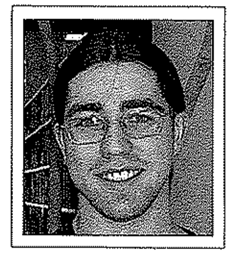
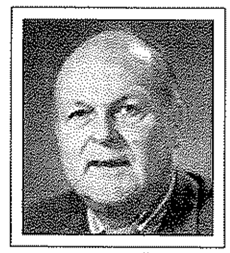
TOWARD AN IMPROVED ORTHOMETRIC HEIGHT SYSTEM FOR CANADA

Robert Kingdon, Petr Vaníček, Marcelo Santos, Artu Ellmann and Robert Tenzer
Department of Geodesy and Geomatics Engineering, University of New Brunswick

Heights in Canada are defined in the system of orthometric heights, according to a method proposed by Helmert in 1890. However, much development in the theory of heights has been done since then, leading to a more rigorous definition of orthometric heights. The new definition takes into account the effects of terrain roughness, laterally varying anomalous topographical density, and the NT geoid-generated gravity disturbance, which are not considered in the Helmert method. This paper presents a calculation of corrections to Helmert orthometric heights, to update them to the more rigorous definition. The corrections for each effect, as well as a total correction comprising all three effects, are evaluated for a Canadian test area which includes several types of terrain. The correction is found to reach decimetres in some mountainous areas.

Au Canada, les hauteurs sont définies dans le système des hauteurs orthométriques selon une méthode proposée par Helmert en 1890. Toutefois, beaucoup de travail théorique sur les hauteurs a été accompli depuis, conduisant à une définition plus rigoureuse des hauteurs orthométriques. La nouvelle définition tient compte des effets de la rugosité du terrain, de la densité topographique anomale variable latéralement et de la perturbation de la gravité du géoïde de la TN, qui ne sont pas considérés dans la méthode de Helmert. Cet article présente un calcul des corrections des hauteurs orthométriques de Helmert, les mettant à jour selon la définition plus rigoureuse. Les corrections pour chaque effet, ainsi que la correction totale comprenant les trois effets, sont évaluées pour une zone d'essai au Canada qui comprend plusieurs types de terrains. La correction peut atteindre quelques décimètres dans certaines zones montagneuses.


Robert Kingdon


Petr Vaníček


Marcelo Santos


Artu Ellmann


Robert Tenzer

Introduction

An understanding of heights is essential to the study of any field of geomatics, and the Canadian height system—called Helmert Orthometric Heights—is directly related to Earth's gravity field as well. This paper deals with the improvement of the Canadian height system, as a result of variations in its gravity field, and in particular, with the determination of corrections to Helmert orthometric heights to account for these variations. Other errors, such as those involved in levelling operations for height determination, or inaccuracies in definition of the height datum, are not considered.

Orthometric height, geometrically, is the length of a (curved) plumbline intersecting the geoid—an equipotential surface having the same potential as mean sea level—at a right angle, and the corresponding point on the surface of the earth. The heights in Canada were estimated by the 1928 adjustment of levelling networks, resulting in the Canadian Geodetic Vertical Datum 1928 (CGVD28). This readjustment used Helmert orthometric heights in its calculations, as did later incarnations such as the North American Vertical Datum 1988 (NAVD88) [*Zilkoski et al.* 1992].

Mathematically, orthometric height, $H^o(\Omega)$ of a point is defined as its geopotential number divided by the mean gravity along the plumbline between that point and the geoid. The geopotential number, defined as the difference between the potential on the geoid and the potential at a surface point, is determined from observed gravity and height differences between that point and mean sea level, realized at the surface of the earth.

The mean value of gravity along the plumbline, here shortened to *mean gravity*, is not so easily determined since the factors contributing to this value are difficult to quantify and require more complex mathematics for their evaluation. Traditionally, mean gravity has only been approximately calculated according to Helmert's method, which accounts for gravity generated by the ellipsoid, and approximates topography with a plate extending to infinity, known as a Bouguer plate. Since Helmert's method was introduced, however, the attempt to calculate mean gravity has undergone significant evolutions. As pointed out in *Tenzer et al.* [2005], those relevant are the inclusion of local terrain effects in the calculation e.g. *Niethammer* [1932] and *Mader* [1954], the introduction of the effect of lateral anomalous density variations [*Vaníček et al.* 1995; *Allister and Featherstone*, 2001] and an examination of the effect of the geoid-generated gravity disturbance [*Martin et al.* 2003]. This paper presents an evaluation of corrections to Helmert orthometric heights required to account for these effects. Corrections calculated in a Canadian context are presented here, and an assessment of their behaviour is carried out. The theoretical background is presented in *Santos et al.* [2005].

Corrections to Helmert Orthometric Heights

The Helmert method gives mean gravity along a plumbline as [*Heiskanen and Moritz*, 1967, Eq. 4-24]:

$$\bar{g}^{H}(\Omega) = g(r_t,\Omega) + 0.0424H(\Omega) \tag{1}$$

where $g(r_t,\Omega)$ is the value of gravity at the point at the surface of the earth with spherical coordinates Ω and geocentric radius r_t, and $H(\Omega)$ is the orthometric height of the surface at the same point. This in based upon application of the Poincaré-Prey reduction to surface gravity, assuming a linear gravity gradient along the plumbline. It takes into account both normal gravity and the effect of a Bouguer plate having a constant density of 2670 kg/m^3. However, as *Santos et al.* [2005] explain, these effects alone do not provide a sufficiently accurate calculation of mean gravity in all parts of Canada.

Mean gravity may be defined more rigorously as a sum of calculable effects. Those related to topography are shown in Figure 1, where the black hills and valleys represent the contribution of terrain roughness, and the columns of varying shades of grey represent lateral density variations, and their contribution. Vertical density variations and effects of atmospheric masses are not shown, since they are not considered herein.

