Effect of Mathematical Correlation on GPS Network Computation

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Abstract: The precision and accuracy of positions obtained from the least-squares adjustment of Global Positioning System (GPS) data are dependent on the fidelity of the algorithms used to model the data. They are also dependent on the a priori covariance matrix of the data that is used in the adjustment. This matrix describes not only the uncertainty of the data but also the correlation among them—both mathematical and physical. In this paper we present a summary of a study of the effect of mathematical correlations on GPS position determination in network mode using carrier-phase double-difference observations. For our analysis, a network with baselines of hundreds to thousands of kilometers in length was processed, applying three distinct cases of observation correlations: case a—the correlations were ignored; case b—only the correlations within baselines were taken into account; and case c—all correlations, including those between baselines, were considered. This analysis used both broadcast and postfitted orbits. It is shown in this study that the proper modeling of mathematical correlations typically yields better reliability of baseline component estimates and more realistic formal error estimates.

INTRODUCTION

Two types of correlations affect the Global Positioning System (GPS) double-difference carrier-phase observations: the mathematical correlations and the physical correlations. The mathematical correlation is created when, for the sake of removing common errors and reducing partially physically correlated errors, the double-difference observation is formed. The physical correlation is a consequence of the environmental effects common to separate observations, making them spatially and/or temporally correlated. The physical correlations are usually not taken into account when processing GPS observations because they are not easily quantifiable.

The correlations affecting the double-difference observations are accounted for via the covariance matrix of the observations. For a baseline, the mathematical correlation yields a block diagonal structure for the observation's covariance matrix. It, including the physical correlations, would result in a fully populated covariance matrix. Taking into account the correlations yields better position-difference estimates and more realistic formal estimates of errors. It also allows easier resolution of carrier-phase ambiguities. Realistic formal errors are important for the tasks of GPS network densification and merging of GPS and terrestrial networks, as well as for the three-dimensional (3D) processing of GPS networks.

Earlier evaluations of the effect caused by the mathematical correlations

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in GPS network processing have been reported in the literature. Vaníček et al. (1985) investigated the impact of mathematical correlations within single baselines with lengths varying between 13 and 60 km. They concluded that the effect of introducing the mathematical correlations has a statistically insignificant impact on the final results. This study disregarded the mathematical correlations between baselines. Beutler et al. (1987) carried out a similar investigation, using a smaller network, including the mathematical correlation between baselines. They found little difference in the coordinate values when correlations were either ignored or not ignored. Hackman et al. (1989) and Hollmann et al. (1990) reported solutions obtained using different analysis programs that handled the correlations differently. The solutions reflect the differences between the programs and not solely the effect of mathematical correlation. Craymer and Beck (1992) compared two GPS network processing strategies—one including the mathematical correlations between baselines, and another in which the baselines are processed independently and then combined in a 3D adjustment. They concluded that these strategies are equivalent if the covariance matrix used for the second mode is properly scaled, a condition difficult to achieve in practice.

El-Rabbany (1994) investigated the effect of physical correlation on a baseline and its accuracy estimation in GPS differential positioning. His approach can be summarized as follows. First, the least-squares adjustment of a sample of baselines of different lengths, of up to 100 km, was carried out, with only the mathematical correlation included. The resulting adjustment residuals reflect the presence of the unmodeled measurement errors. Then, a general empirical covariance function, representing the physical correlation, is obtained from the residuals. This empirical covariance function is then used to modify the covariance matrix of the double-difference observations, leading to a more realistic, fully populated covariance matrix. The following are among the several important conclusions drawn from his study: "The use of an artificial scale factor to scale the overly optimistic covariance matrix is inappropriate" and "The physical correlation is typically inversely proportional to both observation sampling rate and baseline length."

