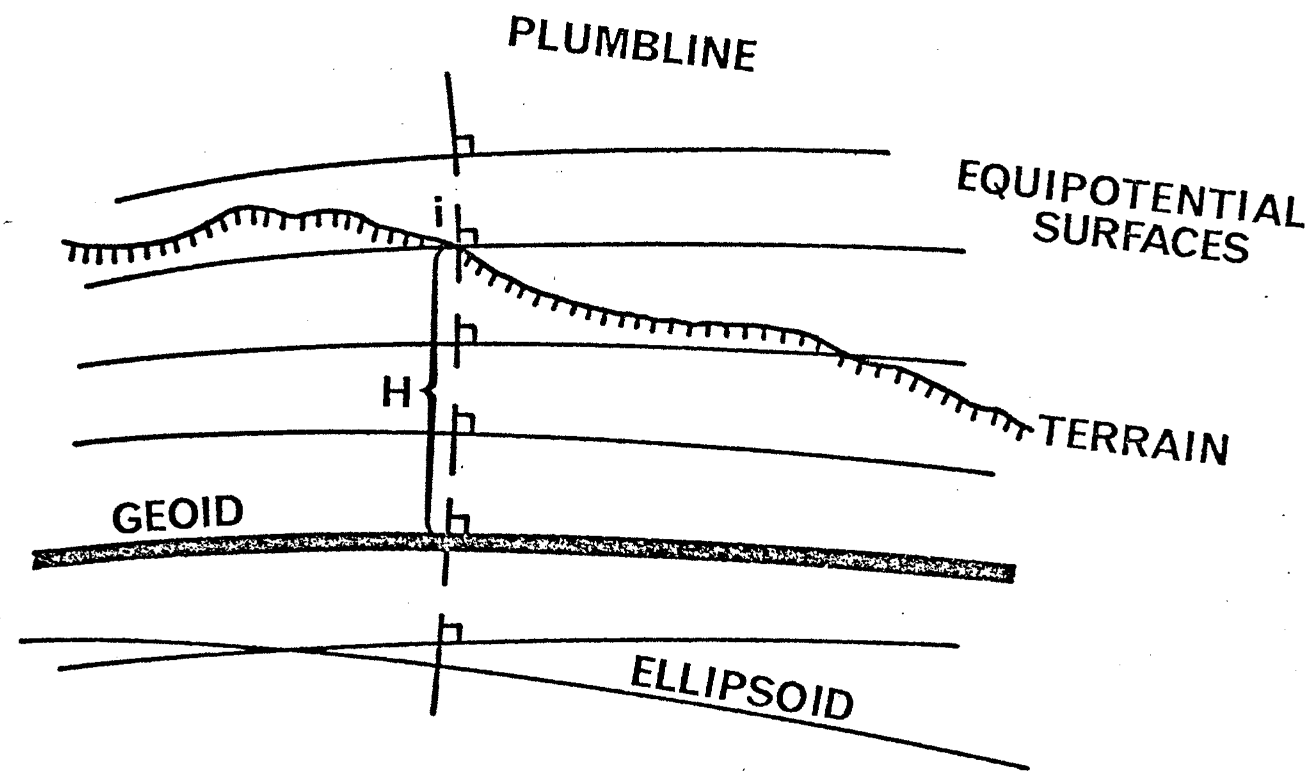


Figure 1-3

Orthometric Height

"real" (physical) properties of the earth.

Astronomic latitude (Φ) is defined as the angle between the astronomic normal (gravity vertical) (tangent to the plumbline at the point of interest) and the plane of the instantaneous equator measured in the astronomic meridian plane (Figure 1-4). Astronomic longitude (Λ) is the angle between the Greenwich Mean Astronomic Meridian and the astronomic meridian plane measured in the plane of the instantaneous equator (Figure 1-4).

The orthometric height (H), is the height of the point of interest above the geoid, measured along the plumbline, as obtained from spirit leveling and en route gravity observations (Figure 1-3). Finally, after some reductions of Φ and Λ for polar motion and plumbline curvature, one obtains the "reduced" astronomic coordinates (Φ, Λ, H) referring to the geoid and the mean rotation axis of the earth (more will be said about this last point in these notes).

We are now in a position to examine the relationship between the Geodetic and Astronomic coordinates. This is an important step for surveyors. Observations are made in the natural system; astronomic coordinates are expressed in the same natural system; therefore, to use this information for computations in a geodetic system, the relationships must be known.

The astro-geodetic (relative) deflection of the vertical (θ) at a point is the angle between the astronomic normal at that point and the normal to the reference ellipsoid at the corresponding point (the point may be on the terrain (θ_t) or on the geoid (θ_g) (Figure 1-5). θ is normally split into two components, ξ - meridian and η - prime vertical (Figure 1-6). Mathematically, the components are given by

$$\xi = \Phi - \phi, \tag{1-1}$$

$$\eta = (\Lambda - \lambda)\cos\phi, \tag{1-2}$$

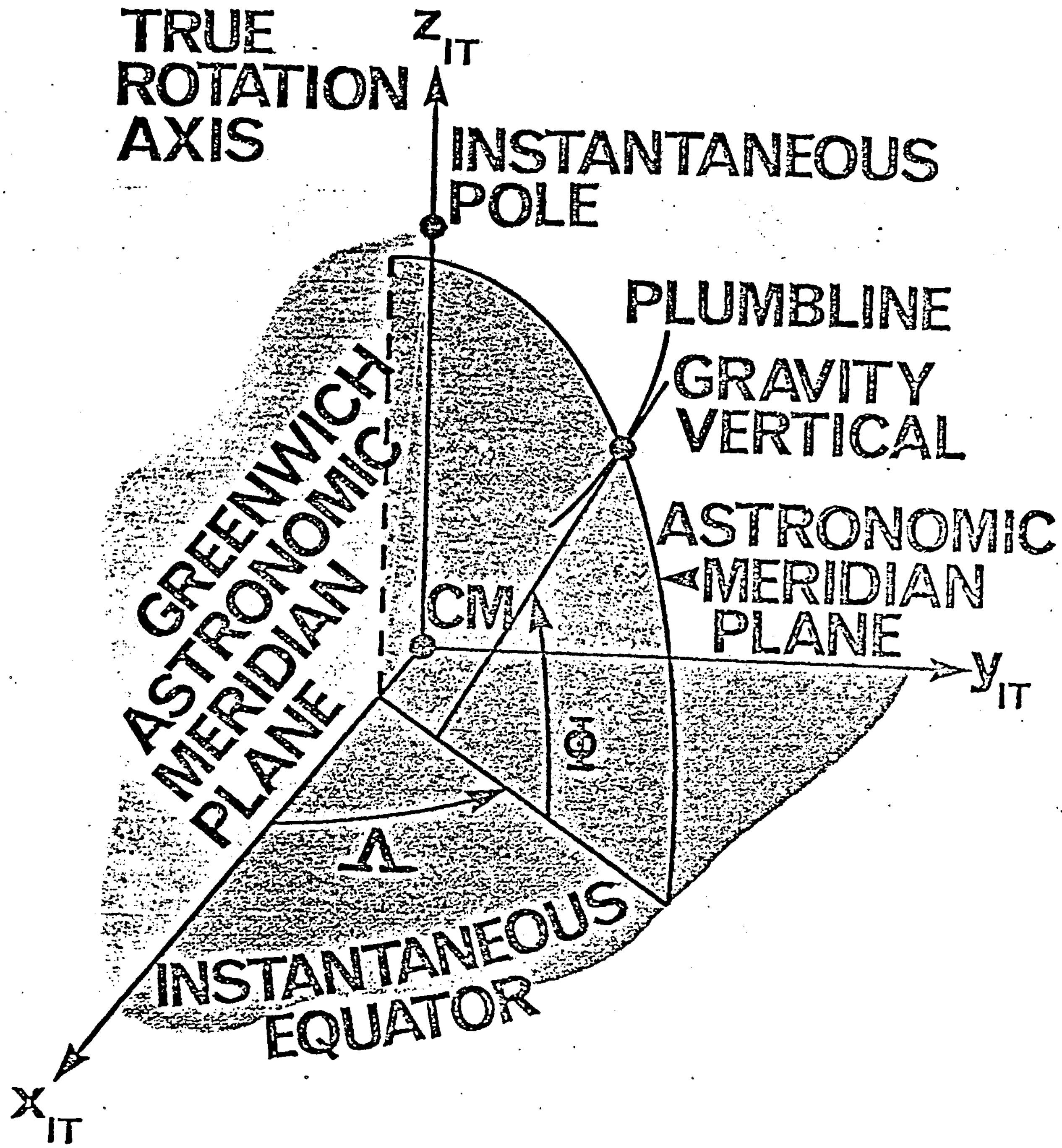


Figure 1-4

Astronomic Latitude (Φ) and Longitude (Λ)

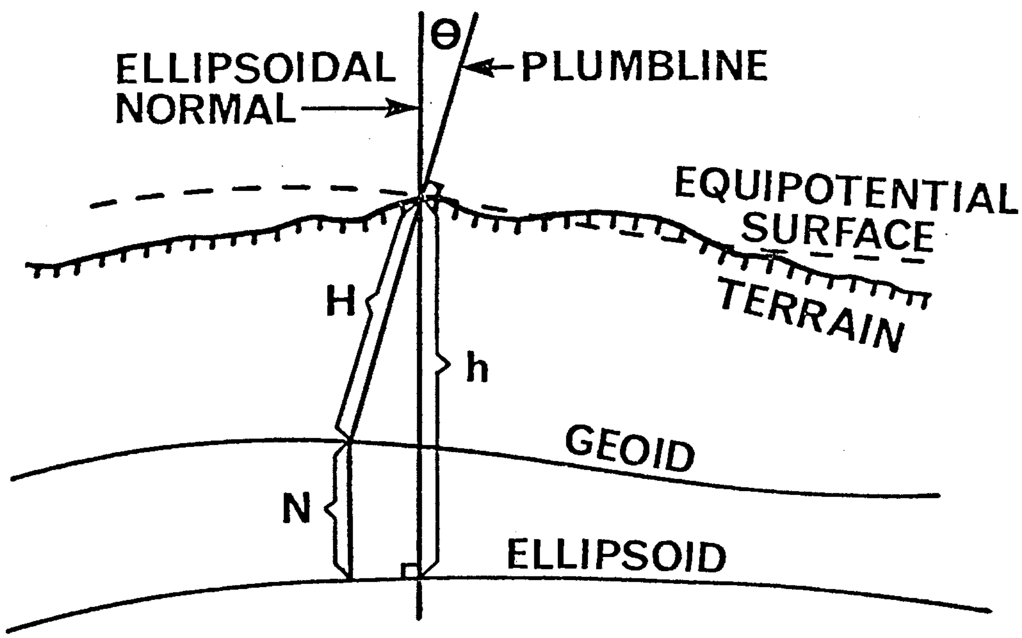
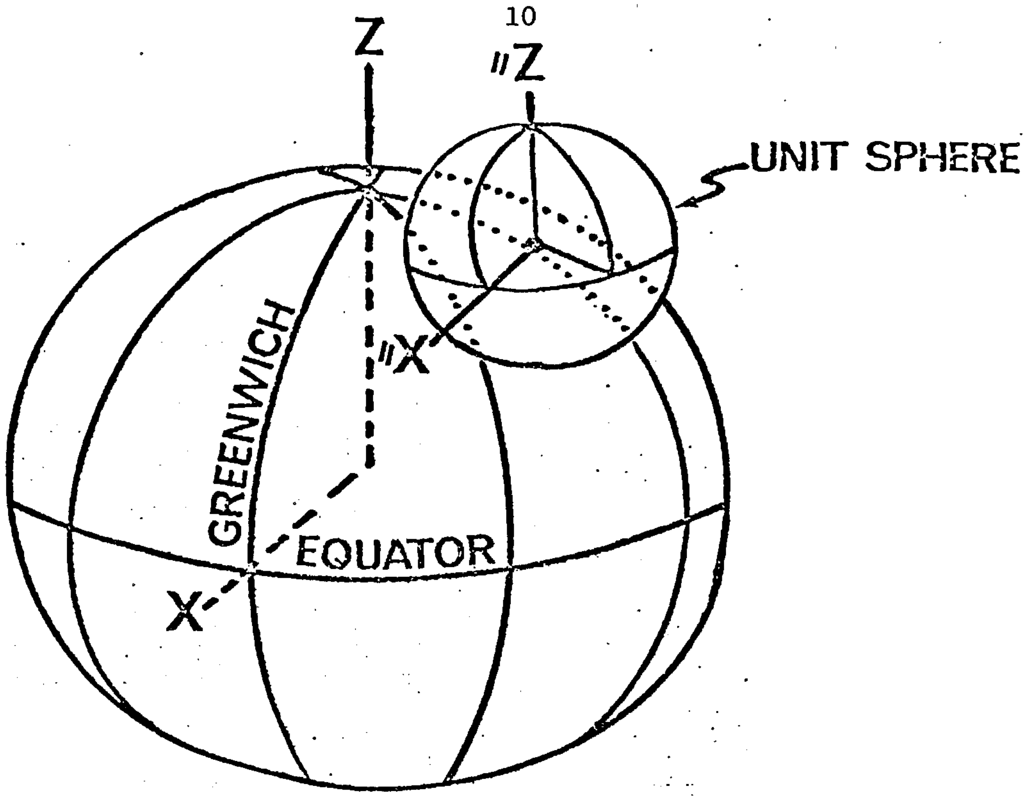
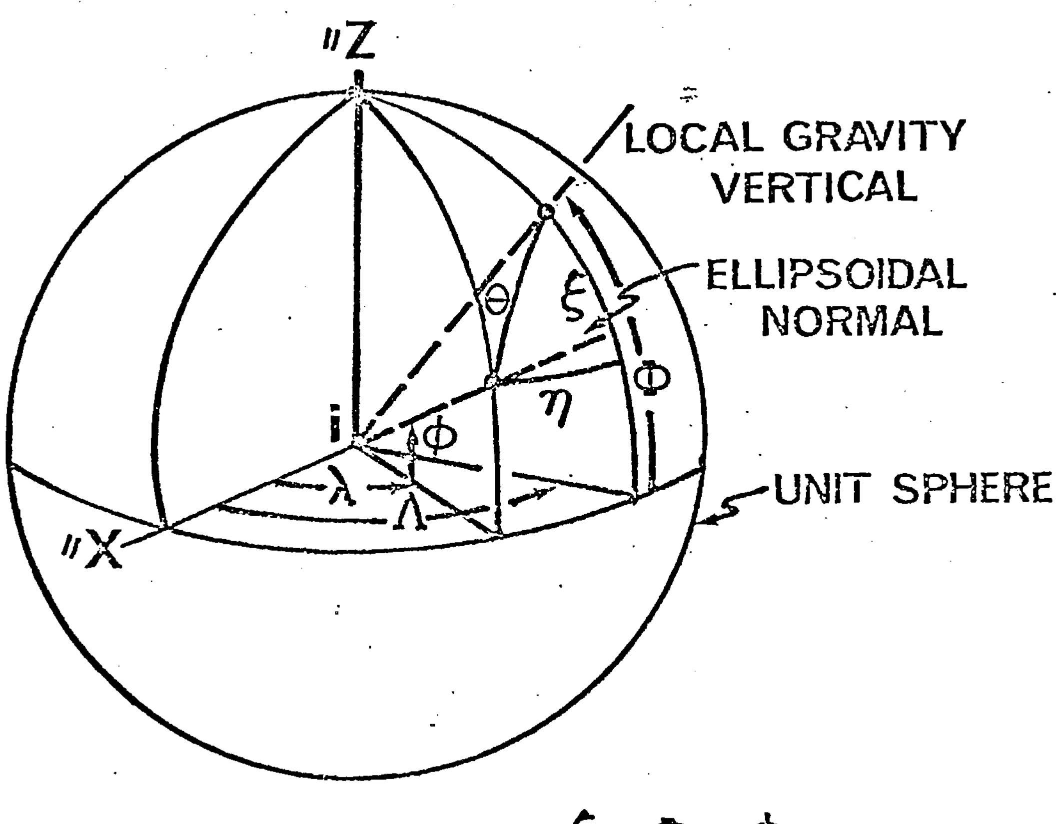


Figure 1-5

Geoid Height & Terrain Deflection of the Vertical





$$\xi = \Phi - \phi$$

$$\eta = (\Lambda - \lambda)\cos\phi$$

Figure 1-6

Components of the Deflection of the Vertical which yields the geodetic-astronomic coordinate relationships we were seeking.

The geoidal height (N) is the distance between the geoid and a reference ellipsoid, measured along an ellipsoidal normal (Figure 1-5). Mathematically, N is given by (with an error of less than 1mm) [Heiskanen and Moritz, 1967]

$$N = h - H. \tag{1-3}$$

Finally, we turn our attention to the azimuths of geodetic lines between points. A geodetic azimuth (α), on the surface of a reference ellipsoid, is the clockwise angle from north between the geodetic meridian of i and the tangent to the ellipsoidal surface curve of shortest distance (the geodesic) between i and j (Figure 1-7). The astronomic azimuth (A) is the angle between the astronomic meridian plane of i and the astronomic normal plane of i through j (Figure 1-8), measured clockwise from north.

The relationship between these A and α is given by the Laplace Azimuth equation [e.g. Heiskanen and Moritz, 1967]

$$(A-\alpha) = \eta \tan\phi + (\xi \sin\alpha - \eta \cos\alpha) \cot z, \tag{1-4}$$

in which z is the zenith distance. Note that the geodetic azimuth, α, must also be corrected for the height of target (skew-normal) and normal section - geodesic separation.

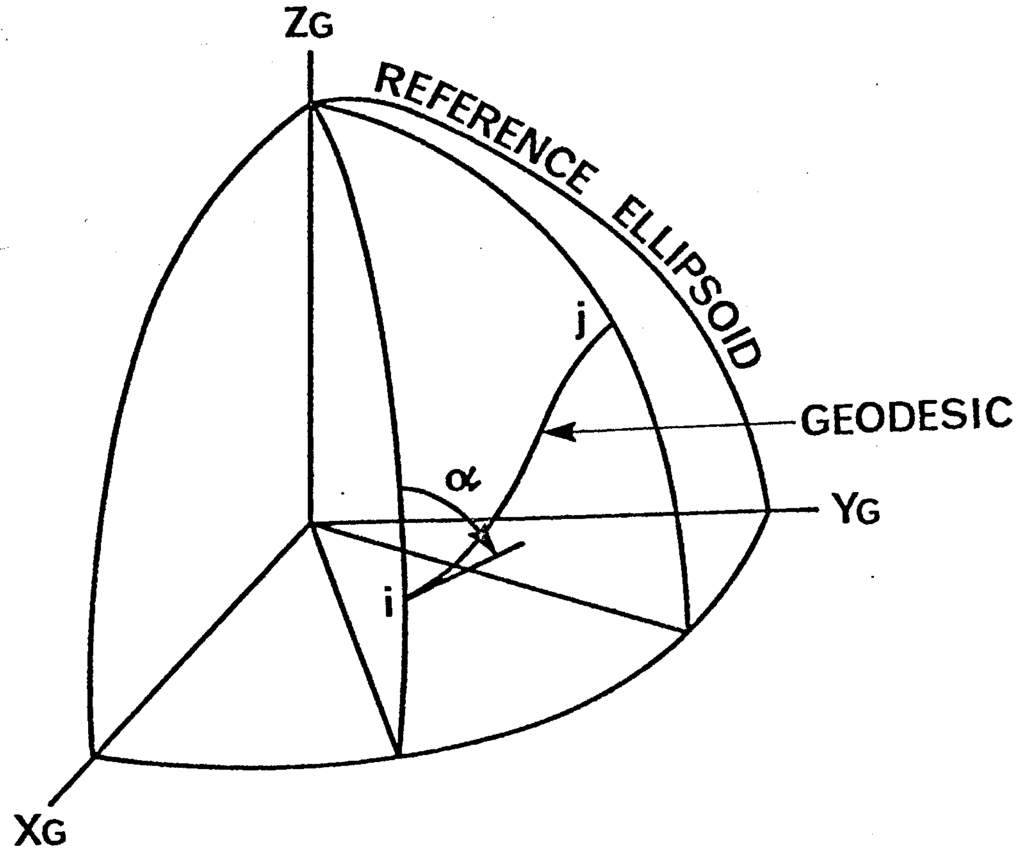


Figure 1-7

Geodetic Azimuth

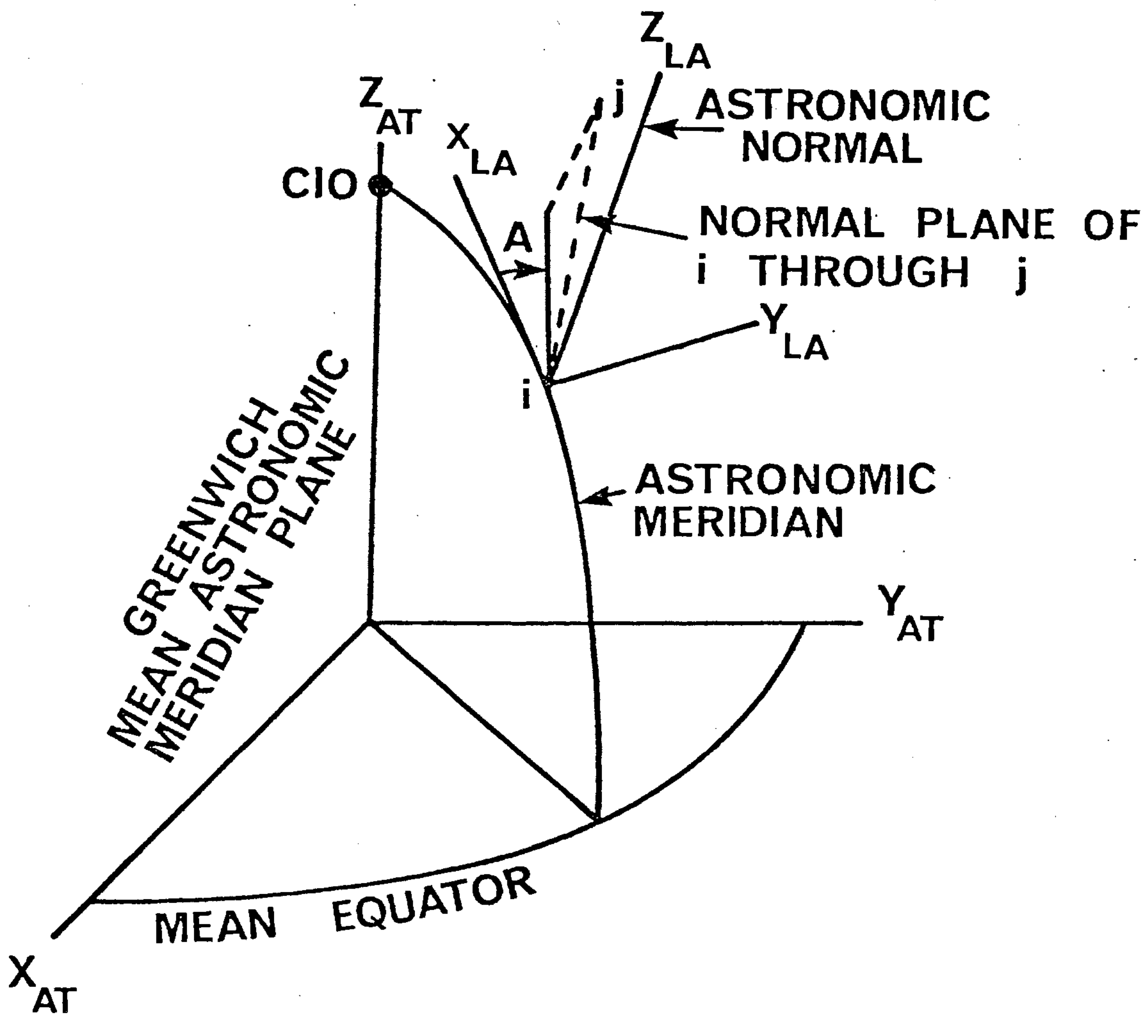


Figure 1-8

Astronomic Azimuth

2. CELESTIAL COORDINATE SYSTEMS

Celestial coordinate systems are used to define the coordinates of celestial bodies such as stars. There are four main celestial coordinate systems - the ecliptic, right ascension, hour angle, and horizon - each of which are examined in detail in this chapter. There are two fundamental differences between celestial coordinate systems and terrestrial and orbital coordinate systems. First, as a consequence of the great distances involved, only directions are considered in celestial coordinate sytems. This means that all vector quantities dealt with can be considered to be unit vectors. The second difference is that celestial geometry is spherical rather than ellipsoidal which simplifies the mathematical relationships involved.

2.1 The Celestial Sphere

The distance from the earth to the nearest star is more than 10^9 earth radii, thus the dimensions of the earth can be considered as negligible compared to the distances to the stars. For example, the closest star is estimated to be 4 light years (40×10^{12} km) from the earth (α CENTAURI), while others are VEGA at 30 light years, α USAE MINORIS (Polaris) at 50 light years. Our sun is only 8.25 light minutes (155×10^6 km) from the earth. As a consequence of those great distances, stars, considered to be moving at near the velocity of light, are perceived by an observer on earth to be moving very little. Therefore, the relationship between the earth and stars can be closely approximated by considering the stars all to be equidistant from the earth and lying on the surface of a celestial sphere, the dimension of which is so large that the earth, and indeed the solar system, can be considered to be a dimensionless point at its centre. Although this point may be considered dimensionless, relationships between directions on the earth and in the solar system can be extended to the celestial sphere.

The instantaneous rotation axis of the earth intersects the celestial sphere at the north and south celestial poles (NCP and SCP respectively) (Figure 2-1). The earth's equitorial plane extented outward intersects the celestial sphere at the celestial equator (Figure 2-1). The vertical (local astronomic normal) intersects the celestial sphere at a point above the observer, the zenith, and a point beneath the observer, the nadir (Figure 2-1). A great circle containing the poles, and is thus perpendicular to the celestial equator, is called an hour circle (Figure 2-1). The observer's vertical plane, containing the poles, is the hour circle through the zenith and is the observer's celestial meridian (Figure 2-1). A small circle parallel to the celestial equator is called a celestial parallel. Another very important plane is that which is normal to the local astronomic vertical and contains the observer (centre of the celestial sphere); it is the celestial horizon (Figure 2-1). The plane normal to the horizon passing through the zenith is the vertical plane. A small circle parallel to the celestial horizon is called an almucantar. The vertical plane normal to the celestial meridian is called the prime vertical. The intersection points of the prime vertical and the celestial horizon are the east and west points.

Due to the rotation of the earth, the zenith (nadir), vertical planes, almucantars, the celestial horizon and meridian continuously change their positions on the celestial sphere, the effects of which will be studied later. If at any instant we select a point S on the celestial sphere (a star), then the celestial meridian and the hour and vertical circles form a spherical triangle called the astronomic triangle of S. Its verticies are the zenith

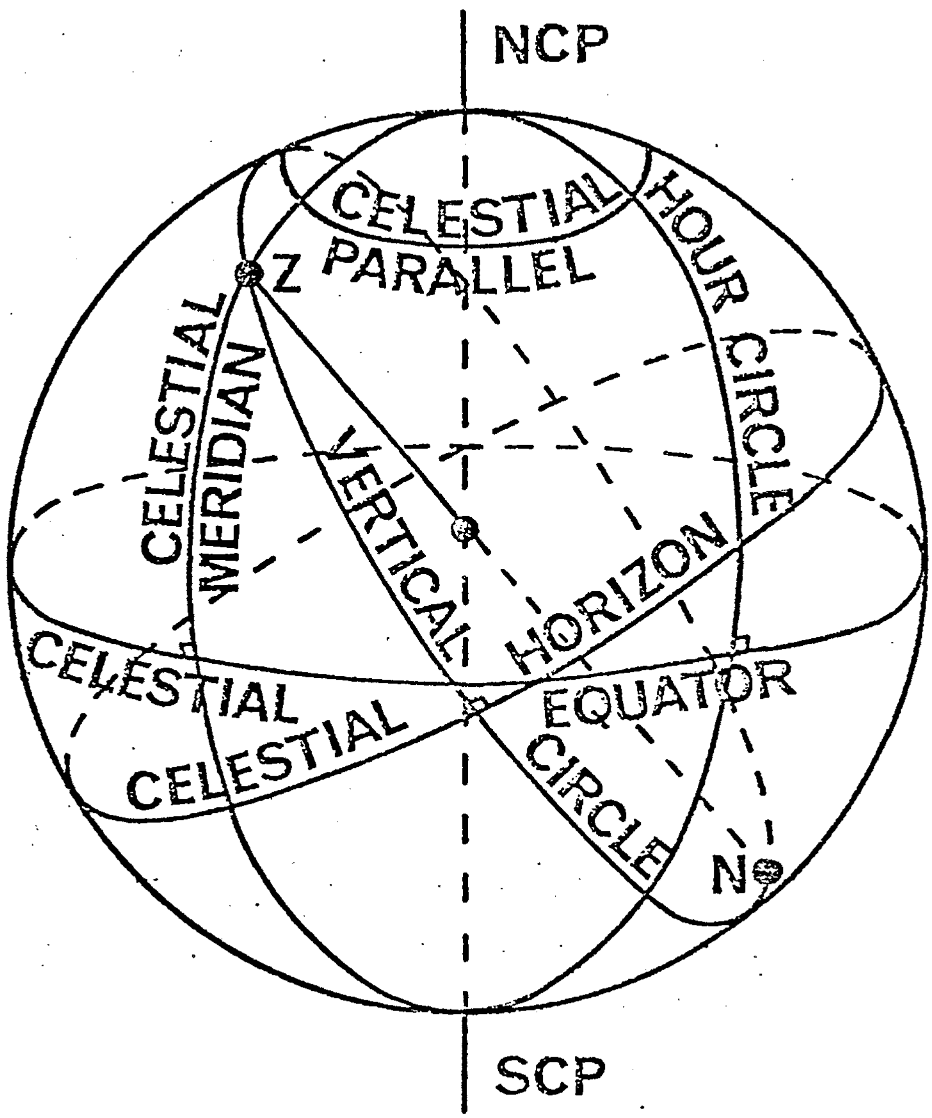


Figure 2-1

Celestial Sphere

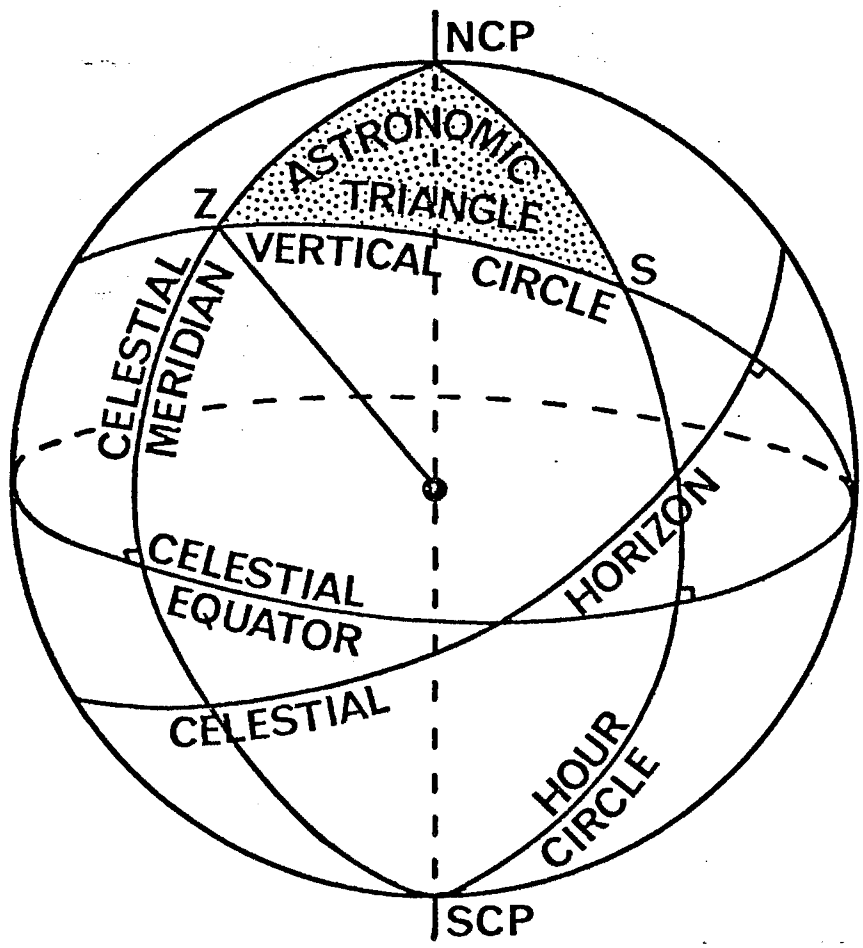


Figure 2-2

Astronomic Triangle

(Z), the north celestial pole (NCP) and S (Figure 2-2).

There are also some important features on the celestial sphere related the revolution of the earth about the sun, or in the reversed concept, the apparent motion of the sun about the earth. The most important of these is the ecliptic, which may be described as the approximate (within 2" [Mueller, 1969]) apparent path of the sun about the earth (Figure 2-3). The ecliptic intersects the celestial equator in a line connecting the equinoxes. The vernal equinox is that point intersection where the apparent sun crosses the celestial equator from south to north. The other equinox is the autumnal equinox. The acute angle between the celestial equator and the ecliptic is termed the obliquity of the ecliptic ($\varepsilon \approx 23°27'$) (Figure 2-3). The points at 90° from either equinox are the points where the sun reaches its greatest angular distance from the celestial equator, and they are the summer solstice (north) and winter solstice (south).

In closing this section we should note that the celestial sphere is only an approximation of the true relationship between the stars and an observer on the earth's surface. Like all approximations, a number of corrections are required to give a precise representation of the true relationship. These corrections represent the facts that: the stars are not stationary points on the celestial sphere but are really moving (proper motion); the earth's rotation axis is not stationary with respect to the stars (precesion, nutation); the earth is displaced from the centre of the celestial sphere (which is taken as the centre of the sun) and the observer is displaced from the mass centre of the earth (parallax); the earth is in motion around the centre of the celestial sphere (aberration); and directions measured through the earth's atmosphere are bent by refraction. All of these effects are discussed in

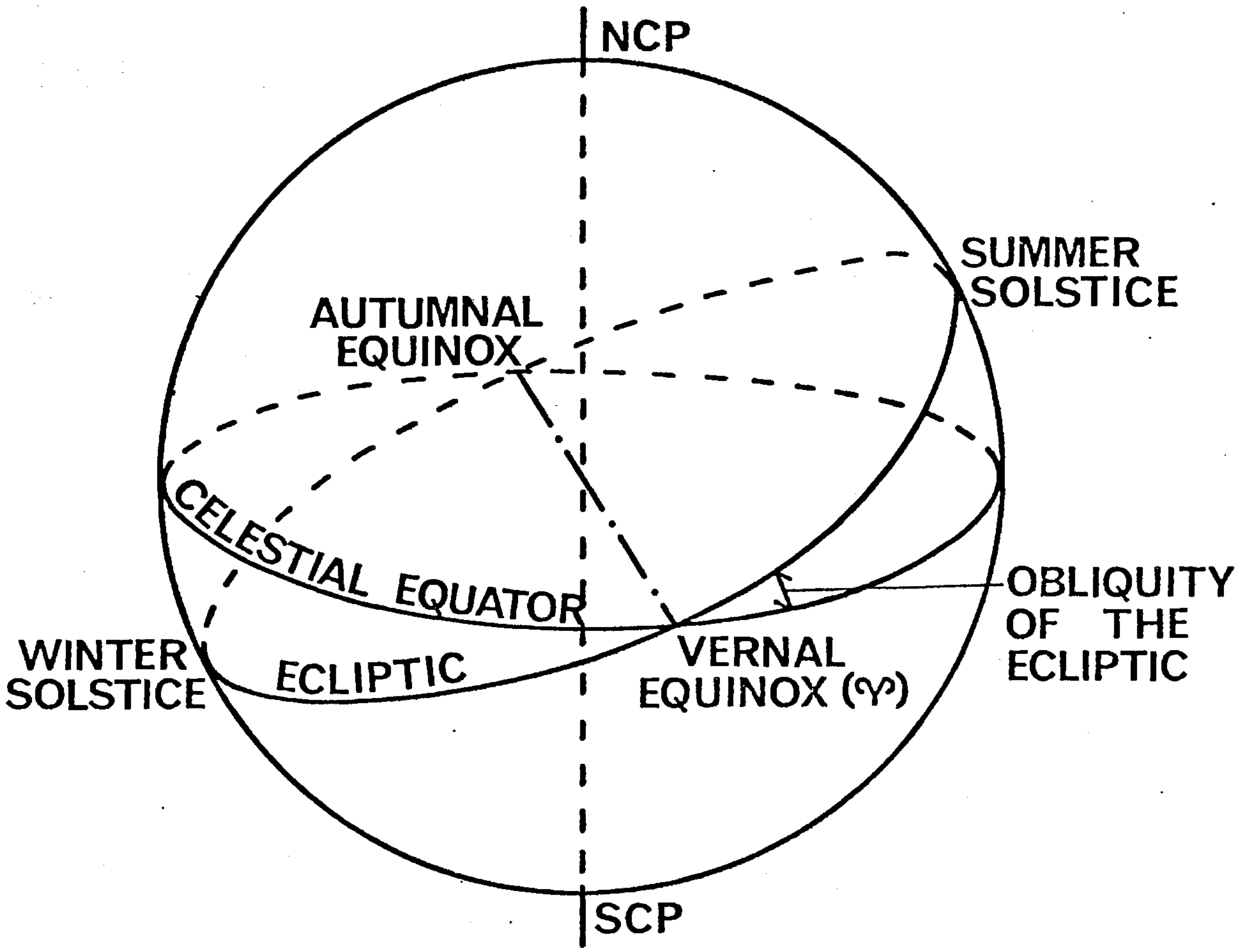


Figure 2-3

Sun's Apparent Motion

detail in these notes.

2.2 Celestial Coordinate Systems

Celestial coordinate systems are used to define the positions of stars on the celestial sphere. Remembering that the distances to the stars are very great, and in fact can be considered equal thus allowing us to treat the celestial sphere as a unit sphere, positions are defined by directions only. One component or curvalinear coordinate is reckoned from a primary reference plane and is measured perpendicular to it, the other from a secondary reference plane and is measured in the primary plane.

In these notes, two methods of describing positions are given. The first is by a set of curvalinear coordinates, the second by a unit vector in three dimensional space expressed as a function of the curvalinear coordinates.

2.2.1 Horizon System

The primary reference plane is the celestial horizon, the secondary is the observer's celestial meridian (Figure 2-4). This system is used to describe the position of a celestial body in a system peculiar to a topographically located observer. The direction to the celestial body S is defined by the altitude(a) and azimuth (A) (Figure 2-4). The altitude is the angle between the celestial horizon and the point S measured in the plane of the vertical circle (0° - 90°). The complimentary angle $z = 90 - a$, is called the zenith distance. The azimuth A is the angle between the observer's celestial meridian and the vertical circle through S measured in a clockwise direction (north to east) in the plane of the celestial horizon (0° - 360°).

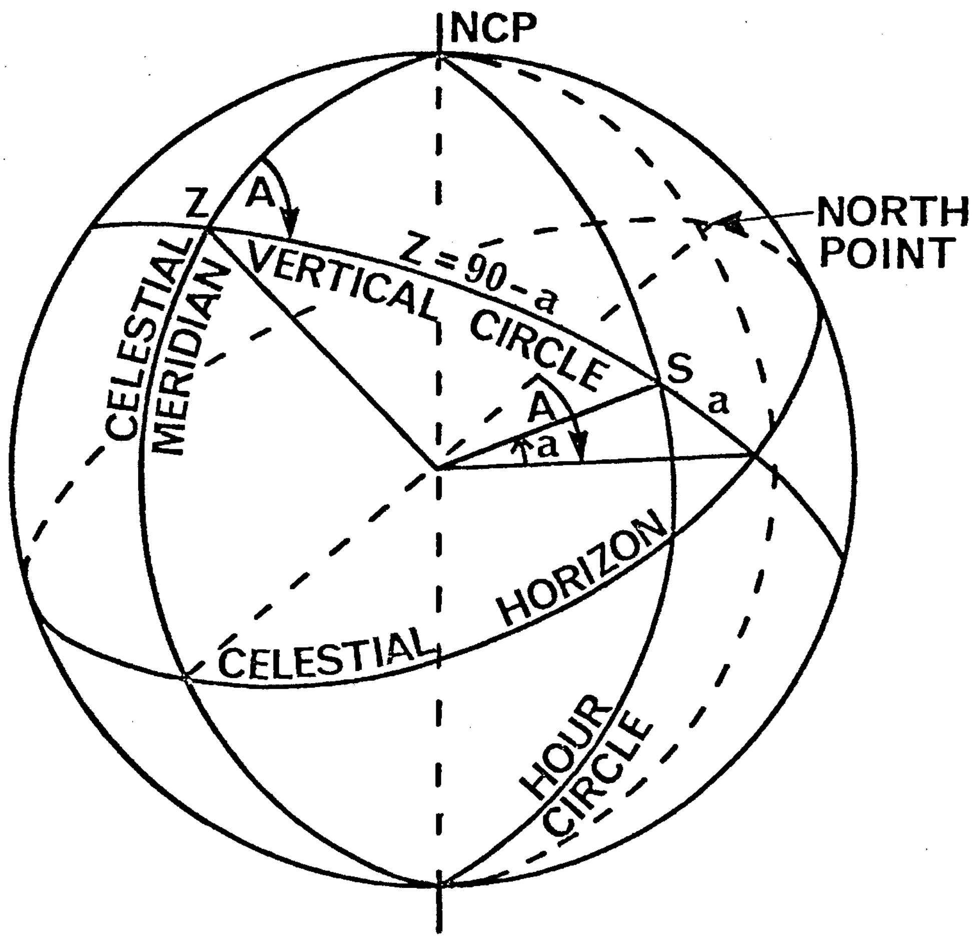
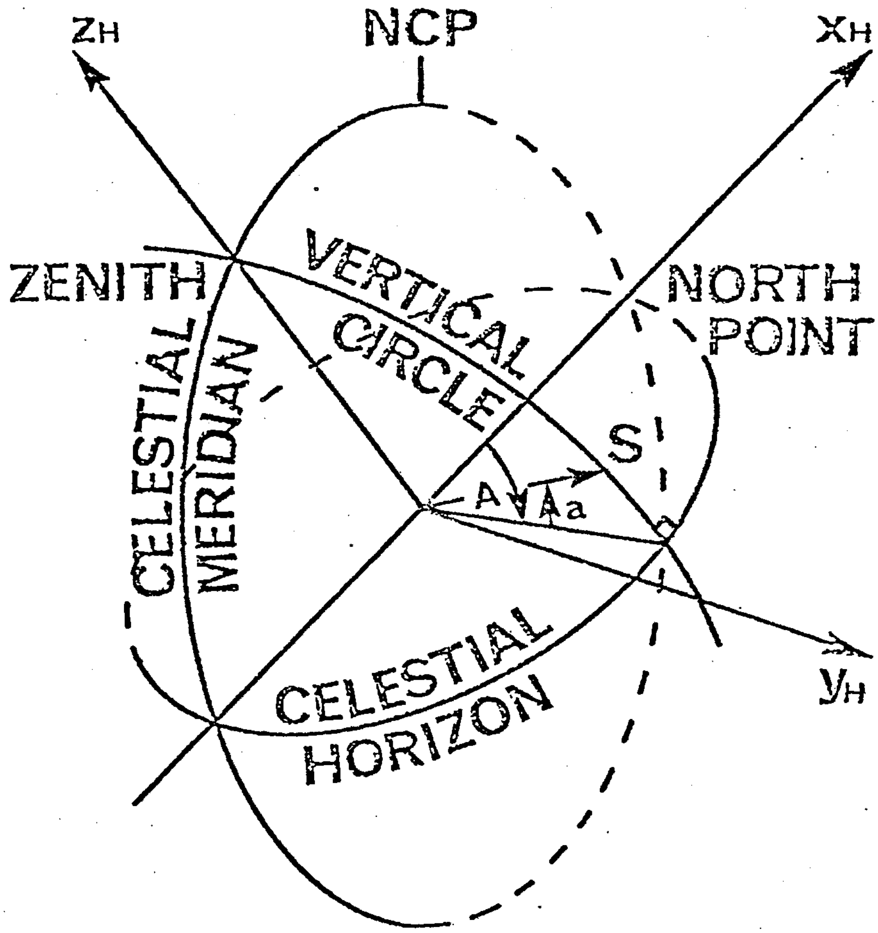
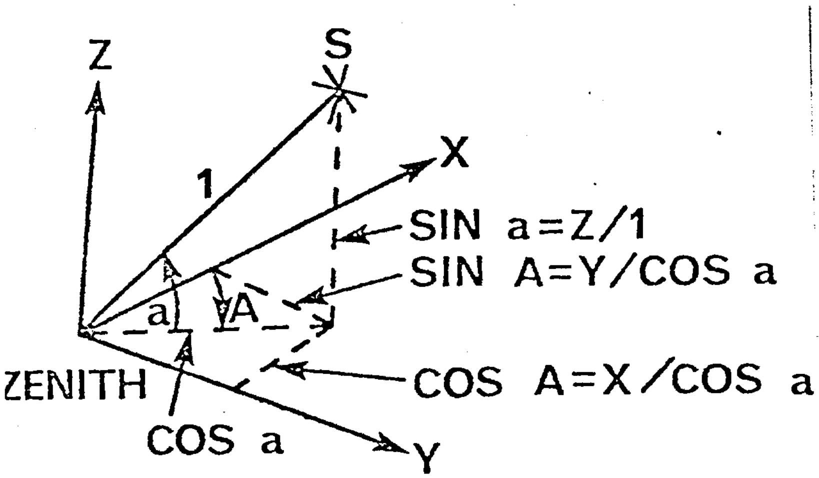


Figure 2-4

Horizon System





$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_H = \begin{bmatrix} \cos a \;\; \cos A \\ \cos a \;\; \sin A \\ \sin a \end{bmatrix}$$

Figure 2-5

Horizon System

To determine the unit vector of the point S in terms of a and A, we must first define the origin and the three axes of the coordinate system. The origin is the heliocentre (centre of mass of the sun [e.g. Eichorn, 1974])*. The primary pole (Z) is the observer's zenith (astronomic normal or gravity vertical). The primary axis (X) is directed towards the north point. The secondary (Y) axis is chosen so that the system is left-handed (Figure 2-5 illustrates this coordinate system). Note that although the horizon system is used to describe the position of a celestial body in a system peculiar to a topographically located observer, the system is heliocentric and not topocentric.

The unit vector describing the position of S is given by

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_H = \begin{bmatrix} \cos a \ \cos A \\ \cos a \ \sin A \\ \sin a \end{bmatrix} \tag{2-1}$$

Conversely, a and A, in terms of $[X,Y,Z]^T$ are (Figure 2-5)

$$a = \sin^{-1} Z \ , \tag{2-2}$$

$$A = \tan^{-1}(Y/X). \tag{2-3}$$

2.2.2 Hour Angle System

The primary reference plane is the celestial equator, the secondary is the hour circle containing the zenith (obserser's celestial meridian). The direction to a celestial body S on the celestial sphere is given by the declination (δ) and hour angle (h). The declination is the angle between the celectial equator and the body S, measured from 0° to 90° in the plane of the hour circle through S. The complement of the declination is called the polar distance. The hour angle is the angle between the hour circle of S

* Throughout these notes, we make the valid approximation heliocentre = barycentre of our solar system.

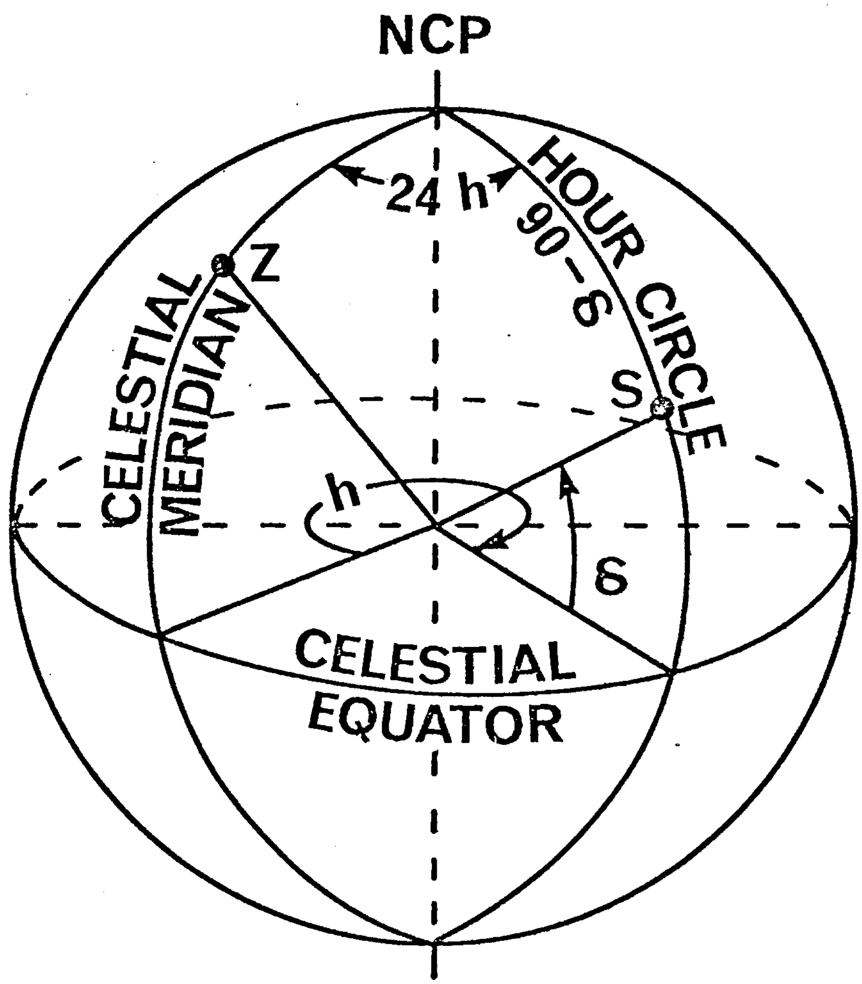
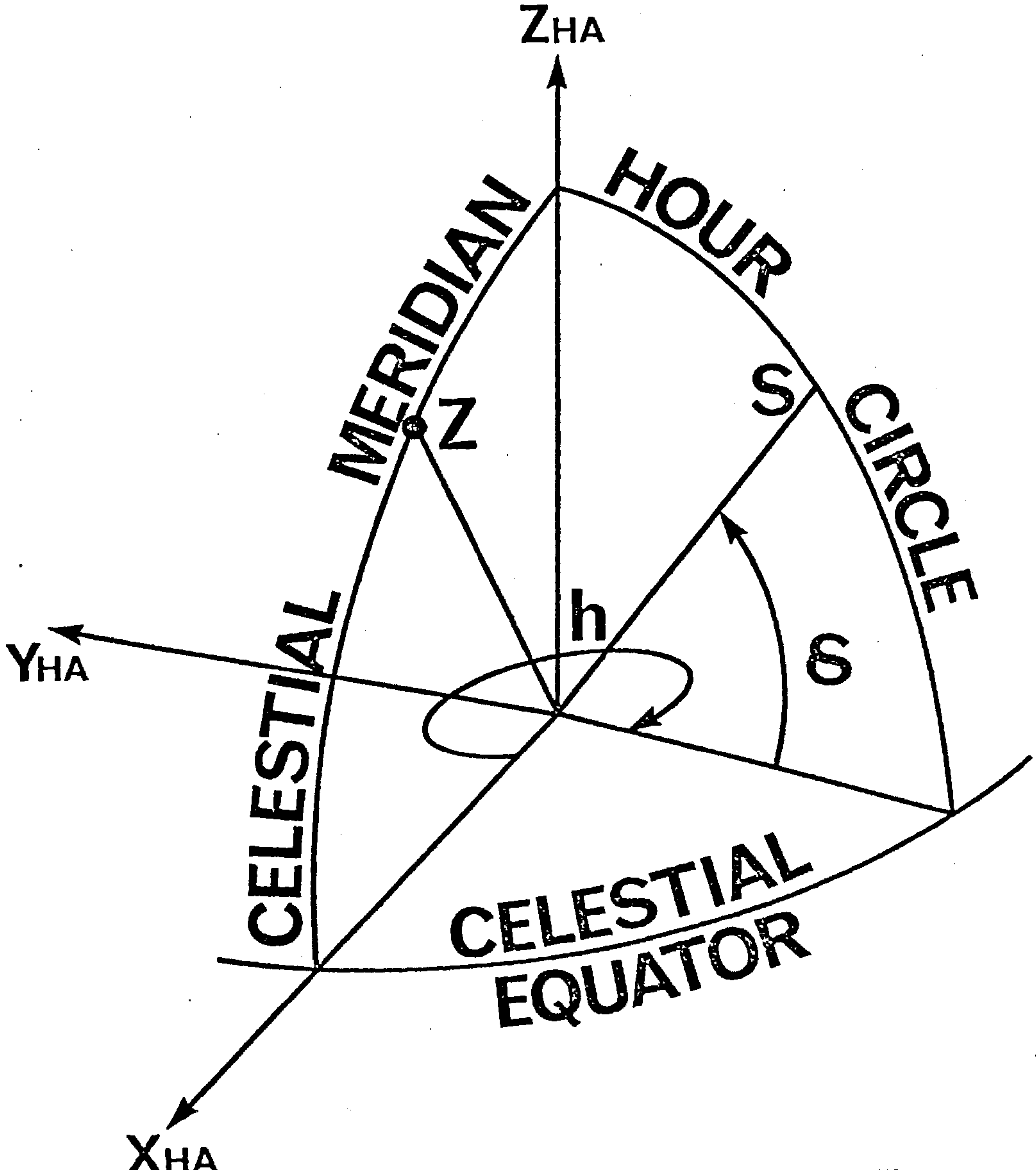


Figure 2-6

Hour Angle System



$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{HA} = \begin{bmatrix} \cos\delta \cos h \\ \cos\delta \sin h \\ \sin\delta \end{bmatrix}$$

Figure 2-7

Hour Angle System

and the observer's celestial meridian (hour circle), and is measured from 0^h to 24^h, in a clockwise direction (west, in the direction of the star's apparent daily motion) in the plane of the celestial equator. Figure 2-6 illustrates the hour angle system's curvalinear coordinates.

