### Generation of a NAD27-NAD83(CSRS) NTv2-type Grid Shift File for New Brunswick

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# Chapter 1 Introduction

This report describes the generation of a NTv2-type grid shift file relating NAD27 and NAD83(CSRS). It contains a brief explanation of why such grid shift file was necessary, the method of least-squares collocation used for the generation of the grid, efforts in quality control and assessment of the quality of the grid.

The Province of New Brunswick faced a tremendous task back in the early 1990's when it decided to adopt the North American Datum of 1983, as used in Canada, NAD83(CSRS)<sup>1</sup>, replacing the Average Terrestrial System of 1977, ATS77 [Hamilton and Doig, 1993; Gillis et al., 2000]. At that time, a decision was made to provide the user community with a computational system to facilitate the conversion of coordinates between both systems. An additional demand appeared, namely, to include the North American Datum of 1927, NAD27, in the computational system in order to satisfy particular applications. This computational system, relating NAD27, NAD83 and ATS77 was christened NB Geocalc. NBGeocalc performs the transformation among those three reference frames handling coordinates of various types, being them geodetic, UTM and Stereographic Double Projection coordinates.

The transformation of coordinates involving the three reference frames handled by NB Geocalc was carried out in different ways. The transformation between NAD27 and ATS77 was made by a routine called "transform," which was based on a residual file, with a nominal precision of  $\pm 5$  cm, 1-sigma. The transformation between ATS77 and NAD83(CSRS) was based on a NTv2 grid (explained below). The transformation between NAD27 and NAD27 and NAD83(CSRS) was made in a two-step solution, from NAD27 to ATS77 and from ATS77 to NAD83(CSRS).

The transformation between ATS77 and NAD83(CSRS) was made following a modelling procedure first described in *Blais* [1979], and latter called the National Transformation (NT). New Brunswick adopted the second version of the modelling, therefore referred to as the National Transformation version 2 (NTv2). The outcome of the modelling was represented in a standardized grid referred to simply as the NTv2 grid. In this way, there is the NTv2 modelling and the NTv2 grid. The grid can be used to represent any other

<sup>&</sup>lt;sup>1</sup>CSRS stands for Canadian Spatial Reference System

modelling approach, such as in the case of the modelling used in this report. As the grid represents the "shift" undergone by the coordinates and is represented as an electronic file, it is sometimes referred to as the "grid shift file."

New Brunswick pioneered the use of the NTv2 grid, setting the stage for it to become a national and international standard, being used not only by other Canadian Provinces but also by several countries, such as the US, Australia and Brazil.

NB Geocalc was made first as a desktop application available to users by several media and latter on as an internet application. NB Geocalc was written by OPTEX, a Frederictonbased company. The original development of NB Geocalc focused on 32-bit computers, which are in a process of becoming obsolete. There are a few incompatibilites that prevent NB Geocalc from running in 64-bit computers. Service New Brunswick is in a process to migrate the whole processing to a web-only application. It has been the desire of Service New Brunswick to have the transformation between NAD27 and NAD83(CSRS) made directly using a NTv2 grid. These two necessities have led to the work described in this report: to model the coordinate transformation between NAD27 and NAD83(CSRS) and represent them in a NTv2 grid shift file that can be used in the new application under development by Service New Brunswick.

The transformation used to model the coordinate transformation between NAD27 and NAD83(CSRS) was based on Least-Squares Collocation and is described in Chapter 2. A description of the data sets used, points that were not included in the final solution and why, and a presentation of the results is found in Chapter 3. Chapter 4 is dedicated to the quality control of the results. An Appendix reviews the structure of the NTv2 grid.

### Chapter 2

## A synopsis of the least-squares collocation procedure

Least-Squares Collocation is an estimation technique that uses a full covariance matrix, which can be built using empirical information derived directly from the input data. It also allows the prediction of parameters (called the "signal") outside the input data. Besides the parameters it allows also prediction of error estimates. This estimation procedure fit well to the problem of building regular grids based on irregular input data, predicting not only the parameters but also the precision at the nodal points. It found its initial applications in Geodesy in the estimation of gravity anomalies [Moritz, 1980].

