THE EARTH TIDES

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THE EARTH TIDES

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PREFACE

In order to make our extensive series of lecture notes more readily available, we have scanned the old master copies and produced electronic versions in Portable Document Format. The quality of the images varies depending on the quality of the originals. The images have not been converted to searchable text.

FORWARD

These lecture notes were written for the graduate students in the Department of Surveying Engineering pursuing the studies in geodesy. The aim of the notes is to acquaint the student with the physical phenomenon of the earth tides and the ways it manifests itselfs. In showing this, it was necessary to make a brief excursion into a realm of geophysics, namely in section 2. A separate section is devoted to the barest description of tidal observation techniques and the treatment of observations.

The pertinence of the earth tides manifestations to geodesy is shown whenever appropriate. Some attention is also given to other than earth-tides phenomena affecting the tidal records. It was felt that, because of the potential of the tidal instrumentation to become a geodetic tool, it is advisable for a future geodesist to be at least aware of the existence of these causes influencing the observations.

It would be unwise to regard the notes as complete in any sense of this word. They can only serve as a guide and the interested student is advised to turn to the listed references for deeper understanding and further knowledge. More earth tides literature is given in [Melchior, 1971].

Comments and critism communicated to the author will be highly appreciated.

P. Vaníček

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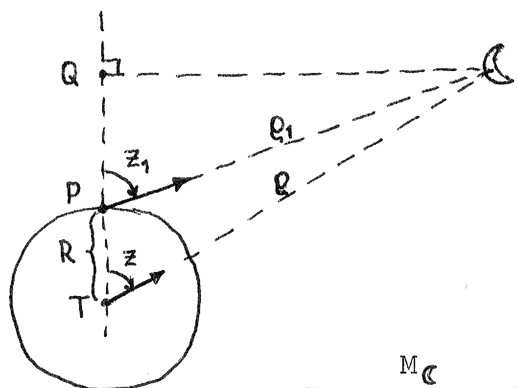
1) TIDES OF THE RIGID EARTH

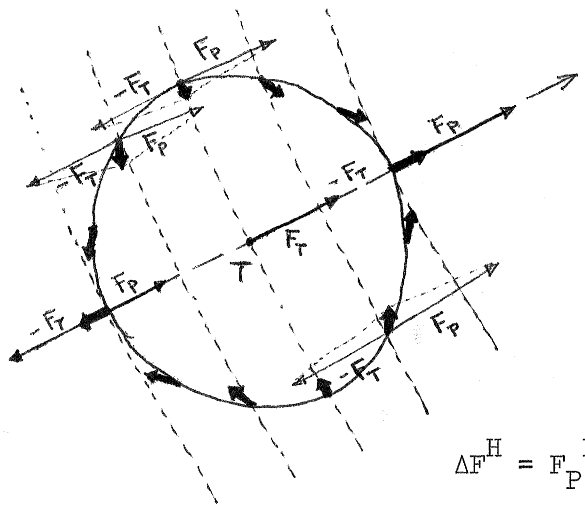
1.1) Tidal Force and Potential

To begin with, let us try to see how does the gravitation attraction of a celestial body, say the moon (☾), affect the earth. To trace the influence let us take two points - the center of gravity of the earth (T) and a point (P) on the surface of the earth. We can write the formulae for the attracting accelerations (in absolute value) in T and P as follows:



$$F_T = \kappa \frac{M_☾}{\rho^2} , \quad F_P = \kappa \frac{M_☾}{\rho_1^2} . \qquad (1)$$

The acceleration F_T, together with similar accelerations excerted by other celestial bodies, govern the motion of the earth in the space. Formulation of this motion constitutes a topic in celestial mechanics and will not be treated here. Our concern in this outline will be the difference of the two accelerations F_T and F_P that can be regarded as the acceleration associated with the deforming (perturbing) force. It provides the tidal strain and stress that try to deform the earth. The strain/stress pattern of the tidal force can be drawn in the following diagram. (The deforming force is depicted by thick arrows.)



It is usual, to express the tidal acceleration in two components - horizontal (ΔF^H) and vertical (ΔF^V). The horizontal component can be written as the difference of the horizontal components of F_T and F_P, i.e.

$$\Delta F^H = F_P{}^H - F_T{}^H = \kappa M_{☾} \left(\frac{\sin Z_1}{\rho_1^2} - \frac{\sin Z}{\rho^2} \right) , \qquad (2)$$

and similarly the vertical component is given by

$$\Delta F^V = F_P{}^V - F_T{}^V = \kappa M_{☾} \left(\frac{\cos Z_1}{\rho_1^2} - \frac{\cos Z}{\rho^2} \right) , \qquad (3)$$

where Z and Z_1 are zenith distances of the celestial body. Considering the earth spherical, in the first approximation, Z_1 and ρ_1 are determined uniquely by Z, ρ and R. Applying the sine law to the triangle ☾TP we get

$$\sin Z_1 = \frac{\rho}{\rho_1} \sin Z.$$

On the other hand, expressing the length PQ twice from the two triangles, TQ and PQ, we obtain

$$\cos Z_1 = \frac{\rho \cos Z - R}{\rho_1} .$$

In order to get rid of ρ_1 in the above formulae, let us express it using the cosine law applied to the triangle ☾TP:

$$\rho_1^2 = \rho^2 + R^2 - 2\rho R \cos Z. \qquad (4)$$

This can be rewritten as

$$\rho_1 = \rho(1 - 2\frac{R}{\rho}\cos Z + (\frac{R}{\rho})^2)^{1/2} = \rho y \qquad (4a)$$

and consequently

$$1/\rho_1 = 1/\rho\ y^{-1}\ . \qquad (4b)$$

Here y^{-1} may be recognized as the generating function for Legendre's polynomials (see [Vaníček, 1971], § 2.22) and we can write

$$y^{-1} = \sum_{m=0}^{\infty} (\frac{R}{\rho})^m P_m(\cos Z) \qquad (5)$$

where $P_m(\cos Z)$ are the Legendre's polynomials of m-th degree. Due to the rapid convergence of this series (R/ρ is of the order of 1/60 for the moon) it is quite sufficient for our purpose, to consider only the first two terms. The error involved in this truncation is of the order of 2% for the moon (and 0.002 % for the sun).