In addition to the contributions shown in Figure 1, gravity includes contributions not generated by topography. These are the gravity generated by the reference ellipsoid being used, and that generated by masses within the geoid. The latter exist in the no-topography (NT) space, a space where the topography of earth has been removed, which is convenient for some calculations. It was introduced in *Vaníček et al.* [2004], which may be referred to for further explanation.

Mathematically, the total gravity may be written:

$$g(r_t,\Omega) \cong \gamma(r,\Omega) + \delta g^{NT}(r,\Omega) + g_B^T(r,\Omega) + g_R^T(r,\Omega) + g^{\delta\rho}(r,\Omega)\,, \tag{2}$$

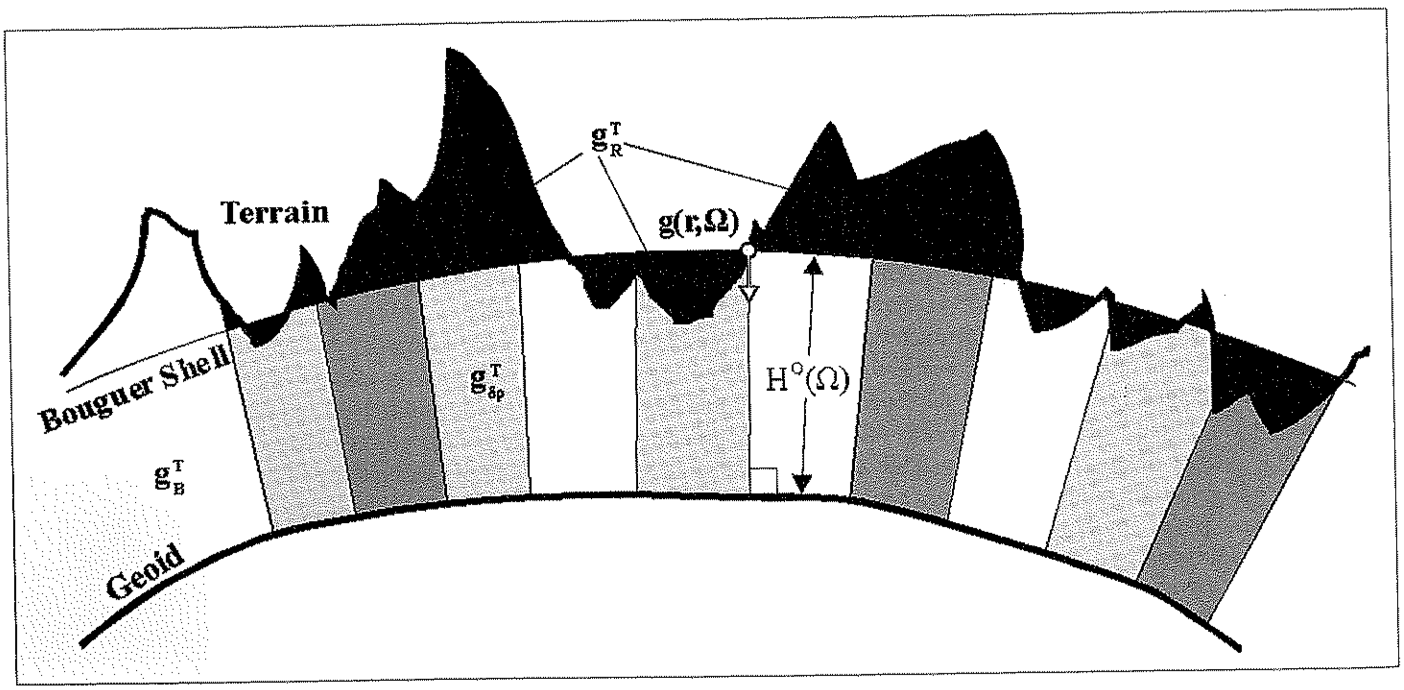


Figure 1: Components of rigorous gravity related to topography.

where $\gamma(r,\Omega)$ represents normal gravity (generated by mass within the geodetic reference ellipsoid, e.g. GRS-80 (e.g. *Moritz* [2000])); $\delta g^{NT}(r,\Omega)$ represents the geoid-generated gravity disturbance (generated by the anomalous mass within the geoid); $g_B^T(r,\Omega)$ represents the effect on gravity of mass within a Bouguer shell of thickness $H^o(\Omega)$, assuming all mass has constant average crustal density; $g_R^T(r,\Omega)$ represents the effect on gravity of terrain variations, or roughness, above and below the Bouguer shell (i.e. hills and valleys) of constant average density; and $g^{\delta\rho}(r,\Omega)$ represents the effect on gravity of lateral density variations from constant average crustal density, within the topography.

Note that effects of atmospheric masses and of radial density variations are neglected throughout this paper. The effect of atmospheric masses is negligible because it is very small [*Tenzer et al.* 2005]. The density of air is around 1.25 kg/m^3, which is less than 0.05% of average crustal density, resulting in a minimal effect of atmospheric masses on gravity. The effect of radial density variations has been neglected because it is difficult to quantify due to insufficient data on the radial distribution of density within the crust.

It follows from Eq. (2) that gravity at a point on the surface, $g(r_t,\Omega)$ is rigorously defined as,

$$g(r_t,\Omega) \cong \gamma(r_t,\Omega) + \delta g^{NT}(r_t,\Omega) + g_B^T(r_t,\Omega) + g_R^T(r_t,\Omega) + g^{\delta\rho}(r_t,\Omega), \quad (3)$$

and mean gravity, $\bar{g}(r,\Omega)$, given by the integral mean of $g(r,\Omega)$, as,

$$\bar{g}(r,\Omega) \cong \bar{\gamma}(\Omega) + \overline{\delta g}^{NT}(\Omega) + \bar{g}_B^T(\Omega) + \bar{g}_R^T(\Omega) \; \bar{g}^{\delta\rho}(\Omega); \quad (4)$$

where the bars over the terms represent integral mean quantities.

