Improved Tropospheric Delay Estimation for Long Baseline, Carrier-Phase Differential GPS Positioning in a Coastal Environment

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BIOGRAPHY

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ABSTRACT

Long baseline, carrier-phase differential GPS positioning in a coastal environment poses unique challenges. It is well known that differential GPS positioning results degrade as baseline length increases due to several sources of error, including the error introduced by differential troposphere. The effect of the troposphere on GPS has been extensively discussed by numerous researchers, either by comparing the resolution of tropospheric prediction models or by assessing the tropospheric delay directly on GPS measurements and results.

In order to improve the estimation of tropospheric delay in the coastal environment, a project has been undertaken by the University of New Brunswick (UNB) and the University of Southern Mississippi (USM). The project includes extensive GPS and meteorological data collection in the Bay of Fundy in Canada. The goal of the research is to examine methods for improving tropospheric delay estimation by employing various sources of data. This includes the use of surface meteorological parameters and Numerical Weather Prediction (NWP) model data. For this research, NWP data are accessed from the Canadian Meteorological Centre's (CMC) regional model and from the National Oceanic and Atmospheric Administration's (NOAA) tropospheric delay product. Tropospheric delays modelled from the NWP model data are compared with those from global prediction models. Results in this paper will demonstrate the effect of using surface meteorological data and NWP model data to estimate the tropospheric delay in a coastal environment.

INTRODUCTION

In long baseline differential GPS positioning there are effective mitigation strategies for all sources of post-processed uncertainty, except tropospheric delay. Clock errors are eliminated by double-differencing the GPS range measurements (Wells et al., 1986). Ionospheric delay uncertainty is almost completely eliminated by twofrequency estimation. GPS satellite orbit errors can be eliminated by post-processing with precise ephemeredes. Multipath uncertainties can be reduced by using special equipment: choke-ring and other multipath-resistant antennas, and receivers with multipath-estimating tracking loops. Multipath is less likely for antennas in motion as on buoys and boats at sea, and has a smaller signature for GPS carrier phase measurements than for GPS code measurements.

In this paper, we look at using various strategies for improving the estimation of tropospheric delay with the goal of improving position solutions for long baselines. This includes the use of surface meteorological parameters in a global tropospheric prediction model and NWP model data in a tropospheric estimation model. These strategies can lead to extended-range marine, ambiguity-resolved, carrier phase, differential GPS positioning measurements.

The goal of our campaign is to advance the science of modeling microwave tropospheric delay over marine areas, and to test, apply, and demonstrate these advances to obtain higher accuracy (centimetre-level) positions at greater distances (10s to 100s of kilometres) from differential reference stations. This may allow for marine vertical positioning accurate enough for vertical control in the measurement and modeling of offshore tidal and other water level variations, offshore determinations of the geoid-ellipsoid separation, hydrographic surveying, calibration of satellite altimetric sensors, navigation, and amphibious and other operations at sea.

MODELLING THE TROPOSPHERE

Tropospheric delay refers to the refraction of the GPS signal as it passes through the neutral atmosphere from the satellite to the earth. The effect causes the distance travelled by the signal to be longer than the actual geometric distance between satellite and receiver.

The neutral atmosphere refers to the non-ionized portion of the atmosphere made up of the stratosphere and the troposphere. Water vapour and dry gases found in the neutral atmosphere affect the propagation of the GPS signal. The troposphere makes up the lower portion of the neutral atmosphere, extending from the earth's surface up to approximately 16 km in altitude. All of the water vapour and the bulk of the dry gases are found in this lower part of the atmosphere. Gradually decreasing quantities of dry gases can extend several hundred kilometers in altitude (Misra and Enge, 2001).