We have developed a collocation-based approach for modelling the distortions of two geodetic networks, capable of building shift grids of any type, including NTv2. Our model was first applied to modelling the distortions of the Brazilian networks under the context of the National Geospatial Framework Project, funded by the Canadian International Development Agency. The same model has been applied for modelling NAD27 into NAD83(CSRS).

The following Section presents a point-to-point summary of the method. A paper with a discussion of the method has been submitted to the Journal of Geodesy [Nievinski et al., 2012] and broader description will be part of the Department of Geodesy and Geomatics Engineering Technical Report series [Nievinski et al., 2012].

#### 2.1 Least-Squares Collocation procedure

- 1. Outlier detection
  - (a) Run adjustment for similarity and spline using diagonal observations covariance matrix
  - (b) Calculate robust standard deviation
  - (c) Plot residuals as histogram or geographical map
  - (d) Inspect plots visually and confirm or not suspected outliers

- (e) List code of outlier points (no need for exclusion, just flagging)
- 2. Covariance modeling
  - (a) Run adjustment for similarity and spline using diagonal observations covariance matrix; retain outliers but enlarge respective observation a priori variance to restrain otherwise undue influence.
  - (b) Obtain empirical covariances
    - i. Form  $n^2$  inter-point distances between n points
    - ii. Form pair-wise covariances
    - iii. Bin covariances by similar distance (irrespective of position, i.e., assuming stationarity)
  - (c) Fit covariance model
    - i. Choose approximate values for covariance model parameters
    - ii. Define objective function to be minimized: modelled minus empirical covariance at each of the binned distances
    - iii. Run non-linear least squares on objective function:
      - A. Obtain parameters design matrix (numerically, via finite differencing using objective function)
      - B. Solve linearized least squares
      - C. If converged, stop.
      - D. Update parameters as per line search criteria, then iterate.

#### 3. Estimation

- (a) Run adjustment for similarity and spline using full observations covariance matrix; retain outliers but enlarge respective observation a priori variance so that they don't exert undue influence; also add signal covariance to a priori observations covariance matrix
  - i. Obtain signal covariance matrix, evaluating model covariance function elementwise on inter-point distance matrix
  - ii. Obtain noise covariance matrix, given nominal observation noise variance and outlier flags
  - iii. Obtain parameters a priori covariance matrix (essential to constrain otherwise ill-determined spline pieces containing no points)
  - iv. Obtain parameters design matrix (analytical manual derivations)
  - v. Solve linear least squares (via Cholesky on normal equations)
- 4. Prediction
  - (a) On test points and on grid nodes

#### Collocation

- (b) Use estimation results (namely, estimates parameters and their a posteriori covariance matrix, as well as pre-whitened residuals)
- (c) Apply prediction equations
- 5. Compare; test

### Chapter 3

### Data sets and results

#### 3.1 Description of the data sets

As many as 23,605 horizontal points were provided by Service New Brunswick, in both NAD27 and NAD83(CSRS). This was the same data set that was used to create the ATS77 to NAD83(CSRS), which should minimise the need to look for outliers.

The area of interest was defined as having the upper right limited by latitude  $48^{0}30'$  and longitude  $63^{0}$  and the lower left limit by latitude  $44^{0}$  and longitude  $69^{0}30'$ .

A look at Figure 3.1 shows that the data points are irregularly distributed, being most of them located at the most densely populated areas along the borders of the province, with a few east-west lines, along roads, e.g., one along the Miramich River and another going through Mount Carleton Provincial Park. The middle region is mostly wooded and mountainous areas. The distribution of points come as something positive for the collocation procedure as it surrounds the whole area of interest and the few east-west lines serve as control. The only two periferal areas that are not covered by data points are the in north-west along the border with the Province of Quebec, and the strip of land that protudes towards the west into the State of Maine. Whatever approximation procedure is applied, these areas will be the ones prone to larger approximation errors.

