Analyzing the Dynamic Behavior of Suspension Bridge Towers Using GPS

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BIOGRAPHY

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Richard B. Langley is a professor in the Department of Geodesy and Geomatics Engineering at UNB, where he has been teaching and conducting research since 1981. He has a B.Sc. in applied physics from the University of Waterloo and a Ph.D. in experimental space science from York University, Toronto. Professor Langley has been active in the development of GPS error models since the early 1980s and is a contributing editor and columnist for GPS World magazine. He is a fellow of the ION.

ABSTRACT

The aim of this work is to characterize the dynamic oscillation of the top of the towers of a suspension bridge with GPS and to analyze the resulting values by Fourier analysis and wavelet transform. It is a complementary research about the analysis of the dynamic movements of the Pierre-Laporte Suspension Bridge in Quebec City, Canada. A previous work [Larocca et al., 2005b] analyzed the deck's movements of this bridge. Suspension bridge fundamentally consists of cables anchored to the earth at their ends and supported by towers at intermediate points. From these cables, a floor or 'deck' is suspended. Therefore, the towers have to be flexible enough to allow for changes in length due to live loads and temperature. Theoretically, the tower can be assumed as a thin beam. GPS data were collected at the towers of the bridge. The data sets were collected by researchers from the Centre de Recherché en Géomatique at Université Laval in July 1996. One GPS receiver was installed on the top of each of the towers, both 110 m in height, whereas a third receiver was placed on the ground, used as reference. Two 3-hour GPS sessions with a data-sampling interval of 2 seconds were collected. As no other sensors were used for measuring the deflections, the conclusions about the results are supported by theoretical values.

INTRODUCTION

Civil infrastructures serve as underpinnings of our present highly industrialized society. It is an important issue how to monitor these widely used infrastructures in order to prevent potential catastrophic events. Bridges are among the important civil infrastructures and are normally designed to have long life spans. Service loads, wind-forces, and accidental actions may cause damage to bridges. Continuous health monitoring is necessary so that early identification and localization of any potential unusual loading conditions or modified structural behavior, which can, in an extreme case, include damage or failure.

In the context of structure monitoring, many different types of excitation methods have been applied to bridge structures. The use of ambient vibration often provides means of evaluating the response of the structure to the actual vibration environment of interest. The responses refer to displacements, accelerations, frequencies of interest, strains and forces on the members of bridge structures, and displacements and stresses of main cables [Ren *et al.,* 2004; Ren and Pen, 2005].

GPS has been extensively used as a reliable tool for monitoring the dynamic behavior of engineering structures, as well as others instruments such as accelerometers and anemometers. In contrast with these instruments, GPS can measure directly the position coordinates, and nowadays relative displacements can be measured at rates of 100 Hz. Some GPS data collection methods are affected by the deficiency in the GPS satellite geometry [Barnes *et al.,* 2003] and complementary instruments, such as pseudolites, are needed to attend applications in surveying, geodesy and structural monitoring.

The Phase Residual Method (PRM) [Schaal and Larocca, 2001a, 2002], in particular, does not need very accurate a priori coordinates of the receivers to determine the amplitude and the frequency values of structures under movements. Because of this characteristic it becomes possible to choose the best place to install the base receiver(s). PRM only requires a proper satellite configuration; i.e., one satellite closely aligned to the direction of the antenna displacement movement and another satellite orthogonal to it. In order to detect a vertical movement, for example, it is necessary to have a satellite close to the zenith and the other close to the horizon. This requirement is not a limiting factor and the technique has been proved by several trials carried out since 2000. As examples, we can mention field tests carried out on environments with buildings and trees around GPS antennas [Schaal and Larocca, 2001b], trials carried out on a cable-stayed footbridge between two building [Larocca and Schaal, 2005], trials carried out on a cable-stayed bridge with reference stations lower than the deck level [Larocca, 2004]. In this study, the aim is to try to detect the tower movements of a suspension bridge.

METHODOLOGY

A very interesting way to detect non-stationary movements is using GPS signal phase variations. Most of the expected dynamic movements of the top of the towers occur in a horizontal plane with a main component aligned with the bridge deck. The phase signal of the lowest satellite detected by the receiver on the top of the tower aligned with the longitudinal deck direction will contain the most contributions due to phase variations antenna displacements. The movement can be extracted using the Phase Residual Method (PRM) in which a high elevation satellite is used as reference with another satellite in the direction of the movements being studied. Since the dynamic movements are not expected to remain unchanged Fourier and wavelets transforms were used for data analyses.

The PRM is based on the analysis of the L1 double difference phase residuals (DDPR) collected from a regular static observation session, under a particular satellite configuration and during a short time span. It is applicable over a short baseline. The residuals incorporate all phase deviations from the adjusted double difference position during the observation. These phase deviations are due to electronic receiver noise, multipath, small dynamic antenna movements and other error sources. Converting the residuals to the frequency domain, it is possible see the different behaviors of the receiver phase noise, multipath and periodic movements allowing the distinction between them. The periodic movement presents a peak due to the oscillation modes of the towers while the receiver noise presents a white noise spectrum and the multipath presents a broad spectrum close to zero frequency.