The delay of the GPS signal can be expressed as the sum of the hydrostatic (N_h) or 'dry' and non-hydrostatic (N_w) or 'wet' refractivities, due to the effects of dry gases and water vapour, respectively.

$$N = N_h + N_w \qquad (1)$$

The zenith total delay (*ztd*) of the signal is determined by integrating the refractivity along the signal path (*dl*)as

$$ztd = 10^{-6}\int N_h dl + \int N_w dl \qquad (2)$$

where refractivity, N, is expressed as (Thayer, 1974)

$$N = k_1\left(\frac{P_d}{T}\right)Z_h^{-1} + \left(k_2\frac{e}{T} + k_3\frac{e}{T^2}\right)Z_w^{-1} \qquad (3)$$

where k_1, k_2, and k_3 are refractivity constants (e.g., Smith and Weintraub, 1953) in Kelvin (K) mbar^{-1} (for k_1 and k_2) and K^2 mbar^{-1} (for k_3), P_d is the partial pressure of dry gases in mbar, e is the partial pressure of water vapour in mbar, T is the temperature in K, Z_h is the compressibility factor for dry air and Z_w is the compressibility factor for water vapour.

Tropospheric delay for GPS positioning is generally accomplished with the use of a global prediction model. Typically, the hydrostatic component of the delay in the zenith direction is in the range of 2.3-2.6 m and represents about 90% of the total delay. As found by Mendes (1999), the hydrostatic component of the delay can be modeled to sub-millimetre accuracy with the use of prediction models such as Saastamoinen (1973). The highly variable non-hydrostatic delay, however, can only be modeled to an accuracy of a few centimetres in the zenith direction. Further error is introduced when the zenith delay is mapped to the elevation angle of the satellite with the use of a mapping function such as, e.g., Niell (1996).

Prediction Models

A variety of global tropospheric delay prediction models exist; each of these varies in how water vapour and temperature changes with altitude (Misra and Enge, 2001). Two of the more common and best performing of the prediction models are Saastamoinen and Hopfield (Mendes, 1999).

The Saastamoinen model (1973) is based on refractivity derived using the gas laws. The hydrostatic (*zhd*) and wet (*zwd*) components of the delay in the zenith direction are expressed as

$$zhd = 0.002277\,(1 + 0.0026\cos 2\phi + 0.00028\,H)P \qquad (4)$$

$$zwd = 0.002277\left(\frac{1255}{T} + 0.05\right)e \qquad (5)$$

where f is the latitude of the receiver, H is the orthometric height of the receiver in km, P is atmospheric pressure in mbar, T is temperature in K, and e is partial pressure of water vapour in mbar.

The Hopfield model (1969) is based on the empirically derived representation of hydrostatic refractivity as a function of height. The hydrostatic and wet components are expressed as

$$zhd = 77.6 \cdot 10^{-6} \frac{P}{T} \frac{\left(h_h - h\right)}{5} \tag{6}$$

$$zwd = 0.373 \frac{e}{T^2} \frac{\left(h_w - h\right)}{5} \tag{7}$$

where P is the pressure in mbar, e is the partial pressure of water vapour, h_h and h_w are the hydrostatic and wet equivalent heights in km, respectively, and h is the height of the receiver in km.

Mendes (1999) finds that the total error in the zenith direction for the Saastamoinen model is on average 0.2 mm for the dry component and about 30 mm for the wet component of the prediction model. The total error for the Hopfield model is 4.3 mm in the dry component and about 30 mm in the wet component in the zenith direction. These total error statistics are based on comparisons to ray traced values from 50 radiosonde stations worldwide.

Estimation model

Tropospheric delays can also be obtained directly by integrating the refractivity along the path of the GPS signal through the neutral atmosphere. One may also integrate vertically to obtain a zenith delay. In this case, a regional NWP model can act as a representation of the neutral atmosphere up to approximately 30 km. The equation for refractivity, N, given in Equation 2 can be expressed in terms of height as

$$\begin{aligned} ztd &= 10^{-6} \int_{h_{ant}}^{h_{top}} k_1 R_d \, \rho \, dh \\ &+ 10^{-6} \int_{h_{ant}}^{h_{top}} \frac{R_d}{\in} \left(k_2 - k_1 \in + \frac{k_3}{T} \right) q \rho \, dh \end{aligned} \tag{8}$$

where R_d is the gas constant for dry air, ρ is the mass density , ∈ is the ratio between the gas constant for dry air and the gas constant for water vapour, and q is the specific humidity in kg/kg.