Substituting now for Z_1 in the original formula expressing ΔF^V we obtain

$$\Delta F^V = \kappa M_☾ (\frac{\rho\cos Z - R}{\rho_1^3} - \frac{\cos Z}{\rho^2}). \qquad (3a)$$

Expressing here ρ_1 in terms of ρ, Z, R, yields:

$$\Delta F_☾^V = \kappa M_☾[\frac{\rho\cos Z - R}{\rho^3}(\sum_{m=0}^{\infty}(\frac{R}{\rho})^m P_m(\cos Z))^3 - \frac{\cos Z}{\rho^2}]$$

$$\doteq \kappa M_☾[\frac{\cos Z}{\rho^2}(1 + \frac{R}{\rho}\cos Z)^3 - \frac{R}{\rho^3}(1 + \frac{R}{\rho}\cos Z)^3 - \frac{\cos Z}{\rho^2}]$$

$$\doteq \kappa M_☾[\frac{\cos Z}{\rho^2}(1 + 3\frac{R}{\rho}\cos Z) - \frac{R}{\rho^3}(1 + 3\frac{R}{\rho}\cos Z) - \frac{\cos Z}{\rho 2}],$$

$$\boxed{\Delta F_☾^V \doteq \kappa M_☾ \frac{R}{\rho^3}(3\cos^2 Z - 1)\ .} \qquad (3b)$$

Note that ΔF^V is positive if $F_P^V > F_T^V$, i.e. if the vertical component of the tidal force points outward. For the horizontal component we get similarly:

$$\Delta F_{☾}^{H} = \kappa M_{☾}\left(\frac{\rho \sin Z}{\rho_1^3} - \frac{\sin Z}{\rho^2}\right)$$

$$= \kappa M_{☾}\left[\frac{\sin Z}{\rho^2}\left(\sum_{m=0}^{\infty}\left(\frac{R}{\rho}\right)^m P_m(\cos Z)\right)^3 - \frac{\sin Z}{\rho^2}\right]$$

$$\doteq \kappa M_{☾}\left[\frac{\sin Z}{\rho^2}\left(1 + 3\frac{R}{\rho}\cos Z\right) - \frac{\sin Z}{\rho^2}\right]$$

$$= \kappa M_{☾}\frac{3R}{\rho^3}\sin Z \cos Z ,$$

$$\boxed{\Delta F_{☾}^{H} \doteq \kappa M_{☾}\frac{3}{2}\frac{R}{\rho^3}\sin 2Z .} \tag{2a}$$

We shall note the following points at this stage. First point is that the horizontal component is directed (in the horizontal plane) towards the attracting celestial body, i.e. to the moon in our case. Second, we can see that the expressions developed above for the moon are valid for any other celestial body as well, when we take the appropriate mass, distance ρ and zenith distance Z. Third, it is conceivable that both components, the vertical and the horizontal, can be obtained from the expression for the tidal potential:

$$\boxed{W \doteq \frac{3}{4}\kappa M\frac{R^2}{\rho^3}\left(\cos 2Z + \frac{1}{3}\right) .} \tag{6}$$

The reader may prove this by direct computation, i.e. take the derivatives of the potential in horizontal and vertical sense.

1.2 Theoretical Magnitude of the Tidal Effects

Let us first consider a second approximation taking the actual distance of the surface point from the center of gravity (a) instead of the mean radius of the earth (R). We can rewrite the formula for the tidal potential as

$$W \doteq \frac{3}{4} \kappa M \frac{a^2}{\rho^3} (\cos 2 Z + \frac{1}{3}) \qquad (6a)$$

with the understanding that if a higher precision is needed Legendre's polynomials of higher order can be taken into account. Introducing the Doodson's tidal constant

$$G (R , C) = \frac{3}{4} \kappa M \frac{R^2}{C^3} , \qquad (7)$$

where C is the mean value of the semi-major axis of the orbit of a celestial body, we get

$$W \doteq (\frac{a}{R})^2 (\frac{C}{\rho})^3 G(R,C) (\cos 2 Z + \frac{1}{3}) . \qquad (6b)$$

The ratios a/R and C/ρ do not deviate from 1 by much and as a first approximation, we may write

$$G (a, \rho) = (\frac{a}{R})^2 (\frac{C}{\rho})^3 G(R,C) \doteq G (R,C). \qquad (7a)$$

Note that the following equation is satisfied:

$$W \doteq G(a, \rho) (\cos 2Z + \frac{1}{3}) . \qquad (6c)$$

The value of the Doodson's constant for the moon is

$$G (R, C) = 26,206 \pm 84 \ [cm^2 \ sec^{-2}].$$

This value has been derived from various astronomic observations. For the sun we get similarly

$$G\ (R\ C\ ) = 0.46051\ G(R\ C\ )\ .$$

This means that the attraction of the sun accounts for about 46% of that of the moon. This ratio persists in all the tidal computations.

Now, we can make a use of the Doodson's constant and write for the vertical component of the tidal acceleration:

$$\Delta F^{V} \doteq \frac{4}{3} \left(\frac{C}{\rho}\right)^{3} \frac{a}{R^{2}}\ G\ (R,C)\ (3 \cos^{2} Z - 1). \qquad (3c)$$

This acceleration can be immediately interpreted as the negative contribution to the gravity. A brief computation shows that gravity hence varies within < -0.054 mgal, 0.108 mgal > due to the varying position of the moon and < -0.025 mgal, 0.050 mgal > due to that of the sun. The span of the combined effect then amounts to 237 μgal. The estimates for other celestial bodies show the following orders of magnitudes.

Venus: 10^{-2} μgal

Jupiter: 10^{-3} μgal

Mars: 10^{-4} μgal

Gallaxie: 10^{-16} μgal

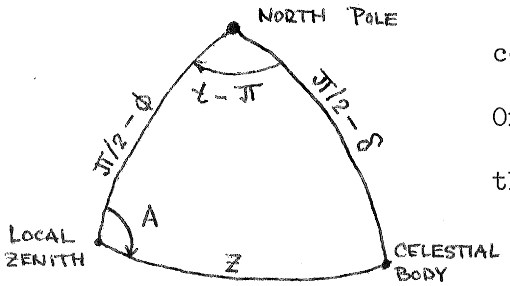
and can therefore be disregarded altogether.

The tidal variations of gravity are known as the <u>gravimetric tide</u>. that evidently has to be taken into consideration in high-precision gravimetric observations. The gravity tide obviously affects the height of the earth equipotential surfaces. The uplift ΔH of these surfaces can be computed from the well known formula, see [Vaníček, 1972]:

$$\boxed{\Delta H = \frac{W}{g}} \qquad (8)$$

1.3) Variations of Tidal Potential with Location and Time

In practice, we are interested not only in the overall magnitude of the tides but also in the variations of the tides with the location on the surface of the earth and with time. To be able to predict these variations for different points and time let us express the zenith distance Z of the celestial body as a function of its position and the point on the earth. Using the nautical triangle, we can write



$$\cos Z = \sin\phi \sin\delta + \cos\delta \cos\phi \cos t.$$

On the other hand, the $(\cos 2Z + \frac{1}{3})$ term in the potential can be rewritten as

$$\cos 2Z + \frac{1}{3} = 2\left(\cos^2 Z - \frac{1}{3}\right) .$$

Now we can try to get the expression for $\cos^2 Z - \frac{1}{3}$ from the cosine formula above. We obtain

$$\cos^2 Z - \frac{1}{3} = \sin^2\phi \sin^2\delta + 2 \sin\phi \cos\phi \sin\delta \cos\delta \cos t + \cos^2\phi \cos^2\delta \cos^2 t - \frac{1}{3} .$$

This can again be rewritten as follows

$$\cos^2 Z - \frac{1}{3} = \sin^2\phi \sin^2\delta + \frac{1}{2}\sin 2\phi \sin 2\delta \cos t + \frac{1}{2}\cos^2\phi \cos^2\delta (1 + \cos 2t) - \frac{1}{3}$$

$$= \frac{1}{2}\cos^2\phi \cos^2\delta \cos 2t + \frac{1}{2}\sin 2\phi \sin 2\delta \cos t + \sin^2\phi \sin^2\delta + \frac{1}{2}\cos^2\phi \cos^2\delta - \frac{1}{3} .$$