In the present study, we revisit the problem of mathematical correlations to carefully elucidate the formulation of the covariance matrix and its implementation in analysis software and demonstrate the effect that proper modeling of the mathematical correlations has on GPS network solutions. For testing, we use baselines of hundreds of kilometers in length and an observation sampling interval of 120 s, and hence assume that the effect of the physical correlation is negligible.

There are three basic ways of dealing with mathematical correlations: (1) to use the undifferenced carrier phases, which are mathematically uncorrelated, resulting in the introduction of satellite and receiver clock parameters that need to be estimated; (2) to decorrelate the double-difference measurements of one epoch by means of Cholesky decomposition (Goad and Müller 1988); or (3) to keep the double difference untouched and compute the corresponding covariance matrix pertaining to each epoch directly from the "differencing operator matrix" used to form the double differences from the undifferenced observations (Beutler et al. 1987). From the mathematical point of view, these approaches are similar. For the analysis reported in the present paper, we use the third approach.

MATHEMATICAL CORRELATION

The double-difference carrier-phase observations have found great use in GPS computations due to the fact that it is capable of greatly reducing or eliminating errors and biases affecting the original (undifferenced) carrier phase observations, such as satellite and receiver clock errors, atmospheric effects, and orbital biases (Langley 1993). A consequence of doubly differencing the carrier phases is that the observations (really pseudoobservations) become mathematically correlated.

When processing data from a network occupied by GPS receivers, three approaches can be applied for handling the mathematical correlations: approach a—ignore them; approach b—consider their effect within each individual baseline (the between-satellite correlation); or approach c—consider their effect both within and between the baselines. Approach c is the most rigorous one, but applies only to simultaneously observed baselines. Approaches a and b do not have such a requirement and can be applied in the processing of single baselines or networks.

Let the double-difference observations for one epoch be represented as

$$\nabla\Delta\mathbf{\Phi} = \mathbf{R}\mathbf{\Phi} \tag{1}$$

where $\mathbf{R}$ = differencing operator matrix (with entries of 0s, + 1s, and −1s); $\mathbf{\Phi}$ = vector of undifferenced carrier-phase observations; and $\nabla\Delta$ = double-difference operator. Applying the law of propagation of variances (Vaníček and Krakiwski 1986), we arrive at the covariance matrix of the double-difference observations

$$\mathbf{C}_{\nabla\Delta\Phi} = \mathbf{R}\mathbf{C}_{\Phi}\mathbf{R}^T \tag{2}$$

where $\mathbf{C}_{\Phi}$ = covariance matrix of vector $\mathbf{\Phi}$. We make the assumption that the undifferenced carrier phases $\mathbf{\Phi}$ have errors that behave randomly, resulting in a homogeneous normal distribution with the expected value equal to zero and variance σ_0^2 equal to one. The undifferenced phases are assumed to be uncorrelated. Thus $\mathbf{C}_{\Phi}$ is equal to the identity matrix. This results in

$$\mathbf{C}_{\nabla\Delta\Phi} = \mathbf{R}\mathbf{R}^T \tag{3}$$

If the mathematical correlation is totally disregarded, $\mathbf{C}_{\nabla\Delta\Phi}$ is equal to the identity matrix. If, in a network mode, only the mathematical correlations of the double-difference observations within individual baselines are considered, the diagonal submatrices, one for each baseline, will have a block diagonal structure in $\mathbf{C}_{\nabla\Delta\Phi}$, and all off-diagonal submatrices will be equal to zero. If all mathematical correlations are taken into account, there will be some nonzero elements in the off-diagonal submatrices, each representing correlations between baseline receivers observing the same satellite at the same time. It goes without saying that matrix $\mathbf{C}_{\nabla\Delta\Phi}$ is scaled by the a priori variance factor of the double-difference observations.