By substituting Eq. (3) into the expression for mean gravity according to Helmert's method, Eq. (1), for a constant crustal density of 2670 kg/m^3, we obtain the following expression for Helmert mean gravity:

$$\bar{g}^H(\Omega) \cong \gamma(r_t,\Omega) + \delta g^{NT}(r_t,\Omega) + g_B^T(r_t,\Omega) + g_R^T(r_t,\Omega) + g^{\delta\rho}(r_t,\Omega) + 0.0424H(\Omega). \quad (5)$$

The correction to Helmert's mean gravity is then given by finding the difference between Eq. (5) and the rigorous mean gravity given by Eq. (4). In this operation, the contributions of normal gravity and gravity generated by mass within the Bouguer shell, along with the last term on the right-hand side of Eq. (5), effectively cancel each other out [*Santos et al.* 2005]. Thus,

$$\begin{aligned} \varepsilon_{\bar{g}}(\Omega) &\cong \bar{g}(\Omega) - \bar{g}^H(\Omega) \\ &\cong \delta g^{NT}(\Omega) - \delta g^{NT}(r_t,\Omega) + \\ &\quad + \bar{g}_R^T(\Omega) - g_R^T(r_t,\Omega) + \\ &\quad + \bar{g}^{\delta\rho}(\Omega) - g^{\delta\rho}(r_t,\Omega) + \end{aligned} \quad (6)$$

and the only terms of the correction which must be calculated are those resulting from the mean and surface effects on gravity of terrain roughness, the laterally varying density distribution, and the geoid-generated gravity disturbance. Once these terms have been calculated, the corresponding correction to Helmert orthometric heights may be determined using (cf. *Heiskanen and Moritz* [1967], chap 4):

$$\varepsilon_{H^o}(\Omega) = -\frac{H^o(\Omega)}{\bar{g}(\Omega)}\varepsilon_{\bar{g}}. \quad (7)$$

Corrections thus calculated may then be applied to levelling benchmarks, or any orthometric heights defined in the Canadian height system.

The Test Area

Software has been written to calculate the corrections to Helmert orthometric heights for the three effects shown in Eq. (6), and the source code is available from the primary author upon request. These calculations were performed using a regular grid of points 5 feet apart, to provide suitable resolution for plots of the area, within a test area stretching from 49° to 54° in latitude, and 235° to 243° east in longitude. This area was chosen because its characteristics produce extreme results for each of the three effects calculated. Although it only represents a small segment of Canada's landmass, it contains both rugged and flat terrain, and seashore, so that the nature of the effects in many other areas may be judged from the results.

Heights in the test area range from 0 m to 3227 m, and laterally varying densities, based on surface geology, range from 2490 kg/m^3 to 2980 kg/m^3. Since the average crustal density is 2670 kg/m^3, assumed constant, this corresponds to laterally varying anomalous densities between -180 kg/m^3 and 310 kg/m^3. Geoid-generated gravity disturbances ranged from -266 mGal to -16 mGal. The distributions of height, anomalous density, and gravity anomalies are shown in Figures 2, 3 and 4.

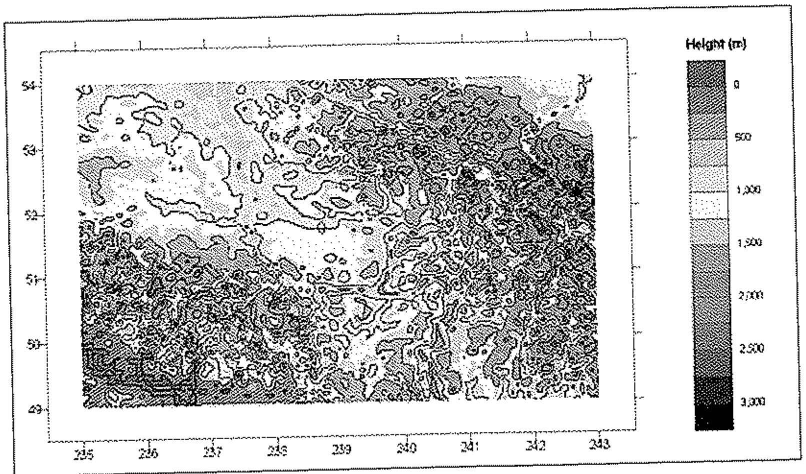


Figure 2: Orthometric heights within the test area, 500 m contours.

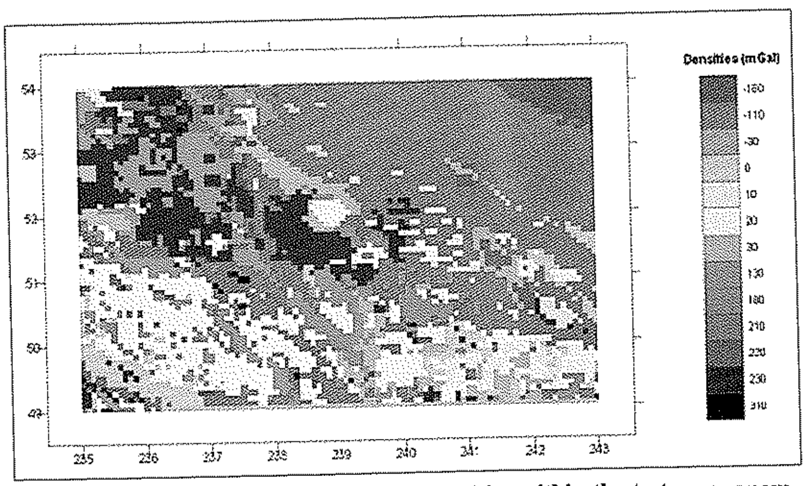


Figure 3: Laterally-varying anomalous densities within the test area, anomalous density indicated by tone. Data from the Geological Survey of Canada.