To define the unit vector of S in the hour angle system, we define the coordinate system as follows (Figure 2-6). The origin is the heliocentre. The primary plane is the equatorial plane, the secondary plane is the celestial meridian plane of the observer. The primary pole (Z) is the NCP, the primary axis (X-axis) is the intersection of the equatorial and observer's celestial meridian planes. The Y-axis is chosen so that the system is left-handed. The hour angle system rotates with the observer. The unit vector is

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{HA} = \begin{bmatrix} \cos\delta \ \cos h \\ \cos\delta \ \sin h \\ \sin\delta \end{bmatrix} , \quad (2\text{-}4)$$

and δ and h are given by

$$\delta = \sin^{-1} Z , \quad (2\text{-}5)$$

$$h = \tan^{-1}(Y/X) . \quad (2\text{-}6)$$

2.2.3 Right Ascension System

The right ascension system is the most important celestial system as it is in this system that star coordinates are published. It also serves as the connection between terrestrial, celestial, and orbital coordinate systems. The primary reference plane is the celestial equator and the secondary is the equinoctal colure (the hour circle passing through the NCP and SCP and the vernal and autumnal equinoxes)(Figure 2-8). The direction to a star S is given by the right ascension (α) and declination (δ), the latter of which

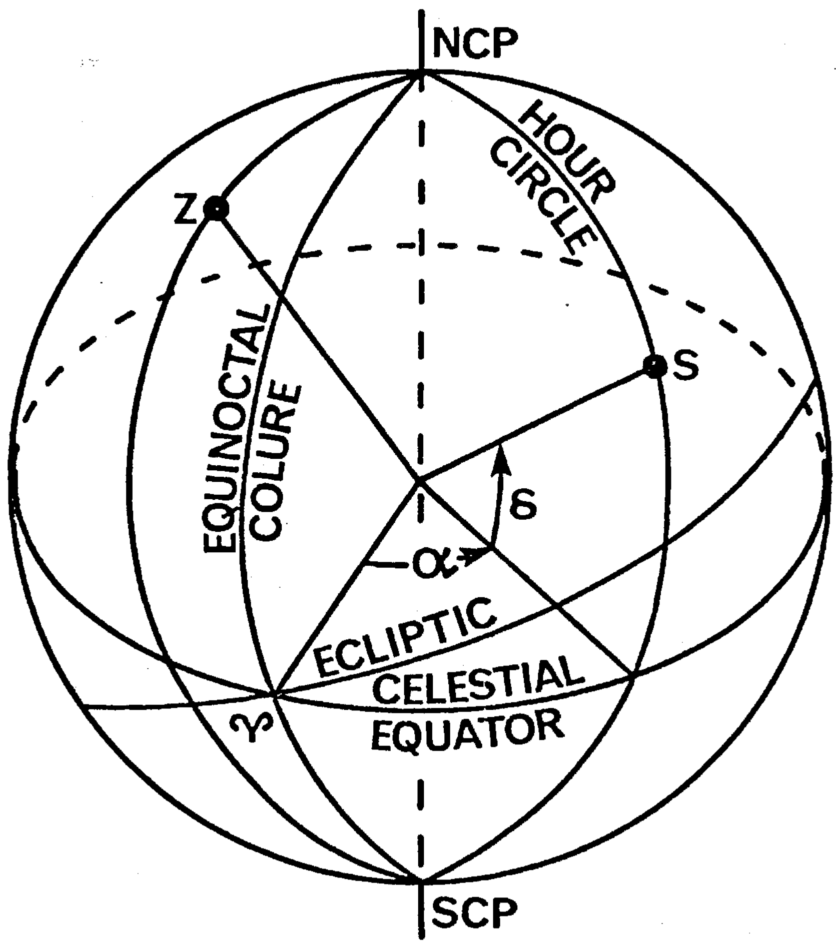
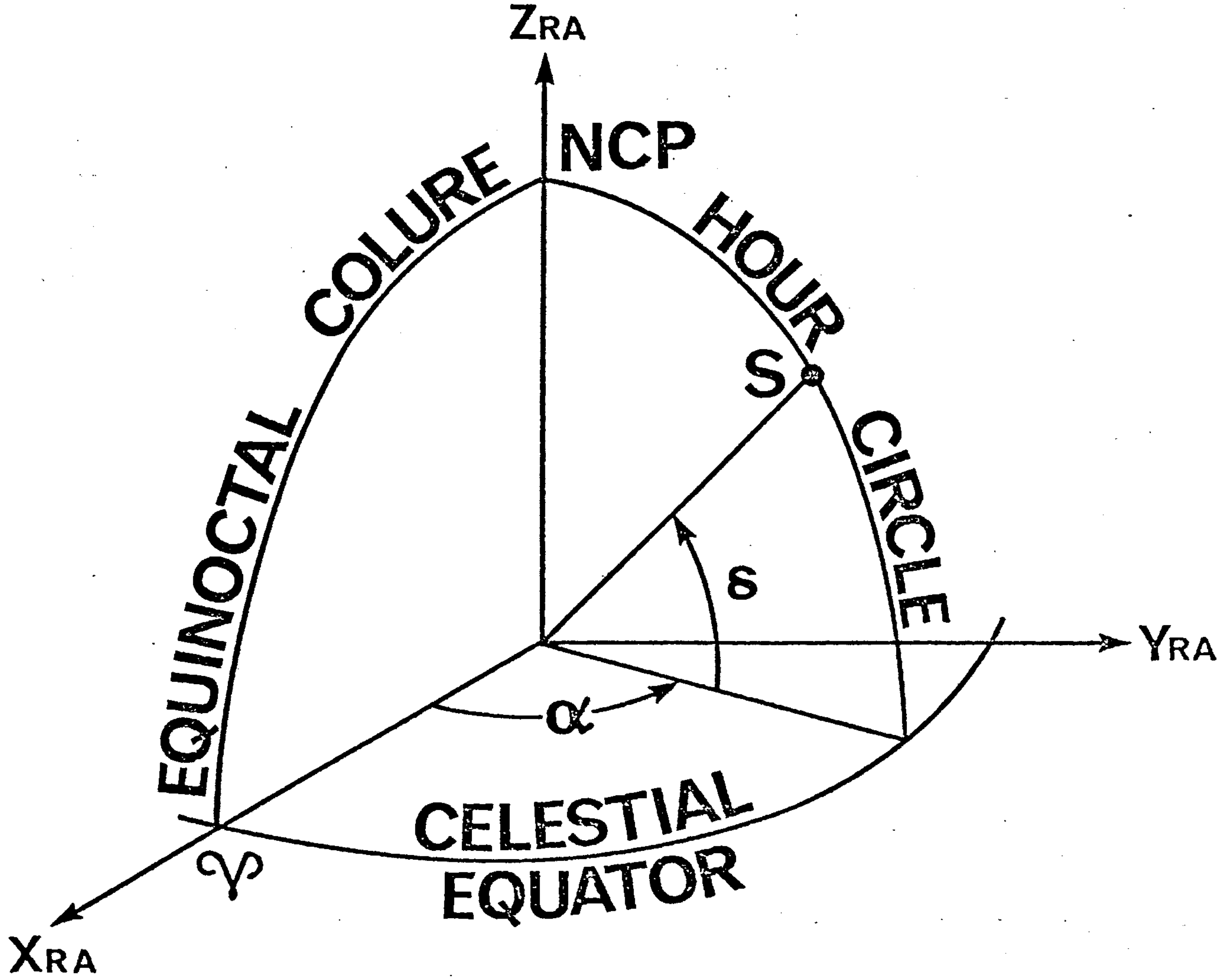


Figure 2-8

Right Ascension System



$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{RA} = \begin{bmatrix} \cos\delta \ \cos\alpha \\ \cos\delta \ \sin\alpha \\ \sin\delta \end{bmatrix}$$

Figure 2-9

Right Ascension System

has already been defined (HA system). The right ascension is the angle between the hour circle of S and the equinoctal colure, measured from the vernal equinox to the east (counter clockwise) in the plane of the celestial equator from 0^h to 24^h.

The right ascension system coordinate axes are defined as having a heliocentric origin, with the equatorial plane as the primary plane, the primary pole (Z) is the NCP, the primary axis (X) is the vernal equinox, and the Y-axis is chosen to make the system right-handed (Figure 2-9). The unit vector describing the direction of a body in the right-ascension system is

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{RA} = \begin{bmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{bmatrix} , \qquad (2\text{-}7)$$

and α and δ are expressed as

$$\alpha = \tan^{-1}(Y/X), \qquad (2\text{-}8)$$

$$\delta = \sin^{-1}Z \qquad (2\text{-}9)$$

2.2.4 Ecliptic System

The ecliptic system is the celestial coordinate system that is closest to being inertial, that is, motionless with respect to the stars. However, due to the effect of the planets on the earth-sun system, the ecliptic plane is slowly rotating (at 0".5 per year) about a slowly moving axis of rotation. The primary reference plane is the ecliptic, the secondary reference plane is the ecliptic meridian of the vernal equinox (contains the north and south ecliptic poles, the vernal and autumnal equinoxes) (Figure 2-10). The direction

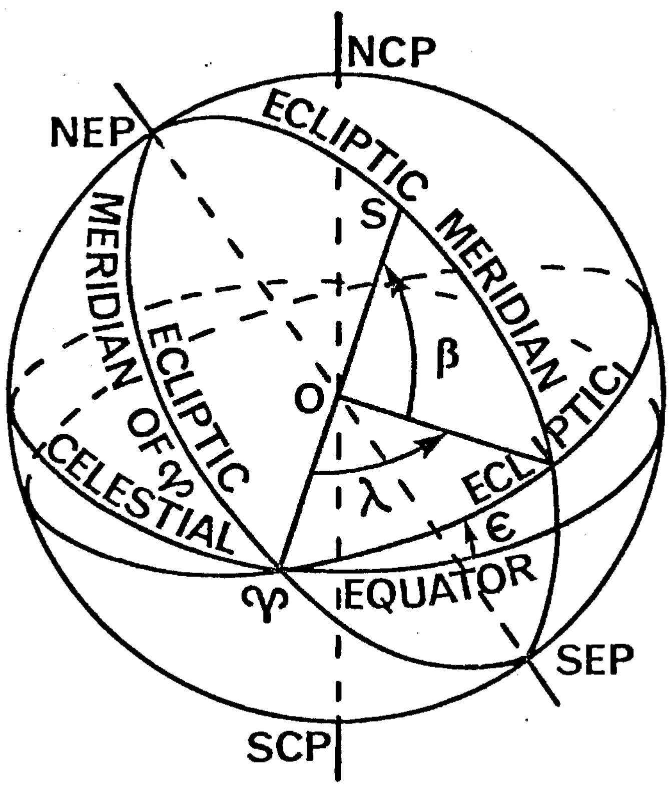
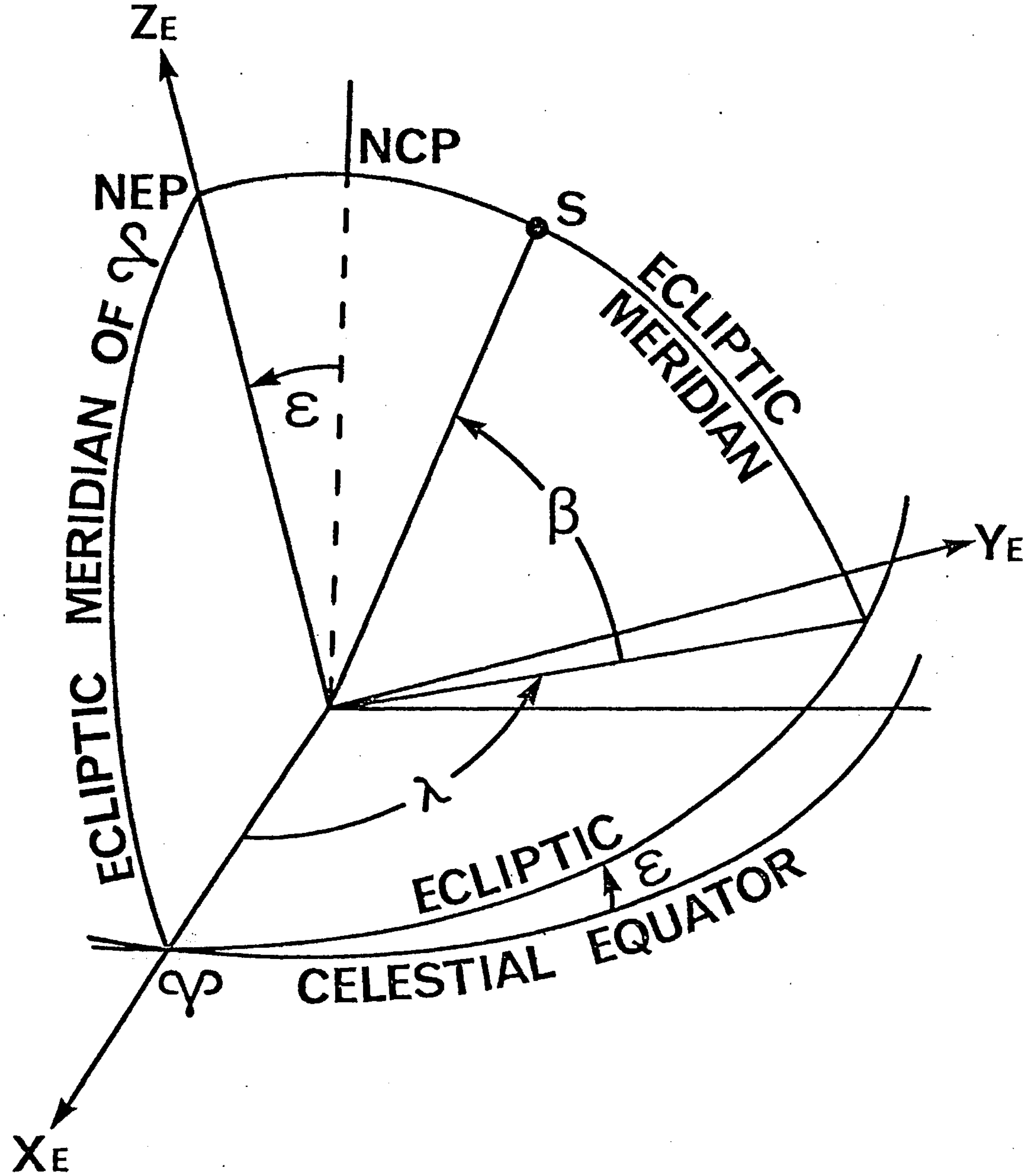


Figure 2-10

Ecliptic System



$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_E = \begin{bmatrix} \cos\beta \cos\lambda \\ \cos\beta \sin\lambda \\ \sin\beta \end{bmatrix}$$

Figure 2-11

Ecliptic System

to a point S on the celestial sphere is given by the ecliptic latitude (β) and ecliptic longitude (λ). The ecliptic latitude is the angle, measured in the ecliptic meridian plane of S, between the ecliptic and the normal OS (Figure 2-10). The ecliptic longitude is measured eastward in the ecliptic plane between the ecliptic meridian of the vernal equinox and the ecliptic meridian of S (Figure 2-10).

The ecliptic system coordinate axes are specified as follows (Figure 2-11). The origin is heliocentric. The primary plane is the ecliptic plane and the primary pole (Z) is the NEP (north ecliptic pole). The primary axis (X) is the normal equinox, and the Y-axis is chosen to make the system right-handed. The unit vector to S is

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_E = \begin{bmatrix} \cos\beta \ \ \cos\lambda \\ \cos\beta \ \ \sin\lambda \\ \sin\beta \end{bmatrix}, \qquad (2\text{-}10)$$

while β and λ are given by

$$\beta = \sin^{-1} Z, \qquad (2\text{-}11)$$

$$\lambda = \tan^{-1}(Y/X) \ . \qquad (2\text{-}12)$$

2.2.5 Summary

The most important characteristics of the coordinate systems, expressed in terms of curvalinear coordinates, are given in Table 2-1. The most important characteristics of the cartesian coordinate systems are shown in Table 2-2 (Note: μ and υ in Table 2-2 denote the curvilinear coordinates measured in the primary reference plane and perpendicular to it respectively).

System	Reference Plane		Parameters Measured from the	
	Primary	Secondary	Primary	Secondary
Horizon	Celestial horizon	Celestial meridian (half containing north pole)	Altitude $-90° \le a \le +90°$ (+toward zenith)	Azimuth $0° \le A \le 360°$ (+east)
Hour Angle	Celestial equator	Hour circle of observer's zenith (half containing zenith)	Declination $-90° \le \delta \le +90°$ (+north)	Hour angle $0^h \le h \le 24^h$ $0° \le h \le 360°$ (+west)
Right Ascension	Celestial equator	Equinoctical colure (half containing vernal equinox)	Declination $-90° \le \delta \le +90°$ (+north)	Right Ascension $0^h \le \alpha \le 24^h$ $0° \le \alpha \le 360°$ (+east)
Ecliptic	Ecliptic	Ecliptic meridan equinox (half containing vernal equinox)	(Ecliptic) Latitude $90° \le \beta \le +90°$ (+north)	(Ecliptic) Longitude $0° \le \lambda \le 360°$ (+east)

CELESTIAL COORDINATE SYSTEMS [Mueller, 1969]

TABLE 2-1

System	Orientation of the Positive Axis			μ	υ	Left or Right handed
	X (Secondary pole)	Y	Z (Primary pole)			
Horizon	North point	$A = 90°$	Zenith	A	a	left
Hour angle	Intersection of the zenith's hour circle with the celestial equator on the zenith's side.	$h = 90° = 6^h$	North celestial pole	h	δ	left
Right ascension	Vernal equinox	$\alpha = 90° = 6^h$	North celestial pole	α	δ	right
Ecliptic	Vernal equinox	$= 90°$	North ecliptic pole	λ	β	right

CARTESIAN CELESTIAL COORDINATE SYSTEMS [Mueller, 1969]

TABLE 2-2

2.3 Transformations Amongst Celestial Coordinate Systems

Transformations amongst celestial coordinate systems is an important aspect of geodetic astronomy for it is through the transformation models that we arrive at the math models for astronomic position and azimuth determination. Two approaches to coordinate systems transformations are dealt with here: (i) the traditional approach, using spherical trigonometry, and (ii) a more general approach using matrices that is particularly applicable to machine computations. The relationships that are developped here are between (i) the horizon and hour angle systems, (ii) the hour angle and right ascension systems, and (iii) the right ascension and ecliptic systems. With these math models, any one system can then be related to any other system (e.g. right ascension and horizon systems).

Before developing the transformation models, several more quantities must be defined. To begin with, the already known quantities relating to the astronomic triangle are (Figure 2-12):

(i) from the horizon system, the astronomic azimuth A and the altitude a or its compliment the zenith distance z=90-a,

(ii) from the hour angle system, the hour angle h or its compliment 24-h,

(iii) from the hour angle or right ascension systems, the declination δ, or its compliment, the polar distance $90-\delta$.

The new quantities required to complete the astronomic triangle are the astronomic latitude Φ or its compliment $90-\Phi$, the difference in astronomic longitude $\Delta\Lambda = \Lambda_s - \Lambda_z$ (=24-h), and the parallactic angle p defined as the angle between the vertical circle and hour circle at S (Figure 2-12).

The last quantity needed is Local Sidereal Time. (Note: this is

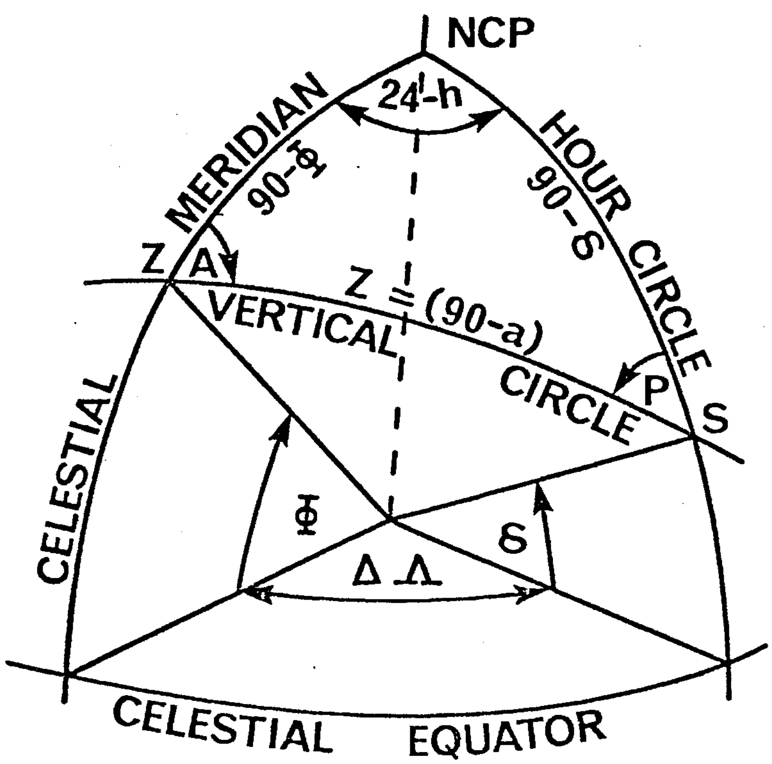


Figure 2-12

Astronomic Triangle

not a complete definition, but is introduced at this juncture to facilitate coordinate transformations. A complete discussion of sidereal time is presented in Chapter 4).

To obtain a definition of Local Sidereal Time, we look at three meridians: (i) the observer's celestial meridian, (ii) the hour circle through S, and (iii) the equinoctal colure. Viewing the celestial sphere from the NCP, we see, on the equatorial plane, the following angles (Figure 2-13): (i) the hour angle (h) between the celestial meridian and the hour circle (measured clockwise), (ii) the right ascension (α) between the equinoctal colure and the hour circle (measured counter clockwise), and (iii) a new quantity, the Local Sidereal Time (LST) measured clockwise from the celestial meridian to the equinoctal colure. LST is defined as the hour angle of the vernal equinox.

2.3.1 Horizon - Hour Angle

Looking first at the Hour Angle to Horizon system transformation, using a spherical trigonometric approach, we know that from the hour angle system we are given h and δ, and we must express these quantities as functions of the horizon system directions, a (or z) and A. Implicit in this transformation is a knowledge of Φ.

From the spherical triangle (Figure 2-14), the law of sines yields

$$\frac{\sin(24-h)}{\sin z} = \frac{\sin A}{\sin(90-\delta)} , \qquad (2\text{-}13)$$

or

$$\frac{-\sin h}{\sin z} = \frac{\sin A}{\cos\delta} , \qquad (2\text{-}14)$$

and finally

$$\sin A \sin z = -\sin h \cos\delta \quad . \qquad (2\text{-}15)$$

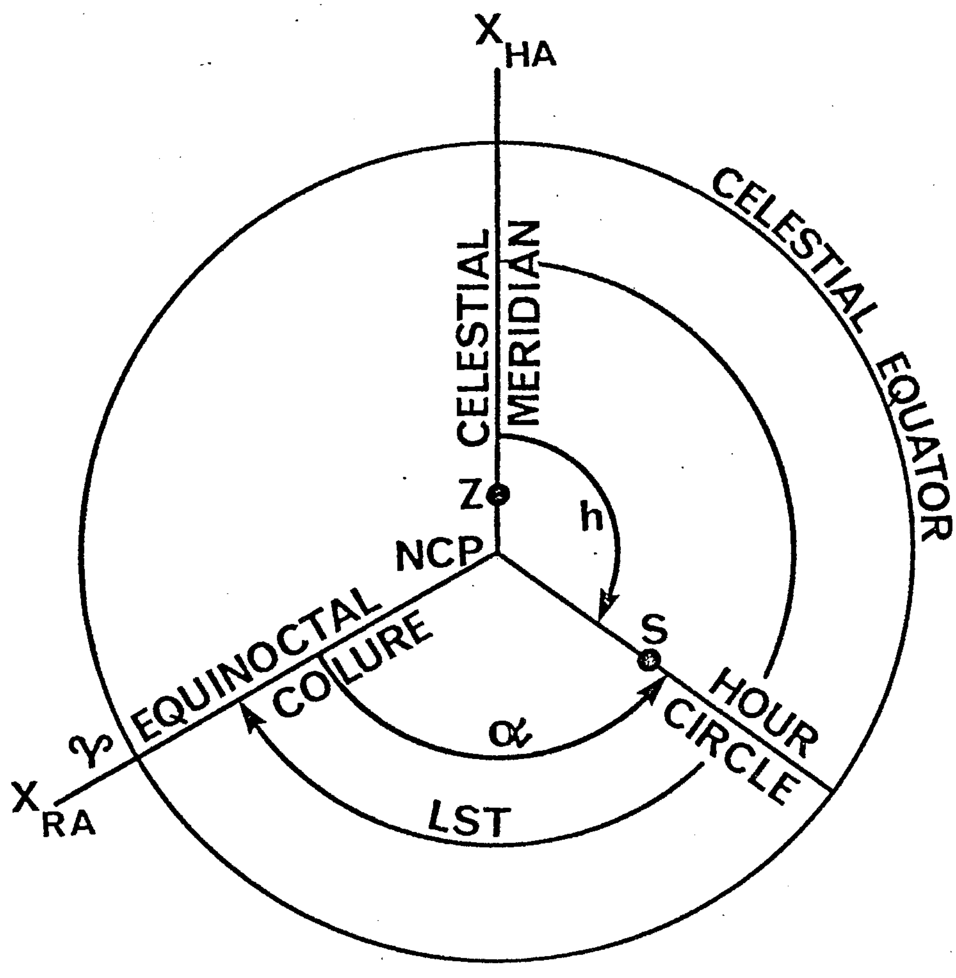


Figure 2-13

Local Sidereal Time

The five parts formula of spherical trigonometry gives

$$\cos A \sin z = \cos(90-\delta)\sin(90-\Phi)-\cos(90-\Phi)\sin(90-\delta)\cos(24-h), \qquad (2\text{-}16)$$

or

$$\cos A \sin z = \sin\delta \cos\Phi - \sin\Phi \cos\delta \cos h \quad . \qquad (2\text{-}17)$$

Now, dividing (2-15) by (2-17) yields

$$\frac{\sin A \sin z}{\cos A \sin z} = \frac{-\sin h \cos\delta}{\sin\delta \cos\Phi - \sin\Phi \cos\delta \cos h} \quad , \qquad (2\text{-}18)$$

which, after cancelling and collecting terms and dividing the numerator and denominator of the right-hand-side by cosδ yields

$$\tan A = \frac{-\sin h}{\tan\delta \cos\Phi - \sin\Phi \cos h} \quad , \qquad (2\text{-}19)$$

or

$$\tan A = \frac{\sin h}{\sin\Phi \cos h - \tan\delta \cos\Phi} \quad . \qquad (2\text{-}20)$$

Finally, the cosine law gives

$$\cos z = \cos(90-\Phi)\cos(90-\delta) + \sin(90-\Phi)\sin(90-\delta)\cos(24-h), \qquad (2\text{-}21)$$

or

$$\cos z = \sin\Phi \sin\delta + \cos\Phi \cos\delta \cos h \quad . \qquad (2\text{-}22)$$

Thus, through equations (2-20) and (2-22), we have the desired results - the quantities a (z) and A expressed as functions of δ and h and a known latitude Φ.

The transformation Horizon to Hour Angle system (given a (=90-z), A, Φ, compute h, δ) is done in a similar way using spherical trigonometry. The sine law yields

$$\cos\delta \sin h = -\sin z \sin A \quad , \qquad (2\text{-}23)$$

and five parts gives,

$$\cos\delta \cos h = \cos z \cos\Phi - \sin z \cos A \sin\Phi \quad . \qquad (2\text{-}24)$$

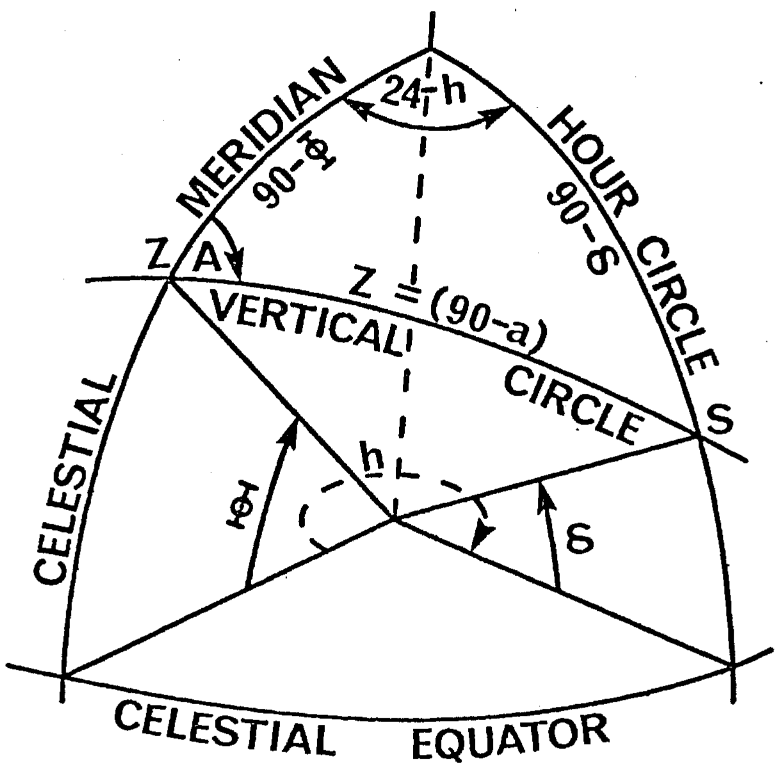


Figure 2-14

Horizon and Hour Angle System Transformations

After dividing (2-23) by (2-24), the result is

$$\tanh = \frac{\sin A}{\cos A \sin\Phi - \cot z \cos\Phi} \quad . \qquad (2\text{-}25)$$

Finally, the cosine law yields

$$\sin\delta = \cos z \sin\Phi + \sin z \cos A \cos\Phi \quad . \qquad (2\text{-}26)$$

Equations (2-25) and (2-26) are the desired results - h and δ expressed solely as functions of a(=90-z),A, and Φ.

Another approach to the solution of these transformations is to use rotation matrices. First, both systems are heliocentric and left-handed. From Figure 2-15, we can see that Y_{HA} is coincident with $-Y_H$ (or Y_H coincident with $-Y_{HA}$), thus they are different by 180°. Also from Figure 2-15, X_{HA}, Z_H, Z_{HA}, and X_H all lie in the plane of the celestial meridian. Z_H and Z_{HA} are also separated by (90-Φ) and are different by 180°. The transformation, then, of HA to H is given simply by

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_H = R_3(180°)\ R_2(90-\Phi) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{HA} , \qquad (2\text{-}27)$$

or giving the fully expanded form of each of the rotation matrices

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_H = \begin{bmatrix} \cos180° & \sin180° & 0 \\ -\sin180° & \cos180° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(90-\Phi) & 0 & -\sin(90-\Phi) \\ 0 & 1 & 0 \\ \sin(90-\Phi) & 0 & \cos(90-\Phi) \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{HA} . \qquad (2\text{-}28)$$

The effect of the first rotation, $R_2(90-\Phi)$, is to bring Z_{HA} into coincidence with Z_H, and X_{HA} into the same plane as X_H (the horizon plane). The second rotation, $R_3(180°)$, brings X_{HA} and Y_{HA} into coincidence with X_H and Y_H respectively, thus completing the transformation.

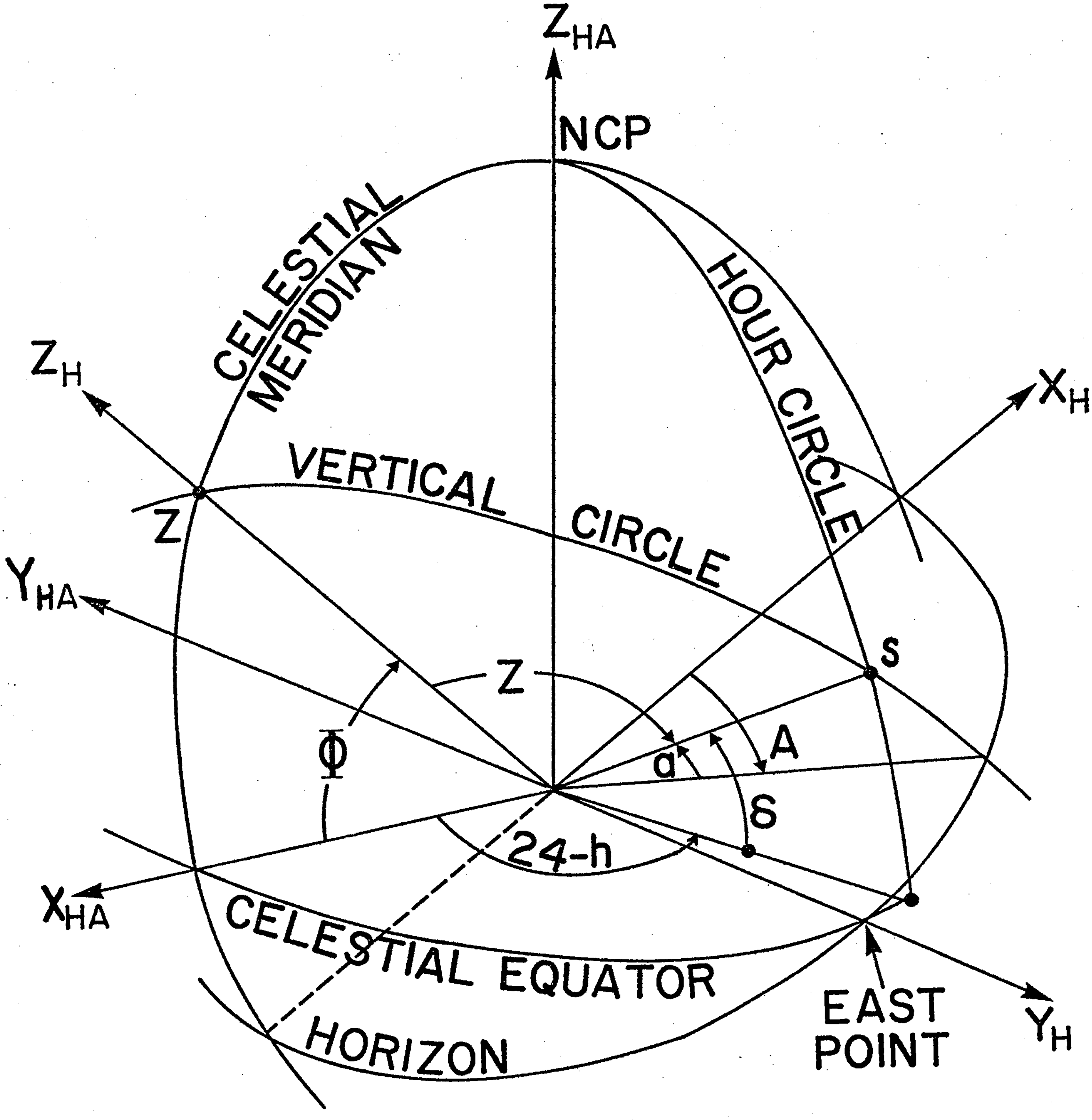


Figure 2-15

Horizon and Hour Angle Systems

The reverse transformation - Horizon to Hour Angle - is simply the inverse* of (2-27), namely

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{HA} = R_2(\Phi-90)R_3(180°) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{H} . \qquad (2\text{-}29)$$

2.3.2 Hour Angle - Right Ascension

Dealing first with the spherical trigonometric approach, it is evident from Figure 2-16 that

$$LST = h + \alpha. \qquad (2\text{-}30)$$

Furthermore, in both the Hour Angle and Right Ascension systems, the declination δ is one of the star coordinates. To transform the Hour Angle coordinates to coordinates in the Right Ascension system (assuming LST is known) we have

$$\alpha = LST-h , \qquad (2\text{-}31)$$

$$\delta=\delta . \qquad (2\text{-}32)$$

Similarly, the hour angle system coordinates expressed as functions of right ascension system coordinates are simply

$$h = LST-\alpha , \qquad (2\text{-}33)$$

$$\delta=\delta . \qquad (2\text{-}34)$$

For a matrix approach, we again examine Figure 2-16. Note that both systems are heliocentric, $Z_{HA} \equiv Z_{RA}$, and X_{HA}, Y_{HA}, X_{RA}, and Y_{RA} all lie in the plane

---

*Note: For rotation matrices, which are orthogonal, the following rules apply. If $x=Ry$, then $y=R^{-1}x$; also $(R_iR_j)^{-1} = R_j^{-1}R_i^{-1}$, and $R^{-1}(\theta) = R(-\theta)$.

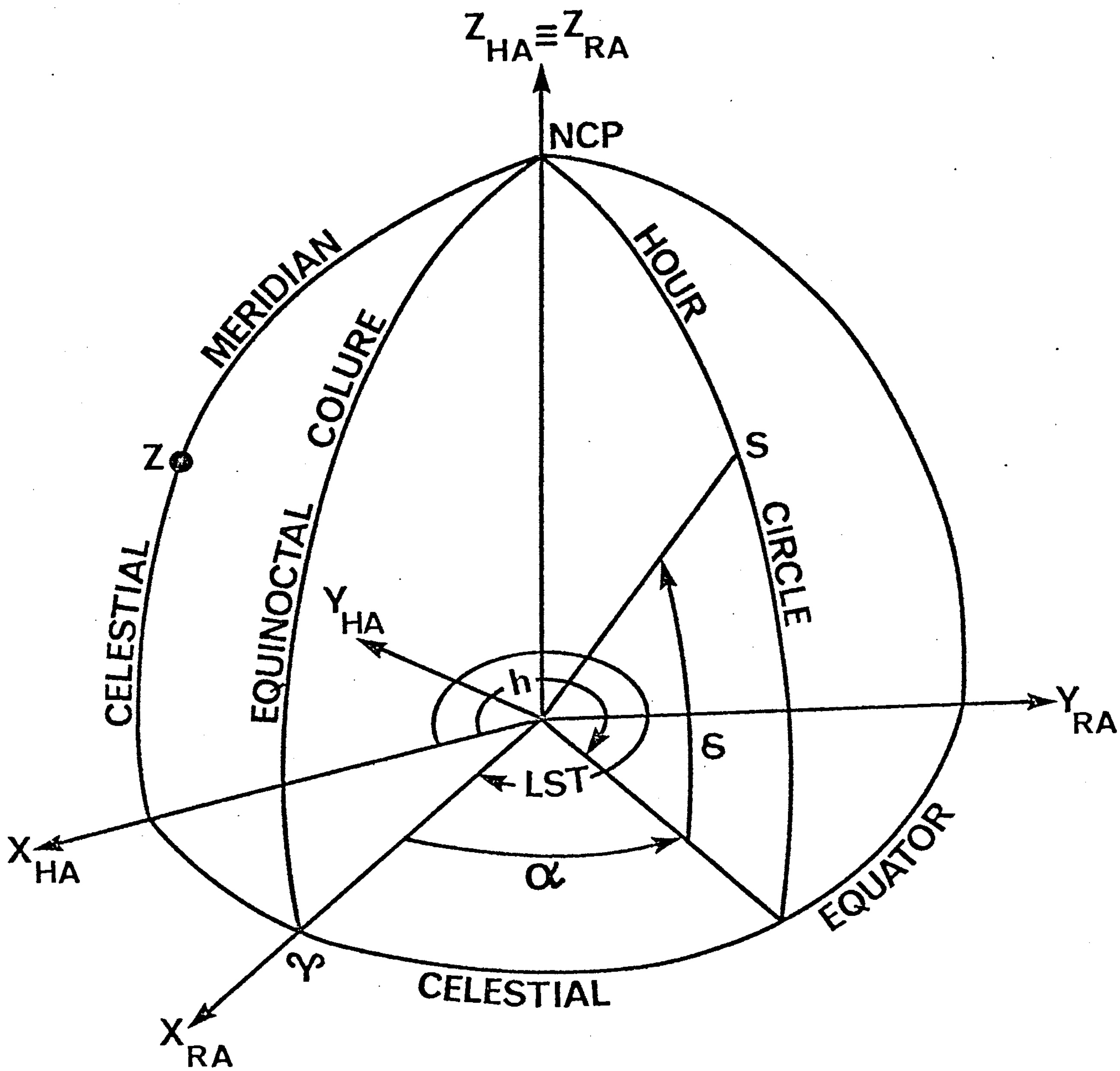


Figure 2-16

Hour Angle - Right Ascension Systems

of the celestial equator. The differences are that the HA system is left-handed, the RA system right-handed, and X_{HA} and X_{RA} are separated by the LST. The Right Ascension system, in terms of the hour angle system, is given by

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{RA} = R_3(-LST)\ P_2 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{HA} , \tag{2-35}$$

or, with $R_3(-LST)$ and the reflection matrix P_2 expanded,

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{RA} = \begin{bmatrix} \cos(-LST) & \sin(-LST) & 0 \\ -\sin(-LST) & \cos(-LST) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{HA} . \tag{2-36}$$

The reflection matrix, P_2, changes the handedness of the hour angle system (from left to right), and the rotation $R_3(-LST)$ brings X_{HA} and Y_{HA} into coincidence with X_{RA} and Y_{RA} respectively.

The inverse transformation is simply

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{HA} = P_2\ R_3(LST) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{RA} . \tag{2-37}$$

2.3.3 Right Ascension - Ecliptic

For the spherical trigonometric approach to the Right Ascension - Ecliptic system transformations, we look at the spherical triangle with vertices NEP, NCP, and S in Figure 2-17. The transformation of the Ecliptic coordinates β, δ (ε assumed known) to the Right Ascension coordinates α, δ, utilizes the same procedure as in 2.3.1.

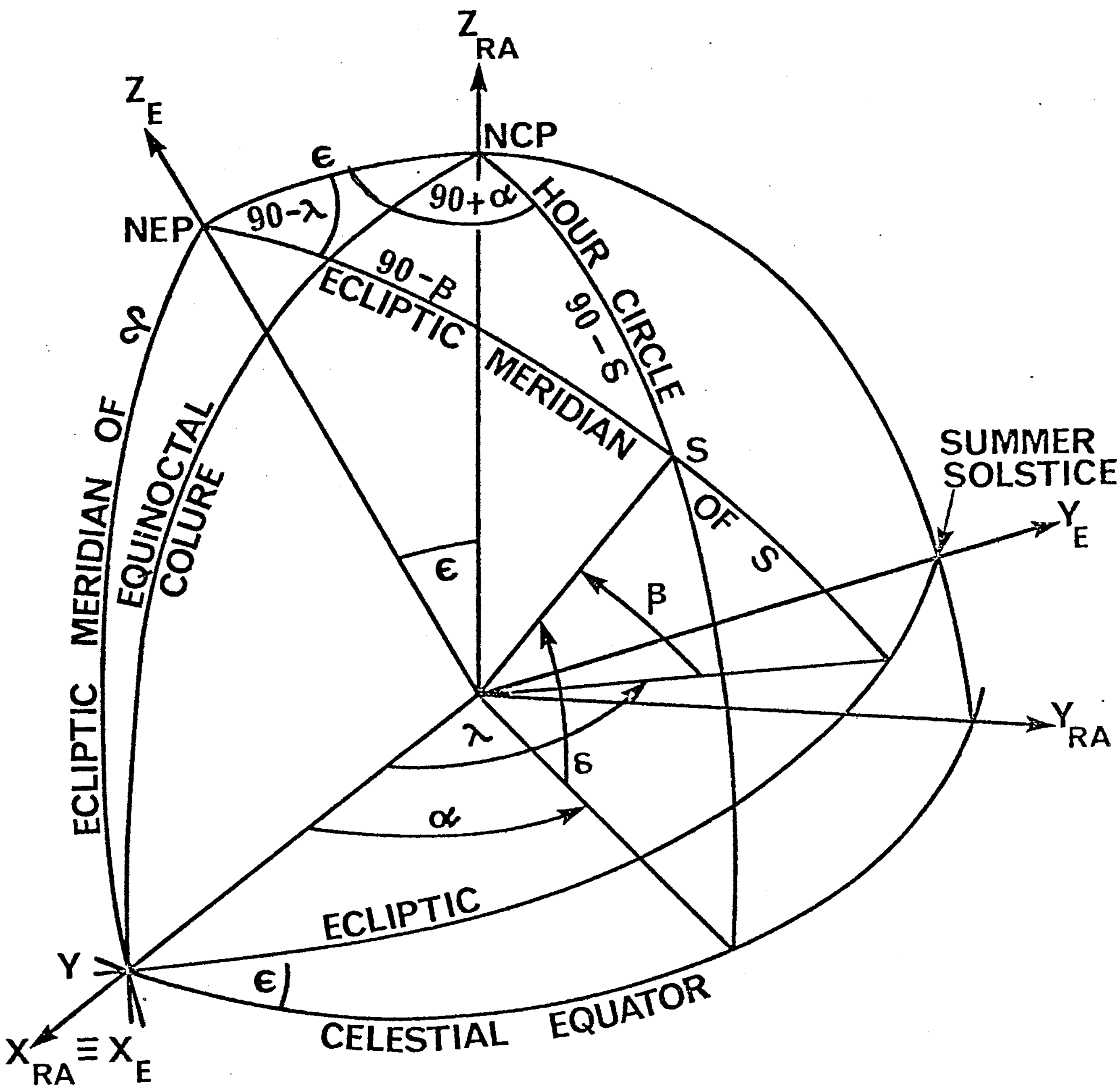


Figure 2-17

Right Ascension-Ecliptic Systems

The sine rule of spherical trigonometry yields

$$\cos\delta \cos\alpha = \cos\beta \cos\lambda \quad , \tag{2-38}$$

and the five parts rule

$$\cos\delta \sin\alpha = \cos\beta \sin\lambda \cos\varepsilon - \sin\beta \sin\varepsilon \quad . \tag{2-39}$$

Dividing (2-39) by (2-38) yields the desired result

$$\tan\alpha = \frac{\sin\lambda \cos\varepsilon - \tan\beta \sin\varepsilon}{\cos\lambda} \quad . \tag{2-40}$$

From the cosine rule

$$\sin\delta = \cos\beta \sin\lambda \sin\varepsilon + \sin\beta \cos\varepsilon \quad , \tag{2-41}$$

which completes the transformation of the Ecliptic system to the Right Ascension system.

Using the same rules of spherical trigonometry (sine, five-parts, cosine), the inverse transformation (Right Ascension to Ecliptic) is given by

$$\cos\beta \cos\lambda = \cos\delta \cos\alpha \quad , \tag{2-42}$$

$$\cos\beta \sin\lambda = \cos\delta \sin\alpha \cos\varepsilon + \sin\delta \sin\varepsilon \ , \tag{2-43}$$

which, after dividing (2-43) by (2-42) yields

$$\tan\lambda = \frac{\sin\alpha \cos\varepsilon + \tan\delta \sin\varepsilon}{\cos\alpha} \quad , \tag{2-44}$$

and

$$\sin\beta = -\cos\delta \sin\alpha \sin\varepsilon + \sin\delta \cos\varepsilon \ . \tag{2-45}$$

From Figure 2-17, it is evident that the difference between the E and RA cartesian systems is simply the obliquity of the ecliptic, ε, which separates the Z_E and Z_{RA}, and Y_E and Y_{RA} axes, the pairs of which lie in the same planes.

The transformations then are given by

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{RA} = R_1(-\varepsilon) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{E} , \qquad (2\text{-}46)$$

and

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{E} = R_1(\varepsilon) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{RA} . \qquad (2\text{-}47)$$

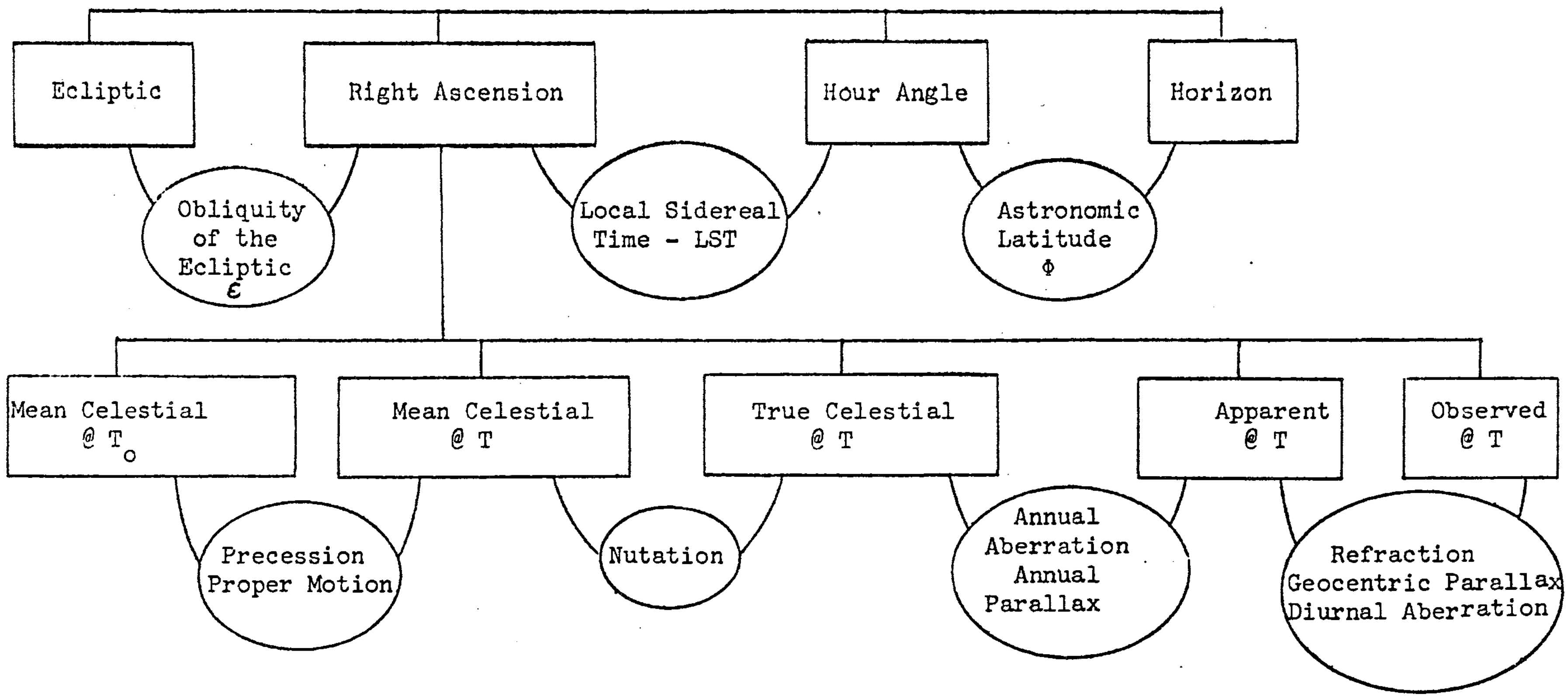
2.3.4 Summary

Figure 2-18 and Table 2-3 summarize the transformations amongst celestial coordinate systems. Note that in Figure 2-18, the quantities that we must know to effect the various transformations are highlighted. In addition, Figure 2-18 highlights the expansion of the Right Ascension system to account for the motions of the coordinate systems in time and space as mentioned in the introduction to this chapter. These effects are covered in detail in Chapter 3.

Table 2-3 highlights the matrix approach to the transformations amongst celestial coordinate systems.

2.4 Special Star Positions

Certain positions of stars on the celestial sphere are given "special" names. As shall be seen later, some of the math models for astronomic position and azimuth determination are based on some of the special positions that stars attain.



CELESTIAL COORDINATE SYSTEM
RELATIONSHIPS [Krakiwsky and Wells, 1971]

FIGURE 2-18

		Original System			
		Ecliptic	Right Ascension	Hour Angle	Horizon
Final System	Ecliptic	$\begin{bmatrix}x\\y\\z\end{bmatrix}_E$	$R_1\ (\varepsilon)$		
	Right Ascension	$R_1\ (-\varepsilon)$	$\begin{bmatrix}x\\y\\z\end{bmatrix}_{R.A.}$	$R_3\ (-LST)\ P_2$	
	Hour Angle		$P_2\ R_3(+LST)$	$\begin{bmatrix}x\\y\\z\end{bmatrix}_{H.A.}$	$R_2(\Phi-90°)\ R_3(180°)$
	Horizon			$R_3(180°)\ R_2(90°-\Phi)$	$\begin{bmatrix}x\\y\\z\end{bmatrix}_{H.}$

TRANSFORMATIONS AMONG CELESTIAL COORDINATE SYSTEMS (Krakiwsky and Wells, 1971).

TABLE 2-3

Figure 2-19 shows the behavior of stars as they move in an apparent east to west path about the earth, as seen by an observer (Z) situated somewhere between the equator and the north pole ($0^\circ \leq \Phi \leq 90^\circ$). Referring to Figure 2-19:

1. North Circumpolar Stars that (a) never cross the prime vertical,
   (b) cross the prime vertical.

   These stars, (a) and (b), are visible at all times to a northern observer.

2. Equatorial stars rise above and set below the celestial horizon
   - (a) more time above the horizon than below,
   - (b) equal times above and below the horizon,
   - (c) more times below the horizon than above.

3. South Circumpolar Stars never rise for a northern observer.

For an observer in the southern hemisphere ($0^\circ \geq \phi \geq -90^\circ$), the explanations of 1, 2, 3 above are reversed. Two special cases are (1) Z located on the equator, thus the observer will see 1/2 the paths of all stars, and (2) Z located at a pole, thus the observer sees all stars in that polar hemisphere as they appear to move in circles about the zenith ($\equiv$pole).

2.4.1 Rising and Setting of Stars

The ability to determine the visibility of any star is fundamental to geodetic astronomy. To establish a star observing program, one must ensure that the chosen stars will be above the horizon during the desired observation period.

<u>Declination</u>

From Figure 2-20, we see that for an observer in the northern hemisphere, a north star, to be on or above the horizon must have a declination given by

$$\delta \geq 90^\circ - \Phi \quad , \qquad (2\text{-}48)$$

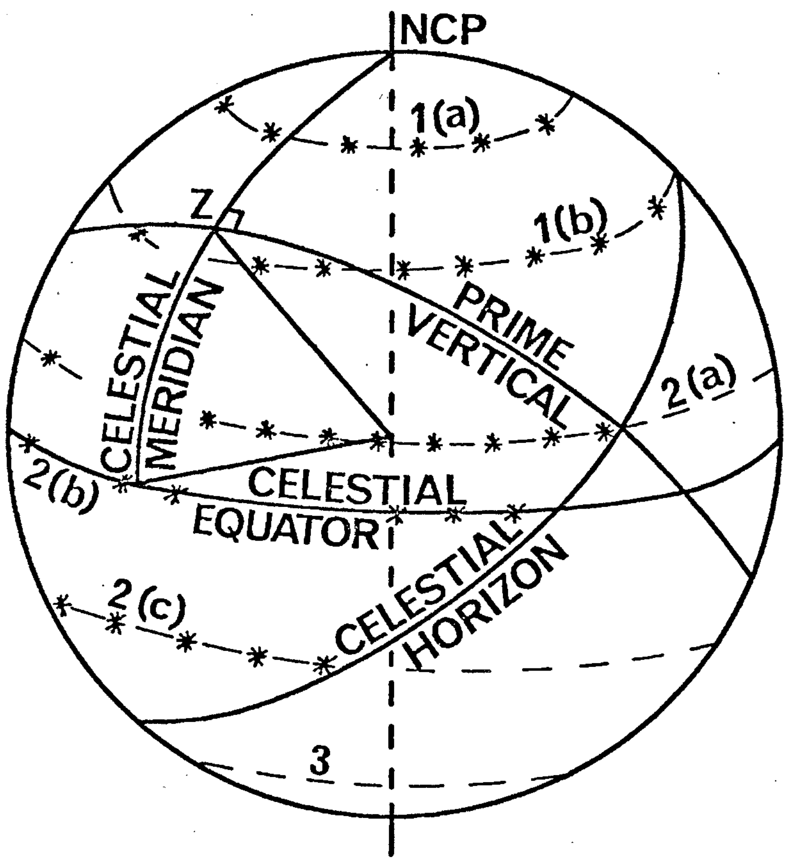


Figure 2-19

Circumpolar and Equatorial Stars

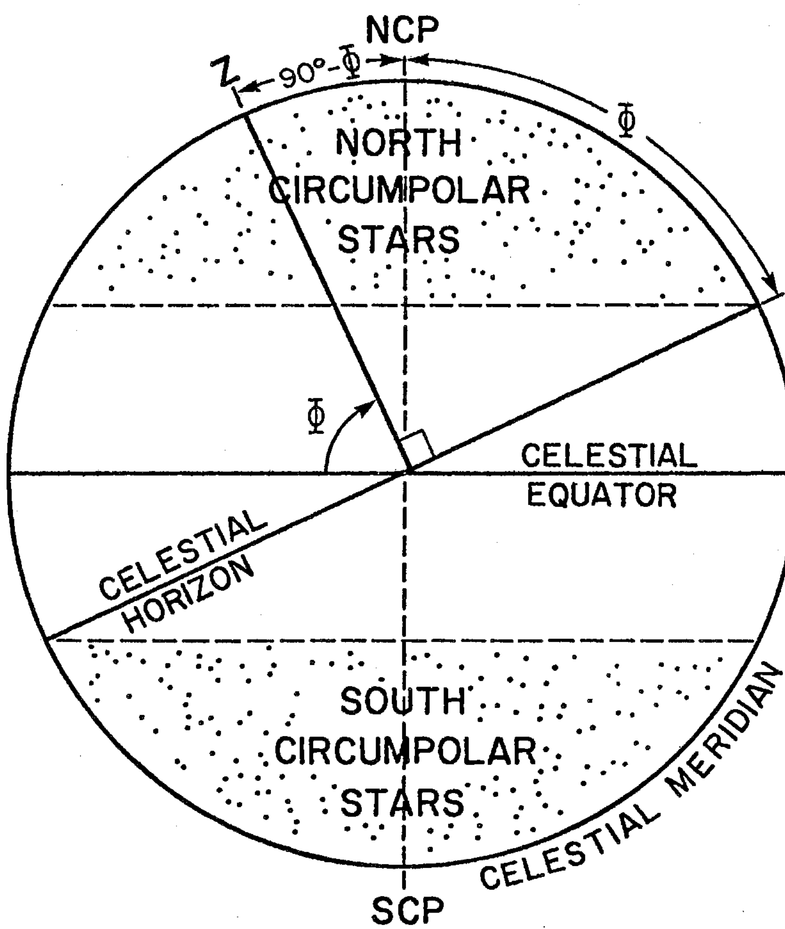


Figure 2-20

Declination for Visibility

and a south star that will rise at some time, must have a declination

$$\delta \geq \Phi - 90^\circ. \tag{2-49}$$

Thus, the condition for rising and setting of a star is

$$90^\circ - \Phi > \delta > \Phi - 90^\circ. \tag{2-50}$$

For example, at Fredericton, where $\Phi \approx 46°N$, the declination of a star must always satisfy the condition (2-50), namely

$$44° > \delta > -44°$$

in order that the star will be visible at some time; that is, the star will rise and set. If $\delta > 44°$, the star will never set (it will always be a visible circumpolar star), and if $\delta < -44°$, the star will never rise (a south circumpolar star).

