An alternative method for detection and mitigation of static multipath in L1 carrier phase measurements.

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BIOGRAPHIES

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ABSTRACT

Multipath affects the Global Positioning System measurements in different ways. For example, in the receiver, multipath distorts the correlation function hampering its peak detection in the delay lock loop, with a consequent error in the pseudorange and its derived products. It also takes a toll in the carrier phase, causing the receiver to measure a distorted phase, with deleterious consequences in differential applications. Maybe for these reasons, multipath remains a major source of error in both static and kinematic high accuracy positioning, and a limiting factor in various applications, such as the Wide Area Augmentation System. Improvements in receiver and antenna technologies, in addition to models based on daily repeatability, have resulted in a better handling of this problem. Some authors have shown that multipath is highly correlated for an array of closely spaced antennas. This paper presents a methodology aimed at evaluating multipath by introducing a temporal factor in the measurements. The methodology makes use of the assumption that multipath parameters and satellite geometry have a slow variation in time and space. The method uses L1 single difference carrier phase measurements that have been collected by two closely spaced antennas. These observables feed a filter that estimates multipath parameters. We believe that he same procedure may be adopted for the L2 carrier phase. Multipath was analysed for various satellites at different azimuth and elevation angles over consecutive days using the same scenario. The high daily repeatability was used to ascertain the presence of multipath. The results show a short variation in the efficiency of the method, i.e., the percentage between the estimated multipath vis-à-vis the measured one. Generally, the efficiency reached 65%. It is concluded that the assumption of the low variation of the multipath parameters over a short period of time can be used to explore the proposed objectives. We believe that very good carrier phase multipath estimates were obtained. This means the method can be an interesting alternative for reference stations, users and also for research, analysis, and modelling of multipath using different scenarios, especially of low cost.

INTRODUCTION

Multipath is a phenomenon in which the signal arrives at a receiver site via two or more different paths [Wells et al., 1986]. A GPS receiver tracks a signal composed of a direct and reflected components. The receiver cannot distinguish between them and then tracks the composite signal. Multipath provokes errors in both pseudorange and carrier phase measurements, depending mostly on the scenario involving the reflecting objects, the antenna, and the satellites.

Presented at the ION National Technical Meeting, January 22-24, 2001, Long Beach, CA

Most of the GPS errors and biases are cancelled out when using differential positioning techniques (respecting limitations due to baseline length), but this is not the case for multipath and thermal noise, or phase noise [Dai et al, 1997].

Multipath propagation can be classified in 3 categories [Moelker, 1997]: specular reflection (coming from the reflection on a smooth surface, being the resulting wavefront a delayed copy of the direct signal, differing from this one only in phase and amplitude), diffraction (due to reflections from the edges or corners of the reflecting objects), and diffuse multipath (due to reflection in rough surfaces, similar to various specular reflections). The two main characteristics of a multipath signal are: it is always weaker than the direct signal due to loss of energy in the reflection (but it can still be strong if the reflecting object is large or if there is no partial obstruction of the signal); and, it is always delayed with respect to the direct signal. Specular multipath limits most applications based on the carrier phase. It is responsible for at least 90% of the errors in carrier phase measurements, being dominant in high accuracy applications [Comp & Axelrad, 1996]. Specular multipath can be characterized by means of an amplitude ratio, propagation delay and phase rotation. Besides the direct signal, several secondary signals may be present. Carrier phase multipath is the major source of error in high accuracy static and kinematic positioning [Ray, 1999]

In recent years several user groups have mentioned multipath as crucial to their applications, such as the WAAS as well as the LASS [Weill, 1997].

With the increase in the use of GPS in Brazil, several institutions have been planning to offer GPS reference stations, having as main objective to yield to the public observation data files. These reference stations may have their sites chosen following other criteria (such as, avoid vandalism) rather than minimize multipath.

There are several ways of dealing with multipath. The first and easiest one is to avoid it by means of an appropriate site selection. Others, are more related to antenna design and receiver hardware multipath approaches. Other ways of dealing with multipath make use of modelling. For example, one can use carrier phase smoothing techniques [Hatch, 1982], use the help of spectral analysis and signal repeatability [Axelrad et al., 1994], analysis of SNR [Reichert and Axelrad, 1999; Kim and Langley, 1999], use of multiple reference stations [Raquet & Lachapelle, 1996].

Some inconvenience in the methods based on modelling are the great number of independent multipath to be resolved, the excess of information to be carried to the processing and the adequability of the model to reality (such as the use of a specular model for the diffuse multipath). According to Comp and Axelrad [1996] the only inconvenient of the methods based on stable environments, i.e., unmodelled (such is this case) is that they can work well if the environment remains indeed unaltered.

According to Ray [1999] there is no efficiency in attenuation of low frequency carrier phase multipath, generally the one with larger intensity, provoked for reflecting surfaces located closely, up to 30 metres from the antenna.

Several geometric aspects, combined with a special antenna configuration, were explored by Becker et al. [1994] to detect multipath in a simulated situation. Another use of this aspect was made by Ray & Cannon [1998] and Ray [1999] using an array of six closely spaced antennas connected to three different receivers, controlled by an external oscillator. Their results are encouraging.

In the present research, we make use of an array of two closely spaced antennas, linked to the same receiver, capable of simultaneously tracking signals from two antennas. A variation would be to use two antennas connected to two different receivers controlled by an external oscillator. We believe that a two-antenna approach can be better used for practical and economical reasons, especially if aimed at a Brazilian reality.