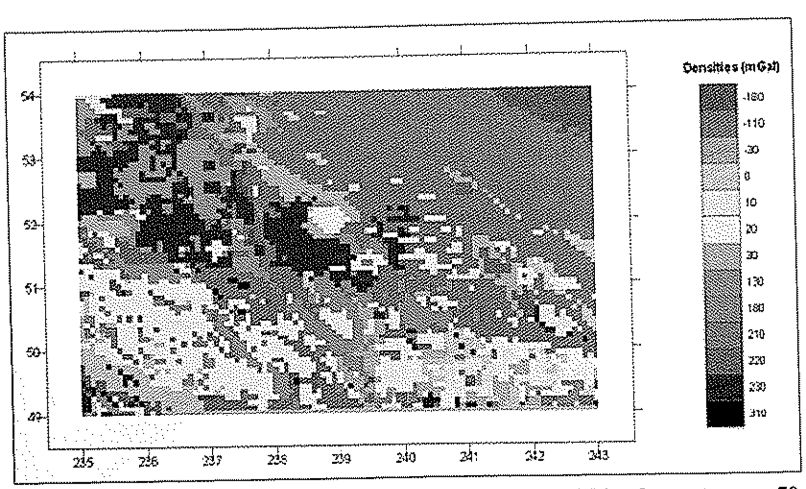


Figure 4: NT Geoid-generated gravity disturbances within the test area, 50 mGal contours.

The Effect of Terrain Roughness

The effect of terrain roughness, sometimes called just "the terrain effect" on gravity may be evaluated by integration of effects of surplus mass above the Bouguer shell, and mass deficit below the Bouguer shell, indicated by the black shaded area in Figure 1.

The terrain effect on mean gravity may be evaluated either as a simple average of the effect on gravity at the Earth's surface and on the geoid, according to the method of *Mader* [1954], or using an integral mean. If the integral mean approach is applied, it may be done using the *Niethammer* [1932] approach, in which the integral mean is approximated by calculating the effects on gravity at a series of points along the plumbline and averaging these results. Alternatively, according to a recent method developed at the University of New Brunswick (UNB), the contribution of terrain roughness to gravitational potential at the surface and on the geoid may be calculated, and divided by orthometric height, to determine an exact value of the effect on mean gravity (e.g. *Tenzer et al.* [2005], Eq. 21). Although results from all three methods are similar, the present study confirms that the UNB method is the most effective due to its speed and accuracy.

For the calculation of both effects, integration of mass surpluses and deficits is performed within a spherical cap of 3°, using the analytical solution for the radially integrated negative radial derivative of the Newton's integral kernel, with the assumption that any masses outside of this area may be neglected in this calculation. The integral kernel is [e.g. *Martinec* 1998]:

$$N\left(r, \psi\left[\Omega, \Omega'\right], r'\right) = \frac{r' \cos\left(\psi[\Omega,\Omega']\right) - r}{\ell^{3}(r, \psi[\Omega,\Omega'] r')}, \qquad (8)$$

where r is the geocentric radius of the computation point, r' is the geocentric radius of the integration point, $\psi[\Omega,\Omega']$ is the spherical angle between the computation point and the integration point, and $l(r,\psi[\Omega,\Omega'],r'$ is the distance between the computation and integration points. The assumption that the effect of terrain beyond 3° is negligible is based on test computations with varying sizes of spherical cap over a test area. DTM data in three integration zones provides a basis for numerical integration. In the innermost zone, a 15-minute by 15-minute square centred on the computation point, 3-second data is used. In the inner zone, a 200-minute square, 30-second data is used. Within the rest of the spherical cap, 5-minute data is used. These parameters were all determined, like the radius of the spherical cap, by variation of each parameter for a test area. Furthermore, interpolation is performed to divide the central 3-second cell into four 1.5-second cells to make the integration accurate enough. Integration is performed to determine both the effect on mean gravity, and the effect on surface gravity. Once these are found, they are subtracted according to Eq. (6) to determine a correction to Helmert's mean gravity for terrain roughness. Note that the total effect on Helmert's mean gravity would have the opposite sign of this correction.

Values of this correction within the test area ranged from 106 mGal to -25 mGal, while the corresponding corrections to Helmert orthometric height, calculated according to Eq. (9), ranged from 4.6 cm to -31.0 cm, and may be seen in Figures 5 and 6.

Effects related to roughness should be expected to peak where the terrain does, and this behaviour is seen in Figures 5 and 6. While it may appear at first glance as though the correction follows the height of terrain almost exactly, Figure 6 shows that this is not the case. For example, notice that the valleys that are distinctly defined in the topographic map of Figure 2 are not easily distinguished in the corrections given in Figures 5 and 6. This behaviour is expected, since the terrain effect on gravity is a function of roughness, rather than height of the computation point. Also, notice that the terrain in the middle of the map, having a more gradual slope, produces lower correction values, while the values peak sharply in rough terrain. While greater heights may amplify the correction, rough terrain is also necessary for it to be significant.