Hour Angle

Now that the limits for the declinations of stars for rising and setting have been defined, we must consider at what hour angle these events will occur.

From the transformation of the Hour Angle to Horizon curvalinear coordinates (Section 2.3.1, (2-22))

$$\cos z = \sin\delta \sin\Phi + \cos\delta \cos h \cos\Phi \ .$$

For rising and setting, $z=90^{\circ}$, and the above equation reduces to

$$\sin\delta \sin\Phi + \cos\delta \cos h \cos\Phi = 0, \tag{2-51}$$

which, after rearrangement of terms yields

$$\cos h = -\tan\delta \tan\Phi \ . \tag{2-52}$$

The star's apparent motion across the celestial sphere is from east to west. For a star that rises and sets, there are two solutions to equation (2-52): the smaller solution designates setting, the larger solution designates rising (Figures 2-21, 2-22). The following example illustrates this point.

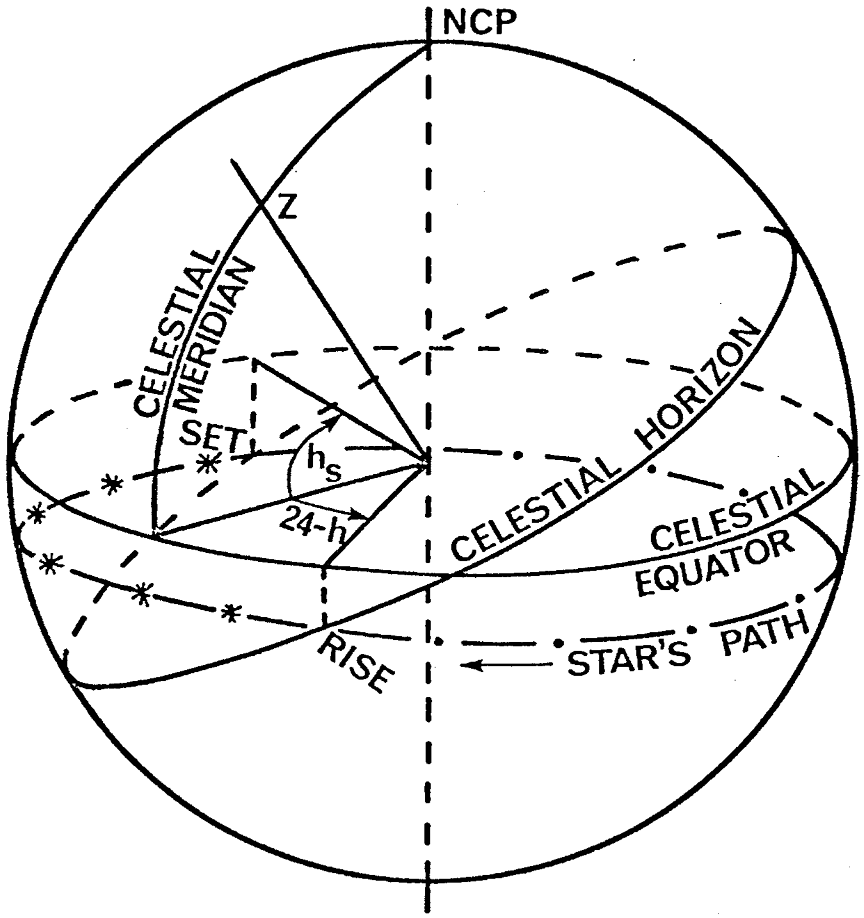


Figure 2-21

Rising and Setting of a Star

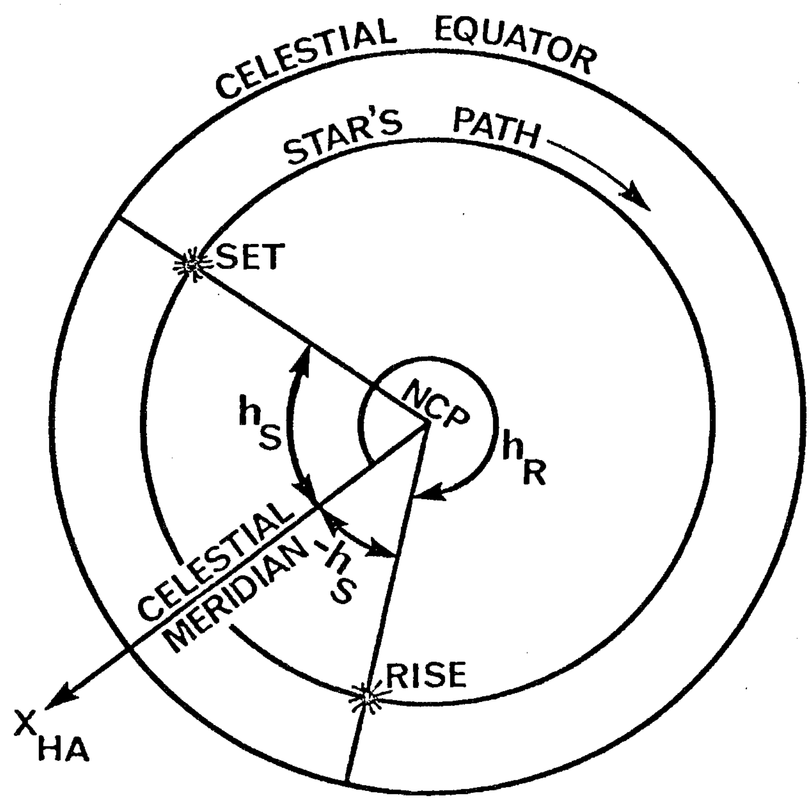


Figure 2-22

Hour Angles of a Star's Rise and Set

At Φ=46°, we wish to investigate the possible use of two stars for an observation program. Their declinations are δ_1=35°, δ_2=50°. From equation (2-50),

$$44° > \delta_1 > -44° ,$$

$$44° < \delta_2 < -44° .$$

The first star, since it satisfies (2-50), will rise and set. The second star, since $\delta_2 >$ 44°, never sets - it is a north circumpolar star to our observer.

Continuing with the first star, equation (2-52) yields

$$\cos h_1 = -\tan 35° \tan 46°$$

or

$$h_1^s = 136.°4759 = 9^h\ 05^m\ 54.^s22,$$

which is the hour angle for setting. The hour angle for rising is

$$h_1^r = (24^h - h_1^s) = 14^h\ 54^m\ 05.^s78.$$

Azimuth

At what azimuth will a star rise or set? From the sine law in the Hour Angle to Horizon system transformation (equation (2-15))

$$\sin z \sin A = -\cos\delta \sin h.$$

With z=90° for rising and setting, then

$$\sin A = -\cos\delta \sin h. \qquad (2\text{-}53)$$

There are, of course, two solutions (Figures 2-23 and 2-24): one using h^r and one using h^s. Using the previous example for star 1 (δ_1=35°, Φ=46°, $h_1^s = 9^h\ 05^m\ 54.^s22$, $h_1^r = 14^h\ 54^m\ 05.^s78$), one gets

$$\sin A_1^r = -\cos 35° \sin 223.^\circ 5241,$$

$$A_1^r = 34°\ 20'\ 27.''64.$$

Similarly

$$A_1^s = 325°\ 39'\ 32.''36.$$

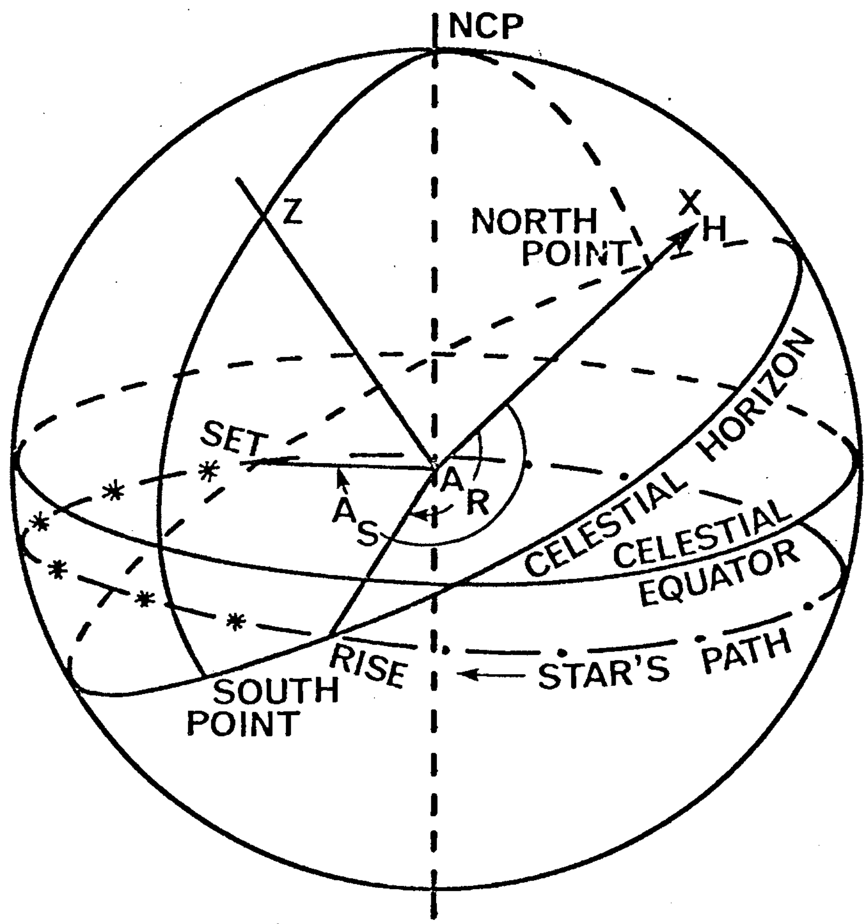


Figure 2-23

Rising and Setting of a Star

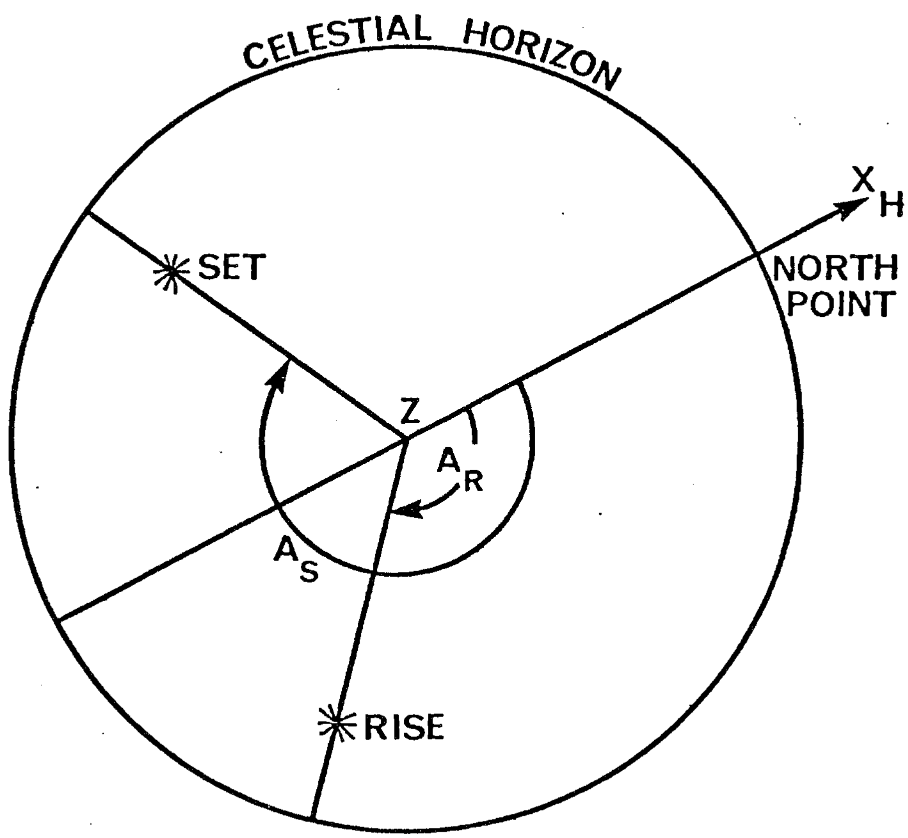


Figure 2-24

Azimuths of a Star's Rise and Set

The following set of rules apply for the hour angles and azimuths of rising and setting stars [Mueller, 1969]:

Northern Stars ($\delta>0°$)	Rise	$12^h<h<18^h$	$0°<A<90°$
	Set	$6^h<h<12^h$	$270°<A<360°$
Equatorial Stars ($\delta=0°$)	Rise	$h=18^h$	$A=90°$
	Set	$h=6^h$	$A=270°$
Southern Stars ($\delta<0°$)	Rise	$18^h<h<24^h$	$90°<A<180°$
	Set	$0^h<h<6^h$	$180°<A<270°$

2.4.2 Culmination (Transit)

When a star's hour circle is coincident with the observer's celestial meridian, it is said to culminate or transit. Upper culmination (UC) is defined as being on the zenith side of the hour circle, and can occur north or south of the zenith (Figure 2-25a and 2-25b). When UC occurs north of Z, the zenith distance is given as (Figure 25a)

$$z = \delta-\Phi \; . \qquad (2-54)$$

The zenith distance of UC south of Z is

$$z = \Phi-\delta \; . \qquad (2-55)$$

Lower Culmination (LC) (Figure 2-25c) is on the nadir side of the hour circle, and for a northern observer, always occurs north of the zenith. The zenith distance at LC is

$$z = 180-(\delta+\Phi). \qquad (2-56)$$

Recalling the examples given in 2.4.1 ($\Phi=46°, \delta_1=35°, \delta_2=50°$), then for the first star

$$z_1^{UC} = \Phi - \delta_1 = 11° \; ,$$

which is south of the zenith, and

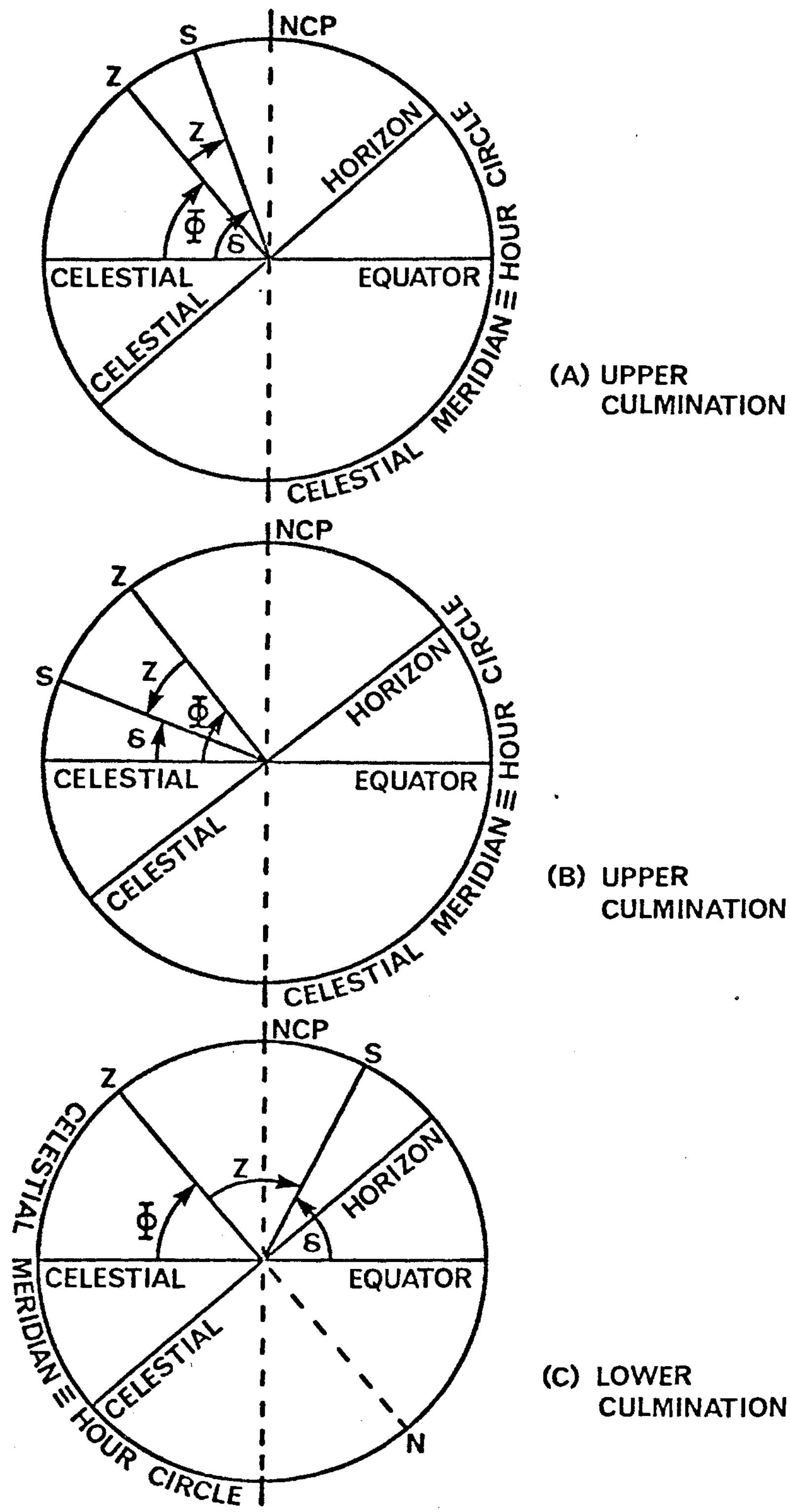


Figure 2-25

Culmination (Transit)

$$z_1^{LC} = 180 -(\delta_1+\Phi) = 99°$$

which means that the star will not be visible ($z>90°$).

For the second star, one obtains

$$z_2^{UC} = \delta_2-\Phi = 4° \ ,$$

$$z_2^{LC} = 180-(\delta_2 +\Phi) = 84° \ ,$$

both of which will be north of the zenith. The hour angle at culmination is $h = 0^h$ for all upper culminations, and $h = 12^h$ for all lower culminations. The azimuths at culmination are as follows: $A = 0°$ for all Upper Culminations north of Z and all Lower Culminations; $A = 180°$ for all Upper Culminations south of the zenith.

Recalling the Hour Angle - Right Ascension coordinate transformations, we had (equation (2-30)

$$LST = h+\alpha \ .$$

Since $h = 0^h$ for Upper Culminations and $h = 12^h$ for Lower Culminations,

$$LST^{UC} = \alpha \ , \qquad (2\text{-}57)$$

$$LST^{LC} = \alpha+ 12^h \ . \qquad (2\text{-}58)$$

2.4.3. Prime Vertical Crossing

For a star to reach the prime vertical (Figure 2-26)

$$0°<\delta<\Phi \ . \qquad (2\text{-}59)$$

To compute the zenith distance of a prime vertical crossing, recall from the Horizon to Hour Angle transformation (via) the cosine rule, equation (2-26) that

$$\sin\delta = \cos z \ \sin\Phi + \sin z \ \cos A \ \cos\Phi \ .$$

For a prime vertical crossing in the east, $A=90°$ (altitude increasing), and for a prime vertical crossing in the west, $A=270°$ (altitude decreasing),

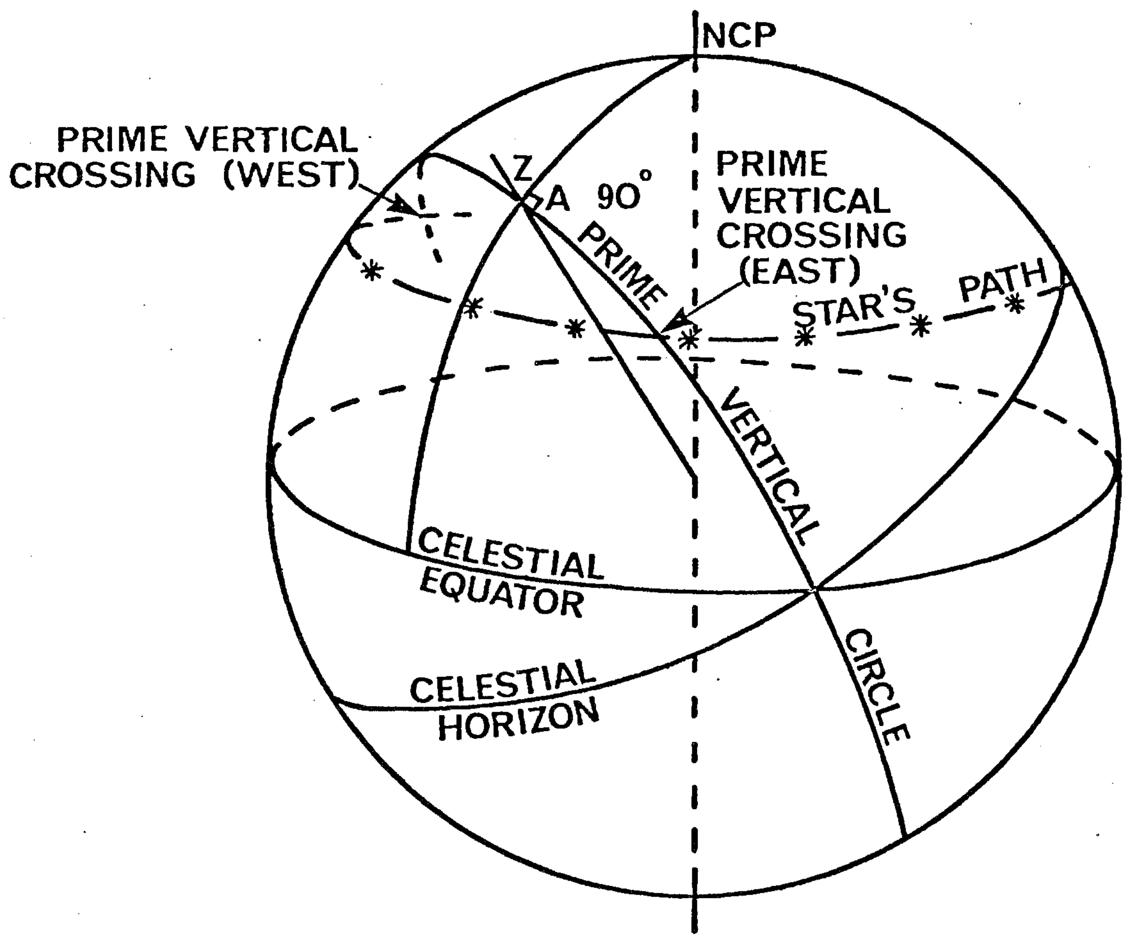


Figure 2-26

Prime Vertical Crossing

we get

$$\cos z = \frac{\sin\delta}{\sin\Phi} = \sin\delta\ \mathrm{cosec}\Phi \quad . \tag{2-60}$$

The hour angle of a prime vertical crossing is computed as follows. From the cosine rule of the Hour Angle to Horizon transformation (equation (2-22))

$$\cos z = \sin\delta \sin\Phi + \cos\delta \cos h \cos\Phi \ .$$

Substituting (2-60) in the above for cosine z yields

$$\frac{\sin\delta}{\sin\Phi} = \sin\delta \sin\Phi + \cos\delta \cos h \cos\Phi \ .$$

or

$$\sin\delta = \sin\delta \sin^2\Phi + \cos\delta \cos h \cos\Phi \sin\Phi \ . \tag{2-61}$$

Now, (2-61) reduces to

$$\cos h = \frac{\sin\delta}{\cos\delta \cos\Phi \sin\Phi} - \frac{\sin\delta \sin^2\Phi}{\cos\delta \cos\Phi \sin\Phi} \ ,$$

or

$$\cos h = \tan\delta \left(\frac{1 - \sin^2\Phi}{\cos\Phi \ \sin\Phi}\right) = \tan\delta \left(\frac{\cos^2\Phi}{\cos\Phi \sin\Phi}\right) \ ,$$

and finally

$$\cos h = \tan\delta \cot\Phi \quad . \tag{2-62}$$

As with the determination of the rising and setting of a star, there are two values for h - prime vertical eastern crossing ($18^h<h<24^h$) and prime vertical western crossing ($0^h<h<6^h$). Continuing with the previous examples ($\delta_1 = 35°$, $\delta_2 = 50°$), it is immediately evident that the second star will not cross the prime vertical ($\delta_2 > 46°$). The first star will cross the prime vertical ($0<\delta_1< 46°$), with azimuths A=90° (eastern) and A=270° (western). The zenith distance for both crossings will be (equation 2-60)

$$\cos z_1 = \frac{\sin 35°}{\sin 46°} \ ,$$

$$z_1 = 37° \ 07' \ 14.''8 .$$

The hour angles of western and eastern crossings are respectively (equation 2-62)

$$\cosh h_2^W = \tan\delta \cot\phi$$
$$= \tan 35° \cot 46° \ ,$$
$$h_1^W = 47.^\circ45397 = 3^h 09^m 48.^s9 \ ,$$

and

$$h_1^E = 24^h\ 00^m\ 00^s - 3^h\ 09^m\ 48.^s9$$
$$= 20^h\ 50^m\ 11.^s0 \ .$$

2.4.4. Elongation

When the parallactic angle (p) is 90°, that is, the hour circle and vertical circle are normal to each other, the star is said to be at elongation (Figure 1-27). Elongation can occur on both sides of the observer's celestial meridian, but only with stars that do not cross the prime vertical. Thus, the condition for elongation is that

$$\delta > \Phi \ . \tag{2-63}$$

From the astronomic triangle (Figure 2-27), the sine law yields

$$\frac{\sin A}{\sin(90-\delta)} = \frac{\sin p}{\sin(90-\Phi)} \ ,$$

or

$$\sin A = \frac{\sin p \cos\delta}{\cos\Phi} \ . \tag{2-64}$$

Since at elongation, p=90°, (2-64) becomes

$$\sin A = \cos\delta \sec\Phi \ . \tag{2-65}$$

For eastern elongation, it is obvious that $0° < A < 90°$, and for western elongation $270° < A < 360°$.

To solve for the zenith distance and hour angle at elongation, one proceeds as follows. Using the cosine law with the astronomic triangle (Figure 1-27), one gets

$$\cos(90-\Phi) = \cos(90-\delta)\cos z + \sin z \sin(90-\delta)\cos p \ , \tag{2-66}$$

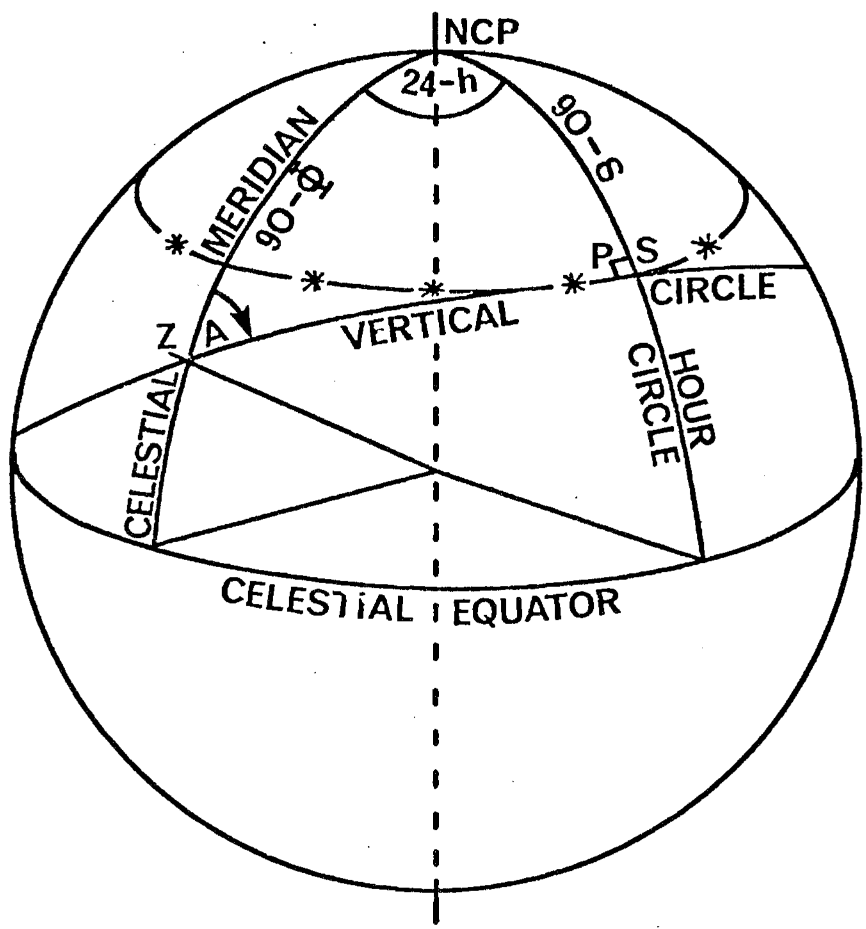


Figure 2-27

Elongation

and when p=90°, cosp=0, then

$$\cos z = \sin\Phi \operatorname{cosec}\delta \quad . \tag{2-67}$$

Now, substituting the above expression for cosz (2-67) in the Hour Angle to Horizon coordinate transformation expression (2-22), namely

$$\cos z = \sin\delta \sin\Phi + \cos\delta \cosh \cos\Phi \quad ,$$

yields

$$\frac{\sin\Phi}{\sin\delta} = \sin\delta \sin\Phi + \cos\delta \cosh \cos\Phi \quad ,$$

which, on rearranging terms gives

$$\cosh = \frac{\sin\Phi - \sin^2\delta \sin\Phi}{\cos\delta \sin\delta \cos\Phi} \quad , \tag{2-68}$$

Further manipulation of (2-68) leads to

$$\cosh = \tan\Phi \left(\frac{1-\sin^2\delta}{\cos\delta\sin\delta}\right) = \tan\Phi \left(\frac{\cos^2\delta}{\cos\delta \sin\delta}\right) \quad ,$$

and finally

$$\cosh = \tan\Phi \cot\delta \quad . \tag{2-69}$$

Note that as with the azimuth at elongation, one will have an eastern and western value for the hour angle.

Continuing with the previous examples - δ_1=35°, δ_2=50°, and Φ=46° - we see that for the first star, $\delta_1<\Phi$, thus it does not elongate. For the second star, $\delta_2>\Phi$ (50°>46°), thus eastern and western elongations will occur. The azimuths, zenith distance, and hour angles for the second star are as follows:

$$\sin A_2^E = \cos\delta \sec\Phi = \cos 50° \sec 46° \quad ,$$

$$A_2^E = 67° \ 43' \ 04''.7 \quad ,$$

$$A_2^W = 360° - A_2^W = 292° \ 16' \ 55''.3 \quad ;$$

$$\cos Z = \sin\Phi \operatorname{cosec}\delta = \sin 46° \operatorname{cosec} 50° \quad ,$$

$$Z_2 = 20° \ 06' \ 37''.3 \quad ;$$

$$\cosh_2^W = \tan\Phi \cot\delta = \tan 46° \cot 50°$$

$$h_2^W = 29°.66733 = 1^h \ 58^m \ 40^s.2 \quad ,$$

$$h_2^E = 24^h - h_2^W = 22^h \ 01^m \ 19^s.8 \quad .$$

3. TIME SYSTEMS

In the beginning of Chapter 2, it was pointed out that the position of a star on the celestial sphere, in any of the four coordinates systems, is valid for only one instant of time T. Due to many factors (e.g. earth's motions, motions of stars), the celestial coordinates are subject to change with time. To fully understand these variations, one must be familiar with the time systems that are used.

To describe time systems, there are three basic definitions that have to be stated. An epoch is a particular instant of time used to define the instant of occurance of some phenomenon or observation. A time interval is the time elapsed between two epochs, and is measured in some time scale. For civil time (the time used for everyday purposes) the units of a time scale (e.g. seconds) are considered fixed in length. With astronomic time systems, the units vary in length for each system. The adopted unit of time should be related to some repetitive physical phenomenon so that it can be easily and reliably established. The phenomenon should be free, or capable of being freed, from short period irregularities to permit interpolation and extrapolation by man-made time-keeping devices.

There are three basic time systems:

(1) Sidereal and Universal Time, based on the diurnal rotation of the earth and determined by star observations,

(2) Atomic Time, based on the period of electro magnetic oscillations produced by the quantum transition of the atom of Caesium 133,

(3) Ephemeris Time, defined via the orbital motion of the earth about the sun.

Ephemeris time is used mainly in the field of celestial mechanics and is of little interest in geodetic astronomy. Sidereal and Universal times are the most useful for geodetic purposes. They are related to each other via rigorous formulae, thus the use of one or the other is purely a matter of convenience. All broadcast time signals are derived from Atomi time, thus the relationship between Atomic time and Sidereal or Universal time is important for geodetic astronomy.

3.1 Sidereal Time

The fundamental unit of the sidereal time interval is the mean sidereal day. This is defined as the interval between two successive upper transits of the mean vernal equinox (the position of ϒ for which uniform precessional motion is accounted for and short period non-uniform nutation is removed) over some meridian (the effects of polar motion on the meridian are removed). The mean sidereal day is reckoned from 0^h at upper transit, which is known as sidereal noon. The units are $1^h_s = 60^m_s$, $1^m_s = 60^s_s$. Apparent (true) sidereal time (the position of ϒ is affected by both precession and nutation), because of its variable (non-uniform) rate, is not used as a measure of time interval. Since the mean equinox is affected only be precession (nutation effects are removed), the mean sidereal day is $0^s_s.0084$ shorter than the actual rotation period of the earth.

From the above definition of the fundamental unit of the sidereal time interval, we see that sidereal time is directly related to the rotation of the earth - equal angles of angular motion correspond to equal intervals of sidereal time. The sidereal epoch is numerically measured by the hour angle of the vernal equinox. The hour angle of the true vernal equinox (position of ϒ affected by precession and nutation) is termed Local Apparent Sidereal Time (LAST). When the hour angle measured is that at the Greenwich mean astronomic meridian (GHA), it is called <u>Greenwich Apparent Sidereal Time</u> (GAST). Note that the use of the Greenwich meridian as a reference meridian for time systems is one of convenience and uniformity. This convenience and uniformity gives us the direct relationships between time and longitude, as well as the simplicity of publishing star coordinates that are independent of, but directly related to, the longitude of an observer. The local hour angle of the mean vernal equinox is called <u>Local Mean Sidereal Time</u> (LMST), and the Greenwich hour angle of the mean vernal equinox is the <u>Greenwich Mean Sidereal Time</u> (GMST). The difference between LAST and LMST, or equivalently, GAST and GMST, is called the <u>Equation of the Equinoxes</u> (Eq. E), namely

$$\text{Eq.E} = \text{LAST} - \text{LMST} = \text{GAST} - \text{GMST}. \tag{3-1}$$

LAST, LMST, GAST, GMST, and Eq.E are all shown in Figure 3-1.

The equation of the equinoxes is due to nutation and varies periodically with a maximum amplitude near 1^s_s (Figure 3-2). It is tabulated for 0^h U.T. (see 3.2) for each day of the year, in the Astromical Almanac (AA), formerly called the American Ephemeris and Nautical Almanac (AENA) [U.S. Naval Observatory, 1980].

To obtain the relationships between Local and Greenwich times, we require the longitude (Λ) of a place. Then, from Figure 3-3, it can be seen that

$$\text{LMST} = \text{GMST} + \Lambda\ , \tag{3-2}$$

$$\text{LAST} = \text{GAST} + \Lambda\ , \tag{3-3}$$

in which Λ is the "reduced" astronomic longitude of the local meridian (corrections for polar motion have been made) measured east from the Greenwich meridian.

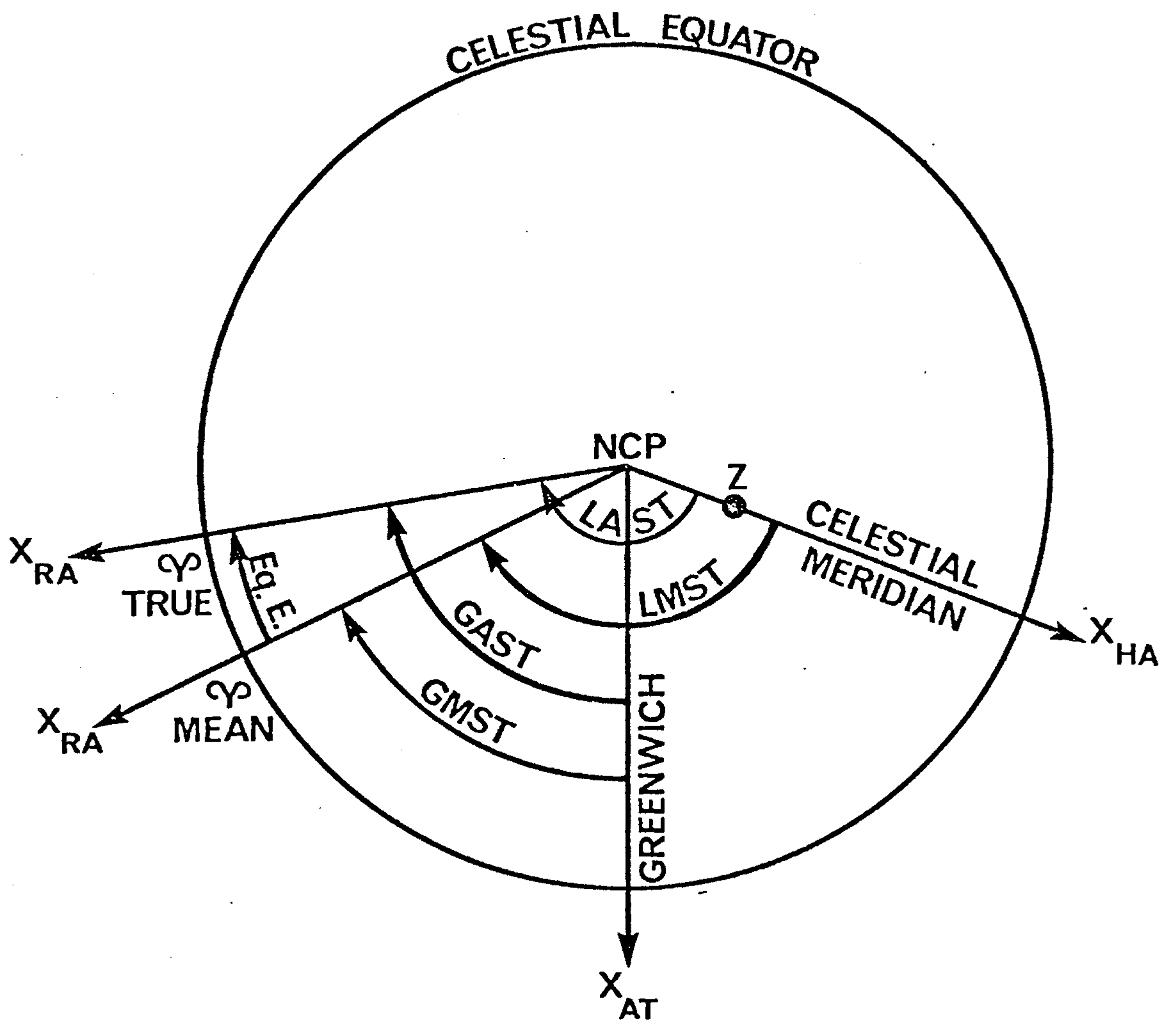


Figure 3-1

Sidereal Time Epochs

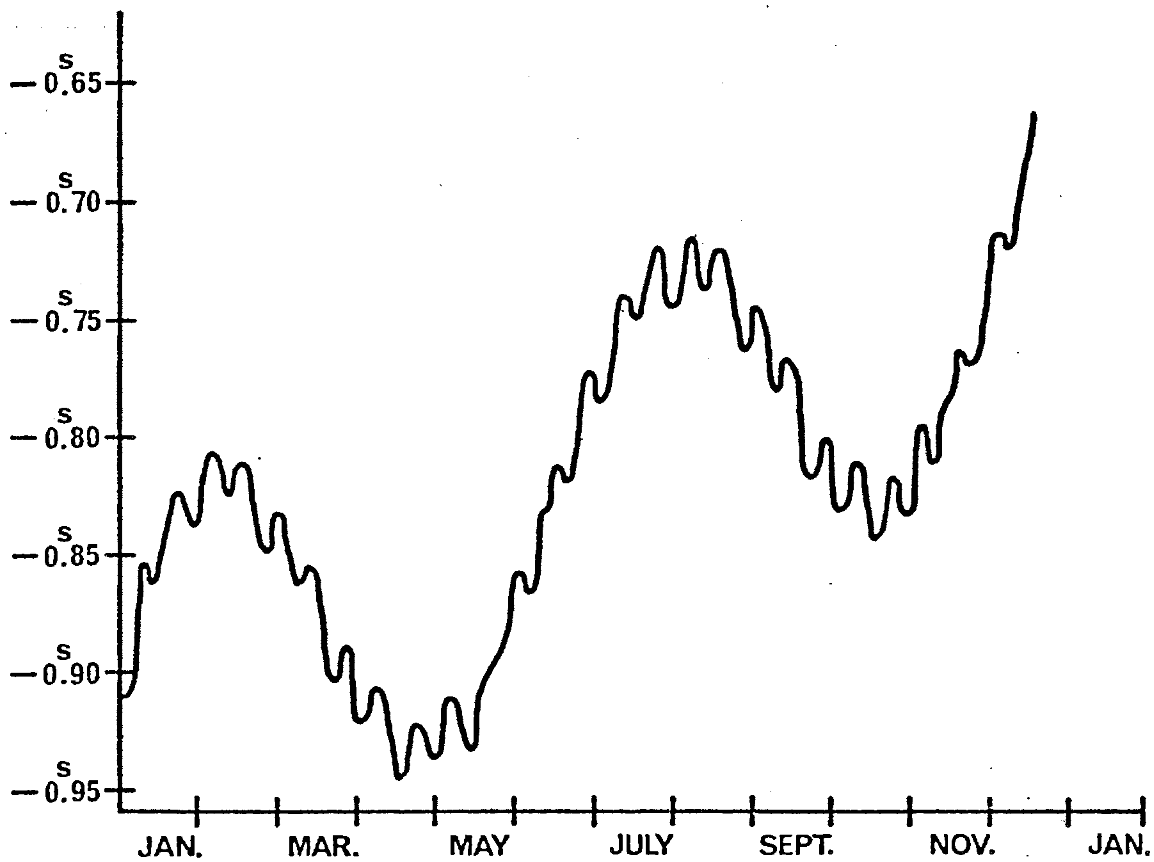


Figure 3-2

Equation of Equinoxes (0^hUT, 1966)
[Mueller, 1969]

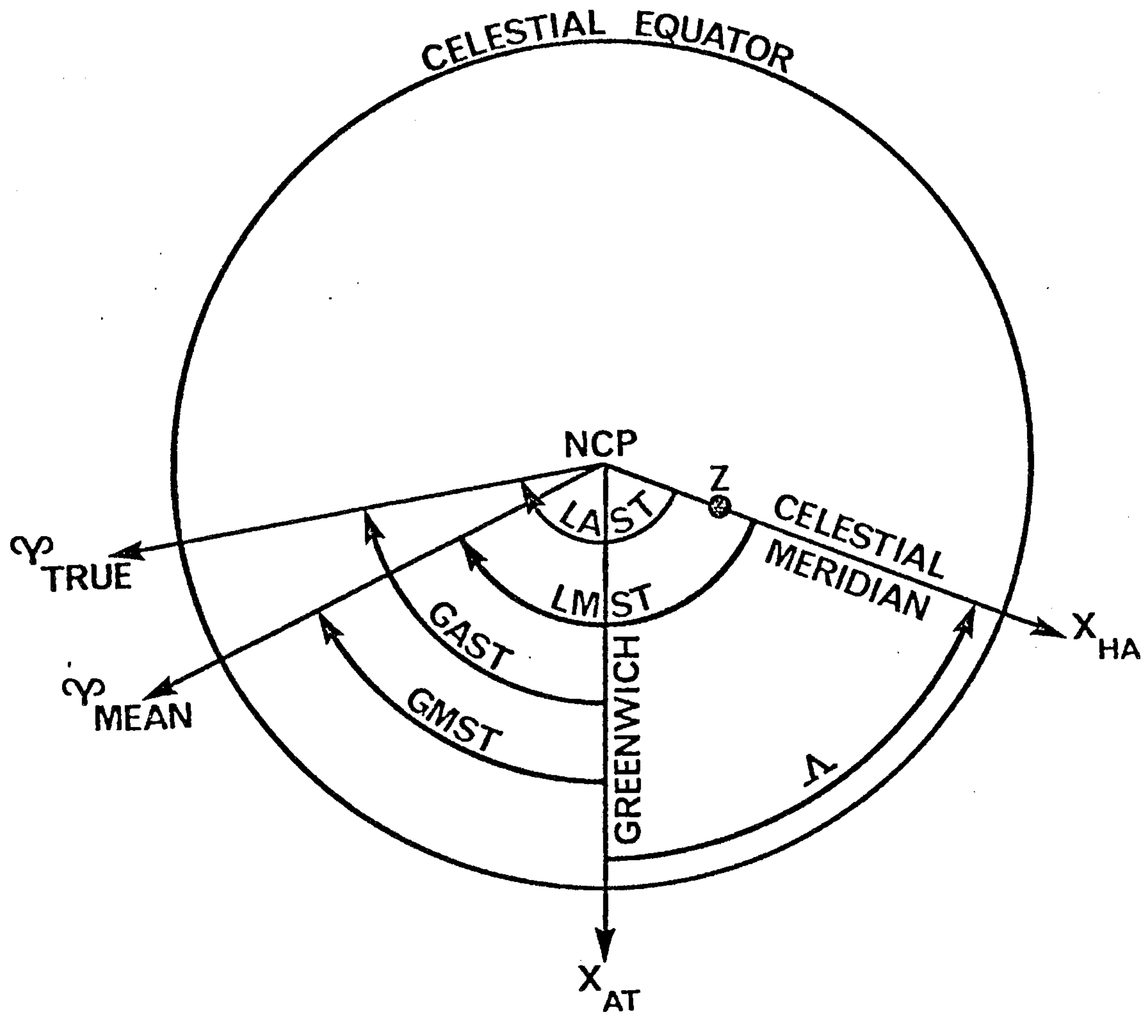


Figure 3-3

Sidereal Time and Longitude

For the purpose of tabulating certain quantities with arguements of sidereal time, the concept of Greenwich Sidereal Date (G.S.D.) and Greenwich Sidereal Day Number are used. The G.S.D. is the number of mean sidereal days that have elapsed on the Greenwich mean astronomic meridian since the beginning of the sidereal day that was in progress at Greenwich noon on Jan. 1, 4713 B.C. The integral part of the G.S.D. is the Greenwich Sidereal Day Number, and the fractional part is the GMST expressed as a fraction of a day. Figure 3-4, which is part of one of the tables from AA shows the G.S.D.

UNIVERSAL AND SIDEREAL TIMES, 1981

Date 0ʰU.T.	Julian Date	G. SIDEREAL TIME (G. H. A. of the Equinox) Apparent	Mean	Equation of Equinoxes at 0ʰU.T.	G.S.D. 0ʰ G.S.T.	U.T. at 0ʰ G.M.S.T. (Greenwich Transit of the Mean Equinox)	
	244	h m s	s	s	245		h m s
Jan. 0	4604.5	6 38 17.1886	17.9594	−0.7708	1299.0	Jan. 0	17 18 51.3829
1	4605.5	6 42 13.7431	14.5148	.7717	1300.0	1	17 14 55.4734
2	4606.5	6 46 10.2993	11.0702	.7708	1301.0	2	17 10 59.5639
3	4607.5	6 50 06.8575	07.6255	.7681	1302.0	3	17 07 03.6545
4	4608.5	6 54 03.4174	04.1809	.7635	1303.0	4	17 03 07.7450
5	4609.5	6 57 59.9787	60.7363	−0.7576	1304.0	5	16 59 11.8356
6	4610.5	7 01 56.5407	57.2916	.7510	1305.0	6	16 55 15.9261
7	4611.5	7 05 53.1023	53.8470	.7447	1306.0	7	16 51 20.0166
8	4612.5	7 09 49.6626	50.4024	.7398	1307.0	8	16 47 24.1072
9	4613.5	7 13 46.2207	46.9577	.7370	1308.0	9	16 43 28.1977
10	4614.5	7 17 42.7763	43.5131	−0.7368	1309.0	10	16 39 32.2882
11	4615.5	7 21 39.3297	40.0685	.7387	1310.0	11	16 35 36.3788
12	4616.5	7 25 35.8817	36.6238	.7421	1311.0	12	16 31 40.4693
13	4617.5	7 29 32.4335	33.1792	.7457	1312.0	13	16 27 44.5598
14	4618.5	7 33 28.9864	29.7346	.7481	1313.0	14	16 23 48.6504
15	4619.5	7 37 25.5415	26.2899	−0.7484	1314.0	15	16 19 52.7409
16	4620.5	7 41 22.0994	22.8453	.7459	1315.0	16	16 15 56.8314
17	4621.5	7 45 18.6598	19.4006	.7409	1316.0	17	16 12 00.9220
18	4622.5	7 49 15.2219	15.9560	.7341	1317.0	18	16 08 05.0125
19	4623.5	7 53 11.7843	12.5114	.7271	1318.0	19	16 04 09.1030
20	4624.5	7 57 08.3457	09.0667	−0.7210	1319.0	20	16 00 13.1936
21	4625.5	8 01 04.9050	05.6221	.7171	1320.0	21	15 56 17.2841
22	4626.5	8 05 01.4618	02.1775	.7157	1321.0	22	15 52 21.3746
23	4627.5	8 08 58.0160	58.7328	.7168	1322.0	23	15 48 25.4652
24	4628.5	8 12 54.5682	55.2882	.7199	1323.0	24	15 44 29.5557
25	4629.5	8 16 51.1192	51.8436	−0.7243	1324.0	25	15 40 33.6462
26	4630.5	8 20 47.6698	48.3989	.7291	1325.0	26	15 36 37.7368
27	4631.5	8 24 44.2208	44.9543	.7335	1326.0	27	15 32 41.8273
28	4632.5	8 28 40.7728	41.5097	.7368	1327.0	28	15 28 45.9178
29	4633.5	8 32 37.3264	38.0650	.7386	1328.0	29	15 24 50.0084
30	4634.5	8 36 33.8818	34.6204	−0.7386	1329.0	30	15 20 54.0989
31	4635.5	8 40 30.4389	31.1758	.7368	1330.0	31	15 16 58.1894
Feb. 1	4636.5	8 44 26.9975	27.7311	.7336	1331.0	Feb. 1	15 13 02.2800
2	4637.5	8 48 23.5571	24.2865	.7294	1332.0	2	15 09 06.3705
3	4638.5	8 52 20.1168	20.8418	.7251	1333.0	3	15 05 10.4610
4	4639.5	8 56 16.6755	17.3972	−0.7217	1334.0	4	15 01 14.5516
5	4640.5	9 00 13.2323	13.9526	.7203	1335.0	5	14 57 18.6421
6	4641.5	9 04 09.7865	10.5079	.7215	1336.0	6	14 53 22.7326
7	4642.5	9 08 06.3381	07.0633	.7252	1337.0	7	14 49 26.8232
8	4643.5	9 12 02.8879	03.6187	.7307	1338.0	8	14 45 30.9137
9	4644.5	9 15 59.4372	60.1740	−0.7369	1339.0	9	14 41 35.0042
10	4645.5	9 19 55.9872	56.7294	.7422	1340.0	10	14 37 39.0948
11	4646.5	9 23 52.5392	53.2848	.7455	1341.0	11	14 33 43.1853
12	4647.5	9 27 49.0938	49.8401	.7463	1342.0	12	14 29 47.2758
13	4648.5	9 31 45.6509	46.3955	.7445	1343.0	13	14 25 51.3664
14	4649.5	9 35 42.2098	42.9509	−0.7411	1344.0	14	14 21 55.4569
15	4650.5	9 39 38.7693	39.5062	−0.7369	1345.0	15	14 17 59.5474

Figure 3-4. [AA, 1981*]
(*This and other dates in figure titles refer to year of application, not year of publication).

3.2 Universal (Solar) Time

The fundamental measure of the universal time interval is the mean solar day defined as the interval between two consecutive transits of a mean (fictitious) sun over a meridian. The mean sun is used in place of the true sun since one of our prerequisites for a time system is the uniformity of the associated physical phenomena. The motion of the true sun is non-uniform due to the varying velocity of the earth in its elliptical orbit about the sun and hence is not used as the physical basis for a precise timekeeping system. The mean sun is characterized by uniform sidereal motion along the equator. The right ascension (α_m) of the mean sun, which characterizes the solar motion through which mean solar time is determined, has been given by Simon Newcomb as [Mueller, 1969]

$$\alpha_m = 18^h\ 38^m\ 45^s\!.836 + 8\ 640\ 184^s\!.542\ t_m + 0^s\!.0929\ t_m^{\ 2} + \ldots \tag{3-4}$$

in which t_m is elapsed time in Julian centuries of 36525 mean solar days which have elapsed since the standard epoch of UT of 1900 January 0.5 UT.

Solar time is related to the apparent diurnal motion of the sun as seen by an observer on the earth. This motion is due in part to our motion in orbit about the sun and in part to the rotation of the earth about its polar axis. The epoch of apparent (true) solar time for any meridian is (Figure 3-5)

$$TT = h_s + 12^h, \tag{3-5}$$

in which h_s is the hour angle of the true sun. The 12^h is added for convenience so that 0^h TT occurs at night (lower transit) to conform with civil timekeeping practice.