METHODOLOGY

The methodology makes use to the fact that signals collected by close by antennas are highly correlated. Also, that the multipath characteristics will not vary over a very short period of time. Two antennas are used, connected to a same receiver. Different observations feed an Extended Kalman Filter (EKF) and four multipath parameters are estimated, representing reflection from all sources. After knowing these parameters, we can compute the multipath effect at each one of the antennas.

THE SCENARIO OF THE EXPERIMENT

An experiment took place on the roof of the Head Hall building, located at the University of New Brunswick campus. Figure 1 shows a diagram. The main reflecting sources are two buildings, composed of wood and brick, 5 metres high, away from the antennas by 12 and 18 m, respectively, the parapets (1.5 m high) away 4 and 15 m from the antennas (one at each side) and three pillars, all 1.3 m high, located 3 and 6 m away from the antennas. The antennas were mounted on the fourth pillar. These reflecting sources must have been responsible by the low frequency and high intensity multipath because they are located closer to the antenna array. Figure 2 shows a picture of the roof, showing the antenna array set up on top of one of the pillars. In its background one of the buildings is seen. Also seen is the window of the office where the notebook that controlled the data collection was situated. Two neighbouring pillars can be seen.

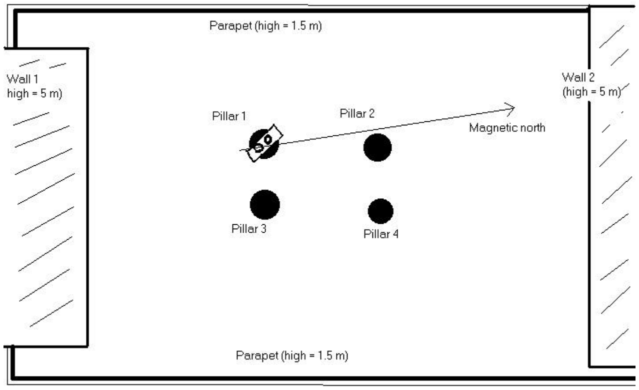


Figure 1: Scheme of Head Hall roof, showing the antennas and the main reflecting sources.


Figure 2: Picture showing the image of the experiment.

MULTIPATH SIGNAL EXPRESSION

Braasch [1996] gives and expression for the multipath signal the way it is tracked by the receiver. This signal is a combination of the direct signal and the various reflecting signals:

$$S(t) = D(t)C(t)\Lambda \sum_{i=0}^{n} a_i \cos(2pf_L + q_0 + \frac{2pd_i}{l}) \ ,$$

(1)

where:
$S(t)$ = composite signal (direct + reflected signal);
$D(t)$ = segments of navigation data superimposed on the signal;
$C(t)$ = C/A code;
L = amplitude of carrier signal;
a_i = direct and reflected signal coefficients, corresponding, in practice, to a direct signal plus one or several reflected signals ($0 \pounds a \pounds 1$);
f_L = carrier frequency (Hz);
d_i = signal path delay with respect to the direct signal;
l = wavelength (m);
q_0 = initial phase(rad).

The direct signal has index equal to zero (i = 0). There is no delay with respect to the direct signal (d_0 = 0), and the reflection coefficient reaches its maximum value (a_0 = 1).

Several authors show an expression for the replicate of this signal, generated internally in the receiver, generally after the DLL, and numerically generated by an oscillator, allowing tracking, by comparison, the signal that arrives from the satellite. This expression, disregarding navigation data, has the form:

$$\Psi = \arctan\left[\frac{\sum_{i=0}^{n} R(t - d_i)a_i \sin(y + \frac{2pd_i}{l})}{\sum_{i=o}^{n} R(t - d_i)a_i \cos(y + \frac{2pd_i}{l})}\right], \quad (2)$$

where:
Ψ = measured signal;
ψ = reflected signal;
R = auto-correlation function;
t = direct signal delay with respect to the code generated inside the receiver;
d = multipath signal delay with respect to the direct signal.

If there is no reflected signal, $\alpha_0 = 1$, $\alpha_1,...,\alpha_n = 0$ and $\delta_0 = 0$. Therefore, $\Psi = \psi$, i.e., the measured phase is equal to the true phase. If there is a reflected signal, $\Psi \neq \psi$, and there will be the need for an expression representing the difference between those two terms. This difference $\Delta\Psi$ represents the error in carrier phase measurement due to multipath. Some authors show an expression for these error [Van Nee, 1995; Braasch, 1996], where the environment's total reflexive effect is represented by a single virtual reflector whose position and intensity relative to the direct signal is a function of time:

$$\Delta\Psi = \arctan\left[\frac{R(t - d)a \sin g}{R(t) + R(t - d)a \cos g}\right],$$

(3)
where:
$\Delta\Psi$ = error due to multipath in a single antenna (rad);
a = reflection coefficient;
g = reflected signal phase (rad).