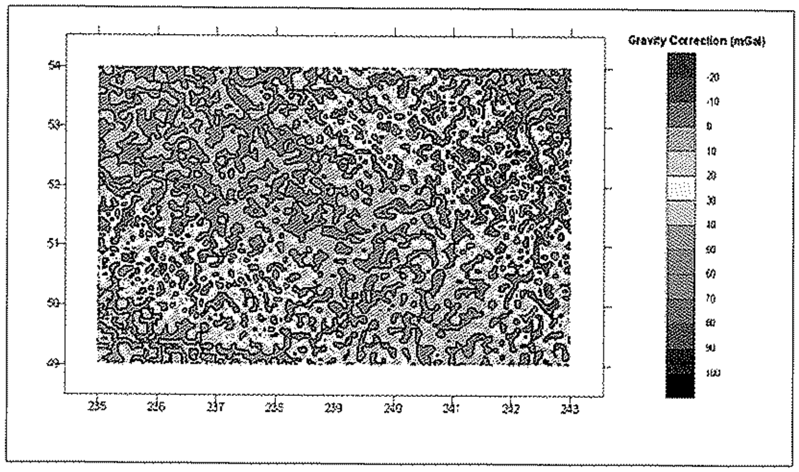


Figure 5: Correction to Helmert mean gravity for terrain roughness within the test area, 40 mGal contours.

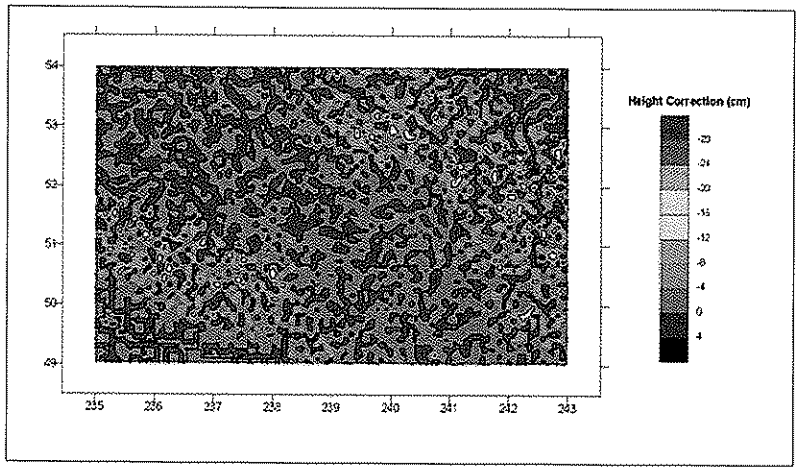


Figure 6: Correction to Helmert orthometric heights for terrain roughness within the test area, 10 cm contours.

The Effect of Laterally-varying Anomalous Density

The effects of the Bouguer shell and of terrain are both evaluated assuming constant topographical density, and thus do not include the effect of density variations. While sufficient data are not available to calculate the effect of radial density variations, the effect of lateral variations may be calculated, thus accounting to some extent for the influence of density variations.

The effect of lateral density variations on gravity is calculated first by obtaining a series of density values for polygons within the area, and then subtracting from these the average crustal density of 2670 kg/m^3. This results in a series of density differences for the polygons, which may be thought of as columns of positive or negative density anomalies, extending from the surface of the geoid to the Earth's surface, as indicated in Figure 1. For a more detailed description, see *Huang et al.* [2001].

To calculate exactly the effect of these anomalous density columns on gravity requires integration of the mass surpluses and deficits within these columns, using the same Newton kernel as for the terrain effect. Practically, however, the variations in height of these columns above and below the Bouguer shell—i.e. their influence on the terrain effect—may be neglected.

Integration was performed over two integration zones, the inner zone and the middle zone. Within the inner zone, which was 5-minute square, integration points were spaced 3-second apart, and density anomaly values were interpolated to these points. Within the middle zone, which was 10-minute square, integration points were spaced 30-second apart and were located in the centre of cells of the input density data. As with the terrain effect calcu-

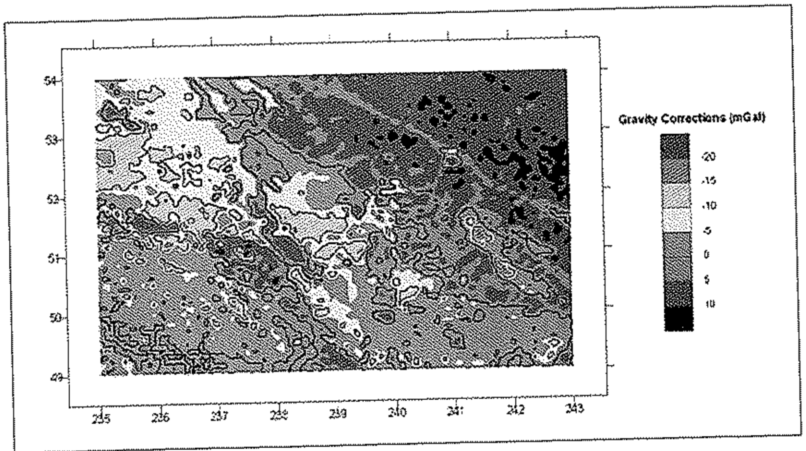


Figure 7: Correction to Helmert mean gravity for laterally-varying anomalous density within the test area, 10 mGal contours.

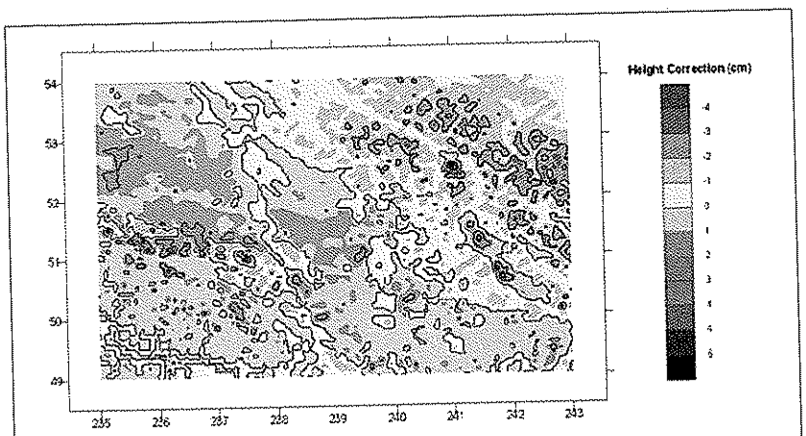


Figure 8: Correction to Helmert orthometric heights for laterally-varying anomalous density within the test area, 2 cm contours.