The epoch of mean solar time for any meridian is (Figure 3-5)

$$MT = h_m + 12^h, \tag{3-6}$$

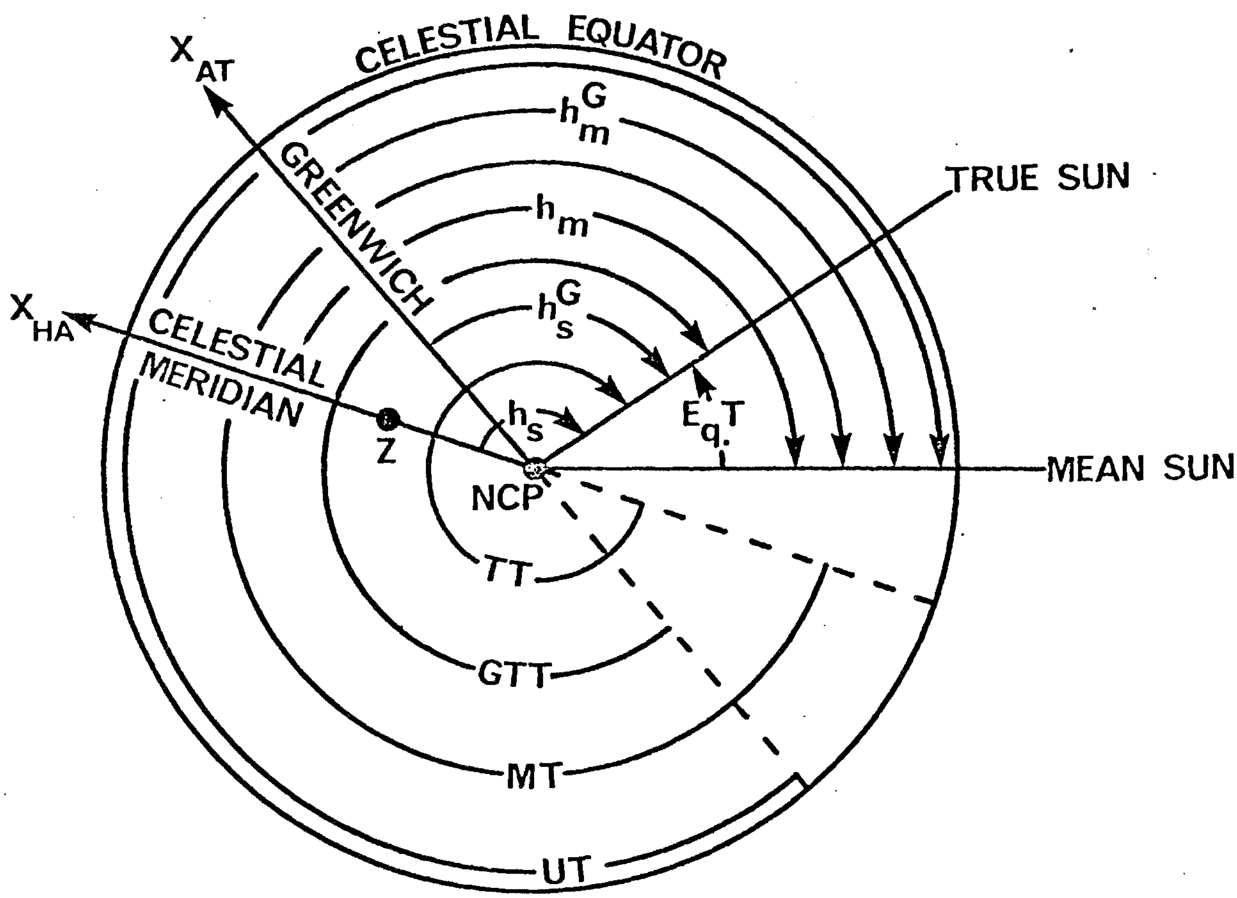


Figure 3-5

Universal (Solar) Time

in which h_m is the hour angle of the mean sun. If the hour angles of the true and mean suns are referred to Greenwich (h_s^G , h_m^G), the times are Greenwich True Time (GTT) and Greenwich Mean Time (GMT) or Universal Time (UT) respectively (Figure 3-5).

The difference between true and mean solar times at a given instant is termed the Equation of Time (Eq.T.). From Figure 3-5

$$\text{Eq.T.} = \text{TT-MT} \qquad (3\text{-}7)$$

and

$$\text{Eq.T.} = \text{GTT} - \text{UT}. \qquad (3\text{-}8)$$

The equation of time reaches absolute values of up to 16^m. Figure 3-6 shows Eq.T. for a one year interval.

Mean and true solar times are related to the astronomic longitude of an observer by (Figure 3-7)

$$\text{MT} = \text{UT} + \Lambda \quad , \qquad (3\text{-}9)$$

and

$$\text{TT} = \text{GTT} + \Lambda \ . \qquad (3\text{-}10)$$

The time required by the mean sun to make two consecutive passages over the mean vernal equinox is the tropical year. The time required by the mean sun to make a complete circuit of the equator is termed the sidereal year. These are given by

1 tropical year = 365.242 198 79 mean solar days,

1 sidereal year = 365.256 360 42 mean solar days.

Since neither of the above years contain and intergral number of mean solar days, civil calendars use either a Julian year in which

1 Juliam year = $365^d.25$ mean solar days = 365^d6^h,

or the presently used Gregorian calendar which has 365.2425 mean solar days. For astronomic purposes, the astronomic year begins at 0^h UT on 31 December of the previous calendar year. This epoch is denoted by the astronomic date January $0^d.0$ UT.

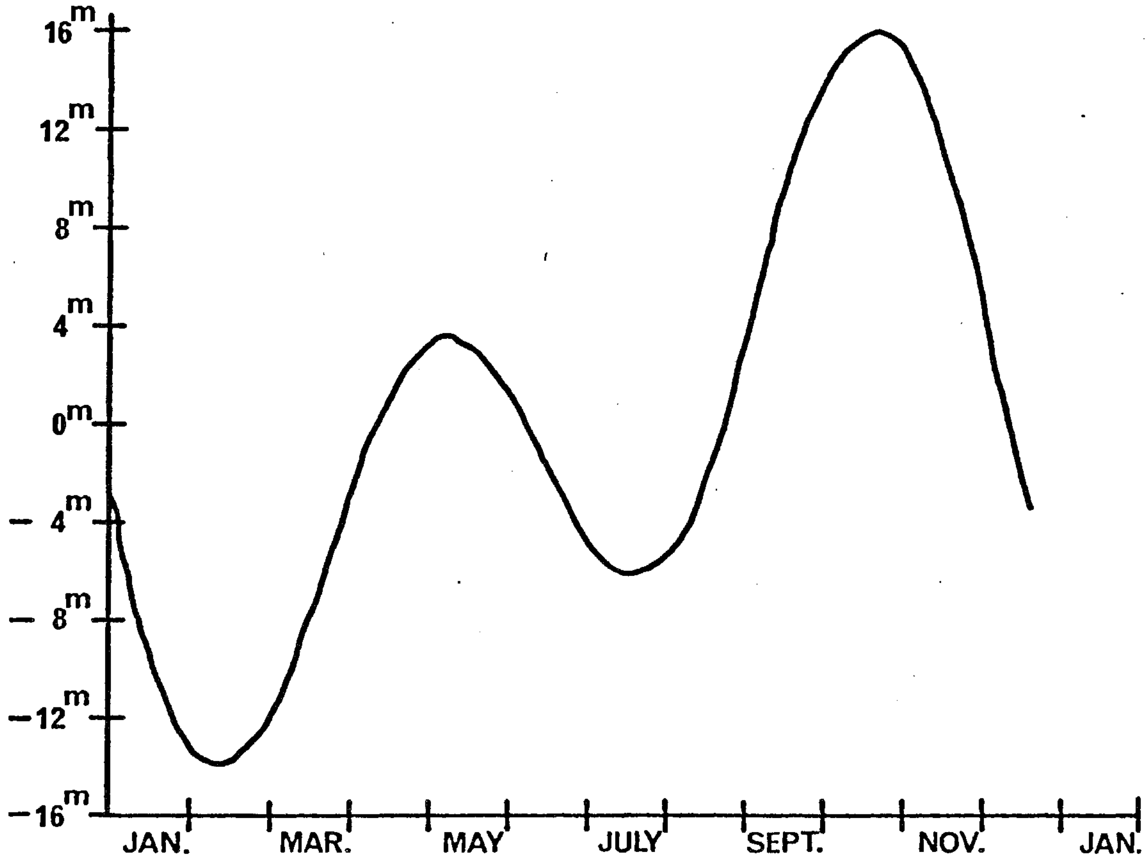


Figure 3-6

Equation of Time (0^h UT, 1966)
[Mueller, 1969]

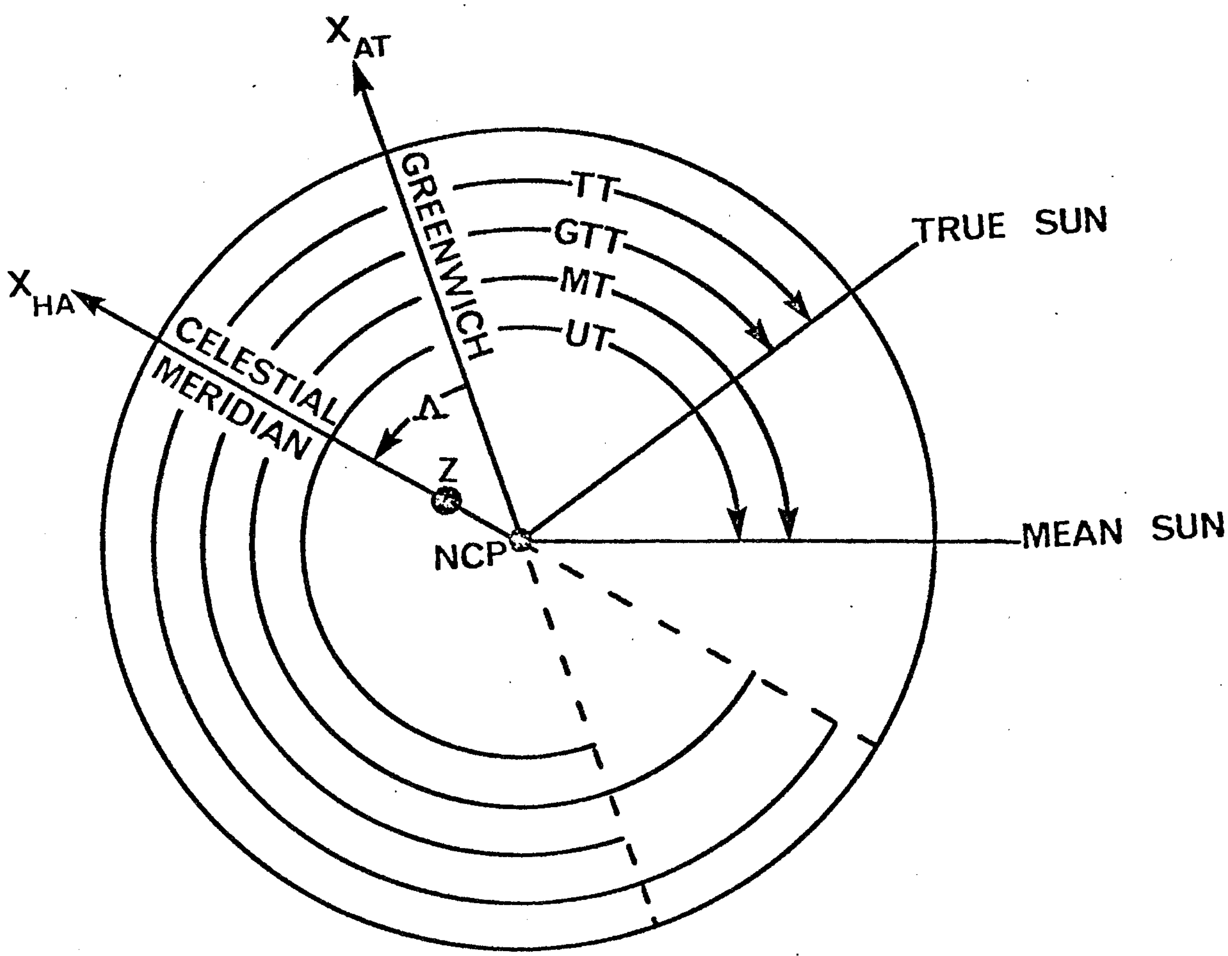


Figure 3-7

Solar Time and Longitude

Corresponding to the Greenwich Sidereal Date (GSD) is the Julian Date (JD). The JD is the number of mean solar days that have elapsed since 12^h UT on January 1 (January $1^d.5$ UT) 4713 BC. For the standard astronomic epoch of 1900 January $0^d.5$ UT, JD = 2 415 020.0. The conversions between GSD and JD are

$$GSD = 0.671 + 1.0027379093\ JD \ , \qquad (3-11)$$

$$JD = -0.669 + 0.9972695664\ GSD \ . \qquad (3-12)$$

3.2.1 Standard (Zone) Time

To avoid the confusion of everyone keeping the mean time of their meridian, civil time is based on a "zone" concept. The standard time over a particular region of longitude corresponds to the mean time of a particular meridian. In general, the world is divided into 24 zones of 15° ($\Delta\Lambda$) each. Zone 0 has the Greenwich meridian as its "standard" meridian, and the zone extends $7\frac{1}{2}^\circ$ on either side of Greenwich. The time zones are numbered -1, -2, ..., -12 east, and +1, +2, ..., +12 west. Note that zone 12 is divided into two parts, $7\frac{1}{2}^\circ$ in extent each, on either side of the International Date Line (180°E). All of this is portrayed in Figure 3-8.

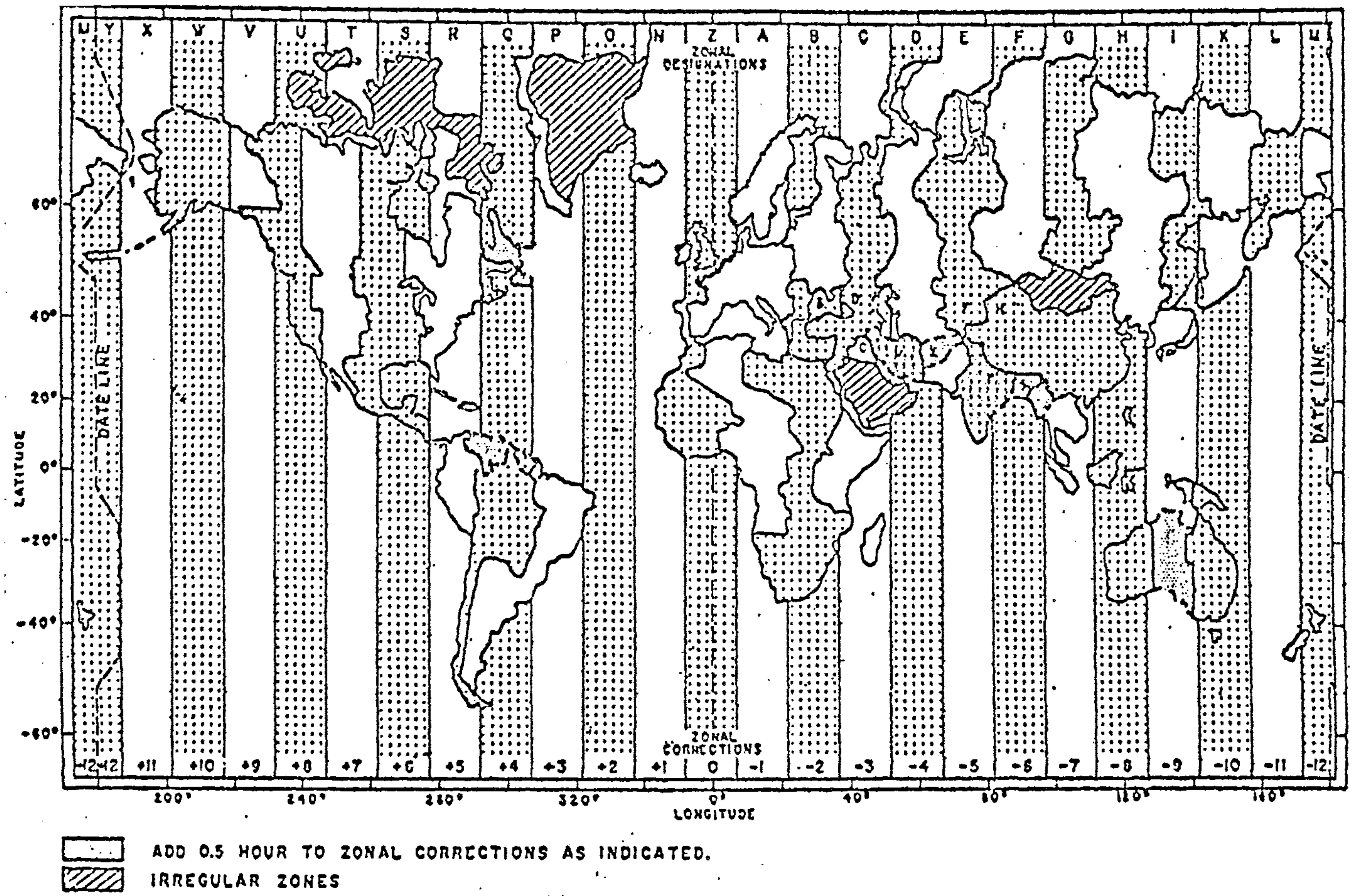
Universal time is related to Zone Time (ZT) by

$$UT = ZT + \Delta Z \ , \qquad (3-13)$$

in which ΔZ is the zonal correction. Care should be taken with ΔZ, particularly in regions where summer time or daylight saving time is used during the spring-summer-fall parts of the year. The effect is a 1^h advance of regular ZT.

3.3 Relationships Between Sidereal and Solar Time Epochs and Intervals

We will deal first with the relationship between time epochs.



STANDARD (ZONE) TIME [Mueller, 1969].

FIGURE 3-8

Note: Since the preparation of this map, the standard time zones in Canada have been changed so that all parts of the Yukon Territory now observe Pacific Standard Time (Zone designation U or +8). See Appendix II.

Equation (3-6) states

$$MT = 12^h + h_m \quad ,$$

where h_m is the hour angle of the mean sun while from Figure 3-9

$$h_m = LMST - \alpha_m , \tag{3-14}$$

which yields

$$MT = LMST - (\alpha_m - 12^h), \tag{3-15}$$

or

$$LMST = MT + (\alpha_m - 12^h) \quad . \tag{3-16}$$

Equations (3-15) and (3-16) represent the transformations of LMST to MT and vice-versa respectively. If we replace MT, h_m, and LMST with GMT, h_m^G , and GMST respectively, then

$$UT = GMST - (\alpha_m - 12^h) \ , \tag{3-17}$$

and

$$GMST = UT + (\alpha_m - 12^h) \quad . \tag{3-18}$$

For practical computations we used tabulated quantities. For example $(\alpha_m - 12^h)$ is tabulated in the Astronomical Almanac (AA); and the quantity $(\alpha_m - 12^h + Eq.\ E)$ is tabulated in the Star Almanac for Land Surveyors (SALS). Thus for simplicity we should always convert MT to UT and LMST to GMST. We should note, of course, that the relationships shown (3-15), (3-16), (3-17), (3-18) relate mean solar and mean sidereal times. To relate true solar time with apparent sidereal time:

(i) compute mean solar time (MT = TT - Eq.T or UT = GTT - Eq.T.),

(ii) compute mean sidereal time ((3-16) or (3-18)),

(iii) compute apparent sidereal time (LAST = Eq.E + LMST or GAST = Eq.E. + GMST).

To relate apparent sidereal time with true solar time, the inverse procedure is used. For illustrative purposes, two numerical examples are used.

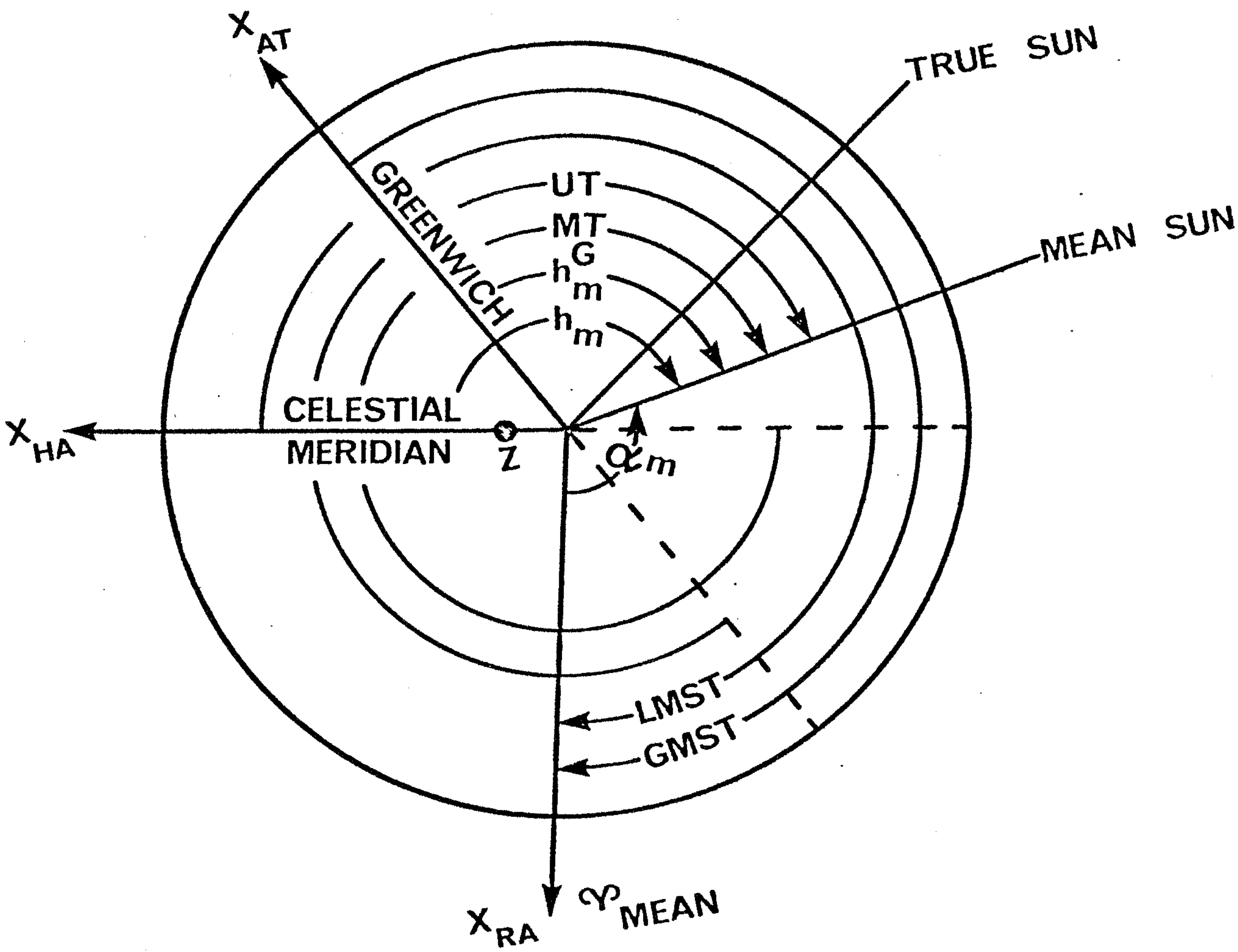


Figure 3-9

Relationships Between Sidereal and Universal Time Epochs (note that the appropriate tabulated values can be found in Figures 3-10, 3-11, 3-12, and 3-13).

Example #1 (using SALS)

Given: MT = $18^h\ 21^m\ 41^s.00$

Feb. 14, 1981

$\Lambda = 66^\circ\ 38'\ 28''$ W

Compute: LAST

MT	$18^h\ 21^m\ 41^s.00$
Λ ($-66^\circ\ 38'\ 28''$)	$-4^h\ 26^m\ 33^s.87$
UT (=MT − Λ)	$22^h\ 48^m\ 14^s.87$
R($\alpha_m - 12^h$ + Eq.E.) at 18^h UT: $9^h\ 38^m\ 39^s.6$	
ΔR for $4^h\ 48^m\ 14^s.87$: $47^s.4$	
($\alpha_m - 12^h$ + Eq.E.)	$9^h\ 39^m\ 27^s.0$
GAST (=UT + ($\alpha_m - 12^h$) + Eq.E)	$32^h\ 27^m\ 41^s.87$
	-24^h
GAST	$8^h\ 27^m\ 41^s.87$
Λ ($-66^\circ\ 38'\ 28''$)	$-4^h\ 26^m\ 33^s.87$
LAST (GAST + Λ)	$4^h\ 01^m\ 8^s.00$

Example #2 (using AA)

Given: MT = $18^h\ 21^m\ 41\overset{s}{.}00$

Feb. 14, 1981

Λ = 66° 38' 28"W

Compute: LAST

MT	$18^h\ 21^m\ 41\overset{s}{.}00$
Λ (-66° 38' 28")	$-4^h\ 26^m\ 33\overset{s}{.}87$
UT (=MT - Λ)	$22^h\ 48^m\ 14\overset{s}{.}87$
$(\alpha_m - 12^h)$ (GMST) at 0^h UT:	$9^h\ 35^m\ 42\overset{s}{.}95$

Add

Sidereal interval equivalent to UT = $22^h\ 48^m\ 14\overset{s}{.}87$ (multiply UT interval by 1.002 737 9093):	$22^h\ 51^m\ 59\overset{s}{.}64$
GMST at $22^h\ 48^m\ 14\overset{s}{.}87$ UT	$32^h\ 27^m\ 42\overset{s}{.}59$
	$-\ 24^h$
GMST	$8^h\ 27^m\ 42\overset{s}{.}59$
Eq. E	$-\ 0\overset{s}{.}74$
GAST (= GMST + Eq. E)	$8^h\ 27^m\ 41\overset{s}{.}85$
Λ (-66° 38' 28")	$-4^h\ 26^m\ 33\overset{s}{.}87$
LAST (= GAST + Λ)	$4^h\ 01^m\ 07\overset{s}{.}98$

SUN—FEBRUARY, 1981

U.T. d h	R h m s	Dec. ° ′	Dec. diff.	E h m s	E diff.
1 0	8 44 27·0	S 17 10·9	42	11 46 25·8	21
Sun. 6	45 26·1	17 06·7	43	46 23·7	20
12	46 25·3	17 02·4	43	46 21·7	20
18	47 24·4	16 58·1	43	46 19·7	19
2 0	8 48 23·6	S 16 53·8	43	11 46 17·8	19
Mon. 6	49 22·7	16 49·5	44	46 15·9	18
12	50 21·8	16 45·1	43	46 14·1	17
18	51 21·0	16 40·8	44	46 12·4	18
3 0	8 52 20·1	S 16 36·4	44	11 46 10·6	16
Tues. 6	53 19·3	16 32·0	45	46 09·0	16
12	54 18·4	16 27·5	44	46 07·4	16
18	55 17·5	16 23·1	44	46 05·8	15
4 0	8 56 16·7	S 16 18·7	45	11 46 04·3	15
Wed. 6	57 15·8	16 14·2	45	46 02·8	14
12	58 15·0	16 09·7	45	46 01·4	13
18	8 59 14·1	16 05·2	45	46 00·1	13
5 0	9 00 13·2	S 16 00·7	46	11 45 58·8	13
Thur. 6	01 12·4	15 56·1	46	45 57·5	12
12	02 11·5	15 51·5	45	45 56·3	12
18	03 10·6	15 47·0	46	45 55·1	11
6 0	9 04 09·8	S 15 42·4	46	11 45 54·0	10
Fri. 6	05 08·9	15 37·8	47	45 53·0	10
12	06 08·1	15 33·1	46	45 52·0	10
18	07 07·2	15 28·5	47	45 51·0	9
7 0	9 08 06·3	S 15 23·8	47	11 45 50·1	8
Sat. 6	09 05·5	15 19·1	46	45 49·3	8
12	10 04·6	15 14·5	48	45 48·5	8
18	11 03·8	15 09·7	47	45 47·7	7
8 0	9 12 02·9	S 15 05·0	47	11 45 47·0	6
Sun. 6	13 02·0	15 00·3	48	45 46·4	6
12	14 01·2	14 55·5	48	45 45·8	6
18	15 00·3	14 50·7	47	45 45·2	5
24	9 15 59·4	S 14 46·0		11 45 44·7	

U.T. d h	R h m s	Dec. ° ′	Dec. diff.	E h m s	E diff.
9 0	9 15 59·4	S 14 46·0	48	11 45 44·7	4
Mon. 6	16 58·6	14 41·2	49	45 44·3	4
12	17 57·7	14 36·3	48	45 43·9	4
18	18 56·8	14 31·5	49	45 43·5	3
10 0	9 19 56·0	S 14 26·6	48	11 45 43·2	2
Tues. 6	20 55·1	14 21·8	49	45 43·0	2
12	21 54·3	14 16·9	49	45 42·8	2
18	22 53·4	14 12·0	49	45 42·6	1
11 0	9 23 52·5	S 14 07·1	49	11 45 42·5	0
Wed. 6	24 51·7	14 02·2	50	45 42·5	0
12	25 50·8	13 57·2	49	45 42·5	0
18	26 50·0	13 52·3	50	45 42·5	1
12 0	9 27 49·1	S 13 47·3	50	11 45 42·6	2
Thur. 6	28 48·2	13 42·3	50	45 42·8	2
12	29 47·4	13 37·3	50	45 43·0	2
18	30 46·5	13 32·3	50	45 43·2	3
13 0	9 31 45·7	S 13 27·3	50	11 45 43·5	3
Fri. 6	32 44·8	13 22·3	51	45 43·8	4
12	33 43·9	13 17·2	50	45 44·2	5
18	34 43·1	13 12·2	51	45 44·7	4
14 0	9 35 42·2	S 13 07·1	51	11 45 45·1	6
Sat. 6	36 41·3	13 02·0	51	45 45·7	6
12	37 40·5	12 56·9	51	45 46·3	6
18	38 39·6	12 51·8	52	45 46·9	7
15 0	9 39 38·8	S 12 46·6	51	11 45 47·6	7
Sun. 6	40 37·9	12 41·5	52	45 48·3	7
12	41 37·0	12 36·3	51	45 49·0	8
18	42 36·2	12 31·2	52	45 49·8	9
16 0	9 43 35·3	S 12 26·0	52	11 45 50·7	9
Mon. 6	44 34·5	12 20·8	52	45 51·6	9
12	45 33·6	12 15·6	52	45 52·5	10
18	46 32·7	12 10·4	52	45 53·5	11
24	9 47 31·9	S 12 05·2		11 45 54·6	

Sun's S.D. 16′·2

SUNRISE

Date	South Latitude									North Latitude							
	60°	55°	50°	45°	40°	30°	20°	10°	0°	10°	20°	30°	40°	45°	50°	55°	60°
Feb.	h	h	h	h	h	h	h	h	h	h	h	h	h	h	h	h	h
1	3·9	4·4	4·7	4·9	5·2	5·5	5·7	6·0	6·2	6·4	6·6	6·8	7·2	7·3	7·6	7·9	8·2
6	4·2	4·5	4·8	5·1	5·2	5·5	5·8	6·0	6·2	6·4	6·6	6·8	7·1	7·2	7·5	7·7	8·1
11	4·4	4·7	5·0	5·2	5·3	5·6	5·8	6·0	6·2	6·3	6·5	6·7	7·0	7·1	7·3	7·5	7·8
16	4·6	4·9	5·1	5·3	5·5	5·7	5·9	6·0	6·2	6·3	6·5	6·7	6·9	7·0	7·2	7·3	7·6
21	4·8	5·1	5·3	5·4	5·5	5·7	5·9	6·0	6·2	6·3	6·4	6·6	6·8	6·9	7·0	7·2	7·4
26	5·0	5·3	5·4	5·5	5·7	5·8	5·9	6·0	6·2	6·3	6·4	6·5	6·7	6·7	6·8	7·0	7·1
31	5·3	5·4	5·5	5·7	5·7	5·9	6·0	6·1	6·2	6·2	6·3	6·4	6·5	6·6	6·7	6·8	6·9

Moon's phases: new moon, $4^d22^h14^m$; first quarter, $11^d17^h49^m$.

Figure 3-10 [SALS, 1981]

INTERPOLATION TABLE FOR *R*

MUTUAL CONVERSION OF INTERVALS OF SOLAR AND SIDEREAL TIME

solar 4^h	ΔR	sidereal 4^h	solar 4^h	ΔR	sidereal 4^h	solar 5^h	ΔR	sidereal 5^h	solar 5^h	ΔR	sidereal 5^h
m s	s	m s	m s	s	m s	m s	s	m s	m s	s	m s
00 08.8	39.5	00 48.2	30 35.0	44.5	31 19.4	00 24.7	49.4	01 14.0	30 14.3	54.3	31 08.6
00 45.3	39.6	01 24.8	31 11.5	44.6	31 56.0	01 01.2	49.5	01 50.6	30 50.9	54.4	31 45.2
01 21.8	39.7	02 01.5	31 48.0	44.7	32 32.7	01 37.7	49.6	02 27.3	31 27.4	54.5	32 21.8
01 58.3	39.8	02 38.1	32 24.5	44.8	33 09.3	02 14.2	49.7	03 03.9	32 03.9	54.6	32 58.5
02 34.9	39.9	03 14.7	33 01.1	44.9	33 45.9	02 50.7	49.8	03 40.5	32 40.4	54.7	33 35.1
03 11.4	40.0	03 51.3	33 37.6	45.0	34 22.5	03 27.3	49.9	04 17.1	33 17.0	54.8	34 11.7
03 47.9	40.1	04 28.0	34 14.1	45.1	34 59.2	04 03.8	50.0	04 53.7	33 53.5	54.9	34 48.3
04 24.4	40.2	05 04.6	34 50.6	45.2	35 35.8	04 40.3	50.1	05 30.4	34 30.0	55.0	35 25.0
05 00.9	40.3	05 41.2	35 27.2	45.3	36 12.4	05 16.8	50.2	06 07.0	35 06.5	55.1	36 01.6
05 37.5	40.4	06 17.8	36 03.7	45.4	36 49.0	05 53.4	50.3	06 43.6	35 43.1	55.2	36 38.2
06 14.0	40.5	06 54.4	36 40.2	45.5	37 25.7	06 29.9	50.4	07 20.2	36 19.6	55.3	37 14.8
06 50.5	40.6	07 31.1	37 16.7	45.6	38 02.3	07 06.4	50.5	07 56.9	36 56.1	55.4	37 51.5
07 27.0	40.7	08 07.7	37 53.3	45.7	38 38.9	07 42.9	50.6	08 33.5	37 32.6	55.5	38 28.1
08 03.6	40.8	08 44.3	38 29.8	45.8	39 15.5	08 19.5	50.7	09 10.1	38 09.2	55.6	39 04.7
08 40.1	40.9	09 20.9	39 06.3	45.9	39 52.2	08 56.0	50.8	09 46.7	38 45.7	55.7	39 41.3
09 16.6	41.0	09 57.6	39 42.8	46.0	40 28.8	09 32.5	50.9	10 23.4	39 22.2	55.8	40 18.0
09 53.1	41.1	10 34.2	40 19.4	46.1	41 05.4	10 09.0	51.0	11 00.0	39 58.7	55.9	40 54.6
10 29.7	41.2	11 10.8	40 55.9	46.2	41 42.0	10 45.6	51.1	11 36.6	40 35.3	56.0	41 31.2
11 06.2	41.3	11 47.4	41 32.4	46.3	42 18.7	11 22.1	51.2	12 13.2	41 11.8	56.1	42 07.8
11 42.7	41.4	12 24.1	42 08.9	46.4	42 55.3	11 58.6	51.3	12 49.9	41 48.3	56.2	42 44.4
12 19.2	41.5	13 00.7	42 45.4	46.5	43 31.9	12 35.1	51.4	13 26.5	42 24.8	56.3	43 21.1
12 55.8	41.6	13 37.3	43 22.0	46.6	44 08.5	13 11.7	51.5	14 03.1	43 01.3	56.4	43 57.7
13 32.3	41.7	14 13.9	43 58.5	46.7	44 45.1	13 48.2	51.6	14 39.7	43 37.9	56.5	44 34.3
14 08.8	41.8	14 50.6	44 35.0	46.8	45 21.8	14 24.7	51.7	15 16.4	44 14.4	56.6	45 10.9
14 45.3	41.9	15 27.2	45 11.5	46.9	45 58.4	15 01.2	51.8	15 53.0	44 50.9	56.7	45 47.6
15 21.9	42.0	16 03.8	45 48.1	47.0	46 35.0	15 37.8	51.9	16 29.6	45 27.4	56.8	46 24.2
15 58.4	42.1	16 40.4	46 24.6	47.1	47 11.6	16 14.3	52.0	17 06.2	46 04.0	56.9	47 00.8
16 34.9	42.2	17 17.1	47 01.1	47.2	47 48.3	16 50.8	52.1	17 42.9	46 40.5	57.0	47 37.4
17 11.4	42.3	17 53.7	47 37.6	47.3	48 24.9	17 27.3	52.2	18 19.5	47 17.0	57.1	48 14.1
17 48.0	42.4	18 30.3	48 14.2	47.4	49 01.5	18 03.9	52.3	18 56.1	47 53.5	57.2	48 50.7
18 24.5	42.5	19 06.9	48 50.7	47.5	49 38.1	18 40.4	52.4	19 32.7	48 30.1	57.3	49 27.3
19 01.0	42.6	19 43.6	49 27.2	47.6	50 14.8	19 16.9	52.5	20 09.4	49 06.6	57.4	50 03.9
19 37.5	42.7	20 20.2	50 03.7	47.7	50 51.4	19 53.4	52.6	20 46.0	49 43.1	57.5	50 40.6
20 14.1	42.8	20 56.8	50 40.3	47.8	51 28.0	20 30.0	52.7	21 22.6	50 19.6	57.6	51 17.2
20 50.6	42.9	21 33.4	51 16.8	47.9	52 04.6	21 06.5	52.8	21 59.2	50 56.2	57.7	51 53.8
21 27.1	43.0	22 10.1	51 53.3	48.0	52 41.3	21 43.0	52.9	22 35.9	51 32.7	57.8	52 30.4
22 03.6	43.1	22 46.7	52 29.8	48.1	53 17.9	22 19.5	53.0	23 12.5	52 09.2	57.9	53 07.1
22 40.2	43.2	23 23.3	53 06.4	48.2	53 54.5	22 56.0	53.1	23 49.1	52 45.7	58.0	53 43.7
23 16.7	43.3	23 59.9	53 42.9	48.3	54 31.1	23 32.6	53.2	24 25.7	53 22.3	58.1	54 20.3
23 53.2	43.4	24 36.5	54 19.4	48.4	55 07.8	24 09.1	53.3	25 02.3	53 58.8	58.2	54 56.9
24 29.7	43.5	25 13.2	54 55.9	48.5	55 44.4	24 45.6	53.4	25 39.0	54 35.3	58.3	55 33.6
25 06.2	43.6	25 49.8	55 32.5	48.6	56 21.0	25 22.1	53.5	26 15.6	55 11.8	58.4	56 10.2
25 42.8	43.7	26 26.4	56 09.0	48.7	56 57.6	25 58.7	53.6	26 52.2	55 48.4	58.5	56 46.8
26 19.3	43.8	27 03.0	56 45.5	48.8	57 34.3	26 35.2	53.7	27 28.8	56 24.9	58.6	57 23.4
26 55.8	43.9	27 39.7	57 22.0	48.9	58 10.9	27 11.7	53.8	28 05.5	57 01.4	58.7	58 00.1
27 32.3	44.0	28 16.3	57 58.6	49.0	58 47.5	27 48.2	53.9	28 42.1	57 37.9	58.8	58 36.7
28 08.9	44.1	28 52.9	58 35.1	49.1	59 24.1	28 24.8	54.0	29 18.7	58 14.5	58.9	59 13.3
28 45.4	44.2	29 29.5	59 11.6	49.2	60 00.8	29 01.3	54.1	29 55.3	58 51.0	59.0	59 49.9
29 21.9	44.3	30 06.2	59 48.1	49.3	60 37.4	29 37.8	54.2	30 32.0	59 27.5	59.1	60 26.6
29 58.4	44.4	30 42.8	60 24.7		61 14.0	30 14.3		31 08.6	60 04.0		61 03.2
30 35.0		31 19.4									

In critical cases ascend.

Add ΔR to solar time interval (left-hand argument) to obtain sidereal time interval.
Subtract ΔR from sidereal time interval (right-hand argument) to obtain solar time interval.

FIGURE 3-11 [SALS, 1981]

UNIVERSAL AND SIDEREAL TIMES, 1981

Date 0ʰU.T.	Julian Date	G. SIDEREAL TIME (G. H. A. of the Equinox) Apparent	Mean	Equation of Equinoxes at 0ʰ U.T.	G.S.D. 0ʰ G.S.T.	U.T. at 0ʰ G.M.S.T. (Greenwich Transit of the Mean Equinox)
	244	h m s	s	s	245	h m s
Jan. 0	4604.5	6 38 17.1886	17.9594	−0.7708	1299.0	Jan. 0 17 18 51.3829
1	4605.5	6 42 13.7431	14.5148	.7717	1300.0	1 17 14 55.4734
2	4606.5	6 46 10.2993	11.0702	.7708	1301.0	2 17 10 59.5639
3	4607.5	6 50 06.8575	07.6255	.7681	1302.0	3 17 07 03.6545
4	4608.5	6 54 03.4174	04.1809	.7635	1303.0	4 17 03 07.7450
5	4609.5	6 57 59.9787	60.7363	−0.7576	1304.0	5 16 59 11.8356
6	4610.5	7 01 56.5407	57.2916	.7510	1305.0	6 16 55 15.9261
7	4611.5	7 05 53.1023	53.8470	.7447	1306.0	7 16 51 20.0166
8	4612.5	7 09 49.6626	50.4024	.7398	1307.0	8 16 47 24.1072
9	4613.5	7 13 46.2207	46.9577	.7370	1308.0	9 16 43 28.1977
10	4614.5	7 17 42.7763	43.5131	−0.7368	1309.0	10 16 39 32.2882
11	4615.5	7 21 39.3297	40.0685	.7387	1310.0	11 16 35 36.3788
12	4616.5	7 25 35.8817	36.6238	.7421	1311.0	12 16 31 40.4693
13	4617.5	7 29 32.4335	33.1792	.7457	1312.0	13 16 27 44.5598
14	4618.5	7 33 28.9864	29.7346	.7481	1313.0	14 16 23 48.6504
15	4619.5	7 37 25.5415	26.2899	−0.7484	1314.0	15 16 19 52.7409
16	4620.5	7 41 22.0994	22.8453	.7459	1315.0	16 16 15 56.8314
17	4621.5	7 45 18.6598	19.4006	.7409	1316.0	17 16 12 00.9220
18	4622.5	7 49 15.2219	15.9560	.7341	1317.0	18 16 08 05.0125
19	4623.5	7 53 11.7843	12.5114	.7271	1318.0	19 16 04 09.1030
20	4624.5	7 57 08.3457	09.0667	−0.7210	1319.0	20 16 00 13.1936
21	4625.5	8 01 04.9050	05.6221	.7171	1320.0	21 15 56 17.2841
22	4626.5	8 05 01.4618	02.1775	.7157	1321.0	22 15 52 21.3746
23	4627.5	8 08 58.0160	58.7328	.7168	1322.0	23 15 48 25.4652
24	4628.5	8 12 54.5682	55.2882	.7199	1323.0	24 15 44 29.5557
25	4629.5	8 16 51.1192	51.8436	−0.7243	1324.0	25 15 40 33.6462
26	4630.5	8 20 47.6698	48.3989	.7291	1325.0	26 15 36 37.7368
27	4631.5	8 24 44.2208	44.9543	.7335	1326.0	27 15 32 41.8273
28	4632.5	8 28 40.7728	41.5097	.7368	1327.0	28 15 28 45.9178
29	4633.5	8 32 37.3264	38.0650	.7386	1328.0	29 15 24 50.0084
30	4634.5	8 36 33.8818	34.6204	−0.7386	1329.0	30 15 20 54.0989
31	4635.5	8 40 30.4389	31.1758	.7368	1330.0	31 15 16 58.1894
Feb. 1	4636.5	8 44 26.9975	27.7311	.7336	1331.0	Feb. 1 15 13 02.2800
2	4637.5	8 48 23.5571	24.2865	.7294	1332.0	2 15 09 06.3705
3	4638.5	8 52 20.1168	20.8418	.7251	1333.0	3 15 05 10.4610
4	4639.5	8 56 16.6755	17.3972	−0.7217	1334.0	4 15 01 14.5516
5	4640.5	9 00 13.2323	13.9526	.7203	1335.0	5 14 57 18.6421
6	4641.5	9 04 09.7865	10.5079	.7215	1336.0	6 14 53 22.7326
7	4642.5	9 08 06.3381	07.0633	.7252	1337.0	7 14 49 26.8232
8	4643.5	9 12 02.8879	03.6187	.7307	1338.0	8 14 45 30.9137
9	4644.5	9 15 59.4372	60.1740	−0.7369	1339.0	9 14 41 35.0042
10	4645.5	9 19 55.9872	56.7294	.7422	1340.0	10 14 37 39.0948
11	4646.5	9 23 52.5392	53.2848	.7455	1341.0	11 14 33 43.1853
12	4647.5	9 27 49.0938	49.8401	.7463	1342.0	12 14 29 47.2758
13	4648.5	9 31 45.6509	46.3955	.7445	1343.0	13 14 25 51.3664
14	4649.5	9 35 42.2098	42.9509	−0.7411	1344.0	14 14 21 55.4569
15	4650.5	9 39 38.7693	39.5062	−0.7369	1345.0	15 14 17 59.5474

Figure 3-12a [AA, 1981]

UNIVERSAL AND SIDEREAL TIMES, 1981

Date 0ʰU.T.	Julian Date	G. SIDEREAL TIME (G. H. A. of the Equinox) Apparent	Mean	Equation of Equinoxes at 0ʰU.T.	G.S.D. 0ʰ G.S.T.	U.T. at 0ʰ G.M.S.T. (Greenwich Transit of the Mean Equinox)
	244	h m s	s	s	245	h m s
Feb. 15	4650.5	9 39 38.7693	39.5062	−0.7369	1345.0	Feb. 15 14 17 59.5474
16	4651.5	9 43 35.3282	36.0616	.7334	1346.0	16 14 14 03.6380
17	4652.5	9 47 31.8854	32.6169	.7315	1347.0	17 14 10 07.7285
18	4653.5	9 51 28.4403	29.1723	.7320	1348.0	18 14 06 11.8191
19	4654.5	9 55 24.9927	25.7277	.7350	1349.0	19 14 02 15.9096
20	4655.5	9 59 21.5429	22.2830	−0.7401	1350.0	20 13 58 20.0001
21	4656.5	10 03 18.0916	18.8384	.7468	1351.0	21 13 54 24.0907
22	4657.5	10 07 14.6396	15.3938	.7541	1352.0	22 13 50 28.1812
23	4658.5	10 11 11.1878	11.9491	.7613	1353.0	23 13 46 32.2717
24	4659.5	10 15 07.7369	08.5045	.7676	1354.0	24 13 42 36.3623
25	4660.5	10 19 04.2874	05.0599	−0.7725	1355.0	25 13 38 40.4528
26	4661.5	10 23 00.8396	01.6152	.7756	1356.0	26 13 34 44.5433
27	4662.5	10 26 57.3936	58.1706	.7770	1357.0	27 13 30 48.6339
28	4663.5	10 30 53.9492	54.7260	.7768	1358.0	28 13 26 52.7244
Mar. 1	4664.5	10 34 50.5059	51.2813	.7754	1359.0	Mar. 1 13 22 56.8149
2	4665.5	10 38 47.0631	47.8367	−0.7735	1360.0	2 13 19 00.9055
3	4666.5	10 42 43.6199	44.3921	.7721	1361.0	3 13 15 04.9960
4	4667.5	10 46 40.1753	40.9474	.7721	1362.0	4 13 11 09.0865
5	4668.5	10 50 36.7284	37.5028	.7743	1363.0	5 13 07 13.1771
6	4669.5	10 54 33.2790	34.0581	.7792	1364.0	6 13 03 17.2676
7	4670.5	10 58 29.8272	30.6135	−0.7863	1365.0	7 12 59 21.3581
8	4671.5	11 02 26.3744	27.1689	.7945	1366.0	8 12 55 25.4487
9	4672.5	11 06 22.9220	23.7242	.8022	1367.0	9 12 51 29.5392
10	4673.5	11 10 19.4715	20.2796	.8081	1368.0	10 12 47 33.6297
11	4674.5	11 14 16.0238	16.8350	.8112	1369.0	11 12 43 37.7203
12	4675.5	11 18 12.5788	13.3903	−0.8115	1370.0	12 12 39 41.8108
13	4676.5	11 22 09.1358	09.9457	.8099	1371.0	13 12 35 45.9013
14	4677.5	11 26 05.6937	06.5011	.8074	1372.0	14 12 31 49.9919
15	4678.5	11 30 02.2512	03.0564	.8053	1373.0	15 12 27 54.0824
16	4679.5	11 33 58.8072	59.6118	.8046	1374.0	16 12 23 58.1729
17	4680.5	11 37 55.3611	56.1672	−0.8060	1375.0	17 12 20 02.2635
18	4681.5	11 41 51.9127	52.7225	.8098	1376.0	18 12 16 06.3540
19	4682.5	11 45 48.4621	49.2779	.8158	1377.0	19 12 12 10.4445
20	4683.5	11 49 45.0099	45.8333	.8234	1378.0	20 12 08 14.5351
21	4684.5	11 53 41.5568	42.3886	.8318	1379.0	21 12 04 18.6256
22	4685.5	11 57 38.1038	38.9440	−0.8402	1380.0	22 12 00 22.7161
23	4686.5	12 01 34.6515	35.4993	.8478	1381.0	23 11 56 26.8067
24	4687.5	12 05 31.2006	32.0547	.8541	1382.0	24 11 52 30.8972
25	4688.5	12 09 27.7515	28.6101	.8586	1383.0	25 11 48 34.9877
26	4689.5	12 13 24.3042	25.1654	.8613	1384.0	26 11 44 39.0783
27	4690.5	12 17 20.8585	21.7208	−0.8623	1385.0	27 11 40 43.1688
28	4691.5	12 21 17.4142	18.2762	.8619	1386.0	28 11 36 47.2593
29	4692.5	12 25 13.9707	14.8315	.8609	1387.0	29 11 32 51.3499
30	4693.5	12 29 10.5270	11.3869	.8599	1388.0	30 11 28 55.4404
31	4694.5	12 33 07.0825	07.9423	.8597	1389.0	31 11 24 59.5309
Apr. 1	4695.5	12 37 03.6363	04.4976	−0.8614	1390.0	Apr. 1 11 21 03.6215
2	4696.5	12 41 00.1877	01.0530	−0.8653	1391.0	2 11 17 07.7120

Figure 3-12b [AA, 1981]

UNIVERSAL AND SIDEREAL TIMES, 1981

Date 0ʰU.T.	Julian Date	G. SIDEREAL TIME (G. H. A. of the Equinox) Apparent	Mean	Equation of Equinoxes at 0ʰ U.T.	G.S.D. 0ʰ G.S.T.	U.T. at 0ʰ G.M.S.T. (Greenwich Transit of the Mean Equinox)
	244	h m s	s	s	245	h m s
Apr. 1	4695.5	12 37 03.6363	04.4976	−0.8614	1390.0	Apr. 1 11 21 03.6215
2	4696.5	12 41 00.1877	01.0530	.8653	1391.0	2 11 17 07.7120
3	4697.5	12 44 56.7368	57.6084	.8716	1392.0	3 11 13 11.8026
4	4698.5	12 48 53.2843	54.1637	.8795	1393.0	4 11 09 15.8931
5	4699.5	12 52 49.8316	50.7191	.8875	1394.0	5 11 05 19.9836
6	4700.5	12 56 46.3804	47.2745	−0.8941	1395.0	6 11 01 24.0742
7	4701.5	13 00 42.9320	43.8298	.8978	1396.0	7 10 57 28.1647
8	4702.5	13 04 39.4869	40.3852	.8983	1397.0	8 10 53 32.2552
9	4703.5	13 08 36.0443	36.9405	.8962	1398.0	9 10 49 36.3458
10	4704.5	13 12 32.6032	33.4959	.8927	1399.0	10 10 45 40.4363
11	4705.5	13 16 29.1620	30.0513	−0.8893	1400.0	11 10 41 44.5268
12	4706.5	13 20 25.7194	26.6066	.8872	1401.0	12 10 37 48.6174
13	4707.5	13 24 22.2748	23.1620	.8872	1402.0	13 10 33 52.7079
14	4708.5	13 28 18.8277	19.7174	.8896	1403.0	14 10 29 56.7984
15	4709.5	13 32 15.3785	16.2727	.8942	1404.0	15 10 26 00.8890
16	4710.5	13 36 11.9276	12.8281	−0.9005	1405.0	16 10 22 04.9795
17	4711.5	13 40 08.4757	09.3835	.9077	1406.0	17 10 18 09.0700
18	4712.5	13 44 05.0237	05.9388	.9151	1407.0	18 10 14 13.1606
19	4713.5	13 48 01.5724	02.4942	.9218	1408.0	19 10 10 17.2511
20	4714.5	13 51 58.1224	59.0496	.9272	1409.0	20 10 06 21.3416
21	4715.5	13 55 54.6740	55.6049	−0.9309	1410.0	21 10 02 25.4322
22	4716.5	13 59 51.2276	52.1603	.9327	1411.0	22 9 58 29.5227
23	4717.5	14 03 47.7830	48.7157	.9326	1412.0	23 9 54 33.6132
24	4718.5	14 07 44.3399	45.2710	.9311	1413.0	24 9 50 37.7038
25	4719.5	14 11 40.8977	41.8264	.9287	1414.0	25 9 46 41.7943
26	4720.5	14 15 37.4556	38.3817	−0.9261	1415.0	26 9 42 45.8848
27	4721.5	14 19 34.0130	34.9371	.9241	1416.0	27 9 38 49.9754
28	4722.5	14 23 30.5689	31.4925	.9236	1417.0	28 9 34 54.0659
29	4723.5	14 27 27.1229	28.0478	.9249	1418.0	29 9 30 58.1564
30	4724.5	14 31 23.6746	24.6032	.9286	1419.0	30 9 27 02.2470
May 1	4725.5	14 35 20.2245	21.1586	−0.9340	1420.0	May 1 9 23 06.3375
2	4726.5	14 39 16.7736	17.7139	.9403	1421.0	2 9 19 10.4280
3	4727.5	14 43 13.3235	14.2693	.9458	1422.0	3 9 15 14.5186
4	4728.5	14 47 09.8758	10.8247	.9488	1423.0	4 9 11 18.6091
5	4729.5	14 51 06.4315	07.3800	.9485	1424.0	5 9 07 22.6996
6	4730.5	14 55 02.9904	03.9354	−0.9450	1425.0	6 9 03 26.7902
7	4731.5	14 58 59.5516	60.4908	.9392	1426.0	7 8 59 30.8807
8	4732.5	15 02 56.1133	57.0461	.9328	1427.0	8 8 55 34.9712
9	4733.5	15 06 52.6741	53.6015	.9274	1428.0	9 8 51 39.0618
10	4734.5	15 10 49.2327	50.1568	.9241	1429.0	10 8 47 43.1523
11	4735.5	15 14 45.7888	46.7122	−0.9234	1430.0	11 8 43 47.2428
12	4736.5	15 18 42.3425	43.2676	.9251	1431.0	12 8 39 51.3334
13	4737.5	15 22 38.8942	39.8229	.9287	1432.0	13 8 35 55.4239
14	4738.5	15 26 35.4447	36.3783	.9336	1433.0	14 8 31 59.5144
15	4739.5	15 30 31.9949	32.9337	.9387	1434.0	15 8 28 03.6050
16	4740.5	15 34 28.5456	29.4890	−0.9434	1435.0	16 8 24 07.6955
17	4741.5	15 38 25.0974	26.0444	−0.9470	1436.0	17 8 20 11.7861

Figure 3-12c [AA, 1981]

We now turn our attention to the relationship of the sidereal and solar time intervals. The direction of the sun in space, relative to the geocentre, varies due to the earth's orbital motion about the sun. The result is that the mean solar day is longer than the mean sidereal day by approximately 4 minutes. After working out rates of change per day, one obtains

$$1^d\ (S) = 23^h\ 56^m\ 04\overset{s}{.}09054\ (M)\ ,\qquad 1^d\ (M) = 24^h\ 03^m\ 56\overset{s}{.}55536\ (S),$$
$$1^h\ (S) = 59^m\ 50\overset{s}{.}17044\ (M)\ ,\qquad 1^h\ (M) = 1^h\ 00^m\ 09\overset{s}{.}85647\ (S),$$
$$1^m\ (S) = 59\overset{s}{.}83617\ (M)\ ,\qquad 1^m\ (M) = 01^m\ 00\overset{s}{.}16427\ (S),$$
$$1^s\ (S) = 0\overset{s}{.}99727\ (M)\ ,\qquad 1^s\ (M) = 01\overset{s}{.}00273\ (S),$$

in which S and M refer to mean sidereal and mean solar times respectively. These numbers are derived from the fact that a tropical (solar) year contains 366.2422 mean sidereal days or 365.2422 mean solar days. The ratio's are then

mean solar time interval = (365.2422/366.2422) = 0.997 269 566 4x mean sidereal time interval,

mean sidereal time interval = (366.2422/365.2422) = 1.002 737 909x mean solar time interval.