Ray et al. [1998] show a simplified form for equation (3) by means of a normalisation and a combination of reflection and path delay coefficients into a single parameter, named modified reflection coefficient a_0, reducing the number of parameters to two:

$$\Delta\Psi = \arctan\left\{\frac{a_0 \sin g}{1 + a_0 \cos g}\right\} . \tag{4}$$

From (4) an expression for the difference in multipath error between two antennas:

$$\Delta\Psi_0 - \Delta\Psi_1 = \arctan\left\{\begin{array}{l} \dfrac{a_0 \sin g_0 - a_0 \sin g_1}{1 + a_0 \cos g_0 - a_0 \cos g_1} \\ \dfrac{+ a_0^2 \sin(g_0 - g_1)}{+ a_0^2 \cos(g_0 - g_1)} \end{array}\right\}.$$

(5)

The reference antenna is referred to by the index 1. The proximity of the antennas allows assuming the same value for α_0 in both of them. The same does not apply for γ which depends on the position of the reflected signal. An expression to compute the antenna phase in antenna 1 was derived based on the space geometry between the antennas by Ray and Canon [1999] as:

$$g_1 = g_0 + \frac{2p\ s_{01} \cos(A_0 - b_{01}) \cos u_0}{l} , \tag{6}$$

where:
g_0 = signal phase at the reference antenna;
s_{01} = distance between antennas 0 and 1;
A_0 = azimuth of reflected signal;
b_{01} = azimuth of the vector formed by the phase centres of antennas 0 and 1;
u_0 = elevation of reflected signal.

The coordinates of the antenna phase centres were computed. These values are of interest as will be seen ahead. Also computed were the distance and azimuth of the baseline formed between them. The distance is 11,43 cm and the azimuth is 301°36′47,8726″ (which corresponds to 5,264145114 radians). An issue to be considered is a possible variation of the antenna phase centres. According to Wells et al [1986], the principal effect due to antenna phase centre variation, when using antennas of the same type, is a scale factor approximately equal to 0.015 ppm. We assume this effect to be negligible due to the antennas being very close together.

PARAMETER ESTIMATION – EKF

We have computed the multipath parameters, which absorb the multipath generated by all reflecting sources. An extended Kalman Filter (EKF) is used [Brown and Hwang, 1992]. The choice for an EKF is due to the non-linearity of the system of partial differential equations of second order of the involved expressions, to the low knowledge of the temporal variation of the parameters and to the high accuracy of the measurement system. According to Brown and Hwang [1992], and EKF is similar to a linearized Kalman Filter with the exception that in the EKF the linearization is carried out on an estimated trajectory instead of on a nominal pre-estimated trajectory. It means that the partial derivatives of the equations, which define the process models and measurements, are evaluated along a trajectory being constantly updated by the filter estimates (which depend on the measurements). This fact makes the sequence of filter gain very dependent on the sequence of samples taken during the experiment steps. In this way, the gain sequence is not pre-determined by the suppositions of the process model, as usually the case. Imprecision in the measurements can lead to filter divergence. This fact shows the EKF requirements in terms of precision of measurements system. In the present work, the measurement precision, as translated in the covariance matrix, is high, and does not constitute a limiting factor.

For the system modelling a Gauss-Markov process is used, due to its simplicity and applicability in various problems related to parameter estimation in the presence of noise. In the Markov theory of dependence, the current estate depends only on itself and on the immediately preceding. The exponential autocorrelation function indicates that the sample values of the process become gradually less correlated as time between samples increase. The autocorrelation function approaches to zero as the time between the samples tend to infinity. In the present work, we assumed that the parameters are independent among themselves, determining this way the diagonality of the estate covariance matrix.

OBSERVATION MODEL

The basic observable used is the single difference between the two antennas [Wells et al., 1986]:

$$\Delta\Phi = \Delta\ r\ + l\ \Delta\ N\ + c\,\Delta dT + \Delta m + \Delta e \tag{7}$$

Several terms of this equation can be eliminated or modelled. Initially, the receiver clock term ΔdT can be eliminated because both antennas are linked to the same receiver, as such, this term is the same for both antennas and cancel out when forming the simple difference. Figure 3 shows the experiment's scheme with the two antennas attached to the same receiver.

The antennas used are the 501 model attached to the BeeLine receiver, both manufactured by the Canadian company NOVATEL. The BeeLine is a 16-channel L1 receiver, in which 8 channels are dedicated to each one of the antennas. Figure 4 shows this equipment.

The geometric term Dr can be calculated from the known antennas and satellites coordinates and then removed for each epoch. Figure 5 illustrates this value.

Figures 6 e 7 show the values of Dr for satellites 9 e 26 representative for the three days used in this paper. For this computation, IGS precise orbits were used. The same pattern would be encountered if broadcast orbits were used due to the fact that between receiver differences are computed.

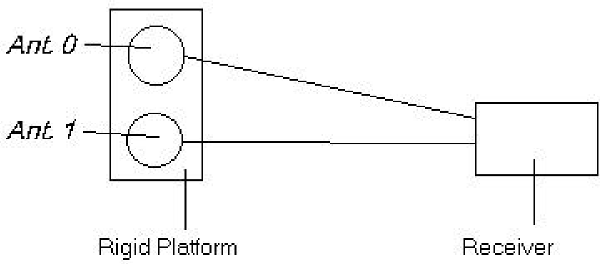


Figure 3: Experiment’s scheme: two antennas connected to the same receiver.

Figure 4: BeeLine receiver, internal card and 501antennas.