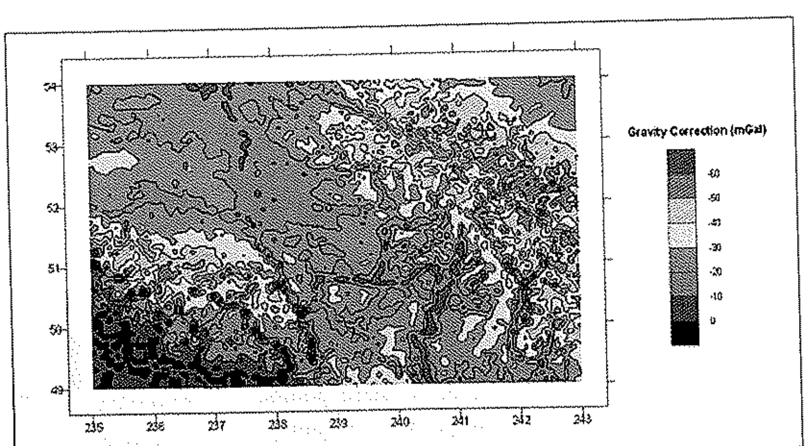


Figure 9: Correction to Helmert mean gravity for the geoid-generated gravity disturbance within the test area, 10 mGal contours.

lations, these values were determined by varying one parameter while holding the others fixed.

Calculation of the effect on mean gravity may be performed using the same three methods as for calculation of the terrain roughness effect. However, the difference in results using these three methods is less than half a centimetre within the test area. Still, the UNB method is preferred and was used for calculations, for the same reasons as with the terrain effect: speed and accuracy.

The correction to Helmert mean gravity for laterally-varying anomalous density over the test area varied from 14 mGal to -24 mGal. Corresponding corrections to height, computed in a regular grid, varied from 6.5 cm to -4.5 cm, with both corrections correlated to both density and height. These results are also similar to those presented in *Tenzer et al.* [2003], whose corrections, calculated for Canadian levelling points, ranged from 3.4 cm to -1.9 cm. Because the actual density in the test area was generally greater than average topographic density, the effect of the laterally-varying density distributions on height was normally positive. It is also noteworthy that the effect on mean gravity is very small, and might normally be negligible, the magnitude of the correction coming mainly from the effect on surface gravity. Results for the test area are shown in Figures 7 and 8.

Figures 7 and 8 show a tendency for both the mean gravity and height corrections to peak where the terrain does, as with the terrain roughness effect; though the tendency is not so pronounced here. This is to be expected, since while the effect of lateral density variations within a Bouguer shell is a function of height, the lateral density variations amount to no more than about 10% of the average crustal density, so that their influence on gravity is relatively small. Also, the magnitude of this effect is generally small, in part because anomalous densities are significantly smaller than average topographical density.

The Geoid-Generated Gravity Disturbance

The geoid-generated gravity disturbance represents the effect of the geoid's mass on gravity. The effect on gravity at the surface may be found by upward continuation of the disturbance, referred to the geoid's surface, to the Earth's surface. This is done using Poisson's integral [*Kellogg* 1929]. Radial integration, using this kernel, is performed over a spherical cap with radius of 7° from the computation point. Disturbances referred to the geoid may be found by removing the secondary indirect topographical effect from the NT gravity anomaly referred to the geoid [*Tenzer et al.* 2005]. Note, however, that this effect may also be calculated directly using downward continuation of gravity disturbances referred to the Earth's surface.

The upward continuation calculation used gravity disturbance data given in a regular grid with 5-minute spacing. Integration was done in two zones, with integration in a 10-minute square inner zone performed separately from that over the rest of the 7° spherical cap. In the inner zone, a planar approximation of distance was applied in evaluation of the Poisson kernel, and the 5-minute input data was interpolated to create a regular 30-second grid. The size of each zone was determined by varying its radius while holding that of the other zone constant. The 5-minute spacing simply reflects the available data, while the 30-second interpolation of data was determined by varying interpolation intervals until the effect on results from any finer interpolation was negligible.

The mean effect on gravity may either be approximated by upward continuation of the disturbance to a point halfway along the plumbline, or directly evaluated using an integral mean. In the latter case, a formula for the indefinite integral of the Poisson kernel has been provided by *Tenzer et al.* [2005], Eq. (9), which is employed, slightly adapted to use a planar approximation of spherical distance in the inner zone, in calculation of the mean geoid-generated gravity disturbance. Both techniques produced results no more than 5 mm apart, but the results from the integral mean approach were accepted as more accurate [*Martin et al.* 2003].

Results for the correction to Helmert mean gravity over the test area vary from -66.1 mGal to 8.9 mGal, and for the correction to Helmert orthometric heights from -0.1 cm to 21.0 cm. As expected, the differences between gravity disturbances on the geoid, and mean or surface gravity disturbances, vary according to the height of the computation point. It also follows that the magnitude of corrections is correlated with height of the computation point, though it is also correlated with the magnitude of the geoid-generated gravity disturbances. Distribution of the results is shown in Figures 9 and 10.

Total Corrections to Helmert Orthometric Height

These three corrections must be added together to determine a final correction to Helmert orthometric heights. The results for the final correction to mean gravity ranges from -0.3 mGal to 58.4 mGal.