The usefulness of a knowledge of the intervals is given in the following numerical example.

Example #3 (using AA)

Given: LAST = $4^h\ 00^m\ 04^s.30$

Feb. 14, 1981

$\Lambda = 66^{\circ}\ 38'\ 28''$W

Compute: ZT

LAST	$4^h\ 00^m\ 04^s.30$
Λ (−66° 38' 28")	$-4^h\ 26^m\ 33^s.87$
GAST (=LAST − Λ)	$8^h\ 26^m\ 38^s.17$
Eq.E. (0^h U.T. Feb. 14, 1981)	$-0^s.74$
GMST (=GAST − Eq.E.)	$8^h\ 26^m\ 38^s.91$
GMST at 0^h U.T., Feb. 14, 1981 $9^h\ 35^m\ 42^s.95$	
Mean Sidereal Interval (GMST − $\text{GMST}_{0^h\,U.T.}$)	$22^h\ 50^m\ 55^s.96$
U.T.(mean sidereal interval x 0.997 269 566 4)	$22^h\ 47^m\ 11^s.37$
ΔZ (round 66°W/15° = 4^h)	4^h
ZT (= UT − ΔZ)	$18^h\ 47^m\ 11^s.37$

3.4 Irregularities of Rotational Time Systems

Sidereal and Universal time systems are based on the rotation of the earth; thus, they are subject to irregularities caused by (i) changes in the rotation rate of the earth, ω_e, and (ii) changes in the position of the rotation axis (polar motion). The differences are expressed in terms of Universal time as follows:

- UT0 is UT deduced from observations, is affected by changes in ω_e and polar motion, thus is an irregular time system.
- UT1 is defined as UT0 corrected for polar motion, thus it represents the true angular motion of the earth and is the system to be used for geodetic astronomy (note that it is an irregular time system due to variations in ω_e).
- UT2 is defined as UT1 corrected for seasonal variations in ω_e. This time system is non-uniform because ω_e is slowly decreasing due to the drag of tidal forces and other reasons.
- UTC Universal Time Coordinated is the internationally agreed upon time system which is transmitted by most radio time stations. UTC has a defined relationship to International Atomic Time (IAT), as well as to UT2 and to UT1.

More information on UTC, UT1, and UT2, particularly applicable to timekeeping for astronomic purposes, is given in chapter 4.

3.5 Atomic Time System

Atomic time is based on the electromagnetic oscillations produced by the quantum transition of an atom. In 1967, the International Committee for Weights and Measures defined the atomic second (time interval) as "the duration of 9 192 631 770 periods of radiation corresponding to the transition between the two hyper-fine levels of the fundamental state of the atom of Caesium 133" [Robbins, 1976]. Atomic frequency standards are the most acurate standards in current use with stabilities of one or two parts in 10^{12} common, and stabilities of two parts in 10^{14} obtained from the mean of sixteen specially selected caesium standards at the US Naval Observatory.

Various atomic time systems with different epochs are in use. The internationally accepted one is called International Atomic Time, which is based on the weighted means of atomic clock systems throughout the world. The work required to define IAT is carried out by BIH (Bureau International de l'Heure) in Paris [Robbins, 1976]. IAT, through its relationship with UTC, is the basis for radio broadcast time signals.

4. TIME DISSEMINATION, TIME-KEEPING, TIME RECORDING

4.1 Time Dissemination

As has been noted several times already, celestial coordinate systems are subject to change with time. To make use of catalogued positions of celestial bodies for position and azimuth determination at any epoch, we require a measure of the epoch relative to the catalogued epoch. In the interests of homogeneity, it is in our best interest if on a world-wide bases everyone uses the same time scale.

Time signals are broadcast by more than 30 stations around the world, with propagation frequencies in either the HF (1 to 50 MHz) or LF (10 to 100 KHz) ranges. The HF (high frequency) tend to be more useful for surveying purposes as a simple commercial radio receiver can be used. The LF (low frequency) signal are more accurate since errors due to propagation delay can be more easily accounted for.

In North America, the two most commonly used time signals are those broadcast by WWV, Boulder, Colorado (2.5 MHz, 5 MHz, 10 MHz, 15 MHz, 20 MHz) and CHU, Ottawa (3.330 MHz, 7.335 MHz, 14.670 MHz).

The broadcast signal is called Universal Time Coordinated (UTC). UTC is a system of seconds of atomic time, that is 1^s UTC is 1^sAT to the accuracy of atomic time and it is constant. At 1972 January 1 0^hUT, UTC was defined to be IAT minus 10^s exactly. The quantity (IAT-UTC), called DAT, will always be an integral number due to the introduction of the "leap second" (see below).

The time we are interested in is UT1. The value, (UT1-UTC) is denoted DUT1 and never exceeds the value $0\overset{s}{.}9$. The value DUT1 is predicted and published by BIH. Precise values are published one month in arears.

In addition, DUT1 can be disseminated from certain time signal transmissions The code for transmission of DUT1 by CHU is given in Figure 4-1.

TAI is currently gaining on UT1 at a rate of about one second per year. To keep DUT1 within the specified $0^s.9$, UTC is decreased by exactly 1^s at certain times. The introduction of this "leap second" is illustrated in Figure 4-2. Note that provisions for a negative leap second are given, should it ever become necessary.

All major observatories of the world transmit UTC with an error of less than $0^s.001$, most with errors less than $0^s.000\ 3$. The propagation delay of high frequency signals reaches values in the order of 0.5ms. Since geodetic observations rarely have an accuracy of better than $0^s.01$ or 10 ms, an error of 0.5 ms is of little consequence. For further information on the subject of propagation delay, the reader is referred to, for example, Robbins [1976]. The various time signals consist essentially of:

(i) short pulses emitted every second, the beginning of the pulse signalling the beginning of the second,

(ii) each minute marked by some type of stress (eg. for CHU, the zero second marker of each minute is longer; for WWV, the 59th second marker is omitted),

(iii) at periodic intervals, station identification, time, and date are announced in morse code, voice, or both,

(iv) DUT1 is indicated by accentuation of an appropriate number of the first 15 seconds of every minute.

A positive value of DUT1 will be indicated by emphasizing a number (n) of consecutive seconds markers following the minute marker from seconds markers one to seconds marker (n) inclusive; (n) being an integer from 1 to 8 inclusive.

$$DUT1 = (n \times 0.1)s$$

A negative value of DUT1 will be indicated by emphasizing a number (m) of consecutive seconds markers following the minute marker from seconds marker nine to seconds marker (8 + m) inclusive; (m) being an integer from 1 to 8 inclusive.

$$DUT1 = -(m \times 0.1)s$$

A zero value of DUT1 will be indicated by the absence of emphasized seconds markers.

The appropriate seconds markers may be emphasized, for example, by lengthening, doubling, splitting, or tone modulation of the normal seconds markers.

EXAMPLES

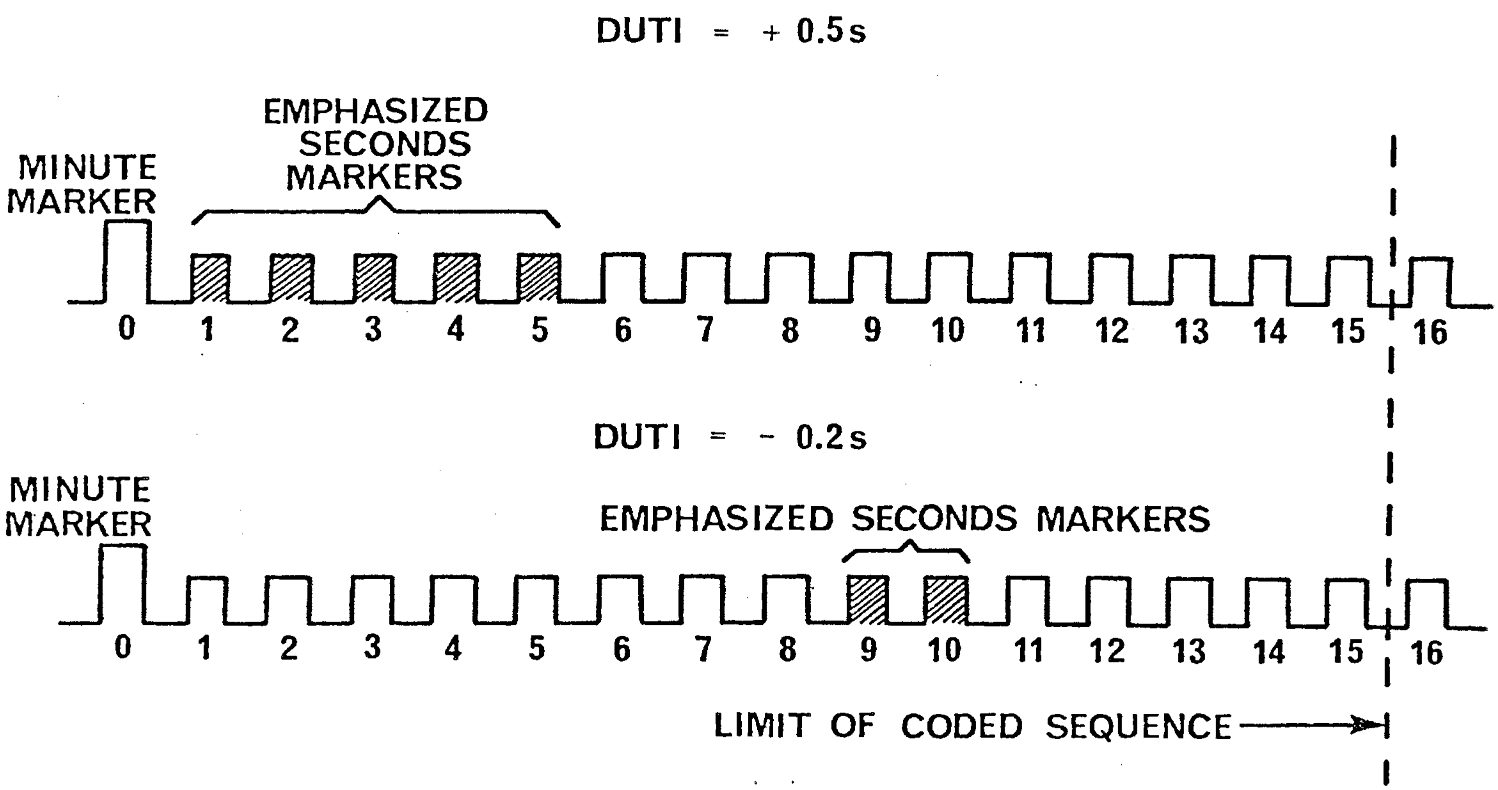


Figure 4-1

Code for the Transmission of DUT1

A positive or negative leap second, when required, should be the last second of a UTC month, but preference should be given to the end of December and June and second preference to the end of March and September. A positive leap second begins at $23^h\ 59^m\ 60^s$ and ends at $0^h\ 0^m\ 0^s$ of the first day of the following month. In the case of a negative leap second, $23^h\ 59^m\ 58^s$ will be followed one second later by $0^h\ 0^m\ 0^s$ of the first day of the following month.

Taking account of what has been said in the preceding paragraph, the dating of events in the vicinity of a leap second shall be effected in the manner indicated in the following figures:

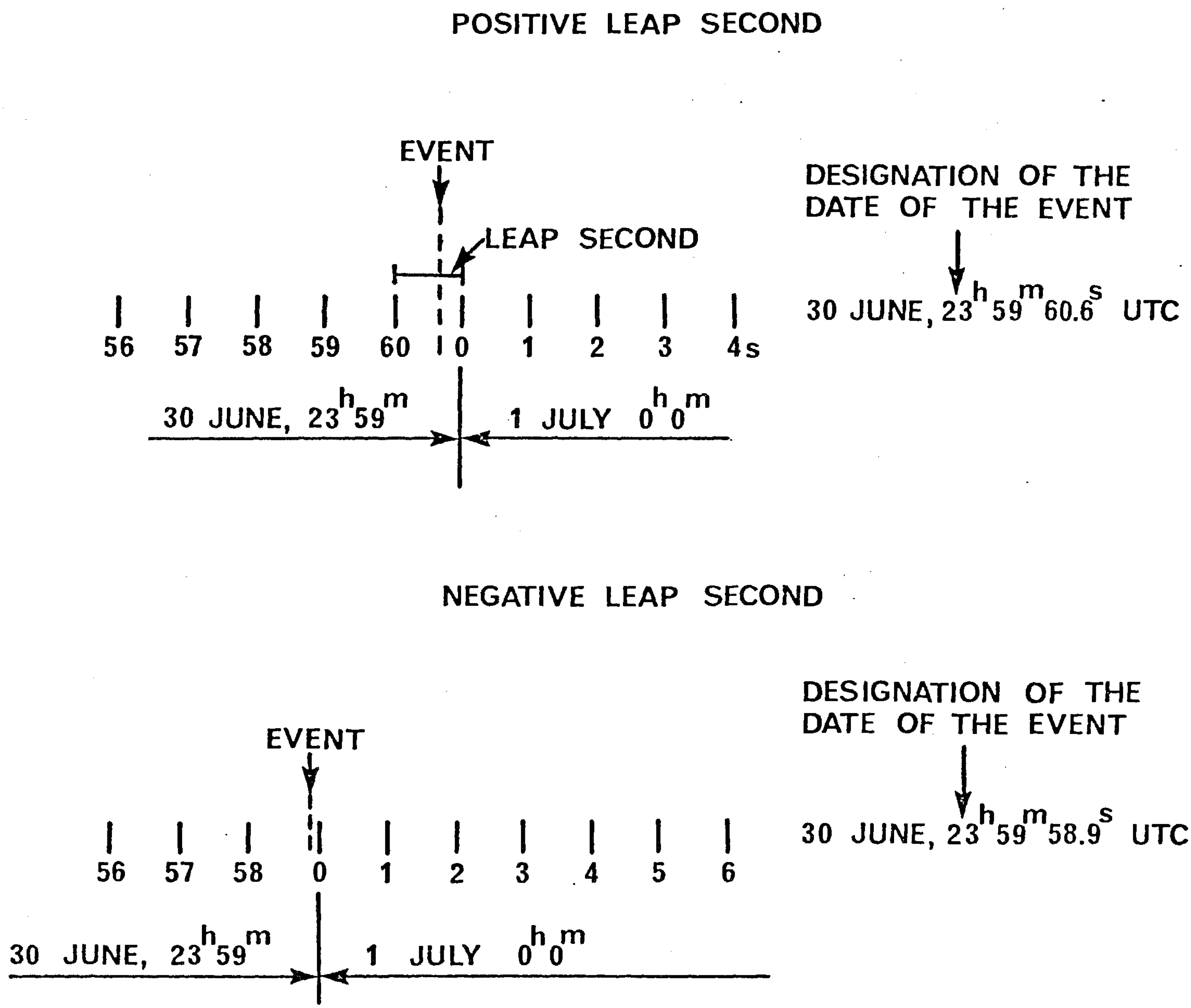


Figure 4-2

Dating of Events in the Vicinity of A Leap Second

4.2 Time-Keeping and Recording

Time reception, before the advent of radio, was by means of the telegraph. Since the 1930's, the radio receiver has been used almost exclusively in geodetic astronomy. Time signals are received and amplified, after which they are used as a means of determining the exact time of an observation. Direct comparison of a time signal with the instant of observation is not made in practice - a clock (chronometer) is used as an intermediate means of timekeeping. Usually, several comparisons of clock time with UTC are made during periods when direction observations are not taking place. The direction observations are subsequently compared to clock time.

The most practical receiver to use is a commercial portable radio with a shortwave (HF) band. In areas where reception is difficult, a marine or aeronautical radio with a good antenna may become necessary.

The chronometer or clock used should be one that is very stable, that is, it should have a constant rate. There are three broad categories of clock that may be used:

(1) Atomic clocks,
(2) Quartz crystal clock,
(3) Mechanical clock.

Atomic clocks are primarily laboratory instruments, are very costly, and have an accuracy that is superfluous for geodetic purposes. Quartz crystal clocks are the preferred clocks. They are portable, stable (accuracy of 1 part in 10^7 to 1 part in 10^{12}), but do require a source of electricity. This type of time-keeper is normally used for geodetic purposes, in conjunction with a time-recorder, (e.g. chronograph) where an accuracy of $0\overset{s}{.}001$ or better is required. Mechanical clocks, with accuracies in the order of 1 part in 10^6, suffice for many surveying needs (eg. where an accuracy of $0^s.1$ is required). They are subject to mechanical failure and sudden variations in rate, but they are portable and independent of an outside source of electricity. The most common and useful mechanical clock is a stop watch, reading to $0^s.1$, with two sweep seconds hands. One should note that many of the new watches - mechanical, electronic, quartz crystal - combine the portable-stable criteria and the stop watch capabilities, and should not be overlooked. With any of these latter devices, time-recording is manual, and the observing procedure is audio-visual.

4.3 Time Observations

In many instances, the set of observations made for the astronomic determination of position and azimuth includes the accurate determination and recording of time. While in many cases,(latitude and azimuth mathematical models and associated star observing programs can be devised to minimize the effects of random and systematic errors in time, the errors in time always have a direct effect on longitude (e.g. if time is in error by $0^s.1$, the error in Λ will be $1''.5$).

The timekeeping device that is most effective for general surveying purposes is the stop watch, most often a stop watch with a mean solar rate (sidereal rate chronometers are available). The five desireable elements for the stop watch are [Robbins, 1976]:

(i) two sweep seconds hands,

(ii) a start-stop-reset button which operates both hands together, resetting them to zero,

(iii) a stop-reset button which operates on the secondary seconds hand only, resetting it to coincidence with the main hand which is unaffected by the operation of this button,

(iv) reading to $0^{s}.1$,

(v) a rate sufficiently uniform over at least one half an hour to ensure that linear interpolation over this period does not give an error greater than $0^{s}.1$.

A recommended time observing procedure is as follows [Robbins, 1976]:

(i) start the watch close to a whole minute of time as given by the radio time signal,

(ii) stop the secondary hand on a whole time signal second, record the reading and reset. A mean of five readings of time differences (see below) will give the ΔT of the stop watch at the time of comparison.

(iii) stop the secondary hand the instant the celestral body is in the desired position in the telescope (this is done by the observer). Record the reading and reset.

(iv) repeat (iii) as direction observations are made.

(v) repeat (ii) periodically (at least every 30^{m}) so that the clock rate for each time observation can be determined.

The clock correction (ΔT) and clock rate ($\Delta_1 T$) are important parts of the determination of time for astronomic position and azimuth determination. Designating the time read on our watch as T, then the zone time (broadcast, for example, by CHU) of an observation can be stated as

$$ZT = T + \Delta T \qquad (4\text{-}1)$$

In (4-1), it is assumed that ΔT is constant. ΔT will usually not be constant but changes should occur at a uniform rate, $\Delta_1 T$. Then ΔT is given by

$$\Delta T = \Delta T_o + (T - T_o)\Delta_1 T , \tag{4-2}$$

where

$\Delta T, \Delta T_o$ are clock corrections at times T, T_o ,

$\Delta_1 T$ is the change in ΔT per unit of time, the clock (chronometer) rate.

The clock correction, ΔT for any time of observation, is determined via a simple linear interpolation as follows:

(i) Using the radio and clock comparisons made before, during, and after the observing program, compute ΔT for each by

$$\Delta T_i = ZT_i - T_i \quad . \tag{4-3}$$

(ii) Compute the time difference, $\Delta ZT_{i,i+1}$ between time comparisons by

$$\Delta ZT_{i,i+1} = ZT_{i+1} - ZT_i \quad . \tag{4-4}$$

(iii) Compute the differences between successive clock corrections corresponding to the differences $\Delta ZT_{i,i+1}$, namely

$$\Delta T_{i,i+1} = \Delta T_{i+1} - \Delta T_i \tag{4-5}$$

(iv) Compute the clock rate per mean solar hour by

$$\Delta_1 T = \frac{\Delta T_{i,i+1}}{\Delta ZT_{i,i+1}} \quad . \tag{4-6}$$

(v) Record the results in a convenient and usable form (e.g. table of values, graph).

This time determination process is illustrated by the following example.

Clock Correction and Clock Rate

(i) Determination of ZT of stop watch

ZT (radio)	Stop Watch (T)	ΔT_i (Radio-Stop watch)
$9^h\ 13^m\ 00^s.0$	$0^h\ 00^m\ 00^s.0$	$9^h\ 13^m\ 00^s.0$
$13^m\ 15^s.0$	$00^m\ 14^s.8$	$9^h\ 13^m\ 00^s.2$
$13^m\ 45^s.0$	$00^m\ 45^s.2$	$9^h\ 12^m\ 59^s.8$
$9^h\ 14^m\ 00^s.0$	$0^h\ 01^m\ 00^s.0$	$9^h\ 13^m\ 00^s.0$
$14^m\ 30^s.0$	$01^m\ 29^s.9$	$9^h\ 13^m\ 00^s.1$

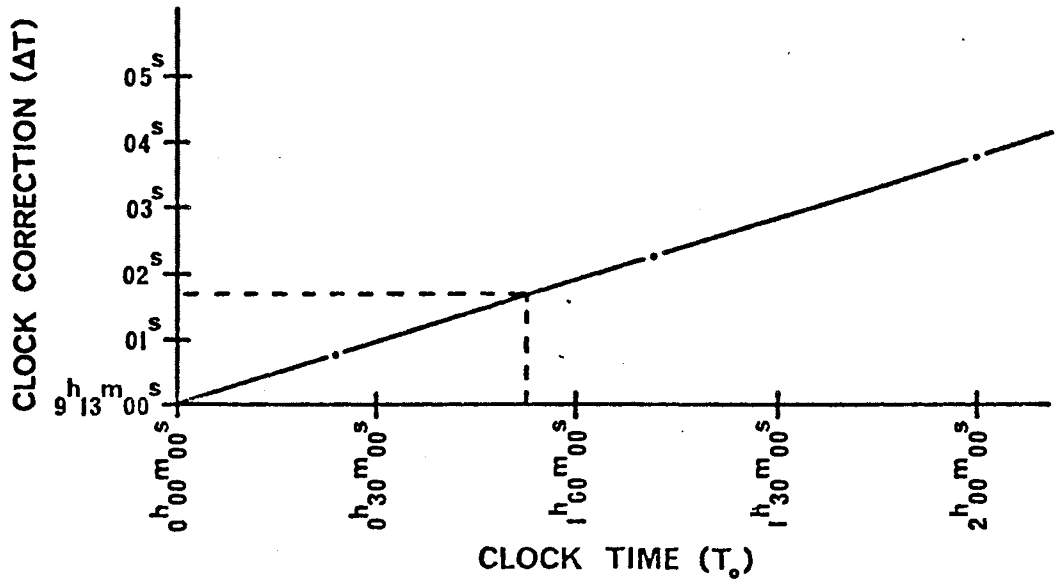
$$\text{ZT of beginning} = (\sum_{i=1}^{5} \Delta T_i)/i = 9^h\ 13^m\ 00^s.0$$

This procedure is to be repeated for each time comparison with the radio time signal.

(ii) Record of time comparisons; determination of clock rate.

ZT (radio)	Clock (stop watch) TIME (T)	$\Delta ZT_{i,i+1}$	ΔT_o	$\Delta T_{i,i+1}$	$\Delta_1 T$ (s/h)
$9^h13^m00^s.0$	$0^h00^m00^s.0$		$9^h13^m00^s.0$		
		$0^h28^m00^s.0$		$00^s.8$	1.7
$9^h41^m00^s.0$	$0^h27^m59^s.2$		$9^h13^m00^s.8$		
		$0^h49^m00^s.0$		$01^s.5$	1.8
$10^h30^m00^s.0$	$1^h16^m57^s.7$		$9^h13^m02^s.3$		
		$1^h03^m00^s.0$		$02^s.0$	1.9
$11^h33^m00^s.0$	$2^h19^m55^s.7$		$9^h13^m04^s.3$		

(iii) Graph of Clock (stop watch) Correction ΔT



(iv) Determination of the true zone time of a direction observation.

Observed Time (stop watch): $0^h\ 52^m\ 28\overset{s}{.}8$

What is true ZT ?

From the graph: ΔT at ZT observed $= 9^h\ 13^m\ 01\overset{s}{.}6$

$$\text{ZT (true)} = 0^h\ 52^m\ 28\overset{s}{.}8 + 9^h\ 13^m\ 01\overset{s}{.}6$$

$$= \underline{10^h\ 05^m\ 30\overset{s}{.}4}$$

Using $\Delta T = \Delta T_o + (T_{i+1} - T_i)\Delta_1 T$ yields

$$\Delta T = 9^h\ 13^m\ 00\overset{s}{.}8 + (0\overset{h}{.}40803)\ 1.8 = 9^h\ 13^m\ 01\overset{s}{.}5$$

$$\text{ZT(true)} = 0^h\ 52^m\ 28\overset{s}{.}8 + 9^h\ 13^m\ 01\overset{s}{.}6$$

$$= \underline{10^h\ 05^m\ 30\overset{s}{.}3}\ .$$

In closing this section, the reader is cautioned that the methods described here for time determination are not adequate for precise (e.g. first-order) determinations of astronomic azimuth and position. Such work requires an accuracy in time of $0^s.01$ or better. For details, the reader is referred to, for example, Mueller [1969] or Robbins [1976].

5. STAR CATALOGUES AND EPHEMERIDES

5.1 Star Catalogues

Star catalogues contain the listing of the positions of stars in a unique coordinate system. The positions are generally given in the mean right ascension coordinate system for a particular epoch T_o. Besides listing the right ascensions and declinations, star catalogues identify each star by number and/or name, and should give their proper motions. Some catalogues also include annual and secular variations. Other pertinent data sometimes given are estimates of the standard errors of the coordinates and their variations, and star magnitudes. (Magnitude is an estimate of a star's brightness given on a numerical scale varying from -2 for a bright star to +15 for a dim star).

There are two main types of catalogues: (i) Observation catalogues containing the results of particular observing programs, and (ii) compilation catalogues containing data from a selection of catalogues (observation and/or other compilation catalogues). The latter group are of interest to us for position and azimuth determination. Several compilation catalogues contain accurate star coordinates for a well-distributed (about the celestial sphere) selection of stars. These catalogues are called fundamental catalogues and the star coordinates contained therein define a fundamental coordinate system. Three of these catalogues are described briefly below.

The Fourth Fundamental Catalogue (FK4) [Fricke and Kopff, 1963] was produced by the Astronomischen Rechen Institute of Heidelberg, Germany in 1963. It was compiled from 158 different observation catalogues and contains coordinates for 1535 stars for the epochs 1950.0 and 1975.0.

A supplement to the FK4, with star coordinates for 1950.0, contains 1987 additional stars. These additions were drawn mainly form the N30 catalogue. It should be noted that the FK4 system has been accepted internationally as being the best available, and all star coordinates should be expressed in this system.

The General or Boss General Catalogue (GC, or BC) [Boss, 1937], compiled from two hundred fifty observation catalogues, contains star coordinates for 33 342 stars for the epoch 1950.0. The present accuracy of the coordinates in this catalogue are somewhat doubtful. GC-FK4 correction tables are available, but are of doubtful value due to the errors in the GC coordinates.

The Normal System N30 (N30) [Morgan, 1952] is a catalogue of 5268 Standard Stars with coordinates for the epoch 1950.0. Although more accurate than the GC, it is not of FK4 quality.

For geodetic purposes, the most useful catalogue is the Apparent Places of Fundamental Stars (APFS) [Astronomische Rechen-Institutes, Heidelberg, 1979]. This is an annual volume containing the apparent places of the FK4 stars, tabulated at 10-day intervals. As the name implies, the published coordinates have not been corrected for short period nutation terms that must be applied before the coordinates are used for position and azimuth determination. For all but latitude observations there are sufficient stars in this publication. For the stringent star-pairing required for latitude determinations there are often insufficient stars to fulfill a first-order observation program. In such cases, the FK4 supplement should be referred to and the coordinates rigorously updated from 1950.0 to the date of observation.

5.2 Ephemerides and Almanacs

Ephemerides and Almanacs are annual volumes containing information of interest to surveyors and others (e.g. mariners) involved with "practical" astronomy. They contain the coordinates of selected stars, tabulated for short intervals of time, and may also contain some or all of the following information: motions of the sun, moon, and planets of our solar system, eclipses, occultations, tides, times of sunrises and sunsets, astronomic refraction, conversions of time and angular measures.

The Astronomical Almanac (AA) [U.S. Naval Observatory, Nautical Almanac Office, 1980] (formerly published in the United States as the American Ephemeris and Nautical Almanac and in the United Kingdom as the Astronomical Ephemeris) is the only one of geodetic interest. It contains, amongst other things, the mean places of 1475 stars catalogued for the beginning of the year of interest (e.g. 1981.0), plus the information required to update the star coordinates to the time of observation. All of the information given is fully explained in a section at the end of each AA. Parts of three tables are given here - Figures 5-1, 5-2, 5-3, - that are of interest to us. Each table is self-explanatory since the information has been previously explained. An extra note concerning Figure 5-1 is as follows. Recall from equation (3-18) that

$$GMST = UT + (\alpha_m - 12^h).$$

Now, since the tabulated values are for 0^hUT, and ST is the hour angle of the vernal equinox, then

$$GAST \text{ at } 0^hUT = (\alpha_m - 12^h + Eq.\ E),$$

$$GMST \text{ at } 0^hUT = (\alpha_m - 12^h) .$$

For more detailed information, the interested reader should study a recent copy of AA.

UNIVERSAL AND SIDEREAL TIMES, 1981

Date 0^hU.T.		Julian Date	G. SIDEREAL TIME (G. H. A. of the Equinox) Apparent	Mean	Equation of Equinoxes at 0^hU.T.	G.S.D. 0^hG.S.T.	U.T. at 0^hG.M.S.T. (Greenwich Transit of the Mean Equinox)		
		244	h m s	s	s	245			h m s
Jan.	0	4604.5	6 38 17.1886	17.9594	−0.7708	1299.0	Jan.	0	17 18 51.3829
	1	4605.5	6 42 13.7431	14.5148	.7717	1300.0		1	17 14 55.4734
	2	4606.5	6 46 10.2993	11.0702	.7708	1301.0		2	17 10 59.5639
	3	4607.5	6 50 06.8575	07.6255	.7681	1302.0		3	17 07 03.6545
	4	4608.5	6 54 03.4174	04.1809	.7635	1303.0		4	17 03 07.7450
	5	4609.5	6 57 59.9787	60.7363	−0.7576	1304.0		5	16 59 11.8356
	6	4610.5	7 01 56.5407	57.2916	.7510	1305.0		6	16 55 15.9261
	7	4611.5	7 05 53.1023	53.8470	.7447	1306.0		7	16 51 20.0166
	8	4612.5	7 09 49.6626	50.4024	.7398	1307.0		8	16 47 24.1072
	9	4613.5	7 13 46.2207	46.9577	.7370	1308.0		9	16 43 28.1977
	10	4614.5	7 17 42.7763	43.5131	−0.7368	1309.0		10	16 39 32.2882
	11	4615.5	7 21 39.3297	40.0685	.7387	1310.0		11	16 35 36.3788
	12	4616.5	7 25 35.8817	36.6238	.7421	1311.0		12	16 31 40.4693
	13	4617.5	7 29 32.4335	33.1792	.7457	1312.0		13	16 27 44.5598
	14	4618.5	7 33 28.9864	29.7346	.7481	1313.0		14	16 23 48.6504
	15	4619.5	7 37 25.5415	26.2899	−0.7484	1314.0		15	16 19 52.7409
	16	4620.5	7 41 22.0994	22.8453	.7459	1315.0		16	16 15 56.8314
	17	4621.5	7 45 18.6598	19.4006	.7409	1316.0		17	16 12 00.9220
	18	4622.5	7 49 15.2219	15.9560	.7341	1317.0		18	16 08 05.0125
	19	4623.5	7 53 11.7843	12.5114	.7271	1318.0		19	16 04 09.1030
	20	4624.5	7 57 08.3457	09.0667	−0.7210	1319.0		20	16 00 13.1936
	21	4625.5	8 01 04.9050	05.6221	.7171	1320.0		21	15 56 17.2841
	22	4626.5	8 05 01.4618	02.1775	.7157	1321.0		22	15 52 21.3746
	23	4627.5	8 08 58.0160	58.7328	.7168	1322.0		23	15 48 25.4652
	24	4628.5	8 12 54.5682	55.2882	.7199	1323.0		24	15 44 29.5557
	25	4629.5	8 16 51.1192	51.8436	−0.7243	1324.0		25	15 40 33.6462
	26	4630.5	8 20 47.6698	48.3989	.7291	1325.0		26	15 36 37.7368
	27	4631.5	8 24 44.2208	44.9543	.7335	1326.0		27	15 32 41.8273
	28	4632.5	8 28 40.7728	41.5097	.7368	1327.0		28	15 28 45.9178
	29	4633.5	8 32 37.3264	38.0650	.7386	1328.0		29	15 24 50.0084
	30	4634.5	8 36 33.8818	34.6204	−0.7386	1329.0		30	15 20 54.0989
	31	4635.5	8 40 30.4389	31.1758	.7368	1330.0		31	15 16 58.1894
Feb.	1	4636.5	8 44 26.9975	27.7311	.7336	1331.0	Feb.	1	15 13 02.2800
	2	4637.5	8 48 23.5571	24.2865	.7294	1332.0		2	15 09 06.3705
	3	4638.5	8 52 20.1168	20.8418	.7251	1333.0		3	15 05 10.4610
	4	4639.5	8 56 16.6755	17.3972	−0.7217	1334.0		4	15 01 14.5516
	5	4640.5	9 00 13.2323	13.9526	.7203	1335.0		5	14 57 18.6421
	6	4641.5	9 04 09.7865	10.5079	.7215	1336.0		6	14 53 22.7326
	7	4642.5	9 08 06.3381	07.0633	.7252	1337.0		7	14 49 26.8232
	8	4643.5	9 12 02.8879	03.6187	.7307	1338.0		8	14 45 30.9137
	9	4644.5	9 15 59.4372	60.1740	−0.7369	1339.0		9	14 41 35.0042
	10	4645.5	9 19 55.9872	56.7294	.7422	1340.0		10	14 37 39.0948
	11	4646.5	9 23 52.5392	53.2848	.7455	1341.0		11	14 33 43.1853
	12	4647.5	9 27 49.0938	49.8401	.7463	1342.0		12	14 29 47.2758
	13	4648.5	9 31 45.6509	46.3955	.7445	1343.0		13	14 25 51.3664
	14	4649.5	9 35 42.2098	42.9509	−0.7411	1344.0		14	14 21 55.4569
	15	4650.5	9 39 38.7693	39.5062	−0.7369	1345.0		15	14 17 59.5474

Figure 5-1 [AA, 1981]

BRIGHT STARS, 1981.0

Name	B.S.	Right Ascension	Declination	Notes	V	B−V	Spectral Type
		h m s	° ′ ″				
θ Oct	9084	0 00 38.2	−77 10 14	F,V	4.78	+1.27	K2 III
30 Psc	9089	0 00 59.1	− 6 07 11	F,V	4.41	+1.63	M3 III
2 Cet	9098	0 02 46.0	−17 26 30	F,V	4.55	−0.05	B9 IV
33 Psc	3	0 04 21.7	− 5 48 50	F	4.61	+1.04	K1 III
21 α And	15	0 07 24.1	+28 59 08	F	2.06	−0.11	B9p
11 β Cas	21	0 08 09.3	+59 02 42	F,S	2.27	+0.34	F2 III-IV
ε Phe	25	0 08 27.0	−45 51 08	F	3.88	+1.03	K0 III
22 And	27	0 09 19.6	+45 58 00	F	5.03	+0.40	F2 II
θ Scl	35	0 10 46.1	−35 14 22	F	5.25	+0.44	dF4
88 γ Peg	39	0 12 15.3	+15 04 41	F,S,V	2.83	−0.23	B2 IV
89 χ Peg	45	0 13 37.0	+20 06 04	F,S	4.80	+1.57	M2 III
7 Cet	48	0 13 40.5	−19 02 17		4.44	+1.66	M1 III
25 σ And	68	0 17 19.8	+36 40 48	F	4.52	+0.05	A2 V
8 ι Cet	74	0 18 27.5	− 8 55 45	F	3.56	+1.22	K1.5 III
ζ Tuc	77	0 19 05.3	−64 59 11	F	4.23	+0.58	F9 V
41 Psc	80	0 19 37.1	+ 8 05 05	F	5.37	+1.34	gK3
27 ρ And	82	0 20 06.9	+37 51 49	F	5.18	+0.42	F6 IV
β Hyi	98	0 24 46.4	−77 21 41	F	2.80	+0.62	G1 IV
κ Phe	100	0 25 16.2	−43 47 07		3.94	+0.17	A5 Vn
α Phe	99	0 25 20.8	−42 24 33	F	2.39	+1.09	K0 IIIb
	118	0 29 25.7	−23 53 34	F	5.19	+0.12	A5 Vn
λ^1 Phe	125	0 30 30.1	−48 54 30	F	4.77	+0.02	A0 V
β^1 Tuc	126	0 30 40.8	−63 03 46		4.37	−0.07	B9 V
15 κ Cas	130	0 31 54.5	+62 49 38	F,S	4.16	+0.14	B1 Ia
29 π And	154	0 35 51.7	+33 36 54	F	4.36	−0.14	B5 V
17 ζ Cas	153	0 35 54.3	+53 47 33	F	3.66	−0.20	B2 IV
	157	0 36 19.8	+35 17 43	S	5.48	+0.88	G3 II
30 ε And	163	0 37 32.8	+29 12 32	F	4.37	+0.87	G8 IIIp
31 δ And	165	0 38 18.5	+30 45 26	F,S	3.27	+1.28	K3 III
18 α Cas	168	0 39 25.2	+56 26 00	F,V	2.23	+1.17	K0- IIIa
μ Phe	180	0 40 25.8	−46 11 21	F	4.59	+0.97	G8 III
η Phe	191	0 42 30.2	−57 34 02	F,D	4.36	+0.00	B9 Vp
16 β Cet	188	0 42 38.1	−18 05 27	F	2.04	+1.02	K1 III
22 ο Cas	193	0 43 39.6	+48 10 50	F	4.54	−0.07	B5 III
34 ζ And	215	0 46 19.7	+24 09 51	F,V	4.06	+1.12	K1 II
63 δ Psc	224	0 47 41.7	+ 7 28 55	F	4.43	+1.50	K5 III
λ Hyi	236	0 47 56.1	−75 01 36	F	5.07	+1.37	K4 III
24 η Cas	219	0 47 56.5	+57 42 55	S	3.44	+0.57	G0 V
64 Psc	225	0 47 58.6	+16 50 18	F	5.07	+0.51	F8 V
35 ν And	226	0 48 45.6	+40 58 33	F	4.53	−0.15	B5 V
19 ϕ^2 Cet	235	0 49 10.4	−10 44 47	F	5.19	+0.50	F8 V
	233	0 49 33.6	+64 08 40	F,C	5.39	+0.49	gG0 + A5
20 Cet	248	0 52 02.1	− 1 14 50	F	4.77	+1.57	M0- IIIa
λ^2 Tuc	270	0 54 18.0	−69 37 47	F	5.45	+1.09	K2 III
27 γ Cas	264	0 55 33.0	+60 36 51	F,V	2.47	−0.15	B0.5 IVel
37 μ And	269	0 55 41.6	+38 23 48	F	3.87	+0.13	A5 V
38 η And	271	0 56 11.3	+23 18 56		4.42	+0.94	G8 III-IV
α Scl	280	0 57 41.4	−29 27 36	F,S	4.31	−0.16	B7 III (C II)
71 ε Psc	294	1 01 57.3	+ 7 47 17	F	4.28	+0.96	K0 III
β Phe	322	1 05 14.2	−46 49 13	2	3.31	+0.89	G8 III

Figure 5-2 [AA, 1981]

BRIGHT STARS, 1981.0

Name	B.S.	Right Ascension	Declination	Notes	V	B−V	Spectral Type
		h m s	° ′ ″				
	285	1 05 54.1	+86 09 22	F	4.25	+1.21	K2 III
ι Tuc	332	1 06 33.7	−61 52 36	F	5.37	+0.88	G5 III
υ Phe	331	1 06 55.8	−41 35 18	F,D	5.21	+0.16	A3 IV/V
30 μ Cas	321	1 06 59.8	+54 49 40	F	5.17	+0.69	G5 Vp
ζ Phe	338	1 07 35.3	−55 20 50	V	3.92	−0.08	B7 V
31 η Cet	334	1 07 38.0	−10 16 58	F	3.45	+1.16	K3 III
42 φ And	335	1 08 23.5	+47 08 27		4.25	−0.07	B7 III
43 β And	337	1 08 39.8	+35 31 13	F	2.06	+1.58	M0 IIIa
33 θ Cas	343	1 09 56.2	+55 02 57		4.33	+0.17	A7 V
84 χ Psc	351	1 10 25.7	+20 56 02	F	4.66	+1.03	G8 III
83 τ Psc	352	1 10 36.6	+29 59 21	F	4.51	+1.09	K0 III-IV
86 ζ Psc	361	1 12 44.2	+ 7 28 31	F	4.86	+0.32	F0 Vn
κ Tuc	377	1 15 07.6	−68 58 37		4.86	+0.47	F6 IV
89 Psc	378	1 16 49.0	+ 3 30 53	F	5.16	+0.07	A3 V
90 υ Psc	383	1 18 25.1	+27 09 53	F	4.76	+0.03	A2 V
34 φ Cas	382	1 18 52.5	+58 07 56	S,M	4.98	+0.68	F0 Ia
46 ξ And	390	1 21 12.9	+45 25 47	F	4.88	+1.08	K0 III-IV
45 θ Cet	402	1 23 04.3	− 8 16 52	F	3.60	+1.06	K0 IIIb
37 δ Cas	403	1 24 33.6	+60 08 13	F,S,V	2.68	+0.13	A5 III-IV
36 ψ Cas	399	1 24 34.4	+68 01 53	F	4.74	+1.05	K0 III
94 Psc	414	1 25 39.9	+19 08 33	F	5.50	+1.11	gK1
48 ω And	417	1 26 30.7	+45 18 33	F	4.83	+0.42	F5 V
γ Phe	429	1 27 32.5	−43 24 55	F	3.41	+1.57	K5+ IIb-IIIa
48 Cet	433	1 28 41.4	−21 43 38	F,7	5.12	+0.02	A1 V
δ Phe	440	1 30 27.7	−49 10 16	F	3.95	+0.99	K0 IIIb
99 η Psc	437	1 30 27.8	+15 14 54	F,3	3.62	+0.97	G8 III
50 υ And	458	1 35 40.5	+41 18 39	F	4.09	+0.54	F8 V
51 And	464	1 36 49.1	+48 31 57	F	3.57	+1.28	K3 III
40 Cas	456	1 36 58.2	+72 56 38	F,D	5.28	+0.96	G8 II-III
α Eri	472	1 37 00.5	−57 19 59	F	0.46	−0.16	B3 Vp
106 ν Psc	489	1 40 26.4	+ 5 23 31	F	4.44	+1.36	K3 III
	490	1 40 57.3	+35 09 01	F	5.40	−0.09	B9 IV-V
π Scl	497	1 41 17.1	−32 25 21	F,M	5.26	+1.04	gK0
	500	1 41 45.8	− 3 47 08	F	4.99	+1.38	K3 II-III
φ Per	496	1 42 27.7	+50 35 37	F,V	4.07	−0.04	B2 Ve4p
52 τ Cet	509	1 43 11.1	−16 02 14	F	3.50	+0.72	G8 Vp
110 ο Psc	510	1 44 23.3	+ 9 03 45	F,S	4.26	+0.96	G8 III
ε Scl	514	1 44 45.3	−25 08 50	F	5.31	+0.39	dF1
	513	1 45 01.9	− 5 49 41	S	5.34	+1.52	K4 III
53 χ Cet	531	1 48 39.0	−10 46 48	F	4.67	+0.33	F2 V
55 ζ Cet	539	1 50 31.3	−10 25 43	F	3.73	+1.14	K2 III
2 α Tri	544	1 51 59.6	+29 29 13	F	3.41	+0.49	F6 IV
111 ξ Psc	549	1 52 34.2	+ 3 05 39	F	4.62	+0.94	K0 III
ψ Phe	555	1 52 53.1	−46 23 43	F	4.41	+1.59	M4 III
45 ε Cas	542	1 53 00.8	+63 34 38	F	3.38	−0.15	B3 Vp
φ Phe	558	1 53 34.7	−42 35 24	F	5.11	−0.06	Ap
6 β Ari	553	1 53 35.2	+20 42 56	F	2.64	+0.13	A5 V
η^2 Hyi	570	1 54 27.1	−67 44 26	F	4.69	+0.95	G8.5 III
χ Eri	566	1 55 13.1	−51 42 11	F	3.70	+0.85	G8 IIIb CN-2
α Hyi	591	1 58 10.3	−61 39 43	F	2.86	+0.28	F0 V

Figure 5-3 [AA, 1981]

The Star Almanac for Land Surveyors (SALS) [H.M. Nautical Almanac Office, 1980], published annually, is expressly designed to meet the surveyor's requirements in astronomy. The main tables included are:

(i) the Sun, in which the right ascension, declination, and equation of time are tabulated for 6-hourly intervals (see explanations below regarding Figure 5-4),

(ii) the Stars, in which the apparent right ascension and declination of 685 stars are given for the beginning of each month (e.g. 0^hUT, March 1, 1981),

(iii) Northern and Southern Circumpolar Stars (five of each), are listed separately, their apparent right ascensions and declinations being given at 10-day intervals for the year,

(iv) the Pole Star Table, a special table for Polaris (α Ursae Minoris) for use in specific math models for latitude and azimuth determinations.

Also included in SALS are several supplementary tables, some of which are companions to those noted above (for interpolation purposes). Examples of three SALS tables are given in Figures 5-4, 5-5, and 5-6. Most of the information given has been explained previously, and the tables are self explanatory. Regarding the terms R and E in Figure 5-4, one should note that

$$R = (\alpha_m - 12^h + Eq.E) = GAST \text{ at } 0^h UT,$$

$$E = Eq.T. - 12^h.$$

The reader is cautioned that SALS is not suitable for use where first-order standards must be met. AA may be used, but common practice is to use one of the fundamental catalogues and compute the updated star coordinates via proceduresoutlined in Chapter 6.

SUN—FEBRUARY, 1981

U.T. d	h	R (h m s)	Dec. (° ′)		E (h m s)		U.T. d	h	R (h m s)	Dec. (° ′)		E (h m s)	
1	0	8 44 27·0	S 17 10·9	42	11 46 25·8	21	9	0	9 15 59·4	S 14 46·0	48	11 45 44·7	4
Sun.	6	45 26·1	17 06·7	43	46 23·7	20	Mon.	6	16 58·6	14 41·2	49	45 44·3	4
	12	46 25·3	17 02·4	43	46 21·7	20		12	17 57·7	14 36·3	48	45 43·9	4
	18	47 24·4	16 58·1	43	46 19·7	19		18	18 56·8	14 31·5	49	45 43·5	3
2	0	8 48 23·6	S 16 53·8	43	11 46 17·8	19	10	0	9 19 56·0	S 14 26·6	48	11 45 43·2	2
Mon.	6	49 22·7	16 49·5	44	46 15·9	18	Tues.	6	20 55·1	14 21·8	49	45 43·0	2
	12	50 21·8	16 45·1	43	46 14·1	17		12	21 54·3	14 16·9	49	45 42·8	2
	18	51 21·0	16 40·8	44	46 12·4	18		18	22 53·4	14 12·0	49	45 42·6	1
3	0	8 52 20·1	S 16 36·4	44	11 46 10·6	16	11	0	9 23 52·5	S 14 07·1	49	11 45 42·5	0
Tues.	6	53 19·3	16 32·0	45	46 09·0	16	Wed.	6	24 51·7	14 02·2	50	45 42·5	0
	12	54 18·4	16 27·5	44	46 07·4	16		12	25 50·8	13 57·2	49	45 42·5	0
	18	55 17·5	16 23·1	44	46 05·8	15		18	26 50·0	13 52·3	50	45 42·5	1
4	0	8 56 16·7	S 16 18·7	45	11 46 04·3	15	12	0	9 27 49·1	S 13 47·3	50	11 45 42·6	2
Wed.	6	57 15·8	16 14·2	45	46 02·8	14	Thur.	6	28 48·2	13 42·3	50	45 42·8	2
	12	58 15·0	16 09·7	45	46 01·4	13		12	29 47·4	13 37·3	50	45 43·0	2
	18	8 59 14·1	16 05·2	45	46 00·1	13		18	30 46·5	13 32·3	50	45 43·2	3
5	0	9 00 13·2	S 16 00·7	46	11 45 58·8	13	13	0	9 31 45·7	S 13 27·3	50	11 45 43·5	3
Thur.	6	01 12·4	15 56·1	46	45 57·5	12	Fri.	6	32 44·8	13 22·3	51	45 43·8	4
	12	02 11·5	15 51·5	45	45 56·3	12		12	33 43·9	13 17·2	50	45 44·2	5
	18	03 10·6	15 47·0	46	45 55·1	11		18	34 43·1	13 12·2	51	45 44·7	4
6	0	9 04 09·8	S 15 42·4	46	11 45 54·0	10	14	0	9 35 42·2	S 13 07·1	51	11 45 45·1	6
Fri.	6	05 08·9	15 37·8	47	45 53·0	10	Sat.	6	36 41·3	13 02·0	51	45 45·7	6
	12	06 08·1	15 33·1	46	45 52·0	10		12	37 40·5	12 56·9	51	45 46·3	6
	18	07 07·2	15 28·5	47	45 51·0	9		18	38 39·6	12 51·8	52	45 46·9	7
7	0	9 08 06·3	S 15 23·8	47	11 45 50·1	8	15	0	9 39 38·8	S 12 46·6	51	11 45 47·6	7
Sat.	6	09 05·5	15 19·1	46	45 49·3	8	Sun.	6	40 37·9	12 41·5	52	45 48·3	7
	12	10 04·6	15 14·5	48	45 48·5	8		12	41 37·0	12 36·3	51	45 49·0	8
	18	11 03·8	15 09·7	47	45 47·7	7		18	42 36·2	12 31·2	52	45 49·8	9
8	0	9 12 02·9	S 15 05·0	47	11 45 47·0	6	16	0	9 43 35·3	S 12 26·0	52	11 45 50·7	9
Sun.	6	13 02·0	15 00·3	48	45 46·4	6	Mon.	6	44 34·5	12 20·8	52	45 51·6	9
	12	14 01·2	14 55·5	48	45 45·8	6		12	45 33·6	12 15·6	52	45 52·5	10
	18	15 00·3	14 50·7	47	45 45·2	5		18	46 32·7	12 10·4	52	45 53·5	11
	24	9 15 59·4	S 14 46·0		11 45 44·7			24	9 47 31·9	S 12 05·2		11 45 54·6	

Sun's S.D. 16′·2

SUNRISE

Date	South Latitude									North Latitude							
	60°	55°	50°	45°	40°	30°	20°	10°	0°	10°	20°	30°	40°	45°	50°	55°	60°
Feb.	h	h	h	h	h	h	h	h	h	h	h	h	h	h	h	h	h
1	3·9	4·4	4·7	4·9	5·2	5·5	5·7	6·0	6·2	6·4	6·6	6·8	7·2	7·3	7·6	7·9	8·2
6	4·2	4·5	4·8	5·1	5·2	5·5	5·8	6·0	6·2	6·4	6·6	6·8	7·1	7·2	7·5	7·7	8·1
11	4·4	4·7	5·0	5·2	5·3	5·6	5·8	6·0	6·2	6·3	6·5	6·7	7·0	7·1	7·3	7·5	7·8
16	4·6	4·9	5·1	5·3	5·5	5·7	5·9	6·0	6·2	6·3	6·5	6·7	6·9	7·0	7·2	7·3	7·6
21	4·8	5·1	5·3	5·4	5·5	5·7	5·9	6·0	6·2	6·3	6·4	6·6	6·8	6·9	7·0	7·2	7·4
26	5·0	5·3	5·4	5·5	5·7	5·8	5·9	6·0	6·2	6·3	6·4	6·5	6·7	6·7	6·8	7·0	7·1
31	5·3	5·4	5·5	5·7	5·7	5·9	6·0	6·1	6·2	6·2	6·3	6·4	6·5	6·6	6·7	6·8	6·9

Moon's phases: new moon, $4^{d}22^{h}14^{m}$; first quarter, $11^{d}17^{h}49^{m}$.