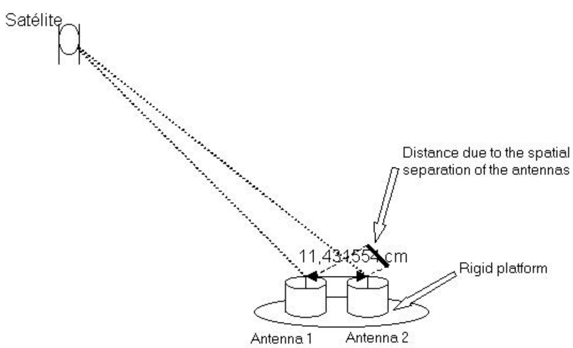


Figure 5: Geometric term Dr due to the spatial separation between the antennas.

The term related to the integer ambiguity is removed from the data series due to the fact that the baseline is shorter than a full cycle. A bias of less than a cycle remains due to the so-called line bias. This bias can be removed because the multipath error cannot be larger than ¼ of a cycle.

What is left from the original single difference are the two terms related to multipath and receiver noise. This residual carrier phase single difference, the observable used to feed the filter, can be written as:

$$\Delta\tilde{\Phi} = \Delta m + \Delta e \ , \tag{8}$$

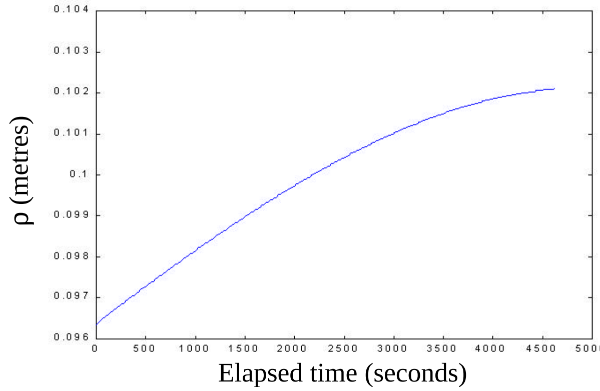


Figure 6: Δρ, in metres, with respect to satellite 9, for days 31 May, and 1 and 2 June 2000 (horizontal axis represents elapsed time since beginning of session).

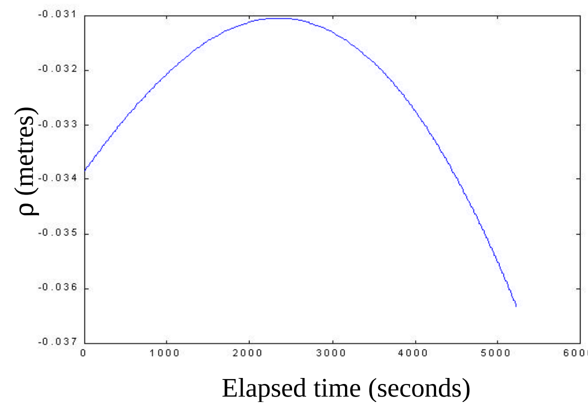


Figure 7: Δρ, in metres, with respect to satellite 26, for days 19, 20 e 21 June 2000 (horizontal axis represents elapsed time since beginning of session).

Figures 8, 9 e 10 show the residual carrier phase single difference for satellite 9, in the three consecutive days referred to before. Figures 11, 12 and 13 show the same quantity for satellite 26, for three consecutive days. The daily repeatability is given by the correlation among those data series and is equal to 76% for satellite 9 and 73,4% for satellite 26. This suggests the presence of multipath. The remnant being due to receiver noise. The elapsed time has been homogeinized by the constant 3 minutes and 36 seconds to account for the difference between UTC and mean solar day.

Another important factor when dealing with multipath is the elevation angle. Figures 14 and 15 show the variation of elevation angle for satellites 9 and 26 during the sessions.

The approximate path on sky of various satellites (besides the two used in this paper) with respect to the baseline formed by the antennas 0 and 1 can be seen in

Figure 16. This path influences the value of $\Delta\rho$, and as such, the EKF input values.

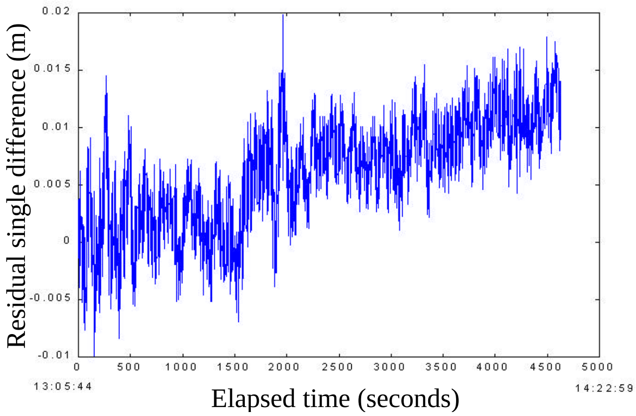


Figure 8: Residual single difference (in metres) for satellite 9, for day 31 May 2000 (horizontal axis represents elapsed time since beginning of session and UTC time).

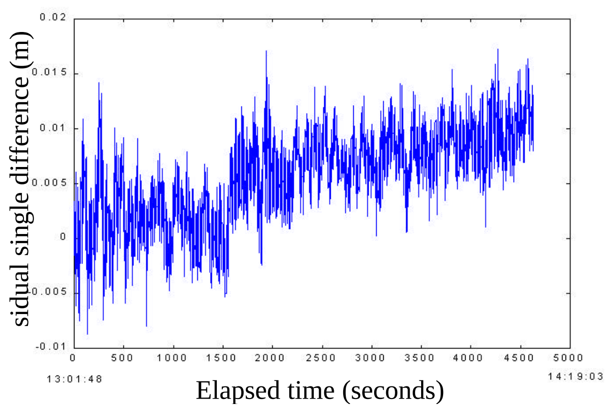


Figure 9: Residual single difference (in metres) for satellite 9, for day 1 June 2000 (horizontal axis indicates elapsed time since beginning of session and UTC time).

