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ON THE APPLICATION OF ROBUSTNESS ANALYSIS TO GEODETIC NETWORKS

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ABSTRACT: Geodetic networks established for engineering construction (e.g., highways, railways, bridges, dams) control typically have coordinates estimated by the method of least-squares and the 'goodness' of the network is measured by a precision analysis based upon the covariance matrix of the estimated parameters. When such a network is designed, traditionally this again is based upon measures derived from the covariance matrix of the estimated parameters. This traditional approach is based upon propagation of random errors.

In addition to this precision analysis, reliability (the detection of outliers/gross errors/blunders among the observations) has been measured using a technique pioneered by the geodesist Baarda. In Baarda's method a statistical test (data-snooping) is used. What happens if one or more observations are burdened with an outlier? It is clear that these outliers will affect the observations and produce incorrect estimates of the parameters. If the outliers are detected by the statistical test then those observations are removed, the network re-adjusted, and we obtain the final results.

In the approach described here, traditional reliability analysis (Baarda's approach) has been augmented with geometrical strength analysis using strain into a technique called Robustness analysis. Robustness analysis is a natural merger of reliability and strain and is defined as the ability to resist deformations induced by the largest undetectable outliers as determined from internal reliability analysis.

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This paper addresses the consequences of what happens when outliers are not detected by Baarda's test. This may happen for two reasons (i) the observation is not sufficiently checked by other independent observations and (ii) the test does not recognize the gross error. By how much can these undetected errors influence the network? If the influence of the undetected errors is small the network is called **robust**, if it is not it is called weak.

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1. INTRODUCTION

The known earliest published description of strain analysis in English seems to be Terada and Miyabe (1929) who used strain to describe real deformation of the earth surface caused by earthquakes. According to Pope (1966) in a series of papers in the Bulletin of the Institute for Earthquake Research of the University of Tokyo, Terada, Miyabe, Tsuboi and others described these techniques and applied them to various areas in Japan and Taiwan. The next scientist interested in strain analysis was Kasahara. In Kasahara (1957), (1958a), (1958b) and (1964), the work of Terada, Miyabe and Tsuboi were referrenced and the analysis of the earlier workers in some respects were extended. Later Burford (1965) followed

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Terada and Miyabe. In Burford (1965) the components of strain for an arc of triangulation in Southern California was computed. Independently Frank (1965) derived methods for computation of strain components and pointed out their advantages and disadvantages. The above authorss are geologists or geophysicists (Pope, 1966). Pope (the first known geodesist to deal with strain analysis) also used this technique for application to repeated geodetic surveys to determine crustal movements.

The use of strain to analyse the strength of a geodetic network was attempted at the University of New Brunswick, this was performed by Thapa (1980). In this study, the impact of incompatible observations in horizontal geodetic networks was investigated using strain analysis. Vaníček et al. (1981) elaborated on this approach. In Dare and Vaníček (1982a) a new method for strain analysis of horizontal geodetic networks based on the measurement of the network deformation was presented. Dare (1982b) developed a method for the strength analysis of geodetic networks using strain; he also studied the effect of scale change, twist or shear. In Craymer et al. (1987) a program package called NETAN for the interactive covariance, strain and strength analysis of networks was introduced. Vaníček et al. (1991) combined the reliability technique introduced by Baarda and the geometrical strength analysis method into one technique called “robustness analysis”. Vaníček and Ong (1992) investigated the datum independence problem in robustness analysis. In Krakiwsky et al. (1993) further developments of robustness analysis were given, such as singularities in robustness, precision of robustness measures and interpretation of robustness measures. Szabo et al. (1993) described robustness analysis of horizontal geodetic networks. Craymer et al. (1993a) and (1993b) presented further findings about robustness analysis. Robustness analysis of horizontal geodetic networks was also studied by Ong (1993) and Amouzgar (1994). Craymer et al. (1995) tried to reduce the computational burden for large geodetic networks by restricting the propagation of the potential biases to points that are within a certain number of connection levels. Vaníček et al. (1996) describe a more economical algorithm for searching for the most influential observations, a more satisfactory definition of the neighborhood in which strain measures are evaluated and a technique for network classification that would take into account both the precision and accuracy in point positions. Krakiwsky et al. (1999) developed and numerically tested in-context absolute and relative confidence regions for geodetic networks. Vaníček et al. (2001) summarized the findings about robustness analysis and gave and explicit proof for the robustness datum independence.

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In this study, further thoughts about robustness analysis are brought forward. In Vaníček et al. (2001) a complete and detailed description of the potential network deformation in terms of three independent measures representing robustness in scale, in orientation and in configuration are given (these are also called ‘robustness primitives’) are given. In practice however, some acceptable threshold values are needed. These threshold values are going to enable us to talk about **the level of robustness** of the network. For instance if a geodetic network is being established for an engineering structure, it must have an acceptable level of robustness. If robustness primitives within the network go beyond the threshold values, we must redesign the network by changing the configuration until we obtain a network of acceptable robustness.

