Effect of terrain on orthometric height

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Structure of presentation

Objective:

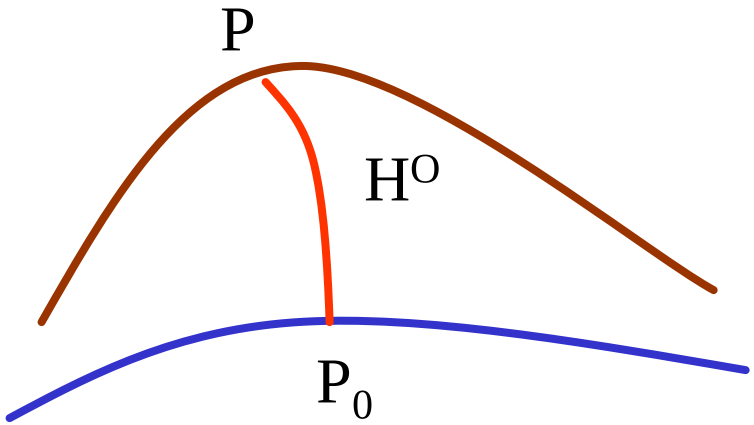
- Investigate the effect of terrain on orthometric height, within the context of the "rigorous" definition of orthometric height.

Contents:

- Show how mean value of gravity along plumbline is expressed within the "rigorous" definition of orthometric heights.
- Show numerical results.
- Review definition of orthometric heights (Helmert, Niethammer, Mader).
- Make comparisons.
- End with concluding remarks.

Orthometric height

- Definition: length of plumbline between the geoid and the Earth's surface.
- For a numerical evaluation, knowledge of mean value of gravity along the plumbline required.
- Mean value of gravity along the plumbline is a function of mass density distribution of Earth and on shape of Earth's surface.



$$H^{O}(\Omega) = \frac{C(r_t(\Omega))}{\bar{g}(\Omega)}$$

Decomposition of actual gravity

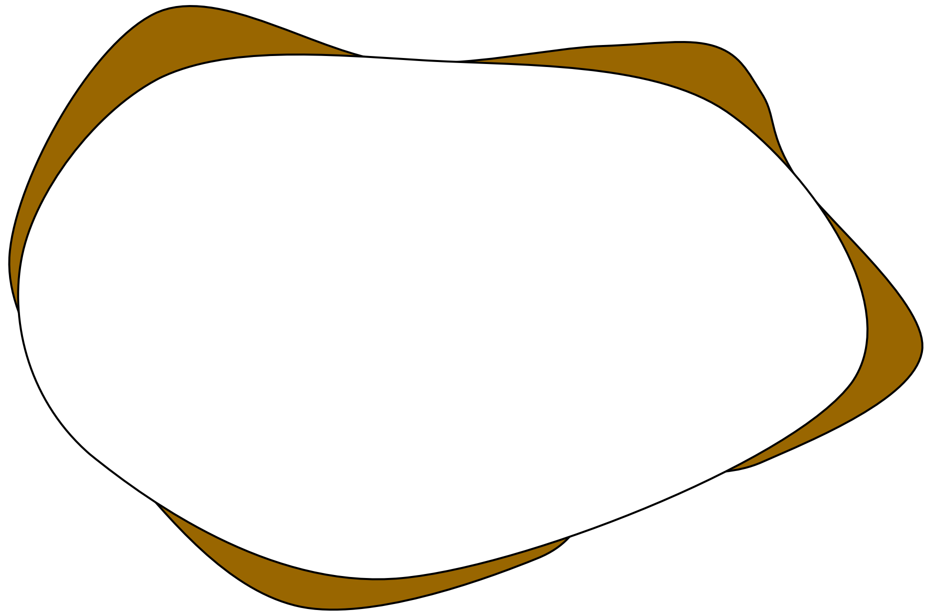
$$g(r,\Omega) = g^{NT}(r,\Omega) + g^{T}(r,\Omega)$$

gravity at a point

gravity generated by the geoid

gravity generated by topography

Effect of topography



Mean gravity generated by topography

- From the definition of integral mean gravity, it follows that:

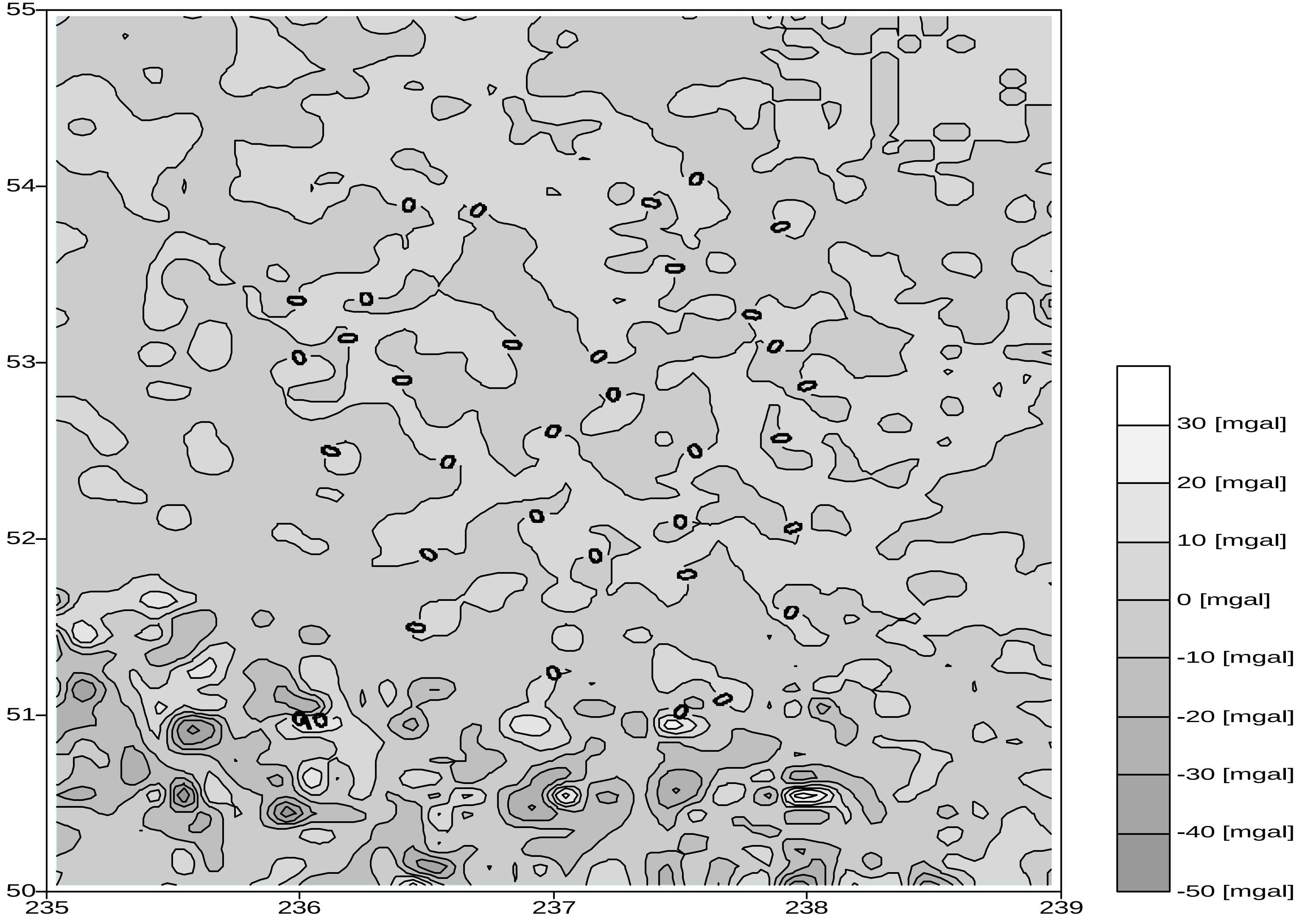
$$\bar{g}^{TC}(\Omega, r_0) \cong \frac{1}{H(\Omega)}\left[V^{TC}(R,\Omega) - V^{TC}(R + H(\Omega),\Omega)\right]$$

- Expressed in terms of difference of potential.