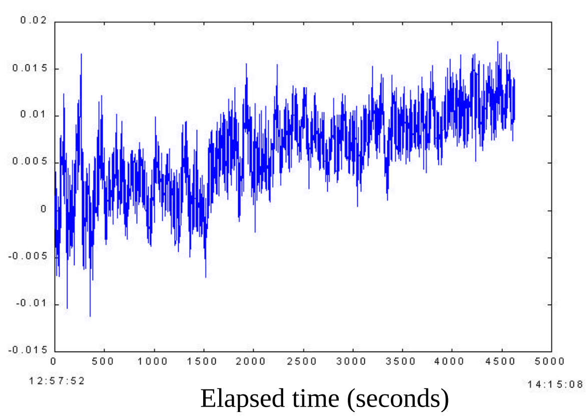


Figure 10: Residual single difference (in metres) for satellite 9, for day 2 June 2000 (horizontal axis indicates elapsed time since beginning of session and UTC time).

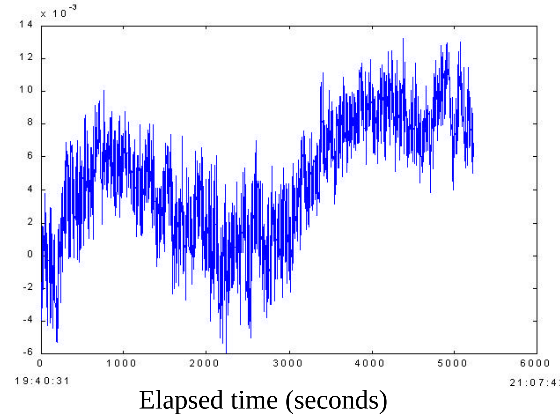


Figure 11: Residual single difference (in metres) for satellite 26, for day 19 June 2000 (horizontal axis indicates elapsed time since beginning of session and UTC time).

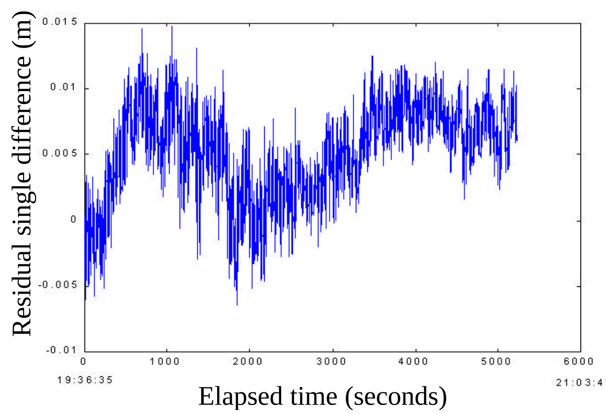


Figure 12: Residual single difference (in metres) for satellite 26, for day 20 June 2000 (horizontal axis indicates elapsed time since beginning of session and UTC time).

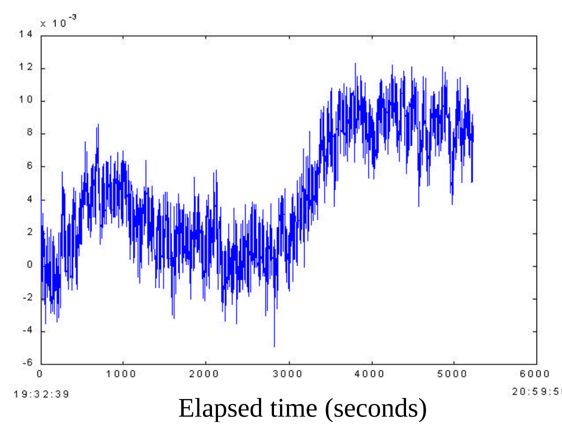


Figure 13: Residual single difference (in metres) for satellite 26, for day 21 June 2000 (horizontal axis elapsed time since beginning of session and UTC time).

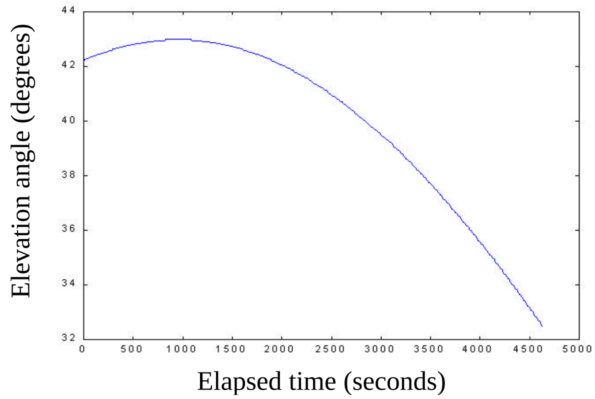


Figure 14: Elevation angle (in metres) for satellite 9 (horizontal axis indicates elapsed time since beginning of session).