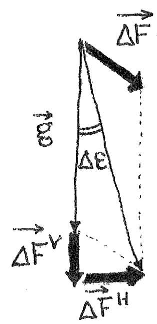
The last three terms may be reformulated again, yielding

$$\sin^2\phi \sin^2\delta + \frac{1}{2}(1 - \sin^2\phi)(1 - \sin^2\delta) - \frac{1}{3} =$$

$$= \sin^2\phi \sin^2\delta + \frac{1}{2} - \frac{1}{2}\sin^2\phi - \frac{1}{2}\sin^2\delta + \frac{1}{2}\sin^2\phi \sin^2\delta - \frac{1}{3}$$

where W is the tidal potential and g is the gravity. The magnitude of the uplift due to the moon is within < -17.8 cm, 35.6 cm > and due to the sun from <-8.2 cm, 16.4 cm >. The total oscilation of the equipotential surfaces can thus be as much as 78.0 cm. As we shall see later, these changes, in case of the actual earth, are even larger.

The horizontal component of the tidal force can be understood as a shear force. It can be measured in terms of the variations of vertical.



From the diagram we can see that the deformed gravity vector $\vec{g} + \Delta\vec{F}$ representing the vertical will be tilted with respect to the undeformed gravity vector $\vec{g}$ by $\Delta\varepsilon$ for which we can write

$$\Delta\varepsilon \doteq \Delta F^H / g \; . \qquad (9)$$

The tidal effect on the vertical is sometimes called the tidal tilt. The maximum tilt will be obviously observed in the vertical plane containing the attracting celestial body. Expressing the horizontal component of the tidal force using the Doodson constant we obtain

$$\boxed{\Delta\varepsilon \doteq \frac{2}{g} \left(\frac{C}{\rho}\right)^3 \frac{a}{R^2} \, G\,(R,\, C) \sin 2Z.} \qquad (9a)$$

Evaluation for the moon gives the value $\Delta\varepsilon$ to be within < -0.017", 0.017"> and for the sun $\Delta\varepsilon \in$ < -0.008", 0.008" >. Hence the maximum variation of the vertical due to the attraction of these two bodies is 0.050". The effect of other celestial bodies can again be neglected.

It has to be pointed out that the tilt is a two-dimensional phenomenon; its magnitude varies with the direction in the horizontal plane. While, for one celestial body, its maximum is seen in the direction towards the body, in the perpendicular direction the tilt goes to zero. In the intermediate directions the tilt is between the maximum and zero.

$$= \frac{3}{2} \sin^2 \phi \sin^2 \delta - \frac{1}{2} \sin^2 \phi - \frac{1}{2} \sin^2 \delta + \frac{1}{6}$$

$$= \frac{1}{2} (3 \sin^2 \phi \sin^2 \delta - \sin^2 \phi - \sin^2 \delta + \frac{1}{3})$$

$$= \frac{3}{2} (\sin^2 \phi - \frac{1}{3}) (\sin^2 \delta - \frac{1}{3}) \quad .$$

Then the tidal potential can be written as

$$W = G\,(a, \rho)\,[\cos^2 \phi \cos^2 \delta \cos 2t + \sin 2\phi \sin 2\delta \cos t + 3\,(\sin^2 \phi - \frac{1}{3})(\sin^2 \delta - \frac{1}{3})\,]. \qquad (6d)$$

The three constituents of the potential can now be dealt with individually. It is convenient to denote W as:

$$W = S + T + Z, \qquad (6e)$$

where

$$S = G(a, \rho) \cos^2 \phi \cos^2 \delta \cos 2t \qquad (10)$$

$$T = G(a, \rho) \sin 2 \phi \sin 2 \delta \cos t \qquad (11)$$

$$Z = 3G(a, \rho) (\sin^2 \phi - \frac{1}{3}) (\sin^2 \delta - \frac{1}{3}) \qquad (12)$$

are known as the <u>sectorial</u>, <u>tesseral</u> and <u>zonal</u> functions respectively.

It can be noted now that the tidal potential contains three variables dependent on time. These are ρ, δ and t. From these, the radius-vector of the celestial body varies with time (around its mean value C) only within a narrow interval. Similarly, the declination δ also oscilates around a mean value for a certain period of time. Hence, replacing ρ and δ, in the frist approximation, by their mean values, we can see that the time variations of the individual tidal constituents are predominently governed by t. t, being the hour angle, undergoes one cycle per day and we can thus see that the sectorial constituent gives raise to mainly semidiurnal variations, the tesseral to diurnal variations and the zonal to long term variations.

1.4) Decomposition of Tidal Potential into Frequencies

The classification of tidal frequencies in the three categories, i.e. semidiurnal diurnal and long-periodic, as used in the previous section is of course valid only in the first approximation. As soon as one starts considering even ρ and δ as varying in time, one discovers that the tidal potential variation is a much more complex phenomenon, requiring a more sophisticated treatment.

We shall show the principle of one such treatment, due basically to G.H. Darwin [1886], for the sectorial constituent.

Rewriting the sectorial constituent

$$S = G\ (a,\ \rho)\ \cos^2 \phi\ \cos^2 \delta\ \cos 2t \qquad (10)$$

as

$$S = G\ (a,\ c)\ \left(\frac{c}{\rho}\right)^3 \cos^2 \phi\ \cos^2 \delta\ \cos 2t\ , \qquad (10a)$$

we can separate the terms depending only on the latitude from those depending on time. Denoting $G(a,c)\cos^2\phi$ by G_ϕ we get

$$S = G_\phi \left(\frac{c}{\rho}\right)^3 \cos^2 \delta\ \cos 2t \qquad (10b)$$

where G_ϕ, known as the geodetic function, does not vary with time. The remaining three terms are all variable in time.

The radius-vector ρ and the declination of the celestial body whose tidal influence we seek (usually only the moon and the sun) can be now expressed in terms of their orbital elements. Expressing even the hour angle in the same terms, we find ourselves dealing with the following periods: [Melchior, 1966]

mean solar day	1 m.s.d.
tropic year	365.242199 m.s.d.
perihelion revolution	20.940 t.y.
lunar nodes revolution	18.613 t.y.
moon perigee revolution	8.847 t.y.
mean anomalistic year	365.25964 m.s.d.
evection period	31.812 m.s.d.
mean synodic month	29.53059 m.s.d.
mean anomalistic month	27.55455 m.s.d.
tropic month	27.321582 m.s.d.
mean draconitic month	27.21222 m.s.d.
variation period	14.76530 m.s.d.
mean lunar day	1.035050 m.s.d.
sidereal day	0.997270 m.s.d.

Table 1

The time-varying terms of the three constituents (sectorial, tesseral, and zonal) can be further rewritten in terms of pure cosine series using some simple trigonometric formulae. This was first done by Doodson [1921] who also introduced symbols for the individual cosine vawes that have been used ever since. The following table summarizes the principal waves as obtained by Doodson [Melchior, 1966].