#### **3.2** Results

To arrive at the final solution two runs of the collocation software were made. The first run allowed to identify a few problematic areas and led to the elimination of one point. The second run led to the final solution. The results of both runs are presented in the following sub-sections.

The collocation method used allows for a detection of outliers following a pre-defined threshold applied to values resulting from tests. Two tests were performed, one on the computation points and another on test points. The test on computation points represents nothing else but an assessment of the residuals of the least-squares collocation. The test using testing points is known as k-fold cross-validation, which involves chosing a number of



Figure 3.1: Input network and bounding box

tests point at time, removing them from the total data set, and comparing the predicted value of the tests point estimated by the least-squares collocation with their known values. Even though expensive from the computational point-of-view because of the large number of times the process canb e repeated, the k-fold cross-validation is a trustworthy technique for assessing the quality of the estimation process because the test points are independent from the data set. In our case, due to the huge amount of points situated very close to each other, we applied the k-fold cross validation only once.

Figure 3.2 shows the test points overlain on the data points. They were chosen arbitrarily in such a way to offer an homogeneous coverage of the whole area. A total of 3900 test points were used in the k-fold cross-validation.

#### 3.2.1 First run

The first statistics of the solution is summarized by Table 3.1, involving all observation points and Table 3.2, involving only the test points. These tables show the values of the mean, the median (i.e., the middle value in the list), the standard deviation (computed with respect to the mean), the root mean square (rms) (computed assuming the mean to be equal to zero), the robust standard deviation (robust to outliers), the robust standard deviation at confidence levels equal to 90%, 95% and 99% (i.e., scaled up by 1.65, 1.95 and 2.58 expansion factors) and the maximum range value.



Figure 3.2: Distribution of test points (in red)

The statistics from the test points are predominantly larger than those involving all observation points, a natural consequence of the smaller number of test points used.

In Table 3.1 both the mean and the median are equal to zero. Since the mean is equal to zero, both standard deviation and rms are the same and a little less than 3 cm for the north component and just 3 mm for the east component. The robust standard deviation, being not affected by outliers, is just 1 mm, remaining less than 5 mm even after being scalled by up to 99%. The maximum ranges in latitude and in longitude are, respectively, 4.057 m and 0.266 m.

Looking now at Table 3.2 the mean and the median are slightly different even though by just a few millimetres (the mean is -8 mm and -2 mm for the north and east compenents, respectively, whereas the median is 1 mm for both). As the mean is very small, the standard deviation and rms are about the same and equal to 0.22 m and 0.07 m for latitude and longitude, respectively. The robust standard deviation for latitude is the same as the standard deviation and the rms, but for longitude it is smaller by 50%. The scaled robust standard deviation gets enlarged, in the latitude and longitude, respectively, to 0.041 m and 0.048 m (at 90% confidence level), to 0.064 m and 0.078 m (at 95% confidence level) and to 0.164 and 0.158 m (at 99% confidence level). The maximum ranges in latitude and in longitude are, respectively, 6.122 m and 0.973 m.

Table 3.3 lists all outlier points greater then one metre. Particular attention should be cast on point 119948. This is the biggest outlier equal to 140.069 m. Looking at Figure 3.3

	N(m)	E(m)
mean	0.0	0.0
standard deviation	0.028	0.003
rms	0.028	0.003
median	0.000	0.000
robust st. dev.	0.001	0.001
90%	0.001	0.001
95%	0.002	0.002
99%	0.004	0.004
Maximum range	4.057	0.266

Table 3.1: Statistics of test on observation points, first run

Table 3.2: Statistics of test on test points, first run

	N(m)	E(m)
mean	-0.008	-0.002
standard deviation	0.223	0.066
rms	0.224	0.066
median	0.001	-0.001
robust st. dev.	0.023	0.030
90%	0.041	0.048
95%	0.064	0.078
99%	0.164	0.158
Maximum range	6.122	0.973

one can see a huge outlier in northeast area of the province (near Bathurst), where point 119948 is located.