To express the delay in terms of pressure, as is appropriate with NWP model data, one must introduce the hydrostatic equation (Wallace and Hobbs, 1977) as

$$dp = -\rho g dh \tag{9}$$

where g is gravity in m/s^2. The final expression of total zenith tropospheric delay is given (Vedel *et al.*, 2001;Jensen, 2002a) as

$$\begin{aligned} ztd &= 10^{-6} \int_{p_{ant}}^{p_{top}} k_1 \frac{R_d}{g} dp \\ &+ 10^{-6} \int_{p_{ant}}^{p_{top}} \frac{R_d}{\in} \left(k_2 - k_1 \in + \frac{k_3}{T} \right) \frac{q}{g} dp \end{aligned} \tag{10}$$

The temperature (*T*), pressure (*P*) and specific humidity (*q*) parameters are extracted from the NWP model at each pressure level. A prediction model is used to estimate the delay for the atmosphere above the top pressure level.

It has been found that the use of regional NWP model data in an estimation model may be beneficial for the modelling of tropospheric delay due to the wet component Accuracies in the 10 to 20 mm range have been obtained as compared to radiosonde data and GPS derived delays (Shueller *et al*., 2000; Bock and Doerflinger, 2000; Pany *et al*., 2001a; Vedel, *et al*., 2001; Jensen, 2002b; Bisnath *et al*., 2004a).

TEST DATA

The data used for this paper have been collected as part of an overarching project aimed at the advancement of the science of modelling microwave tropospheric delay over marine areas. The project is funded by the Office of Naval Research (ONR). Other partners include the Canadian Coast Guard (CCG), the Canadian Meteorological Centre (CMC), National Oceanic and Atmospheric Administration (NOAA), and the Canadian Hydrographic Service (CHS). Detailed project descriptions and early findings are presented in Bisnath *et al*. (2004b) and Santos *et al*. (2004).

The test area is in the Bay of Fundy region on the east coast of Canada. This area provides highly variable weather conditions in a coastal environment with a number of GPS reference stations and meteorological stations in the vicinity.

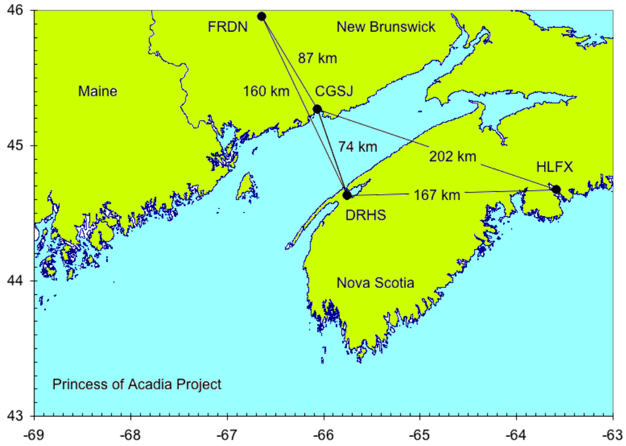

Figure 1 Test area for data collection.

The varying distance between each reference station and the ferry is repeated for each crossing. This allows for sampling under similar geometry, but with widely varying atmospheric conditions. Control for the ferry crossing data comes from long / short baseline solution comparisons. E.g., the short baseline (between the ferry and the near station DRHS) solution allows for verification of the long baseline solutions (between the ferry and the far station CGSJ). Control for static baseline testing comes from known station coordinates.

GPS and Meteorological Data

The GPS and meteorological data are collected on a regularly scheduled ferry travelling a 75 km route between Saint John, New Brunswick, and Digby, Nova Scotia. The data exhibit a great deal of spatial and temporal diversity. GPS and meteorological data are collected near the ferry terminals (stations CGSJ and DRHS) at each end of the route and on the ferry (station BOAT). Data from other continuously operating reference stations (for example the International GPS Service station UNB1, 100 km from Saint John) are also being collected. The data collection began in the fall of 2003 and will continue for one full year.