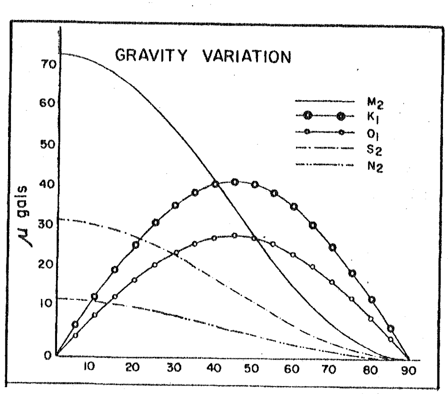
Symbol	Velocity per hour	Amplitude $\cdot 10^5$	Origin (L, lunar; S, solar)
		Long period components	
M_o	0°,000000	+ 50458	L constant flattening
S_o	0°,000000	+ 23411	S constant flattening
S_a	0°,041067	+ 1176	S elliptic wave
S_{sa}	0°,082137	+ 7287	S declinational wave
M_m	0°,544375	+ 8254	L elliptic wave
M_f	1°,098033	+ 15642	L declinational wave
		Diurnal components	
Q_1	13°,398661	+ 7216	L elliptic wave of O_1
O_1	13°,943036	+37689	L principal lunar wave
M_1	14°,496694	- 2964	L elliptic wave of ${}^mK_1$
π_1	14°,917865	+ 1029	S elliptic wave of P_1
P_1	14°,958931	+17554	S solar principal wave
S_1	15°,000002	- 423	S elliptic wave of ${}^sK_1$
${}^mK_1$ }	15°,041069	-36233	L declinational wave
${}^sK_1$ }	15°,041069	-16817	S declinational wave
Ψ_1	15°,082135	- 423	S elliptic wave of ${}^sK_1$
ϕ_1	15°,123206	- 756	S declinational wave
J_1	15°,585443	- 2964	L elliptic wave of ${}^mK_1$
OO_1	16°,139102	- 1623	L declinational wave
		Semi-diurnal components	
$2N_2$	27°,895355	+ 2301	L elliptic wave of M_2
μ_2	27°,968208	+ 2777	L variation wave
N_2	28°,439730	+17387	L major elliptic wave of M_2
ν_2	28°,512583	+ 3303	L evection wave
M_2	28°,984104	+90812	L principal wave
λ_2	29°,455625	- 670	L evection wave
L_2	29°,528479	- 2567	L minor elliptic wave of M_2

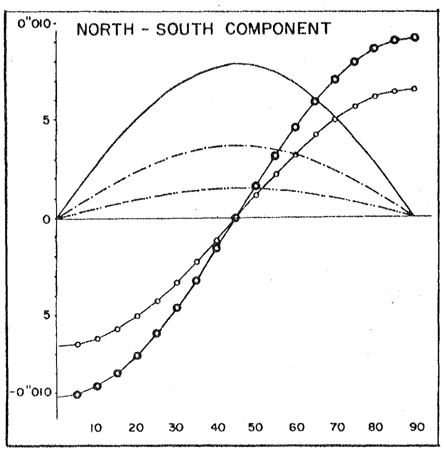
Table 2 continued

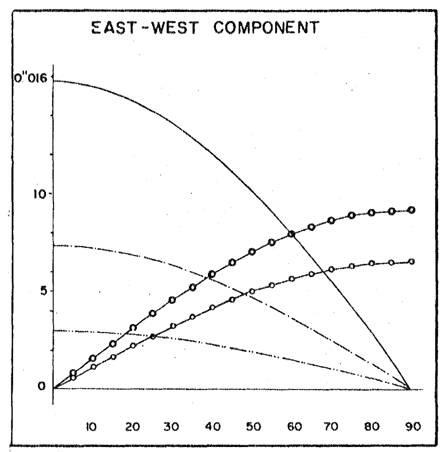
Symbol	Velocity per hour	Amplitude$\cdot 10^5$	Origin (L, lunar; S, solar)
T_2	29°,958933	+ 2479	S major elliptic wave of S_2
S_2	30°,000000	+42286	S principal wave
R_2	30°,041067	- 354	S minor elliptic wave of S_2
${}^{m}K_2$ }	30°,082137	+ 7858	L declinational wave
${}^{s}K_2$ }	30°,082137	+ 3648	S declinational wave
		Ter-diurnal component	
M_3	43°,476156	- 1188	L principal wave

Table 2

We may note that the most predominant waves are the M_2, S_2, N_2 from the semi-diurnal and O_1, K_1 from the diurnal. The actual ratios of amplitudes of the long-period, diurnal, and semi-diurnal will, of course, depend on the values of the appropriate geodetic functions. The diagrams from [Melchior, 1964], illustrate this point.





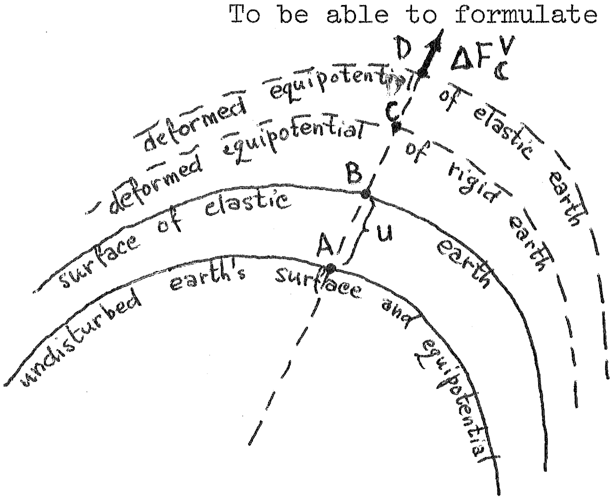


The two-dimensionality of the tilt mentioned in 1.2 should be again noticed here.

2) TIDES OF THE ACTUAL EARTH

2.1) Tidal Deformation of the Earth

So far, we have been assuming the earth to be rigid. In reality the earth is an elastic body and as such deforms in response to the tidal force.



To be able to formulate mathematically the influence of the elasticity of the earth on the observations of gravity and direction of the vertical, let us have a look at the diagram. There we can follow what is happening to the point A on the surface of the real earth coinciding with its own undisturbed equipotential surface . By applying the tidal force $\Delta F_{\leftmoon}^{V}$ due to, say, the moon, the equipotential surface changes so that it passes through the point C. If the earth were completely plastic it would now try to reach the same shape as this equipotential surface. Because the earth is not plastic (more about it will be said later), it does not reach the same shape and conforms only partially to the equipotential. The point A on the surface of the earth gets uplifted (by u) to the position B.

This deformation causes an additional change of the equipotential surface so that it no longer passes through C but D. Hence by applying the tidal force to a point A on the earth and equipotential surface , the physical surface of the earth changes to B and the equipotential surface changes to D.