#### 3.2.2 Second run

SNB decided to have point 119948 removed from the input data considering that keeping it would hurt the integrity of the transformation. After verification in the comparison of the published value for this point in ATS77 and NAD27 it was obvious the NAD27 value was erroneous, thus the need to have it removed so it would not affect its neigbouring points. The second solution can be summarized by Tables 3.4 and 3.5. The removal of point 119948 resulted in a decrease in standard deviation, rms and maximum values, the latter now smaller than 1 m. Figure 3.4 shows that the large outlier near Bathurst is gone, and that smaller ones become visible.

The statistics coming from the second run are better than those of the first run. The removal of point 119948 proved to be worthwhile. In Table 3.4 both the mean and the median are equal to zero. Since the mean is equal to zero, both standard deviation and rms are the same and equal to 2 mm. The robust standard deviation, being not affected



Figure 3.3: Outliers, first run

104285	121945	104841	114301
122120	122115	104752	116937
104743	121944	104754	114330
104284	104282	114310	114331
104286	121980	121940	125261
122119	121947	104703	114090
104283	104753	121697	123561
122099	104288	107151	100434
121946	122114	104730	114332
122116	104281	104742	109124
122136	104728	122092	101056
104287	121941	119948	114309
122118	121979	114174	

Table 3.3: List of points larger then 1 metre

Table 3.4: Statistics of test on observation points, second run

	N (m)	E (m)
mean	0.0	0.0
standard deviation	0.002	0.002
rms	0.002	0.002
median	0.000	0.000
robust st. dev.	0.001	0.001
90%	0.001	0.001
95%	0.002	0.002
99%	0.004	0.004
Maximum range	0.110	0.114

by outliers, is just 1 mm, remaining less than 5 mm even after being scalled by up to 99% confidence level. The maximum ranges in latitude and in longitude are, respectively, 0.110 m and 0.114 m. Looking now at Table 3.5 the mean and the median are at the millimetre level (varying between 2 mm and -1 mm). As the mean is very small, the standard deviation and rms are about the same and equal to 6 cm for both components. The robust standard deviation for latitude is 2 cm and for longitude is equal to 2.8 cm. The scaled robust standard deviation gets enlarged, in the latitude and longitude, respectively, to 0.038 m and 0.047 m (at 90% confidence level), to 0.058 m and 0.076 m (at 95% confidence level) and to 0.149 and 0.160 m (at 99% confidence level). The maximum ranges in latitude and in longitude are, respectively, 0.983 m and 0.970 m.

The statistics of the tests on the results generated after the second run show an improvement with respect to the first run. If we take the worst case scenario, given by the test on the test points, the uncertainty (standard deviation) is at the 6 cm level, at 1-sigma (67%) and at the 12 cm level at a 95% confidence level (scaled by 1.96).



Figure 3.4: Ouliers, second run

N(m)	E (m)
0.002	-0.001
0.057	0.063
0.057	0.063
0.001	-0.001
0.020	0.028
0.038	0.047
0.058	0.076
0.149	0.160
0.983	0.970
	$\begin{array}{c} {\rm N}\ {\rm (m)}\\ 0.002\\ 0.057\\ 0.057\\ 0.001\\ 0.020\\ 0.038\\ 0.058\\ 0.149\\ 0.983 \end{array}$

Table 3.5: Statistics of test on test points, second run

#### 3.2.3 Final grids

From the second run, the grids in latitude, longitude and horizontal resultant were generated following the NTv2 grid shift file format. Figures 3.5, 3.6 and 3.7 show the shifts in latitude, longitude and the resultante horizontal shift. We can see that the shift values in latitude grows from 2 m in the north to 10 m in the south; the shifts in longitude goes from -55 m in the west down to less than -70 m in the east. The total horizontal shift follows the shape of the shifts in longitude, whereas the sign is just a computation artifact. The grid spacing is 30 seconds.