Let us think of one baseline, linking receivers A and B. At a particular epoch, the same five satellites are observed by both receivers. The vector of undifferenced carrier phases $\mathbf{\Phi}$ is

$$\mathbf{\Phi} = [\Phi_A^1, \Phi_A^2, \Phi_A^3, \Phi_A^4, \Phi_A^5, \Phi_B^1, \Phi_B^2, \Phi_B^3, \Phi_B^4, \Phi_B^5]^T \tag{4}$$

The double-difference observations are formed by first differencing between receivers and then differencing these differences between consecutively numbered satellites, i.e., (1-2), (2-3), (3-4), and (4-5). We shall follow this approach throughout the present paper. Therefore, the vector of double-difference observations is

$$\nabla\Delta\Phi = \begin{bmatrix} \nabla\Delta\Phi_{AB}^{12} \\ \nabla\Delta\Phi_{AB}^{23} \\ \nabla\Delta\Phi_{AB}^{34} \\ \nabla\Delta\Phi_{AB}^{45} \end{bmatrix} = \begin{bmatrix} (\Phi_A^1 - \Phi_B^1) - (\Phi_A^2 - \Phi_B^2) \\ (\Phi_A^2 - \Phi_B^2) - (\Phi_A^3 - \Phi_B^3) \\ (\Phi_A^3 - \Phi_B^3) - (\Phi_A^4 - \Phi_B^4) \\ (\Phi_A^4 - \Phi_B^4) - (\Phi_A^5 - \Phi_B^5) \end{bmatrix} \quad (5)$$

The differencing operator matrix **R**, according to (1) and consistent with (4) and (5), is

$$\mathbf{R} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \quad (6)$$

where each row corresponds to a double-difference combination, and each column corresponds to a particular satellite. The first five columns are related to receiver A, whereas the last five columns relate to receiver B. Finally, (3) gives the covariance matrix of the double-difference observations

$$\mathbf{C}_{\nabla\Delta\Phi} = \begin{bmatrix} 4 & -2 & 0 & 0 \\ -2 & 4 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -2 & 4 \end{bmatrix} \quad (7)$$

The differencing operator matrix **R** depends on the way the double differences are formed. For example, if they had been formed as (1-2), (1-3), (1-4), and (1-5), matrix **R** would read

$$\mathbf{R} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

and, consequently, the covariance matrix would be

$$\mathbf{C}_{\nabla\Delta\Phi} = \begin{bmatrix} 4 & 2 & 2 & 2 \\ 2 & 4 & 2 & 2 \\ 2 & 2 & 4 & 2 \\ 2 & 2 & 2 & 4 \end{bmatrix} \quad (9)$$

In the least-squares adjustment of a single baseline, the weight matrix **P** is equal to the inverse of the covariance matrix $\mathbf{C}_{\nabla\Delta\Phi}$. If a network of $n + 1$ receivers is used, the weight matrix **P** is composed of $\mathbf{P}_i$ where $i = 1$, with n submatrices occupying the diagonal; the off-diagonal submatrices are zero, denoting that no mathematical correlation between baselines has been taken into account. Matrix **P**, then, is

$$\mathbf{P} = \begin{bmatrix} P_1 & 0 & \cdots & 0 \\ 0 & P_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & P_n \end{bmatrix} \quad (10)$$

Eq. (10) is not correct, from the theoretical point of view, if any of the baselines have been observed simultaneously.

To take the mathematical correlation between simultaneous double-difference observations, collected from a network of receivers, into account, let us consider the simple case of three stations, occupied simultaneously by receivers A, B, and C, forming two independent baselines, A-B and A-C. Again, assume that the same five satellites have been observed at a particular epoch. The vector of undifferenced carrier phases $\mathbf{\Phi}$ is

$$\mathbf{\Phi} = [\Phi_A^1, \Phi_A^2, \Phi_A^3, \Phi_A^4, \Phi_A^5, \Phi_B^1, \Phi_B^2, \Phi_B^3, \Phi_B^4, \Phi_B^5, \Phi_C^1, \Phi_C^2, \Phi_C^3, \Phi_C^4, \Phi_C^5]^T \quad (11)$$