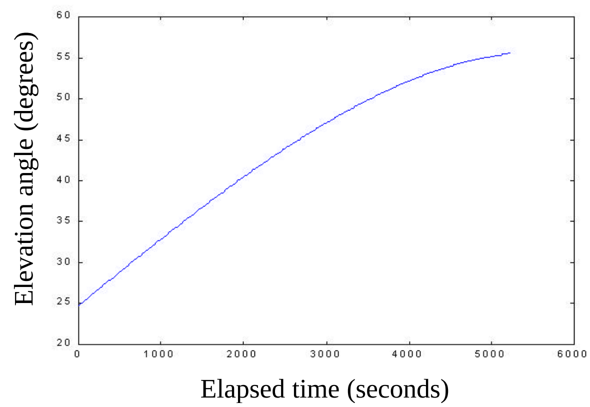


Figure 15: Elevation angle for satellite 26 (horizontal axis indicates elapsed time since beginning of session).

EKF INPUT AND OUTPUT

To form the system of equations used in the EKF, equations (6) and (5) are merged, making explicit the multipath parameters to be estimated: modified reflection coefficient (α_0), reflected sinal phase at the reference station (γ_0), reflected signal azimuth (β_0) and reflected signal elevation angle (υ_0). An important point in this approach is that the equations are not spatially related to different antennas with respect to the reference one. They are temporally related to the reference antenna in different epochs, thus generating one observation vector at every four epochs. The lack of observation equations for multiple antennas is satisfied by more observations with time. As a consequence, the estimated multipath parameters refer to four consecutive epochs. The approach is then based on the assumption that the satellite's angular variation within the time interval of four seconds (2 seconds of arc) is not significant. Therefore, the input vector **z** for the EKF is formed by four consecutive residual single differences taken in four consecutive epochs τ_i, (i=1,4):

$$\mathbf{z} = \left[\Delta\tilde{\Phi}(t_1)_{01}, \Delta\tilde{\Phi}(t_2)_{01}, \Delta\tilde{\Phi}(t_3)_{01}, \Delta\tilde{\Phi}(t_4)_{01}\right]^{\boldsymbol{T}}. \tag{9}$$

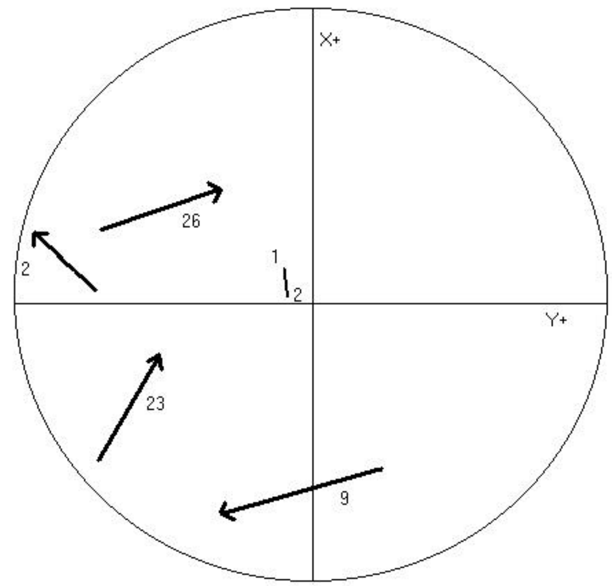


Figure 16: Sky plot of satellites used in the analysis, with respect to the baseline. The baseline between antennas is dislocated from the centre for clarity.

Once estimated the multipath parameters, equations (4) and (6) were used to compute the phase of the reflected signal at antenna 1 and the error due to multipath for both antennas separately.

For the data processing and analysis, routines were written using the Matlab environment.

RESULTS AND ANALYSIS

Let us first concentrate on the multipath effect computed from the estimated parameters. Figures 17 to 20 show the estimated multipath effect at antenna 1 (in metres) for satellites 9 and 26, for the first two days, respectively.

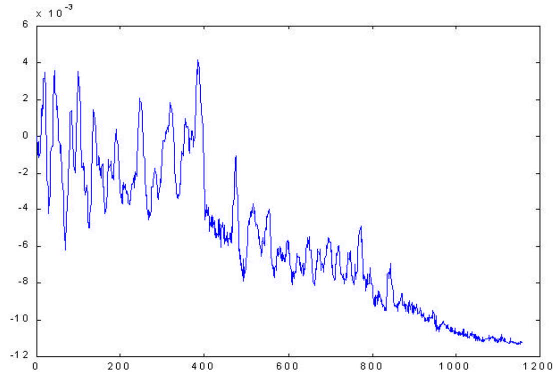
In general, in all analysed aspects, it can be seen that a high correlation between the three days used in this paper exist, followed by a high correlation of the measurement vector among these three days, from which they directly depend.

The effect of multipath in the antenna 1 indicates that it reached values of up to 1.2 cm. Peculiarities in the environment's geometry provoked a not so small high frequency multipath. An agreement between multipath and satellite's elevation angle can be noted.