A process of quality control was applied and it is described in the next chapter.

#### 3.2.4 Uncertainty grids

One of the nice features of the least-squares collocation is that it provides a full covariance matrix of the shift parameters. The estimated standard deviation of the shifts, in latitude and longitude, is represented by an uncertainty field, which is part of the NTv2 grid shift file. Figure 3.8 shows the the uncertainty grid in latitude and Figure 3.9 shows the uncertainty grid in longitude. A look at both figures allows us to see that where there is control (in the regions where the data points are located) the uncertainty associated with the NTv2 is at the order of 20 cm or less. The areas surrounded by the distribution of the data points have a slightly higher uncertainty going up to 40 cm. Inside New Brunwick, the area with the largest uncertainty values is located in the uninhabited area in the north-west portion of the province, in the border with Quebec, where uncertainty values reach higher than half a meter. Figure 3.10 shows similar features.



Figure 3.5: Shift in latitude, in metre



Figure 3.6: Shift in longitude, in metre



Figure 3.7: Resulting horizontal shift



Figure 3.8: Latitude uncertainty, in metre



Figure 3.9: Longitude uncertainty, in metre



Figure 3.10: Resulting uncertainty, in metre

### Chapter 4

### Quality control

#### 4.1 Comparisons with known coordinates

Quality control was performed by comparing the NAD83(CSRS) coordinates computed using the final grid (computed by Collocation) with their known, published values. This comparison would indicate the accuracy of the new grid shift file. Figures 4.1 and 4.2 show this difference, in latitude and longitude, as profiles. The vertical axis represent the difference whereas the horizontal axis shows the individual points in an arbitrary order. The vertical lines inside the plots represent the difference values. In latitude, the largest difference is 3.117 m whereas in longitude it is near 3.972 m. Statistical information from these differences as summarized in Table 4.1. The mean value is equal to 1.3 cm in latitude and to 2.5 cm in longitude, which indicates a small bias that is reflected in the different values of the standard deviation and the rms. The average deviation (the average of the absolute differences) varies between 2 cm and 3.7 cm, respectively for latitude and longitude.

Another way of looking at the differences is by assessing their cumulative density. Table 4.2 shows the cumulative density of the differences between computed and known (published) coordinates in NAD83(CSRS). In latitude, a total of 22574 points lie within a difference of less or equal to 10 cm, which is equivalent to 96.6% of the total points. A total of 23146 points lie within a difference of less or equal to 15 cm, which is equivalent to 98.0% of the total points. A total of 23221 points lie within a difference of less or equal to 20 cm, which is equivalent to 98.3% of the total points. In longitude, a total of 22236 points lie within a difference of less or equal to 10 cm, which is equivalent to 94.2% of the total points. A total of 22685 points lie within a difference of less or equal to 15 cm, which is equivalent to 96.1% of the total points. A total of 23130 points lie within a difference of less or equal to 20 cm, which is equivalent to 97.9% of the total points.



Figure 4.1: Difference between computed and known latitude coordinates, in metre

	$\Delta N(m)$	$\Delta E(m)$
mean	0.013	0.025
standard deviation	0.045	0.071
rms	0.051	0.084
Maximum	3.117	3.972
Average Deviation	0.020	0.037

Table 4.1: Statistics of difference between computed and known coordinates

Table 4.2: Cumulative density of difference between computed and known coordinates

-			
$\Delta N (cm)$	Count	Total	Percentage
$\leq 10$	22574	23605	95.6
$\leq 15$	23146	23605	98.0
$\leq 20$	23211	23605	98.3
$\Delta E (cm)$	Count	Total	Percentage
$\leq 10$	22236	23605	94.2
$\leq 15$	22685	23605	96.1
$\leq 20$	23130	23605	97.9