As no other sensors were used for measuring the deflections, the conclusions about the frequency oscillations detected from GPS data processing for the two towers are supported by theoretical values [Ko *et al.* 2001]. Therefore, the strategy for data analysis was based on the comparison of similar events from the DDPR of both towers by FFT spectra, revealing the occurrence of different frequencies; and by wavelets scalogram, indicating when the particular components occur within the signal [Ogaja *et al.,* 2001]. Finally, for complementing the investigation for the event, a comparison of the FFT spectrum and wavelets scalogram from the longitude coordinates value from each tower was carried out. (see also Ko *et al.* [2001]). The coordinate data values were calculated by researchers from the Université Laval's Centre for Research in Geomatics [Santerre and Lamoureux, 1997].

BRIDGE TRIAL

GPS data were collected on the Pierre-Laporte Suspension Bridge which spans the St. Lawrence River at Quebec City, Quebec, Canada. Opened in 1970, it has 6 lanes, with total length of 1040 m and a width of 27 m. The bridge deck is composed of two end spans of 187 m and one center span with a length of 667 m (Figure 1). Two steel towers, 110 m high support the deck (total weight of 18,000 tons) by vertically hanging suspenders with two main cables of 62 cm diameter (Figure 2) [Labbé, 1997].

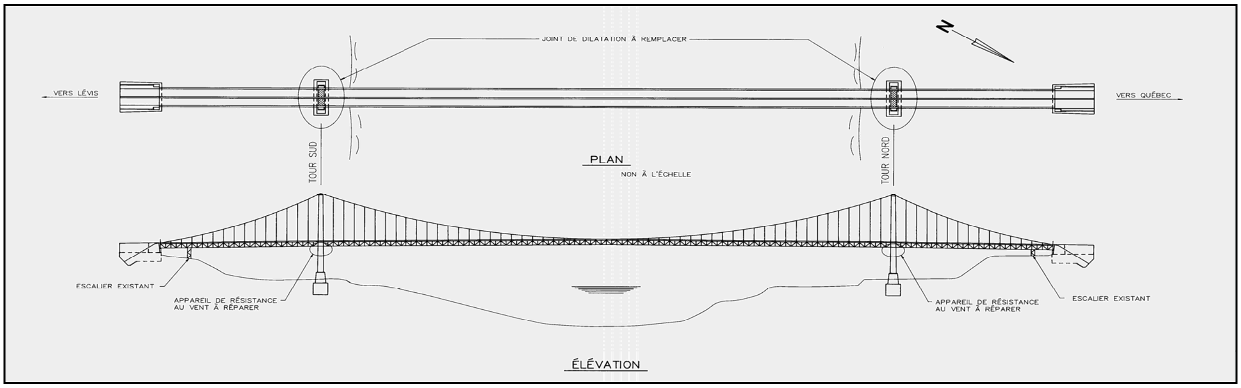

Figure 1. Plan layout and side elevation of Pierre-Laporte Suspension Bridge (Ministère des Transportes du Québec)

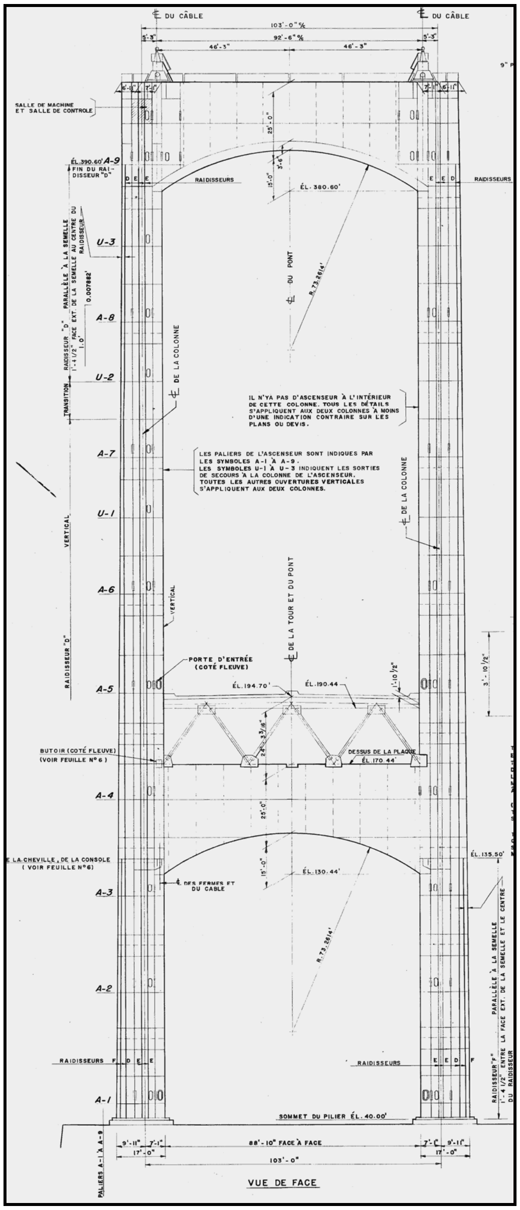

Figure 2. Front elevation layout of towers (Ministère des Transportes du Québec)