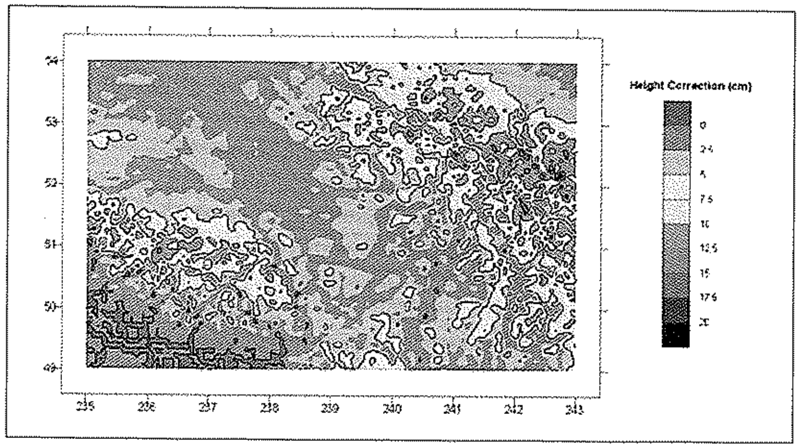


Figure 10: Correction to Helmert orthometric heights for the geoid-generated gravity disturbance within the test area, 5 cm contours.

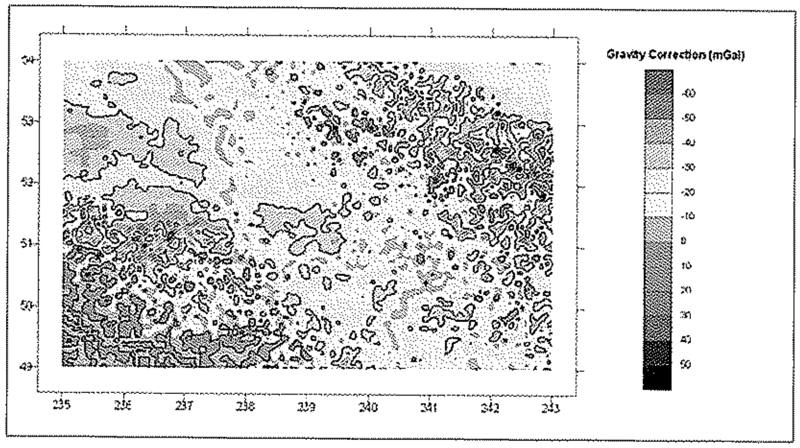


Figure 11: Total corrections to Helmert mean gravity for the spherical terrain effect, the effect of laterally-varying anomalous density, and the geoid-generated gravity disturbance, within the test area. 30 mGal contours.

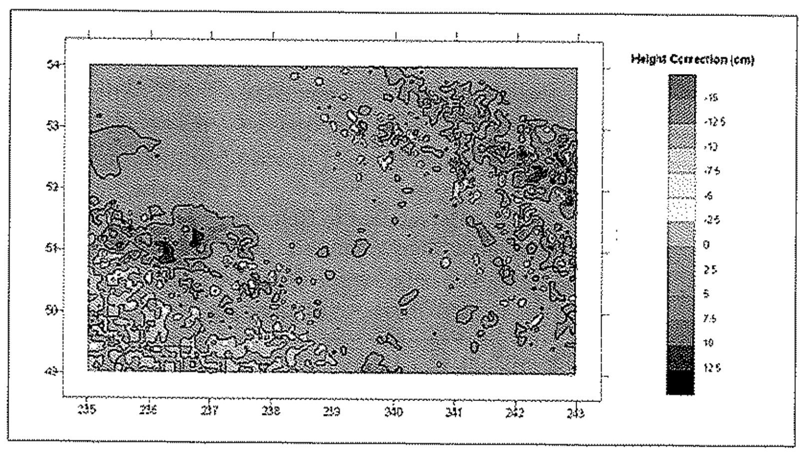


Figure 12: Total corrections to Helmert orthometric height for the spherical terrain effect, the effect of laterally-varying anomalous density, and the geoid-generated gravity disturbance within the test area. 5 cm contours.

These correspond to corrections to Helmert's orthometric height from -17.1 cm to 14.2 cm. In the final summation, the geoid-generated gravity disturbances and the terrain roughness often cancel each other within the test area, where the geoid-generated gravity disturbances are often negative. While the correction resulting from anomalous density occasionally worked in the opposite direction to the terrain effect, in the highest parts of the test area – where the influence of the density correction is amplified – it worked in the same direction as the terrain effect, making the overall correction larger. The corrections to gravity over the test area are given in Figure 11; and the corrections to height in Figure 12.

Figure 11 shows that the correction to gravity has a tendency to peak in areas of rough terrain, where the main factor in these corrections – the terrain roughness effect – is at its greatest, and where the influences of height on the other two corrections is also at a maximum. While these gravity corrections may be the most effective means of seeing which effects have the greatest contribution to the final height corrections, it is the height corrections themselves which this paper sets out to describe. They are generally small, with a tendency to peak in rough terrain at high elevations. In most low-lying areas, the total correction is under 5 cm; while in more rugged areas it reaches as high as 17 cm in magnitude.

A summary of all contributions and total effects is provided in Table 1, below.

Summary and Conclusions

Helmert's standard method for determining orthometric height is not adequate to determine heights having accuracy less than 1 cm in all areas of Canada. In a test area in the Rocky Mountains, corrections for three effects were calculated: the effect of terrain roughness, the effect of laterally-varying anomalous density, and the geoid-generated gravity disturbance. The total correction reached a maximum of decimetres in magnitude. If Canadian heights are to be considered rigorous, these effects must be taken into consideration—especially in mountainous areas, like the Rockies.

Acknowledgements

First, we would like to thank the Geodetic Survey of Division of Natural Resources Canada for supplying the topographic data used in these calculations. We would also like to thank the GEOIDE (GEOmatics for Informed DEcisions) Network of Centres of Excellence of Canada for their financial support of this research. Furthermore, we acknowledge NSERC, the National Science and Engineering Council of Canada, for their funding of our research.