Figure 5-4 [SALS, 1981]

RIGHT ASCENSION OF STARS, 1981

No.	Mag.	R.A.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.
		h m	s	s	s	s	s	s	s	s	s	s	s	s	s
51	4·3	2 21	25·0	23·3	21·7	20·6	20·2	20·8	22·2	24·2	26·1	27·5	28·0	27·6	26·4
52	4·4	2 26	17·3	16·6	15·8	15·3	15·2	15·6	16·5	17·7	18·8	19·7	20·1	20·0	19·6
53	4·3	2 27	08·6	08·3	07·8	07·5	07·6	08·0	08·8	09·7	10·6	11·3	11·7	11·8	11·6
54	4·0	2 38	30·3	29·9	29·5	29·2	29·1	29·5	30·3	31·2	32·1	32·8	33·2	33·3	33·2
55	4·3	2 39	18·5	16·8	15·2	14·0	13·5	13·9	15·2	17·0	19·0	20·5	21·1	20·8	19·7
56	4·1	2 39	55·0	54·4	53·8	53·3	53·2	53·5	54·3	55·4	56·5	57·3	57·7	57·7	57·4
57	3·6	2 42	18·8	18·4	18·0	17·6	17·6	18·0	18·8	19·7	20·6	21·2	21·7	21·8	21·7
58	4·2	2 42	54·1	53·4	52·7	52·1	52·1	52·6	53·7	55·0	56·3	57·2	57·9	58·1	57·8
59	4·4	2 43	12·9	12·5	12·1	11·7	11·7	12·0	12·8	13·7	14·6	15·3	15·7	15·8	15·7
60	4·4	2 43	54·7	54·3	53·9	53·6	53·5	53·9	54·7	55·6	56·5	57·2	57·7	57·9	57·8
61	3·7	2 48	51·8	51·4	50·8	50·5	50·4	50·9	51·7	52·7	53·7	54·4	55·0	55·2	55·1
62	3·9	2 49	18·8	17·9	17·0	16·3	16·2	16·8	18·0	19·5	20·9	22·0	22·8	23·0	22·7
63	4·1	2 52	54·7	54·0	53·1	52·5	52·4	52·9	54·0	55·4	56·8	57·8	58·6	58·8	58·6
64	4·0	2 55	29·8	29·5	29·0	28·6	28·6	28·9	29·6	30·5	31·4	32·1	32·6	32·7	32·6
65	3·4	2 57	32·6	32·0	31·3	30·7	30·5	30·8	31·6	32·6	33·7	34·5	35·0	35·1	34·8
66	2·8	3 01	17·1	16·7	16·3	15·9	15·8	16·2	16·9	17·8	18·7	19·4	19·9	20·1	20·0
67	4·2	3 01	33·3	32·8	32·3	31·8	31·7	32·0	32·7	33·6	34·6	35·3	35·8	36·0	35·8
68	3·1	3 03	25·4	24·6	23·8	23·1	22·9	23·4	24·5	25·9	27·3	28·4	29·2	29·5	29·3
69	3·7	3 03	57·5	57·0	56·4	55·9	55·8	56·2	57·1	58·3	59·4	60·3	60·9	61·2	61·1
70	2–3	3 06	56·0	55·5	54·8	54·3	54·2	54·6	55·5	56·7	57·8	58·7	59·4	59·7	59·6
71	4·2	3 07	41·8	41·2	40·4	39·8	39·7	40·1	41·2	42·5	43·8	44·9	45·6	46·0	45·8
72	3·9	3 11	15·9	15·5	14·9	14·4	14·2	14·5	15·2	16·1	17·1	17·9	18·4	18·6	18·4
73	3·9	3 18	40·4	40·0	39·5	39·0	38·8	39·1	39·7	40·6	41·6	42·3	42·9	43·1	43·0
74	4·3	3 19	10·5	09·9	09·2	08·5	08·3	08·5	09·2	10·3	11·4	12·4	13·0	13·2	12·9
75	1·9	3 22	58·2	57·6	56·8	56·1	55·9	56·3	57·3	58·6	59·9	61·0	61·8	62·2	62·1
76	3·8	3 23	47·4	47·1	46·6	46·2	46·1	46·4	47·1	48·0	48·9	49·6	50·2	50·5	50·5
77	3·7	3 26	08·4	08·0	07·6	07·2	07·0	07·3	08·0	08·9	09·8	10·6	11·1	11·4	11·4
78	4·4	3 27	32·2	31·4	30·4	29·4	29·1	29·5	30·7	32·3	33·9	35·3	36·3	36·9	36·7
79	4·3	3 29	49·5	49·1	48·7	48·2	48·1	48·4	49·0	50·0	50·9	51·7	52·2	52·6	52·6
80	3·8	3 32	02·2	01·8	01·3	00·9	00·7	01·0	01·6	02·5	03·3	04·1	04·6	04·9	04·9
81	4·3	3 32	57·1	56·7	56·1	55·6	55·4	55·6	56·2	57·1	58·1	58·8	59·4	59·7	59·6
82	4·4	3 35	54·3	53·9	53·5	53·0	52·9	53·1	53·7	54·6	55·5	56·2	56·8	57·1	57·1
83	3·1	3 41	34·6	34·1	33·3	32·7	32·4	32·7	33·6	34·8	36·1	37·2	38·0	38·5	38·5
84	3·7	3 42	20·4	20·1	19·6	19·1	18·9	19·1	19·7	20·6	21·5	22·2	22·8	23·1	23·1
85	3·9	3 43	07·8	07·4	06·8	06·3	06·1	06·4	07·1	08·1	09·2	10·1	10·8	11·2	11·3
86	3·8	3 43	44·9	44·6	44·1	43·6	43·4	43·7	44·4	45·3	46·3	47·1	47·8	48·2	48·3
87	3·9	3 43	54·4	54·0	53·3	52·7	52·4	52·7	53·5	54·7	55·8	56·9	57·7	58·2	58·2
88	3·8	3 43	59·1	57·8	56·3	55·0	54·1	54·1	54·8	56·2	58·0	59·5	60·5	60·7	60·1
89	4·3	3 46	02·1	01·7	01·1	00·6	00·4	00·5	01·1	01·9	02·9	03·7	04·3	04·6	04·6
90	3·0	3 46	21·4	21·1	20·6	20·1	19·9	20·1	20·8	21·8	22·7	23·6	24·3	24·7	24·7
91	3·2	3 47	34·4	32·2	29·7	27·4	25·9	25·6	26·6	28·6	31·1	33·5	34·9	35·1	33·9
92	3·8	3 48	02·1	01·7	01·2	00·7	00·5	00·8	01·5	02·4	03·4	04·2	04·9	05·3	05·4
93	4·2	3 48	45·0	44·5	43·8	43·2	42·9	43·0	43·6	44·5	45·5	46·4	47·0	47·3	47·2
94	2·9	3 52	56·4	56·1	55·5	55·0	54·8	55·0	55·7	56·7	57·7	58·7	59·4	59·9	59·9
95	3·0	3 56	35·0	34·6	33·9	33·3	33·1	33·3	34·0	35·1	36·3	37·3	38·1	38·6	38·7
96	3·2	3 57	08·8	08·5	08·0	07·5	07·2	07·4	07·9	08·7	09·6	10·4	11·1	11·4	11·5
97	4·0	3 57	44·1	43·8	43·2	42·6	42·4	42·6	43·3	44·3	45·4	46·4	47·2	47·7	47·8
98	4·4	3 58	28·1	27·0	25·7	24·4	23·6	23·5	24·1	25·4	26·9	28·3	29·3	29·6	29·2
99	3·9	3 59	37·8	37·5	37·0	36·6	36·4	36·5	37·1	38·0	38·9	39·7	40·4	40·8	40·9
100	3·9	4 02	08·9	08·6	08·1	07·7	07·4	07·6	08·2	09·0	09·9	10·7	11·4	11·8	11·9

The figures given refer to the beginning of the month, and should be interpolated to the actual date by means of the table on page 73.

Figure 5-5a [SALS, 1981]

DECLINATION OF STARS, 1981

No.	Name	Dec.	Jan.	F.	M.	Apr.	M.	J.	July	A.	S.	Oct.	N.	D.	Jan.
		° ′	″	″	″	″	″	″	″	″	″	″	″	″	″
51	δ Hydri	S 68 44	70	71	66	58	47	36	27	22	22	29	38	47	53
52	κ Eridani	S 47 46	102	104	101	95	85	75	65	60	59	63	71	79	85
53	ξ² Ceti	N 8 22	25	23	22	22	23	26	31	36	41	43	44	43	42
54	δ Ceti	N 0 14	40	38	37	38	41	45	51	56	61	62	61	59	57
55	ε Hydri	S 68 20	78	80	76	68	58	46	37	31	31	37	46	55	62
56	ι Eridani	S 39 55	91	94	92	86	78	68	59	52	50	54	60	68	75
57	γ Ceti	N 3 09	14	12	11	11	14	17	23	28	32	34	34	32	30
58	θ Persei	N 49 08	61	62	59	54	49	45	44	46	52	58	65	72	77
59	π Ceti	S 13 56	33	35	35	33	28	21	14	08	04	05	08	12	17
60	μ Ceti	N 10 01	58	56	55	54	55	58	62	68	72	75	76	75	74
61	41 Arietis	N 27 10	57	57	55	52	50	50	52	56	60	65	69	72	73
62	η Persei	N 55 48	69	70	68	63	57	52	50	51	56	63	71	79	84
63	τ Persei	N 52 40	73	75	72	68	62	57	55	57	62	69	76	83	88
64	η Eridani	S 8 58	36	38	39	37	33	27	20	14	11	10	13	17	20
65*	θ Eridani	S 40 22	69	73	71	66	58	48	39	32	29	32	39	47	54
66	α Ceti	N 4 00	49	47	46	46	48	51	56	62	66	68	68	66	64
67	τ³ Eridani	S 23 41	70	73	73	70	64	56	47	41	38	39	43	50	55
68	γ Persei	N 53 25	63	65	63	59	53	48	46	47	52	58	65	73	78
69	ρ Persei	N 38 45	64	65	63	60	56	53	53	56	60	66	71	76	79
70	*Algol* (β Persei)	N 40 52	62	63	61	57	53	50	50	52	57	62	68	73	76
71	ι Persei	N 49 32	34	35	34	30	24	20	18	20	24	30	37	43	48
72	α Fornacis	S 29 03	58	62	61	57	50	42	33	26	23	24	30	37	43
73	16 Eridani	S 21 49	50	54	54	51	45	38	30	23	19	20	24	30	36
74	BS 1008 (Eri.)	S 43 08	50	54	53	48	40	30	20	13	10	13	20	28	35
75	α Persei	N 49 47	43	45	44	40	35	31	29	30	33	39	45	52	57
76	ο Tauri	N 8 57	39	37	36	36	37	39	43	48	52	55	55	54	53
77	ξ Tauri	N 9 39	55	54	52	52	53	55	59	64	68	71	71	70	69
78	BS 1035 (Cam.)	N 59 52	35	38	38	33	27	21	17	17	21	26	34	42	49
79	5 Tauri	N 12 52	14	13	12	11	12	13	17	21	25	28	29	29	28
80	ε Eridani	S 9 31	31	34	35	33	29	23	17	11	07	06	09	13	17
81	τ⁵ Eridani	S 21 41	60	64	65	62	56	49	41	34	30	31	35	41	47
82	10 Tauri	N 0 20	22	20	19	19	21	25	31	36	40	41	40	38	35
83	δ Persei	N 47 43	42	44	44	40	36	31	29	30	33	38	43	50	55
84	δ Eridani	S 9 49	49	52	53	52	48	42	35	29	25	24	27	31	36
85	ο Persei	N 32 13	43	43	43	40	38	36	36	38	42	46	50	53	55
86	17 Tauri	N 24 03	13	13	12	10	09	08	10	13	17	20	22	24	25
87	ν Persei	N 42 31	11	13	12	09	05	02	00	01	04	09	14	19	23
88	β Reticuli	S 64 51	81	86	85	80	71	60	49	41	39	42	50	60	68
89	τ⁶ Eridani	S 23 18	34	38	39	36	30	23	15	08	04	04	09	15	21
90	η Tauri	N 24 02	46	46	45	44	43	42	44	47	50	54	56	58	59
91	γ Hydri	S 74 17	73	77	76	71	61	50	40	32	30	33	42	52	60
92	27 Tauri	N 23 59	43	43	42	40	39	39	40	43	47	50	52	54	55
93	BS 1195 (Eri.)	S 36 15	43	48	49	45	38	29	19	12	08	09	15	23	31
94	ζ Persei	N 31 49	39	40	39	37	35	33	33	35	39	42	46	49	51
95	ε Persei	N 39 57	22	24	23	21	17	14	13	14	17	21	25	30	34
96	γ Eridani	S 13 33	55	59	60	58	54	48	41	35	30	30	33	38	43
97	ξ Persei	N 35 44	13	14	14	12	09	06	06	07	10	14	18	22	25
98	δ Reticuli	S 61 26	94	99	99	94	86	75	64	56	53	55	63	73	82
99	λ Tauri	N 12 26	08	07	06	06	06	08	11	15	19	21	22	21	20
100	ν Tauri	N 5 56	07	05	04	04	05	08	12	17	21	23	22	20	18

* No., mag., dist. and p.a. of companion star: 65, 4·4, 8″, 88°

Figure 5-5b [SALS, 1981]

POLE STAR TABLE, 1981

L.S.T.	6^h		7^h		8^h		9^h		10^h		11^h	
	a_0	b_0	a_0	b_0	a_0	b_0	a_0	b_0	a_0	b_0	a_0	b_0
m	′	′	′	′	′	′	′	′	′	′	′	′
0	−26.5	−41.6	−14.8	−47.0	− 2.2	−49.2	+10.6	−48.0	+22.7	−43.5	+33.2	−36.1
3	26.0	42.0	14.2	47.2	1.5	49.2	11.3	47.8	23.2	43.2	33.6	35.7
6	25.4	42.3	13.6	47.4	0.9	49.2	11.9	47.7	23.8	42.9	34.1	35.3
9	24.8	42.7	13.0	47.6	− 0.2	49.2	12.5	47.5	24.4	42.6	34.5	34.8
12	24.3	43.0	12.3	47.7	+ 0.4	49.2	13.1	47.3	24.9	42.2	35.0	34.4
15	−23.7	−43.3	−11.7	−47.9	+ 1.1	−49.2	+13.7	−47.1	+25.5	−41.9	+35.4	−33.9
18	23.2	43.6	11.1	48.0	1.7	49.2	14.4	46.9	26.0	41.6	35.9	33.4
21	22.6	43.9	10.5	48.2	2.4	49.1	15.0	46.7	26.6	41.2	36.3	33.0
24	22.0	44.2	9.8	48.3	3.0	49.1	15.6	46.5	27.1	40.9	36.8	32.5
27	21.4	44.5	9.2	48.4	3.6	49.0	16.2	46.3	27.6	40.5	37.2	32.0
30	−20.8	−44.7	− 8.6	−48.5	+ 4.3	−49.0	+16.8	−46.1	+28.2	−40.2	+37.6	−31.5
33	20.3	45.0	7.9	48.6	4.9	48.9	17.4	45.9	28.7	39.8	38.0	31.0
36	19.7	45.3	7.3	48.7	5.6	48.8	18.0	45.7	29.2	39.4	38.4	30.6
39	19.1	45.5	6.6	48.8	6.2	48.8	18.6	45.4	29.7	39.0	38.8	30.1
42	18.5	45.8	6.0	48.9	6.8	48.7	19.2	45.2	30.2	38.6	39.2	29.5
45	−17.9	−46.0	− 5.4	−49.0	+ 7.5	−48.6	+19.8	−44.9	+30.7	−38.2	+39.6	−29.0
48	17.3	46.2	4.7	49.0	8.1	48.5	20.4	44.6	31.2	37.8	39.9	28.5
51	16.7	46.4	4.1	49.1	8.7	48.3	20.9	44.4	31.7	37.4	40.3	28.0
54	16.1	46.6	3.4	49.1	9.4	48.2	21.5	44.1	32.2	37.0	40.7	27.5
57	15.4	46.8	2.8	49.1	10.0	48.1	22.1	43.8	32.7	36.6	41.0	26.9
60	−14.8	−47.0	− 2.2	−49.2	+10.6	−48.0	+22.7	−43.5	+33.2	−36.1	+41.4	−26.4
Lat.	a_1	b_1	a_1	b_1	a_1	b_1	a_1	b_1	a_1	b_1	a_1	b_1
°												
0	−.3	+.3	−.4	+.2	−.4	−.1	−.4	−.3	−.3	−.4	−.2	−.4
10	−.3	+.3	−.3	+.1	−.4	−.1	−.3	−.2	−.2	−.3	−.2	−.4
20	−.2	+.2	−.3	+.1	−.3	.0	−.3	−.2	−.2	−.3	−.1	−.3
30	−.2	+.2	−.2	+.1	−.2	.0	−.2	−.1	−.1	−.2	−.1	−.2
40	−.1	+.1	−.1	.0	−.1	.0	−.1	−.1	−.1	−.1	−.1	−.1
45	−.1	+.1	−.1	.0	−.1	.0	−.1	.0	.0	−.1	.0	−.1
50	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
55	+.1	−.1	+.1	.0	+.1	.0	+.1	+.1	+.1	+.1	.0	+.1
60	+.2	−.1	+.2	−.1	+.2	.0	+.2	+.1	+.1	+.2	+.1	+.2
62	+.2	−.2	+.2	−.1	+.2	.0	+.2	+.2	+.2	+.2	+.1	+.2
64	+.2	−.2	+.3	−.1	+.3	.0	+.3	+.2	+.2	+.3	+.1	+.3
66	+.3	−.3	+.4	−.1	+.4	+.1	+.3	+.2	+.3	+.3	+.2	+.4
Month	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2	a_2	b_2
Jan.	−.2	+.2	−.2	+.1	−.3	+.1	−.3	.0	−.3	−.1	−.2	−.1
Feb.	.0	+.1	−.1	+.1	−.1	+.1	−.1	+.1	−.1	.0	−.1	.0
Mar.	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
Apr.	.0	−.2	+.1	−.1	+.1	−.1	+.1	−.1	+.1	−.1	+.2	.0
May	−.1	−.3	.0	−.3	+.1	−.3	+.1	−.2	+.2	−.2	+.2	−.1
June	−.2	−.3	−.1	−.4	.0	−.4	+.1	−.4	+.2	−.3	+.3	−.3
July	−.4	−.3	−.3	−.4	−.2	−.4	−.1	−.5	+.1	−.5	+.2	−.4
Aug.	−.5	−.2	−.4	−.3	−.3	−.4	−.2	−.5	−.1	−.5	.0	−.5
Sept.	−.6	.0	−.5	−.1	−.5	−.3	−.4	−.4	−.3	−.5	−.2	−.5
Oct.	−.6	+.2	−.6	.0	−.6	−.1	−.5	−.3	−.5	−.4	−.3	−.5
Nov.	−.5	+.4	−.6	+.2	−.6	+.1	−.6	−.1	−.6	−.2	−.5	−.4
Dec.	−.4	+.5	−.5	+.4	−.6	+.2	−.6	+.1	−.6	−.1	−.6	−.2

Latitude = Corrected observed altitude of *Polaris* + $a_0 + a_1 + a_2$

Azimuth of *Polaris* = $(b_0 + b_1 + b_2)$ sec (latitude)

Figure 5-6 [SALS, 1981]

6. VARIATIONS IN CELESTIAL COORDINATES

The mathematical models for the determination of astronomic latitude, longitude, and azimuth require the use of a celestial bodies apparent (true) coordinates for the epoch of observation. The coordinates (α,δ) of the stars and the sun (the most often observed celestial bodies for surveying purposes) are given in star catalogues, ephemerides, and almanacs for certain predicted epochs that do not (except in exceptional circumstances) coincide with the epoch of observation. To fulfill the requirements stated previously, the coordinates (α,δ) must be updated. As was indicated in Chapter 5, the updating procedure has been made very simple for users of annual publications such as AA, SALS, and APFS. The question is, how are the coordinates in the annual publications derived, and how should one proceed when using one of the fundamental star catalogues (eg. FK4). The aim of this chapter is to answer this question.

In previous discussions in these notes, coordinates in the Right Ascension system have been considered constant with respect to time, and in the Hour Angle and Horizon systems, changes occurred only as a result of earth rotation. In this chapter, we consider the following motions in the context of the Right Ascension system:

(i) precession and nutation: motions of the coordinate system relative to the stars;

(ii) proper motion: relative motion of the stars with respect to each other;

(iii) refraction, aberration, parallax: apparent displacement of stars due to physical phenomena;

(iv) polar motion: motion of the coordinate system with respect to the solid earth.

Except for polar motion, the above factors are discussed in terms of their effects on α and δ of a celestial body. This leads to variations in the Right Ascension coordinate system. The definition of the variations are given below and Figure 6-1 outlines their interrelationships.

(i) Mean Place: heliocentric, referred to some specified mean equator and equinox.

(ii) True Place: heliocentric, referred to the true equator and equinox of date when the celestial body is actually observed.

(iii) Apparent Place: geocentric, referred to the true equator and equinox of date when the celestial body is actually observed.

(iv) Observed Place: topocentric, as determined by means of direct readings on some instrument corrected for systematic instrumental errors (e.g. dislevellment, collimation).

The definitions are amplified in context in the following sections.

6.1 Precession, Nutation, and Proper Motion

The resultant of the attractive forces of the sun, moon, and other planets on our non-symmetrical, non-homogeneous earth, causes a moment which tries to rotate the equatorial plane into the ecliptic plane. We view this as the motion of the earth's axis of rotation about the ecliptic pole. This motion is referred to as precession. The predominant effect of luni-solar precession is a westerly motion of the equinox along the equator of $50''.3$ per year. The period of the motion is approximately 26000 years, and its amplitude (obliquity of the ecliptic) is approximately $23^\circ.5$. Superimposed on this planetary precession, which

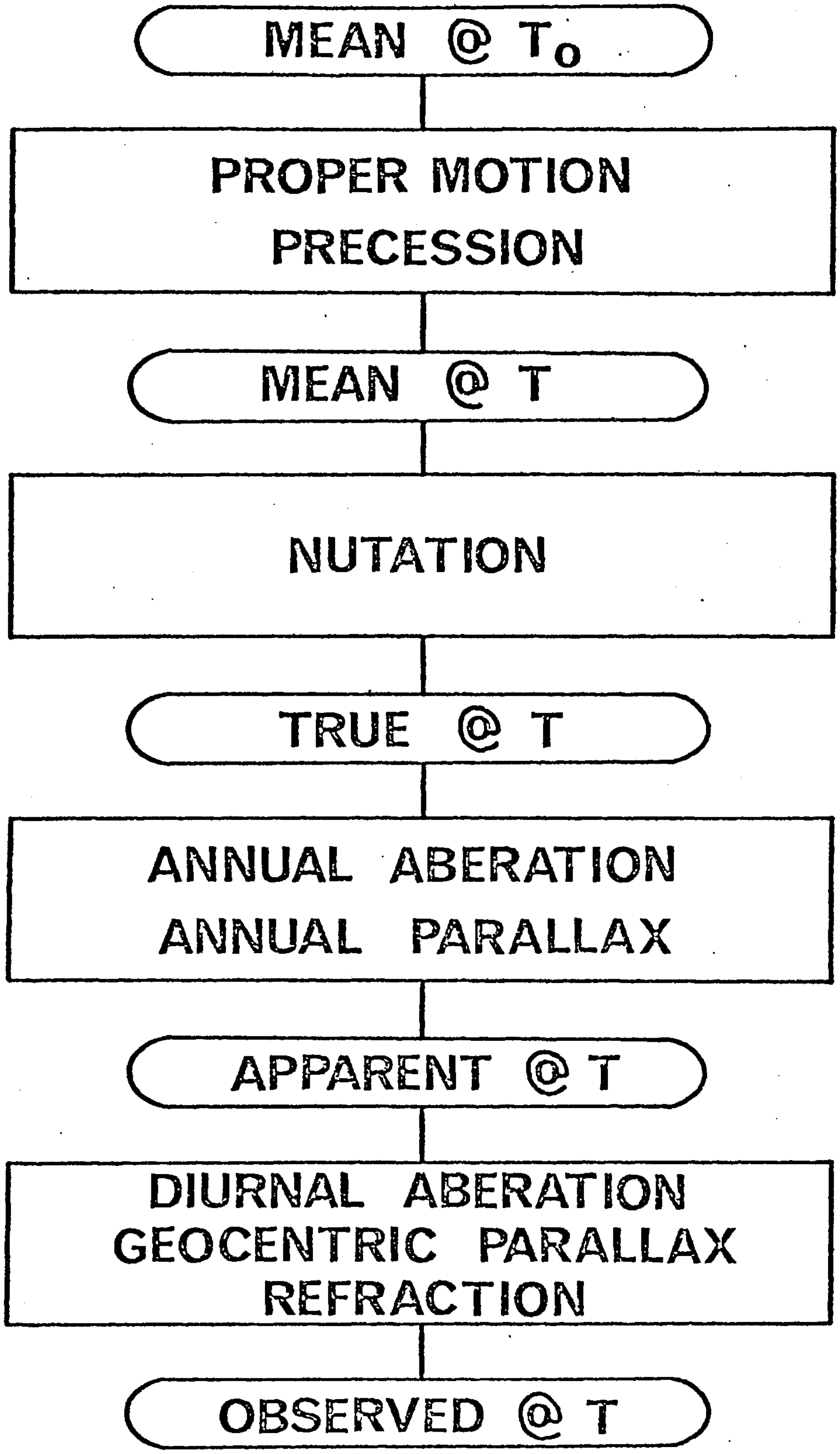


Figure 6-1

Variations of the Right Ascension System causes a westerly motion of the equinox of 12".5 per century, and a change in the obliquity of the ecliptic of 47" per century. General precession is then the sum of luni-solar and planetary precession.

Within the long period precession is the shorter period astronomic nutation. The latter is the result of the earth's motion about the sun, the moon about the earth, and the moon's orbit not lying in the ecliptic plane. The period of this motion is about 19 years, with an amplitude of 9".

General precession, with astronomic nutation superimposed, is shown in Figure 6-2.

Each star appears to have a small motion of its own, designated as its proper motion. This motion is the resultant of the actual motion of the star in space and of its apparent motion due to the changing direction arising from the motion of the sun.

In Chapter 5, it was stated that certain epochs T_o have been chosen as standard epochs to which tabulated mean celestial coordinates (α_o, δ_o) of celestial bodies refer. The mean celestial system is completely defined by:

- a heliocentric origin,
- a primary (Z) pole that is precessing (not nutating), and is called the mean celestial pole,
- a primary axis (X) that is precessing (not nutating), following the motion of the mean vernal equinox,
- a Y-axis that makes the system right-handed.

Now, to define a set of coordinates (α, δ) for an epoch T, we must update from T_o to T, since for every epoch T, a different mean celestial system

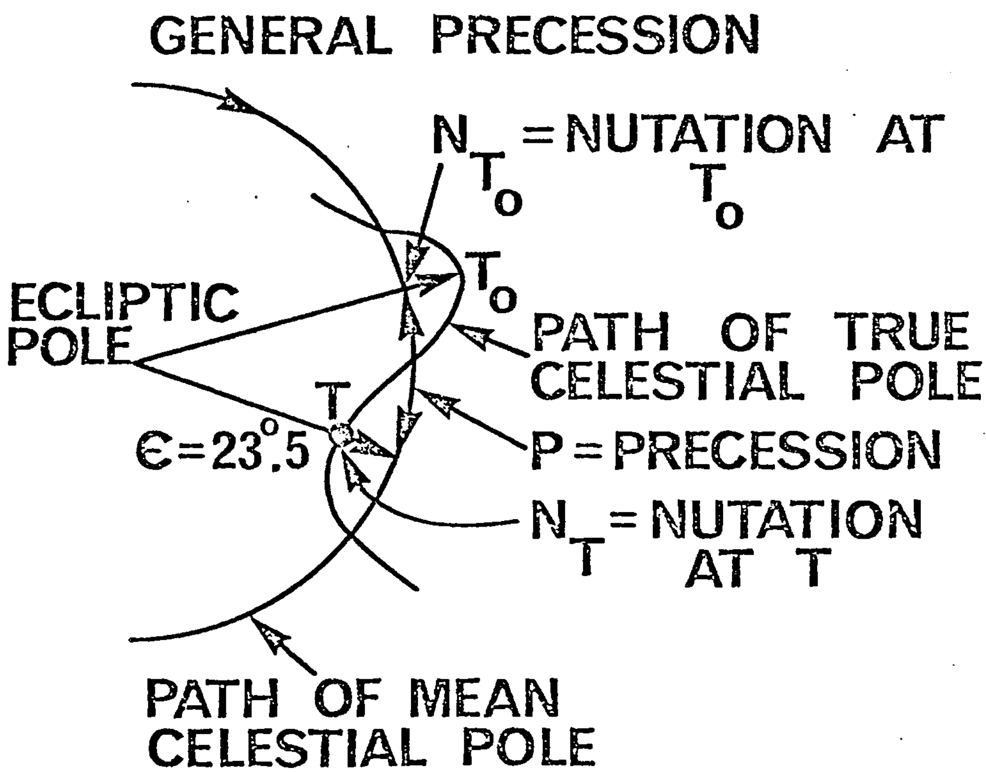


Figure 6-2

Motion of the Celestial Pole

is defined. The relationship between mean celestial systems is defined in terms of <u>precessional elements</u> (ζ_o, θ, z) (Figure 6-3) and <u>proper motion</u> elements (μ_o^a, μ_o^d).

Expressions for the precessional elements were derived by Simon Newcomb around 1900, and are [e.g. Mueller, 1969]

$$\zeta_o = (2304''.250 + 1''.396 t_o)\, t + 0''.302 t^2 + 0''.018 t^3 \quad , \tag{6-1}$$

$$z = \zeta_o + 0''.791 t^2 + 0''.001 t^3 \quad , \tag{6-2}$$

$$\theta = (2004''.682 - 0''.853 t_o) t - 0''.426 t^2 - 0''.042 t^3 \tag{6-3}$$

in which the initial epoch is $T_o = 1900.0 + t_o$, and the final epoch $T = 1900.0 + t_o + t$, in which t_o and t are measured in tropical centuries. The precessional elements (ζ_o, z, θ) are tabulated for the beginning of the current year in AA. From Figure 6-3, it can be seen that

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_T} = R_3(-z) R_2(\theta) R_3(-\zeta_o) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_{T_o}} \quad , \tag{6-4}$$

or, setting

$$P = R_3(-z) R_2(\theta) R_3(-\delta_o) \quad , \tag{6-5}$$

then

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_T} = P \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_{T_o}} \quad . \tag{6-6}$$

Proper motion of a star is accounted for in the mean right ascension system, and its effects on (α_o, δ_o) are part of the update from a mean place at T_o to a mean place at T. The transformation (given here without proof) is

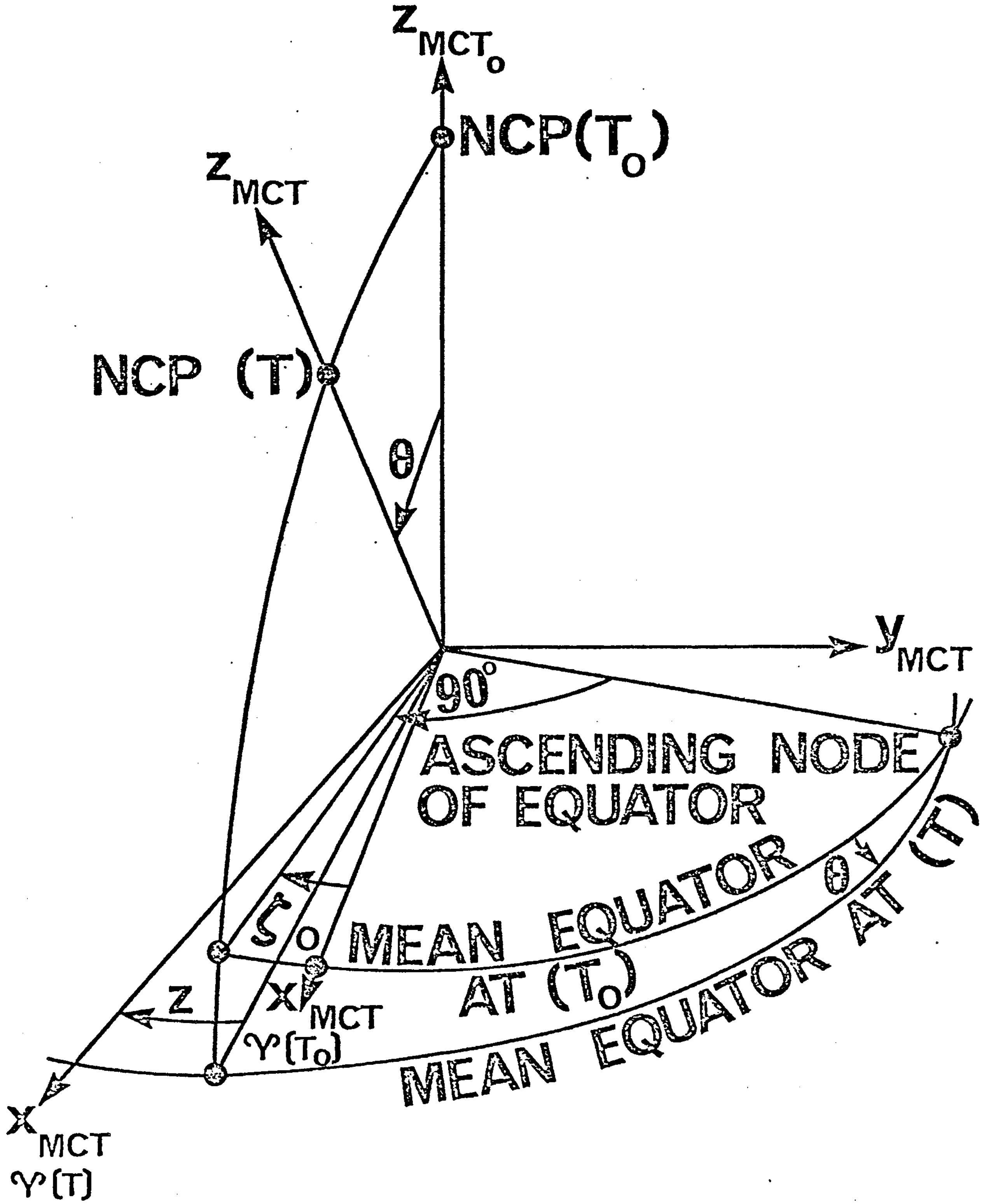


Figure 6-3

Mean Celestial Coordinate Systems

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_T} = R_3(-\alpha_o)R_2(\delta_o-90)R_3(\Psi_o)R_2(\mu t + \frac{1}{2}\frac{d\mu}{dt}t^2)R_3(-\Psi_o)R_2(90-\delta_o)R_3(\alpha_o) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_{T_o}}, \quad (6\text{-}7)$$

in which $t = T - T_o$ (in years), μ and $\frac{d\mu}{dt}$ are the annual tangential component of proper motion and rate of change of that component, and Ψ_o is the direction (azimuth) of proper motion at T_o. The quantities tabulated in a Fundamental Catalogue such as the FK4 are μ_o^α, μ_o^δ, the annual components of proper motion in right ascension and declination, and their rates of change per one hundred years, $d\mu_o^\alpha / dt$, $d\mu_o^\delta/dt$. The annual μ, $d\mu/dt$, and Ψ_o are then given by

$$\mu = ((\mu_o^\alpha \cos \delta_o)^2 + (\mu_o^\delta)^2)^{\frac{1}{2}} ,$$

$$d\mu/dt = \frac{\mu_o^\alpha \cos^2\delta_o \frac{d\mu_o^\alpha}{dt} + \mu_o^\delta \frac{d\mu_o^\delta}{dt}}{100((\mu_o^\alpha \cos \delta_o)^2 + (\mu_o^\delta)^2)^{\frac{1}{2}}} ,$$

$$\Psi_o = \sin^{-1} \left(\frac{\mu_o^\alpha \cos\delta_o}{\mu} \right) = \cos^{-1} \frac{\mu_o^\delta}{\mu} .$$

Setting

$$M = R_3(-\alpha_o)R_2(\delta_o - 90)R_3(\Psi_o)R_2(\mu t + \frac{1}{2} \frac{d\mu}{dt} t^2)R_3(-\Psi_o)R_2(90-\delta_o)R_3(\alpha_o) , \quad (6\text{-}8)$$

(6-7) is rewritten as

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_T} = M \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_{T_o}} \quad (6\text{-}9)$$

Now, combining (6-6) and (6-9), the complete update from a mean place at T_o to a mean place at T is given by

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_T} = PM \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_{T_o}} . \quad (6\text{-}10)$$

The effects of astronomic nutation, which must be accounted for to update from the mean celestial system at T to the true celestial system at T (Figure 6-1), are expressed as <u>nutation in longitude</u> ($\Delta\Psi$) and <u>nutation in the obliquity</u> ($\Delta\varepsilon$). Expressions for $\Delta\Psi$ and $\Delta\varepsilon$ have been developed, and for our purposes, values are tabulated (e.g. AA). The true celestial system at T to which (α_o, δ_o) are updated is defined by:

- a heliocentric origin,
- a primary pole (Z) that is the true celestial pole following the precessing and nutating axis of rotation,
- a primary axis (X) that is the true precessing and nutating vernal equinox,
- a Y-axis that makes the system right-handed.

The true and mean celestial systems are shown in Figure 6-4, and from this, we can deduce the transformation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{TC_T} = R_1(-\varepsilon-\Delta\varepsilon)R_3(-\Delta\Psi)R_1(\varepsilon) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_T} . \quad (6\text{-}11)$$

Designating

$$N = R_1(-\varepsilon-\Delta\varepsilon)R_3(-\Delta\Psi)R_1(\varepsilon) \quad , \quad (6\text{-}12)$$

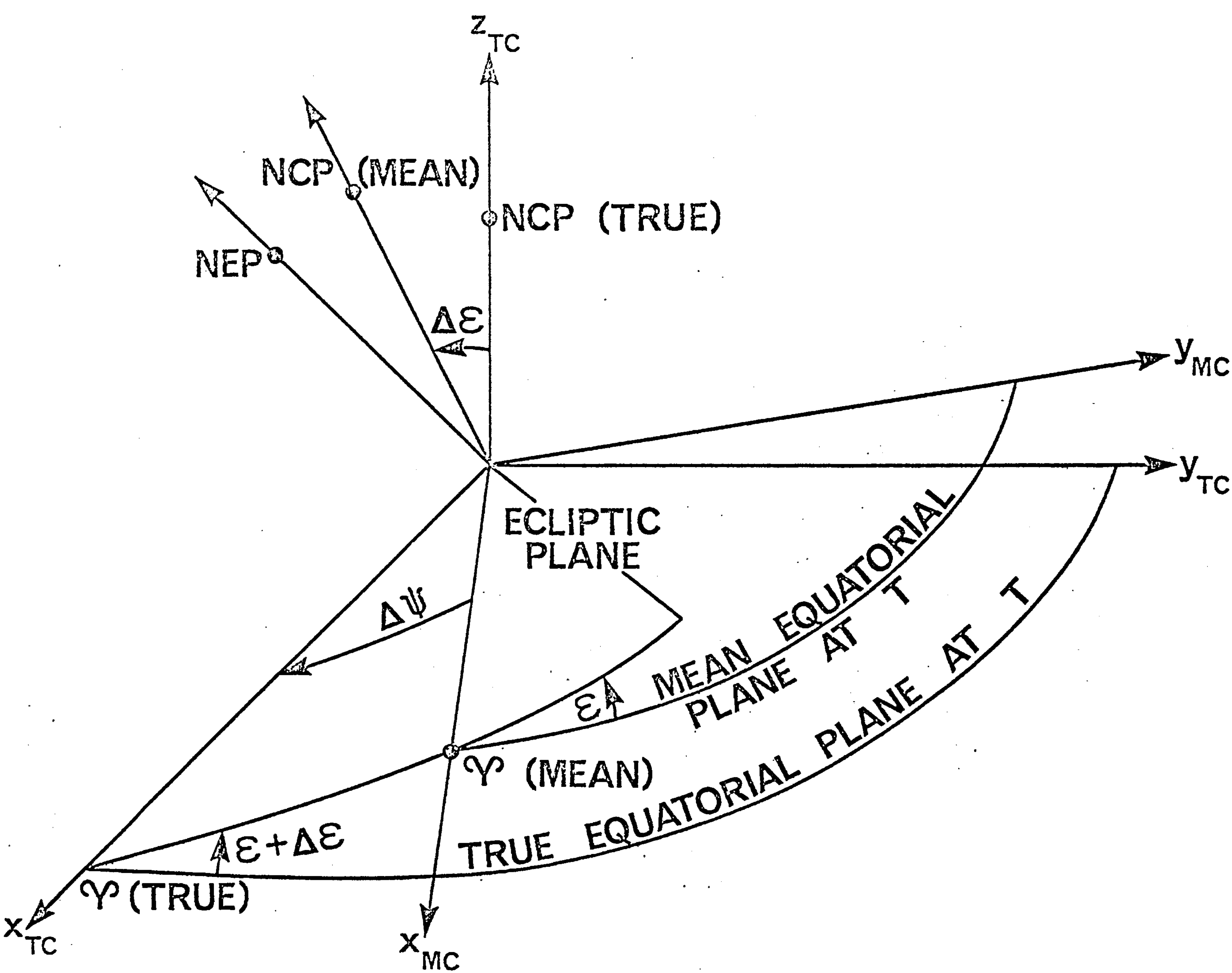


Figure 6-4

True and Mean Celestial Coordinate Systems

(6-11) is written

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{TC_T} = N \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_T} . \tag{6-13}$$

Combining (6-6), (6-9), and (6-11), the update from mean celestial at T_o to true celestial at T is given in one expression, namely

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{TC_T} = NPM \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_{T_o}} . \tag{6-14}$$

6.2 Annual Aberration and Parallax

The apparent place celestial system is the one in which the astronomic coordinates (Φ,Λ) are expressed, thus it is in this system that the mathematical models (relating observables, known and unknown quantities) are formulated. This requires that the (α_o,δ_o) of a celestial body be updated to the apparent place system, defined as having:

- an origin coincident with the earth's centre of gravity,
- a primary pole (Z) coincident with the earth's instantaneous rotation axis,
- a primary axis (X) coincident with the instantaneous vernal equinox,
- a Y-axis that makes the system right-handed.

Evidently, the main change is a shift in origin. This gives rise to two physical effects (i) annual parallax due to the shift in origin, and (ii) annual aberration due to the revolution of the new origin about the heliocentre.

Aberration is the apparent displacement of a celestial body caused by the finite velocity of light propagation combined with the relative motion of the observer and the body. The aberration we are concerned with here is a subset of planetary aberration, namely annual aberration. From the general law of aberration depicted in Figure 6-5, and our knowledge of the earth's motion about the heliocentre, the changes in coordinates due to annual aberration ($\Delta\alpha_A, \Delta\delta_A$) are given by [e.g. Mueller, 1969]

$$\Delta\alpha_A = -\kappa \sec\delta \; (\cos\alpha \; \cos\lambda_s \; \cos\varepsilon + \sin\alpha \; \sin\lambda_s), \qquad (6\text{-}15)$$

$$\Delta\delta_A = -\kappa(\cos\lambda_s \cos\varepsilon(\tan\varepsilon \cos\delta - \sin\alpha \; \sin\lambda_s). \qquad (6\text{-}16)$$

In (6-15) and (6-16), λ_s is the ecliptic longitude of the sun, α,δ are in the true celestial system at T, and κ is the constant of aberration for the earth's orbit (assumed circular for our purposes), given numerically as $\kappa = 20''\!.4958$. In matrix form, the change in the position vector is given by

$$A = \begin{bmatrix} -D \\ C \\ C \tan\varepsilon \end{bmatrix}, \qquad (6\text{-}16)$$

in which C and D are referred to as the aberrational (Besselian) day numbers (Note: $C = -\kappa \cos\varepsilon \cos\lambda_s$; $D = -\kappa \sin\lambda_s$). These quantities are tabulated, for example in AA.

Parallactic displacement is defined as the angle between the directions of a celestial object as seen from an observer and from some standard point of reference. Annual (Stellar) Parallax occurs due to the separation of the earth and sun, and is the difference between a geocentric direction and a heliocentric direction to a celestial body (Figure 6-6). Expressed as changes to (α,δ) we have

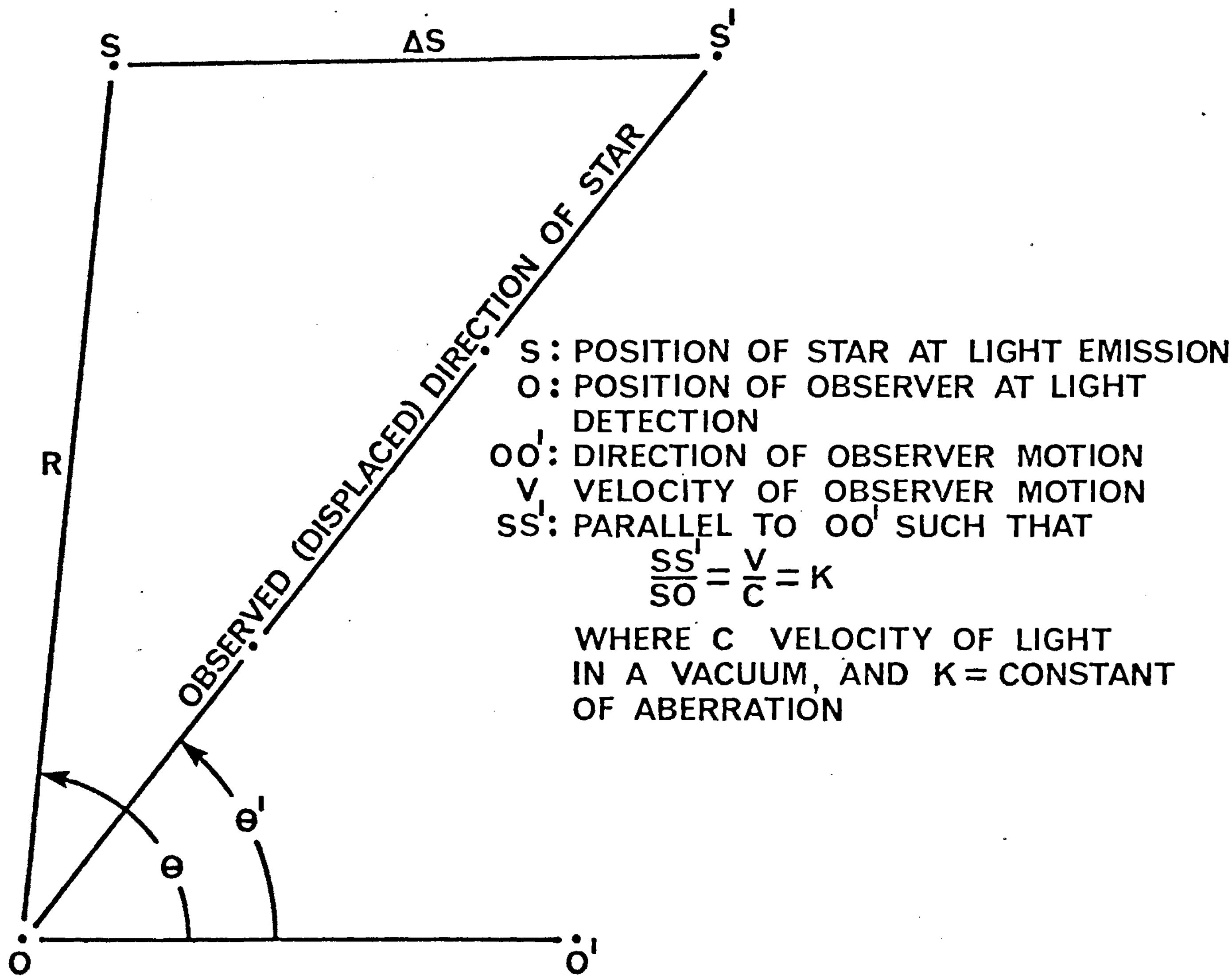
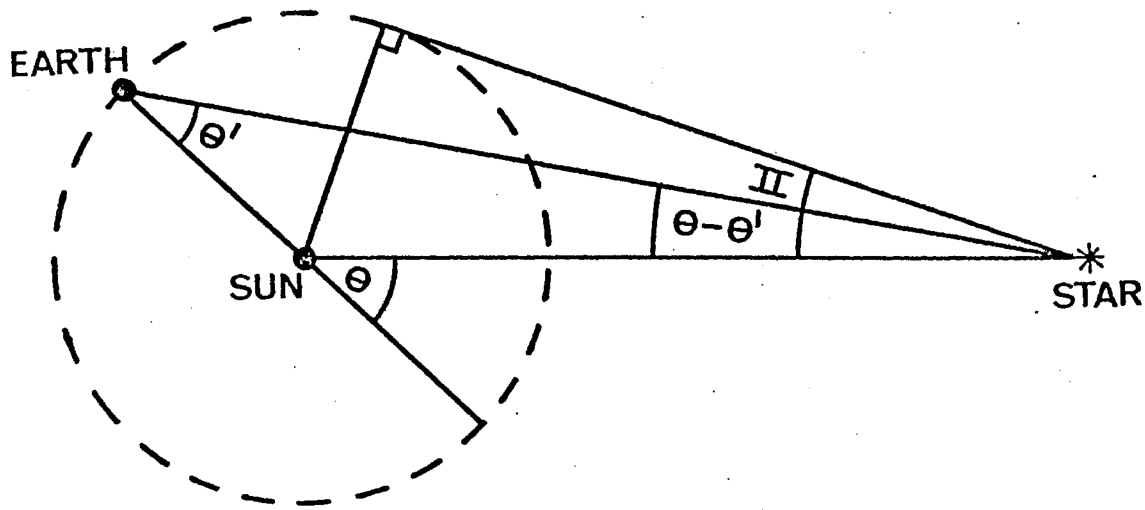


Figure 6-5

Aberration



θ : HELIOCENTRIC DIRECTION
θ' : GEOCENTRIC DIRECTION
Π : ANNUAL (STELLAR) PARALLAX

Figure 6-6

Annual (Stellar) Parallax

$$\Delta\alpha_p = \Pi \ (\cos\alpha \ \cos\varepsilon \ \sin\lambda_s - \sin\alpha \ \cos\lambda_s) \ \sec\delta \quad , \tag{6-17}$$

$$\Delta\delta_p = \Pi \ (\cos\delta \ \sin\varepsilon \ \sin\lambda_s - \cos\alpha \ \sin\delta \ \cos\lambda_s - \sin\alpha \ \sin\delta \ \cos\varepsilon \ \sin\lambda_s), \tag{6-18}$$

in which the new quantity Π is the annual (stellar) parallax. Π is a small quantity - for the nearest star it is 0".76. As for the computation of the effects of aberration, α,δ in (6-17) and (6-18) are in the true celestial system at T. Using the Besselian Day Numbers C and D, the changes in the position vector are given by

$$R = \begin{bmatrix} -C & \sec\varepsilon \\ -D & \cos\varepsilon \\ -D & \sin\varepsilon \end{bmatrix} (\Pi/\kappa) \quad . \tag{6-19}$$

To update (α,δ) from the True Celestial system at T to the Apparent Celestial system at T, we write the expression

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{AC_T} = A + R + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{TC_T} \tag{6-20}$$

Finally, the complete update from Mean Celestial system at T_o to Apparent Celestial at T is given by the expression

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{A.P._T} = A + R + NPM \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{MC_{T_o}} \quad . \tag{6-21}$$

This entire process is summarised in Figure 6-7, and all symbols used in Figure 6-7 are explained in Table 6-1.

CATALOGUED (Heliocentric) $\alpha_o, \delta_o, (T_o)$

CATALOGUED X_o, Y_o, Z_o

$$\begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix} = \begin{bmatrix} \cos\delta_o \cos\alpha_o \\ \cos\delta_o \sin\alpha_o \\ \sin\delta_o \end{bmatrix}$$

PROPER MOTION

$$M = R_3(-\alpha_o) R_2(\delta_o - 90) R_3(\psi_o) R_2(\mu t + 1/2 \tfrac{d\mu}{dt} t^2) R_3(-\psi_o) R_2(90 - \delta_o) R_3(\alpha_o)$$

PRECESSION

$$P = R_3(-z) R_2(\theta) R_3(-\zeta_o)$$

NUTATION

$$N = R_1(-\varepsilon - \Delta\varepsilon) R_3(-\Delta\psi) R_1(\varepsilon)$$

ANNUAL ABERRATION

$$A = \begin{bmatrix} -D \\ C \\ C\tan\varepsilon \end{bmatrix}$$

ANNUAL PARALLAX

$$R = -\begin{bmatrix} C\sec\varepsilon \\ D\cos\varepsilon \\ D\sin\varepsilon \end{bmatrix} \pi/k$$

APPARENT (X, Y, Z)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R + A + NPM \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix}$$

APPARENT (Geocentric) $\alpha', \delta', (T)$

$$\alpha' = \tan^{-1} y/x$$

$$\delta' = \sin^{-1} Z = \tan^{-1} Z/(X^2+Y^2)^{1/2}$$

FIGURE 6-7

POSITION UPDATING $M.C._{T_o}$ to $A.P._T$

Description of Terms

Symbol	Description	Where Obtained
(α_o, δ_o)	Right ascension and declination of the star at epoch T_o	From Fundamental Catalogue
(X_o, Y_o, Z_o)	Direction numbers of star at T_o	From (α_o, δ_o)
μ $\frac{d\mu}{dt}$	Annual Tangential component of proper motion of star at T_o. Rate of change of proper motion	From Fundamental Catalogue
t	Time interval in years	$t=T-T_o$
ψ_o	Direction (azimuth) of proper motion at T_o	From Fundamental Catalogue
(ζ_o, z, θ)	Precessional Elements	From Astronomical
$(\Delta\psi, \Delta\varepsilon)$	Nutational Elements	Ephemeris
ε	Obliquity of the ecliptic	From Astronomical
(C,D)	Aberrational Day Numbers	Ephemeris
II	Stellar Parallax of Star	From Fundamental Catalogue
κ	Constant of annual Aberration	κ=20".4958
(X,Y,Z)	Direction numbers of the star at epoch T	Computed from all the above parameters
(α^1, δ^1)	Right ascension and declination of the star at epoch T, referred to the Apparent (Geocentric) System.	Computed from (X,Y,Z) at time T

TABLE 6-1

6.3 Diurnal Aberration, Geocentric Parallax, and Astronomic Refraction.

At the conclusion of the previous section (6.2), we had (α,δ) expressed in the Apparent Place system at T. At time T, an observer observes a celestial body relative to the Observed Place system. The latter is defined by

- an origin defined by the observers position,
- a primary pole (Z) parallel to the true instantaneous celestial pole,
- a primary axis (X) parallel to the true vernal equinox,
- a Y-axis that makes the system right-handed.

Evidently, the Observed Place is simply a translated Apparent Place system. Our task now is to move from the Observed Place to the Apparent Place where our mathematical models for position determination are formulated. This requires corrections for (i) diurnal aberration, (ii) geocentric parallax, and (iii) astronomic refraction.

Diurnal aberration is the displacement of the direction to a celestial body due to the rotation of the earth. The diurnal constant of aberration, k, is expressed as a function of the earth's rotation (ω_e), the geocentric latitude of an observer (Ψ), and the length of the observer's geocentric position vector (ρ). This yields [e.g. Mueller, 1969]

$$k = 0.320\ \rho\ \cos\Psi = 0.0213\ \rho\ \cos\Psi \qquad (6\text{-}22)$$

The changes in a celestial objects coordinates are given by

$$\Delta\alpha_D = k^s \cosh\ \sec\delta^1, \qquad (6\text{-}23)$$

$$\Delta\delta_D = k'' \sinh\ \sin\delta^1. \qquad (6\text{-}24)$$

The convention adopted here for signs is

$$\Delta\alpha_D = \alpha_{AP_T} - \alpha_{OP_T}, \quad (6\text{-}25)$$

$$\Delta\delta_D = \delta_{AP_T} - \delta_{OP_T}. \quad (6\text{-}26)$$

This same convention applies to the corrections for geocentric parallax and astronomic refraction that are treated next.

Geocentric parallax is the difference between the observed topocentric direction and the required geocentric direction (Figure 6-8). The changes, expressed in terms of the celestial objects right ascension and declination are

$$\Delta\alpha_G = -\pi \sinh \operatorname{cosec} z \cos\Phi \sec\delta' \ , \quad (6\text{-}27)$$

$$\Delta\delta_G = \pi (\sin\Phi \operatorname{cosec} z \sec\delta' - \tan\delta' \cot z). \quad (6\text{-}28)$$

It should be noted that for celestial bodies other than the sun, $\Delta\alpha_G$, $\Delta\delta_G$ are so small as to be negligible.