Three, 48-hour GPS sessions were conducted during the months of July and October 1996 and February 1997 by researchers from Université Laval's Centre for Research in Geomatics. For each session, 5 geodetic-quality GPS receivers were used, observing with a data sampling interval of 2 seconds. The layout of the stations composing the deformation monitoring network can be seen in Figure 3. It shows the stations of the deformation monitoring network. The baseline length (D) and the height difference (h) between the stations are given in Table 1. Two reference stations (RIN1 and RIN2) were set up on bedrock on the north river bank close to the bridge. Stations TON (north direction) and TOS (south direction) were located on the top of the north and south towers, respectively. Station TACE was located on the deck of the bridge. No other sensors were used for measuring the deflections [Santerre and Lamoureux, 1997].

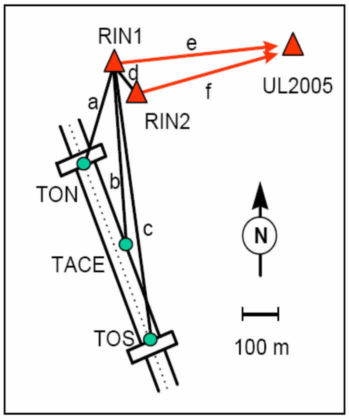

Figure 3. Instrument configuration of Pierre-Laporte Bridge [Santerre and Lamoureux, 1997]

Table 1. Approximate baseline lengths and station height differences. (See Fig. 3)

Baseline	**a**	**b**	**c**	**d**	**e**	**f**
D (km)	0.3	0.7	1.0	0.1	3.5	3.5
Δh (m)	59	-4	59	-2	17	19

[Santerre and Lamoureux, 1997]

GPS DATA ANALYSIS

Although the receivers were L1/L2 capable, only L1 data were used in the PRM analyses. Data were processed from two different observation sub-sessions. These sub-sessions were chosen according to the particular satellite geometry required for the PRM technique. The RIN1-TOS and RIN1-TON baseline were processed using the OMNI software from the U.S. National Geodetic Survey, which provides ASCII data files of the double-difference phase residuals.

The maximum frequency that can be detected by GPS with the data rate used in the experiment according to Nyquist Theorem is 0.25Hz [Brigham, 1974].

First session: RIN1-TOS GPS data analyzes

Initially, the DDPR of the RIN1–TOS baseline during the GPS session between 09h00min and 10h00min [on 17 July 1996] was calculated. The reference satellite (PRN17) was at 79 degrees elevation angle. The measuring satellite (PRN03) was at 13 degrees which was aligned with the front elevation tower leg directions because this side has more area exposed to wind forces. Figure 4 shows the DDPR of all satellites with respect to the reference one and it can be concluded that any information about frequency is not obvious to the unaided eye because of noise and multipath.

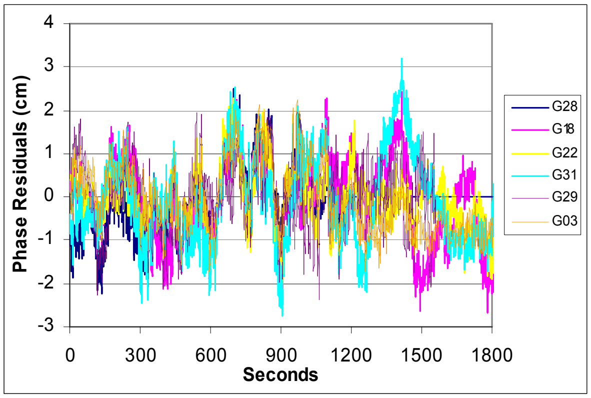

Figure 4. DDPR of the RIN1-TOS baseline

Otherwise, the apparent tower amplitude displacement values are constant during the data span processed ranging around 1.5 cm mainly due to wind effects. During the trials, the wind was not strong with wind speed around 20 km/h (from the east direction). Figure 5 presents only the DDPR of the lowest satellite (PRN03) where amplitude displacements values are clearer.