The vector of double-difference observations is

$$\nabla\Delta\mathbf{\Phi} = \begin{bmatrix} \nabla\Delta\Phi_{AB}^{12} \\ \nabla\Delta\Phi_{AB}^{23} \\ \nabla\Delta\Phi_{AB}^{34} \\ \nabla\Delta\Phi_{AB}^{45} \\ \nabla\Delta\Phi_{AC}^{12} \\ \nabla\Delta\Phi_{AC}^{23} \\ \nabla\Delta\Phi_{AC}^{34} \\ \nabla\Delta\Phi_{AC}^{45} \end{bmatrix} = \begin{bmatrix} (\Phi_A^1 - \Phi_B^1) - (\Phi_A^2 - \Phi_B^2) \\ (\Phi_A^2 - \Phi_B^2) - (\Phi_A^3 - \Phi_B^3) \\ (\Phi_A^3 - \Phi_B^3) - (\Phi_A^4 - \Phi_B^4) \\ (\Phi_A^4 - \Phi_B^4) - (\Phi_A^5 - \Phi_B^5) \\ (\Phi_A^1 - \Phi_C^1) - (\Phi_A^2 - \Phi_C^2) \\ (\Phi_A^2 - \Phi_C^2) - (\Phi_A^3 - \Phi_C^3) \\ (\Phi_A^3 - \Phi_C^3) - (\Phi_A^4 - \Phi_C^4) \\ (\Phi_A^4 - \Phi_C^4) - (\Phi_A^5 - \Phi_C^5) \end{bmatrix} \quad (12)$$

The differencing operator matrix $\mathbf{R}$ is

$$\mathbf{R} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \quad (13)$$

where the first four rows correspond to baseline A-B and the last four rows correspond to baseline A-C. Every consecutive five columns correspond to one station. Of course, this matrix would look different if the independent baselines were chosen differently, such as A-B and B-C. The covariance matrix for our selected baselines is

$$\mathbf{C}_{\nabla\Delta\Phi} = \begin{bmatrix} 4 & -2 & 0 & 0 & -2 & 1 & 0 & 0 \\ -2 & 4 & -2 & 0 & 1 & -2 & 1 & 0 \\ 0 & -2 & 4 & -2 & 0 & 1 & -2 & 1 \\ 0 & 0 & -2 & 4 & 0 & 0 & 1 & -2 \\ -2 & 1 & 0 & 0 & 4 & -2 & 0 & 0 \\ 1 & -2 & 1 & 0 & -2 & 4 & -2 & 0 \\ 0 & 1 & -2 & 1 & 0 & -2 & 4 & -2 \\ 0 & 0 & 1 & -2 & 0 & 0 & -2 & 4 \end{bmatrix} \quad (14)$$

where the diagonal (4×4) submatrices correspond to baselines A-B and A-C, respectively, and the (4×4) off-diagonal matrices correspond to the correlations between them.

Let us alter the previous example to reflect a real-case scenario, when not all satellites observed are common to all three receivers. For example, consider a case when the double-difference vector is

$$\nabla\Delta\Phi = \begin{bmatrix} \nabla\Delta\Phi_{AB}^{12} \\ \nabla\Delta\Phi_{AB}^{24} \\ \nabla\Delta\Phi_{AB}^{46} \\ \nabla\Delta\Phi_{AC}^{13} \\ \nabla\Delta\Phi_{AC}^{35} \\ \nabla\Delta\Phi_{AC}^{56} \end{bmatrix} \tag{15}$$

i.e., only satellites one and six are observed by all three receivers. Matrix **R** is then given by

$$\mathbf{R} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \tag{16}$$

and the covariance matrix of the double-difference observations is given by

$$\mathbf{C}_{\nabla\Delta\Phi} = \begin{bmatrix} 4 & -2 & 0 & -1 & 0 & 0 \\ -2 & 4 & -2 & 0 & 0 & 0 \\ 0 & -2 & 4 & 0 & 0 & -1 \\ -1 & 0 & 0 & 4 & -2 & 0 \\ 0 & 0 & 0 & -2 & 4 & -2 \\ 0 & 0 & -1 & 0 & -2 & 4 \end{bmatrix} \tag{17}$$

with the dimension equal to the total number of double-difference observations. This real-case simulation shows that the differencing operator matrix **R**, and, as a consequence, the covariance matrix $\mathbf{C}_{\nabla\Delta\Phi}$ and the weight matrix