What change in the potential W would now an observer on the surface of the earth observe? The potential W(A) obviously changes to W(B) so that one would observe the difference W(B) - W(A). This difference can be expressed as a summation of three terms: (i) $W_☾$, the lunar tidal potential (same as for the rigid earth);

(ii) $-\Delta W(u)$, the loss in potential due to the uplift u;

(iii) $\Delta W(u)_{def}$, the additional potential due to the tidal deformation of the earth, called the deformation potential.

These three terms can now be evaluated separately:

(i) The lunar tidal potential is given by the eqs. 6 to 6e.

(ii) The loss of potential can be expressed as (see [Vaníček, 1971; 1972]):

$$\Delta W(u) = ug \ , \qquad (13)$$

where u can be interpreted as a linear deformation of an elastic body caused by the force $\Delta F_☾^V$. Applying the Hook's law for a deformation of an elastic body, we can write

$$u = \text{const.}\ \Delta F_☾^V \qquad (14)$$

or

$$u = \text{const.}\ \frac{\partial W_☾}{\partial a} \ . \qquad (14a)$$

Recalling eq. 7a we obtain

$$\frac{\partial W_☾}{\partial a} \doteq \frac{\partial}{\partial a}\left[2\,\frac{a^2}{R^2}\ G(R,\rho_☾)\ (\cos^2 Z_☾ - \tfrac{1}{3})\right]$$

$$= 4\,\frac{a}{R^2}\,G(R,\rho_☾)\ (\cos^2 Z_☾ - \tfrac{1}{3}) = \frac{2}{a}\,W_☾ \ .$$

Hence

$$\Delta W(u) \doteq g \text{ const. } \frac{2}{a} W_{☾} . \qquad (13a)$$

Here, the expression g const. $\frac{2}{a}$ is a function of a and is known as the first Love's "number" h(a) [Love, 1927]:

$$h(a) = g \text{ const. } \frac{2}{a} . \qquad (15)$$

(iii) The deformation potential can be evaluated as the potential of the masses enclosed in between the original surface (A) and the deformed surface (B) of the earth. It is evidently proportionate to the uplift u and we can therefore write

$$\Delta W_{def}(u) = \text{const.* } u . \qquad (16)$$

Substituting for u from 14a we obtain

$$\Delta W_{def}(u) \doteq \text{const.* const. } \frac{2}{a} W_{☾} . \qquad (16a)$$

Here, the expression by $W_{☾}$ is again a function of a and is called the second Love's "number" k(a) [Love, 1927]:

$$k(a) = \text{const.* const. } \frac{2}{a} . \qquad (17)$$

Putting the three terms together we finally end up with the equation for the potential difference:

$$W(B) - W(A) \doteq (1 + k(a) - h(a)) W_{☾} . \qquad (18)$$

It is not difficult to see, that exactly the same equation holds even for the Sun so that the complete potential difference reads:

$$\boxed{W(B) - W(A) \doteq (1 + k - h)(W_{☾} + W_{\odot}) = (1 + k - h) W.} \qquad (18a)$$

It has to bee mentioned now that the elastic response of the earth is different for different frequencies of the deforming force. The higher the frequency of the tidal force, the more "rigidly" the earth responds.

Inversely, if the applied force is constant or secular, the earth responds as an almost plastic body. This is, of course, reflected in the fact the the Love's numbers are also frequency dependent. In the existing literature this functional relationship is not usually expressed explicitely with the understanding that the symbols h, k mean the values for a = R and semidiurnal frequency (corresponding to M_2 constituent).

2.2) Tidal Variations of Other Quantities

Having described how the earth and its equipotential surfaces change in response to the tidal force we can now attempt to answer questions concerning other problems. We may now, for instance, answer the question: How much does the observer actually move (with respect to the centre of gravity of the earth) with the earth-tide? The answer is given by the value of u (AB). Combining 14a and 15 we get

$$\boxed{u = h \frac{W_{☾} + W_{\odot}}{g} = h \frac{W}{g} .} \tag{14b}$$

The deformation of the equipotential surface, given by the displacement AD, can be evaluated from the following formula

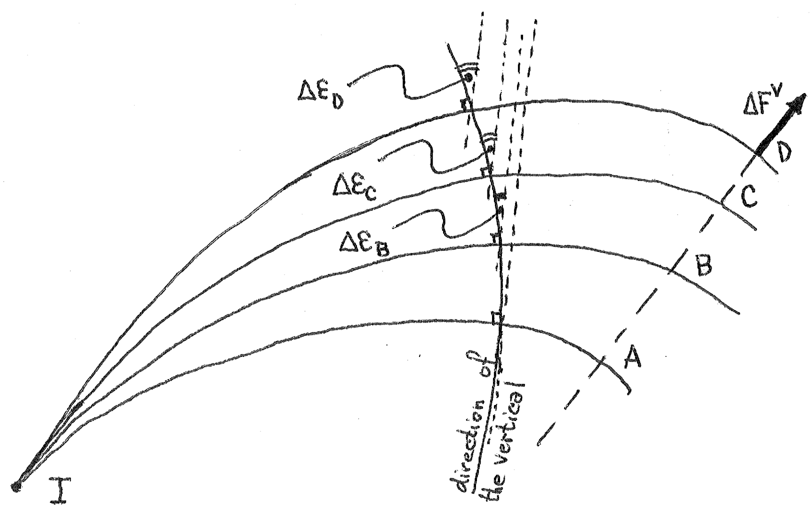
$$\boxed{AD \doteq (1 + k) \frac{W}{g} .} \tag{19}$$

Similarly, we can derive the expression for the theoretical height of the sea-tide. Assuming that the water does reach the equilibrium i.e. the water level coincides with the equipotential surface, the sea-tide height as observed on the shore by an observer moving with the earth would be given by the displacement BD. Thus

$$BD \doteq (1 + k - h)\,\frac{W}{g} \tag{20}$$

which is (1 + k - h) - times smaller than the theoretical value of the sea-tide observed on rigid earth. In reality the sea-tide height is always different from the theoretical value because of various dynamic effects such as the resonance.

Let us consider now the tidal tilt that would be observed by an observer moving with the earth surface. The situation depicted on the diagram will occur.



Instead of observing the magnitude derived for rigid earth, $\Delta\varepsilon_C$ see eq. 9, we would observe the quantity $\Delta\varepsilon_D - \Delta\varepsilon_B$. Assuming that all four surfaces intersect approximately in a line, seen as point I here, we can expect the relative tilts to be proportionate to the appropriate displacements.

Hence

$$\Delta\varepsilon_D - \Delta\varepsilon_B \doteq \lambda BD, \quad \Delta\varepsilon_C \doteq \lambda AC$$

where λ is a constant of proportionality.