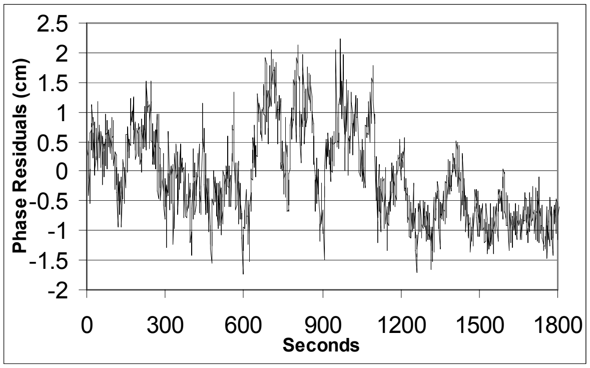

Figure 5. DDPR of the RIN1-TOS baseline from the PRN03

It is interesting to note that the deck's amplitude displacement values are not constant during the period mainly because of traffic on the bridge changing in type and quantity (normal/heavy traffic conditions) and the deck's length is 6 times bigger than tower height. This becomes more evident by looking at Figure 6 that illustrates the DDPR of the RIN1–TACE baseline. The station on the deck (TACE) provides a better visualization of the deflection amplitude caused by traffic load. The traffic-generated deflections range from 4 to 8 cm [Larocca and Schaal, 2005]. Please note the different ranges in the vertical axes of Figures 5 and 6.

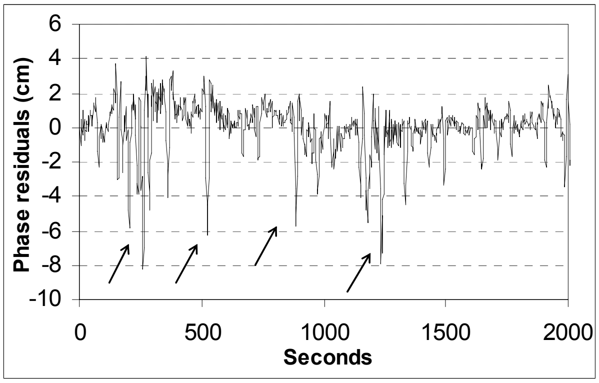

Figure 6. Detail of DDPR of RIN1-TACE baseline showing the sudden phase deviations caused on deck by traffic [indicated by arrows]

First Session: Time-Frequency Signal Representation

Figure 7 presents the corresponding spectrum of 1024 data values with a sampling interval of 2 seconds from the residual data (PRN03) shown in Figure 5. In the spectrum, it is possible to observe information about frequencies in the "critical" region that corresponds to frequencies below 0.1 Hz. This region is generally highly affected by multipath effects but in this case, subsequent analysis and theory permits us to conclude that the critical region also has low frequency longitudinal oscillations tower that range from 0 to 3.5 Hz caused by wind forces.

But this range depends on the shape of the tower and its frequency response. And because of that it is interesting for civil engineering study what happens in this region.

The spectrum in Figure 6 shows peaks that are due to multipath, some noise and also peaks due to tower oscillations. The peak around the 0.04 Hz value stands out because it is far from most of the peaks due to multipath.

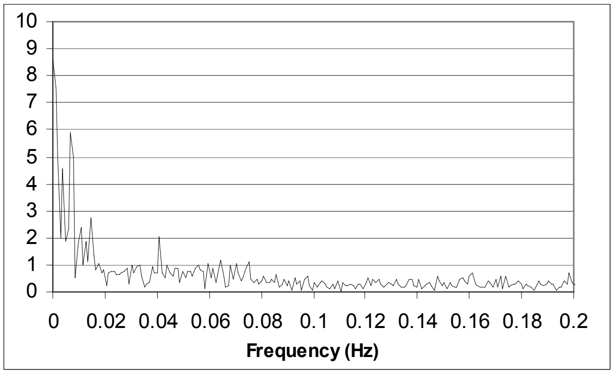

Figure 6. Frequency spectrum from the DDPR of PRN03

Continuing the investigation, the same steps were done for PRN18 which was at 45 degrees elevation and also aligned with the front of the elevation towers' leg directions. Its spectrum also shows an intensity peak around 0.04 Hz, as presented the Figure 7.

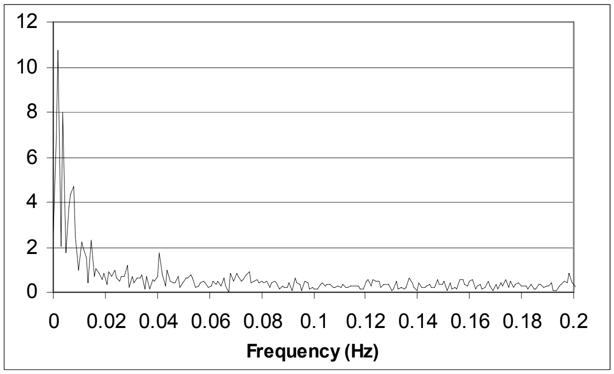

Figure 7. Frequency spectrum from the DDPR of PRN18

As this peak is the only one that occurs far from the critical region affected by multipath, it was chosen for analysis. It can be observed comparing Figures 6 and 7 the random noise in the spectrum of PRN18 has a smaller level than the spectrum of PRN03 which has a lower elevation angle than PRN18. As is well known, lower satellite signals are more affected by multipath than higher ones.