Astronomic refraction is the apparent displacement of a celestial object lying outside our atmosphere that results from light rays being bent in passing through the atmosphere. In general, the light rays bend downward, thus a celestial object appears at a higher altitude than is really true (Figure 6-9). The astronomic refraction angle is defined by

$$\Delta z_R = z - z' \ , \quad (6\text{-}29)$$

in which z' is the observed zenith distance, z is the corrected zenith distance. The effects on α, δ are

$$\Delta\alpha_R = -\Delta z_R \sinh \operatorname{cosec} z' \cos\Phi \sec\delta' \ , \quad (6\text{-}30)$$

$$\Delta\delta_R = -\Delta z_R (\sin\Phi \operatorname{cosec} z' \sec\delta' - \tan\delta' \cot z'). \quad (6\text{-}31)$$

Δz_R values are usually tabulated for some standard temperature and pressure (usually 760mm Hg, $T = 10^{\circ}C$, relative humidity 60%) . Corrections to those values are obtained through determinations of temperature, pressure, and

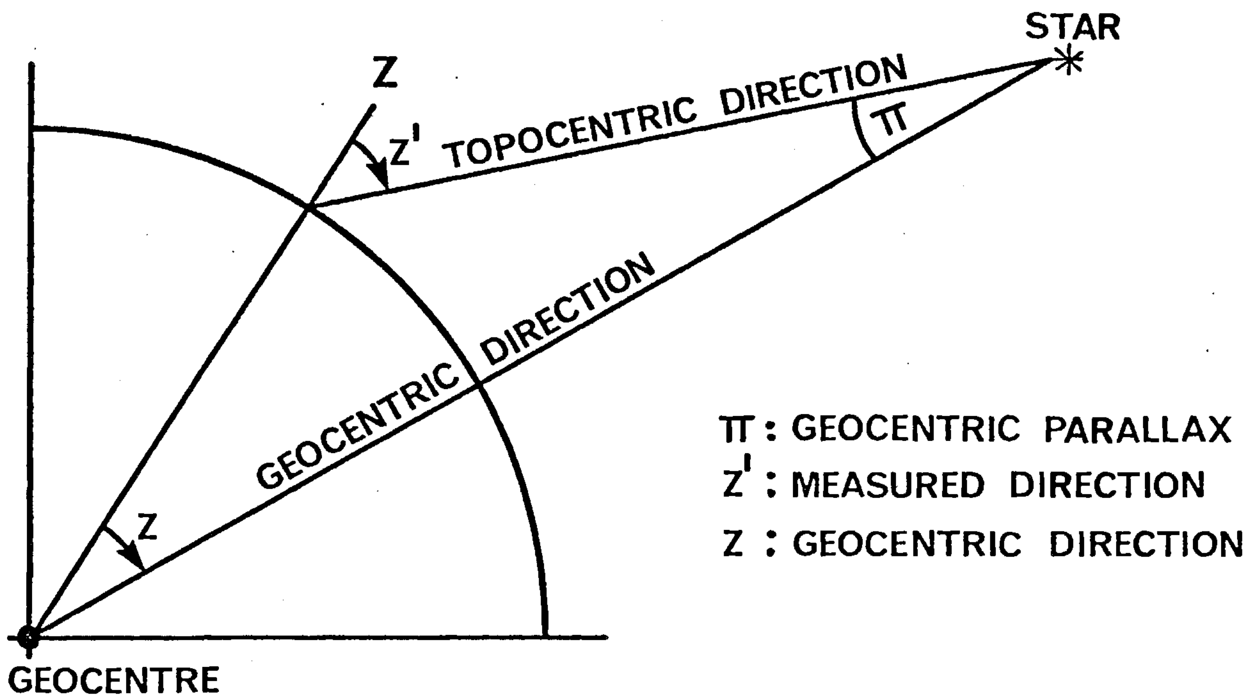


Figure 6-8

Geocentric Parallax

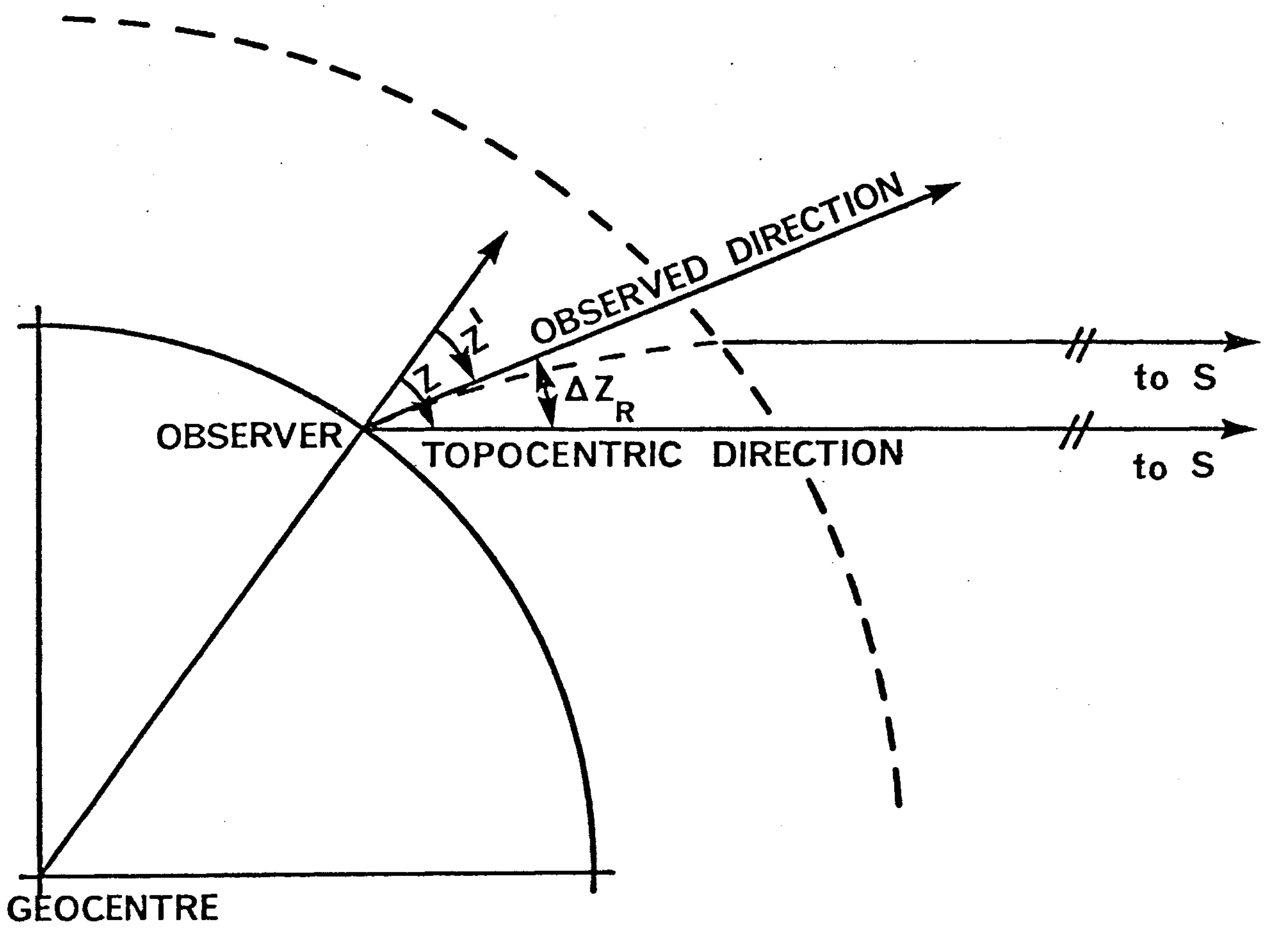


Figure 6-9

Astronomic Refraction

relative humidity at the time of observation (T).

Finally, we can write the expressions for the right-ascension and declination of a celestial body as

$$\alpha_{A.P._T} = \alpha'_{A.P._T} - (\Delta\alpha_D + \Delta\alpha_P + \Delta\alpha_R) \, , \tag{6-32}$$

$$\delta_{A.P._T} = \delta'_{A.P._T} - (\Delta\delta_D + \Delta\delta_P + \Delta\delta_R) \, , \tag{6-33}$$

in which α'_{AP_T} and δ'_{AP_T} refer to the Apparent Place coordinates obtained from equation (6-21). This process is summarized in Figure 6-10.

In closing this section, the reader should note the following. It is common practice in lower-order astronomical position and azimuth determinations to correct the observed direction z' for both geocentric parallax and refraction rather than use (6-32) and (6-33) above. Thus, in our math models we use

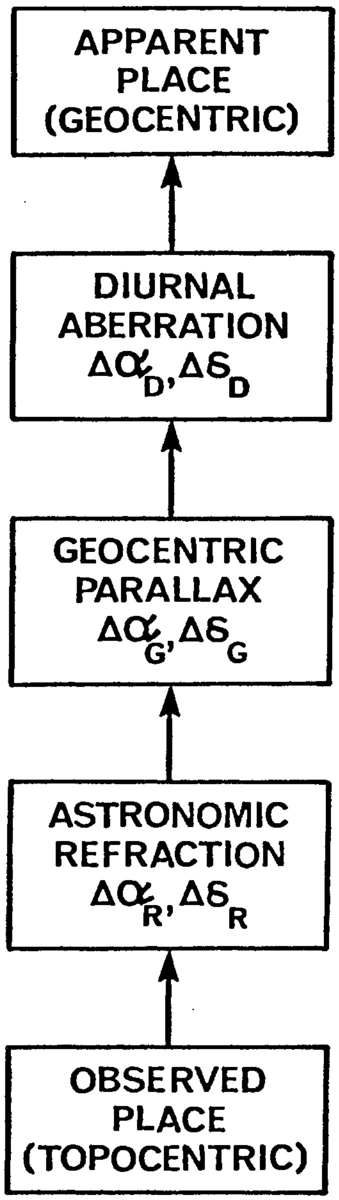
$$z = z' - (\Delta z_R + \Delta z_P) \quad . \tag{6-34}$$

Δz_P, the correction to a zenith distance due to geocentric parallax, is tabulated (e.g. SALS). Note that the effects of diurnal aberration are neglected.

6.4 Polar Motion

The final problem to be solved is the transformation of astronomically determined positional coordinates (Φ, Λ) from the Apparent Place celestial system to the Average Terrestrial coordinate system. This involves two steps: (i) transformation of α_{AP} , δ_{AP} (Apparent Place) to Instantaneous Terrestrial and (ii) transformation of Φ_{IT}, Λ_{IT} (Instantaneous Terrestrial) to Average Terrestrial.

In the first step, we are moving from a non-rotating system to a coordinate system that is rotating with the earth. The Instantaneous



$$\alpha_{AP} = \alpha'_{AP} - (\Delta\alpha_G + \Delta\alpha_D + \Delta\alpha_R)$$

$$\delta_{AP} = \delta'_{AP} - (\Delta\delta_G + \Delta\delta_D + \Delta\delta_R)$$

Figure 6-10

Observed to Apparent Place

Terrestrial (IT) system is defined by

- -a geocentric origin,
- -a primary pole (Z) that coincides with the instantaneous terrestrial pole,
- -a primary axis (X) that is the intersection of the Greenwich Mean Astronomic Meridian and Instantaneous equatorial planes,
- -a Y-axis that makes the system right-handed.

Comparing the definitions of the A.P. and I.T. systems, we see that the only difference is in the location of the primary axis, and futhermore, that X_{IT} is in motion. Recalling the definition of sidereal time, we see that the A.P. and I.T. systems are related through GAST (Figure 6-11), namely

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{IT} = R_3(\text{GAST}) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{AP} , \qquad (6\text{-}35)$$

in which

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{AP_T} = \begin{bmatrix} \cos\delta \cos\alpha \\ \cos\delta \sin\alpha \\ \sin\delta \end{bmatrix}_{AP_T} , \quad \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{IT_T} = \begin{bmatrix} \cos\delta \cos\alpha \\ \cos\delta \sin\alpha \\ \sin\delta \end{bmatrix}_{IT_T} , \qquad (6.36)$$

or, using a spherical approximation of the earth

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{IT} = \begin{bmatrix} \cos\Phi \cos\Lambda \\ \cos\Phi \sin\Lambda \\ \sin\Phi \end{bmatrix}_{I.T.}$$

The reader should note that this transformation is usually (6-35) carried out in an implicit fashion within the mathematical models for position and azimuth determination rather than explicitly as is given here (see, for example, Chapters 8 and 9). The results that one obtains are then

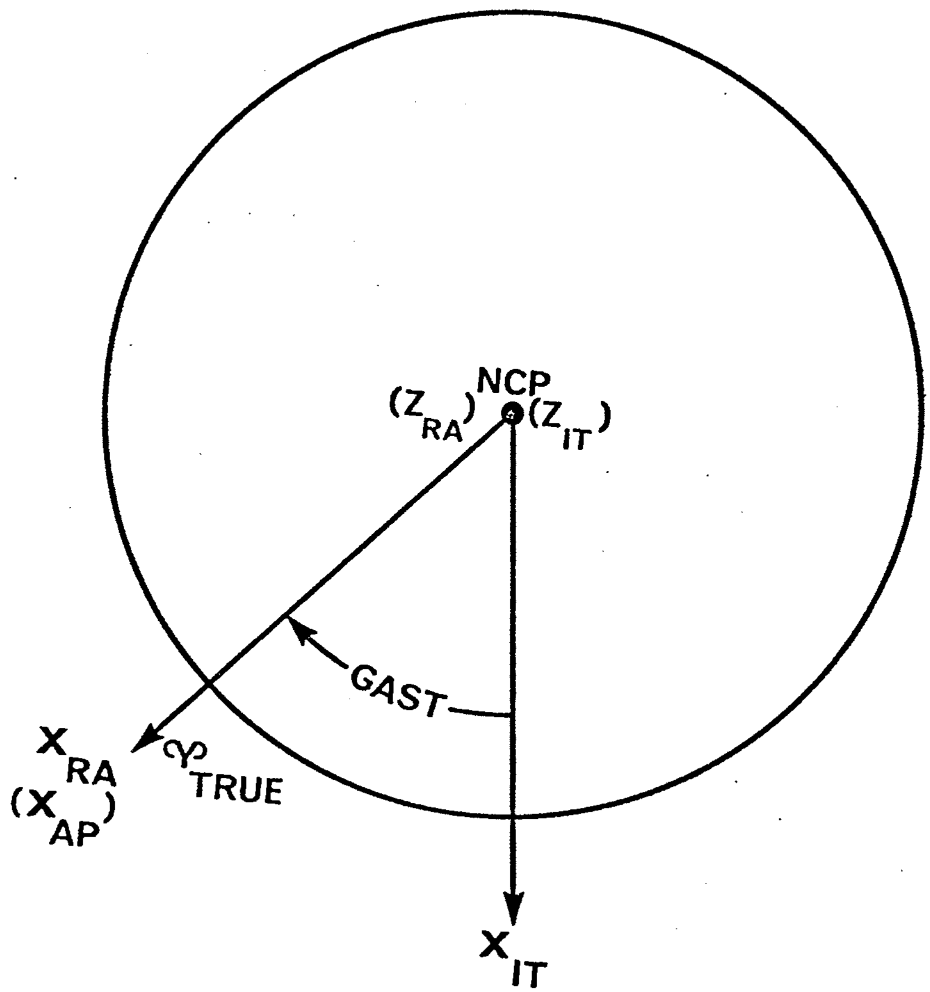


Figure 6-11

Transformation of Apparent Place
To Instantaneous Terrestrial
System

Φ, Λ, A, all expressed in an Instantaneous Terrestrial coordinate system. The final step is to refer these quantities to the Average Terrestrial coordinate system.

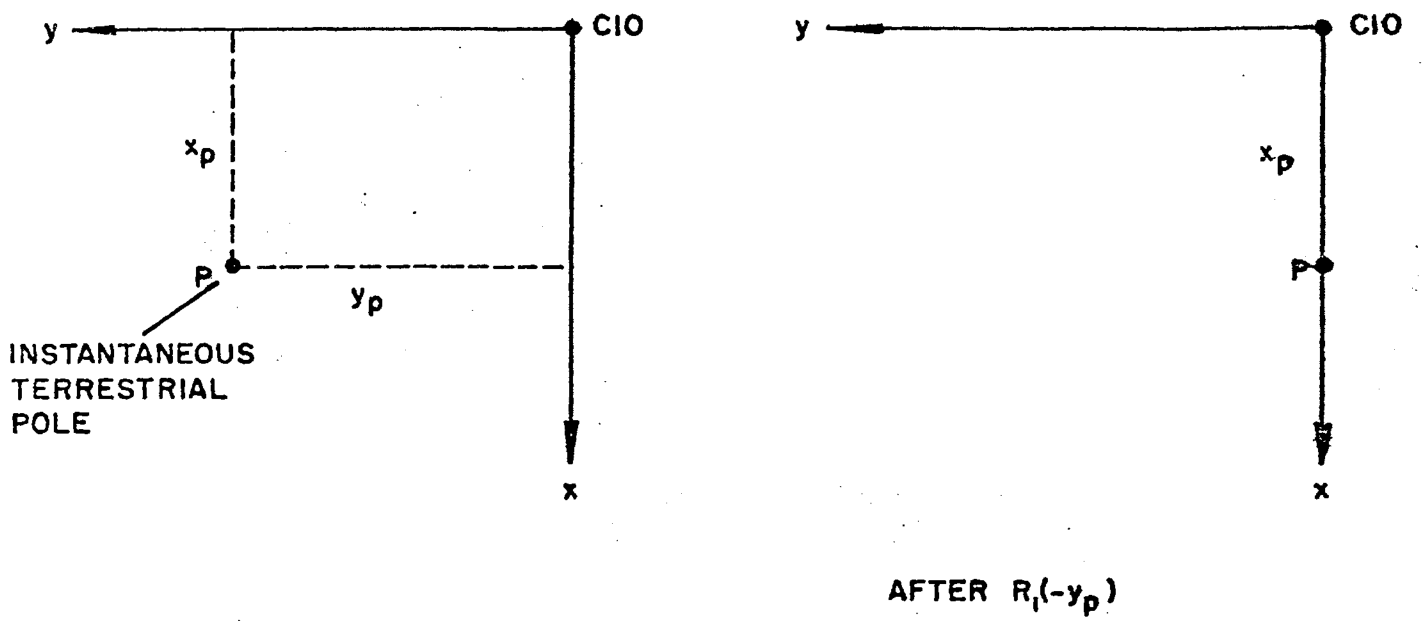
The direction of the earth's instantaneous rotation axis is moving with respect to the earth's surface. The motion, called polar motion, is counter-clockwise, quasi-irregular, has an amplitude of about 5 m, and a period of approximately 430 m.s.d. The motion is expressed in terms of the position of the instantaneous rotation axis with respect to a reference point fixed on the earth's crust. The point used is the mean terrestrial pole for the interval 1900-1905, and is called the Conventional International Origin (C.I.O.).

The Average Terrestrial (AT) coordinate system is the one which we would like to refer astronomic positions (Φ, Λ). This system differs in definition from the I.T. system only in the definition of the primary pole - the primary A.T. pole is the C.I.O. Referring to figure 6-12, we can easily see that

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{AT} = R_2\,(-x_p)\; R_1\,(-y_p) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{I.T.} \qquad (6\text{-}37)$$

Using a spherical approximation for the earth which is adequate for this transformation, one obtains [Mueller, 1969]

$$\begin{bmatrix} \cos\Phi\ \cos\Lambda \\ \cos\Phi\ \sin\Lambda \\ \sin\Phi \end{bmatrix}_{A.T.} = R_2(-x_p)\; R_1(-y_p) \begin{bmatrix} \cos\Phi\ \cos\Lambda \\ \cos\Phi\ \sin\Lambda \\ \sin\Phi \end{bmatrix}_{I.T.} . \qquad (6\text{-}38)$$



TRANSFORMATION FROM INSTANTANEOUS TO AVERAGE TERRESTRIAL SYSTEM.

FIGURE 6-12

Specifically, one obtains after some manipulations, the two equations [Mueller, 1969]

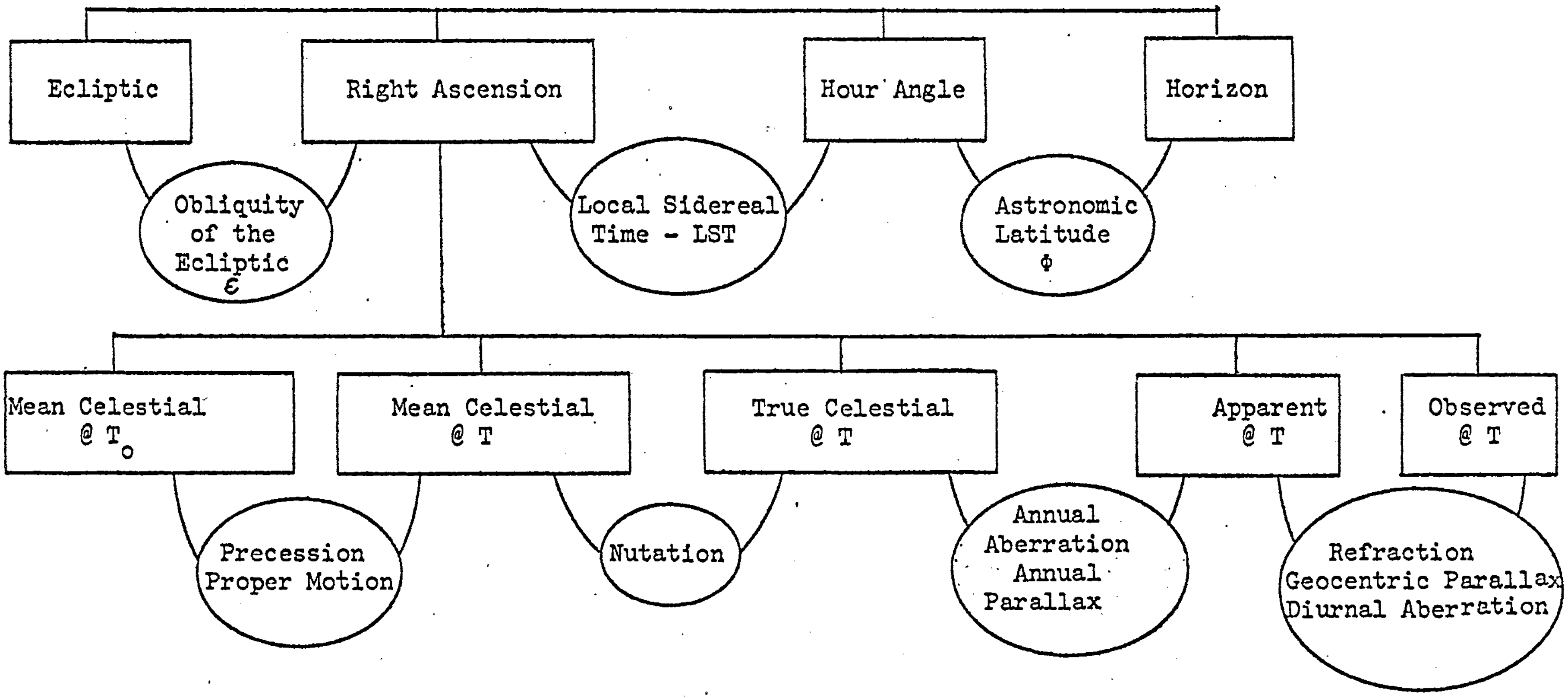
$$\Delta\Phi = \Phi_{AT} - \Phi_{IT} = y_p \sin\Lambda_{IT} - x_p \cos\Lambda_{IT} \quad , \tag{6-39}$$

$$\Delta\Lambda = \Lambda_{AT} - \Lambda_{IT} = -(x_p \sin\Lambda_{IT} + y_p \cos\Lambda_{IT}) \tan\Phi_{IT} \quad . \tag{6-40}$$

The effect of polar motion on the astronomic azimuth is expressed as [Mueller, 1969]

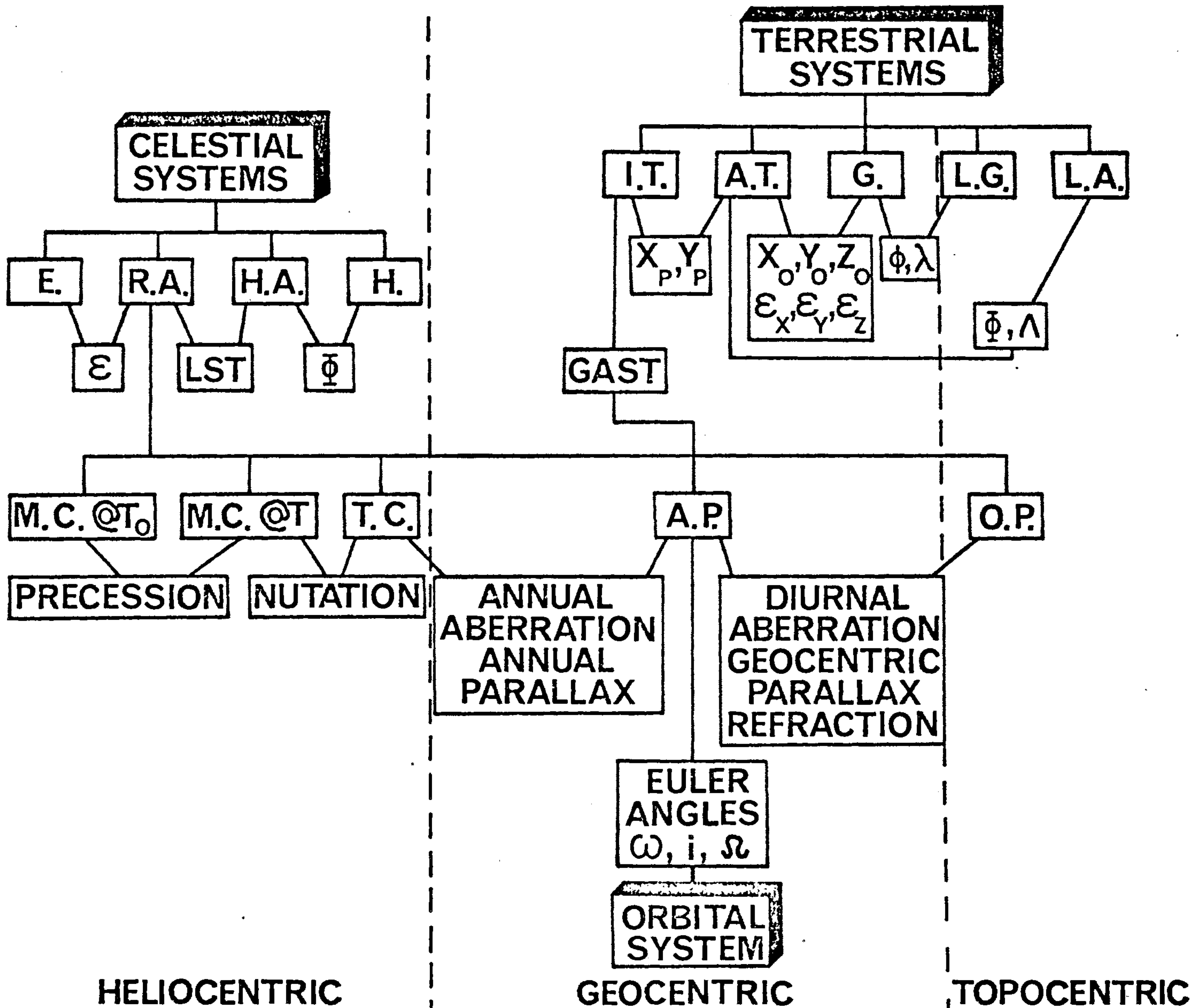
$$\Delta A_p = A_{AT} - A_{IT} = -\,(x_p \sin\Lambda_{IT} + y_p \cos\Lambda_{IT}) \sec\Phi_{IT} \quad . \tag{6-41}$$

The complete process of position updating, and the relationship with all celestial coordinate systems is depicted in Figure 6-13. The relationships amongst all of the coordinate systems used in geodesy, with celestial systems in perspective, is given in Figure 6-14.



CELESTIAL COORDINATE SYSTEMS AND POSITION UPDATING
[Krakiwsky and Wells, 1971]

FIGURE 6-13



COORDINATE SYSTEMS USED IN GEODESY

Figure 6-14

7. DETERMINATION OF ASTRONOMIC AZIMUTH

In Chapter 1, the astronomic azimuth was defined as the angle between the astronomic meridian plane of a point i and the astronomic normal plane of i through another point j. In Chapter 2 (Section 2.2.1), this angle was defined with respect to the Horizon celestial coordinate system in which the point j was a celestial object. Obviously, to determine the astronomic azimuth of a line ij on the earth, we must (i) take sufficient observations to determine the azimuth to a celestial body at an instant of time T, and (ii) measure the horizontal angle between the star and the terrestrial reference object (R.O.).

In these notes, two approaches to astronomic azimuth determination are studied: (i) the hour angle method, in which observations of Polaris (α Ursae Minoris) at any hour angle is treated as a special case, and (ii) the altitude method, in which observation of the sun is treated as a special case. Several alternative azimuth determination methods are treated extensively in Mueller [1969] and Robbins [1976].

The instruments used for azimuth determination are a geodetic theodolite, a chronometer (e.g. stop watch), a HF radio receiver, a thermometer and a barometer. To obtain optimum accuracy in horizontal direction measurements, a striding level should be used to measure the inclination of the horizontal axis.

Finally, the reader should note that the astronomic azimuth determination procedures described here yield azimuths that are classified as being second or lower order. This means that the internal standard deviation of several determinations of the astronomic azimuth would be, at best, $0\overset{''}{.}5$ to $1\overset{''}{.}5$ [e.g. Mueller, 1969]. Most often, using a 1" theodolite with a striding level attachment, the standard deviation of the astronomic azimuth of a terrestrial line would be of the order of 5" to 10" [e.g. Robbins, 1976].

7.1 Azimuth by Star Hour Angles

From the transformation of Hour Angle celestial coordinates (h,δ) to Horizon celestial coordinates (A,a), we have the equation (2-20)

$$\tan A = \frac{\sin h}{\sin\Phi \cos h - \tan\delta \cos\Phi} \qquad (7\text{-}1)$$

To solve this equation for A, we must know the latitude Φ of the place of observation. The declination of the observed star, δ, can be obtained from a star catalogue, ephemeris, or almanac (e.g. FK4, AA, SALS) and updated to the epoch of observation T. The hour angle of a star, h, can not be observed, but it can be determined if time is observed at the instant the star crosses the vertical wire of the observer's telescope. Assuming one observes zone time (ZT), then the hour angle is given by (combining equations (2-30), (3-11), (3-3), (3-13), (3-18))

$$h = ZT + \Delta Z + (\alpha_m - 12^h) + Eq.E. + \Lambda - \alpha , \qquad (7\text{-}2)$$

in which ZT is observed, ΔZ and Λ are assumed known, and α, Eq. E and (α_m-12^h) are catalogued in AA (or $(\alpha_m-12^h+Eq.E)$ is catalogued in SALS), and must be updated to the epoch of observation T.

The minimization of the effects of systematic errors are important for any azimuth determination. This can be done, in part, through the selection of certain stars for an observing program. Assuming that the sources of systematic errors in (7-1) occur in the knowledge of latitude and the determination of the hour angle, we can proceed as follows.

Differentiation of (7-1) yields.

$$dA = \sin A \cot z \, d\Phi + \cos\Phi \, (\tan\Phi - \cos A \cot z) dh. \tag{7-3}$$

Examination of (7-3) yields the following:

(i) when $A = 0^{\circ}$ or 180°, the effects of $d\Phi$ are eliminated,

(ii) when $\tan\Phi = \cos A \cot z$, the effects of dh are eliminated.

Thus, if a star is observed at transit (culmination), the effects of $d\Phi$ are minimized, while if a star is observed at elongation (parallactic angle $p = 90^{\circ}$), the effects of dh are minimized. Of course, it is not possible to satisfy both conditions simultaneously; however, the effects will be elliminated if two stars are observed such that

$$\sin A_1 \cot z_1 = -\sin A_2 \cot z_2 \quad , \tag{7-4}$$

$$2\tan\Phi - (\cos A_1 \cot z_1 + \cos A_2 \cot z_2) = 0 \quad . \tag{7-5}$$

As a general rule, then, the determination of astronomic azimuth using the hour angle method must use a series of star pairs that fulfill conditions (7-4) and (7-5). This procedure is discussed in detail in, for example, Mueller [1969] and Robbins [1976].

A special case can be easily made for cicumpolar stars, the most well-known of which in the northern hemisphere is Polaris (α Ursae Minoris). When $\Phi > 15^{\circ}$ Polaris is easily visible and directions to it are not affected unduly by atmospheric refraction. Since $A \simeq 0^{\circ}$, the error $d\Phi$ is eliminated. Furthermore, since in (7-3), the term [Robbins, 1976]

$$\cos\Phi \, (\tan\Phi - \cos A \cot z) = \cos\delta \cos z \cos p, \tag{7-6}$$

then when $\delta \simeq 90^{\circ}$, the effects of dh are elliminated. Then Polaris can be observed at any hour angle for the determination of astronomic azimuth by the hour angle method.

A suggested observing sequence for Polaris is as follows [Mueller, 1969]:

(i) Direct on R.O., record H.C.R.,

(ii) Direct on Polaris, record H.C.R. and T,

(iii) Repeat (ii),

(iv) Repeat (i),

(v) Reverse telescope and repeat (i) through (iv).

The above observing sequence constitutes one azimuth determination; eight sets are suggested. Note that if a striding level is used, readings of both ends of the level (e.g. e and w for direct, e' and w' for reverse) should be made and recorded after the pointings on the star. An azimuth correction, for each observed set, is then given by [Mueller, 1969]

$$\Delta A'' = \frac{d''}{4} \left((w + w') - (e + e') \right) \cot z \qquad (7\text{-}7)$$

in which d is the value in arc-seconds of each division of the striding level.

Briefly, the computation of astronomic azimuth proceeds as follows:

(i) for each of the mean direct and reverse zone time (ZT) readings of each set on Polaris, compute the hour angle (h) using (7-2),

(ii) using (7-1), compute the astronomic azimuth A of Polaris for each of the mean direct and reverse readings of each set,

(iii) using the mean direct and mean reverse H.C.R.'s of each set on Polaris and the R.O., compute the astronomic azimuth of the terrestrial line,

(iv) the mean of all computed azimuths (two for each observing sequence, eight sets of observations) is the azimuth of the terrestrial line,

(v) computation of the standard deviation of a single azimuth determination and of the mean azimuth completes the computations.

Computations may be shortened somewhat by (i) using the mean readings direct and reverse for each set of observed times and H.C.R.'s (e.g. only 8 azimuth determinations) or (ii) using the mean of all time and H.C.R.'s, make one azimuth determination. If either of these approaches are used, an azimuth correction due to the non-linearity of the star's path is required. This correction (ΔA_c) is termed a second order curvature correction [Mueller, 1969]. It is given by the expression

$$A_c'' = \frac{C_A}{n} \sum_{i=1}^{n} m_i \qquad (7\text{-}8)$$

in which n is the number of observations that have been meaned (e.g. 2 for each observing sequence (direct and reverse), 16 for the total set of observations (8 direct, 8 reverse)). The term C_A is given by

$$C_A = \frac{\tan A}{\sin^2 h} \frac{\cos^2 h - \cos^2 A}{\cos^2 A} \qquad (7\text{-}9)$$

in which the azimuth A is the azimuth of the star (Polaris) computed without the correction. The term m_i is given as

$$m_i = \tfrac{1}{2}\, \tau_i^2 \sin 2'' \quad , \qquad (7\text{-}10)$$

where

$$\tau_i = (T_i - T_o) \quad , \qquad (7\text{-}11)$$

and

$$T_o = \frac{T_1 + T_2 + \dots + T_n}{n} \quad . \qquad (7\text{-}12)$$

If, when observing Polaris, the direct and reverse readings are made within 2^m to 3^m , the curvature correction ΔA_c will be negligible and

Observer: A. Gonzalez-Fletcher	Instruments: Kern DKM3-A theodolite (No. 82514); Hamilton sidereal chronometer (No. 2E12304); Favag chronograph with manual key; Zenith transoceanic radio.
Computer: A. Gonzalez-Fletcher	
Local Date: 3-31-1965	
Location: OSU old astro pillar $\Phi \simeq 40°00'00''$ $\Lambda \simeq -5^h32^m10^s$	
Star Observed: FK4 No. 907 (α Ursae Minoris [Polaris])	Azimuth Mark: West Stadium

1. Level Corrections (Sample)

$$\Delta A = \frac{d}{4}[(w+w') - (e+e')] \cot z$$

	Determination No.		
	1	2	3
d/4 cot z	0″.347	0″.347	0″.347
w + w′	41.8	41.7	41.9
e + e′	43.3	43.3	42.8
ΔA	-0″.52	-0″.59	-0″.31

d = 1″.6/division

2. Azimuth Computation (Sample)

$$\tan A = \sin h / (\sin \Phi \cos h - \cos \Phi \tan \delta)$$

		Determination No.		
		1	2	3
1	Mean chronometer reading of direct and reverse pointings	$9^h22^m02^s.35$	$9^h36^m13^s.85$	$10^h11^m38^s.16$
2	Chronometer correction (computed similar to Example 8.4)	$-5^m51^s.25$	$-5^m51^s.28$	$-5^m51^s.33$
3	AST	$9^h16^m11^s.10$	$9^h30^m22^s.57$	$10^h05^m46^s.83$
4	α (apparent;)	$1^h57^m53^s.46$	$1^h57^m53^s.46$	$1^h57^m53^s.46$
5	h = AST − α	$7^h18^m17^s.64$	$7^h32^m29^s.11$	$8^h07^m53^s.37$
6	h (arc)	109°34′24″.60	113°07′16″.65	121°58′20″.55
7	δ (apparent)	89°06′12″.92	89°06′12″.92	89°06′12″.92
8	cos h	-0.33501582	-0.39267890	-0.52951032
9	tan δ	63.91162817	63.91162817	63.91162812
10	sin Φ cos h	-0.21534402	-0.25240913	-0.34036267
11	cos Φ tan δ	48.95914742	48.95914742	48.95914742
12	(10) - (11)	-49.17449144	-49.21155655	-49.29951009
13	sin h	0.94221251	0.91967535	0.84830350
14	tan A = (13)/(12)	-0.01916059	-0.01868820	-0.01720714
15	A (at the average h)	358°54′08″.33	358°55′45″.73	359°00′51″.12

AZIMUTH BY THE HOUR ANGLE OF POLARIS [Mueller, 1969]

FIGURE 7-1

16	Time difference between direct and reversed pointings (2τ)	$7^m12^s.4$	$2^m44^s.5$	$2^m36^s.5$
17	Curvature correction (equation (9.17))	0".5	0".1	0".1
18	A (Polaris) = (15) + (17)	358°54'08".8	358°55'45".8	359°00'51".2
19	Circle (horizontal) reading on Polaris	258°25'48".9	258°27'28".2	258°32'39".0
20	Correction for dislevelment (ΔA)	-0".5	-0".6	-0".3
21	Corrected circle reading = (19) + (20)	258°25'48".4	258°27'27".6	258°32'38".7
22	Circle reading on Mark	00°01'13".7	00°01'14".7	00°01'17".5
23	Angle between Mark and Polaris = (22) - (21)	101°35'25".3	101°33'47".1	101°28'38".8
24	Azimuth of Mark = (18) + (23)	100°29'34".1	100°29'32".9	100°29'30".0

3. <u>Final Observed Azimuths from Eight Determinations</u>

Determination No.	Azimuth of Mark	v	vv
1	100°29'34".1	-2".39	5.71
2	32 .9	-1.19	1.42
3	30 .0	1.71	2.92
4	31 .3	0.41	0.17
5	32 .0	-0.29	0.08
6	30 .3	1.41	1.99
7	31 .6	0.11	0.01
8	31 .5	0.21	0.04
Final (mean)	100°29'31".71	[v] = -0".02	[vv] = 12.35

Standard deviation of an azimuth determination:

$$m_A = \sqrt{\frac{[vv]}{n-1}} = \sqrt{\frac{12.35}{8-1}} = 1''.33$$

Standard deviation of the mean azimuth:

$$M_A = \frac{m_A}{\sqrt{n}} = \frac{1''.33}{\sqrt{8}} = 0''.47$$

Result:

A_T(West Stadium) = 100°29'31".7 ± 0".5

FIGURE 7-1 (continued)

could therefore be neglected [Robbins, 1976]. An example of azimuth determination by the hour angle of Polaris is given in Figure 7-1.

Azimuth by hour angles is used for all orders of astronomic work. The main advantages of this method are that the observer has only to observe the star as it coincides with the vertical wire of the telescope and since no zenith distance measurement is made, astronomic refraction has no effect. The main disadvantages are the need for a precise time-keeping device and a good knowledge of the observer's longitude.

Table 7-1 summerizes, for several situations, the sources of errors and their magnitudes in the determination of astronomic azimuth by stars hour angles.

7.2 Azimuth by Star Altitudes

Azimuth by star altitudes yields less accurate results than azimuth by star hour angles. The two reasons for this are (i) the star must be observed as it coincides with both the horizontal and vertical wires of the telescope, and (ii) the altitude observation is subject to the effects of astronomic refraction. The method does have the advantage, however, that a precise knowledge of the observer's longitude is not required and an accurate time-keeping device is not needed. Azimuths determined by star altitudes are not adequate for work that requires σ_A to be 5" or less.

Stars at	Elongation	Polaris	Prime Vertical		Lower Transit
a	35^o	ϕ	15^o	$22^o.5$	15^o
δ	60^o				75^o
σ_3 sec a (vertical wire on star)	2".4	2".3	2".6	2".7	2".6
σ_4 (measurement of horizontal angle)	2".5	2".5	2".5	2".5	2".5
σ_5 tan a (reading of plate level)	3".5	2".9	1".3	2".1	1".3
σ_T (time)	0".0	0".1	1".5	1".5	0".8
Sum (plate level).. ..	4".9	4".5	4".1	4".5	3".9
σ_6 tan a (reading of striding level)	0".9	0".7	0".3	0".5	0".3
Sum (striding level) ..	3".6	3".5	3".9	4".0	3".7

Azimuth by hour angle: random errors in latitude 30^o

Stars at	Elongation	Polaris	Prime Vertical	Lower Transit		
a	64^o	ϕ	$22^o.5$	20^o	30^o	40^o
δ	75^o			50^o	60^o	70^o
σ_3 sec a(vertical wire on star)	4".6	4".0	2".7	2".7	2".9	3".3
σ_4 (measurement of horizontal angle)	2".5	2".5	2".5	2".5	2".5	2".5
σ_5 tan a(reading of plate level)	10".2	8".7	2".1	1".8	2".9	4".2
σ_T (time . ..	0".0	0".1	2".6	2".1	1".7	1".3
Sum (Plate level)	11".5	9".9	5".0	4".6	5".1	6".0
σ_6 tan a (reading of striding level	2".6	2".2	0".5	0".5	0".7	1".0
Sum (Striding level)	5".9	5".2	4".5	3".3	4".2	4".5

Azimuth by hour angle: random errors in latitude 60^o
Table 7-1 [Robbins, 1976]

From a relationship between the Horizon and Hour Angle coordinate systems (egn. (2-26), rearranged)

$$\cos A = \frac{\sin\delta - \cos z \sin\Phi}{\sin z \cos\Phi} , \qquad (7\text{-}13)$$

or with z = 90 - a,

$$\cos A = \frac{\sin\delta - \sin a \sin\Phi}{\cos a \cos\Phi} . \qquad (7\text{-}14)$$

To solve either (7-13) or (7-14) for A, the latitude Φ of the observer must be known. The declination, δ, of the observed celestial body is obtained from a catalogue, ephemeris, or almanac and updated to the time of observation. The zenith distance (or altitude) is the measured quantity.

The main sources of systematic error with this method include error in latitude (assumed) and error in the reduced altitude due to uncorrected refraction. Differentiating (7-14) yields

$$dA \sin A = (\tan a - \cos A \tan\Phi)\, d\Phi + (\tan\Phi - \cos A \tan a)\, da . \qquad (7\text{-}15)$$

The effects of $d\Phi$ are zero when

$$\tan a = \cos A \tan\Phi \qquad (7\text{-}16)$$

which occurs when $h = 90^{\circ}$ (6^{h}) or 270° (18^{h}). The effects of da are zero when

$$\tan\Phi = \cos A \tan a \qquad (7\text{-}17)$$

which occurs when the observed celestial body is at elongation. To minimise the effects of systematic errors, two stars are observed such that

$$a_1 = a_2 , \qquad (7\text{-}18)$$

$$A_1 = 360^{\circ} - A_2 . \qquad (7\text{-}19)$$

Further details on the selection of star pairs observing procedure and the computation of azimuths, can be found in, for example, Mueller [1969] and Robbins [1976]. Using Polaris, observations should be made at elongation. Note that no star should have an altitude of less than 15^{o} when using this method, thus for Polaris, $\Phi > 15^{o}$. Estimates of achievable accuracy are given in Table 7-2.

The method of azimuth determination by altitudes is often used in conjunction with sun observations. Using a 1" theodolite and the observation and computation procedures described here, σ_A will be of the order of 20". When special solar attachments are used on an instrument, such as a Roelofs solar prism [Roelofs, 1950], and accuracy (σ_A) of 5" may be attained.

A complete azimuth determination via sun observations consists of <u>two morning</u> and <u>two afternoon</u> determinations. It is best if $30^{o} \le a \le 40^{o}$, and it should never be outside the range $20^{o} \le a \le 50^{o}$. Figure 7-1 illustrates, approximately, the optimum observing times. A suggested observing procedure for one azimuth determination is as follows [Robbins, 1976]:

(i) Direct on RO, record H.C.R. and level (plate or striding) readings,

(ii) Direct on Sun, record H.C.R. and V.C.R., and time to nearest 1^{m} (for methods of observing the sun, see, for example, Roelofs [1950]), record level readings,

(iii) Reverse on Sun, record H.C.R. and V.C.R., and time to nearest 1^{m}, record level readings,

Stars at	Elongation	h = 90°	Prime vertical
a	40°	25°	35°
δ	51°	58°	17°
σ_1 (pointing of horizontal wire on star)	0"	1".0	1".7
σ_2 (measuring of altitude)	0"	1".3	2".3
σ_R (determination of refraction)	0"	0".7	1".2
σ_3 sec a (pointing of vertical wire)	2".6	3".3	3".7
σ_4 (horizontal angle measurement)	2".5	2".5	2".5
σ_5 tan a (reading plate level)	4".2	2".3	3".5
Sum (Plate level) ..	5".5	5".1	6".5
σ_6 tan a (reading striding level)	1".0	0".6	0".9
Sum (Striding level)	3".7	4".5	5".5

Azimuth by altitude: random errors in latitude 30°

Stars at	Elongation	h = 90°	Prime vertical
a	64°	35°	35°
δ	75°	41°	30°
σ_1 (pointing of horizontal wire on star)	± 0"	± 4".8	± 5".2
σ_2 (measuring of altitude)	± 0"	± 6".2	± 6".8
σ_R (determination of refraction)	± 0"	± 3".2	± 3".5
σ_3 sec a (pointing of vertical wire)	± 4".6	± 3".7	± 3".7
σ_4 (horizontal angle measurement)	± 2".5	± 2".5	± 2".5
σ_5 tan a (reading plate level)	± 10".3	± 3".5	± 3".5
Sum (plate level) ..	± 11".6	± 10".2	± 10".8
σ_6 tan a (reading striding level)	± 2".6	± 0".9	± 0".9
Sum (striding level)	± 5".8	± 9".6	± 10".3

Azimuth by altitude: random errors in latitude 60°

Table 7-2 [Robbins, 1976]

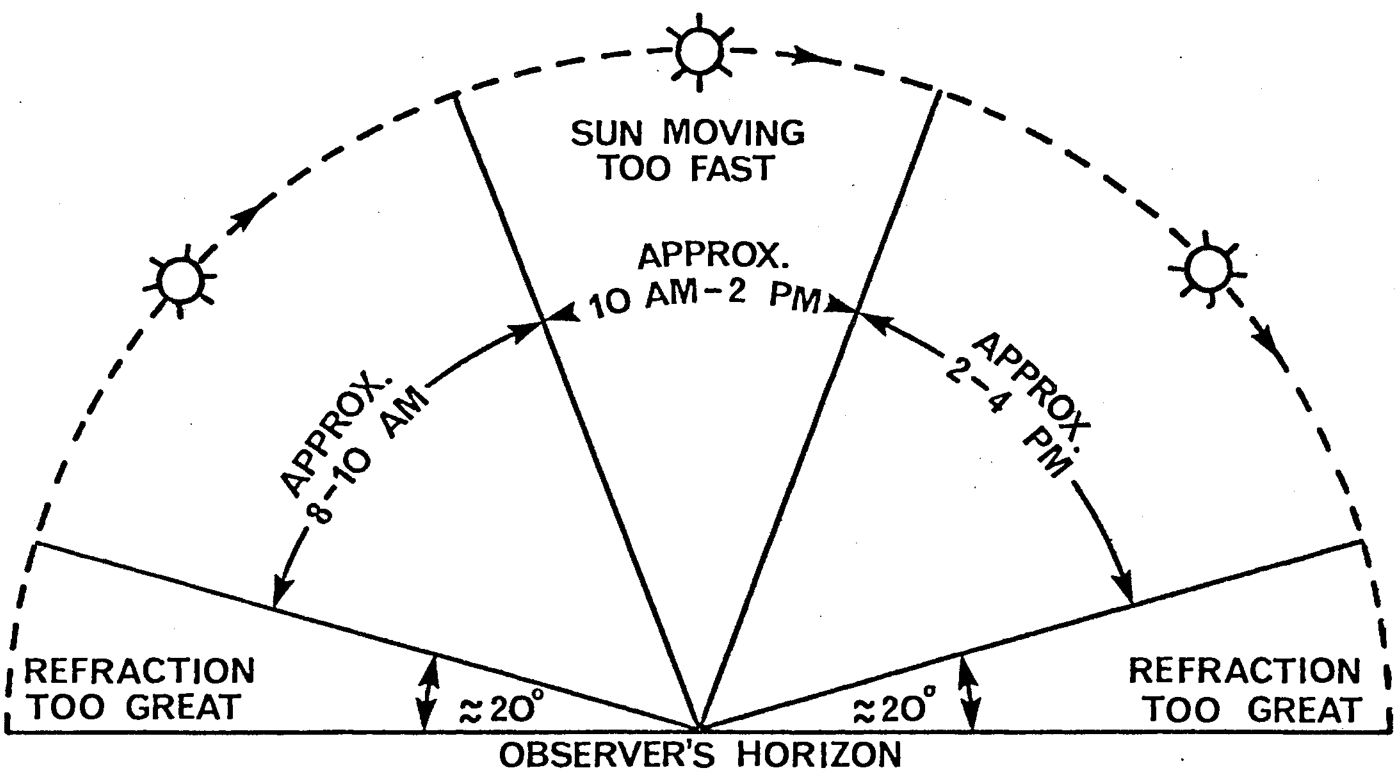


Figure 7-2

Azimuth by the Sun's Altitude

(iv) Reverse on R.O., record H.C.R., record level readings,

(v) Record temperature and pressure.

The computation procedure resulting from the above observations is:

(i) Mean Direct and Reverse horizontal readings to the R.O. and to the Sun,

(ii) correct the horizontal directions in (i) for horizontal axis dislevelment using equation (7-7),

(iii) compute the angle Sun to R.O.,

(iv) mean direct and reverse zenith distance (or altitude) measurements to the Sun, then correct the mean for refraction and parallax,

(v) determine the time of observation in the time system required to obtain a tabulated value of δ of the sun,

(vi) compute the updated value of δ for the time of observation (tabulated δ plus some correction for time),

(vii) using either (7-13) or (7-14), compute the azimuth to the sun,

(viii) using the angle determined in (iii), compute the azimuth to the R.O.

The following example illustrates the observing and computation procedures for an azimuth determination via the altitude of the sun.

Example: Azimuth via Sun's Altitude (using SALS).

Instrument: A Date: May 6, 1977

R.O.: B Time: Central Daylight Time (90°W)

Obs. Procedure: Sun observed in quadrants. Latitude (Φ): 38° 10' 10"

Inst.	Sight	H.C.R.	V.C.R.	Time	Remarks
	B	0° 00' 02".0			
Direct	Sun	157° 54' 56".0	56° 12' 00"	15^{h} 40^{m}	Temp. 20°C
Reverse	Sun	339° 05' 30".0	57° 10' 00"	15^{h} 46^{m}	Pres. 1000 mb.
	B	180° 00' 04".0			

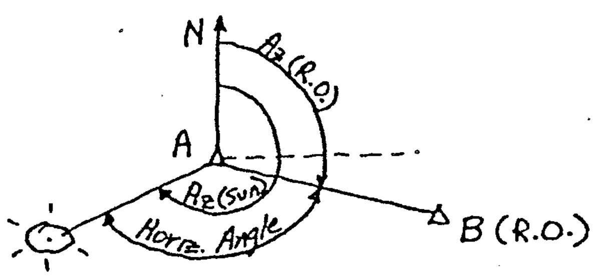
Note: No level readings required.

Means: H.C.R. Sun: 158° 30' 13".0 V.C.R. 56° 41' 00" Time: 15^{h} 43^{m}

H.C.R. B: 0° 00' 03".0

Horiz. Angle: 158° 30' 10".0

Sketch:



Reduction of Altitude

Mean Altitude (a) = 90° − 56° 41' 00" = 33° 19' 00"

Mean Refraction (r_0) = 88"

Correcting factor (f) = 0.95

Refraction = $r_0$xf = 88" x0.95 = − 1' 24"

Parallax = + 06"

Corrected Altitude = 33° 17' 42"

Computation of Sun's Declination

Observed (Mean) Time	15^h	43^m
Clock Correction		-01^m
Correction for Daylight Time	$-\ 1^h$	00^m
Time Zone	$+\ 6^h$	00^m
U.T.	20^h	42^m
Sun's δ at 6 May 18^h U.T.	(N) 16°	39!0
Change in δ since 18^h U.T. ($2^h 42^m$)	+	1!9
Sun's δ at 6 May 20^h 42^m U.T.	16°	40!9

Azimuth Computation

$$\cos A = \frac{\sin \delta - \sin a \sin \Phi}{\cos a \cos \Phi} \qquad (7\text{-}14)$$

$$\cos A = \frac{\sin(16^\circ\ 40!9) - \sin(33^\circ\ 17'\ 42'')\sin(38^\circ\ 10'\ 10'')}{\cos(38^\circ\ 17'\ 42'')\cos(38^\circ\ 10'\ 10'')}$$

$$= \frac{-\ 0.52191}{+\ 0.657391} = -\ 0.0794215$$

Azimuth of Sun $= \cos^{-1}A = 360^\circ - 94^\circ\ 33'\ 19''$	=	265°	26'	41"
Mean Horiz. Angle	=	158°	30'	10"
Azimuth to B (R.O.)	=	106°	56'	31"

8. DETERMINATION OF ASTRONOMIC LATITUDE

Astronomic latitude was defined in Chapter 1. To deduce procedures for determining astronomic latitude from star observations, we must examine an expression that relates observable, tabulated, and known quantities. Equation (2-22), involving the transformation of Hour Angle coordinates to Horizon coordinates, namely

$$\cos z = \sin\Phi \sin\delta + \cos\Phi \cos\delta \cos h \quad ,$$

requires that zenith distance (z) and time (h) be observed, that declination (δ) and right ascension (α) be tabulated, and that longitude (Λ) be known to solve for an unknown latitude (Φ). The effects of systematic errors in zenith distance and time, dz and dh respectively, are shown via the total derivative of (2-22), namely

$$d\Phi = -\sec A\, dz - \cos\Phi \tan A\, dh \quad . \qquad (8\text{-}1)$$

When $A = 0$, then $d\Phi = -dz$ ($\sec A = 1$, $\tan A = 0$), and when $A = 180^{\circ}$, then $d\Phi = dz$ ($\sec A = -1$, $\tan A = 0$). For this reason, most latitude determinations are based on zenith distance measurements of pairs of stars (one north and one south of the zenith such that $z_n = -z_s$) as they transit the observer's meridian.