References

Allister, N.A. and W.E. Featherstone. 2001. Estimation of Helmert Orthometric Heights Using Digital Barcode Levelling, Observed Gravity and Topographic Mass-density Data Over Part of the Darling Scarp, Western Australia. *Geomatics Research Australasia*, 75, pp. 25-52.

Heiskanen, W. and H. Mortiz. 1967. *Physical Geodesy*. Institute of Physical Geodesy, Technical University, Graz, Austria.

Helmert, F.R. 1890. Die *Schwerkraft im Hochgebirge, insbesondere in den Tyroler Alpen*. Veröff. Königl. Preuss. Geod. Inst., No. 1.

Huang, J., P. Vaníček, S. Pagiatakis and W. Brink. 2001. Effect of topographical mass density variation on gravity and the geoid in the Canadian Rocky mountains. *Journal of Geodesy* 74 (11-12), pp. 805-815.

Kellogg O.D. 1929. *Foundations of potential theory*. Springer. Berlin.

Mader, K. 1954. *Die orthometrische Schwerekorrektion des Präzisions-Nivellements in den Hohen Tauern*. Österreichische Zeitschrift für Vermessungswesen, Sonderheft 15.

Table 1: Corrections to Helmert orthometric height for the spherical terrain effect, the effect of laterally-varying anomalous density, and the geoid-generated gravity disturbance within the test area.

Correction	Minimum	Maximum	Mean
Terrain roughness	-31.0 cm	4.6 cm	-1.3 cm
Laterally-varying anomalous density	-4.5 cm	6.5 cm	-0.1 cm
Geoid-generated gravity disturbance	-0.1 cm	21.0 cm	4.0 cm
Total correction	-17.1 cm	14.2 cm	3.7 cm

Martin, B.-A., C. MacPhee, R. Tenzer, P. Vaníček and M. Santos. 2003. Mean geoid-generated gravity disturbance along plumbline. CGU annual meeting, Banff, May 10-14.

Martinec, Z. 1998. Boundary value problems for gravimetric determination of a precise geoid. Lecture notes in *Earth Sciences*, Vol. 73, Springer, Berlin.

Moritz, H. 2000. Geodetic Reference System 1980. *Journal of Geodesy*, 74(1), pp. 1432-1394.

Niethammer, T. 1932. *Nivellement und Schwere als Mittel zur Berechnung wahrer Meereshöhen*. Schweizerische Geodätische Kommission.

Santos, M., P. Vaníček, W.E. Featherstone, R. Kingdon, B.-A. Martin, M.Kuhn and R.Tenzer. 2005. Relation between the rigorous and Helmert's definitions of orthometric heights. *Journal of Geodesy* (in preparation)

Tenzer, R. and P. Vaníček. 2003. The correction to Helmert's orthometric height due to actual lateral variation of topographical density. *Revista Brasileira de Cartografia,* 55(2), pp. 44-47.

Tenzer, R , P. Vaníček, M. Santos, W. E. Featherstone, and M. Kuhn. 2005. The rigorous determination of orthometric heights. *Journal of Geodesy,* 79(1-3), pp. 82-92. Published on line DOI 10.007/s00190-005-0445-2.

Vaníček P., A. Kleusberg, Z. Martinec W. Sun, P. Ong, M. Najafi, P. Vajda, L. Harrie, P. Tomášе, B. Horst. 1995. Compilation of a precise regional geoid. Final report on research done for the Geodetic Survey Division. Fredericton.

Vaníček, P., R.Tenzer, L.E. Sjöberg, Z. Martinec and W.E. Featherstone. 2004. New views of the spherical Bouguer gravity anomaly. *Journal of Geophysics International* 159(2), pp. 460-472.

Zilkoski, D., J. Richards, and G. Young. 1992. Results of the General Adjustment of the North American Datum of 1988. Surveying and Land Information Systems, 52(3), pp. 133-149.

Authors

Robert Kingdon is a Research Assistant at the Department of Geodesy and Geomatics Engineering, University of New Brunswick, Canada. Has completed a BSc in Geodesy and Geomatics Engineering, and is now beginning his studies toward his MSc also at UNB. He has been heavily involved during the summers with UNB's work with rigorous orthometric heights, and in calculating corrections to Helmert's orthometric heights in particular.

Petr Vaníček, Professor Emeritus, Dipl.Ing.(CVUT, Prague); PhD(CSAV, Prague); DrSc(AVCR, Prague); PEng. Petr specializes in geodesy, the more mathematical, the better. He has authored and co-authored some 390 publications, including Geodesy: The Concepts, a textbook used all over the world. Petr joined the Department in 1971 and designed and has taught, at one time or another, all the undergraduate courses in geodesy and adjustment calculus. He retired at the end of the 1998/99 academic year.

Marcelo Santos is Associate Professor at the Department of Geodesy and Geomatics Engineering, University of New Brunswick, Canada. He holds an MSc in Geophysics from the National Observatory, in Rio de Janeiro, and a PhD in Geodesy from the University of New Brunswick. He has been involved in research in the fields of geodesy and navigation.

Artu Ellmann is a postdoctoral fellow at UNB. He holds an MSc (1993) in geodesy from the Moscow State University of Geodesy and Cartography (formerly MIIGAiK) and a PhD (geodesy) from the Royal Institute of Technology (KTH) in Stockholm. He moved to Canada recently after accomplishing his PhD studies in Sweden in 2004. His research interests are in the field of physical geodesy, i.e. geoid modelling and national geodetic networks.

Robert Tenzer, PhD, is currently working at the University of Newcastle upon Tyne as a Research Associate. He was previously employed by the University of New Brunswick, where he was involved in research on geoid determination, datum transformation, and accuracy estimation of geopotential models. ❑