The inability of conventional Fourier analysis to preserve the time dependence and describe the evolutionary spectral characteristics of non-stationary processes requires tools which allow time and frequency localization. The spectral analysis of non-stationary signal cannot describe local transient features due to the averaging over the duration of the signal.

The essential difference between wavelet and Fourier analysis is that the wavelet basis function for any frequency band consists of a number of local functions strung together, each with its own amplitude, and can thus distinguish local events at different times at the same frequency. The wavelet coefficients in a particular band represent the energy at equally spaced time intervals over the duration of the signal. When the squared coefficients are plotted on a time-scale grid, the transfer of energy from one band to the next may be observed along the time axis. This is called the scalogram or mean square map. The volume bounded by the surface is the mean square value of the signal [Daubechies, 1990; Chui, 1992; Gurley and Kareem, 1999].

A program was developed in MATLAB 6.1 software for analyzing DDPR, testing the appropriate wavelet, and calculating the coefficients and frequencies. We chose CWT (continuous wavelet transform) which is a convenient and efficient method of monitoring the performance of time dependent dynamic systems when it is desired to differentiate between smaller frequency bands rather than the discrete wavelet [Newland, 1993; Gurley and Kareem, 1999].

Figure 8 below presents the CWT scalogram of the DDPR of PRN03, where the horizontal axis is time and the vertical axis is the scale of the frequency band. Note that frequency is inversely proportional to the scale value, thus high frequency is seen at the bottom of the scalogram at low scale values.

The unavoidable presence of noise in measured signals tends to reduce the clarity of scalograms, and in some cases may hide fine structure in coefficient amplitude variation. In this case it is possible to observe periodic occurrence of very low frequencies during the data time span from 600 to 1200 sec through the observation of the pockets of higher and lower levels of energy. The estimated longitudinal TOS frequency value indicated is 0.0389 Hz at scale 51 which we will round off to 0.039 Hz.

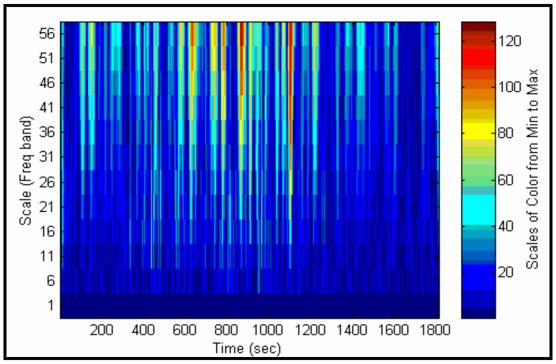

Figure 8. CWT scalogram from the DDPR of PRN03

The same procedure was carried out for DDPR of PRN18 which was at 45 degrees elevation. Figure 9 presents the CWT scalogram. In this case, it is also possible to observe more occurrences of the very low frequency oscillations from 600 to 1200 sec, the same region for PRN03 as stated before. For both analyses, no frequencies higher than 0.039 Hz could be identified.

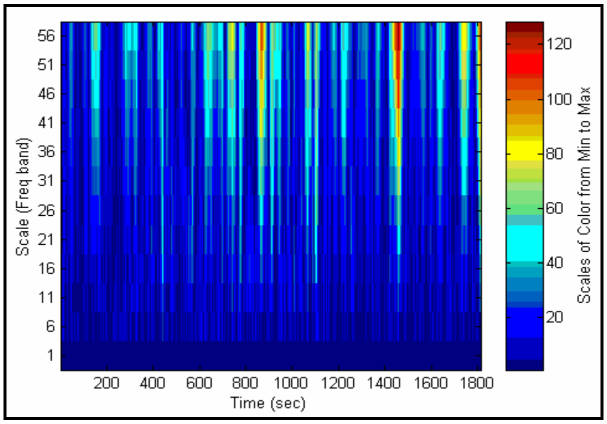

Figure 9. CWT scalogram from the DDPR of PRN18

Second Session: RIN1-TON GPS Data Analysis

After the analysis based on the GPS data of tower TOS, it was decided to process the data of tower TON to compare their dynamic behavior since both have the same structural layout.

We calculated the DDPR of the RIN1–TON baseline during the GPS sub-session between 09h00min and 10h00 min [on 17 July 1996]. The reference satellite (PRN17) was at 79 degrees elevation angle. The measuring satellite (PRN03) was at 13 degrees. Figure 10 presents the DDPR of the lowest satellite (PRN03). The amplitude value is close to 1.5 cm.