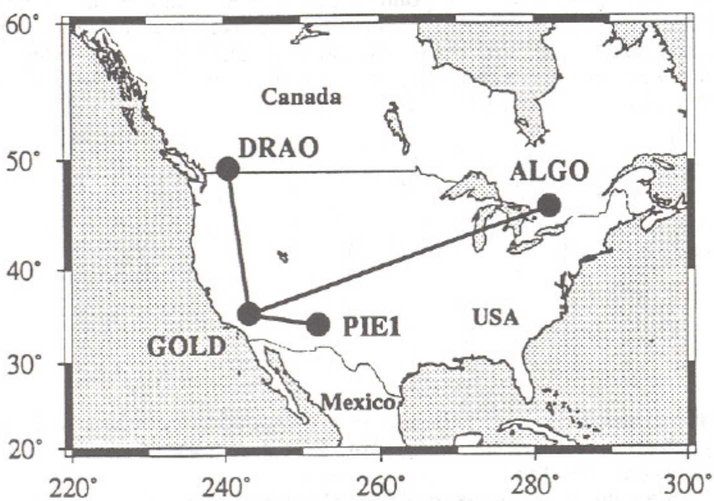

FIG. 1. Geographical Distribution of Network of Stations Used to Assess Effect of Mathematical Correlation

P, depend on the baseline definition as well as on the number and identity of satellites observed simultaneously.

The rigorous implementation of the covariance matrix, including the correlations between baselines, adds a great deal of computation to the processing of GPS observations. Efficient methods are needed, such as the one described by Beutler et al. (1987).

EFFECT OF MATHEMATICAL CORRELATION ON NETWORK

The effect of mathematical correlations, in practice, was assessed using a network composed of four stations of the International GPS Service for Geodynamics network: Goldstone (GOLD), Algonquin (ALGO), Penticton (DRAO), and Pie Town (PIE1). The geographical distribution of this network is shown in Fig. 1. Three independent baselines were formed, with station GOLD being common to all of them. The baselines, and their respective lengths, are as follows: Goldstone-Algonquin (3,402 km), Goldstone-Pie Town (810 km), and Goldstone-Penticton (1,556 km). The observational data set used for the processing shown in the present paper covers a period of three days (days 003, 004, and 005 of GPS week 730). The data from each

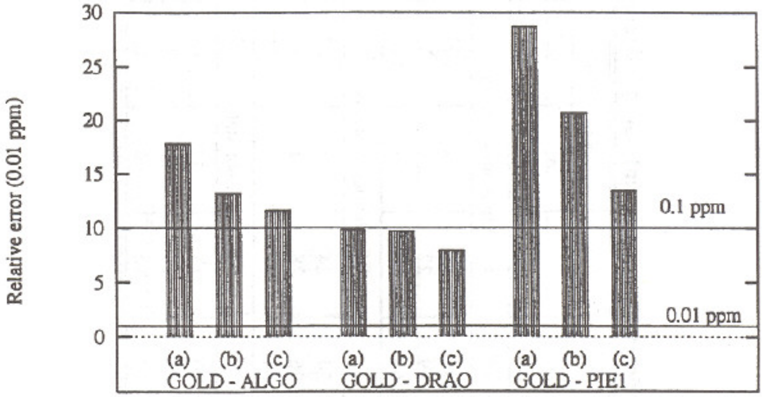

FIG. 2. Relative Error in Baseline Length (Using Broadcast Ephemerides)