Denoting $\Delta\varepsilon_D - \Delta\varepsilon_B$ by $\Delta\varepsilon^*$ we obtain

$$\Delta\varepsilon^* \doteq \Delta\varepsilon_C \frac{BD}{AC} . \tag{21}$$

Here BD is given by eq. 20 and AC by eq. 8. Thus the equation linking the rigid earth tidal tilt $\Delta\varepsilon_C$ with the elastic earth tidal tilt $\Delta\varepsilon^*$ reads

$$\boxed{\Delta\varepsilon^* \doteq (1 + k - h)\, \Delta\varepsilon_C \quad .} \tag{21a}$$

Finally, we can derive an expression for the gravimetric tide observed on the surface of the actual earth. According to section 2.1 we can write

$$W(B) - W(A) = W - \Delta W(u) + \Delta W_{def}(u). \tag{18b}$$

Differentiating this with respect to a yields

$$\frac{\partial}{\partial a}\,(W(B) - W(A)) = \frac{\partial W}{\partial a} - \frac{\partial}{\partial a}\,\Delta W(u) + \frac{\partial}{\partial a}\,\Delta W_{def}(u) \tag{22}$$

which in turn equals

$$\frac{\partial}{\partial a}\,(W(B) - W(A)) = -(g_B - g_A) = g_A - g_B = -\Delta g_{AB}. \tag{22a}$$

It has been shown [Melchior, 1966] that

$$\Delta W(u) \propto a^{-2}, \quad \Delta W_{def}\,(u) \propto a^{-3} \; .$$

Hence

$$\frac{\partial}{\partial a}\,\Delta W(u) = -\frac{2}{a}\,\Delta W(u)$$

and

$$\frac{\partial}{\partial a}\,\Delta W_{def}(u) = -\frac{3}{a}\,\Delta W_{def}(u) \; .$$

Substituting for $\Delta W(u)$ and $\Delta W_{def}(u)$ from (18a) we obtain

$$\frac{\partial}{\partial a}\,\Delta W(u) \doteq -\frac{2}{a}\,hW \doteq -h\,\frac{\partial W}{\partial a}$$

and

$$\frac{\partial}{\partial a} \Delta W_{def}(u) \doteq -\frac{3}{a} kW \doteq -\frac{3}{2} k \frac{\partial W}{\partial a} .$$

Substituting these results back in (22) we get finally

$$\boxed{\Delta g_{AB} \doteq -(1 + h - \frac{3}{2} k) \frac{\partial W}{\partial a}} , \qquad (22b)$$

where $\partial W/\partial a$ can be interpreted as the negatively taken gravimetric tide for the rigid earth.

Tidal variations of other quantities, like linear strain, cubic expansion etc. can also be determined. They however, require an introduction of another function called the Shida's number and denoted by ℓ. Since these quantities are of less interest to geodesist we are not going to talk about them here.

2.3) Experimental Results

The two Love's numbers and the Shida's number are one of the fundamental parameters characterizing the elastic properties of the earth. They are intimately connected with the modulae of compressibility λ and elasticity μ (known also as Lamé's coefficients), Poisson's coefficient ν and Young's modulus E. As such they are of the utmost interest to the geophysicists to understand the physical properties of the earth. Hence a great deal of effort had been put into experiments leading to the evaluation of the above numbers.

From the quantities discussed in 2.2 only the last two i.e. the tilt and gravity variations, lend themselves readily to observation. Hence two different linear combinations of the Love's numbers can be readily observed making use of the phenomenon of earth tides. Denoting the theoretical values, for the rigid earth using formulae 9a and 3c, by $\Delta\varepsilon_C$ and Δg_C, and the corresponding observed values by $\Delta\varepsilon$ and Δg, we obtain from 21a and 22b:

$$(1 + k - h) = \Delta\varepsilon/\Delta\varepsilon_C \tag{23}$$

$$(1 - \frac{3}{2}k + h) = \Delta g/\Delta g_C \ . \tag{24}$$

These two particular linear combinations are known as diminishing and gravimetric factors and denoted by either D and G or γ and δ respectively. Since both k and h vary with the frequency of the tidal force, so do the factors D, G.

From years of tidal observations the following values of D and G have been deduced [Melchior, 1966]:

$$D \doteq 0.706$$

$$G \doteq 1.149$$

for semidiurnal frequencies, namely M_2 component. Solving the equations 23 and 24 yields

$$k = 4 - 2D - 2G \doteq 0.290$$

$$h = 5 - 3D - 2G \doteq 0.584 \ .$$

The values for diurnal frequencies because of smaller amplitudes of diurnal terms in our latitudes and higher degree of their contamination by noise (see section 4) can be determined only with lower degree of certainty.

The extensive analyses of results of tilt observations from various tidal observatories show that there are considerable local biases present. The causes for these biases are not yet understood. The hypothesis that is presently gaining some recognition, [King and Bilham, 1973], postulates that the tilt observations are influenced by coupling of the tilt with stress deformations of underground chambers where the observations are usually carried out. In spite of these local anomalies, the derived values for k and h agree fairly well with the astronomically determined as well as theoretically predicted values.

2.4) Actual Magnitude of the Tidal Effects

In 1.2 we have discussed the theoretical magnitudes of some of the tidal effects. Now, we are finally in a position to evaluate the actual observable effects.

Taking first the tidal tilt, we would not observe the variations of the direction of vertical within $< -0.025'', 0.025'' >$ but D-times smaller, i.e.

$$\Delta\varepsilon \in < -0.018'', 0.018'' > .$$

This having a systematic character in time should be taken into account for precise astronomic and levelling works [Kukkamäkki, 1949; Rune, 1950; Simonsen, 1950].

On the other hand the gravity variations observed on the surface of the actual earth will be G-times larger than the theoretical, namely

$$\Delta g \in < -0.117 \text{ mgal}, 0.234 \text{ mgal} > .$$

Since these variations are larger than the sensitivity of most of the modern gravimeters, the tidal effect is usually taken into account in the precise gravity work.

The tidal deformation of the equipotential surfaces close to the surface of the earth (including thus the geoid) is given by formula 19.

Substituting for W/g the values obtained in 1.2 from eq. 8, we discover that the actual equipotential surface can oscillate within

$$AD \; \varepsilon < -33.6 \text{ cm}, 67.2 \text{ cm} > .$$

Thus the span of the tidal oscillations of the geoid can be over 1 metre.

We can also evaluate the actual uplift of an observer on the surface of the earth. It is given by eq. 14 b that yields

$$u \; \varepsilon < -15.1 \text{ cm}, 30.2 \text{ cm} > .$$

Although this uplift is not observable by terrestrial means, it may play some role in measurements on satellites and other extraterrestrial objects.

Finally, if we disregard the dynamic effects on the sea level then we would observe the sea-tide height given by eq. 20. The oscillations would be

$$BD \; \varepsilon < -18.3 \text{ cm.}, 36.3 \text{ cm} > .$$

As a matter of fact there is a fallacy in the argument contained in this section. We are using, on the one hand, the full span of the tidal potential regardless of the frequencies of its constituents and, on the other hand, we use the values of k and h determined for the semidiurnal frequency, namely that of M_2. However, the stated maximum spans are probably not too far away from the real maximum values. It also must be borne in mind that these maxima are applicable only in certain locations at specific times. The usually observed magnitudes will be somewhat lower.