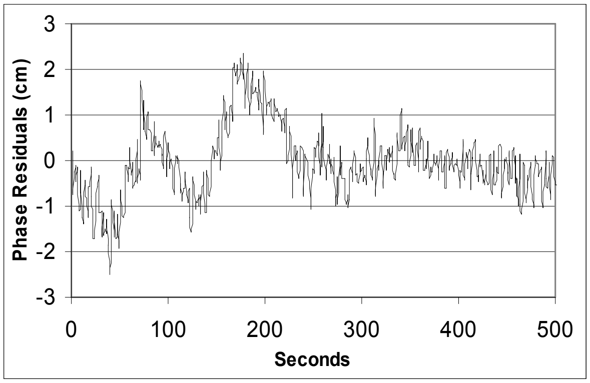

Figure 10. DDPR of the RIN1-TON baseline from the PRN03

Second Session: Time-Frequency Signal Representation

Figure 11 presents the DDPR spectrum of PRN03 with a peak at the frequency around 0.04 Hz. In addition, for comparison purposes the same steps were carried out for PRN22 that was at 49 degrees elevation. In its spectrum it also was observed an intensity peak around 0.04Hz as shown in Figure 12. This peak is in the region affect by multipath.

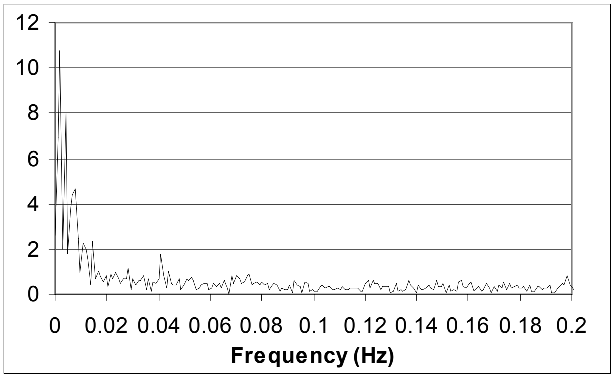

Figure 11. Frequency spectrum from the DDPR of PRN03

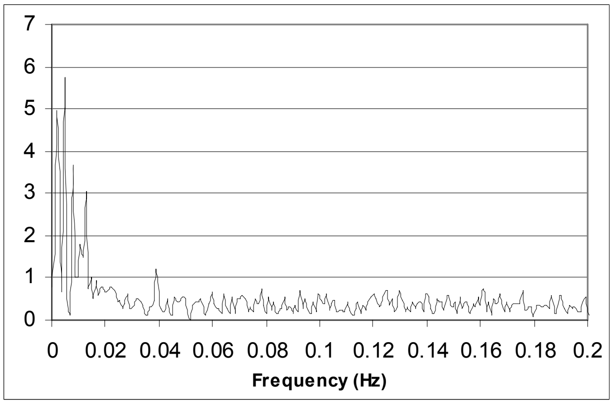

Figure 12. Frequency spectrum from the DDPR of PRN22

Figure 13 presents the CWT scalogram of the DDPR of PRN03, where the horizontal axis is time and the vertical axis is frequency scale. It is possible to observe periodic presence of pockets of higher levels of energy during different times. The estimated longitudinal TON frequency value indicated is 0.0389 Hz at scale 51.

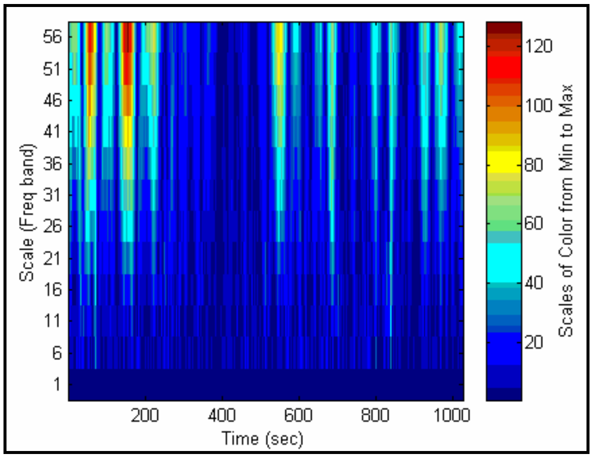

Figure 13. CWT scalogram from the DDPR of PRN03

A similar analysis was carried out for DDPR of PRN22 which was at 49 degrees elevation. Figure 14 presents the CWT scalogram. In this case it is also possible to observe more occurrences of very low frequencies from 600 to 1200 sec, the same region for PRN03 as expected because the towers must present the same dynamic behavior. In both analyses it was verified that there were no frequency values other than 0.039 Hz.

It is possible to observe that the time of low frequency occurrences is similar for the lowest satellite PRN03 and for PRN22 which was at 49 degrees elevation. This fact can be observed mainly around 200, 600 and 1000 sec with other pockets of energy occurring.