Let us now concentrate on a way to assess the efficiency of the proposed methodology. In this case, by efficiency we mean the capacity in effectively identify and attenuate multipath. For this purpose, we have computed a signal ***r*** composed by the difference between the residual carrier phase single difference (input of the

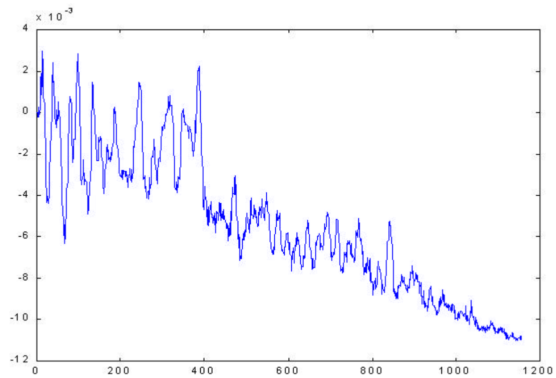
EKF) and multipath signal obtained using the estimated parameters. In other words:

$$r = \Delta\tilde{\Phi} - (\Delta\hat{\Psi}_0 - \Delta\hat{\Psi}_1)\,. \qquad (10)$$



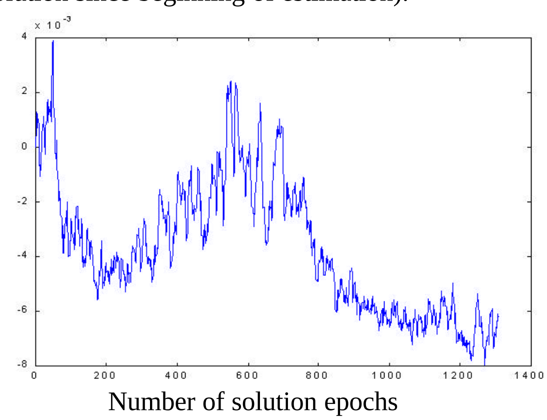
Number of solution epochs

Figure 17: Effect of multipath at antenna 2, in metres, for satellite 9, for 31 May 2000 (horizontal axis show number of solution since beginning of estimation).



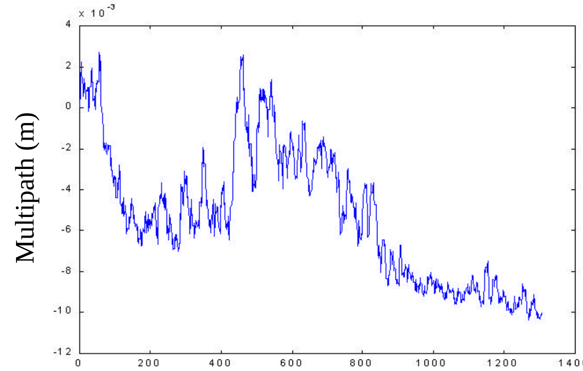
Number of solution epochs

Figure 18: Effect of multipath at antenna 2, in metres, for satellite 9, for 1 June 2000 (horizontal axis show number of solution since beginning of estimation).



Number of solution epochs

Figure 19: Effect of multipath at antenna 2, in metres, for satellite 26, for 19 June 2000 (horizontal axis show number of solution since beginning of estimation).



Number of solution epochs

Figure 20: Effect of multipath at antenna 2, in metres, for satellite 26, for 20 June 2000 (horizontal axis show number of solution since beginning of estimation).

The degree of smoothness of this signal indicates how much both multipath values coincide, giving an indication on the efficiency of the method in detecting the multipath in each one of the two antennas from the measurements. Figures 21 to 24 show these results for satellites 9 and 26.

For an analysis on the degree of smoothness of the signal, the standard deviation of both measured and smoothed signals was used as metric. The average standard deviation of the input signal for the three days, for satellite 9, is equal to 0.0043 m, being the corresponding smoothed value equal to 0.0023 m. Considering the average repeatability for the three days to be equal to 76%, a smoothing of 60,3% is observed, being this value a measure of the efficiency of the method. For satellite 26, the input average standard deviation is equal to 0.0036 m, and 0.0018 m for the smoothed signal. The average repeatability is equal to 73.4%, with a gain equal to 62.6%, being this measure indicating the efficiency of the method.

ON THE USE OF PHASES CORRECTED FROM THE EFFECT OF MULTIPATH

The estimate of the error due to multipath in each one of the antennas subtracted by the phases originally measured yield the values for the carrier phase mitigated from multipath. These corrected values would translate into higher accuracy and when used in the context of a reference station would yield better results at the user's level [Ray, 1999]. If we consider the length of a session (e.g., for 31 May, 1h17min16s) when processed in a Pentium II 266 MHz took less than 3.8 minutes. We believe that as far as processing time is concerned, the approach can be used in real-time, probably entailing the transmission of either real-time corrections or the corrected observations per se (real-time applications are beyond the scope of the current study).

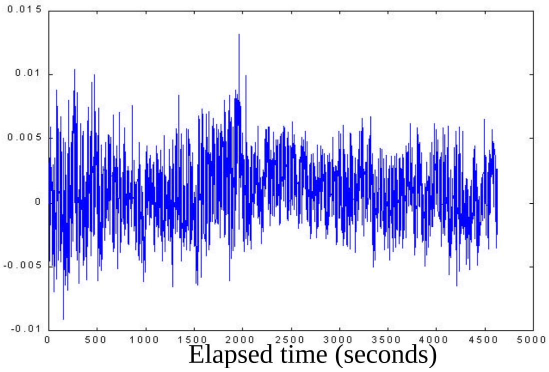


Figure 21: Smoothed signal, in metres, for satellite 9, 31 May 2000 (horizontal axis in elapsed time, in seconds).