Equipment at CGSJ, DRHS and BOAT stations consists of Novatel OEM4 GPS receivers with 600 series antennas and Campbell Scientific meteorological stations. The GPS receivers log at a rate of 1 Hz and the data are stored on anon-site computer. Each of the meteorological stations is equipped with temperature and relative humidity probe, barometric pressure sensor, datalogger and storage device. Data is scanned every 15 seconds and logged every ten minutes for temperature and relative humidity. Barometric pressure is logged every hour.

Equipment at the UNB1 station consists of a TPS LEGACY receiver with a JPSREGANT_DD_E antenna logging at a rate of 30 seconds. Meteorological data are from Suominet station UNB2, a Paroscientific MET3A instrument, logging at a rate of 3 minutes.

NWP Model Data

Numerical Weather Prediction is the forecasting of the state of the atmosphere through the use of numerical models. The models are based on a three-dimensional layered grid system that extends from the earth's surface to approximately 30 km in altitude. A variety of meteorological parameters are represented in the forecast fields making it useful for a wide range of applications (Canadian Meterological Centre, 2002). The models used in this project are the regional configuration of the CMC's Global Environmental Multiscale (GEM) model and the 20 km grid Rapid Update Cycle (RUC) model produced by NOAA.

The regional configuration GEM model was used to estimate the tropospheric delays presented in this paper. It operates on 24 km grid cells and is composed of 27 vertical layers. The model contains the parameters of temperature, barometric pressure, and relative humidity required to derive the total zenith delay. The model extends beyond the borders of Canada excepting some portions of the far north. Data assimilation for the model is performed twice daily at 0 and 12 UTC. Forecasts are available for a 48 hour period in 3 hour increments. Analysis and forecasts data from both models were used in the following tests. Recently, the model grid was reduced to 15 km; no delays have been generated from these data. Further information about the GEM model can be accessed on the Environment Canada website (Canadian Meteorological Centre, 2002).

Zenith tropospheric delays generated with data from the RUC model by the Forecast Systems Laboratory (FSL) at NOAA are available to the public. Wet and hydrostatic delays can be generated via FTP and client software for a particular date and time. The product is available in a data-assimilated hindcast and in 2 hour forecast. Further information regarding the RUC model and the tools required to access the delays can be found on the Ground-based GPS Meteorology website (NOAA Forecast Systems Laboratory, 2001).

TEST METHODOLOGY

Tests were undertaken to assess the effect of using time series of surface meteorological parameters and numerical weather prediction data to estimate zenith tropospheric delays. The first test is in the measurement domain and the second in the position domain.

The first test consists of an analysis of several methods for the estimation of zenith tropospheric delays. Comparing the delays to GPS derived zenith delays accessed from the International GPS Service (IGS) for station UNB1 assesses this test.

The second test consists of improvements in the position solution due to the estimation of tropospheric delays. The second analysis comprises two parts: static and kinematic data series. The static baseline is between station CGSJ and DRHS, a 74 km baseline that is processed as kinematic data, and is a comparison between the position solution and known coordinates. The coordinates for the reference stations are estimated to be correct within 2 cm. The kinematic test takes place while the vessel is in dock and is a comparison between a short and long baseline solutions. The short baseline was determined by using a narrow lane fixed ambiguity linear combination. The long baselines were determined using a ionospheric delay free linear combination. The short baseline has an internal precision of less than 1 cm and is therefore a reasonable estimate of the "true" solution. The long baseline solution will be relatively free of other sources of spatially correlated errors, i.e., the ionospheric delay, therefore most of the remaining error will be due to the effect of the troposphere.

Zenith tropospheric delays using the GEM NWP data were estimated for every 3 hours. Delays using the prediction model were determined for every hour. The GPS data was logged at 1 hertz; delays were estimated at 1 hertz with a cubic spline interpolator. The Niell mapping function was used to map the zenith delays to the data at the appropriate elevation angle. These slant delays were subtracted from the GPS raw code and phase observations in RINEX format. Commercial software, Dynapos by The XYZ's of GPS, Inc., that could be configured not to apply tropospheric modelling was used for all processing. A 5 degree elevation angle mask was applied. Broadcast orbits were used in all cases. In our campaign, we are not attempting to use a real-time data link to supply differential corrections to measurements made on platforms at sea. All raw data are recorded and all processing done after the fact.