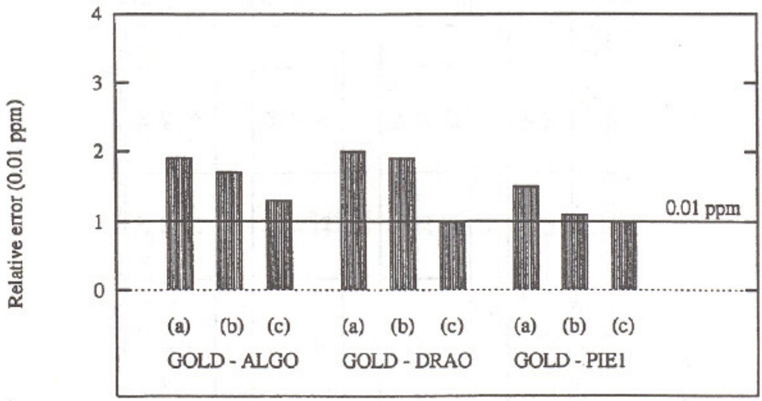

FIG. 3. Relative Error in Baseline Length (Using Postfitted Orbits)

TABLE 1. Formal Errors (in mm) Using Broadcast Ephemerides

Correlation mode (1)	Day 003				Day 004				Day 005			
	σ_ϕ (2)	σ_λ (3)	σ_h (4)	σ_l (5)	σ_ϕ (6)	σ_λ (7)	σ_h (8)	σ_l (9)	σ_ϕ (10)	σ_λ (11)	σ_h (12)	σ_l (13)
(*a*) Algonquin station												
a	22	55	111	44	21	52	108	41	25	62	127	49
b	22	61	123	49	22	61	122	48	26	69	140	55
c	27	85	150	64	27	84	150	62	29	90	163	67
(*b*) Penticton station												
a	27	37	87	24	24	35	77	22	30	45	102	28
b	28	41	94	26	28	41	91	25	33	53	118	31
c	41	49	128	34	39	49	123	33	43	60	147	37
(*c*) Pie town station												
a	14	38	93	40	13	35	86	35	17	40	99	39
b	14	42	99	44	13	41	99	41	16	46	112	46
c	16	49	121	50	15	46	115	45	18	52	133	51

TABLE 2. Formal Errors (in mm) Using Postfitted Ephemerides

Correlation mode (1)	Day 003				Day 004				Day 005			
	σ_ϕ (2)	σ_λ (3)	σ_h (4)	σ_l (5)	σ_ϕ (6)	σ_λ (7)	σ_h (8)	σ_l (9)	σ_ϕ (10)	σ_λ (11)	σ_h (12)	σ_l (13)
(*a*) Algonquin station												
a	1.9	4.9	9.7	4.0	1.8	4.4	8.9	3.7	2.1	5.3	10.1	4.3
b	1.8	5.0	10.0	4.2	1.7	4.6	9.0	3.9	1.9	5.3	10.2	4.5
c	2.2	6.9	12.0	5.3	2.0	6.3	10.7	4.9	2.2	7.3	12.4	5.6
(*b*) Penticton station												
a	2.2	2.8	7.1	2.1	2.0	2.6	6.5	1.9	2.4	3.1	7.9	2.3
b	2.2	2.8	7.0	2.0	2.0	2.6	6.5	1.8	2.4	3.1	7.7	2.3
c	3.2	3.3	9.6	2.6	2.9	3.1	8.8	2.4	3.4	3.6	10.5	2.9
(*c*) Pie town station												
a	1.0	2.8	7.9	2.9	0.9	2.5	6.7	2.5	1.1	2.8	7.6	2.8
b	1.1	3.0	8.0	3.1	1.1	2.4	6.6	2.5	1.1	2.9	7.8	2.9
c	1.3	3.5	9.9	3.5	1.0	2.7	7.6	2.6	1.1	2.8	7.9	2.7

day were processed independently. We used the software *DIPOP* (Santos 1995) to process the data.