The instruments used for latitude determination are the same as those for azimuth determination: a geodetic theodolite with a striding level attachment, a chronometer, an HF radio receiver, a thermometer and a barometer.

Two astronomic latitude determination procedures are given in these notes: (i) Latitude by Meridian Zenith Distances, (ii) Latitude by Polaris at any Hour Angle. The accuracy (σ_Φ) of a latitude determination by either method, using the procedures outlined, is 2" or better [Robbins, 1976]. For alternative latitude determination procedures, the reader is referred to, for example, Mueller [1969] and Robbins [1976].

8.1 Latitude by Meridian Zenith Distances

Equations (2-54), (2-55), and (2-56), rewritten with subscripts to refer to north and south of zenith stars at transit are

$$\Phi = \delta_N - z_N \quad \text{(UC north of zenith)}, \tag{8-2}$$

$$\Phi = z_S - \delta_S \quad \text{(UC south of zenith)}, \tag{8-3}$$

$$\Phi = 180° - \delta_N - z_N \quad \text{(LC north of zenith)}. \tag{8-4}$$

If a star pair UC north of zenith - UC south of zenith are observed, then a combination of (8-2) and (8-3) yields

$$\Phi = \tfrac{1}{2}(\delta_S + \delta_N) + \tfrac{1}{2}(z_S - z_N) , \tag{8-5}$$

while a star pair LC north of zenith -UC south of zenith gives

$$\Phi = \tfrac{1}{2}(\delta_S - \delta_N) + \tfrac{1}{2}(z_S - z_N) + 90° . \tag{8-6}$$

Equations (8-5) and (8-6) are the mathematical models used for latitude determination via meridian zenith distances.

An important aspect of this method of latitude determination is the apriori selection of star-pairs to be observed or the selection of a star programme. Several general points to note regarding a star programme are as follows:

(i) more stars should be listed than are required to be observed (to allow for missed observations due to equipment problems, temporary cloud cover, etc.),

(ii) the two stars in any pair should not differ in zenith distance by more than about 5^{o},

(iii) the stars in any pair, and star pairs, should be selected at time intervals suitable to the capabilities of the observer.

For details on star programmes for latitude determination by meridian zenith distances, the reader is referred to, for example, Robbins [1976].

A suggested observing procedure is:

(i) set the vertical wire of the instrument in the meridian (this may be done using terrestrial information e.g. the known azimuth of a line, or one may determine the meridian via an azimuth determination by Polaris at any Hour Angle),

(ii) set the zenith distance for the first north star; when the star enters the field of view, track it with the horizontal wire until it reaches the vertical wire,

(iii) record the V.C.R., temperature, and pressure,

(iv) repeat (i) to (iii) for the south star of the pair.

This constitutes one observation set for the determination of Φ. Six to eight sets are required to obtain $\sigma_{\Phi} = 2''$ or less. The computations are as follows:

(i) correct each observed zenith distance for refraction, namely

$z = z_{obs.} + \Delta z_R$,

(ii) for each star pair, use either equation (8-5) or (8-6) and compute Φ,

(iii) compute the final Φ as the mean of the six to eight determinations.

An example of this approach can be found, for instance, in Mueller [1969].

8.2 Latitude by Polaris at any Hour Angle

Since Polaris is very near the north celestial pole, the polar distance ($P = 90-\delta$, Figure 8-1) is very small ($P<1^o$), and the azimuth is very close to 0^o. Rewriting equation (2-22) (Hour Angle to Horizon coordinate transformation) with $z = (90 - a)$ and $P = 90 - \delta$ yields

$$\sin a = \sin\Phi \cos P + \cos\Phi \sin P \cosh \qquad (8\text{-}7)$$

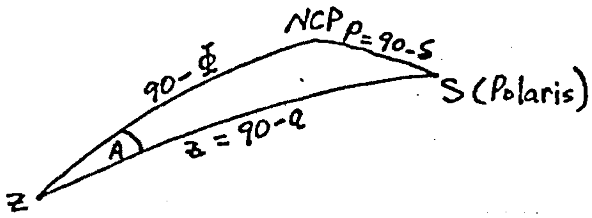


Figure 8-1

Astronomic Triangle for Polaris

Now, setting $\Phi = (a + \delta a)$, in which δa and P are differentially small, and substituting in (8-7) gives

$$\sin a = \sin(a + \delta a)\cos P + \cos(a + \delta a)\sin P \cos h$$
$$= \sin a \cos\delta a \cos P + \cos a \sin\delta a \cos P$$
$$+ \cos a \cos\delta a \sin P \cos h - \sin a \sin\delta a \sin P \cos h. \qquad (8\text{-}8)$$

Then, replacing the small angular quantities (δ_a and P) by their power series (up to and including 4th order terms), namely

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \ldots\ldots ,$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots\ldots ,$$

(8-8) becomes

$$\sin a = (\sin a(1 - \frac{\delta a^2}{2!} + \frac{\delta a^4}{4!} + \cos a\,(\delta a - \frac{\delta a^3}{3!}))\,(1 - \frac{P^2}{2!} + \frac{P^4}{4!})$$

$$+ (\cos a\,(1 - \frac{\delta a^2}{2!} + \frac{\delta a^4}{4!} - \sin a\,(\delta a - \frac{\delta a^3}{3!}))\,(P - \frac{P^3}{3!})\cos h. \qquad (8\text{-}9)$$

Now, taking only the first-order terms of (8-9) into account yields

$$\sin a = \sin a + \delta a \cos a + P\cos a \cos h \qquad (8\text{-}10)$$

or

$$\delta a = -P\cos h\,. \qquad (8\text{-}11)$$

Replacing the second-order δa terms in (8-9) with (8-11) above, and neglecting all terms of higher order gives

$$\delta a = -P\cosh + \frac{P^2}{2} \tan a \sin^2 h \quad . \tag{8-12}$$

Repeating the above process for third and fourth order terms yields the final equation

$$\Phi = a + \delta a = a - P\cosh + \frac{P}{2} \sin P \tan a \sin^2 h$$

$$- \frac{P}{3} \sin^2 P \cosh \sin^2 h + \frac{P}{8} \sin^3 P \sin^4 h \tan^3 a$$

$$- \frac{P}{24} \sin^3 P \sin^2 h \tan a (4-9\sin^2 h) + \ldots\ldots \quad . \tag{8-13}$$

Except in very high latitudes, the series (8-13) may be truncated at the third term.

A suggested observation procedure is as follows:

(i) observe polaris (horizontal wire) and record V.C.R., time, temperature and pressure; repeat this three times.

(ii) reverse the telescope and repeat (i).

This constitutes the observations for one latitude determination.

It should be noted that if azimuth is to be determined simultaneously (using Polaris at any hour angle), then the appropriate H.C.R.'s must be recorded (such a programme requires that the observer place Polaris at the intersection of the horizontal and vertical wires of the telescope).

The computation of latitude proceeds as follows:

(i) compute mean zenith distance measurement for six measurements,

(ii) correct mean zenith distance for refraction and compute the mean altitude ($a = 90 - z$),

(iii) compute the mean time for a, then using (7-2), compute the mean h (note that mean α and ($\alpha_m - 12^h$) must be computed for this part of the latitude determination),

(iv) compute a mean δ, then $P = 90 - \delta$,

(v) compute Φ .

A mean of 12 to 20 determinations will yield an astronomic latitude with $\sigma_\Phi = 2''$ or less [Robbins, 1976].

The reader should note that the use of a special SALS table (Pole Star Tables) leads to a simple computation of latitude using this method, namely

$$\Phi = a + a_o + a_1 + a_2 , \tag{8-14}$$

in which a is the corrected, meaned altitude, and a_o, a_1, a_2 are tabulated values. Using this approach results will not normally differ from those using (8-13) by more than $0\overset{'}{.}2$ (12"), while in most cases the difference is within $0\overset{'}{.}1$ (6") [Robbins, 1976]. Both computation procedures are illustrated in the following example taken from Robbins [1976].

Example: Latitude from Observations of Polaris at any Hour Angle

(i) with SALS tables

Date: 23 August 1969

$\phi \approx 57^o 03'$

$\lambda \approx 7^o\ 27'\ W = -\ 0^h\ 29^m\ 48^s$

Mean of Observations: $a = 56^o\ 57'\ 24''$

$UT = 22^h\ 01^m\ 41\overset{s}{.}8$

$Temp. = 10^oC$

Press. = 1009mb.

Computations:

$R(\alpha_m - 12^h + Eq.E)$ at 18^h $UT = 22^h\ 07^m\ 23\overset{s}{.}9$

ΔR for $4^h\ 01^m\ 42^s$ = $39\overset{s}{.}7$

$$GAST = UT + R + R = 20^h\ 09^m\ 45^s.4$$

$$\lambda \qquad - 0^h\ 29^m\ 48^s$$

$$LAST \qquad 19^h\ 39^m\ 57^s.4$$

$a = 56^\circ\ 57'\ 24''$

r_0 (mean refraction) = $38''$

f (refraction factor) = 1.00

$$a_{corr} = a - (r_0 \times f) = 56^\circ\ 56'\ 46''$$

$$\Phi = a_{corr} + a_0 + a_1 + a_2 = 56^\circ\ 56'\ 46'' + 5'48'' + 09'' + 00''$$

$$\underline{\Phi = 57^\circ\ 02'\ 43''}$$

(ii) with equation (8-13) (first three terms only)

as in (i) plus:

$\alpha = 2^h\ 03^m\ 09^s$, $\delta = 89^\circ\ 07'\ 17''$

$LAST = 19^h\ 39^m\ 57^s.4$

$\alpha = 2^h\ 03^m\ 09^s.0$

$h = 17^h\ 36^m\ 48^s.4 = 264^\circ\ 12'\ 06''$

$a = 56^\circ\ 56'\ 46''$

$P = 3163''$

$\Phi = 56^\circ\ 56'\ 46'' + 319''.5 + 36''.9$

$$\underline{\Phi = 57^\circ\ 02'\ 42''.4}$$

9. DETERMINATION OF ASTRONOMIC LONGITUDE

In chapter 3, we saw that longitude is related directly with sidereal time, namely (egn. 3-3)

$$\Lambda = LAST - GAST \qquad (9\text{-}1)$$

From the Hour Angle - Right Ascension systems coordinate transformations (egn. 2-31)

$$LAST = \alpha + h \; . \qquad (9\text{-}2)$$

Replacing LAST in (9-1) with (9-2) yields the expression

$$\Lambda = \alpha + h - GAST \quad . \qquad (9\text{-}3)$$

This equation (9-3) is the basic relationship used to determine astronomic longitude via star observations. The right ascension (α) is catalogued, and the hour angle (h) and GAST are determined respectively via direction and time observations.

Let us first examine the determination of GAST. Setting T_M as the observed chronometer time (say UTC), and ΔT as the total chronometer correction, then

$$GAST = T_M + \Delta T \quad . \qquad (9\text{-}4)$$

The total correction (ΔT) consists of the following:

(i) the epoch difference ΔT_o at the time of synchronization of UTC and T_M times,

(ii) chronometer drift $\Delta_1 T$,

(iii) DUT1, given by DUT1 = UT1 - UTC (see Chapter 4),

(iv) the difference between UT1 and GAST at the epoch of observation given by

$$LMST = MT + (\alpha_M - 12^h) \; ,$$

$$LAST = LMST + Eq.E.$$

Finally, equation (9-3) reads

$$\Lambda = \alpha + h - (T_M + \Delta T) \qquad (9\text{-}5)$$

The determination of the hour angle (h) is dependent on the astronomic observations made. From the Hour Angle - Horizon systems coordinate transformation (eqn. 2-22),

$$\cosh = \frac{\cos z - \sin\delta \sin\Phi}{\cos\delta \cos\Phi} , \qquad (9\text{-}6)$$

can be used for the determination of a star's hour angle. Now, taking z and T_M as the observed quantities, Φ as a known quantity, and δ as catalogued, the sources of systematic errors affecting longitude determination will be in zenith distance (dz), the assumed latitude ($d\Phi$), the observed time (T_M) and the chronometer correction ($d\Delta T$). Replacing h in (9-5) by (9-6), and taking the total derivative yields

$$d\Lambda = -(\sec\Phi \cot A\, d\Phi + \sec\Phi \operatorname{cosec} A\, dz + dT + d\Delta T). \qquad (9\text{-}7)$$

Examining (9-7), it is obvious that the errors dT and $d\Delta T$ contribute directly to $d\Lambda$ and can not be elliminated by virtue of some star selections. On the other hand, $d\Phi$ and dz can be treated as follows:

(i) when the observed star is on the prime vertical ($A = 90^{\circ}$ or 270°) , then $d\Phi = 0$,

(ii) the effects of dz can be elliminated by observing pairs of stars such that $z_1 \approx z_2$.

As with azimuth and latitude determination procedures in which pairs of stars have to be observed under certain conditions, a star program (observing list) must be compiled prior to making observations. For details, the reader is referred, for example, to Mueller [1969] and Robbins [1976].

The primary equipment requirements for second-order(σ_λ = 3" or less) longitude determination are a geodetic theodolite, a good mechanical or quartz chronometer, an HF radio receiver, and a chronograph equipped with a hand tappet.

Finally, before describing a longitude determination procedure, the reader should note the following. Since the GAST used is free from polar motion, LAST must be freed from polar motion effects. This is accomplished by adding a polar motion correction ($\Delta\Lambda_p$: see Chapter 6) to the final determined longitude (not each observation), such that the final form of equation (9-5) becomes

$$\Lambda = \alpha + h + \Delta\Lambda_p - (T_M + \Delta T) \quad . \tag{9-8}$$

9.1 Longitude by Meridian Transit Times

When a star transits the meridian, $h = 0^h$ (or 12^h) and α = LAST. If pairs of stars are observed such that $z_1 = z_2$, then the effects of dz in (9-7) are elliminated. The observation of time (T_M) as a star transits the meridian enables longitude computation using (9-8). The main sources of error will be $d\Phi$ and the observation of the stars transit of the meridian (due partly to inaccurate meridian setting and partly to collimation). The latter errors cause a timing error. A correction for this effect is computed from [Mueller, 1969]

$$\frac{dA}{dh} = \cos\delta \operatorname{cosec} z \tag{9-9}$$

or

$$dh = \frac{\sin z}{\cos\delta} dA \tag{9-10}$$

where dA is the rate for the star to travel 1' (4^s) of arc in azimuth, yielding dh in seconds.

<u>Example</u>

$\Phi = 40^o$, $\delta_N = 70^o$, $\delta_s = 30^o$

For $dA = 1' = 4^s$

$dh_N = 5^s\!.85$, $dh_s = 0^s\!.80$.

Then, if the longitude difference determined from north and south stars is $2^s\!.0$, the correctionsto Λ are as follows:

$$d\Lambda_N = \frac{(2.0 \times 5.85)}{5.85 + 0.80} = 1\overset{s}{.}76 \quad ,$$

$$d\Lambda_s = \frac{(2.0 \times 0.80)}{(5.85 + 0.80)} = 0\overset{s}{.}24 \quad .$$

A suggested field observation procedure for the determination of longitude by meridian transit times is [Mueller, 1969]:

(i) make radio-chronometer comparisons before, during, and after each set of observations on a star pair,

(ii) set the instrument (vertical wire) in the meridian for the north star and set the zenith distance for the star; when the star enters the field of view, track it until it coincides with the vertical wire and record the time (hand tappet is pressed),

(iii) set the instrument for the south star of the pair (do not reverse the instrument) and repeat (ii),

(iv) repeat (ii) and (iii) for 12-16 pairs of stars.

The associated computation procedure is as follows:

(i) compute the corrected time for each observed transit (e.g. $T_M + \Delta T$),

(ii) compute the apparent right ascension for each transit,

(iii) compute the longitude for each meridian transit using (9-8),

(iv) compute dh for each star; compute $\Delta\Lambda = \Lambda_N - \Lambda_S$ for each star pair; compute the correction to longitude ($d\Lambda$) for each star of a pair (see example),

(v) compute the mean longitude from the 12-16 pairs,

(vi) apply the correction $\Delta\Lambda_p$ to get the final value of longitude.

An example of this procedure is given in Mueller [1969; Example 11.2].

In closing, the reader should be aware that there are several other methods for second order longitude determination. For complete coverage of this topic, the reader is referred to Mueller [1969] and Robbins [1976].

REFERENCES

Astronomisches Rechen-Institut, Heidelberg [1979]. Apparent Places of Fundamental Stars 1981 Containing the 1535 Stars in the Fourth Fundamental Catalogue, (APFS). Verlag G. Braun, Karlsruhe.

Boss, B. [1937]. General Catalogue of 33342 stars for the epoch 1950.0, (GC). Publications of the Carnegie Institute of Washington, 468. Reissued by Johnson Reprint Co., New York.

Eichorn, H. [1974]. Astronomy of Star Positions. Frederick Ungar Publishing Co., New York.

Fricke, W. and A. Kopff [1963]. Fourth Fundamental Catalogue resulting from the revision of the Third Fundamental Catalogue (FK3) carried out under the supervision of W. Fricke and A. Kopff in collaboration with W. Gliese, F. Gondolatsch, T. Lederke, H. Nowacki, W. Strobel and P. Stumpff, (FK4). Veroeffentlichungen des Astronomischen Recken-Instituts, Heidelberg, 10.

H.M. Nautical Almanac Office [1980]. The Star Almanac for Land Surveyors for the year 1981. Her Majesty's Stationery Office, London. (in Canada: Pendragon House Ltd. 2525 Dunwin Drive, Mississauga, Ontario, L5L 1T2.)

Heiskanen, W.A. and H. Moritz [1967]. Physical Geodesy. W.H. Freeman and Company, San Francisco.

Krakiwsky, E.J. and D.E. Wells [1971]. Coordinate Systems in Geodesy. Department of Surveying Engineering Lecture Notes No. 16, University of New Brunswick, Fredericton.

Morgan, H.R. [1952]. Catalog of 5268 Standard Stars, 1950.0. Based on the Norman System N30. Astronomical Papers Prepared for the Use of the American Ephemeris and Nautical Almanac, 13, part 3. Government Printing Office, Washington, D.C.

Morris, William (editor) [1975]. The Heritage Illustrated Dictionary of the English Language. American Heritage Publishing Co., Inc.

Mueller, Ivan I. [1969]. Spherical and Practical Astronomy as Applied to Geodesy. Frederick Ungar Publishing Co., New York.

Robbins, A.R. [1976]. Military Engineering, Volume XIII-Part IX, Field and Geodetic Astronomy. School of Military Survey, Hermitage, Newbury, Berkshire, U.K.

Roelofs, R. [1950]. Astronomy Applied to Land Surveying. N.V. Wed. J. Ahrend & Zoon, Amsterdam.

U.S. Naval Observatory, Nautical Almanac Office [1980]. The Astronomical Almanac for the Year 1981. U.S. Government Printing Office, Washington, D.C.

APPENDIX A

REVIEW OF SPHERICAL TRIGONOMETRY

In this Appendix the various relationships between the six elements of a spherical triangle are derived, using the simple and compact approach of A.R. Clarke, as given in Todhunter and Leathem ("Spherical Trigonometry", Macmillan, 1943).

The 27 relations derived are listed at the end of the Appendix.

A.1 Derivation of Relationships

In Figure A-1, the centre of the unit sphere, O, is joined to the vertices A, B, C of the spherical triangle. Q and R are the projections of C onto OA and OB, hence OQC and ORC are right angles. P is the projection of C onto the plane AOB, hence CP makes a right angle with every line meeting it in that plane. Thus QPC, RPC, and OPC are right angles. Note that the angles CQP and CRP are equal to the angles A and B of the spherical triangle.

We show that OQP is also a right angle by using right triangles COP, COQ, CQP to obtain

$$CO^2 = OP^2 + PC^2 \tag{A-1}$$

$$CO^2 = OQ^2 + QC^2 \tag{A-2}$$

$$QC^2 = QP^2 + PC^2 \quad . \tag{A-3}$$

Equating (A-1) and (A-2), and substituting for QC^2 from (A-3)

$$OP^2 = OQ^2 + QP^2$$

that is OQP is a right angle. Similarly it can be shown ORP is also a right angle.

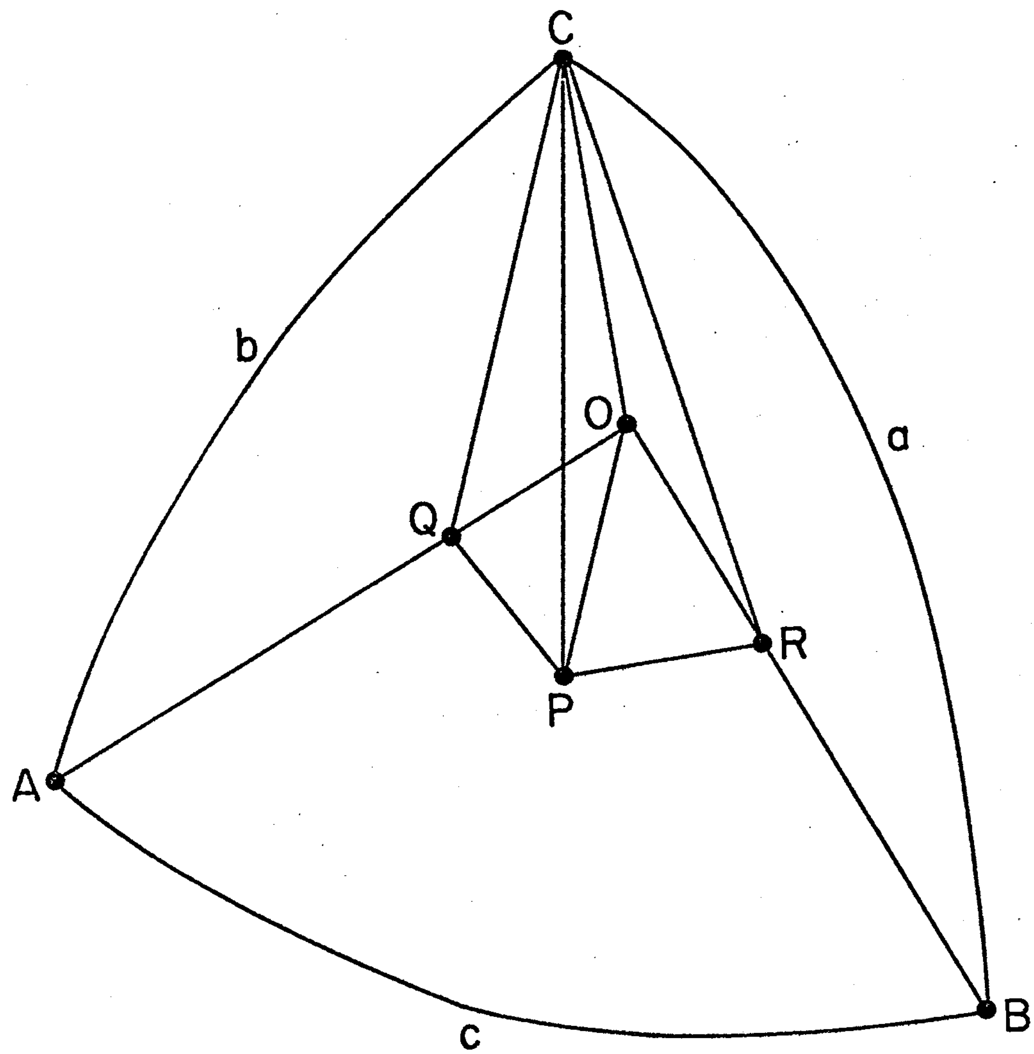


Figure A-1

In Figure A-2, on the plane AOB, S is the projection of Q on OB and T is the projection of R on OA. Note that the angle AOB is equal to the side c of the spherical triangle, and that angles AOB = SQP = TRP. Then from Figure A-1

$$PC = RC \sin B = QC \sin A \qquad (A\text{-}4)$$

and from Figure A-2

$$OR = OS + QP \sin c \qquad (A\text{-}5)$$

$$RP = SQ - QP \cos c \qquad (A\text{-}6)$$

$$QP = TR - RP \cos c \quad . \qquad (A\text{-}7)$$

We can now make the substitutions

$$OR = \cos a$$

$$RC = \sin a$$

$$OQ = \cos b$$

$$QC = \sin b$$

$$QP = QC \cos A = \sin b \cos A$$

$$RP = RC \cos B = \sin a \cos B$$

$$OS = OQ \cos c = \cos b \cos c$$

$$SQ = OQ \sin c = \cos b \sin c$$

$$OT = OR \cos c = \cos a \cos c$$

$$TR = OR \sin c = \cos a \sin c$$

to obtain from (A-4) to (A-7)

$$\sin a \sin B = \sin b \sin A \qquad (A\text{-}8)$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \qquad (A\text{-}9)$$

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A \qquad (A\text{-}10)$$

$$\sin b \cos A = \cos a \sin c - \sin a \cos c \cos B \quad . \qquad (A\text{-}11)$$

Equation (A-8) is one of the three Laws of Sines. Equation (A-9) is one of the three Laws of Cosines for Sides. Equations (A-10) and

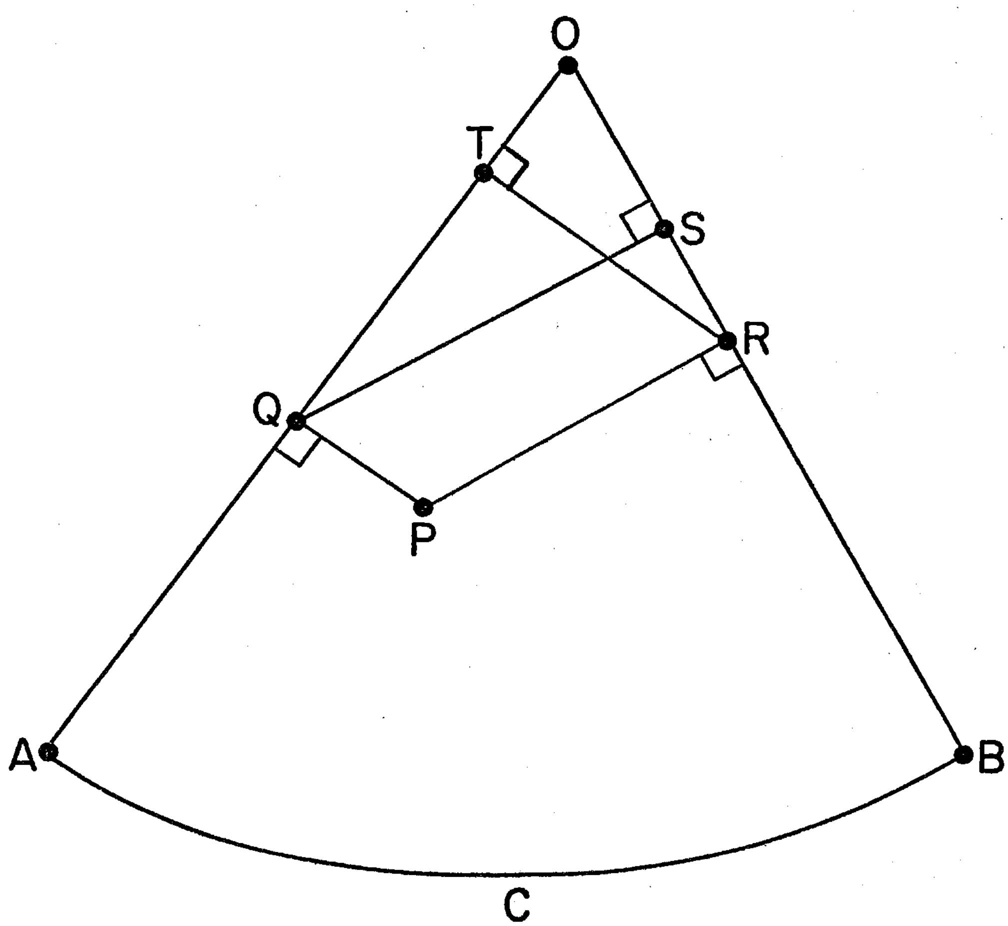


Figure A-2

(A-11) are two of the twelve Five-Element Formulae.

The Four-Element Formulae can be derived by multiplying Equation (A-10) by sin A, and dividing it by Equation (A-8) to obtain

$$\sin A \cot B = \sin c \cot b - \cos A \cos c$$

rearranged as

$$\cos A \cos c = \sin c \cot b - \sin A \cot B \quad . \qquad \text{(A-12)}$$

Similarly multiplying Equation (A-11) by sin B and dividing by the transposed Equation (A-8) we obtain

$$\cos B \cos c = \sin c \cot a - \sin B \cot A \quad . \qquad \text{(A-13)}$$

Equations (A-12) and (A-13) are two of the six Four-Element Formulae.

It can be shown that all the above Laws remain true when the angles are changed into the supplements of the corresponding sides and the sides into the supplements of the corresponding angles. Applying this to Equations (A-8), (A-12), and (A-13) will not generate new equations. However, when applied to Equation (A-9)

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

it becomes

$$\cos(\pi-A) = \cos(\pi-B)\cos(\pi-C)+\sin(\pi-B)\sin(\pi-C)\cos(\pi-a) \qquad \text{(A-14)}$$

or

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

which is one of the three Laws of Cosines for Angles. Similarly Equations (A-10) and (A-11) become

$$\sin A \cos b = \cos B \sin C + \sin B \cos C \cos a \qquad \text{(A-15)}$$

$$\sin B \cos a = \cos A \sin C + \sin A \cos C \cos b \qquad \text{(A-16)}$$

which are two more of the twelve Five-Element Formulae.

Equations (A-8) to (A-16) represent one-third of the relationships. The other two-thirds are obtained by simultaneous cyclic permutation of a, b, c and A, B, C. For example, from Equation (A-8)

$$\sin a \sin B = \sin b \sin A$$

we obtain

$$\sin b \sin C = \sin c \sin B$$

$$\sin c \sin A = \sin a \sin C \quad .$$

The entire set of 27 relations obtained this way are stated below.

A.2 Summary of Relationships

A.2.1 Law of Sines

$$\sin a \sin B = \sin b \sin A$$

$$\sin b \sin C = \sin c \sin B$$

$$\sin c \sin A = \sin a \sin C$$

A.2.2 Law of Cosines (sides)

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos b = \cos c \cos a + \sin c \sin a \cos B$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

A.2.3 Law of Cosines (angles)

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

$$\cos B = -\cos C \cos A + \sin C \sin A \cos b$$

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

A.2.4 Four-Element Formulae

$$\cos A \cos c = \sin c \cot b - \sin A \cot B$$

$$\cos B \cos a = \sin a \cot c - \sin B \cot C$$

$$\cos C \cos b = \sin b \cot a - \sin C \cot A$$

$$\cos B \cos c = \sin c \cot a - \sin B \cot A$$

$$\cos C \cos a = \sin a \cot b - \sin C \cot B$$

$$\cos A \cos b = \sin b \cot c - \sin A \cot C$$

A.2.5 Five-Element Formulae

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$$

$$\sin b \cos C = \cos c \sin a - \sin c \cos a \cos B$$

$$\sin c \cos A = \cos a \sin b - \sin a \cos b \cos C$$

$$\sin b \cos A = \cos a \sin c - \sin a \cos c \cos B$$

$$\sin c \cos B = \cos b \sin a - \sin b \cos a \cos C$$

$$\sin a \cos C = \cos c \sin b - \sin c \cos b \cos A$$

$$\sin A \cos b = \cos B \sin C + \sin B \cos C \cos a$$

$$\sin B \cos c = \cos C \sin A + \sin C \cos A \cos b$$

$$\sin C \cos a = \cos A \sin B + \sin A \cos B \cos c$$

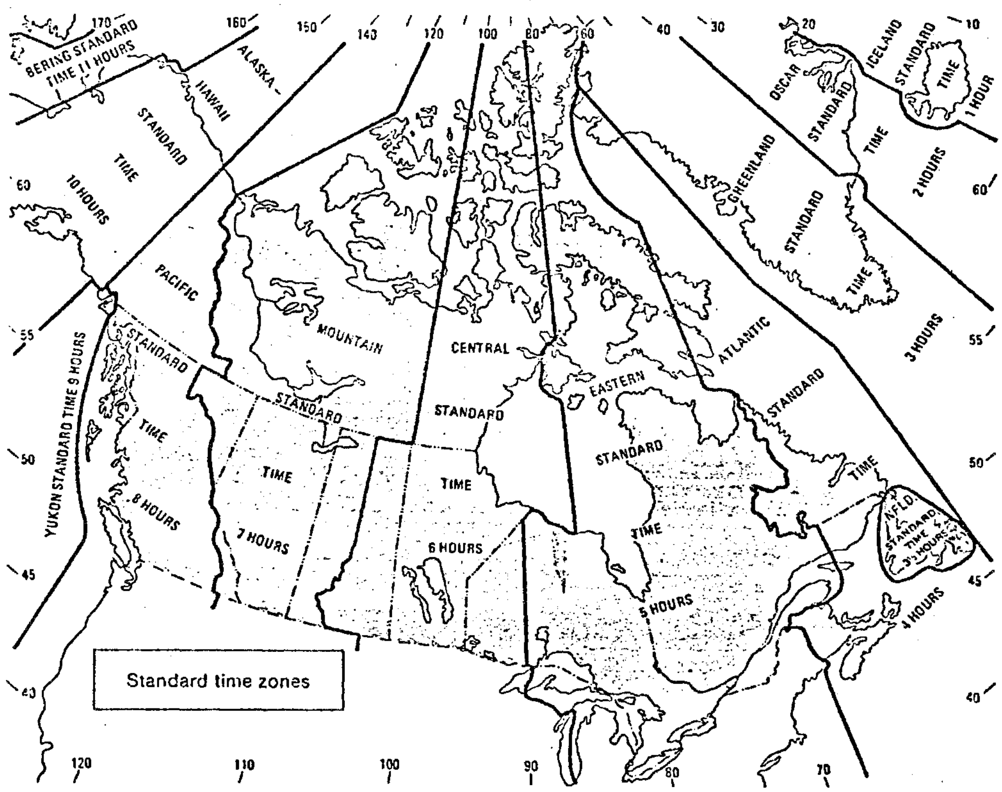
$$\sin B \cos a = \cos A \sin C + \sin A \cos C \cos b$$

$$\sin C \cos b = \cos B \sin A + \sin B \cos A \cos c$$

$$\sin A \cos c = \cos C \sin B + \sin C \cos B \cos a$$

APPENDIX B

CANADIAN TIME ZONES



(Canada Year Book 1978-79, Dept. of Supply and Services, Ottawa)

APPENDIX C

EXCERPT FROM THE FOURTH FUNDAMENTAL CATALOGUE (FK4)

EQUINOX AND EPOCH 1950.0 AND 1975.0

No.	Name	Mag.	Sp.	α	$\frac{d\alpha}{dT}$	$\frac{1}{2}\frac{d^2\alpha}{dT^2}$	μ	$\frac{d\mu}{dT}$	Ep. (α)	m (α)	m (μ)
1253	+19°2254 Leo	6.92	K o	$9^h 43^m 5^s.444$ 9 44 28.710	+ 333^s.241 + 332.887	− 0^s.709 − 0.705	+ 0^s.160 + 0.160	−0^s.001 −0.001	27.44	2.8	18
1254	l Car	3.6-4.8	G o	9 43 52.354 9 44 33.553	+ 164.799 + 164.796	− 0.006 − 0.005	− 0.234 − 0.235	−0.003 −0.003	10.42	9.5	41
1255	Br 1369 UMa	5.20	G o	9 45 22.444 9 46 59.021	+ 386.859 + 385.761	− 2.201 − 2.190	+ 2.162 + 2.156	−0.026 −0.026	26.08	2.5	12
368	υ UMa	3.89	F o	9 47 27.102 9 49 13.454	+ 426.403 + 424.418	− 3.985 − 3.955	− 3.817 − 3.809	+0.032 +0.033	16.50	2.6	10
370	6 Sex	6.00	A 2	9 48 42.826 9 49 58.396	+ 302.309 + 302.251	− 0.118 − 0.113	+ 0.057 + 0.057	−0.001 −0.001	17.29	1.4	9
1256	162 G. Vel	5.72	K o	9 49 23.441 9 50 21.571	+ 232.400 + 232.643	+ 0.482 + 0.490	− 0.380 − 0.380	−0.001 −0.001	21.43	7.6	40
371	μ Leo	4.10	K o	9 49 55.434 9 51 20.659	+ 341.140 + 340.657	− 0.969 − 0.963	− 1.599 − 1.598	+0.004 +0.004	14.55	1.4	6
373	183 G. Hya	5.16	M o	9 52 30.602 9 53 41.360	+ 282.989 + 283.075	+ 0.168 + 0.174	− 0.358 − 0.359	−0.003 −0.003	19.27	2.1	13
1257	18 G. Sex	7.03	K o	9 53 38.894 9 54 53.407	+ 298.062 + 298.043	− 0.039 − 0.035	− 0.166 − 0.166	0.000 0.000	26.42	2.7	16
372	Grb 1586 UMa	5.96	K o	9 53 57.521 9 56 10.786	+ 535.718 + 530.415	−10.691 −10.523	− 1.733 − 1.725	+0.031 +0.032	20.57	4.6	21
374	19 LMi	5.19	F 5	9 54 37.760 9 56 9.479	+ 367.318 + 366.437	− 1.767 − 1.758	− 1.056 − 1.055	+0.006 +0.006	20.46	2.0	10
375	φ Vel	3.70	B 5	9 55 6.249 9 55 58.954	+ 210.696 + 210.941	+ 0.487 + 0.495	− 0.135 − 0.135	−0.001 −0.001	13.38	5.6	30
377	η Ant	5.25	F o	9 56 43.347 9 57 47.729	+ 257.419 + 257.639	+ 0.438 + 0.445	− 0.806 − 0.808	−0.006 −0.006	18.47	3.6	22
376	12 Sex	6.63	A 5	9 57 7.479 9 58 25.259	+ 311.188 + 311.051	− 0.276 − 0.271	− 0.463 − 0.463	+0.001 +0.001	22.33	1.9	11
378	π Leo	4.89	M o	9 57 34.334 9 58 53.564	+ 317.018 + 316.823	− 0.393 − 0.389	− 0.215 − 0.215	−0.001 −0.001	13.03	1.2	5
1258	20 LMi	5.60	G 5	9 58 8.015 9 59 34.389	+ 345.802 + 345.189	− 1.230 − 1.223	− 4.140 − 4.139	+0.006 +0.006	24.37	2.5	11
1259	Pi 9^h229 UMa	5.74	F 5	10 1 17.747 10 2 57.196	+ 398.561 + 397.034	− 3.066 − 3.044	− 0.269 − 0.268	+0.003 +0.003	36.41	3.9	19
1260	193 G. Hya	5.80	F o	10 2 2.261 10 3 11.563	+ 277.137 + 277.283	+ 0.288 + 0.294	− 0.845 − 0.846	−0.002 −0.003	19.07	2.9	18
1261	υ² Hya	4.72	B 8	10 2 41.299 10 3 54.340	+ 292.143 + 292.187	+ 0.086 + 0.092	− 0.289 − 0.289	0.000 0.000	12.88	1.8	10
379	η Leo	3.58	A o p	10 4 36.533 10 5 58.252	+ 327.034 + 326.717	− 0.637 − 0.632	− 0.012 − 0.012	0.000 0.000	16.51	1.4	6
380	α Leo	1.34	B 8	10 5 42.647 10 7 2.482	+ 319.462 + 319.219	− 0.489 − 0.484	− 1.696 − 1.695	+0.003 +0.003	08.52	1.0	4
381	λ Hya	3.83	K o	10 8 8.945 10 9 22.070	+ 292.481 + 292.522	+ 0.080 + 0.085	− 1.407 − 1.408	−0.006 −0.006	15.45	1.5	7
385	ω Car	3.56	B 8	10 12 33.047 10 13 8.698	+ 142.703 + 142.506	− 0.391 − 0.397	− 0.625 − 0.629	−0.014 −0.014	11.62	9.3	49
382	191 G. Vel	4.09	A 2	10 12 37.998 10 13 41.011	+ 251.899 + 252.206	+ 0.609 + 0.618	− 1.345 − 1.347	−0.008 −0.008	16.57	4.4	20
384	ζ Leo	3.65	F o	10 13 54.758 10 15 18.107	+ 333.612 + 333.184	− 0.858 − 0.852	+ 0.134 + 0.134	−0.001 −0.001	18.97	1.4	7
383	λ UMa	3.52	A 2	10 14 5.353 10 15 35.058	+ 361.689 + 360.754	− 1.877 − 1.864	− 1.512 − 1.509	+0.010 +0.010	15.98	1.9	8
1262	32 UMa	5.74	A 3	10 14 25.657 10 16 14.171	+ 435.434 + 432.680	− 5.542 − 5.475	− 1.380 − 1.374	+0.024 +0.024	23.42	4.5	18
1263	ε Sex	5.40	F o	10 15 8.659 10 16 23.189	+ 298.118 + 298.125	+ 0.011 + 0.016	− 1.104 − 1.104	−0.001 −0.001	17.18	1.7	9
1264	187 G. Car	3.44	K 5	10 15 24.581 10 16 14.710	+ 200.367 + 200.608	+ 0.597 + 0.608	− 0.373 − 0.374	−0.006 −0.006	14.19	9.2	39
1265	59 G. Ant	5.62	B 9	10 15 49.908 10 16 58.695	+ 275.039 + 275.234	+ 0.425 + 0.433	− 0.116 − 0.116	0.000 0.000	19.84	3.2	18

EQUINOX AND EPOCH 1950.0 AND 1975.0

No.	δ	$\frac{d\delta}{dT}$	$\frac{1}{2}\frac{d^2\delta}{dT^2}$	μ'	$\frac{d\mu'}{dT}$	Ep. (δ)	m (δ)	m (μ')	GC	N30
1253	+18° 54′ 53.″42 +18 47 57.86	−1658.″91 −1665.53	−13.″31 −13.17	− 1.″76 − 1.76	−0.″01 −0.01	26.00	4.0	26	13444	2335
1254	−62 16 36.42 −62 23 31.98	−1660.65 −1663.82	− 6.36 − 6.33	+ 0.33 + 0.33	+0.02 +0.02	11.64	5.9	25	13462	2339
1255	+46 15 18.13 +46 8 17.72	−1677.87 −1685.44	−15.36 −15.14	− 9.52 − 9.56	−0.18 −0.17	19.20	3.2	12	13497	2345
368	+59 16 30.31 +59 9 25.82	−1693.85 −1702.02	−16.48 −16.20	− 15.55 − 15.48	+0.30 +0.29	09.51	2.2	8	13540	2355
370	− 4 0 29.52 − 4 7 32.11	−1687.48 −1693.25	−11.59 −11.48	− 3.18 − 3.18	0.00 0.00	17.74	2.4	13	13558	2357
1256	−45 57 33.54 −46 4 35.21	−1684.48 −1688.84	− 8.76 − 8.71	+ 3.03 + 3.04	+0.03 +0.03	20.69	7.2	40	13574	2362
371	+26 14 36.08 +26 7 31.27	−1696.01 −1702.44	−12.94 −12.79	− 5.99 − 5.96	+0.12 +0.12	08.03	2.1	8	13590	2364
373	−18 46 18.11 −18 53 25.34	−1706.29 −1711.52	−10.51 −10.42	− 4.22 − 4.21	+0.03 +0.03	21.81	3.8	21	13644	2367
1257	− 7 24 26.90 − 7 31 34.44	−1707.41 −1712.89	−11.01 −10.90	− 0.11 − 0.11	+0.01 +0.01	24.43	4.1	26	13674	2371
372	+73 7 7.17 +72 59 57.74	−1712.77 −1722.63	−19.98 −19.45	− 4.05 − 4.02	+0.13 +0.13	14.87	2.5	12	13684	2373
374	+41 17 40.83 +41 10 31.32	−1714.69 −1721.40	−13.52 −13.33	− 2.91 − 2.89	+0.08 +0.08	16.42	2.7	12	13700	2376
375	−54 19 44.86 −54 26 53.70	−1713.48 −1717.26	− 7.59 − 7.55	+ 0.46 + 0.46	+0.01 +0.01	10.61	4.6	22	13711	2377
377	−35 39 3.66 −35 46 15.14	−1723.64 −1728.23	− 9.21 − 9.15	− 2.41 − 2.39	+0.06 +0.06	19.37	4.5	26	13741	2387
376	+ 3 37 28.41 + 3 30 17.38	−1721.34 −1726.91	−11.20 −11.08	+ 1.69 + 1.70	+0.03 +0.03	19.14	3.3	16	13746	2389
378	+ 8 17 5.76 + 8 9 53.13	−1727.67 −1733.33	−11.39 −11.26	− 2.64 − 2.64	+0.02 +0.02	09.71	1.9	7	13755	2390
1258	+32 10 13.66 +32 2 50.19	−1770.84 −1776.92	−12.25 −12.09	− 43.32 − 43.24	+0.32 +0.32	17.73	3.2	13	13763	2391
1259	+54 8 4.53 +54 0 48.15	−1742.04 −1748.98	−14.00 −13.76	− 0.66 − 0.66	+0.02 +0.02	26.17	4.4	16	13827	2397
1260	−24 2 34.25 −24 9 50.53	−1742.75 −1747.50	− 9.54 − 9.46	+ 1.83 + 1.85	+0.06 +0.06	18.20	4.0	26	13848	2402
1261	−12 49 17.50 −12 56 34.73	−1746.42 −1751.42	−10.05 − 9.95	+ 0.95 + 0.96	+0.02 +0.02	12.09	2.7	14	13861	2408
379	+17 0 26.17 +16 53 6.45	−1756.12 −1761.65	−11.12 −10.99	− 0.58 − 0.58	0.00 0.00	12.83	2.2	9	13899	2412
380	+12 12 44.54 +12 5 23.90	−1759.91 −1765.23	−10.70 −10.57	+ 0.26 + 0.29	+0.12 +0.12	01.35	1.7	5	13926	2414
381	−12 6 22.55 −12 13 48.08	−1779.73 −1784.49	− 9.57 − 9.47	− 9.46 − 9.44	+0.10 +0.10	14.05	2.3	11	13982	2424
385	−69 47 21.35 −69 54 48.54	−1787.70 −1789.84	− 4.30 − 4.27	+ 0.29 + 0.30	+0.04 +0.04	12.97	4.6	23	14074	2434
382	−41 52 25.23 −41 59 51.84	−1784.47 −1788.39	− 7.86 − 7.80	+ 3.84 + 3.86	+0.09 +0.09	14.27	4.7	22	14076	2435
384	+23 40 1.94 +23 32 32.65	−1794.55 −1799.75	−10.48 −10.33	− 1.21 − 1.21	−0.01 −0.01	15.37	2.2	10	14107	2440
383	+43 9 53.54 +43 2 23.26	−1798.29 −1803.90	−11.32 −11.13	− 4.26 − 4.24	+0.10 +0.10	06.91	2.4	8	14113	2442
1262	+65 21 31.58 +65 14 1.60	−1796.54 −1803.30	−13.68 −13.36	− 1.19 − 1.17	+0.09 +0.09	13.14	3.0	12	14123	2445
1263	− 7 49 6.78 − 7 56 36.91	−1798.22 −1802.78	− 9.18 − 9.07	− 0.10 − 0.08	+0.07 +0.07	16.67	2.7	15	14129	2446
1264	−61 4 55.05 −61 12 25.17	−1798.86 −1801.87	− 6.04 − 6.00	+ 0.29 + 0.30	+0.02 +0.02	13.84	5.9	24	14133	2448
1265	−28 44 29.04 −28 51 59.54	−1799.92 −1804.11	− 8.41 − 8.53	+ 0.85 + 0.85	+0.01 +0.01	19.09	4.5	25	14144	2450

APPENDIX D

EXCERPT FROM APPARENT PLACES OF FUNDAMENTAL STARS

APPARENT PLACES OF STARS, 1981

AT UPPER TRANSIT AT GREENWICH

No.	1261				379			
Name	υ^2 Hydrae				η Leonis			
Mag.Spect.	4.72		B8		3.58		A0p	
U.T.	R.A.		Dec.		R.A.		Dec.	
	h m 10 04		° ′ −12 58		h m 10 06		° ′ +16 50	
d	s		″		s		″	
1 −7.8	11.861	+306	14.55	−230	17.535	+323	76.96	−157
1 2.1	12.145	+284	16.94	−239	17.839	+304	75.60	−136
1 12.1	12.397	+252	19.34	−240	18.112	+273	74.48	−112
1 22.1	12.606	+209	21.67	−233	18.344	+232	73.64	−84
2 1.1	12.772	+166	23.87	−220	18.531	+187	73.08	−56
2 11.0	12.868	+116	25.90	−203	18.667	+136	72.82	−26
2 21.0	12.955	+67	27.70	−180	18.751	+84	72.84	+2
3 3.0	12.976	+21	29.27	−157	18.786	+35	73.08	+24
3 12.9	12.952	−24	30.57	−130	18.774	−12	73.54	+46
3 22.9	12.891	−61	31.59	−102	18.722	−52	74.14	+60
4 1.9	12.802	−89	32.36	−77	18.638	−84	74.85	+71
4 11.9	12.687	−115	32.85	−49	18.527	−111	75.63	+78
4 21.8	12.559	−128	33.09	−24	18.401	−126	76.42	+79
5 1.8	12.422	−137	33.10	−1	18.267	−134	77.20	+78
5 11.8	12.282	−140	32.86	+24	18.128	−139	77.94	+74
5 21.8	12.148	−134	32.40	+46	17.997	−131	78.60	+66
5 31.7	12.020	−128	31.74	+66	17.874	−123	79.18	+58
6 10.7	11.905	−115	30.88	+86	17.764	−110	79.66	+48
6 20.7	11.806	−99	29.86	+102	17.674	−90	80.03	+37
6 30.6	11.723	−83	28.70	+116	17.601	−73	80.30	+27
7 10.6	11.660	−63	27.42	+128	17.551	−50	80.44	+14
7 20.6	11.621	−39	26.09	+133	17.525	−26	80.44	+0
7 30.6	11.604	−17	24.71	+138	17.521	−4	80.32	−12
8 9.5	11.613	+9	23.36	+135	17.545	+24	80.04	−28
8 19.5	11.651	+38	22.09	+127	17.595	+50	79.60	−44
8 29.5	11.717	+66	20.94	+115	17.673	+78	79.00	−60
9 8.5	11.816	+99	19.99	+95	17.784	+111	78.21	−79
9 18.4	11.947	+131	19.29	+70	17.925	+141	77.23	−98
9 28.4	12.112	+165	18.87	+42	18.098	+173	76.07	−116
10 8.4	12.313	+201	18.80	+7	18.306	+208	74.72	−135
10 18.3	12.545	+232	19.10	−30	18.545	+239	73.21	−151
10 28.3	12.809	+264	19.77	−67	18.816	+271	71.55	−166
11 7.3	13.099	+290	20.84	−107	19.114	+298	69.78	−177
11 17.3	13.408	+309	22.26	−142	19.433	+319	67.96	−182
11 27.2	13.731	+323	24.01	−175	19.767	+334	66.11	−185
12 7.2	14.057	+326	26.03	−202	20.108	+341	64.32	−179
12 17.2	14.375	+318	28.25	−222	20.443	+335	62.65	−167
12 27.2	14.678	+303	30.61	−236	20.765	+322	61.13	−152
12 37.1	14.952	+274	33.02	−241	21.061	+296	59.85	−128
		+239		−238		+261		−102
Mean Place	11.872		19.85		17.852		80.71	
sec δ, tan δ	+1.026		−0.230		+1.045		+0.303	
dα(ψ), dδ(ψ)	−0.058		−0.35		−0.065		−0.35	
dα(ε), dδ(ε)	−0.013		+0.48		−0.018		+0.48	
Dble.Trans.	February 21				February 21			

No.	380				381			
Name	α Leonis *(Regulus)*				λ Hydrae			
Mag.Spect.	1.34		B8		3.83		K0	
U.T.	R.A.		Dec.		R.A.		Dec.	
	h m 10 07		° ′ +12 03		h m 10 09		° ′ −12 15	
d	s		″		s		″	
1 −7.8	21.350	+317	35.65	−171	39.573	+307	29.85	−229
1 2.1	21.647	+297	34.10	−156	39.860	+287	32.21	−236
1 12.1	21.914	+267	32.75	−135	40.115	+255	34.59	−238
1 22.1	22.140	+226	31.65	−110	40.330	+215	36.89	−230
2 1.1	22.322	+182	30.81	−84	40.500	+170	39.07	−218
2 11.0	22.454	+132	30.26	−55	40.622	+122	41.07	−200
2 21.0	22.535	+81	29.97	−29	40.694	+72	42.84	−177
3 3.0	22.570	+35	29.92	−5	40.721	+27	44.37	−153
3 12.9	22.557	−13	30.10	+18	40.702	−19	45.65	−128
3 22.9	22.507	−50	30.44	+34	40.647	−55	46.65	−100
4 1.9	22.425	−82	30.92	+48	40.563	−84	47.40	−75
4 11.9	22.317	−108	31.51	+59	40.453	−110	47.88	−48
4 21.8	22.194	−123	32.16	+65	40.328	−125	48.11	−23
5 1.8	22.063	−131	32.83	+67	40.194	−134	48.11	+0
5 11.8	21.928	−135	33.51	+68	40.056	−138	47.88	+23
5 21.8	21.800	−128	34.16	+65	39.923	−133	47.43	+45
5 31.7	21.680	−120	34.78	+62	39.796	−127	46.80	+63
6 10.7	21.573	−107	35.34	+56	39.680	−116	45.96	+84
6 20.7	21.483	−90	35.84	+50	39.580	−100	44.98	+98
6 30.6	21.411	−72	36.27	+43	39.496	−84	43.87	+111
7 10.6	21.361	−50	36.61	+34	39.431	−65	42.63	+124
7 20.6	21.334	−27	36.84	+23	39.388	−43	41.35	+128
7 30.6	21.328	−6	36.97	+13	39.366	−22	40.03	+132
8 9.5	21.349	+21	36.96	−1	39.372	+6	38.73	+130
8 19.5	21.397	+48	36.80	−16	39.404	+32	37.52	+121
8 29.5	21.472	+75	36.48	−32	39.465	+61	36.42	+110
9 8.5	21.578	+106	35.97	−51	39.559	+94	35.52	+90
9 18.4	21.714	+136	35.26	−71	39.685	+126	34.86	+66
9 28.4	21.883	+169	34.34	−92	39.844	+159	34.48	+38
10 8.4	22.085	+202	33.21	−113	40.040	+196	34.44	+4
10 18.3	22.319	+234	31.88	−133	40.268	+228	34.76	−32
10 28.3	22.583	+264	30.35	−153	40.528	+260	35.45	−69
11 7.3	22.876	+293	28.67	−168	40.816	+288	36.53	−108
11 17.3	23.187	+311	26.89	−178	41.122	+306	37.96	−143
11 27.2	23.515	+328	25.03	−186	41.445	+323	39.70	−174
12 7.2	23.849	+334	23.18	−185	41.771	+326	41.72	−202
12 17.2	24.177	+328	21.39	−179	42.090	+319	43.92	−220
12 27.2	24.492	+315	19.72	−167	42.395	+305	46.26	−234
12 37.1	24.781	+289	18.23	−149	42.673	+278	48.66	−240
		+255		−127		+242		−235
Mean Place	21.633		37.95		39.622		35.18	
sec δ, tan δ	+1.023		+0.214		+1.023		−0.217	
dα(ψ), dδ(ψ)	+0.064		−0.35		−0.058		−0.35	
dα(ε), dδ(ε)	−0.013		+0.47		−0.013		+0.46	
Dble.Trans.	February 22				February 22			