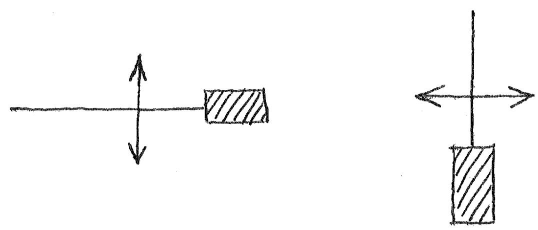
3) INSTRUMENTATION AND ANALYSIS OF OBSERVATIONS

3.1) Tilt Measurements

To detect the tidal tilt one has to observe the variations of the spatial angle between the local vertical (direction of gravity) and a line thought to represent the position of the ground. Hence any instrument sensing the direction of gravity whose frame can be rigidly connected with the ground (bed-rock) can be used.

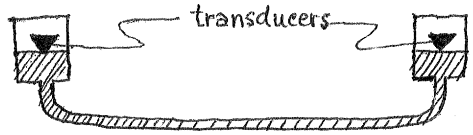
There are two basic designs available. First uses a beam (pendulum) suspended from the frame as a sensor, the second uses a liquid level. Many variations on these two themes have been devised with the common aim to obtain a very high sensitivity. Since the magnitude of the phenomenon we want to observe is extremely small, at most 0.018" in aboslute value, the instruments have to be very sensitive to give some meaningfull results. The typical requirement for a tiltmeter is a sensitivity of $\pm$ 0".0002, i.e. 0.2 msec of arc or 1 nrad (nanoradian), or better.

It is obvious that this accuracy can be achieved with a pendulum only when it is either very long or its position is recorded very precisely. The displacement of the end of a vertical pendulum say 1 metre long corresponding to 1 nrad that has to be recorded is 10^{-3} μm, i.e. 10Å. This is naturally very difficult and thus "longer" pendulums are usually thought to be a better answer. The necessary sizeable length is achieved by mounting the pendulum almost horizontally. Even a short, horizontally mounted beam, can be made effectively equivalent to a very long vertical pendulum by making its angle with the vertical close to $\pi/2$. Such pendulums are known as <u>horizontal pendulums</u> and are used in most tiltmeters. A well designed and manufactured horizontal pendulum can be operated under almost any sensitivity we want. There are two horizontal pendulums necessary to record the tilt in two dimensions, see the figure



The arrows show the directions of tilt sensed by the pendulums. (View from the top).

Various types of tiltmeters are described for instance in [Melchior, 1966]. The second category of tiltmeters, based on the recording of a liquid level variations, are known as <u>hydrostatic tiltmeters</u>. They can be of various shapes, using various liquids and dimensions. The most popular systems use two containers connected by a pipe and measure the difference in the levels by means of transducers. The direction of the vertical plane passing through the containers coincides with the direction of the tilt sensed by the instrument. Hence two pairs of containers are again required to get the tilt in two dimensions.



A good discussion of the principles of these instruments is given in [Bower, 1973].

3.2) Gravity Measurements

To detect the tidal variations of gravity very sensitive gravimeters are used. Since the gravity varies at most by some 0.2 mgal, it is usually required that the gravimeter be accurate enough to sense changes within the μgal range. This sensitivity is normally obtained by most of the modern instruments. [Lennon and Baker, 1973].

The actual design of the tidal gravity - meters is basically the same as that of the ordinary meters. The differences are in a more sophisticated and more precise read-out systems. Their installation does not require such an excessive care as the installation of tiltmeters. This is because the gravity meters do not have to be connected with the bed-rock, providing the ground it stays on is stable, within the mm range; change of 1 μgal corresponds to approximately 3 mm displacement in the vertical sense. One problem is, however, common to both tilt and gravity variation measurements. This is the calibration of the instruments.

3.3) Analysis of Observations

As we have seen in 2.3, the diminishing and gravimetric factors vary with the frequency of the deforming force. Hence, theroetically, we should obtain different results for D and G from equations 23 and 24 using different tidal vawes (see 1.4) in the ratios $\Delta\varepsilon/\Delta\varepsilon_c$ and $\Delta g/\Delta g_c$. We therefore compute the two factors separately for every tidal frequency and get $D(M_2)$, $G(M_2)$, $D(O_1)$, $G(O_1)$ etc. While the frequency decomposition of the theoretical values $\Delta\varepsilon_c$ and Δg_c was treated already in 1.4, we have to say something about the decompositon of the observed values $\Delta\varepsilon$ and Δg.

At the first view, the problem of such a decomposition known as harmonic analysis is a simple one. (Note that the term harmonic analysis has here a different meaning than in mathematics). Given the observed time series (normally values for every hour), we want to determine the amplitudes of the individual periodic constituents of known frequencies (Table 2). This can conceivably be achieved by a straighforward least-squares fit of a trigonometric polynomial

$$T(t) = \sum_{i=1}^{n} (a_i \cos \omega_i t + b_i \sin \omega_i t) \tag{25}$$

with predetermined frequencies ω_i. Unfortunately, however, the actual observed time series are contaminated by a "noise" of periodic as well non-periodic nature (see the next section). The amplitude of the noise is often much larger than the amplitudes of most of the tidal components so that the signal to noise ratio is usually adverse.

It is known from the theory of least squares approximation (see for instance [Vaníček and Wells, 1972], that the lower is the signal to noise ratio, the less reliable results we can expect to obtain of the fitting. For this reason, the signal, i.e., the sum of tidal components, has to be enhanced first. This can be done in a variety of ways. The most obvious one seems to be the use of various numerical "filters" designed to suppress unwanted frequencies in the time series.

Once the signal has been enhanced, we can use the above described least-squares fitting and obtain the amplitudes (A) and phases (p)

$$A_i = \sqrt{(a_i^2 + b_i^2)} \text{ , } p_i = \text{arctg } b_i/a_i \tag{26}$$

of the individual components. The amplitudes then can be compared to the amplitudes of the theoretical tidal vawes and the values D and G computed. Similarly, we can also compare the phases of the individual components to get the phase lags of the theoretical and observed vawes. These phase lags should reflect the velocity of gravitation propagation and the speed of the elastic response of the earth. They should be very nearly equal to zero.

There has been a number of techniques developed for the "harmonic analysis". Some of them combine the analysis with the comparisons in one operation. Customarily, only the higher frequency components (diurnal, semidiurnal, terdiurnal) are sought. A comprehensive account of these techniques is given in [Godin, 1972].

4) TILT AND GRAVITY VARIATIONS DUE TO OTHER CAUSES (NOISE)

4.1) Noise with Tidal Frequencies

We have mentioned at several occasions that the records of both the tidal tilt as well as the gravimetric tide are generally strongly contaminated by noise. This noise is generated by a whole gamily of causes. Because of the necessary difference in techniques used to suppress or separate the noise it is convenient to distinguish the noise:

i) with tidal frequencies

ii) with non-tidal frequencies

iii) non-periodic.