2. RELIABILITY ANALYSIS

After geodetic networks for engineering construction (e.g., highways, railways, bridges, dams) control are physically established they are measured and point coordinates are estimated by the method of least-squares. What happens if one or more observations are burdened with gross errors, blunders or outliers? It is clear that these outliers will affect the observations and produce incorrect estimates of the parameters. Therefore they must be detected and corrected. Generally in practice they are removed and the network is re-adjusted. To detect the outliers among the observations Baarda’s method of statistical testing (data-snooping) or another equivalent method is used. What happens if outliers are not detected by the test? This may happen for two reasons (i) the observation is not sufficiently checked by other independent observations and (ii) the test does not recognize the gross error. These situations were first investigated by Baarda (1968).

Baarda’s reliability theory is given in (Baarda 1968). By using the null hypothesis (H_0) testing a statistical decision concerning postulated population parameters (mean μ and variance σ^2 etc.) is made. For every

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null hypothesis there exists an infinite number of alternative hypothesis (H_1), each of which states that the population parameters have some other particular values. The probability α_0 of rejecting H_0 when in fact H_0 is true (the Type I error) is called the significance level. The complementary probability ($1-\alpha_0$) is called the confidence level. Likewise, a situation might arise such that H_0 is false but it is accepted by the test. This is called the Type II error. The probability of making this (wrong) decision is β_0. ($1-\beta_0$) is called the power of test (Vaníček et al. 1991).

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By using Baarda's theory of reliability, Δl_i (the maximum value of the outliers among the observations which would not be detected by a statistical test with significance level α_0) can be estimated as follows:

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[1] $$\Delta l_i = \lambda_0(\alpha_0, \beta_0)\frac{\sigma_{l_i}}{\sqrt{r_i}}$$

where λ_0 is the value of the shift (non-centrality parameter) of the postulated distribution in the alternative hypothesis as a function of selected probabilities α_0, β_0. σ_{l_i} is the a priori value of standard deviation of the i-th observation, r_i is Baarda's redundancy number, which expresses the degree of influence on the estimated positions of the i-th observation (Vaníček et al. 1991, Vaníček et al. 2001). Figure 1 illustrates the relation between α_0, β_0 and λ_0.

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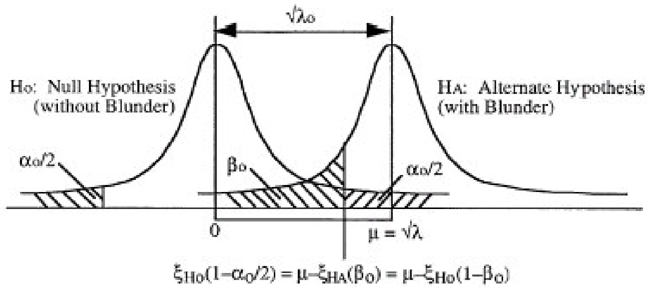

Figure 1. Relationship between α_0, β_0 and λ_0 (from Vaníček et al. 2001).

3. DESCRIPTION OF NETWORK DEFORMATION

To be able to measure the degree of robustness of a network, its degree of potential deformation has to be measured. The potential degree of deformation is described by means of displacements of individual points of the network. The estimates for displacements caused by outliers are given as follows (Vaníček and Krakiwsky, 1986).

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[2] $$\mathbf{\Delta\hat{x}} = (\mathbf{A}^T\mathbf{C}_l^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{C}_l^{-1}\mathbf{\Delta l}$$

where **A** is the design matrix, $\mathbf{C}_l^{-1}$ is the covariance matrix of the observations, $\mathbf{\Delta l}$ is the maximum undetectable error vector and $\mathbf{\Delta\hat{x}}$ is the displacement vector.

The problem with displacements is that their estimates are « datum dependent ». That is, these estimates depend not only on the geometry of the network, and accuracy of the observations but also on the selection of constraints for the adjustment. However deformation description must reflect only network geometry, the type and accuracy of the observations. Therefore the strain technique must be used (Vaníček et al. 2001).