Figure 4.2: Difference between computed and known longitude coordinates, in metre



Figure 4.3: Difference between computed and Geocalc latitude coordinates, in metre

#### 4.2 Comparison with NB Geocalc

Quality control was also performed by comparing the NAD83(CSRS) coordinates computed using the final shifts (computed by Collocation) with their counterparts computed using NBGeocalc. This comparison would indicate how much difference a user, who has been using NB Geocalc will feel when the new grid shift files start to be used. Figures 4.3 and 4.4 show this difference, in latitude and longitude, as profiles. The vertical lines inside the plots represent the difference values. In latitude, the largest difference is 2.569 m whereas in longitude it is 4.749 m. Statistical information from these differences as summarized in Table 4.3. The mean value is equal to 1.1 cm in latitude and to 1.9 cm in longitude, which indicates a bias that is reflected in the different values of the standard deviation (7.7 cm and 8.4 cm) and the rms (8.0 cm and 9.2 cm). The average deviation (the average of the absolute differences) varies between 2.1 cm and 3.4 cm, respectively for latitude and longitude.

Let us compare Table 4.1 with Table 4.3 since they allow us to shed some light on the final grid, how much different the transformation using it will differ from the one based on NBGeocalc and whether using the grid will bring any benefit. First of all, the comparison of the tables indicate that the solution generated with the grid and with NB Geocalc



Figure 4.4: Difference between computed and Geocalc longitude coordinates, in metre

Table 4.3:	Statistics	of	difference	between	computed	and	Geocalc	coordinates
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	$\Delta$ N (m)	$\Delta E (m)$
mean	0.011	0.019
standard deviation	0.077	0.084
rms	0.080	0.092
Maximum	2.569	4.749
Average Deviation	0.021	0.034

are similar, a reasuring fact. What is better, the differences with respect to the known coordinates are less than the differences with respect to NB Geocalc, indicating that there is even an improvement when using the grid. In other words, the use of the grid will result in transformed NAD83(CSRS) closer to the actual NAD83(CSRS) than if using the current NB Geocalc solution.

Figures 4.2 and 4.2 are an attempt to ilustrate the resultant of the mean horizontal coordinates and the resultante of theis respective standard deviation (coming from Tables 4.1 and 4.3) differ from each other. Figure 4.2 illustrates that the mean horizontal differences of coordinates differs by just 0.6 cm. If we consider the standard deviation of each individual solution we can atest that this difference is statistically insignificant at 99% confidence level. Figure 4.2 shows that the spread of the resultant differences is 12.2 cm with respect to the GeoCalc solution and 8.4 cm with respect to the known coordinates. Their difference is equal to 3.8 cm. This allows us to conclude that the new, Collocation-based solution is on average more precise than the previous GeoCalc-based one.



Figure 4.5: Relation between computed, Geocalc and known mean coordinates resultant



Figure 4.6: Relation between computed, Geocalc and known standard deviation resultant

# Chapter 5 Conclusions

This report presents the results related to the generation of a grid shift file relating NAD27 and NAD83(CSRS) in New Brunswick. This grid shift file will be used in a new application being developed by Service New Brunswick with the intention to replace NB Geocalc. A data set composed of 23,605 horizontal coordinates, in both frames, was treated in a least-squares collocation algorithm which resulted in the prediction of a transformation set covering the whole province of New Brunswick as an equally spaced, 30 seconds grid. This grid was later written following the NTv2 format.

During the least-squares collocation process, both the data set and the final results passed through a process of quality control. One input data (point 119948) was eliminated, therefore not used in the final solution.

The formal error of the estimation indicates that transformations between NAD27 and NAD83(CSRS) using the new grid will have a precision of up to 20 cm if done in an area covered by the input data set, and up to 40 cm if done anywhere else, with the exception of the north-west area of the province (in the border with Quebec) where the geometry of the data points is not favourable. In this later region, the precision deteriorates up to 80 cm.