Noise with non-tidal frequencies as wll as non-periodic noise can be quite efficiently filtered out of the observed record, or time series, be it tilt or gravity. On the other hand, noise with tidal frequencise cannot. It combines with the true tidal signal into a time series that again contains nothing but components with the same frequencies as the signal. This can be acertained by the following simple computation, dealing with just one such component of frequency ω. Let us write for the signal S(t) and the noise N(t):

$$S(t) = A_s \cos(\omega t - p_s) = a_s \cos \omega t + b_s \sin \omega t$$

$$N(t) = A_N \cos(\omega t - p_N) = a_N \cos \omega t + b_N \sin \omega t ,$$

where the parameters A, p, a, b are again linked via relations 26. Adding the signal and the noise together, we obtain

$$T(t) = S(t) + N(t) = (a_s + a_N) \cos \omega t + (b_s + b_N) \sin \omega t$$

$$= A_T \cos (\omega t - p_T) ,$$

where
$$A_T = [(a_s + a_N)^2 + (b_s + b_N)^2] \tag{26}$$

$$p_T = \operatorname{arctg} \frac{b_s + b_N}{a_s + a_N} . \tag{27}$$

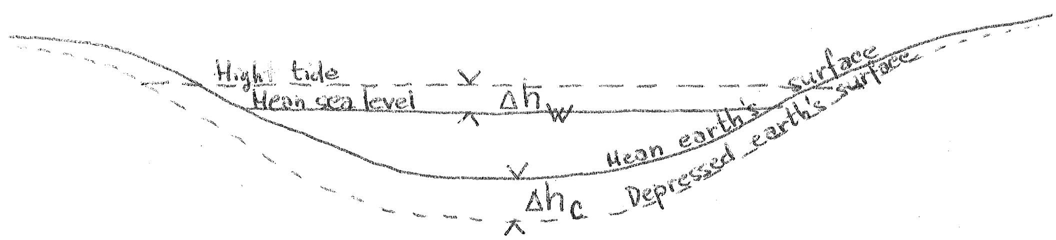
Hence the sum is again a purely periodic vawe with the same frequency ω. We can therefore conclude that the frequency (harmonic) analysis of the tidal records reveals the amplitudes and phases of the components which correspond to the combined effects.

4.2 Sea Tides Effect

Disregarding the effect of the atmospheric tides, negligible for all practical purposes, and the tidal stress, whose role is not yet properly understood, the only remaining effect with tidal frequencies is the effect of the sea tides. This effect is particularly strong in coastal areas but can be traced even deep inland. Because of its importance, we devote a separate section to this problem alone.

As we have already mentioned, the sea tides can be decomposed into components of the same frequencies as the earth tides. However, the phenomenon of sea tides is more complicated than that of the earth tides because of the dynamic effects of the sea. These dynamic effects are especially severe in shallow coastal basins (shallow-sea effect) where they can distort the amplitude of the tides several tens of times and the phases of the individual components by several tens of degrees. In the Bay of Fundy, Bristol Channel, Saint-Malo Bay (southwest part of the English Channel) and the Liverpool Bay, to name just a few locations, the actually observed magnitudes of the tides are over 10 metres.

The incoming tidal water influences the tilt and the gravity in two ways:



(i) - The load of the water layer of thickness Δh_W depresses the earth crust by Δh_c. This depression is, in turn, reflected in the change of the equipotential surface.

(ii) - The water layer, representing a considerable amount of matter, also exerts some gravitational attraction on the masses around.
It has been shown by various authors, e.g. [Bower, 1969; Lennon and Vaníček, 1969; Beaumont and Lambert, 1972], that the sea tides combined effect can reach a magnitude of several times that of the earth tides.

From the geodetic point of view, one thus has to be doubly careful when carrying out a precise work in the vicinity (several tens of miles) of high tides sea basins. From the geophysical view, it is conceivable that the sea tides effect can be utlized to learn about the physical properties of the earth crust.

Referring to equations 26 and 27 we can see that they can be used in this context. Taking for any tidal vawe A_T and p_T as obtained from the harmonic analysis and assuming a_s and b_s as sufficiently well defined by the tidal theory and the derived values of k and h, we can solve for the parameters a_a, b_a. After some development we obtain

$$a_N = A_T \cos p_T - a_s \tag{28}$$

and

$$b_N = A_T \sin p_t - b_s \ . \tag{29}$$

These two parameters obviously define the magnitude and phase of the sea tides effect in the appropriate frequency as excerted by all the seas around the observation site. They have to be then compared to similar qualities obtained from a model of the phenomenon. Any such model requires the knowledge of the sea tides characteristics (distribution) and the knowledge, or working hypothesis, of the parameters characterizing the elastic response of the earth crust to the water loading.

The major hindrance in studying the sea tides effect is the fact that the sea tides characteristics are not yet very well known, particularly on the open seas. Also, the "cotidal charts", i.e. charts depicting the sea tides in terms of iso-tide lines, are to any reasonable extent - available only for the M_2 constituent. And even these are derived practically exclusively from the information given by the land based tide gauges.

4.3) Other Periodic Effects

Having dealt briefly with the phenomena posessing all the tidal frequencies, we shall mention other periodic causes. These include basically all the atmospheric variations i.e. variations of temperature, barometric pressure, rain and snow fall, subsurface water level, etc.

From these generally the temperature variations have the most pronounced effect because they influence not only the movement of the ground but also the operation of the instruments. For this reason a protection against temperature changes is sought and the instruments are usually placed in well insulated underground chambers in natural or man-made cavities. A fairly thick cover is needed, particularly for tiltmeters, to shelter them from spurious temperature effects.

Barometric pressure, surface water and snow loads act in much the same way as the sea water load. They tend to induce local tilts that depend on the local and regional patterns of inhomogeneity and fractures. Very little is known about the effect of the subsurface water level variations. However, it is conceivable that in at least certain areas it is not negligible.

All the atmospheric variations have a pronounced annual period, common with the tides. This is the reason why the annual component of the earth tides is too contaminated to be of any value for determining the elastic response of the earth at this frequency. From the frequency point of view, the temperature variations again are the most dangerous because of their diurnal component that tends to combine with the diurnal tidal constituents.

The annual component of the observed time series offers some interesting geophysical interpretations and has been investigated by several researchers, e.g. [Pícha,1966; Vaníček, 1971]. It has been shown that its magnitude in tilt observations can reach as much as a few seconds of arc. The International Association of Geodesy recommended as early as in 1950 to investigate its effect on precise levelling networks.

4.4) Non-periodic Effects

The last group of effects are the effects that do not have a periodic character, at least within the space of several years. These effects are caused by non-periodic movements of the ground, natural as well as man-made, eustatic water level changes, secular climatic changes, etc.

Generally, noise of this nature called also drift, does not present a serious problem for the harmonic analysis and can be, as we have already mentioned in 4.1, separated from the tidal signal. More difficult problem, however, is the interpretation. The drift in fact consists of three distinctly different contributions:

i) instrumental drift (instability),

ii) instability of the instrument mounting,

iii) non-periodic effects.

The first contribution is generally larger when gravimeters are used, the second is more serious in tilt measurements.

In spite of the instrument and mounting instability, it is believed by some scholars that the continous tilt and gravity observations can be used for the crustal movement detection. It has been shown, e.g., [Lennon and Vaníček, 1969], that given a proper care the instabilities of the instrument and its mounting can be minimized and the drift then interpreted as showing the real movement of the ground. Hence, tiltmeters, and to a lesser degree the gravimeters (because of their lower sensitivity to vertical displacements), present a potentially powerful tool even in the field of crustal movements.

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