Calculation of the spherical terrain correction to Helmert's orthometric height

Robert Kingdon, Robert Tenzer, Marcelo Santos and Petr Vanícek

University of New Brunswick, Department of Geodesy and Geomatics Engineering,
Fredericton, N.B., E3B 5A3, Canada

Basic concepts, principles and definitions:

Definition of Orthometric Height: $H^{O}(\Omega)=\dfrac{C(\Omega)}{\overline{g}(\Omega)}$

Helmert's Mean Gravity: $\overline{g}^{H}=g(r_t)+dg(r_t)+g^{TC}(r_t)+g^{BS}(r_t)+g^{dr}(r_t)-2pGr_0H^0(\Omega)-\dfrac{\partial g}{\partial r}\dfrac{H^0(\Omega)}{2}$

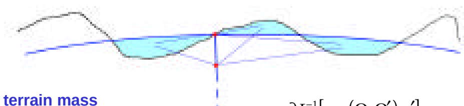
Rigorous Mean Gravity: $\overline{g}=\overline{g}+\overline{d}g+\overline{g}^{BS}+\overline{g}^{TC}+\overline{g}^{dr}$

Given: $\overline{g}\approx g(r_t)-\dfrac{\partial g}{\partial r}\dfrac{H^0(\Omega)}{2}$, $\overline{g}^{BS}\approx 2pGr_0H^0(\Omega)$, and $g^{BS}\approx 4pGr_0H^0(\Omega)$,

Correction to Helmert's Mean Gravity: $\overline{g}^{H}-\overline{g}\approx dg(r_t)-\overline{d}g+g^{TC}(r_t)-\overline{g}^{TC}+g^{dr}(r_t)-\overline{g}^{dr}.$

Effect of terrain roughness on gravity at a point:

This effect may be calculated using an analytical integration of the effect of the additional mass above the spherical Bouguer shell, or missing mass below the spherical Bouguer shell. For the numerical integration over the surface domain, the analytical solution for the radial integrated negative radial derivative of the Newton's integral kernel is used according to Martinec (1998).

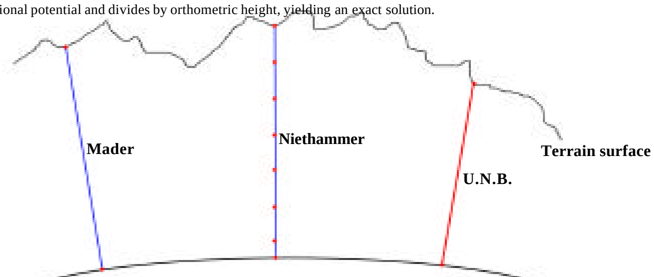


Integration of terrain mass around computation point:

$$g^{TC}(r)=G\,?_0\iint_{\Omega\in\Omega_0}\frac{\partial L^{-1}[r,\psi(\Omega,\Omega'),r']}{\partial r}\,d\Omega'\ \text{(Martinec, 1998)},$$

Effect of terrain roughness on mean gravity:

This effect may be computed using three methods: Mader (1954), Niethammer (1932) and U.N.B. The Mader method approximates the mean value of the gravity effect using an average of the gravity effect values at the surface and on the geoid. The Niethammer method approximates the mean value using a an average of gravity effect values computed at various points along the plumbline. The U.N.B. method computes the surface and geoid values of the effect on gravitational potential and divides by orthometric height, yielding an exact solution.



Three methods used to compute effects on mean gravity

Niethammer (1932) Method: $\overline{g}^{N}(\Omega)=\overline{g}^{H}(\Omega)-g^{TC}(r_t(\Omega))+\dfrac{1}{H^0(\Omega)}\displaystyle\int_{r=R}^{R+H^0(\Omega)}g(r,\Omega)\,dr$

Mader (1954) Method: $\overline{g}^{M}(\Omega)=\overline{g}^{H}(\Omega)-g^{TC}(r_t(\Omega))+\dfrac{g^{TC}(r_t(\Omega))+g^{TC}(r_g(\Omega))}{2}$

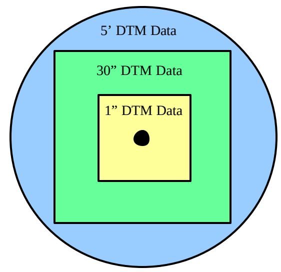
U.N.B. Method: $\overline{g}(\Omega)=\overline{g}^{H}(\Omega)-g^{TC}(r_t(\Omega))+\dfrac{V^{TC}(r_g(\Omega))-V^{TC}(r_t(\Omega))}{H^0(\Omega)}$

Correction to Helmert orthometric height:

Correction to Helmert mean gravity: $corr_{\overline{g}^H}=\overline{g}-\overline{g}^{H}$

Correction to Helmert orthometric height: $corr_{H^{OH}}=-\dfrac{corr_{\overline{g}^H}\times H^{O}}{\overline{g}}$

Necessary DTM Data:



Only near-zone data was necessary for the computation, and more dense data for areas nearer to the computation point. The densities chosen were found effective for computations.

Calculation Time:

The Niethammer method, using 100m intervals between points on the plumbline, took far longer than the other methods. Depending on the spacing of the points along the plumbline, this method could take more or less than this amount of time. Although 100m intervals were used for comparisons, 20m was found necessary to bring results within 1mm of the more accurate U.N.B. method – but this requires approximately 5 times as long.

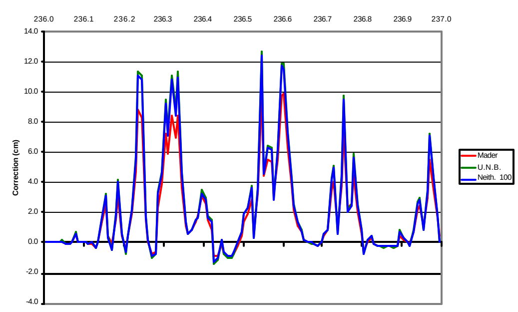
Calculation times using the three methods

Method	Computation Time (Approximate)
Niethammer (100m)	~20h
Mader	~2h
U.N.B.	~2h

Comparison of corrections to Helmert orthometric heights:

As the chart to the right shows, the correction values from the Niethammer method computed using 100m intervals closely coincide with those computed using the U.N.B. method. Those of the Mader method, however, are significantly different – by as much as 2cm in some places.

Some correction values are negative, as the plot shows, but the majority of these are small. The lowest value in the test area was only -1.6cm. This may result from corrections being calculated from parts of the integration area below the horizon.



Statistics of results by method

Method	Min. (cm)	Max. (cm)	Mean (cm)
Niethammer	-1.6	25.8	0.7
Mader	-1.1	19.7	0.8
U.N.B.	-1.6	25.9	0.8

Conclusions:

1. The correction to Helmert orthometric height resulting from terrain roughness may be as great as 26cm in areas of very rough terrain. Generally, however, it is small – around 1cm in the test area. Though the correction may sometimes be negative, negative values are quite small.

2. The correction is best computed using the U.N.B. method, which yields an exact result, and is much faster than the Niethammer method, which yields comparably good results.

Acknowledgments:

Funds partly provided by the GEOIDE Network and by a National Science Engineering and Research Council of Canada (NSERC) USRA Scholarship.

AGU/CGU Joint Assembly, Montréal, 2004