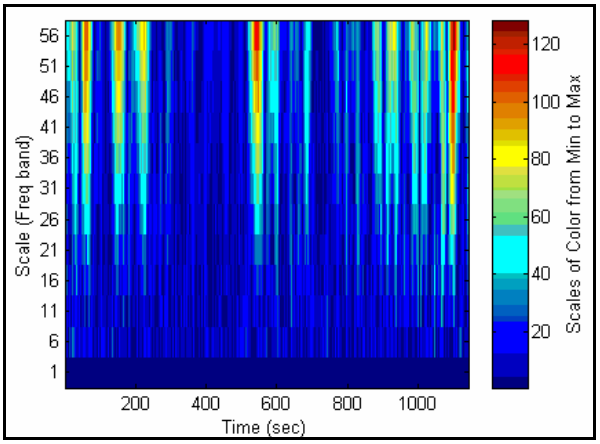

Figure 14. CWT scalogram from the DDPR of PRN22

COORDINATE ANALYSIS

To verify the results indicated by FFT and CWT analysis for the GPS data collected at the TON and TOS towers, a comparison between the FFT and wavelets results and that of the longitude coordinates values of both towers was carried out. The coordinate values were obtained by a different method, the Modified GPS-OTF Algorithm (being kindly supplied by the Université Laval's Centre for Research in Geomatics).

TOS Tower

The longitude coordinate values from the tower TOS are shown in Figure 15. Applying the FFT, the spectrum revealed the occurrence of a frequency close to the occurrence registered for TOS spectrum from DDPR, as shown in Figure 16. The other peaks at frequencies lower than 0.04 Hz, probably also due to tower oscillation, will not be investigated in this work because in the DDPR spectrum obtained by PRM they are quite degraded by the multipath effect.

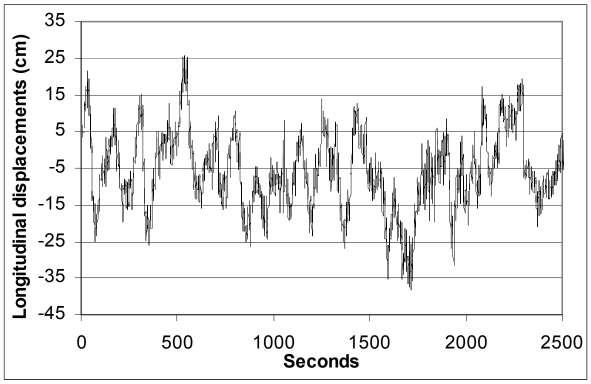

Figure 15. Longitude coordinate values of station TOS

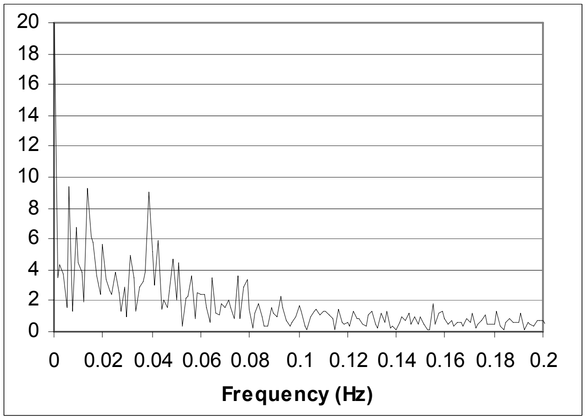

Figure 16. Frequency spectrum from the longitude coordinate values of station TOS obtained by Santerre and Lamoureux [1997]

Next, CWT was applied to the coordinate values. The resulting scalogram is presented in Figure 17. In this case it is possible to observe more occurrences of very low longitudinal frequency oscillations around 500, 1000 and 1500 sec coinciding with the region obtained for the PRN03 and PRN18 DDPR analyses and the 51 scale. The value for highest frequency agreed with 0.039 Hz.

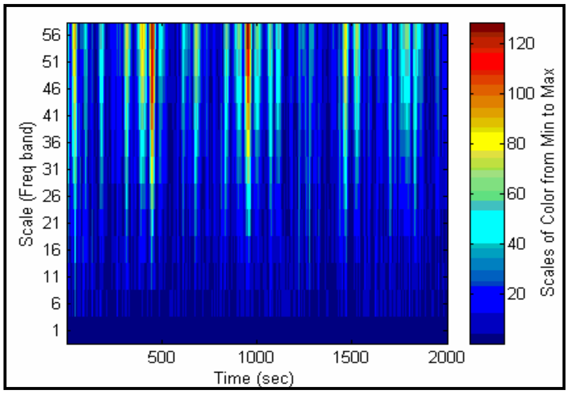

Figure 17. CWT scalogram from the longitude coordinate values of station TOS

TON Tower

The same procedure was applied to tower TON. Figure 18 presents its longitude coordinate values.