Ideally, the delays would be applied to the raw data during processing and not applied to the data in the RINEX file. With the latter comes an increased chance of cycle slips during processing (Jensen, 2002a).

ZENITH DELAY RESULTS

The results of the zenith tropospheric delay comparison are shown in terms of total delay. Statistics are given as standard deviation in latitude, longitude and height. The standard deviation has been determined as the difference between the predicted or estimated delay when compared to GPS derived delays from the IGS SINEX product. The delay product has an estimated accuracy of 4 mm, though accuracies may vary at individual stations. The statistics have been generated with six full days of 1 Hertz data, spanning hour 0 UTC May 23 to hour 24 UTC May 28, 2004.

The various sources of data and models included in the test are described in Table 1.

Table 1 Description of data and models used in tests.

IGS	GPS derived delays from IGS SINEX product
GEM	Delay product from NOAA based on RUC model
NOAA	Delays estimated from CMC's GEM model
SAAS	Delays predicted by Saastamoinen model using time series of surface meteorological parameters
SAAS std	Delays predicted by Saastamoinen model using standard surface meteorological parameters
HOP	Delays predicted by Saastamoinen model using time series of surface meteorological parameters
HOP std	Delays predicted by Saastamoinen model standard surface meteorological parameters

In this case, standard meteorological parameters are considered to be 20 degrees Celsius, 1013.25 mbars pressure, and 50 % relative humidity.

All models perform adequately for the estimation of the hydrostatic component, but not for the wet component of the delay. The zenith tropospheric total delays for time series from hour 0 UTC on 23 May to hour 24 UTC on 28 May, 2004 are shown in Figure 2.

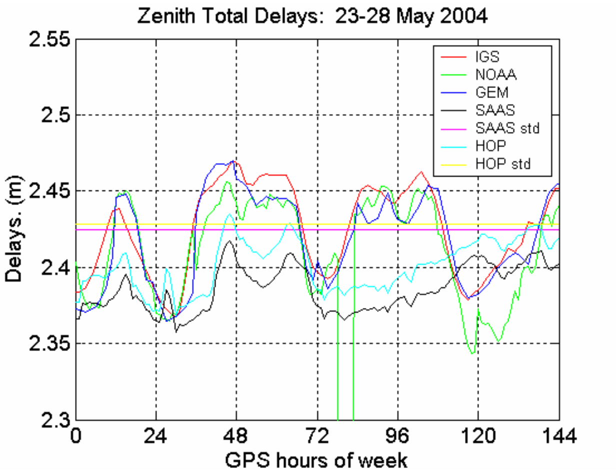

Figure 2 Zenith total tropospheric delays for 23-28 May, 2004 at station UNB1.

Biases are introduced in the wet component of the total delay results when surface meteorological parameters are used in the Saastamoinen and Hopfield models. The section of the NOAA delays that seems to drop to zero value is not due to incorrect delay estimates, but to missing data.

The mean and standard deviations for all the modelled delays determined at UNB1 compared to GPS derived total tropospheric delays computed by the IGS are given in Table 2. The standard deviation of the differences between the estimation model with the GEM data and the GPS derived delays from the IGS are 13 mm over the 6 day period. Standard deviation results for the tropospheric delay product from NOAA are similar at 15 mm. The standard deviation results for the prediction models, Saastamoinen and Hopfield, were all 29 mm, though the mean varied due to the bias in the models where measured surface meteorological values were used.

Table 2 Mean and standard deviation (m) of differences in zenith delays for station UNB1. IGS minus model delays.

Description	Mean (m)	Std. Dev. (m)
IGS - GEM	0.006	0.013
IGS - NOAA	0.012	0.015
IGS - SAAS	0.039	0.029
IGS – SAAS std	0.001	0.029
IGS - HOP	0.024	0.029
IGS – HOP std	-0.004	0.029

The results in Table 2 are consistent with previous findings from other researchers and represent a significant improvement in modelling of total tropospheric delay with the use of NWP model data.