The accuracy of the transformations between NAD27 and NAD83(CSRS) using the new grid was evaluated by comparing the transformed coordinates with the known, published values in NAD83(CSRS). This assessment is valid only in the area covered by the input data set. It indicates an accuracy of 4.5 cm in latitude and 7.1 cm in longitude, at a 67% confidence

Tests indicate that the final grid shift file generated by least-squares Collocation provides more precise results with respect to the known NAD83(CSRS) coordinates than if using the transformation currently embedded into NB Geocalc.

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# Appendix A NTv2 format

The NTv2 format is presented in the Appendix using as example a portion of the final grid shift file relating NAD27 and NAD83(CSRS). Table A.1 shows the header and the description of each one of the entries. It is important to emphsize a few fields. A grid shift file provides the shift parameters used to transform coordinates "fom" one frame "to" another. The final grid provided transforms from NAD27 to NAD83(CSRS). The inverse transformation (from NAD83(CSRS) to NAD27) takes place by simply changing the sign of the shift parameters. The semi-major and semi-minor axis of the reference ellipsoid used by each one of the frames are given in units of metre. The limits of the grid is given in arcs-of-second. To retrive them in units of degree a simple operation is required.

Table A.2 shows the first two lines of the file containing the parameters used in the transformation. The first two entries of each line are the latitude and longitude shift parameters. They are provided in arcs-of-second. The last two entries are the uncertainty (standard deviation) of the shift parameters, given in metres, as estimated by the Collocation. It can be seen that the uncertainties are quite high (over a meter). That is because the first entry relates to the South-East position of the grid. Looking back at Figure 3.8 and 3.9 we can see that the uncertainties for that position is indeed quite high because this point is far away from the area of interest where data exist (this point is somewhere in the Atlantic Ocean). The order of the shift parameters grows, starting at the South, from East to West. As the westmost point is reached it goes back to the East, moving up North by the grid size (30 arcs-of-second).

The overall idea on the use of the NTv2 grid is quite simple. To transform the coordinates of one point from NAD27 to NDA83(CSRS) an algorith seach for the cell in which the point is located and computes the shifts to be applied to the latitude and longitude of that point by a plain bi-linear interpolation.

NTv2 element	NTv2 value	Meaning		
NUM_OREC	11	number of header records in overview		
NUM_SREC	11	number of header records in sub grid		
NUM_FILE	1	number of sub grids		
GS_TYPE	SECONDS	unit of grid size		
VERSION	01Jun11	version and date		
SYSTEM_	Fnad27	system"from"		
SYSTEM_	Tnad83	system"to"		
MAJOR_F	6378206.400	semi-major axis of the reference ellipsoid of the "from" system		
MINOR_F	6356583.800	semi-major axis of the reference ellipsoid of the "to" system		
MAJOR_T	6378137.000	semi-major axis of the reference ellipsoid of the "to" system		
MINOR_T	6356752.314	semi-major axis of the reference ellipsoid of the "to" system		
SUB_NAME		file name		
PARENT	none	directory name		
CREATED	06/2011	date when the grid was generated		
UPDATED	06/2011	date of last update		
S_LAT	158400.000000	Southern grid limit (in arcs-of-second)		
N_LAT	174600.000000	Northern grid limit (in arcs-of-second)		
E_LONG	226800.000000	Eastern grid limit (in arcs-of-second)		
W_LONG	250200.000000	Western grid limit (in arcs-of-second)		
LAT_INC	30.000000	grid size in latitude		
LONG_INC	30.000000	grid size in longitude		
GS_	COUNT422521	number of grid nodes		

Table A.1: Meaning of NTv2 format header elements

shift in latitude	shift in longitude	uncertainty in latitude	uncertainty in longitude
0.342011	-2.302077	1.258068	1.250061
0.342019	-2.301362	1.256903	1.248840

Table A.2: Meaning of NTv2 format, shift (arcs-of-second) and uncertainties (metre)