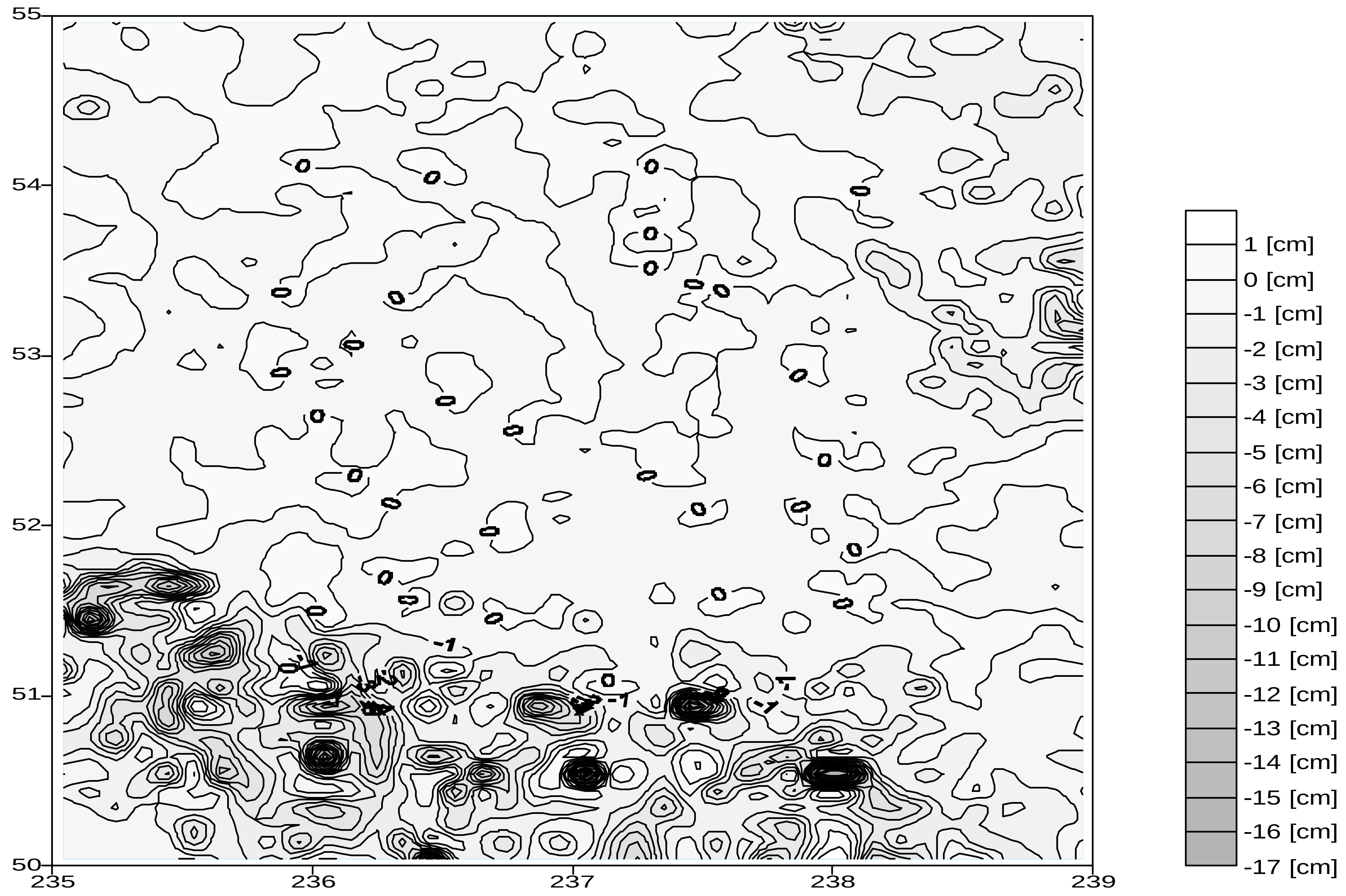
$$\overline{g}^{TC}(\Omega) = \overline{g}^{TC}(r_0;\Omega) + \overline{g}^{TC}(dr;\Omega)$$

- Contribution coming from the mean mass density plus a correction due to density variations.