The next test will demonstrate whether a significant improvement can be obtained in the positioning results due to this improvement in the measurement domain.

POSITIONING RESULTS

The GPS data files used for the positioning tests incorporate delays estimated using the GEM model (GEM), delays predicted with the Hopfield model with measured surface meteorological parameters (HOP), and delays predicted with the Hopfield model using standard surface meteorological parameters (HOP std). The time series for the position domain tests is from hour 5 UTC to hour 10 UTC on 27 May, 2004. The weather conditions during this time period are stable with high relative humidity, temperatures in the 5 to 10 degree Celsius range, and low barometric pressure.

The first part of the position domain results is for the static baseline processed as kinematic data. The standard deviations appear to be larger than one would expect if the data had been processed as static data. The statistics given in Table 3 are based on solution differences between the known coordinates of the station (CGSJ) and each solution after convergence.

Table 3 Standard Deviation (m) for static baseline DRHS to CGSJ. Known coordinates for CGSJ minus kinematic solution.

Comparison	Standard Deviation (m)		
	ΔLat. (m)	ΔLon. (m)	ΔHgt. (m)
CGSJ - GEM	0.048	0.059	0.100
CGSJ - HOP	0.061	0.057	0.115
CGSJ - HOP std	0.063	0.048	0.109

Statistics show slight improvement in the latitude and height components but not in the longitude. There seems to be no benefit during this time period to using measured surface meteorological parameters in the Hopfield prediction model.

The second part of the positioning test is the kinematic data series assessment via processing of the narrow lane fixed integer ambiguity short baseline solution (Nlfixed) and comparing it against the ionospheric delay free long baseline solution. The statistics given in Table 4 are based on solution differences between the short baseline solution between DRHS and BOAT (approximate distance of 4 km) and the long baseline solution between CGSJ and BOAT (approximate distance of 74 km). The vessel is in dock in Digby during this time period.

Table 4 Standard Deviation (m) for kinematic data set. Short baseline solution minus long baseline solution.

Comparison	Standard Deviation (m)		
	ΔLat. (m)	ΔLon. (m)	ΔHgt. (m)
Nlfixed - GEM	0.060	0.087	0.088
Nlfixed - HOP	0.052	0.052	0.114
Nlfixed - HOP std	0.064	0.056	0.096

Statistics for the short / long baseline solution shows a slight improvement in the latitude and height, but not in the longitude. Again, there seems to be no benefit during this time to use measured surface meteorological parameters in the prediction model. The results for the static baseline and kinematic data are similar with little improvement in either case.

The improvement found in the measurement domain does not translate significantly into the position domain for this time series of data. This may be due to the spatial consistency in weather condition in the region resulting in little differential troposphere. Results also seem to be related to the particular geometry of the sites chosen for the kinematic study. Reference stations in Fredericton and Halifax will be included in future work to provide variety in geometry and baseline length.

CONCLUSIONS AND RECOMMENDATIONS

Improved estimation of tropospheric delay may lead to improvements in positioning accuracy for long baseline GPS positioning.

In order to improve tropospheric delay estimation NWP model data and surface meteorological parameters were used in the modelling of the delay. The results show significant improvement in zenith tropospheric delay estimation with use of NWP model data. In a comparison with GPS derived delays from the IGS, the delays estimated with GEM model data and the tropospheric delay product from NOAA shows standard deviations of 13 mm and 15 mm, respectively. This is an improvement on the delays modelled with global tropospheric prediction models with standard deviations of 29 mm.

A slight improvement in position solution was found with the use of the NWP derived delays, particularly in vertical component. The minimal improvement can perhaps be attributed to the high spatial correlation over area resulting in a lack of differential troposphere between the stations.

There was, during the test period, no apparent improvement with use of surface meteorological data in the global tropospheric prediction model over the use of standard meteorological parameters.

Results in zenith delay estimation and positioning are encouraging but require further study. The study is to be expanded to include stations in Fredericton and Halifax giving baseline distances up to 200 km. The study is also to be expanded to include wider variety of weather conditions especially times of high variability, i.e., during the passage of weather fronts.

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