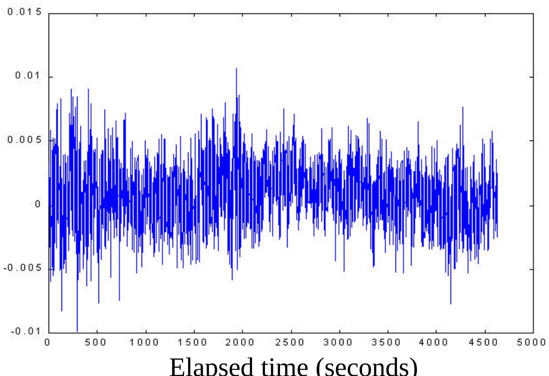


Figure 22: Smoothed signal, in metres, for satellite 9, 1 June 2000 (horizontal axis in elapsed time, in seconds).

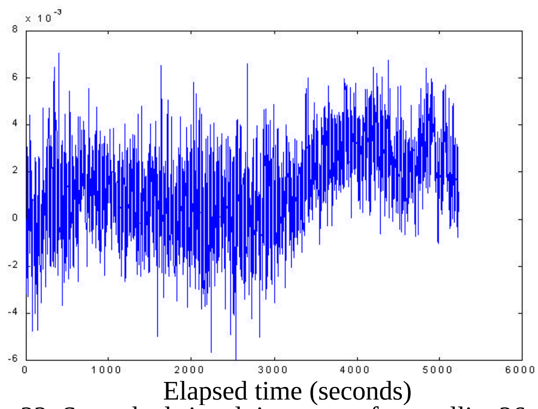


Figure 23: Smoothed signal, in metres, for satellite 26, 19 June 2000 (horizontal axis in elapsed time, in seconds).

Table 1 shows the results coming from the application of the methodology for satellites 9 and 26.

Sate-llite (PRN)	Mean multipath (cm)	Smoothed signal before (std.)	Smoothed signal after (std.)	Corre-lation (%)	Gain (%)
9	0.569	0.433	0.226	76.0	60.3
26	0.525	0.357	0.182	73.4	62.6

Table 1: Results in the application of the methodology for satellites 9 e 26.

The mean input signal is used, being both for the input signal (measured multipath) and by means of the standard deviation of the smoothed signal (measured multipath minus computed multipath). All three days were considered. The correlation and gain are also indicated in the table.

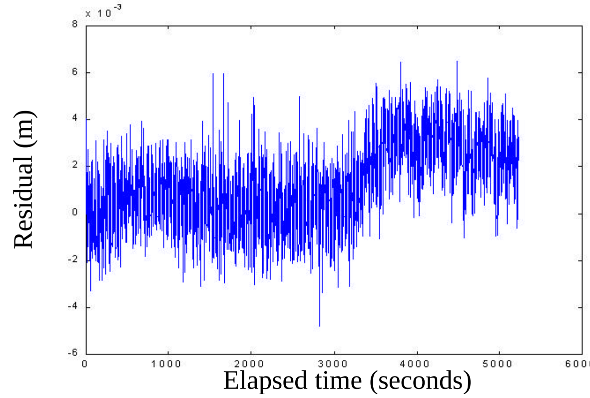


Figure 24: Smoothed signal, in metres, for satellite 26, 20 June 2000 (horizontal axis in elapsed time, in seconds).

CONCLUSIONS

The results allow us to say that it is possible to use the low angular variation of the satellites in a short period of time along with the high correlation of signals collected by nearby antennas to generate an efficient methodology capable of identifying and mitigating carrier phase multipath.

The angular variation of the satellites with respect to the antennas can be used to feed an EKF. It can then be used to form the system of equations used by the estimator.

In practical and economical terms, the methodology showed to be satisfactory, since it uses only one receiver and two antennas to generate all frame of identification and mitigation of multipath for all tracked satellites, without the need of external clocks or additional antennas.

As far as the accuracy of the methodology is concerned, an average 65% gain was obtained. This number should improve with further improvements in the estimator.

For different satellites, there is a similarity in the correlation between signals with multipath in the different days, with corresponding session times with identical geometry. This indicates a real correlation, being that the value that does not repeat (25%) due to another reason (thermal noise). An estimate of its value can be derived, even though this has been the object of the present study.

Even though the methodology has only been tested in L1 carrier phases, we believe that the same approach can be used for L2 carrier phase.

The processing time is less than the time used for the data collection, which indicates that the algorithm can be used in a real-time case scenario, specially in the context of a reference station, which is the interest one of the current work.

The methodology seems also interesting for the study of multipath in different scenarios and for a longer period of time, allowing the study of long term behaviour of this phenomenon.

The method is also well behaved in terms of performance of the estimator: solution at the channel level, has no limitation in terms of nature and number of signals with multipath, seems possible for real-time application and is not heavy computationally speaking.

There is the intention to keep on with the investigation, in different scenarios, varying the distance between the antennas and reflecting objects, in order to confirm the level of efficiency obtained.

Improvements in the estimator may be possible, such as in the covariance matrices, process modelling and better initial values (with the possibility of using previous ones, in an recursive manner). These improvements should result in an improvement of the current efficiency level. If possible to generate corrections to other time intervals (say, every 15 seconds), an iteration of the EKF can be used (the so called Iterated Extended Kalman Filtering) in these intervals. Eventual improvement should be studied.

The same study can be extended to dual-frequency receivers. Finally, additional studies related to data transmission may be made, concerned to the applicability of the method in real-time applications.

ACKNOWLEDGMENTS

The BeeLine receiver and 501 antennas used in the equipment were made available by NovaTel Inc., by means of a Field Trial Agreement with the University of New Brunswick. The first author was partly funded by CAPES, a Brazilian research financing agency.

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