Let us denote a displacement of a point as follows

$$[3]\quad \mathbf{\Delta x}_i = \begin{bmatrix} \Delta x_i \\ \Delta y_i \end{bmatrix} = \begin{bmatrix} u_i \\ v_i \end{bmatrix}$$

then the deformation or gradient matrix for the point is given as

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$$[4]\quad \mathbf{E}_i = \begin{bmatrix} \frac{\partial u_i}{\partial x} & \frac{\partial u_i}{\partial y} \\ \frac{\partial v_i}{\partial x} & \frac{\partial v_i}{\partial y} \end{bmatrix}$$

$$[5]\quad \mathbf{E} = \frac{1}{2}(\mathbf{E} + \mathbf{E}^T) + \frac{1}{2}(\mathbf{E} - \mathbf{E}^T)$$

$$[6]\quad \mathbf{E} = \mathbf{S} + \mathbf{A}$$

The matrix **S** describes symmetrical differential deformation and the matrix **A** (it should not be confused with design matrix already introduced) (Mustafa, call it something else. This looks silly.) describes anti-symmetrical differential deformation at a point.

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$$[7]\quad \mathbf{S} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) \\ \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) & \frac{\partial v}{\partial y} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{xy} & \varepsilon_{yy} \end{bmatrix}$$

$$[8]\quad \mathbf{A} = \begin{bmatrix} 0 & \frac{1}{2}(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}) \\ \frac{1}{2}(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}) & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}$$

ω describes a differential rotation at the point of interest. As mentioned above, network deformation should not depend on the choice of a datum. In Vaníček et al. (2001) it is shown that scale change has only a second order and thus negligible effect on the deformation matrix, while translations of the datum origin and rotations of the coordinate system have no effect at all.

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4. COMPUTATION OF DEFORMATION MATRIX AND ROBUSTNESS PRIMITIVES

The computation of deformation matrix is given in detail in Vaníček et al. (2001). Therefore here only the resulting formulae are given.

$$[9]\quad \mathbf{E}_i = \mathbf{T}_i(\mathbf{A}^T\mathbf{C}_l^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{C}_l^{-1}\mathbf{\Delta l}$$

If Eq. 1 and Eq. 4 are substituted in Eq. 2, we obtain

$$[10]\quad \begin{bmatrix} \frac{\partial u_i}{\partial x} \\ \frac{\partial u_i}{\partial y} \\ \frac{\partial v_i}{\partial x} \\ \frac{\partial v_i}{\partial y} \end{bmatrix} = \mathbf{T}_i(\mathbf{A}^T\mathbf{C}_l^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{C}_l^{-1}\left(\lambda_0(\alpha_0,\beta_0)\frac{\sigma_{l_i}}{\sqrt{r_i}}\right)$$

where $\mathbf{T_i}$ is a matrix based upon coordinates of points and connections: -its computation is discussed by Vaníček et al. (2001).

Then the primitives are obtained as follows (Vaníček et al. 1991; Vaníček et al. 2001).

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$$[11]\quad \sigma = \frac{1}{2}(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y})\ ,\ \tau = \frac{1}{2}(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y})\ ,\ \nu = \frac{1}{2}(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x})\ ,\ \omega = \frac{1}{2}(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y})$$

5. COMPUTATIONS OF THRESHOLD VALUES FOR ROBUSTNESS PRIMITIVES

After calculating robustness primitives and setting up initial conditions (derivations are not given here) for solving the boundary value problem for the network the displacements, the displacement for each point can be computed as follows:

$$[12]\quad \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \sigma+\tau & \upsilon-\omega \\ \upsilon+\omega & \sigma-\tau \end{bmatrix}\begin{bmatrix} x_i - x_0 \\ y_i - y_0 \end{bmatrix}$$

If we examine the right side of the formula all components of the matrices are known. In this case if we assign reasonable values for displacements for the points, and each time set three of the robustness primitives to zero, we can calculate a threshold value for each primitive. (Mustafa, this is gibberish! You must describe properly how to set up the BVP and how to solve it.) For example, if we let σ=0, ω=0 and ν=0 and assume the allowable displacement in x (the direction) is10 cm, we can calculate the value for σ as follows:

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$$[13]\quad \sigma = \frac{10}{x_i - x_0}$$

If we apply the same approach for each primitive, a threshold value for each primitive can be computed.

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6. CONCLUDING REMARKS

To be able to construct and monitor engineering structures (e.g., highways, railways, bridges, dam) geodetic networks must be established, measured and evaluated. To obtain reliable results the networks has to be adjusted. For this purpose Baarda's statistical testing method (data-snooping) or another technique is used. To see the effect of outliers that are not detected by Baarda's test, robustness is analysed, is in terms of three independent measures representing robustness in scale, in orientation and in configuration called robustness primitives; however, to assess networks properly some acceptable threshold values are needed. For this purpose the gradient matrix is defined using robustness primitives and initial conditions are formulated. By using these means computing threshold values for robustness primitives seems realistic. Calculating threshold values would enable us to talk about the degree of robustness of networks. Moreover they should help to design the network. If robustness primitives within the designed network go beyond threshold values, we must consider redesigning the network by changing the configuration until we obtain a sufficiently robust network.

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