Mean values of the gravitational attraction caused by the spherical terrain roughness term



Correction due to the spherical terrain correction



The inclusion of terrain term in our approach

- Mean gravity generated by topography expressed in terms of potential. Solution more accurate.
- It is composed of a contribution coming from the mean mass density plus a correction due to density variations.
- Dominant term represents the change in the roughness part of the Secondary Indirect Topographical Effects keeping a direct relationship with the topography of constant density of ρ_0, from the geoid to the surface of the earth, divided by the height of the point of interest.
- Numerical evaluation is similar to the one applied in the geoid computation, and is rather simple.

Mean value of gravity along plumbline

- Several prescriptions:

  - ✓ Helmert's: (1890)

$$\bar{g}^{H}(\Omega) = g(r_t(\Omega)) + \frac{\partial g(H, f)}{\partial H}\frac{\mathrm{H}^{\mathrm{O}}(\Omega)}{2} - 2pGr_0\mathrm{H}^{\mathrm{O}}(\Omega)$$

  - ✓ Niethammer: (1932)

$$\bar{g}^{N}(\Omega) = \bar{g}^{H}(\Omega) - g^{TC}(r_t(\Omega)) + \bar{g}^{TC}(\Omega)$$

  - ✓ Mader: (1950)

$$\bar{g}^{M}(\Omega) = \bar{g}^{H}(\Omega) - \frac{1}{2}(g^{TC}(r_t(\Omega)) - g^{TC}(r_g, \Omega))$$

(some) Characteristics of previous approaches

- … dealing with a terrain term …
- Mader orthometric height:
  - ✓ assumes linear variation in gravity above geoid.
  - ✓ uncertainty increases in mountainous area
  - ✓ computationally intensive (requires computation of terrain effects at topograpic surface and geoid)
- Niethammer orthometric height:
  - ✓ Greater compatibility with GPS-derived heights from a gravimetric geoid that includes terrain correction.
  - ✓ More computationally intensive than Mader's

Mean value of gravity along plumbline

$$\bar{g}(\Omega) \cong \bar{g}^{H}(\Omega) + corr(\overline{dg}^{NT}(\Omega)) + corr(\bar{g}^{TC}(\Omega))$$

$$\bar{g}(\Omega) \cong \bar{g}^{H}(\Omega) - dg^{NT}(r_t(\Omega)) + \overline{dg}^{NT}(\Omega) - g^{TC}(r_t(\Omega)) + \bar{g}^{TC}(\Omega)$$

Martin et al, 2003 (poster)

This presentation

Comparisons

- Niethammer: (1932)

$$\bar{g}^{N}(\Omega) = \bar{g}^{H}(\Omega) - g^{TC}(r_t(\Omega)) + \bar{g}^{TC}(\Omega)$$

- Mader: (1950)

$$\bar{g}^{M}(\Omega) = \bar{g}^{H}(\Omega) - \frac{1}{2}(g^{TC}(r_t(\Omega)) - g^{TC}(r_g,\Omega))$$

- Our approach:

$$\bar{g}(\Omega) \cong \bar{g}^{H}(\Omega) - dg^{NT}(r_t(\Omega)) + \overline{dg}^{NT}(\Omega) - g^{TC}(r_t(\Omega)) + \bar{g}^{TC}(\Omega)$$

Concluding remarks

- Mean gravity generated by topography expressed in terms of potential ⇒ Solution smoother therefore can evaluated more accurately.
- Terrain effect reached 17 cm in higher peaks, between 0 and 2 cm in the plateau, in test area.
- Numerical evaluation similar to the one applied in the geoid computation ⇒ closer to the geoid.
- Comparison shows that terrain effect missing in Helmert's and that effect of gravity disturbance (and also term due to irregularities in density) also missing in Niethammer's and Mader's orthometric heights.
- Numerical comparison to be carried out using synthetic gravity field.