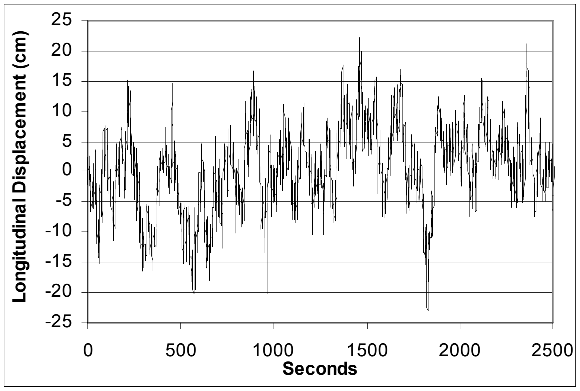

Figure 18. Longitude coordinates values of TON station

The FFT spectrum revealed the occurrence of a frequency close to the occurrence registered for TON spectrum from DDPR, around 0.04 Hz, as show Figure 19.

Figure 20 presents the scalogram. It is also possible to observe occurrences of low frequency oscillations mostly around 500 and 1000, coinciding with the region obtained for the PRN03 and PRN22 DDPR analyses.

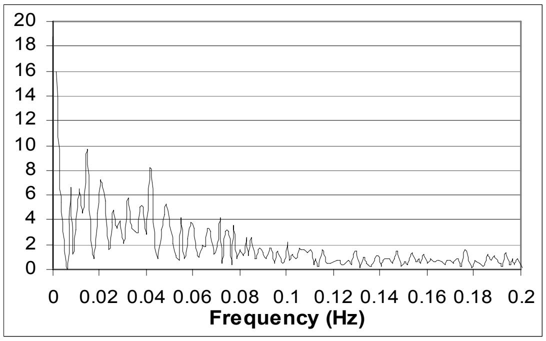

Figure 19. Frequency spectrum from the longitude coordinate values of station TON obtained by Santerre and Lamoureux [1997]

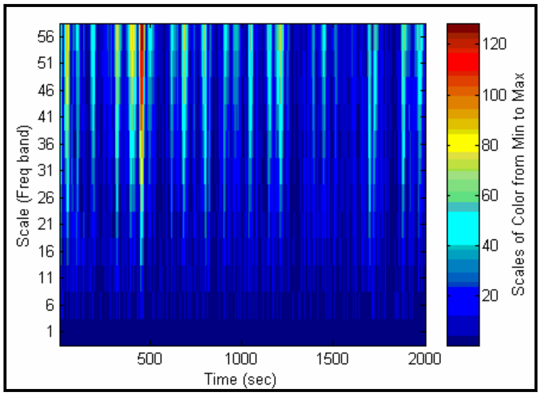

Figure 20. CWT scalogram from the longitude coordinate values of station TON

CONCLUDING REMARKS

The wavelet transform consists of a number of local functions strung together, each with its own amplitude, and can thus distinguish local events at different times at the same frequency; it can provide the detection of some very low frequency oscillations that must be analyzed even in the region highly affected by multipath, the frequency band below 0.08 Hertz. The upper frequency is limited by the sampling rate of the receiver. In our case, the data rate was 0.5 Hz.

Advances in sensor systems, measurement techniques, communications, information processing and computational technology have promoted the applications of health monitoring techniques to civil engineering structures.

The strategy chosen for the investigation of the GPS data collected at both towers (TON and TOS) used the fact that similar events would be observed from the double difference phase residuals obtained by the L1 Phase Residual Method, by the FFT spectrum (looking for revealing occurrences of different frequencies) and by wavelets scalogram (indicating when a particular component occurs within the signal). Good results were obtained. In the scalogram in the pockets of higher and lower levels of energy, a longitudinal frequency at 0.039 Hz was identified, due to oscillations caused by wind forces and/or by the motion of the deck as the bridge was open to normal traffic during the trial. Considering that we don't have access to a finite-element-model analysis of this bridge, we can not excluded either of these forces.

Finally, for complementing the investigation, a comparison of the FFT spectrum and wavelets scalogram of the longitude coordinate values from each of the towers was carried out. These values were obtained by other method, the Modified GPS-OTF Algorithm developed by Laval's Centre for Research in Geomatics. According to the results obtained it was verified that they agree very well in the determination of the low frequency longitudinal oscillation of the towers even though they use different algorithms and programs for data processing and satellite selection.

This fact confirms the potential of the PRM as a technique which permits us to determine the longitudinal frequency of the towers of the Pierre-Laporte Bridge without the use of any conventional sensor. Additionally, results obtained by PRM are not affected by the deficiencies of GPS satellite geometry in the northern sky quadrant. Results verify that GPS L1 double difference phase residuals obtained from only two satellites chosen according to the expected tower movement directions to be measured can be used providing sources of information for dynamic behavior of bridge towers.

Nevertheless, like any other developing technology, GPS positioning has its limits when it is applied to precise engineering needs and multipath is still one of the major degradation sources.

ACKNOWLEDGMENTS

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