As far as the mathematical correlations were concerned, the network was processed in three different modes: mode a—totally disregarding the mathematical correlation, i.e., assuming $\mathbf{C}_{\nabla\Delta\Phi}$ to be equal to a uniformly scaled identity matrix; mode b—modeling the mathematical correlation within each baseline, according to (10); and mode c—modeling the mathematical correlations between baselines as well as within baselines. The adjusted baselines were compared to their published International Earth Rotation Service Terrestrial Reference Frame 1992 (ITRF92) values, in an attempt to gauge the accuracy of the three modes. Also, the results were compared in order to assess how the formal errors (precision) of the different solutions behaved.

The effect of mathematical correlations on estimated baseline lengths is seen in Fig. 2 for the solution using the broadcast ephemerides, and in Fig. 3 for the solution using the postfitted ephemerides. These figures show the average relative error in baseline length, for the three days involved in the processing, by comparing the baselines resulting from the adjustment with the published ITRF92 values (Altamini and Boucher 1993). Again, the three correlation modes, a, b, and c, were tested. It can be clearly seen that the results improve with the proper modeling of the mathematical correlation by as much as 50%.

The effect of mathematical correlations on the precision estimation of the network solution, for the three correlation modes, for all three days, for all three baselines, is summarized in Table 1, for a solution using the broadcast satellite ephemerides, and in Table 2, for a solution using postfitted ephemerides. In these tables, σ_ϕ represents the formal error in latitude, σ_λ represents the formal error in longitude, σ_h represents the formal error in height, and σ_l represents the formal error in baseline length. The postfitted ephemerides were produced as a batch orbital solution for the three days used (Santos 1995). It can be seen that the formal errors without the mathematical correlation (compare in mode a) are generally the smallest among the three correlation modes. The formal errors of the results when the mathematical correlations between baselines are fully taken into account are the largest ones. The formal errors using mathematical correlations within baselines are, most of the time, in between. This indicates that by taking the mathematical correlations into account, more realistic formal errors of the estimated position differences are obtained. The two tables also show that the formal errors using the broadcast ephemerides are larger than those using postfitted ephemerides.

Another interesting feature to be noticed is the difference in the quality of the solution obtained using the broadcast and the postfitted orbits. According to the rule of thumb (Vaníček et al. 1985), the broadcast orbits used seem to have an orbital bias in the 3 to 4 m range, whereas the postfitted orbit used is certainly below the 1 m level. The use of postfitted orbits results in a final solution that is one order of magnitude more accurate than the solution obtained from the use of broadcast orbits.

CONCLUSIONS

The present paper describes an investigation into the effect of mathematical correlations in the processing of observational data from a GPS network. The modes of mathematical correlation considered were: mode a—correlations were ignored; made b—correlations only within baselines were taken into account; and mode c—correlations between baselines were also accounted for. The conclusions are as follows: (1) better (more accurate) results are obtained using modes c, b, and a, in that order; (2) the proper modeling of mathematical correlations yields a more realistic precision estimation typically, $\sigma_{(c)} > \sigma_{(b)} > \sigma_{(a)}$; and (3) the effect of orbit quality seems to be greater than the proper modeling of the mathematical correlation itself. The writers intend to follow up this study with further testing of the effect of mathematical correlation on networks with a mixture of baseline lengths.

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APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

$\mathbf{C}_{\nabla\Delta\Phi}$ = covariance matrix of double difference observations;

$\mathbf{C}_\Phi$ = covariance matrix of undifferenced observations;
$\mathbf{P}$ = weight matrix;
$\mathbf{R}$ = differencing operator matrix;
$\nabla\Delta$ = double difference operator;
σ_o^2 = a priori variance; and
$\mathbf{\Phi}$ = vector of undifferenced carrier phase observations.